On the effectiveness of climate policies

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February 24, 2020

Abstract

We present an quantitative integrated assessment model (IAM) designed as a dynamic, multi-region general-equilibrium model coupled with climate and carbon-cycle modules. The energy input into production comes from an array of different sources, including those not based on fossil fuel. The IAM setup is aimed toward policy evaluation, with a focus on policies that are (i) not necessarily optimal and (ii) potentially different quantitatively and qualitatively across regions. We conduct three key exercises. We first compare policies that have the right design—global carbon taxes—but the wrong magnitude: a tax that is set based on worries about climate change that *ex post* turn out to be overly pessimistic and a tax based on the reverse mistake (an optimistic view that turns out to vastly understate the climate challenge ex post). We find a sharp asymmetry: the former is not very costly at all to human welfare whereas the latter is very costly. Second, we examine taxes that differ significantly by region and discuss the cost of implementing them instead of an optimal—uniform—scheme; here we record welfare costs that potentially are very high. Third, we look at efforts to promote green energy—a suboptimal policy in isolation—and argue that reliance on such efforts is highly hazardous. In addition to addressing these policy issues, we show that the model, which is rather tractable, can be extended in interesting directions.

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The present paper circulated in an earlier version with a different name ("Integrated Assessment in a Multi-region World with Multiple Energy Sources and Endogenous Technical Change").

1 Introduction

According to IPCC (2013), it is extremely likely that human activity is the dominant cause of the observed roughly 1 degree global warming since the mid-20th century. By bringing about more frequent and severe extreme-weather events, sea level rise, and impacts on ecosystems, climate change will affect the everyday life of substantial amounts of people around the world; in many regions, it will affect food and water security, human lives and health, and productivity. The effects on the economy are complex, long lasting, and regionally very different, but the estimates suggest that the effects are negative on average: global warming causes damages.[1]

These facts are what we consider to be a scientific consensus, spanning natural and social sciences. Moreover, it is clear that the human activity leading to global warming—carbon emissions—takes the form of a pure externality. For such occurrences, there is a straightforward recipe in economic textbooks: tax the activity that causes the damage by an amount equal to the externality damage, and "efficiency" would be obtained: human welfare (somehow weighted across time and space) would be maximized subject to the restrictions nature places on us (i.e., our resource constraints).

There is significant uncertainty on the natural-science mechanisms as well as on the economic transmissions involved, and the weighting across people is a philosophical decision, making it hard to be very precise on a quantitative recommendation, i.e., a level for the tax rate on carbon emissions. However, the principle is clear. What, then, remains to be done for economists, aside from providing better precision on the side of damage estimates? This question motivates our paper. In fact, we suspect that many leading economists view the climate-change problem as a trivial one conceptually: we know how to solve it, and therefore the role for high-level research is limited. Perhaps this is what lies behind the observation recently made by Oswald and Stern (2019): economists, they argue, have been largely missing from this area of research, especially as represented by publications in our top journals. Our message in this paper is an optimistic one from this perspective: we argue that there are important, and highly nontrivial, research questions at least within the area that where climate change overlaps with macroeconomic analysis.

The key idea we emphasize here is that whereas we all know how to find the optimum—we apply Pigou (1920)—we have so far done very little in terms of providing quantitative comparisons between various suboptimal policies. What we offer here is precisely a frame-

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[1] IPCC (2018) and Dell, Jones, and Olken (2012).

work for this kind of analysis. We think such a framework is urgently needed, as we are currently very far from an implementation of the optimal global policy package: the global tax on carbon is negative on average (due largely to large subsidies to coal-generated energy in many parts of the world), not possible, and taxes differ very widely across regions. More to the point, it unfortunately appears unrealistic to jump straight to the best policy any time soon, so in order to move forward, we need to provide policymakers with quantitative guidance in comparing different possible paths forward for policies around the world. And such comparisons require solidly tested—empirically grounded, based on historical economic and climate data—integrated assessment models (IAMs) allowing us to assess welfare outcomes (over time and space) as functions of these different policy combinations. The economics is key here: this is where our expertise is both unique and sorely needed. Natural scientists may, for example, have views on what outcomes for emission paths are desirable, but they do not have methods for computing the costs of arriving at such paths by different means: this requires an understanding of how markets work and, in particular, how they respond to policy.

The construction of IAMs of the sort needed was pioneered by Nordhaus and we follow in his footsteps. In particular, we build further on Golosov et al. (2014), which proposes a quantitative model very much like Nordhaus's DICE and RICE models but emphasizes dynamic general equilibrium analysis, in a number of directions allowing us to conduct multiregional analysis of policy heterogeneity. The specific setting is one where the world consists of one oil-producing region and arbitrarily many oil-importing regions. Each oil-importing region features a representative firm that employs capital, labor, and an energy composite consisting of an array of imperfectly substitutable energy sources to produce final output. One of the energy sources is oil that is sold on a world market, and the remaining sources are all produced regionally. Finally, each oil-importing region features a government that governs the climate policy within that region. As all IAM:s, our model also contains representations of the climate as well as a carbon cycle.

Despite this rich model structure, all variables except the oil price have closed-form solutions, which allows the analysis to be highly tractable and transparent. It is important to stress, however, that this tractability does not mean that the model is less suitable for quantitative analysis: it is a straightforward extension of the neoclassical growth model (in its market implementation) and hence consistent with long-run macroeconomic facts. We run the model in Matlab, but it also solves easily in Excel in under a second.

In the calibration, we allow for eight regions: Europe, U.S., China, India, Africa, South

America, Oceania, and an oil producer. Our considered energy inputs include conventional oil, coal, green (renewable) energy, and the output from hydraulic fracturing ("fracking"), which is highly substitutable with conventional oil.[2] The costs of producing coal and green energy are allowed to differ across regions and we acknowledge that China subsidizes fossil-fuel use very heavily. The model is then calibrated with regional TFP levels to match current world emissions as well as each region's share of world emissions and GDP.

We use the model to address three sets of specific questions. The first departures from the insight that that any climate policy today has to be implemented under substantial imperfect knowledge about both the climate system and the economic damages. We specifically ask how large the respective costs are from implementing climate policies that *ex post*, i.e., when the uncertainties have been resolved, turn out to have been too ambitions or too lax.

To answer this question we focus on the imperfect knowledge that characterizes two sets of crucial parameters: the climate sensitivity and the parameters that determines the regional damages as functions of the global temperature. The climate sensitivity quantifies how much the temperature increases from a doubling of the concentration of CO_2 in the atmosphere, and IPCC (2013) reports a wide range of values for this parameter. Regarding the economic damages, we rely on the recent meta-study by Nordhaus and Moffat (2017). Rather than focusing on the uncertainty surrounding these parameters, we focus on the extremes as given by as upper and lower bounds of intervals given in the literature.

Our results show a substantial asymmetry between the costs of policies that *ex post* turn out to have been too lax and too ambitious: the costs of underestimating climate change are roughly one order of magnitude larger than the costs of overestimating it. For Africa and India, the consumption losses—relative to a case with an optimal tax—from underestimation increase from about 1% per year to a colossal 6% per year up to the year 2100. The losses from too ambitious a policy, however, remain below 0.6% into the far future. The results are qualitatively similar for other regions, except for the U.S., where the costs of overestimation are larger. The reason for the higher costs in the U.S. is that the higher tax hurts the fracking industry significantly, leading to a substantially higher energy price in the United States.

The intuition for the limited costs of overestimating climate change is first and foremost that they arise as a result of a policy that has the right design (a uniform global carbon tax) but simply is set too high. I.e., as long as a smart policy tool is used, it is not so costly to over-employ it. Second, we find that two substitution elasticities are key. The

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[2] Since fracking is virtually non-existing in most of the considered regions, we only allow fracking in the United States.

different energy sources are far from perfectly substitutable, but according the the estimates in the literature it seems possible for green energy to replace a large part of the reduction in coal that the high tax is generating. Furthermore, the evidence points to energy as relatively substitutable with capital and labor over the medium run at least—which is the time horizon our model focuses on.[3] Hence, any reduction in the supply of energy from the tax can also be compensated by increases in capital without too high a cost. Thus, our overall conclusion from the first policy experiment is that an ambitions climate policy is a cheap insurance against possibly very high damages from climate change.

Our second policy question concerns the design of the policy. In particular, an efficient climate policy requires the marginal cost of emission reductions to be the same everywhere—across sectors and regions of the world. Many departures from this principle can be considered and here we consider one: when some countries/regions do not contribute by taxing emissions, then, what are the costs of achieving a given desired total reduction in the temperature (compared to an optimal, uniform tax design)? We can, in particular, use our IAM to compare economic welfare under an optimal policy to that when some regions have zero carbon taxes. We consider two cases: one where Africa and India do not implement the carbon tax and one where China does not tax carbon. The results show that the costs of achieving a certain temperature increase substantially when some regions do not participate. The NPV loss in world consumption is about 4% when Africa and India do not participate and about 7% when China does not participate. The results also show that while Africa and India do not gain substantially from being exempt from the tax, the NPV increase in Chinese consumption is considerable at about 15% from not having the tax. In both cases, the U.S. is substantially worse off from the less cost-efficient tax, again because the higher tax specifically hurts the fracking industry and raises the price of energy in the United States.

The third question that we address relates to the relative prices of green and fossil fuels. We know that an appropriately set carbon tax corrects the market failure by increasing the price of fossil fuel, which also would induce firms and households to substitute green for fossil energy. The question we ask here is: what if policy instead were used directly to re-orient R&D toward green energy? Acemoglu et al. (2012) argue precisely that even a temporary subsidy to green energy could be very potent under some conditions. What we do in this case is use our calibrated IAM to compare scenarios that differ in the future productivity developments within the green and fossil sectors. In brief, we conclude that what is key is

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[3] We argue that unitary elasticity, which is entirely unrealistic at an annual frequency, is a reasonable assumption at medium-run frequencies.

not to make green energy cheaper but to make fossil fuel, in particular coal, more expensive. Given the increasingly popularity of subsidies to green energy around the world (and in the EU in particular), this result is rather shocking. Improvements in green energy are good, to be sure, because they allow us to use more energy in total. However, they are strikingly impotent at preventing climate change.

More precisely, we find that a policy that makes the relative price of green energy fall 2% faster per year has close to indistinguishable effects on global temperature relative to having implemented no policy at all. An important parameter influencing this result is the elasticity of substitution between fossil and green energy, which we calibrate to match the empirical meta-estimate of 0.95, as reported in Stern (2012). With green and fossil energy sources being gross complements, a lower price of green translates into a lower price for energy in general. Cheaper energy then induces increased consumption also of other energy sources, such as coal, which results in higher temperature increases and more climate damages. Policies that make green energy cheaper are therefore not only ineffective in mitigating global warming but, in fact, marginally worse than the complete absence of policy.

We also consider higher values for the elasticity of substitution between green and fossil energy sources since, after all, it is possible that the available estimates do not factor in the nature of future technical change. Policies that make green energy cheaper are then no longer worse than the absence of policy, but for what we consider high but imaginable values of the elasticity, they are insufficient in mitigating climate change. Only with elasticities that are more than five times higher than empirical estimates can a policy that makes green energy cheaper reduce global warming sufficiently to make an important difference. We supplement these insights with an experiment where fossil energy has stagnant technology growth relative to the rest of the economy: there is no technological change in fossil production and, hence, fossil energy prices rise by two percent per year. We show that this “intervention” is similar to the use of a sizable tax on carbon, i.e., it reduces climate change to an important extent. That is, to the extent that subsidies to green energy can redirect R&D resources so that the fossil sector experiences stagnant productivity, such subsidies would be valuable, but only then: boosting green technology per se does not solve the climate problem. We are skeptical that mere subsidies to green could cause stagnation in the fossil sector, but cannot rule it out. However, to merely hope that this is the case appears very unwise, at least to compare to the direct approach of taxing the fossil sector.

