The Role of Wage Expectations in the Labor Market

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Abstract

High volatility in the U.S. labor market, coupled with a low correlation between labor market variables and productivity, presents a challenge for the traditional search and matching models. This paper develops a search and matching model with internally rational agents who hold subjective wage expectations. This approach significantly improves alignment with U.S. labor market data, outperforming the standard rational expectations model. The model's wage expectations are consistent with data from European Commission professional forecasters, adding a dynamic source that enhances the model's fit to labor market moments.

Keywords: Internal Rationality, Wage Expectations, Labor Markets, Subjective Expectations, Belief Shock.

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1 Introduction

The Search and Matching Model (DMP) has become the standard equilibrium unemployment theory. However, several studies question the model's ability to accurately represent labor market fluctuations in the United States.[1] In particular, the standard DMP struggles to replicate observed fluctuations in the labor market and the propagation of productivity shocks. Targeting the ratio of standard deviations between labor market variables and productivity has been the focus of much research. However, the near zero correlation between productivity and labor market tightness post-1989 has been largely neglected in the literature. In this paper, I show that a DMP model is able to reproduce these observations if one allows for small deviations from rational expectations (RE).

I study how to introduce internal rationality (IR) in a DMP model.[2] I relax the standard assumption that agents have perfect knowledge about the wage function obtained from the standard Nash bargaining process. Agents have limited foresight and can not perfectly predict the outcome of wage bargaining, instead workers and firms have subjective beliefs about wages, and they maximize their objective functions subject to their constraints. I call such agents "internal rational" because they know all internal aspects of their problem and maximize their respective objective functions given their knowledge about the wage process. I consider systems of beliefs implying only a small deviation from RE, and that match some aspects of survey wage expectations. The model has a self-referential mechanism: shifts in beliefs about future returns to labor affect current wages, and agents use realized wages to update their beliefs. This generates an additional source of dynamics that helps to match the data. Framing the model under IR provides a microfoundation to previous adaptive learning papers on unemployment.[3]

Moreover, I present a formal econometric test of the null hypothesis that survey evidence is consistent with RE, demonstrating that this hypothesis is rejected by the survey data on real wage expectation. This finding introduces an additional puzzle for the standard version of the DMP model. The data for this analysis is sourced from the European Commission's professional forecasters. A notable feature of this test lies in its ability to reveal the underlying reasons for the rejection of the RE hypothesis: the failure arises because survey-based expectations and rational expectations exhibit different covariation with real GDP growth. Specifically, wage expectations display a significant lower covariation with GDP growth than actual wage realizations, indicating that agents perceive wages to be more rigid than they are in reality. This finding is useful to discipline expectations in the model of IR.

To quantitatively evaluate the learning and the RE models, I consider how well each framework replicates key labor market moments. Using a formal structural estimation approach based on simulated moments (MSM), I adapt the results of

[1]See Shimer (2005), Hall (2005), Fujita and Ramey (2003), Costain and Reiter (2008).
[2]See Adam and Marcet (2011).
[3]See Schaefer and Singleton (2018) and Di Pace et al. (2021).

Duffie and Singleton (1990) to estimate certain model parameters. Subsequently, I conduct a formal test to determine whether the model statistics significantly deviate from their empirical counterparts. The learning model provides a notably more accurate representation of U.S. labor market data compared to the RE model.A central finding is its capacity to generate a low contemporaneous correlation between labor market tightness and productivity, along with elevated relative volatilities across labor market variables. For instance, the learning model produces relative volatilities of unemployment and the vacancy-unemployment ratio that are approximately 32 and 9 times higher, respectively, than those generated under rational expectations.

Most RE models require wages to exhibit minimal responsiveness to productivity changes to achieve higher volatility, leading to a wage volatility that is less than that of productivity—a feature inconsistent with empirical observations. In my approach, wages are not rigid, they adjust in response to both productivity fluctuations and agents' evolving expectations, resulting in wage volatility that exceeds productivity volatility, aligning more closely with empirical data. Furthermore, the model replicates additional labor market dynamics, capturing the low comovement of both vacancies and unemployment with productivity. Moreover, the IR model matches the low comovement between wage expectations and GDP found in surveys of professional forecasters, while simultaneously capturing a stronger comovement between actual wage realizations and GDP. Additionally, the model matches the autocorrelation of forecast errors and produces a positive standard deviation of forecast errors, consistent with survey evidence. This flexibility in wage expectations enhances the model's ability to mirror real labor market dynamics without relying on imposed wage rigidities.

The reduction in the correlation between labor market variables and labor productivity arises from the additional source of variability introduced by the learning mechanism, which significantly influences job creation dynamics. Under RE, labor market tightness is determined almost exclusively by current productivity, resulting in a correlation close to one. In contrast, under IR, labor market tightness is affected not only by productivity but also by time-varying coefficients governing wage expectations. This added complexity reduces the correlation between labor market variables and productivity, allowing the model to more accurately reflect observed labor market behavior.

The presence of subjective expectations introduces an endogenous mechanism that amplifies productivity shocks in the labor market. To understand how this mechanism generates volatility, imagine a recession arrives. Firms are uncertain about how much wages will decrease, but as survey data suggests and the model match, they believe that wages will only adjust slightly to changes in productivity. Hiring decisions, which are based on projections of future profits per hire, require a forecast of both future productivity and wages over an indefinite horizon. In the IR model, when productivity falls during a recession, firms belief that future wages will not decline significantly. Consequently, they expect lower profits compared to the RE scenario, where firms anticipate a stronger wage adjustment. As a result, under

IR, firms post fewer vacancies during downturns, amplifying the negative impact of the shock. Following the initial shock, pessimistic outlooks drive firms and workers to negotiate wages downwards—more than firms initially expected when making their hiring decisions. This creates a disconnect: firms initially reduced job postings due to anticipated high wage costs, but realized wages turn out to be lower than expected. However, due to the adaptive learning process in IR, firms do not immediately revise their expectations; instead, they adjust them gradually over time as they receive new information. This slow adjustment means that it takes several periods for wage expectations to align with reality, sustaining low vacancy postings and leading to persistently high unemployment.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 tests the RE assumption with data from professional forecasters. Section 4 describes the model. Section 5 presents the calibration of the model and summarizes the main results. Section 6 performs some robustness exercises. Lastly, section 7 concludes.

2 Related Literature

This model aligns with efforts to address the Shimer puzzle in the search and matching model literature. Two primary solutions stand out in the literature. (I) Change in wage formation, as suggested by Shimer (2005), Hall (2005), Gertler and Trigari (2009), where wages do not fully adjust to productivity shifts, thereby spurring job creation. (II) Calibration changes, as proposed by Hagedorn and Manovskii (2008), which enhance firm bargaining power and unemployment benefits, thereby inducing endogenous wage rigidities. However, these methods have faced criticism, and there remains no consensus in the literature on how to solve the puzzle.[4] Although these models generate volatility in the labor market, they fall short in explaining the near-zero correlation between labor market variables and productivity, and the slightly higher wage volatility relative to productivity. Departing slightly from RE, I introduce more rigid expectations rather than rigid wages. To the best of my knowledge, this is the first paper to propose a model that is capable of generating high volatility in the labor market, a subdued correlation between vacancy-unemployment ratio and labor productivity, flexible wages, and a rationale for wage expectation surveys.

Recent studies have examined the DMP model while departing from the full information rational expectations (FIRE) assumption. For example Morales-Jiménez (2022) and Menzio (2022) explore scenarios where workers misperceive the true process of productivity. In this papers, workers misperceive the true process for productivity. Moreover, workers are assumed to know the mapping from productivity to wages. In these models, agents still require immense knowledge of market behavior. Alternatively, I endow agents with uncertainty regarding how wages are

[4]These approaches were criticized by Pissarides (2009), Haefke et al. (2013), Mortensen and Nagypal (2007) and Costain and Reiter (2008). There are more solutions to generate volatility in the labor market; see Costain and Reiter (2008), Silva and Toledo (2009), Reiter (2007), and Menzio (2005) among others.

linked to productivity. This assumption is validated using wage expectation surveys. My model effectively captures the observed correlation between wage forecast errors and GDP, as documented in survey data. Specifically, it generates a significantly reduced covariance between wage expectations and GDP, which stands in contrast to the one implied by rational expectations models. In contrast, Morales-Jiménez (2022) and Menzio (2022) do not use survey data to test the productivity expectations hypothesis, a gap that my paper addresses.

This paper extends the adaptive learning literature, with applications outlined in Evans and Honkapohja (2001), Bullard and Mitra (2002) and Eusepi and Preston (2011). Recently, the introduction of a standard adaptive learning approach in the search and matching model has been studied. Schaefer and Singleton (2018) find that when agents make one-step-ahead forecast of labor market tightness, the learning model struggles to capture labor market volatility. Conversely,Di Pace et al. (2021) find that when agents use a misspecified model for wage expectations, while this approach amplifies labor market dynamics, it overstates wage fluctuations and does not appreciably adjust the correlation between labor market variables and productivity. Among these studies, Di Pace et al. (2021) is the most closely related to this work. The main difference lies in the way agents form wage expectations. In my paper, agents use productivity directly to form wage expectations, a fact I test with survey data, while in their paper they form wage expectations using an autoregressive model, implying that agents have an inaccurate model to form such expectations. This paper builds on the adaptive learning literature, but maintains the rationality of the agents. Importantly, it is also specific about beliefs system that the agents have in the economy.[5] Both these modeling features are the hallmark of the Internal Rationality framework developed by Adam and Marcet (2011). This approach has not been applied to the search and matching model before and can provide a micro-foundation for adaptive learning models.

A vibrant literature has recently developed studying the behavior of expectation surveys. Several papers show that there is a significant discrepancy between the expectations implicit in the macroeconomic model under RE and those coming from survey data; see Conlon et al. (2018), Greenwood and Shleifer (2014), Adam et al. (2017), Malmendier and Nagel (2016), Coibion and Gorodnichenko (2012),Coibion and Gorodnichenko (2015); among others. I applied the statistical test proposed by Adam et al. (2017) to see whether the data support the rational expectation assumption regarding the formation of future wages. The results indicate that professional forecasters do not form wage forecast in accordance with rational expectations. Furthermore, the test provides insights into why the RE hypothesis fails, which I used as a guide to validate the model.

[5]The adaptive learning literature does not specify what agents' views are on the evolution of macro-variables. They only equip them with a recursion, which tracks some moments of the variable. If beliefs are not fully specified in the model, then why, exactly, agents must form expectations according to a given recursion and how this relates to rational behaviour is unclear.

