The Network Origins of Bank Influence: Evidence from Bank–to–Firm and Firm–to–Firm Linkages*

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Abstract

Bank lending shocks affect firm behaviour and permeate, via firm inter linkages, throughout the real the economy. In this paper, we investigate how the structure of the real economy determines the aggregate real influence of individual banks. Data on the universe of (a) firm–to–firm transactions and (b) bank–firm borrowing relations of the Belgian economy allows us to reconstruct (a) the firm–level input–output production architecture of the Belgian economy and (b) the credit network supporting it. These objects allow us to structurally quantify and decompose the network origins of individual bank influence. We then study how the current structure of the Belgian economy affects the size of aggregate real GDP fluctuations induced by shocks from individual banks. Lastly, we revisit the Lucas (1976) argument – i.e. the rate at which this aggregate effect vanishes as the number of banks in the economy increases. We show that our analysis speaks to key research questions related to financial sector competition policy, strategic bank lending and various macro prudential topics.

Keywords: Shock propagation · Input–output linkages · Bank–level influence
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1 Introduction

There is a long and well–established tradition in the macroeconomic literature of adding financial frictions to standard macroeconomic models in order to show the importance of the banking sector for real business cycle fluctuations (Bernanke and Gertler, 1989; Bernanke and Blinder, 1988). In most models, the banking sector acts either (*i*) as an amplification mechanism of shocks originating elsewhere in the economy or (*ii*) as an exogenous source of shocks.[1] In the latter case, these models typically appeal to aggregate shocks, common across all banks, in order to explain aggregate fluctuations (e.g. Gerali et al. (2010); Andrés et al. (2013); Andrés and Arce (2012)). Idiosyncratic – bank specific – shocks to credit supply to firms are not taken into account as a material source of business cycles. Algebraically, this assumption enters these models through the inclusion of a representative bank (or atomistic banks) instead of a finite set of heterogeneous banks. The *diversification argument* by Lucas (1977)[2] justifies this approach: the real effects of idiosyncratic variation in lending from individual banks cancel out, implying that bank specific shocks are irrelevant for the study of fluctuations in macroeconomic aggregates.

Although aggregate shocks to credit supply are without any doubt important, recent empirical research identifies idiosyncratic credit–supply shocks of individual banks as important drivers of macroeconomic variables, e.g. (*i*) aggregate investment (Amiti and Weinstein, 2016; Amador and Nagengast, 2016), (*ii*) gross domestic product (Buch et al., 2014; Buch and Neugebauer, 2011), (*iii*) international trade patterns (Dewachter et al.,

[1]When financial frictions act as amplification mechanisms, a shock originates in the non–financial sector (e.g. exogenous changes in productivity, preferences, etc.) and is consequently aggravated by financial frictions. Such models include Bernanke and Gertler (1989); Bernanke et al. (1999); Carlstrom and Fuerst (1997). Alternatively, the initial disruption arises in the financial sector without initial changes in the non-financial sector. Typically, financial frictions cause fewer funds to be channelled from banks to borrowers which in turn affects the real economy. Standard models include Kiyotaki and Moore (2012); Del Negro et al. (2010); Gertler and Karadi (2011); Christiano et al. (2010).

[2]Not to be confused with the well-known Lucas (1977) critique about behavioural changes in macroeconomic models.

2017; Niepmann and Eisenlohr, 2014), (*iv*) aggregate employment (Greenstone et al., 2014), (*v*) economywide total factor productivity growth (Manaresi and Pierri, 2018), etc.

In view of this disconnect between (*i*) traditional macro models and (*ii*) the emerging empirical evidence, we revisit the scale of aggregate real GDP fluctuations due to bank specific credit supply shocks in a New–Keynesian (NK) set–up. Our model features three sources of heterogeneity that are shown to attenuate the extent to which bank–specific credit supply shocks cancel out in the aggregate. (*i*) First, we embed a heterogeneous firm–level input–output structure into the model (in the spirit of Long and Plosser (1983)) where firms rely on each other for intermediate input requirements. (*ii*) Second, we allow for heterogeneity in the level of value added that individual firms produce. (*iii*) Finally, we introduce a monopolistic banking sector (a variant to Gerali et al. (2010); Cuciniello and Signoretti (2015)) featuring a firm–bank credit network and heterogeneous collateral constraints. The model is set up such that – under extreme parametrizations – the framework nests the NK models of Iacoviello (2015, 2005); Pasten et al. (2018a,b); Carvalho and Lee (2011) and IO–like models of Bremus et al. (2018); Gabaix (2011); Acemoglu et al. (2012).

The general equilibrium of the model is analytically tractable and delivers an *influence measure* for individual banks. The latter summarizes the extent to which credit of an individual bank supports value added creation in the economy. Key to our framework is that the effect of a shock to credit conditions from an individual bank (in casu, cost of debt and collateral requirements) does not remain confined to its borrowing firms, but permeates across the production architecture of the real economy and endogenously affects credit constraints elsewhere in the economy. The total influence of an individual bank on real gdp is then determined by the extent the bank, both directly and indirectly (via the input–output structure), affects value added creation by individual firms in the real economy.

We show that two interlocked networks (in casu individual firm credit portfolios and the production architecture of the real economy) potentially introduce large asymmetries in the influence of individual banks. Stated differently, some banks intensively support – directly and/or indirectly – value added of the non–financial sector, whereas other banks are only marginally connected to firms involved in value added creation. Consequently, when some banks are asymmetrically influential, their credit supply shocks do not easily cancel out with that of other banks – even when the number of banks in the economy is large. We derive closed form expressions to characterize this relationship between (*i*) the asymmetry in bank influences, (*ii*) the number of banks in the economy and (*iii*) aggregate real GDP fluctuations.

The model is calibrated to the Belgian economy. The backbone of our calibration exercise relies on three confidential databases provided by the National Bank of Belgium; (*i*) the *corporate credit register*, (*ii*) the *Business–to–Business* database and (*iii*) *value added tax declarations*. The first datasource provides detailed information on the population of loans from banks to firms (as well as underlying collateral). It allows us to calibrate the credit network that ties banks to firms and loan–to–value ratio's that banks demand from firms. The second contains a quasi exhaustive list of all business–to–business transactions between VAT liable Belgian firms. We use it to calibrate an input–output matrix at the firm level.[3] Finally, the VAT declarations allow us to directly observe value added from individual firms. In sum, the three datasets allow us to reconstruct the production architecture of the Belgian economy as well as the credit network supporting it.

We find that the true structure of the Belgian economy, bank specific

[3]While there has been an increased interest in shock propagation across networks, the available firm–to–firm data is often restricted in terms of the (*i*) sectoral coverage (see e.g. Acemoglu et al. (2015a)), (*ii*) geographical coverage (see e.g. Carvalho et al. (2015)), (*iii*) reporting thresholds (see e.g. Atalay et al. (2011); Bernard et al. (2015)), (*iv*) nominal quantification of business relationships (see e.g. Kelly et al. (2014); Carvalho et al. (2015)) or (*v*) level of aggregation (see e.g. Acemoglu et al. (2012); Shea (2002)). The $B2B$ is unrestricted in aforementioned dimensions.

shocks have material aggregate effects; Their aggregate effect is 60% as large as that of a common shock of similar size that affects all banks. This underscores that the widespread practice of discarding bank–specific shocks in macro models is highly restrictive. In the full symmetric Belgian economy, this multiplier is only 10% of what an aggregate bank shock would generate. We show that asymmetries in the credit network and the production economy reinforce each other, i.e. banks that disproportionally lend to firms that create a lot of value added are also indirectly – through IO interactions – very supportive to value added creation of firms elsewhere in the economy to which they do not lend directly.

We quantify and decompose the influence of individual banks to disentangle the (network) origins of individual bank influence in the Belgian economy. We show that the most banks derive their real influence from lending directly to firms that produce value added ($\sim 70\%$). The remaining 30% of bank–level influence is obtained from indirect shock propagation throughout the real economy. Nonetheless, this influence structure of banks is very heterogeneous, e.g. some banks obtain most of their influence from shock propagation whereas some banks derive all of their influence from lending to large firms. Importantly, we show that bank size does not map one–to–one with our influence measure. The reason is that a small bank (in terms of credit volume) can have a significant impact on the aggregate economy if it supports key activities in the real economy whereas a large bank might not.

We distill three important policy implications from the analysis. First, the input–output structure of the real economy as a vehicle through which bank shocks propagate is an important feature in the context of skewed sector presence. Skewed sector presence implies that some individual banks are dominant credit providers to firms in specific (non–financial) sectors (e.g. Boeve et al. (2010); De Jonghe et al. (2016); Paravisini et al. (2014)).[4] An important takeaway from our framework is that special-

[4]Skewed sector presence, arising from bank specialization, is widespread. E.g. it is not uncommon for specific industries to be exclusively served by a single bank.

ization of banks can lead to increased macroeconomic volatility as bank shocks propagate, via their respective sector of specialization, to all other other firms that rely on inputs from this sector. E.g. we show that, for Belgium, in the aftermath of the financial crisis, such shifts in strategic bank lending have caused an increase in macroeconomic volatility.

The Herfindahl–Hirschmann index (HHI) is a popular policy measure to quantify financial sectoral competition, concentration and financial sector stability. It is often used to identify potentially disruptive M&A's given that an increase in the HHI is often associated with increased macroeconomic volatility Bremus (2015); Bremus and Buch (2017); Bremus et al. (2018). We show that the HHI is only marginally informative about macroeconomic volatility. The reason is that the HHI focuses exclusively on market shares. Our framework, instead, focuses on the (in)direct role that the borrowing NFCs in the banks' portfolio play in the real economy. M&A activity in the banking sector might lead to an increase in the HHI, but might enhance symmetry in the real economy, which lowers volatility. We show that the HHI has effectively remained relatively stable in Belgium over the last decade whereas macroeconomic volatility from bank–specific shocks has increased.

Finally, the role of shock propagation through production networks also speaks to the identification of significant banks. In a European context, following SSM regulation, such identification is i.a. based on concepts such as size – "too big to fail" – and interconnections – "too interconnected to fail" (Freixas et al. (2000); Allen and Gale (2000); Acemoglu et al. (2015b)). The latter principle refers to connections arising from inter bank claims and connections to highly leveraged shadow financial institutions. Our view on interconnectivity complements this more traditional notion of financial sector connectedness and focuses on the way the production architecture of the economy is matched to the banking sector.

Literature & Contribution. Most directly, this paper contributes to the emerging literature on the micro origins of aggregate fluctuations.

Although it has been a topic of interest in both the real business cycle literature and the financial economics literature, both research areas developed largely in isolation from each other. Our focus on the *financial* micro origins of aggregate *real* fluctuations bridges the gap between these two research areas.

For the real economy, Gabaix (2011) was the first to develop the view that a large part of aggregate fluctuations arises from idiosyncratic productivity shocks to large individual firms. Empirical evidence strongly supports this view in the case of aggregate exports (Di Giovanni et al., 2014), GDP (Gabaix, 2011), the trade balance (Canals et al., 2007), etc. A related literature, originating from the seminal contribution of Long and Plosser (1983), explores the role of production networks in generating aggregate fluctuations in GDP. Idiosyncratic productivity shocks to individual sectors are found to propagate throughout the economy via input-output linkages, causing comovement across sectors and ultimately leading to aggregate fluctuations in GDP (see e.g. Shea (2002); Horvath (1998); Foerster et al. (2011); Dupor (1999); Conley and Dupor (2003); Acemoglu et al. (2012)). More recently, this network view has been taken to the firm level (e.g. Kelly et al. (2014), Carvalho et al. (2015), Carvalho (2014), Baqaee (2016), Stella (2015), Magerman et al. (2015), Dewachter et al. (2017), etc.).

A parallel, but distinct, literature has focused on how the architecture of the financial system works as an amplification mechanism of shocks to individual banks. Early contributions include Freixas et al. (2000) and Allen and Gale (2000) who investigate how inter–bank claims affect the resilience of the aggregate financial system to the insolvency of any individual bank. This network view has been central in subsequent contributions as well (e.g. Dasgupta (2004); Elliott et al. (2014); Allen et al. (2012); Acemoglu et al. (2015b)).[5]

[5] Anecdotal evidence of the financial crisis (e.g. Brunnermeier and Pedersen (2009)) and the empirical banking literature (e.g. Degryse and Nguyen (2007); Haldane and May (2011)) establish the relevance of these mechanisms.

More generally, this paper is part of a larger agenda that investigates the impact of credit frictions on the real economy. Credit frictions are found to distort key economic activities, e.g. investment (Almeida and Campello, 2007), export (Paravisini et al., 2015; Amiti and Weinstein, 2011; Strasser, 2013), pricing behaviour (Strasser, 2013), etc. While most of these studies offer compelling micro-level evidence that individual banks matter for individual firms, they have not addressed the question of how important individual intermediaries are in determining their aggregate counterpart. An emerging (but exclusively empirical) literature aims to fills this gap (e.g. Amiti and Weinstein (2016); Buch and Neugebauer (2011); Niepmann and Eisenlohr (2014)). This paper is the first to formally identify the exact mechanisms and preconditions that drive these empirical results.

Although this contribution is not the first paper to show that bank shocks spread through a network, our paper is the first to show how these shocks propagate and amplify through interlocked networks (in casu individual firm credit portfolios and the production architecture of the real economy). Our empirical results reveal patterns that remain hidden when using isolated datasets.

The remainder of the paper is organized as follows. Section 2 presents a basic economic environment. Section 3 analyses the log linearised model and relates the size of aggregate real volatility to the structure of the economy. Section 4 discusses the data sources and calibration exercise underlying the analysis of the model in section 5 and 6. Section 7 concludes. All proofs and mathematical details are relegated to the appendix.

2 The model

The model is a variant of the standard New Keynesian (NK) model from which we make three key departures; (i) a heterogeneous firm–level input–output structure in the spirit of Long and Plosser (1983) where firms rely on each other for intermediate input requirements, (ii) heterogeneity in the level of value added individual firms produce, (iii) a monopolistic banking sector featuring a firm–bank credit network and heterogeneous collateral constraints. The model is set up such that, under extreme parametrizations, the framework nests the NK models of Pasten et al. (2018a,b) and Iacoviello (2005) and IO models analyzed in Bremus et al. (2018); Acemoglu et al. (2012); Gabaix (2011).

2.1 Households

A large number of infinitely lived households exist. Households derive utility from non–durable consumption, land holdings and leisure. The representative household maximizes the additive separable utility function

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\big(\log(c_t)+\iota h_t-\sum_{j=1}^{J}g_j\frac{l_{jt}^{1+\varphi}}{1+\varphi}\big)\Big]$$

where c_t is an aggregate household consumption bundle, h_t denotes holding of land, and l_{jt} denotes the hours of labour supplied to firm j. The parameters β, φ, ι and $\{g_j\}_{j=1}^{J}$ denote the discount factor, the inverse of the (Frisch) elasticity of labor supply, relative land utility and the relative disutilities of supplying labour to individual firms, respectively. The aggregate consumption bundle is defined as

$$c_t=\Big(\sum_{j=1}^{J}\theta_j^{\frac{1}{\eta}}c_{jt}^{1-\frac{1}{\eta}}\Big)^{\frac{\eta}{\eta-1}} \qquad \sum_{j=1}^{J}\theta_j=1\wedge\theta_j\in[0,1]\ \forall k \tag{1}$$

where c_{jt} denotes consumption of goods produced by firm j and $\boldsymbol{\theta}$ collects the steady state share of good j in household consumption. If $\theta_j=0$, firm j does not sell to households directly. The price index associated with aggregate consumption, P_t, is defined as

$$P_t=\Big(\sum_{j=1}^{J}\theta_jP_{jt}^{1-\eta}\Big)^{\frac{1}{1-\eta}}$$

where P_{jt} denotes the nominal price of goods from firm j. The households' demand schedule for goods from firm j is

$$c_{jt} = \theta_j \big(\frac{P_{jt}}{P_t}\big)^{-\eta} c_t$$

The budget constraint, in real terms, is given by

$$c_t + q_t(h_t - h_{t-1}) + \frac{R_{t-1}d_{t-1}}{\pi_t} = d_t + \sum_{j=1}^{J} w_{jt} l_{jt} + \sum_{j=1}^{J} \Delta_{jt} + \sum_{b=1}^{B} \Delta_{bt}$$

Where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate, $q_t \equiv \frac{Q_t}{P_t}$ is the real land price, $w_{jt} \equiv W_{jt}/P_t$ is the real wage paid by firm j. h_t denotes the stock of land owned by the household. We assume that nominal, risk–free, one–period bank deposits $d_t \equiv D_t/P_t$ are the only financial asset available to households. D_t pays an interest rate R_t set by the monetary authority. π_{jt} and π_{bt} are lump sum firm and bank profits, respectively, which are channelled to households.

The first order conditions w.r.t. $\{c_t, d_t, l_{jt}, h_t\}_{t=0}^{\infty}$ deliver firm–level labour supply schedules (2a), Euler equation (2b) and demand for land (2c)

$$w_{jt} = g_j l_{jt}^{\varphi} c_t \tag{2a}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \big(\frac{R_t}{c_{t+1}}\big) \tag{2b}$$

$$\frac{q_t}{c_t} = \frac{\iota}{h_t} + \beta \mathbb{E}_t \Big(\frac{q_{t+1}}{c_{t+1}}\Big) \tag{2c}$$

2.2 Production

Production in the economy is shaped by J monopolistic competitive firms. Firm j produces quantity y_{jt} according to the constant returns to scale Cobb–Douglas technology

$$y_{jt} = A_j \big(n_{jt}^{\phi_j} m_{jt}^{1-\phi_j}\big)^{\delta_j} k_{jt}^{1-\delta_j} \quad s.t. \;\; \phi_j, \delta_j \in [0,1] \;\; and \;\; \delta_j < 1$$

where n_{jt}, m_{jt} and k_{jt} denote (i) hired household labour, (ii) an intermediate input bundle and (iii) capital services, respectively. The first constraint imposes constant returns to scale in variable inputs. The second ensures the existence of a stable equilibrium. A_j is an innocuous technology parameter.[6] The intermediate input bundle m_{jt}

$$m_{jt} = \Big(\sum_{j'=1}^{J} \omega_{jj'}^{\frac{1}{\eta}} m_{jj't}^{1-\frac{1}{\eta}} \Big)^{\frac{\eta}{\eta-1}} \qquad \sum_{j'=1}^{J} \omega_{jj'} = 1 \wedge \omega_{jj'} \in [0,1] \ \forall k, k'$$

is a Dixit–Stiglitz aggregate over intermediate inputs sourced from other firms; $m_{jj't}$ denotes the amount of goods that firm j procures from firm j'. Optimal demand schedules for intermediate goods are given by

$$m_{jj't} = \omega_{jj'} \Big(\frac{P_{j't}}{P_t^{\omega}} \Big)^{-\eta} m_{jt}$$

where $P_t^{\omega} = \Big(\sum_{j=1}^{J} \omega_{jj'} P_{jt}^{1-\eta} \Big)^{\frac{1}{1-\eta}}$ is the input–cost index of firm j. $\mathbf{\Omega} \in \mathbb{R}^{J \times J}$ with generic element $\omega_{jj'}$ denotes the input–output matrix of the economy at the firm level, describing the flow of intermediates across the economy. $\omega_{jj'} = 0$ if firm j does not directly buy from firm j' and the larger $\omega_{jj'}$, the more important firm j' is as an input supplier to firm j. This interconnected production architecture leaves firms exposed to distress originating with other firms.

Firms are further characterized by staggered price setting consistent with a Calvo (1983)–Yun (1996) framework. Firm j has a probability of $1 - \alpha_j$ to reset its price each period and is thus faced with the following dynamic price setting problem

$$\underset{P_{jt}^*}{\text{Max}} \ \mathbb{E}_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} (P_{jt} y_{jt+s} - MC_{jt+s}(W_{jt+s}, P_{jt+s}^{\omega}, F_{jt+s}) y_{jt+s})$$

[6]It is a normalization constant introduced for convenience when loglinearizing the model. Its value does not affect volatility of gross domestic product, the main quantity of interest in this paper.