It should be noted that it is clear from throughout our analysis that what is key for mitigating global warming is a tax on coal; whether a global carbon tax is applied to conventional oil or not is, actually, immaterial. [4] It is, however, critical that an effective carbon tax also is implemented on the non-conventional oil that is produced with the fracking technology. The reason is that fracking has the potential to dramatically increase the stock of nonconventional oil, which may generate substantial additional global warming.

We finally point to interesting potential model extensions. One is to introduce endogenous technical change explicitly: energy-producing firms can invest to lower the marginal cost of producing one form of energy at the expense of the marginal costs of other energy sources. This allows for an analysis of how energy taxes interact with cost-reducing technical change. The results from this exercise show that taxes that are proportional to the price of a specific energy good are completely impotent in reducing the demand for this good. The intuition for this potentially surprising result comes from the fact that a proportional tax generates two effects with different signs. On the one hand, a proportional tax increases the marginal value of cost reductions. On the other, the proportional tax reduces the demand for the energy good, which also reduces the marginal value of cost reductions. In our setting, these two effects exactly balances. Research and development, thus, effectively nullifies the effects of taxes on the after-tax price.

Even though proportional taxes are ineffective in dealing with global warming, this is not true for per-unit taxes (i.e., taxes per ton of the energy source). With per-unit taxes, endogenous technical change actually reinforces the effectiveness of carbon taxes.

The paper is organized as follows: Section 2 sets up the basic model with exogenous growth. Section 3 describes calibration of the model, and the benchmark results are then presented in Section 4. In Section 5, the direction of technical change is endogenized and Section 6, finally, discusses the main assumptions and concludes.

2 The model

2.1 The economy

The world consists of r regions. Region 1 features a representative oil producer that is endowed with a finite amount of conventional oil that it extracts and sells to the rest of the world in a competitive manner. This region will be referred to as the *oil producer*. The remaining $r-1$ regions have no endowments of conventional oil. Instead, they import oil from

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[4] In contrast to a coal tax, however, an tax on conventional oil has large consequences for the distribution of income across regions.

the oil producer and pay for this import with a common final good that is identical in all countries. These regions share a number of features but are different in sizes, productivity, and initial capital stocks. We will refer to them as *oil consumers.* There are are no international capital markets. The absence of capital markets is a simplification but likely not a major one: if the model is calibrated such that marginal products of capital are similar at time 0—as it will be here—and preferences are the same—which we assume they are—then the long-run marginal products of capital will also be the same. Hence, at these points in time there is no need for capital to move across regions. During the transition, the marginal products will in general differ across regions, but these differences are not major.[5]

Each region $i \in \{1, 2, ..., r\}$ features a representative consumer with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log(C_{i,t}). \tag{1}$$

2.1.1 Oil consumers

Each oil-consuming region has access to an aggregate production function for the final good, $Y_{i,t}$, given by

$$Y_{i,t} = A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu} \tag{2}$$

where A is total factor productivity, L is labor used in final-good production, K is the capital stock, and E denotes energy services. In Hassler, Krusell, and Olovsson (2017), we discuss the nature of the aggregate production function in capital/labor and energy and find that unitary elasticity is not unreasonable for periods as long as ten years, our calibration target here.

Energy services in each region are produced by local competitive firms that combine different energy sources as inputs. One energy input is the oil that is imported from the oil-producing region, whereas the other energy sources are all produced locally. We consider $n+l$ different energy inputs, and we allow the elasticity of substitution to differ between some energy inputs. Specifically, there are n different imperfectly substitutable energy inputs that are available to all regions. In recent years, however, hydraulic fracturing—or fracking—has made it possible to produce unconventional oil and gas in substantial quantities in the United States, and potentially in other regions. This motivates the inclusion of l additional unconventional fossil fuels that are highly substitutable with conventional oil.

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[5] In Krusell and Smith (2020) a similar setting is examined under both intertemporal trade across regions and autarky and very small differences are found there.

In regions that engage in fracking, the supply of oil is a composite of the imported conventional oil and the l locally produced unconventional substitutes, i.e., the outputs from fracking. Denoting conventional oil used by region i in period t by $e_{1,i,t}$, and the l different variants of unconventional oil by e_{n+1}, e_{n+2},...,e_{n+l}, respectively, the oil composite is given by

$$O_{i,t} \equiv \widetilde{l} \left(\lambda_1^{oil} e_{1,i,t}^{\rho_h} + \sum_{j=1}^{l} \lambda_{j+1}^{oil} e_{n+j,i,t}^{\rho_h} \right)^{\frac{1}{\rho_h}}, \tag{3}$$

where ρ_h determines the the elasticity of subsitution between the inputs, $\sum_{j=1}^{l+1} \lambda_j^{oil} = 1$, and $\widetilde{l} \equiv 1 + l$. In regions that do not engage in fracking, $O_{i,t}$ is just given by $e_{1,i,t}$.

The total amount of energy services in region i in period t is then an aggregate of the oil and the $n-1$ remaining energy inputs

$$E_{i,t} \equiv \mathcal{E}(O_{i,t}, e_{2,i,t}, \ldots, e_{n,i,t}) = \left(\lambda_1 O_{i,t}^{\rho} + \sum_{\kappa=2}^{n} \lambda_\kappa e_{\kappa,i,t}^{\rho} \right)^{\frac{1}{\rho}}, \tag{4}$$

where ρ determines the elasticity of subsitution, and $\sum_{k=1}^{n} \lambda_k = 1$.

Except for conventional oil, all energy sources are locally produced with a production technology that is linear in the final good; this assumption is identical to one where there are explicit energy-source production sectors, between which all production factors can be moved freely within the period, that all have the same isoquants and only differ in TFP levels. Specifically, $p_{k,i,t}$ units of the final good are required to produce $e_{k,i,t}$ units of energy source $k \in \{2, ..., n+l\}$ in region i and period t. In the benchmark specification of the model, the productivity in energy production is exogenous, but it is made endogenous in Section 5.

Final goods that are not used for energy production are consumed or invested, and capital is, for analytical tractability, assumed to fully depreciate between periods. In business-cycle models, where a time period is a quarter or a year, this is a wholly inappropriate assumption, but in models where a period is ten years or even longer, this assumption is not so unreasonable and we do not think that considering a depreciation rate $\delta < 1$ would materially change our findings. The resource constraint for the final good is then given by

$$C_{i,t} + K_{i,t+1} = A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu} - p_{1,t} e_{1,i,t} - \sum_{k=2}^{n+l} p_{k,i,t} e_{k,i,t}, \tag{5}$$

where $p_{1,t}$ denotes the world market price for conventional oil. The assumption of an identical final good allows us to express this price in terms of the global final good.

2.1.2 Oil producers

Region 1 extracts oil without any resource cost, and the total stock of oil in the ground at time t has size R_t. With extraction in period t given by $\sum_{i=2}^{r} e_{1,i,t}$, the law of motion for the stock of oil is given by

$$R_{t+1} = R_t - \sum_{i=2}^{r} e_{1,i,t}, \text{ s.t. } R_t \geq 0, \forall t. \tag{6}$$

Since the oil producer derives all its income from oil, its budget/resource constraint is given by

$$C_{1,t} = p_{1,t} \left(R_t - R_{t+1} \right). \tag{7}$$

A key assumption that was mentioned above and that is implicit in this equation is that the oil producer cannot invest the proceeds from oil sales abroad; it also cannot store oil. These assumptions are clearly not met in the data but greatly simplify the analysis, as we shall see shortly.

2.2 Carbon circulation

Usage of fossil energy sources generates CO_2 emissions. The parameter g_j measures how dirty energy source j is. Because we will measure fossil energy sources by their carbon content, the fossil energy sources all have $g_j = 1$. Purely green energy sources, in contrast, have $g_j = 0$.[6] Total emissions from region i in period t are then given by

$$M_{i,t} = \sum_{j=1}^{n+l} g_j e_{j,i,t}.$$

Following Golosov et al. (2014), the law of motion for the atmospheric excess stock of carbon S_t is given by

$$S_t = \sum_{s=0}^{\infty} (1 - d_s) \sum_{i=2}^{r} M_{i,t-s},$$

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[6] Allowing intermediate cases, i.e., emissions from non-fossil energy sources is straightforward.

where

$$1 - d_s = \varphi_L + (1 - \varphi_L)\, \varphi_0\, (1 - \varphi)^s$$

measures carbon depreciation from the atmosphere.

Specifically, the share of emissions that remains in the atmosphere forever is φ_L, the share that leaves the atmosphere within a period is $1 - \varphi_0$, and the remainder $(1 - \varphi_L)\, \varphi_0$ depreciates geometrically at rate φ.

2.3 Climate and damages

The climate is affected by the concentration of CO_2 in the atmosphere via the greenhouse effect. Golosov et al. (2014) gives arguments to the effect that the effect of the CO_2 concentration on productivity is well captured by a log-linear specification. We therefore assume that

$$A_{i,t} = \exp\left(z_{i,t} - \gamma_{i,t} S_{t-1}\right), \tag{8}$$

where $z_{i,t}$ is a potentially stochastic productivity trend, and $\gamma_{i,t}$ is a region-specific and possibly time-varying parameter that determines how climate-related damages depend on the level of the atmospheric CO_2 concentration. Note that this specification implies that the marginal damage per unit of excess carbon in the atmosphere is a constant share of net-of-damage output given by the parameter $\gamma_{i,t.}$. The value $\gamma_{i,t}$ is positively affected by i) the sensitivity of the global mean temperature to changes in the CO_2 concentration, ii) the sensitivity of the regional climate to global mean temperature, and iii) the sensitivity of the regional economy to climate change.

The specification in (8) implies that climate damages materialize with a lag of one period. Given that climate change is a slow-moving process and the evolution of the atmospheric CO_2-concentration is sluggish, this is immaterial for the dynamics of climate damages. As we will see below, however, the computation of the equilibrium allocation is much simplified by the introduction of this lag.

The climate system follows the energy budget model in DICE/RICE:

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left(\frac{\eta}{\ln 2} \ln\left(\frac{S_{t-1}}{S_0}\right) - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T^L_{t-1}\right) \right) \\ T^L_t &= T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1}\right) \end{aligned}$$

where T_t is the global mean temperature in the atmosphere (and upper layers of the oceans) and T_t^L is the mean temperature in the deep oceans. Both these temperatures are measured as deviations from their pre-industrial levels. Note that temperature does not appear in the model equations anywhere but here; hence, the are solved for residually.

2.4 Governments

A key purpose of the analysis is to analyze the consequences of climate policies that are not necessarily optimal and potentially may be quantitatively and qualitatively different across regions. To this end, we allow each oil-consuming region to set a carbon tax, $\tau_{i,t}$, and we consider two cases: *ad valorem* taxes and per-unit taxes. With *ad valorem* taxes, the cost for the energy-service provider of using energy source j in region i is $(1+\tau_{i,t}g_j)\,p_{j,i,t}$, whereas it is $\tau_{i,t}g_j + p_{\kappa,i,t}$ with per-unit taxes. For simplicity, tax revenues are recycled back to the household within the period in the form of a negative income tax rate, $\Gamma_{i,t}$, applying to total household income $w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$.[7] The government budget constraint is then given by

$$\Gamma_{i,t}\left(w_{i,t}L_{i,t} + r_{i,t}K_{i,t}\right) = \tau_{i,t}\sum_{\kappa=1}^{n+l} g_\kappa e_{\kappa,i,t} p_{\kappa,i,t}$$

with *ad valorem* taxes, and

$$\Gamma_{i,t}\left(w_{i,t}L_{i,t} + r_{i,t}K_{i,t}\right) = \tau_{i,t}\sum_{\kappa=1}^{n+l} g_\kappa e_{\kappa,i,t}$$

with per-unit taxes.

2.5 Markets and equilibrium

All agents are price takers and markets within regions are assumed to be perfect and complete.

