

Portfolio Rebalancing and Asset Pricing

with Heterogeneous Inattention*

Omar Rachedi

Banco de España

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Abstract

Can households' inattention to the stock market quantitatively account for the inertia in portfolio rebalancing? I address this question by introducing an observation cost into a production economy with heterogeneous agents. In this environment inattention changes endogenously over time and across agents. I find that inattention explains the inertia in portfolio rebalancing and its heterogeneity across households. Inattention also rationalizes the limited stock market participation observed in the data, and improves the asset pricing performance of the model. Finally, I present a novel testable implication linking the effects of inattention on portfolio choices and asset prices to households' funding liquidity.

Keywords: Observation cost, limited stock market participation, equity premium.

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1 Introduction

Can households' limited attention to the stock market quantitatively account for the inertia in households' portfolio rebalancing? I address this question by introducing an observation cost into a production economy with heterogeneous agents, idiosyncratic labor income risk, and borrowing constraints. In this environment inattention changes endogenously over time and across agents. I discipline the quantitative analysis by calibrating the observation cost to match the duration of inattention of the median household observed in the data.

I find that inattention accounts for half of the inertia in portfolio rebalancing and its heterogeneity across stockholders. In the model, as it is in the data, wealthy stockholders invest much more actively than poor stockholders. Inattention also reconciles the amount of limited stock market participation observed in the data with a low per-period participation cost. In addition, I show that inattention improves the asset pricing performance of the model. Importantly, I highlight a novel testable implication that links households' inattention to households' funding liquidity: inattention matters quantitatively on the dynamics of portfolio rebalancing and asset prices only if borrowing constraints are tight enough.

This paper studies the role of households' inattention by relaxing the assumption that agents are always aware of the state of the economy. Despite standard models postulate that households continuously collect information on the stock market and derive optimal consumption/savings plans, in the data we observe a different pattern. For example, Ameriks *et al.* (2003) show that households plan infrequently, and wealthy agents plan more often than poor ones. Alvarez *et al.* (2012) use data from two Italian surveys and find that the median household pays attention to the stock market every 3 months. Furthermore, there is a large heterogeneity in inattention across households: 24% of households observe their financial portfolios less often than twice per year, while 20% of them do it more often than once per week. Finally, Rossi

(2010), Da et al. (2011), and Sichermann et al. (2016) find that the allocation of attention comoves with stock returns.¹

This evidence has motivated a new strand of the literature, which studies the implications of households' limited attention on portfolio choices and asset prices. Nevertheless, the results are still inconclusive. Gabaix and Laibson (2002) and Rossi (2010) show that models with inattention can account for the lumpiness in portfolio adjustments and the dynamics of asset prices. Conversely, Chen (2006) and Finocchiaro (2011) find that inattention has negligible effects on portfolio choices and the level of the equity premium.²

In this paper I evaluate whether the observed duration of households' inattention can quantitatively account for the inertia in households' portfolio rebalancing. I build on Reis (2006) and develop a tractable theory of endogenous inattention with heterogeneity across agents. I propose a model that introduces an observation cost into the environment of Krusell and Smith (1997, 1998). Namely, I consider a production economy with incomplete markets, idiosyncratic labor income risk, and heterogeneous agents, who incur in an observation cost whenever they collect information on the aggregate states of the economy and formulate a new plan for financial investment. This feature generates a novel trade-off: attentive households take better decisions, but also bear higher costs. As a result, households decide to plan at infrequent dates and stay inattentive meanwhile. Inattentive agents do not gather new information on the aggregate states of the economy and their financial portfolios change by inertia following the realizations of stock and bond returns.

When I bring the model to the data, I discipline the role of inattention by calibrating the observation cost to match the duration of inattention of the median household

¹Few other papers show that investors' attention affects stock prices and portfolio choices, e.g. Barber and Odean (2008), Brunnermeier and Nagel (2008), Della Vigna and Pollet (2009), Hirshleifer et al. (2009), and Mondria et al. (2010).

²Lynch (1996), Duffie (2010), and Chien et al. (2011, 2012) study the implications of *exogenous* infrequent portfolio rebalancing on asset prices. In the Supplementary Appendix, I study how the results of the model change when inattention is considered as either an endogenous variable or an exogenous one.

estimated by Alvarez *et al.* (2012). I also discipline the interaction of inattention with households' funding liquidity by calibrating the tightness of the borrowing constraints to match the share of households with negative wealth. These choices imply that the aim of the paper is not to use inattention to match portfolio rebalancing and asset prices. Rather, the model can be used to address the following question: once the observation cost and the tightness of the borrowing constraints are pinned down by the data, how much of the dynamics of portfolio rebalancing and asset prices can be accounted for *only* by households' inattention?

Looking at the results of the model, I find that the duration of inattention depends negatively on households' wealth - in line with the evidence of Ameriks *et al.* (2003) - because observation costs are relatively higher for poor agents. The cyclicity of inattention depends on the marginal gain and the marginal cost of being attentive and actively investing in the stock market. Both forces are countercyclical, but they asymmetrically affect different agents. Poor households plan in expansions because they cannot afford the observation cost in bad times. Instead, wealthy agents plan in recession to benefit of the higher expected return to equity. Overall the level of inattention is countercyclical.

Second, households' portfolio rebalancing displays substantial inertia. As long as inattentive agents do not invest actively, their financial positions follow passively the realizations of the aggregate shock. On average, households actively offset around 73% of the passive variations in the risky share. Hence, inertia drives 27% of the changes in the financial portfolios. Since in the data inertia characterizes 50% of the variations in the risky share, as documented in Calvet *et al.* (2009), the model can account for 54% of the observed inertia in portfolio rebalancing. In the model, the inertia is entirely determined by the observation cost. Indeed, when the observation cost is set to zero, households always actively adjust their financial positions.

The model generates a large heterogeneity in the degree of portfolio rebalancing

across households. Wealthy agents are attentive often enough to actively rebalance their portfolios period-by-period. Instead, poor stockholders are very inattentive and offset just 43% of the passive variation in their portfolios. This result is consistent with the evidence of Calvet et al. (2009), who find that although on average portfolio rebalancing is rather inertial, wealthy households invest very actively.

Third, inattention provides a rationale to the limited stock market participation. The model can account for the share of market participants observed in the data with a low per-period participation cost. Without inattention, matching the same share of stockholders requires a participation cost which is four times larger. Inattention is a barrier to financial investment because households anticipate that during the periods of inattention they end up investing sub-optimally.

Fourth, inattention improves the asset pricing performance of the model. On the one hand, inattention raises the volatility of stock returns, by boosting the movements in the marginal productivity of capital. As inattentive agents cannot immediately adjust their portfolios to the realizations of the aggregate shock, individual financial investment alternates between periods of inaction and periods of sharp adjustments. As a result, aggregate investment becomes more volatile and less correlated with the realizations of the aggregate productivity shock. These two channels raise the volatility of the marginal productivity of capital. On the other hand, inattention induces countercyclical variations in the equity premium. This result is usually obtained through consumption habits or long run risk. Instead, here it is just the by-product of the observation cost, that concentrates the aggregate risk on a small measure of agents. As in Chien et al. (2012), at each point of time there are few attentive investors that actively trade stocks and bear the whole aggregate risk of the economy, commanding a higher return rate on equity.

Finally, I provide a novel testable implication that links households' inattention to households' funding liquidity. I find that inattention matters quantitatively on

the dynamics of portfolio rebalancing and asset prices only if borrowing constraints are tight enough. When borrowing constraints are loose, households can borrow and smooth away any investment mistake made during inattention. Moreover, all households participate in the stock market with buy-and-hold positions, as in Chen (2006). In this case, households dilute the observation cost by trading very infrequently, portfolio rebalancing is passive, and inattention does not affect asset prices.

2 Related Literature

This paper studies households' inattention to the stock market. In the literature, inattention is usually achieved either by making agents gathering information and planning financial investment at discrete dates (e.g., Duffie and Sun, 1990; Lynch, 1996; Gabaix and Laibson, 2002; Chen, 2006; Reis, 2006; Rossi, 2010; Finocchiaro, 2011; Chien et al., 2011, 2012), or through learning with capacity constraints (e.g., Sims, 2003; Peng, 2005; Huan and Liu, 2007).³ I follow the first strand of the literature because of my emphasis on the effects of inattention on agents' portfolio choices. Indeed, I study a heterogeneous agent economy, in which the individual portfolio choice is not trivially determined. This feature avoids having a representative agent which in equilibrium holds the portfolio of the market. Models featuring learning with capacity constraint can be extended to the case of heterogeneous agents and idiosyncratic shocks only by neglecting the existence of higher-order beliefs, as discussed in Porapakkarm and Young (2008).⁴ Yet, Angeletos and La'O (2009) show that higher-order beliefs do play a crucial role in the dissemination of information across agents.

