

Monetary Policy and Sovereign Debt Sustainability*

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Abstract

We analyze the consequences of monetary policy for sovereign debt sustainability and welfare, in a model of a small open economy where the government issues long-term nominal debt without commitment not to default on it or erode its real value through (costly) inflation. While giving rise to a costly 'inflationary bias', discretionary inflation also *improves* sovereign debt sustainability by shifting out the default region, precisely because the inflationary bias to be incurred upon reentry into capital markets after each default episode makes default most costly *ex ante*. Compared to a scenario in which the government effectively renounces the use of discretionary inflation, the latter actually achieve *higher* welfare when the economy is sufficiently close to default.

Keywords: monetary and fiscal policy, discretion, fundamental sovereign default, inflationary bias, continuous time.

JEL codes: E5, E62, F34

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1 Introduction

One of the main legacies of the 2007-9 financial crisis and the subsequent recession has been the emergence of large fiscal deficits across the industrialized world, resulting in government debt-to-GDP ratios near or above record levels in countries such as the United States, the United Kingdom, or the Euro area periphery. Before the summer of 2012, the peripheral Euro area economies experienced dramatic spikes in their sovereign default premia, whereas other highly indebted countries such as the US and the UK did not. Many observers emphasized that a key difference between both groups of countries was that, whereas the US and the UK controlled the supply of the currency in which they issued their debt and hence had the option to reduce its real value by creating inflation, such an option was not available to the peripheral Euro area economies. In recent years, sovereign debt troubles have lingered on (and at times intensified) in countries such as Greece, with many voices asking again whether those countries would have been better off with an independent monetary policy. These developments raise the question as to how the ability to inflate debt away affects sovereign debt sustainability and welfare outcomes.

In this paper, we address the above question by studying the implications of inflationary policy when the government cannot commit not to default explicitly on its debt, but also not to reduce its real value through inflation. We do so in the context of a standard quantitative model of optimal sovereign default *à la* Aguiar and Gopinath (2006) and Arellano (2008). As in Hatchondo and Martínez (2009) and Chatterjee and Eyigungor (2012), we consider a small open economy in which a benevolent government issues long-term noncontingent bonds to foreign investors. The government may default on its debt at any time, but at the expense of temporary exclusion from capital markets (thus losing the ability to smooth consumption) and a drop in output during the exclusion period. In order to introduce a role for monetary policy, we depart from the standard literature by assuming that debt is noncontingent in *nominal* terms, such that inflation erodes its real value; inflation however entails welfare costs, thus creating a meaningful trade-off for monetary policy. The government chooses optimal fiscal and monetary policy under discretion, i.e. without commitment on the future path of primary deficits or inflation.

Our analysis first highlights the properties of the optimal default and inflation policies. As in the standard literature, the default frontier is upward sloping in debt-income space: sovereign default is optimal when debt is high and income is low. As regards inflation, we show it depends on two factors. First, the real value of debt. Indeed, as long as there is debt outstanding, the government will have an incentive to inflate it away. This is a reflection of the 'inflationary bias' that arises in the presence of nominal government debt.¹ Second, inflation depends on the welfare gain from a marginal reduction in the real value of debt. We find that a marginal increase in

¹See e.g. Díaz-Giménez et al. (2008), Martín (2009), Niemann (2011), and Niemann, Pichler and Sorger (2013).

inflation is rather ineffective at improving welfare when the economy is close to sovereign default. This is because in that situation there is little difference between the social value of repaying and that of defaulting: since the latter value is independent of how indebted the government is, the former value is also fairly insensitive to the debt burden. We show that these two forces amount to an inflation policy that increases roughly linearly with debt until the latter approaches sufficiently its default threshold, at which point it starts declining towards zero.

The optimal inflation policy has a key impact on sovereign debt sustainability. We illustrate this by comparing our model with a counterfactual economy in which inflation is zero at all times ('no-inflation regime').² We show that inflation *improves* sovereign debt sustainability by shifting out the default frontier: at any income level, the debt level above which the government prefers default to repayment is higher. The reason is the following. Following a default and exclusion spell, the government reenters capital markets and starts issuing debt again. But by the 'inflationary bias' explained above, it also starts creating inflation and hence incurring the associated welfare costs. These costs make the option to default more costly *ex ante*, leading the government to postpone default at debt levels for which it would prefer to default in the no-inflation regime.

The strong non-linearity in the optimal inflation policy implies that the monetary policy response to aggregate shocks depends crucially on the state of the economy at the time of the shock. We show that, following a negative income shock, optimal inflation increases only when debt (and the new income level) are sufficiently far away from the default frontier. As we have seen, it is only when default is sufficiently distant that inflating away the debt is welfare-improving enough to justify the associated welfare cost. Conversely, negative income shocks may justify a reduction in inflation if the new state of the economy is sufficiently close to the default frontier, i.e. precisely when default is more of a concern.

Finally, we show that the welfare implications of discretionary inflation depend crucially on the state of the economy: for all income levels except very low ones, welfare is *higher* in the *inflationary* regime when debt is sufficiently close to the default threshold. The reason goes back to the two major effects of discretionary inflation: the inflationary bias, and the improvement of debt sustainability *vis-à-vis* the no-inflation regime. The welfare costs from the inflationary bias are relatively constant across debt and income levels. By contrast, the welfare gains (through higher consumption utility flows) from the improvement in debt sustainability are felt more strongly in the vicinity of default and for higher income levels. As a result, provided income is not too low, it is precisely when debt approaches the default threshold that the inflationary regime dominates in welfare terms.

Literature review. Our paper is related to the literature on quantitative models of optimal

²We may interpret the no-inflation regime as a scenario in which the government issues foreign currency (real) debt or joins an anti-inflationary monetary union, thus effectively renouncing the use of inflation.

sovereign default in small open economies initiated by Aguiar and Gopinath (2006) and Arellano (2008), who in turn built on the seminal qualitative framework of Eaton and Gersovitz (1981).³ Our framework is closest to the models with long-term bonds developed by Hatchondo and Martínez (2009) and Chatterjee and Eyigungor (2012).⁴ We contribute to this literature by proposing a model with nominal debt and costly inflation and analyzing the implications of optimal discretionary monetary policy for sovereign debt sustainability and welfare. In this regard, our analysis is related to Sunder-Plassmann (2017) and Röttger (2017), who introduce optimal default *à la* Arellano (2008) into closed-economy frameworks with monetary frictions *à la* Díaz-Giménez et al. (2008) and Martin (2009). Apart from differences in modelling and in the relevant channels,⁵ our papers differ largely in focus. Sunder-Plassmann (2017) studies how the denomination of sovereign debt (nominal vs. real) affects the government’s incentives to inflate or default on its debt over the long run. Röttger (2017) focuses on how the ability to default changes the conduct of monetary and fiscal policy in the short and long run. By contrast, we analyze how a government’s ability to inflate away its nominal, local-currency-denominated debt affects not only its sustainability but also social welfare.⁶

Our paper is more loosely related to a recent literature that analyzes, in the context of sovereign debt models with multiple equilibria in the tradition of Calvo (1988) and Cole and Kehoe (2000), to what extent monetary policy can eliminate the possibility of self-fulfilling debt crises. Examples of this research are Aguiar et al. (2013, 2015), Reis (2013), Da Rocha, Giménez and Lores (2013), Araujo, León and Santos (2013), Corsetti and Dedola (2016), Camous and Cooper (2015), and Bacchetta, Perazzi and van Wincoop (2018). Unlike these contributions, we do not consider self-fulfilling debt crises, focusing instead on sovereign default due only to bad fundamentals.⁷ Conversely, none of the above papers analyze the consequences of monetary policy for debt sustainability and welfare in a quantitative, fully dynamic economy with recurrent aggregate

³For an in-depth review of the literature on quantitative models of sovereign default and more generally of sovereign debt crises, see Aguiar et al. (2016).

⁴In fact, our model is essentially a continuous-time version of Chatterjee and Eyigungor (2012). We assume continuous time because it offers considerable computational advantages, as discussed later on.

⁵In Sunder-Plassmann’s (2017) and Röttger’s (2017) closed-economy setups with monetary frictions, inflating away the debt (or reducing it through outright default) allows the government to reduce future distortionary taxation, including the inflation tax on consumption goods purchased with cash. In our cashless, open-economy framework, by contrast, inflation produces a redistribution from foreign investors to the domestic economy, which is closer in spirit to the channel through which default favors welfare in the standard open-economy model of sovereign default (e.g. Arellano, 2008; Aguiar and Gopinath, 2006).

⁶Also related is the work of Du and Schreger (2017), who analyze how the denomination of *corporate* debt determines the sovereign’s incentive to inflate or default on its (local-currency-denominated) debt, in an Aguiar-Gopinath-Arellano economy extended to allow for firms that face borrowing constraints and a currency mismatch between revenues and liabilities.

⁷By contrast, the papers above consider only default due to self-fulfilling expectations. An exception is Corsetti and Dedola’s (2016) qualitative framework, where default can also be due to weak fundamentals.

shocks and optimal fundamental default, as we do.⁸ A key insight from our analysis is that discretionary inflation not only improves sovereign debt sustainability –by pushing out the debt-income frontier beyond which the country suffers (fundamental) default–, but also improves welfare when the economy is sufficiently close to default. While not directly comparable, this stands somewhat in contrast with one of the key lessons from the above literature, according to which the use of discretionary inflation policy generally backfires by not avoiding self-fulfilling debt crisis and yet causing welfare losses (see e.g. Calvo, 1988, or more recently Corsetti and Dedola, 2016).⁹

In modelling the choice of inflation without commitment as a trade-off between the reduction in the real debt burden *vis-à-vis* foreign investors and the utility costs of inflation, our model bears some resemblance with Aguiar et al. (2013). Apart from this aspect, both papers differ notably in modelling, focus and findings. Aguiar et al. (2013) study the effects of the utility costs of inflation –which the authors refer to as the government’s ‘inflation credibility’– on the potential for self-fulfilling debt crises, in a qualitative model without fundamental uncertainty where failure by investors to roll over the debt may lead the government to choose outright default over full principal repayment. Aguiar et al. (2013) find that, if inflation costs are below a certain threshold, inflationary policy (i) makes the economy *more vulnerable* by reducing the debt threshold above which the economy is exposed to self-fulfilling crises and (ii) achieves strictly *lower* welfare for any debt level, *vis-à-vis* a scenario with foreign currency debt (analogous to our ‘no-inflation’ regime). By contrast, we evaluate the consequences of discretionary inflation for fundamentally-driven default in a quantitative stochastic framework. We find that the inflationary regime (i) always *improves* debt sustainability by shifting out the default frontier, regardless of the size of inflation costs, and (ii) may indeed achieve *higher* welfare when the economy is sufficiently close to the default frontier.

Finally, we make a technical contribution by laying out a quantitative optimal sovereign default model in continuous time and introducing a new numerical method to compute the equilibrium. As discussed in Achdou et al. (2017), the computational burden is reduced in continuous-time dynamic programming compared to discrete-time methods. A number of recent works have built on the methodological innovations introduced in this paper. Toure (2017) considers an extended real version of the model that leads to semi-closed form solutions to disentangle which of the model fea-

⁸Many of the above contributions are qualitative, working in environments with two periods or two-period-lived agents (e.g. Corsetti and Dedola, 2016; Camous and Cooper, 2015) or without fundamental uncertainty (e.g. Aguiar et al., 2013, 2015). Bacchetta et al. (2018) propose a dynamic framework where fundamental uncertainty is restricted to the value of primary deficit at some future date. Da Rocha et al. (2013) and Araujo et al. (2013) study fully dynamic, stochastic environments with self-fulfilling debt crisis; however, they do not discuss the role of discretionary inflation for debt sustainability and welfare that is central to our analysis.

⁹Corsetti and Dedola (2016) qualify the Calvo (1988) result by showing that, if inflation costs are convex, then multiplicity disappears and the equilibrium is unique. Camous and Cooper (2015), and Bacchetta, Perazzi and van Wincoop (2018) show that optimal monetary policy under *commitment* can be successful at eliminating self-fulfilling debt crisis.

tures influences credit spreads, expected returns and cross-country correlations. Bornstein (2018) compares, in a model based on ours, the equilibrium dynamics and computing times to those of discrete-time models, finding that continuous-time techniques are faster.

The rest of the paper is structured as follows. Section 2 lays out the model and proves some analytical results. Section 3 compares the equilibrium, dynamic and welfare properties of the inflationary and no-inflation regimes. Section 4 concludes.

2 Model

We consider a continuous-time model of a small open economy.

2.1 Output, price level and sovereign debt

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ be a filtered probability space. There is a single, freely traded consumption good which has an international price normalized to one. The economy is endowed with y_t units of the good each period (real GDP). The evolution of $z_t = \log(y_t)$ is given by a bounded Ornstein–Uhlenbeck process¹⁰

$$dz_t = -\mu z_t dt + \sigma dW_t, \quad (1)$$

The local currency price relative to the World price at time t is denoted P_t . It evolves according to

$$dP_t = \pi_t P_t dt, \quad (2)$$

where π_t is the instantaneous inflation rate.

The government trades a nominal non-contingent bond with risk-neutral competitive foreign investors. Let B_t denote the outstanding stock of nominal government bonds; assuming that each bond has a nominal value of one unit of domestic currency, B_t also represents the total nominal value of outstanding debt. We assume that outstanding debt is amortized at rate $\lambda > 0$ per unit of time. The nominal value of outstanding debt thus evolves as follows,

$$dB_t = B_t^{new} dt - \lambda B_t dt,$$

where B_t^{new} is the flow of new debt issued at time t . Each bond pays a proportional coupon δ per unit of time.¹¹ The nominal market price of government bonds at time t is Q_t . Also, the

¹⁰This is the continuous-time counterpart of the AR(1).

¹¹Our modelling of long-term nominal debt, with bonds amortized at a constant rate and with fixed coupon rate, is similar to the nominal perpetual bonds with geometrically decaying coupons introduced by Woodford (2001) in a discrete-time macroeconomic framework. In fact, the latter bonds can be interpreted as a particular case of the bonds considered here, with the amortization and coupon rate adding up to one ($\lambda + \delta = 1$). See also Hatchondo

government incurs a nominal primary deficit $P_t(c_t - y_t)$, where c_t is aggregate consumption.¹² The government's flow of funds constraint is then

$$Q_t B_t^{new} = (\lambda + \delta) B_t + P_t(c_t - y_t).$$

That is, the proceeds from issuance of new bonds must cover amortization and coupon payments plus the primary deficit. Combining the last two equations, we obtain the following dynamics for nominal debt outstanding,

$$dB_t = \left[\frac{(\lambda + \delta) B_t + P_t(c_t - y_t)}{Q_t} - \lambda B_t \right] dt. \quad (3)$$

We define real debt in face value terms as $b_t \equiv B_t/P_t$. Its dynamics are given by

$$db_t = \left[\underbrace{\frac{(\lambda + \delta) b_t + c_t - y_t}{Q_t}}_{\equiv b_t^{new}} - (\lambda + \pi_t) b_t \right] dt, \quad (4)$$

where $b_t^{new} = B_t^{new}/P_t$ is the real face value of new bond issuances.

