Owners' Labor Supply and Firm Dynamics*

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Abstract

Owner hours form a large part of US firms' labor input, and they have a systematic positive relationship with firm size and output. We introduce owners' endogenous labor supply into a model of entrepreneurial choice with managerial and blue-collar labor and financial frictions. The model matches well the positive relationship between owners' effort and contemporaneous and future firm performance. Owner hours allow the majority of firms to stay small, yet, a small fraction of firms to quickly overcome financial frictions and have large employment growth. Reducing financial frictions increases wealth inequality at the bottom of the entrepreneurial wealth distribution and decreases wage inequality between blue-collar and managerial workers.

JEL: E23, J22, J23, L26.

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1 Introduction

A staggering feature of US business dynamics is the large heterogeneity among firms. Only 50% of newly found businesses survive until year eight. Among the ones that survive, many remain small, but a few sustain rapid employment growth. Initial seminal works on entrepreneurship and firm dynamics such as Lucas (1978) and Hopenhayn (1992) use the factors related to productivity and talent as the determinants of firm success. Quadrini (2000), Cagetti and De Nardi (2006), Buera and Shin (2013), and Midrigan and Xu (2014) extend these models to capture the role of financial frictions in shaping firm outcomes and an ownerÂŽs wealth as a tool to overcome such frictions.

This paper studies owners' labor supply as an additional factor in shaping heterogeneity in firm survival, output, and entrepreneurs' wealth. There are two motivating features in the data regarding owner hours: high levels on average and large cross-sectional variation. Table 1 highlights the facts using the Survey of Business Owners (SBO). An average owner works around 40 hours in her own firm and 45 hours conditional on employing a worker.[1] What is more, 70 percent of firms do not employ any workers, in line with the findings of Hurst and Pugsley (2011). Though, even for employers, owner hours are on average 52 percent of the hired labor within a firm. Regarding the cross-sectional variation, the top 25 percent of owner hours are around 1.6 times as long as the bottom 25 percent, both for employers and non-employers.[2]

To understand the interplay between owner hours, financial frictions, and firm success, we extend the standard general equilibrium model of entrepreneurial choice with an endogenous labor supply decision of owners. Entrepreneurs differ in their productivity, wealth, and disutility of working, and they face financial constraints. Similar to Bhattacharya et al. (2013) and Lee (2017), the factors of production are capital, managerial labor, and blue-

[1] These levels are consistent with the Survey of Income and Program Participation (SIPP) analyzed in Yurdagul (2017).

[2] The data does not contain information on the hours spent outside of the business. In order to assure that entrepreneurs working little in their business do not work large hours as an employee, we consult to the SIPP, which has information on the amount of hours spent in one's own business and as an employee. We find that on average, business owners work less than 3 percent of their hours outside of their business (and earn less than 3 percent of total income from outside of their business). Even owners who spend less than 20 hours per week in their business, work on average only 25 percent of their total hours outside of this business. Taken together, working outside of one's own business is not a major revenue source for the vast majority of business owners.

Moment	Non-employer	Employer
Owner hours, mean	39.3	45.9
Fraction of firms	0.70	0.30
Owner hours / hired labor, mean		0.52
Owner hours, P75/P25	1.7	1.6

Table 1: Summary statistics

Note: The statistics are from the Survey of Business Owners (SBO). Owner hours are reported in weekly units. P75/P25 denotes the interquartile range. The Appendix C gives details on the variables and selection from the data.

collar labor. The entrepreneur chooses the capital to rent, and the managerial and blue-collar labor to hire from the market. She can also supply her own hours into the firm and choose how to allocate these hours between managerial and blue-collar tasks.[3]

We calibrate the model to match hours and labor decision of firms in the SBO and labor market moments from the Survey of Income and Program Participation (SIPP). The model replicates well several relevant non-targeted features in the data through the interaction of financial frictions, disutility of work, and endongenous labor supply. First, it matches closely the rapid growth in the average number of employees during firms' first years of operation. At entry, a significant share of entrepreneurs has a high disutility of work and, despite non-binding financial frictions, operate their business as non-employers without any desire to grow. These businesses have a relatively high exit rate providing growth in average hired labor through the extensive margin. At the intensive margin, firms with a good business idea aim to grow but financial frictions slow down this growth. As returns are large, owners work long hours in these firms. Moreover, long working hours help entrepreneurs to overcome financial frictions, thereby, sustain rapid employment growth. Thus, as in the data, working hours are positively correlated with contemporaneous and future firm performance. Regarding the type of task owners perform, we show that low wealth owners perform both blue-collar and managerial tasks. Because managerial hiring is relatively costly, owners specialize in this task with rising wealth levels. As managerial work complements the hired blue-collar labor, the model is able to generate owners' long working hours even at medium size firms. These forces are amplified for owners with a low disutility of working. We show

[3]Some recent papers study the role of management in shaping firm dynamics across countries. See, for instance, Bloom et al. (2016) for an overall review of the role of management practices; and Akcigit et al. (2016) specifically for the role of the ease of delegation to hired managers.

that an entry firm in the bottom 17% of the distribution of disutility of working grows almost twice as fast in terms of labor as a firm in the top 17% during the first eight years of operation.

We use counterfactual simulations to understand the impact on the entrepreneurial sector from owners providing their own hours. We differentiate between the ability to supply their own hours, from the ability of doing it flexibly instead of having fixed hours, as in Allub and Erosa (2014). Restricting owner hours to zero decreases the share of entrepreneurs in the economy from 7.6 percent to 1.8 percent. Particularly the least productive, those that are non-employers when an owner supplies her hours, exit the market, but also high productivity firms do not enter the market any longer. For them, supplying their own hours, thereby, overcoming financial frictions, is particularly important. As a result, total output in the entrepreneurial sector is almost 80% lower than in the baseline economy. Fixing owner hours to their average goes a long way towards the output in the baseline economy. In that case, about 5 percent of the population operate as entrepreneur and total entrepreneurial output is about 80% of the baseline economy. Yet, in the absence of long working hours of a few entrepreneurs, firm growth is still suppressed. After eight years, firms are on average 10% smaller than in the baseline model.

Long working hours of the highest productivity entrepreneurs and the presence of the low productivity non-employers that operate at their optimal size implies that firms are on average less borrowing constrained than in a model without endogenous labor supply. As a consequence, our model predicts a much milder increase in entrepreneurial production when relaxing the financial constraints than a comparable model without endogenous labor supply.

These mechanisms also enable our model to generate a thicker left and right tail in the wealth distribution among entrepreneurs, thereby, match the large inequality we observe in the data. Regarding the left tail, the non-employers hold very little wealth. Moreover, high productivity firms enter with little wealth because they know that they can use their own effort to overcome financial frictions. With respect to the right tail, the rapid growth of firms facilitated by owners' effort allows these firms to quickly reach high output levels, thereby, accumulate large wealth holdings.

These mechanisms also provide new insights for the dynamics in wealth inequality when reducing financial frictions. The ratio of the median to the 10th percentile of the wealth distribution increases with lower frictions. Because the least productive firms, which tend to be at the bottom of the distribution, already have sufficient assets to operate close to their optimal size, they reduce their wealth holdings by almost 1-to-1 when frictions are reduced. In contrast, the median firm is more severely borrowing constrained and decreases its wealth holdings by less than 1-to-1. The ratio of the 99th percentile to the median firm changes very little, and much less than in a model without owner hours. The reason is that the most productive firms, those at the top of the wealth distribution, are effectively less constrained in our model than in in a model without owner hours, because they can exert their own effort to mitigate financial constraints. Hence, they reduce their wealth holdings to a similar degree as the median firm in response to a decrease in financial frictions.

Besides wealth inequality, owner hours also have implications for the interaction of financial frictions with workers' earnings inequality. We find that the wage premium of managers relative to blue-collar workers decreases as a result of a decrease in financial frictions. Lower financial frictions imply that entrepreneurs need to work less as blue-collar workers at low wealth levels which increases the demand for these tasks. In addition, owners increase their managerial hours to complement the hired blue-collar workers, therefore, decrease the demand for managers and their wages.

The paper is organized as follows. Section 2 describes the model. Section 3 explains the calibration and Section 4 discusses the model's predictions for relevant empirical features. Section 5 shows the role of owner hours in shaping their life-cycle patterns and studies the interlink between owner hours, financial frictions, and inequality. Section 6 concludes.

2 Model

We study an infinite horizon closed economy with a continuum of households. Time is discrete, and the discount factor is β. Households derive utility from consumption and leisure, over which they have separable preferences. We assume log preferences for the consumption. The objective of each household i is to maximize its life-time utility given by

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{log(c_{it}) - v_{it}\frac{h_{it}^{1+\phi}}{1+\phi}\right\},$$

where c_{it} is consumption, h_{it} are the number of working hours, ϕ governs the individual labor supply elasticity, and v governs the disutility of work. At the beginning of a period, a household is characterized by her asset holdings, a_{it}, her entrepreneurial ability, z_{it}, and her disutility of working, v_{it}. The latter two follow independent AR(1) processes:

$$\begin{aligned}\log z_{it} &= (1-\rho_z)\log z_0 + \rho_z \log z_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0,\sigma_\epsilon) \\ \log v_{it} &= (1-\rho_v)\log v_0 + \rho_v \log v_{it-1} + u_{it}, \quad u_{it} \sim N(0,\sigma_v).\end{aligned} \tag{1}$$

A household is either working as an entrepreneur or as a salaried worker. We assume that a salaried worker works fixed 40 hours per week, $\bar{h}$. While this is certainly a simplification, it is consistent with a large literature in labor economics suggesting that working hours in salaried work are inflexible. (See Dickens and Lundberg (1993), Stewart and Swaffield (1997), Boheim and Taylor (2004), Aaronson and French (2004), among others.) A household employed as salaried worker performs a task, j, that is either managerial or blue-collar, $j \in \{m, b\}$ and earns a hourly wage w_j. Representing labor market risk, tasks evolve stochastically. We denote the probability to be a manager next period by ϵ_m^j.

In general, a household can decide at the end of each period whether to continue as salaried worker or start a business. However, with probability χ_W she is forced to start a business. This additional margin of entering entrepreneurship allows the model to generate the lower tail in the income and productivity distribution of observed entrepreneurs. It is a reduced form of modeling the fact documented by Hurst and Pugsley (2011) that only one-third of entrepreneurs report “Having a good business idea or creating a new product” as the first reason to start a business. Reasons related to the necessity to generate income or non-pecuniary reasons are other major factors behind start-up decisions.