Where $\Lambda_{t,t+s} \equiv \frac{\beta^s c_t}{c_{t+s}} \frac{P_t}{P_{t+s}}$ is the stochastic discount kernel between period t and $t+s$. F_{jt} is the capital rental rate and $MC_{jt}(\cdot)$ denotes nominal marginal costs and is defined as

$$MC_{jt} = (\frac{W_{jt}}{\delta_j \phi_j})^{\delta_j \phi_j} (\frac{P^{\omega}_{jt}}{\delta_j(1-\phi_j)})^{\delta_j(1-\phi_j)} (\frac{F_{jt}}{1-\delta_j})^{(1-\delta_j)}$$

after imposing the optimal mix of variable inputs.

The production process of firm j is overseen by an entrepreneur (which we consider as integrally part of firm j and thus indexed as such). Entrepreneur j does not receive wages, but receives income from combining its own land holdings $\widetilde{h}_{jt}$ with labour from the household $\widetilde{n}_{jt}$ to produce capital services k_{jt} for firm j

$$k_{jt} = (\widetilde{n}'_{jt})^{1-\nu_j} (\widetilde{h}'_{jt-1})^{1-\nu_j}$$

We assume entrepreneur j to consume and to maximize the utility function

$$\mathbb{E}_0 \sum_{t=1}^{\infty} \gamma^t \widetilde{c}_{jt} \qquad (\gamma < \beta)$$

subject the flow of funds in real terms

$$f_{jt} k_{jt} + s_{jt} = \widetilde{c}_{jt} + q_t(\widetilde{h}_{jt} - \widetilde{h}_{jt-1}) + \frac{R_{jt-1} s_{jt-1}}{\pi_t} + w_{jt} \widetilde{n}_{jt}$$

where for simplicity, but w.l.o.g., we assume that entrepreneur j has the same taste for varieties as households: $\widetilde{c}_{jt} = \left(\sum_{j=1}^{J} \theta_j^{\frac{1}{\eta}} (\widetilde{c}_{jt})^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$.

Along the lines of Iacoviello (2015) and Kiyotaki and Moore (1997), we cap the amount of bank debt firm j can obtain. More precisely, banks impose a collateral constraint on entrepreneur j: the nominal loan gross of interest payments cannot exceed a certain fraction (the pledgeability ratio) of the expected nominal resale value of the entrepreneurs land holdings.

The collateral constraint can be expressed in real terms as,

$$s_{jt} \leq \ell_{jt}\mathbb{E}_t \frac{q_t \widetilde{h}_{jt}\pi_t}{R_t}$$

Define λ_{jt} as the shadow value of the borrowing constraint. Optimizing w.r.t. $\{\widetilde{c}_{jt}, \widetilde{h}_{jt}, s_{jt}, \widetilde{n}_{jt}\}_{t=0}^{\infty}$ then yields an (i) Euler equation, (ii) real estate demand and (iii) labor demand schedule

$$\begin{aligned}
\frac{1}{\widetilde{c}_{jt}} &= \mathbb{E}_t\Big(\frac{\gamma R_{jt}}{\widetilde{c}_{jt+1}}\Big) + \lambda_{jt}R_{jt} \\
\frac{q_t}{\widetilde{c}_{jt}} &= \mathbb{E}_t\Big(\frac{\gamma}{\widetilde{c}_{t+1}}\big(\nu_j \frac{k_{jt}}{\widetilde{h}_{jt}} + q_{t+1}\Big) + \lambda_{jt}\ell_{jt}\pi_{t+1}q_{t+1} \\
w_{jt} &= (1-\nu_j)k_{jt}f_{jt}/\widetilde{n}_{jt})
\end{aligned}$$

Where the equations show that a tightening of the loan–to–value ratio (or a drop in the value of pledgaeble collateral) curtails the amount of capital services and thus production capacities, y_{jt}.

2.3 Banks

Banks are assumed to intermediate all credit flows between households (savers) and firms (borrowers). We assume that banks are perfectly competitive on the deposits market, and so they take the nominal deposit rate, R_t, which is set by the central bank, as given. However, competition in the loans market is imperfect, so each bank enjoys some monopolistic power when providing working credit to firm k.

Following Gerali et al. (2010), we assume that units of loan contracts are a composite constant elasticity of substitution basket of slightly differentiated financial products – each supplied by a different bank

$$S_{jt} = \Big(\sum_{b=1}^{B} \psi_{jb}^{\frac{1}{\mu_{bt}}} S_{jbt}^{1-\frac{1}{\mu_{bt}}}\Big)^{\frac{\mu_{bt}}{\mu_{bt}-1}}$$

Where S_{jbt} denotes demand for credit of firm j from bank b. Optimal demand schedules of firm j

$$S_{jbt} = \psi_{jb}\Big(\frac{R_{jbt}}{R_{jt}}\Big)^{-\mu_{bt}} S_{jt}$$

$\mathbf{\Psi} \in \mathbb{R}^{J\times B}$, with generic element ψ_{jb} introduces a bank–firm credit network into the model. $\mathbf{\Psi}$ fixes the extensive margin; $\psi_{jb} > 0$ ($\psi_{jb} = 0$) if bank b lends (does not lend) to firm j. The CES specification allows for substitution in the intensive margin, i.e. across existing bank—firms relations. In the steady state, $\mathbf{\Psi}$ introduces the feature that some banks are large/small or intensively/marginally connected to firms in the real economy.

Bank b sets the interest rates on credit to firm j, r_{jbt}, in order to maximize a discounted stream of profits

$$\mathbb{E}_0 \sum_{s=0}^{\infty} \Lambda_{t,t+s} P_{t+s} \Delta_{bt+s}$$

subject to credit demand schedules and a flow of funds

$$\Delta_{bt} + R_{t-1} d_{bt-1}\pi_t + \sum_{j=1}^{J} s_{jbt} = \sum_{j=1}^{J} r_{jbt-1} s_{jbt-1}\pi_t + d_{bt}$$

In addition, bank b has to abide its balance sheet identity, $\sum_{j=1}^{J} s_{jbt} \equiv d_{bt}$, which implies that real profits simplify to $\pi_{bt+1} = (\sum_{j=1}^{J} R_{jbt} s_{jbt} - R_t) s_{jbt}$. Optimal interest rate set to firm k are then $r_{jbt} = \frac{\mu_{bt}}{\mu_{bt}-1} R_t$.

2.4 Monetary Authority

The monetary authority sets the gross short–term nominal interest rate, R_t, according to a Taylor rule

$$R_t = \frac{1}{R}\Big(\frac{P_t}{P_{t-1}}\Big)^{\phi_\pi}\Big(\frac{gdp_t}{gdp}\Big)^{\phi_{gdp}}$$

where R and GDP are the steady state policy rate and real gross domestic product. Note that, in our model with intermediate inputs, gdp_t and economy wide production $\sum_{j=1}^{J} y_{jt}$ do not collapse.

2.5 Market Clearing

The model equilibrium is characterized by an allocation of quantities and prices that satisfy (i) the households/entrepreneurs optimality conditions and budget constraint, (ii) the firms optimality conditions, (iii) the monetary policy rule, (iv) the bank balance sheet identity and (v) market clearing conditions; $\{d_t = \sum_{b=1}^{B} d_{bt}\}$, $\{l_t = \sum_{j=1}^{J}(n_{jt} + \widetilde{n}_{jt})\}$, $\{h = h_t + \sum_{j=1}^{J} h'_{jt}\}$, $\{y_{jt} = c_{jt} + \sum_{j=1}^{J} \widetilde{c}_{jt} + \sum_{j'=1}^{J} m_{j'jt}\}_{j=1}^{J}$. Where the expressions represent, respectively, the asset market clearing condition, the labour market–clearing condition, the fixed stock of land, market clearing and Walras' law.

2.6 Exogenous processes

We assume the following processes on the shocks of the model

$$\log(\ell_{bt}) = \epsilon_t^{(\ell)} + \varepsilon_{bt}^{(\ell)}$$
$$\log(\frac{\mu_{bt}}{\mu_{bt} - 1}) = \epsilon_t^{(r)} + \varepsilon_{bt}^{(r)}$$

where $\epsilon_t^{(\ell)}, \epsilon_t^{(r)}$ represent an aggregate shock to LTV and borrowing rates, respectively. $\varepsilon_{bt}^{(\ell)}$ and $\varepsilon_{bt}^{(r)}$ capture the bank–specific idiosyncratic shocks to aforementioned variables. Aggregate and bank–specific shocks follow an $AR(1)$ process and share lag coefficients ρ^ℓ and ρ^r, respectively.

2.7 Log linearised model

We solve the model by log–linearizing the equilibrium conditions around the deterministic zero–inflation steady state. The appendix E provides a detailed derivation of the steady–state equilibrium as well as the full set of log–linearized equations.

2.8 Relation to the literature

In general, our model combines a standard amplification mechanism brought about by financial frictions (such as analysed by Kiyotaki and Moore (1997)) with an amplification mechanism in the real economy caused by firm–to–firm interactions (such as analysed by e.g. Long and Plosser (1983)). Both amplification mechanisms are often analysed in isolation. E.g. as shown in appendix E, the one–sector version of our model collapses to that analyzed in Iacoviello (2005). Alternatively, if we discard the role of entrepreneurs, housing and financial frictions, the model tightly tracks that elaborated by Pasten et al. (2018a,b). Mappings to other models are further discussed in appendix E.

3 Theoretical results in a simplified framework

We now investigate how (i) the production architecture of the economy $(\mathbf{\Omega})$, the credit network supporting it $(\mathbf{\Psi})$ and (iii) the concentration of value added in the economy $(\boldsymbol{\theta})$ give rise to macroeconomic fluctuations from small idiosyncratic bank shocks. We also assess the rate at which this aggregate effect decays as the number of banks in the economy increases. The theoretical results in this section hold under a set of simplifying assumptions, which we gradually relax in the last subsection.

3.1 Simplifying assumptions

Assumption 1. *Households have linear disutility of labor* $(\varphi = 0)$.

Assumption 2. *Monetary policy targets nominal gross domestic product*

$$P_t C_t = PC$$

Assumption 3. *All firms are equally capital intensive ($\{\delta_j = \delta\}_{j=1}^J$).*

Assumption 4. *Entrepreneurs have zero consumption mass, $\{C'_{jt} = 0\}_{j=1}^J$.*

Assumption 5. *We replace the Calvo (1983)–Yun (1996) framework of staggerd price/rate setting of firms/banks by a simple rule. All prices are flexible, but with a probability α, a firm has to set its price before observing shocks. Thus,*

$$P_{kt} = \begin{cases} \mathbb{E}_{t-1}[P^*_{kt}] & \text{with probability } \alpha \\ P^*_{kt} & \text{with probability } 1-\alpha \end{cases}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set.

Assumption 6. *We make the following assumptions on the shock structure. Zero mean: $\mathbb{E}_t\epsilon_t^{(\ell)} = \mathbb{E}_t\epsilon_t^{(r)} = \mathbb{E}_t\varepsilon_{bt}^{(\ell)} = \mathbb{E}_t\varepsilon_{bt}^{(r)} = 0$, all shocks are orthogonal and have a finite second moment, $\mathbb{V}(\epsilon_t^{(\ell)}) = \mathbb{V}(\epsilon_t^{(r)}) = \mathbb{V}(\varepsilon_{bt}^{(\ell)}) = \mathbb{V}(\varepsilon_{bt}^{(r)}) = \sigma < infty$.*

Assumption 1 and 2 jointly fix labour supply and pin down wages. Assumption 3 implies that all firms are equally credit intensive. Assumption 4 shuts down an amplification mechanism via collateral constraints. Assumption 5 collapses the time dimension of the model impulse response functions. Assumption 6 is not restrictive and is maintained throughout the text.

3.2 Equilibrium and Propagation Mechanism

In our simplified model, aggregate real household consumption, c_t, equals total real value added (real gross domestic product). As shown in appendix B, under simplifying assumptions (1)–(6), the expression for log linearised real GDP is analytically tractable and given by (henceforth, hat notation signals log linearised variables)

$$\widehat{c}_{t|B} = \boldsymbol{\nu}'_B \boldsymbol{\iota}_B \epsilon_t + \boldsymbol{\nu}'_B \boldsymbol{\varepsilon}_{t|B} \tag{3}$$

where $\boldsymbol{\iota}_B$ is a unity vector of size B, ϵ_t is a common banks shock and $\boldsymbol{\varepsilon}_{t|B}$ is the vector capturing bank specific shocks ε_{bt} defined in assumption 6. Shocks to LTV ratios do not enter the simplified framework. Subscript B (or $|B$) is henceforth included to denote the number of banks in the economy. $\boldsymbol{\nu}_B$ is the *influence vector* defined as

$$\boldsymbol{\nu}_B \equiv \kappa \boldsymbol{\Psi}'_B [\mathbb{I} - \widetilde{\boldsymbol{\Omega}}']^{-1} \boldsymbol{\theta} \quad ; \quad \widetilde{\boldsymbol{\Omega}} \equiv \delta(1-\alpha)\boldsymbol{\Phi}\boldsymbol{\Omega}, \kappa \equiv (1-\delta)(1-\alpha) \tag{4}$$

where $\boldsymbol{\Phi}$ is a diagonal matrix with ϕ_j on the diagonal. $\boldsymbol{\nu}_B \equiv [\nu_{1|B}, \ldots, \nu_{B|B}]' \in \mathbb{R}^{B\times 1}$ is a vector of multipliers (elasticities) that maps the vector of bank shocks, $\boldsymbol{\varepsilon}_{t|B}$, to aggregate GDP fluctuations. In that respect, $\boldsymbol{\nu}_B$ quantifes the aggregate real influence of B individual banks in the economy.

To develop more intuition w.r.t. the structure of $\boldsymbol{\nu}_B$, it is useful to rewrite it as a converging Neumann series

$$\boldsymbol{\nu}_B = \kappa \boldsymbol{\Psi}'_B \Big(\sum_{n=0}^{\infty} \widetilde{\boldsymbol{\Omega}}^n \Big) \boldsymbol{\theta} = \sum_{n=0}^{\infty} \boldsymbol{\nu}_B^{(n)} \tag{5}$$

The summands in the infinite sum capture the separate steps through which shocks propagate throughout the real economy. Hence, the expression identifies the channels through which individual banks affects the real economy. (4) is then interpreted as a collapsed impulse response function (IRF) whereas (5) disentangles the steps of this IRF.[7]

[7] To see this more clearly, the first-order effect of $\varepsilon_{bt|B}$ on real gdp is obtained for $n = 0$

$$\nu_{b|B}^{(n=0)} = \kappa \boldsymbol{\theta}' \boldsymbol{\psi}_{b|B}$$

3.3 Aggregate volatility

From assumption 6 it is possible to decompose the standard deviation of real gdp into an aggregate source and a bank–level source:

$$\sqrt{Var(\widehat{c}_{t|B})} = \underbrace{\|\boldsymbol{\nu}_B\|_2\sigma}_{\substack{\text{Bank–specific} \\ \text{origin}}} + \underbrace{\|\boldsymbol{\nu}_B\|_1\sigma}_{\substack{\text{Common bank} \\ \text{origin}}} \tag{6}$$

where $\|\boldsymbol{\nu}_B\|_2 = \sqrt{\sum_{b=1}^{B} \nu_{b|B}^2}$ is the Euclidean norm of $\boldsymbol{\nu}_B$. In addition, $\|\boldsymbol{\nu}_B\|_1 = \sum_{b=1}^{B} \nu_{b|B}$ is the Manhattan norm of $\boldsymbol{\nu}_B$ – i.e. the sum of the elements in $\boldsymbol{\nu}_B$.

Note that $\|\boldsymbol{\nu}_B\|_1$ is a constant, the size of which depends on the structure of the economy but invariant to the number of banks in the economy.[8] In contrast, as shown later, the size of $\|\boldsymbol{\nu}_B\|_2$ also depends on the number of banks in the economy. This means that, as the number of banks in the economy varies, the aggregate impact of bank–specific shocks varies, whereas the impact of a common shock remains constant. Moreover, $\|\boldsymbol{\nu}_B\|_2$ depends on the structure of the economy ($\boldsymbol{\Psi}$,$\boldsymbol{\delta}$, $\boldsymbol{\phi}$, $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$). This means that, as we consider alternative structures of the economy, the aggregate impact of bank–specific shocks varies.

In the remaining sections we thus study how the structure of the bank–to–firm network ($\boldsymbol{\Psi}$), firm–to–firm network ($\boldsymbol{\Omega}$) and heterogeneity in value added ($\boldsymbol{\theta}$) act as mechanisms through which individual bank shocks lead

where and $\boldsymbol{\psi}_{b|B}$ is the b'th column of the $\boldsymbol{\Psi}_B$ matrix. This expression shows that *gdp* is affected because the firms borrowing from bank b are forced to downscale production. However, aforementioned expression is not the end of the adjustment process. There is a second–order effect as these contractions additionally affect all firms (including those not borrowing from bank b) that rely on inputs from these firms. This is captured by the second term in (5)

$$\nu_{b|B}^{(n=0)} = \kappa\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\psi}_{b|B}$$

where $\widetilde{\boldsymbol{\Omega}}$ quantifies the spillovers from affected firms to their immediate firm customers. Continuing the propagation in this fashion, with higher order effects ($n > 1$), we have that the total real effect of the shock from bank b is given by (4).

[8]More precisely, $\|\boldsymbol{\nu}_B\|_1 = \kappa(\sum_{n=0}^{n} \widetilde{\boldsymbol{\Omega}^n})\boldsymbol{\iota}$.

to aggregate fluctuations. To this end, we first investigate the size of aggregate volatility generated by micro level volatility σ in an economy with a given number of banks B and network structure ($\boldsymbol{\Psi}$, $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$). Second, we assess the rate at which this aggregate effect decays as the number of banks increases. That is, we investigate how fast $\|\boldsymbol{\nu}_B\|_2$ vanishes as B increases.

To achieve these two objectives, we do not analyse $\boldsymbol{\nu}_B$ directly. Recall from (5) that $\boldsymbol{\nu}_B$ can be rewritten as a Neumann series. The elements of this infinite sum are positive. Hence, one can provide lower bounds on the size and decay of $\|\boldsymbol{\nu}_B\|_2$ by incremently analyzing $\nu_{b|B}^{(n=0)}, \nu_{b|B}^{(n=1)}, \ldots$. Not only is this analytically more tractable, it also delivers direct insight into the reason why some banks are more influential than others.

3.4 Disentangling the Network Origins of Bank Influence

3.4.1 First–Order Connections

This section analyses $\boldsymbol{\nu}_B^{(n=0)} \equiv [\boldsymbol{\nu}_{1|B}^{(n=0)}, \ldots, \boldsymbol{\nu}_{B|B}^{(n=0)}]'$

$$\boldsymbol{\nu}_B \geq \boldsymbol{\nu}_B^{(n=0)} = \kappa \underbrace{\boldsymbol{\Psi}'\boldsymbol{\theta}_B}_{\substack{\text{First–order} \\ \text{outdegree of} \\ \text{banks}}} = \kappa \begin{pmatrix} d_{1|B}^{(1)} \\ \vdots \\ d_{B|B}^{(1)} \end{pmatrix} \tag{7}$$

where $\geq$ and $=$ hold elementwise.

$\boldsymbol{\nu}_B^{(n=0)}$ does not depend on the production structure of the real economy since shock propagation through production chains in the real economy is muted in the first order approximation. In $\boldsymbol{\nu}_{b|B}^{(n=0)}$, the influence of bank b on real gdp depends only on its connection with firms that produce value added. In order to quantify this notion, we define the *first–order outdegree firm k*.