³Inattention is also closely tied to the concepts of information acquisition, e.g. Grossman and Stiglitz (1980) and Peress (2004), and uncertainty, see Veronesi (1999) and Andrei and Hasler (2015).

⁴When agents have imperfect common knowledge and differ in their information set, they need to forecast other agents' forecast, and so on so forth. In this case, equilibrium prices do not depend only on the infinite-dimensional distribution of agents across wealth, but also on the infinite-dimensional distribution of beliefs.

My paper differs from the literature on inattention on two main dimensions. First, I discipline the role of inattention by calibrating the observation cost to match the actual duration of inattention for the median household. In this way, I can evaluate whether the observed level of inattention can quantitatively account for the heterogeneous dynamics in portfolio rebalancing across households. Second, I highlight a novel testable mechanism that links inattention - and its quantitative effects on portfolio choices and asset prices - to the tightness of households' borrowing constraints.

3 The Model

In the economy there is a representative firm that uses capital and labor to produce a consumption good. On the other side, there is a unit measure of ex-ante identical agents. Households are ex-post heterogeneous because they bear an uninsurable idiosyncratic labor income risk. Moreover, they incur in a monetary observation cost whenever they collect information on the aggregate states of the economy and take the optimal decisions on portfolio choices. Households can invest in three assets: a risk-free bond, risky capital, and a transaction account that yields no interest payment. The transaction account is liquid: inattentive households finance consumption expenditure only using the transaction account.

3.1 The Firm

The production sector of the economy consists of a representative firm, which produces a consumption good $Y_t \in \mathbf{Y} \subset \mathbb{R}_+$ using a Cobb-Douglas production function

$$Y_t = z_t N_t^{1-\eta} K_t^\eta \quad (1)$$

where $\eta \in (0, 1)$ denotes the capital income share. The variable $z_t \in \mathbf{Z} \subset R_+$ follows a stationary Markov process with transition probabilities $\Gamma_z(z', z) = \Pr(z_{t+1} = z' | z_t = z)$.

The firm hires $N_t \in \mathbf{N} \subset \mathbb{R}_+$ workers at the wage w_t , and rents from the households the stock of physical capital $K_t \in \mathbf{K} \subset \mathbb{R}_+$ at the interest rate r_t^s . Physical capital depreciates at a rate $\delta \in (0, 1)$ after production. The firm chooses capital and labor to equate the marginal productivities to prices, as follows

$$r_t^s = \eta z_t N_t^{1-\eta} K_t^{\eta-1} - \delta \quad (2)$$

$$w_t = (1 - \eta) z_t N_t^{-\eta} K_t^\eta. \quad (3)$$

3.2 Households

The economy is populated by a measure one of ex-ante identical households. They are infinitely lived, discount the future at the rate $\beta \in (0, 1)$, and maximize lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad (4)$$

where $c_t \in \mathbf{C} \subset \mathbb{R}_+$ denotes consumption at time t . I consider a CRRA utility function $U(c) = \frac{c^{1-\theta}}{1-\theta}$, where θ denotes the risk aversion of the households.

3.2.1 Idiosyncratic Shocks

Households bear an idiosyncratic labor income risk which consists of two components. First, households are hit by a shock $e_t \in \mathbf{E} \subset \{0, 1\}$, which determines their employment status. A household has a job when $e_t = 1$ and is unemployed when $e_t = 0$. I assume that e_t follows a stationary Markov process with transition probabilities

$$\Gamma_e(z, z', e, e') = \Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z'). \quad (5)$$

Although the shock is idiosyncratic and washes out in the aggregate, its transition probabilities depend on the aggregate productivity shock. In this way, both the id-

iosyncratic uncertainty and the unemployment rate of the economy rise in recessions.⁵ Second, when a household is employed, it faces a further shock $\xi_t \in \Xi \subset \mathbb{R}_+$, which determines the efficiency units of hours worked. This shock is orthogonal to the aggregate productivity shock and follows a stationary Markov process with transitional probabilities

$$\Gamma_\xi(\xi, \xi') = \Pr(\xi_{t+1} = \xi' | \xi_t = \xi). \quad (6)$$

When a household is unemployed, it receives a constant unemployment benefit $\bar{w} > 0$, which is financed through a lump sum tax τ applied to employed agents. Households' labor income l_t is then

$$l_t = w_t \xi_t e_t + \bar{w} (1 - e_t) - \tau e_t. \quad (7)$$

As in Pijoan-Mas (2007), I consider two sources of idiosyncratic uncertainty just for quantitative reasons. In the calibration of the model, the employment shock disciplines the correlation between the individual labor income risk and the aggregate shock, whereas I use the shock to the efficiency units of labor to match the cross-sectional distribution of labor income observed in the data.

3.2.2 Market Arrangements

Households can allocate their overall wealth $\omega_t \in \Omega \subset \mathbb{R}$ between consumption and savings. Households can save in three different ways. First, households own the capital of the economy. Each household holds $s_t \in \mathbf{S} \equiv [\underline{s}, \infty]$ units of capital, which are either rented to the firm or traded among households. Capital is risky and yields the rate r_t^s , as defined in (2). Second, households can also invest in a one-period non-contingent bond $b_t \in \mathbf{B} \equiv [\underline{b}, \infty]$, which is in zero net supply. The bond yields a risk-free rate r_t^b . Households face exogenous borrowing constraints for both assets and cannot go shorter than \underline{s} in risky equity and \underline{b} in risk-free bonds. When these

⁵Mankiw (1986) and Krueger and Lustig (2010) show that a countercyclical idiosyncratic uncertainty raises the price of risk. Storesletten et al. (2007) find that in the data labor income risk peaks in recessions.

values equal zero, households cannot take short positions at all. Third, as in Abel et al. (2007, 2013), households can also save in a transaction account $a_t \in \mathbf{A} \equiv [0, \infty]$. The transaction account yields no interest payment.⁶

In addition, households incur in a fixed per-period participation cost ϕ whenever investing in the stock market, that is, whenever $s_t \neq 0$. This cost may prevent households from investing in the stocks. In the quantitative analysis, I use this cost to match the observed amount of limited stock market participation, and evaluate to what extent inattention can reconcile a large share of non-participants with a low per-period participation cost.

In this framework, markets are incomplete because households cannot trade claims which are contingent on the realizations of the idiosyncratic shock. As long as the labor income risk cannot be fully insured, households are ex post heterogeneous in wealth, consumption, and portfolio choices.

3.2.3 Observation Cost

Households incur in a monetary observation cost proportional to their labor income χl_t whenever they acquire information on the aggregate states of the economy and define their optimal choices on stocks and bonds. This cost is a reduced form for the financial and time opportunity expenditures bore by households to figure out the optimal composition of the financial portfolio.

The observation cost induces the households to plan infrequently and stay inattentive meanwhile. Planning dates are defined as dates $d_i \in \mathbf{D} \subset \mathbb{N}$ such that $d_{i+1} \geq d_i$ for any i . At a planning date d_i , households pay the cost χl_{d_i} , collect the information on the aggregate states of the economy, and decide the next planning date d_{i+1} . Moreover, households decide the stream of consumption throughout the period of inattention $[c(e_i, \xi_i), c(e_{d_{i+1}-1}, \xi_{d_{i+1}-1})]$, and the investment in the transaction

⁶In equilibrium households save in the transaction account only if the observation cost is positive. The only rationale of this account is to provide liquid funds to inattentive households for financing their consumption.

account a_{d_i+1} , risky capital s_{d_i+1} , and risk-free bonds b_{d_i+1} . Importantly, the stream of consumption throughout the period of inattention is set conditional on the realizations of the idiosyncratic shocks e_t and ξ_t . Instead, at non planning dates, households are inattentive and follow the pre-determined plan for consumption set in the previous planning date. I assume that inattentive households finance consumption using the transaction account and the stream of labor income. Throughout inattention, interest payments r_t^s and r_t^b are reinvested in equity and bonds, respectively.

In the model, attentive households observe all the states of the economy, while inattentive households have a limited amount of information. I assume that inattentive households do not observe the aggregate states of the economy, although they are always fully aware of the realizations of the idiosyncratic shocks e_t and ξ_t . In the benchmark economy, I assume that throughout inattention only the choice of consumption - and *not* the choices on the composition of the financial portfolio - depends on the realizations of the idiosyncratic shocks. This assumption is consistent with the empirical evidence of Alvarez et al. (2012), who find that just 6% of the households adjust their portfolio more often than they observe it.⁷

I further characterize the conditions governing inattention in the model. To maintain the existence of credit imperfections, I postulate that inattention breaks out exogenously whenever households hit the borrowing constraints. In such a case, an unmodeled financial intermediary calls the attention of the households, which pay the observation cost and become attentive. Moreover, I assume that households become attentive when their consumption plan cannot be financed anymore by the transaction account and the labor income.

