Zombie Lending and Policy Traps[*]

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Abstract

We build a model with heterogeneous firms and banks to analyze how policy affects credit allocation and long-term economic outcomes. When firms are hit by small negative shocks, conventional monetary policy can restore efficient production through a standard bank lending channel as long as interest rates remain above their effective lower bound. Large shocks necessitate unconventional policy such as regulatory forbearance towards banks to stabilize the economy. Excessive accommodation, however, is contractionary as it induces zombie lending and a "diabolical sorting", whereby low-capitalization banks extend new credit or evergreen existing loans to low-productivity firms. In a dynamic setting, policy aimed at avoiding short-term recessions can be trapped into protracted low rates and excessive forbearance, due to congestion externalities imposed by zombie lending on healthier firms. The resulting economic sclerosis delays the recovery from transitory shocks, and can even lead to permanent output losses, highlighting the importance of maintaining upfront a well-capitalized banking system.

Keywords: Bank capital, Credit misallocation, Evergreening, Forbearance, Conventional and unconventional monetary policy

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1 Introduction

Since the housing and banking crisis in Japan in the early 1990s, regulatory forbearance towards banks has been increasingly combined with accommodative monetary policy in a bid to restore economic growth. Such forbearance typically consists of supporting depositors and other creditors of banks in the form of explicit or implicit government guarantees as well as liquidity support from the central bank, while simultaneously allowing a delayed recognition of stressed or non-performing loans on bank balance-sheets. This policy combination also found favor in the eurozone periphery countries following the global financial crisis of 2007-08 and especially after the sovereign debt crisis in 2011-12. The operative period of this policy combination seemed to have become protracted relative to the initial intentions and expectations, while the impact on economic growth has remained relatively muted.

Starting with Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008), the literature has attributed this ineffectiveness of policy in improving long-term economic outcomes (at least in part) to credit misallocation, in particular, to the phenomenon of *zombie lending*: weakly capitalized banks using regulatory forbearance to extend new credit or evergreen existing loans to their stressed borrowers, even as healthier firms in the economy experience adverse spillovers from the resulting proliferation of "zombie" firms (Section 2 provides a detailed summary of this empirical evidence).[1] The global policy response to the COVID-19 pandemic has also featured a combination of ultra-loose monetary policy and regulatory forbearance, raising the spectre of world-wide zombification and stagnation of economies and in turn of whether and how policy exit can be structured (Group of Thirty, 2020).

In this paper, we build a model with heterogeneous firms and banks to analyze how policy can affect the efficiency of credit allocation and long-term economic outcomes. The model makes three important contributions. First, it helps understand why in the face of large shocks, the policy response to restore economic growth may feature a combination of conventional policy in the form of monetary accommodation and unconventional policy in the form of regulatory forbearance towards banks. Unconventional policy arises in our model only when the conventional policy hits an effective or zero lower bound, distinct from the modeling of regulatory forbearance in the banking literature as arising from a

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[1] Several empirical studies document that the misallocation of credit by undercapitalized banks is a feature of developed and emerging economies alike. Chari, Jain and Kulkarni (2021) and Cong et al. (2019) provide evidence for large emerging economies such as India and China, where the public sector ownership of banks creates additional considerations linked to the political economy of bank lending and recapitalization.

time-inconsistency problem of regulation (Mailath and Mester, 1994).

Secondly, the model derives in equilibrium the empirically documented phenomenon that regulatory forbearance leads to zombie lending and a "diabolical sorting", whereby low-capitalization banks extend new credit or evergreen existing loans to low-productivity firms.[2] It is this positive implication of the model that then allows for a meaningful normative analysis of the policies affecting bank incentives to engage in such lending.

Thirdly, by examining a dynamic setting in which zombie lending imposes congestion externalities in the form of adverse productivity spillovers on healthier firms, the model explains why economies facing large, but only transitory, shocks may jointly feature thereafter (i) a phase of delayed recovery and potentially permanent output losses; and, (ii) a policy trap whereby monetary accommodation and regulatory forbearance aimed at avoiding short-term recessions become entrenched even as they persistently fail to restore long-term economic health. This last result of economic sclerosis, which transforms transitory shocks into potentially permanent stagnation, is the most salient feature of our analysis and highlights the importance of maintaining a well-capitalized banking system for avoiding a proliferation of zombie firms in the economy.

Let us describe our model, designed to be as tractable as possible while remaining consistent with the empirical features of zombie lending. We start with a static setting that describes what happens within a period, before turning to the full dynamic model. The economy is populated by heterogeneous firms that differ in their productivity and risk. Firms' investments require credit, which is provided by banks that are themselves heterogeneous in their level of capitalization. Banks face a portfolio problem, whose solution will depend on their capital: they decide whether to invest in safe assets (meant to capture a wide range of non-loan assets, such as central bank reserves, government bonds, mortgage-backed securities, or foreign assets) or lend to the productive sector, and if so, to which type of firms.

Policies play a crucial role in banks' incentives, and thereby the equilibrium allocation of credit. We summarize all the components of policy that affect bank decisions into two simple instruments: the risk-free rate R^f set by conventional monetary policy, and an "unconventional policy" or "forbearance" parameter p, that determines the level of government guarantees granted to banks willing to lend. Accommodative conventional monetary policy makes lending more attractive by lowering the return on safe assets R^f. This is a standard bank lending channel. Increasing forbearance also stimulates lending, by compressing the

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[2] See, e.g., Peek and Rosengren (2005), Okamura (2011), and Storz et al. (2017).

cost of funds associated with lending: a higher p lowers the cost of funds because a larger part of the loans' risk is borne by the government. But excessive forbearance can tilt banks' portfolios towards riskier loans to less productive firms: this is the "zombie lending channel". Our baseline model, featuring no regulation and therefore no regulatory arbitrage, treats risk-shifting as the primitive economic force pushing towards zombie lending. Later on, we incorporate a complementary evergreening motive for zombie lending, that arises in the presence of capital requirements.

The two-sided heterogeneity opens the door to the diabolical sorting documented in the data: banks with low capital and high leverage end up lending to less productive firms, even though aggregate output would be raised by letting these firms exit and be replaced by more productive entrants. The reason is that the subsidy arising from forbearance is increasing in both banks' asset risk and bank leverage. This sorting between banks and firms leads to a delicate policy trade-off. While zombie lending and depressed creative destruction are the main perils on the side of poorly capitalized banks, policymakers must also encourage well-capitalized banks to lend. The latter are not tempted by zombie lending, but they may invest in safe assets instead of lending to good firms. This tension is at the heart of our analysis of the optimal policy mix in response to exogenous shocks.

The output of good firms depends on an aggregate productivity or demand shock. Since zombie loans and safe assets are always less productive than loans to good firms, output reaches its potential if and only if all banks lend and there is no zombie lending. As long as the risk-free rate is not constrained, conventional monetary policy alone without any forbearance can achieve this objective. Without forbearance, there is no zombie lending by weak banks, while a sufficiently low risk-free rate encourages healthy banks to lend. Furthermore, larger negative shocks to fundamentals must be accommodated by a lower interest rate. Hence, if the shocks are large enough, conventional monetary policy runs into an effective lower bound on interest rates (ELB). This is where unconventional policy and its unintended consequences come into play.

A small amount of forbearance is beneficial, as it can substitute for the impaired or constrained conventional monetary policy and help lower banks' funding costs, thereby stimulating lending and output. Pushing on the forbearance string, however, will spur zombie lending by weak banks and hurt aggregate output. Surprisingly, we show that the optimal unconventional policy is non-monotonic in the size of the shocks: when shocks are moderate, forbearance should increase with the size of the shock as expected; but in the face of large shocks, policymakers should actually backtrack and reduce forbearance to avoid triggering zombie lending, even though this entails letting some banks invest in safe assets instead of lending. For large shocks, the output loss from zombie lending by banks can far exceed the opportunity cost of not lending to some healthy firms. Thus our result shows that there exists a “reversal” level of unconventional policy, a counterpart to the “reversal interest rate” below which conventional monetary policy turns contractionary (Brunnermeier and Koby, 2019).

Our full dynamic model adds two realistic features. First, we acknowledge that in the short run, keeping some of the less productive firms alive might be desirable to avoid reallocation frictions in capital and labor markets. Second, we allow zombie lending to cause negative spillovers on the productivity of healthy firms in future periods, to capture the congestion externalities in input or output markets documented in the empirical literature. These two ingredients further complicate the design of optimal policy, because there is now an additional dynamic trade-off. We show that the crucial parameter is the horizon of policymakers and consider two polar cases: “patient” policymakers, seeking to avoid future output losses; and, “myopic” policymakers willing to preserve incumbent firms at the expense of future productivity, due to term limits or reputational concerns that shorten their effective horizon.

In the main policy experiment we consider, the economy suffers a transitory exogenous shock to fundamentals, as in the static model. With a long policy horizon, the optimal response is exactly as in the static model: conventional monetary policy without forbearance achieves potential output for small shocks; some forbearance is optimal once the ELB binds when shocks remain moderate; and forbearance should decrease for large shocks to avoid any zombie lending and the associated congestion externalities. Myopic policymakers, on the other hand, implement the same joint policies for small and moderate shocks, but respond very differently to large shocks. Since they are focused on the short-term benefits of zombie lending (e.g., avoiding a painful reallocation of labor), they keep increasing forbearance as shocks deepen.

The dynamic consequences of myopic policy can be dire: we find that if spillovers from zombie lending to the productivity of healthy firms are strong enough, the optimal myopic policy response precipitates the economy into the following *policy trap*. Although the exogenous shock is transitory, future policymakers face an endogenously low productivity due to the congestion externalities, and continue responding in the same accommodating way, keeping interest rates low and forbearance high. This keeps zombies alive, and productivity low, for at least another period. At the very least, this negative dynamic feed-back generates endogenous persistence. In the extreme, for large enough initial shocks, the pattern repeats itself until the economy converges to a *sclerosis* steady state, defined as featuring a permanent combination of interest rates stuck at the ELB, high forbearance, zombie lending, and low output, reminiscent of the Japanese lost decades and the post-global-financial-crisis stagnation in the eurozone. In our theory, forward-looking policymakers should accept a “V-shaped” recession (i.e., sharp but transitory) precisely when fundamental shocks are large, which is exactly the opposite of what is argued in practice. While this argument resembles some of the classic “liquidationist” views of, e.g., Hayek and Schumpeter, in our framework this conclusion is contingent on many factors such as the size of the shock, the policy space available to address the shock with conventional tools, and the state of the banking sector when the shock hits.

Our theoretical framework builds on the seminal contribution of Caballero, Hoshi and Kashyap (2008) and extends it in two key dimensions. First, while their work highlights the negative spillovers generated by zombie lending due to congestion in input and output markets, it does not model financial intermediaries and their incentives to extend credit to low productivity firms. By contrast, banks and their capital structure are front and center in our framework. Second, our model stresses the nexus between policy, credit allocation, and aggregate outcomes. To the best of our knowledge, our model represents the first comprehensive theoretical treatment of zombie lending and policy traps. We put a central focus on bank capital, and how it affects – and is dynamically affected by – regulatory forbearance, to induce credit misallocation and output losses.

The central role of bank capital in our analysis raises important questions: Do under-capitalized banks have incentives to issue more equity? And if not, can regulators eliminate the zombie lending problem by simply increasing capital requirements? We address these questions in an extension of our model in the Online Appendix C, allowing for costly equity issuance, legacy lending, and capital requirements. We find that the same risk-shifting incentives affecting lending decisions also apply to capital structure decisions, preventing under-capitalized banks from raising enough capital to avoid zombie lending. Imposing high capital requirements can then deter zombie lending if the costs of breaking the relationship with a legacy zombie borrower (e.g., recognizing losses) are low enough. However, if these costs are high, we show that zombie lending becomes inevitable, in the sense that some banks will evergreen (lend to legacy zombie firms) for *any* level of capital requirement. Furthermore, there is a zombie-minimizing level of capital requirement and going beyond this level leads to even more zombie lending. The reason is that in order to satisfy the capital requirement, some banks must make up for switching costs by issuing equity. It becomes cheaper to roll over the zombie loans to economize switching costs; in other words, higher capital requirements lead more banks to choose this evergreening. The implication is that to avoid zombie lending, bank capital requirements need to be raised upfront rather than upon realization of economic shocks.

The remainder of the paper is organized as follows. In Section 2, we review key empirical facts about zombie lending which drive our modeling choices and relate our paper to other theoretical contributions. Section 3 develops our baseline model. In Section 4 we analyze optimal policy and turn to the dynamic model in Section 5. Section 6 and the Online Appendix present extensions around the role of bank capital. Section 7 concludes with some directions for future research.

2 Empirical Motivation and Related Literature

The existing empirical studies on zombie lending document four main facts which guide the construction of our theoretical model.[3] First, starting from the seminal contributions of Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008) related to the Japanese stagnation that began in the early 1990s, the literature has identified low levels of bank capital as the main motive for zombie lending. Several papers find evidence that the weakest banks have an incentive to allocate their credit to zombie firms in order to avoid the realization of losses on their balance sheets, thus adopting an evergreening behavior which causes a misallocation of credit away from healthier, more productive firms. In addition to the evidence coming from Japan (which also includes Giannetti and Simonov 2013 and Okamura 2011), other contributions have found similar evidence when analyzing different contexts such as the eurozone post the global financial crisis (Acharya et al., 2021a), during the sovereign debt crisis (Acharya et al. 2019), and in particular, peripheral European countries (Storz et al., 2017), such as Italy (Passalacqua et al. 2020 and Schivardi, Sette and Tabellini 2021) and Portugal (Blattner, Farinha and Rebelo 2020 and Bonfim et al. 2020).

Secondly, banks engage in zombie lending by both increasing their credit supply to the weakest borrowers, and by charging them subsidized interest rates. Indeed, Caballero, Hoshi and Kashyap (2008) and most of the following literature use subsidized bank credit as a criterion to identify zombie firms empirically. Evidence for subsidizing behavior from

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[3] See also Acharya et al. (2021b) for a recent survey of the empirical and theoretical literatures on zombie lending.

banks is provided by, for instance, (i) Acharya et al. (2019), who show – in the context of the "whatever it takes" (Outright Monetary Transactions) announcement by the European Central Bank in July 2012 – that weakly capitalized banks significantly reduced the interest rates for low-quality firms while leaving the rates for high-quality firms unchanged; and (ii) Schivardi, Sette and Tabellini (2021), who provide evidence that unhealthy banks in Italy did not charge higher interest rates to low-quality zombie borrowers compared to healthier firms. These findings might furthermore suggest that a low interest rate environment could help the proliferation of zombie firms by reducing the opportunity cost of evergreening and encouraging risk-taking behavior (Banerjee and Hofmann 2018). Relatedly, Asriyan et al. (2021) provide evidence from the US and Spain that declining interest rates, by raising the aggregate demand for and thus the cost of capital, can crowd out the investment of the more productive firms (see also the evidence in Gopinath et al. 2017).

Thirdly, there is a large body of evidence suggesting that the practice of zombie lending can have broad adverse effects on the real economy. In addition to causing a misallocation of credit, the presence of zombie firms can also induce congestion externalities in both input and output markets. This can induce misallocation of other resources and negative spillovers on healthier firms, which translate into lower economic outcomes such as depressed employment, productivity, innovation, investment, markups, and sales growth. While a full, general equilibrium, quantification would require a more structural approach, the empirical estimates available so far suggest that not only spillover effects can be substantial, but also that the output losses due to spillover effects might be persistent and even compound over time. In Japan, Caballero, Hoshi and Kashyap (2008) find that between 1993 and 2002, depending on the industry, the presence of zombies reduced other firms' cumulative investment and employment by 14 to 50 percentage points and 5 to 19 percentage points, respectively. In Europe, Acharya et al. (2019) find that between 2009 and 2014 non-zombie firms experienced 0.7 to 2.6 years of investment years lost (again depending on the industry) and an employment loss of 3 to 11 percentage points due to the presence of zombies. Following an exacerbation of evergreening practices by poorly capitalized banks in Portugal, Blattner, Farinha and Rebelo (2020) estimate that the reallocation of credit toward zombies can explain up to 13 percent of the observed decline of aggregate TFP. [4] A closely related literature shows positive spillover effects of banking deregulations (which can be

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[4] Other recent papers on this topic include Banerjee and Hofmann (2018), Adalet McGowan, Andrews and Millot (2018), Acharya et al. (2020a), Schmidt et al. (2020). Schivardi, Sette and Tabellini (2020, 2021), however, suggest that goods market spillovers might be limited in Italy in the short to medium run, while acknowledging that zombie lending could impair long-run growth.

viewed as the mirror image of the negative spillovers from zombie lending), in particular on creative destruction and allocative efficiency (Bertrand, Schoar and Thesmar, 2007).