Definition 1. *The first–order outdegree of firm j is defined as $d_{j|J}^{(1)} \equiv \theta_j$.*

The set $d_J^{(1)} = \{d_{1|J}^{(1)}, ..., d_{J|J}^{(1)}\}$ *is called the first–order outdegree sequence of the real economy* $\mathcal{E}_B$*.*

In $d_{j|J}^{(1)}$, the superscript (1) signals it is a *first–order* outdegree. The subscript $|J$ designates it is an outdegree for firms (not banks). The first–order outdegree of firm j trivially equals its share in aggregate real gdp. The motivation for including this trivial definition is to illustrate the analogy with higher order outdegrees to be defined below. The *first–order outdegree of bank* b maps this firm–level influence to banks, as is evident from the following definition.

Definition 2. *The first–order outdegree of bank* b *is defined as:* $d_{b|B}^{(1)} \equiv \sum_{j=1}^{J} d_{j|J}^{(1)}(1-\delta)\psi_{jb|B}$*. The set* $d_B^{(1)} = \{d_{1|B}^{(1)}, ..., d_{B|B}^{(1)}\}$*, is called the first–order outdegree sequence of the financial sector in economy* $\mathcal{E}_B$*.*

Since $d_{j|J}^{(1)}$ quantifies the contribution of firm j in real gdp, $d_{b|B}^{(1)}$ quantifies the extent to which bank b lends to such firms. In general, here and below, the credit network $\mathbf{\Psi}$ maps the firm–level influence to individual banks.

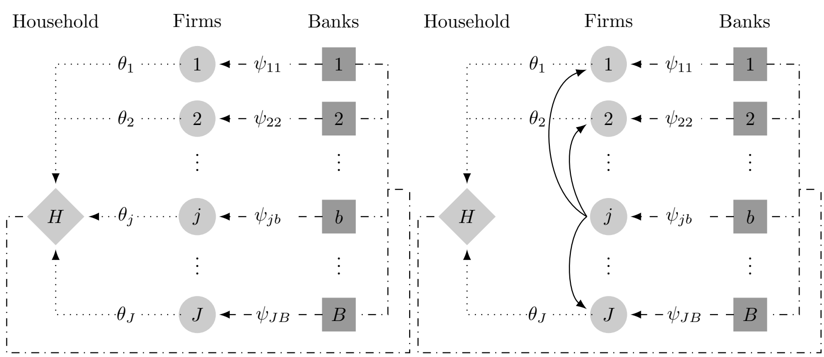


Figure 1: A graphical representation of two hypothetical economies.

We next define the *coefficient of variation*, which measures the asymmetry in $d_B^{(1)}$

Definition 3. *Provided an economy $\mathcal{E}_B$ with first–order outdegree sequence of the financial sector $d_B^{(1)}$, the coefficient of variation of $d_B^{(1)}$ is*

$$CV_{d_B^{(1)}} \equiv \frac{\sqrt{\mathbb{V}(d_B^{(1)})}}{\bar{d}_B^{(1)}} \tag{8}$$

where $\bar{d}_B^{(1)} \equiv \frac{1}{B}\sum_{b=1}^{B} d_{b|B}^{(1)}$ is the average bank outdegree and $\mathbb{V}(d_B^{(1)}) = \left(\frac{1}{B}\sum_{b=1}^{B}(d_{b|B}^{(1)} - \bar{d}_B^{(1)})^2\right)^{\frac{1}{2}}$ is the population variance of $d_B^{(1)}$.

To ground the intuition of $CV_{d_B^{(1)}}$, reconsider stylized economies $\mathcal{E}_B^{(i)}$. If $\{\theta_j = J^{-1}\}_{j=1}^{J}$, each bank supports an equal share of gdp, in which case $CV_{d_B^{(1)}} = 0$. To the extent that $\theta_1 \neq \theta_2 \neq ...$, some banks are better tied to value added production than other, $CV_{d_B^{(1)}}$ increases.

The following proposition documents the impact of such an asymmetric structure on the size of aggregate volatility caused by bank–level shocks

Proposition 1. *In economy $\mathcal{E}_B$, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq (1-\alpha)\left(\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2}\right)\sigma + \|\boldsymbol{\nu}_B\|_1\sigma$$

or, equivalently,

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \frac{\kappa}{\sqrt{B}}\sqrt{1 + CV^2_{d_B^{(1)}}}\sigma + \|\boldsymbol{\nu}_B\|_1\sigma$$

Proof: See appendix.

Proposition 1 posits that, for a fixed number of banks B, a larger asymmetry in the first–order outdegree distribution of the financial sector implies a larger aggregate volatility that originates from bank–level shocks. Intuitively, for a fixed B, the shocks from the influential banks are not easily set off by shocks from other banks.

In addition, proposition 1 posits that, for a constant $CV_{d_B^{(1)}}$, a larger number of banks in the economy implies a lower aggregate volatility that originates from bank–level shocks. The reason is that, when B is small, bank level idiosyncrasies generally have a smaller tendency to cancel out. When B increases, following a law of large numbers type of intuition, banks shocks tend to average each other out, resulting in smaller aggregate fluctuations. The rate at which this happens depends on the distribution of the first–order outdegree of the financial sector. It is then interesting to verify what the bracketed expression in proposition 1 looks like for relevant distributions of the first–order outdegree of the financial sector.

In general, the first–order outdegree sequence of a network is often found to have power law tails (Clauset et al., 2009).[9] In keeping with Gabaix (2008), we define a power law as

Definition 4. *The random variable X follows a power-law distribution with shape parameter ϱ when $Pr(X > x) = (\frac{x}{x_0})^{-\varrho}$ for $x \geq x_0$ and $\varrho > 0$.*

As shown in the appendix, the following proposition quantifies the rate of decay of aggregate volatility if the first–order outdegree of the financial sector follows a power law distribution

Corollary 1. *Consider a sequence of economies $\{\mathcal{E}_B\}_{B\in\mathbb{N}}$ with a power law first–order outdegree sequence of the financial sector and shape parameter $\varrho \in (1, 2)$. Then aggregate volatility due to bank–specific shocks satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} = \Omega\Big(\frac{1}{B^{\frac{\varrho-1}{\varrho}}}\Big)$$

where the Landau notation $f(B) = \Omega(g(B))$ implies $f(B)$ is bounded below by $g(B)$ asymptotically.

The corollary establishes that, if the distribution of the first-order outdegree the financial sector is governed by a power law distribution,

[9] Stiglitz et al. (2011) finds that the number of clients per bank follows a power law. Ennis (2001) finds that bank sizes can be characterized by a power law.

then aggregate volatility decays at a rate $\frac{1}{B^{\frac{\varrho-1}{\varrho}}}$. This rate becomes slower as $\varrho \to 1$, where ϱ close to 1 means that the distribution of the first–order outdegree which is very asymetric. Intuitively, when ϱ is close to 1, only a few banks have a large first–order outdegree (compared to the bulk of other banks, which have a small first–order outdegree). Then, even when the number of banks in the economy is very large, shocks to this minority of influential banks do not easily cancel out with the bulk of less influential banks.

Corrollary 1 is therefore useful when comparing the relative size of aggregate volatility between economies with the same number of banks B but a different asymmetry ϱ, or vice versa. An important assumption underlying corrollary 1 is that throughout the increase in B, the asymmetry as governed by ϱ should stay constant. E.g. if the number of banks in the economy decreases because two banks merge, the net effect on aggregate volatility depends on (i) the move of B to $B-1$ and on (ii) whether $CV_{d_B^{(1)}}$ increases or decreases.

Lastly, note that the standard diversification argument, based on the central limit theorem, rate in the corollary allows for decay rates significantly slower than the one predicted by the diversification argument: $\sqrt{B}$.

The arguments in this subsection relied on a first–order approximation of the influence vector and ignored propagation of shocks through production linkages. We take this step in the next subsection.

3.4.2 Second Order Connections

This section analyses $\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)}$

$$\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)} = \kappa \underbrace{\boldsymbol{\theta}'\boldsymbol{\Psi}_B}_{\substack{\text{First–Order}\\\text{Outdegree of}\\\text{banks}}} + \kappa \underbrace{\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\Psi}_B}_{\substack{\text{Second–Order}\\\text{Outdegree of}\\\text{banks}}} = \kappa \begin{pmatrix} d_{1|B}^{(1)} \\ \vdots \\ d_{B|B}^{(1)} \end{pmatrix} + \kappa \begin{pmatrix} d_{1|B}^{(2)} \\ \vdots \\ d_{B|B}^{(2)} \end{pmatrix} \tag{9}$$

$\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)}$ not only takes into account that banks are influential because of their first–order interconnectedness. It also accounts for the fact that the impact of a shock from bank b to its borrowers does not remain confined with its borrowers. In a first step, the shock also propagates from the corporate borrowers to their corporate customers. E.g. although an electricity distributor typically represents a material share in gdp (large $d^{(1)}_{j|J}$), this direct share underestimates its importance for aggregate gdp given that a material share of other firms directly rely on this electricity distributor for their production (i.e. $d^{(2)}_{j|J}$ is large).

In order to formalize this notion, we define the second–order outdegree of firm j.

Definition 5. *The second–order outdegree of firm j is defined as $d^{(2)}_{j|J} \equiv \sum_{j'=1}^{J} \theta_{j'}\widetilde{\omega}_{j'j} = \boldsymbol{\Omega}$. The set $d^{(2)}_J = \{d^{(2)}_{1|J}, ..., d^{(2)}_{J|J}\}$ is called the second–order outdegree sequence of the real economy $\mathcal{E}_B$.*

In general, $d^{(2)}_{j|J}$ quantifies the extent to which firm j is a direct input supplier to firms in the economy that sell to households. The *second–order outdegree of bank b* captures the extent to which bank b lends to firms with a high second–order outdegree

Definition 6. *The second–order outdegree of bank b is defined as: $d^{(2)}_{b|B} \equiv \sum_{j=1}^{J} d^{(2)}_{j|J}(1-\delta)\psi_{jb|B} = \boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\Psi}_{\boldsymbol{b|B}}$. The set $d^{(2)}_B = \{d^{(2)}_{1|B}, ..., d^{(2)}_{B|B}\}$, is called the second–order outdegree sequence of the financial sector in economy $\mathcal{E}_B$.*

$d^{(2)}_{b|B}$ captures the fact that, for a bank to be influential for gdp, it need not necessarily borrow to those firms that produce a lot of value added (proposition 1). It can also be influential if it provides credit to these firms that are important input suppliers to the firms that produce much value added. Hence, the the information contained in $d^{(2)}_{b|B}$ is fundamentally different from that contained in $d^{(1)}_{b|B}$.

The next proposition refines proposition 1 using the additional information of the firm–to–firm production network:

Proposition 2. *In economy $\mathcal{E}_B$, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \sqrt{\sum_{b=1}^{B}(d^{(1)}_{b|B})^2(1-\alpha)\sigma} + \sqrt{\sum_{b=1}^{B}(d^{(2)}_{b|B})^2(1-\alpha)\sigma} + \|\boldsymbol{\nu}_B\|_1\sigma$$

or, equivalently,

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \big(\frac{1}{\sqrt{B}}\sqrt{1+CV_{d^{(1)}_{b|B}}} + \frac{\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\iota}}{\sqrt{B}}\sqrt{1+CV_{d^{(2)}_{b|B}}}\big)\kappa\sigma + \|\boldsymbol{\nu}_B\|_1\sigma$$

Proof: See appendix.

Proposition 2 shows how second–order interconnections, captured by $CV_{d^{(2)}_B}$, affect aggregate volatility. Proposition 2 is economically a more interesting result as it captures not only the fact that some banks are important credit providers to producers of the household good, but also the more subtle notion that some firms are key input suppliers to aforementioned firms, and banks gain influence when lending to these firms. E.g. in economy $\mathcal{E}^{(II)}_B$, bank b has a zero first–order outdegree yet it is influential since its borrower is a key input suppliers to all other firms.

The input–output structure of the real economy as a vehicle through which bank shocks propagate is an interesting feature in the context of skewed sector presence. Skewed sector presence implies that some individual banks are dominant credit providers to firms in specific (non–financial) sectors (e.g. De Jonghe et al. (2016)). Following Proposition 2, this can lead to increased macroeconomic volatility as bank shocks propagate, via their respective sector of specialization, to all other other firms that rely on inputs from this sector. In addition, it also alludes to the identification of economic significance of banks (e.g. in the context of macroprudential policy, state aid, etc.) and competition policy concepts (such as the Herfindahl Index). We come back to these policy related questions at the end of the paper.

We have the following counterpart to corrollary 1

Corollary 2. *Consider a sequence of economies* $\{\mathcal{E}_B\}_{B\in\mathbb{N}}$ *where (i) the first-order outdegree of the financial sector* $d^{(1)}_{b|B}$ *follows a power law outdegree sequence with shape parameter* $\varrho \in (1,2)$ *and (ii) the second-order outdegree of the financial sector* $d^{(2)}_{b|B}$ *follows a power law outdegree sequence with shape parameter* $\zeta \in (1,2)$*, then aggregate volatility due to bank–specific shocks satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} = \Omega\big(B^{-\frac{(\nu-1)}{\nu}}\big) \quad \text{where } \nu = Min\{\varrho, \zeta\}$$

where the Landau notation $f(B) = \Omega(g(B))$ *implies* $f(B)$ *is bounded below by* $g(B)$ *asymptotically.*

As before, this corollary establishes that, if the distribution of second-order outdegree of the financial sector has a power law tail, then aggregate volatility decays at a slower rate than predicted by the standard diversification argument.

The *dominance property* (see. Jessen and Mikosch (2006)) of the power law bridges corrolary 1 and corrolary 2. For a sequence of economies in which the empirical distributions of the first and second order outdegrees of the financial sector have power law tails with shape parameters ϱ and ζ, the tighter bound for the decay rate of aggregate volatility is determined by $min\{\varrho, \zeta\}$.[10]

3.4.3 Higher Order Bank Shocks

We now allow for shock propagation beyond the buyer–supplier relationship and account for higher order relationships (e.g. customers of customers). These interconnections in the real economy imply that bank shocks are not confined to its immediate borrowers (as in section 3.2) or corporate customers of these borrowers (as in section 3.3), but propagate

[10] In addition, if the first-order outdegree sequence of the financial sector does not have a power law tail but the second-order does (or vice versa), then the power law tail will dominate and the rate of decay is determined by the shape parameter of the power law tail.

further through the economy across the value chain.

To formally capture these complex patterns of interconnections, we define the p'th order outdegree of firm j.

Definition 7. *The p'th-order outdegree of firm j is defined as* $d^{(p)}_{j|J} \equiv \sum_{j'=1}^{J} \theta_{j'} \left[\mathbf{\Omega}^{p-1}\right]_{j'j}$. *The set* $d^{(p)}_{J} = \{d^{(p)}_{1|J}, ..., d^{(p)}_{J|J}\}$ *is called the* p'*th–order outdegree sequence of the real economy* $\mathcal{E}_B$.

where $\left[\mathbf{\Omega}^{p}\right]_{j'j}$ designates the (j', j)'th element of $\mathbf{\Omega}^p$. This (j', j)'th element quantifies the indirect share of firm j of order p in the production function of firm j'. Note that for $p = \{1, 2\}$, this definition collapses to $\{d^{(1)}_{j|J}, d^{(2)}_{j|J}\}$, respectively. Analogously, the p'th order outdegree of bank b is

Definition 8. *The p'th–order outdegree of bank b is defined as:* $d^{(p)}_{b|B} \equiv \sum_{j=1}^{J} d^{(p)}_{j|J} \xi_{kb}$. *The set* $d^{(p)}_{B} = \{d^{(p)}_{1|B}, ..., d^{(p)}_{B|B}\}$, *is called the p'th–order outdegree sequence of the financial sector in economy* $\mathcal{E}_B$.

The statistic captures the extent to which individual banks affect influential firms in the economy indirectly through input-output linkages in the real economy. In general $d^{(p)}_{b|B}$ captures these higher order interconnections.

In general, for $m \geq 0$ we have the following decomposition in outdegrees

$$\boldsymbol{\nu}^{(N=m)}_{B} = \begin{pmatrix} \nu_{1|B} \\ \vdots \\ \nu_{B|B} \end{pmatrix} = \kappa \sum_{n=0}^{m} \underbrace{\begin{pmatrix} d^{(n+1)}_{1|B} \\ \vdots \\ d^{(n+1)}_{B|B} \end{pmatrix}}_{\text{(n+1)'th–Order outdegree}} \tag{10}$$

where the expression collapses to (7) and (9) for $m = 0$ and $m = 1$, respectively. The generalization of proposition 1 and proposition 2 is

Proposition 3. *In economy $\mathcal{E}_B$, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \Big(\sum_{m=0}^{M} \frac{\widetilde{\boldsymbol{\theta}}(\widetilde{\boldsymbol{\Omega}}')^m \boldsymbol{\iota}}{\sqrt{B}} \sqrt{1 + CV_{d_{b|B}^{(m)}}} \Big) \kappa\sigma + \|\boldsymbol{\nu}_B\|_1 \sigma$$

Proof: See appendix.

Proposition 3 captures the role of higher order interconnections. It captures the impact of *cascade effects*, where a bank induced firm shock propagates, not only to its immediate corporate customers, but also to customers of customers and higher order relationships in the real economy. The proposition echoes the intution of the *network–based financial accelerator* in Stiglitz et al. (2010), in which the failure of a single firm propagates via firm–to–firm linkages causing an avalanche of firm failures (although Proposition 3 provides a closed form expression whereas Stiglitz et al. (2010) provide no closed form solution but instead rely on a simulation exercise).

Proposition x also speaks to a recent empirical literature that emphasize bank size (total credit volume) as the relevant statistic for bank–specific contributions to real aggregate fluctuations Bremus et al. (2018). Although a correlation with bank size is apparent, as we demonstrate in our application below, a focus on bank–size tells is potentially misleading whereas proper identification of the banks' borrowers and their role in the production architecture is more informative.

3.5 Relaxing assumptions

We now discuss the implications of relaxing the model assumptions 1–6. Throughout, we focus on the intuition.

3.5.1 Relaxing assumption 3

We first allow for heterogeneous capital intensity. This requires modifying definition 2

Definition 9. *The modified first–order outdegree of firm* j *is defined as* $d^{(1)}_{j|J} \equiv (1-\delta_j)\theta_j$. *The set* $d^{(1)}_J = \{d^{(1)}_{1|J}, ..., d^{(1)}_{J|J}\}$ *is called the modified first–order outdegree sequence of the real economy* $\mathcal{E}_B$.

The modified first–order outdegree of bank b, $\widetilde{d}^{(1)}_{b|B}$ now take into account the fact that production of some of its borrowers are relatively more/less credit intensive. Proposition x then generalizes as follows

Proposition 4. *In economy* $\mathcal{E}_B$, *aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \Big(\sum_{b=1}^{B}(d^{(1)}_{b|B})^2 + \sum_{b=1}^{B}(d^{(1)}_{b|B} - \widetilde{d}^{(1)}_{b|B})^2\Big) + \Big(Cov(d^{(1)}_{b|B}, d^{(1)}_{b|B} - \widetilde{d}^{(1)}_{b|B}) + \mathbb{E}[d^{(1)}_{b|B}]\mathbb{E}[d$$

or

$$\sqrt{Var\ (\widehat{c}_{t|B})} \geq \frac{\kappa}{\sqrt{B}}\sqrt{1 + CV^2_{\widetilde{d}^{(1)}_B}}\sigma + \|\boldsymbol{\nu}_B\|_1\sigma$$

where Cov *is the population covariance operator. Proof: See appendix.*

As per proposition x, the volatility of gdp depends on an *asymmetry* effect and a *level* effect. The role of asymmetry is similar to before, but potentially exacerbated/moderated by a covariance term. If banks with a large first order outdegree typically lend to firms that are relatively less credit intensive, the covariance term dampens aggregate volatility. The level effect (last term) does not affect the asymmetry of banks, but dampens aggregate volatility as less of the value added activities in the economy require bank debt.[11]

3.5.2 Relaxing assumption 5

First, relaxing assumption 5 brings a time dimension to the story. That is, under assumption 5 the impulse response function is collapsed to the single period in which the shock occurs. The Calvo (1983)–Yun (1996)

[11] Generalizing proposition x beyond for higher order terms is straightforward, albeit the notation becomes very thorny and cumbersome. They are available upon request.

framework requires us to take into account the time dimensions. Following DeJong and Dave (2011)

$$c_{t|B} = \sum_{b=1}^{B} \sum_{s=t}^{\infty} \nu_{b,t-s} \epsilon_{bt-s} + \sum_{s=t}^{\infty} \nu_{b,t-s}$$

such that $Var(c_{t|B}) = \sqrt{\sum_{s=t}^{\infty} \boldsymbol{\nu}_{t-s}' \boldsymbol{\nu}_{t-s}} + \sum_{s=t}^{\infty} \boldsymbol{\iota}_B' \boldsymbol{\nu}_{t-s} = \|\boldsymbol{\nu}_B\|_2 \sigma + \|\boldsymbol{\nu}_B\|_1 \sigma$.