2.2 Preferences

The representative household has preferences over paths for consumption and domestic inflation given by

$$U_0 \equiv \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u(c_t) - x(\pi_t) dt \right]. \quad (5)$$

Instantaneous utility takes the form

$$u(c) = \begin{cases} \log(c), & \text{if } \gamma = 1 \\ \frac{c^{1-\gamma}-1}{1-\gamma}, & \text{if } \gamma \neq 1 \end{cases}, \quad x(\pi) = \frac{\psi}{2} \pi^2, \quad (6)$$

where $\psi > 0$. The functional form for the utility costs of inflation, $\psi\pi^2/2$, can be justified on the grounds of costly price adjustment by firms. In particular, in Appendix A we lay out an economy where firms are explicitly modelled, and where a subset of them are price-setters but incur a standard quadratic cost of price adjustment *à la* Rotemberg (1982). As we show there, social welfare in such an economy can be expressed as in equations (5) and (6) with $\gamma = 1$, and

and Martinez (2009) and Chatterjee and Eyigungor (2012) for recent uses of similar modelling devices for long term (real) bonds in discrete-time open economy setups.

¹²As in Arellano (2008), we assume that the government rebates back to households all the net proceedings from its international credit operations (i.e. its primary deficit) in a lump-sum fashion. Denoting by \tilde{T}_t the primary deficit, we thus have $P_t c_t = P_t y_t + \tilde{T}_t$. This implies $\tilde{T}_t = P_t(c_t - y_t)$.

the relevant equilibrium conditions are identical to those in the simple model described here.¹³

2.3 Fiscal and monetary policy

The government chooses fiscal policy at each point in time along two dimensions: it sets optimally consumption c_t , and it chooses whether to continue honoring debt repayments or else to default. In addition, the government implements monetary policy by choosing the inflation rate π_t at each point in time. Before analyzing the government's problem, we present first the sovereign default scenario.

2.3.1 The default scenario

Following most of the literature on quantitative sovereign default models (e.g. Aguiar and Gopinath, 2006; Arellano, 2008), we assume that a default entails two types of costs. First, the government is excluded from international capital markets temporarily. The duration of this exclusion period, τ , is random and follows an exponential distribution with average duration $1/\chi$. Second, during the exclusion period the country's output endowment declines. Suppose the government defaults at an arbitrary debt ratio b . Then during the exclusion period the country's output endowment is given by $y_t^{def} = y_t - \epsilon(y_t)$, with $\epsilon(\cdot)$ being the output loss. This specification of output loss is similar to the one in Arellano (2008) or Chatterjee and Eyigungor (2012). During the exclusion phase, households simply consume the output endowment, $c_t = y_t^{def}$. Upon reentry into capital markets, debt starts at zero. It follows that the government has no incentive to create inflation during the exclusion period, as that would generate direct welfare costs while not reducing the debt level upon reentry; we thus have $\pi_t = 0$ for $t \in (\tilde{t}, \tilde{t} + \tau)$, where \tilde{t} denotes the time of the most recent default.

The main benefit of defaulting is of course the possibility of eliminating the debt burden. We can express the value of defaulting at a time $\tilde{t} = 0$ as $U_0^{def} = V_0^{def}$, where $V_0^{def} \equiv V_{def}(y_0)$ is the value of defaulting, given by

$$V_{def}(z) = \mathbb{E} \left\{ \int_0^\tau e^{-\rho t} u(e^{z_t} - \epsilon(e^{z_t}), 0) dt + e^{-\rho\tau} V(0, z) \mid z_0 = z \right\}, \quad (7)$$

where $y_t = e^{z_t}$. Applying the Feynman-Kac formula, we obtain the following representation as a partial differential equation (PDE)

$$\rho V_{def}(z) = u_{def}(z) - \mu z \frac{\partial V_{def}}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_{def}}{\partial z^2} + \chi [V(0, z) - V_{def}(z)], \quad (8)$$

¹³To be precise, the quadratic utility cost $\psi\pi_t^2/2$ is an approximation to the exact utility cost of inflation in the model of Appendix A. As a robustness test, we have simulated the model using the exact inflation utility cost and verified that the results are virtually identical to those in the paper.

where $u_{def}(z) = \frac{[e^z - \epsilon(e^z)]^{1-\gamma} - 1}{1-\gamma}$.

2.3.2 The general problem

At every point in time the government decides optimally whether to default or not, in addition to choosing consumption and inflation. Following a default, and once the government regains access to capital markets, it starts accumulating debt and is confronted again with the choice of defaulting. This is a sequence of *optimal stopping* problems, as one of the policy instruments is a sequence of stopping times. We denote by T the *time to default*. The latter is a stopping time with respect to the filtration $\{\mathcal{F}_t\}$, defined as the smallest time t' such that the government decides to default.¹⁴ The government maximizes social welfare under discretion. When doing so, it takes as given the bond price schedule $Q(b, z)$, which determines how investors price government bonds in each state and which is characterized below. The government thus maximizes households' utility (5) subject to the laws of motion of income (1) and debt (4). The value function of the government is defined as

$$V(b, z) = \max_{T, \{c_t, \pi_t\}_{t \in [0, T]}} \mathbb{E} \left\{ \int_0^T e^{-\rho t} (u(c_t) - x(\pi_t)) dt + e^{-\rho T} V_{def}(z_T) \mid b_0 = b, z_0 = z \right\}. \quad (9)$$

The value function must satisfy a so-called ‘‘HJB Variational Inequality’’ (Øksendal, 1995; Pham, 2009):

$$0 = \max \left\{ V_{def}(z) - V(b, z), \max_{c, \pi} u(c) - x(\pi) + s(b, z, c, \pi) \frac{\partial V}{\partial b} - \mu z \frac{\partial V}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial z^2} - \rho V(b, z) \right\}, \quad (10)$$

where

$$s(b, z, c, \pi) = \frac{c - e^z + (\lambda + \delta)b}{Q(b, z)} - (\lambda + \pi)b \quad (11)$$

is the *drift* of the state variable b_t (see equation 4) The first order conditions of this problem imply the following policy functions for consumption and inflation,

$$u'(c(b, y)) = -\frac{\partial V}{\partial b} \frac{1}{Q(b, z)}, \quad (12)$$

$$\pi(b, z) = -\frac{1}{\psi} b \frac{\partial V}{\partial b}. \quad (13)$$

Therefore, the optimal consumption increases with bond prices and decreases with the slope of the value function (in absolute value). The intuition is straightforward. Higher bond prices (equivalently, lower bond yields) make it cheaper for the government to finance primary deficits.

¹⁴Therefore, the time of default in absolute time is $\tilde{t} = t + T$.

Likewise, a steeper value function makes it more costly to increase the debt burden by incurring in primary deficits. As regards optimal inflation, the latter increases both with debt and the slope (in absolute value) of the value function. Intuitively, the higher the debt level the larger the reduction in the debt burden that can be achieved through a marginal increase in inflation. Similarly, a steeper value function increases the incentive to use inflation so as to reduce the debt burden.

The optimal default policy $d(b, z)$ may take only two values: 1 (default) or 0 (no default),

$$d(b, z) = \begin{cases} 1, & \text{if } V_{def}(b, z) > V(b, z), \\ 0, & \text{if } V_{def}(b, z) \leq V(b, z), \end{cases} \quad (14)$$

that is, default only happens when the value of defaulting is higher than that of repayment.

2.4 Foreign investors and bond pricing

The government sells bonds to competitive risk-neutral foreign investors that can invest elsewhere at the risk-free real rate \bar{r} . As explained before, during repayment spells bonds pay a coupon rate δ and are amortized at rate λ . But following a default (at some time $\tilde{t} = t + T$) the price of a bond goes to zero, as the government renounces to make any further repayment. The nominal price of the bond at an arbitrary time $t = 0$ is given by

$$Q(b, z) = \mathbb{E} \left[\int_0^T e^{-(\bar{r}+\lambda)t - \int_0^t \pi_s ds} (\lambda + \delta) dt \mid b_0 = b, z_0 = z \right], \quad (15)$$

where b and z follow the laws of motion (4) and (1), respectively, given the optimal policies $c(b, z)$, $\pi(b, z)$ and $d(b, z)$.

Applying the Feynman-Kac formula, we obtain the following PDE

$$(\bar{r} + \pi(b, z) + \lambda) Q(b, z) = (\lambda + \delta) + s(b, z) \frac{\partial Q}{\partial b} - \mu z \frac{\partial Q}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 Q}{\partial z^2}, \quad \text{if } d(b, z) = 0, \quad (16)$$

$$Q(b, z) = 0, \quad \text{if } d(b, z) = 1, \quad (17)$$

for all (b, z) , where the drift function $s(b, z) \equiv s(b, z, c(b, z), \pi(b, z))$ is given by (11).

Given a current nominal bond price $Q(b, z)$, the implicit nominal *bond yield* $r(b, z)$ is the discount rate for which the discounted future promised cash flows from the bond equal its price. The discounted future promised payments are $\int_0^\infty e^{-(r(b, z) + \lambda)t} (\lambda + \delta) dt = \frac{\lambda + \delta}{r(b, z) + \lambda}$. Therefore, the bond yield function is

$$r(b, z) = \frac{\lambda + \delta}{Q(b, z)} - \lambda. \quad (18)$$

The gap between the nominal yield $r(b, z)$ and the riskless real rate \bar{r} is called the interest-rate

spread.

2.5 Equilibrium

We define our equilibrium concept:

Definition 1 (MPE) *A Markov Perfect Equilibrium is a value function $V(b, z)$, a consumption policy $c(b, z)$, and inflation policy $\pi(b, z)$ and default policy $d(b, z)$ and a bond price function $Q(b, z)$ such that:*

1. *Given prices Q , the value function V solves the government problem (10); the optimal inflation is π , the optimal consumption is c , and the optimal default policy is d .*
2. *Given the optimal inflation π , consumption c and default policy d , bond prices solve the pricing equation (16).*

The government takes the bond price function as given and chooses inflation and consumption (continuous policies) and whether to default or not (stopping policy) to maximize its value function. The investors take these policies as given and price government bonds accordingly. Equilibrium in the no inflation regime is defined analogously, with $\pi = 0$ replacing the inflation policy function.

The definition of Markov Perfect Equilibrium (MPE) is a particular case of a Markov equilibrium in continuous-time games. It is composed by a coupled system of two partial differential equations (PDEs): the HJB equation and the bond pricing equation.¹⁵

2.6 Discussion: the effects of inflation

It is instructive at this point to discuss the effects of inflation on debt accumulation and hence on equilibrium dynamics. Inflation has two key effects in this environment, which can be readily seen from the drift function for debt accumulation and the bond pricing condition, equations (11) and (15) respectively. On the one hand, *instantaneous* inflation π erodes the real value of debt through the classic Fisherian channel. On the other hand, *expected future* inflation during the life of the long-term bond reduces its price Q through a higher inflation premium; this by equation (11) forces the government to issue more new bonds in order to cover its financing needs, thus accelerating debt accumulation (in face value terms).

A no-inflation regime. In what follows, in order to better understand the consequences of discretionary inflation for macroeconomic and welfare outcomes, we will be comparing our baseline model –which we refer to as the ‘inflationary regime’– with a counterfactual scenario in

¹⁵See Başar and Oldser (1999) or Dockner et al. (2000) for references on continuous-time (also known as differential) game theory.

which inflation is zero in all states: $\pi(b, z) = 0$. This ‘no inflation regime’ represents a scenario in which the government has effectively renounced the use of discretionary inflation, for instance by issuing foreign currency debt –in which case creating inflation is pointless in this framework– or by joining a monetary union with a strong anti-inflation mandate.

2.7 The inflation bias

Even if a complete analytical characterization of equilibrium is out of our reach, it is worthwhile to provide an analytical insight before moving to the numerical analysis in the following sections.

Proposition 1 (Inflation bias) *Inflation is always positive at positive debt levels:*

$$\pi(b, z) > 0, \text{ for all } b > 0.$$

Proof. The policy function for consumption (equation 12) is $[c(b, y)]^{-\gamma} = \frac{-1}{Q(b, z)} \frac{\partial V}{\partial b}$. Given that $Q(b, z) > 0$, consumption utility is well-defined and finite only if $\frac{\partial V}{\partial b} > 0$. Using this in the inflation policy function (equation 13), we have $\pi(b, z) = \frac{\partial V}{\partial b} \frac{b}{\psi} > 0$ for all $b > 0$. ■

The result in Proposition 1 is reminiscent of the classical ‘inflationary bias’ of discretionary monetary policy originally emphasized by Kydland and Prescott (1977) and Barro and Gordon (1983). In those papers, the source of the inflation bias is a persistent attempt by the monetary authority to raise output above its natural level. Here, by contrast, it arises from the existence of a positive stock of non-contingent nominal sovereign debt (such that $b > 0$) and from the welfare gains that can be achieved by reducing the real value of such nominal debt ($-\frac{\partial V}{\partial b} > 0$) at the expense of foreign investors. In this regard, it is more closely related to the one arising in analyses of discretionary monetary policy in models with nominal non-contingent debt, such as Díaz-Giménez et al. (2008), Martin (2009), and Niemann et al. (2013).

3 Numerical analysis

3.1 Computation

Having laid out our theoretical model, we now use it in order to analyze its equilibrium properties. Unfortunately, we are not able to solve the model analytically.¹⁶ We thus resort to numerical

¹⁶Analytical solutions are seldom found in Markov Perfect equilibrium models, not even in the deterministic case. In fact, in the deterministic case of Markov Perfect Equilibrium, even existence of a solution is not guaranteed in most cases, as discussed in Bressan (2010, Section 5). The stochastic case typically has a solution, but very restrictive assumptions (e.g. linear-quadratic structures) need to be imposed in order to be able to find it analytically; see e.g. the examples in Dockner et al. (2000). In a more stylized model of optimal default, for instance, Bressan

solutions. To this end we introduce a new numerical algorithm to analyze continuous-time default models, described in Appendix B.

Our numerical algorithm is based on an augmented model which assumes that the government may only default when it receives an exogenous option to default. We assume that the option to default follows a Poisson process with arrival rate ϕ . This model nests the case of continuous default choice by taking the limit as the arrival rate tends to infinity, $\phi \rightarrow \infty$. The advantage of this formulation is that both the HJB equations in the “normal” and autarky regions as well as the bond pricing equation can be efficiently solved using an upwind finite difference scheme similar to the one introduced in Achdou et al. (2017). The complete algorithm has a “two-loop” structure: the inner loop computes value functions and bond prices given a default policy and the outer loop updates the default policy.

Compared to discrete-time methods, working in continuous time has several advantages. First, the computational burden is reduced: while solving the discrete-time Bellman equation requires computation of expectations over all possible future states, in the continuous-time Hamilton-Jacobi-Bellman equation expectations are replaced by the first- and second-order derivatives of the value function. Second, the ergodic distributions can be efficiently computed using the Kolmogorov Forward (KF) equation (also known as Fokker-Planck equation), thus making it unnecessary to use more time-consuming and less precise methods such as Monte Carlo simulation, as typically done in discrete-time models.

3.2 Calibration

The calibration is intended to be mainly illustrative. As mentioned before, our model is essentially a continuous-time version of Chatterjee and Eyigungor (2012; henceforth CE) extended with nominal debt and inflation costs. In order to facilitate the comparability of our analysis with the latter paper and other closely related studies, we take most of our parameter values from CE.¹⁷ Let the unit of time be 1 year, such that all rates are in annual terms.

The parameters of the endowment process are set to $\mu = 0.28$ and $\sigma = 0.054$, such that the quarterly discrete-time approximation of the output process is ($\Delta t = 1/4$)

$$z_t = (1 - \mu\Delta t) z_{t-1} + \sigma\sqrt{\Delta t}\varepsilon_t = 0.93z_{t-1} + 0.027\varepsilon_t,$$

which are the values reported in footnote 23 of CE for linearly detrended quarterly real GDP in

and Nguyen (2016) are able to prove the existence of a solution in the open-loop Nash equilibrium but not in the Markov Perfect one. In the latter they can only show that, if a smooth solution exists, it should satisfy a nonlinear partial differential equation, a result analogous to the definition of equilibrium in our model.

