Term Premium and Quantitative Easing in a Fractionally Cointegrated Yield Curve*

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Abstract

The co-movement of US sovereign rates suggests a long-run equilibrium relationship. Traditional cointegrated systems need to assume that interest rates are unit roots and thus implying non-stationary and non-mean-reverting dynamics. We postulate and estimate a fractional cointegrated model (FCVAR) which allows for mean reverting though persistent patterns. Our results point at the existence of such mean-reverting fractional cointegration among sovereign rates. The implied US term premium –across different maturities– is less volatile than the classical I(0) stationary and I(1) unit root models. Our analysis highlights the role of real factors in shaping term premium dynamics. We further identify the dynamic effects of quantitative easing policies on our identified term premium. In contrast to the stationary-implied term premium, we find a significant term premium decline following these large-scale asset purchase programs.

JEL Classification: C2, C3, E4, G1

Keywords: U.S. yield curve; long-run relation; fractional cointegration; term premium; quantitative easing

1 Introduction

Understanding sovereign yield curve dynamics remains a fundamental topic for investors, bankers, policy makers, media and academics. This explains why it keeps receiving so much interest across discussions in all these quarters. A specific source of term structure attention is the joint co-movement of interest rates across maturities. As Figure 1 shows, US sovereign rates track each other quite closely despite their different maturities. Why is this the case? Many equilibrium models, such as those based on no-arbitrage, propose the existence of common factors (level, slope and curvature) driving yield dynamics across all maturities. At the same time, researchers and policy makers have long pointed to long-rates embedding expectations of short-rates. As a result, both producing the correct short-term forecasts and capturing the common dependence of rates across maturities is of utmost importance. This is why empirical models keep trying to improve both the characterization and estimation of joint bond yield dynamics. Indeed, correctly exploiting this cross-sectional term structure co-movement has relevant economic implications for both fiscal and monetary policy, term premium identification, predictability of future macro variables as well as banking management.

[Insert Figure 1: US Sovereign Interest Rates]

Figure 1 also suggests a potential long-run dependence across the different interest rates. In the term structure literature, this behaviour has been traditionally characterized via cointegration techniques (see Campbell and Shiller (1987) for a seminal study). In short, traditional cointegration imposes that all interest rates are unit roots or I(1) processes and that they cannot wander away from each other during long periods of time. While this methodology has advantages, such as exploiting this long-term relation across rates, this structure imposes an un-appealing non-mean reversion in rates. As explained by Campbell, Lo and MacKinlay (1997) and Diebold and Rudebusch (2013), this implies that shocks to interest rates have permanent effects, despite the fact that sovereign interest rates, at least in most industrialized economies, do not exhibit such behavior. Moreover, though most standard unit root methods cannot reject the presence of unit root tests individually in the interest rate series, it is well known that these methods have very low power against fractional alternatives as those used in this work (Diebold and Rudebusch, 1991, Hassler and Wolters, 1993, Lee and Schmidt, 1996, etc.). On the other hand, by jointly estimating the order of integration, we allow for the cross-correlation across interest rates.

Therefore, capturing this joint co-movement across maturities and at least allowing for mean-reversion dynamics should be in the agenda of any natural term structure model. This is what we explore and test in this paper, where we apply multivariate fractional cointegration techniques which allow for a flexible estimation of short and long-run dynamics in the term structure of interest rates. This econometric model simultaneously identifies the order of integration of rates (one, zero or a fractional number) and the potential existence of one (or several) cointegration relationships. Indeed, whether interest rates are cointegrated, fractionally cointegrated or not cointegrated is an empirical question which we tackle in this paper. To this end, we estimate a fractional cointegration vector auto-regressive (FCVAR) model (Johansen and Nielsen, (2012)), with US sovereign rates of different maturities.

We estimate the FCVAR using four sovereign interest rates capturing the short, medium and long ends of the yield curve for the US. Our findings point at a single long-run cointegration relation among the four interest rates. Our estimation results show that the order of integration of the interest rates is 0.756 with monthly data and statistically different from zero and one. Our results thus reject modeling sovereign rates in a unit-root cointegration framework. An implication of this result is that the common macro-finance shocks affecting the yield curve turn out to have transitory –rather than permanent– though long lasting effects on the term structure. Our results also reject the joint modeling of interest rates in standard stationary vector auto-regressive systems, given that we estimate the order of integration to be well and significantly above zero and that we find that there exists a long-run equilibrium relationship along the term structure of interest rates. We perform separate analogous FCVAR estimations for the UK and Germany and find similar results for the yields' order of integration and a similar long-run cointegration relation.

