A Monetary Policy Asset Pricing Model

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Abstract

We propose a model in which macroeconomic needs as interpreted by the central bank ("the Fed") drive aggregate asset prices. In our model, monetary policy influences macroeconomic activity by changing aggregate asset prices (financial conditions). Thus, an optimizing Fed adjusts its policy tools to target the aggregate asset price per potential output that delivers future macroeconomic balance under its beliefs ("pystar"). This perspective has several implications, among which we highlight: (i) the Fed induces "excess" asset price volatility to offset aggregate demand shocks, but also stabilizes asset price fluctuations due to financial market shocks ("the Fed put/call"); (ii) macroeconomic news that improves the Fed's ability to predict future conditions reduces output volatility but increases asset price volatility; (iii) with aggregate demand inertia, the Fed overshoots asset prices upward (resp. downward) when the output gap is negative (resp. positive), (iv) inflation is negatively correlated with aggregate asset prices, regardless of whether inflation is driven by demand shocks or supply shocks; and (v) when the market and the Fed have different beliefs, the market perceives monetary policy "mistakes" that generate a policy risk premium and may induce "behind-the-curve" dynamics.

JEL Codes: G12, E43, E44, E52, E32

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1. Introduction

"So, of course, monetary policy does, famously, work with long and variable lags. The way I think of it is, our policy decisions affect financial conditions immediately. In fact, financial conditions have usually been affected well before we actually announce our decisions. Then, changes in financial conditions begin to affect economic activity... within a few months." (Chair Jerome Powell's Press Conference, September 21, 2022)

Monetary policy works by changing financial conditions—a summary measure of aggregate asset prices—which then transmits to the real economy with a lag.[1] In this paper, we turn these observations into an asset pricing model. The key idea is to reverse engineer the Fed's policy problem to solve for the aggregate asset price per potential output that ensures future macroeconomic balance under the Fed's beliefs ("pystar"). When the Fed is unconstrained and acts optimally, asset prices cannot deviate much from "pystar." For example, during the late stages of the Covid-19 recovery, we saw several episodes where markets attempted to rebound. However, these rallies were quickly reversed by a Fed speech or a policy announcement, since the Fed believed the economy needed tight financial conditions to reduce inflation.

Our model features a two-speed economy: a slow and unsophisticated macroeconomic side and a fast and sophisticated financial market side. The two-speed is a realistic feature, as emphasized by Chair Powell's quote, and it enables us to introduce policy transmission lags and other frictions that complicate monetary policy in practice. Specifically, in our model spending decisions are made by a group of agents ("households") that respond to asset prices, but with noise, delays, and inertia.[2] Asset pricing is determined by a different group of agents ("the market"), who are forward looking, and immediately react to economic shocks and the likely monetary policy response to those shocks. The Fed *wants* to influence the behavior of households, but it *needs* to operate through the market. Moreover, the market and the Fed have their own sets of beliefs about the future state of the economy and the corresponding policy response. This means that the Fed needs to closely monitor and "cooperate" with the market to control asset prices, and the market needs to monitor the Fed to determine asset prices and the risk premium.

[1] One of the most popular financial conditions index followed by practitioners is Goldman's GSUSFCI Index (see Hatzius and Stehn (2018)). Excluding house prices (for which there is no high frequency data), the equity market dominates fluctuations in financial conditions in the US since 2000. The equity market (measured as the Shiller's P/E ratio) accounts for about 40 percent of the average annual absolute change in financial conditions. This compares with less than 20% (each) for corporate spreads and riskless long rates. See Hatzius et al. (2017).

[2] The model also has "hand-to-mouth" agents, but these only play a technical role, as they simplify the labor supply side of the model and generate a Keynesian multiplier.

As a baseline, we start with a relatively standard model without transmission lags. Specifically, households mostly follow the optimal consumption rule with log utility. They respond to aggregate wealth immediately, with a constant marginal propensity to consume (MPC) out of wealth. We allow for aggregate demand shocks, which we capture with noisy deviations from the optimal consumption rule. We show that, even in this relatively standard setup, "pystar" is driven by macroeconomic needs rather than by financial forces such as cash-flow expectations or risk premia. We also show that, on the one hand, aggregate demand shocks induce *opposite* fluctuations in "pystar." When there is a positive demand shock, the Fed lowers asset prices to offset the positive output gap the shock would otherwise induce (and vice versa for a negative demand shock). This creates the appearance of policy-induced "excess" volatility in asset prices. However, this volatility shields the economy from shocks that would otherwise exacerbate business cycles. On the other hand, the Fed stabilizes asset price fluctuations driven by financial shocks, such as expectations or risk premia ("the Fed put/call"), in order to insulate macroeconomic activity from these shocks.

We then introduce *transmission lags*, which make the Fed's *beliefs* drive "pystar." In addition to acting with noise, households are inertial and respond to asset prices with a lag. These lags are empirically well-documented and make monetary policy difficult. The Fed needs to forecast future macroeconomic conditions because it effectively sets policy for a *future* period. When the Fed expects aggregate supply to increase (as in the Covid-19 recovery) or aggregate demand to decrease, it targets higher asset prices. Conversely, when the Fed expects higher demand or lower supply, it sets lower asset prices. In this context, asset prices fluctuate with macroeconomic news that shifts the Fed's beliefs. More precise news makes output less volatile, but it increases asset price volatility.

In practice, transmission lags come from microeconomic frictions that generate inertial behavior, such as adjustment costs or habit formation. The same frictions imply *internal* demand inertia: that is, along with responding to asset prices with a lag, households partly repeat their own *past* spending behavior. When we allow for internal inertia, current output persists into the future, even if the driving shocks are not persistent. The Fed then targets a "pystar" that neutralizes *the future effects of current output.* When output is below (resp. above) its potential, the Fed overshoots asset prices upward (resp. downward) to achieve macroeconomic balance faster. This overshooting seemingly creates a disconnect between the performance of the economy and financial markets, but it also accelerates the recovery.

For simplicity, in most of the paper we assume fully sticky good prices. When we endogenize inflation via a standard New Keynesian Phillips Curve (NKPC), we find that inflation is *negatively* correlated with aggregate asset prices, regardless of whether inflation is driven by demand shocks or supply shocks. This result is driven by two observations. First, transmission lags imply the Fed stabilizes the *expected* future output gaps and inflation, but it cannot stabilize the current output gap. Therefore, inflation depends *only* on the *current* output gap. Second, both demand and supply shocks induce a *negative* covariance between the current output gap and aggregate asset prices. A positive demand shock raises the output gap (and inflation) and induces the Fed to overshoot asset prices in the *downward* direction. A negative supply shock also raises the output gap (and inflation) and induces the Fed to target a lower asset price to align the future demand with the lower level of supply. It follows that in our model inflation is bad news for asset prices. This also implies that the inflation risk premium is typically *positive*: the expected *real* return on the nominal risk-free asset (which is subject to inflation risk) usually exceeds the return on the *real* risk-free asset (which is inflation-protected).

Since the Fed's beliefs about the future state of the economy drive asset prices, our final set of results investigate what happens when the Fed and the market have belief disagreements—as we routinely see in practice. Our earlier results are robust to disagreements: that is, the Fed still implements the "pystar" that is appropriate under its own belief. However, disagreements affect the risk premium and the policy interest rate. When the market disagrees with the Fed, it perceives policy "mistakes." The market's anticipation of *future* disagreements and "mistakes" increases the aggregate risk premium—we refer to this as a policy risk premium. In addition, *current* disagreements create a "behind-the-curve" phenomenon where the market expects the Fed to reverse course. For instance, a demand-optimistic market (that expects higher aggregate demand than the Fed) thinks a dovish Fed will induce a positive output gap, after which it will have to reverse course and overshoot asset prices downward. We further show that disagreements affect the interest rate the Fed needs to set to achieve "pystar." The market's perception that the Fed is "behind-the-curve" and will make future "mistakes" exerts pressure on aggregate asset prices. In equilibrium, the Fed adjusts the interest rate to absorb this pressure and keep the asset price at "pystar" (in line with "the Fed put/call").

Literature review. This paper continues our investigation of the interaction between monetary policy, financial markets, and business cycles. Our earlier work focused on spillover effects from financial markets to macroeconomic outcomes. When monetary policy is constrained, financial market shocks or frictions—such as time-varying risk premia or financial speculation—can cause aggregate demand recessions and motivate prudential policies (see, e.g., Caballero and Simsek (2020, 2021c); Pflueger et al. (2020); Caballero and Farhi (2018)). Likewise, policy constraints and financial frictions can amplify supply shocks and motivate unconventional monetary policy (see, e.g., Caballero and Simsek (2021a)). This paper uses a similar framework but focuses on the spillback effects from the needs of the macroeconomy to financial markets. To make these needs realistic, we enrich the macroeconomics side of our earlier model with ingredients such as demand shocks, transmission lags, and demand inertia. We focus on the asset pricing implications of a monetary policy framework aimed at stabilizing this richer economy by influencing financial conditions.

In terms of the specific modeling ingredients, we build on some of the insights in our recent work. In Caballero and Simsek (2021b), we showed that aggregate demand inertia induces the Fed to generate a temporary disconnect between the real economy and asset prices. In Caballero and Simsek (2022a), we began our exploration of the consequences of disagreements between the Fed and the market for optimal monetary policy. The former paper studies a one-off shock, while the latter paper's analysis is conducted within a standard log-linearized New Keynesian model. This paper integrates the monetary policy insights of both papers into a proper asset pricing model with risk and risk-premia. This integration enables us to obtain several new results that have no counterparts in our earlier work. Among other results, we show that the Fed's beliefs drive asset prices, inflation is negatively correlated with aggregate asset prices, and the Fed-market disagreements induce a policy risk premium and a behind-the-curve phenomenon.

The idea that asset prices are influenced by macroeconomic conditions is familiar from consumption-based asset pricing models (e.g., Lucas (1978)). Relative to this literature, our model has two distinct features. First, we assume output is determined by aggregate demand (due to nominal rigidities). This feature creates a central role for the Fed: in our model, asset prices are driven by macroeconomic conditions filtered through the Fed's beliefs. Second, we assume aggregate consumption reacts to asset prices with noise, delays, and inertia. These features allow for richer dynamics between asset prices and consumption than typically emphasized in the literature.[3]

The connection between the Fed and asset prices is also present in an emergent New Keynesian literature with explicit risk markets (Caballero and Simsek (2020); Kekre and Lenel (2021); Pflueger and Rinaldi (2020)). That literature focuses on risk-market shocks or monetary policy shocks, whereas we focus on macroeconomic shocks and highlight how they can spill back to risk markets through the Fed's response to these shocks. Also,

[3]In recent work, Anderson (2021) shows that allowing for consumption mistakes can improve the empirical success of the consumption CAPM, but he does not analyze nominal rigidities or monetary policy (see also Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)).

in that literature the Fed is often embedded in a Taylor-type rule, rather than being an optimizing agent with its own set of beliefs.

Our results on the inflation risk premium are related to a large empirical literature that studies the relationship between inflation and asset prices (see Cieslak and Pflueger (2022) for a survey). In recent work, Fang et al. (2022) show that *core* inflation (excluding food and energy) comoves negatively with most asset returns, which is consistent with our analysis. Fang et al. (2022) explain this comovement result using a New Keynesian model with cost-push (markup) shocks. In contrast, we show that, with realistic policy lags and inertia, demand or supply shocks also induce a negative correlation between inflation and asset prices. The effect of demand shocks is particularly noteworthy, because these shocks simultaneously induce *high output* (a demand boom) along with *high inflation*, as in the late stages of the Covid-19 recovery. In contrast, a cost-push shock would induce the Fed to target low output (a demand recession) to fight high inflation.

Contemporaneously, Bianchi et al. (2022a,b) build and estimate models in which asset prices, like in our model, are determined by forward-looking agents ("investors"), whereas the macroeconomic dynamics are driven by less sophisticated agents with inertial beliefs ("households"). They emphasize that investors' beliefs about monetary policy regimes affect asset prices. A key substantive difference between our models is that in ours macroeconomic outcomes are affected by asset prices. This channel drives our results, as it provides the rationale for the Fed to target asset prices.

There is an extensive finance literature documenting "excess" volatility in asset prices, such as the stock market (see, e.g., Shiller (2014)). The literature has emphasized a number of financial-market shocks that could induce asset price volatility, e.g., time-varying risk premia, time-varying beliefs, or supply-demand effects (see, e.g., Cochrane (2011); Campbell (2014); Gabaix and Koijen (2021)). We complement this literature by showing that macroeconomic shocks, along with the optimal monetary policy response to these shocks, can cause the appearance of "excess" volatility in asset prices. In our model, an activist Fed trying to stabilize the economy in response to (non-financial) aggregate demand shocks will deliberately generate asset price fluctuations not linked to underlying productivity.

Our model is also consistent with the "excess" volatility in long-term bonds observed by Van Binsbergen (2020). In our model, when there is a positive financial shock, the Fed reduces bond prices to insulate aggregate asset prices from this shock (the Fed put/call). When there is a positive aggregate demand shock, the Fed once again reduces bond prices—this time to reduce aggregate asset prices to insulate the economy from the demand shock. We show that the two types of shocks have different effects on the covariance of aggregate bond and stock prices. A positive financial shock reduces bond prices while *increasing* stock prices, whereas a positive demand shock reduces both stock and bond prices (see Remark 3). Assuming the composition of these shocks changes over time, our model can help explain the changes in the covariance between bond and stock returns observed in the data (see, e.g., Pflueger and Viceira (2011); Campbell et al. (2009, 2020)).

Finally, the idea that monetary policy affects and operates through asset prices is well supported empirically by Jensen et al. (1996), Thorbecke (1997), Jensen and Mercer (2002), Rigobon and Sack (2004), Ehrmann and Fratzscher (2004), Bernanke and Kuttner (2005), Bauer and Swanson (2020), among others. Moreover, Cieslak and Vissing-Jorgensen (2020) conduct a textual analysis of FOMC documents and find strong support for the idea that the Fed pays attention to stock prices and cuts interest rates after stock price declines ("the Fed put"). We build on these insights and turn them into an asset pricing framework. If monetary policy operates through financial markets, then the Fed would ideally like asset prices to be consistent with monetary policy objectives. These objectives depend on the nature of the shocks that hit the economy and on the macroeconomic frictions. In our model, the Fed provides a put (and a call) for the market's belief (valuation) shocks and "uses" the market to offset non-financial aggregate demand shocks.

The rest of the paper is organized as follows. Section 2 introduces our baseline model without transmission delays or inertia. Section 3 establishes our results for transmission lags and internal inertia. Section 4 endogenizes inflation. Section 5 introduces disagreements between the market and the Fed. Section 6 provides final remarks. The appendix contains the omitted derivations and extensions.

2. The baseline model without transmission lags

In this section, we develop a baseline version of our model without transmission lags. This model is a variant of the textbook New Keynesian model with the main difference that we allow for a financial market block with a non-trivial risk premium, and we characterize the implications of the model for aggregate asset prices. Even in this relatively standard setup, *aggregate* asset prices are determined by macroeconomic needs, rather than by financial forces such as cash-flow expectations or risk premia. In particular, aggregate demand shocks induce seemingly "excess" (policy-induced) asset price volatility. In contrast, the market's belief shocks about future cash flows do not affect aggregate asset prices because they are stabilized by monetary policy ("the Fed put/call").

2.1. Environment

The economy is set in discrete time $t \in \{0, 1, ..\}$. There are four types of agents: "asset-holding households" (the households), "hand-to-mouth agents," "portfolio managers (the market)," and "the central bank (the Fed)." Hand-to-mouth agents do not play an important role beyond decoupling the labor supply from the households' consumption behavior. Households make consumption-savings decisions (possibly with frictions) that drive aggregate demand. The market makes a portfolio choice decision on behalf of the households and determines asset prices. The Fed sets monetary policy to close the output gap.

Supply side and nominal rigidities. The supply side features a competitive final goods sector and monopolistically competitive intermediate goods firms that produce according to

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}}, \text{ where } Y_t(\nu) = A_t L_t(\nu)^{1-\alpha}.$$

For now, the intermediate good firms have fully sticky nominal prices (we endogenize inflation in Section 4). Since these firms operate with a markup, they find it optimal to meet the demand for their good (for relatively small demand shocks, which we assume). Therefore, output is determined by aggregate demand, which depends on the consumption of households, C_t^H, and hand-to-mouth agents, C_t^{HM}:

$$Y_t = C_t^H + C_t^{HM}. \tag{1}$$

Labor is supplied by the hand-to-mouth agents. They have the per-period utility function

$$\log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi},$$

which leads to a standard labor supply curve (see Appendix A.1).

With these production technologies, if the model was fully competitive, labor's share of output would be constant and given by $(1-\alpha) Y_t$. However, since the intermediate good firms have monopoly power and make pure profits, labor's share is smaller than $(1-\alpha) Y_t$. To simplify the exposition, we assume the government taxes part of the firms' profits (lump-sum) and redistributes to workers (lump-sum), so that labor's share is as in the fully competitive case (see Appendix A.1 for details). This implies the spending of hand-to-mouth agents (who supply all labor) is

$$C_t^{HM} = (1-\alpha) Y_t. \tag{2}$$

Combining Eqs. (1) and (2) yields

$$Y_t = \frac{C_t^H}{\alpha}. \tag{3}$$

Hand-to-mouth agents create a Keynesian multiplier effect, but output is ultimately determined by (asset-holding) *households'* spending, C_t^H.

Potential output and aggregate supply shocks. Consider a flexible-price benchmark economy without nominal rigidities (the same setup except the intermediate good firms have fully flexible prices). In this benchmark, the equilibrium labor supply is constant and solves $\chi (L^*)^{1+\varphi} = \frac{\varepsilon-1}{\varepsilon}$ (see Appendix A.1). Output is given by $Y_t^* = A_t (L^*)^{1-\alpha}$. We refer to Y_t^* as *potential* output. Log potential output, $y_t^* = \log Y_t^*$, is driven by A_t and evolves according to

$$y_{t+1}^* = y_t^* + z_{t+1}, \qquad \text{where } z_{t+1} \sim N\left(0, \sigma_z^2\right). \tag{4}$$

For simplicity, supply shocks are permanent and follow a log-normal distribution.