2.5.1 The oil-producing region

The problem for the representative oil producer is to choose how much oil to keep in the ground for next period, R_{t+1}, while taking the world market price of oil as given. Substituting

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[7] If taxes were paid back as lump-sum transfers, the model would be somewhat more difficult to solve. Given that the potential revenues are very small and, hence, the implied tax rate is small, this assumption will not be of quantitative importance.

(7) into (1) and taking the first-order condition with respect to R_{t+1} yields

$$\frac{1}{R_t - R_{t+1}} = \frac{\beta}{R_{t+1} - R_{t+2}},$$

The above equation delivers $R_{t+1} = \beta R_t$, which implies a constant depletion rate. Consumption of the oil producer is then given by $C_{1,t} = p_{1,t}\left(1-\beta\right)R_t$.[8]

Our setting, then, delivers a path of oil use that is entirely supply-determined. This is convenient for solving the model, but it also means that the price of oil does not need to satisfy a typical Hotelling equation, which in this case would require the price of oil to rise at the rate of interest (prevailing in the oil-consuming regions). Instead, the price of oil will be demand-determined entirely: given the supply of oil that the oil producers choose, higher or lower demand for oil would directly be reflected in prices without any change in oil use.[9]

2.5.2 Energy providers in oil-consuming regions

In regions that engage in fracking, the problem for the energy-service provider can be solved in two steps. First, the regional relative demands of conventional and unconventional oil can be determined from the following cost-minimization problem:

$$\begin{aligned} \min_{e_{1,i,t},(e_{n+j,i,t})_{j=1}^{l}} \quad & \hat{p}_{1,i,t}e_{1,i,t} + \sum_{j=1}^{l} \hat{p}_{n+j,i,t}e_{n+j,i,t} \\ -P_{i,t}^{O} & \left(\widetilde{l}\left(\lambda_1^{oil} e_{1,i,t}^{\rho_h} + \sum_{j=1}^{l} \lambda_{j+1}^{oil} e_{n+j,i,t}^{\rho_h} \right)^{\frac{1}{\rho_h}} - O_{i,t} \right), \end{aligned} \tag{9}$$

where $\hat{p}_{1,i,t}$ denotes the tax-inclusive price of conventional oil, and with corresponding notation for the unconventional substitutes.[10] By construction, the Lagrange multiplier, $P_{i,t}^{O}$, defines the exact price index of the oil composite.

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[8] Note that even if $p_{1,t}$ were to be stochastic, this would have no effect on the oil supply, since income and substitution effects exactly cancel under logarithmic preferences.

[9] A model where oil use is determined jointly by supply and demand would be preferable, as would a setting where oil producers could conduct in some intertemporal arbitrage based on the level of the world interest rate.

[10] Note that assuming that all regions would have the same cost of producing unconventional oil products would be isomorphic to allowing global markets for “fracked” oil.

The first-order conditions to the problem defined by (9) yields

$$e_{j,i,t} = \frac{O_{i,t}}{\widetilde{l}} \left(\widetilde{l} \lambda_j^{oil} \frac{P_{i,t}^O}{\hat{p}_{j,i,t}} \right)^{\frac{1}{1-\rho_h}}, \tag{10}$$

with $j \in \{1, n+1, ..., n+l\}$. Inserting (10) into the energy-service provider's expenditure function for oil goods, the exact price index for oil becomes

$$P_{i,t}^O = \widetilde{l}^{-1} \left(\left(\lambda_1^{oil}\right)^{\frac{1}{1-\rho_h}} \hat{p}_{1,i,t}^{\frac{\rho_h}{\rho_h - 1}} + \sum_{j=1}^{l} \left(\lambda_{j+1}^{oil}\right)^{\frac{1}{1-\rho_h}} \hat{p}_{n+j,i,t}^{\frac{\rho_h}{\rho_h - 1}} \right)^{\frac{\rho_h - 1}{\rho_h}}. \tag{11}$$

With the relative demands for the different oil substitutes given by (10), the second step for the energy-service provider is to derive the demand functions for all fuels. This implies solving the following cost-minimization problem:

$$\min_{O_{i,t}, (e_{j,i,t})_{j=2}^n} P_{i,t}^O O_{i,t} + \sum_{j=2}^{n} \hat{p}_{j,i,t} e_{k,i,t} - \Lambda_{i,t} \left(\left(\lambda_1 O_{i,t}^{\rho} + \sum_{j=2}^{n} \lambda_j e_{j,i,t}^{\rho} \right)^{\frac{1}{\rho}} - E_{i,t} \right). \tag{12}$$

The first-order conditions deliver

$$O_{i,t} = \left(\lambda_1 \frac{P_{i,t}}{P_{i,t}^O} \right)^{\frac{1}{1-\rho}} E_{i,t}, \tag{13}$$

and

$$e_{j,i,t} = \left(\lambda_j \frac{P_{i,t}}{\hat{p}_{j,i,t}} \right)^{\frac{1}{1-\rho}} E_{i,t}, \quad j \in \{2, ..., n\}. \tag{14}$$

The price index of energy services can then be shown to be given by

$$P_{i,t} = \left(\lambda_1^{\frac{1}{1-\rho}} \left(P_{i,t}^O\right)^{\frac{\rho}{\rho-1}} + \sum_{j=2}^{n} \lambda_j^{\frac{1}{1-\rho}} \hat{p}_{j,i,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}. \tag{15}$$

2.5.3 Final-good producers in oil-consuming regions

Producers of the final good maximize profits while taking $P_{i,t}$ as given. Taking the first-order condition in region i with respect to energy use and rearranging delivers

$$E_{i,t} = \left(\nu \frac{A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha}}{P_{i,t}} \right)^{\frac{1}{1-\nu}}. \tag{16}$$

The price-taking behavior of the final-good producing firm also implies that

$$\begin{aligned} w_{i,t} &\equiv \frac{\partial Y_{i,t}}{\partial L_{i,t}} = (1-\alpha-\nu) \frac{Y_{i,t}}{L_{i,t}} && (17) \\ r_{i,t} &\equiv \frac{\partial Y_{i,t}}{\partial K_{i,t}} = \alpha \frac{Y_{i,t}}{K_{i,t}}. && (18) \end{aligned}$$

The assumption of constant returns to scale implies that profits are zero in equilibrium, and that output net of energy expenses is given by $(1-\nu) Y_{i,t} \equiv \hat{Y}_{i,t}$. Note, however, that the shares of spending on different energy sources are not necessarily constant, unless $\rho = 0$, i.e., when the overall production function is Cobb-Douglas in all inputs.

2.5.4 Households in oil-consuming regions

Households supply labor inelastically and maximize (1) subject to the budget constraint

$$C_{i,t} + K_{i,t+1} = (1 + \Gamma_{i,t}) (w_{i,t} L_{i,t} + r_{i,t} K_{i,t}) = (1 + \Gamma_{i,t}) \hat{Y}_{i,t}. \tag{19}$$

The result from this optimization is the Euler equation, which is given by

$$\frac{C_{i,t+1}}{C_{i,t}} = \beta (1 + \Gamma_{i,t+1}) r_{i,t+1}. \tag{20}$$

Defining the savings rate out of net output as $s_{i,t} = \left(\hat{Y}_{i,t} - C_{i,t} \right) / \hat{Y}_{i,t}$, we obtain $C_{i,t} = (1 - s_{i,t}) (1 + \Gamma_{i,t}) \hat{Y}_{i,t}$ and $K_{i,t+1} = s_{i,t} (1 + \Gamma_{i,t}) \hat{Y}_{i,t}$. Inserting these expressions and (18) into (20) yields

$$\frac{1 - s_{i,t+1}}{1 - s_{i,t}} = \beta \frac{\alpha}{s_{i,t} (1-\nu)}. \tag{21}$$

The difference equation in $s_{i,t}$ defined by (21) only has one non-explosive solution: $s_{i,t} =$

$\frac{\alpha\beta}{1-\nu} \equiv s$, $\forall t$. This savings rate defines optimal household behavior.[11]

2.5.5 The equilibrium allocation

We leave out the formal equilibrium definition—it is straightforward given the discussion above. The key properties of the market allocation can now be summarized in the following proposition.

Proposition 1 *In each period, the equilibrium allocation is determined by state variables $K_{i,t}, R_t$ and S_{t-1} such that*
i) the capital savings rate is constant at $s = \frac{\alpha\beta}{1-\nu}$,
ii) the supply of conventional oil is given by $(1-\beta)R_t$,
iii) the price of the oil composite is $P^O_{i,t} = \widetilde{l}^{-1} \left(\left(\lambda_1^{oil}\right)^{\frac{1}{1-\rho_h}} \hat{p}_{1,i,t}^{\frac{\rho_h}{\rho_h-1}} + \sum_{j=1}^{l} \left(\lambda_{j+1}^{oil}\right)^{\frac{1}{1-\rho_h}} \hat{p}_{n+j,i,t}^{\frac{\rho_h}{\rho_h-1}} \right)^{\frac{\rho_h-1}{\rho_h}}$,
iv) the price of energy services is $P_{i,t} = \left(\lambda_1^{\frac{1}{1-\rho}} \left(P^O_{i,t}\right)^{\frac{\rho}{\rho-1}} + \sum_{j=2}^{n} \lambda_j^{\frac{1}{1-\rho}} \hat{p}_{j,i,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}$,
v) energy-service demand is $E_{i,t} = \left(\nu \frac{e^{\left(z_{i,t}-\gamma_{i,t}S_{t-1}\right)} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha}}{P_{t,i}} \right)^{\frac{1}{1-\nu}}$,
vi) regional demand for composite oil is given by $O_{i,t} = \left(\lambda_1 \frac{P_{i,t}}{P^O_{i,t}}\right)^{\frac{1}{1-\rho}} E_{i,t}$,
vii) regional demands conventional and unconventional oil is $e_{j,i,t} = \frac{O_{i,t}}{\widetilde{l}} \left(\widetilde{l} \lambda_j^{oil} \frac{P^O_{i,t}}{\hat{p}_{j,i,t}} \right)^{\frac{1}{1-\rho_h}}$, $j \in \{1, n+1, ..., n+l\}$,
viii) regional demands for the remaining $n-1$ fuels are $e_{j,i,t} = \left(\lambda_j \frac{P_{t,i}}{\hat{p}_{j,i,t}}\right)^{\frac{1}{1-\rho}} E_{i,t}$, $j \in \{2, ..., n\}$, and
ix) net output is $\hat{Y}_{i,t} = (1-\nu) A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}$.
The price of oil is determined from market clearing in the world oil market $\sum_{i=2}^{r} e_{1,i,t} = (1-\beta) R_t$. The state variables evolve according to $K_{i,t} = \frac{\alpha\beta}{1-\nu}(1+\Gamma_{i,t}) \hat{Y}_{i,t}$, $R_{t+1} = \beta R_t$ and $S_t = \sum_{v=0}^{t} (1-d_{t-v}) \sum_i M_{i,t}$.

Two things should be noted. First, the allocation is determined sequentially without any forward-looking terms; this is due to the combination of logarithmic utility, Cobb-Douglas production, and full depreciation. Second, given a world market price of oil ($p_{1,t}$), all equilibrium conditions have closed-form solutions. Hence, in each period, finding the

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[11] This convenient result is, among other things, a result of letting fossil-fuel taxes be proportional to income.

equilibrium is only a matter of finding the equilibrium oil price where supply is predetermined at $(1-\beta)\,R_t$. As a result, the model typically solves in under a second.