These assumptions affect the realized duration of inattention. A household that at time d_i decides not to observe the states of the economy until d_{i+1} ceases to be inattentive at the realized new planning date $\lambda(d_{i+1})$, which is the minimum between

⁷The Supplementary Appendix relaxes this assumption, by considering a version of the model in which during inattention also the choices of bonds and stocks depend on the realizations of the idiosyncratic shocks.

the desired new planning date d_{i+1} and the periods in which either the household hits the borrowing constraint, $\{j \in [d_i, d_{i+1}] : b_{j+1} < \underline{b} \text{ or } s_{j+1} < \underline{s}\}$, or consumption cannot be financed anymore by the liquid funds, $\{j \in [d_i, d_{i+1}] : c_j > a_j + l_j\}$.

3.2.4 Value Function

To define the aggregate states of the households' problem, I introduce the distribution of the agents γ_t - defined over households' idiosyncratic states, the decisions of inattention, the portfolio choices, and the consumption path $\{\omega_t, e_t, \xi_t, d_t, a_t, b_t, c_t\}$ - which characterizes the probability measure on the σ -algebra generated by the Borel set $J \equiv \Omega \times \mathbf{E} \times \Xi \times \mathbf{D} \times \mathbf{A} \times \mathbf{S} \times \mathbf{B} \times \mathbf{C}$.

Roughly speaking, γ_t keeps track of all the heterogeneity among agents. In this environment γ_t is an aggregate state. Indeed, Krusell and Smith (1997, 1998) show that prices depend on the entire distribution of agents across their idiosyncratic states. The distribution γ_t evolves over time following a law of motion

$$\gamma_{t+1} = H(\gamma_t, z_t, z_{t+1}). \quad (8)$$

The operator $H(\cdot)$ pins down the changes in the measure γ_t taking as given an initial value and the realizations of the aggregate shock z_t .

The structure of the problem takes also into account how the information is revealed to the agents. The state variables of this economy $x_t \equiv \{\omega_t, e_t, \xi_t; z_t, \gamma_t\}$ are random variable defined on a filtered probability space (X, F, P) . X denotes the set including all the possible realizations of x_t , F is the filtration $\{F_t, t \geq 0\}$ consisting of the σ -algebra that controls how the information on the states of the economy is disclosed to the agents, and P is the probability measure defined on F . Hereafter, I define the expectation of a variable v_t conditional on the information set at time k as $\mathbb{E}_k[v_t] = \int v_t dP(F_k) = \int v(x_t) dP(F_k)$. The state vector $P(v_t | F_k) = P(v_t | x_k)$

is a sufficient statistics for the probability of any variable v_t because of the Markov structure of x_t .

The presence of observation costs and inattentive agents implies some measurability constraints on the expectations of the households. Namely, a planning date d_i defines a new filtration \mathcal{F}_s such that $\mathcal{F}_s = F_{d_i}$ for $s \in [d_i, \lambda(d_{i+1})]$. Hence, any decision made throughout the duration of inattention is conditional on the information at time d_i , because households do not update their information on the aggregate states of the economy until the new planning date $\lambda(d_{i+1})$. Taking into account this measurability constraint, I write agents' recursive problem as

$$V(\omega_t, e_t, \xi_t; z_t, \gamma_t) = \max_{d, [c(e_t, \xi_t), c(e_{\lambda(d)-1}, \xi_{\lambda(d)-1})], a_{t+1}, s_{t+1}, b_{t+1}} \mathbb{E}_t \left[\sum_{j=t}^{\lambda(d)} \beta^{j-t} U(c(e_j, \xi_j)) \right. \\ \left. + \beta^{\lambda(d)-t} V(\omega_{\lambda(d)}, e_{\lambda(d)}, \xi_{\lambda(d)}; z_{\lambda(d)}, \gamma_{\lambda(d)}) \right] \quad (9)$$

$$\text{s.t. } \omega_t = c(e_t, \xi_t) + a_{t+1} + s_{t+1} + b_{t+1} + \phi \mathbb{I}_{\{s_{t+1} \neq 0\}} \quad (10)$$

$$\omega_{\lambda(d)} = s_{t+1} \prod_{j=t+1}^{\lambda(d)} \left(1 + r_j^s(z_j, \gamma_j) \right) + b_{t+1} \prod_{j=t+1}^{\lambda(d)} \left(1 + r_j^b(z_j, \gamma_j) \right) + a_{t+1} + \dots \\ \dots + \sum_{j=t+1}^{\lambda(d)} l_j(z_j, \gamma_j) - \sum_{j=t+1}^{\lambda(d)-1} c(e_j, \xi_j) - \sum_{j=t+2}^{\lambda(d)-1} \phi \mathbb{I}_{\{s_j \neq 0\}} - \chi l_{\lambda(d)}(z_{\lambda(d)}, \gamma_{\lambda(d)}) \quad (11)$$

$$\gamma_{\lambda(d)} = H(\gamma_t, z^{\lambda(d)}) \quad (12)$$

$$s_{t+1} \geq \underline{s}, \quad b_{t+1} \geq \underline{b}, \quad a_{t+1} \geq 0, \quad c(e_t, \xi_t) \geq 0 \quad (13)$$

$$\lambda(d) = \min_{j \in [t+1, d]} \left\{ d, b_{t+1} \prod_{k=t+1}^j [1 + r_k^b(z_k, \gamma_k)] < \underline{b}, s_{t+1} \prod_{k=t+1}^j [1 + r_k^s(z_k, \gamma_k)] < \underline{s}, \right. \\ \left. \dots, \sum_{k=t+1}^j c(e_k, \xi_k) > a_{t+1} + \sum_{k=t+1}^j l_k(z_k, \gamma_k) \right\}. \quad (14)$$

Equation (10) denotes the budget constraint of the agents, who use their wealth to consume, invest in the two assets, save in the transaction account, and pay the participation cost in case they own stocks. Equation (11) shows the evolution over time

of wealth, which depends on the consumption stream and the returns to investment throughout inattention. At the realized new planning date $\lambda(d)$ agents incur in the observation cost $\chi l_{\lambda(d)}$. Equation (12) defines the law of motion of the distribution of agents γ_t conditional on the history of aggregate shocks $z^{\lambda(d)}$. Finally, Equation (13) denotes the borrowing constraints faced by the households, whereas Equation (14) describes the new realized planning date $\lambda(d)$.

3.3 Equilibrium

3.3.1 Definition of Equilibrium.

A competitive equilibrium for this economy is a value function V , a set of policy functions $\{g^c, g^b, g^s, g^a, g^d\}$, a set of prices $\{r^b, r^s, w\}$, and a law of motion $H(\cdot)$ for the measure of agents γ such that

- Given the prices $\{r^b, r^s, w\}$, the law of motion $H(\cdot)$, the exogenous transition matrices $\{\Gamma^z, \Gamma^e, \Gamma^\xi\}$, the value function V , and the set of policy functions $\{g^c, g^b, g^s, g^a, g^d\}$ solve the household's problem;
- The bonds market clears, $\int g^b d\gamma = 0$;
- The capital market clears, $\int g^s d\gamma = K'$;
- The labor market clears, $\int e \xi d\gamma = N$;
- The unemployment benefit is financed by a lump sum tax on employed households, $\int \bar{w} (1 - e) d\gamma = \int \tau e d\gamma$;
- The law of motion $H(\cdot)$ is generated by the optimal decisions $\{g^c, g^b, g^s, g^a, g^d\}$, the transition matrices $\{\Gamma^z, \Gamma^e, \Gamma^\xi\}$, and the history of aggregate shocks z .

3.3.2 First-Order Conditions.

Gabaix and Laibson (2002) consider an environment in which agents are exogenously inattentive for a fixed number of periods. In their model, the Euler equation for con-

sumption holds only for the mass of attentive agents because inattentive households are off their equilibrium condition. Instead, here the Euler equations of both attentive and inattentive agents hold in equilibrium. At a planning date t the Euler equation is a standard stochastic inter-temporal condition that reads

$$\mathbb{E}_t \left[M_{\lambda(d),t} \prod_{k=t+1}^{\lambda(d)} \left(r_k^s(z_k, \gamma_k) - r_k^b(z_k, \gamma_k) \right) \right] = 0 \quad (15)$$

where $M_{\lambda(d),t} = \beta^{\lambda(d)-t} \frac{U'(c_{\lambda(d)})}{U'(c_t)}$ denotes households' stochastic discount factor. This condition posits that the optimal share of stocks in the portfolio equalizes the compounded expected discounted returns from stocks and bonds throughout the period of inattention.