¹⁷The only two exceptions are the inflation cost parameter ψ , which is absent from CE’s real model, and the risk aversion coefficient γ .

Argentina for the period 1980-2001.

The bond duration parameter λ is set to 0.20 so that the mean duration is 5 years (20 quarters). We set coupon payments δ to 0.12. We set χ to 0.1538, so that the average period of exclusion results in 6.5 years (26 quarters). The risk-free rate \bar{r} is set to 0.04. These parameters are all annual counterparts of the quarterly parameters in CE.

The discount rate is set to $\rho = \log\left(\frac{1}{0.954}\right)^4 = 0.1884$. As in CE, Output costs take the form

$$\epsilon(y) = \max\{0, d_0 + d_1 y^2\},$$

with $d_0 = -0.18$ and $d_1 = 0.2456$. These three parameters (ρ, d_0, d_1) are selected by CE to match (i) an average external debt-to-output ratio of 0.7; (ii) an average default spread of 0.08; and (iii) an standard deviation of spread of 0.04. While our model is not exactly the same as CE, using their parameter values produces roughly similar moments to theirs.¹⁸ Moreover, we have done robustness analysis with respect to these three parameters and found that our qualitative results are broadly preserved.¹⁹

In order to calibrate the scale of inflation utility costs, ψ , we turn to our microfoundations for inflation costs based on price adjustment costs *à la* Rotemberg (1982). As it is well known, the Rotemberg (1982) and Calvo (1983) pricing models deliver isomorphic (linearized) inflation equations -the so-called 'New Keynesian Phillips curve'- that differ only in how their respective slope depend on the structural parameters. As shown in Appendix A, when risk aversion $\gamma = 1$ (log utility) the slope of the inflation equation under Rotemberg pricing is $\frac{\epsilon-1}{\psi}$, where ϵ is the elasticity of individual firm demand curves. It can also be shown that, in a continuous-time setup, the equivalent slope under Calvo pricing is $\xi(\xi + \rho)$, where ξ is the price adjustment rate in that model.²⁰ It follows that, for the slope to be the same in both models, we need $\psi = \frac{\epsilon-1}{\xi(\xi+\rho)}$. Setting ϵ to 11 (such that the gross markup $\epsilon/(\epsilon - 1)$ equals 1.10) and ξ to 1 (such that prices last on average for 1 year, which is broadly consistent with the micro evidence) and given our calibration for the discount factor ρ , we obtain $\psi = 8.4$.

Table 1 summarizes the calibration.²¹

¹⁸Beyond the fact that ours is a continuous-time model in which default may happen on a continuous-time basis, whereas CE's is a discrete-time model at quarterly frequency, our (no-inflation) model differs from CE in that we do not consider any transitory income shock and we calibrate $\gamma = 1$ (for the reason explained below). In Section 3.6 we compute moments and compare them to those in CE.

¹⁹These results are available upon request.

²⁰The proof is available upon request.

²¹The computational parameters are as follow. We consider 400 points in the debt space, ranging from 0 to -1 and 100 points in the income space, from -0.3 to 0.3 . We have conducted several robustness analyses by modifying these parameters and results remain unchanged.

Table 1. Baseline calibration

Parameter	Value	Description
\bar{r}	0.04	world real interest rate
ρ	0.1884	subjective discount rate
μ	0.28	drift parameter output
σ	0.054	diffusion parameter output
λ	0.20	bond amortization rate
δ	0.12	bond coupon rate
χ	0.1538	reentry rate
d_0	-0.18	default cost parameter
d_1	0.2456	default cost parameter
γ	1	risk aversion
ψ	8.4	inflation disutility parameter

3.3 Equilibrium

We begin by analyzing the equilibrium policy functions in each policy regime.

Inflationary regime. The solid blue lines in Figure 1 show, for three different income levels, the inflationary regime’s value and policy functions in the repayment segment of debt.²² As shown by the first row, the value function declines gently and almost linearly with the country’s debt burden, except for debt levels very close to default when the slope declines in absolute value. As regards the bond price (fourth row), its gentle decline reflects mostly expectations of higher inflation during the life of the bond, except for debt levels close to the default threshold when the sharper price decline reflects the increase in default risk.²³

Armed with the value and bond price functions, one can then use the optimality conditions (13) and (12) to analyze the optimal inflation and consumption policies (second and third rows in Figure 1). According to equation (13), optimal inflation is proportional to the product of the slope of the value function (in absolute value) and the debt level: $\pi(b, \cdot) = \psi^{-1}b[-V_b(b, \cdot)]$. Since the value function is approximately linear for low and medium debt levels, over that range the welfare gain per unit of debt reduction is roughly constant, and thus inflation increases approximately linearly with debt. This reflects the *inflationary bias* of monetary policy under discretion: as long as there is debt outstanding, the central bank will try to erode its real value by inflating it away. In the vicinity of the default threshold, by contrast, the value function starts *flattening*, such that a marginal reduction in debt yields a lower and lower marginal welfare gain, and optimal inflation

²²The optimal default threshold is marked by the upper limit of the x-axis in each panel.

²³In the figure we display the values in the debt grid corresponding to repayment states. Notice that bond prices and inflation are both *zero* at the exact point of default (not included in the repayment grid).

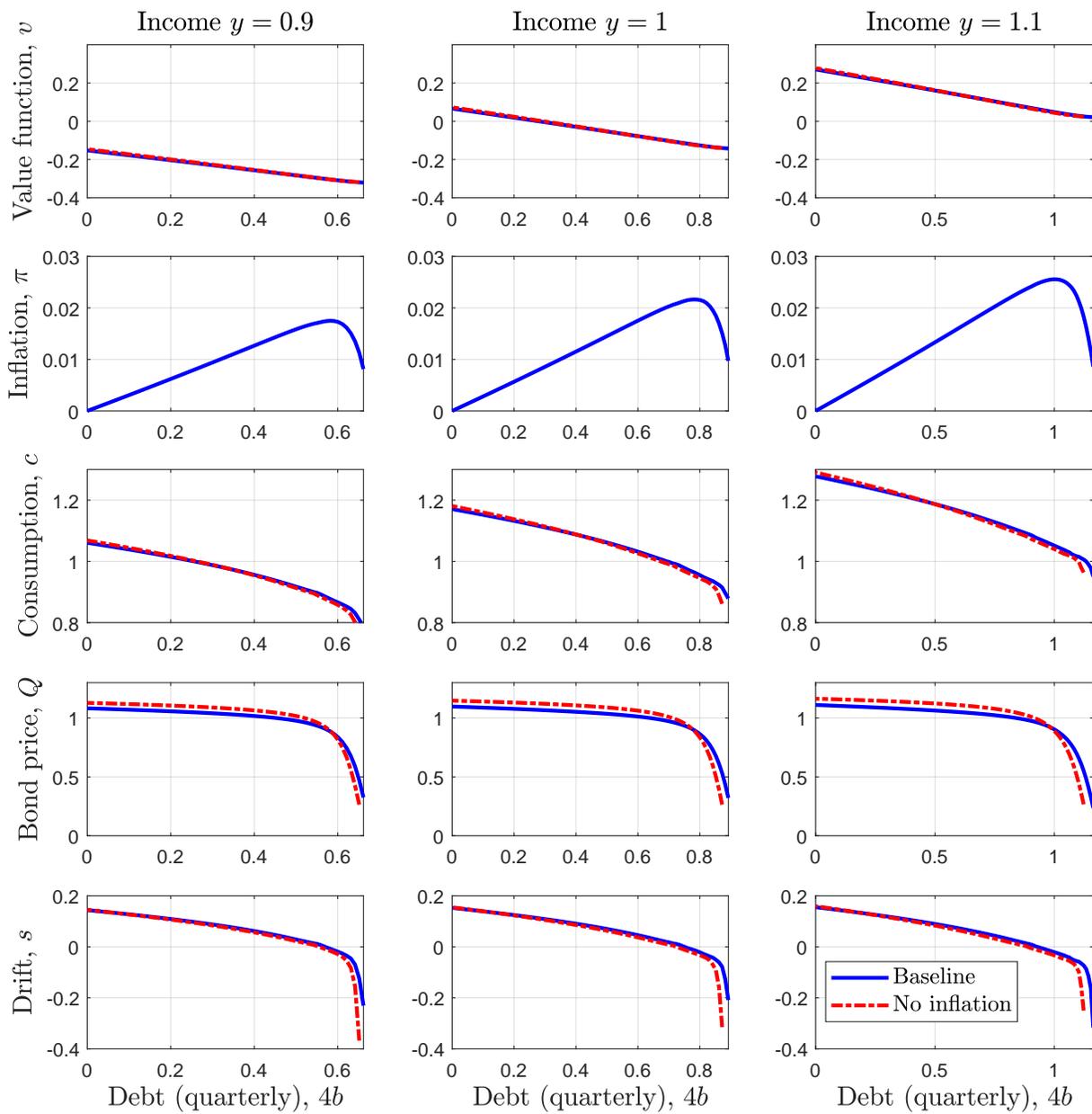


Figure 1: Equilibrium objects.

decreases as a result.²⁴ Intuitively, when default is imminent, the government understands there is little to be gained from inflating away the debt, because welfare after default is unaffected by how much (real) debt has been defaulted upon.

Consumption declines too in a roughly linear fashion, reflecting –as per equation (12)– the gentle decline in bond prices and the nearly constant slope of the value function in most of the debt space. As debt approaches the default threshold, however, the sharp decline in bond prices raises the cost of financing external deficits, which leads the government to reduce consumption more aggressively.

Finally, the last row in Figure 1 displays the drift function for debt accumulation, $s(b, z) = \frac{c(b, z) - e^z + (\lambda + \delta)b}{Q(b, z)} - (\lambda + \pi)b$. For moderate debt levels, the drift is mostly driven by the behavior of consumption, declining alongside the latter as debt increases. In the vicinity of default, however, the fall in the drift is reinforced by the steep decline in bond prices, which forces the government to drastically reduce –in fact, to turn negative– its pace of debt accumulation.

No-inflation regime. Consider now the equilibrium in the ‘no-inflation regime’, depicted by the red dashed lines in Figure 1.²⁵ Notice how in this case bond prices are higher for low and medium debt levels, reflecting the lack of any inflation premium associated with positive expected inflation during the life of the bond. However, as debt approaches the default threshold the situation reverses and bond prices are lower than in the inflationary regime, due to the increase in the default premium.

The consumption policy is similarly to the one in the inflationary case. Whether consumption is higher or lower than in the latter case is largely determined by bond prices. For low and medium debt levels, consumption in the no-inflation regime is higher due to higher bond prices. But as explained above, as debt approaches the default region bond prices in the no-inflation regime fall below their inflationary-regime counterparts, and hence so does consumption. This reflects the fact that the default threshold is also lower in the no-inflation case, for the reasons we explain next.

Default regions and debt sustainability. We have just seen that default thresholds are higher in the inflationary regime for different income levels. In fact, this is true for any income level. Figure 2 displays the default policy $d(b, y)$ in debt-income space. As is customary in real sovereign default models of this kind, in the no-inflation regime the default frontier is upward-sloping in debt-output space: the government defaults when debt is sufficiently high (for given output) or when output

²⁴The reason why the value function flattens near default is the following. In equilibrium, the slope of the repayment value function must equal that of the default value function at the default threshold, a property known as ‘smooth pasting’ (see e.g. Dixit and Pindyck (1994), Øksendal (1995) or Achdou et al. (2017) for a discussion of this property in continuous-time models). Since the default value function $V_{def}(z)$ is independent of debt, it must be the case that $\partial V(b, z)/\partial b = 0$ at the default threshold (for each z). This forces the (downward-sloping) value function to become flatter as debt approaches the default threshold.

²⁵The default threshold in this case is marked by the value of debt at the end of each red dashed line.

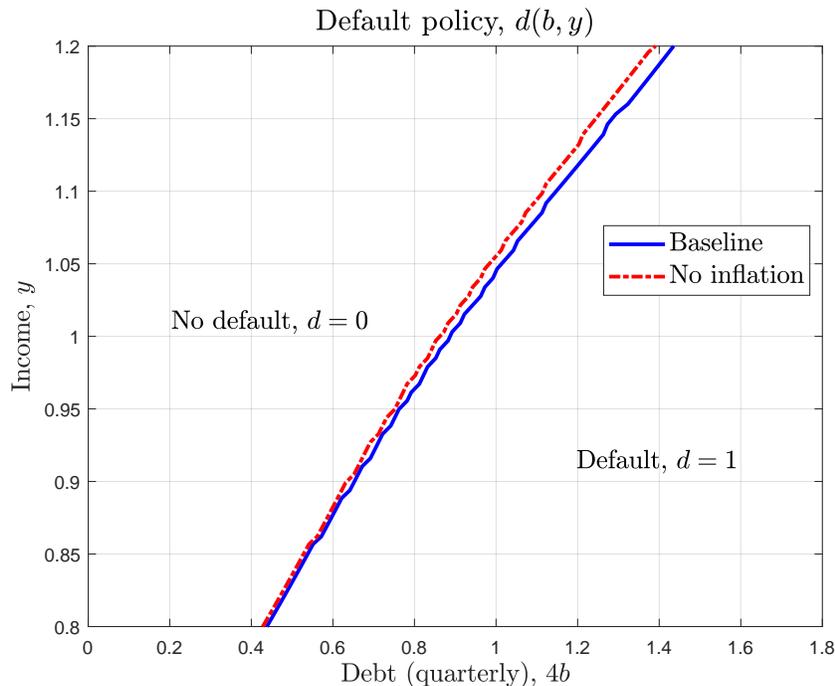


Figure 2: Default frontier.

is sufficiently low (for given debt). The same is true in the inflationary case. Importantly, the availability of the inflationary policy tool allows the government to *shift out* the default frontier, i.e. inflation improves the sustainability of sovereign debt.

The intuition is as follows. Following a default and exclusion spell, the government reenters financial markets with zero debt. At such reentry point welfare is slightly lower in the inflationary regime, reflecting the welfare costs of the inflationary bias to be incurred during the ensuing repayment spell. The anticipation of these costs makes the option to default more costly *ex ante*.²⁶ *Ceteris paribus*, this implies that the debt level at which the value of repaying equals that of defaulting is higher in the inflationary case. To summarize, the anticipation of the cost from the inflationary bias in future repayment spells makes default more costly, and debt more sustainable, when discretionary inflation is available for the government.

Our analysis should not be interpreted as suggesting that governments should inflate away their debt instead of defaulting on it, or that inflation is effective at deterring default. Default is an optimal choice that the government may take (and indeed takes in equilibrium) if and when fundamentals deteriorate sufficiently.

²⁶The value of defaulting also depends on the output loss and the exclusion from financial markets. But both the output loss and the (expected) duration of the exclusion spell are the same in both policy regimes. Therefore, any difference in $V_{def}(\cdot)$ across both regimes must come from the continuation value upon reentry into capital markets, $V(0, \cdot)$.