In our model with sticky prices, output is given by (3) and can deviate from its potential. We let $y_t = \log Y_t$ denote log output and $\tilde{y}_t = y_t - y_t^*$ denote the *output gap.*

Financial assets. There are two assets. There is a market portfolio, which is a claim on firms' profits αY_t (the firms' share of output). We let P_t denote the ex-dividend price of the market portfolio. Its gross return is given by

$$R_{t+1} = \frac{\alpha Y_{t+1} + P_{t+1}}{P_t}. \tag{5}$$

There is also a risk-free asset in zero net supply. Its gross return R_t^f is set by the Fed, as we describe subsequently.

Households' consumption-savings decisions and demand shocks. Households have standard preferences:

$$E_t \left[\sum_{h=0}^{\infty} \beta^{t+h} \log C_{t+h}^H \right], \tag{6}$$

along with the budget constraint

$$\begin{aligned}
W_{t+1} + C^H_{t+1} &= W_t \left((1-\omega_t) R^f_t + \omega_t R_{t+1} \right) \\
&= D_{t+1} + K_{t+1}, \qquad (7) \\
\text{where } D_{t+1} &= W_t \left[(1-\omega_t) \left(R^f_t - 1 \right) + \omega_t \frac{\alpha Y_{t+1}}{P_t} \right] \\
\text{and } K_{t+1} &= W_t \left[1 - \omega_t + \omega_t \frac{P_{t+1}}{P_t} \right].
\end{aligned}$$

W_t denotes the end-of-period wealth and ω_t denotes the market portfolio weight in period t. The term $W_t \left((1-\omega_t) R^f_t + \omega_t R_{t+1} \right)$ is the beginning-of-period wealth in period $t+1$. The second line breaks this term into a component that captures the interest and dividend income (D_{t+1}) and a residual component that captures the capital (K_{t+1}). This distinction will facilitate our exposition.

Households make a consumption-savings decision. However, they do not *necessarily* make an optimal decision. Rather, we assume households follow consumption rules. To impose some discipline on these rules, we start with the optimal rule with the preferences in (6), which is given by

$$C^H_t = (1-\beta)(D_t + K_t).$$

According to the optimal rule, households spend a fraction of their beginning-of-period wealth. We consider empirically-grounded deviations from this rule. In the benchmark model, we assume that consumption instead follows:

$$C^H_t = (1-\beta)(D_t + K_t \exp(\delta_t)), \qquad \text{where } \delta_t \sim N\left(0, \sigma^2_\delta\right). \qquad (8)$$

Here, δ_t captures aggregate demand shocks; all else equal, a higher δ_t means households spend more than predicted by the optimal rule. The exact functional form does not play an important role beyond simplifying the expressions. The special case $\sigma^2_\delta = 0$ corresponds to the textbook model in which households' consumption is fully optimal.

The demand shock captures various behavioral or informational frictions that affect households' spending in practice, e.g., a consumer sentiment shock. It can also be viewed as a simple modeling device to capture a variety of shocks that affect aggregate demand, e.g., fiscal policy shocks. We assume the demand shocks are transitory, although our analysis is flexible and can accommodate more persistent shocks. In subsequent sections, we will modify the rule in (8) to introduce transmission lags and demand inertia.

The portfolio managers (the market) and the portfolio allocation. Households delegate their portfolio choice to portfolio managers (the market), who invest on their behalf. The portfolio managers are infinitesimal and they do not consume themselves. They make a portfolio allocation to maximize expected log household *wealth*,

$$\max_{\omega_t} E_t^M \left[\log \left(W_t \left(R_t^f + \omega_t \left(R_{t+1} - R_t^f \right) \right) \right) \right]. \tag{9}$$

We formulate the portfolio problem in terms of wealth, rather than consumption, because we allow consumption to deviate from the optimal rule. In our setup, wealth is a more accurate representation of welfare, as it captures the *ideal* consumption a household could choose if she followed the optimal rule. We assume portfolio managers maximize log-wealth in line with the households' preferences in (6). In the special case where households follow the optimal rule ($\sigma_\delta^2 = 0$), problem (9) results in portfolio allocations that maximize the households' utility. The superscript M captures the market's belief.

Problem (9) implies a standard optimality condition,

$$E_t^M \left[\left(R_{t+1} - R_t^f \right) \frac{1}{R_t^f + \omega_t \left(R_{t+1} - R_t^f \right)} \right] = 0. \tag{10}$$

Asset market clearing and the equilibrium return. Financial markets are in equilibrium when the households hold the market portfolio, both before and after the portfolio allocation:

$$W_t = P_t \quad \text{and} \quad \omega_t = 1. \tag{11}$$

Substituting $\omega_t = 1$ into the optimality condition (10), we obtain $E_t^M \left[\frac{R_t^f}{R_{t+1}} \right] = 1$. Assuming R_{t+1} is (approximately) log-normally distributed, this implies a *financial market equilibrium condition,*

$$E_t^M \left[r_{t+1} \right] + \frac{1}{2} var_t^M \left[r_{t+1} \right] - i_t = rp_t \equiv var_t^M \left[r_{t+1} \right]. \tag{12}$$

We use lower-case letters to represent the log of the corresponding variable and $i_t = \log R_t^f$ to denote the log risk-free interest rate. In equilibrium, the expected excess return on the market portfolio is equal to the required risk premium, which is determined by the variance of the aggregate return.

Campbell-Shiller approximation to the equilibrium return. We use a log-normal approximation to the equilibrium return on the market portfolio, $R_{t+1} = \frac{\alpha Y_{t+1} + P_{t+1}}{P_t}$, that facilitates closed-form solutions. In Appendix A.2, we show that absent shocks the dividend price ratio is constant and given by $\frac{\alpha Y_t}{P_t} = \frac{1-\beta}{\beta}$. We then log-linearize (5) around this ratio to obtain

$$\begin{aligned} r_{t+1} &= \kappa + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t, \\ \text{where } \kappa &\equiv -\beta \log \beta - (1-\beta) \log \left(\frac{1-\beta}{\alpha} \right). \end{aligned} \tag{13}$$

This is the Campbell-Shiller approximation applied to our model (see Campbell (2017)).

The central bank (the Fed) and monetary policy. In each period, the Fed sets the risk-free interest rate (without commitment) to minimize the discounted sum of quadratic log output gaps:

$$\min_{R_t^f} E_t^F \left[\sum_{h=0}^{\infty} \beta^h \tilde{y}_{t+h}^2 \right]. \tag{14}$$

The superscript F captures the Fed's belief. In the baseline model, the solution to problem (14) is simple; the Fed always sets the interest rate that closes the output gap

$$Y_t = Y_t^*, \quad \text{which implies } \tilde{y}_t = y_t - y_t^* = 0. \tag{15}$$

2.2. Macroeconomic needs drive the aggregate asset price

We next characterize the equilibrium and establish the main result in this section. We do this in a slight extension of the baseline setup in which the market thinks the supply shocks are drawn from

$$z_{t+1} \sim N\left(b_t, \sigma_z^2\right), \quad \text{where } b_t \sim N\left(0, \sigma_b^2\right). \tag{16}$$

Here, b_t is a *belief shock for future cash flows.* This enables us to capture standard financial forces, such as time-varying expectations (or risk premia), that *can* drive asset prices without a change in *current* cash flows. The special case $\sigma_b^2 = 0$ is the baseline model (see (4)).

To solve for the equilibrium, we first combine Eqs. (7) and (11) to obtain $D_t = \alpha Y_t, K_t = P_t$. In equilibrium, dividends are equal to the firms' share of output. Capital is equal to the (ex-dividend) value of the market portfolio. Substituting these observations into the consumption rule in (8), we obtain

$$C_t^H = (1-\beta)(\alpha Y_t + P_t \exp(\delta_t)).$$

Substituting Eq. (3) ($C_t^H = \alpha Y_t$) into this expression yields *an output-asset price relation*

$$\begin{aligned} Y_t &= (1-\beta)\frac{1}{\alpha\beta}P_t \exp(\delta_t) \\ \Longrightarrow & \quad y_t = m + p_t + \delta_t, \quad \text{where } m \equiv \log\left(\frac{1-\beta}{\alpha\beta}\right). \end{aligned} \tag{17}$$

Output depends on aggregate wealth, P_t, the MPC out of wealth, $1-\beta$, the demand shock, δ_t, and the Keynesian multiplier, $1/(\alpha\beta)$. The second line describes the relation in logs and obtains the derived parameter m.

The output-asset price relation in (17) and its variants play a central role in our analysis. Specifically, we invert this equation to find the asset price that solves the Fed's policy problem. In this section, the Fed sets output equal to its potential at all times (see (15)). Therefore, the Fed targets asset prices

$$p_t = p_t^* \equiv y_t^* - m - \delta_t,$$

which, from the output-asset price relation, ensures that $y_t = y_t^*$. We normalize p_t^* by potential output to define *"pystar"—the Fed's target (log) aggregate asset price per potential output:*

$$(py)_t^* \equiv p_t^* - y_t^* = -m - \delta_t. \tag{18}$$

The Fed targets asset prices such that households spend just enough to ensure that aggregate demand is equal to aggregate supply.

How does the Fed achieve "pystar"? This depends on the financial market side of the model. Using (13), along with $y_t = y_t^*$ and $p_t = p_t^*$, we calculate

$$\begin{aligned} r_{t+1} &= \rho + (1-\beta) y_{t+1}^* + \beta\left(y_{t+1}^* - \delta_{t+1}\right) - (y_t^* - \delta_t) \\ &= \rho + \delta_t + z_{t+1} - \beta\delta_{t+1}. \end{aligned} \tag{19}$$

The equilibrium return is affected by supply *and* demand shocks. A positive future supply shock increases the return. A positive future demand shock δ_{t+1} *reduces* the realized return (due to the policy response it triggers). Combining (19) and (12), and using (16), we solve for the equilibrium interest rate the Fed needs to set to achieve "pystar," and for the equilibrium risk premium

$$i_t = \rho + \delta_t + b_t - \frac{1}{2} rp_t \quad \text{and} \quad rp_t = \sigma_z^2 + \beta^2 \sigma_\delta^2. \tag{20}$$

Both supply and demand shocks raise the risk premium because they contribute to asset price volatility. The interest rate is decreasing in the risk premium and increasing in the demand shock and in the belief shock. The following result summarizes the equilibrium.

Proposition 1 (Macroeconomic needs drive the aggregate asset price)**.** *Consider the model without transmission lags and with belief shocks for future cash flows. In equilibrium, the Fed targets a* "pystar" *given by* (18)*. The equilibrium return on the market portfolio is given by* (19)*. The policy interest rate and the risk premium are given by* (20)*.*

This result shows that the *aggregate* asset price is driven by *macroeconomic needs* rather than by standard financial forces. In particular, Eq. (18) shows that the aggregate asset price depends on macroeconomic variables and shocks (m and δ_t), but *not* on financial forces such as beliefs for future cash flows (b_t). Eq. (20) shows that these belief shocks (as well as risk premia shocks) are absorbed by the interest rate. The result also describes *when* the Fed destabilizes or stabilizes financial markets, as we note in the following corollaries.

Corollary 1 (Demand shocks and policy-induced "excess" volatility)**.** *A positive demand shock increases the interest rate and* reduces *"pystar" (and vice versa for a negative demand shock). Thus, the volatility of the price of the market portfolio and the risk premium are increasing in the volatility of the demand shock, σ_δ.*

The Fed successfully mitigates the output effect of demand shocks, but in doing so it increases the aggregate asset price volatility and the risk premium. Thus, demand shocks can create the appearance of policy-induced "excess" asset price volatility. To an outside observer, the aggregate asset price might appear to be "excessively" volatile, but this volatility plays a useful macroeconomic stabilization role.

Corollary 2 (The Fed put/call)**.** *A positive belief shock for future cash flows increases the policy interest rate (and vice versa for a negative belief shock) but does not affect "pystar." Thus, the volatility of the price of the market portfolio and the risk premium do not depend on the volatility of the belief shock, σ_b.*

The Fed insulates the aggregate asset price from beliefs about future cash flows. For instance, when the market becomes more pessimistic about future cash flows, it exerts downward pressure on asset prices. If the Fed left the interest rate unchanged, the aggregate asset price would decline and induce a demand recession. Therefore, the Fed reduces the interest rate to absorb the downward price pressure and keep the aggregate asset price unchanged—providing an explanation for *"the Fed put."* Consequently, standard drivers of asset prices, such as time-varying beliefs (or risk premia), do *not* contribute to *aggregate* asset price volatility. Note that the Fed does not minimize asset price volatility *per se*—the Fed simply mitigates the output (gap) volatility that the (beliefs-driven) asset price volatility would otherwise induce.

Taken together, these corollaries imply that the Fed stabilizes *aggregate* asset price fluctuations that result from purely financial shocks, but it *destabilizes* asset prices in response to macroeconomic (demand) shocks. These seemingly opposing results follow from the common principle that the *aggregate* asset price is primarily driven by *macroeconomic* needs.

Remark 1 ("pystar" vs "rstar")**.** *The Fed's target aggregate asset price per potential output, "pystar," resembles the "rstar" in the textbook New Keynesian model—the interest rate that closes the output gaps. In our model, as Eq.* (17) *makes clear, monetary policy works through aggregate asset prices rather than through the short-term interest rate—the latter is simply the Fed's policy tool to achieve its target asset price. Therefore, our model makes more precise predictions for "pystar" than it does for "rstar" (see also the concluding section).*

Remark 2 (Output-asset price relation)**.** *The output-asset price relation* (17) *can be interpreted more broadly as a reduced form for various channels that link asset prices and aggregate demand. For example, in Caballero and Simsek (2020) we show that adding investment also leaves the output-asset price relation qualitatively unchanged (due to a Q-theory mechanism).*

Remark 3 (Stocks and bonds)**.** *We highlight that "pystar" refers to the (normalized) target price of the market portfolio, which includes a broad collection of assets. Our results constrain asset prices collectively, but allow for traditional financial factors to play a role at the individual asset (or asset-class) level.*[4] *What does our model imply for the price*

[4] To see this, suppose we allow the portfolio managers to also trade a risky asset j with payoff $\left\{X_t^j\right\}$. Problem (9) then implies that the price of this asset satisfies a standard formula

$$P_{t,j} = E_t^M \left[M_{t+1}\left(X_{t+1}^j + P_{t+1}^j\right)\right] \text{ where } M_{t+1} = \frac{1}{R_{t+1}}.$$

Here, M_{t+1} is the stochastic discount factor, driven by the return on the market portfolio.

of aggregate *stocks and bonds? We investigate this question in Appendix B.1, where we allow production firms to issue (risk-free) debt. Thus, there are two claims on the market portfolio: $P_t = P_t^s + P_t^b$, where P_t^s is the price of the equity claim ("aggregate stocks") and P_t^b is the price of the debt claim ("aggregate bonds"). In that context, we show that the price of the market portfolio and the risk-free rate are the same as in this section (see Proposition 6). However, the shocks have richer effects on the price of the equity and debt claims. In particular, a positive belief shock raises the price of the equity claim and reduces the price of the debt claim (see Corollary 11). While the Fed stabilizes the sum, $P_t = P_t^s + P_t^b$, an increase in expected earnings raises the price of the equity claim* relative *to the debt claim, since the equity claim is more exposed to future earnings. In contrast, a positive demand shock uniformly reduces the price of both equity and debt claims, since it affects asset prices primarily by raising the interest rate (see Corollary 10).*

3. Asset pricing with lags and inertia

So far, we have assumed that monetary policy affects asset prices instantaneously *and* that asset prices affect aggregate demand instantaneously. These assumptions imply that monetary policy is very powerful: it can set output to its potential at all times. In practice, monetary policy has much less control over aggregate demand. A major reason for this lack of control is *transmission lags*—a large empirical literature documents that the full effect of monetary policy on output builds over several quarters. These lags are an implication of *aggregate demand inertia,* the idea that agents tend to repeat their past spending behavior and thus respond to exogenous disturbances (such as monetary policy or asset prices) gradually. Inertia is a realistic feature that may arise from microeconomic frictions such as adjustment costs or habit formation (see Caballero and Simsek (2021b, 2022b) and Woodford (2005), Chapter 5 for further discussion). Quantitative New-Keynesian models typically assume inertia, because it helps match the observed delayed response of aggregate demand to a variety of shocks. In this section, we investigate the implications of inertia for the effect of monetary policy on asset prices.

Suppose households follow a modified version of the rule in (8),

$$C_t^H = (1-\beta) D_t + \eta\beta C_{t-1}^H + (1-\eta)(1-\beta) K_{t-1} \exp(\delta_t), \tag{21}$$

where $\delta_t \sim N(0, \sigma_\delta^2)$ as before and $\eta \in [0, 1)$.

This rule captures two distinct types of inertia. First, there are *transmission lags*: households respond to the *lagged* value of the capital portion of their wealth. In fact, when $\eta = 0$ the rule, $C_t^H = (1-\beta)(D_t + K_{t-1}\exp(\delta_t))$, is the same as before except that K_t is replaced by K_{t-1}. To simplify the equations, we assume households respond to capital with a lag of one period, but they respond to dividend and interest income immediately (see Remark 4 at the end of the section on how to quantify the length of a period).