The Euler equation of an inattentive agent between time v and q , with $t < v < q < \lambda$ is deterministic and equals

$$M_{q,v} \prod_{k=v+1}^q \left(r_k^s(z_k, \gamma_k) - r_k^b(z_k, \gamma_k) \right) = 0. \quad (16)$$

Inattentive agents do not gather any new information on the states of the economy, and therefore they behave as if there were no uncertainty. Agents get back to the stochastic inter-temporal conditions as soon as they reach a new planning date and update their information set. As agents alternate between attention and inattention, they also shift from stochastic to deterministic Euler equations.⁸

3.4 Inattention in the Model and in the Data

In the model inattentive households do not observe the aggregate states of the economy, whereas they are always fully aware of the realizations of the idiosyncratic shocks e_t and ξ_t . The model represents a tractable extension to the case of het-

⁸In either case, the Euler equations are not satisfied with equality for borrowing constrained agents.

erogeneous agents of the inattentiveness proposed by Reis (2006). Although in Reis (2006) inattentive agents do not receive any flow of information, I relax this condition by allowing inattentive households to observe at least their idiosyncratic sources of uncertainty. From this point of view, this model bridges the gap between the inattentiveness of Reis (2006) and the rational inattention of Sims (2003). In Sims (2003), households choose how to allocate their limited capacity of information acquisition, by deciding the noise up to which they observe all the relevant variables of the economy. My model can be considered a limiting case of Sims (2003), in which households decide to allocate their entire capacity to observe the idiosyncratic shocks, up to the point that the noise around the idiosyncratic shocks disappears, while the noise on the aggregate states goes to infinity.

The definition of inattention of the model slightly differs from the definition of inattention of the survey studied by Alvarez et al. (2012). In their data, inattention refers to the frequency with which households observe their financial portfolio. Instead, the model considers a broader definition of inattention, by focusing on the frequency of observation of the aggregate states of the economy.

Furthermore, in the model households always adjust their portfolios upon paying the observation cost. Although this is not always the case in the data, the correlation between observing and adjusting the financial portfolio is very high and equals 0.45. Alvarez et al. (2012) show that such a correlation can be rationalized with the presence of both observation costs and portfolio transaction costs. Since I study an economy in which households do not face the additional friction of the transaction cost, my model implies a lower bound on the effects of inattention on portfolio inertia and asset prices.⁹

⁹I abstract from the portfolio transaction costs for purely computational reasons. The introduction of portfolio transaction costs requires the addition of yet another state variable, which would limit substantially the computational tractability of the model. For instance, following the choices I made in the calibration exercise, this further state variable would inflate the grid points from 5,184,000 up to 311,040,000.

4 Calibration

The calibration strategy follows Krusell and Smith (1997, 1998) and Pijoan-Mas (2007). Some parameters are calibrated to match salient facts of the U.S. economy, while others (e.g., the risk aversion of the household) are set to values estimated in the literature. Throughout the quantitative analysis, I set one period of the model to correspond to one month. Nevertheless, I report the asset pricing statistics aggregated at the annual frequency to be consistent with the literature.

First of all, I calibrate the aggregate shock to match the volatility of output growth. The idiosyncratic labor income risk is defined to target the cross-sectional distribution of labor income, and its correlation with the aggregate unemployment rate. It is important to have a realistic variation in labor income because the choice of inattention, and consequently the effect of the observation cost on portfolio rebalancing and asset prices, depends on the budget of the households. The observation cost is defined to replicate the duration of inattention of the median household, while the participation cost is set to match the share of stockholders observed in the data. Finally, I calibrate three parameters that capture the amount of wealth in the economy: the time discount factor β and the borrowing constraint on stocks s and bonds b . The discount factor is set to match the U.S. annual capital to output ratio of 2.5, which yields a value of $\beta = 0.9951$. The calibration of the borrowing constraint is very important because I show that the quantitative implications of inattention depends crucially on how tighten borrowing constraints are. First, I equalize the level of the constraint on stocks and bonds, that is, $s = b$. Second, I pin down the level of both constraints by matching the fraction of households with negative wealth, which in the data is around 10%, as shown in Diaz-Gimenez et al. (2011). This choice implies a value of $s = b = -5.96$, which is equivalent to around three times the monthly income of the median household.

The parameters set to values estimated in the literature are the capital share of

the production function η , the capital depreciation rate δ , and the risk aversion of the household θ . I choose a capital share $\eta = 0.40$, as suggested by Cooley and Prescott (1995). The depreciation rate equals $\delta = 0.0066$ to match a 2% quarterly depreciation. The risk aversion of the household is $\theta = 5$, which gives an inter-temporal elasticity of substitution of 0.2.

4.1 Aggregate Productivity Shock

I assume that the aggregate productivity shock z_t follows a two-state first-order Markov chain, with values z_g and z_b denoting the realizations in good and bad times, respectively. The two parameters of the transition function are calibrated targeting a duration of 2.5 quarters for both states. The values z_g and z_b are instead defined to match the standard deviation of the Hodrick-Prescott filtered quarterly aggregate output, which is 1.89% in the data. These values are model dependent, and vary with the specification of the environment.

4.2 Idiosyncratic Labor Income Shock

Employment Status. The employment shock e_t follows a two-state first-order Markov chain, which requires the calibration of ten parameters that define four transition matrices two by two. I consider the ten targets of Krusell and Smith (1997, 1998). I first define four conditions that create a one-to-one mapping between the state of the aggregate shock and the level of unemployment: regardless of the previous realizations of the shock, the good productivity shock z_g comes always with an unemployment rate u_g , and the bad productivity shock z_b with an unemployment rate u_b . In this way, the realization of the aggregate shock pins down the unemployment rate of the economy. The four conditions are

$$1 - u_g = u_g \Gamma_e(z_g, z_g, 0, 1) + (1 - u_g) \Gamma_e(z_g, z_g, 1, 1)$$

$$1 - u_g = u_b \Gamma_e(z_b, z_g, 0, 1) + (1 - u_b) \Gamma_e(z_b, z_g, 1, 1)$$

$$1 - u_b = u_g \Gamma_e(z_g, z_b, 0, 1) + (1 - u_g) \Gamma_e(z_g, z_b, 1, 1)$$

$$1 - u_b = u_b \Gamma_e(z_b, z_b, 0, 1) + (1 - u_b) \Gamma_e(z_b, z_b, 1, 1).$$

The levels of the unemployment rate in good time and bad time are defined to match the actual average and standard deviation of the unemployment rate. Using data from the Bureau of Labor Statistics from 1948 to 2012, I obtain that the two moments equal 5.67% and 1.68%, respectively. Under the assumption that the unemployment rate fluctuates symmetrically around its mean, I find $u_g = 0.0402$ and $u_b = 0.0732$. Two further conditions come by matching the expected duration of unemployment in good times (6 months) and bad times (10 months). Finally, I set both the job finding probability when moving from the good state to the bad one and the probability of losing the job in the transition from the bad state to the good one to zero.

Unemployment Benefit. I set the monthly unemployment benefit \bar{w} to be 5% of the average monthly labor earning. Although different values of the benefit affect the lower end of the wealth distribution, they have no sizable effect on the dynamics of portfolio rebalancing and asset prices.

Efficiency Units of Hour. The shock to the efficiency unit of hour ξ_t follows a three-state first-order Markov chain. The values of the shock and the transition function are calibrated to match three facts on the cross-sectional dispersion of labor earnings across households: the share of labor earnings held by the top 20%, the share of labor earnings held by the bottom 40%, and the Gini coefficient of labor earnings. Table 1 reports the calibrated values and the transition function of the shock ξ_t , while Table 2 compares the three statistics on the distribution of labor earnings in the data and in the model.

Table 1: Parameters of the shock to the efficiency units of hour

	$\xi_1 = 6$	$\xi_2 = 2$	$\xi_3 = 1$
$\Gamma_\xi(\xi_1, \cdot)$		$\Gamma_\xi(\xi_2, \cdot)$	$\Gamma_\xi(\xi_3, \cdot)$
$\Gamma_\xi(\cdot, \xi_1)$	0.9850	0.0025	0.0050
$\Gamma_\xi(\cdot, \xi_2)$	0.0100	0.9850	0.0100
$\Gamma_\xi(\cdot, \xi_3)$	0.0050	0.0125	0.9850

Note: The efficiency unit of hours ξ_t follows a first-order Markov chain with transition function Γ_ξ .

Table 2: The distribution of labor earnings

Target	Model	Data
Share earnings top 20%	62.1%	63.5%
Share earnings bottom 40%	4.4%	4.2%
Gini index	0.57	0.64

Note: the data is from Díaz-Giménez et al. (2011).

4.3 Participation Cost

I calibrate the fixed per-period participation cost ϕ to match the amount of limited stock market participation observed in the data. Fafilukis (2013) reports that in the U.S. in 2007 the share of stockholders is 59.4%. The model matches this moment with a participation cost that equals $\phi = 0.019$, which amounts to 0.8% of households' monthly labor earnings. For example, if the average household earns an income of around \$3,000 per month, the cost equals \$24.