3.5.3 Relaxing assumption 4

Allowing for collateral constraints introduces a heterogeneous amplification mechanism to the model. To see this consider the households' Euler equation, which can be solved forward to yield

$$\frac{q_t}{c_t} = \sum_{s=0}^{\infty} \varrho^s j h_{t+s}^{-1} \equiv \mathcal{H}_t$$

Since land does not depreciate, h_t is effectively an idealized durable in the sense of Barsky et al. (2007). As ϱ is close to 1, any short term fluctuations in h_t affects $\mathcal{H}_t$ relatively little. The following approximation then holds

$$\frac{q_t}{c_t} \approx \mathcal{H}$$

This condition means that movements in land prices comove intimately with household consumption. Hence, shocks in the economy that stimulate household consumption will also drive up land prices.

E.g., a shock to the loan premium spills over to the firms marginal cost and is subsequently reflected in firm output prices. This cascades through the IO structure of the economy. Higher prices in the economy lower household consumption and depress housing prices. This decline in housing prices reduces the value of the pledgeable collateral of the firm, which endogenously constrains its ability to borrow and, consequently, its production capabilities. This, in turn, feeds back into rising prices, etc. A similar sequence of events unfolds following shocks to LTV ratios directly.

Aforementioned mechanism only introduces a level effect, i.e. it amplifies the size of individual bank shocks, but it does not affect the incidence to which they cancel each other out. Heterogeneous bank–firm specific LTV ratios do affect the tendency of averaging out. Banks that require higher LTV ratios from their borrowers install stronger financial accelerator mechanism with their firms. Shocks from these banks get amplified more vis–a–vis banks that set lower LTV ratios (see the discussion in Jensen et al. (2018); Walentin (2014)).

3.5.4 Relaxing assumption 1 and 2

As shown in our quantitative results below, a Taylor rule and non–linear labour supply only marginally affects the dynamics of the model variables. The role of relaxing these assumptions is thoroughly discussed in Pasten et al. (2018a).

4 Data sources and calibration

In this section we calibrate our model to the Belgian economy. We first elaborate the used data sources and subsequently discuss the details of our calibration.

4.1 Data sources

The calibration of our model relies on six confidential administrative data sources provided by the National Bank of Belgium (NBB): (*i*) the Business–to–Business transactions database ($B2B$), (*ii*) the Corporate Credit Register (CCR), (*iii*) the International Trade database (ITD), (*iv*) Firm Annual Accounts (AA) (*v*) Vale Added Tax declarations (VAT), and (*vi*) a panel of firm product–level price data underlying the official Belgian producer price index (PPI) statistics. All datasets are linked by VAT numbers and all minimally span the time frame $2002 - 2014$.

The backbone of our analysis constitutes the NBB B2B transactions database. This dataset documents both the extensive and the intensive margins of domestic buyer–supplier relationships in Belgium. In particular, an observation is the sum of all sales (in euros, excluding any value–added tax) from VAT–liable firm j to VAT–liable firm j' in a given calendar year. Coverage is quasi universal, as all relationships with annual sales of at least 250 euros must be reported, and pecuniary sanctions on late and erroneous reporting ensure high data quality.[12]

Note that, while the VAT ID is the legal entity of a firm in Belgium, some firms might be owned by other firms, generating intra–firm trade between parents and subsidiaries, which might not be subject to typical market forces. In this paper, we focus on propagation across firm boundaries. Given that the nature of propagation across firms, but within the confinements of a group structure, is likely to be different from across boundaries, we aggregate VAT IDs up to the group–level using ownership filings in the annual accounts. We carry out this aggregation in all data sources.[13]

The CCR contains monthly updated information on the population of loans granted from banks to non–financial firms. The banks are established in Belgium and licensed by the NBB. This concerns both (a) branches incorporated under foreign law established in Belgium as well as (b) banks incorporated under Belgian law. An observation in the CCR

[12]While there has been an increased interest in shock propagation across production networks, the available firm–to–firm data is often restricted in terms of the (a) sectoral coverage (see e.g. Acemoglu et al. (2015a)), (b) geographical coverage (see e.g. Carvalho et al. (2015)), (c) reporting thresholds (see e.g. Atalay et al. (2011); Bernard et al. (2015)), (d) nominal quantification of business relationships (see e.g. (Kelly et al., 2014; Carvalho et al., 2015)) or (e) level of aggregation (see e.g. **?**Acemoglu et al. (2012); Shea (2002)). The $B2B$ is unrestricted in these dimensions.

[13]We aggregate variables across multiple VAT IDs owned by the same firm. Inputs (B2B), Credit lines (CCR), Exports and Imports (ITD), labor costs (AA) are summed across subsidiaries to the group level. Intra–group transactions in the B2B database are dropped.

documents: (a) the bank, (b) the firm and (c) the nominal value (denominated in Euros) (d) and the collateral value of the granted loan at time t. The CCR is available at the monthly level but for our purpose converted to the annual level (i.e. the frequency of the other data sources). As of 2011, there is no reporting threshold (before, a threshold of $25,000$ euro was installed).

The AA reports firm–level costs attributed to day–to–day business operations. Firm level sales are also documented in the AA, but since their coverage is incomplete, we rely on VAT declarations to obtain firm–level sales. Given that both costs and sales also reflect cross–border trade we rely on the ITD to strip these numbers from imports and exports. The latter are obtained from the ITD database. This harmonizes our calibration with our closed economy set–up of our model.

Finally, the Belgian National Institute of Statistics collects, on a monthly basis, prices of industrial products, which are used to compute the Belgian producer price index. The price information is collected through a monthly phone survey of around $1,500$ firms. The firms participate on a voluntary basis. We use this data source to measure price stickiness.

To significantly reduce the dimensions (and computational complexities) of our model, we impose a set of mild sample restrictions at the firm–level. We restrict our analysis to firms in the private and non–financial sector that report (i) fixed assets larger than 100 Euro, (ii) more than one full time equivalent employee and (iii) non–negative wages at least once during the sample period. In addition, we require firms to participate in the Belgian production network (i.e. have at least one trade flow reported in the $B2B$ database) and borrow from at least one bank.

Applying these criteria reduces the number of firms significantly. Table 1 documents the dimensions of the used sample. In 2014, for example, only $217,973$ firms satisfy the above criteria (representing 22.10% of the then active firms in Belgium). The large reduction in sample size is mostly driven by the exclusion of very small firms that do not submit annual accounts to the Central Balance Sheet Office. Although these firms account for a sizeable fraction of firms in the Belgian economy, they only represent a small fraction of Belgian economic activity. E.g. in 2002, our subset of firms represent 74.01% of aggregate value added, 90.01% of total private employment. Although it captures only 66.08% of all observed firm–to–firm linkages, it does captures 82.27% of its volume.

4.2 Calibration

In this section we discuss the calibration of our model. The calibration of parameters not specific to our model set–up $\{\beta, \gamma, \varphi, j, \phi_{\pi}, \phi_{gdp}, \rho^{\ell}, \rho^{r}\}$ are taken from the literature and their values are documented in table 3. The calibration of the parameters $\{\delta_j, \phi_j, \alpha_j, \ell_j, \omega_{jj'}, \theta_j, \psi_{jb}\}$ are elaborated upon below. Unless stated otherwise, parametrization relies on the 2014 vintage of the available datasets.

$\widehat{\boldsymbol{\Omega}}$ is constructed using the $B2B$ database. From the first–order conditions $\omega_{jj'} = P'_j M_{jj'}/(P^{\omega}_j M_j)$, i.e. $\widehat{\omega}_{jj'}$ is the share of firm j expenditures on j' inputs in total inputs sourced by firm j. Both $P_{j'} M_{jj'}$ and $P^{\omega}_j M_j$ are directly inferred from the $B2B$ database. Some firms do not buy intermediates, in which case $\phi_j = 1$ and $\sum_{j'=1}^{J} \widehat{\omega}_{jj'} = 0$. For all other firms, $\sum_{j'=1}^{J} \widehat{\omega}_{jj'} = 1$.

$\widehat{\boldsymbol{\Psi}}$, the empirical counterpart for $\boldsymbol{\Psi}$, makes use of the CCR. In keeping with the model, $\widehat{\psi}_{jb}$ is calculated as the share of bank b credit to firm j in the total credit portfolio of firm j. The lTV values of individual firms, ℓ_{jb}, is inferred from the CCR as the total amount of pledged collateral of credit from bank b to firm j. Furthermore, $\ell_j = \sum_{j=1}^{J} \psi_{jb} \ell_{jb}$.

$\widehat{\boldsymbol{\theta}}$ captures the steady state shares of firm sales to final demand. We obtain total firm–level sales from the VAT declarations, from which we subtract (i) intermediate sales to other firms (from the B2B) and (ii) exports (from the ITD). The remaining sum captures firm–level sales for final domestic consumption.

For simplicity, we assume that entrepreneurs do not hire labour, $\nu_j : 1$. In that case, entrepreneur capital services purely reflect variation in credit supply. Then, in order to arrive at a model consistent estimate for δ_j and ϕ_j, note that, under CRS Cobb–Douglas technologies, $\delta_j \phi_j$, $\delta_j(1 - \phi_j)$ and δ_j reflect the share of wages, intermediates and capital services in total costs of firm j, respectively. Total costs, are observed from the AA. The wage bill, $W_t N_{jt}$ is directly reported in the AA, total intermediate inputs, $P^{\omega}_{jt} M_{jt}$, is observed from the $B2B$ database.

We use the confidential microdata underlying the producer price data (PPI) to calculate the frequency of price adjustment at the firm–level; α_j. We calculate the frequency of price changes at the firm level, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is 10 Euro for two months and then 15 Euro for another three months, one price change occurs during five months, and the frequency is 1/5. For firms that are not in the micro price sample, we set α_j as the average α_j of the sector in which firm j is active.

Table 2 documents summary statistics related to our calibration. The average capital share of production is 0.22, which is close to macroeconomic estimates (**?**). The average Calvo parameteres is 0.88, which implies that the average firms keeps its prices fixed for 2.77 quarters. In addition, the average ω_{ij} is small, meaning that most firms have a disaggregate input portfolio. The mean of LTV ratio's, here directly calibrated from micro–level data, is close to that used in related models (e.g. 0.89 in Iacoviello (2005), 0.9 Iacoviello (2015)). Nonetheless, significant dispersion in LTV ratios is present. Note that our model assumes a fixed extensive margin of firm–firm and bank–firm relations. In appendix E, we provide evidence that such a simplified view on the economy provides a good approximation of the Belgian production architecture and credit network supporting it.

5 Size of aggregate fluctuations

In this section we estimate the size of aggregate real volatility: $\|\boldsymbol{\nu}_B\|_2$. Throughout, we report this multiplier relative to $\|\boldsymbol{\nu}_B\|_1$, i.e. relative to the multiplier that maps an aggregate (common) bank shock of the same size to aggregate GDP volatility. $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ then quantifies the relative size of aggregate fluctuations caused by bank specific shocks vis–à–vis common shocks.

In addition to the calibrated true heterogeneous scenario, we consider various hypothetical economies where we impose symmetry on $\widehat{\boldsymbol{\Psi}}, \widehat{\boldsymbol{\Omega}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}}, \widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\delta}}$ respectively. More precisely, we define these as $\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_B, \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_J, \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\delta}}^S = \bar{\delta}\boldsymbol{\iota}_J$. $\boldsymbol{\iota}$ are unit vectors and the superscript S is shorthand notation for symmetry. In words; $\widehat{\boldsymbol{\Psi}}^S$ shuts down all heterogeneity in the bank–firm network and considers an economy where all banks are equally tied to the real economy. $\widehat{\boldsymbol{\theta}}^S$ represents an economy where all firms are equally important contributors to GDP. $\widehat{\boldsymbol{\Omega}}^S$ captures a set–up where all firms operate in isolation (i.e. no production network). $\widehat{\boldsymbol{\alpha}}^S, \widehat{\boldsymbol{\phi}}^S, \widehat{\boldsymbol{\delta}}^S$ impose homogeneous price stickiness, homogeneous labour and capital shares for all firms in the economy, respectively.

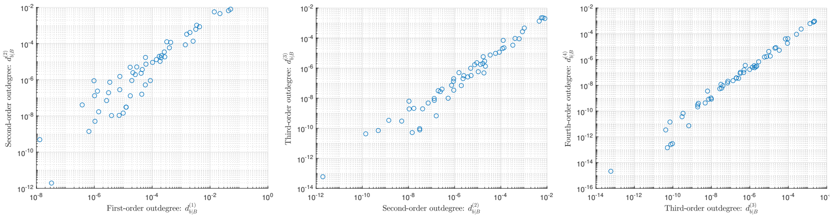
5.1 Simplified framework

The calibration results from our simplified framework (abiding assumptions 1–6) are documented in table (A). Across all rows and columns, aggregate volatility from idiosyncratic bank shocks is the smallest in scenario's involving $\widehat{\boldsymbol{\Psi}}$ (scenario $1, 3, 6$). From the discussion in the theory section, this scenario equalizes all $\{\nu_{b|B}\}_{b=1}^B$, implying that $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1} = \frac{1}{\sqrt{B}} = 0.18$. Intuitively, if all banks are tied to the Belgian economy in the same way, their idiosyncratic shocks maximally cancel each other out relative to an economywide shock, which does not cancel out. The role of individual banks is the largest in the full heterogeneous set–up, in which the impact of bank–level shock is 44% of what an aggregate shock of the same magnitude would cause. As per the discussion above, in this scenario the dominance of a minority of banks is most outspoken and their shocks do not easily cancel out with that of the other banks.

Which dimension of the structure of the economy introduces this large asymmetry in the influence of banks? First, consider the role of a heterogeneous value added structure. $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ increases, meaning that banks that are better tied to the real economy are also – on average – more tied to firms that create a lot of value added. A similar story is true for scenario 3, where some banks are very intimately connected to firms that take up a central role in the production process. Moreover, both asymmetries brought about by $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$ reinforce each other in the full heterogeneous set up. Banks that are disproportionally lend to firms that create a lot of value added are also indirectly – through IO interactions – very supportive to value added creation of firms to which they do not borrow directly. In the wording of the discussion in section 3, in figure 6, panel (a), we see that banks with a high (low) first order outdegree typically also have a high (low) second order outdegree, etc.

Across columns, we see that aggregate volatility is slightly attenuated while allowing for heterogeneous δ. This means that influential banks are, on average, tied to firms that are less credit intensive than the average firm. On the other hand, allowing for heterogeneous Calvo probabilities and labour shares, influential banks are tied (in)directly more to firms that have flexible prices and less labour intensive.

In conclusion, throughout, the magnitude of aggregate volatility induced by individual bank shocks is sizeable compared to that of an aggregate shock common to all banks. Throughout all scenarios, the multiplier associated with idiosyncratic bank shocks is almost half as large as the multiplier associated with one single aggregate (common) shock that affects all banks. This implies that the common practice of ignoring bank specific volatility is a strong restriction when explaining aggregate volatility. In appendix E we document similar results for other years.



(a) $d^{(1)}_{b|B}$ vs. $d^{(2)}_{b|B}$ (b) $d^{(2)}_{b|B}$ vs. $d^{(3)}_{b|B}$. (c) $d^{(3)}_{b|B}$ vs. $d^{(4)}_{b|B}$

Figure 2: The panels maps the $k'th$ order outdegree of bank b to the $k+1'th$ outdegree of bank b to establish a positive correlation. Data are for 2012.

5.2 Full framework

Compared to the simplified framework, the full model allows for (i) a collateral constraint amplification mechanism, (ii) the Calvo (1983)–Yun (1996) framework of staggered price setting, (iii) nonlinear labour disutility, (iv) a Taylor rule. The introduction of the Calvo Yun framework gives rise to impulse response functions whereas, in the previous section, all adjustments were instantaneous and the impulse response functions were collapsed to the period of the shock.

We first relax (i) and (ii). Figure x plots impulse response functions when in the presence of (i) the Calvo (1983)–Yun (1996) framework and (ii) collateral constraints amplification mechanism. We focus on a unit standard deviation in bank specific LTV requirements (similar graphs for a standard deviation in borrowing rates is included in appendix E). We focus on two banks, one in the top and bottom 10 banks according to the size of $\nu_{b|B}$ in figure x. On impact, a contraction in LTV ratios depresses the amount of debt its borrowers can take out, d_{kt}. This constrains production of firms borrowing from bank b. A drop in output drives up prices, which – in the context of an interlinked production structure – spill over to other firms (conditional on the role of intermediates). The latter firms will pass this marginal cost increase into their own prices, giving rise to a cascade effect on price. This, in turn, affects consumption and real gdp. In addition, a drop in the value of collateral endogenously tightens the the ability of firms to borrow, which sets in motion a second round of amplification effects.

In table x, we replicate table x. In panel A, we focus on homogeneous LTV ratios. In panel B, we allow for bank–firm specific LTV ratios. In panel A, Confirming our results from above, the processes that induce asymmetry in the simplified framework are still at work in the full framework. Appendix E gives similar results for an interst rate shock.

Heterogeneous collateral requirements introduce an additional source of asymmetry into the model. LTVs requirements of particular banks required by individual borrowers are heterogeneous.

In figure x we relax (iii) and (iv). and assess volatility in table x and y. A comparison with figure x and table x indicates that the introduction of (iii) non–linear labour disutilty ($\varphi = 1$) and a (iv) a Taylor rule does not bring quantitatively different dynamics to the table.

6 Decay of aggregate fluctuations

In section 5 we quantified the aggregate effect of bank–level shocks for a fixed number of B banks. In this section we investigate the rate at which this multiplier would decay should the number of banks in the economy vary. Following proposition 1, 2 and 3 in our simplified framework this rate depends on the asymmetry of the constituent elements of the influence vector.

We derived approximate (proposition 1, 2) and exact (proposition 3) rates of decay and focused on the particular case of power law distributions (corrollary 1, 2). In view of these propositions, table 6 fits a power law to the first four out-degrees of the banks.

The first order outdegree is found to follow a power law distribution with shape parameter 1.57, such that corrollary 1 implies that the multiplier in section 5.3 should not decay faster than the rate $B^{\frac{\beta}{\beta-1}} = B^{0.36}$.

The law or large number agument as posited by Lucas (1977), would suggest a rate of decay of $B^{0.5}$. The rate we identify is much slower. Intuitively, when there is a large asymmetry, shocks from banks with a large first–order outdegree are not easily offset by shocks from other banks, even in the face of a large number of other banks. Table 5 reveals that that the asymmetry in the higher order effects is even larger. Hence, by the dominance property (supra), these higher order effects place an even tighter bound of the rate of decay. E.g. using corrollary 2, the second–order outdegree suggests $\|\boldsymbol{\nu}_2\|$ to decay at the rate $B^{0.33}$.