Hence, the value of a household as a worker in a given period, if she is assigned to a task $j \in \{m, b\}$ is given by:

$$V^w(z,j,v,a) = \max_{a'}\Big\{log(c) - v\frac{\bar{h}^{1+\phi}}{1+\phi} + \beta E_{z',v'|z,v}\big[\chi_W\, V^e(z',v',a')$$
$$+ \quad (1-\chi_W)\max\{V^e(z',v',a'), \epsilon^j_m V^w(z',m,v',a') + (1-\epsilon^j_m)V^w(z',b,v',a')\}\big]\Big\}$$

s.t.

$$c = w_j\bar{h} + (1+r)a - a',$$

where $V^e(z',v',a')$ is the value of being an entrepreneur next period.

Entrepreneurs' production function depends on whether or not they employ workers. Similar to Allub and Erosa (2014), Bhattacharya et al. (2013), and Lee (2017), production of employer entrepreneurs requires both managerial and blue-collar components, as well as capital. Importantly, these entrepreneurs can exert their own labor into their firms, allocating their hours into the two labor units:

$$Y_{it} = z_{it}\left[K^{\alpha}_{it}\Big(h_{m,it} + L_{m,it}\Big)^{\lambda}\Big(h_{b,it} + L_{b,it}\Big)^{\mu}\right]^{\eta},$$

where Y is value added of firm, K is the employed capital, h_m and h_b are the managerial and blue-collar hours of the owner, L_m and L_b are the hired labor for managerial and blue-collar tasks. Parameters α, λ, and μ govern the factor shares with $\alpha + \lambda + \mu = 1$, and $\eta < 1$ gives the span-of-control.[4]

For non-employers the distinction between managerial and blue-collar worker becomes idle. We assume a production function that is similar to the one for the employing firms:

$$Y_{it} = z_{it}\left(K^{\alpha}_{it}(\tilde{\kappa}h_{it})^{\tilde{\lambda}}\right)^{\eta},$$

where h is the total hours of the owner worked in the firm. Non-employers share the same span of control with the rest of the entrepreneurs, but their labor input is shifted with a parameter $\tilde{\kappa}$. Parameter $\tilde{\lambda}$ captures the weight of owners' hours in their production.

An entrepreneur decides on her future wealth, a', her own labor supply (h_m and h_b) in her firm, how much outside labor to hire at market prices (w_m and w_b), and how much capital

[4] For simplicity, we assume that one hour worked in a unit by an owner is equally productive as one efficiency unit hired from others.

to rent at market price $r + \delta$, where δ represents the capital depreciation rate. Yurdagul (2017) argues that the inflexibility in the working hours in salaried work can account for some of the observed cases of entrepreneurship. Accordingly, the flexible hours capture one item within the "Non-pecuniary" motives mentioned above.

An entrepreneur has the option to become a worker with probability $(1-\chi_E)$, i.e., receive a job offer as salaried worker. When switching to salaried work she draws with probability ϵ_m^e $(1-\epsilon_m^e)$ a managerial (blue-collar) task. Define the upper envelope over quitting into employment given productivity and disutility of working realizations:

$$W(z', v', a') = \max\left\{V^e(z', v', a'), (1-\epsilon_m^e)V^w(z', b, v', a') + \epsilon_m^e V^w(z', m, v', a')\right\}.$$

Then the value of entrepreneurship is

$$V^e(z, v, a) = \max_{a', K, h_m, h_b, L_m, L_b} \left\{ \log(c) - v\frac{h^{1+\phi}}{1+\phi} + \beta E_{z', v'|z, v}\left[(1-\chi_E)W(z', v', a') + \chi_E V^e(z', v', a')\right]\right\}$$

s.t.

$$c = \max\{\Pi_E, \Pi_{NE}\} + (1+r)a - a'$$

$$\Pi_E = z\left[K^\alpha \Big(h_m + L_m\Big)^\lambda \Big(h_b + L_b\Big)^\mu\right]^\eta - w_m L_m - w_b L_b - (r+\delta)K$$

$$\Pi_{NE} = z\tilde{\kappa}\left[K^\alpha h^{\tilde{\lambda}}\right]^\eta - (r+\delta)K$$

$$w_m L_m + w_b L_b + (r+\delta)K \leq \theta a$$

$$h = h_m + h_b$$

$$h_j \in \{0, h_j^2, .., h_j^N\},\ L_j \geq 0,\ j \in \{m, b\}$$

where we assume the firm faces a borrowing constraint that requires it to prefund its operational expenses by using a fraction of owners' wealth, θ, and restrict the optimal choices of owner hours in each task (total hours in case of non-employers) to be in a discrete set with N choices.

Finally, following Quadrini (2000) and Cagetti and De Nardi (2006), there is a non-entrepreneurial sector represented by a production function exhibiting constant returns to scale, with relative factor shares equal to those in the production function of employer entrepreneurs that hires any resources not used in the entrepreneurial sector:

$$Y_n = K_n^{\alpha} L_{b,n}^{\lambda} L_{m,n}^{\mu}.$$

The model does not allow for a complete analytical characterization, and we solve it numerically given the first order conditions derived in Appendix A.[5] Nevertheless, it is useful to discuss the implications of the model for decisions on owner hours and hired labor, before moving on to the calibration section.

Discussion: Wealth, financial frictions, and owner hours. Figure 1 plots the optimal choices of an entrepreneur for allocating her own time, and hired labor into managerial and blue-collar tasks given a particular level of productivity and disutility of work within the calibration described in the next section. On the x-axis, we plot the level of wealth (relative to the average) to highlight the role of financial frictions in these policies.

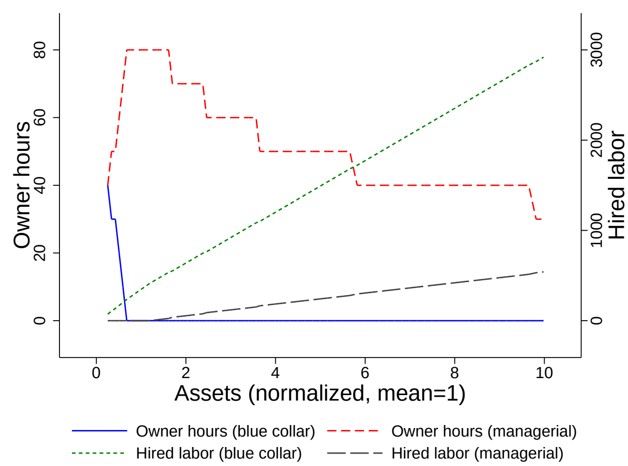

Figure 1: Policies on owner hours and hired labor across wealth

Note: The figure shows the policy functions for the 98th percentile productivity and median disutility of working in the underlying distribution of shocks. In these plots we use the parameterizations that we highlight in the next section. Hours are given in weekly units. Owner hours in each task are restricted to be on a discrete grid, as detailed in Appendix B.

For low levels of wealth, the firm does not hire any managers, and hires very few blue-collar workers due to the financial constraints. Accordingly, the owner shoulders most of the tasks in the firm, allocating a big share of her time into blue-collar work. As the wealth level increases, the firm chooses to hire only blue-collar workers initially. Since, managerial

[5] Appendix B gives the details of our numerical strategy for solving the model.

labor is relatively expensive, the owner shifts her hours gradually into managerial tasks to accompany the blue-collar labor. Eventually, she fully devotes her time into managerial tasks to keep up with the increasing number of workers to manage. Meanwhile, as the level of wealth increases further, the disutility costs of additional work are larger than the managerial wage, and there is a threshold wealth level after which the owner starts hiring managers from the market. As more managerial tasks are outsourced, the owner reduces her own working hours.

3 Calibration

We calibrate the model taking a period as a year. We set the discount factor, β, to 0.92 following Midrigan and Xu (2014). For the parameter governing the labor supply elasticity, ϕ, we use a value of 2 corresponding to an intermediate degree commonly estimated in the literature. We follow Cagetti and De Nardi (2006) in setting the capital share, α, to 0.33, the depreciation rate to 0.06, and the span of control, η, to 0.88.[6]

We calibrate the rest of the parameters to match moments from the US data. We use two data sources in the calibration. The SBO has the advantage of a large representative sample of US firms, with detailed information on their employment and sales. The SIPP, differently from the SBO, has information from all households, including salaried workers and business owners, and it has a panel structure. However, the number of observations on owners is much smaller than in the SBO and the employment information gives the firm size over only three categories.[7] Accordingly we use the SIPP to obtain targets that require panel data, or those that involve information from the salaried workers. For the rest of the moments, we use the SBO. Appendix C provides a more detailed description of the data samples.

We calibrate the degree of financial frictions, θ, to match the average hours of labor hired by firms. With the dispersion of productivity shocks, σ_z, we target the right tail

[6] Their population of interest, entrepreneurs with a desire to grow are similar to employers in our model. The span of control, η, determines their size with unconstrained policies.

[7] In the SIPP, we count people as entrepreneurs whenever they report having a business, the business income is above their income from being an employee, business income is at least \$50 per quarter, and the business size is at most 100 employees.

Parameter	Value	Basis
		Panel A: Literature or data directly
β	0.92	Midrigan and Xu (2014)
ϕ	2	Inverse of labor supply elasticity
α	0.33	Cagetti and De Nardi (2006)
δ	0.06	‘’
η	0.88	‘’
		Panel B: Calibrated
θ	0.5	Mean labor hired = 74.7
σ_z	0.106	90/75 labor ratio, employers = 2.0
z_0	0.50	Fraction of owners = 0.072
$\tilde{\kappa}$	0.59	Fraction of non-employers = 0.70
ρ_z	0.98	Average firm age = 12.2
χ_E	0.805	Number of 5-year-old, rel. to 1-year-old firms= 0.68
σ_v	0.8	Std. employer hours = 0.47
ρ_v	0.67	Persistence owner hours = 0.77
v_0	0.26	Mean owner hours, employers = 45.9
$\tilde{\lambda}$	0.62	Mean owner hours, non-employers = 39.3
χ_W	0.002	Mean output employers rel. to non-employers = 11.1
λ	0.165	Mean wage of managers rel. to workers = 1.68
ϵ_m^b	3.75%	Rate of transition blue-collar to manager = 3.75%
ϵ_m^m	78.8%	Rate of remaining a manager = 78.8%
ϵ_m^e	12.3%	Rate of transition owner to manager = 12.3%

Table 2: Parameters

of the distribution, i.e., the 90/75 labor ratio of employers. We target with the mean of log productivity that seven percent of the population works on their own account, as we observe in the SIPP. Similarly, we set the productivity of non-employer hours, $\tilde{\kappa}$, to assure that 70 percent of all entrepreneurs are non-employers. We target two moments regarding firm survival which we view as crucial to understand the role of owners' effort in firms' life-cycle. With the persistence of productivity shocks, we match that the average firm age is 12.2. The probability to be able to leave entrepreneurship, χ_E, governs the exit of young entrepreneurs and we target the fraction of firms of age 5 in the data relative to those of age 1.