3 Calibration

We now turn to the calibration of the model. We set $r = 8$ to consider a world consisting of seven oil-consuming regions. Specifically, these regions represent Europe, the U.S., China, South America, India, Africa, and what we refer to as Oceania. The oil producing region consists broadly of OPEC and Russia. South America includes all countries in South America except the countries that are part of OPEC, and the same is true for Africa. The region referred to as India also includes Pakistan and Bangladesh. Our definition of Oceania, finally, is also wider and somewhat different from the geographical region: it includes Australia, Japan, Indonesia, Malaysia, Myanmar, New Zeeland, Phillipines, Thailand, and Vietnam.[12]

3.1 Energy sources and their properties

The number of different fuel inputs into energy production that are available in all regions is set to $n = 3$. The first fuel represents oil, whereas fuel of types two and three represent coal and green (renewable) energy, respectively. Given that the amount of non-conventional reserves of fossil fuel that are extractable by fracking and other existing or future technologies is hard to assess, we only allow fracking in the United States and we only consider one type of unconventional oil. Hence, we set $l = 1$ and let the fourth fuel represent the output from fracking in the United States. Measuring oil and coal in carbon units imply that $g_{j,i,t} = 1$ for $j \in 1, 2, 4$ and 0 for $j = 3$.

The production of energy services is calibrated in the following way. The elasticity of substitution between the $n = 3$ energy sources is set to 0.95 to match the unweighted mean of the oil-coal, oil-electricity and coal-electricity elasticities that is found in the metastudy by Stern (2012).[13] This implies setting ρ to -0.058, which we use as the main case. In Section 5, where we discuss the case of endogenous direction of technical change, we will return to how results depend on this parameter.

In the benchmark model, the cost of producing coal and green fuel is constant in terms of the final good. To calibrate the $\lambda' s$, prices and quantities of the three types of fuel are

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[12] Together, the considered regions account for somewhat less than 90% of global emissions in 2016.

[13] More specifically, Stern (2012) is based on 47 studies of interfuel substitution.

needed. Abstracting from fracking and combining the demand equations (14) and (13), we derive the following relationships

$$\frac{\lambda_1}{\lambda_k} = \left(\frac{e_{1,t}}{e_{k,t}}\right)^{1-\rho} \frac{p_{1,t}}{p_{j,t}}, \quad j = 2, 3.$$

Using world market prices from Golosov at al. (2014), the coal price is set to (USD) \$74/ton and the (pre-financial crises) oil price is set to \$70/barrel, corresponding to \$70·7.33 per ton.[14] These prices are assumed to apply in all regions. For coal, it also represents the cost of production and transportation. The relative price between oil and coal in units of carbon is then 5.87. Using the same source for the ratio of global oil to coal use in carbon units we find that $\lambda_1/\lambda_2 = 5.348$. For green energy, we use data for the sum of nuclear, hydro, wind, waste, and other renewables from Golosov et al. (2014) and adopt with their (somewhat arbitrary) assumption of a unitary relative price between oil and renewables. This gives $\lambda_1/\lambda_3 = 1.527$. Together with the normalization $1 = \lambda_1 + \lambda_2 + \lambda_3$, the λ's are then given by $\lambda_1 = 0.543$, $\lambda_2 = 0.102$, and $\lambda_3 = 0.356$.

Fracking costs in the U.S. are roughly \$40 per barrel, corresponding to \$347 per ton of carbon. We impose a high degree of substitutability between the output from fracking and conventional fossil fuels and set the elasticity of substitution between the different components of the oil composite to 10, implying $\rho_h = 0.9$.[15]

3.2 Fossil-fuel reserves

Turning now to existing stocks of fossil fuels, BP (2010) reports that global proved reserves of oil are 181.7 Gigatons. However, this number only includes the aggregate reserves that are economically profitable to extract at current economic and technical conditions. In particular, it does not take technical progress, or that new profitable oil reserves may be discovered, into account. An alternative study is Rogner (1997) that does take technical progress into account. This study estimates global fossil reserves to be larger than 5,000 Gigatons of oil equivalents (Gtoe).[16] About 16% of these reserves constitute oil, i.e., 800 Gtoe. Against this background, we set the existing stock of oil to 330 Gtoe, i.e., somewhere

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[14] Taxes and subsidies of fossil fuel and other energy sources are disregarded.

[15] For related studies on the U.S. shale oil boom, see Çakir Melek, Plante, and Yücel (2017) and Bornstein et al. (2017).

[16] By expressing quantities in oil equivalents, the difference in energy content between natural gas, oil, and various grades of coal is accounted for.

well within the range of these two estimates. The carbon content of oil and coal is 84.6% and 71% respectively in weight.

3.3 Output and emissions

The discount factor is set to 0.985[10] with the understanding that a period is a decade. The production parameters α and ν respectively determine the income shares for capital and energy. We set α to 0.3, and ν to 0.031 to match a capital share equal of 0.3 and an energy share of 0.031.

Initial global GDP is set to about \$ 80 trillion, which matches the observed value for 2017. We then calibrate the TFP levels in the different regions to match each region's share of global PPP-adjusted GDP as reported by the World Bank.[17] The calibrated distribution implies that roughly 60% of world GDP is produced in equal shares in the U.S., EU and China, whereas Oceania accounts for 13%, India 9%, South America 6%, Oil countries 12%, and Africa 6%. Based on Stefanski (2017), we acknowledge that China subsidizes coal production and we set the subsidy to match China's share of world CO_2 emissions.[18] Given the close connection between GDP and emissions, the model then well matches emission shares in all regions. GDP and emission share in the model and data are plotted in Figure 1.

The U.S. and the E.U. are both assumed to start out on their respective balanced growth paths, where the constant technological growth rate is given by $\Delta z_{i,t} = 1.5\%$ per year. Based on the findings in Caselli and Feyrer (2007), the initial capital stock is set to equalize real interest rates across all regions. Together these assumptions imply a GDP growth rate of 2.1% per year, and they pin down initial productivities and capital stocks in all regions.[19]

Due to catching-up effects, China, India, Africa, South America, and Oceania are all assumed to initially grow faster than the European Union and the United States.[20] This is implemented by allowing TFP in those regions to converge to a growth path that is 60% higher than their current ones. The implication is that China converges to a BGP with approximately twice the GDP of the E.U. and the U.S., whereas India and Africa both

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[17] World Bank: World Development Indicators. The numbers are for 2016.

[18] The implied subsidy is \$83 per ton of carbon. The share coming from the oil producer is also targeted directly by assuming that a fraction of their oil extraction gives rise to CO_2 emissions. This share is set at 0.32.

[19] See the appendix for details.

[20] It is unclear whether this is a good approximation for Oceania, given that this region as we define it includes both developed and developing countries.

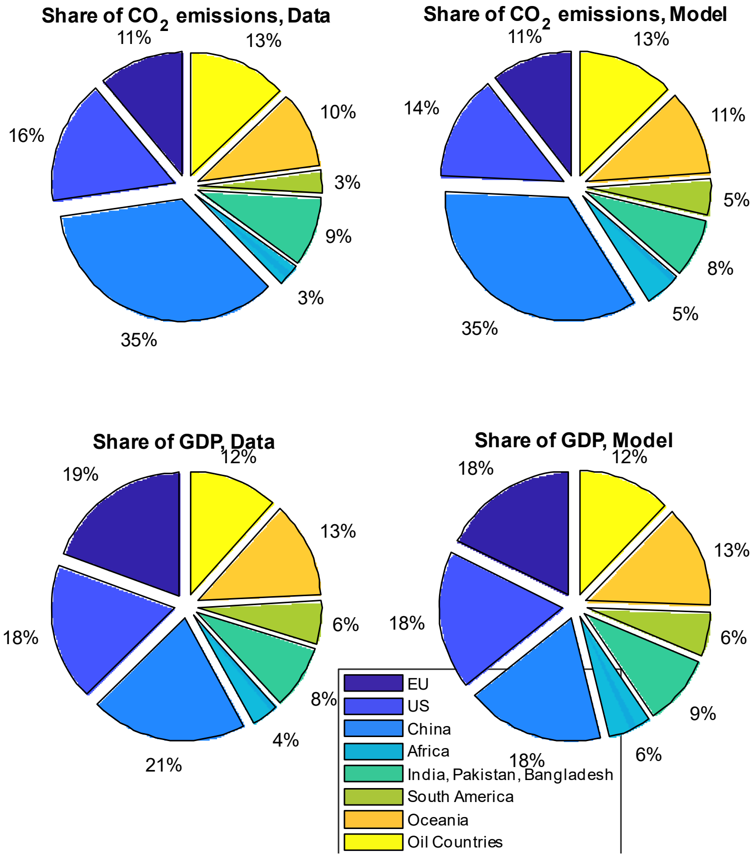

Figure 1: Regional shares of world output and world CO$_2$ emissions in the model and in the data.

converge to a path with the same GDP as the E.U. and the U.S. Formally, we assume that for these regions log (TFP) follows the following process:

$$\begin{aligned} z_{i,t+1} &= z_{i,t} + 10 \log 1.015 + \frac{1}{4}\left(\hat{z}_{i,t} - z_{i,t}\right), \qquad (22) \\ \hat{z}_{i,t+1} &= \hat{z}_{i,t} + 10 \log 1.015, \\ \hat{z}_{i,0} &= z_{i,0} + \log 1.6, \end{aligned}$$

where $e^{\hat{z}_{i,t}}$ is the TFP path that actual TFP converges to, and $t = 0$ is the initial period. As seen in equation (22), 1/4 of the productivity gap $(\hat{z}_{i,t} - z_{i,t})$ is removed each decade.

3.4 Climate damages

The climate sensitivity, ξ, is set to $3°C$ per doubling of atmospheric CO_2 concentration, and the initial stock of atmospheric carbon is set to 586 gigatons of carbon (GtC). The region-specific damages are then calibrated using the latest estimates from RICE (Nordhaus, 2010). Specifically, this study provides linear-quadratic regional damage functions that are stating the share of GDP lost due to changes in the global mean temperature. The parameters of these damage functions are, for each region, provided in Table 1 in Appendix A.1. Using the Arrhenius equation, we can then for each set of damage parameters express the damage elasticity as a function of the global mean temperature

$$\gamma_i = -\frac{\ln\left(1 - \left(\phi_{1,i} T + \phi_{2,i} T^2\right)\right)}{S_0\left(e^{\frac{T \ln(2)}{\xi}} - 1\right)}. \qquad (23)$$

Given that the functions implied by (23) are relatively flat, the γ_i-functions are approximated with the value of γ_i evaluated at $3.5°C$, yielding $\gamma_{US} = 2.395$, $\gamma_{Eu} = 2.698$, $\gamma_{Ch} = 2.514$, $\gamma_{In} = 5.058$, $\gamma_{Af} = 5.031$, $\gamma_{SA} = 2.57$, and $\gamma_{OC} = 2.74$, with all these numbers multiplied by 10^{-5}.[21,22] For the carbon-cycle parameters, we follow Golosov et al. (2014) by setting $\varphi_L = 0.2$, $\varphi_0 = 0.393$, and $\varphi = 0.0228$.

The full calibration is summarized in Tables 2-3 in Appendix A.1.

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[21] See Figure X in Appendix A.1.

[22] See Hassler, Krusell, and Olovsson (2018) for an analysis that evaluates ranges of plausible estimates for both the climate sensitivity and the sensitivity of the economy to climate change.

4 Results

4.1 Global warming with regionally different taxes

We are now ready to present simulation results and analyze the costs and benefits of sub-optimal policies. We start by considering a few policy scenarios that differ in terms of coverage and in the level of the tax. This gives some preliminary insights into the effectiveness of the different policies in mitigating climate change. The first scenario imposes a global Pigouvian tax. This implies setting the initial global tax to \$76.8 per ton of carbon, which then increases by 2.2% per year, i.e., approximately at the growth rate of GDP. This tax fully internalizes the climate externality given the chosen parameters, as is shown in Golosov et al. (2014).[23] The second assumes that only Europe implements the fossil-fuel tax. The third scenario features a global tax, but only on coal. The fourth scenario considers a unilateral European coal tax only. Finally, we consider a more ambitious tax policy that features the current Swedish carbon tax. This tax is close to seven times the Pigouvian tax that is imposed globally on coal.[24] As a benchmark, we also present results for the *laissez-faire* scenario.