3 Wages and Wage Forecast

Wage expectations play an important role in the labor market decisions. In the search and matching framework, they affect the match surplus and therefore, current wages and also, the hiring decisions made by firms. In the standard DMP model, workers and firms bargain about the wage and the equilibrium wage equation is known by them. More precisely, all agents are assumed to know the mapping from observed productivity shocks to equilibrium wages. This "complete information" assumption is commonly made, although rarely proven, because expectations are very rarely observed.[6]

This section shows that wage forecasts are inconsistent with the notion that agents hold rational wage expectations. I present a formal econometric test following Adam et al. (2017) which shows that wage expectations and rational expectations exhibit different covariation with real GDP. The test also provides insights into the reasons why the RE hypothesis is rejected, helping to identify specific discrepancies between expected and realized economic outcomes. To conduct this test, I use survey data from professional forecasters provided by the European Commission. In this dataset, the observed covariance between wage expectations and real GDP is significantly lower than the one implied by rational expectations.

3.1 Professional Forecasters

This section conducts a test for rational expectations using survey data that comprises the average annual wage growth forecast per employee in the United States, reported by the European Commission for the period of 1999 to 2020.[7]

Let $E_t^{\mathcal{S}}$ denote the agent's subjective expectation operator based on information up to time t, which can differ from the rational expectation operator E_t. Let $\hat{w}_{t+4}$ denote the four-period ahead realized annual growth of wages, and let s_k be a measure of agent's subjective beliefs regarding future growth of wages that are possibly subject to measurement error, ν_t, obtained from survey data. Therefore, $s_{t+4} = E_t^{\mathcal{S}}(\hat{w}_{t+4}) + \nu_t$ represents an estimate of the agents' subjective beliefs about annual wage growth four quarters ahead. Given the forecast horizon of professional forecasters, t stands for quarters.

$$\hat{w}_{t+4} = c^R + b^R \hat{y}_t + \eta_t, \tag{1}$$

$$s_{t+4} = c^E + b^E \hat{y}_t + \epsilon_t, \tag{2}$$

[6] Conlon et al. (2018), using the Survey of Consumer Expectations (SCE) Labor Market Survey from the New York FED, found a significant correlation between labor force's revisions of wage offer forecasts and their forecast errors. This finding supports the existence of information rigidities in forming expectations about future wage offers. Also Di Pace et al. (2021) find that the forecast error of professional forecasters is correlated with the gdp growth.

[7] The forecast is reported twice a year in Autumn and Spring. They just report the average forecast. Link reports: https://ec.europa.eu/economy_finance/publications/european_economy/forecasts/index_en.htm

Indep. variable	b^R	b^E	P-value H$_0 : b^R = b^E$	P-value H$_0 : b^R \leq b^E$
$\hat{y}_t$	0.984***	0.192***	0.002	0.001
	(4.407)	(3.349)		
$\hat{y}_{t-1}$	0.961***	0.984***	0.049	0.024
	(2.753)	(4.407)		

Table 1: RE test

Note: ****,**,* denote sig. at 1%, 5% and 10% levels, respectively. t statistics in parentheses. The Table presents the results of the test $b^R = b^E$. The third row shows the results of the test where I include the independent variable with a lag of half a year. The p-values for the test are constructed using bootstrapping, 1000 bootstrap samples. The number of observations is 42.*

where $\hat{y}$ represents annual real gdp growth. Under the null hypothesis of RE ($H_0 : E_t = E_t^{\mathcal{S}}$), if $\hat{y}_t$ is in the informational set of agents for time period t, the prediction error must be orthogonal to $\hat{y}_t$. $\hat{b}^R$ and $\hat{b}^E$ must be estimates of the same regression coefficient because $b^R = b^E$.[8] If the coefficients across the two equations differ, the RE hypothesis is rejected.[9]

Table (1) shows the result of the test. Column 4 displays the p-values, while column 5 presents the p-values for the right-tailed test. As a robustness exercise, in the third row, I report the results when the test is performed with annual productivity growth lagged, ensuring that the variable is in the informational set of the professional forecaster. The results provide strong evidence against the hypothesis that survey expectations of wages are compatible with RE. This rejection arises because survey expectations and rational expectations covary differently with real GDP growth, which implies that professional forecasters perceive wages as being more sticky and less sensitive to changes in economic conditions compared to what the RE hypothesis would suggest. As a result, the forecast error of wages is correlated with real GDP growth, contrary to what RE would predict.

An intriguing observation emerges from the data: during recessions, professional forecasters tend to overestimate wage growth, whereas during expansionary peri-

[8] Di Pace et al. (2021) employ the same survey data up to 2018Q3. However, rather than employing regression (1) and (2) to test Rational Expectations (RE), they examine the correlation between the forecast error, $\hat{w}_{t+4} - s_{t+4}$, and GDP growth. While this is a valid approach, as supported by Coibion and Gorodnichenko (2012, 2015), it does not allow for an exploration into the potential association between GDP growth and the forecast of wage growth.

[9] Coibion and Gorodnichenko (2012, 2015) bring evidence in favor of information rigidity in expectation formation described by a significant correlation between forecast revisions and forecast error.

$$s_{t+2/t} - s_{t+2/t-1} = c + b(\hat{w}_{t+2} - s_{t+2/t}) + \epsilon_t$$

Using my data, b=0.047, the non-significance (p-value of 0.171) can be due to the fact that the measurement error of the survey data makes the explanatory variables correlated with the residual and gives a bias coefficient.

ods, they underestimate it. For instance, amidst the Great Recession, forecasters predicted an average annual wage growth of 0.99%. In contrast, the actual average annual growth in the correspondent period declined by 4.21%. Between Q1-2011 and Q3-2016, a period of economic expansion, the pattern reversed. Forecasts anticipated a growth of 0.76%, yet the actual realization was an impressive 3.88%. Such disparities in wage growth predictions could potentially account for the pronounced fluctuations observed in the labor market. For example, during an expansionary period, a firm that anticipates lower future wages might be inclined to post more job vacancies.

Additionally, I find that the forecast error, defined as $\hat{w}_{t+4} - s_{t+4}$, exhibits serial autocorrelation, with an autocorrelation coefficient of 0.68 and a p-value of 0.000. This finding also challenges the rational expectations theory, further supporting the presence of rigidities in how agents form expectations about wage growth..

4 The Model

I propose a model featuring labor market search and matching friction as in Mortensen and Pissarides (1994) applied to the business cycle. Under the standard setting of RE, agents understand how productivity maps to wages. Instead, I assume the lack of common knowledge of general equilibrium wage mapping and equip agents with a fully specified system of beliefs. Agents form their expectations about the future path of wages based on their respective perceived law of motion (PLM) and update their beliefs as new information becomes available. Given their expectations, agents take optimal decisions. Two shocks can hit the economy: a productivity shock and a shock that affects the agents' beliefs about their expected wages. At the start of a period, shocks occur. Agents forecast future wages, influencing employment surplus of workers, firms' hiring surplus and vacancy decisions. Should a match occur, wages are then bargained over. The period concludes with certain jobs destroyed exogenously.

4.1 The Labor Market

Following the standard literature, this economy is characterized by frictions in the labor market. There is a time-consuming and costly process of matching workers and job vacancies, which is capture by a standard constant returns to scale matching function *m(u,v)* where u denotes the unemployment rate and v is the vacancy rate. I refer to $\theta_t = \frac{v_t}{u_t}$ as the market tightness at time t. Hence, the rate at which unemployed workers find a jobs, $f(\theta)$, and vacancies are filled $q(\theta)$ depend of the vacancy-unemployment ratio, where $f(\theta) = \theta q(\theta)$ and $f(\theta)' > 0$, $q(\theta)' < 0$. The unemployment rate increases when jobs are destroyed at a exogenous rate, λ, and decreases when workers find jobs. Thus,employment evolves according

$$n_{t+1} = (1 - \lambda)n_t + q(\theta_t)v_t. \tag{3}$$

The labor productivity takes the form of stationary AR(1) in logs:

$$\ln(y_t) = (1-\rho)\ln(\overline{y}) + \rho\ln(y_{t-1}) + \epsilon_t,\ 0 < \rho < 1. \tag{4}$$

Where $\epsilon_t \sim N(0, \sigma^2)$ and ρ measures the persistence.

4.2 Worker's Problem

There is a continuum of identical, risk neutral workers with total measure one and an infinite horizon. These workers can either be employed or unemployed in each period.

An employed worker earns a wage w_t at t, and faces a probability λ of losing his job in the subsequent period. Conversely, an unemployed worker receives unemployment benefits b and has a probability $f(\theta_t)$ of finding a job in the next period. The wage process, w_t, and the tightness of the labor market, θ_t, are given by individual workers. Individual workers have nothing to choose, whether they are employed or not is determined exogenously. The primary calculation where their expectations will play a role is the net surplus of the match that is used to bargain the wage with the firm if the match is realized. This surplus is the difference between the value of being employed and unemployed.

Deriving the standard surplus of the worker hides many assumptions that I wish to bring out in this section. The worker surplus depends on expectations and expectations are determined with a probability measure $\mathcal{P}^w$. The definition of $\mathcal{P}^w$ depends on exactly how much workers are assumed to know about the equilibrium process for n, θ and w and about the properties of these variables. So, I start with a general definition of $\mathcal{P}^w$ that is consistent with the above setup and that encompasses a number of standard equilibrium concepts that are found in the literature. This will be useful, first, to unveil some assumptions in the adaptive learning literature that are often not explicitly stated and it will allow me to extend those equilibrium concepts. Then, I obtain step by step some familiar derivations in the literature and explain how each derivation depends on an increasing amount of assumptions. This provides a clear comparison of the IR equilibrium studied in the paper with RE and with some adaptive learning versions of the model.

4.2.1 A generic worker problem under Internal Rationality

Consider first the case where I do not make any assumption about the relation between workers' beliefs and actual equilibrium. The next subsection will cover the case of RE as well as the case of Bayesian/RE.

If workers are rational, at the very least, the state space for the measure $\mathcal{P}^w$ has to contain the payoff relevant variables for individual workers that are beyond the agent's control, therefore $\mathcal{P}^w$ puts probabilities on sequences $\{(w, \theta, n)^t\}_{t=0}^{\infty}$.[10]

[10] From the point of view of probability theory I should also state that the probabilities $\mathcal{P}^w$ are defined on the sets of a sigma algebra of the mentioned sequence space, but since, it is obvious

$(w,\theta,n)^t$ is the usual notation describing sequences up to t, and it is understood that $E_t^{\mathcal{P}^w}$ responds to the usual definition meaning "conditional expectation given $(w,\theta,n)^t$".

Following, I state the first assumption on beliefs

Assumption 1. The belief system $\mathcal{P}^w$ is Markov up to a state vector m. More precisely,

$$\begin{aligned} Prob^{\mathcal{P}^w}(w_t,\theta_t,n_t \mid (w,\theta,n)^{t-1}) &= \mu(m_{t-1}), \\ m_t &= g(m_{t-1},y_t,w_t,\theta_t,n_t). \end{aligned} \tag{5}$$

For some given functions μ, g conformable to their arguments and for a vector m_t that contains θ_t, w_t, n_t. In standard IR models, m will also contain variables that in the workers' mind summarize the best forecast of future wages, as is the case in the main sections of this paper.

Now, I can formulate the value functions for the worker.