We use the stochastic process for the disutility of working to match moments of entrepreneurs in the data. In particular, we set the autocorrelation parameter, ρ_v, and the dispersion σ_v, to match the autocorrelation and standard deviation in the data. We use the mean of the log disutility of labor, v_0, to match that employers work 46 hours per week. We use the weight of owner hours in non-employer production, $\tilde{\lambda}$, to match their average hours worked of 39 hours. With the probability of being forced out of salaried work, χ_W, we target that the average output of employers is 11 times as large as the output of an average non-employer.

We calibrate the managers' weight in production, λ, to get a wage premium over blue-collar workers of 1.68. We set the probabilities of being a manager, conditional on continuing in salaried work, for managers and blue-collar workers of last period, ϵ_m^m and ϵ_m^b, directly to the to annualized rates in the SIPP. Similarly, we calibrate the probability to become a manager after exiting entrepreneurship, ϵ_m^e, to the value observed in the data.[8] Targeting the flow rates suffices to get the relative population shares correct. The fraction of blue-collar workers is 0.786 (0.793) and the fraction of managers is 0.139 (0.135) in the model (data).

We show in Appendix D that the model matches the targeted moments well. In the next section, we compare the model implications in non-targeted moments to those in the data. Moreover, Appendix E increases some of the less standard parameters by 10% and shows that the moments presented in the next section are not very sensitive to these.

4 Results

In our calibration, the only measure on the dispersion of firm size that we target is the upper tail of the employment distribution among employer firms. Nevertheless, the model can account for the observed overall dispersion of hired labor, with a standard deviation of log-hired labor at 1.2 compared to 1.03 in the data. Importantly, the model generates firms as big as we observe in the SBO. In particular, the 90th and 95th percentiles of the employment distribution in our model simulations hire 599 and 912 weekly hours externally, where in the data these thresholds are at 490 and 1190 hours.

The model also generates a thick right tail in the output distribution; though somewhat less than in the data. The 90/75 percentile ratio is 1.8, compared to 2.5 in the data. Since the model has fewer employer firms of very low output levels, it falls short in generating the overall variation in output, with a standard deviation of log-output at 0.86 compared to the

[8]Since we assume that the endogenous entry and exit into entrepreneurship is decided before realizing the tasks in case of salaried work, we can directly pin these transition probabilities from the data.

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
Hired labor			Output			Owner hours		
Sd (logs)	1.0	1.2	Sd (logs)	1.36	0.86	P90/P75	1.0	1.3
P90	490	599	P90/P75	2.5	1.8	P75/P25	1.6	2.0
P95	1190	912	P75/P25	5.8	3.1			

Table 3: Summary of non-targeted moments

Note: Statistics correspond to employer firms. In the data we also discard firms employing more than 100 workers to be consistent with our model.

value of 1.36 in the data.

The calibration, targeting the standard deviation in owner hours among employer firms, generates an interquartile ratio of 2.0, slightly above the corresponding ratio of 1.6 observed in the data. Meanwhile, the right tail of the owner hours distribution in the model is not very thick, with the 90th percentile working only 30 percent more than the owners working at the 75th percentile. This is in line with our observation in the data that 90/75 percentile ratio is equal to 1.

In our calibration, we do not target the growth patterns of firms observed in the data. Accordingly, we find it useful to compare our model's implications for how firms grow with age, as a test of the model's performance. Figure 2a shows that firms of age 5 in the SBO hire about twice as many hours as firms of age 1, with total labor around 200 weekly hours compared to 100 hours. Similarly, firms of age 18-27 hire more than 150 percent more labor than firms of age 1. This is partially driven by the fraction of non-employer firms going down from 90 percent to 60 percent between age 1 to 18-27. Our model replicates these patterns well, both in terms of the rates of growth and also in terms of the levels. An average firm of age 1 in our model simulations hire around 90 hours, which becomes around 140 hours at age 5. Firms of age 18 to 27, on average, hire 300 hours in the model. The fraction of non-employers among firms decreases from around 90 percent to 55 percent in our model.

The comparisons of averages across age groups reflect both the selection among firms across years, and also the growth performance of the survivors. At entry, the distribution is already tilted to more productive firms and survivors are on average those with good productivity outcomes. With regard to the value of leisure, we highlight in the next section that the entry distribution is U-shaped: Particularly low and high disutility types are more likely to start a business. Those with a low disutility are those working many hours and growing their company. Those with a high disutility are predominantly non-employers, whose share declines in age.

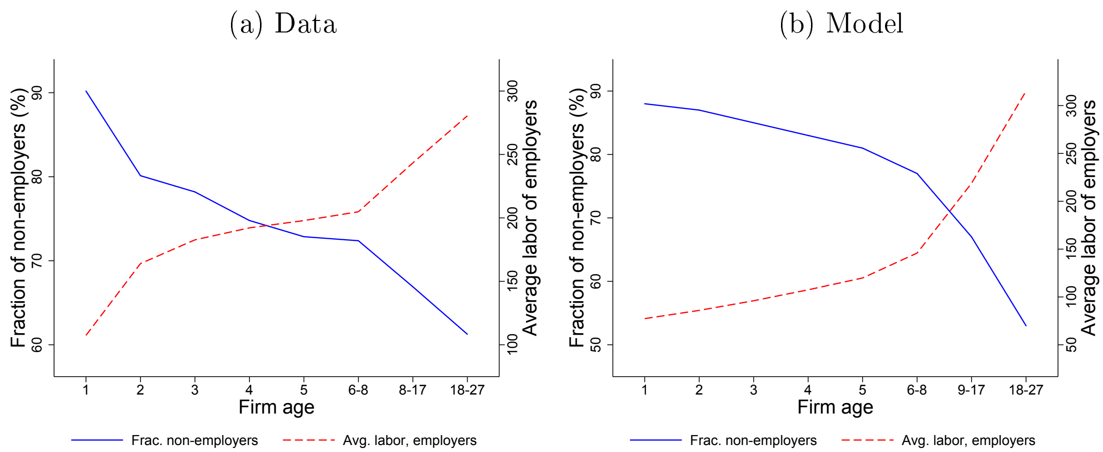

Figure 2: Employment patterns over age

Note: The figure plots the averages for each age group in the model and in the data. We provide details on data variable definitions in Appendix C.

We now study the model performance in generating the relationship between owner hours and firm size. By calibration, employers work close to 6 hours more on average than the non-employers. It is instructive to understand the source of this difference in our model. For a non-employer, her shadow value of working one more hour is her marginal product which absent financial frictions must be smaller than the wage of a blue-collar worker. For an employer, this shadow value is the wage of hiring a manager, in case she is hiring managers, and the wage of blue-collar workers otherwise. We find that in equilibrium, the shadow value for employers is much larger than for a non-employer. At the same time, the average disutility of work is smaller for the former. Both factors lead to longer hours of work for employers.

Figure 3a shows that in the SBO, the owner hours increase from a level of 40 hours per week for the non-employers, to a level around 46 for the firms of middle size. This pattern reverses for firms that are even larger, with about 44 hours per week for employers of 50 to 99 workers. Figure 3c shows that the model replicates the observed hump-shaped pattern of owner hours with respect to firm size. Owner hours increase from non-employers to small employer firms, and remains similar for firms employing 1 to 9 workers. For larger size groups, average hours worked by owners goes down to about 25 hours. This is in line with our discussion on the interplay between financial frictions, owner hours and firm size in Section 2. Owners in our model shoulder the managerial duties for the low to medium levels of firm size. This makes their hours increase with the number of workers, which are typically hired for blue-collar tasks. For large enough firms, owners outsource the managerial tasks, and enjoy more leisure generating the hump-shaped relationship in our model.