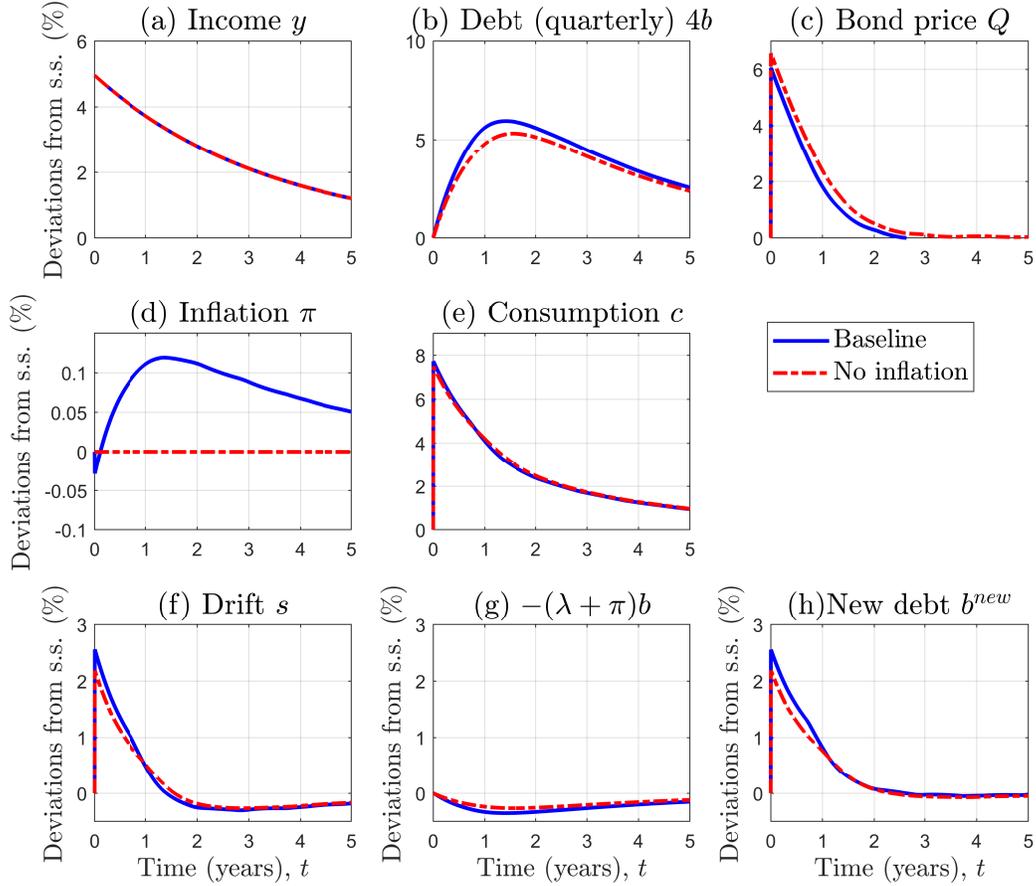


Figure 3: Generalized impulse response functions (positive shock).

3.4 Comparative dynamics: impulse-responses

In order to further illustrate the comparative properties of both regimes, we analyze the economy's dynamic response to income shocks. Figure 3 displays the generalized impulse-response functions following a positive income shock that increases income by 5 percent, both for the baseline (blue solid lines) and the no-inflation regime (red dotted lines).²⁷ In each case, the initial condition is the stochastic steady state, $(b_0, y_0) = (b_{ss}, 1)$, where b_{ss} is defined as the debt level for which the drift is zero: $s(b_{ss}, 1) = 0$.²⁸

Panel (b) displays the response of the debt stock. Its dynamics are governed by the drift

²⁷We use the standard discrete-time approximation of the law of motion of the state, given by $b_{t+\Delta t} - b_t = s(b_t, z_t) \Delta t$ and $z_{t+\Delta t} - z_t = -\mu z_t \Delta t + \sigma \sqrt{\Delta t} \varepsilon_{t+\Delta t}$, with $\varepsilon_{t+\Delta t} \sim N(0, 1)$. We choose a daily frequency, $\Delta t = 1/360$.

²⁸The stochastic steady-state debt is roughly similar in both regimes: $4b = 74.5\%$ for the baseline and 71.9% for the no-inflation one. Figure 3 shows deviations from the respective steady state, to which all variables return asymptotically in the absence of further shocks.

function, equation (11), reproduced here for convenience,

$$\dot{b}_t = s(b_t, z_t) = \frac{(\lambda + \delta) b_t + c(b_t, z_t) - e^{z_t}}{Q(b_t, z_t)} - [\lambda + \pi(b_t, z_t)] b_t.$$

Let us start first with the *inflationary* regime. On impact, the rise in income $y = e^z$ raises bond prices (reflecting a lower default risk) and this in turn stimulates consumption. *A priori*, the effect on new bond issuance, $\frac{(\lambda+\delta)b+c(\cdot)-e^z}{Q(\cdot)} = b^{new}(\cdot)$, is ambiguous. According to the figure, the consumption increase dominates the fall in the bond price, and as a result bond issuance jumps up and debt initially increases. Mirroring the latter, inflation increases gradually, following a slight decline on impact. As time goes by, the increase in debt and the gradual fall in income back to its steady state level bring the economy closer to the default region. This can be seen in the phase diagram in Figure 4, which portrays the adjustment in both state variables. The resulting increase in the default premium, together with the gradual fall in income, start pushing bond prices back to steady state right after the initial increase. This in turn is mirrored by the gradual fall in consumption, which again dominates the adjustment in bond prices and initiates the gradual reduction in new bond issuance. In parallel to these developments, the increase in inflation accelerates the erosion of real debt through the classic Fisherian effect (panel g). After about one and a half years, the latter effect dominates the new bond issuance (which by then is almost back to baseline), and debt starts falling slowly back to its steady state level.

It is worthwhile to analyze the optimal inflation response more closely. As mentioned above, inflation falls slightly on impact. To understand this, combine equations (13) and (12) to obtain

$$\frac{\psi \pi(b, z)}{u'(c(b, z))} = \frac{b}{1/Q(b, z)}.$$

This equation simply says that the marginal rate of substitution between consumption and inflation in household preferences must equal the marginal effectiveness of consumption and inflation at reducing the pace of debt accumulation. In other words, at the margin the government must be indifferent between reducing debt accumulation (and hence raising welfare) by inflating it away and by reducing real spending. Indeed, inflation reduces debt accumulation by the factor b , as it deflates the entire stock of debt, whereas consumption cuts reduce bond issuance needs in the amount $1/Q(b, z)$ –see equation (4). On impact, the rise in bond prices makes debt reduction through consumption cuts less effective vis-a-vis inflation, but the fall in the marginal utility of consumption makes such strategy less costly in terms of welfare. Quantitatively, the latter effect dominates slightly, hence the small fall in inflation. After the impact, however, with both u' and $1/Q$ moving in tandem, the increase in debt becomes the dominant force and produces the gradual increase in inflation shown before.

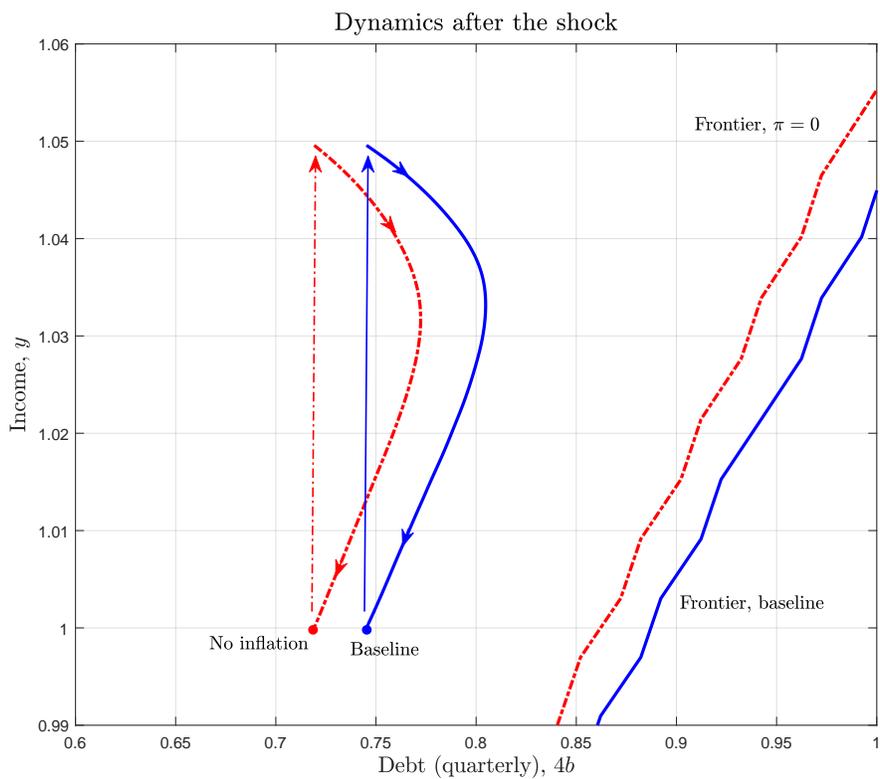


Figure 4: Phase diagram (positive shock). Notes: The circles mark the stochastic steady states (initial points).

Consider now the adjustment to the same income shock in the *no-inflation* regime (red dashed lines Figure 3). On impact bond prices increase by more than in the inflationary regime due precisely to the absence of an inflation premium. At the same time, the consumption increase is very similar. As a result, new bond issuance increases by less and so does the stock of debt.

Figure 5 shows the economy's response to a *negative* income shock of the same size as the one analyzed in Figure 3, again for both regimes and starting from the respective stochastic steady state. Symmetrically to the case of the positive shock, the negative shock reduces debt. As in the case of the positive shock, the (negative) change in debt is slightly more pronounced in the inflationary regime than in the no-inflation regime, but for a different reason. As shown in the phase diagram in Figure 11 in the appendix, after the shock the inflationary regime is closer to its own default frontier than the no-inflation regime is. As a result, the default premium increases slightly more in the inflationary case, thus giving the government the incentive to reduce its debt somewhat more.

Perhaps more interestingly, *on impact* the inflation response is strongly *negative*. To better understand this result, Figure 6 shows the consumption policy function $\pi(b, y) \equiv \pi(b, z = \log(y))$ for the steady-state level of income ($y = 1$), as well as for a lower and a higher income level ($y = 0.95, 1.05$). Starting again from the steady-state, b_{ss} and $y = 1$, and in response to a negative shock, on impact inflation falls vertically to the point given by the lower policy function evaluated at $b = b_{ss}$. Such drastic fall in inflation is due to the fact that the negative income shock brings the economy closer to the default frontier (see again Figure 11). This makes default more imminent which, as argued above, reduces the government's incentive to inflate.

Figure 6 also illustrates how the initial debt level shapes the inflation impact response to income shocks. For instance, starting from steady-state income, a negative income shock may justify an *increase* in inflation provided debt is sufficiently *low*, for it is when default is sufficiently distant that debt reductions are more effective at raising welfare and hence justify the utility losses of creating inflation. Likewise, a positive income shock may justify an increase in inflation on impact provided debt is sufficiently high, for basically the same reason: the positive shock pushes back the default debt threshold, making default more distant and inflation more attractive.

In summary, the impact inflation response to shocks depends crucially on the prevailing debt level when the shock hits, responding positively to negative shocks (and vice versa) only for relatively low debt, i.e. precisely when default is less of a concern. After the impact, by contrast, inflation is mostly determined by, and basically commoves with, the evolution of debt.

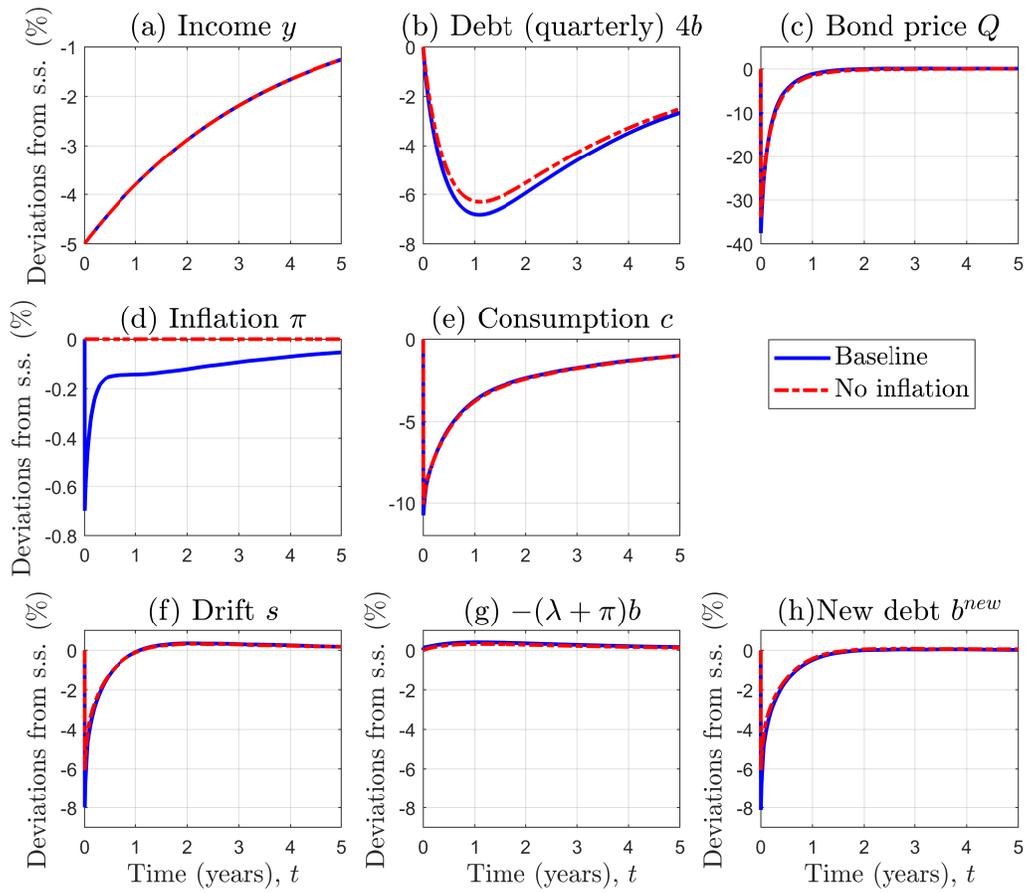


Figure 5: Generalized impulse response functions (negative shock).

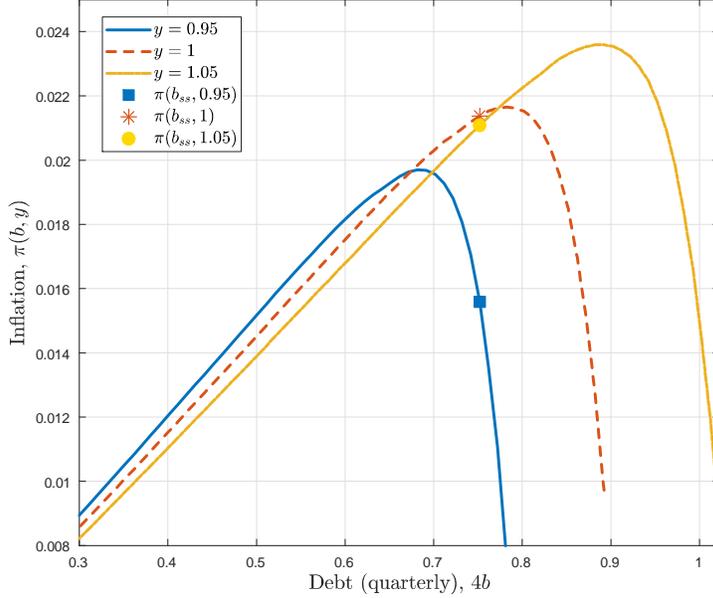


Figure 6: Inflation policy.