Second, for $\eta > 0$, there is also *internal inertia*: in addition to responding to asset prices with a lag, households partly repeat their own (past) spending behavior. All else equal, strong current spending implies strong spending in the future (and vice versa for weak spending). Formally, households respond to a weighted-average of their past spending and lagged aggregate wealth. The parameter η captures the extent of internal inertia. We multiply the coefficient on lagged spending by β so that the equation holds in a steady state.[5]

To characterize the equilibrium, observe that Eq. (21) along with $D_t = \alpha Y_t, K_t = P_t, C_t^H = \alpha Y_t$ implies a modified output-asset price relation (cf. (17))

$$Y_t = \left(\eta Y_{t-1} + (1-\eta)\frac{1-\beta}{\alpha\beta}P_{t-1}\right)\exp(\delta_t).$$

In Appendix A.3, we approximate this relation (around the steady state for Y_t/P_t) to obtain

$$y_t = (1-\eta)m + \eta y_{t-1} + (1-\eta)p_{t-1} + \delta_t. \tag{22}$$

Asset prices affect output as before, but the effects operate with a lag. In view of internal inertia, output also depends on past output.

An immediate implication of Eq. (22) is that output gaps can no longer be zero at all times and states. To see this, consider the equilibrium in period t. Since p_{t-1} is predetermined, output fluctuates with demand shocks δ_t. However, potential output still evolves according to (4) and fluctuates according to supply shocks z_t. Since δ_t and z_t are uncorrelated (by assumption), the output gap is non-zero except for a measure zero set of events. Because output responds to asset prices with a lag, *both supply and demand shocks* lead to output gaps, which the Fed cannot offset.

In this case, the Fed minimizes the same quadratic objective function (14) as before, but subject to the constraint (22). Then, the optimal policy (without commitment)

[5]In Caballero and Simsek (2021b), we derive a version of the rule in (21) by assuming that in every period only a fraction of agents adjust their spending. Here, we simply assume the equation as an aggregate "rule" and derive its implications for asset prices.

implies

$$E_t^F\left[y_{t+1}\right]=E_t^F\left[y_{t+1}^*\right]. \tag{23}$$

That is, the Fed sets *expected* demand equal to *expected* supply, *under its belief.* The rest of the model is the same as in Section 2.1.

In the rest of this section, we derive the asset pricing implications of $(22-23)$. We start with the case, $\eta=0$, which isolates the role of transmission lags. We show that transmission lags imply that *the Fed's beliefs* about future demand and supply drive monetary policy decisions *and* the aggregate asset price. We then consider the case, $\eta>0$, and show that it implies policy-induced *asset price overshooting.*

3.1. Transmission lags and the Fed's beliefs

Suppose $\eta=0$. In this case, the output-asset price relation (22) becomes

$$y_t=m+p_{t-1}+\delta_t. \tag{24}$$

Combining this with the policy rule in (23), and using $y_{t+1}^*=y_t^*+z_{t+1}$, we obtain

$$m+p_t^*+E_t^F\left[\delta_{t+1}\right]=y_t^*+E_t^F\left[z_{t+1}\right].$$

Expected demand depends on the Fed's target asset price and the Fed's expectation for the demand shock. Expected supply depends on the Fed's expectation for the supply shock. We can then solve for the Fed's target aggregate asset price

$$p_t=p_t^*\equiv y_t^*-E_t^F\left[\tilde{\delta}_{t+1}\right]-m,$$

where $\tilde{\delta}_{t+1}\equiv\delta_{t+1}-z_{t+1}$ is *the net demand shock.* As before, we normalize this price with the current potential output, which yields "pystar":

$$(py)_t^*=p_t^*-y_t^*=-E_t^F\left[\tilde{\delta}_{t+1}\right]-m. \tag{25}$$

Now "pystar" depends on the Fed's *expectation* about future macroeconomic needs. The Fed sets a higher "pystar" when it expects lower future demand or higher future supply. Conversely, the Fed sets a lower "pystar" when it expects higher future demand or lower future supply.

Substituting Eq. (25) into (24) (with $p_t = p_t^*$) we solve for future output and its gap:

$$y_{t+1} = y_t^* + \delta_{t+1} - E_t^F\left[\tilde{\delta}_{t+1}\right] \tag{26}$$

$$\tilde{y}_{t+1} = \tilde{\delta}_{t+1} - E_t^F\left[\tilde{\delta}_{t+1}\right]. \tag{27}$$

The first equation says that output is driven by *demand* shocks relative to the Fed's forecast of *net* demand, while supply shocks do not affect output contemporaneously. The second equation says that the output gap is driven by the unforecastable component of net demand shocks. If demand is realized to be higher than (or supply is realized to be lower than) what the Fed forecasted, then the output gap is positive. The following proposition summarizes this discussion.

Proposition 2 (Transmission lags and the Fed's beliefs)**.** *Consider the model with pure transmission lags ($\eta = 0$). In equilibrium, "pystar" is given by* (25) *and is* decreasing *in the Fed's beliefs about future* net *aggregate demand. Output and its gap are given by* $(26-27)$*. The output gap is driven by net demand shocks relative to the Fed's forecast.*

To characterize the equilibrium further, we need to specify the Fed's beliefs about net demand. We do this in a slight variant of the baseline model in which the agents receive *macroeconomic news* about future aggregate demand—news about future supply leads to similar results. In addition to closing the model, this setup enables us to investigate how macroeconomic news affects asset prices and output volatility.

3.1.1. Macroeconomic news

Suppose the agents receive a signal about the next period's demand shock:

$$s_t = \delta_{t+1} + e_t, \quad \text{where } e_t \sim N\left(0, \sigma_e^2\right).$$

For now, the Fed and the market agree on the interpretation of the signal. We consider the implications of disagreements between the Fed and the market in Section 5. Throughout the paper, when agents have common beliefs, we drop the superscript on the expectations and the variance operators. For notational simplicity, we assume that agents do not receive a signal about the future supply shock and thus believe $E_t[z_{t+1}] = 0$.

Recall that demand shocks are drawn from the i.i.d. distribution, $N(0, \sigma_\delta^2)$. Therefore, after observing s_t, the Fed and the market have common posterior beliefs given by

$$\begin{aligned} \delta_{t+1} &\sim N\left(\gamma s_t, \sigma^2_{\overline{\delta}}\right) \quad \text{where} \\ \gamma &= \frac{1/\sigma^2_e}{1/\sigma^2_e + 1/\sigma^2_\delta} \quad \text{and} \quad \sigma^2_{\overline{\delta}} = \frac{1}{1/\sigma^2_e + 1/\sigma^2_\delta}. \end{aligned} \tag{28}$$

The posterior mean is a dampened version of the signal, and the posterior variance is smaller than the prior variance.

With this setup, agents' common belief for the expected net demand in the next period is $E_t\left[\tilde{\delta}_{t+1}\right] = E_t\left[\delta_{t+1}\right] = \gamma s_t$ (since $E_t\left[z_{t+1}\right] = 0$). The following corollary characterizes the equilibrium with these beliefs, It also shows that macroeconomic news has opposite effects on output and asset price volatilities.

Corollary 3 (Macroeconomic news and volatility)**.** *Consider the setup in Proposition 2 with news about future demand. The equilibrium is given by:*

$$p_t = p_t^* \equiv y_t^* - \gamma s_t - m \tag{29}$$

$$y_{t+1} = y_t^* + \delta_{t+1} - \gamma s_t \tag{30}$$

$$\tilde{y}_{t+1} = \delta_{t+1} - \gamma s_t - z_{t+1} \tag{31}$$

$$r_{t+1} = \rho + \gamma s_t + (1-\beta)\left(\delta_{t+1} - \gamma s_t\right) + \beta\left(z_{t+1} - \gamma s_{t+1}\right) \tag{32}$$

$$i_t = \rho + E_t\left[r_{t+1}\right] - \frac{1}{2} r p_t, \text{ with } E_t\left[r_{t+1}\right] = \gamma s_t \tag{33}$$

$$r p_t = var_t\left(r_{t+1}\right) = (1-\beta)^2 \sigma^2_{\overline{\delta}} + \beta^2\left(\sigma^2_z + \sigma^2_\delta - \sigma^2_{\overline{\delta}}\right). \tag{34}$$

The conditional volatility of output and asset prices are given by

$$var_t\left(y_{t+1}\right) = \sigma^2_{\overline{\delta}} \quad \text{and} \quad var_t\left(p_{t+1}\right) = \sigma^2_z + \left(\sigma^2_\delta - \sigma^2_{\overline{\delta}}\right). \tag{35}$$

More precise news (lower σ^2_e and $\sigma^2_{\overline{\delta}}$) reduces the volatility of output but increases the volatility of the price of the market portfolio. When $\beta > 1-\beta$ (which holds for reasonable calibrations), more precise news also increases the return volatility and the risk premium.

Eqs. $(29-31)$ follow from substituting the agents' beliefs into $(25-27)$. Eq. (35) uses the equilibrium characterization to calculate the volatility of the output and the aggregate asset price. Output volatility depends on the *unforecastable* demand variance, $var_t\left(\delta_{t+1} - \gamma s_t\right) = \sigma^2_{\overline{\delta}}$, whereas the aggregate asset price volatility depends on the *forecastable* demand variance, $var_t\left(\gamma s_{t+1}\right) = \sigma^2_\delta - \sigma^2_{\overline{\delta}}$ and on the supply variance σ^2_z. With more precise signals, the Fed becomes more "activist" and preemptively responds to demand shocks. Hence, more precise news lowers the volatility of output but *raises* the volatility of the aggregate asset price.

Corollary 3 characterizes the rest of the equilibrium and yields additional results. To understand these results, first consider Eq. (32) that describes the equilibrium return. This expression follows by substituting the equilibrium output and asset price into (13). In equilibrium, the return is increasing in the future demand shock relative to expectations, $\delta_{t+1} - \gamma s_t$, because this raises future output, y_{t+1}. The return is also increasing in the future supply shock z_{t+1} (resp. decreasing in the future demand signal, s_{t+1}), because this increases (reps. decreases) future asset prices, p_{t+1}.

Next consider Eqs. $(33-34)$ that describe the interest rate and the risk premium. These expressions follow from combining (32) with (12). In equilibrium, the interest rate is increasing in the demand news; after a positive demand signal, $s_t > 0$, the Fed reduces the aggregate asset price by *increasing* the interest rate. The risk premium (the return volatility) reflects a weighted average of future output (cash flow) volatility and future asset price volatility. Since more precise news reduces output volatility but raises asset price volatility, it exerts counteracting effects on the risk premium. When $\beta > 1-\beta$ (which holds for reasonable calibrations), more precise news also *increases* the risk premium. That is, since conditional asset returns depend on future asset prices relatively more than on future cash flows, a higher asset price volatility translates into a higher risk premium, despite greater macroeconomic stability and lower output (cash flow) volatility.

3.2. Internal inertia and asset price overshooting

Next consider the output-asset price relation (22) with internal inertia ($\eta > 0$)

$$y_t = (1-\eta)\, m + \eta y_{t-1} + (1-\eta)\, p_{t-1} + \delta_t.$$

Combining this with (23), and using $y^*_{t+1} = y^*_t + z_{t+1}$, we obtain

$$(1-\eta)\, m + \eta y_t + (1-\eta)\, p^*_t + E^F_t [\delta_{t+1}] = y^*_t + E^F_t [z_{t+1}] \, .$$

As before, we invert this equation to solve for "pystar"

$$(py)^*_t = p^*_t - y^*_t = -\frac{\eta}{1-\eta}\tilde{y}_t - \frac{E^F_t\left[\tilde{\delta}_{t+1}\right]}{1-\eta} - m, \quad \text{where } \tilde{\delta}_{t+1} \equiv \delta_{t+1} - z_{t+1}. \tag{36}$$

Compared to Eq. (25), the equilibrium features *asset price overshooting*—when the output gap is negative (in a demand recession), "pystar" is higher than usual (and vice versa for a positive output gap). With inertia, in a demand recession the Fed realizes that the current weakness in economic activity will persist into the feature. Therefore, the Fed overshoots asset prices upward to neutralize the future effects of current weakness. Conversely, in a demand boom, the Fed overshoots asset prices downward to neutralize the future effects of strong spending in the current period. The overshooting mechanism creates a seeming disconnect between the performance of the economy and the financial markets, but this disconnect is useful in closing the output gap.

Eq. (36) also implies that inertia *amplifies* the Fed's response to the current output gap and to its net demand forecast. Since inertia reduces the MPC out of wealth in a given period (controlling for the cumulative impact), the Fed "turns up" the signal to compensate for inertia and induce a faster recovery.

Substituting (36) back into (22) (with $p_t = p_t^*$) we solve for output and its gap

$$y_{t+1} = y_t^* + \delta_{t+1} - E_t^F \left[\tilde{\delta}_{t+1} \right] \tag{37}$$

$$\tilde{y}_{t+1} = \tilde{\delta}_{t+1} - E_t^F \left[\tilde{\delta}_{t+1} \right]. \tag{38}$$

These expressions are the same as before [see (26)]. Since the Fed overshoots asset prices to neutralize the effects of the current output gap, future output and its gap are driven by unforecastable shocks, as before. The following proposition summarizes this discussion.

Proposition 3 (Internal inertia and asset price overshooting)**.** *Consider the model with internal inertia and transmission lags ($\eta > 0$). In equilibrium, "pystar," output, and the output gap are given by Eqs.* $(36 - 38)$*. The equilibrium features a Fed-induced asset price overshooting: in response to a negative current output gap, the Fed targets a* higher-than-average *"pystar" (and vice versa for a positive output gap).*

We next adopt the belief structure in Section 3.1.1 where agents' (common) beliefs satisfy $E_t[\delta_{t+1}] = \gamma s_t$ and $E_t[z_{t+1}] = 0$. The following result completes the characterization and describes the covariance of output and asset prices. This covariance plays a key role in our analysis of inflation in the next section.

Corollary 4 (Covariance between the output gap and asset prices)**.** *Consider setup in*

Proposition 4 with the belief structure in Section 3.1.1. The equilibrium is given by

$$p_t = p_t^* = y_{t-1}^* + z_t - \frac{\eta}{1-\eta}\left(\delta_t - \gamma s_{t-1} - z_t\right) - \frac{\gamma s_t}{1-\eta} - m \tag{39}$$

$$y_{t+1} = y_t^* + \delta_{t+1} - \gamma s_t \tag{40}$$

$$\tilde{y}_{t+1} = \delta_{t+1} - \gamma s_t - z_{t+1} \tag{41}$$

$$\begin{aligned} r_{t+1} &= \rho + \frac{\gamma s_t}{1-\eta} + \frac{\eta}{1-\eta}\left(\delta_t - \gamma s_{t-1} - z_t\right) \\ &\quad + \left((1-\beta) - \beta\frac{\eta}{1-\eta}\right)\left(\delta_{t+1} - \gamma s_t\right) + \frac{\beta}{1-\eta}\left(z_{t+1} - \gamma s_{t+1}\right) \end{aligned} \tag{42}$$

$$i_t = E_t\left[r_{t+1}\right] - \frac{1}{2} rp_t, \text{ with } E_t\left[r_{t+1}\right] = \rho + \frac{\gamma s_t}{1-\eta} + \frac{\eta\left(\delta_t - \gamma s_{t-1} - z_t\right)}{1-\eta} \tag{43}$$

$$rp_t = var_t\left(r_{t+1}\right) = \left(\frac{1-\eta-\beta}{1-\eta}\right)^2 \sigma_{\tilde{\delta}}^2 + \left(\frac{\beta}{1-\eta}\right)^2\left(\sigma_z^2 + \sigma_\delta^2 - \sigma_{\tilde{\delta}}^2\right). \tag{44}$$

The output gap and the price of the market portfolio are negatively correlated:

$$cov_t\left(\tilde{y}_{t+1}, p_{t+1}\right) = -\frac{\eta}{1-\eta}\sigma_{\tilde{\delta}}^2 - \frac{1}{1-\eta}\sigma_z^2, \tag{45}$$

where $\sigma_{\tilde{\delta}}^2 = var_t\left(\delta_{t+1} - \gamma s_t\right)$ *is the unforecastable demand variance [see* (28)*].*

Eqs. $(39-44)$ generalize Eqs. $(29-34)$ to the setting with inertia. Eq. (45) uses the equilibrium characterization to show that *demand and supply shocks both* induce a *negative* conditional covariance between the output gap and asset prices.

To understand (45), first consider a negative supply shock, $z_{t+1} < 0$. This shock increases the output gap $\tilde{y}_{t+1}$, by reducing potential output y_{t+1}^*. At the same time, since the decline in the potential output is persistent, the shock induces the Fed to target *lower* asset prices. Note that this mechanism is driven by transmission lags but it does not rely on internal inertia. Supply shocks induce a negative covariance between the output gap and asset prices for any $\eta \geq 0$.

Next consider a positive demand shock, $\delta_{t+1} > 0$. This shock also drives up the output gap $\tilde{y}_{t+1}$, by raising actual output y_{t+1}. In view internal inertia, the Fed then overshoots asset prices downward. This mechanism *does* rely on inertia ($\eta > 0$). The effect is also more surprising than the effect of supply shocks, because an *increase* in output is associated with a *decrease* in asset prices. This is another manifestation of the disconnect between the performance of the economy and financial markets.

Remark 4 (Transmission lags and inertia in practice)**.** *We capture transmission lags by assuming that spending responds to asset prices with a delay of one period. How should we* *think of the length of a period? We envision the period length as* the planning horizon of the Fed*: a period is sufficiently long that the Fed can expect its current decisions to have a meaningful impact on the real economic activity in the next period. In practice, while the impact of monetary policy on output begins to be felt within months, its peak cumulative effect can take up to two years (see, e.g., Romer and Romer (2004) and Chodorow-Reich et al. (2021)). Based on this evidence, we can think of the period length as somewhere between a quarter and two years and calibrate the internal inertia accordingly,* $\eta > 0$*, so that Eq.* (22) *implies asset prices have a meaningful impact in the next period,* $(1-\eta)\, p_t$*, and have their cumulative impact in subsequent periods,* $(1-\eta)\,(1+\eta+\eta^2+...)\, p_t = p_t$.[6]

4. Inflation and asset prices

We next extend our model to allow for partially flexible prices and inflation. We adopt the textbook setup in which inflation is determined by a New-Keynesian Phillips Curve (NKPC). We show that in equilibrium inflation depends *only* on the *current* output gap. Therefore, Corollary 4 from the last section implies that *inflation* and asset prices are also negatively correlated. To simplify the exposition, we relegate the details of this section to Appendix A.4 and describe the changes introduced by inflation.