The results are presented in Figure 2, which plots the increases in global mean temperature over the next 200 years that result from the different scenarios. A couple of important results are immediate from the figure. First, global fossil-fuel taxes are effective in mitigating climate change. Towards the end of this century, the difference in temperature between the *laissez faire* and global Pigouvian taxation is 0.65 degrees Celsius, and this difference increases substantially thereafter. A global Swedish tax would, in fact, stop global warming almost completely and keep the global mean temperature just above 1.5 degrees Celsius at the end of century. Second, what matters for the resulting change in the temperature is taxes on coal. Both the cases with global taxes and with taxes only in the E.U. are identical in terms of climate change, i.e., regardless of whether oil is taxed or not. The intuition for this result is straightforward. The supply of oil is inelastic and does not respond to taxes, while the opposite is true for coal. Finally, and maybe not too surprisingly, it is not effective to impose taxes only in the EU. In fact, the difference in temperature between the case with only taxes in the EU and *laissez faire* are negligible.

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[23] Compared to European taxes on gasoline, this tax is modest. Given a carbon content of around 0.65 Kg/liter for gasoline, this implies a tax of 5 cents per liter of gasoline.

[24] The Swedish CO_2 tax is roughly \$523 dollars per ton carbon. This amounts to \$0.34 per liter gasoline.

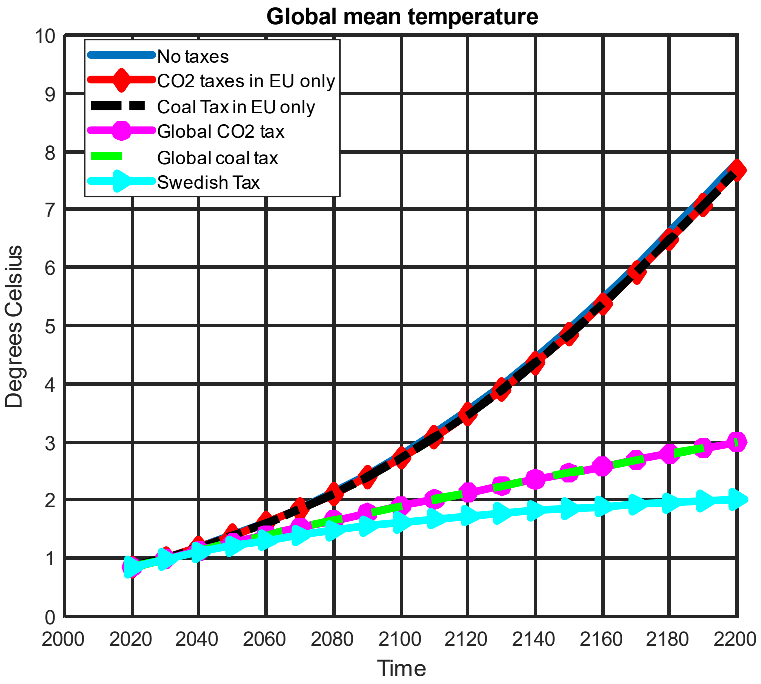

Figure 2: Global warming with different carbon tax schemes.

4.2 Ex-post policy errors

The second question that we ask relates to the fact that any climate policy today has to be implemented under substantial imperfect knowledge about both the climate system and the economic damages. We specifically ask how large the respective costs are from implementing climate policies that are inappropriate *ex post*, i.e., when the uncertainties have been resolved. That is, we consider policies that *ex post* will turn out to have been too ambitions or too lax.

To answer this question we focus on the imperfect knowledge that characterizes two sets of crucial parameters: the climate sensitivity and the parameters that determine the regional damages as functions of the global temperature. The climate sensitivity quantifies how much the temperature increases from a doubling of the concentration of CO_2 in the atmosphere, and here the IPCC (2013) reports a wide range of values. Regarding the economic damages, we rely on the recent meta-study by Nordhaus and Moffat (2017). Rather than focusing on the uncertainty surrounding these parameters, we focus on the extremes as given by as upper and lower bounds of intervals given in these documents. The results are presented in

Figure 3.

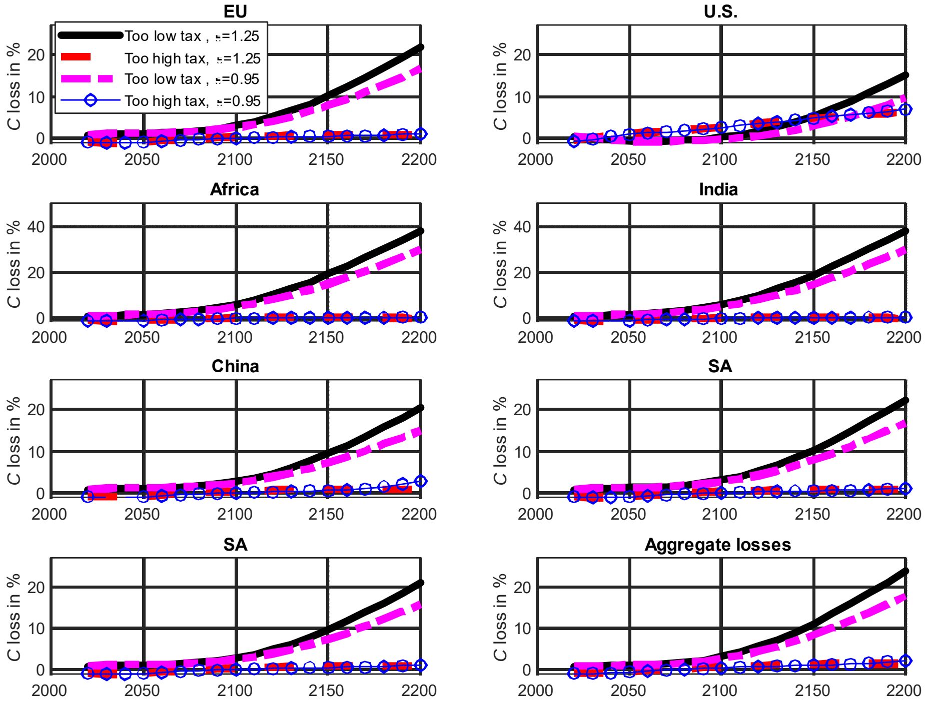

Figure 3: Welfare costs in different regions from under and overestimating climate change.

As can be seen, the costs of underestimating climate change are one order of magnitude larger than the costs of overestimating it for all regions except for the United States. In addition, Figure 3 shows that the costs of underestimating climate change are higher with a higher elasticity of substitution between energy goods. The aggregate consumption losses are close to 20 percent in 2200 and they increase rapidly after that. For Africa, the costs are even larger: a more than 40 percent reduction in consumption in the year 2200 relative to the optimal policy.

The intuition for why the costs are increasing in the elasticity are as follows. When climate change is underestimated, the carbon tax is low and coal is cheap. With a higher ϵ, more coal will thus be used. Even though the level of the tax rate is too low in this scenario, the tax increases over time. Eventually, coal therefore becomes more expensive and coal use then falls faster in relative terms the higher ϵ is.

The cost of overestimating the effects of global warming are also higher with a higher elasticity, but this effect is just marginal. Hence, we conclude that a higher elasticity of substitution between energy goods creates a larger asymmetry: it increases the cost of underestimating climate change more than it increases the costs of overestimating climate change.

Of course, the limited costs of following a "prudent", i.e., high-tax policy crucially also depend on the design of the policy: it is a global (uniformly applied) carbon tax. If we considered a prudent stance under a less well-designed policy scheme, the costs would rise significantly. We turn to this issue next.

4.3 Cost efficiency: departures from uniformity

We now quantify the importance of cost efficiency in climate mitigation by considering several sub-optimal policy experiments and comparing them to the first best. The particular sub-optimal policies all involve carbon taxes, but set differently in different parts of the world. Within each country, however, we assume that taxes are set in a uniform way.[25]

Since non-optimal policies produce different paths for the global temperature than the optimal policy, we design the alternative policies so that they all result in the same temperature increase as the optimal policy in a specific year. Somewhat arbitrarily, we choose the year 2165, which is 150 years from the starting year in the model. With the optimal policy, the temperature increases by 2.6^oC by 2165. It is important to point out that the alternative policies all have temperature trajectories that may differ substantially from that in the first best. In particular, the temperature increase will generally only be the same as the first best in 2165.

To evaluate the potential importance of cost efficiency, we compute the discounted region-specific cost of a specific policy relative to the optimal policy as

$$\Delta_i = 100 \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{c_{i,t}}{c_{i,t}^{opt}} - 1 \right), \tag{24}$$

where $c_{i,t}^{opt}$ is consumption under the optimal tax, $c_{i,t}$ is consumption with the alternative policy, and we multiply by 100 to express numbers in percent. The region-specific costs are then aggregated by weighing the cost for each region by the income share in that region, i.e.,

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[25] Clearly, to the extent that countries use carbon taxes, they do not seem to employ uniform carbon taxes across sectors and across firms and households. The implied inefficiencies are likely high, but we know of no estimates to use here and must leave this important question to future research.

$$\Delta = \sum_{i=1}^{R} \Delta_i \frac{GDP_i}{GDP_W}, \tag{25}$$

where is GDP_W is global GDP.

The first experiment we consider is one where the developing regions, i.e., Africa and India do not implement any policy at all. In order for the temperature increase to be exactly $2.6^o C$ in 2165, all other regions must then implement a carbon tax that is 5.3 times higher than the optimal tax.[26]

The other experiment that we consider is one where China only participates to a limited extent in mitigating global warming. We do not set Chinese taxes to zero, because if we did it would be impossible to limit the temperature increase to $2.6^o C$ by 2165. China is thus assumed to implement a CO_2 tax that is about 15% of the optimal tax rate. The remaining regions implement a CO_2-tax that is 20 times higher than optimal. The results from these experiments are presented in Figure 4.

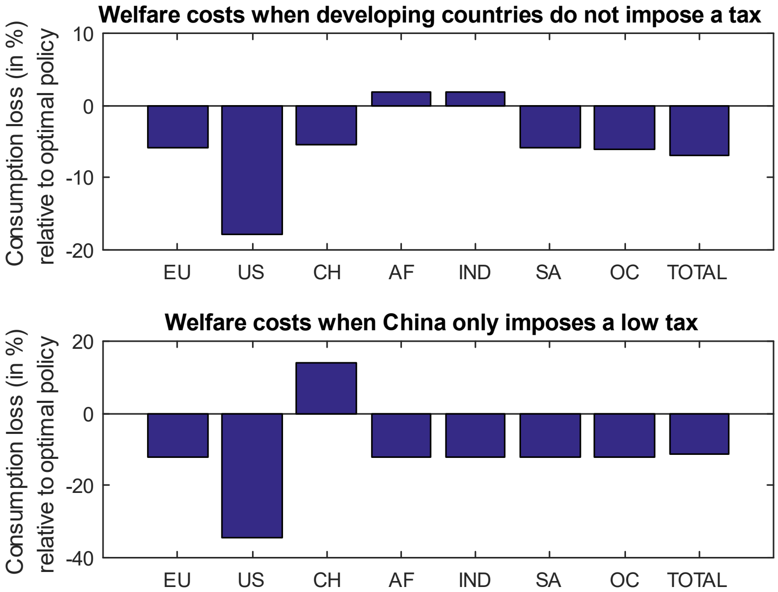

Figure 4: Distributional and total effects of suboptimal climate policies.

Not surprisingly, the regions that do not participate in mitigating climate change gain

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[26] Even though this indeed implies is a high tax rate rate, it is useful to recall that the Swedish carbon tax is about 7 times higher than the optimal tax.

in net present value terms. In the first experiment, Africa and India free-ride on the other regions, but still only experience welfare gains just below 2 percent. The reason for these mild gains is that the global temperature increases relatively fast after the initial 150 years with the alternative policy. This hurts Africa and India, which are both sensitive to climate change. The other regions all lose about 6 percent, except the U.S., which loses about 18 percent of consumption relative to the optimal policy (i.e., not relative to laissez faire). The reason for this large loss again has to do with fracking. The higher taxes in the U.S. specifically hurts the fracking industry, which implies a substantially higher energy price in the U.S. with the alternative policies. This effect is not present for the other regions, since they do not have access to fracking.