The present value of working for an agent is as follows:

$$\mathcal{W}(m_t) = w_t + \beta E_t^{\mathcal{P}^w}\left[(1-\lambda)\mathcal{W}(m_{t+1}) + \lambda\mathcal{U}(m_{t+1})\right]. \tag{6}$$

On the other hand, workers can be unemployed. The present value of unemployment is given by:

$$\mathcal{U}(m_t) = b + \beta E_t^{sw}[f(\theta_t)\mathcal{W}(m_{t+1}) + (1-f(\theta_t))\mathcal{U}(m_{t+1})]. \tag{7}$$

Where $\mathcal{W}$ and $\mathcal{U}$ are time-invariant functions.

It may seem that this is enough to arrive at a standard equation for worker's surplus, $\mathcal{W}-\mathcal{U}$. But since I have not given any market knowledge to agents, they still do not necessarily know the equilibrium process of θ unless I make the following additional assumption.

Assumption 2. Individual workers have a model that forecast correctly the true evolution of θ. Formally, $Prob^{\mathcal{P}^w}(\theta_t=\theta|m_t=m) = Prob^{\mathcal{P}}(\theta_t=\theta|m_t=m)\ \forall(\theta,m)$.

Only under all these assumptions I get the workers' share of the total surplus is:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = w_t - b + \beta\,(1-\lambda-f(\theta_t))\,E_t^{\mathcal{P}^w}\left(\mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right). \tag{8}$$

This equation would be satisfied when agents learn about wages, as long as Assumptions 1-2 hold. Learning problem remains hidden in the belief structure $\mathcal{P}^w$. In section 4.4, I provide an explicit system of beliefs $\mathcal{P}^w$.

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how to set this up and it does not have an impact on any application to search models we will not mention sigma algebras anywhere else in the paper.

4.2.2 The individual problem under RE

Assume now that agents are endowed with the knowledge that wages are a function of the productivity, y_t, that is I include in (5) an equation giving w_t as an exact function of y_t. This is summarize in assumption 3.

Assumption 3. The system of equations (5) includes

$$w_t = \mu_w(y_t). \tag{9}$$

In addition, assume that agents know the law of motion of productivity, i.e. they know equation (4). In this paper, I focus on the RE equilibrium that takes the form of the fundamental or minimum state variable solution (MSV).[11] With these additional assumptions then, indeed, we have that $m_t = (y_t)$. In this case, market wages carry only redundant information. This allows to exclude wages from the state space without loss of generality.

Additionally, I have to assume the following.

Assumption 4. Agents' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_w(y_t)$.

Then workers have RE.

4.3 Firms Problem

Consider an economy populated by a mass of infinity firms. Firms' revenues are $y_t n_t$, where n_t and y_t are exogenous to the firms. The productivity, y_t follows a AR(1) process (4). Firms pay a total of $w_t n_t$ at t, the wage process is taken as given by firms. Each period firms choose the number of vacancies v to post at a constant ongoing cost c. Their period-t profits, Π_t, is $y_t n_t - w_t n_t - cv_t$.

A key feature of equilibria will be the firms' expected discounted profits from period t onwards, given by

$$\Pi_t \equiv E_t^{\mathcal{P}^f}\left(\sum_{j=0}^{\infty}\beta^j[y_{t+j}n_{t+j} - w_{t+j}n_{t+j} - cv_{t+j})]\right), \tag{10}$$

where $\mathcal{P}^f$ is the firm' probability measure about relevant future variables.

To derive the standard job creation condition, it is common in the literature to appeal to dynamic programming to write Π_t in a forward recursive form. In the next subsection, I set out the necessary assumptions to derive this equation. The definition of $\mathcal{P}^f$ depends on exactly how much firms are assumed to know about the

[11] While there may be RE equilibria contradicting this assumption, with added lags in wage determination, Campbell (1994) shows that the RE solution has w_t as an ARMA(2,1) process. Adhering to McCallum (1983), I select the minimal state variable set that's indispensable for a solution.

equilibrium process for w, y, θ, n, and about the properties of these variables. Therefore, as in the workers problem, I start with a general definition of $\mathcal{P}^f$ and derive step by step some familiar derivations and explain how each derivation depends on a large amount of assumptions.

4.3.1 A generic firm problem under Internal Rationality

This subsection will cover the case of RE as well as the case of Bayesian/RE for the firms' problem. I do not make any assumption about the process for equilibrium variables nor about the relation between firms' beliefs and actual equilibrium.

The state space for the measure $\mathcal{P}^f$ has to contain all payoff-relevant variables for individual firms. Hence, $\mathcal{P}^f$ puts probabilities on sequences $\{(w, \theta, y)^t\}_{t=0}^{\infty}$. $(w, \theta, y)^t$ is the sequences up to t. $E_t^{\mathcal{P}^f}$ in (10) represents the "conditional expectation given $(w, \theta, y)^t$".

Since Π_t is still a function of the whole sequence $(w, \theta, y)^t$, to obtain a recursive formulation, I need to add assumptions 1 of the workers' problem, that set that the belief system is a Markov up to a state vector, together with the transversality condition, $E_t^{\mathcal{P}^f} \beta^j \Pi_{t+j} \to 0$ as $j \to \infty$ almost surely in $\mathcal{P}^f$.

Additionally, to set the problem and derive the job creation condition, I have to add *assumption* 2 from the worker' problem, that firms forecast correctly the labor market tightness, and the following *assumption* 5.[12]

Assumption 5. Individual firms know the law of motion of n.

Taking into account previous assumptions, firms make contingent plans for vacancy posting subject to the evolution of employment (3). Now, I can state the maximization problem of the firms:

$$\Pi(m_t) = \max_{v_t \geq 0} \quad y_t n_t - w_t n_t - c v_t + E_t^{\mathcal{P}f} \ \Pi(m_{t+1}) \tag{11}$$

subject to

$$n_{t+1} = (1 - \lambda) n_t + q(\theta_t) v_t. \tag{12}$$

Below, I will specify the probability measure through some perceived law of motion describing the firm's view about the evolution of (w_t, y_t) over time, together with a prior distribution about the parameters governing this law of motion. Optimal behavior will then entail learning about these parameters, in the sense that agents update their posterior beliefs about the unknown parameters in the line of new wage, and productivity observations. For the moment, this learning problem remains hidden in the belief structure $\mathcal{P}^f$.

[12]In Garcia-Rodriguez and Pinilla-Torremocha (2021), we relax assumption 2 in the DMP model.

Optimality Conditions. The firm's optimal plan is characterized by the first order condition, together with the envelop condition with respect to n_t.

$$E_t^{\mathcal{P}^f} \mathcal{J}_{t+1} = \frac{c}{\beta q(\theta_t)}, \tag{13}$$

$$\mathcal{J}_t = y_t - w_t + \beta(1-\lambda) E_t^{\mathcal{P}^f} \mathcal{J}_{t+1}. \tag{14}$$

where $\mathcal{J}_t = \frac{\partial \Pi(m_t)}{\partial n_t}$ represents the marginal value of having an additional worker employed at the firm. Therefore, equation (14) gives the surplus of the firm coming from a match. Combining (13) and (14) and iterated forward, I come up with the job creation condition

$$\frac{c}{q(\theta_t)} = E_t^{\mathcal{P}^f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j \left[\frac{y_{t+j} - w_{t+j}}{1-\lambda} \right]. \tag{15}$$

This equation would be satisfied when agents learn about wages, as long as all previous assumptions hold. Therefore, in this case I have that the usual job creation condition, but I still need a generic m in (13) and (14).

4.3.2 The individual firm problem under RE

Analogous to section 4.2.2 of the worker's problem, assume now that firms are endowed with some knowledge of how wages are formed, i.e., assumption 4 holds. Also, firms know the law of motion of y and firms' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_f(y_t)$.

Then, firms have RE.

4.4 Agents' Belief System

Once one departs from rational expectations, beliefs become part of the microfoundations of the model. Previous sections left open how $\mathcal{P}^w$ and $\mathcal{P}^f$ incorporate wage beliefs. In this section, I introduce a fully specified probability measure $\mathcal{P}$ and derive the optimal belief updating equation it implies. For simplicity, I assume that this part of beliefs is common to $\mathcal{P}^w$ and $\mathcal{P}^f$. Nevertheless, agents may not know that this is true prior to wage bargaining. It is important to understand how agents view the wage process to specify an internally consistent rational agent model. The belief system of internally rational agents requires that they do not make obvious mistakes while learning.

Agents have the following perceived law of motion (PLM) which they use to make forecast of wages:

$$\begin{aligned} w_t &= d_t^c + d_t^y y_{t-1} + \epsilon_t, \\ D_t &= D_{t-1} + \nu_t. \end{aligned} \tag{16}$$

Where $D_t = [d_t^c \ \ d_t^y]$. Shocks $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other. This PLM considers a fundamental or minimal state variable solution with unobserved coefficients.

Consider the case where agents' prior beliefs are centered at the REE with the prior variance $\sigma_{D,0}^2$:

$$D_0 \sim \mathcal{N}(D^{REE}, \sigma_{D,0}^2 I_2). \tag{17}$$

Where $\sigma_{D,0}^2$ is set to the steady-state Kalman filter variance. Note that the agents' beliefs (16) encompass the REE of the model. In particular, when agents believe $\sigma_\nu^2 = 0$ and assign probability 1 to $D_0 = D^{REE}$, I have that $D_t = D^{REE}$ for all $t \geq 0$ and wages are given by RE equilibrium wages in all periods. Alternatively, if (17) is combined with a belief that σ_ν^2 is small, even though the resulting dynamics of the economy are not going to be precisely given by REE, it will be close to REE.

Agents' posterior beliefs at any time t are given by

$$D_t \sim \mathcal{N}(\hat{D}_t, \sigma_{D,t}^2 I_2). \tag{18}$$

Given that agents are rational, they update $\hat{D}_t$ according to the recursive least squares (RLS) algorithm:

$$\begin{aligned} \hat{D}_t &= \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[\mathbf{w}_{t-1} - \hat{D}_{t-1}' z_{t-1}] + \epsilon_t^\beta, \\ R_t &= R_{t-1} + \gamma(z_{t-1} z_{t-1}' - R_{t-1}). \end{aligned} \tag{19}$$

Where $\hat{D}_t = [\hat{d}_t^c \ \ \hat{d}_t^y]'$ represent the estimated coefficients, R_t denotes the moment matrix for $z_{t-1} = [1 \ \ y_{t-1}]$ and $\mathbf{w}_t$ denotes the realized previous wage. $\epsilon_t^\beta \sim \mathcal{N}(0_{2,1}, \sigma^{\beta 2} I_2)$ is a shock to wage beliefs and γ denotes the steady state Kalman gain $\in$ (0,1) that determines the rate at which older observations are discounted.[13] Strictly speaking, given the above information structure the Kalman filter requires $\sigma^{\beta 2} = 0$. This shock to beliefs can be interpreted as additional information about ν_t available to agents or as a departure from fully rational belief formation.