We adopt the standard Calvo setup: at each instant a randomly selected fraction of intermediate firms reset their nominal price, with a constant hazard. This price remains unchanged until the firm gets to adjust again. In the appendix, we show that this leads to the standard NKPC:

$$\pi_t = \kappa \tilde{y}_t + \beta E_t^S \left[\pi_{t+1}\right]. \tag{46}$$

Here, π_t denotes the log-linearized nominal inflation realized in period t. The parameter, κ, is a composite price flexibility parameter (see $(A.26)$). The superscript S denotes the price-setters' (firms') beliefs.

We keep the macroeconomic side of the model the same as in Section 3. In particular, the output-asset price relation (22) still holds.

We change the financial market side of the model slightly to allow for a *nominal* interest

[6] In Caballero and Simsek (2021b), we use a model that features aggregate demand inertia but no explicit transmission lags, because the model is set in continuous time. In that environment, if there is no cost to overshooting asset prices, the Fed closes a negative output gap immediately by increasing (and subsequently decreasing) asset prices *by an infinite amount* to compensate for inertia. However, once we add costs to asset price overshooting, we recover the analogue of Proposition 3. Hence, transmission lags can also be viewed as capturing unmodeled costs to asset price overshooting in an environment with inertia. While the Fed might be able to shorten transmission lags by increasing asset price overshooting, there are natural limits to this alternative policy.

rate (which is what the Fed sets) in addition to the real interest rate. Specifically, there is a *nominal* risk-free asset with nominal interest rate denoted by R_t^{fn}, in addition to the real risk-free asset with real interest rate R_t^f, and the market portfolio with real return R_{t+1}. Both risk-free assets are in zero net supply. Assuming the return and inflation are (approximately) jointly log-normally distributed, we have the following *two* financial market equilibrium conditions (see the appendix):

$$E_t^M[r_{t+1}] + \frac{1}{2} var_t^M[r_{t+1}] - i_t = rp_t \equiv var_t^M[r_{t+1}] \tag{47}$$

$$\left[i_t^n - E_t^M[\pi_{t+1}] + \frac{1}{2} var(\pi_{t+1})\right] - i_t = irp_t \equiv -cov(\pi_{t+1}, r_{t+1}), \tag{48}$$

where $i_t = \log R_t^f$ denotes the *real* risk-free interest rate (as before) and $i_t^n = \log R_t^{fn}$ denotes the nominal risk-free interest rate.

Eq. (47) is the same as the earlier financial market equilibrium condition (12). Eq. (48) is new and describes the difference between the expected *real* return on the nominal risk-free rate and the real risk-free rate (the variance term $\frac{1}{2} var(\pi_{t+1})$ is a Jensen's inequality adjustment). This difference corresponds to *the inflation risk premium:* the additional return investors require from holding the nominal bond due to the fact that its *real* return declines with inflation. Eq. (48) says that the inflation risk premium depends negatively on the covariance between inflation and the (real) return on the market portfolio. If $cov(\pi_{t+1}, r_{t+1}) < 0$, i.e. inflation is high when the market portfolio generates a low return, the inflation risk premium is positive. If instead $cov(\pi_{t+1}, r_{t+1}) > 0$, the inflation risk premium is negative.[7]

We also adjust the Fed's problem to incorporate the costs of inflation gaps [cf. (14)]

$$\min_{i_t^n} E_t^F \left[\sum_{h=0}^{\infty} \beta^h \left(\tilde{y}_{t+h}^2 + \psi \pi_{t+h}^2\right)\right].$$

Here, ψ denotes the relative welfare weight for the inflation gaps. We normalize the inflation target to zero so the inflation gap is equal to inflation. Note also that the Fed sets the *nominal* interest rate i_t^n (which is no longer the same as the real rate i_t).

Finally, we assume all agents (the Fed , the market, and the price setters) have common beliefs.[8] The following results generalize Proposition 3 and Corollary 4 to this setup.

[7]Eq. (48) is a *generalized* Fisher equation that accounts for the inflation risk premium. In the textbook New-Keynesian model, the variance and covariance terms vanish (due to log-linearization) and this becomes the standard Fisher equation, $i_t^n - E_t^M[\pi_{t+1}] = i_t$.

[8]In Caballero and Simsek (2022a), we show that disagreements between the Fed and the price setters affect the price setters' expected inflation and induce a policy trade-off similar to "cost-push" shocks.

Proposition 4 (Asset pricing with inflation)**.** *Consider the setup in Proposition 3 but with nominal prices that are partially flexible. Suppose agents have common beliefs. There is an equilibrium in which the Fed achieves a zero* expected *inflation and a zero* expected *output gap*

$$E_t[\pi_{t+1}] = 0 \quad \text{and} \quad E_t[\tilde{y}_{t+1}] = 0. \tag{49}$$

In equilibrium, the real *variables are the same as in Proposition 3: Eqs.* $(36-38)$ *still apply. Inflation is given by*

$$\pi_t = \kappa \tilde{y}_t. \tag{50}$$

In equilibrium, the Fed still targets a zero *expected* output gap. By doing this, the Fed also achieves zero *expected* inflation. That is, in this setup, the "divine coincidence" applies in *expectation*—the Fed does not face a trade-off between stabilizing the output gap and inflation.

Since the Fed still targets a zero output gap on average, $E_t[\tilde{y}_{t+1}] = 0$, the *real* side of the model is the same as before. Hence, the equilibrium with inflation has a block-recursive structure. We solve for the *real* variables using Proposition 3 (ignoring inflation). We then solve for inflation using Eq. (50). Inflation depends *only* on the current output gap, because the Fed stabilizes future inflation on average, $E_t[\pi_{t+1}] = 0$ (and the price setters know this fact and have the same beliefs as the Fed).

Corollary 5 (Covariance between inflation and asset prices)**.** *Consider the setup in Proposition 4 with the belief structure in Section 3.1.1. In equilibrium, the* real *variables are the same as in Corollary 4: Eqs.* $(39-44)$ *still apply. Inflation is given by*

$$\pi_{t+1} = \kappa \tilde{y}_{t+1} = \kappa\left(\delta_{t+1} - \gamma s_t - z_{t+1}\right). \tag{51}$$

Inflation and the price of the market portfolio are negatively correlated (see (45)*):*

$$cov_t(\pi_{t+1}, p_{t+1}) = \kappa cov_t(\tilde{y}_{t+1}, p_{t+1}) = -\kappa\left(\frac{\eta}{1-\eta}\sigma_\delta^2 + \frac{1}{1-\eta}\sigma_z^2\right).$$

Since inflation depends on the current output gap, Corollary 4 implies that the aggregate asset price is negatively correlated, not only with the output gap, but also with inflation. As we described earlier, an unexpected positive demand shock increases the output gap *and inflation.* With inertia, a positive output gap induces the Fed to overshoot asset prices in the downward direction. Consequently, *inflation* and asset prices are negatively correlated. Recall also that a negative supply shock increases the output gap,

Here, we abstract from these effects and focus on the asset pricing effects of inflation.

while reducing asset prices (regardless of the degree of inertia). Therefore, *regardless of whether it is driven by demand shocks or supply shocks, inflation is bad news for asset prices.* This observation implies that in our setup the inflation risk premium is likely to be positive. The following result describes the conditions under which this is the case.

Corollary 6 (Inflation risk premium)**.** *Consider the setup in Corollary 5. The inflation risk premium and the nominal interest rate are given by*

$$irp_t = -cov_t\left(\pi_{t+1}, r_{t+1}\right) = \kappa \frac{\beta}{1-\eta}\sigma_z^2 + \kappa\left(\beta\frac{\eta}{1-\eta} - (1-\beta)\right)\sigma_{\overline{\delta}}^2 \tag{52}$$

$$i_t^n = i_t + irp_t - \frac{1}{2}var\left(\pi_{t+1}\right) \tag{53}$$

where the real interest rate i_t *is given by* (43) *and* $var_t\left(\pi_{t+1}\right) = \kappa^2\left(\sigma_{\overline{\delta}}^2 + \sigma_z^2\right)$*. The inflation risk premium is strictly positive iff*

$$\frac{\frac{\sigma_z^2}{\sigma_{\overline{\delta}}^2} + \eta}{1-\eta} > \frac{1-\beta}{\beta}. \tag{54}$$

The result follows from combining our earlier characterization of equilibrium with the financial market equilibrium condition (48). In equilibrium, the inflation risk premium depends on the covariance of inflation with the aggregate asset price. The nominal interest rate depends on the inflation risk premium and the real interest rate, which is the same as before (along with a Jensen adjustment term).

Eq. (54) describes the conditions under which the inflation risk premium is positive. With typical calibrations, the term on the right side of this condition is likely to be small. Thus, this condition fails *only if* the supply shocks are rare (relative to demand shocks) *and* aggregate demand inertia is small. Put differently, *either* a sizeable frequency of supply shocks *or* a sizeable aggregate demand inertia is sufficient for the inflation risk premium to be positive.

To see the intuition for Eq. (54), recall that the equilibrium return satisfies

$$r_{t+1} = \kappa + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t.$$

Inflation is negatively correlated with the aggregate asset price, $cov_t\left(\pi_{t+1}, p_{t+1}\right) < 0$ [see Corollary 5]. Hence, inflation can be positively correlated with the return only if it is driven by a shock that has counteracting effects on y_{t+1} and p_{t+1}, *and* its effect on y_{t+1} is stronger than its effect on p_{t+1}. A positive supply shock ($z_{t+1} > 0$) does not affect y_{t+1} and raises p_{t+1}, so it always induces a negative correlation between inflation and the return.

A positive demand shock $(\delta_{t+1} - \gamma s_t)$ raises y_{t+1} and decreases p_{t+1}, so it *can* induce a positive correlation between inflation and the return. The effect on y_{t+1} dominates the effect on p_{t+1} only if inertia is sufficiently low so that the Fed does not overshoot asset prices by much. Thus, the inflation risk premium can be negative only if demand surprises are very frequent relative to supply shocks, *and* inertia is low. Aside from these cases, the inflation risk premium is positive.

5. Asset pricing with Fed-market disagreements

The previous sections showed the importance of the Fed's beliefs for monetary policy and asset prices. In practice, market participants have their own opinionated beliefs and routinely disagree with the Fed on macroeconomic conditions and appropriate policy (see Caballero and Simsek (2022a)). We next derive the asset pricing implications of belief disagreements between the market and the Fed. In this context, the Fed still implements the "pystar" that is appropriate under its *own* belief. However, with disagreements, the market anticipates policy "mistakes" that have important implications for the risk premium and the interest rate. First, the anticipation of future disagreements and "mistakes" increases the risk premium—we refer to this as a *policy risk premium.* Second, current disagreements induce a "behind-the-curve" phenomenon in which the market expects the Fed to reverse course. Third, both current and anticipated future disagreements affect the policy interest rate the Fed needs to set to achieve "pystar."

Throughout this section, we use the model from Section 3.2 that features inertia $(\eta > 0)$ and fully sticky prices $(\kappa = 0)$. The latter assumption simplifies the exposition and abstracts from the effects of disagreements on inflation (see Footnote 8).

We introduce belief disagreements by modifying the signal environment from Section 3.1.1. As before, agents receive a public signal about aggregate demand. Unlike before, the Fed and the market disagree about the interpretation of this signal. After observing the public signal, each agent $j \in \{F, M\}$ forms an idiosyncratic interpretation, μ_t^j. Given this interpretation, the agent believes the public signal is drawn from

$$s_t =^j \delta_{t+1} - \mu_t^j + e_t, \quad \text{where } e_t \sim N\left(0, \sigma_e^2\right).$$

The noise term e_t is i.i.d. across periods and independent from other random variables. The notation $=^j$ captures that the equality holds under agent j's belief. Given their interpretations, agents form posterior mean-beliefs:

$$E_t^F[\delta_{t+1}] = \gamma\left(s_t + \mu_t^F\right) \quad \text{and} \quad E_t^M[\delta_{t+1}] = \gamma\left(s_t + \mu_t^M\right), \tag{55}$$

where γ is the same as before (see (28)). Each agent thinks its interpretation is correct. Hence, when agents interpret the signal differently, they develop belief disagreements about the future aggregate demand shock. For now, we assume agents observe the others' interpretations (and beliefs).

We also assume that agents' interpretations follow a joint Normal distribution that is i.i.d. across periods (and both agents know this distribution):

$$\mu_t^F, \mu_t^M \sim N\left(0, \sigma_\mu^2\right) \quad \text{and} \quad corr\left(\mu_t^F, \mu_t^M\right) = 1 - \frac{D}{2} \text{ with } D \in [0, 2]. \tag{56}$$

The parameter D captures the scope of disagreement. When $D = 0$, interpretations are the same and there are no disagreements. Eq. (56) also implies:

$$E_t^j\left[\mu_{t+1}^F - \mu_{t+1}^M\right] = 0 \quad \text{and} \quad var_t^j\left[\mu_{t+1}^F - \mu_{t+1}^M\right] = D\sigma_\mu^2. \tag{57}$$

Agents think interpretation differences have mean zero and variance increasing with D.

A key implication of this setup is that each agent thinks the other agent's posterior belief is a "noisy" version of her own belief. To see this, consider the Fed's posterior belief

$$\gamma\left(s_{t+1} + \mu_{t+1}^F\right) = \gamma\left(s_{t+1} + \mu_{t+1}^M\right) + \gamma\left(\mu_{t+1}^F - \mu_{t+1}^M\right). \tag{58}$$

The market thinks its own belief, $\gamma\left(s_{t+1} + \mu_{t+1}^M\right)$, is correct. Therefore, the market thinks the Fed's belief is a noisier version of its own belief. Specifically, the market's perceived variance of the Fed's future belief is the sum of the forecastable demand variance ($\sigma_\delta^2 - \sigma_{\overline{\delta}}^2$) and a noise term that increases with the scope of disagreement,

$$var_t^M\left(\gamma\left(s_{t+1} + \mu_{t+1}^F\right)\right) = \left(\sigma_\delta^2 - \sigma_{\overline{\delta}}^2\right) + \gamma^2 D\sigma_\mu^2. \tag{59}$$

The rest of the model is the same as in Section 3.2. The following proposition characterizes the equilibrium and generalizes Corollary 4 to the case with disagreements.

Proposition 5 (Fed-market disagreements)**.** *Consider the setup in Proposition 3 and*

Fed-market disagreements. The equilibrium is given by

$$
\begin{aligned}
(py_t)^* &= p_t^* - y_t^* = -\frac{\eta}{1-\eta}\tilde{y}_t - \frac{\gamma\left(s_t + \mu_t^F\right)}{1-\eta} - m && (60)\\
y_t &= y_{t-1}^* + \delta_t - \gamma\left(s_{t-1} + \mu_{t-1}^F\right) && (61)\\
\tilde{y}_t &= \delta_t - \gamma\left(s_{t-1} + \mu_{t-1}^F\right) - z_t && (62)\\
r_{t+1} &= \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} \\
&\quad + \frac{1-\eta-\beta}{1-\eta}\left(\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right) + \frac{\beta}{1-\eta}\left(z_{t+1} - \gamma\left(s_{t+1} + \mu_{t+1}^F\right)\right) && (63)\\
i_t &= E_t^M\left[r_{t+1}\right] - \frac{rp_t}{2} && (64)\\
&\quad \textit{where } E_t^M\left[r_{t+1}\right] = \rho + \frac{\eta\tilde{y}_t}{1-\eta} + (\beta+\eta)\frac{\gamma\left(s_t + \mu_t^F\right)}{1-\eta} + (1-\beta-\eta)\frac{\gamma\left(s_t + \mu_t^M\right)}{1-\eta} - \frac{rp_t}{2} \\
rp_t &= var_t\left(r_{t+1}\right) = rp_t^{com} + \beta^2\gamma^2 D\sigma_\mu^2, && (65)
\end{aligned}
$$

where rp_t^{com} *is the risk premium with common beliefs characterized in* (44)*.*

Eqs. $(60-62)$ show that the Fed-market disagreements *do not* affect the equilibrium "pystar," output, or the output gap, which are still determined by *the Fed's* belief. In fact, these equations follow from $(36-38)$ in Section 3.2 after substituting the Fed's belief, $E_t^F\left[\delta_{t+1}\right] = \gamma\left(s_t + \mu_t^F\right)$. The Fed still shields the economy from forecasted demand shocks *under its belief.*

In contrast, Eqs. $(64-65)$ show that disagreements *do* affect the interest rate and the risk premium. Disagreements matter because the market has a different belief than the Fed and thinks the Fed should be targeting a different "pystar." Therefore, the market thinks the Fed is making a policy "mistake." These perceived "mistakes" affect the interest rate and the risk premium. In the rest of this section, we present three corollaries that unpack these effects.

Corollary 7 (Policy risk premium)**.** *The risk premium,* $rp_t = var_t^M\left[r_{t+1}\right] = rp_t^{com} + \beta^2\gamma^2 D\sigma_\mu^2$*, is* increasing *in the scope of disagreement between the Fed and the market (D).*

The result follows from Eq. (65). We sketch the proof of this equation, which helps develop the intuition. Note that the aggregate asset price in the next period is

$$p_{t+1} = p_{t+1}^* = y_{t+1}^* - \frac{\eta}{1-\eta}\tilde{y}_{t+1} - \frac{\gamma\left(s_{t+1} + \mu_{t+1}^F\right)}{1-\eta} - m. \tag{66}$$

Combining this expression with $(61-62)$, we obtain

$$var_t^M(p_{t+1}) = var_t^{com}(p_{t+1}) + \gamma^2 D\sigma_\mu^2,$$

where $var_t^{com}(p_{t+1})$ is the asset price volatility that would obtain if the beliefs were common. Hence, disagreements increase the market's *perceived* future asset price volatility $var_t^M(p_{t+1})$. Thus, disagreements also increase the market's *perceived* return volatility $var_t^M(r_{t+1})$ (see the appendix for details).