Finally, a last set of findings is related to the effects of possible policy measures which might affect zombie lending. Regulatory forbearance and unconventional monetary policies implemented by central banks can be effective in refinancing banks in periods of crisis, even without explicit capital injections (Acharya et al. 2019; Acharya et al. 2020b). Regulatory forbearance and other forms of bank guarantees, however, may also lead to an increase in zombie lending practices (Gropp, Guettler and Saadi 2020). Similarly, bank recapitalizations are not effective at reducing zombie lending unless they are able to substantially improve bank's balance sheets. If this is not the case, these policies can instead exacerbate the zombie lending problem (Giannetti and Simonov 2013; Acharya et al. 2019; Blattner, Farinha and Rebelo 2020). Unexpected inspections conducted by the regulators, on the other hand, seem to be an effective tool for reducing banks' risk-taking behavior and evergreening practices (Bonfim et al. 2020; Passalacqua et al. 2020).

Our model seeks to incorporate all of these empirically documented features of zombie lending and regulatory policies that induce it, as well as the attendant spillovers to other firms in the economy. Theoretically, Bruche and Llobet (2013) and Hu and Varas (2021) also investigate banks' incentives for zombie lending. Similarly to our paper, Bruche and Llobet (2013) focus on the perverse incentives of banks with weak balance sheets, and propose a screening mechanism that can induce some of these institutions to liquidate their bad loans. This kind of mechanism depends on the specific nature of asymmetric information between banks and regulators, and thus may not perform as well in other settings (Chan, Greenbaum and Thakor, 1992); we study instead how standard non-targeted policies affect zombie lending. Hu and Varas (2021) propose a complementary explanation unrelated to bank capital for banks' evergreening behavior based on dynamic information revelation. Relative to this literature, our goal is to take seriously microeconomic incentives while preserving the tractability required to analyze general equilibrium outcomes. Therefore we incorporate bank heterogeneity and risk-taking behavior but do not model asymmetric information explicitly.

Our paper is also related to the macroeconomic literature on financial frictions and misallocation (e.g., Midrigan and Xu 2014 and Buera and Moll 2015). Buera, Moll and Shin (2013) also show that policies aimed at boosting short-run output can lead to long-run productivity losses. They focus on targeted industrial policies such as credit subsidies directly aimed at firms. Our focus is on the interplay between stabilization policies and bank lending incentives in response to macroeconomic shocks. Moreover, while the key friction in their framework is the assumed inertia of policies (e.g., it might be politically infeasible to withdraw subsidies from firms that have become unproductive), we show that in the presence of intertemporal congestion externalities, policymakers can fall into a policy trap even if they can update policies optimally at each point in time. Tracey (2021) investigates the effects of forbearance on lending in a quantitative macroeconomic model and argues that this was one of the causes that contributed to the lower output experienced by the euro area in the years following the sovereign debt crisis. Two other papers related to ours are Gopinath et al. (2017) and Asriyan et al. (2021), who argue that a low interest rate environment can induce misallocation of capital, together with output and productivity losses. Our focus is on the central role of banks in credit misallocation and how banks' incentives depend on macroeconomic policies. Our results on sclerosis and policy traps speak to the episodes of stagnation traps analyzed by Benigno and Fornaro (2018), who also highlight the ineffectiveness of conventional monetary policy alone in stimulating the economy.

3 A Model of Zombie Lending

We present a model of zombie lending consistent with the key empirical features highlighted in the previous section. We begin with a static model, which can be viewed as one period of the dynamic model presented in Section 5.

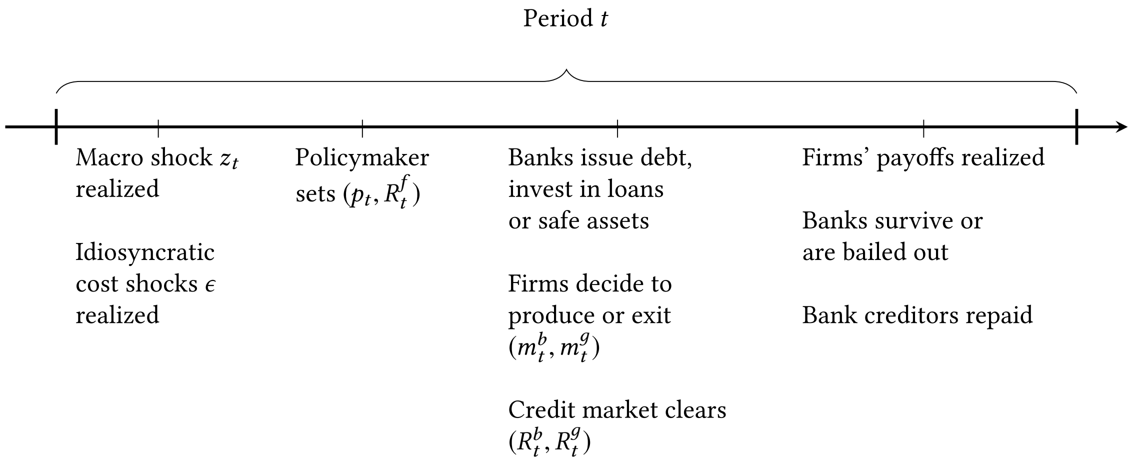
The economy is populated by heterogeneous firms that differ in their productivity and risk. These firms' investments require credit, which is provided by heterogeneous banks that differ in their level of capitalization. This two-sided heterogeneity opens the door to the diabolical sorting that has been documented in the data: poorly capitalized banks end up lending to less productive firms, even though aggregate output would be raised by letting these firms exit and be replaced by more productive entrants. Our model highlights the role of central bank policy, both conventional and unconventional, in determining banks' portfolios, and therefore the equilibrium allocation of credit and aggregate output.

3.1 Environment: Heterogeneous Firms and Banks

Figure 1 shows a timeline of the events in a period.

Firms. There are two types of firms, G or B. Initially, the economy is populated by a unit mass of incumbent firms. A mass $1 - \lambda$ of incumbents are endowed with an indivis-

Figure 1: Timeline of events within a period.



ible project of type G, that yields revenues $y^g(z)$ with probability θ^g and zero otherwise. A mass λ of incumbents are endowed with type B projects, yielding revenues $y^b(z)$ with probability θ^b and zero otherwise. y^g and y^b can depend on an aggregate shock z; we omit the dependence in z until Section 4. There are also potential entrants, each endowed with a type G project. Without loss of generality, we assume the mass of potential entrants to be equal to λ.[5]

Both types of projects require \$1 in capital to be implemented. Firms have no wealth, and need to finance their project entirely via bank debt. Firm types are observable to banks. Therefore, the debt contracts feature type-specific interest rates: G firms borrow at a rate R^g and B firms borrow at a rate R^b.

In addition, all firms incur a production cost $c + \epsilon_i$, where c is common to all firms while $\epsilon_i \in [0, \bar{\epsilon}]$ is an idiosyncratic cost shifter distributed according to the same c.d.f. H for both types of firms. The realization ϵ_i is known to the firm (but not to the bank) before production and financing decisions are made. Potential entrants also draw an idiosyncratic cost shifter ϵ_i, observed before the decision of whether to enter or not, from the same distribution H as incumbents. Relative to incumbents, potential entrants must pay an additional entry cost $\kappa \geq 0$ in order to be able to enter and produce, but they also have a technological advantage $\gamma \geq 0$, that decreases their production cost to $c - \gamma + \epsilon_i$. For simplicity, γ is assumed to be equal to κ, as in Caballero, Hoshi and Kashyap (2008).

Given the binary payoff structure, the project and the loan share the same risk: firms

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[5] This assumption simplifies expressions but allowing the mass of potential entrants to differ from λ does not affect our results.

repay their loan entirely if their project succeeds, and default on the full loan if their project fails. We assume that type B projects are riskier:

$$\Delta\theta = \theta^g - \theta^b > 0$$

to capture the fact that B firms have more outstanding debt and are thus more likely to default on their new loans.[6] Banks' investment in safe assets yields a baseline output $\underline{Y}$ (see below). We make the following assumption on payoffs:

Assumption 1. $\theta^b y^b - c < \underline{Y} < \theta^g y^g - c - \bar{\epsilon}$.

Thus, regardless of the idiosyncratic cost realizations, type B projects are less productive (in the sense of expected output) than safe assets. Safe assets, in turn, are safer but less productive than type G projects. The greater risk and lower profitability of type B projects mirror the characteristics of "zombie firms."

Banks. There is a unit mass of heterogenous financial intermediaries (hereafter, banks) indexed by their equity e. Bank equity is distributed in the interval $[e_{\min}, e_{\max}]$ according to the c.d.f. F, with $0 < e_{\min} \leq e_{\max} < 1$.

We assume a fixed balance sheet scale of \$1.[7] Each bank can invest its entire \$1 in a single asset, which can be either a risky loan or a safe asset. Banks can lend to a type $j = \{b, g\}$ firm at rate R^j, earning an expected return equal to $\theta^j R^j$. Credit markets are competitive: loan rates R^j are taken as given by both firms and banks, and determined in general equilibrium.

Alternatively, banks can invest in safe assets. We interpret safe assets as a broad class of assets held in banks' portfolios that are generally safer than corporate loans, such as mortgages, reserves, Treasuries, or asset-backed securities. Safe assets are supplied elastically and pay a risk-free return R^f set by monetary policy. Each unit invested in safe assets generates a baseline output $\underline{Y}$ through an unmodeled technology (e.g., government spending).

On the liability side, a bank with capital e needs to raise $1 - e$ of debt in order to invest. In equilibrium, debt holders require an expected return equal to R^f. The actual contractual

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[6] Acharya et al. (2019) analyze zombie lending around the eurozone debt crisis and show that zombie firms had numerous characteristics that made them riskier borrowers: higher leverage and lower net worth and profitability ratios, and an interest rate coverage ratio (IC) of 0.2 as opposed to 1.8 for other low-IC firms and 6.6 for high-IC firms. See also the evidence in Hoshi (2006) and Okamura (2011).

[7] Our statements about low capital e banks should be interpreted more broadly as applying to high leverage banks. With fixed scale, the leverage is simply $1 - e$. Our analysis would go through with variable bank scale and convex costs of increasing the size of liabilities.

rate paid to debt holders by each bank, $\tilde{R}^j$, depends on the riskiness of banks' asset choice j and on the degree of government guarantees indexed by a parameter p set by policy, as we describe next. Specifically, we assume that debt holders are able to recover their principal with probability $p \in [0, 1]$ if the bank defaults.[8] Thus the contractual rate $\tilde{R}^j$ on the debt of a bank that invests in asset $j \in \{g, b, f\}$ needs to satisfy

$$R^f = \theta^j \tilde{R}^j + \left(1 - \theta^j\right) p. \tag{1}$$

Policy instruments: R^f and p. Policymakers affect banks decisions through the choice of R^f and p. They directly control the level of the risk-free rate R^f through conventional monetary policy. They also set the parameter p, which influences banks' cost of capital through the debt pricing equation (1): a higher degree of insurance p encourages risky lending by decreasing the associated cost of funds. Equivalently, a low p means a strong market discipline: debt holders respond to bank risk-taking by requiring a higher rate.

There are several complementary interpretations of the policy variable p. A natural one is to view p as capturing the degree of insurance offered to depositors above and beyond the level needed to prevent bank runs (that we normalize to $p = 0$). Another is to think of p as indexing the leniency of bank closure policy: higher p means more regulatory forbearance. More broadly, p can be thought of as an unconventional monetary policy tool, such as the quantitative easing (QE) implemented by central banks starting from the Great Recession and onward. The common thread of these policies that is relevant in our framework is the impact on banks' cost of external financing: in all cases, a higher p undermines market discipline and incentivizes bank risk-taking. [9]

The two variables R^f and p impact banks' decisions—and therefore credit allocation—through two different channels. The first channel is a standard *bank lending channel*, that is, the choice between investing in safe assets versus lending to the productive sector. A lower R^f stimulates lending to both types of firms by decreasing the return of investing in safe assets relative to loans. Government guarantees subsidize riskier investments, thus a higher p also stimulates lending to both types of firms, by lowering the cost of funds.

The second channel is the *zombie lending channel*, operating through the choice be-

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[8] An alternative formulation would assume that the net interest $\tilde{R}^j - 1$ is also guaranteed with probability p. Our formulation yields simpler expressions throughout. Moreover, in the U.S., it is consistent with the insurance scheme offered to depositors by the FDIC.

[9] See Acharya et al. (2019) for empirical evidence of the effect of unconventional monetary policy actions (the OMT program in Europe) on banks' asset composition. They also show that until the ECB stepped in, market discipline was effective at making weaker banks reduce their risk.

tween lending to different types of borrowers. A higher p not only makes lending in general more appealing, but it also increases the profits from loans to B firms relatively more. These loans are riskier, thus a given subsidy p lowers the cost of funds by more when lending to B firms, through the term $\left(1-\theta^{b}\right)p$ in (1). As we will show, the incentives to lend to one type of firm or the other are bank-specific, as they depend on bank capitalization.

3.2 Equilibrium: Diabolical Sorting between Banks and Firms

Since there is a unit mass of banks and each bank lends to at most one firm, in equilibrium we must determine both the aggregate amount of lending (banks who do not lend invest in safe assets) and the composition of lending. As we explain below, the highest level of aggregate output is achieved when there is maximal *creative destruction*. That is, all the type B incumbent firms exit, and are replaced by more productive type G entrants. We model the entry and exit process building on Caballero, Hoshi and Kashyap (2008), with the additional layer of banks' portfolio choices. Equilibrium loan interest rates are the variables that adjust to bring about, or hinder, creative destruction.

Firms' entry and exit decisions. Given the realization of production costs, ϵ_i, and after observing the borrowing rate offered by banks, incumbent firms decide whether to produce or exit, and potential new entrants decide whether to enter or not. Incumbents remain in business and undertake their project if and only if they expect positive profits, which happens if and only if the idiosyncratic cost realization ϵ is lower than a type-specific threshold $\tilde{\epsilon}^{i}$, $i=g,b$. A type G incumbent drawing ϵ_i produces if

$$\epsilon_i \leq \tilde{\epsilon}^{g} = \theta^{g}\left(y^{g}-R^{g}\right)-c \tag{2}$$

and exits otherwise, while a type B incumbent drawing ϵ_i produces if

$$\epsilon_i \leq \tilde{\epsilon}^{b} = \theta^{b}\left(y^{b}-R^{b}\right)-c \tag{3}$$

and exits otherwise. The assumption $\gamma=\kappa$ ensures that the entry decision by potential entrants is exactly the same as for type G incumbents: a potential entrant drawing ϵ_i enters if and only if $\epsilon_i \leq \tilde{\epsilon}^{g}$. Generalizing to $\gamma \neq \kappa$ only requires keeping track of a different threshold for type G incumbents and entrants.

The masses of active firms of type G and B are respectively

$$m^g = \underbrace{(1-\lambda)H(\tilde{\epsilon}^g)}_{\text{incumbents}} + \underbrace{\lambda H(\tilde{\epsilon}^g)}_{\text{entrants}} = H(\theta^g(y^g - R^g) - c),$$

$$m^b = \lambda H\left(\theta^b\left(y^b - R^b\right) - c\right). \tag{4}$$

m^g and m^b are the aggregate loan demands from each type of firm. There is no intensive margin adjustment as projects are all of unit size, but higher loan rates decrease aggregate loan demand at the extensive margin.

Banks' portfolio choice. A bank with equity e chooses among the three investment options (safe assets, lending to type G firms, lending to type B firm) to maximize expected profits. Taking as given p, the loan rates R^g, R^b and the risk-free rate R^f, the bank solves

$$\max_{j\in\{g,b,f\}} \theta^j\left[R^j - \tilde{R}^j(1-e)\right]$$

$$\text{s.t. } \tilde{R}^j = \frac{R^f - \left(1-\theta^j\right)p}{\theta^j}.$$

The following proposition, proved in Appendix A, characterizes the solution of banks' problem as a function of their level of capitalization. To simplify the statement we consider equilibria with a positive amount of lending to G firms, which requires

$$R^f\Delta\theta \leq \theta^g R^g\left(1-\theta^b\right) - \theta^b R^b(1-\theta^g). \tag{5}$$

Proposition 1 (Diabolical sorting)**.** *Define the following equity levels:*

$$e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}$$

$$e^{**} = 1 - \frac{R^f - \theta^g R^g}{p(1-\theta^g)}.$$

Suppose that (5) holds.[10] *Then $e^* \leq e^{**}$ and banks' asset choices are characterized by the following thresholds:*

(i) Banks with equity $e < e^*$ lend to a type B borrower at rate R^b.

(ii) Banks with equity $e^* < e < e^{**}$ lend to a type G borrower at rate R^g.