In principle, it is possible to fit a power law tail to any distribution. Hence, using the goodness–of–fit test described in Clauset et al. (2009), we formally test whether the obtained power law is indeed a good characterization of the data. This test relies on a bootstrapping procedure and generates a p-value that can be used to quantify the plausibility of the hypothesis. If the p-value is large, then any difference between the empirical data and the power law can be explained with statistical fluctuations. If $p \simeq 0$, then the power law does not provide a plausible fit to the data. The goodness-of-fit tests reveal that the p values for the first three outdegrees are well above the 0.1 threshhold suggested by Clauset et al. (2009), confirming that the the distributions indeed follow a power law.

figure 6 then delivers a graphical intuition why higher order outdegrees reinforce the assymetry in the influence vector and further slow down the rate of decay. A similar result holds in figure 6, panel (b) and (c).

7 Financial sector policy

In this section, we show that our framework provides insights into various policy topics that have recently been addressed in the literature.

7.1 Distortion of the Herfindahl–Hirschmann index

Gabaix (2011) develops a theory of granularity for non–financial corporations (NFCs). It states that the Herfindahl–Hirschmann index (HHI) of the non—financial sector is a sufficient statistic to map microlevel volatilty of NFCs to economywide volatility.[14] Intuitively, a larger HHI reflects a larger market share of a subset of NFCs, in which case a non–trivial part of macroeconomic volatility can be traced to shocks to microeconomic volatility originating with large firms.

Bremus et al. (2018) develop a parallel framework for the financial sector, which puts bank size (total credit supplied to NFCs) at the center of the analysis. Under extreme parametrizations of our model, i.e. no inter firm trade $\{\phi_j\}_{j=1}^J = 1, \{\omega_{jj'}\}_{j,j'=1}^J$, equal value added of firms $\{\theta_j\}_{j=1}^J = 1$ on top of assumptions $1-6$, our framework collapses to theirs (see appendix E);

$$\begin{aligned}
\sqrt{Var(c_{t|B})} &= \sqrt{\sum_{b=1}^{B}\Big(\frac{d_b}{\sum_{b=1}^{B} d_b}\Big)^2}\sigma \\
&= \frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}\sigma \\
&= \frac{1}{\sqrt{B}}\sqrt{CV^2_{\frac{d_b}{\sum_{b=1}^{B} d_b}}+1} \\
&= \frac{1}{\sqrt{B}}\sqrt{CV^2_{d_b^{(1)}}+1}
\end{aligned}$$

where d_b is steady state of bank b credit in economywide credit (i.e. market shares), σ is microeconomic volatility and the bracketed terms reflects the HHI index. Under aforementioned assumptions, our relative multiplier $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}$ collapses to the HHI. Also note that the last equation holds exactly. That is, under this extreme parametrization, higher order outdegrees are zero and bank market shares are isomorph to gdp shares.

[14]But see recent contributions of Pasten et al. (2018a) and **?** who falsify this claim in the context of price rigities and nonlinearities, respectively.

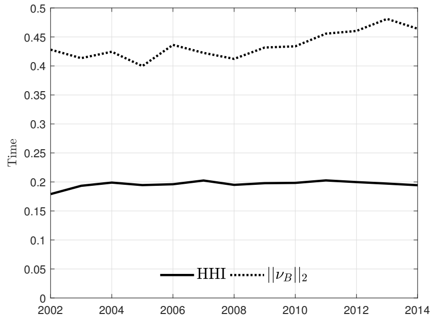


Figure 3: This figure plots the HHI and the norm of the influence vector under assumption $1-6$.

In general $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}$ does not collapse to the HHI. The reason is that the HHI focuses exclusively on market shares. Our framework, instead, focuses on the (in)direct role that the borrowing NFCs in the banks' portfolio play in the real economy. Providing a large volume of credit, but does not necessary make a bank influential if this credit is only marginally used for value added activities. $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}$ instead accounts how banks are tied to value added production in the economy, both directly and indirectly.

Figure x plots the HHI of the Belgian banking sector. In addition, we graph $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}$. We document two interesting insights. First, $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\boldsymbol{\nu}_B\|_1}$ is much larger than the HHI. Using the HHI as a multiplier for microeconomic volatility severly underestimates aggregate volatility given that it does not take into account the heterogeneous amplification mechanisms. Second, given the evolution of the HHI, equation 5 suggests that that macroeconomic volatility from micro–level shocks has remained stable over the last decade, whereas our framework highlights that it has gone up.

7.2 Strategic bank lending

An emerging literature has documented the incidence of bank specialization. E.g. banks have been documented to specialize in particular domestic geographic regions (Boeve et al. (2010)), industrial sectors (De Jonghe et al. (2016)), export activities (Paravisini et al. (2014)), etc. The benefits from such non–random matching of firms and banks include i.a. better screening abilities which reduce the problem of adverse selection and a better assessment of the collateral value (**?**). Specialized banks can detect a deterioration of the borrower's business earlier and may react in a timely manner by risk mitigation, for example, by requesting additional collateral (monitoring in a narrow sense). The large concentration of the banks lending portfolio and the increased sectoral Herfindahl are often cited as the downsides of bank specialization.

A relevant question is to ask whether such bank specialization is desirable from a macroeconomic viewpoint. That is, does sectoral specialization dampen or amplify macroeconomic volatility? In our framework, the question boils down to whether such specialization increases or decreases asymmetry in the economy. E.g. non–influential banks that start to specialize in the steel sector now indirectly impact all sectors that (in)directly rely on steel. As specialization renders this bank more influential, its shocks offset those from other influential banks. On the other hand, if such specialization occurs with an already influential banks, specialization enhances asymmetry and increases volatility.

In order to investigate the effect of bank–specialization in the Belgian economy, we define a hypothetical "specialization–neutral" credit network, which mutes the role of specialization. Define $\widetilde{\mathbf{\Psi}}$ with $\{\widetilde{\psi}_{jb}\}_{j=1}^{J} = \frac{\boldsymbol{\iota}'_J \boldsymbol{\psi}_b}{\boldsymbol{\iota}'_J \mathbf{\Psi} \boldsymbol{\iota}'}$. $\widetilde{\mathbf{\Psi}}$ homogenizes lending patterns of banks across sectors.

Figure 4 plots $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\widetilde{\boldsymbol{\nu}}_B\|_2}$, i.e. the ratio of the norm of the influence vector with the actual credit network and the specialization neutral credit network, respectively. From the plot, sector specialization is shown to have depressed aggregate volatility throughout $2002 - 2009$, but increases

7.3 Big banks vs. influential banks

We next relate our influence measure with banks size. Figure x correlates bank size $\frac{d_b}{\sum_{b=1}^{B} d_b}$ with a measure of bank influence in our framework

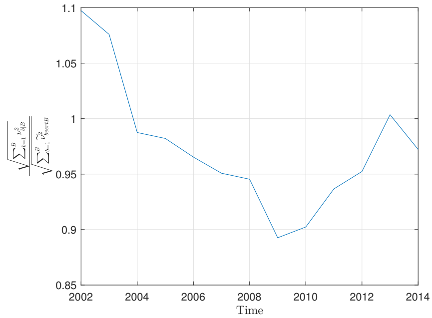


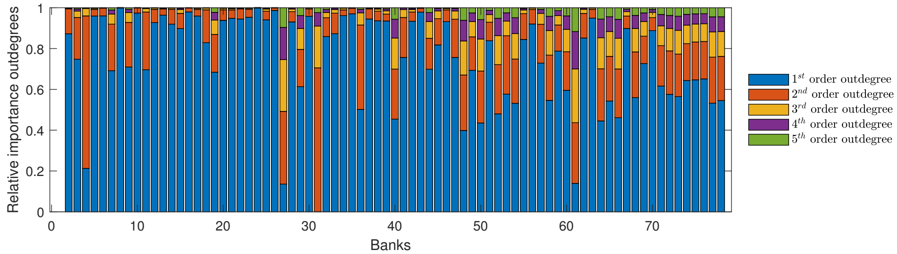
Figure 4: This figure plots the ratio

Figure 5: $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\widetilde{\boldsymbol{\nu}}_B\|_2}$, i.e. the ratio of the norm of the influence vector with the actual credit network and the specialization neutral credit network).

$\nu_{b|B}$. If the production architecture of the economy was irrelevant, all data would collapse on the 45 degree line. However, significant deviations exist and banks are ordered differently – even among the top banks.

One interesting question is to investigate how important are higher order effects in explaining bank influence? In order to answer this question, recall, following (**??**), that individual bank influence $\nu_{b|B}$ can be additively decomposed into its outdegrees $d^{(1)}_{b|B}$, $d^{(2)}_{b|B}$, etc. In figure 4, panel (a), we plot the relative share of each outdegree in total bank influence $\nu_{b|B}$. Banks are ordered according to bank size $\frac{d_b}{\sum_{b=1}^{B} d_b}$ from left to right. A key insight from the emerging pattern is that, within banks, the origins of bank influence is very heterogeneous. Multiple banks derive their influence almost exclusively from lending to central firms in the production network. Other banks are influential because they directly support firms that create a lot of value added. On average, most influence is derived from the size of their immediate borrowers (first–order outdegree). Higher order outdegrees, which capture network propagation effects in the real economy, represent on average 30% of bank influence.

The imperfect correlation of bank size and influence is echoed in art. 4 of the SSM regulation which stipulates that some banks can be significant (and fall under supervision of the ECB) due to its economic importance to a member state – irrespective of its size. Our influence provides guidance on identifying such significant banks.



8 Conclusion

Any modern economy is characterized by an interlinked production architecture in which firms rely on each other for their input requirements. As credit is vital to support this production process, shocks to credit availability of individual firms propagate throughout this production network. In this paper, we study how the structure of the real economy determines the aggregate real impact of lending shocks from individual banks.

To than end, we develop a NK model which combines a standard amplification mechanism brought about by financial frictions with an amplification mechanism in the real economy caused by firm–to–firm interactions. The framework provides a simple but powerful framework to identify how banks are both directly and indirectly supportive of value creation by non financial firms in the real economy.

We show that two interlocked networks (in casu individual firm credit portfolios and the production architecture of the real economy) introduce large asymmetries in the influence of individual banks. Stated differently, some banks intensively support – directly and/or indirectly – value added of the non–financial sector, whereas other banks are only marginally connected to firms involved in value added creation. Consequently, when some banks are asymmetrically influential, their credit supply shocks do not easily cancel out with that of other banks. We show that this has policy implications related to the identification of significant banks, the scope to which one can reliably use the Herfindahl–Hirschmann index and the role of sector specialization of individual banks.

A Tables

Table 1: Dimensions used data

	B2B		CCR		Variables				
Year	Firms (number)	Obs. (number)	Banks (number)	Obs. (number)	Value added (share total)	Employment (share total)	Trade transactions (share total)	Trade volume (share total)	Firms (share total)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2002	113,809	4,091,210	89	165,081	74.01	90.01	66.08	82.27	15.69
2003	117,184	4,271,350	86	154,352	72.89	89.56	65.61	76.91	15.98
2004	120,414	4,318,907	81	153,872	70.26	88.12	66.05	76.44	16.08
2005	123,028	4,468,199	77	156,458	72.14	88.16	66.93	74.91	16.29
2006	126,996	4,677,163	76	161,938	72.22	88.07	67.07	74.38	16.42
2007	132,302	4,824,779	80	169,411	73.78	87.68	68.05	76.34	16.68
2008	134,466	4,982,887	74	173,061	62.35	86.51	67.44	75.26	16.91
2009	138,665	4,934,340	73	178,838	62.91	86.31	67.89	76.81	16.97
2010	139,938	5,089,192	67	179,208	60.03	86.41	38.36	74.56	16.82
2011	146,335	5,299,890	65	187,191	56.93	86.65	68.81	74.75	17.01
2012	214,663	6,697,987	68	289,286	54.22	89.94	74.34	73.33	22.86
2013	218,775	6,662,241	64	198,401	52.2	89.86	74.23	74.99	22.81
2014	217,973	6,708,970	61	194,046	50.24	88.71	72.86	75.41	22.1

This table reports the dimensions of the data sample used and reports the share of our sample in aggregate Belgian statistics. Value added (column (6)) is derived by summing firm–level value added (firm total sales minus imports and intermediate purchases). Employment (column (7)) is total full time equivalent obtained from the annual accounts. Total firms (column (12)) is the total number of individual firms in the raw annual accounts, $B2B$ data and CCR prior to imposing the exclusion restrictions.

Table 2: DESCRIPTIVE STATISTICS STRUCTURAL PARAMETERS

		Percentile				
Variable	**Mean**	**10**	**25**	**50**	**75**	**90**
Calvo probability (α_j)	0.88	0.78	0.83	0.93	0.96	0.98
Capital share ($1-\delta_j$)	0.22	0.01	0.053	0.17	0.31	0.52
Labour share ($\delta_j \phi_j$)	0.47	0.15	0.29	0.46	0.95	0.79
Intermediate input share ($\delta_j(1-\phi_j)$)	0.3	0.05	0.13	0.28	0.45	0.602
Loan–to–value ratio (ℓ_j)	0.91	0.791	0.915	0.929	1	1
Intermediate input share ($\omega_{jj'} \times 100$)	1.28	0.002	0.013	0.073	0.416	2.031
Share credit portfolio ($\psi_{jb} \times 100$)	0.39	0.03	0.12	0.31	0.58	0.78

This table provides summary statistics on the structural parameters used in the model. Summary statistics are based on the pooled sample of firms throughout 2002 − 2014. Descriptives of intermediate input shares and credit portfolio shares are based on the subsample $\omega_{j'j'}, psi_{jb} \in (0,1)$.

Table 3: Parameter values

Parameters	Value	Explanation
β	0.95	Discount factor households
γ	0.95	Discount factor entrepreneurs
η	2	Elasticity of substitution across firms
α	0.2	Price stickiness parameter
ℓ	0.2	Price stickiness parameter
κ	3.5	Elasticity of substitution across banks
ϕ_π	1.5	Taylor rule parameter, consumer inflation
ϕ_{gdp}	0.5	Taylor rule parameter, consumption variation
φ	1.5	Inverse of Frish labour supply
$\rho^{(\ell)}$	0.9	AR parameter LTV shocks
$\rho^{(r)}$	0.9	AR parameter interest shocks

Calibration of model parameters. CES elasticity of substitution is taken from **?**. Bank markups to firms and autoregressive parameters are taken from Gerali et al. (2010).

Table 4: Results in a simplified framework

	$\boldsymbol{\Xi}$	$\boldsymbol{\Omega}$	$\boldsymbol{\theta}$	Het $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Het $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$
(1)	Het, $\widehat{\boldsymbol{\Xi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.464	0.427	0.440	0.473	0.427
(2)	Hom, $\widehat{\boldsymbol{\Xi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.134	0.134	0.134	0.134	0.134
(3)	Het, $\widehat{\boldsymbol{\Xi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.400	0.368	0.380	0.408	0.368
(4)	Het, $\widehat{\boldsymbol{\Xi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.134	0.134	0.134	0.134	0.134
(5)	Hom, $\widehat{\boldsymbol{\Xi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.370	0.341	0.352	0.378	0.341
(6)	Het, $\widehat{\boldsymbol{\Xi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.134	0.134	0.134	0.134	0.134
(7)	Hom, $\widehat{\boldsymbol{\Xi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.330	0.303	0.313	0.336	0.303
(8)	Hom, $\widehat{\boldsymbol{\Xi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.134	0.134	0.134	0.134	0.134

$\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_B, \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_J, \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J$. $\boldsymbol{\iota}$ are unit vectors and the superscript S is shorthand notation for symmetry. In words; $\widehat{\boldsymbol{\Psi}}^S$ shuts down all heterogeneity in the bank–firm network and considers an economy where all banks are equally tied to the real economy. $\widehat{\boldsymbol{\theta}}^S$ represents an economy where all firms are equally important contributors to GDP. $\widehat{\boldsymbol{\Omega}}^S$ captures a set–up where all firms operate in isolation (i.e. no production network). $\widehat{\boldsymbol{\alpha}}^S, \widehat{\boldsymbol{\phi}}^S, \widehat{\boldsymbol{\delta}}^S$ impose homogeneous price stickiness, homogeneous labour and capital shares for all firms in the economy, respectively.

Table 5: RESULTS IN THE FULL FRAMEWORK

	$\boldsymbol{\Psi}$	$\boldsymbol{\Omega}$	$\boldsymbol{\theta}$	Het $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Het $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$
Idiosyncratic interest rate shocks								
(1)	Het, $\widehat{\boldsymbol{\Psi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.617	0.567	0.586	0.629	0.567
(2)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.179	0.179	0.179	0.179	0.179
(3)	Het, $\widehat{\boldsymbol{\Psi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.532	0.490	0.505	0.543	0.490
(4)	Het, $\widehat{\boldsymbol{\Psi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.179	0.179	0.179	0.179	0.179
(5)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.493	0.453	0.468	0.503	0.453
(6)	Het, $\widehat{\boldsymbol{\Psi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.179	0.179	0.179	0.179	0.179
(7)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.439	0.404	0.417	0.448	0.404
(8)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.179	0.179	0.179	0.179	0.179
Idiosyncratic LTV shocks								
(1)	Het, $\widehat{\boldsymbol{\Psi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.893	0.567	0.586	0.629	0.567
(2)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\theta}}$	0.259	0.259	0.259	0.259	0.259
(3)	Het, $\widehat{\boldsymbol{\Psi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.770	0.490	0.505	0.543	0.490
(4)	Het, $\widehat{\boldsymbol{\Psi}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.259	0.259	0.259	0.259	0.259
(5)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.713	0.453	0.468	0.503	0.453
(6)	Het, $\widehat{\boldsymbol{\Psi}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.259	0.259	0.259	0.259	0.259
(7)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\theta}}$	0.635	0.404	0.417	0.448	0.404
(8)	Hom, $\widehat{\boldsymbol{\Psi}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\theta}}^S$	0.259	0.259	0.259	0.259	0.259

$\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_B, \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_J, \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J$. $\boldsymbol{\iota}$ are unit vectors and the superscript S is shorthand notation for symmetry. In words; $\widehat{\boldsymbol{\Psi}}^S$ shuts down all heterogeneity in the bank–firm network and considers an economy where all banks are equally tied to the real economy. $\widehat{\boldsymbol{\theta}}^S$ represents an economy where all firms are equally important contributors to GDP. $\widehat{\boldsymbol{\Omega}}^S$ captures a set–up where all firms operate in isolation (i.e. no production network). $\widehat{\boldsymbol{\alpha}}^S, \widehat{\boldsymbol{\phi}}^S, \widehat{\boldsymbol{\delta}}^S$ impose homogeneous price stickiness, homogeneous labour and capital shares for all firms in the economy, respectively.