With greater D the market thinks the Fed's *future* beliefs will be "noisier" and the Fed will make more frequent policy "mistakes." Therefore, the market also *perceives* greater future price and return volatility and demands a higher risk premium. We refer to the component of risk premium that stems from disagreements, $\beta^2\gamma^2 D\sigma_\mu^2$, as *the policy risk premium.* In practice, we expect this premium to be especially large at times of macroeconomic uncertainty, which are likely to create a greater scope for disagreements.

Corollary 7 shows that the anticipation of future disagreements, $\mu_{t+1}^F - \mu_{t+1}^M$, induces a risk premium. Our next result shows that *current* disagreements, $\mu_t^F - \mu_t^M$, induce a phenomenon that we call *behind-the-curve.*

Corollary 8 ("Behind-the-curve")**.** *Suppose the market is more demand-optimistic than the Fed, $\mu_t^M > \mu_t^F$ (symmetric-opposite results hold when the market is more demand-pessimistic). The market thinks the Fed is "behind-the-curve" and will induce a positive output gap*

$$E_t^M[\tilde{y}_{t+1}] = \gamma\left(\mu_t^M - \mu_t^F\right) > 0, \tag{67}$$

after which it will have to reverse course and implement a lower-than-average "pystar"

$$E_t^M[p_{t+1}] = y_t^* - m - \frac{\eta}{1-\eta}\gamma\left(\mu_t^M - \mu_t^F\right) < y_t^* - m. \tag{68}$$

In terms of the interest rates, the demand-optimistic market thinks the Fed will switch from setting "too low" rates (lower than what is ideal under the market's belief) to "too high" rates (higher than what the Fed expects and higher than the long-run average rate):

$$\begin{aligned} i_t &= i_t^M - \frac{\beta+\eta}{1-\eta}\gamma\left(\mu_t^M - \mu_t^F\right) < i_t^M && (69) \\ E_t^M[i_{t+1}] &= E_t^F[i_{t+1}] + \frac{\eta}{1-\eta}\gamma\left(\mu_t^M - \mu_t^F\right) \text{ with } E_t^F[i_{t+1}] = \rho - \frac{rp_{t+1}}{2}, && (70) \end{aligned}$$

where i_t^M is the equilibrium interest rate that would obtain in period t if the Fed had the same beliefs as the market in this period, $\mu_t^F = \mu_t^M$ (see (64)).

For a sketch proof, first consider the market's expectation for the future output gap, $\tilde{y}_{t+1} = y_{t+1} - y^*_{t+1}$. Using Eq. (62), along with $E^M_t [z_{t+1}] = 0$, we calculate

$$\begin{aligned} E^M_t [\tilde{y}_{t+1}] &= E^M_t \left[\delta_{t+1} - \gamma \left(s_t + \mu^F_t\right)\right] \\ &= E^M_t \left[\delta_{t+1} - \gamma \left(s_t + \mu^M_t\right)\right] + \gamma \left(\mu^M_t - \mu^F_t\right) \\ &= \gamma \left(\mu^M_t - \mu^F_t\right). \end{aligned}$$

The second line uses (58) applied to period t and the last line uses the fact that $E^M_t \left[\delta_{t+1} - \gamma \left(s_t + \mu^M_t\right)\right] = 0$ (the market thinks its belief is unbiased). This proves (67). The market thinks the Fed is making a "mistake" and will not be able to achieve its target output gap on average (recall that the Fed targets a zero output gap, $E^F_t [\tilde{y}_{t+1}] = 0$). Naturally, a demand-optimistic market expects a positive output gap.

Next consider the market's expectation for the future asset price, p_{t+1}. Using Eq. (66), along with $E^M_t \left[y^*_{t+1}\right] = y^*_t$ and $E^M_t \left[s_{t+1} + \mu^F_{t+1}\right] = 0$, we obtain

$$E^M_t [p_{t+1}] = y^*_t - \frac{\eta}{1-\eta} E^M_t [\tilde{y}_{t+1}] - m.$$

Combining this with (67) proves (68). With internal inertia, the market further thinks the Fed will have to reverse course and make a large policy adjustment to address the future output gaps that its "mistake" will induce. A demand-optimistic market thinks: *once the positive output gap develops, the Fed will realize its "mistake" and will have to reverse course, implementing a low future "pystar."*

Finally, consider the market's perception of the current interest rate, i_t vs i^M_t, and its expectation for the future interest rate, $E^M_t [i_{t+1}]$. Eqs. $(69 - 70)$ follow from combining (64) with (67). These expressions describe the implications of "behind-the-curve" for interest rates. A demand-optimistic market thinks the Fed is currently setting "too low" rates. The market further thinks that, once the positive output gap develops, the Fed will switch to setting "too high" rates (to implement a low future "pystar").

Our final corollary explains the effect of (future and current) disagreements on the policy interest rate.

Corollary 9 (Disagreements and the policy interest rate)**.** *(i) An increase in the scope of disagreements between the Fed and the market (D) reduces the interest rate i_t.*

(ii) Let i^F_t denote the interest rate when the market has the same belief as the Fed in the current period, $\mu^M_t = \mu^F_t$ (for a given D). When inertia is relatively low ($\eta < 1-\beta$), a demand optimistic market ($\mu^M_t > \mu^F_t$) induces the (demand-pessimistic) Fed to set a higher

interest rate that partially accommodates the market's belief, $i_t > i_t^F$. Conversely, when inertia is high ($\eta > 1-\beta$), a demand optimistic market induces the (demand-pessimistic) Fed to set a lower interest rate that overweights the Fed's own belief, $i_t < i_t^F$.

The first part follows from Corollary 7; a greater scope of future disagreements increases the risk premium. If the policy did not adjust the interest rate, a greater risk premium would reduce asset prices below "pystar." The Fed reduces the interest rate to keep asset prices at "pystar" (consistent with "the Fed put/call").

The second part is driven by the effect of (current) disagreements on the market's expected return, $E_t^M[r_{t+1}]$ (recall that $i_t = E_t^M[r_{t+1}] - rp_t/2$). This depends on the market's expectations for future cash flows and future asset prices, $E_t^M[y_{t+1}]$ and $E_t^M[p_{t+1}]$ (see (13)). Corollary 8 implies that current disagreements ("behind-the-curve") induce competing effects on $E_t^M[y_{t+1}]$ and $E_t^M[p_{t+1}]$. On the one hand, a demand-optimistic market expects relatively high cash-flows (driven by the output boom that it anticipates). On the other hand, the market also expects relatively low asset prices. The expected return in (64) balances these two forces (see the proof of the proposition for a derivation):

$$E_t^M[r_{t+1}] = \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} + \left[(1-\beta) - \frac{\beta\eta}{1-\eta}\right]\gamma\left(\mu_t^M - \mu_t^F\right).$$

The first term in the square bracket captures the future cash-flow effect of disagreements and the second term captures the future asset-prices effect. When $\eta < 1-\beta$ (relatively low inertia), the cash flow effect dominates and a more demand-optimistic market ($\mu_t^M > \mu_t^F$) expects a higher return. When $\eta > 1-\beta$ (relatively high inertia), the asset-prices effect dominates and a more demand-optimistic market expects a *lower* return.

When inertia is low, a demand-optimistic market expects a high return via the anticipation of high cash flows. This induces a demand-pessimistic Fed to set a relatively high interest rate that partially accommodates the market's view. If the Fed did not adjust the interest rate, the market's expectations of a high return would increase asset prices above "pystar." Conversely, when inertia is high, a demand-optimistic market expects a relatively low return via the anticipation of low future asset prices. This induces a demand-optimistic Fed to cut the rate more aggressively to prevent a decline of asset prices and implement "pystar."

Remark 5 (Fed belief surprises as monetary policy shocks)**.** *We have assumed that the market always knows the Fed's current belief (and vice versa). In practice, the market is often uncertain about the Fed's belief and learns it through a policy speech or announcement. In Caballero and Simsek (2022a), we use this observation to develop a theory of* microfounded *monetary policy shocks. To illustrate the idea, suppose each period has two phases. Initially, the market does not know the Fed's interpretation* μ_t^F. *Later in the period, the market learns* μ_t^F *(before portfolio and consumption decisions). The Fed knows* μ_t^M *throughout. In this setting, the revelation of the Fed's belief to the market affects financial markets like textbook monetary policy shocks (which are often modeled as random interest rate changes). For instance, when the Fed is revealed to be more demand-optimistic than the market expected, the interest rate increases and the aggregate asset price declines (see Appendix B.2 for a formalization). Unlike the textbook policy shocks, these shocks are not random: they are* "optimal" *under the Fed's belief. These shocks affect financial markets* like *random shocks because the market does not share the Fed's beliefs and perceives the policy changes induced by the Fed's beliefs as "mistakes."*[9]

6. Final Remarks

Summary. We developed a framework to analyze the impact of monetary policy on asset prices. The central idea is to reverse engineer the Fed's policy problem to solve for the "pystar" that ensures future macroeconomic balance under the Fed's belief. When the Fed is unconstrained and acts optimally, it keeps asset prices close to this level by adjusting its policy tools. We characterized "pystar" in a two-speed economy where households are slow and respond to asset prices with lags. In this context, we showed that "pystar" is primarily driven by *macroeconomic needs as perceived by the Fed*, rather than by financial market forces such as the market's expectations or risk premia. We analyzed the implications of this idea in several versions of the model which differ in the households' spending behavior. Our analysis uncovered several forces that drive the aggregate asset price and the interest rate.

First, non-financial aggregate demand shocks induce the *opposite* fluctuations in the aggregate asset price, creating the appearance of "excess" asset price volatility. The Fed *uses* the aggregate asset price to insulate the economy from demand shocks. In contrast, purely financial-market shocks (such as time-varying beliefs or risk premia) do not affect the aggregate asset price—they are absorbed by the interest rate. The Fed stabilizes the

[9] One caveat is that, in the data, monetary policy shocks seem to affect stock prices through the risk premium (see Bernanke and Kuttner (2005); Bauer et al. (2023)). In our model, monetary policy shocks operate via the traditional interest rate and cash-flow channels. However, the model features a policy "mistakes" risk-premium that depends on disagreements (see Corollary 7). If the policy announcement provides information about the scope of new disagreements (D), then it can also affect the policy risk premium. In other words, during policy events, the market may not only learn *what* the Fed thinks, but also *how* the Fed thinks—and how much it is likely to deviate from its own view in future periods.

asset price impact of these shocks to prevent excessive macroeconomic fluctuations.

Second, with realistic transmission lags, *the Fed's beliefs* about future demand or supply drive the aggregate asset price. With more precise macroeconomic signals, the Fed preempts future aggregate demand shocks more aggressively, which improves macroeconomic stability but increases asset price volatility.

Third, with internal demand inertia, the Fed overshoots the aggregate asset price upward (resp. downward) to neutralize the recessions (resp. booms) caused by demand shocks. This overshooting creates the appearance of a disconnect between the performance of the economy and the financial markets.

Fourth, inflation is *negatively* correlated with asset prices, *regardless of whether it is driven by demand or supply shocks.* A positive demand shock increases the output gap and inflation, while inducing the Fed to overshoot asset prices downward. A negative supply shock induces the Fed to target lower asset prices to align future demand with the lower level of supply. With either shock, positive inflation surprises are *bad news* for asset prices. This also implies that the inflation risk premium is typically positive.

Fifth, disagreements between the market and the Fed affect the risk premium and interest rate. The market anticipates excessive policy-induced volatility and demands a *policy risk premium*, which is especially high at times of macroeconomic uncertainty and disagreements. The market also thinks the Fed is *"behind-the-curve"* and will reverse course: for instance, a demand-optimistic market thinks the Fed will induce positive output gaps after which it will have to overshoot asset prices downward. The market's perceptions of excessive policy-induced volatility and "behind-the-curve" affect the policy interest rate the Fed needs to set to achieve "pystar."

Robustness and future work. We deliberately kept our analysis stark. Among other things, we assumed that the Fed is willing and able to use its tools with full potency to close the output gap (subject to the restrictions from transmission lags). In practice, the Fed's power is more limited. The Fed may face an effective lower bound or other interest rate constraints. Alternatively, the Fed might be reluctant to act too aggressively in fear of destabilizing the financial sector. Our results are qualitatively robust to allowing for these types of monetary policy constraints. Naturally, these constraints dampen the Fed's put and its response to aggregate demand shocks. Thus, the constraints allow for some asset price volatility driven by financial market forces, while mitigating the policy-induced asset price volatility.

A general theme of our paper is that the Fed targets *the aggregate asset price* (financial conditions) rather than the policy interest rate. The policy interest rate is simply the tool the Fed uses to achieve its asset price target. This observation has two implications. First, our model makes stronger predictions for the aggregate asset price than for the policy rate. The aggregate asset price is driven by the Fed's perception of macroeconomic imbalances. In contrast, the policy interest rate is driven by subtle details of the model, such as disagreements between the Fed and the market, the extent of aggregate demand inertia, and various forces that drive the risk premium. Second, formulating policy rules in terms of the aggregate asset price, rather than in terms of the policy rate, could be helpful. Our model supports *Taylor-like rules in terms of the aggregate asset price*. For instance, Eq. (36) from Section 3.2 describes "pystar" as a function of the current output gap, $\tilde{y}_t$ (and a second term that incorporates the Fed's beliefs about future macroeconomic conditions). In an extension of our model with multiple assets, where different asset prices might have a different impact on aggregate demand, the policy would suggest targeting a *financial conditions index (FCI)* that weights different asset valuations according to *their impact on aggregate demand*. We leave the analysis of the optimal FCI for future work.

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A. Appendix: Omitted derivations

This appendix presents the analytical derivations and proofs omitted from the main text.

A.1. Microfoundations for the baseline environment

In this section, we describe the details of the baseline environment that we describe in Section 2.1 and use throughout the paper.

The supply side is the same as in Caballero and Simsek (2021b), with the difference that here we allow for shocks to potential output. In particular, the real side of the economy features two types of agents: "asset-holding households" (the households) denoted by superscript $i = H$, and "hand-to-mouth agents" denoted by superscript $i = HM$. There is a single factor, labor.

Hand-to-mouth agents supply labor according to standard intra-period preferences. They do not hold financial assets and spend all of their income. We write the hand-to-mouth agents' problem as,

$$\begin{aligned} &\max_{L_t} \log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \\ &Q_t C_t^{HM} = W_t L_t + T_t. \end{aligned} \tag{A.1}$$

Here, φ denotes the Frisch elasticity of labor supply, Q_t denotes the nominal price for the final good, W_t denotes the nominal wage, and T_t denotes lump-sum transfers to labor (described subsequently). Using the optimality condition for problem $(A.1)$, we obtain a standard labor supply curve

$$\frac{W_t}{Q_t} = \chi L_t^{\varphi} C_t^{HM}. \tag{A.2}$$

Households own and spend out of the market portfolio and they supply no labor.

Production is otherwise similar to the standard New Keynesian model. There is a continuum of monopolistically competitive firms, denoted by $\nu \in [0,1]$. These firms produce differentiated intermediate goods, $Y_t(\nu)$, subject to the Cobb-Douglas technology,

$$Y_t(\nu) = A_t L_t(\nu)^{1-\alpha}. \tag{A.3}$$

Here, $1-\alpha$ denotes the share of labor in production and A_t the total factor productivity. We allow A_t to change over time to capture supply shocks [see (4)].

A competitive final goods producer combines the intermediate goods according to the CES technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)}, \tag{A.4}$$

for some $\varepsilon > 1$. This implies the price of the final consumption good is determined by the ideal price index,

$$Q_t = \left(\int_0^1 Q_t(\nu)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)}, \tag{A.5}$$

and the demand for intermediate good firms satisfies,

$$Y_t(\nu) \leq \left(\frac{Q_t(\nu)}{Q_t} \right)^{-\varepsilon} Y_t. \tag{A.6}$$

Here, $Q_t(\nu)$ denotes the nominal price set by the intermediate good firm ν.

The labor market clearing condition is

$$\int_0^1 L_t(\nu) d\nu = L_t. \tag{A.7}$$

The goods market clearing condition is

$$Y_t = C_t^H + C_t^{HM}. \tag{A.8}$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the profits lump-sum and redistributes to workers to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T_t = (1-\alpha) Q_t Y_t - W_t L_t. \tag{A.9}$$

This ensures that in equilibrium hand-to-mouth agents receive and spend their production share of output, $(1-\alpha) Q_t Y_t$, and consume [see $(A.1)$]

$$C_t^{HM} = (1-\alpha) Y_t. \tag{A.10}$$

Households receive the total profits from the intermediate good firms, which amount to the residual share of output, $\Pi_t \equiv \int_0^1 \Pi_t(\nu) d\nu = \alpha Q_t Y_t$.

Flexible-price benchmark and potential output. To characterize the equilibrium, it is useful to start with a benchmark setting without nominal rigidities. In this benchmark, an intermediate good firm ν solves the following problem,

$$\Pi = \max_{Q,L} QY - W_t L - T_t \tag{A.11}$$
$$\text{where } Y = A_t L^{1-\alpha} = \left(\frac{Q}{Q_t}\right)^{-\varepsilon} Y_t.$$

The firm takes as given the aggregate price, wage, and output, Q_t, W_t, Y_t, and chooses its price, labor input, and output Q, L, Y.

The optimal price is given by

$$Q = \frac{\varepsilon}{\varepsilon - 1} W_t \frac{1}{(1-\alpha) A_t L^{-\alpha}}. \tag{A.12}$$

The firm sets an optimal markup over the marginal cost, where the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q_t = Q$ and $L_t = L$. Substituting this into $(A.12)$, we obtain a labor demand equation,

$$\frac{W_t}{Q_t} = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) A_t L_t^{-\alpha}. \tag{A.13}$$

Combining this with the labor supply equation $(A.2)$, and substituting the hand-to-mouth consumption $(A.10)$, we obtain the equilibrium labor as the solution to,

$$\chi (L^*)^{\varphi} (1-\alpha) Y_t^* = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) A_t (L^*)^{-\alpha}.$$

In equilibrium, output is given by $Y_t^* = A_t (L^*)^{1-\alpha}$. Therefore, the equilibrium condition simplifies to,

$$\chi (L^*)^{1+\varphi} = \frac{\varepsilon - 1}{\varepsilon}. \tag{A.14}$$

We refer to L^* as the potential labor supply and $Y^* = A_t (L^*)^{1-\alpha}$ as the potential output.