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[10] The only difference if condition (5) does not hold is that $e^* > e^{**}$, implying that region (ii) does not exist, i.e., no bank lends to a G firm. This leads to an even more extreme diabolical sorting: low equity banks lend to type B borrowers, and high equity banks invest in safe assets.

(iii) Banks with equity $e > e^{**}$ do not lend and invest in safe assets at rate R^f.

Therefore the aggregate loan supplies to G and B firms, respectively, are given by

$$m^g = F(e^{**}) - F(e^*) \qquad \text{and} \qquad m^b = F(e^*), \tag{6}$$

with the remaining mass of banks, $1 - F(e^{**})$, investing in safe assets.

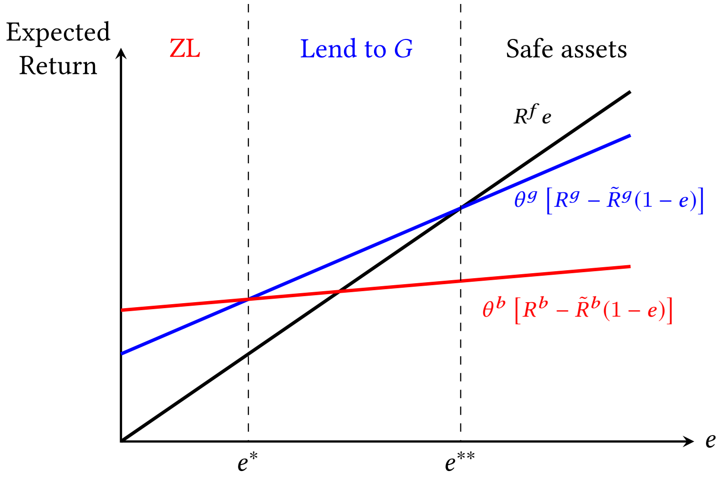
Proposition 1 shows that the solution of banks' problem features a *diabolical sorting* of poorly capitalized banks with low productivity firms. Importantly, while we start by taking loan rates as given, sorting will be a general equilibrium phenomenon that responds to policies. Sorting emerges from the complementarity between leverage, risk-taking, and policy. Regulatory forbearance induces risk-shifting in lending decisions. Crucially, the risk-shifting incentives depend on capitalization. To see this, we can rewrite bank e's expected profits from choosing investment of type j as $\theta^j[R^i - \tilde{R}^j(1-e)] = \theta^j R^j - R^f(1-e) + \gamma^j(p,e)$, where

$$\gamma^j(p,e) = p(1-\theta^j)(1-e)$$

is the subsidy to type j investments. Government guarantees $p > 0$ provide a subsidy only if banks have some leverage, and if they take positive risk; thus γ^j is zero for safe investments $\theta^j = 1$ or for fully equity-funded banks ($e = 1$). Outside these extreme cases, the subsidy is not only increasing in p, in leverage $1 - e$ and in risk $1 - \theta^j$, but also supermodular in these three variables. Thus our diabolical sorting result reflects the strong complementarity between leverage and risk-taking: for any $p > 0$, banks with high leverage are the ones who benefit the most from risk-taking.

Figure 2 offers a graphical intuition for this result, showing the expected profits from the three available investments as a function of bank capital e. While all banks have the option to finance good type of projects (either incumbents or new entrants), a high p incentivizes poorly capitalized banks to engage in *zombie lending* (ZL), financing low productivity firms whose projects have lower expected output but higher private returns for the bank in case of success. Our sorting result is in line with the recent evidence that poorly capitalized banks are more likely to extend credit to risky and unproductive borrowers (e.g., Andrews and Petroulakis 2019 in Europe and Faria e Castro, Pascal and Sánchez 2021 in the U.S.). It is also consistent with the lending behavior of weakly capitalized banks observed during Japan's banking crisis (Giannetti and Simonov, 2013), in Italy during the financial crisis (Schivardi, Sette and Tabellini, 2021), and in the aftermath of the European sovereign crisis following the ECB Outright Monetary Transactions (OMT) program (Acharya et al., 2019).

Figure 2: Expected profits as a function of bank capital e.



Note: Each line shows the expected profit from investing in asset j, $\theta^j \left[R^j - \tilde{R}^j(1-e)\right]$, as a function of e. The red line shows $j = b$ (lending to a type B firm). The blue line shows $j = g$ (lending to a type G firm). The black line shows $j = f$ (investing in safe assets).

The main mechanism through which zombie lending hurts creative destruction, productivity, and output in our model complements the one introduced by the seminal contribution of Caballero, Hoshi and Kashyap (2008). In their framework, zombie lending is detrimental because it creates congestion in input and output markets through, e.g., a higher input costs or lower profits.[11] In our framework, the adverse consequences of zombie lending on healthy firms are due to congestion in credit markets, since bank lending is a scarce resource. The channels featured in Caballero, Hoshi and Kashyap (2008) can be readily incorporated in our framework by making the cost c endogenous, which would only amplify the strength of our credit allocation channel. In the dynamic version of the model presented in Section 5, we explicitly incorporate congestion externalities to account for the harmful effects of zombies on healthy firms that unfold over time, and show that they have dramatic consequences for long-run credit allocation and output.

General equilibrium and aggregate output. Using the equations that define aggregate loan demand in (4) and aggregate loan supply in (6), we can characterize the general equilibrium in credit markets and the resulting aggregate output.

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[11] See also Acharya et al. (2020a) for evidence of congestion effects in product markets driven by the presence of zombie firms.

Definition 1. Given policies $\left(R^f, p\right)$, the static general equilibrium of the model is characterized by loan rates $\left(R^g, R^b\right)$ such that agents optimize and credit markets clear:

$$F\left(e^*\right) = \lambda H\left(\theta^b\left(y^b - R^b\right) - c\right)$$
$$F\left(e^{**}\right) - F\left(e^*\right) = H\left(\theta^g\left(y^g - R^g\right) - c\right),$$

where the thresholds e^* and e^{**} are defined in Proposition 1.

Given equilibrium loan rates, aggregate output can be written as

$$Y = \underline{Y} + \int_0^{\theta^g\left(y^g - R^g\right) - c} \left[\theta^g y^g - c - \epsilon - \underline{Y}\right] dH\left(\epsilon\right) + \lambda \int_0^{\theta^b\left(y^b - R^b\right) - c} \left[\theta^b y^b - c - \epsilon - \underline{Y}\right] dH\left(\epsilon\right). \tag{7}$$

The first term in (7) denotes the baseline output $\underline{Y}$ produced by banks' investments in securities. Relative to this baseline, the second term captures the positive net contribution of type G firms (both incumbents and entrants). The third term is the negative net contribution of type B firms. A low rate R^g increases aggregate output because it stimulates the entry and continuation of highly productive G firms. A low rate R^b has the opposite effect: it depresses aggregate output by deterring the exit of B firms, which are less productive.

3.3 Discussion of Main Assumptions

Bank balance sheets. In our model, banks have degenerate balance sheets and as a result the diabolical sorting is extreme, as each bank loads up on a single asset type. In reality, banks hold a variety of loans and securities in different proportions, and often specialize in lending to particular types of firms or sectors of the economy for which they acquired specific competences or information (see, e.g., Berger, Minnis and Sutherland 2017, Paravisini, Rappoport and Schnabl 2020). Loans in our model can thus be viewed as being portfolios of loans to a sector.[12] One should thus interpret Proposition 1 more broadly, as stating that banks with lower equity have a larger share of zombie loans than banks with higher equity, while banks with higher equity hold a larger share of safe assets.[13]

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[12] The assumption of full specialization could be relaxed by allowing banks to hold a portfolio of projects with correlated risks a la Vasicek (1977) without affecting the key message of the model.

[13] Even within the space of securities, Acharya and Steffen (2015) find a diabolical sorting around the eurozone sovereign crisis, with low-capital banks loading up on the riskier "GIIPS" bonds (Greece, Italy, Ireland, Portugal, Spain) and negatively on German bonds whereas high-capital banks had the opposite behavior.

Uniform forbearance policy p. A risk-sensitive p making banks' cost of funds independent of their assets would eliminate risk-shifting incentives. Although we do not model incomplete information explicitly, our assumption that p is *not* risk-sensitive builds on Chan, Greenbaum and Thakor (1992)'s insight, showing that when there is private information about bank assets and/or ex-post bank moral hazard (e.g., bank monitoring effort is non-contractible), it is impossible to implement risk-sensitive, incentive-compatible deposit insurance pricing in very general environments. In light of this impossibility result, we further simplify the model by assuming that p is the same for all banks.[14]

Other policies and fiscal constraints. We restrict the set of policy instruments to two variables, R^f and p. A natural question is whether other policies, such as bailouts or any form of subsidies that recapitalize the banking sector, could help in preventing zombie lending or even restore the first best allocation. Fiscal space is a key determinant of the feasibility of these policies, and it is itself endogenous to the state of the banking sector due to the "doom-loop" between banks and sovereign debt sustainability (Acharya, Drechsler and Schnabl 2014, Farhi and Tirole 2018). We focus on economies and states of the world in which fiscal capacity is tight and bank undercapitalization must be taken as given ex-post, at least in the short run. In Appendix C we consider ex-ante policies forcing banks to raise capital and show when they can indeed suppress zombie lending, but also when they can backfire by further encouraging zombie lending relative to laissez-faire.

Banks' incentives: risk-shifting and evergreening. In our baseline model, risk-shifting is the primitive economic driver of zombie lending. An alternative and complementary explanation for banks' zombie lending incentives relies on the presence of capital requirements and other forms of regulatory constraints: weak banks may prefer rolling over loans to their economically unviable legacy borrowers if this allows them to delay or avoid the recognition of losses. This commonly known "evergreening channel" of zombie lending is apparent in the data (Peek and Rosengren 2005) and important to study, but we start from a simpler institutional setting, showing that zombie lending arises even without capital requirements. In Appendix C we analyze evergreening and study the design of capital regulation taking the threat of zombie lending into account.

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[14] We also considered an extension in which p is allowed to depend on the observable level of bank capital e; this helps reduce zombie lending to some limited extent, but our results remain unchanged qualitatively.

4 Optimal Policy Response to Aggregate Shocks

In this section, we analyze how policymakers can optimally combine their instruments R^f and p to maximize aggregate output, and how the optimal policy mix should respond to shocks to fundamentals.

4.1 Potential Output and Optimal Policy

We define *potential output* Y^* as the highest possible aggregate output:

$$Y^* = \theta^g y^g - c - \mathbf{E}[\epsilon]. \tag{8}$$

According to equation (7), the economy achieves Y^* when all bank capital is used to finance the productive sector (i.e., there is no investment in bonds) and, within the productive sector, the most productive firms (i.e., there is no zombie lending). Achieving these two objectives requires both R^f and p to be sufficiently low so as to maximize the bank lending channel while preventing the zombie lending channel. A low risk-free rate R^f discourages substitution towards safe assets; a low subsidy p curbs the risk-shifting incentives of poorly capitalized banks.

Proposition 2 (Output-maximizing policies)**.** *There exist a threshold $\bar{p} > 0$ and an increasing function $\bar{R}^f(p)$ such that output reaches its potential ($Y = Y^*$) if and only if*

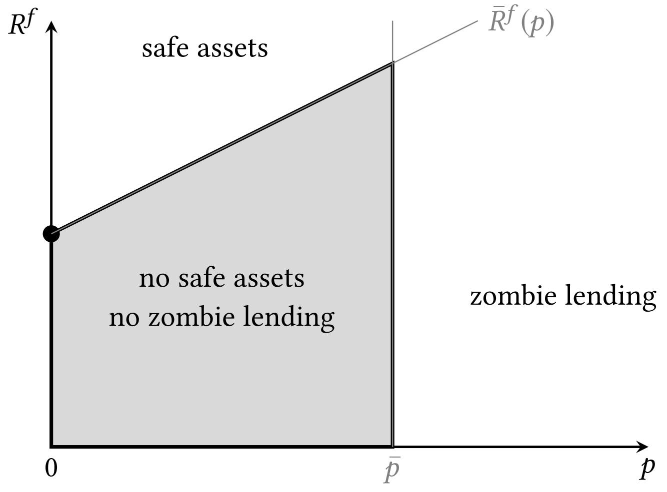
$$R^f \leq \bar{R}^f(p) \qquad \text{and} \qquad p \leq \bar{p}.$$

For any $p > \bar{p}$, zombie lending necessarily emerges in equilibrium and output falls short of Y^*.

Closed-form expressions for the thresholds $\bar{R}^f(p)$ and $\bar{p}$ are provided together with the proof in Appendix A. Figure 3 provides a graphical representation of Proposition 2 in the (p, R^f) space. The condition $R^f \leq \bar{R}^f(p)$ ensures that the return on safe assets is sufficiently low to make lending more attractive even for the banks with the maximal equity $e_{\max}$, because these are the banks who benefit the least from any subsidy p. The second condition $p \leq \bar{p}$ prevents the emergence of zombie lending, by ensuring that even the most leveraged banks, with equity $e_{\min}$, prefer to lend to type G firms.

Different combinations of p and R^f can achieve Y^*, but as long as transfers from taxpayers to banks are costly from the social planner's perspective, policymakers would strictly prefer policies that minimize p. Therefore we define the *optimal* policy as follows:

Figure 3: Optimal monetary policy.



Note: The gray area indicates all policy combinations (p, R^f) that achieve the potential output Y^*. The black dot (on the y-axis) denotes the optimal policy mix that achieves potential output at the minimal insurance cost.

Definition 2. The optimal policy is the combination (p, R^f) that minimizes p among the set of output-maximizing policies.

An immediate consequence of Proposition 2 is that without any constraint on monetary policy, a sufficiently low R^f, together with $p = 0$, achieves potential output at no insurance cost to the taxpayers.[15] In that case, banks' cost of capital fully adjusts for risk ($\tilde{R}^j = \frac{R^f}{\theta^j}$, $j = g, b$). The removal of the subsidy eliminates zombie lending and the corresponding output loss:

Corollary 1. *Absent constraints on conventional monetary policy R^f, potential output Y^* can always be attained and the optimal policy is:*

$$R^f = \bar{R}^f(0) = \theta^g y^g - c - \bar{\epsilon},$$
$$p = 0.$$

$\bar{R}^f(0)$ provides a notion of the "natural interest rate", that is the interest rate required to achieve Y^* with $p = 0$. Crucially, the natural rate fluctuates with fundamentals: negative

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[15] A positive level of insurance may be desirable in order to prevent panic withdrawals, bank runs, and the costly liquidations of financial institutions that might follow. Our variable p should be interpreted as the insurance and forbearance that goes above and beyond the "normal" level of guarantees needed to ensure financial stability.

productivity or demand shocks in the form of declines in y^g must be accommodated by a lower risk-free rate, exactly as in standard macroeconomic models. In the next section we study the optimal policy response to such shocks.

4.2 Optimal Policy Response to Shocks with an ELB Constraint

In this section we study the optimal policy response to shocks to the fundamentals of the economy. If shocks are large enough, the required rate R^f may be quite low. We show that, in the presence of an effective lower bound that constrains conventional monetary policy, some degree of accommodation through a positive level of forbearance p is desirable. Interestingly, we find that the optimal level of forbearance is a non-monotonic function of the shock.

We assume that an aggregate productivity or demand shock z hurts the revenue of good projects without affecting bad projects, whose revenues in case of a success remain at a fixed level y^b. We parametrize

$$y^g(z) = \bar{y}^g (1 - z)$$

where z lies between 0 and $z_{\max}$ such that Assumption 1 holds even for $z = z_{\max}$. More generally, all the results in this section apply to "gap-reducing" shocks z such that

$$\theta^g \frac{dy^g}{dz} < \theta^b \frac{dy^b}{dz}. \tag{9}$$

A central insight from our paper is that this type of shock reducing the profitability gap between good and zombie firms is especially dangerous for bank incentives and therefore policymakers trying to stimulate bank lending; we discuss the empirical relevance of gap-reducing shocks below.