4.4 Observation Cost

I discipline the amount of inattention risk in the model by calibrating the observation cost to match the duration of inattention of the median household. Alvarez et al. (2012) estimate that the median household observes its portfolio every 3 months. The model matches this moment with an observation cost that equals $\chi = 0.029$, which amounts to 2.9% of households' monthly labor earnings. For example, if the average household earns an income of around \$3,000 per month, the cost equals \$87.

4.5 Computation of the Model

The computation of heterogeneous agent models with aggregate uncertainty are cumbersome because the distribution γ_t , a state of the problem, is an infinite-dimensional object. I approximate γ_t using a finite set of moments of the distribution of aggregate capital K_t - as in Krusell and Smith (1997, 1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008) - and the number of inattentive agents in the economy in every period ζ_t . On the one hand, the approximation using a finite set of moments of aggregate capital K_t can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of γ_t . Nevertheless, this class of models generates almost linear economies, in which it is sufficient to consider just the first moment of the distribution of capital to have a perfect fit for the approximation. On the other hand, inattention adds a further term ζ_t , which signals active investors about the degree of the informational frictions in the economy. This condition adds a further law of motion upon which to converge. The presence of inattention implies one further complication. The decision of the agents on the duration of inattention requires the evaluation of their value function over a wide range of different time horizons. I report the details of the computational algorithm in the Supplementary Appendix.

5 Results

I compare the results of the benchmark model with three alternative calibrations. In the first one, the observation cost is zero and there is no inattention. In the second one, the observation cost is more severe and amounts to $\chi = 0.058$. Finally, I consider an economy in which agents are more risk averse, with $\theta = 8$. I calibrate each version of the model to match the volatility of aggregate output growth, the cross-sectional distribution of labor earnings, the amount of limited stock market participation, the level of aggregate wealth, and the fraction of households with negative wealth. Results are computed from a simulated path of 3,000 agents over 10,000 periods.

5.1 Inattention

The observation cost is calibrated to a 3 months duration of inattention for the median household. It turns out that such a cost prevents a large fraction of agents from gathering information on the stock market. Table III shows that in the model, in any given month, the average fraction of inattentive agents in the economy equals 44%. Furthermore, Figure 1 shows that there is a negative correlation between wealth and inattention, in line with the empirical evidence of Ameriks *et al.* (2003) and Alvarez *et al.* (2012).

Consistently with the data, the model generates a sizable heterogeneity in the duration of inattention across households. For example, the wealthiest 20% of households observe the aggregate states of the economy every period. Instead, poor agents cannot afford the observation cost and end up being more inattentive. In the model, the poorest 20% of households stay inattentive for 9 months on average. These results point out that in the model inattention behaves both as a time-dependent and a state-dependent rule. Indeed, at each point of time households set a time-dependent rule, deciding how long to stay inattentive. Yet, when a household becomes wealth-

Table 3: Inattention

Inattention	$\chi = 0.029$	$\chi = 0$	$\chi = 0.058$	$\theta = 8$	Data
A. Duration of inattention (months)					
Median	3.0	0	3.6	4.0	3.0
Median - good times	2.8	0	3.2	3.7	-
Median - bad times	3.2	0	3.9	4.4	-
75th percentile - good times	1.0	0	1.1	1.2	-
75th percentile - bad times	0.6	0	0.9	0.8	-
25th percentile - good times	6.1	0	6.5	6.7	-
25th percentile - bad times	6.7	0	7.0	7.2	-
B. Fraction of inattentive agents					
Median	0.44	0	0.48	0.49	-
Median - good times	0.41	0	0.44	0.47	-
Median - bad times	0.50	0	0.54	0.52	-

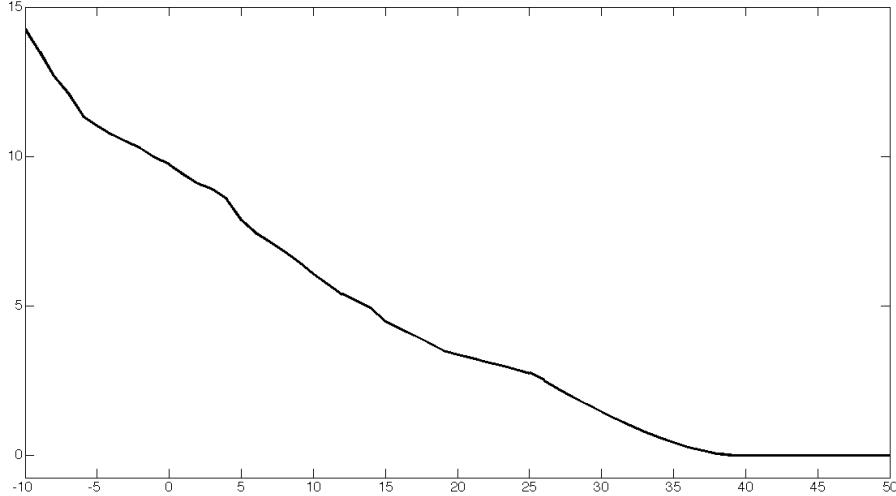
Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. Good times denote the periods in which the aggregate productivity shock is $z_t = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z_t = z_b$. The fraction of inattentive agents are reported in percentage values. Data is from Alvarez et al. (2012).

ier, it opts for shorter periods of inattention. Thus, inattention looks as if it were conditional on wealth.¹⁰

The dynamics of inattention over the business cycle depend on two forces. On the one hand, the countercyclical equity premium induces households to plan in recessions because it is the moment in which the cost of inattention in terms of foregone financial

¹⁰Reis (2006) labels this property of inattention as “recursive time-contingency”. See Alvarez et al. (2012) and Abel et al. (2007, 2013) for further characterizations of the dynamics of inattention over time.

Figure 1: Optimal Choice of Inattention



Note: the figure plots the policy function of inattention g^d as a function of wealth ω . The idiosyncratic shocks are set to $e_t = 1$ and $\xi_t = 2$. The aggregate shock is $z_t = z_g$ and the aggregate capital equals its mean.

returns is highest. On the other hand, the severity of the observation cost fluctuates following households' wealth. In recessions, households are poorer and cannot afford the observation cost. The results show that wealthy agents tend to be attentive in bad times, to profit from the higher equity premium. For example, in the model the agents at the 75-th percentile of the wealth distribution are on average inattentive for 1 month in good times and 0.6 months in bad times. Instead, the direct cost of inattention affects relatively more poor agents, which prefer to plan in expansions. The agents at the 25-th percentile of the wealth distribution are on average inattentive for 6.1 months in good times and 6.7 months in bad times. Overall, inattention is countercyclical: both the duration of inattention for the median agent and the fraction of inattentive agents in the economy rise in recession. The countercyclicality of inattention is consistent with the empirical evidence of Sichermann et al. (2016), who report that 401(k) retirement account logins fall in bearish markets.

Increasing the size of the observation cost to $\chi = 0.058$ extends the duration of inattention for the median agent up to 3.6 months. Also a risk aversion of $\theta = 8$ does

increase the duration of inattention, which goes up to 4 months. This last result is in line with the evidence provided by Alvarez et al. (2012), who show that more risk averse investors observe their portfolio less frequently. This outcome is the net result of two counteracting forces. Households with a higher risk aversion change their portfolio towards risk-free bonds, decreasing the need of observing the stock market. At the same time, more risk averse agents have a stronger desire for consumption smoothing, which induces them to keep track of their investments more frequently. In the model, the first channel offsets the second one, implying a longer duration of inattention for more risk averse agents.

5.2 Stock Market Participation

Favilukis (2013) reports that in 2007 just 59.4% of the households participate in the stock market. I show that inattention can reconcile this amount of limited stock market participation with a low per-period participation cost. Although the model is calibrated to match exactly the participation rate observed in the data, the level of the participation cost required to get the right number of stockholders is endogenous. The amount of this cost is then informative of the extent in which inattention can rationalize the limited stock market participation.

Table 4 shows that, in the benchmark economy with a positive observation cost, the model matches the observed share of stockholders with a participation cost that amounts to 0.8% of households' average monthly income. If the average household earns an income of around \$3,000 per month, the cost of participating in the market for one entire year equals \$288. When I abstract from the observation cost, the model requires a participation cost which is four times larger: the model matches the same share of participants with a participation cost that amounts to 3.0% of households' average monthly income. Following the previous example, in this case the cost of participating in the market for one entire year rises up to \$1,080.