Fully sticky prices. We next describe the equilibrium with nominal rigidities. For simplicity, we focus on the case with full price stickiness. In particular, intermediate good firms have a preset nominal price that remains fixed over time, $Q_t(\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q_t = Q^*$ [see $(A.5)$]. Then, each intermediate good firm ν at time t solves the following version of problem $(A.11)$,

$$\Pi = \max_{L} Q^* Y - W_t L - T_t \tag{A.15}$$
$$\text{where } Y = AL^{1-\alpha} \leq Y_t.$$

For small aggregate demand shocks (which we assume) each firm optimally chooses to meet the demand for its goods, $Y = AL^{1-\alpha} = Y_t$. Therefore, each firm's output is determined by aggregate demand, which is equal to spending by households and hand-to-mouth agents [see $(A.8)$],

$$Y_t = C_t^H + C_t^{HM}.$$

This establishes Eq. (1) in the main text.

Finally, recall that hand-to-mouth agents' spending is given by $C_t^{HM} = (1-\alpha) Y_t$ [see Eq. $(A.10)$]. Combining this with $Y_t = C_t^H + C_t^{HM}$, the aggregate demand for goods is determined by the households' spending,

$$Y_t = \frac{C_t^H}{\alpha}.$$

This establishes Eq. (3) in the main text.

Campbell-Shiller approximation. We finally derive the Campbell-Shiller approximation in (13). First note that Eq. (5) implies

$$\begin{aligned} r_{t+1} &= \log\left(\frac{\alpha Y_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t} + \frac{P_{t+1}}{P_t}\right) \\ &= \log\left(\frac{\alpha Y_{t+1}}{P_{t+1}} + 1\right) + \log\left(\frac{P_{t+1}}{P_t}\right) \\ &= \log\left(1 + X_{t+1}\right) + p_{t+1} - p_t. \end{aligned} \tag{A.16}$$

Here, we have defined the dividend price ratio, $X_t = \alpha Y_t / P_t$.

Next consider the steady-state value of the dividend-price ratio absent shocks, denoted by X^*. Following the same steps as in Section 2.2, and setting the demand shock to zero ($\delta_t = 0$), we obtain the steady-state output-asset price relation $Y^* = (1-\beta)\frac{1}{\alpha\beta}P^*$ (see (17)). This implies $X^* = \alpha Y_t^* / P_t^* = \frac{1-\beta}{\beta}$.

Finally, log-linearize $(A.16)$ around $X_{t+1} = X^*$. Let $x_{t+1} = \log\left(X_{t+1}/X^*\right)$ denote the log deviation of the dividend price ratio from its steady-state level. Consider the term, $\log\left(1 + X_{t+1}\right) = \log\left(1 + X^* \exp\left(x_{t+1}\right)\right)$. Using a Taylor approximation around $x_{t+1} = 0$, we obtain

$$\begin{aligned} \log\left(1 + X_{t+1}\right) &\approx \log\left(1 + X^*\right) + \frac{X^*}{1+X^*} x_{t+1} \\ &\approx \log\left(\frac{1}{\beta}\right) + (1-\beta)\left(\log\left(\frac{\alpha Y_{t+1}}{P_{t+1}}\right) - \log\left(\frac{1-\beta}{\beta}\right)\right). \end{aligned}$$

Substituting this into $(A.16)$ and collecting the constant terms, we obtain Eq. (13) in the main text.

A.2. Omitted derivations in Section 2

Proof of Proposition 1. In the main text, we show that the equilibrium asset price is given by (18)

$$p_t = p_t^* \equiv y_t^* - \delta_t - m \text{ where } m = \log\left(\frac{1-\beta}{\alpha\beta}\right).$$

Substituting this along with $y_{t+1} = y_{t+1}^*$ into (13), we obtain

$$\begin{aligned}
r_{t+1} &= \kappa + (1-\beta)\, y_{t+1}^* + \beta p_{t+1} - p_t \\
&= \kappa + (\beta - 1)\log\left(\frac{\alpha\beta}{1-\beta}\right) + (1-\beta)\, y_{t+1}^* + \beta\left(y_{t+1}^* - \delta_{t+1}\right) - \left(y_t^* - \delta_t\right) \\
&= \rho + (1-\beta)\, y_{t+1}^* + \beta\left(y_{t+1}^* - \delta_{t+1}\right) - \left(y_t^* - \delta_t\right) \\
&= \rho + \delta_t + z_{t+1} - \beta\delta_{t+1}.
\end{aligned}$$

The third line substitutes κ from (13) and $\rho = -\log\beta$ to calculate the constant term. This last line substitutes $y_{t+1}^* = y_t^* + z_{t+1}$ to describe the return in terms of the shocks. This establishes (19). In the main text, we show that this implies (20). Corollaries 1 and 2 immediately follow from the equilibrium characterization, completing the proof. □

A.3. Omitted derivations in Section 3

We first derive Eq. (22). We then present the proofs of the propositions and the corollaries in Section 3.

Output asset price relation with lags and inertia. Recall that in this section we have the modified version of the consumption rule

$$C_t^H = (1-\beta)\, D_t + \beta\left[\eta C_{t-1}^H + (1-\eta)\frac{1-\beta}{\beta} K_{t-1}\right]\exp\left(\delta_t\right).$$

Substituting $D_t = \alpha Y_t$ and $K_{t-1} = P_{t-1}$ and $C_t^H = \alpha Y_t$, we obtain

$$Y_t = \left(\eta Y_{t-1} + (1-\eta)\frac{1-\beta}{\alpha\beta} P_{t-1}\right)\exp\left(\delta_t\right).$$

Dividing by P_{t-1} and taking logs, we obtain

$$\begin{aligned}
y_t &= \log\left(\eta\frac{Y_{t-1}}{P_{t-1}} + (1-\eta)\frac{1-\beta}{\alpha\beta}\right) + p_{t-1} + \delta_t \\
&= \log\left(\eta Z_{t-1} + (1-\eta)\, Z^*\right) + p_{t-1} + \delta_t \\
&= \log\left(1 + \eta\left(\frac{Z_{t-1}}{Z^*} - 1\right)\right) + \log Z^* + p_{t-1} + \delta_t.
\end{aligned} \tag{A.17}$$

Here, the second line substitutes the output price ratio, $Z_t = Y_t/P_t$, and its steady-state level, $Z^* = Y_t^*/P_t^* = \frac{1-\beta}{\alpha\beta}$ (see (18)).

Next, let $z_{t-1} = \log(Z_{t-1}/Z^*)$ denote the log deviation of the output price ratio from its steady-state level. Note that

$$\log\left(1+\eta\left(\frac{Z_{t-1}}{Z^*}-1\right)\right) = \log\left(1+\eta\left(\exp\left(z_{t-1}\right)-1\right)\right) \approx \eta z_{t-1}.$$

Here, the last line applies a Taylor approximation around $z_{t-1} = 0$. Substituting this into $(A.17)$, we obtain

$$\begin{aligned} y_t &= \eta z_{t-1} + \log Z^* + p_{t-1} + \delta_t \\ &= (1-\eta)\log Z^* + \eta \log Z_{t-1} + p_{t-1} + \delta_t \\ &= (1-\eta)\, m + \eta\left(y_{t-1} - p_{t-1}\right) + p_{t-1} + \delta_t \\ &= (1-\eta)\, m + \eta y_{t-1} + (1-\eta)\, p_{t-1} + \delta_t. \end{aligned}$$

Here, the second line substitutes $z_{t-1} = \log(Z_{t-1}/Z^*)$. The third line substitutes $Z_{t-1} = Y_{t-1}/P_{t-1}$ and $m = \log Z^* = \log\left(\frac{1-\beta}{\alpha\beta}\right)$ (see (17)). The last line establishes Eq. (22).

Proof of Proposition 2. Presented in the main text. □

Proof of Corollary 3. Most of the proof is presented in the main text. To calculate the volatility induced by news, note that γs_t and $\delta_{t+1} - \gamma s_t$ capture the *forecastable* and the *unforecastable* components of aggregate demand shocks. These components are uncorrelated with one another and have variance given by

$$var_t\left(\delta_{t+1} - \gamma s_t\right) = \sigma^2_{\tilde{\delta}} \quad \text{ and } \quad var_t\left(\gamma s_{t+1}\right) = \sigma^2_{\delta} - \sigma^2_{\tilde{\delta}}. \tag{A.18}$$

Combining this expression with Eq. (29) establishes Eq. (35) in the main text.

To calculate the risk premium and the interest rate, note that Eq. (13) implies

$$\begin{aligned} r_{t+1} &= \kappa + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t \\ &= \rho + (1-\beta)\left(y_t^* + \delta_{t+1} - \gamma s_t\right) + \beta\left(y_{t+1}^* - \gamma s_{t+1}\right) - \left(y_t^* - \gamma s_t\right) \\ &= \rho + \gamma s_t + (1-\beta)\left(\delta_{t+1} - \gamma s_t\right) + \beta\left(z_{t+1} - \gamma s_{t+1}\right). \end{aligned}$$

Here, the second line substitutes y_{t+1}, p_{t+1}, p_t using (29) and simplifies the constant terms (similar to the proof of Proposition 1). The last line substitutes $y_{t+1}^* = y_t^* + z_{t+1}$ and simplifies the expression. This proves (32).

Finally, combining (32) with $(A.18)$, we obtain Eq. (34). Combining the expression with (12), we also obtain (33), completing the proof. □

Proof of Proposition 3. Presented in the main text. $\square$

Proof of Corollary 4. Under the belief structure in Section 3.1.1, agents' (common) expectation for the demand shock is given by $E_t[\delta_{t+1}] = \gamma s_t$, and their expected supply shock is zero, $E_t[z_{t+1}] = 0$. After substituting these beliefs, Eqs. $(36-37)$ (for period t) imply the closed-form solutions in $(39-41)$.

Next consider the equilibrium return r_{t+1} given by (see (13))

$$r_{t+1} = \kappa + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t.$$

Combining this with Eqs. (39) and (40), we obtain

$$\begin{aligned}
r_{t+1} &= \kappa + (1-\beta)\left(y_t^* + \delta_{t+1} - \gamma s_t\right) \\
&\quad + \beta\left(y_{t+1}^* - \frac{\eta \tilde{y}_{t+1} + \gamma s_{t+1}}{1-\eta} - m\right) - \left(y_t^* - \frac{\eta \tilde{y}_t + \gamma s_t}{1-\eta} - m\right) \\
&= \rho + (1-\beta)\left(y_t^* + \delta_{t+1} - \gamma s_t\right) \\
&\quad + \beta\left(y_t^* + z_{t+1} - \frac{\eta\left(\delta_{t+1} - \gamma s_t - z_{t+1}\right) + \gamma s_{t+1}}{1-\eta}\right) \\
&\quad - \left(y_t^* - \frac{\eta\left(\delta_t - \gamma s_{t-1} - z_t\right) + \gamma s_t}{1-\eta}\right) \\
&= \rho + \frac{\gamma s_t}{1-\eta} + \frac{\eta}{1-\eta}\left(\delta_t - \gamma s_{t-1} - z_t\right) \\
&\quad + \left((1-\beta) - \beta\frac{\eta}{1-\eta}\right)\left(\delta_{t+1} - \gamma s_t\right) + \frac{\beta}{1-\eta}\left(z_{t+1} - \gamma s_{t+1}\right).
\end{aligned}$$

Here, the second equality simplifies the constant terms and substitutes $\tilde{y}_{t+1} = \delta_{t+1} - \gamma s_t - z_{t+1}$ (see (41)) and $y_{t+1}^* = y_t^* + z_{t+1}$. The last equality collects similar terms together. This proves (42).

Next consider the equilibrium interest rate i_t and the risk premium rp_t. Using (12) and (42), we obtain (43)

$$i_t = E_t[r_{t+1}] - \frac{1}{2} rp_t \text{ where } E_t[r_{t+1}] = \rho + \frac{\gamma s_t}{1-\eta} + \frac{\eta}{1-\eta}\left(\delta_t - \gamma s_{t-1} - z_t\right).$$

Using (42), we further obtain (44),

$$rp_t = var_t\left(r_{t+1}\right) = \left(\frac{1-\eta-\beta}{1-\eta}\right)^2 \sigma_{\tilde{\delta}}^2 + \left(\frac{\beta}{1-\eta}\right)^2\left(\sigma_z^2 + \sigma_\delta^2 - \sigma_{\tilde{\delta}}^2\right).$$

Finally, we use this characterization to calculate the conditional covariance $cov_{t-1}(y_t, p_t)$. Note that the unforecastable component of the demand shock, $\delta_t - \gamma s_{t-1}$, is uncorrelated with the supply shock, z_t. It is also uncorrelated with the signal for the next period's demand, s_t (since the demand shocks are i.i.d.). Combining these observations with (39) implies $cov_{t-1}(y_t, p_t) =$

$-\frac{\eta}{1-\eta}\sigma_{\delta}^{2}$ where $\sigma_{\delta}^{2} = var_{t-1}\left(\delta_t - \gamma s_{t-1}\right)$. This establishes (45) and completes the proof. $\square$

A.4. Omitted derivations in Section 4

We first present the details of the model with inflation that we use in Section 4. We then present the proof of Proposition 4. Throughout, we adopt the same notation as before for the *real* (inflation-adjusted) variables and introduce new notation for the nominal variables. In particular, Y_t, P_t, R_t^f denote the *real* real output, the *real* aggregate asset price, and the *real* interest rate, respectively.

New-Keynesian Phillips Curve (NKPC). Consider the supply side described in Appendix A.1. Recall that there is a continuum of monopolistically competitive firms, denoted by $\nu \in [0,1]$, that produce according to the Cobb-Douglas technology $(A.3)$. A final good sector aggregates the output from these firms according to $(A.4)$. The labor supply is provided by hand-to-mouth agents according to $(A.2)$.

An intermediate good firm's price is denoted by $Q_t(\nu)$. So far, we have assumed that these prices are permanently fixed. We now assume that in each period, a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period t, set their labor input to meet the demand for their goods (since firms operate with a markup and we focus on small shocks).

Consider the firms that adjust their price in period t. Let Q_t^{adj} denote the optimal price set by these firms. We assume Q_t^{adj} solves the following version of problem $(A.11)$

$$\max_{Q_t^{adj}} \sum_{h=0}^{\infty} \theta^h E_t^S \left\{ M_{t,t+h} \left(Y_{t+h|t} Q_t^{adj} - W_{t+h} L_{t+h|t} - T_t \right) \right\} \tag{A.19}$$

$$\text{where } Y_{t+h|t} = A_{t+h} L_{t+h|t}^{1-\alpha} = \left(\frac{Q_t^{adj}}{Q_{t+h}} \right)^{-\varepsilon} Y_{t+h}$$

$$\text{and } M_{t,t+h} = \beta^h \frac{1/P_{t+h}}{1/P_t} \frac{Q_t}{Q_{t+h}}.$$

The terms, $L_{t+h|t}, Y_{t+h|t}$, denote the input and the output of the firm (that resets its price in period t) in a future period $t+h$. The term, $M_{t,t+h}$, is the stochastic discount factor (SDF) between periods t and $t+h$. Recall that P_t denotes the end-of-period price of the market portfolio. Consistent with the financial market side of our model, we assume the SDF is determined by asset-holding households' wealth rather than their consumption. In equilibrium, asset-holding households' wealth is equal to the value of the market portfolio. We use $E_t^S[\cdot]$ to denote the firms' (price–setters') expectations.

The optimality condition for problem $(A.19)$ is given by

$$\sum_{h=0}^{\infty}\theta^h E_t^S\left\{M_{t,t+h}Q_{t+h}^{\varepsilon}Y_{t+h}\left(Q_t^{adj}-\frac{\varepsilon}{\varepsilon-1}\frac{W_{t+h}}{(1-\alpha)A_{t+h}L_{t+h|t}^{-\alpha}}\right)\right\}=0 \tag{A.20}$$

$$\text{where } L_{t+h|t}=\left(\frac{Q_t^{adj}}{Q_{t+h}}\right)^{\frac{-\varepsilon}{1-\alpha}}\left(\frac{Y_{t+h}}{A_{t+h}}\right)^{\frac{1}{1-\alpha}}.$$

We next combine Eq. $(A.20)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real potential outcomes and zero inflation, that is, $L_t = L^*, Y_t = Y_t^*$ and $Q_t = Q^*$ for each t, where recall that L^* is given by $(A.14)$ and $Y^* = A_t L^*$. Throughout, we use the notation $\tilde{x}_t = \log(X_t/X_t^*)$ to denote the log-linearized version of the corresponding variable X_t. We also let $Z_t = \frac{W_t}{A_t Q_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize the labor-supply equilibrium condition $(A.2)$ and use $C_t^{HM} = (1-\alpha)Y_t$ to obtain

$$\tilde{z}_t = \varphi\tilde{l}_t + \tilde{y}_t. \tag{A.21}$$

Log-linearizing Eqs. $(A.3-A.4)$ and $(A.7)$, we also obtain

$$\tilde{y}_t = (1-\alpha)\,\tilde{l}_t. \tag{A.22}$$

Finally, we log-linearize Eq. $(A.20)$ to obtain

$$\sum_{h=0}^{\infty}(\theta\beta)^h E_t^S\left\{\tilde{q}_t^{adj}-\left(\tilde{z}_{t+h}+\alpha\tilde{l}_{t+h|t}+\tilde{q}_{t+h}\right)\right\}=0, \tag{A.23}$$

$$\text{where } \tilde{l}_{t|t+h}=\frac{-\varepsilon\left(\tilde{q}_t^{adj}-\tilde{q}_{t+h}\right)}{1-\alpha}+\tilde{l}_{t+h}.$$

The second line uses $\tilde{y}_t = (1-\alpha)\,\tilde{l}_t$.