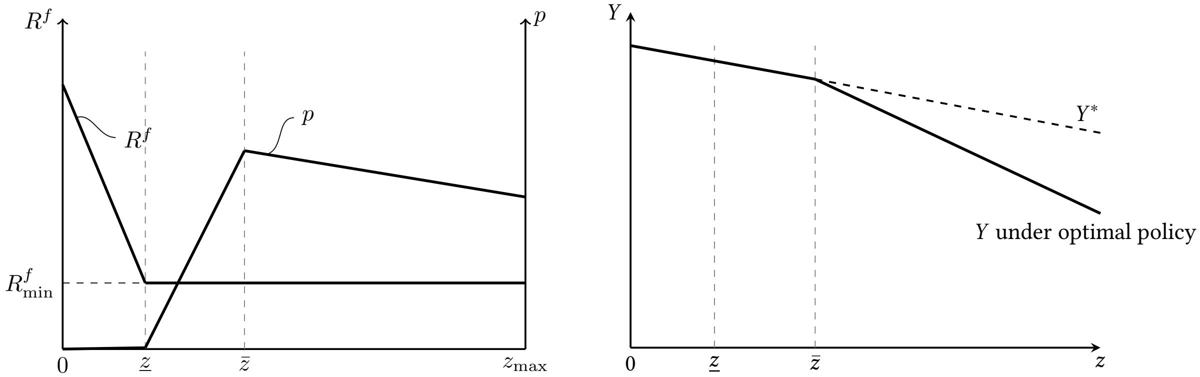
As discussed above, potential output can in principle be attained by conventional monetary policy alone by setting R^f to a sufficiently low level. However, an "effective lower bound" (ELB) may prevent the central bank from implementing Y^* if the required risk-free rate is too low. We now suppose there is an exogenous lower bound on the risk-free rate

$$R^f \geq R^f_{\min}. \tag{10}$$

A natural example is a zero lower bound $R^f_{\min} = 1$ (supposing inflation is zero) that arises because investors can always choose to save in cash instead of other negative-yield safe assets. $R^f_{\min}$ could be slightly lower than 1 if there are some costs of storing cash.

Before turning to the formal statement of the results, we describe the joint optimal

Figure 4: Optimal policy as a function of shock z.



Note: This figure illustrates the optimal joint policy response (R^f and p) to aggregate shocks z and the corresponding output and potential output (Y and Y^*).

policy response R^f and p to shocks z graphically in Figure 4. The striking result is that the optimal forbearance policy $p(z)$ is non-monotonic in the size of the shock. There are two thresholds $\underline{z}$ and $\bar{z}$. In line with Proposition 2, following small shocks $z \leq \underline{z}$, an accommodative conventional monetary policy can achieve Y^* at no costs ($p = 0$). Moderate shocks $z \in \left[\underline{z}, \bar{z}\right]$, however, require combining conventional monetary policy and forbearance policy in order to keep the economy at its full capacity. Specifically, a positive p helps stabilize output once the full swing of conventional monetary policy is constrained by the lower bound ($R^f = R^f_{\min}$). In this region, the optimal unconventional policy is to expand regulatory forbearance in response to more severe shocks. The increase in p subsidizes bank lending as much as possible, subject to the constraint of not triggering any zombie lending. If shocks are moderate, some forbearance $p > 0$ is sufficient to attain Y^*.

The effectiveness and desirability of unconventional policy actions is different when the economy is hit by severe aggregate shocks. In the region $z > \bar{z}$, conventional monetary policy is still constrained by the effective lower bound, but now the optimal unconventional policy needs to balance two opposite forces. On the one hand, an increase in regulatory forbearance (higher p) spurs lending at the expense of investment in safe assets. On the other hand, if forbearance p is too high, poorly capitalized banks engage in risk shifting and zombie lending.

As a result, for large enough shocks, policymakers must optimally *reduce* the degree of regulatory forbearance p as shock size z increases, and allow some banks to retrench from lending and invest in safe assets instead. Aggregate output Y necessarily falls short of its potential Y^* (which is itself already low due to the fundamental shock z, as shown on the right panel of Figure 4). Put differently, when severe aggregate shocks hit the economy, policy should allow healthy banks to start hoarding safe assets, rather than "pushing on a string": more accommodation would only trigger more zombie lending by the poorly capitalized banks. Our result shows that there exists a "reversal" level of unconventional policy above which further accommodation becomes contractionary, a counterpart to the "reversal interest rate" for conventional monetary policy (Brunnermeier and Koby, 2019).

Proposition 3 formalizes these results. The proof, including the definition of the thresholds $\underline{z}$ and $\bar{z}$, is in Appendix A.

Proposition 3 (Optimal policy with ELB)**.** *There exist thresholds* $\underline{z} > 0$ *and* $\bar{z} > \underline{z}$ *such that the optimal policy response to an aggregate shock* z *is the following:*

(i) *For small shocks* $z \leq \underline{z}$*, conventional monetary policy alone achieves* Y^**. The optimal policy features* $p = 0$ *and an interest rate* $R^f(z)$ *that is decreasing in the size of the shock, given by* $R^f(z) = \theta^g \bar{y}^g (1 - z) - c - \bar{\epsilon}$*.*

(ii) *For moderate shocks* z *such that* $\underline{z} < z \leq \bar{z}$*, unconventional policy* $p > 0$ *can achieve* Y^**. The ELB binds,* $R^f = R^f_{\text{min}}$*, and the optimal unconventional policy* $p(z)$ *is increasing in the size of the shock, given by* $p(z) = \frac{R^f_{\text{min}} + c + \bar{\epsilon} - \theta^g \bar{y}^g (1-z)}{(1-e_{\text{max}})(1-\theta^g)}$*.*

(iii) *For larger shocks* $z > \bar{z}$*,* Y^* *is not attainable. The ELB binds and the optimal unconventional policy* $p(z)$ *is decreasing in the size of the shock.*

Proposition 3 highlights the role of *large shocks*. A lack of profitable investment opportunities for good firms is not only detrimental per se, but it also makes zombie lending more attractive to banks. Thus zombie lending tends to emerge after large "gap-reducing" shocks that hit economies with a weak banking sector. By reducing the profitability gap between zombie and good firms, such shocks make good firms look temporarily closer to zombies. As a result zombie lending becomes more appealing, since the subsidy $p\left(1 - \theta^b\right)(1 - e)$ now accounts for a relatively larger share of the expected profit.

Our results abstract from the frictions in the bankruptcy system that may also follow such large shocks. A massive wave of bankruptcies may lead to court congestion, fire sales, and widespread financial stress, calling for a richer set of policies than those we consider, such as those analyzed in Gourinchas et al. (2020) and Greenwood, Iverson and Thesmar (2020).

Gap-reducing shocks. The empirical literature reviewed in Section 2 shows a lower investment and employment growth of healthy firms in sectors with a larger share of zombie firms. The leading interpretation is that they reflect the congestion externalities we discuss in Section 5. But these findings could also be in part driven by the "gap-reducing shocks" we focused on, that is shocks reducing the profitability gap between healthy and zombie firms according to condition (9). For instance, our model predicts that with multiple sectors s facing different gap-reducing shocks z_s but subject to a common forbearance policy p, the sectors suffering from larger shocks z_s would display more zombie lending in equilibrium.

These considerations suggest important directions for future empirical research. Firstly, whether accommodative policies trigger zombie lending depends on the nature of the recession, e.g., on the capitalization of the banking sector at the time of the shock and how the shock affects the profitability gap between zombie firms and healthy firms.[16] Secondly, it would be valuable to disentangle congestion externalities and gap-reducing shocks.

The role of bank capital. Our model also highlights that the capitalization of the banking system not only plays a crucial role in determining the allocation of credit—as illustrated in Proposition 1—but also mediates the effectiveness of policy interventions following real economic shocks. In fact, the threshold $\bar{z}$ depends on the equity distribution, and in particular on the minimal level of equity $e_{\min}$:

$$\operatorname{sign}\left(\frac{\partial \bar{z}}{\partial e_{\min}}\right) = \operatorname{sign}\left(R^f_{\min} + c - \theta^b y^b\right).$$

Hence $\partial \bar{z}/\partial e_{\min}$ is positive whenever the ELB binds. Therefore we have:

Corollary 2. *An improvement in the health of weak banks (higher $e_{\min}$) leads to a more resilient economy, in the sense that policy can achieve Y^* in response to a larger range of shocks $z \in [0, \bar{z}]$.*

This result links the potency of monetary policy to the level of capitalization of the banking system, and is consistent with Acharya et al. (2020b).

To summarize, the theoretical framework in the previous section reproduces some key

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[16] In the particular case of the COVID-19 shock, the massive government support to firms and banks happened at a time in which U.S. banks were well-capitalized while the shock triggered a reallocation of consumption from services to goods, without necessarily reducing the gap between healthy firms and zombies in each sector. This may explain why zombie lending has not been viewed as a prominent concern in the U.S. so far (Favara, Minoiu and Perez-Orive, 2021) although there is also recent evidence of evergreening by weaker banks (Faria e Castro, Pascal and Sánchez, 2021).

empirical findings relating the allocative efficiency of credit markets, optimal policy actions, and the capitalization of the banking system. Another important fact documented by the literature is that zombie lending has real spillover effects. Not only does it depress current output by taking up resources that could be utilized more efficiently elsewhere, but it can also erode the fundamentals of the economy due to negative externalities imposed by unproductive firms on the other firms in the economy. The following section studies the dynamic implications of zombie lending in the presence of these externalities.

5 Dynamic Model: Policy Traps and Sclerosis

Zombie lending is far from being a temporary problem. In fact, it has been proposed as one of the leading channels behind the Japanese stagnation taking place since the 1990s and the slow European recovery following the financial and sovereign debt crises (Hoshi and Kashyap 2015). To incorporate these features we turn to a dynamic version of our model that emphasizes how the interplay of accommodative policies and zombie lending can lead to persistent output losses and policy traps. Our main result shows that in response to even transitory shocks, the economy can get stuck in a state of permanent low productivity and output (which we call "sclerosis") with policymakers forced to implement a combination of low interest rates and high forbearance (which we call a "policy trap").

5.1 Dynamic Environment

To analyze the dynamic implications of zombie lending on the real economy, we introduce a simple modification of our framework that captures the negative externalities that the presence of zombie firms can impose on healthy firms in the economy over time.

Empirical studies highlight the dynamic effects of zombie lending on both zombie firms and good firms. At the same time, it has been argued that forcing zombie firms out of the market "too quickly" might entail significant short-term costs due to reallocation frictions in capital and labor markets.[17] Thus keeping some unproductive firms alive, at least temporarily, might be desirable. We model this trade-off by unpacking the output effect of zombies into a short-run component and a long-run component as follows. As in Section 4.2, we assume the economy is hit by adverse aggregate shock z at time $t = 0$, which affects the productivity of type G firms: $y_0^g = \bar{y}^g(1 - z_0)$. Like before, the shock is "gap-reducing":

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[17] See, e.g., Alvarez and Veracierto (2001), Buera, Jaef and Shin (2015), and Schivardi et al. (2021).

the productivity of type B firms is unaffected by the shock and lower than the productivity of type G firms. However, we now assume that the expected output of type B firms is higher than the real output produced by the investment in bonds, $\underline{Y}$. That is, we replace Assumption 1 with the following assumption:

Assumption 2. *$\underline{Y} < \theta^b y^b - c - \bar{\epsilon}$, and $\theta^b y^b < \theta^g y^g - \bar{\epsilon}$.*

A natural justification for Assumption 1 is that reallocation of labor and capital may take time and resources, so that keeping type B firms alive may yield short-term gains. The exit of type B firms (even if they get replaced by type G entrants) may also entail labor market externalities or redistributive effects that policymakers seek to avoid. Assumption 1 embeds these concerns directly into the realized "output" $\theta^b y^b$ instead of modeling the details of the labor market and how they enter social welfare.

The full cost of keeping zombie firms alive materializes over time. We assume that the presence of type B firms produces negative externalities, hurting the productivity of healthy firms in the next period:

$$y^g_{t+1} = \bar{y}^g \left(1 - z_{t+1}\right) \quad t \geq 0$$

where z_{t+1} increases with the extent of zombie lending in the previous period

$$z_{t+1} = \alpha m^b_t + \eta^Z_{t+1}. \tag{11}$$

η^Z_{t+1} is an exogenous aggregate shock, exactly as the shock z in the previous sections. In addition, productivity is now affected by an endogenous component αm^b_t. The parameter $\alpha \geq 0$ captures *congestion externalities*, that is the various channels through which zombies impact the performance of good firms, for instance labor and input market congestion, as highlighted by Caballero, Hoshi and Kashyap (2008), or in output markets due to price competition, as documented by Acharya et al. (2020a).

Bank and firm dynamics. In the dynamic model, we need to specify how banks evolve over time. Bank returns are stochastic, with some banks failing and others making large profits. In general, accounting for bank entry and exit and tracking the evolution of the full distribution of bank equity presents significant technical challenges, similar to the ones encountered in macroeconomic models with heterogeneous households and incomplete markets. We thus make the following assumptions to make the dynamic model tractable:

Assumption 3 (Bank dynamics)**.** *There are overlapping generations of bankers: bank managers at t are replaced after one period and earn a fraction ρ of the income accruing at $t + 1$.*

The manager of a bank with date-t equity e_t chooses project $i \in \{b, g, f\}$ to maximize

$$\rho\theta^i \left[R_t^i - \tilde{R}_t^i (1 - e_t)\right].$$

At the beginning of each period $t+1$, after date-t bank managers have been paid and replaced, failing banks are replaced by new banks and the profits of all surviving banks are pooled together and redistributed to all banks equally and banks raise equity $\iota > 0$.

Under these assumptions, denoting by m_t^i the mass of banks investing in asset class $i \in \{b, g, f\}$ at t, each bank starts period $t+1$ with equity

$$e_{t+1} = \iota + (1-\rho)\left[m_t^f R_t^f + m_t^g \theta^g \left[R_t^g - \tilde{R}_t^g (1-e_t)\right] + m_t^b \theta^b \left[R_t^b - \tilde{R}_t^b (1-e_t)\right]\right], \tag{12}$$

which also corresponds to the capitalization of the banking sector as a whole. This simplification allows us to keep track of the evolution of the aggregate capitalization of the banking system, rather than the entire distribution of bank equity. Since banks are indistinguishable, they will be indifferent between different investment options in equilibrium. Even though the portfolio of individual banks is indeterminate, the aggregate portfolio of the banking system is well-defined (as in a Miller (1977) equilibrium where bank capital structure is only determinate the aggregate level), which is all we need to study the output effects of zombie lending.

The short-term nature of bank managers' contracts implies that banks' franchise value does not enter the bank investment problem, therefore banks' portfolio choice is the same as static problem of Section 3. In particular, given date-t equilibrium rates, the optimal portfolio choice is characterized by the same thresholds e_t^* and e_t^{**} stated in Proposition 1. In a more general setting, banks would have to consider their franchise value when choosing their portfolios, which would then feed back into the equilibrium thresholds e_t^* and e_t^{**}. Accounting for the effect of the franchise value on bank's portfolio choices is an interesting extension that we leave for future research.[18]

Equilibrium. We can now define a dynamic equilibrium:

Definition 3. Given a path of policies $\left\{R_t^f, p_t\right\}_{t\geq 0}$ and fundamentals $\left\{y_t^g, y_t^b\right\}_{t\geq 0}$, a dynamic equilibrium is a sequence of masses $\left\{m_t^b, m_t^g, m_t^f\right\}_{t\geq 0}$, equity e_t, and loan rates $\left\{R_t^g, R_t^b\right\}$ such

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[18] We also assume that firms are focused on short-term profits, hence their entry and exit decisions are the same as in the static model; unlike the assumption on the bank side which simplifies the dynamic model considerably, the assumption on firms is mostly for exposition and can be relaxed to allow for forward-looking firms, see Appendix B.

that for all t, banks sort optimally:

$$m_t^b > 0 \Rightarrow e_t \leq e_t^* = 1 - \frac{\theta^g R_t^g - \theta^b R_t^b}{p_t\left(\theta^g - \theta^b\right)},$$
$$m_t^g + m_t^b < 1 \Rightarrow e_t \geq e_t^{**} = 1 - \frac{R_t^f - \theta^g R_t^g}{p_t\left(1 - \theta^g\right)},$$

bank equity e_t follows (12), markets clear

$$\begin{aligned} m_t^b &= \lambda H\left(\theta^b\left(y_t^b - R_t^b\right) - c_t\right), \\ m_t^g - m_t^b &= H\left(\theta^g\left(y_t^g - R_t^g\right) - c_t\right), \\ m_t^f &= 1 - H\left(\theta^g\left(y_t^g - R_t^g\right) - c_t\right), \end{aligned}$$

and productivity follows (11).

Next, we describe how policies are determined depending on policymakers' objectives, and characterize the resulting equilibria.

5.2 Policymakers' Horizon and Policy Rules

The dynamic equilibrium depends on the path of policies $\left\{p_t, R_t^f\right\}$, which in turn are set by policymakers depending on their objective function. We assume that the policy objective is to maximize the present discounted value of aggregate output

$$\max_{\left\{p_t, R_t^f\right\}_{t\geq 0}} \sum_t \beta^t Y_t,$$

where β denotes the policymakers' discount factor, which may or may not coincide with the "social discount factor" of the households consuming the output. As in the static model, policy affects equilibrium outcomes by influencing banks lending decisions through the choice of R^f and p. Because lending to type B firms has short-term benefits but possible long-term costs, the choice of the policy mix depends on how much weight policymakers put on current output relative to future productivity. A lower value of β puts more weight on current output, tolerating more zombie lending and thus larger future output losses caused by congestion externalities.