Table 4: Participation to the stock market

Variable	$\chi = 0.029$	$\chi = 0$	$\chi = 0.058$	$\theta = 8$	Data
Fraction of Shareholders	59.4%	59.4%	59.4%	59.4%	59.4%
Per-Period Participation Cost (in terms of avg. monthly labor income)	0.8%	3.0%	0.6%	0.7%	-
$\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$	0.86	0.39	0.91	0.98	1.60
Gini Index of Wealth	0.66	0.42	0.69	0.71	0.82

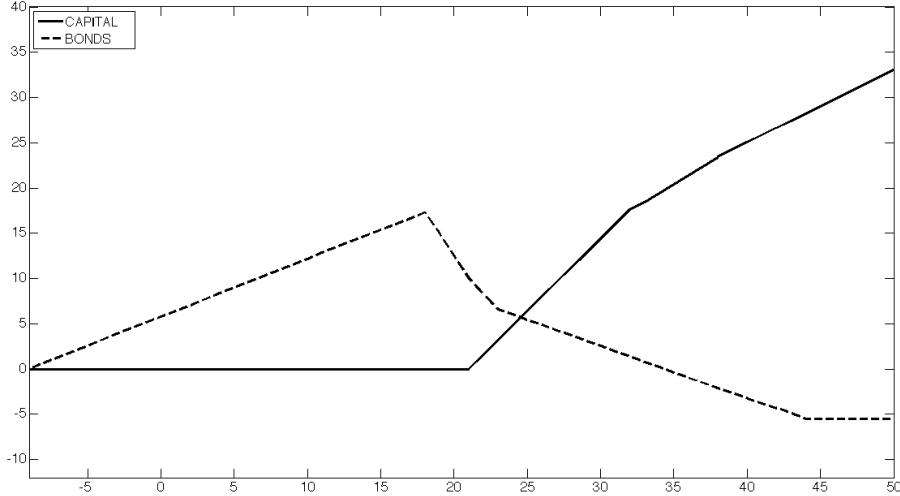
Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. All the economies are calibrated to match the share of stock market participants observed in the data. The ratio $\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$ compares the standard deviation of the consumption growth of stockholders $\sigma(\Delta \log c_S)$ with the standard deviation of consumption growth of non-stockholders $\sigma(\Delta \log c_{NS})$. Data is from Fabilukis (2013), Mankiw and Zeldes (1991) and Díaz-Giménez et al. (2011).

This exercise highlights that - unlike in Saito (1996), Basak and Cuoco (1998), and Guvenen (2009) - here the limited participation does not arise exogenously: it is jointly determined by the presence of the observation cost and the participation cost. In the model, inattention is a barrier to financial investment because households anticipate that, during their inattention periods, they cannot actively manage their portfolios and end up investing sub-optimally. This result points out to a new rationale to the limited stock market participation: not only trading costs matter, as in Gomes and Michaelides (2008), but also the fact that processing all the information required to invest optimally in the financial markets is not a trivial task at all.

The model also successfully predicts that stockholders are on average wealthier than non-stockholders. Figure 2 shows that stockholders tend to be the wealthiest agents of the economy. For example, the poorest 8% of the households do not hold any risky capital because they are inattentive very often.

The model fails in reproducing the higher consumption growth volatility of stock-

Figure 2: Optimal Portfolio Choices



Note: the figure plots the policy functions of investment in risky assets g^a (continuous line) and risk free bonds g^b (dashed line) as a function of wealth ω . The idiosyncratic shocks are set to $e_t = 1$ and $\xi_t = 2$. The aggregate shock is $z_t = z_g$ and the aggregate capital equals its mean.

holders with respect to non-stockholders. In the model, stockholders turn out to be wealthy agents that are able to self-insure their consumption stream, experiencing thereby a lower volatility than non-stockholders. The rationale of this counterfactual implication of the model is twofold. First, Guvenen (2009) finds that a low participation rate is not enough to generate a higher volatility of consumption for stockholders, unless stockholders have a higher inter-temporal elasticity of substitution than non-stockholders, as it is assumed in Gomes and Michaelides (2008). Second, the model is not able to account for the dispersion in the wealth distribution observed in the data. In the model the Gini index of wealth equals 0.66, while the empirical counterpart is 0.82. In the model, the wealth of rich households is not fully invested in stocks, as they keep bonds in their portfolios. In this way, the volatility of consumption of stockholders is not entirely determined by the fluctuations of the stock market. As a result, the counterfactual distribution of wealth leads to the underestimation of the volatility of consumption of wealthy agents. Nevertheless, Table 4 shows that with-

out inattention the model generates even a more compressed distribution of wealth: abstracting from the observation cost reduces the Gini index from 0.66 down to 0.42.

5.3 Portfolio Rebalancing

In this Section, I study to what extent stockholders manage to actively offset the passive variations in their portfolios. To understand the implications of inattention on portfolio rebalancing, I take the simulated data of my model and replicate the regression that Calvet et al. (2009) run on a panel of Swedish households. In this way I quantify whether - and to what extent - households' inattention can account for the dynamics of portfolio rebalancing observed in the data.

Let me first define the risky share $\alpha_{i,t}$ of the household i at time t as $\alpha_{i,t} = \frac{s_{i,t+1}}{s_{i,t+1} + b_{i,t+1}}$, that is, the ratio of risky capital over the sum of risky capital and risk free bonds. I decompose the variations over time in the risky share $\alpha_{i,t+1} - \alpha_{i,t}$ in two components: the passive change $P_{i,t+1}$ and the active change $A_{i,t+1}$.

Consider a stockholder that at time t invests in stocks $s_{i,t+1}$ and bonds $b_{i,t+1}$. In the next period, her positions amount to $(1 + r_{t+1}^s) s_{i,t+1}$ stocks and $(1 + r_{t+1}^b) b_{i,t+1}$ bonds. If the stockholder does not adjust the portfolio, the new risky share equals

$$\alpha_{i,t+1}^P = \frac{(1 + r_{t+1}^s) s_{i,t+1}}{(1 + r_{t+1}^s) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1}} = \frac{(1 + r_{t+1}^s) \alpha_{i,t+1}}{(r_{t+1}^s - r_{t+1}^b) \alpha_{i,t} + (1 + r_{t+1}^b)}.$$

I refer to the variable $\alpha_{i,t+1}^P$ as the passive risky share, that is, the risky share that is expected in the case stockholders do not rebalance at all their portfolios.

I define the passive change $P_{i,t+1}$ as the change in the risky share for a stockholder that does not adjust her financial portfolio

$$P_{i,t+1} = \alpha_{i,t+1}^P - \alpha_{i,t}.$$

Then, I define the active change $A_{i,t+1}$ as the residual change in the risky share that

is not accounted for by the passive risky share $\alpha_{i,t+1}^P$, that is

$$A_{i,t+1} = \alpha_{i,t+1} - \alpha_{i,t+1}^P.$$

$A_{i,t+1}$ does not capture any mechanic change in the risky share and therefore quantifies the amount of active rebalancing of a stockholder. This measure is defined such that the overall change in the risky share is the sum of the passive and active change

$$\alpha_{i,t+1} - \alpha_{i,t} = A_{i,t+1} + P_{i,t+1}.$$

Following Calvet et al. (2009), I study the dynamics of active and passive portfolio rebalancing across households by estimating the panel regression

$$A_{i,t+1} = \text{constant} + \psi * P_{i,t+1} + \mu * \alpha_{i,t} + \epsilon_{i,t+1}.$$

The coefficient ψ defines the amount of passive change which is offset by the active rebalancing of stockholders. Instead, the coefficient μ captures the dependence of the adjustments in the financial portfolio on the previous share invested in stocks. A fully passive stockholder would have both $\psi = 0$ and $\mu = 0$. I estimate the regressions at the yearly frequency, over a sample of about 800 periods. I consider only stockholders that maintain a position in stocks for at least two consecutive years. The regression is estimated over a panel of households over a total of around 1,200,000 stockholder-period observations. To be consistent with Calvet et al. (2009), I compute the annual risky share and related variables from the simulated data taken from the last month of each year.

Table 5.3 reports the estimates of the parameter ψ of the regression above in four different cases: I consider all the years of my simulated data (Panel A), I consider only the expansionary periods (Panel B), I consider all the years focusing only on stock-

Table 5: Panel Regression of Active and Passive Portfolio Rebalancing

	$\chi = 0.029$	$\chi = 0$	$\chi = 0.058$	$\theta = 8$	Data
Active Change $i,t+1$ = Constant + $\psi \times$ Passive Change $i,t+1$ + $\mu \times$ Risky Share i,t + $\epsilon_{i,t+1}$					
					A. All Years
$\hat{\psi}$ (Passive Change)	-0.730	-0.997	-0.703	-0.712	-0.504
Adjusted R^2	0.70	0.76	0.72	0.71	0.12
B. Expansions					
$\hat{\psi}$ (Passive Change)	-0.774	-0.998	-0.756	-0.766	-
Adjusted R^2	0.71	0.77	0.73	0.70	-
C. All Years - Stockholders in the 75th Percentile of the Wealth Distribution					
$\hat{\psi}$ (Passive Change)	-0.876	-0.999	-0.868	-0.860	-
Adjusted R^2	0.73	0.77	0.71	0.73	-
D. All Years - Stockholders in the 25th Percentile of the Wealth Distribution					
$\hat{\psi}$ (Passive Change)	-0.425	-0.996	-0.419	-0.406	-
Adjusted R^2	0.74	0.80	0.79	0.79	-

Note: the table reports the yearly panel regressions of the active change $A_{i,t+1}$ in portfolio rebalancing on a constant, the passive change $P_{i,t+1}$ in portfolio rebalancing and the share of the portfolio invested in stocks in the previous year $\alpha_{i,t}$. I consider households that participate in the stock market over two consecutive years (t and $t + 1$). Passive Change $\hat{\psi}$ is the estimated amount of passive change which is actively offset by the households by adjusting their portfolios. The variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. Data is from Calvet et al. (2009).

holders in the 75th percentile of the wealth distribution (Panel C), I consider all the years focusing only on stockholders in the 25th percentile of the wealth distribution (Panel D).