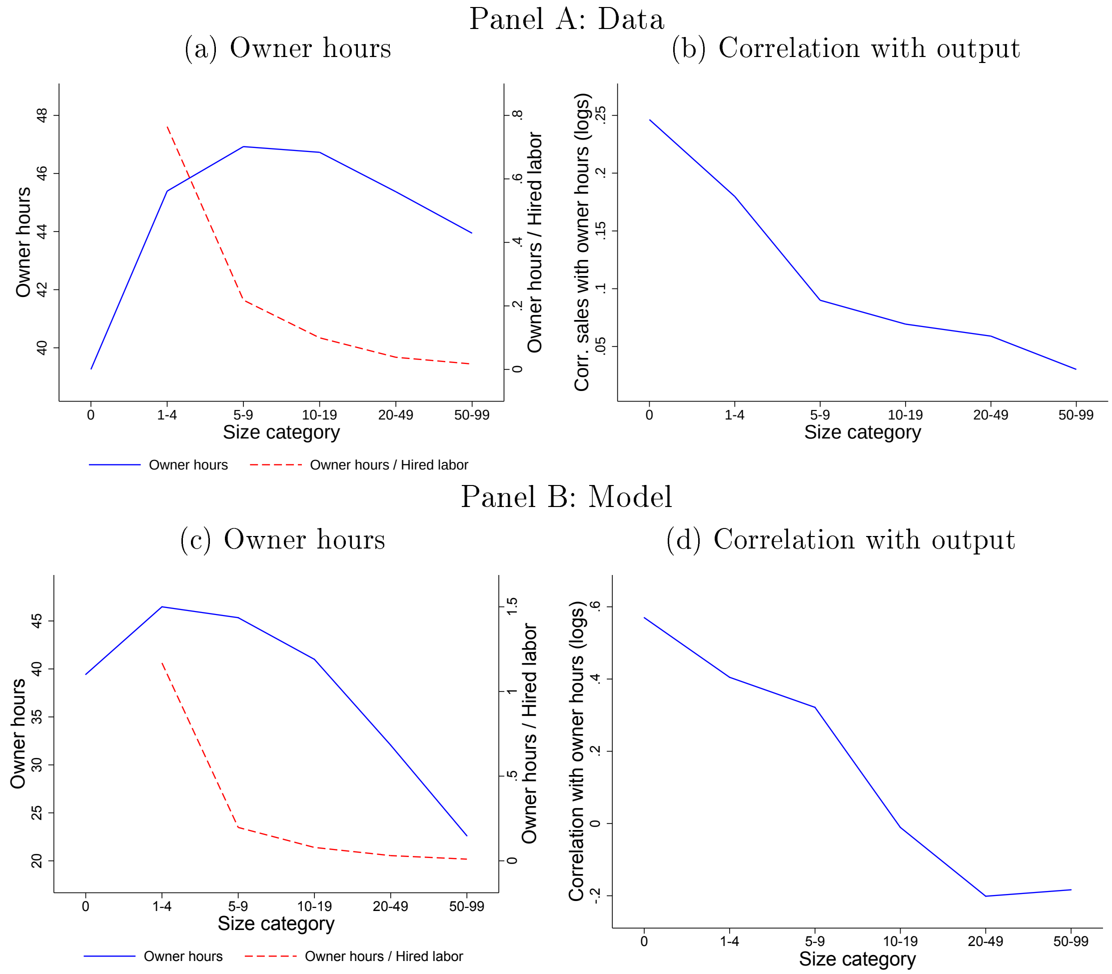

Figure 3: Owner hours, labor and output

Note: We pool all the firms both for the data and for the model. Profits in the model is the output net of all costs of externally hired or rented factors. Hours are given in weekly units. We provide details on data variable definitions in Appendix C.

Figures 3a and 3c show that the patterns of average owner hours to hired labor ratio against firm size are also similar in the data and in the model. In the SBO, owner hours are about 80 percent of the hired hours at firms employing 1 to 4 workers. Meanwhile, in larger firms, the owner hours relative to hired hours decline in a monotone way, though, even at medium-size firms (10 to 19 employees) owner hours contribute to around 10 percent of the total hired labor. At larger firms, this weight converges to zero due to the increase in the total hired labor. In the model, the smallest employer firms have this ratio around 100 percent, which gradually approaches to zero at larger firms with a shape similar to the one observed in the data.

Without financial frictions, total labor in managerial and blue-collar units would be at the optimal size given the entrepreneurial productivity. Owner hours would help increase firm profits by saving payroll expenses, but this would not alter the firm output. The only link between firm output and owner hours would be a negative relationship that arises due to the concavity in marginal utility of consumption. In particular, owners of firms with high output would have a low propensity to exert effort to substitute hired labor and to increase firm profits. Yet, Figure 3b shows that owner hours (in logs) are positively correlated with output in the data. This correlation is particularly strong at small firms (around 0.20) and falls towards zero at firms with 50 to 99 employees. Importantly, the model replicates this qualitative feature. Figure 3d shows that this correlation is about 0.6 for firms with no employees, and goes down with size. The mechanism through which the model generates the decreasing pattern in these correlations is the interaction of financial frictions with the high degree of substitutability between owner hours and hired labor within each task. With the frictions, owner hours help increase the total hours within each unit. With more workers, this role is more limited due to diminishing returns to labor. This implies that correlations decrease with size, as it does in the data.

Besides a positive correlation between owner hours and contemporaneous firm output, entrepreneurial effort can also be positively correlated with future performance of their firms. In the next exercise, we use the Kauffman Firm Survey (KFS) that follows a single cohort of entering firms over time to study how the differences in owner hours in the initial phases of the life-cycle can be related to the differences in firms' performance later on, and compare those with the implications of the model. Life-cycles of firms whose owners work very long and short hours initially can look different for two reasons. First, owners that work long hours in the first years of the business will work more in the following years, if the factors that make them work long hours (low disutility of working, high firm productivity) in the beginning will persist. Figure 4a shows that in the data, owners that work particularly long (short) hours in the beginning of the operations typically work long (short) hours in the years to come, though we observe some mean reversion. Second, the initial effort of the owners itself helps them build the capital they need to increase the output levels in the future. In fact, figures 4b and 4e illustrate that the differences in the output of the two groups of firms remain at around 2 log points and 1.5 log points in the model. Similar patterns are also visible for employment in figures 4c and 4f. In the data, firms the differences in the hired labor between the two groups are initially around 50 hours and after eighth years, this difference actually increases to around 150 hours. In the model, the corresponding difference starts at around 5 hours and in 8 years becomes as large as 40 hours. Large persistent differences in firm performance that are related to firm observables during the first year of operation also links to recent evidence by Pugsley et al. (2017) who show that most post-entry firm growth is predictable by founding year observables.

Overall, the model-implied patterns of owner hours and their relationship with the hired labor suggest that this model presents an appropriate laboratory to study the owners' labor supply. In what follows, we pursue the implications of this model for firm dynamics.

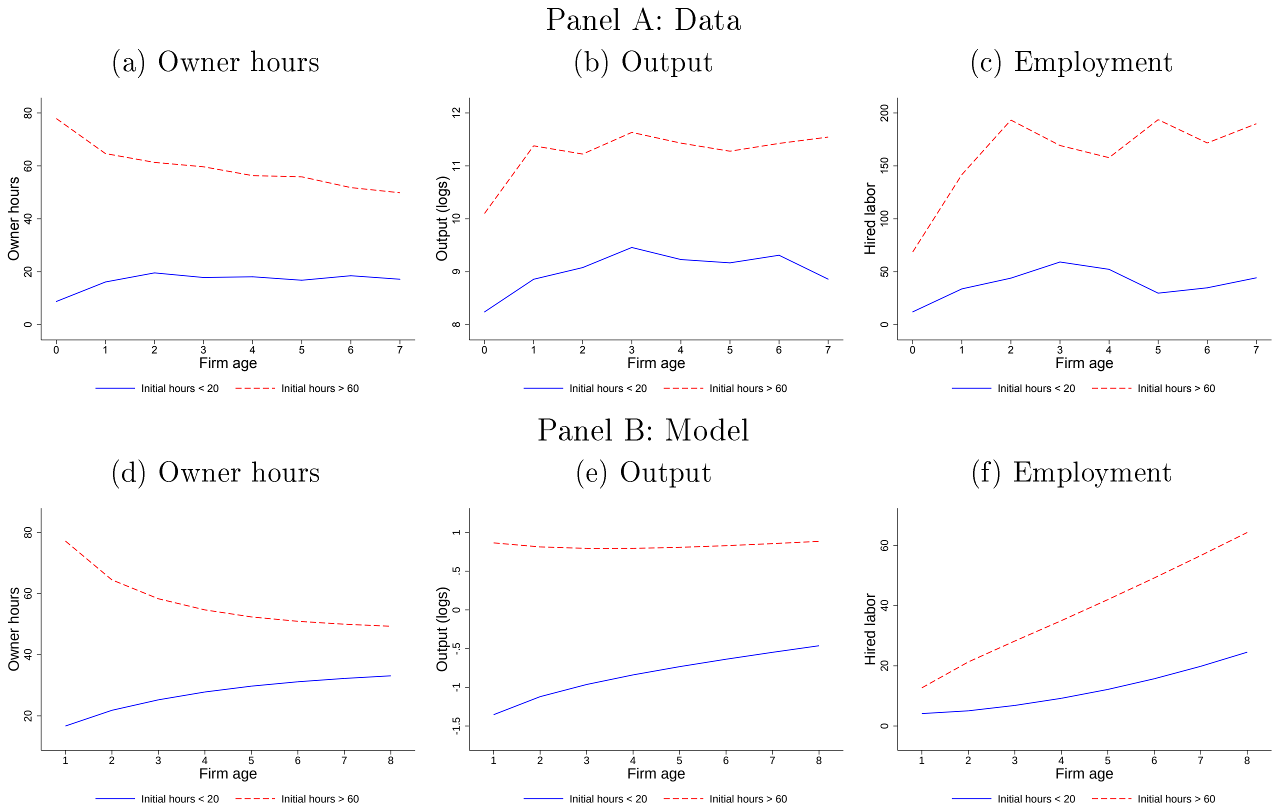

Figure 4: Initial hours and the life-cycle

Note: The figures in Panel A use two subsamples of the population of firms that survive all the waves of the Kauffman Firm Survey. The figures in Panel B use two subsamples of the population of firms from our model simulation. For each panel, one group consists of the firms whose owners work less than 20 hours per week, and the other by those whose owners work more than 60 hours per week, during the first year of operations. Then they plot the averages for these firms in each period.

5 Role of entrepreneurial hours

In this section, our objective is to highlight the role of owners' effort in shaping firm performance and inequality. There are three questions we answer: (1) How does heterogeneity in the disutility of working shape firms' life-cycle patterns? (2) What is the role of owner hours in aggregates such as total output of the entrepreneurial sector and firm survival? (3) How does inequality in the economy and its interaction with financial frictions change once we allow for owner hours? We use our general equilibrium framework to go over each of the three questions separately.

5.1 Heterogeneous disutility of work and firm performance

In Section 4, we emphasize that initially high (low) owner hours are followed by particularly good (poor) firm performances. We can use the variation in initial (exogenous) disutility of working to provide a more causal relationship between owner hours and life-cycle differences. In Figure 5, we show a comparison of two different samples of firms whose owners have particularly high (top 17% of the ergodic distribution) and low (bottom 17%) disutility of work at entry.