We consider two polar cases: a "No Zombie lending" policy, chosen by policymakers with high enough β, and a short-termist or "myopic" policy, chosen by policymakers with low enough β. We interpret a low policy horizon as arising from term limits, regulatory capture by incumbents, or reputational concerns that create a wedge between the public and regulatory objectives, as analyzed, for example, by Boot and Thakor (1993).

Long horizon: No Zombie lending policy. The *No Zombie lending (NZ) policy*

$$p_t = p^{NZ}(z_t, e_t)$$

is exactly the same as the optimal policy in the static model described in Proposition 3, in the special case of a degenerate equity distribution with $e_{\min} = e_{\max} = e_t$. In particular, p^{NZ} is non-monotonic in z_t. For moderate shocks (as long as Y^* can be reached), regulatory forbearance p increases with the shock z_t; for large shocks, the optimal p decreases with z_t (see Figure 4). While the optimal policy is the same, the rationale for lowering the degree of forbearance is different in the two settings. In the static model, type B firms were so unproductive that it was overall output-maximizing to prevent any form of zombie lending. Here, financing type B firms dominates investing in safe assets in the short run. Therefore, preventing zombie lending has a cost: it leads to a lower short-run output Y_t than under the policy that maximizes short-run output (described next), as some healthy banks end up investing in safe assets instead of lending. A policymaker with a high enough discount factor β is willing to bear this cost to maintain future productivity.

Short horizon: Myopic policy. Conversely, a policymaker with a sufficiently low discount factor β chooses to minimize the short-term costs of the shock z. This might require allowing zombie lending in equilibrium, even if doing so jeopardizes future productivity and output. Specifically, the optimal *myopic policy*

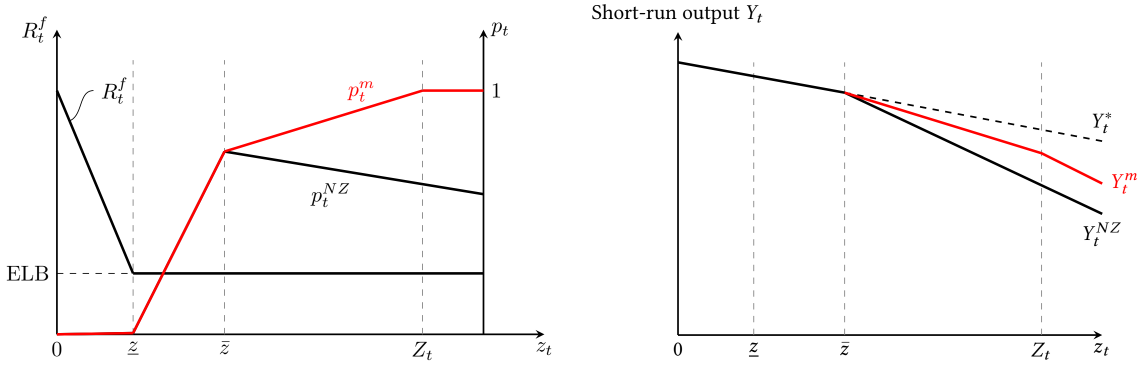
$$p_t = p^m(z_t, e_t)$$

maximizes short-run output at each point in time by ensuring that $m_t^g + m_t^b = 1$ at all times. Its key feature is that it will always seek to maximize aggregate lending using unconventional instruments such as regulatory forbearance, even though, for large enough shocks, this means tolerating some zombie lending. As a result, unlike the no zombie lending policy, the optimal myopic p is increasing in z: larger shocks are accommodated with higher p, until p reaches its upper bound of 1. Formally, we have:

Lemma 1. $p^m(z_t, e_t) = \min\left\{1, \frac{P^m(z_t)}{1-e_t}\right\}$ *where* P^m *is increasing in* z_t.

Denote Y_t^{NZ} and Y_t^m the levels of output arising from the NZ and myopic policies, re-

Figure 5: Optimal policy as a function of shock z_t under different policy regimes.



Note: The left panel illustrates the optimal joint policy response (R_t and p_t) in a No Zombie lending policy regime (black) and myopic policy regime as a function of the size of the shock z_t. The right panel illustrates the associated aggregate output achieved in the short run under the policy regimes.

spectively.[19] Figure 5 contrasts the two policy regimes and the level of output achieved by the two policies in the short run. First, we see that on the conventional monetary policy side, the optimal interest rate does not depend on β. In both cases, policymakers set the risk-free rate at its natural interest rate $R_t^f = \bar{R}^f(0)$ defined in Proposition 2 for $z \leq \underline{z}$, and set it at its lower bound $R_t^f = R_{\min}^f$ for $z > \underline{z}$. Second, the optimal forbearance p is also identical under the two policy regimes, as long as the shock is low enough so that Y^* can be achieved by increasing p without incentivizing banks to engage in zombie lending.

The only difference arises for large shocks $z > \bar{z}$. The NZ policy backtracks and reduces forbearance p as shocks grow larger, whereas the myopic policy keeps accommodating more and more until it hits the upper bound $p = 1$. The right panel shows the short-run output gains from this accommodation: while it remains impossible to perfectly stabilize the economy and achieve Y^*, output is much closer to Y^* under the myopic policy. But as we shall see next, this may come at a heavy cost.

5.3 Persistence of Output Losses under Different Policy Regimes

We now turn to our main dynamic experiment and result: transitory shocks can generate permanent output losses and policy traps due to the dynamic externalities imposed by zombie lending. Suppose the economy starts in a "good" steady state in which the zero lower bound is not binding ($R^f = \theta^g \bar{y}^g - c - \bar{\epsilon} > R_{\min}^f$). Thus no forbearance is needed ($p = 0$), there is no zombie lending, aggregate output is $Y = Y^*$, and equity is $e_0 = \frac{\iota}{1-(1-\rho)R^f}$.

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[19] Appendix A contains analytical expressions for Y_t^{NZ} and Y_t^m.

At date-0 a transitory shock $z_0 = \eta_0^Z > 0$ hits, so that $y_0^g = \bar{y}^g (1 - z_0)$. The shock only lasts for one period, hence we have $\eta_t^Z = 0$ for $t \geq 1$. We contrast the paths of the economy under the No Zombie lending and myopic policy rules. Recall from Proposition 3 that there exists a threshold $\bar{z}$ such that for shocks $z_0 \leq \bar{z}$, optimal policy can still attain the potential output Y^* without triggering any zombie lending. Therefore the NZ and myopic policies only differ for large shocks $z_0 > \bar{z}$. Let us then restrict attention to large enough shocks $z_0 > \bar{z}$. Under both policy stances, the optimal conventional policy implies setting the minimal risk-free rate $R_t^f = R_{\min}^f$ as long as as $z_t > \bar{z}$. However, the paths of p_t will differ across markedly. In fact, we show that seemingly small within-period differences between the NZ and myopic policies can lead to completely different long-run outcomes.

No Zombie lending policy: Transitory recession and full recovery. Under the NZ policy (high β), congestion externalities never materialize since there is no zombie lending in any period in equilibrium. The endogenous component of productivity losses is always zero, and since there are no further exogenous shocks, z reverts immediately to zero starting from date-1 ($z_t = 0 \quad \forall t \geq 1$). The date-0 recession is "V-shaped": it can be quite deep, but remains short-lived. Output recovers immediately from the transitory aggregate shock. The following proposition formally describes the full equilibrium path:

Proposition 4. *Under the No Zombie lending policy, the transitional dynamics for policies and aggregate output following the shock z_0 are given by*

$$\begin{array}{ll} \underline{t = 0} & \underline{\textit{for all } t \geq 1} \\ R_0^f = R_{\min}^f & R_t^f = \theta^g \bar{y}^g - c - \bar{\epsilon} \\ p_0 = \dfrac{R_{\min}^f + c - \theta^b y^b}{(1 - e_0)\left(1 - \theta^b\right)} & p_t = 0 \\ Y_0 = Y_0^{NZ} < Y^*(z_0) & Y_t = Y^*(0) \end{array}$$

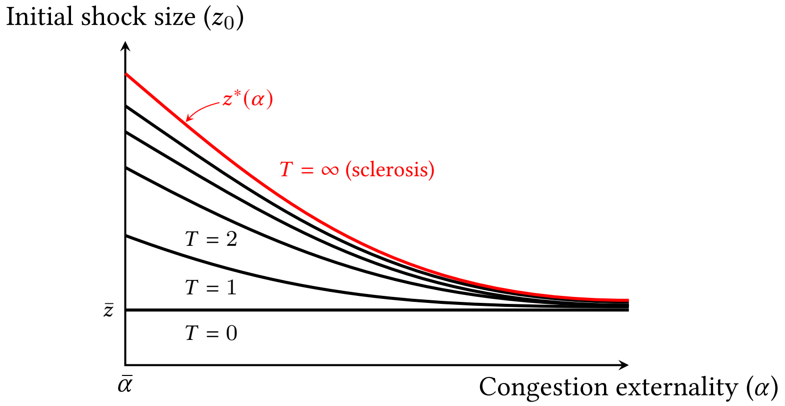
Myopic Policy: Policy Trap and Sclerosis. Under a myopic policy regime (low β), policymakers accommodate using regulatory forbearance, and allow some zombie lending at any date t, in spite of the potential long-term costs on the productivity of healthy firms.

The mass of zombies at date-t is

$$m_t^b = \lambda H \left(\theta^b y^b - R_t^f + p^m(z_t, e_t)(1 - e_t)\left(1 - \theta^b\right) - c\right).$$

In particular, since $z_0 > \bar{z}$ the date-0 mass of zombies m_0^b will be positive, which hurts the

Figure 6: Congestion externalities and persistence of output losses.



Note: This figure illustrates the persistence of the output losses as a function of the strength of congestion externalities due to zombie lending (α) and the size of the initial shock (z_0). Each label $T = 0$, $T = 1$, . . . corresponds to an area in the (α, z_0) space such that $T = \max\{t \text{ s.t. } z_t > 0\}$. In particular, the bottom rectangle $T = 0$ corresponds to purely transitory output losses due to the exogenous shock η_0^Z, while the upper red region $T = \infty$ corresponds to permanent output losses, i.e., sclerosis.

productivity of good firms at date-1 through $z_1 > 0$, and so on. The form of congestion externalities (11) implies that z_t follows the first-order Markov process

$$z_{t+1} = \alpha\lambda H\left(\theta^b y^b - R^f_{opt}(z_t) + p^m(z_t, e_t)(1 - e_t)\left(1 - \theta^b\right) - c\right).$$

The myopic policy creates an endogenous "hysteresis" channel: current accommodation leads to endogenous persistence of the initial shock, that worsens when congestion externalities α are larger.[20] In fact, as shown in Figure 6, if α is high enough, the myopic policy response to a sufficiently severe transitory shock z_0 pushes the economy to a steady state with *permanently* lower output, defined as follows:

Definition 4 (Sclerosis steady state)**.** A sclerosis steady state is a steady state equilibrium with the interest rate at the ELB ($R^f = R^f_{\min}$), permanent forbearance ($p > 0$) and potential output permanently depressed ($z > 0$).

Unlike in standard macroeconomic models, the natural rate becomes an endogenous variable. Sclerosis is associated with a *policy trap*: present policies aimed at minimizing short-term losses tie the hands of future policymakers through their effect on future pro-

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[20] This negative hysteresis effect is absent from "one-sector models", as it would be unrealistic to assume that future productivity and potential output depend negatively on current output. But we are highlighting one way in which this can happen, due to zombies.

ductivity. As a result, the economy may be stuck at the ELB forever even though the natural interest rate would recover to a positive level under a different policy rule.

We can now express our main dynamic result. We assume a technical condition on the distribution H of idiosyncratic cost shocks ϵ,

$$\sup_{e\in[0,1]} \frac{h\left(\theta^b y^b - R^f_{\min} + (1-e)\left(1-\theta^b\right) - c\right)}{h\left(H^{-1}\left(1-\lambda H\left(\theta^b y^b - R^f_{\min} + (1-e)\left(1-\theta^b\right) - c\right)\right)\right)} \geq 1 - \frac{\Delta\theta}{1-\theta^b} \tag{13}$$

which is satisfied when H is uniform, for instance.

Proposition 5 (Myopic policy and sclerosis)**.** *Suppose that congestion externalities are large enough, $\alpha \geq \bar{\alpha}$, for some positive $\bar{\alpha}$ (given in Appendix A) and let $z^*(\alpha) \geq \bar{z}$ be the smallest positive solution to $z = \alpha\lambda H\left(\theta^b y^b - R^f_{\min} + P^m(z)\left(1-\theta^b\right) - c\right)$. Then:*

1. *$z^*(\alpha)$ is increasing in α.*
2. *There exists a unique stable sclerosis steady state. It features maximal forbearance $p = 1$ and permanent output losses $z_\infty > 0$ such that*

   $$z_\infty = \alpha\lambda H\left(\theta^b y^b - R^f_{\min} + (1-e_\infty)\left(1-\theta^b\right) - c\right)$$

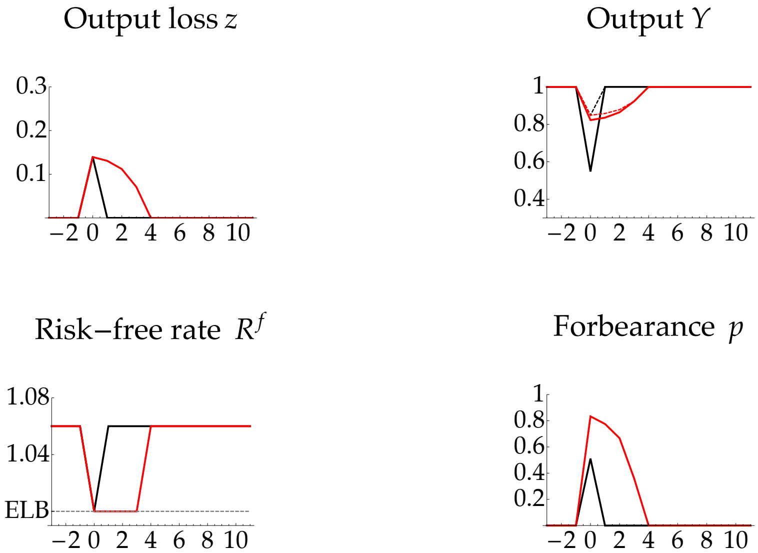
   where $e_\infty = \frac{\iota}{1-(1-\rho)R^f_{\min}} < e_0$ denotes steady state bank equity.
3. *For initial shocks $z_0 < z^*(\alpha)$, the economy converges to the no-zombie steady state, while for initial shocks $z_0 > z^*(\alpha)$ the economy converges to the stable sclerosis steady state with $z_t > 0, p_t > 0$ and a binding ELB $R^f_t = R^f_{\min}$ for all t along the transition.*

Figure 7 displays the impulse responses of output losses z_t, aggregate output Y_t, and the optimal policies R^f_t and p_t under the two policy regimes (NZ policy, in black, and myopic policy, in red). Panel A shows equilibrium paths following a shock z_0 that is above $\bar{z}$ but below the threshold $z^*(\alpha)$ defined in Proposition 4. The ELB binds at the time of the shock under both policy regimes. Forbearance also increases in both cases, but by much more under the myopic policy.

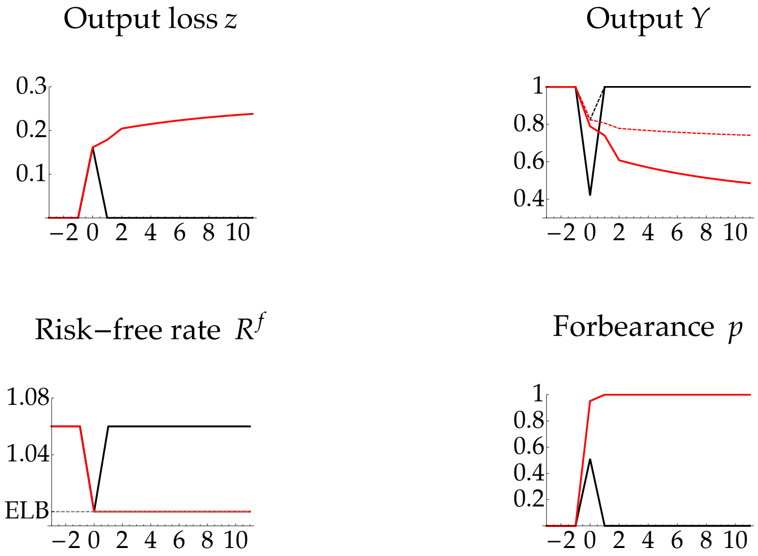
As a result, output drops sharply under the NZ policy, but recovers immediately to its pre-shock level at $t = 1$. The interest rate also recovers after the initial shock. By contrast, the myopic policy succeeds in stabilizing date-0 output at a higher level thanks to the more generous forbearance policy that keeps some zombie firms alive. The stabilization of short-term output comes at the cost of a protracted output loss for multiple periods, with

Figure 7: Impulse responses under the NZ policy (black) and the myopic policy (red).