Our analysis yields a natural estimate of the term premium on long-term bonds, an important object of analysis for policy makers. Higher term premia reveal that investors require higher returns for long-term bonds, which point at a number of macro-finance or policy risks for the economy. The term premium associated with our fractional cointegrated system displays a marked degree of persistence and is clearly counter-cyclical. We analyze the sources of our term premium dynamics and show that they diverge with respect to term premia implied by stationary I(0) and unit-root I(1) models. In particular, unemployment is key to understand its counter-cyclical dynamics. Moreover, we show that the term premium response to recent quantitative easing (QE) policies is significantly different from that coming from stationary models. In particular, we identify a dynamic decline of the term premium following these large expansions in asset purchases, whereas no significant effect is found on the stationary model. QE shocks are also shown to increase both economic activity and inflation. Our results thus pose relevant policy implications in the recent debate on the effects of monetary stimulus withdrawal (see Yellen, 2017).

The paper proceeds as follows. Section 2 summarizes the fractional cointegration econometric framework and describes the economic implications of this modeling strategy for the term structure of interest rates. Section 3 discusses the data, empirical strategy and estimation procedure. Section 4 presents the empirical results of the paper. It shows the structure of the US yield curve, the implied term premium and its economic sources – comparing to I(0) and I(1) alternatives–, provides estimations for other international yield curves (UK and Germany) and studies the effects of unconventional monetary actions across macro-finance variables. Section 5 concludes.

2 Fractional Cointegration

In this section, we first briefly outline the multivariate fractional cointegration framework and lay out some of its general economic implications. Then we go on to motivate why fractional cointegration can be an appropriate modeling technique for the term structure of interest rates.

2.1 Econometric Setting

Our methodology to model term structure dynamics is based on the concept of long memory behavior. Given a covariance stationary process $x_t, t = 0, \pm 1, \ldots$, a series has long memory if its spectral density function contains a pole or singularity at least at one frequency in the spectrum. Alternatively, it can be defined in the time domain by saying that x_t displays the property of long memory if the infinite sum of the auto-covariances is infinite. A typical model satisfying the above two properties is the fractionally integrated or $I(d)$ model, where d is a positive value and can be formulated as:

$$(1-L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{1}$$

with $x_t = 0$ for $t \leq 0$, where L represents the lag-operator, i.e. $L^k x_t = x_{t-k}$, and u_t is an $I(0)$ or short-memory process, defined in the frequency domain as a process with a spectral density function that is positive and bounded at all frequencies. Note that in this context, if $d > 0$, the spectral density function of x_t is unbounded at the smallest (zero) frequency, and the polynomial in the left hand side of equation (1) can be written for all real d as:

$$(1-L)^d = \sum_{j=0}^{\infty} \begin{pmatrix} d \\ j \end{pmatrix} (-1)^j L^j = \left(1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 \ldots \right), \tag{2}$$

and thus:

$$(1-L)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \frac{d(d-1)(d-2)}{6} x_{t-3} + \ldots, \tag{3}$$

so that equation (1) can be expressed as:

$$x_t = d x_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \frac{d(d-1)(d-2)}{6} x_{t-3} + \ldots + u_t. \tag{4}$$

Thus, the differencing parameter d plays a crucial role in describing the degree of dependence (persistence) in the data: The higher the value of d is, the higher the level of dependence between observations is. Three values of d are of particular interest. First, the case of $d = 0$ that implies short memory behaviour as opposed to the case of long memory with $d > 0$. Second, $d = 0.5$, since x_t becomes non-stationary as long as $d \geq 0.5$.[1] Finally, if $d < 1$ x_t is mean reverting with the effect of the shocks disappearing in the long-run, contrary to what happens if $d \geq 1$ with shocks having permanent effects and lasting forever.

The natural generalization of the concept of fractional integration to the multivariate case is the idea of fractional cointegration. In this paper, we employ the Fractionally Cointegrated Vector AutoRegressive (FCVAR) model introduced by Johansen and Nielsen (2012). This method is used to determine the long-run equilibrium relationship between series. Given two real numbers d, b, the components of the vector z_t are said to be cointegrated of order d, b, denoted $z_t \sim CI(d, b)$, if all the components of z_t are $I(d)$ and there exists a vector $a \neq 0$ such that $s_t = a'z_t \sim I(\lambda) = I(d-b), b > 0$.[2] The Fractionally Cointegrated Vector AutoRegressive (FCVAR) model introduced by Johansen (2008) and further expanded by Johansen and Nielsen (2010, 2012) is a generalization of Johansen (1995) Cointegrated Vector AutoRegressive (CVAR) model which allows for fractional processes of order d that cointegrate to order $d - b$ ($b > 0$). In order to introduce the FCVAR model, we refer first to the well-known, non-fractional, CVAR model. Let $Y_t, t = 1, ...T$ be a p-dimensional $I(1)$ time series vector. The CVAR model is:

$$\Delta Y_t = \alpha^* \beta^{*\prime} Y_{t-1} + \sum_{i=1}^{k} \Gamma_i^* \Delta Y_{t-i} + \varepsilon_t = \alpha^* \beta^{*\prime} L Y_t + \sum_{i=1}^{k} \Gamma_i^* \Delta L^i Y_t + \varepsilon_t, \tag{5}$$

where Δ refers to the first difference operator, i.e., $\Delta = (1 - L)$, α^* is the vector or

[1] It is non-stationary in the sense that the variance of the partial sums increases in magnitude with d.

[2] A more general definition of fractional cointegration allows different orders of integration for each individual series. See, e.g., Robinson and Marinucci (2001), Robinson and Hualde (2003) and others.

matrix of adjustment parameters, β^* is the vector or matrix of cointegrating vectors and the sequence of matrices Γ_i^* governs the short-run I(0) VAR dynamics. The simplest way to derive the FCVAR model is to replace the difference and lag operators Δ and L in (5) by their fractional counterparts, Δ^b and $L_b = 1 - \Delta^b$, respectively. We then obtain:

$$\Delta^b Y_t = \alpha\beta' L_b Y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t, \tag{6}$$

which is applied to $Y_t = \Delta^{d-b}(X_t - \mu)$, where X_t is the $p \times 1$ vector of our series of interest and μ is a level parameter vector which accommodates a non-zero starting point for the first observation on the process.[3] α, β, Γ have an analogous interpretation to $\alpha^*, \beta^*, \Gamma^*$ in (5) but they only coincide under $d = b = 1$. We therefore have that:

$$\Delta^d (X_t - \mu) = \alpha\beta' L_b \Delta^{d-b}(X_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i (X_t - \mu) + \varepsilon_t, \tag{7}$$

where ε_t is p-dimensional independent and identically distributed with mean zero and covariance matrix Ω. The parameters have the usual interpretations known from the CVAR model. In particular, α and β are $p \times r$ matrices, where $0 \leq r \leq p$. The columns of β are the cointegrating relationships in the system, that is to say the long-run equilibria. The parameters Γ_i govern the short-run behavior of the variables –with k being the lag length of the VAR– and the coefficients in α represent the speed of adjustment towards equilibrium for each of the variables. Thus, the FCVAR model permits simultaneous modelling of the long-run equilibria, the adjustment responses to deviations from the equilibria and the short-run dynamics of the system. Notice that the cointegration intuition in the fractional case is analogous to the I(1) case, ie that there

[3]The use of a long-run mean term μ in the FCVAR model and its interaction with the initial values is not yet well established in the literature. Recent papers discussing this issue are Johansen and Nielsen (2016) and Nielsen and Shibaev (2018).

is a long-run relation among the variables. In Johansen and Nielsen (2012) and Nielsen and Popiel (2016) one can find estimation and inference explanations of the model, and the latter provides Matlab computer programs for the calculation of estimators and test statistics.

2.2 Why an FCVAR Model for the Yield Curve

We now discuss several features of the FCVAR model to model yield curve dynamics. The first one is model generality and flexibility. The FCVAR model lets the data at the same time (i) determine the order of integration of interest rates -without having to resort to unit root or fractional integration pre-testing on the order of integration ex-ante (with the well-known lack of statistical power of unit root tests)– and (ii) estimate the joint multivariate model dynamics allowing for the combination of short and long-memory dynamics. In particular, the FCVAR can accommodate fractional integration. Empirically, several authors have shown that interest rates tend to display significant fractional integration dynamics, as found in Backus and Zin (1993), Gil-Alana and Moreno (2011), Osterrieder (2013), Abbritti, Gil-Alana, Lovcha and Moreno (2016) and Golinski and Zaffaroni (2016), among many others. Theoretically, and inspired by the work of Robinson (1978) and Granger (1980), Altissimo, Mojon and Zaffaroni (2009) show that aggregation of sub-indices can explain inflation persistence. If aggregation explains fractional integration in inflation, then interest rates can all inherit fractional integration due to standard inflation targeting strategies by monetary policy makers.