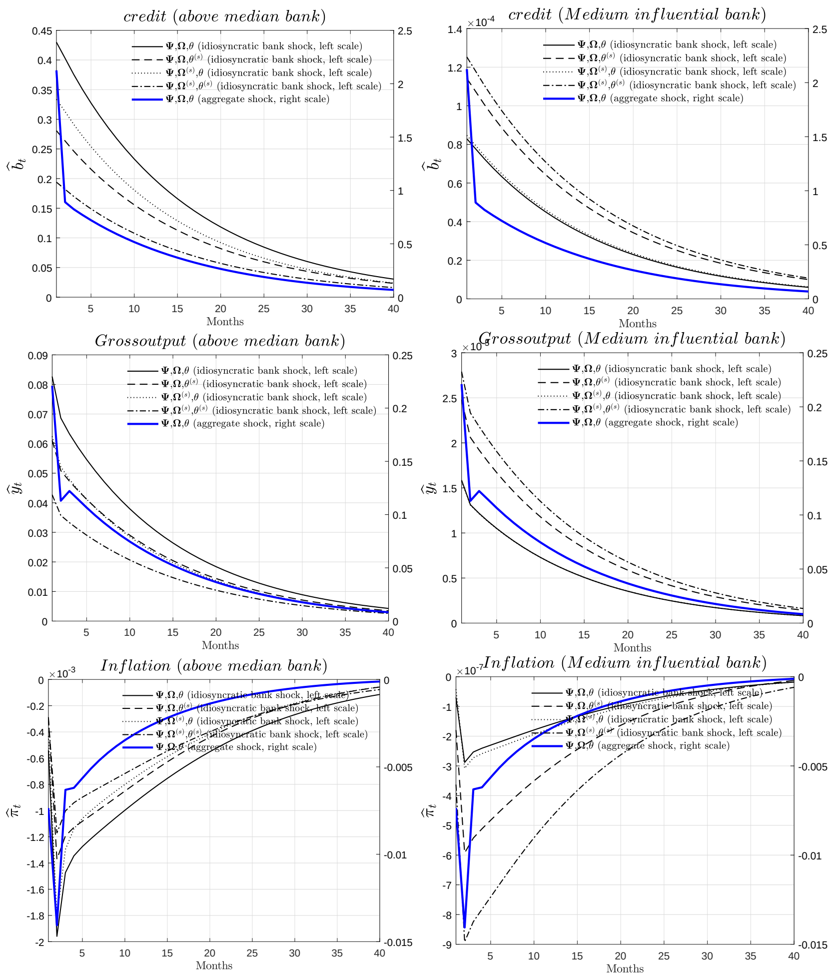
Table 6: Power law fit to outdegrees

	First order outdegree $d_B^{(1)}$	Second order outdegree $d_B^{(2)}$	Third order outdegree $d_B^{(3)}$	Fourth order outdegree $d_B^{(4)}$
Shape parameter	1.37	1.31	1.23	1.21
Diversification $B^{\frac{\beta-1}{\beta}}$	$B^{0.27}$	$B^{0.23}$	$B^{0.18}$	$B^{0.17}$
Goodness-of-fit test	$p = 0.22$	$p = 0.14$	$p = 0.23$	$p = 0.12$

Power law fit using the methodology in Clauset et al. (2009). Power law fit using the rank correction methodology discussed in **?** delivers similar results.

B Figures

B.1 Impulse response functions



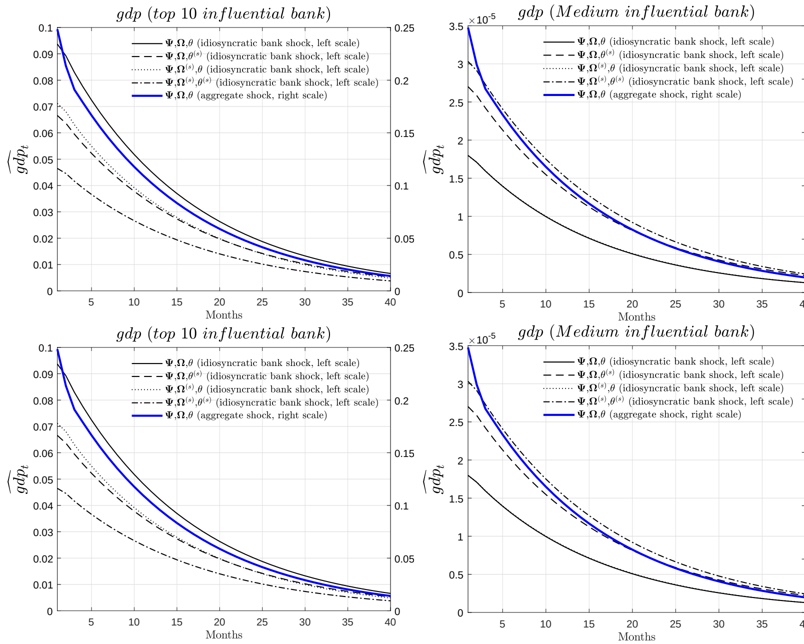
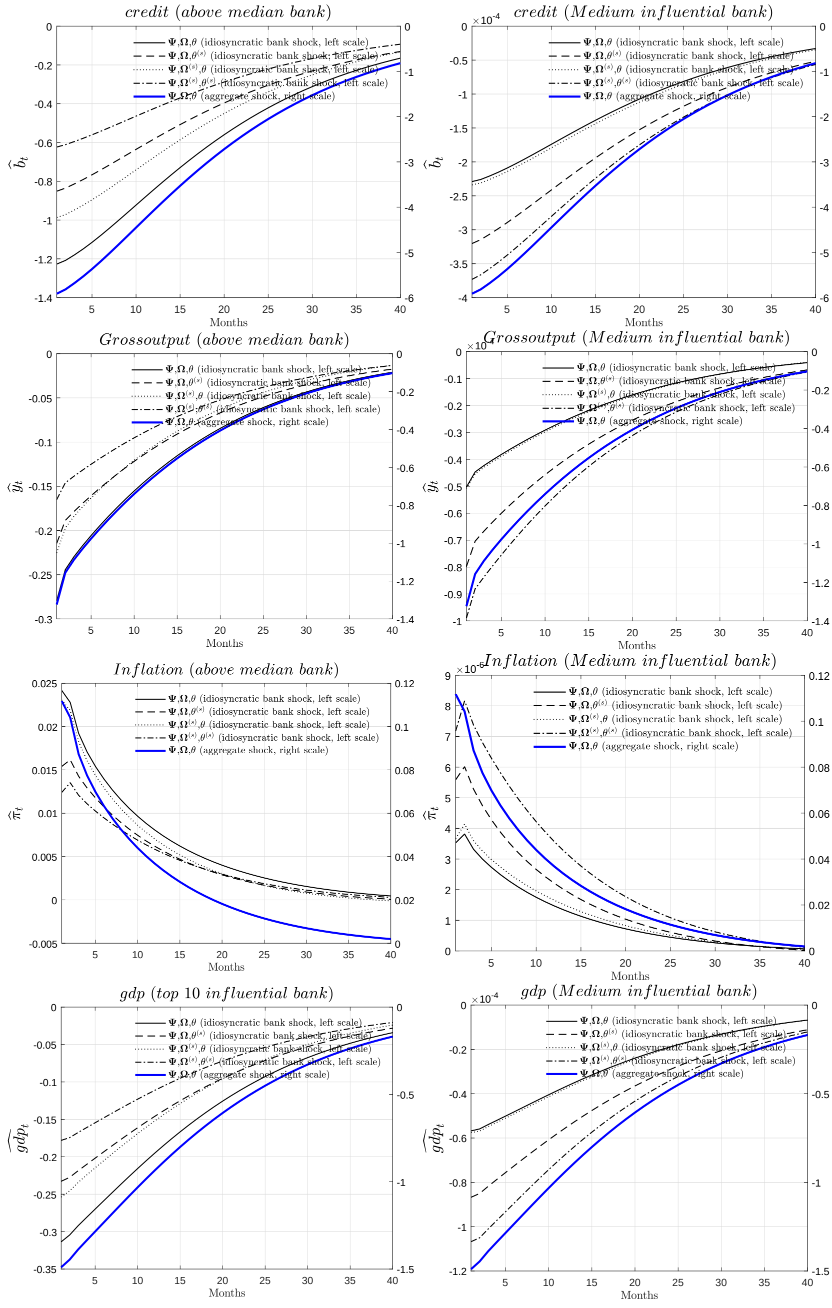


Figure 6: Impulse response function of the model assuming $\varphi = 0$ and monetary policy targetting $P_t C_t = PC$.


credit (above median bank)
credit (Medium influential bank)
Grossoutput (above median bank)
Grossoutput (Medium influential bank)
Inflation (above median bank)
Inflation (Medium influential bank)
gdp (top 10 influential bank)
gdp (Medium influential bank)
Ψ,Ω,θ (idiosyncratic bank shock, left scale)
Ψ,Ω,θ(s) (idiosyncratic bank shock, left scale)
Ψ,Ω(s),θ (idiosyncratic bank shock, left scale)
Ψ,Ω(s),θ(s) (idiosyncratic bank shock, left scale)
Ψ,Ω,θ (aggregate shock, right scale)
Months

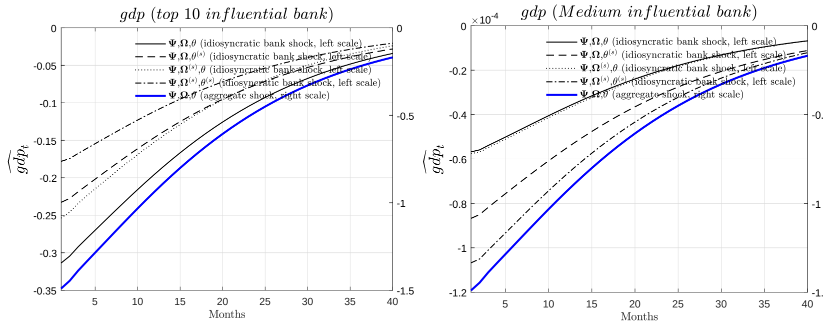


Figure 7: Impulse response function of the model assuming $\varphi = 0$ and monetary policy targetting $P_t C_t = PC$.

C Proofs

Most of the proofs are modest variations to the proofs included in Gabaix (2011) and Acemoglu et al. (2012). Consequently we will also rely heavily on Levy's Theorem, which for completeness is restated here (taken from (**?**, p. 153))

Theorem 1 (Levy's theorem.)**.** *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

1. $\lim_{x\to\infty} \mathbb{P}(X_1 > x)/\mathbb{P}(|X_1| > x) = \theta \in (0, 1)$

2. $\mathbb{P}(X_1 > x) = x^{-\zeta} L_x$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L_{tx}/Lx = 1$.

   Let $S_K = \sum_{k=1}^{K} X_k, a_K = inf\{x : \mathbb{P}(|X_1| > x) \leq 1/K\}$ *and* $b_k = K\mathbb{E}[X_1 \mathbf{1}_{abs X_1 \leq a_K}]$

   As $K \to \infty, (S_K - b_K)/a_K \xrightarrow{d} u$ *where* u *has a nondegenerate distribution.*

Proof of Proposition 1. The first part of the proposition readily follows without proof. For the second part, it is verifiable that

$$\boldsymbol{\nu}_B \geq \boldsymbol{\nu}_B^{(n=0)} = \kappa \boldsymbol{\theta}' \boldsymbol{\Psi}_B$$

where $\geq$ and $=$ hold element-wise. Hence, it must be true that

$$\|\boldsymbol{\nu}'_B\|_2^2 \geq \|\kappa\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2 \sum_{b=1}^{B} (d_{b|B}^{(1)})^2$$

Recall from definition 3 that $CV_{d_B^{(1)}} \equiv \frac{1}{\bar{d}_B^{(1)}} \Big(\frac{1}{B} \sum_{b=1}^{B} (d_{b|B}^{(1)} - \bar{d}_B^{(1)})^2 \Big)^{\frac{1}{2}}$. Rewrite the r.h.s. using $\bar{d}_B^{(1)} = (1-\delta)B^{-1}$, $\sum_{b=1}^{B} \bar{d}_B^{(1)} = 1-\delta$ and $\sum_{b=1}^{B} d_{b|B}^{(1)} = 1-\delta$ in order to obtain

$$CV_{d_B^{(1)}} = B\sqrt{\frac{1}{B}\Big(\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 - \frac{(1-\delta)^2}{B}\Big)}$$

$$\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 = \frac{(1-delta)^2 B}{B^2} CV^2_{d_B^{(1)}} + \frac{(1-\delta)^2}{B}$$

Use this expression to rewrite the first part in the proposition

$$\begin{aligned}
\sqrt{Var\ (\widehat{c}_{t|B})} &\geq \kappa\Big(\sum_{b=1}^{B}(d_{b|B}^{(1)})^2\Big)\sigma + \|\boldsymbol{\nu}_B\|_1\sigma \\
&\geq \kappa\Big(\sqrt{\frac{1}{B}CV^2_{d_B^{(1)}} + \frac{1}{B}}\Big)\sigma + \|\boldsymbol{\nu}_B\|_1\sigma \\
&\geq \frac{\kappa}{\sqrt{B}}\sqrt{1 + CV^2_{d_B^{(1)}}}\sigma + \|\boldsymbol{\nu}_B\|_1\sigma
\end{aligned}$$

which delivers the result.

Q.E.D.

Proof of Corollary 1. For $\boldsymbol{\nu}_B^{(n=0)}$ we have that

$$\begin{aligned}
\|\boldsymbol{\nu}_B^{(n=0)}\|_2 &= \|\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2 \\
&= \frac{\mathbb{E}\bar{d}_B^{(1)}}{\mathbb{E}\bar{d}_B^{(1)}}\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2}
\end{aligned}$$

if $d^{(1)}_{b|B}$ follows a power law with coefficient $\beta \in (1,2)$ we have that $(d^{(1)}_{b|B})^2$ follows a power law with coefficient $\beta/2$ since

$$\mathbb{P}(d^{(1)}_{b|B})^2 > x) = \mathbb{P}(d^{(1)}_{b|B} > x^{\frac{1}{2}}) = a(x^{\frac{1}{2}})^{-\beta} = ax^{\frac{-\beta}{2}}$$

Using Levy's theorem we have that

$$B^{-2/\beta} \sum_{b=1}^{B} (d^{(1)}_{b|B})^2 \xrightarrow{d} u$$

where u is a Levy distributed variable with exponent $\beta/2$. Hence,

$$B\|\boldsymbol{\nu}^{(k=0)}_B\|_2 = \frac{\sqrt{\sum_{b=1}^{B}(d^{(1)}_{b|B})^2}}{\mathbb{E}\bar{d}^{(1)}_B}$$

Such that

$$B^{1-\frac{1}{\beta}}\|\boldsymbol{\nu}^{(k=0)}_B\|_2 = \frac{B^{-1/\beta}\sqrt{\sum_{b=1}^{B}(d^{(1)}_{b|B})^2}}{\mathbb{E}\bar{d}^{(1)}_B} \xrightarrow{d} \frac{u^{1/2}}{\mathbb{E}\bar{d}^{(1)}_B}$$

Q.E.D.

Proof of Proposition 2. It is readily verifiable that

$$\boldsymbol{\nu}_B \geq \sum_{n}^{1} \boldsymbol{\nu}^{(n=0)}_B = \kappa\boldsymbol{\theta}'(\mathbb{I} + \widetilde{\Omega})\boldsymbol{\Psi}_B$$

where $\geq$ and $=$ hold element–wise. It must be true that

$$\begin{aligned}\|\sum_{n=0}^{1} \boldsymbol{\nu}^{(n)}_B\|_2^2 &= \big(\kappa\boldsymbol{\theta}'(\mathbb{I} + \widetilde{\boldsymbol{\Omega}})\boldsymbol{\Psi}_B\big)\big(\kappa\boldsymbol{\Psi}'_B(\mathbb{I} + \widetilde{\boldsymbol{\Omega}}')\boldsymbol{\theta}\big) \\ &= \underbrace{\|\kappa\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2^2}_{(1)} + \underbrace{\|\kappa\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\Psi}_B\|_2^2}_{(2)} + \underbrace{2\kappa^2\boldsymbol{\theta}'\boldsymbol{\Psi}_B\boldsymbol{\Psi}'_B\widetilde{\boldsymbol{\Omega}}'\boldsymbol{\theta}}_{(3)}\end{aligned}$$

Or, after rewriting

$$(1) : \|\kappa\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2 \sum_{b=1}^{B} (d_{b|B}^{(1)})^2$$

$$(2) : \|\kappa\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2 \sum_{b=1}^{B} (d_{b|B}^{(2)})^2$$

$$(3) : 2\kappa^2\boldsymbol{\theta}'\boldsymbol{\Psi}_B\boldsymbol{\Psi}_B'\boldsymbol{\Omega}'\boldsymbol{\theta} = 2(1-\alpha)^2 \sum_{b=1}^{B} d_{b|B}^{(2)} d_{b|B}^{(1)}$$

Note that the following expression holds:

$$\sum_{b=1}^{B}((1-\alpha)d_{b|B}^{(1)} - (1-\alpha)d_{b|B}^{(2)})^2 \geq 0$$

$$\underbrace{(1-\alpha)^2 \sum_{b=1}^{B}(d_{b|B}^{(1)})^2_{(1)} + (1-\alpha)^2 \sum_{b=1}^{B}(d_{b|B}^{(2)})^2}_{(2)} \geq \underbrace{2(1-\alpha)^2 \sum_{b=1}^{B} d_{b|B}^{(2)} d_{b|B}^{(1)}}_{(3)}$$

such that the sum of (1) and (2) always dominate (3). As a result:

$$\|\boldsymbol{\nu}_B\|_2 \geq \kappa \sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2} + \kappa \sqrt{\sum_{b=1}^{B}(d_{b|B}^{(2)})^2}$$

From proof 1, $\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 = \frac{(1-\delta)^2}{B} CV^2_{d_B^{(1)}} + \frac{(1-\delta)^2}{B}$. For the second term, using the same strategy as before, it is readily verifiable that $\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(2)})^2} = (1-\delta)^2 \boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\iota}(\frac{1}{B}CV^2_{d_B^{(2)}} + \frac{1}{B})$. Hence,

$$\|\boldsymbol{\nu}_B\|_2 \geq (\frac{1}{\sqrt{B}}\sqrt{1 + CV^2_{d_{b|B}^{(1)}}} + \frac{\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\iota}}{\sqrt{B}}\sqrt{1 + CV^2_{d_{b|B}^{(2)}}})\kappa$$

Q.E.D.

Proof of Corollary 2. Note that from the proof of proposition 2 it holds that

$$Var\ (\widehat{c}_B) = \Omega\Big(\sum_{b=1}^{B}(q_b^B)^2\Big)$$

The rest of the proof is similar to the proof of corollary 1.

Q.E.D.

Proof of Proposition 3. The proof follows the same logical flow as the previous proofs.

1. Rewrite the approximation of the influence vector $\boldsymbol{\nu}_B^{(N)}$ as a finite power series.

2. Show that the squared norm of this finite sum, when the order of approximation is N equals $\|\boldsymbol{\nu}_B^{(n=N)}\|_2^2 = \varsigma^2 \sum\limits_{1\leq i\leq j\leq M} \sum_{b=1}^{B}(\varpi^{1-i}d_{b|B}^{(i)})(\varpi^{1-j}d_{b|B}^{(j)})$.

3. The terms for which $i = j$ dominate all terms for which $i \neq j$ since expansion of $\sum_{b=1}^{B}(\varpi^{1-i}d_{b|B}^{(i)} - \varpi^{1-j}d_{b|B}^{(j)})^2$ delivers $\sum_{b=1}^{B}(\varpi^{1-i}d_{b|B}^{(i)})^2 + \sum_{b=1}^{B}(\varpi^{1-j}d_{b|B}^{(j)})^2 \geq 2\varpi^{1-i}\varpi^{1-j}\sum_{b=1}^{B} d_{b|B}^{(i)}d_{b|B}^{(j)}$ for all $i \neq j$.

Q.E.D.

D Steady State Solution and Log-linear System

This section elaborates the first–order conditions and other relevant equations used to log linearize the model.