Panel A: Small initial shock $z_0 < z^*(\alpha)$



Panel B: Large initial shock $z_0 > z^*(\alpha)$



Note: This figure displays the impulse responses of output losses z_t, aggregate output Y_t (solid lines) and potential output Y_t^* (dashed lines), and the optimal policies R_t^f and p_t under the the two policy regimes (No Zombie lending, in black, and the myopic policy, in red). Panel A focuses on the case of a small initial shock $z_0 < z^*(\alpha)$, showing that myopic policy softens the initial output loss but leads to endogenous persistence in the shock. Panel B focuses on the case of a large initial shock $z_0 > z^*(\alpha)$, showing that the myopic policy leads to a policy trap and eventually to a sclerosis steady state.

interest rates stuck at the ELB and forbearance p at a high level. While this path features endogenous persistence of the initial shock, the economy eventually converges back to its pre-shock steady state.

Panel B shows the equilibrium paths following a large initial shock $z_0 > z^*(\alpha)$. While initially the paths under the two policies regimes are similar to the ones following a smaller initial shock, they soon start diverging from each other. Like before, the economy experiences a sharp but short-lived output loss under the No Zombie lending regime. But under the myopic policy, the date-1 output loss z_1 stemming from congestion externalities is even larger than the initial shock z_0. This puts the economy on a dangerous path: at $t = 1$, the endogenously weaker fundamentals induce myopic policymakers to accommodate even further, by keeping interest rates as low as possible and allowing even higher forbearance ($p_1^m > p_0^m$), which, in turn, hurts date-2 productivity, and so on. For a while, this myopic policy manages to stabilize output Y_t close to potential output Y_t^*, albeit with a major side effect: potential output Y_t^* itself (dashed red line) starts falling because the presence of zombie firms reduces the productivity other firms in the economy. Moreover, once zombie lending becomes a permanent feature of the economy, all policymakers can do is exert maximal accommodation to stimulate output ($R^f = R^f_{\min}$, $p^m = 1$), which however is not sufficient to prevent a large gap between output and its potential. The economy snowballs towards sclerosis and monetary policy is trapped.

5.4 Discussion

Comparative statics. Transitory shocks are more likely to push the economy into the policy trap and a permanent sclerosis when policymakers' horizon β is low and congestion externalities α are high. Other economic factors also play a role, by affecting the threshold $z^*(\alpha)$ in Proposition 4. For instance, a low baseline productivity of good firms $\theta^g \bar{y}^g$ and a high rate of firm distress λ (and hence a high required rate of creative destruction to maintain potential output) also decrease z^*, which increases the risk of sclerosis. A low growth environment is thus particularly dangerous: not only is potential output already low, but the economy is also more fragile and output is more susceptible to fall below potential due to zombie lending.

Is sclerosis inefficient? We characterized policy regimes based on the policymakers' horizon. Whether the equilibrium path under the two policy regimes is efficient or not depends on the discount factor of the households who end up consuming the goods produced.

In particular, if the low discount factor β reflected the common impatience of policymakers and households, the sclerosis steady state would actually be an efficient outcome. But if instead, as is more likely, the policymakers' discount factor β accounts for short-termism due to term limits or political incentives, then the economy could end up in a sclerosis steady state even though an efficient allocation (from households' standpoint) would put less weight on immediate stabilization and more on future output.

Measuring the evolution of misallocation. Our analysis offers a novel perspective on the dynamics of resource misallocation. Cross-sectional dispersion in firm-level revenue productivity (TFPR) and marginal revenue product of capital (MRPK) are the most common empirical indicators of misallocation (e.g., Hsieh and Klenow 2009 and Gopinath et al. 2017). Our model focuses on the *extensive* margin of credit misallocation (i.e., which firms have access to credit), abstracting from *intensive* margin effects (i.e., whether firms are borrowing the right amount of capital). All the firms of a given type $i = G, B$ produce the same expected output $\theta^i y^i$, which captures both TFPR and MRPK.

In this context, dispersion in θy is inversely U-shaped in the mass of zombies, although aggregate productivity is strictly decreasing in the mass of zombies. In an equilibrium without zombie lending, there is no dispersion in θy (equal to $\theta^g y^g$ for all active firms) and no misallocation. A large shock that increases zombie lending also increases the dispersion in θy on impact. Notably, however, one might then observe a gradual decrease in dispersion in θy while in fact aggregate productivity keeps falling. Along a transition path with a growing share of zombie firms—such as the transition towards a sclerosis steady state—congestion externalities reduce the productivity gap between zombie and healthy firms, resulting in a decline of cross-sectional dispersion over time. Moreover, once zombie firms are already prevalent, a further zombification of the economy decreases dispersion. These considerations suggest that, in the presence of extensive margin effects and congestion externalities, dispersion-based measures of misallocation might fail to capture the true extent of misallocation imputable to credit market frictions.

6 Model Extensions

Equity Issuance, Capital Requirements, and Evergreening. In the Online Appendix C we return to the role of bank capital, extending the model to allow for equity issuance, capital loss recognition and capital requirements. We show that on their own, poorly capitalized banks have no incentives to issue enough equity to escape the zombie lending region, because the equity issuance decision suffers from the same risk-shifting incentives as the lending decision. Accommodative monetary policy can even worsen the zombie lending problem by reducing banks' equity issuance, or equivalently, increasing bank payouts.

Since banks will not recapitalize sufficiently by themselves, can regulators mitigate zombie lending by forcing banks to raise enough capital? We show that the answer depends on the "stickiness" of lending relationships. While it may be socially efficient for a bank to terminate a relationship with a legacy borrower turned bad, the bank may suffer from a variety of private switching costs. For instance, the bank has to set aside loss provisions, undertake a costly restructuring process, and spend time and money to screen for new borrowers. If these switching costs are low enough, regulators can indeed set high enough capital requirements to deter zombie lending altogether. If switching costs are high, however, capital requirements can backfire, as higher capital requirements ends up increasing zombie lending, in the form of evergreening of loans to legacy zombie firms.

Therefore our model highlights the importance of maintaining a well-capitalized banking system to avoid such policy traps, as not raising capital requirements upfront but raising them significantly upon the arrival of shocks can also backfire by encouraging zombie lending. In practice, capital requirements are not raised in a timely manner partly because they may end up being a burden on the public exchequer in the face of *en masse* banking sector undercapitalization. Similarly, there may be fiscal costs associated with excessive regulatory forbearance or there might be redistributive consequences of adopting forbearance rules that are tied to bank asset quality. Factoring in these costs and frictions, and analyzing their interactions with political economy considerations such as government myopia, seem important issues to model in the future; in particular, they have a bearing on sovereign risk which in turn can amplify banking sector undercapitalization (Acharya, Drechsler and Schnabl 2014; Brunnermeier et al. 2016; Farhi and Tirole 2018).

Ex-Post Interventions. We conclude this section with a brief discussion of what can be done ex post, once banks' balance sheets are already plagued by zombie loans, but leave a full treatment for a separate paper. We showed that the policies we already considered (conventional monetary policy, unconventional policy, and capital requirements) are unlikely to suffice, and may even make things worse.

Since zombie lending is a consequence of bank under-capitalization, a natural alternative policy is to recapitalize the banking sector. Historically, government capital has indeed been the most successful response to zombie lending episodes, either through direct equity injections as in Japan in the 2000s, or indirectly through asset purchases by a "bad bank", i.e., a public entity designed to absorb zombie or non-performing loans, such as the Korea Asset Management Corporation (KAMCO) in South Korea following the 1997-1998 financial crisis, or the Resolution Trust Corporation in the U.S. following the Savings and Loans crisis in the 1980s. Even when the government finally injects capital, a timid intervention that leaves some banks under-capitalized may spur further zombie lending, as shown by Peek and Rosengren (2005) and Giannetti and Simonov (2013) in Japan, and Acharya et al. (2019) in Europe.

Implicitly, our analysis assumes that the government lacks the ability or the willingness to recapitalize the banks sufficiently, at least in the short run. This can stem from fiscal constraints, or from the fact that just like the instruments we already considered, bank recapitalizations are subject to policy myopia. Hence a government may optimally delay injecting equity if the costs of doing so (e.g., political backlash, heightened sovereign credit risk) are borne immediately while the benefits only materialize over time.

Studying the optimal mix of policies as a function of the government's fiscal capacity is an important extension for future research. For instance, once we take into account both the costs of subsidies (through p) and the costs of recapitalization, how should the government balance the two policies? Another consideration is the resulting moral hazard: while it is ex-post optimal to target equity injections towards banks below the threshold e^*, the anticipation of such interventions would undermine bank incentives ex ante.

7 Conclusion

Our goal in this paper was to provide a theoretical framework with heterogeneous firms and banks that is consistent with the empirical evidence and helps understand better zombie lending and associated policy traps. The most salient findings from our model are that (i) aggressive unconventional policy runs the risk of introducing credit misallocation via a diabolical sorting, whereby low-capitalization banks extend new credit or evergreen existing loans to low-productivity firms; and (ii) policy aimed at avoiding short-term recessions can be trapped into protracted excessive forbearance due to congestion externalities imposed by such zombie lending on healthier firms.

Viewed through the lens of our model, it becomes paramount for efficient policy to avoid economic sclerosis precisely when shocks are large, as addressing such shocks with aggressive regulatory forbearance in order to secure short-term gains runs a high risk of zombie lending; conversely, when shocks are large, it may be necessary to embrace short-term recessions to prevent a delayed recovery and potentially permanent output losses. This trade-off is receiving increasing attention in the policy debate in the aftermath of the pandemic. The banking sectors of many countries have been recapitalized to absorb stressed level of losses; nevertheless, the unprecedented amount of distress and the adoption of ultra-loose monetary policy combined with regulatory forbearance has inevitably raised the specter of long-term economic stagnation from a zombification of the economy.

Our results suggest several directions for further empirical research. The implication that evergreening of existing bad loans and gambling on risky (but apparently "safe") securities are both manifestations of bank undercapitalization is worthy of detailed investigation in terms of their relative importance and substitution versus complementarity properties. Finally, a key aspect of our model is how monetary and banking policy interact with bank-firm quality over time, potentially converting transitory shocks into lost decades. This risk receives much discussion but needs to be studied with data on regulatory choices to deepen our understanding of policy evolution in response to large shocks and its long-term economic consequences.

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A Proofs

Proof of Proposition 1. There are two cases to consider:
Case 1. A bank prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g \left(R^g - \tilde{R}^g (1-e)\right) > \theta^b \left(R^b - \tilde{R}^b (1-e)\right).$$

Using the definition of $\tilde{R}^j$, $j = g, b$, this condition is met for banks with level of capitalization above the following threshold:

$$e > e^* = 1 - \frac{\left(\theta^g R^g - \theta^b R^b\right)}{p\left(\theta^g - \theta^b\right)}.$$

Case 2. A bank prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f - R^d(1-e) > \theta^g \left(R^g - \tilde{R}^g (1-e)\right)$$

Using the definition of $\tilde{R}^g = \frac{R^d - (1-\theta^g)p}{\theta^g}$ and under the assumption that $R^d = R^f$, this condition is met for banks with level of capitalization above the following threshold:

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{p\,(1-\theta^g)}.$$

As long as $e^{**} > e^*$, a bank that prefers investing in safe assets over lending to type G firms *a fortiori* prefers investing in safe assets over lending to type B firms. The following conditions ensured that $e^* < e^{**}$:

$$\frac{R^f - \theta^g R^g}{1-\theta^g} \leq \frac{\theta^g R^g - \theta^b R^b}{\theta^g - \theta^b},$$

or, equivalently,

$$R^f \leq R^g \frac{\theta^g \left(1-\theta^b\right)}{\left(\theta^g - \theta^b\right)} - R^b \frac{\theta^b\,(1-\theta^g)}{\left(\theta^g - \theta^b\right)}.$$

Proof of Proposition 2. $Y = Y^*$ is achieved when all banks lend and there is no zombie lending, hence $m^g = 1$ and $m^b = 0$. Relative to this composition of firms, any substitution towards bonds decreases output because $\theta^g y^g - c - \bar{\epsilon} > 0$, and any increase in zombie lending decreases output because $\theta^b y^b < \theta^g y^g - \bar{\epsilon}$.

In an equilibrium with $Y = Y^*$ loan rates are given by

$$R^b = y^b - \frac{c}{\theta^b}$$
$$R^g = y^g - \frac{1}{\theta^g}(c + \bar{\epsilon})$$

Given these equilibrium loan rates, we verify that there is indeed no zombie lending, that is $e^* \leq e_{\min}$, and that all banks lend, that is $e^{**} \geq e_{\max}$. These conditions can be rewritten respectively as

$$1 - \frac{\theta^g R^g - \theta^b R^b}{p\left(\theta^g - \theta^b\right)} = 1 - \frac{\theta^g y^g - \theta^b y^b - \bar{\epsilon}}{p\left(\theta^g - \theta^b\right)} \leq e_{\min} \Leftrightarrow p \leq \bar{p}$$

and

$$1 - \frac{R^f - \theta^g R^g}{p\left(1 - \theta^g\right)} = 1 - \frac{R^f + c + \bar{\epsilon} - \theta^g y^g}{p\left(1 - \theta^g\right)} \geq e_{\max} \Leftrightarrow R^f \leq \bar{R}^f(p)$$

where $\bar{p} = \frac{\theta^g y^g - \theta^b y^b - \bar{\epsilon}}{(1-e_{\min})\left(\theta^g - \theta^b\right)}$ and $\bar{R}^f(p) = \theta^g y^g - c - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g)p$.

Moreover, if R^f is lower than the type G project with the lowest net present value, i.e. $R^f < \theta^g y^g - c - \bar{\epsilon}$, then all banks lend and with $p \leq \bar{p}$ the economy reaches Y^* because there is also no zombie lending.

Finally, if $p > \bar{p}$ then there is necessarily zombie lending in equilibrium and $Y < Y^*$, regardless of the level of R^f.

Proof of Proposition 3. When the shock z_t is small, the economy's fundamentals remain strong and an accommodating conventional monetary policy alone can achieve $Y = Y^*$ at no costs ($p = 0$), without violating the ELB constraints. Adapting the results of Proposition 2, the monetary policy rate that achieves $m_g = 1$ with $p = 0$ is

$$R^f(z) = \theta^g \bar{y}^g(1 - z) - c - \bar{\epsilon}.$$

This equation satisfies the ELB constraint if $\theta^g \bar{y}^g(1 - z) - c - \bar{\epsilon} > R^f_{\min}$ or

$$z_t < \underline{z} = 1 - \frac{R^f_{\min} + c + \bar{\epsilon}}{\theta^g \bar{y}^g}.$$

For moderate shocks, $z_t > \underline{z}$, a combination of conventional and a lax forbearance policy, $p(z)$, can still achieve $Y = Y^*$ even if the ELB binds. Adapting the results of Proposition 2, this requires

$$R^f(z) = \theta^g \bar{y}^g (1 - z) - c - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g)\, p(z).$$

Exhausting the stimulus from conventional monetary policy, the optimal policy sets $R^f(z) = R^f_{\min}$, so p must satisfy $R^f_{\min} = \theta^g \bar{y}^g (1 - z) - c - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g)\, p(z)$, or

$$p(z) = \frac{R^f_{\min} + c + \bar{\epsilon} - \theta^g \bar{y}^g (1 - z)}{(1 - e_{\max})(1 - \theta^g)}$$

which is an increasing function of z_t. This equation holds as long as

$$z_t < \bar{z} = 1 - \frac{1}{\theta^g \bar{y}^g} \left[\frac{\left(\theta^b y^b + \bar{\epsilon}\right)(1 - e_{\max})(1 - \theta^g) + \left(R^f_{\min} + c + \bar{\epsilon}\right)(1 - e_{\min})\left(\theta^g - \theta^b\right)}{(1 - e_{\max})(1 - \theta^g) + (1 - e_{\min})\left(\theta^g - \theta^b\right)} \right].$$

For large shocks, $z_t > \bar{z}$, conventional monetary policy is constrained by the lower bound and increasing the level of forbearance exacerbates the output loss by inducing credit misallocation. Thus the optimal policy response ensures $m_b = 1$ but $0 < m_g < 1$ and Y^* is not attainable. The output-maximizing policy response is $R^f(z) = R^f_{\min}$ and, according to Proposition 2, the forbearance policy $p(z) > 0$ that solves:

$$F\left(1 - \frac{(1 - e_{\min})\left(\theta^g - \theta^b\right)}{1 - \theta^g} - \frac{R^f_{\min} + c - \theta^b y^b}{p(1 - \theta^g)}\right) = H\left(\theta^g \bar{y}^g (1 - z) - \theta^b y^b - p(1 - e_{\min})\left(\theta^g - \theta^b\right)\right),$$

which implies that the optimal $p(z)$ is decreasing in the size of the shock.