3.5 Welfare analysis

We now investigate the welfare consequences of discretionary inflation. To this aim, we decompose the value function (eq. 9) as

$$V(b, z) = V_c(b, z) + V_\pi(b, z), \quad (19)$$

where

$$V_c(b, z) = \max_{T, \{c_t, \pi_t\}_{t \in [0, T]}} \mathbb{E} \left\{ \int_0^T e^{-\rho t} u(c_t) dt + e^{-\rho T} V_{def, c}(z) \mid b_0 = b, z_0 = z \right\}, \quad (20)$$

$$V_\pi(b, z) = \max_{T, \{c_t, \pi_t\}_{t \in [0, T]}} \mathbb{E} \left\{ - \int_0^T e^{-\rho t} x(\pi_t) dt + e^{-\rho T} V_{def, \pi}(z) \mid b_0 = b, z_0 = z \right\}, \quad (21)$$

and we have analogously decomposed the default value function as $V_{def}(z_T) = V_{def, c}(b, z) + V_{def, \pi}(b, z)$, where

$$V_{def, c}(z) = \mathbb{E} \left\{ \int_0^\tau e^{-\rho t} u(e^{z_t} - \epsilon(e^{z_t}), 0) dt + e^{-\rho \tau} V_c(0, z) \mid z_0 = z \right\},$$

$$V_{def, \pi}(z) = \mathbb{E} \left\{ 0 + e^{-\rho \tau} V_\pi(0, z) \mid z_0 = z \right\}.$$

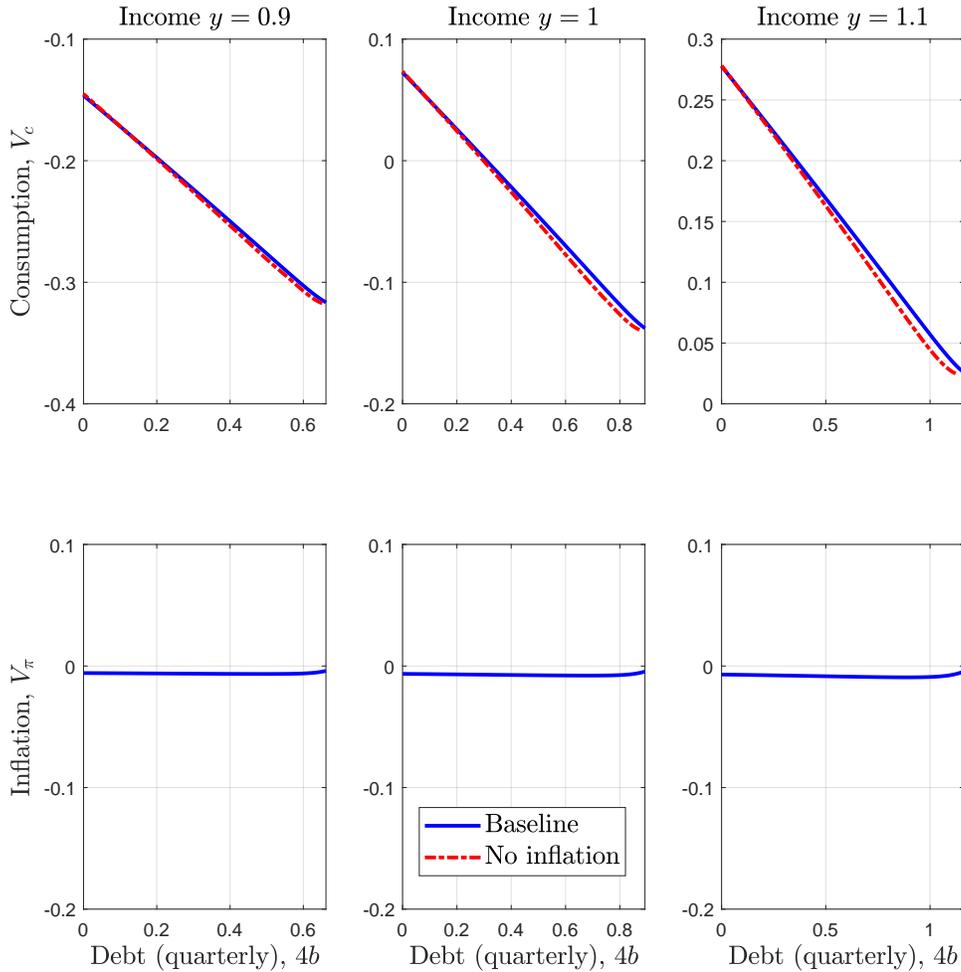


Figure 7: Welfare decomposition.

Therefore, V_c and V_π capture the expected discounted stream of consumption utility flows and inflation disutility flows, respectively.²⁹

Figure 7 shows the welfare decomposition in both regimes, for three different income levels. Notice first that the welfare losses from the inflationary bias in the inflationary regime, V_π , are relatively small, reflecting the fact that inflation always remains at first-order levels and hence causes at most second-order utility losses. Importantly though, the utility costs of inflation explain most of the welfare advantage of the no-inflation regime at zero debt, i.e. the point of reentry into capital markets following an exclusion spell. This justifies our earlier claim that it is the welfare cost of the inflationary bias that makes default more costly ex-ante *vis-à-vis* the no-inflation regime.

By contrast, the inflationary regime achieves a higher stream of consumption utility flows, V_c ,

²⁹See Appendix C for an explanation of how V_c and V_π can be computed. Obviously, $V_\pi = 0$ in the no-inflation regime.

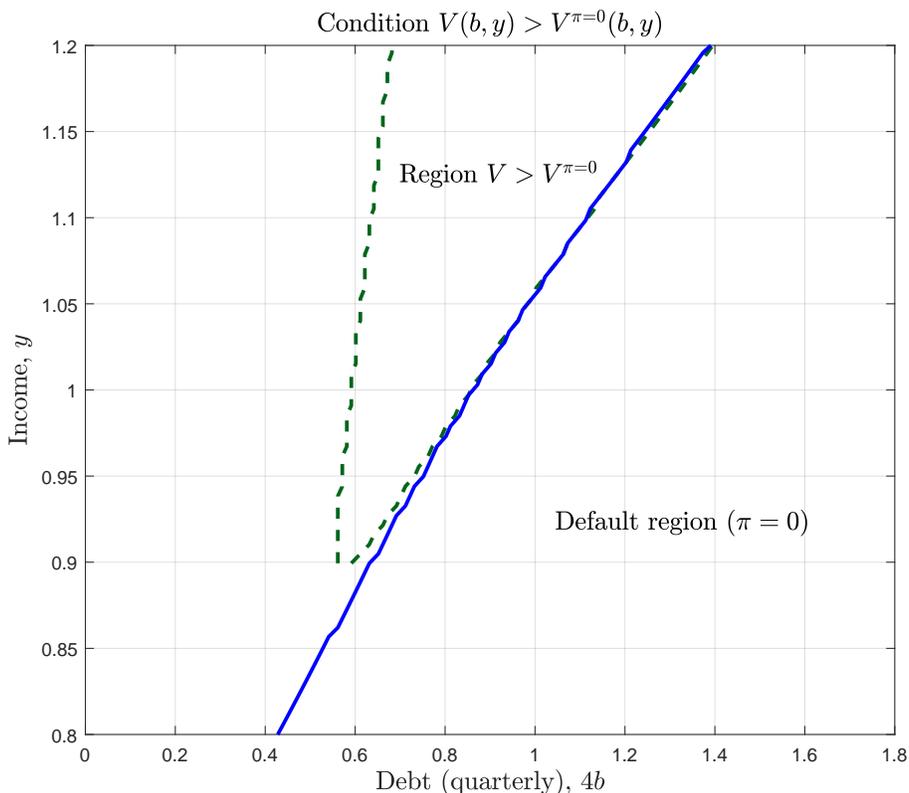


Figure 8: Isowelfare curve ($V = V^{\pi=0}$) and default frontier.

the more so the closer the economy is to default. We have already seen that the inflationary regime raises the default threshold *vis-à-vis* the no-inflation regime, which in turn is reflected in higher bond prices, and consequently higher consumption, in the vicinity of the default threshold of the no-inflation regime (see Figure 1). Higher consumption in this debt segment is then reflected in a higher value of V_c .

A closer look at the welfare comparison of both regimes is provided by Figure 8, which portrays the region of the state space where the inflationary regime dominates the no-inflation one. For all income levels except very low ones, the inflationary regime achieves better welfare outcomes when debt is close to the default frontier. It is only for very low income levels that the no-inflation regime dominates in welfare terms for any debt level. The welfare decomposition in Figure 7 tells us why. While the welfare cost of the inflationary bias V_π is fairly stable across debt and income levels, the consumption welfare gain of the inflationary regime, $V_c - V_c^{\pi=0}$, increases with debt, for the reasons explained above. Moreover, near the respective default threshold, such consumption welfare gain increases with income too.³⁰ As a result, for debt levels close to the default frontier

³⁰Intuitively, the fact that income is mean reverting implies that, starting from a high income level, income will tend to fall to steady state. In terms of the phase-diagram in Figure 4, starting from high income (and a common

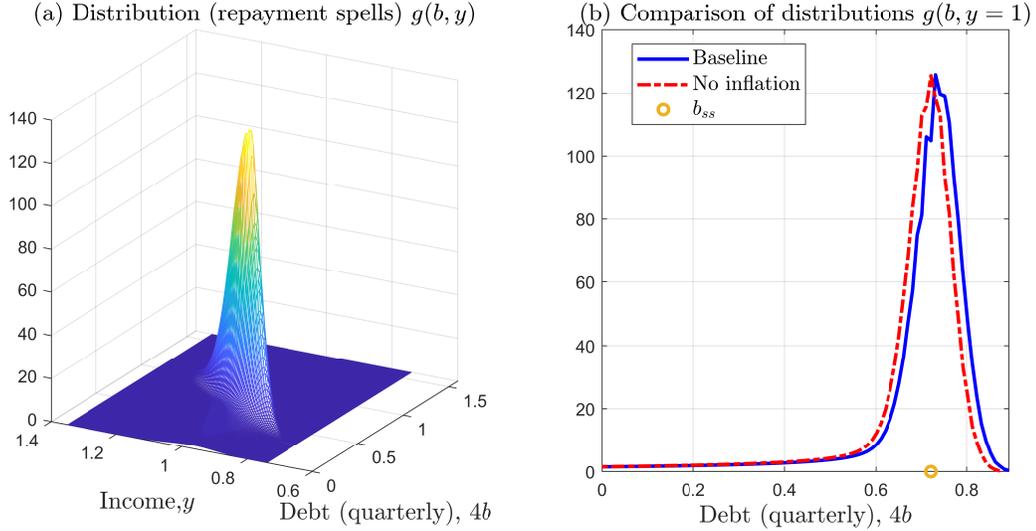


Figure 9: Stationary distribution. Note: the circle marks the stochastic steady state b_{ss} in the baseline model.

the inflationary regime dominates the no-inflation one, except for very low income levels, for which the welfare cost of the inflationary bias dominates the consumption welfare gains from inflation regardless of the debt level.

3.6 Average performance

So far we have compared both regimes in terms of their equilibrium behavior at each point of the state space. It is also interesting to compare their *average* behavior. In order to compute unconditional averages, we first need to solve for the ergodic debt-income distribution. For this purpose, it is useful to distinguish between (a) repayment spells and (b) the exclusion periods that follow each default. The stationary distribution conditional on being in a repayment spell is denoted by $g(b, y)$ and the autarky stationary distribution is $g^{def}(b, y)$. They satisfy the *Kolmogorov Forward Equation* (KFE) and can be efficiently computed as described in Appendix B.

Panel (a) in Figure 9 displays $g(b, y)$ in the baseline inflationary regime. During repayment spells, the economy stays most of the time within a narrow region that covers roughly diagonally the state space, from a high-income-high-debt zone to a low-income-low-debt one. Panel (b) displays, for both regimes, a slice of the respective distribution conditional on $y = 1$. It shows how the

debt level) the economy will move south in both regimes and hence closer to the respective default frontier. But since such frontier is *closer* in the no-inflation regime, agents anticipate a faster reduction in bond prices and consumption compared to the inflationary case, resulting in a larger gap in consumption-related welfare V_c .

inflationary regime is able to sustain slightly higher debt levels than the no-inflation one.

Having computed the ergodic debt-income distribution, we can then calculate a number of relevant moments in the model. Table 2 displays average values of key variables for both monetary regimes. Notice first that our model produces debt and spread levels comparable to those in the literature.³¹ The average inflation rate in the baseline regime, 1.4%, speaks of a relatively moderate inflationary bias.

Table 2. Moments

Variable	units	Policy regime	
		No inflation	Inflationary
inflation, π	%	0	1.4
debt-to-GDP, b	%	46.3	48.2
spread, $r - \bar{r}$	%	5.7	6.9
spread (std. dev.)	bp	6.2	6.5
Welfare loss, $V - V_{\pi=0}$	% cons.	0	-0.07

Finally, the inflationary regime incurs a small average welfare loss relative to the no-inflation regime, of 0.07% of permanent consumption. This reflects two main forces. On the one hand, we have seen that the inflationary regime sustains higher debt levels. Since value functions are strictly decreasing in debt, this results in lower average welfare in the inflationary case. On the other hand, we have also seen that, every time the government reenters capital markets after an exclusion spell, it does so at zero debt, for which welfare is (slightly) lower in the inflationary case due to the welfare costs of the inflationary bias. The ensuing transition period with lower welfare works in detriment of the inflationary regime when computing average welfare.

3.7 Sensitivity: utility costs of inflation

Finally, we assess the sensitivity of our results to alternative values of what is arguably the most important parameter in the model: the scale of inflation utility costs ψ . Panel (b) in Figure 10 displays, for a relatively wide range of income levels, the corresponding default threshold for debt as a function of ψ . In order to consider as wide a range as possible, we consider values in between two extremes: the lowest possible value of ψ that allows our solution algorithm to converge, and

³¹Our averages for debt-to-GDP and spreads are somewhat lower than those (70 and 8.15, respectively) reported by CE, on which our calibration is based. Beyond some modelling differences, one key reason for this is that, when computing moments in their model, they discard the first 5 years of each repayment spell, precisely when debt (which starts each repayment spell at zero) and the spread are lowest. Following the same approach would bring our model's moments closer to those of Chatterjee and Eyigungor (2012).

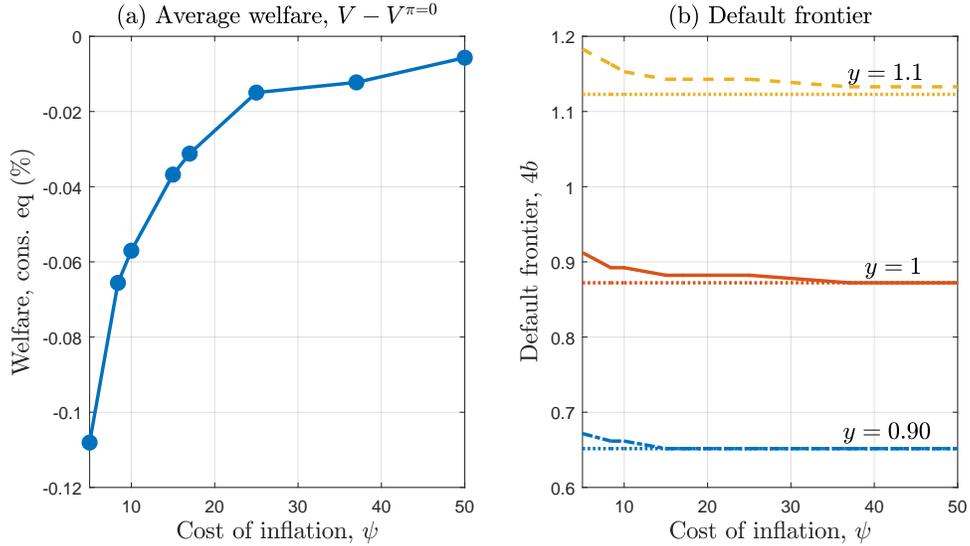


Figure 10: Average welfare and default frontier for different values of ψ . Note: dotted lines in panel (b) indicate asymptotic values at $\psi \rightarrow \infty$ ($\pi = 0$).

a value of $\psi = 50$ for which the inflationary regime is virtually indistinguishable from the no-inflation one (equivalently, from the limiting case $\psi \rightarrow \infty$). The key message is that, regardless of the size of inflation costs and for any income level, the inflationary regime always improves debt sustainability, by pushing up the default threshold *vis-à-vis* the no-inflation regime.