The life-cycle prospects of these firms differ for two main reasons. First, disutility of working is a persistent process; thus, the amount of hours worked is persistently different among the two groups even without any further differences. Figure 5a confirms this. Due to the mean-reversion in the disutility of work, the difference in the averages between the two groups goes down from 0.9 to 0.2 log points in 5 years, and becomes negligible after 8 years. Figure 5b shows that the hours of owners in the two samples converge to each other in a fashion similar to their disutilities of working. The second reason for the different patterns in the two samples is that the owners' initial effort determines how fast they accumulate resources to grow out of the financial constraints. The faster pace of accumulating wealth for the sample with more hardworking owners counteracts the narrowing gap in the disutilities of working. In fact, Figure 5c shows that the level of log-output for the two samples do not converge to each other. The differences in output remain around 6 log points after 8 years. The employment patterns for these two groups actually diverge from each other with time. Initially, the sample with higher disutility of working hire more labor due to the substitutability between owner hours and hired labor. Meanwhile, as firms age, the owners with initially lower disutility of working manage to increase their firm size much more than the other group, due to the faster accumulation of wealth, hence, higher ability to grow.

Among firm entrants, the model exhibits a large amount of heterogeneity in the value of leisure. At age one, the lowest and highest sextile of the ergodic distribution of the disutility of working are represented by 17 and 24 percent, respectively. In order to quantify the role that the variation in entrepreneurs' working attitude play on firms' life-cycle, we give in Table 4 the effects of shutting down this variation ($\sigma_v = 0$) in our benchmark calibration.[9]

[9] In this exercise, we adjust the v_0 to maintain the baseline mean disutility in the ergodic distribution.

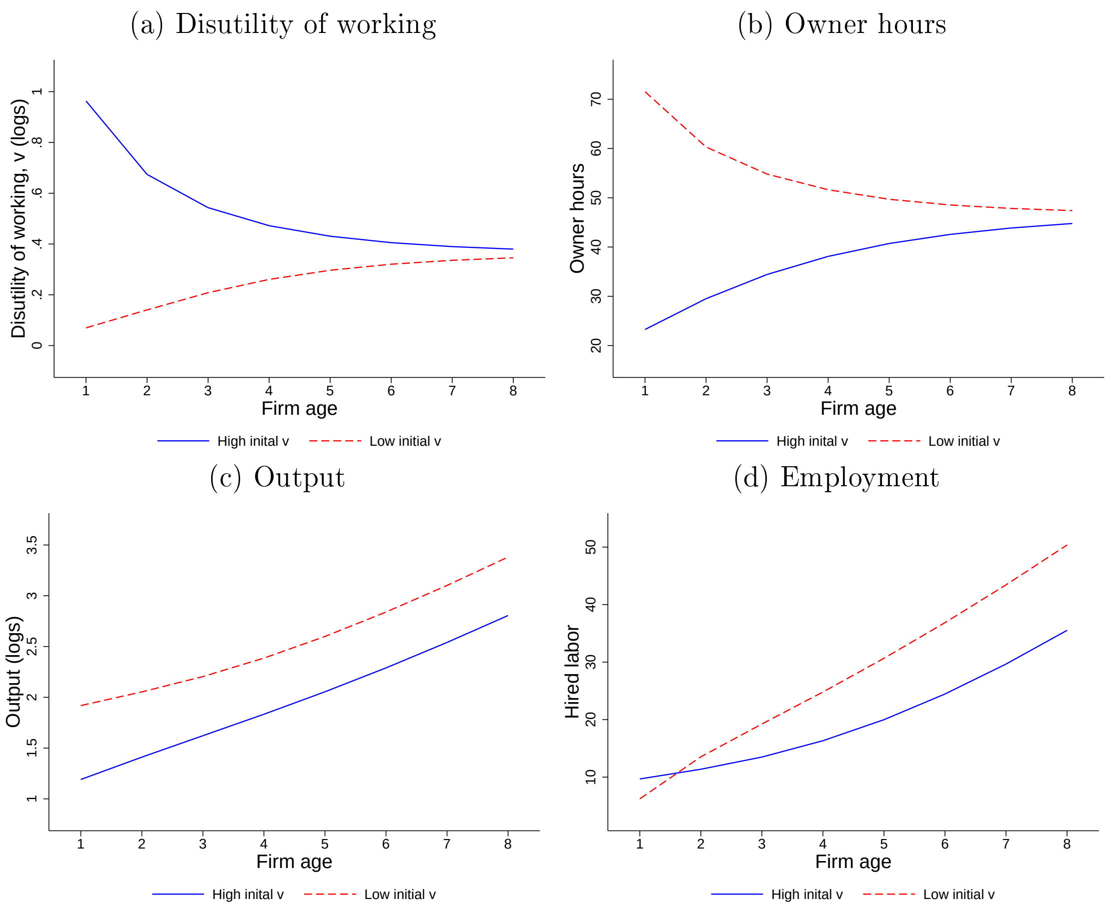

Figure 5: Initial disutility of working and the life-cycle (model)

Note: The construct the figures, we follow two different samples of firms with different disutility of working. One sample consists of those of the bottom 17% of the ergodic distribution, and for the other we use the top 17%. We then plot the following averages for each age of these groups. For year 1, these samples account for 17 and 24 percent of all 1-year-old firms. Hours are in weekly units.

The patterns of average labor hired across age groups of firms flatten significantly as a result. Specifically, the average labor of employer firms of age 4 relative to age 1 goes down from 1.39 to 1, and the same ratio for age 6-8 relative to 1 goes down from 1.89 to 1.30. The rationale is the one highlighted in Figure 5d. First, firms start with more employees as their owners provide less effort. Second, firms miss the funds to finance rapid growth. The table also highlights that the high average working hours of entrepreneurs are driven by their right tail. Once we eliminate low disutility of working, the working hours of the observed business owners go down by about 20 percent. Eliminating variation in the disutility of work also affects the desirability of entrepreneurship. There is a slightly lower survival prospects of firms in this case compared to the benchmark, and a decline in the fraction of entrepreneurs.

	Benchmark	$\sigma_v = 0$
Owner hours	44.6	36.0
Hired labor	255.4	274.6
Firm survival	0.50	0.48
No. owners	0.08	0.06
Ratio of avg. labor		
Age 4/1, overall	2.01	1.82
Age 6-8/1, overall	3.78	3.44
Age 4/1, emp.	1.39	1.00
Age 6-8/1, emp.	1.89	1.30

Table 4: **Effects of shutting down the variation in disutility of working**

Note: The first column gives the statistics from our benchmark calibration. The last column gives the results from shutting down the variation in disutility of working, i.e., $\sigma_v = 0$, holding the rest of the parameter values in the benchmark calibration and solving for the general equilibrium.

5.2 Counterfactual: Restricting owner hours

In this subsection, we quantify the importance of owners supplying their own labor effort into their firms. We distinguish between the ability to supply their own hours, from the ability of doing it flexibly. Accordingly, we first ask what would happen if we had prohibited the entrepreneurs in our benchmark model to supply their own hours to their firm. Then we study the implications of forcing the owners to supply the levels of hours equal to the averages in the benchmark simulation.

Table 5 shows the implications of ruling out owner hours in firms' production in the panel labeled "$h = 0$". The equilibrium number of owners decreases from 7.6 percent of the population to 1.8 percent. The average labor hired by an employer goes down from 255 hours to 93 hours suggesting that particularly fewer high productive firms are active. For those, the value of entrepreneurship declines because growth prospects are worse. In particular, the average labor hired by 4-year-old firms relative to new start-ups goes down from 2 to 1.7, and the same average for 6 to 8 year-old firms goes down from 3.8 to 2.7. Reflecting a worsened pool of entrepreneurs, the survival rate to year 8 also decrease from 50 to 30 percent. Less survival and slower growth impact the total amount of production and labor input in the entrepreneurial sector. Total output by 1-year-old firms goes down to a one-third relative to the baseline. The comparison gets starker across age groups, making the total output of firms in this alternative world as low as one-fifth of the benchmark. Due to the lack of owner hours, 1-year-old firms hire more in total than the benchmark, but these aggregates go down substantially with further ages. Total hours hired by the entrepreneurs in the alternative world without owner hours is 28 percent of that in the benchmark.

In order to control for the effect of the number of entrepreneurs in our comparison, we also solve the equilibrium in this alternative economy but force owners to follow the entry and exit policy functions that are optimal in the benchmark model.[10] We find that the output and hired labor by the entrepreneurial firms go down even when we control for the entry-exit rules. For instance, the total output in age 1 goes down by 20 percent. With higher ages, this decline is starker due to reduced possibilities of wealth accumulation. Total hired labor in this exercise is larger for 1-year-old firms, again due to the lack of owner hours to substitute hired labor, but even with this channel, the total labor hired goes down in older firms relative to the benchmark. The average labor hired by a 4-year-old firm relative to a new start-up in this counterfactual is 0.92, much lower than the rate in the benchmark that points to a doubling of average labor between these ages. Similarly, the average labor of firms of age 6 to 8, relative to that of start-ups, decreases from 3.8 to around 1.

The exercise above does not show how the ability of adjusting their hours in the intensive margin helps owners improve their firms' prospects. We tackle this issue in the last panel labeled "$h = \bar{h}$" of Table 5, which shows the implications of forcing the entrepreneurs to supply the average managerial hours (i.e., $h_m = 38.5$) and the average blue-collar hours (i.e., $h_b = 2.4$) worked by owners in the baseline. This exercise shows that even this milder experiment, firms' growth patterns are flatter, and their contribution to the economy is lower, than in the baseline. The total output of the entrepreneurial sector diverges from the totals in the baseline as we look at latter phases of the life-cycle. Total production of firms of age 8 is about 10 percent lower, and the total output is 20 percent lower than the benchmark in which owners adjust their hours optimally. The total hours hired in age 1 goes up because in the benchmark owners work long hours in the first year themselves, but the hired labor gets reduced for the latter parts of the life-cycle if owners cannot adjust their hours. Comparisons of average labor across ages also reflect this flattening. For instance,

[10] In solving this problem, the owners know they have to abide by the benchmark entry-exit policy functions (as a function of productivity, disutility of working and wealth) and choose their factor inputs and wealth accumulation optimally. Accordingly, the implied number of entrepreneurs and survival rates can still be different than the benchmark through differences in the wealth accumulation.