D.1 First–order conditions

D.1.1 Household

$$w_{jt} = g_j l_{jt}^{\varphi} c_t \tag{B-1}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t\Big(\frac{R_t}{\pi_{t+1} c_{t+1}}\Big) \tag{B-2}$$

$$\frac{q_t}{c_t} = \frac{\iota}{h_t} + \beta \mathbb{E}_t\Big(\frac{q_{t+1}}{c_{t+1}}\Big) \tag{B-3}$$

D.1.2 Firms

Let $\Lambda_{t,t+s} \equiv \frac{\beta^s c_t}{c_{t+s}} \frac{P_t}{P_{t+s}}$ denote the stochastic discount factor of nominal profits of firm j with $\Lambda_{t,t} = 1$ and $\Lambda_{t+s,t+s+1} = R_{t+s}^{-1}$. Firm j maximizes the present value of profits

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(P_{jt+s} y_{jt+s} - (W_{jt+s} n_{jt+s} + P^{\omega}_{jt+s} m_{jt+s}) - F_{jt+s} k_{jt+s})$$

s.t.

$$\begin{cases} y_{jt+s} &= \sum_{j'=1}^{J} m_{j'jt+s} + c_{jt+s} + \widetilde{c}_{jt+s} \\ y_{jt+s} &= A_j \big(n_{jt+s}^{\phi_j} m_{jt+s}^{1-\phi_j}\big)^{\delta_j} k_{jt+s}^{1-\delta_j} \\ y_{jt+s} &= \Big(\sum_{j'=1}^{J} \omega_{j'j} \big(\frac{P_{jt+s}}{P^{\omega}_{j't+s}}\big)^{-\eta} m_{j't+s}\Big) + \theta_j \big(\frac{P_{jt+s}}{P_{t+s}}\big)^{-\eta} (c_{t+s} + \widetilde{c}_{t+s}) \\ P_{jt+s} &= \begin{cases} P^{*}_{jt+s} & \text{with probability } 1-\alpha_j \\ P_{jt+s-1} & \text{with probability } \alpha_j \end{cases} \end{cases}$$

The first order conditions w.r.t. n_{jt}, m_{jt} and k_{jt} deliver

$$n_{jt}: \qquad W_{jt} = MC_{jt}(\partial y_{jt}/\partial n_{jt})$$

$$m_{jt}: \qquad P^{\omega}_{jt} = MC_{jt}(\partial y_{jt}/\partial m_{jt})$$

$$k_{jt}: \qquad F_{jt} = MC_{jt}(\partial y_{jt}/\partial k_{jt})$$

Where MC_{jt} denotes nominal marginal costs. Furthermore

$$\begin{cases} \frac{m_{jt}}{n_{jt}} &= \frac{\delta_j(1-\phi_j)}{\delta_j \phi_j} \frac{W_{jt}}{P^{\omega}_{jt}} \\ \frac{n_{jt}}{k_{jt}} &= \frac{\delta_j \phi_j}{\delta_j} \frac{F_{jt}}{W_{jt}} \end{cases}$$

Nominal marginal costs are obtained using

$$y_{jt} = A_j\Big[\Big(\frac{\delta_j(1-\phi_j)}{\delta_j\phi_j}\Big)\Big(\frac{W_{jt}}{P^{\omega}_{jt}}\Big)\Big]^{\delta_j(1-\phi_j)}\Big[\Big(\frac{\delta_j}{(1-\delta_j)\phi_j}\Big)\Big(\frac{W_{jt}}{F_{jt}}\Big)\Big]^{1-\delta_j} n_{jt} - \Phi_j \tag{B-4}$$

$$y_{jt} = A_j\Big[\Big(\frac{\delta_j\phi_j}{(1-\delta_j)\phi_j}\Big)\Big(\frac{P^{\omega}_{jt}}{W_{jt}}\Big)\Big]^{\delta_j\phi_j}\Big[\Big(\frac{(1-\delta_j)}{\delta_j(1-\phi_j)}\Big)\Big(\frac{P^{\omega}_{jt}}{F_{jt}}\Big)\Big]^{(1-\delta_j)} m_{jt} - \Phi_j \tag{B-5}$$

$$y_{jt} = A_j\Big[\Big(\frac{\delta_j\phi_j}{(1-\delta_j)}\Big)\Big(\frac{F_{jt}}{W_{jt}}\Big)\Big]^{\delta_j\phi_j}\Big[\Big(\frac{\delta_j(1-\phi_j)}{(1-\delta_j)}\Big)\Big(\frac{F_{jt}}{P^{\omega}_{jt}}\Big)\Big]^{\delta_j(1-\phi_j)} k_{jt} - \Phi_j \tag{B-6}$$

such that

$$MC_{jt} = \frac{1}{A_j}\Big(\frac{W_{jt}}{\delta_j\phi_j}\Big)^{\delta_j\phi_j}\Big(\frac{P^{\omega}_{jt}}{\delta_j(1-\phi_j)}\Big)^{\delta_j(1-\phi_j)}\Big(\frac{F_{jt}}{\delta_j}\Big)^{1-\delta_j}$$

or in real terms $mc_{jt} \equiv \frac{MC_{jt}}{P_t}$

$$mc_{jt} = \frac{1}{A_j}\Big(\frac{w_{jt}}{\delta_j\phi_j}\Big)^{\delta_j\phi_j}\Big(\frac{p^{\omega}_{jt}}{\delta_j(1-\phi_j)}\Big)^{\delta_j(1-\phi_j)}\Big(\frac{f_{jt}}{\delta_j}\Big)^{1-\delta_j}$$

with $w_{jt} \equiv \frac{W_{jt}}{P_t}$, $p^{\omega}_{jt} \equiv \frac{P^{\omega}_{jt}}{P_t}$ and $f_{jt} \equiv \frac{F_{jt}}{P_t}$.

D.1.3 Entrepreneurs

$$\{\frac{1}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma R_{jt}}{\widetilde{c}_{jt+1}\pi_{t+1}} + \widetilde{\lambda}_{jt}R_{jt}\Big)\}_{j=1}^{J} \tag{B-7}$$

$$\{\frac{q_t}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma}{\widetilde{c}_{jt+1}}\big(\nu_j\frac{f_{jt+1}k_{jt+1}}{\widetilde{h}_{jt}} + q_{t+1}\Big) + \widetilde{\lambda}_{jt}\ell_{jt}\pi_{t+1}q_{t+1}\}_{j=1}^{J} \tag{B-8}$$

$$\{w_{jt}\widetilde{n}_{jt} = (1-\nu_j)k_{jt}f_{jt}\}_{j=1}^{J} \tag{B-9}$$

$$\{F_{jt}K_{jt} + S_{jt} = P_t\widetilde{C}_{jt} + Q_t(\widetilde{h}_{jt} - \widetilde{h}_{jt-1}) + \frac{R_{jt-1}S_{jt-1}}{\pi_t} + W_{jt}\widetilde{n}_{jt}\}_{j=1}^{J} \tag{B-10}$$

$$\{k_{jt} = \widetilde{n}_{jt}^{1-\nu_j} \widetilde{h}_{jt-1}^{\nu_j}\}_{j=1}^{J} \tag{B-11}$$

$$\{S_{jt} = \ell_{jt} \mathbb{E}_t \frac{q_{t+1} \widetilde{h}_{jt} \pi_t}{R_{jt}}\}_{j=1}^{J} \tag{B-12}$$

D.1.4 Market clearing

$$\{D_{bt} = \sum_{j=1}^{J} S_{jbt}\}_{b=1}^{B}$$

$$D_t = \sum_{b=1}^{B} D_{bt}$$

$$l_t = \sum_{j=1}^{J} (n_{jt} + \widetilde{n}_{jt})$$

$$h = h_t + \sum_{j=1}^{J} \widetilde{h}_{jt}$$

$$\{y_{jt} = c_{jt} + \widetilde{c}_{jt} + \sum_{j'=1}^{J} m_{j'jt}\}_{j=1}^{J}$$

D.2 Steady state

For simplicity, we make three assumptions that deliver a symmetric steady state.

Assumption A-1. *$gdp \equiv c + \widetilde{c} = 1$.*

Assumption A-2. *As in Carvalho and Lee (2011); Pasten et al. (2018a) we take $g_j = s_j^{-\varphi}$ where $s_j \equiv \frac{l_j}{l}$ denotes the share of household labour employed by firm k.*

Assumption A-3. *We normalize $\{A_j\}_{j=1}^{J}$ such that steady state prices of goods/capital services equal the aggregate price level; i.e. $\frac{P_j}{P} = p_j = \frac{P_j^{\omega}}{P} = p_j^{\omega} = \frac{F_j}{P} = f_j = 1$.*

Assumption A-1, comes w.l.o.g. and merely pins down the size of the economy.Assumption A-2, equalizes steady state wages across firms. Assumption A-3 allows for a symmetric steady state.

From the household Euler equation, $R = \frac{1}{\beta}$. From the bank first order conditions, $R_{jb} = \frac{\epsilon}{\epsilon-1}R$, such that $\frac{R_j}{R} = \frac{\epsilon}{\epsilon-1}$. From the firm first–order conditions (B-4)–(B-6),

$$\begin{aligned} n_j w_j &= mc_j \delta_j \phi_j (y_j + \Phi_j) \\ m_j p_j^{\omega} &= mc_j \delta_j (1-\phi_j)(y_j + \Phi_j) \\ k_j f_j &= mc_j (1-\delta_j)(y_j + \Phi_j) \end{aligned}$$

Real firm profit is then defined as

$$\begin{aligned} \pi_j &= y_j - \left(n_j w_j + m_j p_j^{\omega} + k_j f_j\right) \\ &= y_j - mc_j (y_j + \Phi_j) \end{aligned}$$

In order to rule out entry, $\pi_j = 0$, we pin down the fixed costs

$$\Phi_j = \frac{1 - mc_j}{mc_j} y_j$$

and since $mc_j = \frac{\eta-1}{\eta}$, we have that $\Phi_j = (\eta-1)^{-1} y_j$. Consequently, from the first–order conditions of intermediate goods producers, we have that

$$\begin{aligned} n_j w_j &= \frac{\eta-1}{\eta} \delta_j \phi_j (y_j + \Phi_j) = \delta_j \phi_j y_j \\ m_j p_j^{m} &= \frac{\eta-1}{\eta} \delta_j (1-\phi_j)(y_j + \Phi_j) = \delta_j (1-\phi_j) y_j \\ k_j f_j &= \frac{\eta-1}{\eta} (1-\delta_j)(y_j + \Phi_j) = (1-\delta_j) y_j \end{aligned}$$

Under Assumption A-3, it is true that

$$\begin{aligned} c_j &= \theta_j c \\ \widetilde{c}_j &= \theta_j \widetilde{c} \end{aligned}$$

$$m_{jk'} = \omega_{jk'} m_j$$

Furthermore, from Assumption A-2,

$$\begin{aligned} y_j &= c_j + \widetilde{c}_j + \sum_{j'=1}^{J} m_{j'k} \\ &= \theta_j + \sum_{j=1}^{J} \delta_{j'}(1-\phi_{j'})\omega_{j'k} y_{j'} \end{aligned}$$

implying that $\boldsymbol{y} = (\mathbb{I} - \widetilde{\boldsymbol{\Omega}}')^{-1}\boldsymbol{\iota}_J = \boldsymbol{\tau}$, where $\boldsymbol{y} = [y_1, ..., y_j]$.

Next, from the entrepreneur Euler equation (B-7)

$$\widetilde{\lambda}_j = \frac{1}{\widetilde{c}_j}\Big(\beta(\frac{\epsilon - 1}{\epsilon}) - \gamma)$$

From the entrepreneur first–order condition (B-8)

$$\frac{q\widetilde{h}_j}{k_j} = \frac{\gamma\nu_j}{1 - \gamma - \ell_j(\beta(\frac{\epsilon-1}{\epsilon}) - \gamma)} = \frac{\gamma\nu_j}{1 - \widetilde{\gamma}_j}$$

with $\widetilde{\gamma}_j \equiv (1-\ell_j)\gamma + \ell_j\beta(\frac{\epsilon-1}{\epsilon})$. From the entrepreneur budget constraint, we have that

$$\frac{\widetilde{b}_j}{\widetilde{k}_j} = \frac{\epsilon - 1}{\epsilon}\frac{\beta\ell_j\gamma\nu_j}{1 - \widetilde{\gamma}_j}$$

From the budget constraint of entrepreneur k

$$\widetilde{c}_j = \nu_j\tau_j\Big(1 + \big(1 - (\frac{\epsilon - 1}{\epsilon})(\frac{1}{\beta})\big)\big(\frac{\beta\frac{\epsilon-1}{\epsilon}\ell_j\gamma\nu_j}{1 - \widetilde{\gamma}_j}) - (1 - \nu_j)\Big)$$

which can be used to determine $c = 1 - \sum_{j=1}^{J} \widetilde{c}_j$ and $c_j = \theta_j c$. From the labour supply schedule and assumption (ii)

$$w_j = l^{\varphi} c$$

Note that $q\sum_{j=1}^{J}\widetilde{h}_j = \sum_{j=1}^{J}\frac{\gamma\nu_j\tau_j(1-\delta_j)}{1-\widetilde{\gamma}_j}$, such that

$$\frac{\sum_{j=1}^{J}\widetilde{h}_j}{1-\sum_{j=1}^{J}\widetilde{h}_j} = \sum_{j=1}^{J}\frac{\gamma\nu_j\tau_j(1-\delta_j)(1-\beta)}{(1-\widetilde{\gamma}_j)jc}$$

D.3 Log linearization

Here we present full set of log–linearized equations. Hats denotes deviations from steady state.

Household

$$\begin{aligned}
\widehat{c}_t &= \widehat{c}_{t+1} - \widehat{r}\widehat{r}_t \\
\widehat{c}_t &= \sum_{j=1}^{J}\theta_j\widehat{c}_{jt} \\
\{\widehat{c}_{jt} &= \widehat{c}_t + \eta\widehat{p}_{jt}\}_{j=1}^{J} \\
\{\widehat{w}_{jt} &= \varphi\widehat{l}_{jt} + \widehat{c}_t\}_{j=1}^{J} \\
\{\widehat{l}_{jt} &= \zeta_j^n\widehat{n}_{jt} + (1-\zeta_j^n)\widehat{\widetilde{n}}_{jt}\}_{j=1}^{J} \\
\widehat{q}_t &= \beta\widehat{q}_{t+1} + \zeta^h\widehat{h}_t + \widehat{c}_t - \beta\widehat{c}_{t+1}
\end{aligned}$$

Firms & entrepreneurs

$$\begin{aligned}
\{\widehat{p}_{jt}^{\omega} &= \sum_{j'=1}^{J}\omega_{jk'}\widehat{p}_{jt}\}_{j=1}^{J} \\
\{\widehat{y}_{jt} &= \zeta_j^c\widehat{c}_{jt} + \sum_{j=1}^{J}\zeta_j^{\widetilde{c}}\widehat{\widetilde{c}}_{jt} + \sum_{j'=1}^{J}\zeta_{j'k}^m\widehat{m}_{j'kt}\}_{j=1}^{J} \\
\{\widehat{\pi}_{jt} &= \widehat{p}_{jt} - \widehat{p}_{jt-1} + \widehat{\pi}_t\}_{j=1}^{J} \\
\{\widehat{\pi}_{jt} &= \beta\widehat{\pi}_{jt+1} + \zeta^{\pi}(\widehat{mc}_{jt} - \widehat{p}_{jt})\}_{j=1}^{J} \\
\{\widehat{mc}_{jt} &= \delta_j\phi_j\widehat{w}_{jt} + \delta_j(1-\phi_j)\widehat{p}_{jt}^{\omega} + (1-\delta_j)\widehat{f}_{jt} + \delta_j(\widehat{r}_{jt} - \widehat{r}_t)\}_{j=1}^{J} \\
\{\widehat{m}_{jt} &= \sum_{j'=1}^{J}\omega_{jk'}\widehat{m}_{jk't}\}_{j=1}^{J} \\
\{\widehat{m}_{jt} &= \widehat{z}_{jt} - \eta(\widehat{p}_{jt} - \widehat{p}_{jt}^{\omega})\}_{j=1}^{J}
\end{aligned}$$

$$\{\widehat{y}_{jt} = \delta_j(\phi_j\widehat{l}_{jt} + (1-\phi_j)\widehat{m}_{jt} + (1-\delta_j)\widehat{k}_{jt}\}_{j=1}^J$$

$$\{\widehat{w}_{jt} - \widehat{p}_t^\omega = \widehat{m}_{jt} - \widehat{l}_{jt}\}_{j=1}^J$$

$$\{\widehat{f}_{jt} - \widehat{p}_t^\omega = \widehat{k}_{jt} - \widehat{l}_{jt}\}_{j=1}^J$$

$$\{\widehat{m}_{jk't} = m_{jt} + \eta(\widehat{p}_{jt} - \widehat{p}_{jt}^\omega\}_{j=1}^J$$

$$\{\widehat{\widetilde{c}}_{jt} = \widehat{\widetilde{c}}_t + \eta\widehat{p}_{jt}\}_{j=1}^J$$

$$\{\widehat{k}_{jt} = \nu_j\widehat{\widetilde{n}}_{jt} + (1-\nu_j)\widehat{h}_{jt-1}\}_{j=1}^J$$

$$\{\widehat{\widetilde{b}}_{jt} = \widehat{\widetilde{h}}_{jt} + \widehat{\ell}_{jt} + \widehat{q}_{t+1} - \widehat{rr}_t\}_{j=1}^J$$

$$\{\widehat{w}_{jt} + \widehat{\widetilde{n}}_{jt} = \widehat{f}_{jt} + \widehat{\widetilde{k}}_{jt}\}_{j=1}^J$$

$$\{\widehat{q}_t = \widetilde{\gamma}_j\widehat{q}_{t+1} + (1-\widetilde{\gamma}_j)(\widehat{k}_{jt+1} + \widehat{f}_{jt+1} - \widehat{\widetilde{h}}_{jt}) - \widehat{\ell}_j\beta\frac{\epsilon-1}{\epsilon}\widehat{rr}_t - (1-\beta\frac{\epsilon-1}{\epsilon}(\widetilde{c}_{jt+1} -$$

Monetary policy

$$\widehat{R}_t = \phi_\pi\widehat{\pi}_t + \phi_c\widehat{c}_t$$

$$\widehat{rr}_t = \widehat{r}_t - \widehat{\pi}_{t+1}$$

Exogenous processes

$$\{\widehat{\ell}_{jt} = \sum_{b=1}^{B}\psi_{jb}(\varepsilon_{bt}^\ell + \varepsilon_t^\ell)\}_{j=1}^J$$

$$\{\widehat{r}_{jt} = \widehat{r}_t + \sum_{b=1}^{B}\psi_{jb}(\varepsilon_{bt}^r + \varepsilon_t^r)\}_{j=1}^J$$

Structural composite parameters

$$\zeta_j^n = \frac{1}{1 + (\frac{\epsilon}{\epsilon-1})\frac{(1-\nu_j)(1-\delta_j)}{\delta_j\phi_j}}$$

$$\zeta_j^c = c_j/\tau_j$$

$$\zeta_j^{\widetilde{c}} = \widetilde{c}_j/\tau_j$$

$$\zeta_{j'k}^m = \frac{\epsilon-1}{\epsilon}\delta_{j'}(1-\phi_{j'})\frac{\tau_j}{\tau_{j'}}$$

$$\zeta^\pi = \frac{(1-\beta\alpha_j)(1-\alpha_j)}{\alpha_j}$$

$$\zeta^h = \sum_{j=1}^{J}\frac{\gamma\nu_j\tau_j(1-\beta)}{(1-\widetilde{\gamma}_j)jc}$$

D.4 Equilibrium in a Simplified Framework

First, under assumptions (1) and (2), household labour supply to firm k yields

$$\frac{W_{jt}}{P_t} = g_j C_t = 0$$

such that $\widehat{W}_{jt} = 0$. Second, by assumption assumptions (3) and (4), normal marginal cost collapses to

$$MC_{jt} = \frac{1}{A_j}\Big(\frac{W_{jt}}{\delta\phi_j}\Big)^{\phi_j}\Big(\frac{P^{\omega}_{jt}}{\delta(1-\phi_{j)}}\Big)^{(1-\phi_j)}\Big(\frac{R_{kt}}{1-\delta}\Big)^{(1-\delta)}$$

or in log linear form $\widehat{MC}_{jt} = \delta(1-\phi_j)\widetilde{P}^{\omega}_{jt} + (1-\delta)\sum_{b=1}^{B}\psi_{jb}\epsilon^{r}_{bt}$. Third, under assumption (5),

$$\widehat{P}_{jt} = \alpha\widehat{MC}_{jt}$$

This implies that

$$\widehat{P}_{jt} = \delta(1-\phi_j)\widehat{P}^{\omega}_{t} + (1-\delta)\sum_{b=1}^{B}\psi_{jb}\epsilon^{r}_{bt}$$

Or in matrix form with $\boldsymbol{P}_t = [P_{1t}, ..., P_{jt}]'$, $\boldsymbol{P}_t = [\mathbb{I} - \widetilde{\boldsymbol{\Omega}}']^{-1}\boldsymbol{\Psi\epsilon}$. Given that $\widehat{P}_t = \boldsymbol{\theta}'\boldsymbol{P}_t$ and $\widehat{GDP_t} = \widehat{C}_t = -\widehat{P}_t$,

$$\widehat{P}_t = -\boldsymbol{\theta}'[\mathbb{I} - \widetilde{\boldsymbol{\Omega}}']^{-1}\boldsymbol{\Psi\epsilon}_t$$

D.5 Nesting the literature

D.5.1 Iacoviello (2005)

If we take (1) $J = 1$, (2) $\epsilon_b \to \infty$, (3) $\delta_j = 0$, (4) $\phi_j = 1$. Then,

$$\widehat{k}_t = \frac{c}{k}\widehat{c}_t + \frac{\widetilde{c}}{k}\widehat{c}_t$$

$$
\begin{aligned}
\widehat{c}_t &= \mathbb{E}_t \widehat{c}_{t+1} - \widehat{rr}_t \\
\frac{c}{y}\widehat{\tilde{c}}_t &= \frac{b}{y}\widehat{\tilde{b}}_t + \frac{b}{\beta k}(\widehat{\pi}_t - \widehat{R}_{t-1} - \widehat{\tilde{b}}_{t-1}) + \nu(\widehat{k}_t + \widehat{f}_t) - qh\Delta\widehat{h}_t \\
\widehat{q}_t &= \tilde{\gamma}\mathbb{E}_t\widehat{q}_{t+1} + (1-\tilde{\gamma})(\widehat{k}_{t+1} - \widehat{h}_t + \widehat{f}_{t+1}) - \ell\beta\widehat{rr}_t - (1-\ell\beta)\mathbb{E}_t(\widehat{\tilde{c}}_{t+1} - \widehat{\tilde{c}}_t) \\
\widehat{q}_t &= \beta\widehat{q}_{t+1} + \zeta^h\widehat{h}_t + \widehat{c}_t - \beta\widehat{c}_{t+1} \\
\widehat{\tilde{b}}_t &= \widehat{q}_{t+1} + \widehat{\tilde{h}}_t - \widehat{rr}_t \\
\widehat{k}_t &= \frac{\eta\nu}{\eta-(1-\nu)}\widehat{\tilde{h}}_{t-1} - \frac{1-\nu}{\eta-(1-\nu)}(\widehat{\tilde{c}}_t - \widehat{f}_t) \\
\widehat{\pi}_t &= \beta\widehat{\pi}_{t+1} + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\widehat{f}_t \\
\widehat{R}_t &= \phi_\pi\widehat{\pi}_t + \phi_c\widehat{c}_t
\end{aligned}
$$

with $\tilde{\gamma} \equiv (1-\ell)\gamma + \ell\beta$, $\zeta^h =$ which is the simplified model analyzed by Iacoviello (2005).