Panel (a) of Figure 10 displays average welfare, $\mathbb{E}(V)$, in both regimes as a function again of ψ . The inflationary regime always achieves worse average welfare outcomes –with the gap closing as ψ goes to infinity and both regimes become equivalent. This is despite the fact that the inflationary regime continues to achieve higher welfare in certain *states* even for extreme calibrations. This can be seen in Figure 12, which shows that the iso-welfare frontier (i.e. the debt and income levels for which $V = V^{\pi=0}$) between both regimes for the two extreme values of ψ are relatively similar to the frontier under the baseline calibration: the inflationary regime dominates for relatively high debt levels, provided income is not too low. The dominance of the no-inflation regime in *average* welfare terms, regardless of ψ , reflects again the two forces explained in the previous subsection: (i) the conjunction of welfare functions that decrease in debt and lower debt levels in the no-inflation regime, and (ii) the welfare losses incurred by the inflationary regime upon reentry into capital markets after every exclusion spell.

While not directly comparable, our results stand somewhat in contrast to those of Farhi et al. (2013), who analyze the effect of inflation policy without commitment in a qualitative model

of self-fulfilling sovereign debt crises. They find that, for inflation utility costs below a certain threshold, *vis-à-vis* a scenario with foreign currency debt (analogous to our no-inflation regime) domestic currency debt makes the economy more vulnerable by lowering the debt threshold above which the economy is exposed to self-fulfilling crises. Instead, we find that for any scale of inflation costs our inflationary regime improves sovereign debt sustainability by raising the debt threshold above which the economy suffers fundamentally-driven default.

4 Conclusions

This paper has addressed the consequences of discretionary inflation policy for sovereign debt sustainability and welfare, in a standard quantitative model of optimal sovereign debt default extended with nominal long-term debt and costly inflation. Two key results stand out. First, inflation improves the sustainability of sovereign debt by enlarging the set of debt and income levels in which the government prefers repayment to default, relative to a counterfactual scenario where inflation is zero at all times. Such 'blessing' stems from the main 'curse' of discretionary inflation: the welfare costs created by the 'inflationary bias' that the government incurs by attempting to inflate away its debt. These costs are incurred soon after the government reenters capital markets following a default and exclusion spell and starts issuing debt again, and their anticipation makes default more costly *ex ante* relative to the zero-inflation regime.

Second, inflation may also improve welfare relative to zero inflation, but only in certain states of the world: when debt is sufficiently close to the default threshold. The reason lies again in the interplay between the inflationary bias and debt sustainability. The welfare costs from the former are relatively stable across income and debt levels. By contrast, the gains in consumption utility from the improvement in debt sustainability are felt more strongly in the vicinity of the default frontier. As a result, it is precisely when the economy is close to default that discretionary inflation may be welfare-improving.

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Online appendix (not for publication)

A. An economy with costly price adjustment

In this appendix, we lay out a model economy with the following characteristics: (i) firms are explicitly modelled, (ii) a subset of them are price-setters but incur a convex cost for changing their nominal price, and (iii) the social welfare function and the equilibrium conditions are the same as in the model economy in the main text.

Final good producer

In the model laid out in the main text, we assumed that output of the single consumption good Y_t is exogenous. Consider now an alternative setup in which the single consumption good is produced by a representative, perfectly competitive final good producer with the following Dixit-Stiglitz technology,

$$Y_t = \left(\int_0^1 y_{it}^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}, \quad (22)$$

where $\{y_{it}\}$ is a continuum of intermediate goods and $\varepsilon > 1$. Let P_{it} denote the nominal price of intermediate good $i \in [0, 1]$. The firm chooses $\{y_{it}\}$ to maximize profits, $P_t Y_t - \int_0^1 P_{it} y_{it} di$, subject to (22). The first order conditions are

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t, \quad (23)$$

for each $i \in [0, 1]$. Assuming free entry, the zero profit condition and equations (23) imply $P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}$.

Intermediate goods producers

Each intermediate good i is produced by a monopolistically competitive intermediate-good producer, which we will refer to as 'firm i ' henceforth for brevity. Firm i operates a linear production technology,

$$y_{it} = Z_t n_{it}, \quad (24)$$

where n_{it} is labor input and $Z_t = e^{z_t}$ is productivity, where z_t follows equation (1) in the main text. At each point in time, firms can change the price of their product but face quadratic price adjustment cost as in Rotemberg (1982). Letting $\dot{P}_{it} \equiv dP_{it}/dt$ denote the change in the firm's

price, price adjustment costs in units of the final good are given by

$$\Psi_t \left(\frac{\dot{P}_{it}}{P_{it}} \right) \equiv \frac{\psi}{2} \left(\frac{\dot{P}_{it}}{P_{it}} \right)^2 \tilde{C}_t, \quad (25)$$

where \tilde{C}_t is aggregate consumption. Let $\pi_{it} \equiv \dot{P}_{it}/P_{it}$ denote the rate of increase in the firm's price. The instantaneous profit function in units of the final good is given by

$$\begin{aligned} \Pi_{it} &= \frac{P_{it}}{P_t} y_{it} - w_t n_{it} - \Psi_t(\pi_{it}) \\ &= \left(\frac{P_{it}}{P_t} - \frac{w_t}{Z_t} \right) \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t - \Psi_t(\pi_{it}), \end{aligned} \quad (26)$$

where w_t is the perfectly competitive real wage and in the second equality we have used (23) and (24). Without loss of generality, firms are assumed to be risk neutral and have the same discount factor as households, ρ . Then firm i 's objective function is

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \Pi_{it} di,$$

with Π_{it} given by (26). Notice that the firm's optimization *problem* is not affected by sovereign defaults, although of course default does affect the aggregate variables that enter the firm's problem (Y_t, P_t , etc.). The state variable specific to firm i , P_{it} , evolves according to $dP_{it} = \pi_{it} P_{it} dt$. We conjecture that the aggregate state relevant to the firm's decisions can be summarized by $(b_t, z_t, P_t) \equiv S_t$.³² Then firm i 's *value function* $J(P_{it}, S_t)$ must satisfy the following Hamilton-Jacobi-Bellman (HJB) equation,

$$\begin{aligned} \rho J(P_i, S) &= \max_{\pi_i} \left\{ \left(\frac{P_i}{P} - \frac{w}{e^z} \right) \left(\frac{P_i}{P} \right)^{-\varepsilon} Y - \Psi(\pi_i) + \pi_i P_i \frac{\partial J}{\partial P_i}(P_i, S) \right\} \\ &\quad + \mu'_S(S) D_S J(P_i, S) + \frac{1}{2} \sigma'_S(S) (D_{SS} J(P_i, S)) \sigma_S(S), \end{aligned}$$

where the vectors $(\mu_S(S), \sigma_S(S))$ collect the drift and diffusion terms, respectively, of the aggregate states S , and (D_S, D_{SS}) are the gradient and Hessian operators, respectively, with respect to S .³³ The first order and envelope conditions of this problem are (we omit the arguments of J to

³²In particular, we later show that in equilibrium $Y_t = Z_t$, whereas w_t and \tilde{C}_t are also functions of (b_t, Z_t, P_t) . The states P_t and b_t follow the same laws of motion as in the main text, equations (2) and (4) respectively, whereas $z_t = \log Z_t$ follows equation (1).

³³In particular, $\mu_S(S) = [s(b, z), -\mu z, \pi P]'$, where $s(b, z)$ is the drift of a as defined in section 2 of the main text; and $\sigma_S(S) = [0, \sigma, 0]'$.

ease the notation),

$$\psi \pi_i \tilde{C} = P_i \frac{\partial J}{\partial P_i},$$

$$\begin{aligned} \rho \frac{\partial J}{\partial P_i} &= \left[\varepsilon \frac{w}{Z} - (\varepsilon - 1) \frac{P_i}{P} \right] \left(\frac{P_i}{P} \right)^{-\varepsilon} \frac{Y}{P_i} + \pi_i \left(\frac{\partial J}{\partial P_i} + P_i \frac{\partial^2 J}{\partial P_i^2} \right) \\ &\quad + \frac{\partial}{\partial P_i} \left[\mu'_S(S) D_S J + \frac{1}{2} \sigma'_S(S) (D_{SS} J) \sigma_S(S) \right]. \end{aligned}$$

In what follows, we will consider a symmetric equilibrium in which all firms choose the same price: $P_i = P, \pi_i = \pi$ for all i . After some algebra, it can be shown that the above conditions imply the following pricing Euler equation,³⁴

$$\begin{aligned} \left(\rho - \frac{\tilde{C}_b(b, z) s(b, z) - \tilde{C}_z(b, z) \mu z}{\tilde{C}(b, z)} \right) \pi(b, z) &= \frac{\varepsilon - 1}{\psi} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{w}{e^z} - 1 \right) \frac{e^z}{\tilde{C}(b, z)} \\ &\quad + s(b, z) \pi_b(b, z) - \mu z \pi_z(b, z) + \sigma^2 F(S). \end{aligned} \quad (27)$$

where $\tilde{C}(b, z)$ and $\pi(b, z)$ denote the equilibrium policy functions for total spending and inflation, and $F(S)$ is a function of the aggregate state capturing the effect of aggregate uncertainty (σ) on firms' pricing decision. Equation (27) determines the market clearing wage w as a function of S .

Households and the utility costs of inflation

The representative household's preferences are given by

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log(\tilde{C}_t) dt,$$

where \tilde{C}_t is household consumption of the final good. Define *total real spending* as the sum of household consumption and price adjustment costs,

$$\begin{aligned} C_t &\equiv \tilde{C}_t + \int_0^1 \Psi_t(\pi_{it}) di \\ &= \tilde{C}_t + \frac{\psi}{2} \pi_t^2 \tilde{C}_t, \end{aligned} \quad (28)$$

³⁴The proof is available upon request.

where in the second equality we have used the definition of Ψ_t (eq. 25) and the symmetry across firms in equilibrium. Instantaneous utility can then be expressed as

$$\begin{aligned}\log(\tilde{C}_t) &= \log(C_t) - \log\left(1 + \frac{\psi}{2}\pi_t^2\right) \\ &= \log(C_t) - \frac{\psi}{2}\pi_t^2 + O\left(\left\|\frac{\psi}{2}\pi_t^2\right\|^2\right),\end{aligned}\tag{29}$$

where $O(\|x\|^2)$ denotes terms of order second and higher in x . Expression (29) is the same as the utility function in the main text (eq. 6), up to a first order approximation of $\log(1+x)$ around $x=0$, where $x \equiv \frac{\psi}{2}\pi^2$ represents the percentage of aggregate spending that is lost to price adjustment. For our baseline calibration ($\psi = 8.4$), the latter object is relatively small even for relatively high inflation rates, and therefore so is the error in computing the utility losses from price adjustment.³⁵ Therefore, the utility function used in the main text provides a fairly accurate approximation of the welfare losses caused by inflation in the economy with costly price adjustment described here. We have solved the equilibrium implied by the exact inflation disutility function in the first line of equation (29) and found that the results are virtually identical to those in the main text.

As in the model in the main text, the government rebates to the household all the net proceedings from its international credit operations, denoted by \tilde{T}_t in nominal terms. We assume that the household supplies one unit of labor input inelastically: $n_t = 1$. It also receives firms' profits in a lump-sum manner. Thus the household's nominal budget constraint is

$$P_t\tilde{C}_t = P_t w_t + P_t \int_0^1 \Pi_{it} di + \tilde{T}_t.$$

In the symmetric equilibrium, each firm's labor demand is $n_{it} = y_{it}/Z_t = Y_t/Z_t$. Since labor supply equals one, labor market clearing requires

$$\int_0^1 n_{it} di = Y_t/Z_t = 1 \Leftrightarrow Y_t = Z_t.$$

Therefore, in equilibrium output is simply equal to exogenous productivity Z_t . Each firm's real profits equal $\Pi_{it} = Y_t - w_t - \frac{\psi}{2}\pi_t^2\tilde{C}_t$. Using this in the household's budget constraint, we obtain

$$\tilde{T}_t = P_t \left(\tilde{C}_t + \frac{\psi}{2}\pi_t^2\tilde{C}_t - Y_t \right) = P_t (C_t - Y_t),$$

³⁵For our baseline calibration, inflation never reaches 3.4%. At this rate, the exact and the approximated price adjustment cost are $\log(1 + \frac{\psi}{2}\pi^2) = 0.4843\%$ and $\frac{\psi}{2}\pi^2 = 0.4855\%$ of aggregate spending, respectively.

where in the second equality we have used (28).

Fiscal and monetary policy

The government maximizes household welfare subject to the laws of motion of the aggregate state variables. The default scenario is the same as in the main text, with one qualification: upon default and during the subsequent exclusion period, productivity equals $(Z_t - \epsilon(Z_t))$. This, together with the fact that in equilibrium $Y_t = Z_t$, implies that the default scenario is exactly as in the main text. It is then trivial to show that the government's maximization problem is exactly the same as in the main text, once we take into account that (i) the welfare criterion is the same (equation 9), and (ii) the law of motion of the debt ratio is the same (equation 4). As a result, the policy functions for inflation and primary deficit ratio will also be the same: $\pi_t = \pi(b_t, z_t)$, $c_t = c(b_t, z_t)$.

Notice finally that, since $Y_t = Z_t$, in equilibrium we have $C_t = Z_t \equiv C(b_t, z_t)$, and therefore $\tilde{C}_t = C(b_t, z_t) / [1 + \frac{\psi}{2}\pi(b_t, z_t)^2] \equiv \tilde{C}(b_t, z_t)$. Likewise, the pricing Euler equation derived above (equation 27) determines the market clearing wage given the aggregate state: $w_t = w(b_t, z_t, P_t)$. We thus verify our previous conjecture that (b_t, z_t, P_t) are the relevant aggregate states for firms.

Appendix B: description of the numerical algorithm

The augmented model with an option to default

Here we present an algorithm to compute the equilibrium of the model with inflation. Our numerical algorithm is based on an augmented model which assumes that the government may only default when it receives an exogenous option to default. We assume that the option to default follows a Poisson process with parameter ϕ , that is, there is a number of random times $\{\tilde{t}_i\}_{i=1}^{\infty}$ at which the government decides whether to continue repaying its debt or to default. If the arrival rate tends to infinite $\phi \rightarrow \infty$ the government can default at any point in time, whereas when $\phi = 0$ default is not available. Therefore for a large enough value of ϕ this specification nests the case of continuous default discussed in the main body of the text. The advantage of this method is that it prevents some numerical problems associated with the numerical solution of default problems in continuous time discussed below. Instead of the level of debt, we employ the level of net assets $a = -b$.