We next combine Eqs. $(A.21-A.23)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms

$$\begin{aligned}\tilde{q}_t^{adj} &= (1-\theta\beta)\sum_{h=0}^{\infty}(\theta\beta)^h E_t^S\left[\Theta\tilde{y}_{t+h}+\tilde{q}_{t+h}\right],\\ \text{where } \Theta &= \frac{1+\varphi}{1-\alpha+\alpha\varepsilon}\end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{q}_t^{adj} = (1-\theta\beta)\left(\Theta\tilde{y}_t+\tilde{q}_t\right)+\theta\beta E_t^S\left[\tilde{q}_{t+1}^{adj}\right]. \tag{A.24}$$

Here, we have used the law of iterated expectations, $E_t^S[\cdot] = E_t^S\left[E_{t+1}^S[\cdot]\right]$.

Next, we consider the aggregate price index $(A.5)$

$$
\begin{aligned}
Q_t &= \left((1-\theta)\left(Q_t^{adj}\right)^{1-\varepsilon} + \int_{S_t} \left(Q_{t-1}(\nu)\right)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)} \\
&= \left((1-\theta)\left(Q_t^{adj}\right)^{1-\varepsilon} + \theta Q_{t-1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)},
\end{aligned}
$$

where we have used the observation that a fraction θ of prices are the same as in the last period. The term, S_t, denotes the set of sticky firms in period t, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{q}_t = (1-\theta)\,\tilde{q}_t^{adj} + \theta\tilde{q}_{t-1}$. After substituting inflation, $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$, this implies

$$
\pi_t = (1-\theta)\left(\tilde{q}_t^{adj} - \tilde{q}_{t-1}\right). \tag{A.25}
$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.24)$ can be written in terms of the price change of adjusting firms as

$$
\tilde{q}_t^{adj} - \tilde{q}_{t-1} = (1-\theta\beta)\,\Theta\tilde{y}_t + \tilde{q}_t - \tilde{q}_{t-1} + \theta\beta E_t^S\left[\tilde{q}_{t+1}^{adj} - \tilde{q}_t\right].
$$

Substituting $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$ and combining with Eq. $(A.25)$, we obtain the New-Keynesian Phillips curve (46) that we use in the main text

$$
{}_t\left[\pi_{t+1}\right]
$$

$$
\text{where } \kappa = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1+\varphi}{1-\alpha+\alpha\varepsilon}. \tag{A.26}
$$

Output-asset price relation with inflation. We keep the macroeconomic side of the model the same as in Section 3. Specifically, households' optimality condition is still given by (21). Following the same steps as in Section 3, the output-asset price relation (22) still holds

$$
y_t = (1-\eta)\,m + \eta y_{t-1} + (1-\eta)\,p_{t-1} + \delta_t.
$$

Here, $y_t = \log Y_t$ and $p_t = \log P_t$ denote the log of *real* output and the log of the *real* aggregate asset price.

Financial market equilibrium conditions with inflation. We adjust the financial market side of the model to allow for a nominal interest rate in addition to the real interest rate. Specifically, financial markets feature *three* types of assets: a market portfolio, a *real* risk-free asset, and a *nominal* risk-free asset. Both risk-free assets are in zero net supply. As before, we let R_{t+1} denote the *real* return on the market portfolio and R_t^f denote the *real* risk-free interest rate. We also let R_t^{fn} denote the *nominal* risk-free interest rate. The Fed sets the *nominal* interest rate, R_t^{fn}, which is no longer the same as the real interest rate, R_t^f.

With these assumptions, portfolio managers (the market) solve the following version of problem (9)

$$\max_{\omega_t} E_t^M \left[\log \left(W_t \left(R_t^f + \omega_t \left(R_{t+1} - R_t^f \right) + \omega_t^n \left(\frac{R_t^{fn}}{Q_{t+1}/Q_t} - R_t^f \right) \right) \right) \right].$$

Here, recall that Q_t denotes the aggregate nominal price index. Therefore, Q_{t+1}/Q_t denotes the realized inflation and $\frac{R_t^{fn}}{Q_{t+1}/Q_t}$ denotes the *real* return on the nominal risk-free asset. Note that the nominal asset is risky in *real* terms because its return does not scale with inflation.

In equilibrium, we have $\omega_t = 1$ and $\omega_t^n = 0$. Following the same steps as before, we obtain *two* optimality conditions

$$E_t^M \left[\frac{R_t^f}{R_{t+1}} \right] = 1 \text{ and } E_t^M \left[\frac{R_t^{fn}}{R_{t+1} Q_{t+1}/Q_t} \right] = 1.$$

Assuming R_{t+1} is (approximately) log-normally distributed, the first optimality condition implies Eq. (47) in the main text

$$E_t^M \left[r_{t+1} \right] + \frac{1}{2} var_t^M \left[r_{t+1} \right] - i_t = rp_t \equiv var_t^M \left[r_{t+1} \right].$$

Assuming R_{t+1} and inflation Q_{t+1}/Q_t are (approximately) jointly log-normally distributed, the second optimality condition implies

$$i_t^n - E_t^M \left[\pi_{t+1} \right] - i_t + \frac{1}{2} var_t \left[\pi_{t+1} \right] + cov_t \left(\pi_{t+1}, r_{t+1} \right) + \frac{1}{2} var_t \left[r_{t+1} \right] = 0.$$

After combining this with (12) and rearranging terms, we obtain Eq. (48) in the main text

$$\left[i_t^n - E_t^M \left[\pi_{t+1} \right] + \frac{1}{2} var \left(\pi_{t+1} \right) \right] - i_t = irp_t \equiv -cov \left(\pi_{t+1}, r_{t+1} \right).$$

The Fed's policy problem with inflation. We adjust the Fed's problem to incorporate the costs of inflation gaps [cf. (14)]

$$\max_{i_t^n} -\frac{1}{2} E_t^F \left[\sum_{h=0}^{\infty} \beta^h \left(\tilde{y}_{t+h}^2 + \psi \pi_{t+h}^2 \right) \right] \tag{A.27}$$

Here, ψ denotes the relative welfare weight for the inflation gaps. We normalize the inflation target to zero so the inflation gap is equal to inflation. Note also that the Fed sets the *nominal* interest rate i_t^n, which is no longer the same as the real rate i_t. As before, the Fed sets policy without commitment.

Finally, we assume all agents (the firm, the market, and the price setters) have common beliefs. In Caballero and Simsek (2022a), we show that disagreements between the Fed and the price setters affect the market's expected inflation and induce a policy trade-off similar to "cost-push" shocks. Here, we abstract from these effects to focus on other drivers of inflation.

This completes the description of the model with inflation. We next prove Proposition 4, which characterizes the equilibrium.

Proof of Proposition 4. We conjecture and verify that there is an equilibrium in which Eqs. (49) hold

$$E_t[\pi_{t+1}] = 0 \text{ and } E_t[\tilde{y}_{t+1}] = 0,$$

along with Eqs. $(36-38)$ from Section 3.2. In particular, the Fed still targets a zero *expected* output gap. By doing this, the Fed also achieves zero *expected* inflation.

As before, the Fed effectively controls the *real* aggregate asset price p_t. Therefore, we write the Fed's problem as:

$$\begin{aligned} & \max_{p_t} -\frac{1}{2} E_t\left[\sum_{h=0}^{\infty} \beta^h \left(\tilde{y}_{t+h}^2 + \psi \pi_{t+h}^2\right)\right] \qquad \text{(A.28)} \\ y_t &= (1-\eta) m + \eta y_{t-1} + (1-\eta) p_{t-1} + \delta_t \\ \pi_t &= \kappa \tilde{y}_t + \beta E_t[\pi_{t+1}]. \end{aligned}$$

Here, the last two lines follow from Eqs. (22) and (46), respectively.

Next note that our conjecture for expected inflation, $E_t[\pi_{t+1}] = 0$, implies that inflation is given by $\pi_{t+1} = \kappa \tilde{y}_{t+1}$. Substituting this expression, the Fed's problem becomes:

$$\begin{aligned} & \max_{p_t} -\frac{1}{2}\left(1+\psi\kappa^2\right)\left[\tilde{y}_t + E_t\left[\tilde{y}_{t+1}^2\right] + E_t\left[\sum_{h=2}^{\infty} \beta^h \tilde{y}_{t+h}^2\right]\right] \\ & y_{t+1} = (1-\eta) m + \eta y_t + (1-\eta) p_t + \delta_{t+1} \\ & \text{and } \tilde{y}_{t+h} = \tilde{\delta}_{t+h} - E_t\left[\tilde{\delta}_{t+h}\right] \text{ for } h \geq 2. \end{aligned}$$

Here, the last line uses our conjecture for the future output gaps (see (38)). The current output gap $\tilde{y}_t$ is predetermined and not influenced by the current Fed decision. The future output gaps $\{\tilde{y}_{t+2}, \tilde{y}_{t+3}..\}$ are driven by unforecastable future shocks and therefore they are also not influenced by the current Fed decision. Using these observations, the optimality condition for problem $(A.28)$ implies

$$E_t[\tilde{y}_{t+1}] = 0. \qquad \text{(A.29)}$$

That is, the Fed targets a zero output gap on average as before.

We next verify our conjecture that the expected inflation is zero, $E_t[\pi_{t+1}] = 0$. First we take the period t expectations of the NKPC Eq. (46) for period $t+1$ to obtain

$$E_t\left[\pi_{t+1}\right] = \kappa E_t\left[\tilde{y}_{t+1}\right] + \beta E_t\left[\pi_{t+2}\right].$$

We then solve this equation forward (and assume inflation remains bounded in the limit) to obtain

$$E_t\left[\pi_{t+1}\right] = \kappa \sum_{h=1}^{\infty} \beta^h E_{t+h-1}\left[\tilde{y}_{t+h}\right] = \kappa E_t\left[\sum_{h=1}^{\infty} \beta^h E_{t+h-1}\left[\tilde{y}_{t+h}\right]\right] = 0. \tag{A.30}$$

Here, the second equality uses the law of iterated expectations and the last equality substitutes $(A.29)$. This verifies $E_t\left[\pi_{t+1}\right] = 0$.

Since the Fed's optimality condition from Section 3.2 still holds ($E_t\left[\tilde{y}_{t+1}\right] = 0$), the rest of the equilibrium is the same as in Section 3.2. In particular, "pystar," the output, and the output gap are given by $(36-38)$. This verifies our conjecture that there is an equilibrium that satisfies the equations in (49) along with along with Eqs. $(36-38)$.

Finally, note that Eq. (38) implies the output gap is given by

$$\tilde{y}_t = \tilde{\delta}_t - E_{t-1}\left[\tilde{\delta}_t\right].$$

Combining this observation with NKPC (46), inflation is given by Eq. (50) in the main text

$$\pi_t = \kappa \tilde{y}_t = \kappa\left(\tilde{\delta}_t - E_{t-1}\left[\tilde{\delta}_t\right]\right).$$

This completes the proof. □

Proof of Corollary 5. Assume the belief structure in Section 3.1.1. Following the same steps as in the proof of Corollary 4, we find that Eqs. $(39-44)$ still apply. Applying the expression for the output gap along with Eq. (50) implies Eq. (51), completing the proof. □

Proof of Corollary 6. Using Corollary 4 (which still applies), the return is given by

$$\begin{aligned} r_{t+1} &= \rho + \frac{\gamma s_t}{1-\eta} + \frac{\eta}{1-\eta}\left(\delta_t - \gamma s_{t-1} - z_t\right) \\ &\quad + \left((1-\beta) - \beta\frac{\eta}{1-\eta}\right)\left(\delta_{t+1} - \gamma s_t\right) + \frac{\beta}{1-\eta}\left(z_{t+1} - \gamma s_{t+1}\right) \end{aligned}$$

Using (51), inflation is given by

$$\pi_{t+1} = \kappa\left(\delta_{t+1} - \gamma s_t - z_{t+1}\right).$$

Combining these expressions, we obtain (52). Combining this with the financial market equilibrium condition (48), we further obtain (53), completing the proof. □

A.5. Omitted derivations in Section 5

Proof of Proposition 5. Consider the model with disagreements and aggregate demand inertia described in Section 5. With disagreements, the equilibrium price, output, and output gap still satisfy (36). The Fed's expected demand is given by $E_t^F\left[\tilde{\delta}_{t+1}\right] = E_t^F\left[\delta_{t+1}\right] = \gamma\left(s_t + \mu_t^F\right)$. Combining these observations, we obtain Eqs. $(60-62)$.

Next consider the equilibrium return r_{t+1}, given by (see (13))

$$r_{t+1} = \kappa + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t.$$

Substituting for the equilibrium output and the price from $(60-61)$, we obtain

$$\begin{aligned}
r_{t+1} &= \kappa + (1-\beta)\left(y_t^* + \delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right) \\
&\quad + \beta\left(y_{t+1}^* - \frac{\eta\tilde{y}_{t+1} + \gamma\left(s_{t+1} + \mu_{t+1}^F\right)}{1-\eta} - m\right) - \left(y_t^* - \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} - m\right) \\
&= \begin{array}{l} \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} + (1-\beta)\left(\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right) \\ \beta\left(\frac{z_{t+1}}{1-\eta} - \frac{\eta\left(\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right)}{1-\eta} - \frac{\gamma\left(s_{t+1} + \mu_{t+1}^F\right)}{1-\eta}\right) \end{array} \\
&= \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} \\
&\quad + \frac{1-\eta-\beta}{1-\eta}\left(\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right) + \frac{\beta}{1-\eta}\left(z_{t+1} - \gamma\left(s_{t+1} + \mu_{t+1}^F\right)\right).
\end{aligned}$$

Here, the second equality simplifies the constant terms and substitutes $\tilde{y}_{t+1} = \delta_{t+1} - \gamma\left(s_t + \mu_t^F\right) - z_{t+1}$ and $y_{t+1}^* = y_t^* + z_{t+1}$. The last equality collects similar terms together. This proves (63).

Next consider the equilibrium interest rate i_t. Taking the expectation of (63) under the *market's* belief, we obtain

$$\begin{aligned}
E_t^M\left[r_{t+1}\right] &= \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} + \frac{1-\eta-\beta}{1-\eta} E_t^M\left[\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right] \\
&= \rho + \frac{\eta\tilde{y}_t + \gamma\left(s_t + \mu_t^F\right)}{1-\eta} + \frac{1-\eta-\beta}{1-\eta}\gamma\left(\mu_t^M - \mu_t^F\right) \\
&= \rho + \frac{\eta\tilde{y}_t + \gamma s_t}{1-\eta} + (\beta+\eta)\frac{\gamma\mu_t^F}{1-\eta} + (1-\beta-\eta)\frac{\gamma\mu_t^M}{1-\eta}
\end{aligned}$$

Here, the first line uses $E_t^M\left[z_{t+1}\right] = 0$ and $E_t^M\left[s_{t+1} + \mu_{t+1}^F\right] = 0$ (the market thinks the Fed's future signal will be unbiased on average). The second line substitutes $E_t^M\left[\delta_{t+1} - \gamma\left(s_t + \mu_t^F\right)\right] = \gamma\left(\mu_t^M - \mu_t^F\right)$, which follows from (67). The last line rearranges terms. Combining this expression with (12) and rearranging terms, we obtain (64).

Finally consider the risk premium rp_t. Using (63), we obtain

$$\begin{aligned}
rp_t &= var_t^M\left[r_{t+1}\right] \\
&= var_t^M\left[\frac{1-\eta-\beta}{1-\eta}\delta_{t+1}+\frac{\beta}{1-\eta}\left(z_{t+1}-\gamma\left(s_{t+1}+\mu_{t+1}^F\right)\right)\right] \\
&= \left(\frac{1-\eta-\beta}{1-\eta}\right)^2\sigma_{\bar{\delta}}^2+\left(\frac{\beta}{1-\eta}\right)^2\left[\sigma_z^2+\sigma_\delta^2-\sigma_{\bar{\delta}}^2+\gamma^2 D\sigma_\mu^2\right]. \qquad \text{(A.31)}
\end{aligned}$$

Here, we have used (59) and the analogue of (A.18). Combining this with (34), we obtain

$$\begin{aligned}
var_t^M\left[r_{t+1}\right]=rp_t &= rp_t^{com}+\beta^2\gamma^2 D\sigma_\mu^2 \\
\text{where } rp_t^{com} &= \left(\frac{1-\eta-\beta}{1-\eta}\right)^2\sigma_{\bar{\delta}}^2+\left(\frac{\beta}{1-\eta}\right)^2\left(\sigma_z^2+\sigma_\delta^2-\sigma_{\bar{\delta}}^2\right).
\end{aligned}$$

This establishes (65) and completes the proof. □

Proof of Corollary 7. Follows from Eq. (65). □

Proof of Corollary 8. Most of the proof is presented in the main text. Here, we derive Eqs. $(69-70)$.

Consider the market's perception of the equilibrium interest rate i_t. Recall that the interest rate is given by (64). Substituting $\mu_t^F=\mu_t^M$ into this expression, we obtain

$$i_t^M=\rho+\frac{\eta\tilde{y}_t+\gamma\left(s_t+\mu_t^M\right)}{1-\eta}-\frac{rp_t}{2}.$$

Subtracting this from (64), we obtain

$$i_t-i_t^M=-\left(\beta+\eta\right)\frac{\gamma\left(\mu_t^M-\mu_t^F\right)}{1-\eta}.$$

When $\mu_t^M>\mu_t^F$, we also have $i_t<i_t^M$. The market thinks the interest rate is "too low": lower than what would obtain if the Fed shared the same belief as the market. This proves (69).