	Benchmark	$h=0$		$h=\bar{h}$	
		Full	BM churning	Full	BM churning
Employer hours	44.6	0.0	0.0	40.9	40.9
Hired labor	255.4	93.2	36.1	296.2	284.3
Total output (bm=1)					
Age 1	1.00	0.35	0.79	0.99	0.88
Age 8	1.00	0.30	0.44	0.89	0.82
Overall	1.00	0.21	0.52	0.79	0.86
Tot. hired labor (bm=1)					
Age 1	1.00	1.23	3.28	1.58	0.93
Age 8	1.00	0.52	0.76	1.00	0.79
Overall	1.00	0.28	0.70	0.82	0.85
Ratio avg. labor					
Age 4/1	2.01	1.66	0.92	1.54	1.69
Age 6-8/1	3.78	2.66	0.99	2.62	3.22
Firm survival to age 8	0.50	0.31	0.50	0.48	0.50
No. owners	0.08	0.02	0.11	0.05	0.07

Table 5: Counterfactuals on owner hours

Note: Output and labor statistics are the aggregates of the entrepreneurial sector. Output, hired labor, and total labor are given relative to their values in the first year of the benchmark. The first column is the benchmark economy with optimally chosen owner hours. The second column fixes owner hours at zero, and reoptimizes the rest of the choice variables (wealth, entry, exit, labor and capital). The third column uses entry-exit decision rules in the benchmark and lets owners reoptimize on their wealth, labor and capital when they cannot supply their own hours. The fourth and the fifth columns do the same as in the second and third, but by forcing owners supply managerial (h_m) and blue-collar (h_b) hours fixed at the benchmark averages.

the average labor hired by a 4-year-old firm relative to a recent start-up goes from 2 to 1.5. This inflexibility in owner hours slightly reduces the survival prospects of start-ups, decreases the equilibrium number of owners in the economy from 8 percent to 5 percent of the population. Nevertheless, the flattening of patterns remains true if we use the benchmark policy functions for entry and exit decisions to control for the role of selection in these differences.

Our findings suggest that the ability of working in one's own firm is crucial in determining their firms' life-cycle performance. Moreover, the intensive margin adjustments in owner hours, and their flexibility in allocating their effort between managerial and blue-collar tasks help owners steepen the growth patterns of their firms, and make entrepreneurship a more desirable option ex-ante.

Moment	Benchmark	$h = 0$ calibration
	(rel. to corresponding baseline)	
Output	1.55	3.10
Hired labor	1.72	3.12
Wealth, sd (logs)	1.00	1.09
Wealth, P99/50 ratio	1.04	1.18
Wealth, P50/10 ratio	1.39	0.99
$\Delta r\%$	-0.1	-0.01
$\Delta w_b\%$	1.05	0.01
$\Delta w_m\%$	-3.49	-0.16

Table 6: Impact of relaxing the financial constraints (doubling θ)

Note: The panel shows moments in the benchmark model setting the θ at the double of its calibration (from 0.5 to 1), keeping the rest of the parameters unchanged. The right panel does the same for the calibration with $h = 0$, doubling the θ (from 0.57 to 1.14). Except for equilibrium prices, the statistics are given relative to the baseline (original θ) of the corresponding calibration.

5.3 Relevance of owner hours in understanding the role of financial frictions and inequality

The ability of high productive owners to work long hours make these firms effectively less borrowing constrained. Moreover, borrowing constraints are non-binding for low productivity non-employers that have no desire to grow. To understand the quantitative importance of owner hours in our understanding of financial frictions, we compare the baseline model to a more standard model that does not allow owners to supply effort ($h = 0$). To give the latter a fair shot in matching the data, we recalibrate the model to match (as far as possible) the targets from our calibration. Appendix D.1 describes the calibration in detail.[11]

Our first counterfactual experiment doubles the parameter θ; thus decreases the strength of financial frictions, in the baseline and the alternative model.[12] Then we compute the aggregate output and hired labor for firms. Table 6 summarizes the results of this experiment in the two models. In our benchmark model, the output effects of relaxing the financial constraints are milder than in the alternative that features no owner hours. Doubling

[11] There are a few caveats in the calibration of this model to the data. First, by construction, the process of the disutility of working loses relevance in shaping firms' trajectories. Second, this model cannot capture the non-employer firms in our sample, hence, the target population of this calibration excludes the firms without any employees.

[12] The calibrated θ in the model without owner hours is equal to 0.57. As a result, our exercise actually implies a larger absolute change in the benchmark model than the alternative without owner hours.

Moment	Data	Benchmark	$h = 0$ calibration
Sd. (logs)	1.8	2.2	1.1
P99/50	77	31	4.2

Table 7: Wealth distribution among owners

Note: Data refers to net wealth from the SIPP. We impute topcoded business equity from a pareto distribution fitted to non-topcoded business equity. We discard firms employing more than 100 workers to be consistent with our model.

the θ generates a 55 percent increase in total entrepreneurial output in the benchmark model, and it triples the output in the $h = 0$ model. The differences are similar in terms of hired labor, which increases by 70 percent in the benchmark and again triples in the alternative model. The differences in the two models stem from the role of owner hours in the production of firms. In the presence of borrowing constraints, the owners insert their own effort to substitute the external labor which can be infeasible to hire. This additional input mitigates the effects of financial constraints on their firms. When the constraints are relaxed, this naturally leads to a bigger boost in output in the model that does not take into account this cushion.

The same mechanisms also enable our model to generate a longer left and right tail in the wealth distribution among entrepreneurs. Table 7 shows that wealth is highly dispersed among entrepreneurs. The log standard deviation of wealth is 1.8 in the SIPP data. Table 7 shows how the benchmark model exhibits a much wider range of wealth compared to the alternative without owner hours. Allowing for owner hours brings the ratio of top 1 percentile of wealth to the median from 4.2 to 31, hence, much closer to the data counterpart of 77. In the overall variation, our model actually exceeds the data by having a standard deviation of log-wealth at 2.2, compared to 1.8. In the calibration without owner hours, this dispersion remains at 1.1. The left-tail of the wealth distribution is thicker in the baseline model because it allows for non-employers that have no desire to accumulate large wealth because their income is relatively low and they have no desire to grow. Moreover, high productivity firms enter with little wealth because they know that they can use their own effort to overcome financial frictions. Regarding the right tail, the rapid growth of firms made possible by owners' effort allows these firms to reach high output levels quickly; thereby, accumulate large wealth holdings.

To understand the interaction of wealth inequality with financial frictions, we use again a counterfactual simulation that reduces these frictions. Table 6 shows that the median wealth of owners, relative to the bottom 10 percentile, goes up by 40 percent in our model; much more than in the model without owner hours. In the baseline model, those at the bottom of the wealth distribution tend to be the non-employers that have no desire to grow. Given that they operate already at their optimum size, they decrease their wealth holdings close to 1-to-1 when financial frictions decrease. In contrast, those in the middle of the distribution are more severely borrowing constrained and decrease their wealth by less. The ratio of the 99th percentile to the median of the wealth distribution changes little with lower frictions, and much less than in a model without owner hours. In particular, the wealth of the top 1 percent relative to the median goes up only by 4 percent in our model, while it increases by 18 percent in the model without owner hours. The reason is that the highest productivity firms, those at the top of the wealth distribution, are effectively less constrained in our model than in the alternative, because they can exert their own effort to mitigate financial constraints. Hence, they reduce their wealth holdings to a similar degree as the median firm in response to a decrease in financial frictions.

Besides wealth inequality, owner hours also have implications for the interaction of financial frictions with workers' earnings inequality. Table 6 shows that these effects are close to zero in the $h = 0$ model. Total labor input in this model depends only on the number of entrepreneurs, therefore, is almost fixed. Firms' capital demand increases as a result of the decrease in financial frictions, but households are willing to increase savings by almost the same amount. In the model with owner hours, the increase in blue-collar wages and the decrease in managerial wages is much larger. This results from financial frictions affecting owners differently across the wealth distribution. For those with low wealth, relaxing the financial constraints pushes them to the right in Figure 1 leading to less blue-collar hours; therefore, an increase in blue-collar labor demand. These firms start being able to hire outside blue-collar workers which creates an incentive to increase managerial hours. Total managerial hours increase which decreases their wage.

6 Conclusion

This paper studies owners' labor supply as a factor in shaping heterogeneity in firm survival, output, and entrepreneurs' wealth. For the majority of US firms, owner hours are the predominant labor input. We find that particularly workers with a high disutility of work find it attractive to start relatively unproductive business and operate them as non-employers. Owners of high productivity firms find it optimal to work long hours in managerial tasks to complement their hired blue-collar labor creating a positive relationship between owner hours and firm performance. These long hours allow the firm to overcome financial frictions quickly and obtain rapid employment growth.

To understand the quantitative importance of owners providing their own hours, we study the implications of restricting these hours to zero. Output of the entrepreneurial sector would decrease by about 80%. In part, this reflects the previous low productivity non-employers exiting the market. However, there would be also less high productivity firms active because those firms would no longer find it optimal to start operation given the difficulty of overcoming the financial frictions. These large effects suggest that labor laws that place restriction on owners in supplying their own work to productive tasks may have large quantitative effects. Examples include restrictions on opening hours and regulations requiring bookkeeping and other reporting standards that divert the owners' time from productive tasks.

Owner hours are also decisive in understanding wealth inequality among owners by changing our understanding of which firms are borrowing constrained. Low productivity non-employers hold little wealth because their income is low and they have no desire to grow. In contrast, high productivity firms are able to overcome the financial friction relatively quickly; thereby, generate high income and achieve large wealth accumulation. Put differently, our results suggest that subsidies on capital may be largely wasted at the smallest firms and have less importance for growing firms than previously suggested.

We find that a decrease in financial frictions lead to rich dynamics in wealth and wage inequality. Low productivity firms that tend to have little wealth to start with respond by decreasing wealth almost one-to-one. In contrast, firm higher up the productivity ladder are, though mildly, financially constrained and decrease their wealth less than one-to-one. As a consequence, wealth dispersion among entrepreneurs increases. At the same time, decreasing financial frictions decreases wage inequality between blue-collar workers and managers. Lower financial frictions imply that entrepreneurs need to work less as blue-collar workers at low wealth levels which increases the demand for these tasks. In addition, owners increase their managerial hours to complement the hired blue-collar workers, therefore, decrease demand for managers and their wages.