The HJB in this case results in

$$\begin{aligned} \rho V(a, z) = & \max_{c, \pi, d \in \{0, 1\}} \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{\psi}{2}\pi^2 + s(a, z, c, \pi) \frac{\partial V}{\partial a} - \mu z \frac{\partial V}{\partial z} \\ & + \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial z^2} + \phi d(a, z) [V_{def}(z) - V(a, z)]. \end{aligned} \quad (30)$$

Notice that the variational inequality has been replaced by a HJB equation including the term

$$\phi d(a, z) [V_{def}(z) - V(a, z)].$$

If the default policy d at the state (a, z) is one then the term indicates how in the case of the arrival of an option to default the value function jumps to the case with default, $V_{def}(z)$. The HJB for the default value (8) function remains the same.

Bond prices in this case are given by

$$(\bar{r} + \pi(a, z) + \lambda) Q(a, z) = (\lambda + \delta) + s(a, z) \frac{\partial Q}{\partial a} - \mu z \frac{\partial Q}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 Q}{\partial z^2} - \phi d(a, z) Q(a, z), \quad (31)$$

where the term $-\phi d(a, z) Q(a, z)$ indicates that in the case of default the price of the bond is zero. The advantage of this formulation, with a finite ϕ is that bond prices in the no-default region are never zero. There is a lower bound

$$Q^{\min}(a, z) = \frac{(\lambda + \delta)}{\bar{r} + \pi(a, z) + \lambda + \phi} > 0,$$

which can be made arbitrarily small by increasing ϕ . Having a non-zero value of Q avoids the problem that the drift function $s(a, z)$ (defined in equation 11) becomes infinite at the default frontier as it is a function of the inverse of the bond price.

For values of ϕ larger than 4 – corresponding to an average default option per quarter – the equilibrium objects remain constant up to the considered precision. In any case, we calibrate $\phi = 50$ to avoid any possible error associated with the numerical scheme.

Solution to the no default Hamilton-Jacobi-Bellman equation

The HJB equation is solved by a finite difference scheme following Achdou et al. (2017). It approximates the value function $V(a, z)$ on a finite grid with steps Δa and $\Delta z : a \in \{a_1, \dots, a_I\}$, $z \in \{z_1, \dots, z_J\}$. We use the notation $V_{i,j} := V(a_i, z_j)$, $i = 1, \dots, I$; $j = 1, \dots, J$. The derivative of V with respect to a can be approximated with either a forward or a backward approximation:

$$\frac{\partial V(a_i, z_j)}{\partial a} \approx \partial_{a,F} V_{i,j} := \frac{V_{i+1,j} - V_{i,j}}{\Delta a}, \quad (32)$$

$$\frac{\partial V(a_i, z_j)}{\partial a} \approx \partial_{a,B} V_{i,j} := \frac{V_{i,j} - V_{i-1,j}}{\Delta a}, \quad (33)$$

where the decision between one approximation or the other depends on the sign of the savings function $s_{i,j} = \left(\frac{\lambda + \delta}{Q(a_i, z_j)} - \lambda - \pi(a_i, z_j) \right) a_i + \frac{e^{z_j} - c_{i,j}}{Q(a_i, z_j)}$ through an “upwind scheme” described below.

The derivative of V with respect to z is approximated using a forward approximation

$$\frac{\partial V(a_i, z_j)}{\partial z} \approx \partial_z V_{i,j} := \frac{V_{i,j+1} - V_{i,j}}{\Delta z}, \quad (34)$$

$$\frac{\partial^2 V(a_i, z_j)}{\partial z^2} \approx \partial_{zz} V_{i,j} := \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{(\Delta z)^2}. \quad (35)$$

The HJB equation (30) is

$$\begin{aligned} \rho V(a, z) = & u(c) - x(\pi) + \left[\left(\frac{\lambda + \delta}{Q(a, z)} - \lambda \right) a + \frac{e^z - c}{Q(a, z)} \right] \frac{\partial V}{\partial a} - \mu z \frac{\partial V}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial z^2} \\ & + \phi d(a, z) [V_{def}(a, z) - V(a, z)], \end{aligned}$$

where

$$\begin{aligned} c &= (u')^{-1} \left[\frac{1}{Q(a, z)} \frac{\partial V}{\partial a} \right] = \left[\frac{1}{Q(a, z)} \frac{\partial V}{\partial a} \right]^{-1/\gamma} \\ \pi &= -\frac{a}{\psi} \frac{\partial V}{\partial a} = -\frac{a}{\psi} c^{-\gamma} Q(a, z). \end{aligned}$$

The HJB equation is approximated by an upwind scheme

$$\begin{aligned} \frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta} + \rho V_{i,j}^{n+1} = & u(c_{i,j}^n) - x(\pi_{i,j}^n) + \phi d_{i,j}^n V_{def,i,j}^n + \partial_{a,F} V_{i,j}^{n+1} s_{i,j,F}^n \mathbf{1}_{s_{i,j,F}^n > 0} + \partial_{a,B} V_{i,j}^{n+1} s_{i,j,B}^n \mathbf{1}_{s_{i,j,B}^n < 0} \\ & - \mu z_j \partial_z V_{i,j}^{n+1} + \frac{\sigma_z^2}{2} \partial_{zz} V_{i,j}^{n+1} - \phi d_{i,j}^n V_{i,j}^{n+1}, \end{aligned}$$

where

$$\begin{aligned} s_{i,j,F}^n &= \left[\left(\frac{\lambda + \delta}{Q_{i,j}^n} - \lambda - \frac{-a}{\psi} \partial_{a,F} V_{i,j}^n \right) a_i + \frac{e^{z_j} - (u')^{-1} \left[\frac{\partial_{a,F} V_{i,j}^n}{Q_{i,j}^n} \right]}{Q_{i,j}^n} \right], \\ s_{i,j,B}^n &= \left[\left(\frac{\lambda + \delta}{Q_{i,j}^n} - \lambda - \frac{-a}{\psi} \partial_{a,B} V_{i,j}^n \right) a_i + \frac{e^{z_j} - (u')^{-1} \left[\frac{\partial_{a,B} V_{i,j}^n}{Q_{i,j}^n} \right]}{Q_{i,j}^n} \right]. \end{aligned}$$

This can be expressed as:

$$\frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta} + \rho V_{i,j}^{n+1} = u(c_{i,j}^n) - x(\pi_{i,j}^n) + \phi d_{i,j}^n V_{def,i,j}^n + V_{i-1,j}^{n+1} \varrho_{i,j} + V_{i,j}^{n+1} \beta_{i,j} + V_{i+1,j}^{n+1} \eta_{i,j} + V_{i,j-1}^{n+1} \xi + V_{i,j+1}^{n+1} \varsigma_j, \quad (36)$$

where

$$c_{i,j}^n = (u')^{-1} \left[\frac{1}{Q_{i,j}^n} \left(\partial_{a,F} V_{i,j}^n \mathbf{1}_{s_{i,j,F}^n > 0} + \partial_{a,B} V_{i,j}^n \mathbf{1}_{s_{i,j,B}^n < 0} + Q_{i,j}^n u' (c_{i,j}^{0,n}) \mathbf{1}_{s_{i,j,F}^n < 0, s_{i,j,B}^n > 0} \right) \right], \quad (37)$$

$$\pi_{i,j}^n = -\frac{a_i}{\psi} (c_{i,j}^n)^{-\gamma} Q_{i,j}^n, \quad (38)$$

$$\varrho_{i,j} = -\frac{s_{i,j,B}^n \mathbf{1}_{s_{i,j,B}^n < 0}}{\Delta a},$$

$$\beta_{i,j} = -\frac{s_{i,j,F}^n \mathbf{1}_{s_{i,j,F}^n > 0}}{\Delta a} + \frac{s_{i,j,B}^n \mathbf{1}_{s_{i,j,B}^n < 0}}{\Delta a} + \frac{\mu z_j}{\Delta z} - \frac{\sigma^2}{(\Delta z)^2} - d_{i,j}^n \phi,$$

$$\eta_{i,j} = \frac{s_{i,j,F}^n \mathbf{1}_{s_{i,j,F}^n > 0}}{\Delta a},$$

$$\xi = \frac{\sigma^2}{2(\Delta z)^2},$$

$$\varsigma_j = \frac{\sigma^2}{2(\Delta z)^2} - \frac{\mu z_j}{\Delta z},$$

where

$$c_{i,j}^{0,n} = Q_{i,j}^n \left(\frac{\lambda + \delta}{Q(a_i, z_j)} - \lambda - \pi_{i,j}^{0,n} \right) a_i + e^{z_j},$$

$$\pi_{i,j}^{0,n} = \left(\frac{-a}{\psi} \partial_{a,F} V_{i,j}^n + \frac{-a}{\psi} \partial_{a,B} V_{i,j}^n \right) / 2.$$

The state constraint $a \leq 0$ is enforced by setting $s_{i,j,F}^n = 0$. Similarly, we impose $a \geq a^{\min}$ for a value of a^{\min} large enough, which requires $s_{i,j,B}^n = 0$. Therefore, the values $V_{0,j}^{n+1}$ and $V_{I+1,j}^{n+1}$ are never used. The boundary conditions with respect to z are

$$\frac{\partial V(a, \underline{z})}{\partial z} = \frac{\partial V(a, \bar{z})}{\partial z} = 0,$$

as the process is reflected. At the boundaries in the j dimension, equation (36) becomes

$$\frac{V_{i,1}^{n+1} - V_{i,1}^n}{\Delta} + \rho V_{i,j}^{n+1} = u(c_{i,1}^n) - x(\pi_{i,j}^n) + \phi d_{i,1}^n V_{def,i,1}^n + V_{i-1,1}^{n+1} \varrho_{i,1} + V_{i,1}^{n+1} (\beta_{i,1} + \xi) + V_{i+1,1}^{n+1} \eta_{i,1} + V_{i,2}^{n+1} \varsigma_1,$$

$$\frac{V_{i,J}^{n+1} - V_{i,J}^n}{\Delta} + \rho V_{i,j}^{n+1} = u(c_{i,J}^n) - x(\pi_{i,j}^n) + \phi d_{i,J}^n V_{def,i,J}^n + V_{i-1,J}^{n+1} \varrho_{i,J} + V_{i,J}^{n+1} (\beta_{i,J} + \varsigma_J) + V_{i+1,J}^{n+1} \eta_{i,J} + V_{i,J-1}^{n+1} \xi.$$

Equation (36) is a system of $I \times J$ linear equations which can be written in matrix notation as:

$$\frac{\mathbf{V}^{n+1} - \mathbf{V}^n}{\Delta} + \rho \mathbf{V}^{n+1} = \mathbf{u}^n + \mathbf{A}^n \mathbf{V}^{n+1}, \quad (39)$$

where the matrix \mathbf{A}^n and the vectors \mathbf{V}^{n+1} and \mathbf{u}^n are defined by:

$$\mathbf{A}^n = \begin{bmatrix} \beta_{1,1} + \xi & \eta_{1,1} & 0 & \cdots & 0 & \varsigma_1 & 0 & 0 & \cdots & 0 \\ \varrho_{2,1} & \beta_{2,1} + \xi & \eta_{2,1} & 0 & \cdots & 0 & \varsigma_1 & 0 & \cdots & 0 \\ 0 & \varrho_{3,1} & \beta_{3,1} + \xi & \eta_{3,1} & 0 & \cdots & 0 & \varsigma_1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \varrho_{I,1} & \beta_{I,1} + \xi & \eta_{I,1} & 0 & 0 & \cdots & 0 \\ \xi & 0 & \cdots & 0 & \varrho_{1,2} & \beta_{1,2} & \eta_{1,2} & 0 & \cdots & 0 \\ 0 & \xi & \cdots & 0 & 0 & \varrho_{2,2} & \beta_{2,2} & \eta_{2,2} & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \varrho_{I-1,J} & \beta_{I-1,J} + \varsigma_J & \eta_{I-1,J} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \varrho_{I,J} & \beta_{I,I} + \varsigma_J \end{bmatrix},$$

$$\mathbf{V}^{n+1} = \begin{bmatrix} V_{1,1}^{n+1} \\ V_{2,1}^{n+1} \\ \vdots \\ V_{1,2}^{n+1} \\ V_{2,2}^{n+1} \\ \vdots \\ V_{I-1,J}^{n+1} \\ V_{I,J}^{n+1} \end{bmatrix}, \quad \mathbf{u}^n = \begin{bmatrix} u(c_{1,1}^n) - x(\pi_{1,1}^n) + \phi d_{1,1}^n V_{def,1,1}^n \\ u(c_{2,1}^n) - x(\pi_{2,1}^n) + \phi d_{2,1}^n V_{def,2,1}^n \\ \vdots \\ u(c_{1,2}^n) - x(\pi_{1,2}^n) + \phi d_{1,2}^n V_{def,1,2}^n \\ u(c_{2,2}^n) - x(\pi_{2,2}^n) + \phi d_{2,2}^n V_{def,2,2}^n \\ \vdots \\ u(c_{I-1,J}^n) - x(\pi_{I-1,J}^n) + \phi d_{I-1,J}^n V_{def,I-1,J}^n \\ u(c_{I,J}^n) - x(\pi_{I,J}^n) + \phi d_{I,J}^n V_{def,I,J}^n \end{bmatrix}.$$

The system can in turn be written as

$$\mathbf{B}^n \mathbf{V}^{n+1} = \mathbf{d}^n,$$

where $\mathbf{B}^n = (\frac{1}{\Delta} + \rho) \mathbf{I} - \mathbf{A}^n$ and $\mathbf{d}^n = \mathbf{u}^n + \frac{\mathbf{V}^n}{\Delta}$. \mathbf{I} is the identity matrix. Matrix \mathbf{B}^n is a sparse matrix, and the system can be efficiently solved in Matlab.