Proof of Lemma 1. Given the degenerate distribution of equity e_t, if

$$H\left(\theta^g y^g (1 - z_t) - R^f_t + (1 - e_t)(1 - \theta^g) - c\right) + \lambda H\left(\theta^b y^b - R^f_t + (1 - e_t)\left(1 - \theta^b\right) - c\right) < 1 \tag{14}$$

then even the maximal forbearance $p_t = 1$ cannot prevent some banks from investing in safe assets, and so the myopic policy sets $p^m(z_t, e_t) = 1$. Otherwise, if ((14)) doesn't hold, the optimal myopic policy $p^m(z_t) = P^m(z_t)$ that solves

$$H\left(\theta^g y^g (1 - z_t) - R^f_t + P^m(z_t)(1 - \theta^g) - c\right) + \lambda H\left(\theta^b y^b - R^f_t + P^m(z_t)\left(1 - \theta^b\right) - c\right) = 1$$

with

$$e_t^* = 1 - \frac{\theta^g R_t^g - \theta^b R_t^b}{p_t \left(\theta^g - \theta^b\right)} \leq e_t$$
$$e_t^{**} = 1 - \frac{R_t^f - \theta^g R_t^g}{p_t \left(1 - \theta^g\right)} \geq e_t$$

These conditions ensure that all banks lend (some to good firms and some to bad firms). Moreover, the forbearance policy $P^m(z_t)$ is an increasing function of z_t.

Proof of Proposition 5. A stable sclerosis steady state must have

$$p^m\left(z, e_\infty\right) = 1$$

i.e.

$$H\left(\theta^g y^g \left(1 - z_0\right) - R_{\min}^f + \left(1 - e_0\right)\left(1 - \theta^g\right) - c\right) + \lambda H\left(\theta^b y^b - R_{\min}^f + \left(1 - e_0\right)\left(1 - \theta^b\right) - c\right) < 1$$

This can be written concisely as

$$z > Z\left(e_\infty\right)$$

where

$$\zeta\left(e\right) = 1 - \frac{R_{\min}^f + c - \left(1 - e\right)\left(1 - \theta^g\right) + H^{-1}\left(1 - \lambda H\left(\theta^b y^b - R_{\min}^f + \left(1 - e\right)\left(1 - \theta^b\right) - c\right)\right)}{\theta^g \bar{y}^g}$$
$$Z\left(e\right) = \max\left\{\bar{z}, \zeta\left(e\right)\right\}$$

are decreasing functions of e by (13).

At any t the zero lower bound binds and $p^m\left(z_t, e_t\right) > 0$ if and only if $z_t \geq \bar{z}$. Moreover, if $z_t \geq Z\left(e_t\right)$ then the optimal myopic policy sets $p^m\left(z_t, e_t\right) = 1$ and therefore

$$z_{t+1} = z\left(\lambda H\left(\theta^b y^b - R_{\min}^f + \left(1 - e_t\right)\left(1 - \theta^b\right) - c\right)\right)$$

Thus we have a permanent sclerosis equilibrium (defined below) if for each t $z_{t+1} \geq Z\left(e_{t+1}\right)$ or

$$z\left(\lambda H\left(\theta^b y^b - R_{\min}^f + \left(1 - e_t\right)\left(1 - \theta^b\right) - c\right)\right) \geq \max\left\{\bar{z}, \zeta\left(\iota + \left(1 - \rho\right) R_{\min}^f e_t\right)\right\}$$

that is for all t

$$\alpha \geq \frac{\max\left\{\bar{z}, \zeta\left(\iota + (1-\rho) R^{f}_{\min} e_t\right)\right\}}{\lambda H\left(\theta^b y^b - R^{f}_{\min} + (1-e_t)\left(1-\theta^b\right) - c\right)}$$

ζ is decreasing in e but the denominator is also decreasing in e_t. We always have

$$\frac{\iota}{1-(1-\rho) R^{f}_{\min}} = \underline{e}_{\infty} \leq e_t \leq e_0 = \frac{\iota}{1-(1-\rho)\left[\theta^g \bar{y}^g - c - \bar{\epsilon}\right]}$$

Therefore an upper bound on the right-hand side is

$$\hat{\alpha} = \frac{\max\left\{\bar{z}, \zeta\left(\iota + (1-\rho) R^{f}_{\min} \underline{e}_{\infty}\right)\right\}}{\lambda H\left(\theta^b y^b - R^{f}_{\min} + (1-e_0)\left(1-\theta^b\right) - c\right)}$$

and a sufficient condition for permanent sclerosis to happen is $\alpha \geq \hat{\alpha}$.

B Forward-looking firm dynamics

Incumbent firms draw a cost shock ϵ in each period. If they do not exit they earn current expected profit

$$\pi^i_t(\epsilon) = \theta^i\left(y^i_t - R^i_t\right) - c_t - \epsilon$$

Assume firms exit when their project fails. A forward-looking incumbent firm's value function if it does not exit is

$$\Pi^i_t(\epsilon) = \pi^i_t(\epsilon) + \beta\theta^i \underbrace{\mathrm{E_t}\left[\left(1-\lambda^i\right)\max\left\{\Pi^i_{t+1}(\epsilon_{t+1}), 0\right\} + \lambda^i \max\left\{\Pi^{-i}_{t+1}(\epsilon_{t+1}), 0\right\}\right]}_{=W^i_{t+1}}$$

where with a probability λ^i the firm can change type to $-i$ next period. Then the firm does not exit if and only if

$$\Pi^i_t(\epsilon) \geq 0 \Leftrightarrow \epsilon \leq \bar{\epsilon}^i_t = \theta^i\left(y^i_t + \beta W^i_{t+1} - R^i_t\right) - c_t$$

A myopic firm ignores the W^i_{t+1} part, hence does not exit if and only if $\pi^i_t(\epsilon) \geq 0$, i.e., $\epsilon \leq \theta^i\left(y^i_t - R^i_t\right) - c_t$.

Potential entrants are all of the $i = g$ type, and have cost $c_t - \gamma - \epsilon$. If they enter they must pay an entry cost κ, hence they earn current expected profit

$$\pi_t^n(\epsilon) = \theta^g \left(y^g - R_t^g\right) - c_t + \gamma - \epsilon - \kappa$$

in the first period. After one period they become incumbents and lose their productivity advantage γ (it is straightforward but inconvenient to generalize to γ lasting multiple periods). Thus a potential entrant enters if and only if

$$\epsilon \leq \bar{\epsilon}_t^n = \bar{\epsilon}_t^g + \gamma - \kappa$$

Incumbents' value functions satisfy

$$\Pi_t^i(\epsilon) = \pi_t^i(\epsilon) + \beta\theta^i \left[\left(1-\lambda^i\right) \int_0^{\bar{\epsilon}_{t+1}^i} \Pi_{t+1}^i(\epsilon')\, dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \Pi_{t+1}^{-i}(\epsilon')\, dH(\epsilon')\right]$$

Since ϵ is additive and iid, $\Pi_t^i(\epsilon) = \Pi_t^i(0) - \epsilon$ and by definition (in the case of an interior solution which we will check)

$$\Pi_t^i(0) = \bar{\epsilon}_t^i$$

Thus we need only keep track of the two paths of the two thresholds $\left\{\bar{\epsilon}_t^g, \bar{\epsilon}_t^b\right\}_t$. Rearranging the Bellman equation, they solve

$$\bar{\epsilon}_t^i = \pi_t^i(0) + \beta\theta^i \left[\left(1-\lambda^i\right) \int_0^{\bar{\epsilon}_{t+1}^i} \left(\bar{\epsilon}_{t+1}^{i,o} - \epsilon'\right) dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \left(\bar{\epsilon}_t^{-i,o} - \epsilon'\right) dH(\epsilon')\right]$$

If H is uniform between 0 and 1, this simplifies to two quadratic equations.

Online Appendix

C Extensions on the Role of Bank Capital

Our paper highlights how an undercapitalized banking sector constrains policymakers, thereby making the economy more fragile in response to fundamental shocks. In our baseline model, we made this point taking the distribution of bank equity as given; we now consider how the distribution of bank equity itself responds to monetary policy, forbearance, and regulation. How do the conclusions reached so far change when banks can choose their capital structure? And if capital is endogenous, can regulators solve the misallocation of credit by forcing banks to raise more capital?

To examine these questions, we enrich our static framework along several successive dimensions. First, we allow banks to issue equity at a cost. Second, we introduce another policy instrument, capital requirements, that allows regulators to force banks to issue more equity than they would on their own. Third, we draw a distinction between legacy lending (rolling over preexisting loans) and new lending. Breaking a lending relationship may force banks to recognize losses that bring them closer to the capital requirement, leading to an additional evergreening motive for zombie lending.

C.1 Endogenous Bank Capital

We first extend the static environment described in Section 3 by allowing for the distribution of bank equity, F, to be endogenously determined in equilibrium and thus potentially respond to the policy variables R^f and p. The main result in this section is that tightening conventional monetary policy can reduce zombie lending, through a bank equity issuance channel.

Specifically, banks start with a pre-issuance equity level e. They then decide simultaneously how much equity they want to issue ($\Delta \geq 0$) and in which asset to invest (type G loans, type B loans, or safe assets):

$$\begin{aligned} \max_{j \in \{g,b,f\},\Delta} \; & \theta^j \left(R^j - \tilde{R}^j \left(1 - e'\right)\right) - \kappa\left(\Delta\right) \\ \text{s.t. } & e' = e + \Delta \end{aligned}$$

where κ is an increasing, convex, differentiable equity issuance cost function. Conditional on choosing project j, the optimal equity issuance is

$$\Delta^j = (\kappa')^{-1}\left(\theta^j \tilde{R}^j\right) \tag{A.1}$$

Accounting for their optimal equity issuance decisions, banks sort themselves into projects j. The optimal equity issuance policy does not depend directly on a bank's pre-issuance equity e because the cost κ is additive. Yet, in equilibrium, the amount of issuance issued by different banks varies with e. Intuitively, e determines banks' asset choices, which in turn affect the optimal equity issuance. Hence risk-shifting acts as a double whammy: banks that start with a lower level of capitalization issue less equity, because they will be the ones lending to relatively riskier borrowers even after issuing more equity. By contrast, banks that start with high capital internalize that they will be the ones lending to safer borrower or even investing in safe assets, and thus have incentives to issue more equity.

As in the baseline model, there is a diabolical sorting: poorly capitalized banks engage in risk shifting and zombie lending. There exist again thresholds e^* and e^{**} such that banks with pre-issuance equity $e < e^*$ lend to a type B borrower, banks with $e^* < e < e^{**}$ lend to a type G borrower, and banks with $e > e^{**}$ do not lend and invest in safe assets. But now the equity thresholds depend on the equity issuance cost (we give expressions in the Appendix), which leads to an important difference with to the baseline model: even conditional on loan rates (i.e., in partial equilibrium), conventional monetary policy can affect the threshold e^*.

Proposition 6. *Given loan rates R^g, R^b, an increase in R^f decreases e^*. An increase in p raises e^* more than without equity issuance.*

A sufficient condition for these comparative statics to also hold in general equilibrium (taking into account the adjustment of loan rates R^g and R^b) is that all banks lend, i.e., $e^{**} > e_{\max}$.

Proposition 6 uncovers a new relationship between zombie lending and conventional monetary policy when equity is endogenous. As previously discussed, when banks cannot choose their leverage—or, equivalently, when equity issuance costs are infinitely high— the level of R^f has no bite on banks' relative returns from lending to good versus bad types of firms. Once equity issuance costs are introduced, however, a reduction in the monetary policy rate R^f increases the threshold e^*, thereby increasing zombie lending. The intuition is that a higher interest rate increases the returns on all assets and therefore encourages banks to issue more equity to take advantage of these higher returns. Our reduced-form formulation in which equity is limited by an issuance cost function κ makes this point particularly stark and simple. More generally, higher interest rates will increase equity issuance if the required return on bank equity does not adjust fully with the risk-free rate, as is the case empirically, so that higher interest rates make the cost of equity relatively lower.[21]

C.2 Capital Requirements and Evergreening

Next, we extend our model to examine the evergreening motive for zombie lending, and understand how it complements the risk-shifting motive that we have focused on so far. A key policy question in the face of prevalent zombie lending is whether tightening capital requirements is a good remedy. Improving the distribution of bank capital appears to be a natural solution to tilt credit allocation towards safer and more productive lending; but the counterargument is that tighter regulation may backfire, by generating incentives for banks to extend and pretend out of fear of having to recapitalize to satisfy the requirement. We propose a novel framework to think about these issues.

Extended model with switching costs. Our baseline model treats old and new borrowers symmetrically: in each period, banks choose which borrower to lend to independently of their previous lending relationships. This simplification abstracts from the empirical finding that weak banks are willing to "extend and pretend", by rolling over cheap loans to *legacy* borrowers that should be declared as non-performing. We now incorporate this complementary driver of zombie lending, by breaking the symmetry between old and new borrowers in a parsimonious way:

Assumption 4. *If a bank switches from its legacy B borrower to a new borrower, its equity falls from e to $e - \delta$, for some switching cost $\delta \geq 0$.*

The presence of a positive switching cost will prolong some borrower-lender relationships that would have been broken under our baseline model, which assumes $\delta = 0$. The switching cost δ captures first and foremost the loss provisions that banks must put aside when declaring loans as non-performing; but δ is also meant to include the screening ef-

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[21] An important consequence is that the endogenous response of banks' capital structure imposes an additional constraint on monetary policy. Moderate interest rates are needed to prevent banks from investing in safe assets instead of lending, as in the baseline model with exogenous equity. But there is a new force: lowering interest rates "too much" makes zombie lending more likely, by deterring equity issuance. Hence achieving potential output Y^* requires, as in Proposition 2, to set p and R^f low enough, together with a novel restriction that the risk-free rate R^f cannot be set too low either. Proposition 9 in the Appendix formalizes this result.

fort that the bank must spend when creating a relationship with a new borrower.[22] Indeed, banks will never want to switch from a legacy B borrower to a new B borrower, so the only switches that could observed in equilibrium are towards a new G borrower. This presumes some costly information gathering to learn which borrowers are indeed good. As our focus is on the effect of switching costs on zombie lending, we only impose the cost on banks matched with legacy B borrowers and assume that switching is costless for all other configurations. For instance, switching from a good legacy borrower to a bad new borrower is costless because there is no need to screen the new borrower, and no loss from liquidating the legacy loan. Our results extend easily to a more general switching cost structure, with costs δ_{ij} depending on both the legacy match i and the new match j.

The distinction between legacy and new borrowers requires us to model some salient aspects of lending relationships. First, we need to determine which outstanding borrower-lender pairs are continued, and which of them are broken so that the bank can lend to a new borrower. Second, we must specify the loan rates offered to legacy borrowers, as those will differ from the rates offered to new borrowers due to the hold-up problem.

We model the renegotiation between banks and legacy borrowers as follows. At the beginning of a period, before the idiosyncratic cost shock ϵ of the borrower is realized, a bank and its legacy borrower choose whether to stay matched or not, and what loan rate $\bar{R}^i$ the legacy borrower must pay to the bank if they do remain matched, as follows:

- *Privately efficient separations:* We assume that continuation and separation decisions are privately efficient from the borrower-lender pair's perspective. The pair separates if and only if the joint surplus of remaining matched is lower than the joint surplus outside the relationship, in which case the bank lends to a new borrower and the borrower seeks to borrow from a new bank. Formally, denote

  $$\Delta S^i(e) = \bar{S}^i(e) - S^i(e)$$

  the difference between the joint surplus inside and outside the relationship, respectively, for a legacy borrower of type i and a bank with capital e. All the surpluses depend on δ, policies, and equilibrium rates, but we leave these dependences implicit. The relationship is broken if and only if $\Delta S^i(e) < 0$. Note that separation may not be *socially* efficient: for instance, the borrower-lender pair ignores the cost of in-

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[22] The efficiency of the debt resolution system affects the cost of insolvencies and the magnitudes of loan loss provisions. Therefore, bankruptcy reforms may alleviate the incidence of zombie lending (Becker and Ivashina, 2021). However, the benefits of such reforms depend on the level of bank capitalization, which determines the strength of banks' zombie-lending incentives (Kulkarni et al., 2021).

surance borne by the government, or the welfare of the new borrower that the bank would have lent to conditional on breaking up.