These beliefs constitute a small deviation from RE beliefs in the limiting case with vanishing innovation to the random walk process. Agents' prior uncertainty then vanishes, and the optimal gain goes to zero. As a result, one recovers the RE equilibrium value for wages.

4.5 Wage Bargaining

Wages are negotiated according to a Nash bargaining process. Each agent calculates its respective surplus from its problem, taking into account its system of beliefs of

[13]The variable w_t is *not* introduced with a delay in the estimation of $\hat{D}$, is a standard assumption in the learning literature. This approach conveniently avoids the simultaneous determination of forecasts and endogenous variables. As proved by Marcet and Sargent (1989a), this does not alter the asymptotic results obtained in the following as compared to the algorithm allowing for simultaneity.

wages, before going to the bargaining process. The wage w_t maximizes the joint surplus of a match between workers and firms,

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha} \mathcal{J}_t^{1-\alpha} \tag{20}$$

where α represents the bargaining power of the worker. The first order condition of this problem gives the standard sharing rule that characterizes the optimal split of the aggregate surplus,

$$(1-\alpha)(\mathcal{W}(m_t) - \mathcal{U}(m_t)) = \alpha(\mathcal{J}_t). \tag{21}$$

Assuming that agents know that (21) holds in expectations, the equilibrium wage mapping $\mathbf{w}_t$ is given by

$$\mathbf{w}_t = \alpha(y_t + c\theta_t) + (1-\alpha)b. \tag{22}$$

Since agents do not hold rational wage expectations, I need to distinguish between the stochastic process for equilibrium wages $\mathbf{w}_t$ and agents' perceived wage process w_t. The wage equation is the weighted average of the marginal product of employment, the cost of replacing the worker, and the opportunity cost of working, b. Labor market tightness is a function of expectations; therefore, expectations play an important role in determining wages in equilibrium.

4.6 Equilibrium Dynamics under Learning

Under internal rationality, the solution of the model is summarized by (15), (22) and (19). It follows from (16) and (4) that beliefs about wages k periods ahead are given by

$$E_t^{\mathcal{P}}(w_{t+k}) = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{k-1}) + \rho^{k-1}y_t). \tag{23}$$

Inserting equation (23) into equation (15) and, then the resulting one into (22), one can write the actual law of motion (ALM) of wages as follows:

$$\mathbf{w}_t = T_c(\hat{d}_t^c \ \hat{d}_t^y) + T_y(\hat{d}_t^y)y_{t-1} + T_\epsilon(\hat{d}_t^y)\epsilon_t, \tag{24}$$

where T_c, T_y and T_ϵ are functions of the estimated coefficients of the PLM.[14] T_c, T_y and T_ϵ represent the coefficients of the the equilibrium wage equation and therefore, implicitly defines the mapping from the PLM to the ALM. The interpretation of the ALM is that describes the stochastic process followed by wages if forecasts are made under the fixed rule given by the PLM. To formulate the T-mapping, $T(\hat{D}) = (T_c, T_y)$, I following the method of Marcet and Sargent (1989b) and Evans and Honkapohja (2001). This function maps the agents' perceptions about wage coefficients $(\hat{D})$ to their realized values $(T(\hat{D}))$. The T-mapping is not know to agents.

The fixed point of this mapping is the REE of the model.

[14]For exact formula for T_c, T_y and T_ϵ and the derivations see Appendix B.3.

$Definition$: A rational expectations equilibrium is a matrix $D = [d^c, d^y]$ that satisfies $D = T(D)$. Thus a rational expectations equilibrium is a fixed point of the mapping T. Let me denote such equilibrium by $d^{c,RE}$ and $d^{y,RE}$.

T-mapping determines the evolution of beliefs in the transition to long-run equilibrium. The fact that agents learn about D_t introduces a different dynamic behavior. In particular, if firms believe that wages are going to be high tomorrow, this expectation will be transmitted to the actual realized wage through (24), and wages respond to this belief. This is a key feature of self-referential learning models that are absent in Bayesian learning models. Wage expectations affect realized wages, and agents use wages to update their expectations and so on.

Intuitively, the reason learning matters is the following. The higher the wage expectation, the lower the number of vacancies that firms open up, because their expected profits are lower. This makes the labor market tighter, which in turn reduces the probability of finding a job. When firms and workers negotiate wages, -through the bargaining process- in the presence of lower expected profits and a lower probability of finding a job, wages tend to fall. Figures (1) and (2) shows the $T_c(\hat{d}_t^c\ \hat{d}_t^{y,RE})$ and $T_y(\hat{d}_t^y)$, respectively, represented by the dashed line, which are linear decreasing functions. Values of the coefficient $\hat{d}_t^c$ and $\hat{d}_t^y$, on the right hand side of the fixed point, which is the intersection between the 45 degree line and the T-mapping, indicate that agents expect wages above their realization and vice versa. The negative slopes of the dashed lines reflects the negative relationship between wage expectations and wages present in the model.

Because the agent's equation of wages can differ from the truth, his beliefs evolve over time. To understand the dynamic behavior of $\hat{D}$, it helps to analyze whether the learning rule induces instability in the state evolution. Using the theorems of Sargent and Williams (2005), if g is small enough, to analyze local stability, I need to check the following condition, known as E-stability condition.[15] Accordingly, the stability of the systems (19) is governed by the following ordinary differential equations (o.d.e.):

$$\begin{bmatrix} \dot{\hat{d}}^c \\ \dot{\hat{d}}^y \end{bmatrix} = \begin{bmatrix} T_c(\hat{d}_t^c\ \hat{d}_t^c) - \hat{d}^c \\ T_y(\hat{d}_t^y) - \hat{d}^y \end{bmatrix}. \tag{25}$$

For local stability, I need all eigenvalues of Ω are less than 0 in real part:

$$\Omega = \left.\frac{\partial[T(D) - D]}{\partial D}\right|_{D=D^{RE}} < 0. \tag{26}$$

The eigenvalues are real and negative, because the derivative of the T-mapping with respect to D is negative as one can see in figures (1) and (2), so that the condition for local stability of the learning mechanisms is satisfied.[16] Therefore, one may expect

[15] If g is small enough, the local stability conditions are the same than assuming decreasing gain, $g = \frac{1}{t}$.

[16] Table 5 in the Appendix shows the first point of the T-mapping and the eigenvalues of Ω under the calibration of learning and RE of Tables 2 and 3.

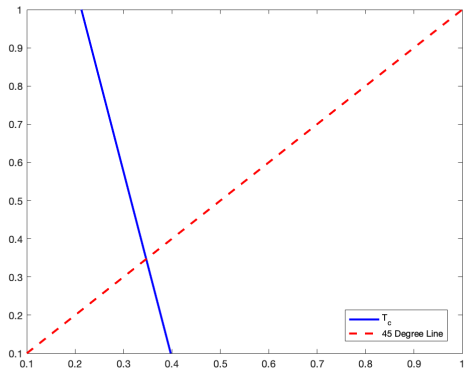

Figure 1: Operator T-mapping for the constant coefficient

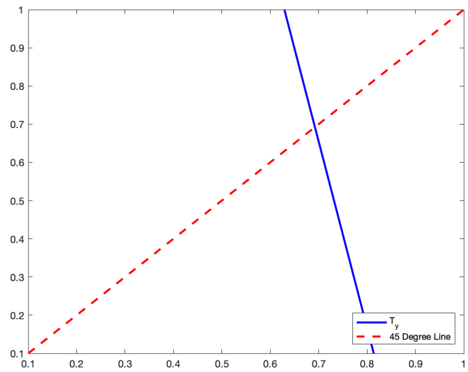

Figure 2: Operator T-mapping for the productivity coefficient

Note: The dashed line represents the operator, T-mapping for each coefficient . The thick black line represents the 45 degree line. The ellipse represents the RE point -which is the fixed point of the T-mapping. In the first subplot, I assume that the coefficient of the productivity is at its RE point. T-mapping is obtained under the calibration of the learning model especified in section 5.1.

constant gain models fluctuate around the REE, and least squared learning would converge.

5 Quantitative Analysis

In macroeconomics, search and matching models are essential tools for evaluating a range of labor market policies, both existing and prospective. Therefore, it's crucial for the selected model to accurately reflect observed moments in the data. However, the textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the US economy in response to productivity shocks of plausible magnitude. Additionally, the model demonstrates a lack of propagation, evidenced by an almost 1 contemporaneous correlation between the vacancy-unemployment ratio and productivity, a stark contrast to the near-zero correlation observed in empirical data. The model proposed by Di Pace et al. (2021) does not account for the latter fact.

This section evaluates the quantitative performance of the search and matching model with subjective wage beliefs. I formally estimate and test the model using a mixed strategy calibration that includes the Method of Simulated Moments (MSM). Testing helps me to focus on the ability of the model to explain the specific moments of the data described in Table 4.

5.1 Estimation of the Model

This section describes the calibration/estimation of the model parameters. The parameterization strategy is threefold. The model has 11 parameters: a subset is selected from the literature, another subset is picked from the US data, and the rest is estimated following the Method of Simulated Moments (MSM). [17]

Specifically, the vector $\hat{Z} = [\beta, \lambda, \alpha, \overline{y}, \nu]$ is obtained directly from the literature. I normalize time to one-quarter. Following the literature, I assume that the matching function is Cobb-Douglas. Without loss of generality, the steady state of productivity is normalized to 1. The value of the discount factor β is set to generate an annual real interest rate of approximately 5%. The value of the separation rate is set following Shimer (2005), who suggests a quarterly separation rate of 0.10. Hence, on average, jobs last for approximately 2.5 years.

Parameter	Description	Value	Source
β	discount factor	0.99	r=0.05
λ	separation rate	0.10	Shimer (2005)
α	bargaining power worker	0.50	Hosios rule: $\alpha = 1 - \nu$
ν	elasticity of matching function	0.50	Petrongolo and Pissarides (2001)
$\overline{y}$	steady state productivity	1.00	Normalization
σ_ϵ	st. dev. of productivity shocks	0.0058	Data (1989Q4-2019Q4)
ρ	persistency of productivity	0.73	Data (1989Q4-2019Q4)

Table 2: Calibrated quarterly parameters from literature and data

I set the value of the elasticity of the matching function at 0.5 in line with the literature. This value lies within the plausible interval of [0.5 0.7] as surveyed by Petrongolo and Pissarides (2001). Following Hosios (1990), I set the bargaining power of the worker to 0.5. Using US data, I set the standard deviation and persistence of the productivity process to match the empirical behavior of labor productivity from 1990 to 2020. I find a quarterly autocorrelation and standard deviation of 0.7518 and 0.0058, respectively.

Defining $Z = [c, A, g, \sigma^\beta, b]$ as the vector of parameters to be estimated using an extension of the Simulated Method of Moments. These parameters are estimated to match the first 11 moments reported in table 4, are standardly used in the search and matching literature to summarize the main features of the labor market.[18] The MSM estimator is given by

$$\min_{Z} \quad (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z))' \hat{\Sigma}_{\mathcal{S}}^{-1} (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z)) \ . \tag{27}$$

[17] This constitutes another difference with respect to the paper of Di Pace et al. (2021), they do not use MSM to estimate some parameters of the model.