D.5.2 Pasten et al. (2018a)

If we take (1) $\delta_{j=1} = 1$, (2) $A_j = A_{jt}$, (3) zero consumption mass on entrepreneurs, $\tilde{c}_j = 0$, (4) $h = \iota = 0$. Then,

Household

$$
\begin{aligned}
\widehat{c}_t &= \widehat{c}_{t+1} - \widehat{rr}_t \\
\widehat{c}_t &= \sum_{j=1}^{J}\theta_j\widehat{c}_{jt} \\
\{\widehat{c}_{jt} &= \widehat{c}_t + \eta\widehat{p}_{jt}\}_{j=1}^{J} \\
\{\widehat{w}_{jt} &= \varphi\widehat{l}_{jt} + \widehat{c}_t\}_{j=1}^{J}
\end{aligned}
$$

Firms

$$
\{\widehat{p}^{\omega}_{jt} = \sum_{j'=1}^{J}\omega_{jk'}\widehat{p}_{jt}\}_{j=1}^{J}
$$

$$\{\widehat{y}_{jt} = \zeta^c_j \widehat{c}_{jt} + \sum_{j'=1}^{J} \zeta^m_{j'k} \widehat{m}_{j'kt}\}_{j=1}^J$$
$$\{\widehat{\pi}_{jt} = \widehat{p}_{jt} - \widehat{p}_{jt-1} + \widehat{\pi}_t\}_{j=1}^J$$
$$\{\widehat{\pi}_{jt} = \beta \widehat{\pi}_{jt+1} + \zeta^\pi(\widehat{mc}_{jt} - \widehat{p}_{jt})\}_{j=1}^J$$
$$\{\widehat{mc}_{jt} = \phi \widehat{w}_{jt} + (1-\phi)\widehat{p}^\omega_{jt}$$
$$\{\widehat{m}_{jt} = \sum_{j'=1}^{J} \omega_{jk'} \widehat{m}_{jk't}\}_{j=1}^J$$
$$\{\widehat{m}_{jt} = \widehat{m}_{jt} - \eta(\widehat{p}_{jt} - \widehat{p}^\omega_{jt})\}_{j=1}^J$$
$$\{\widehat{y}_{jt} = (\phi \widehat{l}_{jt} + (1-\phi)\widehat{m}_{jt}\}_{j=1}^J$$
$$\{\widehat{w}_{jt} - \widehat{p}^\omega_t = \widehat{m}_{jt} - \widehat{l}_{jt}\}_{j=1}^J$$
$$\{\widehat{m}_{jk't} = m_{jt} + \eta(\widehat{p}_{jt} - \widehat{p}^\omega_{jt}\}_{j=1}^J$$

Monetary policy

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_c \widehat{c}_t$$
$$\widehat{rr}_t = \widehat{r}_t - \widehat{\pi}_{t+1}$$

D.5.3 Acemoglu et al. (2012)

If we take (1) $\delta_{j=1} = 1$, (2) $A_j = A_{jt}$, (3) zero consumption mass on entrepreneurs, $\widetilde{c}_j = 0$, (4) $h = \iota = 0$, (5) $\{\alpha_j = 0\}_{j=0}^J$, (6) $\{\phi_j = \phi\}_{j=0}^J$, (7) $\{\theta_j = 1/J\}_{j=0}^J$. (8) $\eta = 1$ Then, $\widehat{c}_t = \frac{1}{J}[\mathbb{I} - (1-\phi)\mathbf{\Omega}]^{-1}\boldsymbol{\varepsilon}^a_t$, where $\boldsymbol{\varepsilon}^a_t$ is a vector capturing Harrod neutral shocks to productivity.

D.5.4 Gabaix (2011)

The framework of Acemoglu et al. (2012) can easily be reformulated to the Gabaix (2011) framework (Acemoglu et al., 2012, p. 1983).

D.5.5 Bremus et al. (2018)

If we take (1) Assumptions 1 – 6 body text. (2) $\{\phi_j\}_{j=1}^J = 1$, (3) $\{\omega_{jj'}\}_{j,j'=1}^J$, (4) $\{\theta_j\}_{j=1}^J = 1$. Then, $Var(\widehat{c}_t) = \sum_{b=1}^B \left(\frac{d_b}{\sum_{b=1}^B d_b}\right)^2 \sigma$.

E Additional results

E.1 Modelling assumptions: supplier switching (extensive margin)

Our model does not allow firms to adjust their supplier portfolio in the extensive margin (adding and dropping suppliers) in order to offset shocks from suppliers. In order to gauge the role of the extensive margin, we probe the stability of the real economy network for the period $2002-2014$.

We find that on average 44.57% of the firm-to-firm linkages at time t did not exist in the previous period $t-1$. Reversely, 42.89% of the firm-to-firm contracts at time t were discontinued in the following period $t+1$. These high levels of variation in the extensive margin conceal that added (discontinued) supplier relations only represent small shares in the overall input portfolio of firms. At the firm level, new and discontinued purchases from input suppliers in total intermediate purchases have an average input weight (in total intermediates) of 8.45% and 8.81%, respectively. In addition to genuine supplier switching, these figures also reflect (*i*) firm entry, (*ii*) firm exit, (*iii*) one–time capital purchases, (*iv*) transactions crossing the reporting threshold. In sum, firms actively discontinue supplier relationships and establish new ones but the size of these relationships are found to be small.

Table E.1: Firm switching

	Average	Percentiles				
		$p=90$	$p=75$	$p=50$	$p=25$	$p=10$
Firm–to–Firm level						
New firm–to–firm contracts at t (% of total firm–to–firm contracts active at t)	44.57	100	46.77	37.5	31.25	26.26
Number of firm-to-firm contracts discontinued at $t+1$ (% of total firm-to-firm contracts active at t)	42.89	100	44.89	36.04	30.00	25.27
Firm level						
Total weight of new supplier contracts in firm portfolio	8.45	17.69	10.85	6.75	3.40	0.00
Total weight of discontinued supplier contracts in firm portfolio	8.81	18.22	11.28	7.15	3.86	0.00

t refers to years. The reported statistics are averages across $2002-2014$. Numbers are denoted in percentages.

E.2 Modelling assumptions: Bank switching (extensive margin)

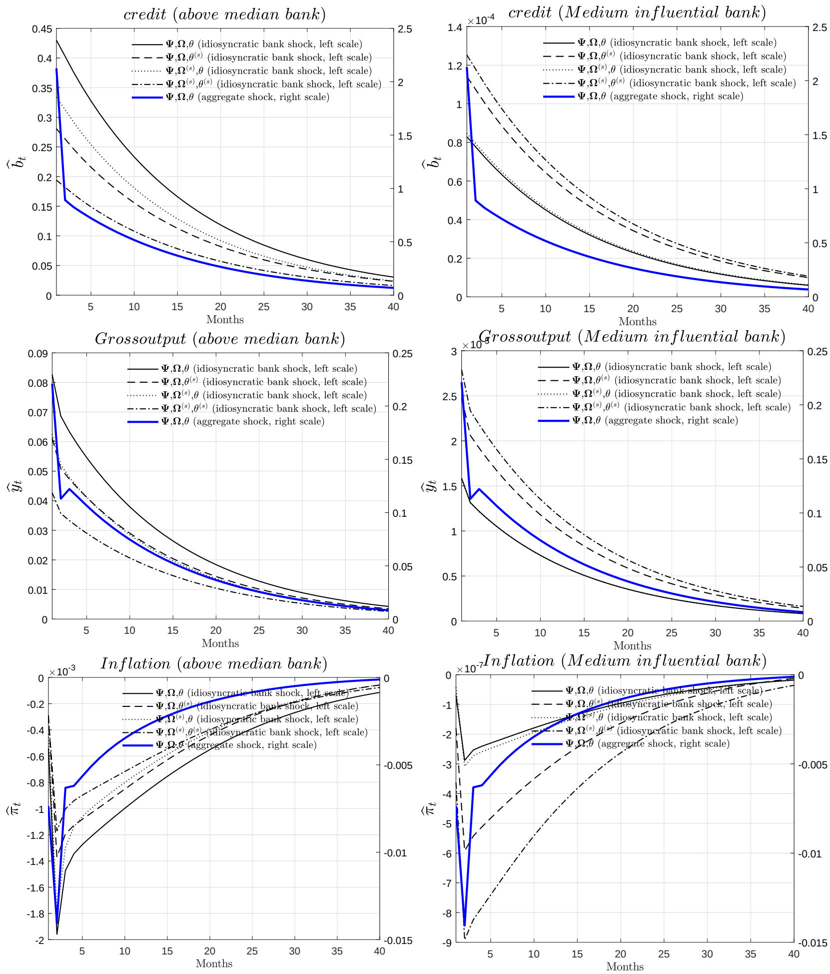
We study the impact of bank shocks taking the credit network $\mathbf{\Psi}$ as given. Our model does not allow firms to establish new banking relationships in order to offset adverse credit–supply shocks. In order to gauge the restrictiveness of this assumption, we exploit the high frequency panel structure of the CCR (monthly periods from $2002m1 - 2011m3$). Figure x plots the fraction of firms that add (drop) a bank at time $t+1$ (at time t) to their portfolio. We observe that the extensive margin of the credit network is very stable. On average, only 1.23% of the bank firm-relationships are discontinued in the next month. Similarly, x% if firms do not add a bank to their portfolio. These numbers are corrected for changes of the bank identity due to M&A). This evidence suggests that active bank switching is not a feature of the Belgian credit network. Various theoretical arguments explain why we observe this high stability in the bank-to-firm network; the existence of fixed costs **?**, relationship lending (**?**), bank specialization (Paravisini et al., 2014), etc.

Table E.2: Bank switching of firms

	2004	2006	2008	2010	2012	2014
Firm–to–Firm level						
Firms that add a bank to their portfolio at time t	1.54	1.56	1.57	1.36	2.57	2.68
Firms that drop a bank to their portfolio at time t	1.91	1.80	1.71	1.65	2.62	2.70

t refers to years and are 12 month averages. Numbers are denoted in percentages.

E.3 Impulse response functions



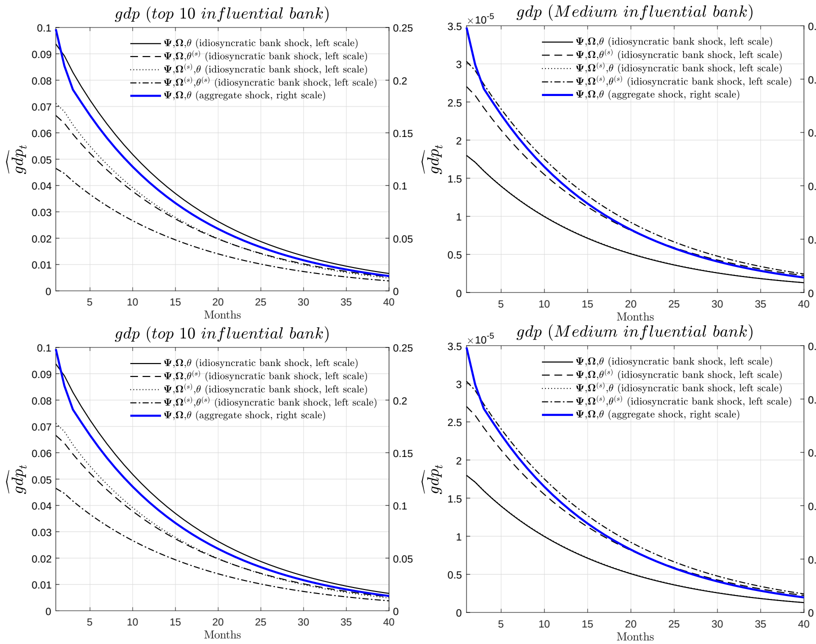
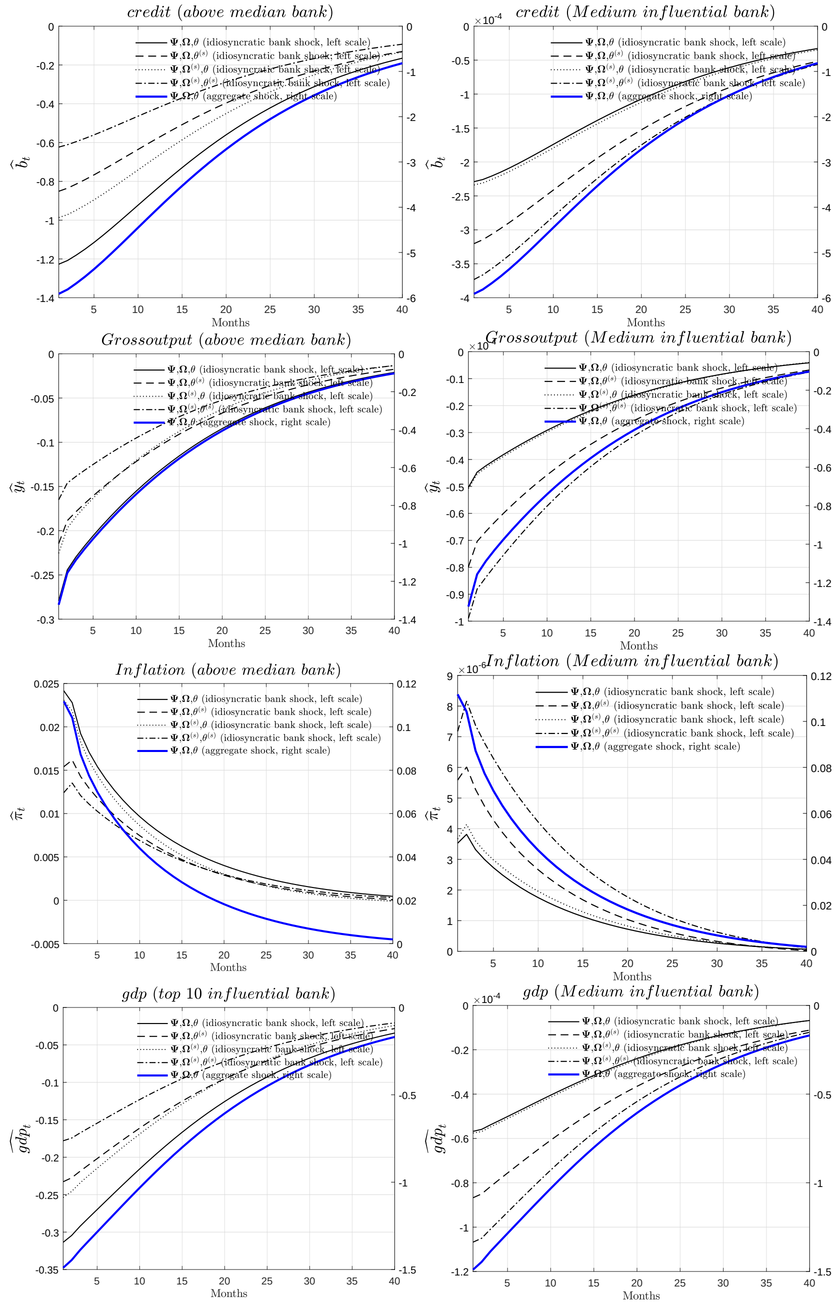


Figure 8: Impulse response function of the model assuming $\varphi = 0$ and monetary policy targetting $P_tC_t = PC$.


credit (above median bank)
credit (Medium influential bank)
Grossoutput (above median bank)
Grossoutput (Medium influential bank)
Inflation (above median bank)
Inflation (Medium influential bank)
gdp (top 10 influential bank)
gdp (Medium influential bank)
Ψ,Ω,θ (idiosyncratic bank shock, left scale)
$\Psi,\Omega,\theta^{(s)}$ (idiosyncratic bank shock, left scale)
$\Psi,\Omega^{(s)},\theta$ (idiosyncratic bank shock, left scale)
$\Psi,\Omega^{(s)},\theta^{(s)}$ (idiosyncratic bank shock, left scale)
Ψ,Ω,θ (aggregate shock, right scale)
Months
$\widehat{b}_t$
$\widehat{y}_t$
$\widehat{\pi}_t$
$\widehat{gdp}_t$

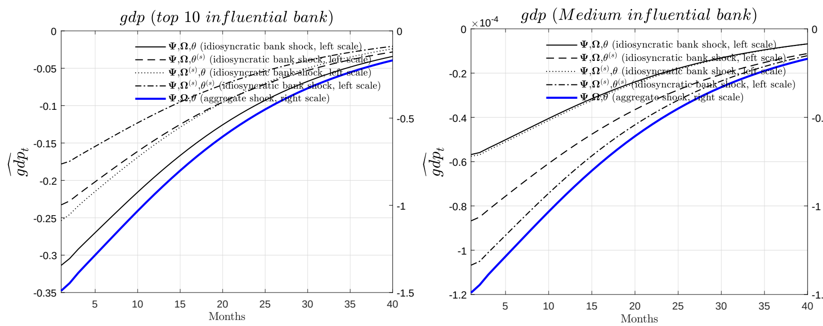


Figure 9: Impulse response function of the model assuming $\varphi = 0$ and monetary policy targetting $P_tC_t = PC$.

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