A Web appendix: Solving entrepreneurs' problem

Here we focus on the first order conditions of an entrepreneurs' problem, taking as given the productivity level z, the disutility of work v, wealth a, and also her choice on asset holdings a'. The first order conditions, given the corresponding variable is positive, are:

$$(K) \qquad \eta\alpha\frac{Y}{K} = (r+\delta)(1+C) \tag{2}$$

$$(L_m, L_b) \qquad \eta\lambda\frac{Y}{\kappa h_m + L_m} = w_m(1+C), \quad \eta\mu\frac{Y}{h_b + L_b} = w_b(1+C) \tag{3}$$

There are 4 different cases that can happen:

Case 1: $L_m = 0, L_b > 0$**.**

$$\eta\alpha\frac{Y}{K} = (r+\delta)(1+C), \qquad \eta\mu\frac{Y}{h_b + L} = w_b(1+C)$$

If financial constraints do not bind, these give:

$$Y = \left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}(\kappa h_m)^{\lambda\eta}\left(\frac{\eta\mu}{w_b}\right)^{\mu\eta}\right]^{\frac{1}{1-\alpha\eta-\mu\eta}}$$

$$\Pi = Y(1-\alpha\eta-\mu\eta) + w_b h - \xi$$

Then we get:
$$K = \eta\alpha\frac{Y}{r+\delta}, \; L_b = \eta\mu\frac{Y}{w_b} - h_b.$$

Meanwhile, if the financial constraints bind:

$$Y = z\left(\frac{\theta a - \xi + w_b h_b}{\alpha\eta + \mu\eta}\right)^{\alpha\eta+\mu\eta}\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}(\kappa h_m)^{\lambda}\left(\frac{\eta\mu}{w_b}\right)^{\mu\eta}$$

$$\Pi = Y - \theta a$$

Then
$$L = \frac{\mu}{\alpha+\mu}\frac{\theta a - \xi}{w_b} - h_b\frac{\alpha}{\alpha+\mu}, \quad K = \frac{\theta a + w_b h_b - \xi}{r+\delta}\frac{\alpha}{\alpha+\mu}$$

Case 2: $L_m > 0, L_b > 0$**.**

$$\eta\alpha\frac{Y}{K} = (r+\delta)(1+C), \quad \eta\lambda\frac{Y}{\kappa h_m + L_m} = w_m(1+C), \quad \eta\mu\frac{Y}{h_b + L_b} = w_b(1+C)$$

If financial constraints do not bind (i.e. $C = 0$), then:

$$Y = \left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}\left(\frac{\lambda}{w_m}\right)^{\lambda}\left(\frac{\eta\mu}{w_b}\right)^{\eta\mu}\right]^{\frac{1}{1-\eta}}$$

$$\Pi = Y(1-\eta) + w_m\kappa h_m + w_b h_b - \xi$$

$$K = \alpha\eta\frac{Y}{r+\delta}, \quad L_b = \frac{\mu\eta Y}{w_b} - h_b, \quad L_m = \frac{\lambda\eta Y}{w_m} - \kappa h_m.$$

If financial constraints bind, then:
$$Y = z\left(\frac{\theta a - \xi + \kappa w_m h_m + w_b h_b}{\eta}\right)^{\eta}\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}\left(\frac{\lambda\eta}{w_m}\right)^{\lambda\eta}\left(\frac{\mu\eta}{w_b}\right)^{\mu\eta}$$

$$\Pi = Y - \theta a$$

$$L_b = \frac{\theta a - \xi + \kappa w_m h_m + w_b h_b}{w_b}\mu - h_b, \quad L_m = \frac{\theta a - \xi + \kappa w_m h_m + w_b h_b}{w_m}\lambda - \kappa h_m$$

$$K = \frac{\theta a - \xi + \kappa w_m h_m + w_b h_b}{r+\delta}\alpha.$$

Case 3: $L_m = 0, L_b = 0$**.**
$$\alpha\eta\frac{Y}{K} = (r+\delta)(1+C)$$

If the financial constraints do not bind:

$$Y = \left[z \left(\frac{\alpha\eta}{r+\delta} \right)^{\alpha\eta} (\tilde{\kappa} h)^{\tilde{\lambda}\eta} \right]^{\frac{1}{1-\alpha\eta}}$$

$$\Pi = Y(1-\alpha\eta)$$

and

$$K = \alpha\eta \frac{Y}{r+\delta}$$

If the financial constraints bind, then:

$$Y = z \left(\frac{\theta a}{r+\delta} \right)^{\alpha\eta} (\tilde{\kappa} h)^{\tilde{\lambda}\eta}$$

$$\Pi = Y - \theta a$$

$$K = \frac{\theta a}{r+\delta}$$

For all cases except Case 3, we have a fixed cost of ξ in the profits.

B Web appendix: Computational details

We discretize the worker state into 100 asset states, 21 productivity states, and 6 states for the disutility of work. We allow for off grid asset choices with a total of 892 possible choices. Regarding productivity, given a share of only 7% of entrepreneurs in society, we are particularly interested in its right tail. Thus, we choose a non-symetric grid with 67% of the grid points lying in the top 10% of the ergodic productivity distribution. For the discrete hours choices, we allow for 10 grid points for managerial work and 10 grid points for blue-collar work.

The algorithm guesses an interest rate, the managerial wage, and the blue-collar wage. Given prices, we solve the value functions at the grid points and use a distribution function iteration to compute the stationary distribution of the economy. Given the stationary distribution, we compute total asset supply, managerial hours supply, blue-collar hours supply, and capital and labor demand of entrepreneurs. Given these quantities, we compute the aggregate supply levels not absorbed by the entrepreneurial sector of each factor, $K_j^{s,n}$, $L_{m,j}^{s,n}$, $L_{b,j}^{s,n}$ and use the first order conditions from the non-entrepreneurial sector to compute an update for equilibrium prices:

$$\frac{K}{L_m} = \frac{\alpha}{\lambda}\frac{w_m}{r+\delta},\ \frac{L_b}{L_m} = \frac{\mu}{\lambda}\frac{w_m}{w_b},\ \alpha^\alpha\lambda^\lambda\mu^\mu = (r+\delta)^\alpha w_m^\lambda w_b^\mu.$$

$$r_{j+1} = \epsilon r_j + (1-\epsilon)\left[\frac{\alpha}{\lambda}w_{m,j}\frac{L_{m,j}^{s,n}}{K_j^{s,n}} - \delta\right]$$

$$w_{b,j+1} = \epsilon w_{b,j} + (1-\epsilon)\left[\frac{\mu}{\lambda}w_{m,j}\frac{L_{m,j}^{s,n}}{L_{b,j}^{s,n}}\right]$$

$$w_{m,j+1} = \left[\alpha^\alpha\lambda^\lambda\mu^\mu(r_{j+1}+\delta)^{-\alpha}w_{b,j+1}^{-\mu}\right]^{\frac{1}{\lambda}},$$

where ϵ is a relaxation parameter to make the updates smoother.

C Web appendix: Data details

In our analysis we use three data sources.

Survey of Business Owners (SBO). SBO data comes from the Public Use Microdata Sample (PUMS) file for 2007, provided by the US Census. We exclude firms with multiple owners in our analysis. The sample also excludes owners of businesses larger than or equal to 100 employees, and those generating a sales volume of less than 10 thousand dollars annually. We weight observations with their tabular weights (*tabwgt*). To avoid disclosure, SBO categorizes the information on receipts and employment. We take the median integer in the range of a category as the corresponding continuous value of the variable.[13]

For output, we use the total receipts (*receipts_noisy*). For hired labor hours, we multiply the number of workers (*employment_noisy*) by 35, as we do not observe part time

[13] See http://www.census.gov/econ/sbo/ for complete data description.

and full-time workers separately. Owner hours are given within 6 categories. We replace the category "None", with zero hours; category "Less than 20" as 10 hours; category "20 to 39" as 30 hours; "40 hours" as is; "41 to 59 hours" as 50 hours; and "60 and above" as 65 hours.

Kauffman Firm Survey (KFS). This data is provided by the Kauffman Foundation. Our sample exludes firms with multiple owners. We weight the firms with their initial sampling weights. We use the private version of the KFS accessed through the NORC Data Enclave.[14]

Owner hours is the hours worked by the owner for the business on average per week ($g1b1\_hours\_owner\_01\_X$). To construct the total hired labor per week, we sum the number of full time employees ($c6\_num\_ft\_employees\_X$) multiplied by 40 and the number of part-time employees ($c7\_num\_pt\_employees\_X$) multiplied by 20. For yearly profits and the payroll, we use profits ($f24\_profit\_amt\_X$) and the amount paid to wages, salaries and benefits in the wave of reference ($f18a\_wage\_exp\_Y\_amt\_X$).

We construct our measure of output using the profits, payroll and the total value of capital owned by the business. Since we do not observe the costs of capital rental, we obtain an opportunity cost of capital by multiplying different types of assets by the sum of their depreciation rates and the risk-free interest rate. The depreciation rates for each item comes from the Bureau of Economic Analysis estimates for year 2004.[15] The risk-free interest rate is set at 5 percent following our calibration of the model. In specific, we construct the cost of capital by adding the value of business-owned land and buildings ($f28e\_asset\_landbuild\_X$) by $0.05 + 0.02$, value of equipment ($f28d\_asset\_equip\_X$) by $0.05 + 0.06$ and value of vehicles ($f28f\_asset\_veh\_X$) by $0.05 + 0.12$.[16] We compute output by adding profits, payroll costs and the costs of capital.[17]

Survey of Income and Program Participation (SIPP). The SIPP is a representative survey of US non-institutionalized households with a panel dimension of up to 4 years. Every 4 months, the Census conducts an interview with all adult members of participating

[14]For the definitions of the raw variables, see http://www1.kauffman.org/kfs/KFSWiki/Data-Dictionary.aspx.
[15]See https://www.bea.gov/national/FA2004/Tablecandtext.pdf.
[16]The implied cost shares of capital are very much in line with the usual estimates of 0.33.
[17]Unfortunately, we do not observe the costs of intermediary goods.

households asking them about their work and household characteristics during the preceding 4 months. In order to account for the seam-bias effect generated by the recollection period, we aggregate the monthly information to quarterly observations. We use the panel from 1996, 2000, and 2008. We deflate all data using the CPI.