[18] I include functions of moments, instead of pure moments. I target 13 functions of moments. See appendix C for more details.

where $\tilde{\mathcal{S}}(Z)$ is the vector of empirical moments to be matched, $\hat{\mathcal{S}}$ is the model moments counterpart and $\hat{\Sigma}_{\mathcal{S}}$ is the weighting matrix, which determines the relative importance of each statistic deviation from its target. I use a diagonal weighting matrix whose diagonal is composed of the inverse of the estimated variances of the data statistics.[19] Model-implied statistics are generated through a Montecarlo experiment with 10,000 realizations. I formally test the hypothesis that any individual model statistics differ from its empirical counterpart.

The calibrated gain is inside the values found in the literature, which range from 0.002-0.05. Additionally, when compared against wage forecasts from the European Commission, the estimated gain is 0.086.[20] Meanwhile, the standard deviation of the belief shock, introduced in a variant of the learning model, is notably conservative. It is markedly smaller than the empirical standard deviation, which I estimated using the survey of the European Commission, standing at 0.01. These values estimated by survey data can be interpreted as upper bounds.

Parameter	Description	Values Learning	Values RE
c	cost of open a vacancy	0.0881	0.3000
A	efficiency matching technology	0.2600	0.1986
g	constant gain	0.0036	0.0000
σ^{β}	Std. wage belief shocks	0.0010	0.0000
b	unemployment benefits	0.7016	0.8500

Table 3: Estimated quarterly parameters from SMM

5.2 Statistical Properties

In this section, the estimation results are reported. Table 4 contains statistics from the US labor market data and those implied by the model under rational expectations and learning dynamics.[21] The sample length of one simulation is T=120 quarters. I simulate the model 10,000 times and report the mean values of the statistics of interest as deviations from the steady state, facilitating comparison to earlier studies.[22] The first panel of Table 4 shows moments targeted during the esti-

[19] In practice the estimated variances of the data moments, $\hat{\mathcal{S}}$ is used. The variances are obtained using a Newey-West estimator and the delta method as in Adam et al. (2016).

[20] Learning in the model is about wage level, therefore I have transformed the annual wage growth forecasts into de-trended levels to estimate the gain, ensuring the forecast generated by the European Commission remains parallel to forecasts implied by the model's learning mechanism. I estimate the gain parameter using a nonlinear least squares to minimize the distance between expectations implied by a constant gain algorithm and the survey expectations.

[21] The data sources are listed in Appendix A. All variables are transformed into real terms. All wage-related moments are calculated using the real wage rate, which is defined as $\frac{RealWages}{1000*Employment}$, following the literature.

[22] The initial values of the employment, n_t, unemployment, u_t, productivity, y_t and wages w_t needed to initialize the algorithm are set to the steady state values. The initial value R is given by $R_0 = T^{-1}z_T'z_T$ where T is 155 quarters that represent a pre-sample period before 1989Q4. The initial D_0 are set to the RE values under the learning calibration.

mation process. The statistics considered are the relative standard deviation of each labor market variable with respect to the standard deviation of labor productivity, the correlation between each labor variable and labor market tightness, the latter's autocorrelation and wages, and the Beveridge curve represented by the correlation between unemployment and vacancies. The second horizontal panel displays a set of non-targeted moments, encompassing additional autocorrelations and correlations, coefficients from regressions (1) and (2) in Section 3 used to test Rational Expectations with forecast data from Professional Forecasters at the European Commission, and the correlation and standard deviation of their forecast errors.

The second column of Table 4 lists empirical moments obtained from the data, while the third and fourth columns present the learning model's moments and corresponding t-statistics. The fifth and sixth columns provide the moments and t-statistics for the RE model.[23]

The simplest version of the DMP model with learning performs remarkably well in quantitative terms. Most of the statistics of the model pass the t-tests, successfully replicating observed labor market dynamics. Notably, it achieves a low contemporaneous correlation between labor market tightness and productivity while also exhibiting high relative volatilities in labor market variables, thereby solving the amplification and propagation puzzles. this outcome is attained without generating rigid wages. This represents a significant success, being problematic for the standard real business cycle model.

Job creation is determined by the difference between expected productivity and the expected cost of labor for new hires. In my model, the learning mechanism dampens the responsiveness of wage expectations to productivity changes, which generates significant amplification effects in labor market outcomes. Specifically, when agents believe that future wages are less responsive to productivity shocks, firms adjust vacancy postings more dramatically, as perceived labor costs do not move proportionally to changes in productivity. This results in larger fluctuations in employment and vacancies, thereby amplifying the impact of productivity shocks on the labor market.

Crucially, this amplification effect does not imply conventional wage rigidities. Unlike models that directly assume wage rigidity to explain limited wage responsiveness, my approach allows wage expectations to adapt through a learning process. This means wages can still adjust, but their adjustments are influenced by evolving expectations rather than being constrained by fixed rules. Thus, equilibrium wages depend on both productivity and agents' expectations, introducing additional dynamics that make wage volatility greater than productivity volatility. This additional dynamics generated by learning about D as described in section 4.6, lead to wages fluctuating more extensively than productivity.

[23]Note that, since the model is over-identified—having more targeted moments than parameters—the t-statistics are not expected to be exactly zero, reflecting some degree of approximation error.

Moment's Symbol	Data	Learning Model	t-stat	RE model Re-est	t-stat
Targeted moments					
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.946	7.349	2.204	0.230	5.617
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.203	14.604	-0.747	2.101	5.923
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.687	20.354	1.093	2.197	5.672
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	2.642	2.107	1.287	0.566	4.995
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.047	0.075	-0.284	1.000	-2.441
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.983	0.966	1.914	0.995	-1.283
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.959	-1.878	-0.669	-28.230
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.756	0.960	-1.656	1.000	-1.984
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.942	0.872	-1.021	0.703	-1.338
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.883	0.854	2.044	0.703	12.541
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.853	-3.234	-0.594	-14.570
Non Targeted moments					
$\rho(\tilde{u}_{t-1}, \tilde{u}_t)$	0.941	0.958	-0.908	0.963	-1.166
$\rho(\tilde{v}_{t-1}, \tilde{v}_t)$	0.923	0.742	2.074	0.670	2.900
$\rho(\tilde{u}, \tilde{y})$	0.127	-0.059	2.963	-0.669	12.682
$\rho(\tilde{v}, \tilde{y})$	0.027	0.084	-0.944	0.995	-16.022
b^E	0.192	0.270	-1.390	1.342	-20.548
b^R	0.984	0.906	0.354	1.325	-1.554
$\rho(\tilde{FE}_t, \tilde{FE}_{t-1})$	0.683	0.799	-	0.000	-
$\sigma(FE)$	0.039	0.072	-	0.000	-

Table 4: Labor Market Statistics

Note: *Data moments are computed over the period 1989Q4 to 2019Q4. Moments have been computed as averages over 10,000 simulations.* b^R *is the coefficientes of regression (1) and* b^E *is the coefficientes of regression (2) running in Section 3. Survey data: European Commision from 1999Q3 to 2020Q1. Forecast real wage growth and forecast errors in the model are computed as* $E_t^{\mathcal{P}}(\tilde{w}_{t+4} - \tilde{w}_t)$ *and* $(\tilde{w}_{t+4} - \tilde{w}_t) - E_t^{\mathcal{P}}(\tilde{w}_{t+4} - \tilde{w}_t)$*, respectively. t-ratios are defined as* $\sqrt{T}$*(data moment-model moment)/(estimated standard deviation of the model moment).*

Additionally, the model provides an explanation for why labor market tightness is not strongly correlated with productivity, a feature often overlooked in the literature addressing Shimer's puzzle. This occurs due to the introduction of fluctuations driven by wage learning. In this model, labor market tightness depends not only on productivity but also on agents' evolving expectations of wage coefficients, which introduces additional dynamics. As a result, the model effectively reduces the correlation between productivity and other labor market variables, such as unemployment and vacancies, as demonstrated by the third and fourth non-targeted moments. In contrast, the RE version of the model does not capture these dynamics, emphasizing the distinctive value of incorporating learning into the model framework.

The model also provides insight into wage forecasts by professional forecasters at the European Commission. Specifically, it closely aligns with the empirical coefficient from regression (2), even though this coefficient was not a direct target of the estimation process. As seen in the data, the model captures the empirical finding that wage expectations are less sensitive to GDP fluctuations. Furthermore, the learning mechanism generates positive correlations and a standard deviation of forecast errors greater than zero, providing further support for the departure from rational expectations in explaining these stylized facts.

Despite its strengths, the model is not without limitations. Some large t-ratios indicate areas where improvements could be made, though it is important to consider the models relative simplicity compared to other models in the DMP literature.

To further investigate, I consider a scenario in which agents learn about two key coefficients affecting wages and introduce a belief shock. There might be speculation about which coefficient drives the models key results, or whether the outcomes are attributable to learning or simply to shocks in expectations. In Section 7, focused on robustness, I demonstrate that the predominant factor is indeed the learning about the coefficient that goes with productivity. This re-calibrated model with just learning about d_t^y, achieves a relative standard deviation of the vacancy-unemployment ratio and unemployment at XX and XX, respectively, and the correlation between the vacancy-unemployment ratio and productivity drops to 0.34.

The key to understanding where the volatility in the learning model of d_t^y comes from, lies in the job creation equation (15). This equation is a function of the discounted presented value of profits, the difference of the infinite sums of expected revenues (Θ_y) and expected labor costs (Θ_w). That different can be written as follows:

$$\Theta_y - \Theta_w = \underbrace{C}_{R1} + \underbrace{\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho} y_t}_{R2} + \underbrace{\left(\frac{1}{1-\beta(1-\lambda)\rho} - \frac{1}{1-\beta(1-\lambda)}\right)\hat{d}_t^y}_{R3}, \quad (28)$$

where $C = \frac{1-d^{c,RE}}{1-\beta(1-\lambda)} - \frac{\rho}{1-\beta(1-\lambda)\rho}$ is a constant.[24] Under RE, the volatility in the discounted present value of profits arises solely from term R2, while term R3 remains constant, implying zero variance for that part. Therefore, under RE $var(\Theta_y - \Theta_w)^{RE} = (\frac{\rho - d^{y,RE}}{1-\beta(1-\lambda)\rho})^2 var(y_t)$. Solutions that seek to solve the volatility puzzle keeping RE, try to make the coefficient $d^{y,RE}$ smaller to increase the value of $\frac{\rho-\hat{d}_t^y}{1-\beta(1-\lambda)\rho}$. The coefficient $d^{y,RE}$ is the one that governs the response of wages to changes in productivity. A common approach to address this issue is to introduce wage rigidities.