Panel A shows that in the model stockholders actively offset around 73% of the passive change in their portfolio share. This result implies that inertia accounts for the remaining 27% of the movements in the financial portfolios. Since Calvet *et al.* (2009) find that inertia characterizes 50% of the changes in the risky share of Swedish households, the model is able to account for 54% of the inertia in portfolio rebalancing observed in the data. In the model, the inertia in portfolio rebalancing is entirely driven by inattention. Indeed, when I shut down the observation cost, households always actively manage their financial positions.

Panel B shows that the amount of active rebalancing increases during expansions, going up to 77%. This result is consistent with the evidence of Calvet *et al.* (2009), who find that the amount of active rebalancing decreased in Sweden from 2000 to 2002, a period of bearish stock market. In the model, stockholders are on average more attentive during expansions, and therefore the average amount of active rebalancing rises in good times.

Panel C and D compare the amount of active rebalancing across wealthy and poor stockholders. Panel C shows that wealthy households offset 88% of the passive change in the risky share. This value is twice as large as the amount of rebalancing estimated across poor households, which equals 43%. Again, wealthy stockholders can afford to be attentive often enough to have a very active management of their portfolios. Instead, poor households end up being inattentive, and their portfolios follow by inertia the realizations of stock and bond returns. This result highlights that the model is able to capture the heterogeneity in the inertia of portfolio rebalancing across households.

5.4 Asset Pricing Moments

5.4.1 Stock and Bond Returns

Panel A of Table 6 reports the results of the model on the level and the dynamics of stock returns, bond returns, and the equity premium. First, I discuss the standard deviations because the observation cost almost doubles the volatility of stock returns. In the benchmark model the standard deviation of returns is 11.5%, which accounts for around 60% of the volatility observed in the data, that is 19.3%. Nonetheless, without inattention the standard deviation would be just 6.3%.

The observation cost boosts the volatility of returns because it amplifies the movements in the marginal productivity of capital. Since inattentive agents cannot immediately adjust their portfolios to the realizations of the aggregate shock, individual financial investment alternates between periods of inaction and periods of sharp adjustments. This mechanism raises the fluctuations in the marginal productivity of capital through two channels, by hiking up the volatility of aggregate investment, and by reducing the correlation of aggregate investment with the realizations of the aggregate productivity shock. In the economy with inattention, the standard deviation of investment at the quarterly frequency equals 2.5%. Instead, when I abstract from the observation cost, the volatility of investment drops to 1.4%. Both values are lower than the empirical counterpart, that equals 4.5%. In addition, in the economy with inattention, the correlation between investment and output is 0.93, whereas in the economy without inattention the correlation is much higher and equals 0.99. In the data, the correlation is 0.95.

As far as the volatility of the risk-free rate is concerned, I find a standard deviation of 3.7%, which is lower than its empirical counterpart, that equals 5.4%. Note that standard models usually deliver risk-free rates which fluctuate too much. For example, Jermann (1998) and Boldrin et *al.* (2001) report a standard deviation between

Table 6: Asset pricing moments

Variable	Moment	$\chi = 0.029$	$\chi = 0$	$\chi = 0.058$	$\theta = 8$	Data
A. Stock and bond returns						
Stock return	Mean	4.47	1.91	4.81	5.08	8.11
	Std. dev.	11.51	6.31	11.97	12.23	19.30
Risk-free return	Mean	1.83	1.10	1.84	1.98	1.94
	Std. dev.	3.65	2.62	4.00	4.39	5.44
B. Equity premium						
Equity premium	Mean	2.64	0.81	2.97	3.10	6.17
	Std. dev.	11.35	6.27	11.81	12.02	19.49
Sharpe ratio	Mean	0.23	0.13	0.25	0.26	0.32
C. Cyclical dynamics						
Stock returns	Std. dev. - good times	11.33	6.29	8.45	12.07	-
	Std. dev. - bad times	11.70	6.34	8.69	12.44	-
Equity premium	Mean - good times	2.48	0.80	2.75	2.83	-
	Mean - bad times	2.97	0.82	3.22	3.40	-

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in expectation and reported in annualized percentage values. Annual returns are defined as the sum of log monthly returns. The equity premium is the $r_{t+1}^e = \mathbb{E}[r_{t+1}^a - r_{t+1}^b]$. The Sharpe ratio is defined as the ratio between the equity premium and its standard deviation. Good times denote the periods in which the aggregate productivity shock is $z_t = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z_t = z_b$. Data is from Campbell (1999) and Guvenen (2009).

10% and 20%. The mechanism that prevents volatility to surge is similar to the one explored by Guvenen (2009). Poor agents have a strong desire to smooth consumption, and their high demand of precautionary savings offsets any large movements in bond returns. Although in Guvenen (2009) the strong desire for consumption smoothing is achieved through a low elasticity of inter-temporal substitution, here it

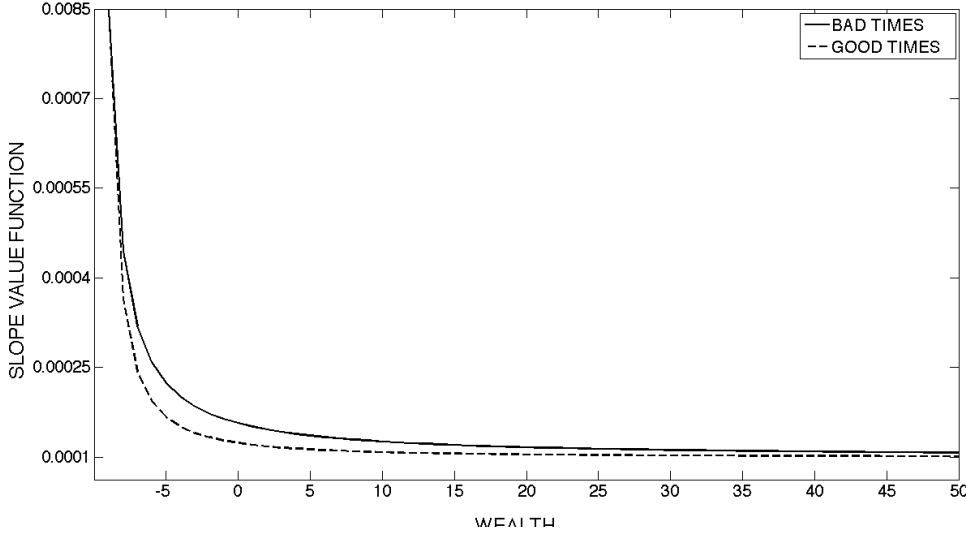
is the observation cost that forces poor agents to insure against the risk of infrequent planning.

When looking at the level of the equity premium reported in Panel B of Table 6, I find that the model generates a wedge between stock returns and bond yields which equals 2.6%. In the economy without the observation cost, the equity premium is just 0.8%. Hence, in the model households' inattention to the stock market accounts for around 30% of the observed level of the equity premium.

The observation cost raises the equity premium through two channels. First, as in Guvenen (2009), the limited participation in the stock market concentrates the entire aggregate risk of the economy on a small measure of stockholders, who accordingly demand a higher compensation for holding equity. Second, inattention exacerbates the curvature of the value function of the agents. Figures 3 - 4 show that inattention raises the implied risk aversion of the households. Although in an economy without observation costs households' value function is rather flat, with inattention the value function becomes both more concave and more responsive to aggregate conditions. In this way, inattention amplifies the risk associated to holding stocks, especially in bad times.