In order to avoid abrupt jumps in bond prices we combine it with a relaxation scheme such that, given a constant $\varkappa \in (0, 1)$, if we denote the result of the linear system of equations (39) as $\hat{V}_{i,j}^{n+1}$, then

$$V_{i,j}^{n+1} = \varkappa \hat{V}_{i,j}^{n+1} + (1 - \varkappa) V_{i,j}^n. \quad (40)$$

Solution to the default Hamilton-Jacobi-Bellman equation

The HJB equation in this case (8) is

$$\rho V_{def}(z) = u_{def}(z) - \mu z \frac{\partial V_{def}}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_{def}}{\partial z^2} + \chi [V(0, z) - V_{def}(z)],$$

which, using a finite difference method identical to the one described above (this time no upwind is necessary as there is no control), yields

$$\frac{V_{def,i,j}^{n+1} - V_{def,i,j}^n}{\Delta} + \rho V_{i,j}^{n+1} = u_{def}(z_j) + \chi V_{I,j}^n + V_{def,i,j}^{n+1} \hat{\beta}_j + V_{def,i,j-1}^{n+1} \xi + V_{def,i,j+1}^{n+1} \varsigma_j,$$

where $\hat{\beta}_j = \frac{\mu z_j}{\Delta z} - \frac{\sigma^2}{(\Delta z)^2} - \chi$, which can be written in matrix notation as:

$$\frac{\mathbf{V}_{def}^{n+1} - \mathbf{V}_{def}^n}{\Delta} + \rho \mathbf{V}_{def}^{n+1} = \mathbf{u}_{def}^n + \mathbf{A}_{def}^n \mathbf{V}_{def}^{n+1}, \quad (41)$$

where

$$\mathbf{A}_{def}^n = \begin{bmatrix} \beta_1 + \xi & 0 & 0 & \cdots & 0 & \varsigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \beta_1 + \xi & 0 & 0 & \cdots & 0 & \varsigma_1 & 0 & \cdots & 0 \\ 0 & 0 & \beta_1 + \xi & 0 & 0 & \cdots & 0 & \varsigma_1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \beta_1 + \xi & 0 & 0 & 0 & \cdots & 0 \\ \xi & 0 & \cdots & 0 & 0 & \beta_2 & 0 & 0 & \cdots & 0 \\ 0 & \xi & \cdots & 0 & 0 & 0 & \beta_2 & 0 & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \beta_J + \varsigma_J & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \beta_J + \varsigma_J \end{bmatrix},$$

$$\mathbf{V}_{def}^{n+1} = \begin{bmatrix} V_{def,1,1}^{n+1} \\ V_{def,2,1}^{n+1} \\ \vdots \\ V_{def,1,2}^{n+1} \\ V_{def,2,2}^{n+1} \\ \vdots \\ V_{def,I-1,J}^{n+1} \\ V_{def,I,J}^{n+1} \end{bmatrix}, \quad \mathbf{u}_{def}^n = \begin{bmatrix} u_{def}(z_1) + \chi V_{I,1}^n \\ u_{def}(z_1) + \chi V_{I,1}^n \\ \vdots \\ u_{def}(z_2) + \chi V_{I,2}^n \\ u_{def}(z_2) + \chi V_{I,2}^n \\ \vdots \\ u_{def}(z_J) + \chi V_{I,J}^n \\ u_{def}(z_J) + \chi V_{I,J}^n \end{bmatrix}.$$

Solution to the bond price equation The bond price equation (16) is

$$Q(a, z) (\bar{r} + \lambda + \pi(a, z)) = (\lambda + \delta) + s(a, z) \frac{\partial Q}{\partial a} - \mu z \frac{\partial Q}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 Q}{\partial z^2} - \phi d(a, z) Q(a, z)$$

which, using again a finite difference method identical to the ones already described above (this time no upwind is necessary either), yields

$$\frac{Q_{i,j}^{n+1} - Q_{i,j}^n}{\Delta} + (\bar{r} + \lambda + \pi_{i,j}^n) Q_{i,j}^{n+1} = (\lambda + \delta) + Q_{i-1,j}^{n+1} \varrho_{i,j} + Q_{i,j}^{n+1} \beta_{i,j} + Q_{i+1,j}^{n+1} \chi_{i,j} + Q_{i,j-1}^{n+1} \xi_j + Q_{i,j+1}^{n+1} \varsigma_j,$$

or

$$\frac{\mathbf{Q}^{n+1} - \mathbf{Q}^n}{\Delta} + (\bar{r} + \lambda + \mathbf{\Pi}^n) \mathbf{Q}^{n+1} = (\lambda + \delta) \mathbf{1}_{I \times J} + \mathbf{A}^n \mathbf{Q}^{n+1}, \quad (42)$$

where $\mathbf{1}_{I \times J}$ is a column vector of $I \times J$ ones, $\mathbf{\Pi}^n$ is a diagonal matrix with elements $\pi_{i,j}^n$ and

$$\mathbf{Q}^{n+1} = \begin{bmatrix} Q_{1,1}^{n+1} \\ Q_{2,1}^{n+1} \\ \vdots \\ Q_{1,2}^{n+1} \\ Q_{2,2}^{n+1} \\ \vdots \\ Q_{I-1,J}^{n+1} \\ Q_{I,J}^{n+1} \end{bmatrix}.$$

We also combine it with a relaxation scheme such that, if we denote the result of the linear system of equations (42) as $\hat{Q}_{i,j}^{n+1}$, then

$$Q_{i,j}^{n+1} = \varkappa \hat{Q}_{i,j}^{n+1} + (1 - \varkappa) Q_{i,j}^n. \quad (43)$$

Complete algorithm

The algorithm to solve the model is based on two loops, an inner loop that finds the value functions and bond prices given the optimal default, and an outer one which computes the optimal default.

Outer loop. Begin with an initial guess $V_{i,j}^0 = u(r a_i + z_j) / \rho$, $V_{def,i,j}^0 = [u_{def}(1) + \chi V_{I,j}^0] / (\rho + \chi)$, $Q_{i,j}^0 = 1$, $d_{i,j} = 0$ and set $m = 0$. Then:

1. Given $d_{i,j}^m$, run the inner loop to find $V_{i,j}^{m+1}$, $V_{def,i,j}^{m+1}$, $Q_{i,j}^{m+1}$ using as an initial guess $V_{i,j}^m$, $V_{def,i,j}^m$, $Q_{i,j}^m$.
2. Compute $d_{i,j}^{m+1}$ according to (14).

3. If $d_{i,j}^{m+1}$ is close enough to $d_{i,j}^m$ stop. If not set $m := m + 1$ and go to step 1.

Inner loop. Given a default policy $d_{i,j}^m = 0$, begin with an initial guess $V_{i,j}^0 = V_{i,j}^m$, $V_{def,i,j}^0 = V_{def,i,j}^m$, $Q_{i,j}^0 = Q_{i,j}^m$ and set $n = 0$. Then:

1. Compute $\partial_{a,F}V_{i,j}^n$, $\partial_{a,B}V_{i,j}^n$, $\partial_zV_{i,j}^n$ and $\partial_{zz}V_{i,j}^n$ using (32)-(35).
2. Compute $c_{i,j}^n$ and $\pi_{i,j}^n$ using (37) and (38), respectively.
3. Find $V_{i,j}^{n+1}$ solving the linear system of equations (39) plus the relaxation scheme (40).
4. Find $V_{def,i,j}^{n+1}$ solving the linear system of equations (41).
5. Compute $d_{i,j}^{n+1} = 1_{V_{def,i,j}^{n+1} > V_{i,j}^{n+1}}$.
6. Find $Q_{i,j}^{n+1}$ solving the linear system of equations (42) plus the relaxation scheme (43).
7. If $V_{i,j}^{n+1}$ is close enough to $V_{i,j}^n$ and $Q_{i,j}^{n+1}$ is close enough to $Q_{i,j}^n$, stop. If not set $n := n + 1$ and go to step 1.

Solution to the Kolmogorov Forward equation

Finally, we describe here the algorithm to solve the Kolmogorov Forward equation. Let $g(a, z)$ denote no-default ergodic distribution, that is the stationary share of time spent at debt a and log-income z while in good credit standing. It satisfies the following Kolmogorov Forward equation:

$$0 = -\frac{\partial}{\partial a} [s(a, z)g(a, z)] + \frac{\partial}{\partial z} [\mu z g(a, z)] + \frac{\sigma^2}{2} \frac{\partial^2 g}{\partial z^2} - \phi d(a, z)g(a, z) + \chi g^{def}(z) \delta(a), \quad (44)$$

where $s(a, z)$ is the drift function given by (4), g^{def} is the autarky ergodic distribution, defined as the stationary share of time spent in exclusion at debt a and log-income z and $\delta(\cdot)$ is the Dirac 'delta'.³⁶

The autarky ergodic distribution satisfies the following Kolmogorov Forward equation:

$$0 = -\frac{\partial}{\partial a} [s(a, z)g^{def}(a, z)] + \frac{\partial}{\partial z} [\mu z g^{def}(a, z)] + \frac{\sigma^2}{2} \frac{\partial^2 g^{def}}{\partial z^2} + \phi d(a, z)g^{def}(a, z) - \chi g^{def}(z) \delta(a). \quad (45)$$

³⁶The *Dirac delta* is a distribution or generalized function such that $\delta[f] = \int_{-\varepsilon}^{\varepsilon} f(x)\delta(x) dx = f(0)$, $\forall \varepsilon > 0$, $f \in L^1(-\varepsilon, \varepsilon)$. A heuristic characterization is the following:

$$\int_{-\infty}^{\infty} \delta(x) dx = 1, \\ \delta(x) = \begin{cases} \infty, & x = 0, \\ 0, & x \neq 0. \end{cases}$$

We solve the above equation using an upwind finite difference scheme as in Achdou et al. (2017). We use the notation $g_{i,j} \equiv g(a_i, z_j)$. The solution in matrix form is

$$\begin{aligned}\mathbf{A}^T \mathbf{g} + \chi \mathbf{g}_I^{def} &= 0, \\ \mathbf{A}_{def}^T \mathbf{g}^{def} + \phi \mathbf{D} \mathbf{g} &= 0,\end{aligned}$$

where \mathbf{g}_I^{def} is a $I \times J$ matrix of zeros at all positions except at $i = I$ where the j elements are $g^{def}(a_I, z_j)$ and

$$\mathbf{D}^n = \begin{bmatrix} d_{1,1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & d_{2,1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & d_{3,1} & 0 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & d_{I-1,J} & 0 \\ 0 & 0 & \cdots & 0 & 0 & d_{I,J} \end{bmatrix}.$$

In order to find the solution of this system we run an iterative algorithm with an initial guess $\mathbf{g}_I^{def} = \mathbf{0}$. We solve the system iteratively using a standard linear equation solver. Finally, we normalize the joint distribution $g + g^{def}$ to one.

Appendix C. Welfare decomposition

We first show how one can express the two components of the welfare decomposition (equations 20 and 21) recursively. We start by expressing the HJB equation of the repayment value function (equation 30) as

$$\begin{aligned}[\rho + \phi d(b, z)] V(b, z) &= u(c(b, z)) - \frac{\psi}{2} \pi(b, z)^2 + \mathbf{s}(b, z) \frac{\partial V}{\partial b} - \mu z \frac{\partial V}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial z^2} \\ &\quad + \phi d(b, z) V_{def}(z).\end{aligned}$$

Similarly, we express the HJB equation of the default value function (equation 8) as

$$(\rho + \chi) V_{def}(z) = u_{def}(z) - \mu z \frac{\partial V_{def}}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_{def}}{\partial z^2} + \chi V(0, z).$$

Using our postulated decompositions, $V(b, z) = V_c(b, z) + V_\pi(b, z)$ and $V_{def}(b, z) = V_{def,c}(b, z) + V_{def,\pi}(b, z)$, in the above two expressions, and ignoring function arguments except where needed,

we obtain

$$\begin{aligned}
[\rho + \phi d] (V_c + V_\pi) &= u(c) - \frac{\psi}{2} \pi^2 + \mathbf{s} \left(\frac{\partial V_c}{\partial b} + \frac{\partial V_\pi}{\partial b} \right) - \mu z \left(\frac{\partial V_c}{\partial z} + \frac{\partial V_\pi}{\partial z} \right) \\
&\quad + \frac{\sigma^2}{2} \left(\frac{\partial^2 V_c}{\partial z^2} + \frac{\partial^2 V_\pi}{\partial z^2} \right) + \phi d (V_{def,c} + V_{def,\pi}),
\end{aligned}$$

$$\begin{aligned}
(\rho + \chi) (V_{def,c} + V_{def,\pi}) &= u_{def} - \mu z \left(\frac{\partial V_{def,c}}{\partial z} + \frac{\partial V_{def,\pi}}{\partial z} \right) + \frac{\sigma^2}{2} \left(\frac{\partial^2 V_{def,c}}{\partial z^2} + \frac{\partial^2 V_{def,\pi}}{\partial z^2} \right) \\
&\quad + \chi [V_c(0, z) + V_\pi(0, z)].
\end{aligned}$$

We can then write

$$\begin{aligned}
[\rho + \phi d] V_c &= u(c) + \mathbf{s} \frac{\partial V_c}{\partial b} - \mu z \frac{\partial V_c}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_c}{\partial z^2} + \phi d V_{def,c}, \\
[\rho + \phi d] V_\pi &= -\frac{\psi}{2} \pi^2 + \mathbf{s} \frac{\partial V_\pi}{\partial b} - \mu z \frac{\partial V_\pi}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_\pi}{\partial z^2} + \phi d V_{def,\pi}, \\
(\rho + \chi) V_{def,c} &= u_{def} - \mu z \frac{\partial V_{def,c}}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_{def,c}}{\partial z^2} + \chi V_c(0, z), \\
(\rho + \chi) V_{def,\pi} &= 0 - \mu z \frac{\partial V_{def,\pi}}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_{def,\pi}}{\partial z^2} + \chi V_\pi(0, z).
\end{aligned}$$

These four value functions can be then be solved using finite-difference methods similar to those described in Appendix B.

Appendix D. Additional figures

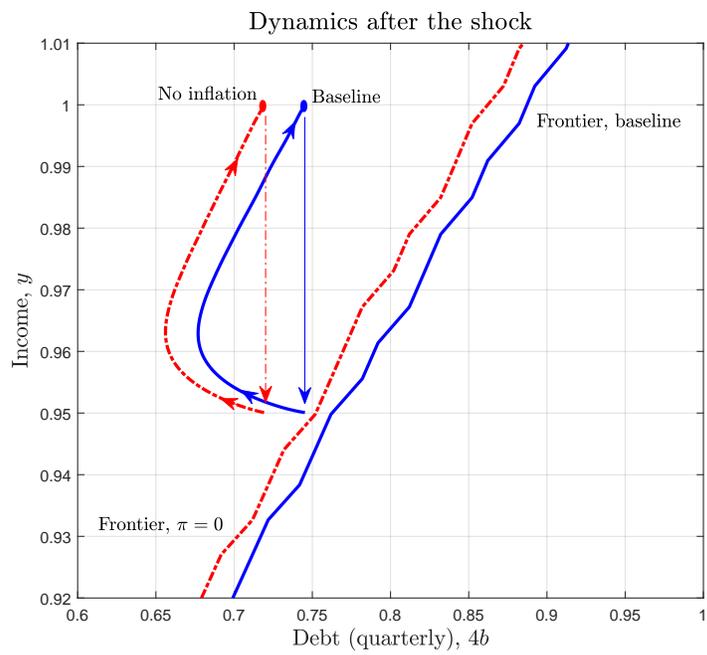


Figure 11: Phase diagram (negative shock). Notes: The circles mark the stochastic steady states (initial points).

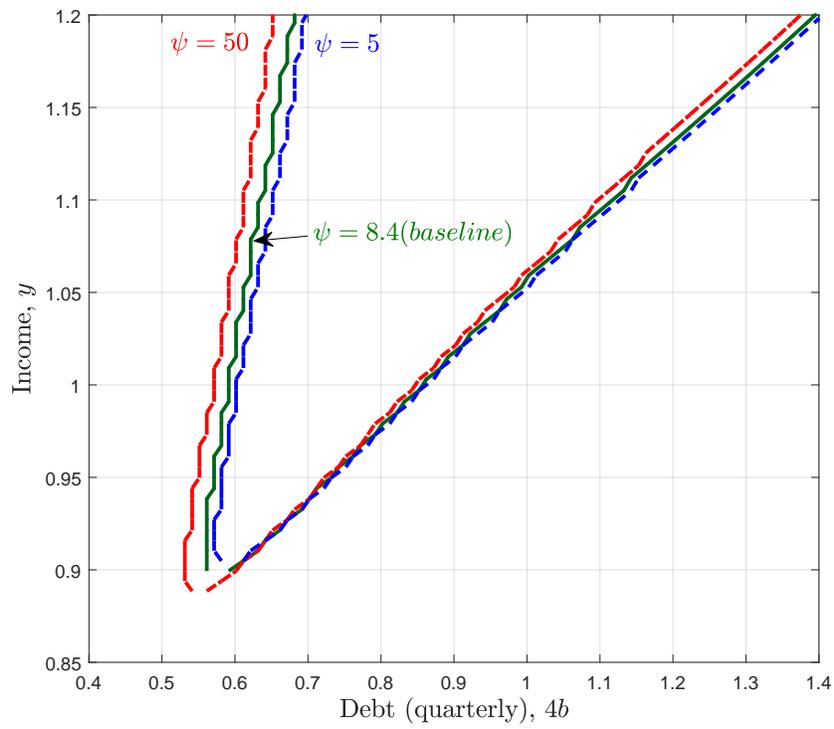


Figure 12: Isowelfare curves ($V = V^{\pi=0}$) for different values of ψ .