Next consider the market's expectation of the future interest rate $E_t^M\left[i_{t+1}\right]$. Consider Eq. (64) for period $t+1$,

$$i_{t+1}=\rho+\frac{\eta\tilde{y}_{t+1}+\gamma s_{t+1}}{1-\eta}+\left(\beta+\eta\right)\frac{\gamma\mu_{t+1}^F}{1-\eta}+\left(1-\beta-\eta\right)\frac{\gamma\mu_{t+1}^M}{1-\eta}-\frac{rp_{t+1}}{2}.$$

Taking the expectation under the Fed's belief in period t, we obtain

$$\begin{aligned}
E_t^F\left[i_{t+1}\right] &= \rho+\frac{\eta E_t^F\left[\tilde{y}_{t+1}\right]}{1-\eta}-\frac{rp_{t+1}}{2} \\
&= \rho-\frac{rp_{t+1}}{2}.
\end{aligned}$$

The Fed expects future output gaps to be zero. Thus, the Fed expects the future interest rate to be centered around its long-run level. Taking the expectation under the market's belief in period t, we instead obtain

$$\begin{aligned} E_t^M\left[i_{t+1}\right] &= \rho+\frac{\eta E_t^M\left[\tilde{y}_{t+1}\right]}{1-\eta}-\frac{rp_{t+1}}{2} \\ &= \rho+\frac{\eta\gamma\left(\mu_t^M-\mu_t^F\right)}{1-\eta}-\frac{rp_{t+1}}{2}. \end{aligned}$$

Since the market expects future output gaps to be non-zero, it also expects the future interest rate to react to these output gaps. In particular, a demand-optimistic market ($\mu_t^M > \mu_t^F$) expects the Fed to induce a positive output gap, which will then force the Fed to aggressively raise the interest rate. This establishes (70) and completes the proof. □

Proof of Corollary 9. The proof of the first part is presented in the main text. For the second part, note that the interest rate is given by (64). Substituting $\mu_t^M = \mu_t^F$ into this expression, we obtain

$$i_t^F = \rho+\frac{\eta\tilde{y}_t+\gamma\left(s_t+\mu_t^F\right)}{1-\eta}-\frac{rp_t}{2}.$$

Subtracting this from (64), we obtain

$$i_t - i_t^F = (1-\beta-\eta)\,\frac{\gamma\left(\mu_t^M-\mu_t^F\right)}{1-\eta}.$$

When $\mu_t^M > \mu_t^F$, we have $i_t > i_t^F$ if $\eta < 1-\beta$ and $i_t < i_t^F$ if $\eta > 1-\beta$. The main text describes the intuition behind these effects. This completes the proof. □

B. Appendix: Omitted extensions

This appendix presents the model extensions omitted from the main text.

B.1. Asset pricing for aggregate stocks and bonds

In the main text, we assume the market portfolio is the *only* financial claim on the production firms. In this appendix, we analyze the extension we discuss in Remark 3 where production firms can also issue risk-free debt. Thus, there are in general two claims on production firms: the equity claim ("aggregate stocks") and the risk-free debt claim ("aggregate bonds"). We characterize asset prices and show that the price of the market portfolio and the risk-free interest rate are *the same* as in the main text. We further show that a positive demand shock reduces the price of both equity and debt claims, but a positive belief shock for future earnings raises the price of the equity claim while reducing the price of the debt claim.

Formally, consider the baseline model without transmission lags we analyze in Section 2 (the analysis could be extended to the setup with lags and inertia). Suppose at the end of period t (and only in this period) the representative production firm issues short-term debt and uses the proceeds to buy back equity shares. Let D denote the debt due in period $t+1$. For simplicity, we assume the debt issuance is sufficiently small that the firm never defaults (technically we consider the limit as $D \to 0$). We let P_t^b and P_t^s denote the price of the debt and equity claims, respectively. The one-period-ahead return on these claims are given by

$$R_{t+1}^b = \frac{D}{P_t^b} \text{ and } R_{t+1}^s = \frac{\alpha Y_{t+1} + P_{t+1} - D}{P_t^s}. \tag{B.1}$$

As before, the market portfolio represents a claim on all financial assets. Its price and return are given by

$$P_t = P_t^b + P_t^s \tag{B.2}$$

$$R_t = \frac{P_t^b}{P_t} R_{t+1}^b + \frac{P_t^s}{P_t} R_{t+1}^s = \frac{\alpha Y_{t+1} + P_{t+1}}{P_t}. \tag{B.3}$$

We adapt the portfolio managers' (the market's) problem (9) to allow for investment in equity and debt claims. Since debt is safe, by no arbitrage its *equilibrium* return is given by the risk-free interest rate

$$R_{t+1}^b = R_t^f. \tag{B.4}$$

We can then formulate the managers' problem as follows

$$\max_{\omega_t^s} E_t^M \left[\log \left(W_t \left(R_t^f + \omega_t^s \left(R_{t+1}^s - R_t^f \right) \right) \right) \right].$$

This is the same as problem (9) with the difference that the managers invest in the equity claim (rather than in the market portfolio). Finally, we adapt the asset market clearing condition (11) as follows

$$W_t = P_t \quad \text{and} \quad \omega_t^s = \frac{P_t^s}{P_t}. \tag{B.5}$$

In equilibrium, the portfolio weight on the equity claim is equal to its value relative to the market portfolio.

The following results characterize the asset prices in equilibrium and generalizes the results from Section 2 to this setting (the proofs are at the end of the section).

Proposition 6 (Asset pricing with stocks and bonds)**.** *Consider the baseline model from Section 2 with the difference that firms issue risk-free debt in (only) period t. There is an equilibrium in which the price of the market portfolio $P_t = \exp(p_t)$ and the interest rate $R_t^f = \exp(i_t)$ are the same as in Proposition 1 and given by* (18) *and* (20)*.*

In period t, the price of the debt and the equity claims are given by, respectively,

$$P_t^b = \frac{D}{R_t^f} \tag{B.6}$$

$$P_t^s = P_t - P_t^b. \tag{B.7}$$

For small shocks, these prices approximately satisfy

$$\tilde{p}_t^b = -(\delta_t + b_t) \tag{B.8}$$

$$\tilde{p}_t^s \frac{\overline{P}_t^s}{\overline{P}_t} = z_t - \delta_t - \tilde{p}_t^b \frac{\overline{P}_t^b}{\overline{P}_t}. \tag{B.9}$$

Here, $\overline{P}_t^b, \overline{P}_t^s$ and $\overline{P}_t = \overline{P}_t^b + \overline{P}_t^s$ denote the asset prices in a benchmark with no shocks $\delta_t = b_t = z_t = 0$ and $\tilde{p}_t^b = \log\left(P_t^b/\overline{P}^b\right), \tilde{p}_t^s = \log\left(P_t^s/\overline{P}_t^s\right)$ denote the log deviations of the debt and the equity claims around the benchmark.

Corollary 10 (Demand shocks with stocks and bonds)**.** *A positive demand shock increases the interest rate R_t^f and reduces the price of the market portfolio P_t. For small shocks, the shock also reduces the price of the debt and equity claims:*

$$\frac{d\tilde{p}_t^b}{d\delta_t} = -1 \text{ and } \frac{d\tilde{p}_t^s}{d\delta_t} = -1.$$

Corollary 11 (The Fed put/call with stocks and bonds)**.** *A positive belief shock for future cash flows increases the policy interest rate R_t^f and does not affect the price of the market portfolio P_t. This shock also reduces the price of the debt claim and* increases *the price of the equity claim:*

$$\frac{d\tilde{p}_t^b}{db_t} = -1 \text{ and } \frac{d\tilde{p}_t^s}{db_t} = \frac{\overline{P}_t^b}{\overline{P}_t} > 0.$$

Proposition 6 is an application of the Modigliani-Miller Theorem. Since there are no financial frictions, firms' value with leverage is the same as their value without leverage. This in turn implies that the equilibrium without leverage, characterized in Proposition 1, remains an equilibrium with leverage. With leverage, we additionally obtain the price of the debt and equity claims. Eqs. $(B.6 - B.7)$ characterize these prices and Eqs. $(B.8 - B.9)$ characterize the log-linearized prices for small shocks. The debt price depends on its face value and the interest rate. Since equity is a levered claim on firms, its price is equal to the price of the market portfolio net of the debt claim.

Corollary 10 generalizes Corollary 1 to this setting. A demand shock reduces the price of the debt and the equity claim, as well as the price of the market portfolio. This shock affects the prices in period t by raising the interest rate, which reduces the value of most financial assets.

Finally, Corollary 2 generalizes Corollary 2 to this setting. A belief shock for future cash flows generates richer effects than a demand shock: it reduces the price of the debt claim and raises the price of the equity claim, while leaving the price of the market portfolio unchanged (as before). While the Fed stabilizes the aggregate asset price, $P_t = P_t^s + P_t^b$, it induces relative price effects between equity and debt claims. Since the debt claim is mainly driven by the Fed's interest rate decision, its price decreases. In contrast, since the equity claim is partly driven by the beliefs about future earnings, the belief shock increases its price despite the Fed's interest rate response.

Proof of Proposition 6. For periods $t+1$ onward, the model is the same as before so the equilibrium is also unchanged. Consider the equilibrium in period t. We show that there is an equilibrium in which $P_t = \exp(p_t)$ and $R_t^f = \exp(i_t)$ are the same as in Proposition 1. To prove this, we claim that the financial equilibrium condition for the market portfolio is the same as before,

$$E_t^M \left[\frac{R_t^f}{R_{t+1}} \right] = 1. \tag{B.10}$$

This in turn implies that the approximate equilibrium condition (12) still applies. Note also that the output price relation (17) still holds. These equations imply that p_t and i_t are the same as in Proposition 1.

It remains to prove the claim in $(B.10)$. To this end, first observe that problem (9) implies the following analogue of (10)

$$E_t^M \left[\left(R_{t+1}^s - R_t^f \right) \frac{1}{R_t^f + \omega_t^s \left(R_{t+1}^s - R_t^f \right)} \right] = 0. \tag{B.11}$$

Next note that in equilibrium we have

$$R_t^f + \omega_t^s \left(R_{t+1}^s - R_t^f \right) = (1 - \omega_t^s) R_t^b + \omega_t^s R_{t+1}^s = R_{t+1}.$$

Here, we have used $(B.4), (B.5)$ and $(B.3)$. Likewise, we also have

$$\begin{aligned}
R^s_{t+1} - R^f_t &= \frac{\alpha Y_{t+1} + P_{t+1}}{P^s_t} - \frac{D}{P^s_t} - \frac{D}{P^b_t} \\
&= \frac{P^s_t + P^b_t}{P^s_t} \left(\frac{\alpha Y_{t+1} + P_{t+1}}{P^s_t + P^b_t} - \frac{D}{P^b_t} \right) \\
&= \frac{P^s_t + P^b_t}{P^s_t} \left(R_{t+1} - R^f_t \right).
\end{aligned}$$

Here, we have used $(B.3)$ and $R^f_t = R^b_{t+1} = \frac{D}{P^b_t}$. Using $(B.11)$, and substituting the expressions for $R^f_t + \omega^s_t \left(R^s_{t+1} - R^f_t \right)$ and $R^s_{t+1} - R^f_t$, we further obtain

$$\frac{P^s_t + P^b_t}{P^s_t} E^M_t \left[\left(R_{t+1} - R^f_t \right) \frac{1}{R_{t+1}} \right] = 0.$$

Rearranging this expression, we prove $(B.10)$.

Why is the financial market equilibrium condition the same as before? In equilibrium agents hold the market portfolio, which implies that the stochastic discount factor is the same as before $(1/R_{t+1})$. In addition, stocks are a levered claim on the market portfolio, which implies that the optimality condition for stocks implies the optimality condition for the market portfolio. Consequently, the financial market side of the model is unchanged.

Next note that Eqs. $(B.6 - B.7)$ follow from Eqs. $(B.1), (B.4)$ and $(B.2)$. To log-linearize these equations, we first write them as

$$\begin{aligned}
\overline{P}^b_t \exp\left(\tilde{p}^b_t\right) &= D \exp\left(-\left(\rho + \delta_t + b_t - \frac{1}{2} rp \right) \right) \\
\overline{P}^s_t \exp\left(\tilde{p}^s_t\right) &= \exp\left(y^*_{t-1} + z_t - m - \delta_t \right) - \overline{P}^b_t \exp\left(\tilde{p}^b_t\right).
\end{aligned}$$

Here, we substituted i_t and p_t from Eqs. (18) and (20). We then linearize around $\delta_t = b_t = z_t = 0$ and substitute $\overline{P}^b_t = D \exp\left(-\left(\rho - \frac{1}{2} rp\right)\right), \overline{P}_t = \exp\left(y^*_{t-1} - m\right)$ to obtain

$$\begin{aligned}
\tilde{p}^b_t \overline{P}^b_t &= -\left(\delta_t + b_t\right) \overline{P}^b_t \\
\tilde{p}^s_t \overline{P}^s_t &= \left(z_t - \delta_t\right) \overline{P}_t - \tilde{p}^b_t \overline{P}^b_t.
\end{aligned}$$

Rearranging these expressions proves Eqs. $(B.8 - B.9)$. □

Proof of Corollary 10. The effect on the market portfolio and the interest rate follow from Corollary 1. For the effect on the debt and equity claims, Eqs. $(B.8 - B.9)$ imply

$$\frac{d\tilde{p}^b_t}{d\delta_t} = -1 \text{ and } \frac{d\tilde{p}^s_t}{d\delta_t} = \frac{-1 + \frac{\overline{P}^b_t}{\overline{P}_t}}{\frac{\overline{P}^s_t}{\overline{P}_t}} = -1,$$

where the last equality follows since $\overline{P}_t = \overline{P}_t^b + \overline{P}_t^s$. $\square$

Proof of Corollary 10. The effect on the market portfolio and the interest rate follow from Corollary 2. The effect of the debt and equity claims follow by taking the derivative of Eqs. $(B.8 - B.9)$ with respect to b_t. $\square$

B.2. Fed belief surprises and monetary policy shocks

In Section 5, we analyze Fed-market disagreements in a setting in which the market always knows the Fed's current belief (and vice versa). In practice, the market is often uncertain about the Fed's belief and learns it through a policy speech or announcement. In Caballero and Simsek (2022a), we use this observation to develop a theory of *microfounded* monetary policy shocks driven by Fed belief surprises. In this appendix, we extend our model in Section 5 to formally illustrate these shocks that we briefly discuss in Remark 5.

To capture Fed belief surprises, consider the setup in Section 5 with the difference that each period has two phases. Initially, the market does not know the Fed's interpretation μ_t^F. Later in the period, the market learns μ_t^F (before portfolio and consumption decisions). Our goal is to understand how the revelation of the Fed's interpretation to the market affects asset prices. For simplicity, suppose the Fed knows the market's interpretation μ_t^M throughout.

Initially, the market does not know the Fed's interpretation and needs to form an expectation about it. Using (56), the market thinks

$$\mu_t^F = \tilde{\beta}\mu_t^M + \tilde{\varepsilon}_t^F, \tag{B.12}$$

where $\tilde{\beta} = corr\left(\mu_t^F, \mu_t^M\right) = 1 - \frac{D}{2}$ and $\tilde{\varepsilon}_t^F$ has a zero mean. Given μ_t^M, the market expects the Fed's interpretation to be $\tilde{E}_t^M\left[\mu_t^F\right] = \tilde{\beta}\mu_t^M$. Here, we use $\tilde{E}_t^M[\cdot]$ to denote the expectations operator before the revelation of the Fed's actual belief μ_t^F. Therefore, the market also expects the aggregate asset price to be [see (60)]

$$\tilde{E}_t^M[p_t] = y_t^* - \frac{\eta}{1-\eta}\tilde{y}_t - \frac{\gamma\left(s_t + \tilde{\beta}\mu_t^M\right)}{1-\eta} - m.$$

Later in the period, the market learns μ_t^F and the aggregate price is realized to be

$$p_t = y_t^* - \frac{\eta}{1-\eta}\tilde{y}_t - \frac{\gamma\left(s_t + \mu_t^F\right)}{1-\eta} - m.$$

Combining these observations, we obtain

$$p_t - \tilde{E}_t^M[p_t] = -\frac{\gamma\left(\mu_t^F - \tilde{\beta}\mu_t^M\right)}{1-\eta} = -\frac{\gamma\tilde{\varepsilon}_t^F}{1-\eta}. \tag{B.13}$$

The *surprise change in the Fed's belief* (driven by its residual interpretation given the market's interpretation) affects asset prices. When the Fed is revealed to be more demand-optimistic than the market expected, asset prices decline. Conversely, when the Fed is revealed to be more demand-pessimistic than expected, asset prices increase.

Using (64), it is also easy to check that the revelation of the Fed's belief affects the interest rate:

$$i_t - \tilde{E}_t^M [i_t] = \frac{\beta + \eta}{1 - \eta} \gamma \tilde{\varepsilon}_t^F. \tag{B.14}$$

This surprise increase in the interest rate (partly) drives the valuation decline in (B.13). The following result summarizes this discussion.

Proposition 7 (Fed belief surprises and monetary policy shocks)**.** *When the Fed is revealed to be more demand-optimistic than the market expected, $\mu_t^F > \tilde{E}_t^M \left[\mu_t^F\right] = \tilde{\beta}\mu_t^M$, the interest rate increases and the price of the market portfolio declines (and vice-versa when the Fed is revealed to be more-demand pessimistic than the market expected, $\mu_t^F < \tilde{\beta}\mu_t^M$).*

One caveat is that we have assumed the market learns the Fed's belief μ_t^F automatically (in the second phase of the period). In practice, the Fed's beliefs are usually revealed to the market through a monetary policy announcement. Our model can also capture this feature because, as illustrated by (B.14), there is a one-to-one mapping between the policy interest rate and the Fed's belief surprise. In particular, when the Fed announces a higher interest rate than the market expected, this decision can reveal the Fed to be more demand-optimistic than what the market expected and trigger a monetary policy shock. In Caballero and Simsek (2022a), we formalize this idea and show that it is optimal for the Fed to set the rate in (64) and fully reveal its belief. Once the market learns the Fed's belief, an analogue of Proposition 7 holds.