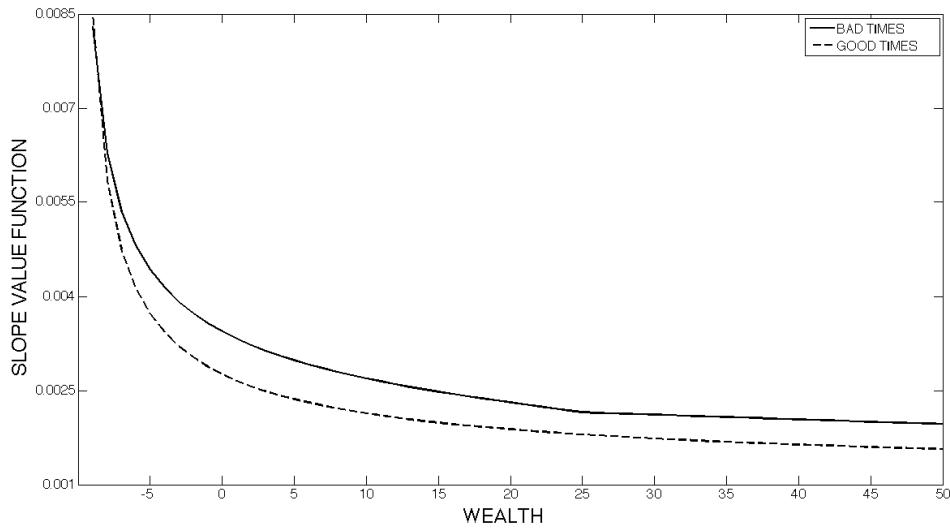
Overall the model falls shorter in accounting for asset prices when compared to other papers that in the literature study the role of inattention. For instance Chien et al. (2011, 2012) consider a model with inattentive agents which delivers asset prices moments much closer to the data. Although these papers follow different calibrations strategies, the main difference with my paper lies in the modeling of inattention. Chien et al. (2011, 2012) consider an economy with an *exogenous* measure of agents which trade at *exogenously* fixed intermittent dates. In the Supplementary Appendix, I study a version of my model with exogenous inattention, in which the duration of inattention is calibrated to match exactly the cross-sectional duration of inattention obtained in the economy with observation costs. I find that moving

Figure 3: Slope of the Value Function - Attentive Economy



Note: the figure plots the slope of agents' value function as a function of wealth ω in an economy no observation cost, i.e. $\chi = 0$. Bad times (continuous line) and good times (dashed line) denote periods in which the aggregate productivity shock is $z_t = z_b$ and $z_t = z_g$. The idiosyncratic shocks are set to $e_t = 1$ and $\xi_t = 2$.

Figure 4: Slope of the Value Function - Inattentive Economy



Note: the figure plots the slope of agents' value function as a function of wealth ω in an economy with observation costs, i.e. $\chi = 0.029$. Bad times (continuous line) and good times (dashed line) denote periods in which the aggregate productivity shock is $z_t = z_b$ and $z_t = z_g$. The idiosyncratic shocks are set to $e_t = 1$ and $\xi_t = 2$.

from an endogenous to an exogenous inattention leads to an overstatement of both the inertia in portfolio rebalancing and the level of the equity premium. Intuitively, when inattention is endogenous, households can choose optimally when to observe the aggregate states of the economy. As a result, households have yet another choice for smoothing their consumption stream, which leads to a more frequent rebalancing of their portfolio and to a lower price of risk.

5.4.2 Cyclical Dynamics

Inattention generates countercyclical variations in both the stock returns volatility and the equity premium, as shown in Panel C of Table 6. Since the observation cost bites more strongly in recessions, there are very few active investors in the economy, which implies that the investment in physical capital is low and very responsive to the decision of the marginal attentive stockholder. Instead, when the observation cost goes to zero the volatility becomes rather acyclical.

Inattention leads to an equity premium which is countercyclical and displays large variations over the cycle, a result which is usually obtained through consumption habits (Campbell and Cochrane, 1999) or long-run risk (Bansal and Yaron, 2004). In the model, the equity premium equals 2.48% in good times and 2.97% in bad times. This result is in line with the empirical evidence on a positive risk-return trade-off. Again, the cyclical disappears when I shut down the observation costs.

5.5 The Role of Borrowing Constraints

Chen (2006) considers a Lucas-tree economy where heterogeneous agents face an observation cost, finding that any household owns stock, the portfolio rebalancing is mostly passive, and the equity premium is zero. Instead, in my model there is a vast heterogeneity in the degree of portfolio rebalancing across households, the equity premium is 2.64%, and the observation cost reconciles the amount of limited stock

market participation observed in the data with a low participation cost. What is the main feature of the model that allows households' inattention to matter quantitatively on the dynamics of both portfolio rebalancing and asset prices? In this Section, I show a novel testable mechanism that links households' inattention to households' funding liquidity. I find that the tightness of the borrowing constraints shapes the implications of households' inattention: inattention affects the dynamics of portfolio rebalancing and asset prices only if borrowing constraints are tight enough.

In what follows, I compare three economies which differ only for the level of the borrowing constraints. The first one is the benchmark model, where the borrowing constraints equal minus three times the monthly income of the median household. In the second case, I consider an economy in which agents cannot borrow at all, while in the last case the constraints are loose and equal minus six times the monthly income of the median household. In this way, I can identify how the effects of households' inattention depend on credit market frictions. Furthermore, I keep constant the participation cost across the three alternative economies to disentangle the contribution of the borrowing constraints - and their interaction with inattention - on the stock market participation decisions of the households.

Panel A of Table 7 reports three moments from the three economies: the fraction of stockholders, the amount of passive rebalancing that households offset by actively changing their financial portfolio, and the level of the equity premium.

Panel A shows that when I consider an environment with very tight borrowing constraints, the fraction of stockholders falls down dramatically to 50.7%. Portfolio rebalancing becomes more active: now the households can offset 78.9% of the passive change in the risky share. This increase in the active management of financial portfolios is mainly due to a composition effect. The drop in the participation rate implies that the few stockholders of the economy are very wealthy and can afford to incur in the observation cost very often. Hence, stockholders manage on average

more actively their portfolios.

Table 7: The role of borrowing constraints

Variable	Benchmark	Tight Constraints	Loose Constraints
A. Inattentive economy - $\chi = 0.029$			
% Stockholders	59.4	50.7	77.1
$\hat{\psi}$ (Passive Change)	-0.730	-0.789	-0.186
Equity premium	2.64	6.31	0.36
B. Attentive economy - $\chi = 0$			
% Stockholders	89.8	88.2	91.5
$\hat{\psi}$ (Passive Change)	-0.998	-0.989	-0.996
Equity premium	0.81	5.01	0.04

Note: The variable χ defines the observation cost. In the “Benchmark” model, borrowing constraints on bonds and stocks equal around minus three times the monthly income of the median households, that is, $\underline{b} = \underline{s} = -5.96$. The “Tight Constraints” model does not allow short sales in neither stocks nor bonds, that is, $\underline{s} = \underline{b} = 0$. In the “Loose Constraints” model borrowing constraints on bonds and stocks equal around minus three times the monthly income of the median households, that is, $\underline{b} = \underline{s} = -11.92$. Passive Change $\hat{\psi}$ is the estimated amount of passive change which is actively offset by the households by adjusting their portfolios. The parameter is estimated in a panel regression of the active change in portfolio rebalancing on the passive change in portfolio rebalancing

$$ActiveChange_{i,t+1} = Constant + \psi \times PassiveChange_{i,t+1} + \mu \times RiskyShare_{i,t} + \epsilon_{i,t+1}.$$

When I consider loose borrowing constraints, both the share of stockholders and the amount of inertia in portfolio rebalancing rise substantially. In this economy, the households decide to participate in the stock market and they dilute the observation cost by trading very infrequently. Anyway, the loose borrowing constraints allow agents to borrow sufficiently to smooth away any eventual mistake made throughout inattention. As a result, inattention does not affect the price of risk and the equity

premium is zero, as in Chen (2006).

Importantly, these differences across economies are not driven *only* by the changes in the tightness of the borrowing constraints, because most of the action hinges on the interaction between borrowing constraints and the observation cost. To show this mechanism, I replicate the exercise using the three economies above *without* the observation cost. Panel B of Table 7 reports the results. In this case, the differences in the tightness of the borrowing constraint per se can alter the equity premium, but have no effect whatsoever on the dynamics of portfolio rebalancing.

6 Conclusion

This paper studies whether households' inattention to the stock market explains the inertia in households' portfolio rebalancing. To answer this question, I introduce an observation cost into an otherwise standard production economy with heterogeneous agents, idiosyncratic labor income risk, and borrowing constraints. In this model inattention changes endogenously over time and across agents. To discipline the quantitative analysis, I calibrate the observation cost to match the duration of inattention of the median household estimated by Alvarez *et al.* (2012).

The quantitative results show that inattention accounts for half of the inertia in portfolio rebalancing, and explains its heterogeneity across households. In the model, as it is in the data, wealthy households invest much more actively than poor households. Moreover, inattention can rationalize the limited stock market participation observed in the data, and improves the asset pricing performance of the model. Finally, I highlight a novel testable implication that links households' inattention to households' funding liquidity: inattention matters quantitatively on the dynamics of portfolio rebalancing and asset prices only if borrowing constraints are tight enough.

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