4.4 Technical change: green vs. fossil

The previous section revealed that a global coal tax is an efficient way of mitigating climate change. Specifically, while coal use increases by a factor of 40 over the coming 200 years in the *laissez faire* economy, a coal tax that increases by 2% per year reduces the increase in coal use to a factor of two. Total emissions then fall over time. Higher taxes than the Pigouvian tax we consider imply to a substantial reduction in emissions. The growth rate of green energy is, however, largely unaffected in the different scenarios. Let us now look into consider technical change that can potentially differ across the green and fossil industries. This is an indirect way of studying whether subsidies to the development of green technology can be an efficient substitute for the Pigouvian tax.

If we use equation (14) to compute the relative use of coal and green energy, we obtain

$$\frac{e_{2,i,t}}{e_{3,i,t}} = \left(\frac{\lambda_2}{\lambda_3} \frac{p_{3,i,t}}{(1+\tau_{i,t})\, p_{2,i,t}} \right)^{\frac{1}{1-\rho}} .$$

The above equation shows that the relative usage of coal and green fuel is driven by the relative price including taxes. It suggests that if technology were to develop in a way that increases the relative price of coal, this would have similar effects as a tax on coal. We therefore consider two scenarios. First, we look at one where green energy becomes 2% cheaper per year to produce (i.e., $p_{3,i,t}$ falls by 2% per year in all regions) relative to the overall price level; such an outcome could presumably come about as a result of subsidies. In a second case, we adds to the first scenario an assumption that coal becomes more expensive over time (i.e., $p_{2,i,t}$ increases over time): the technological development in the coal industry is less fast than for the economy as a whole.[27]

The benchmark results of the simulations are presented in the top graph in Figure 5. They show, first, that the fact that the price of green energy falls over time by 2% has no positive consequences for the climate. In fact, emissions are even larger in this scenario than in the baseline case. The reason is that lower green-energy prices imply lower energy prices in general, which increases the demand for all energy services, including for coal. The figure also shows that the second scenario, where $p_{2,i,t}$ increases and $p_{3,i,t}$ falls, produces a path of the global mean temperature that is virtually indistinguishable from that with global Pigouvian taxes.

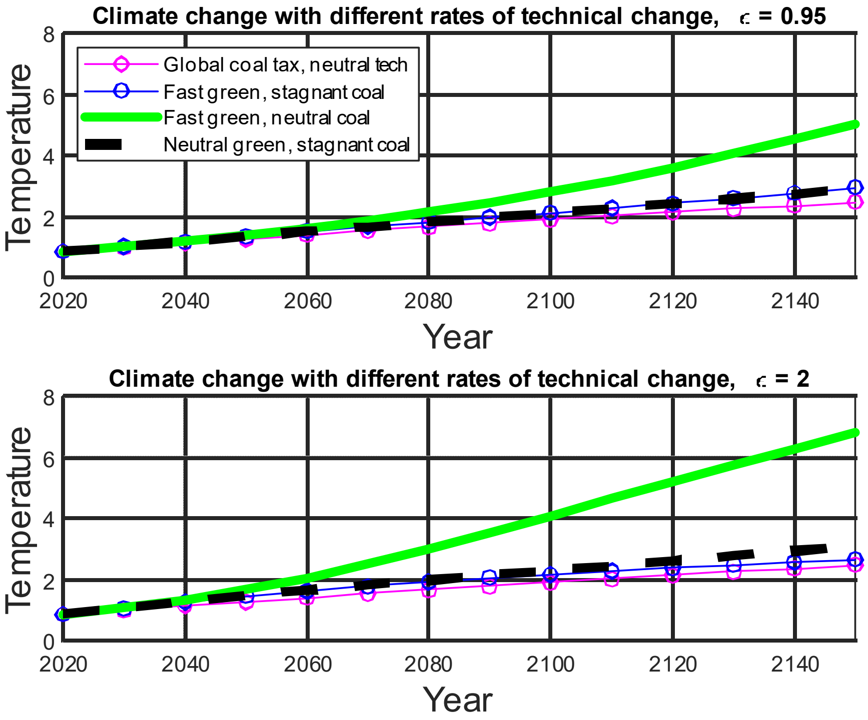

Figure 5: Top graph: global mean temperatures with different rates of technical change with an elasticity of substitution between green and fossil energy sources set to 0.95. Bottom graph: same as in the top graph but with the elasticity of substitution between green and fossil energy sources set to 2.

The bottom graph shows that these results do not depend crucially on that the substitu-

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[27] I.e., we do not assume technological regress: think of a case where the production of coal-based industry does not improve whereas the final-goods sector sees TFP growth of 2 percent per year.

tion elasticity between green and fossil energy is less than one. Also with an elasticity of 2 is a lower relative price of green energy insufficient in mitigating global warming. In fact, a high elasticity produces even worse outcomes in the cases where coal does not become more expensive. The intuition for this result comes from that coal initially is cheaper to produce than green energy. A higher elasticity then induces relatively more coal use for an extensive period, with higher temperature increases as a result.

The observation that stagnant technology in coal energy production is powerful in mitigating climate change is interesting, but it leaves open whether such an outcome would be possible to attain by the use of policy. Acemoglu et al. (2012) make such an argument by assuming that green subsidies draw research efforts away from coal, essentially building in highly mobile research efforts and abstracting away from spillovers from green to fossil. Whether green subsidies are this powerful is, to us, far from obvious in practice, and green policy alone is therefore a hazardous way to go for that reason: if this policy only ends up affecting green technology development itself, it does not help us slow down climate change (and may even speed it up).

The finding that directing technical change away from the production of fossil fuel is a powerful means of overcoming the problems associated with climate change suggests that we need to better understand the determinants of technical change. Next, we show that the present model can be extended in this direction.

5 Extensions: endogenous technical change

We now turn to an analysis of endogenous technical change. The purpose of this section is to provide a simple framework for endogenizing the cost of producing the different sources of energy. The analysis is different from the one in previous sections in that it is stylized: we do not build on empirical insights into the nature of R&D technology. But given more quantitative information on this technology, we believe that the present setting could be put to productive applied usse.

For simplicity, we abstract from fracking here. We maintain the assumption that oil is imported, whereas the other energy sources are domestically produced at costs $p_{k,i,t}$. We allow a tax $\tau_{k,i,t}$ on each fuel. Specifically, we will consider both per-unit taxes and proportional *ad-valorem* taxes.

As in the previous sections, there is an energy-producing representative firm selling energy services on a competitive market. The important difference is that the energy producer now also has the possibility to improve the technologies for producing the different domestic energy inputs. Specifically, all energy inputs except oil can be reduced at a cost that is determined by the constraint $RD_i\left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}},...,\frac{p_{n,i,t}}{\bar{p}_{n,i,t-1}}\right)\geq 0$.[28] Here, $\bar{p}_{j,i,t-1}$ denotes production costs if no cost reductions occur. These costs are determined by aggregate innovation decisions in the previous period, either in the region i itself or globally. Cost reductions then spill over with a one-period lag. Firms thus choose technology here without internalizing its effect on technologies in the future, a standard approach in the endogenous-growth literature.[29]

The problem of the representative energy-service provider with *ad-valorem* taxes is then given by

$$\min_{(e_{j,i,t})_1^n,(p_{j,i,t})_2^n} \sum_{j=1}^{n}\left(1+\tau_{j,i,t}\right)p_{j,i,t}e_{j,i,t} \tag{26}$$

$$-P_{i,t}\left(\mathcal{E}(e_{1,i,t},\ldots,e_{n,i,t})-E_{i,t}\right)-\Lambda_{i,t}RD_i\left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}},...,\frac{p_{n,i,t}}{\bar{p}_{n,i,t-1}}\right).$$

The problem defined by (26) differs from that in (12) in that the former problem includes a set of new choice variables $\{p_{j,i,t}\}_2^n$, as well as a new constraint $RD_i\left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}},...,\frac{p_{n,i,t}}{\bar{p}_{n,i,t-1}}\right)$ with a Lagrange multiplier $\Lambda_{i,t}$. Hence, the energy-producer takes the oil price, $p_{1,i,t}$ as given, but can affect $p_{j,i,t}$ for $j\neq 1$. It is straightforward to verify that all features of Proposition 1 still hold.

The first-order conditions with respect to $e_{j,i,t}$ and $p_{j,i,t}, k\neq 1$ are given by

$$e^*_{j,i,t}=E_{i,t}\left(\frac{P_{t,i}\lambda_j}{\left(1+\tau_{j,i,t}\right)p_{j,i,t}}\right)^{\frac{1}{1-\rho}}, \tag{27}$$

and

$$\left(1+\tau_{j,i,t}\right)e^*_{j,i,t}=\Lambda_{i,t}\frac{\partial RD_i\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{j,i,t}}, \tag{28}$$

respectively. Note that (27) is identical to (14) in the case of exogenous technical change.

With per-unit taxes imposed on the energy sources, the objective function (26) of the

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[28] In the specification of the R&D technology, we follow the assumptions in Hassler et al. (2017), where the focus is on energy-saving.

[29] It appears reasonable to include some degree of internalization of the dynamic benefits of R&D, because these efforts are often firm-specific to some extent.

energy-service provider becomes

$$\sum_{j=1}^{n} \left(\tau_{j,i,t} + p_{j,i,t}\right) e_{j,i,t}.$$

The resulting first-order conditions are then given by

$$e^*_{j,i,t} = E_{i,t} \left(\frac{P_{i,t}\lambda_j}{\tau_{j,i,t} + p_{j,i,t}} \right)^{\frac{1}{1-\rho}}$$

and

$$e^*_{j,i,t} = \Lambda_{i,t} \frac{\partial RD_i \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}, \ldots, \frac{p_{n,i,t}}{\bar{p}_{n,i,t-1}} \right)}{\partial p_{j,i,t}}. \tag{29}$$

Comparing (28) and (29), we find an important difference. The left-hand side of both equations represent the value of reducing costs of producing a particular fuel. In the case of *ad-valorem* taxes, this value increases in the tax for given quantity $e^*_{j,i,t}$. This is not the case with taxes per unit. We will return to this feature below.

5.0.1 A parametric example

We now make specific assumptions on the technology that is available to the energy-service producers. In particular, consider

$$RD_i \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}, \frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}} \right) = \min \left(\varepsilon_{2,i} \log \frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}, 0 \right) + \min \left(\varepsilon_{3,i} \log \frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}}, 0 \right) \geq a, \tag{30}$$

$$\sum_j \varepsilon_{j,i} = 1, \tag{31}$$

where a is negative and we assume, for simplicity, that there is a lower bound on the rate of technical change: $p_{j,i,t}$ will not increase. (Recall that an increase means technological retardation relative to that in the final-goods sector; another bound would be an absolute one.)

The R&D production technology defined by (30) can be thought of as an interesting starting point in the spirit of Romer (1986), where the number of researchers that are active in R&D determines the rate of technological change. Our specification can then be seen as an extension to directed technical change where the number of R&D workers is fixed, and the productivity in improving the technology differs between the energy sources.

The specification in (30) now implies, in interior cases for $j = 2, 3$,

$$\frac{\partial RD_i\left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}, \frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}}\right)}{\partial p_{j,i,t}} = \frac{\varepsilon_j}{p_{j,i,t}},$$

which, in turn, implies

$$\left(1 + \tau_{j,i,t}\right) p^*_{j,i,t} e^*_{j,i,t} = \varepsilon_j \Lambda_{i,t}. \tag{32}$$

This allows us to deduce the following.