In contrast, the mechanism in my model operates in a fundamentally different way. Even with a small standard deviation in the learning parameter $\hat{d}_t^y$, term R3 can contribute significantly to the volatility of the discounted present value of profits. This is because the coefficient that multiplies $\hat{d}_t^y$ is large, resulting in amplified volatility from even minor changes in expectations. The learning process makes R3, which is constant under RE, become a significant source of variability. Moreover, these

[24]See Appendix B.1. for the details.

small deviations from rational expectations also lead to an increase in the volatility of term R2. For example, under the learning calibration, R2 exhibits three times more volatility than in the RE model.

This key distinction, where learning induces volatility in both R2 and R3, demonstrates how the learning dynamics generate much greater overall variability in the labor market compared to a model that assumes rational expectations. The incorporation of learning into agents' beliefs about productivity not only allows for more realistic labor market fluctuations but also eliminates the need to impose wage rigidity.

5.3 Impulse Responses

In this section, I provide an intuitive explanation of why the learning model successfully generates large fluctuations in labor market variables through the expectational channel of wages. Observing the empirical data, such as the survey of wage expectations and the regression coefficient on wage sensitivity to GDP presented earlier, it seems that agents expect lower wage adjustments in response to economic changes. This supports the assumption of perceived wage rigidity, where agents believe that future wages are less sensitive to economic changes than they truly are. Here, I show the implications of this phenomenon when a negative shock hits the economy, comparing the scenario where agents have subjective learning-based expectations against the scenario where agents operate under rational expectations , fully aware of the wage process and the precise influence of productivity changes on wages.

To examine the different dynamics, I study how labor market variables respond to a temporary productivity shock under both RE and learning models. The impulse responses (IRFs) in Figure 3 represent the results from simulating both models over 300 periods, with a one-standard deviation negative productivity shock introduced in period 50. The IRFs are plotted from the period of the shock onward. They show the differences in model variable behaviors between the shock and non-shock scenarios, averaged over 100 repetitions, as percentage deviations from steady-state values.

In the learning model, agents enter the recession with wage expectations that are slightly misaligned with the RE benchmark. In particular, agents believe that future wages are higher and less responsive to economic changes than in reality. This assumption seeks to capture the tendency of professional forecasters to overestimate wages during recessionary periods. Thus, the wage expectation coefficients, $\hat{d}_t^c$ and $\hat{d}_t^y$, are set one standard deviation of the belief shock higher than their RE counterparts. Notably, the standard deviation of the sentiment shock is very small, approximately one-sixth of the standard deviation of the productivity shock. This small deviation represents the tendency of agents to overestimate wages during recessionary periods, while still acknowledging a realistic degree of uncertainty.

Figure 3 shows the impulse responses of unemployment, vacancies, labor market tightness, wages, GDP, and one-period-ahead wage expectations to a negative TFP innovation. Under rational expectations, the response of labor market variables is relatively muted. Agents with RE beliefs are fully aware of the equilibrium relationships that determine future wages, and therefore, their wage expectations absorb most of the impact of the productivity shock. This limits the reaction of other labor market variables like vacancies and unemployment, resulting in negligible amplification.

By contrast, the learning model displays distinct dynamic responses, driven by the belief that wages are less reactive to economic fluctuations. This is evident in the subplot for wage expectations (bottom right panel of Figure 3), where agents expect that wages will decrease less compared to the RE case, indicating that agents anticipate minimal wage adjustments. Following the negative productivity shock, the perceived rigidity in future wages leads to significantly reduced incentives for vacancy creation. Firms, expecting higher future labor costs compared to the RE case, decide to cut back on job postings, causing vacancies to fall sharply and unemployment to rise dramatically. This effect is much more pronounced compared to the RE model, which explains the observed differences in amplification.

In the top left panel of Figure 3, the unemployment IRF shows a substantial and persistent increase under the learning model, which is absent under RE. Vacancies, depicted in the top middle panel, also show a steep decline, consistent with the decreased job creation incentives. Labor market tightness, shown in the top right panel, drops significantly under the learning scenario, indicating reduced competition for workers, unlike the RE case, which shows only minor adjustments.

The wage IRFs (bottom left panel) provide insight into the source of this amplification. Under RE, wages immediately adjust to the new economic conditions, while under learning, wages adjust more gradually. Even if agents think that future wages are rigid, this does not translate into rigid realized wages, but rather exacerbates the decline in other labor market variables. This distinction is one of the main things that differentiates my mechanism from models that retain RE but assume wage rigidity. The GDP panel (bottom middle) similarly illustrates how output declines more sharply in the learning model due to the greater impact on employment and firm output decisions.

Lastly, the wage expectations one quarter ahead (bottom right panel) demonstrate the core mechanism behind the divergence between the two models, highlighting the differences in expectation alignment. The initial misalignment creates a disconnect between expected and realized wages, prolonging the adverse labor market effects and reflecting perceived wage rigidity. Under RE, wage expectations and realized wages are closely aligned, resulting in less persistent labor market disturbances compared to the learning case.

In summary, the IRFs illustrate that subjective beliefs, in contrast to rational expectations, introduces an additional layer of dynamics. Agents' mistaken belief that wages are rigid during downturns leads to greater labor market fluctuations, amplified through decreased vacancy creation and prolonged unemployment. This additional propagation mechanism allows the model to align more closely with the observed volatility in labor market data, solving both the amplification and propagation puzzles identified in the literature.

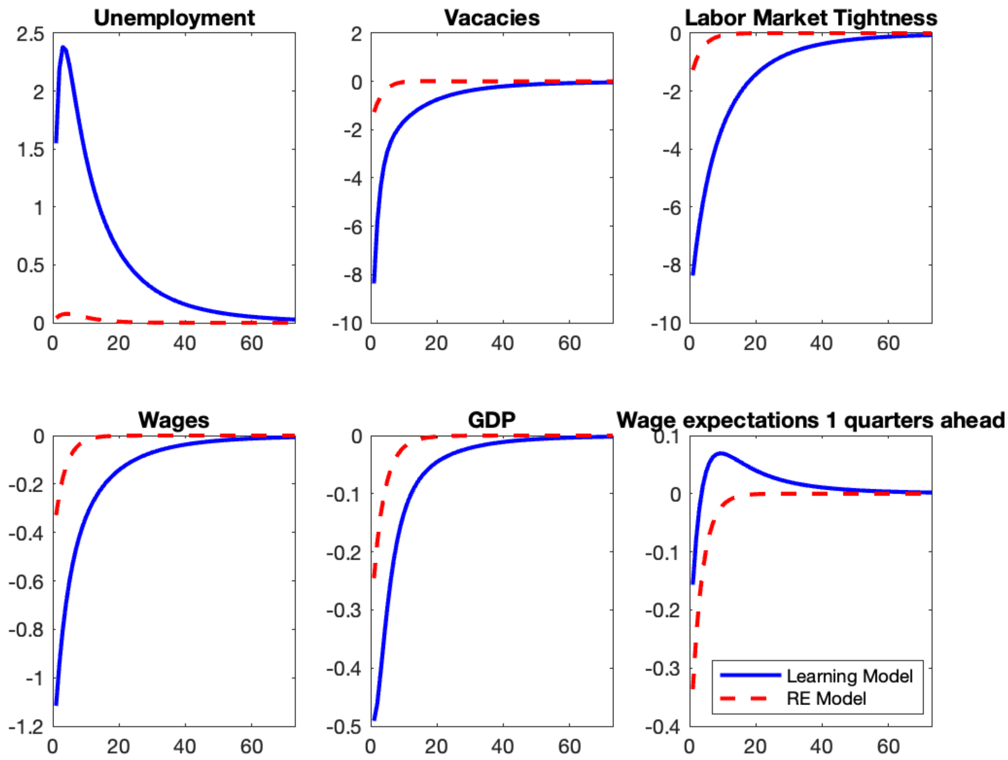

Figure 3: Impulse Responses

Impulse responses to a one negative standard deviation productivity shock. The dotted red line is RE, solid blue line is the learning model. Percentage deviations from steady-state values reported along the vertical axis. The horizontal axis displays the number of quarters after the shock. The coefficients of the learning model are initialized such as $d_0^i = d^{i,RE} + 0.001$ *for* $i = c, y$*, slightly away from the RE equilibrium.*

6 Robustness

In this section, I examine the performance of the learning mechanism under alternative assumptions and extensions. All tables I refer to in Section X in the Appendix.

6.1 Alternative Calibration

I assess the robustness of model proposed in the paper by introducing changes to the calibration of certain parameter values. Results are collected in the Table 6.

First, I examine the performance of the baseline model under RE with the solution proposed by Hagedorn and Manovskii (2008) to compare both solutions under the same framework. According to them, the standard DMP model fails to match the data due to incorrect parametrization of two parameters: the instantaneous utility of being unemployment and workers' bargaining power. By calibrating the unemployment benefits at a higher, ($b = 0.955$), closed to the steady-state of wages, and assigning a low bargaining power to worker, close to zero ($\alpha = 0.05$), the model generates endogenous wage rigidities.[25] This is evident in the significant drop in the RE value of the coefficient that goes with productivity in the linear wage equation, $d^{y,RE}$.

Under the learning model, that coefficient moves around 0.6923, whereas under the calibration of Hagedorn and Manovskii (2008) it is reduced to 0.05. The rigid wages increase second moment of the vacancy-unemployment ratio, as the forth column of Table 6 shows. However, the RE model, under the calibration proposed by Hagedorn and Manovskii (2008), fails to generate a relative standard deviation between wages and productivity greater than 1. Furthermore, it is unable to significantly reduce the correlation between the vacancy-unemployment ratio and productivity.

Second, I simulate the learning model under the calibration by Shimer (2005), keeping the gain parameter equal to the estimated value reported in Table 3, fifth column. This exercise is performed to verify that the amplification is a result of the learning process, rather than an alternative calibration of other parameters, such us a higher bargaining power of the worker, $\alpha = 0.72$ or lower unemployment benefits, $b = 0.4$. Column 5 of Table 6 shows that the model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model.

6.2 Learning about d_t^y

In the main paper, I introduce a framework in which agents learn about two coefficients that affect wages and incorporate a belief shock. This may rise questions about which specific coefficient yields these results, or whether the outcomes are driven by the learning process itself or simply by the shock to expectations.

In this subsection, I evaluate the performance of the learning model when agents learn about how productivity correlates with wages, represented in the model by d_t^y. In this scenario, d_t^c is fixed at the RE value, and belief shocks are excluded during the model's simulation.

Table 6, column 3, presents the moments derived from this learning model along with the respective t-statistics. The model in which agents learn about d_t^y demonstrates impressive quantitative performance. The model is able to generate a low contem-

[25]The calibration of the remaining parameters follows table 2 and the fourth column of table 3, except for ν, which is set such that $\nu = 1 - \alpha$, following the Hosios rule.

poraneous correlation between labor market tightness and productivity, along with high relative volatilities in the labor market. As a result, it successfully addresses both the propagation and amplification puzzles.