We keep individuals between ages 22 and 65. A worker is counted to be an entrepreneur whenever they report having a business, the business income is above their income from being an employee, business income is at least $50 per quarter, and the firm size is at most 100 employees. Individuals may report to have up to two businesses and we aggregate all business income. Business equity is top-coded and we impute top-coded values using a Pareto distribution fitted to the upper tail of non-top-coded values. Employees report up to two jobs and we use the information from the job with higher earnings. We separate employees into managerial and blue-collar workers. Managers are executives, managerial occupations, and managerial related occupations according to the 1990 occupation dictionary definitions.

D Calibration information

In Section 3, we omitted the documentation of the benchmark model's fit to the targeted moments. Here in Table 8, we provide this comparison. The upper panel gives the moments targeted from the SBO, and the lower panel gives the targets from the SIPP.

D.1 Calibration with $h = 0$.

In our Subsection 5.3, we compare the implications of our benchmark calibration with those of a calibration omitting the owner hours in the production function. Table 9 gives the calibrated parameters in this model. In calibrating these parameters, we follow the same strategy as in the benchmark, with two exceptions. First, the process of disutility of working is not identified in this model. Second, we focus only on employer firms, as non-employer firms cannot exist when owner hours are assumed away.

Moment	Data	Model
Mean labor hired (SBO)	74.7	76.6
90/75 labor ratio, employers	2.0	2.0
Fraction of owners	0.072	0.076
Fraction of non-employers	0.70	0.71
Average firm age	12.2	11.6
Number of 5-year-old, rel. to 1-year-old firms	0.68	0.67
Std. employer hours	0.47	0.51
Persistence owner hours	0.77	0.77
Mean owner hours, employers	45.9	44.6
Mean owner hours, non-employers	39.3	39.4
Mean output employers rel. to non-employers	11.1	11.4
Mean wage of managers rel. to workers (SIPP)	1.68	1.68
Rate of transition blue-collar to manager	3.75%	3.75%
Rate of remaining a manager	21.2%	21.2%
Rate of transition owner to manager	12.3%	12.3%

Table 8: Summary of targeted moments

Parameter	Value	Basis
		Panel A: Literature or data directly
β	0.92	Midrigan and Xu (2014)
α	0.33	Cagetti and De Nardi (2006)
δ	0.06	‘’
η	0.88	‘’
		Panel B: Calibrated
θ	0.57	Mean labor hired = 249 (SBO)
σ_z	0.065	90/75 labor ratio, employers = 2.0
z_0	-0.05	Fraction of owners = 0.022
ρ_z	0.98	Average firm age = 15.28
χ_E	0.243	Number of 5-year-old, rel. to 1-year-old firms= 0.68
χ_W	0.0	No forced entry
λ	0.154	Mean wage of managers rel. to workers = 1.68 (SIPP)
ϵ_m^b	3.75%	Rate of transition blue-collar to manager = 3.75%
ϵ_m^m	21.2%	Rate of remaining a manager = 21.2%
ϵ_m^e	12.3%	Rate of transition owner to manager = 12.3%

Table 9: Parameters

E Robustness

In this section, we highlight that the mechanisms of our model are not particular sensitive to what might be called non-standard parameters. To this end, we simulate the model moments from Section 4 when increasing by 10% the volatility of shocks to leisure, σ_v, the relative productivity of the self employed, $\tilde{\kappa}$, the probability to have to start a business, χ_W, and the probability not to have an option to go back to salaried work, χ_E. Figure 6 shows that none of these parameters has a strong impact on average firm growth or the selection of entrepreneurs into non-employer. Moreover, Figure 7 shows that none of these parameters substantially changes the allocation of entrepreneurial hours over firm sizes, or the correlation of owner hours and firm output.

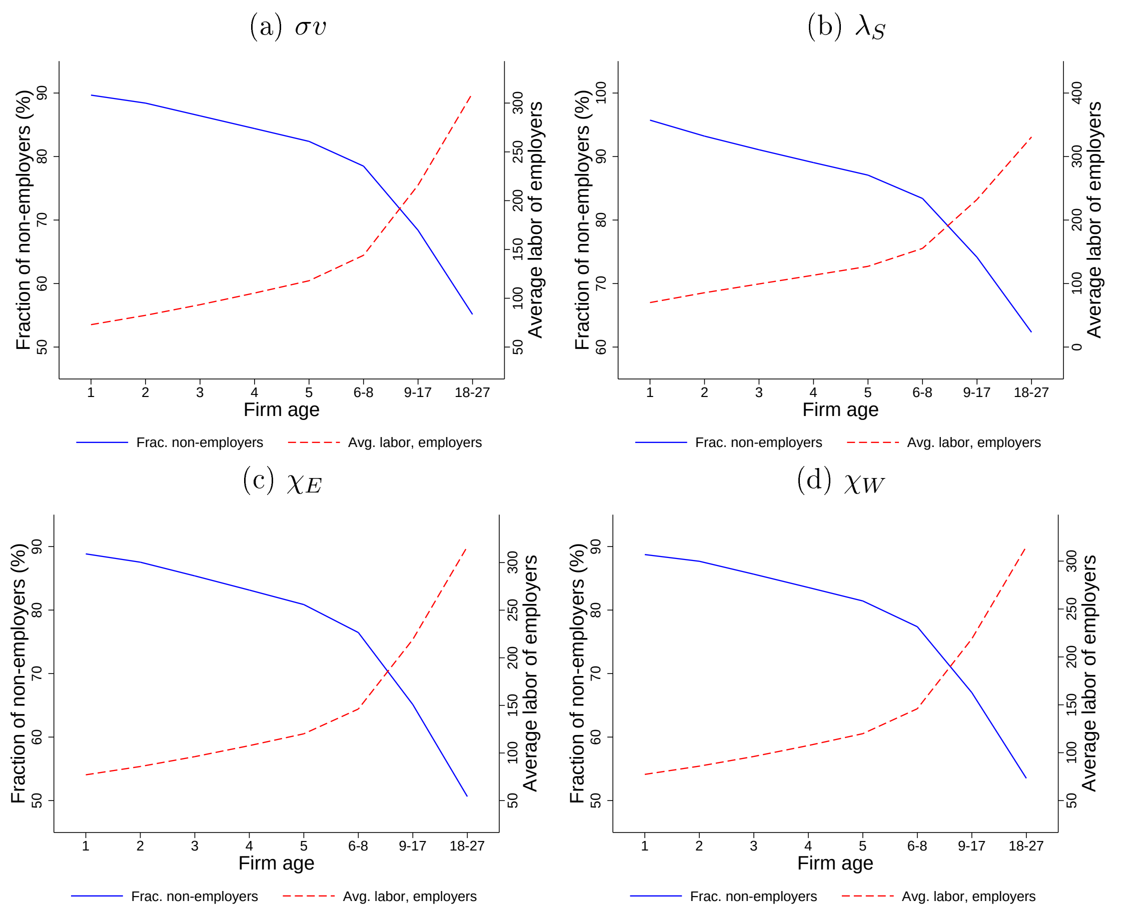

Figure 6: Employment patterns over age; robustness checks

Note: We pool all the firms both for the data and for the model. Hours are given in weekly units.

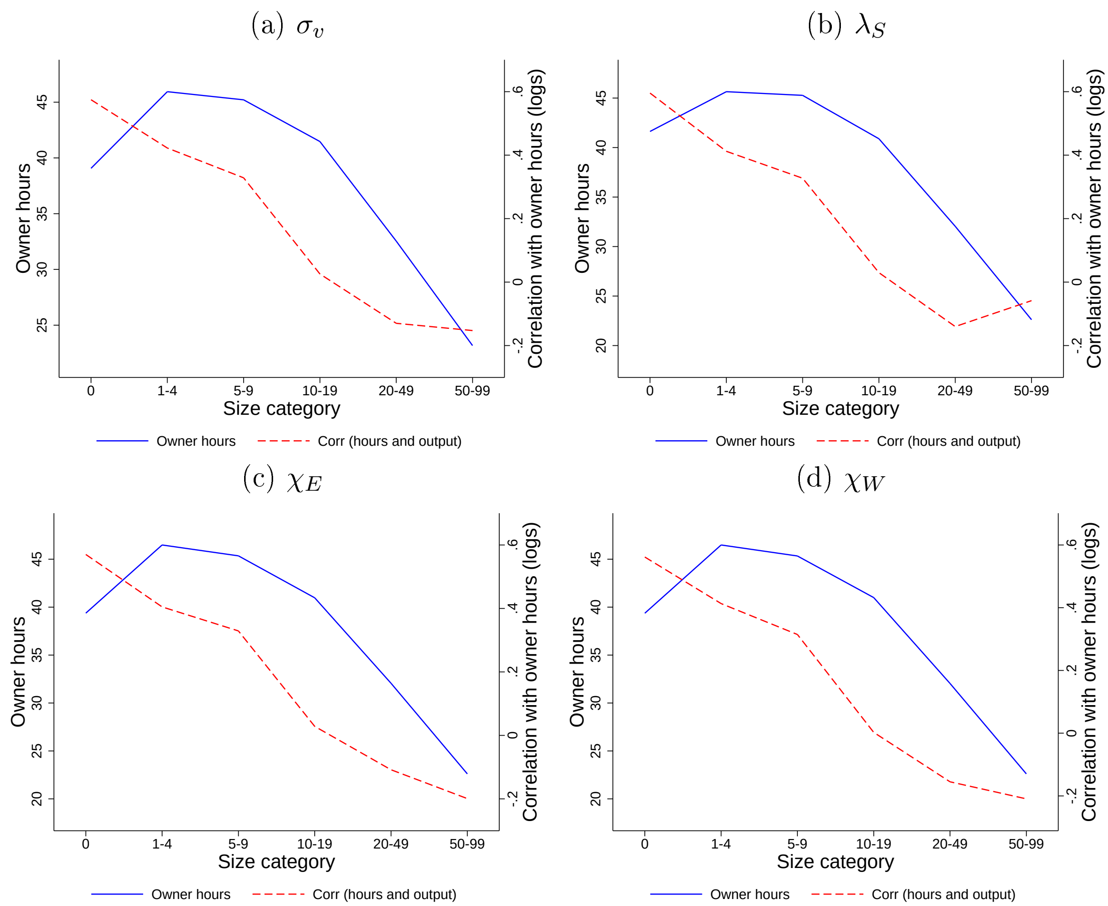

Figure 7: Owner hours, labor and output; robustness checks

Note: We pool all the firms both for the data and for the model. Hours are given in weekly units.

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