Proposition 2 *When first-order conditions for the technology choice are satisfied, and taxes are* ad-valorem*, spending on green energy is a fixed fraction of all spending on domestically produces energy sources, i.e.,*

$$\frac{\left(1 + \tau_{3,i,t}\right) p^*_{3,i,t} e^*_{3,i,t}}{P_{t,i} E_{i,t} - \left(1 + \tau_{1,i,t}\right) p_{1,i,t} e^*_{1,i,t}} = \varepsilon_3.$$

Proof. Follows directly from noting that $\left(1 + \tau_{1,i,t}\right) p_{1,i,t} e^*_{1,i,t} + \sum_{k=2}^{3} \left(1 + \tau_{j,i,t}\right) p^*_{j,i,t} e^*_{j,i,t} = P_{i,t} E_{i,t}$ and using (32). ■

Furthermore, using the expression (27) in (32) for $j = 2$ and 3, we obtain the following two conditions.

$$\left(1 + \tau_{j,i,t}\right) p^*_{j,i,t} = \left(\frac{\varepsilon_j \Lambda_{i,t}}{E_{i,t}}\right)^{\frac{\rho - 1}{\rho}} \left(P_{i,t} \lambda_j\right)^{\frac{1}{\rho}}, \tag{33}$$

and

$$\frac{\left(1 + \tau_{2,i,t}\right) p^*_{2,i,t}}{\left(1 + \tau_{3,i,t}\right) p^*_{3,i,t}} = \left(\frac{\varepsilon_2}{\varepsilon_3}\right)^{\frac{\rho - 1}{\rho}} \left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho}}. \tag{34}$$

Since the right-hand side of (34) only contains technological constants, it follows that taxes cannot affect the after-tax relative price of domestic fuels, provided that the solutions to the first-order conditions all are interior. Furthermore, the growth rates of the two production costs are identical.

With both prices and spending being independent of taxes, also volumes are independent of taxes, i.e.,

$$\frac{e^*_{2,i,t}}{e^*_{3,i,t}} = \left(\frac{\lambda_3}{\lambda_2} \frac{\varepsilon_2}{\varepsilon_3}\right)^{\frac{1}{\rho}}.$$

To obtain some intuition, consider the first-order condition for the choice $p_k, k \neq 1$ given by (28). The left-hand side of the equation represents the marginal value of cost reductions. As can be seen, this value is increasing in the tax rate $\tau_{j,i,t}$ for a given quantity $e^*_{j,i,t}$. The fact that taxes are proportional to the cost of production implies that costs reductions are worth more the higher is the tax, *ceteris paribus.* However, an increased tax also reduces $e^*_{j,i,t}$ and this reduces the value of cost reductions. With the log-linear specification of R&D costs, the two effects exactly balances. Hence, R&D effectively nullifies the effect of taxes on after-tax prices.

This intuition also suggests that per-unit taxes should, in fact, lead to higher costs simply because the positive effect of taxes on the value of cost reductions just described disappears. Let us therefore now turn to the formal analysis on the effects of per-unit taxes. In this case, the first-order condition are given by

$$p_{j,i,t} e^*_{j,i,t} = \Lambda_{i,t} \varepsilon_j,$$

where

$$e^*_{j,i,t} = E_{i,t} \left(\frac{P_{t,i} \lambda_k}{\tau_{j,i,t} + p_{j,i,t}} \right)^{\frac{1}{1-\rho}}.$$

Combining the conditions for $j = 2$ and 3, we obtain the following condition .

$$\frac{p_{2,i,t} \left(\tau_{2,i,t} + p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t} \left(\tau_{3,i,t} + p_{3,i,t}\right)^{\frac{-1}{1-\rho}}} = \frac{1-\varepsilon_3}{\varepsilon_3} \left(\frac{\lambda_2}{\lambda_3} \right)^{\frac{1}{\rho-1}}. \tag{35}$$

Given this, we can consider the effects of changes in the coal tax. Total differentiation of the previous expression, noting that the R&D constraint implies $\frac{dp_{3,i,t}}{dp_{2,i,t}} = -\frac{1-\varepsilon_3}{\varepsilon_3} \frac{p_{3,i,t}}{p_{2,i,t}}$, and evaluating at both taxes being zero, yields

$$\left. \frac{dp_{2,i,t}}{d\tau_{2,i,t}} \right|_{\tau_{3,i,t} = \tau_{2,i,t} = 0} = -\frac{\varepsilon_3}{\rho}.$$

Hence, when ρ is negative but close to zero, an increase in coal taxes leads to a large increase in the relative price of coal.

5.0.2 A numerical example

In this section, we add the assumptions that the choice of extraction and production costs for green energy and coal is interior and that taxes are zero. In other words, we assume that current production prices of coal and green energy are optimally chosen, satisfy $\log \frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}} > 0$ and $\log \frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}} > 0$ and that taxes are zero. These assumptions, in particular the last one, are not necessarily satisfied in reality where both taxes and subsidies to energy production as well as to R&D can coexists. Our calibration, however, serves the purpose of illustrating the quantitative implications of the model.

Expressed per ton of carbon, the price of coal is calibrated to \$103.4 and the price of green in carbon oil equivalents to \$600. Equipped with our previously calibrated values of for λ_2 and λ_3, we can use these prices to calibrate the R&D parameter ε_3 from the following relationship:

$$\frac{p_{2,i,t}\left(0+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(0+p_{3,i,t}\right)^{\frac{-1}{1-\rho}}}=\frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}},$$

delivering $\varepsilon_3 = 0.782$.

The implication of this calibration is that it is easier to improve the productivity in coal production than in green-energy production. If that would not be the case, the model would predict that, currently, all R&D efforts should be directed toward reducing the relatively high cost of green energy.

Consider now the implications of introducing a carbon tax on coal. Using the R&D constraint (30), and initially disregarding the constraint on cost increases ($p_{2,i,t} \leq \bar{p}_{2,i,t-1}$), the relation between the two domestic fuel prices must imply that $\left(p_{2,i,t}\right)^{(1-\varepsilon_3)}\left(p_{3,i,t}\right)^{\varepsilon_3}$ is constant. Using the calibrated value for ε_3, and the initial prices, this constant is 409.0, yielding $p_{3,i,t}$ as a function of $p_{2,i,t}$:

$$p_{3,i,t}=p_3\left(p_{2,i,t}\right)=409.0^{\frac{1}{\varepsilon_3}}\left(p_{2,i,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}. \qquad (36)$$

Using this relation in (35) delivers

$$\frac{p_{2,i,t}\left(\tau_{2,i,t}+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(p_{2,i,t}\right)^{\frac{\rho}{\rho-1}}}=\frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}},$$

which implicitly defines $p_{2,i,t}$ as a function of the coal tax. Having found $p_{2,i,t}$, equation (36) provides $p_{3,i,t}$.

Turning to results, the solid curves in Figure 6 depicts the resulting relations between taxes and prices in an interior solution. As can be seen, the coal price is an increasing function of the tax, whereas the green-energy price is a decreasing function. Thus, R&D strongly amplifies the effect of a tax. The fact that $\varepsilon_3 > 1/2$ implies that it requires more R&D resources to reduce the cost of producing green energy than reducing the cost of producing coal. Both curves are, however, quite steep. Already for a coal tax of around \$35 per ton of carbon will the coal price becomes as high as the price of green energy. For the coal tax used in the previous section, i.e., \$76.8 per ton of carbon the coal price is substantially higher than the price of green energy (\$680 vs. \$400 per ton carbon) rather than only a fourth of the green price without taxes (see Figure 6).

The large effect of taxes on equilibrium prices is due to the fairly high elasticity of substitution. Already a moderately lower elasticity makes taxes a much less potent driver of the R&D direction. The dashed curves are computed under the assumption of an elasticity of substitution of 1/2, i.e., $\rho = -1$, with the λs and εs recalibrated. Clearly, the coal and green-energy prices are then fairly insensitive to taxes. A coal tax of \$100 per ton leads to an increase in the coal price before taxes of approximately \$50. Thus, the R&D feedback mechanism is still positive, but much weaker than with an elasticity of substitution that is close to unity. The intuition for this result is straightforward. The direct effect of a tax is to reduce the volume demanded, the more the larger is the elasticity of substitution. From (29), it follows that the marginal value of directing R&D to reduce costs is given by the volume. Thus, the reduction in the marginal value of R&D falls more the higher is the elasticity of substitution.

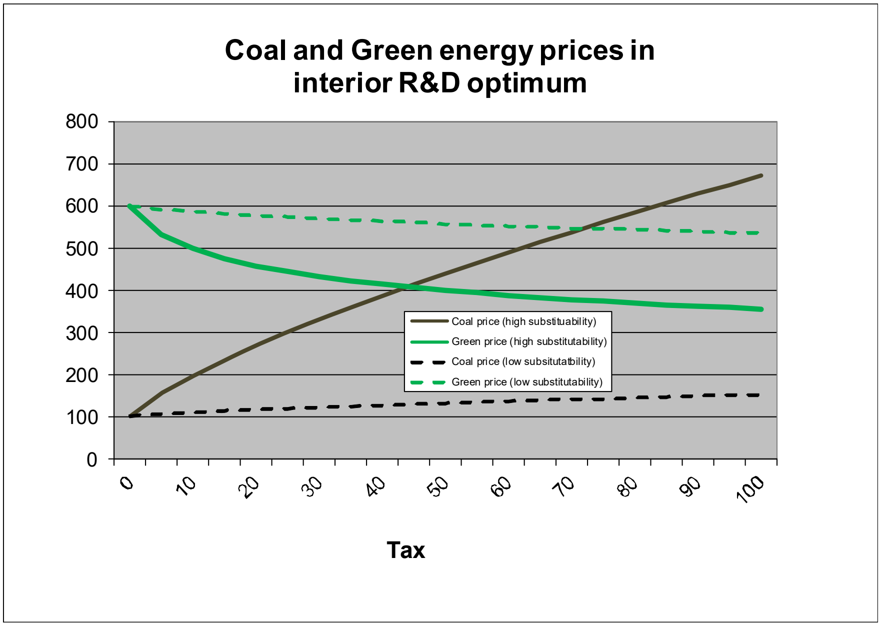

Figure 6: The effect of carbon taxes on energy prices.

Let us now return to the constraint $p_{2,i,t} \leq \bar{p}_{2,i,t-1}$. As discussed above, it seems reasonable to assume that technology cannot regress. Without technical advances in the coal industry, it may be assumed that costs in terms of the final good increases at about the rate of GDP growth. This would amount to somewhat more than 20% over a decade, which can be seen as an upper bound on how much the coal price can increase within a period. The conclusions are then that even a very modest tax on coal would lead to a stagnant coal technology, and that R&D would be directed towards improving the green energy. Let us therefore, finally, consider the implications of a global coal tax of the same size as in the previous section, but noting that this will lead all R&D to focus on reducing the cost of green energy. We use the R&D parameter calibrated for Europe and endogenize the growth rates of coal and green-energy prices from an initial situation where they are both constant in terms of the final good (the same rate of technical advances as in the final good sector). Then, redirecting R&D towards green implies (by assumption) that coal prices grow at 2% per year. The price of green energy instead falls by $((1-\varepsilon_3)/\varepsilon_3)\cdot 2\% = 0.56\%$ per year. It may also be noted that the calibration implies that if R&D instead would be fully directed towards reducing production costs for coal, the relative price of coal would fall by $(\varepsilon_3/(1-\varepsilon_3))\cdot 2 = 7.2\%$ per year. This is a large number but, in fact, not completely out of line with data. On average, the relative price of coal in units of GDP fell by 4.8% per year over the period 1950 to 2000.[30] For most of the decades, the reduction was much faster. If we exclude the the 1970s, i.e., the decade with the two major oil-price shocks, the average decline in the relative coal price was 6.93% per year.