6.3 Learning about the constant coefficient

In the previous subsection, I argued that the primary driver of the labor market fluctuations in the learning model is the learning process about the coefficient that governs the relationship between wages and productivity in the wage linear equation, denoted by d^y.

Subsequently, I investigate the performance of the learning model when agents are provided with the rational expectations coefficient $d^y_t = d^{y,RE}\ \forall t$, and their learning focused solely on the constant term of the wage linear equation, d^c. In this analysis, I assume that the model environment is devoid of belief shocks to isolate the impact of learning about the constant.

The findings, presented in column 7 of Table 6, reveal that the revised model yields a relative standard deviation of labor market tightness, which is 2.5 times the one generated by the RE model. Nevertheless, it is lower compared to the case where agents learn about d^y_t. Notably, the model effectively reduces the contemporaneous correlation between labor market tightness and labor productivity.

6.4 Information Assumption

In the baseline model, I assume that agents do not observe period wages at the time they make their forecasts. This is a standard assumption in the learning literature to avoid the simultaneous determination of forecast and endogenous variables. In this section, I move away from that assumption, and instead assume that the forecast of wages, the decision regarding vacancies, and realized wages are determined simultaneously. Consequently, agents' beliefs evolve according to the following scheme:

$$\begin{aligned} \hat{D}_t &= \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[w_t - \hat{D}'_{t-1} z_t], \\ R_t &= R_{t-1} + \gamma(z_{t-1} z'_{t-1} - R_{t-1}). \end{aligned} \tag{29}$$

Where $\hat{D}^f_t = [\hat{d}^c_t \ \hat{d}^y_t]'$ represents the estimated coefficients, and $z_{t-1} = [1 \ y_t]$. Note that in this case, $(w_t - \hat{D}'_{t-1} z_t)$ represents the most recent forecast error.

As shown in Table 6, column 6, the re-calibrated model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model. However, this is achieved at the cost of making wages excessively volatile.

6.5 Alternative departures from Full Information RE

Deviations from full information rational expectations (FIRE) are increasingly recognized as a mechanism that amplifies the volatility of labor market variables in standard DMP models. In this section, I compare my model with other models that depart from FIRE by analyzing key labor market moments from the US and comparing forecasts of wage growth dynamics.

A prominent theory, proposed by Menzio (2022), posits that workers misperceive the true productivity process, maintaining stubborn beliefs about economic fundamentals. Apart from that, they know the complete model. In Menzio's model, workers erroneously assume that aggregate productivity is constant over time, irrespective of underlying fluctuations. This results in static expectations regarding job prospects and wages, unresponsive to shifts in economic fundamentals. Firms, on the other hand, possess rational expectations and full knowledge of how workers form their expectations. Since the firm knows that the worker's beliefs cannot be changed, it has no choice but to accommodate them. Notably, in this model, workers make systematic errors without engaging in a learning process. However, Menzio's analysis focuses solely on elasticities, without simulating or evaluating broader labor market moments.

On the other hand, agents may operate under limited knowledge of market behavior, exemplified by scenarios where they lack insight about some market outcomes, and try to learn about it using past data. In this paper, agents use the minimum state variable solution to forecast future wages, which nests the REE of interest. However, economic agents, like econometricians, may fail to correctly specify the ALM, even asymptotically. Agents may adopt PLMs for forecasting, which may diverge from ALMs. For example, agents may include only a subset of the state variables in their forecast rule. This case is addressed in this framework by Di Pace et al. (2021). As in this paper, they assume that agents do not know the wage process and propose an adaptive learning framework where restrict PLMs of wages to those which do not depend on productivity. Particularly, they assume that agents use autoregressive models to form wage expectations.

I simulate and compare the performance of these models alongside my own framework. The key results are summarized in Table 7. While the models demonstrate the ability to generate pronounced labor market fluctuations, my model outperforms both the Di Pace et al. (2021) and Menzio (2022) models in some dimensions. Specifically, the model significantly reduces the correlation between productivity and labor market tightness, and accurately replicates observed fluctuations in real wages. This contrasts with the heightened relative wage fluctuations produced by Di Pace et al. (2021) approach and the opposing effect observed in the Menzio's model.

In addition, it is observed that both the Menzio and Di Pace et al. (2021) models fail to replicate the coefficients from the rational expectations test regressions. Under the Menzio model, both coefficients in the RE test regressions are zero, which is rejected by the empirical data on wage expectations and their realizations. Di Pace et al. (2021) show that the prediction error is correlated with GDP growth. Although the difference between the coefficients, $b_E - b_R$, is significantly different from zero in their model, the individual coefficients are notably higher compared to those found in the data.

7 Conclusions

A simple search and matching model applied to the business cycle is able to quantitatively replicate a number of important labor market facts in US, provided that one slightly relaxes the assumption that agents perfectly know how wages are formed in the market. I assume that agents are internally rational, in the sense that they formulate their doubts about market outcomes using a consistent set of subjective beliefs about wages and behave optimally given this set of beliefs. The system of beliefs is internally consistent in the sense that it specifies a proper joint distribution of wages and fundamentals at all dates. Moreover, the perceived distribution of wage behavior, although different from the true distribution, is nevertheless close to it and the discrepancies are hard to detect.

In such a setting, optimal behavior implies that agents learn about equilibrium wage process from past wage behavior. By allowing agents to learn and adjust their beliefs about future wages, the model successfully replicates key empirical features of the U.S. labor market, including the high volatility of unemployment and vacancies, as well as the weak correlation between labor market variables and labor productivity. These features are difficult to explain under a standard rational expectations framework without assuming rigid wages.

The results demonstrate that subjective wage expectations can generate amplification mechanisms that significantly influence labor market dynamics. This is achieved without imposing rigid wages but instead allowing wages to be influenced by agents' evolving beliefs. The endogenous adjustment of expectations introduces persistent deviations in labor market tightness, reducing its correlation with productivity and enhancing the responsiveness of labor market variables to economic shocks.

A formal econometric test using survey data from the European Commission reveals that professional forecasters' wage expectations are inconsistent with the rational expectations assumption, supporting the validity of the learning mechanism used in this model. In particular, the lower covariation between wage expectations and real GDP compared to the covariation with actual wage realizations highlights the perception of wage rigidity among agents, which in turn impacts their decision-making process regarding vacancies and employment.

This model effectively bridges the gap between the theoretical DMP framework and the empirical observations related to labor market fluctuations and the propagation of productivity shocks. It also explains why the correlation between labor market variables and productivity has been historically difficult to capture—introducing subjective wage expectations provides an additional layer of dynamics that aligns more closely with observed labor market behavior.

In conclusion, the model presented offers a compelling alternative to traditional RE-based frameworks, providing insights into labor market fluctuations that better align with empirical evidence. The findings emphasize the importance of considering agents' belief systems and subjective expectations in understanding labor market dynamics. Future research could expand on this framework by exploring different forms of belief evolution or incorporating additional labor market frictions, which might further improve the model's ability to capture the complexities of real-world labor markets.

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A Data Source

The time series are presented as seasonally adjusted quarterly series. The period covered is from 1989-Q4 to 2019-Q4, a total of 277 quarters.

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Consumer Price Index: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis;
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B Theoretical Model

B.1 Equilibrium equations

To determine the labor market tightness of the economy, I have to start with the job creation condition:

$$\frac{c}{q(\theta_t)} = E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j [\frac{y_{t+j} - w_{t+j}}{1-\lambda}] \tag{30}$$

Plugging the expectations of productivity and wages; $E_t y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t w_{t+j} = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{j-1}) + \rho^{j-1} y_t)$, the labor market tightness can be writte as follows:

$$\theta_t = (\frac{A\beta}{c})^{\frac{1}{1-\nu}}[\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}, \tag{31}$$

where Θ_y represents the present discount revenues and Θ_w the present discount labor costs. These two can be writte as

$$\begin{aligned}\Theta_y &= \frac{1}{1-\beta(1-\lambda)} + \frac{\rho}{1-\beta(1-\lambda)\rho}(y_t - 1),\\ \Theta_w &= \frac{\hat{d}_t^c + \hat{d}_t^y}{1-\beta(1-\lambda)} + \frac{\hat{d}_t^y}{1-\beta(1-\lambda)\rho}(y_t - 1).\end{aligned} \tag{32}$$

Therefore, vacancies are determine by

$$v_t = u_t(\frac{A\beta}{c})^{\frac{1}{1-\nu}}[\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}. \tag{33}$$

B.2 Wages

Wages are negotiated according to a Nash bargaining process. The wage maximizes the joint surplus of a match between workers and firms. The maximization problem is the following:

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha}\, \mathcal{J}_t^{1-\alpha} \tag{34}$$

where α is the workers' bargaining power. The first order condition is as follows:

$$\begin{aligned}\alpha\,(\mathcal{W}-\mathcal{U})^{\alpha-1}\,(\mathcal{J}_t)^{1-\alpha} + (\mathcal{W}-\mathcal{U})^{\alpha}\,(1-\alpha)\,(\mathcal{J}_t)^{-\alpha}\,(-1) = 0,\\ \alpha\,(\mathcal{W}-\mathcal{U})^{\alpha-1}\,(\mathcal{J}_t)^{1-\alpha} = (\mathcal{W}-\mathcal{U})^{\alpha}\,(1-\alpha)\,(\mathcal{J}_t)^{-\alpha},\\ \alpha\,(\mathcal{J}_t) = (1-\alpha)\,(\mathcal{W}-\mathcal{U}).\end{aligned} \tag{35}$$

Therefore, the following equalities are satisfied:

$$\mathcal{W} - \mathcal{U} = \alpha S_t, \tag{36}$$

$$\mathcal{J}_t = (1-\alpha)S_t. \tag{37}$$

Where S_t is the total surplus of the match.

$$S_t = (\mathcal{W} - \mathcal{U}) + \mathcal{J}_t. \tag{38}$$

Plugging the surpluses into (35), I come up with:

$$\begin{aligned}\alpha(\Big[y_t - w_t + \beta\Big[(1-\lambda)E_t^{\mathcal{P}^f}(\mathcal{J}_{t+1}))\Big] =\\ (1-\alpha)\left[w_t - b + \beta\left[(1-\lambda-f(\theta_t))E_t^{\mathcal{P}^w}\mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right]\right].\end{aligned} \tag{39}$$

Assuming that agents belief that (62) and (63) hold in expectations,

$$\begin{aligned}\alpha\Big[y_t - w_t + \beta E_t^{\mathcal{P}^f}\left[(1-\lambda)((1-\alpha)S_{t+1})\right]\Big] =\\ (1-\alpha)\left[w_t - b + \beta E_t^{\mathcal{P}^w}\left[(1-\lambda-f(\theta_t))(\alpha S_{t+1})\right]\right]\end{aligned} \tag{40}$$