- *Nash bargaining:* A continuing borrower-lender pair renegotiates the loan rate and splits the surplus according to generalized Nash bargaining, with α denoting the share of the surplus appropriated by the firm and $1-\alpha$ the share accruing to the bank. Therefore, conditional on the relationship remaining in place, that is $\Delta S^i(e) \geq 0$, the legacy rate $\bar{R}^i(e)$ for a borrower of type i matched to a bank with equity e is given by

  $$\bar{R}^i(e) = R^i - \frac{\alpha}{\theta^i}\Delta S^i(e).$$

  Excessively low loan rates are widely used to detect or even define zombie lending empirically since Caballero, Hoshi and Kashyap (2008). This bargaining framework generates exactly this kind of "subsidized rates": intuitively, when loan terms are renegotiated, legacy B borrowers are able to appropriate part of the switching cost that banks economize by continuing the relationship. [23]

Can higher capital requirements prevent zombie lending? Finally, we introduce capital requirements. We build on our equity issuance extension and suppose that in addition, the regulator can impose a capital requirement, which is a floor $\hat{e}$ on *post-issuance* equity

$$e' \geq \hat{e}.$$

Therefore banks' problem becomes

$$\begin{aligned} \max_{j\in\{g,b,f\},\Delta} \; & \theta^j \left(R^j - \tilde{R}^j\left(1-e'\right)\right) - \kappa\left(\Delta\right) \\ \text{s.t. } & e' = e + \Delta \\ & e' \geq \hat{e} \end{aligned}$$

where as before e denotes pre-issuance equity. Our main result in this section is that if switching costs δ are high enough, and capital requirements are already strict, then tightening regulation further can worsen zombie lending through the evergreening channel.

Throughout this section we keep other policies R^f and p fixed (for instance, because

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[23] We explored another complementary explanation for subsidized rates: the default probability is endogenous to loan rates, so particularly low loan rates may be a way to ensure repayment of the zombie loans. We abstract from this explanation as it does not yield significant differences with the case of an exogenous default probability.

the economy has fallen into a policy trap hence these variables cannot adjust anymore) to focus on the effect of capital requirements. It is useful to define

$$\sigma\left(e'\right)=\theta^{g}\left[R^{g}-\tilde{R}^{g}\left(1-e'\right)\right]-\theta^{b}\left[R^{b}-\tilde{R}^{b}\left(1-e'\right)\right]$$

which represents the payoff difference between lending to a G firm and a B firm (ignoring any equity issuance costs) for a bank with post-issuance equity e'. To make things interesting, we restrict attention to parameters such that if the regulator sets a capital requirement low enough that it does not bind even for the bank with the lowest capital $e=e_{\min}$ then that bank prefers to lend to a type-B firm. Formally,

$$\sigma\left(\hat{e}\right)<\kappa\left(\hat{e}-e_{\min}+\delta\right)-\kappa\left(\hat{e}-e_{\min}\right). \tag{A.2}$$

for all $\hat{e}\leq\min\left\{e_{\min}+\Delta^{b},e_{\min}+\Delta^{g}-\delta\right\}$. Condition (A.2) means that there is indeed some zombie lending absent capital requirements. This is the only interesting case to consider, as otherwise capital requirements would be irrelevant for credit allocation and aggregate output, and introducing them would only create a deadweight loss in terms of equity issuance costs.[24]

In the absence of any switching costs ($\delta=0$), it is straightforward to deter zombie lending completely: the regulator can just impose a capital requirement $\hat{e}$ that is sufficiently high, and more precisely, above the equity threshold e^{*} in an equilibrium without zombie lending. Intuitively, the case of low enough switching costs must be similar to when there are no switching costs at all. Indeed, we find that for low enough δ, there always exists a sufficiently tight capital requirement $\hat{e}^{NZ}$ (where NZ stands for No Zombie lending) that suppresses zombie lending altogether.

Proposition 7 (Low switching costs)**.** *Suppose that switching costs are low:* $\delta<\Delta^{g}-\Delta^{b}$ *and let* $\hat{e}^{NZ}$ *solve* $\sigma\left(\hat{e}^{NZ}\right)=\kappa\left(\hat{e}^{NZ}-e_{\min}+\delta\right)-\kappa\left(\hat{e}^{NZ}-e_{\min}\right)$*. Then any capital requirement above* $\hat{e}^{NZ}$ *can suppress zombie lending* ($m^{b}=0$)*.*

Does it mean that we can always solve the zombie lending problem using capital regulation? We find that the answer is no. Surprisingly, when the switching cost δ is high enough, no capital requirement can deter zombie lending completely: some positive equi-

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[24] This is true because we consider a static model. In a dynamic setting, capital requirements could matter for future credit allocation even if they do not bind in the present. This is one rationale behind precautionary cyclical capital requirements.

librium zombie lending is inevitable. In fact, the stronger result is that increasing capital requirements beyond some level can even backfire, by further encouraging zombie lending:

Proposition 8 (High switching costs: Evergreening)**.** *Suppose that switching costs are high:* $\delta > \Delta^g - \Delta^b$*. Then zombie lending is minimized by setting the capital requirement*

$$\hat{e} = 1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}$$

and increasing capital requirements above that level strictly increases zombie lending. No capital requirement can suppress zombie lending.

Proposition 8 captures the evergreening motive of zombie lending. The intuition is as follows. A bank compares two options: recognizing the loss at a cost δ, which allows a fresh start with a new G borrower, or rolling over the loan to the legacy B borrower. The second option allows to economize the switching cost δ, and becomes especially attractive with a high δ. Switching to a new borrower brings an additional cost if the bank is already poorly capitalized: its equity will drop to $e - \delta$, which forces the bank to undertake a costly recapitalization to satisfy the requirement $\hat{e}$. Thus there is a set of banks for which the cost of recapitalization acts as an additional motive to roll over the zombie loan, and the set of such banks expands as the capital requirement $\hat{e}$ increases.

Together, Propositions 7 and 8 highlight a subtle link between capital requirements and zombie lending. In particular, both cases are likely to be relevant because the "switching cost" δ and the threshold $\bar{\delta}$ depend on the country and industry of the borrower, and the history of the lending relationship. For instance, δ will be higher when there is more asymmetric information between banks and potential new borrowers, and when zombie debt has been accumulating for a longer time (as this increases the losses that banks would eventually recognize). Just like in the dynamic model, the longer policymakers wait before tackling the zombie lending problem, the harder it becomes to solve it. The case of high switching costs in Proposition 8 is consistent with some of the empirical evidence on the unintended consequences of capital requirements, for instance following the increase in capital requirements by the European Banking Authority in 2011, as documented by Blattner, Farinha and Rebelo (2020). Relatedly, Chopra, Subramanian and Tantri (2020) show that other regulatory actions such as ex-post bank cleanups can also trigger zombie lending if they are not accompanied by ex-ante bank recapitalization.

C.3 Proofs

Proof of Proposition 6. Following the same steps as without equity issuance costs we find:

$$e^{*} = 1 - \frac{\theta^{g} R^{g} - \theta^{b} R^{b}}{(\theta^{g} - \theta^{b})p} - \frac{\varphi\left(\theta^{g} \tilde{R}^{g}\right) - \varphi\left(\theta^{b} \tilde{R}^{b}\right)}{\theta^{g} \tilde{R}^{g} - \theta^{b} \tilde{R}^{b}}$$

$$e^{**} = 1 - \frac{R^{f} - \theta^{g} R^{g}}{p\left(1 - \theta^{g}\right)} - \frac{\varphi\left(R^{f}\right) - \varphi\left(\theta^{g} \tilde{R}^{g}\right)}{R^{f} - \theta^{g} \tilde{R}^{g}}$$

where $\varphi(x) = x\left(\kappa'\right)^{-1}(x) - \kappa\left(\left(\kappa'\right)^{-1}(x)\right)$. The function φ inherits the properties of κ, as $\varphi'(x) = \left(\kappa'\right)^{-1}(x)$ and $\varphi''(x) = \frac{1}{\kappa''\left(\left(\kappa'\right)^{-1}(x)\right)}$. Since $\theta^{g} \tilde{R}^{g} - \theta^{b} \tilde{R}^{b} = (\theta^{g} - \theta^{b})p > 0$, it follows from the convexity of φ that the slope of $\frac{\varphi\left(\theta^{g} \tilde{R}^{g}\right) - \varphi\left(\theta^{b} \tilde{R}^{b}\right)}{\theta^{g} \tilde{R}^{g} - \theta^{b} \tilde{R}^{b}}$ is increasing with R^{f} and (decreasing with p).

The following result generalizes Proposition 2 and characterizes the optimal policy, in the case of quadratic equity issuance costs $\kappa(x) = \frac{1}{a}\frac{x^{2}}{2}$ that allow for closed-form solutions:

Proposition 9 (Optimal policy with equity issuance). *Output reaches its potential ($Y = Y^{*}$) if and only if*

$$\underline{R}^{f}(p) \leq R^{f} \leq \bar{R}^{f}(p)$$

and

$$p \leq \bar{p}$$

where $\underline{R}^{f}(p)$ and $\bar{R}^{f}(p)$ are given in the Appendix.

The limit case $a \longrightarrow 0$ recovers the no-issuance benchmark from Proposition 2. Under quadratic issuance costs, the optimal policy is characterized by the thresholds

$$\underline{R}^{f}(p) = p\left(1 - \frac{\theta^{g} + \theta^{b}}{2}\right) - \frac{1}{a}\left(1 - e_{\min}\right)\left[\frac{\bar{p}}{p} - 1\right]$$

$$\bar{R}^{f}(p) = \frac{1}{1 + ap\left(1 - \theta^{g}\right)} \bar{R}^{f}_{\text{no issuance}}(p) + \frac{ap^{2}\left(1 - \theta^{g}\right)^{2}}{2\left(1 + ap\left(1 - \theta^{g}\right)\right)}$$

and $\bar{p}$ and $\bar{R}^{f}_{\text{no issuance}}(p)$ are as defined in Proposition 2.

Proof of Proposition 9. Banks choose borrower type based on their post-issuance equity $e' = e + \Delta e$. Define the function $\varphi(x) = x(\kappa')^{-1}(x) - \kappa\left((\kappa')^{-1}(x)\right)$. There are two cases to consider:

Case 1. A bank with pre-issuance equity e prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g\left(R^g - \tilde{R}^g(1-e-\Delta^g)\right) - \kappa(\Delta^g) \geq \theta^b\left(R^b - \tilde{R}^b\left(1-e-\Delta^b\right)\right) - \kappa\left(\Delta^b\right)$$

which can be rewritten as

$$e > e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}.$$

Case 2. A bank with pre-issuance equity e prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f\left(e + \Delta^f\right) - \kappa\left(\Delta^f\right) \geq \theta^g\left(R^g - \tilde{R}^g(1-e-\Delta^g)\right) - \kappa(\Delta^g)$$

which can be rewritten as

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{R^f - \theta^g \tilde{R}^g} - \frac{\varphi\left(R^f\right) - \varphi\left(\theta^g \tilde{R}^g\right)}{R^f - \theta^g \tilde{R}^g}.$$

Proof of Proposition 7. When $\delta > \Delta^g - \Delta^b$, there are three relevant regions for banks initially matched with a bad firm. If $e < \hat{e} - \Delta^b$, then the capital requirement is binding even if the bank remains with its legacy B borrower. If $e > \hat{e} - \Delta^g + \delta$, the capital requirement is never binding, whether the bank switches or not. For intermediate equity $e \in \left[\hat{e} - \Delta^b, \hat{e} - \Delta^g + \delta\right]$, the capital requirement is binding only if the bank switches.

We start with the banks matched to a borrower that turns B.

1. Suppose that $\hat{e}$ is high enough that the bank $e = \hat{e} - \Delta^b$ prefers to switch to a new G borrower and thus issue $\hat{e} - e - \delta = \Delta^b + \delta$, that is

$$\sigma(\hat{e}) \geq \kappa\left(\Delta^b + \delta\right) - \kappa\left(\Delta^b\right) \tag{A.3}$$

or

$$\delta \leq \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

Therefore, all the banks above $e = \hat{e} - \Delta^b$ will prefer to switch, and the only potential for zombie lending is for banks below $\hat{e} - \Delta^b$. In that case, banks lending to zombies are those with pre-issuance equity e below the indifference threshold e^* solving

$$\sigma(\hat{e}) = \kappa(\hat{e} - e^* + \delta) - \kappa(\hat{e} - e^*)$$

Note that $\sigma(\hat{e}) > 0$ implies $\hat{e} > E^*$. From the implicit function theorem, when $\hat{e}$ increases (holding loan rates fixed in this partial equilibrium first step) we have

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma'(\hat{e})}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)} = 1 - \frac{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

This can be rewritten as

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\kappa'(\Delta^g) - \kappa'\left(\Delta^b\right)}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

Since $\delta > \Delta^g - \Delta^b$ and $\hat{e} - e^* \geq \Delta^b$, we necessarily have

$$\frac{\partial e^*}{\partial \hat{e}} > 0$$

and thus in this region, increasing capital requirements worsens legacy zombie lending.

(a) Suppose then that (A.3) doesn't hold:

$$\delta > \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

which implies that the bank with $e = \hat{e} - \Delta^b$ prefers to stay matched with its legacy B borrower.

   i. If the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to switch to a new G borrower, that is

$$\sigma(\hat{e}) > \kappa(\Delta^g) - \kappa\left(\Delta^b\right) + \theta^b \tilde{R}^b\left(\delta - \Delta^g + \Delta^b\right) \tag{A.4}$$

   holds, then all banks with even higher e also switch. Thus the indifference threshold e^* is in the intermediate region $\left[\hat{e} - \Delta^b, \hat{e} - \Delta^g + \delta\right]$ and solves

$$\theta^b\left[R^b - \tilde{R}^b\left(1 - e^* - \Delta^b\right)\right] - \kappa\left(\Delta^b\right) = \theta^g\left[R^g - \tilde{R}^g(1 - \hat{e})\right] - \kappa(\hat{e} - e^* + \delta)$$

or

$$\sigma(\hat{e}) = \theta^b \tilde{R}^b \left(e^* - \hat{e} + \Delta^b\right) + \kappa(\hat{e} - e^* + \delta) - \kappa\left(\Delta^b\right)$$

By the implicit function theorem,

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma'(\hat{e})}{\kappa'(\hat{e} - e + \delta) - \theta^b \tilde{R}^b} = \frac{\kappa'(\hat{e} - e + \delta) - \theta^g \tilde{R}^g}{\kappa'(\hat{e} - e + \delta) - \theta^b \tilde{R}^b} > 0$$

which follows from $\hat{e} - e + \delta \geq \Delta^g > \Delta^b$. Therefore, in this region as well, increasing capital requirements worsens legacy zombie lending.

ii. The last case is when $\hat{e}$ is so low that even the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to lend to its legacy B borrower, that is

$$\sigma(\hat{e}) < \kappa(\Delta^g) - \kappa\left(\Delta^b\right) + \theta^b \tilde{R}^b \left(\delta - \Delta^g + \Delta^b\right) \tag{A.5}$$

holds, and so all the banks with lower equity also rollover the B loan. Then the indifference threshold e^* is above $\hat{e} - \Delta^g + \delta$ and is the same as in the absence of a capital requirement:

$$e^* = \underbrace{1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}}_{E^*} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} + \frac{\theta^g \tilde{R}^g}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}\delta$$

so does not vary with $\hat{e}$. Low enough capital requirements become irrelevant for legacy zombie lending.

For banks matched with a good firm, since we abstract from switching costs δ, they will switch to a new zombie borrower if their post-issuance equity is below

$$E^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$$

hence capital requirements have a knife-edge effect: either $\hat{e} \leq E^*$ and the capital requirement is irrelevant, or $\hat{e} \geq E^*$ and the capital requirement prevents all these banks (matched with a G firm) from switching to a new B borrower. Since we just showed that increasing $\hat{e}$ can never decrease legacy zombie lending, the only potential benefit is to prevent “new” zombie lending.

Next, note that the point $\hat{e}$ such that (A.5) holds with equality, that is

$$\hat{e} = E^{*} + \Delta^{g} - \frac{\varphi\left(\theta^{g}\tilde{R}^{g}\right) - \varphi\left(\theta^{b}\tilde{R}^{b}\right)}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}} + \frac{\theta^{b}\tilde{R}^{b}}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}}\delta$$

is strictly above E^{*} since $\sigma\left(\hat{e}\right) = \kappa\left(\Delta^{g}\right) - \kappa\left(\Delta^{b}\right) + \theta^{b}\tilde{R}^{b}\left(\delta - \Delta^{g} + \Delta^{b}\right) > 0 = \sigma\left(E^{*}\right)$.