In Figure 7, we show the paths for the global mean temperature with taxes and endogenous technical change, along with the *laissez-faire* scenario (which, by assumption, remains identical to the case with exogenous technical change). As can be seen, the temperature rises substantially less with endogenous technical change due to the fact that coal use immediately starts falling. Recall that with exogenous technical change and a global coal tax, coal use increased over the coming century and peaked around 2090.

So far, results have been derived under the assumption that interior solutions to the R&D problem define the optimum. However, if the elasticity of substitution $1/(1-\rho)$ is large enough, the energy-producing firm may specialize in some fuels. The purpose of the next sub-section is to analyze this case.

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[30] This is calculated as the difference between the growth of nominal GDP and the rate of change of the nominal coal price. The GDP data is from the FRED database, and the data on coal prices from EIA (2012).

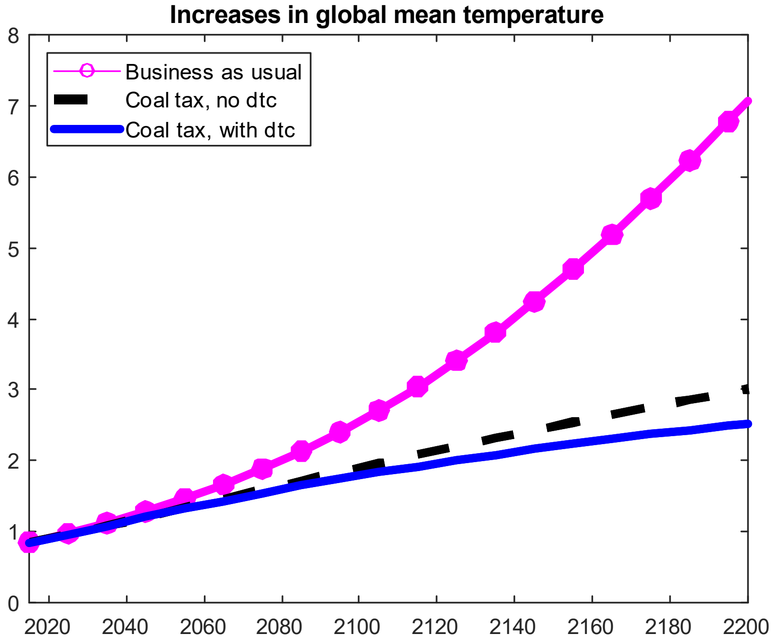

Figure 7: The interaction between carbon taxes and directed technical change.

5.1 Corner solutions with $\rho > 0$

Cost minimization amounts to minimizing the price index $P_{i,t}$ as defined by (15), subject to the relevant constraints. Formally, the problem is

$$\min_{p_{2,i,t} \leq \bar{p}_{2,i,t-1}, p_{3,i,t} \leq \bar{p}_{3,i,t-1}} P_{i,t}$$
$$s.t. \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{\varepsilon_2} \left(\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}}\right)^{\varepsilon_3} = \exp\left(a\right).$$

Using the constraint to replace $p_{3,i,t}$ and maximizing over $p_{2,i,t}$, the second derivative evaluated at the first-order condition is

$$\frac{\rho}{\rho - 1} \frac{\varepsilon_2 + \varepsilon_3}{\varepsilon_3} \frac{\varepsilon_2}{\varepsilon_3} \frac{1}{\left(p_{2,i,t}\right)^2} \left(\left(1 + \tau_{3,i,t}\right) \bar{p}_{3,i,t-1} \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{-\frac{\varepsilon_2}{\varepsilon_3}} \exp\left(-\frac{a}{\varepsilon_3}\right)\right)^{\frac{\rho}{\rho-1}} \lambda_3^{\frac{1}{1-\rho}},$$

which is negative if and only if $\rho > 0$. In this case, the first-order condition identifies a maximal cost. Minimum energy cost then arises at one of the corners, i.e., we obtain the following.[31]

$$\left(\bar{p}_{2,i,t-1}e^{\frac{-a}{\varepsilon_2}}, \bar{p}_{3,i,t-1}\right) \text{ and } \left(\bar{p}_{2,i,t-1}, \bar{p}_{3,i,t-1}e^{\frac{-a}{\varepsilon_3}}\right).$$

It is immediate that if and only if

$$X_{2/3} \equiv \frac{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} \exp\left(\frac{a}{\varepsilon_2}\frac{\rho}{1-\rho}\right) + 1}{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} + \exp\left(\frac{a}{\varepsilon_3}\frac{\rho}{1-\rho}\right)} < 1, \tag{37}$$

where $\hat{p}_{2,3} \equiv \frac{(1+\tau_{2,i,t})\bar{p}_{2,i,t-1}}{(1+\tau_{3,i,t})\bar{p}_{3,i,t-1}}\left(\frac{\lambda_2}{\lambda_3}\right)^{-\frac{1}{\rho}}$, then costs are lower if the first corner is used (i.e., when all resources are employed reducing the cost for producing fuel of type 2). The cut-off where energy firms are indifferent between the corners is

$$\bar{p}_{2,3} \equiv \left(\frac{\exp\left(\frac{\rho}{1-\rho}\frac{a}{\varepsilon_2}\right) - 1}{\exp\left(\frac{\rho}{1-\rho}\frac{a}{\varepsilon_3}\right) - 1}\right)^{\frac{\rho-1}{\rho}}.$$

Since $X_{2/3}$ is increasing in $\hat{p}_{2,3}$ when $\rho > 0$, costs are lower (higher) in the corner where costs of fuel 2 are reduced whenever $\hat{p}_{2,3} < (>)\, \bar{p}_{2,3}$.

A key difference between the result here and in the previous subsection is that the direction of R&D can permanently be shifted by a temporary tax. To see this, note that the base-line costs enter in the optimality condition (37). A temporary tax may be required to make R&D switch to renewables. Over time, this leads $\frac{\bar{p}_{2,i,t-1}}{\bar{p}_{3,i,t-1}}$ to increase and, eventually, no tax is required to keep R&D directed towards reducing the cost of renewables. This is the mechanism in Acemoglu et al. (2012).

6 Concluding remarks

The key message of the present paper is methodological: there is important insight to be gained from evaluating quantitatively designed integrated assessment models of climate and the economy for various policy options available. We think that the framework we present here can be used to answer many important questions and we analyze a number of examples of such questions here: we find rather robustly that (i) "overdoing it"—taxing carbon at rate that *ex post* turns out to be unnecessarily high—is a good insurance mechanism; (ii)

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[31] We have not yet proved global concavity, but that seems straightforward.

cost inefficiencies from not using a uniform carbon tax are major (and have uneven effects across the world); and (iii) that it is hazardous to pin our hopes to the belief that (subsidies to) green technology will improve at a rapid rate, at least from the perspective of climate change.

There is much left to do, however. In fact, the shortcomings of the current framework are arguably its strengths, from the perspective of Oswald and Stern (2019): there is plenty of work to do for economists! Let us therefore comment briefly on some aspects of our model that can be improved upon or extended. To begin with, like the natural-science insights we build on and incorporate into the economic model, we need the economic model to quantitatively capture the key historical developments in a satisfactory manner. Does our model satisfy this criterion? The question amounts to: is the setup we use a reasonable medium- to long-run economic model of the world? Let us point out here that, as far as we can tell, there is no off-the-shelf *quantitative* model of the evolution of the world economy. Perhaps economists did not find it useful to construct such a model? In light of, for example, the recent secular fall in the real interest rate around the world, that answer seems hard to believe. Or has it simply been too difficult to construct it? We are not sure, but our impression is that it is only with the global question of climate change that world-economy models have started to appear. It goes without saying, then, that such models can be improved in a great many ways.

One type of extension is to allow several production sectors, including agriculture and the specific challenges facing this sector in the developing world. In this context, a medium- to long-run model of the world economy should have some mechanisms describing the conditions under which developing countries make successful transitions and converge toward the levels of output per capita in the richer countries; whether such transitions take place in particular must be important for the ability of these countries to adapt to climate change. All of these important issues are side-stepped in our modeling in this paper.

We have also constructed a model that is (almost) solvable in closed form. A closed-form solution is by no means necessary—we believe that the computational methods available today more than suffice for solving models similar to the one here under more general assumptions—but, actually, turns out to be a reasonable starting point quantitatively for the questions at hand here. Checking robustness by altering preferences and technology outside the closed-form realm would clearly be valuable, though. Another obvious limitation of the model is that it does not allow for trade between the oil-importing regions. If trade is allowed, a strict climate policy in one region may lead production of some goods to move to regions with less strict climate policies. Trade thus introduces the possibility of carbon leakage in coal. Since this type of trade is not allowed in the current version of the model, the analysis abstract from the potential importance of carbon leakage in coal. Our results in this paper show that local climate initiatives are insufficient; trade and carbon leakage in coal are only expected to amplify these results. In more recent work, we are currently working on an extension of the model that allows for trade in intermediate goods.

Another convenient assumption we used here is the non-existence of an international bond market. Allowing bond trade between the oil-consuming countries would not change the results as long as these countries all are on a common balanced growth path. We assumed that China, India, and Africa currently are growing significantly faster than the other regions, however. In this setting, the possibility to trade in bonds would induce the faster growing regions to run, potentially very large, current account deficits, and hence our modeling appears inadequate. A somewhat puzzling feature of the data, however, is that we do not observe this, but rather the opposite, even further underlining the need for a global macroeconomic model that fits the main facts. Allowing for trade in bonds between oil consumers and oil producers would also be valuable. Such an extension would complicate the oil-supply decision. In particular, the oil price would then obey the Hotelling rule. As in the case of capital flows, however, the basic model prediction there—that oil prices rise secularly at the rate of interest—does not seem to be borne out in the data. Against this background, although one would like to allow for trade in bonds one would arguably want to add other features that make sure that the model can match the key historical patterns of quantities and prices.

Finally, the way we model the R&D decision is potentially also too stylized. Specifically, we assumed that the production of all energy inputs (except oil) can be reduced at a cost that is determined by a log-linear constraint. The result that proportional taxes are impotent directly relies on that specific functional form. Future research needs to evaluate the sensitivity of this and other results with respect to this assumption. In particular, microeconomic evidence on the R&D process as in Popp (2002) and Aghion et al. (2016) may be possible to put to productive use here.

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A Appendix

A.1 Calibration

Table 1: Parameters of the damage functions for the each region

	Linear ϕ_1	Quadratic ϕ_2
US	$0.000 * 10^{-2}$	$0.1414 * 10^{-2}$
EU	$0.000 * 10^{-2}$	$0.1591 * 10^{-2}$
China	$0.0785 * 10^{-2}$	$0.1259 * 10^{-2}$
Africa	$0.3410 * 10^{-2}$	$0.1983 * 10^{-2}$
India	$0.4385 * 10^{-2}$	$0.1689 * 10^{-2}$
South America		
Oceania		

Source: Nordhaus (2010).

Table 2: The calibration

Parameter	Value
α	0.30
β	0.985^{10}
ν	0.031
ρ	-0.058
ρ_h	0.90
λ_1^{oil}	0.44
R_0	330 Gtoe
λ_1	0.543
λ_2	0.102
λ_3	0.356
$p_{2,t}$	US \$74/ton
$p_{3,t}$	US \$600/ton
$p_{4,t}$	US \$347/ton

Parameter values used in the main calibration of the model.

Table 3: Calibration across regions

	Initial TFP	Initial capital stock	Regional γ
US	52.7250	38.5336	$2.395 * 10^{-5}$
EU	52.7250	38.8080	$2.698 * 10^{-5}$
China	52.7250	39.2000	$2.514 * 10^{-5}$
Africa	24.6050	12.3480	$5.058 * 10^{-5}$
India	34.1050	20.0900	$5.031 * 10^{-5}$
South America	24.6050	12.4460	$2.57 * 10^{-5}$
Oceania	43.6050	29.2040	$2.74 * 10^{-5}$

Regional parameter values used in the main calibration of the model.