Doing some algebra, I come up with

$$\begin{aligned}(1-\alpha)w_t-(1-\alpha)b+(1-\alpha)\beta E_t^{\mathcal{P}^w}\left[(1-\lambda-f(\theta_t))(\alpha S_{t+1})\right]=\\ \alpha y_t-\alpha w_t+\alpha\beta E_t^{\mathcal{P}^f}\left[(1-\lambda)((1-\alpha)S_{t+1})\right]\end{aligned} \tag{41}$$

Let's assume that $E_t^{\mathcal{P}^f}=E_t^{\mathcal{P}^w}=E_t^{\mathcal{P}}$,

$$w_t=\alpha y_t+(1-\alpha)b+\beta f(\theta_t)\alpha E_t^{\mathcal{P}}(S_{t+1})(1-\alpha).$$

Finally, if both agents know that the FOC of firms hold, $(1-\alpha)\beta E_t^{\mathcal{P}}(S_{t+1})=\frac{c\theta_t}{f(\theta_t)}$, I come up with the following expression:

$$\begin{aligned}w_t&=\alpha y_t+(1-\alpha)b+f(\theta_t)\alpha\frac{c\theta_t}{f(\theta_t)}\\ &=\alpha(y_t+c\theta_t)+(1-\alpha)b.\end{aligned} \tag{42}$$

B.3 T-mapping

First all, I linearize the job creation condition applying a first-order Taylor polynomial of this equation at the steady state $\theta=\overline{\theta}$, $w=\overline{w}$ and $y=\overline{y}=1$. The job creation condition is represented by the following equation:

$$\frac{c}{\beta q(\theta_t)}=E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}\left[\beta(1-\lambda)\right]^{j-1}\left[y_{t+j}-w_{t+j}\right] \tag{43}$$

I take the first-order Taylor polynomial of each component of the previous equation:

$$\frac{c}{\beta q(\theta_t)}=\frac{c}{\beta q(\overline{\theta})}-\frac{c}{\beta q(\overline{\theta})^2}\frac{\partial q(\overline{\theta})}{\partial\overline{\theta}}(\theta_t-\overline{\theta}) \tag{44}$$

$$E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}y_{t+j}=\frac{1}{1-\beta(1-\lambda)}+E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}(y_{t+j}-1) \tag{45}$$

$$E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}w_{t+j}=\frac{\overline{w}}{1-\beta(1-\lambda)}+E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}(w_{t+j}-\overline{w}) \tag{46}$$

Therefore, I can write equation (43) as

$$\frac{c}{\beta q(\overline{\theta})}-\frac{c}{\beta q(\overline{\theta})^2}\frac{\partial q(\overline{\theta})}{\partial\overline{\theta}}(\theta_t-\overline{\theta})=E_t^{\mathcal{P}^f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}[y_{t+j}-w_{t+j}], \tag{47}$$

$$\theta_t=\overline{\theta}+\phi E_t^{sf}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}[y_{t+j}-w_{t+j}]. \tag{48}$$

where $\phi = \frac{\beta q(\overline{\theta})^2}{c(q'(\overline{\theta}))}$. I plug the previous equation into the wage equation, I come up with

$$w_t = \alpha\left(y_t + c\left(\overline{\theta} + \phi E_t^{\mathcal{P}f}\sum_{j=1}^{\infty}[\beta(1-\lambda)]^{j-1}[y_{t+j} - w_{t+j}]\right)\right) + (1-\alpha)b. \tag{49}$$

Taking into account the expectation; $E_t^{\mathcal{P}} y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t^{\mathcal{P}} w_{t+j} = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{j-1}) + \rho^{j-1}y_t)$, I come up with:

$$w_t = T_c + T_y y_{t-1} + T_\epsilon \epsilon_t, \tag{50}$$

where

$$T_c = \alpha\left[c\overline{\theta} + \frac{c\phi}{1-\beta(1-\lambda)}\left[1 - (\hat{d}_t^c + \hat{d}_t^y)\right] + (1-\rho) - \rho\left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho}\right]\right] + (1-\alpha)b,$$
$$T_y = \rho\left[\alpha + \phi\alpha c\left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)}\right]\right],$$
$$T_\epsilon = \left[\alpha + \phi\alpha c\left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)}\right]\right]. \tag{51}$$

Model	d^c	d^t	**Eigenvalues of** Ω
Learning Model	0.347	0.692	-5.876, -4.568
RE Model	0.399	0.549	-1.2689, -1.1968

Table 5: Fixed Point Information for Learning and RE Models

Note: *First point of the T-mapping and the eigenvalues of Ω under the calibration of learning and RE from Tables 2 and 3.*

C Method of moments

$$\min_{\theta} \quad (\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta))'\hat{\Sigma}_{\mathcal{S}}^{-1}(\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta)) \ . \tag{52}$$

Where $\hat{\Sigma}_{\mathcal{S}}$ is the varianza of the moments.

C.1 The Statistics and Moment Functions

This section gives explicit expressions for the statistics function S(.) and the moment functions *h(.)*.

The underlying sample moments needed to construct statistics of interes are:

$$\hat{M}_N = \frac{1}{N}\sum_{t=1}^{N} h(y_t), \tag{53}$$

where $h(.)$ is defined as $\begin{bmatrix} \tilde{v}_t \\ \tilde{u}_t \\ \tilde{\theta}_t \\ \tilde{y}_t \\ \tilde{w} \\ \tilde{v}_t^2 \\ \tilde{u}_t^2 \\ \tilde{\theta}_t^2 \\ \tilde{y}_t^2 \\ \tilde{w}_t^2 \\ \tilde{v}_t\tilde{\theta}_t \\ \tilde{u}_t\tilde{\theta}_t \\ \tilde{y}_t\tilde{\theta}_t \\ \tilde{w}_t\tilde{\theta}_t \\ \tilde{v}_t\tilde{u}_t \\ \tilde{\theta}_t\tilde{\theta}_{t-1} \\ \tilde{w}_t\tilde{w}_{t-1} \end{bmatrix}$.

The 13 statistics I consider can be expresed as functions of the moments as follows:

$$\mathcal{S}(M) = \begin{bmatrix} \sigma_{\tilde{v}} \\ \sigma_{\tilde{u}} \\ \sigma_{\tilde{\theta}} \\ \sigma_{\tilde{y}} \\ \sigma_{\tilde{w}} \\ \rho(\tilde{v}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{w}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{v}_t \\ \rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t) \\ \rho(\tilde{w}_{t-1}, \tilde{w}_t) \end{bmatrix} = \begin{bmatrix} \sqrt{M_6 - M_1^2} \\ \sqrt{M_7 - M_2^2} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_9 - M_4^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{11} - M_1 M_3}{\sqrt{(M_6 - M_1^2)(M_8 - M_3^2)}} \\ \frac{M_{12} - M_2 M_3}{\sqrt{(M_7 - M_2^2)(M_8 - M_3^2)}} \\ \frac{M_{13} - M_4 M_3}{\sqrt{(M_9 - M_4^2)(M_8 - M_3^2)}} \\ \frac{M_{14} - M_5 M_3}{\sqrt{(M_{10} - M_5^2)(M_8 - M_3^2)}} \\ \frac{M_{15} - M_1 M_2}{\sqrt{(M_2 - M_1^2)(M_7 - M_2^2)}} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{17} - M_3^2}{S_{11}^2} \\ \frac{M_{18} - M_5^2}{S_{12}^2} \end{bmatrix}, \tag{54}$$

where M_i drenotes the ith element of M.

I compute the t-statistics for a particular statistic i as follows:

$$\sqrt{N} \frac{\mathcal{S}_i - \mathcal{S}_i^M}{\hat{\sum}_{\mathcal{S}}}, \tag{55}$$

where $\mathcal{S}_i$ are the i statistic of the data and $\mathcal{S}_i^M$ is the i statistic coming from the model. $\hat{\sum}_{\mathcal{S}}$ is the variance for the sample statistics $\mathcal{S}$:

$$\hat{\sum}_{\mathcal{S}} = \frac{\partial S}{\partial M'} \hat{\mathcal{S}}_w \frac{\partial S'}{\partial M}. \tag{56}$$

I can test if the ability of the model to explain individual moments using t-statistics based on formal asymptotic distribution:

$$\sqrt{N} \frac{\mathcal{S} - \mathcal{S}^M}{\hat{\sum}_{\mathcal{S}}} \to N(0,1). \tag{57}$$

D Tables of Summary Statistics of Robustness

	Data	Learning[1] model (Re-est)	RE model[2] Hagedorn and Manovskii	Learning[3] Shimer	Learning[4] Simultaneous (Re-est)	Learning[5] Constant (Re-est)
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.687	17.305	40.387	14.456	17.874	5.137
		(1.861)	(-3.959)	(2.580)	(1.718)	(4.930)
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	2.642	1.542	0.052	1.881	12.721	0.657
		(2.647)	(6.231)	(1.831)	(-24.253)	(4.776)
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.047	0.393	1.000	0.032	0.0253	0.652
		(-1.025)	(-2.441)	(-0.183)	(-0.168)	(-1.631)

Table 6: Summary Statistics: Alternative calibration, learning constant coefficient and infortantion assumption

Note: ***1***. *Learning model about* d_t^y *with productivity shocks,* $d_t^c = d^{c,RE} = 0.352$. *Calibration follows tables 2 and the estimated coefficients coming from SMM are:* $c = 0.15$, $A = 0.46$, $g = 0.05$ *and* $b = 0.61$. ***2***. *Calibration follows tables 2 and 3 of the RE model except for parameters* $b = 0.955$, $\alpha = 0.05$ *and* $\nu = 0.95$. ***3***. *Calibration follows tables 2 and 3 of the learning model, expect for parameters* $b = 0.4$, $\alpha = 0.72$ *and* $\nu = 0.28$. ***4***. *Calibration follows tables 2 and the estimated coefficients coming from SMM are:* $c = 0.30$, $A = 0.25$, $g = 0.03$ *and* $b = 0.42$. ***5***. *Learning model about* d_t^c *with productivity shocks,* $d_t^y = d^{y,RE} = 0.616$. *Calibration follows tables 2 and the estimated coefficients coming from SMM are:* $c = 0.30$, $A = 0.13$, $g = 0.041$ *and* $b = 0.53$.*The moments of the data are calculated for the period 1989Q4: 2019Q4. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.*

	Data	Menzio[1]	t-stat	Di Pace et al[2]	t-stat
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.687	24.249	0.110	20.449	1.069
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	2.642	0.000	7.550	3.906	-3.042
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.047	1.000	-2.420	0.975	-2.383
b^E	0.192	0.000	3.423	2.123	-34.482
b^R	0.984	0.000	4.475	3.846	-13.024

Table 7: Summary Statistics: Alternative calibration, learning constant coefficient and infortantion assumption

Note: ***1***. *Calibration follows tables 2 and the estimated coefficients coming from SMM are:* $c = 0.33$, $A = 3.25$, *and* $b = 0.45$. ***2*** *Calibration and simulation follow Di Pace et al. (2021). The moments of the data are calculated for the period 1989Q4: 2019Q4. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.*