Second, the proposed FCVAR model also allows for explicit long-run relations among the yields since it endogenously estimates the (potential) cointegration relationships among the yields. I(1)-cointegration has been proposed in the term structure literature by several authors, such as Campbell and Shiller (1987), Cieslak and Povala (2015) and Bauer and Rudebusch (2019). The advantage of our model is that the FCVAR accommodates cointegration without the need to assume I(1) dynamics for interest rates, since interest rates can be cointegrated and mean-reverting. In sum, given the separate evidence of fractional integration for yields and cointegration, it seems sensible to feature them jointly in order to capture the correct joint yield dynamics.

Third, by identifying the actual integration order of the variables -and the potential cointegration relations-, it avoids potentially important mis-specifications in key policy objects, such as the term premium, as we show in the paper. In particular, letting the data choose the order of integration represents an important advantage, avoiding the risk of over/under-differencing the variables. As shown by Cochrane and Piazzesi (2008), by assuming I(1) cointegration or an I(0) VAR model, we may be mis-specifying the model estimates, parameters, test restrictions and implied dynamics, such as the term premium. This has become an very relevant issue in recent times, as both academics and policy makers strive to understand the effects of quantitative easing policies (and subsequent tapering) on term premium dynamics (see D'Amico, English, López-Salido and Nelson (2012) and Yellen (2017), among others). Indeed, we show below that both term premium interpretation and policy evaluation can crucially differ depending on term premium identification.

One limitation of the FCVAR model is that it is a model for individual interest rates (4 in our case, as we show below) and not for the entire yield curve. It is thus different from popular models, such as the affine term structure models which model the full yield curve. Despite this fact, our model includes information on the short, medium and long ends of the yield curve, thus incorporating a wealth of macro-finance information to identify the expectations of the short-rate and thus the term premium. Moreover, by including the actual interest rates in estimation, the cointegration interpretation can naturally arise.

Of course, there are other alternative techniques for modeling interest rates. One of them is regime switching (see Ang and Bekaert (2002) and Baele, Bekaert, Cho, Inghelbrecht and Moreno (2015), among others). Regime switching has the appealing feature of allowing shifts in meaningful key reduced-form or policy parameters, such as the reaction to inflation deviations from target or changes in interest rate inertia induced by financial stability purposes. These shifts influence the whole term structure, thus shaping joint yield dynamics. While the fractional cointegration approach does not model these parameter shifts, it can be consistent with regime switching dynamics. Indeed, as explained by Diebold and Inoue (2001), the dynamics of fractional integration and regime switching are easily confused, with fractional integration being able to capture some of the embedded autocorrelations derived from regime switching processes.

Another interesting modeling alternative for interest rate is the one based on "near-cointegration" proposed in Jardet, Monfort and Pegoraro (2013). They propose a no-arbitrage term structure model that takes into account the persistence of the variables (short-rate, the spread between the long and the short rates, and GDP growth) by using a near cointegration" approach. Using this approach, they still impose integer degrees of differentiation, not taking into account the possibility of fractional values, unlike in the present work.

3 Data and Estimation

In our empirical work, we employ monthly series corresponding to the U.S. Treasury Yield Curve. The data was obtained online from the work by Gürkaynak, Sack and Wright

(2007). Their yield curve estimates are updated periodically and provide a benchmark US sovereign zero-coupon yield curve. In our international extension, we use publicly available yields from Germany and the UK. Our baseline specification includes four series, namely the one, three, five and ten year sovereign rates. In this way, our data vector X_t includes information about the short, medium and long end of the yield curve. By including different parts of the term structure, our model captures key macro-finance information, including future economic and financial expectations. Our dataset covers observations from August 1971 up to April 2018. Figure 1 shows the dynamics of the four interest rates for our sample period.

In terms of estimation, we proceed as follows: We first assume that a sample of length $T + N$ is available on X_t, where N denotes the number of observations used for conditioning. As shown in Johansen and Nielsen (2016), model (7) can be estimated by conditional maximum likelihood, conditional on N initial values, by maximizing the following function:

$$\log L_T(\lambda) = \frac{T}{2}\left(\log(2\pi) + 1\right) - \frac{T}{2}\log\det\left\{T^{-1}\sum_{t=N+1}^{T+N}\varepsilon_t(\lambda)\,\varepsilon_t(\lambda)'\right\}. \tag{8}$$

For model (7) the residuals are:

$$\varepsilon_t(\lambda) = \Delta^d(X_t - \mu) - \alpha\beta'\Delta^{d-b}L_b(X_t - \mu) - \sum_{i=1}^{k}\Gamma_i\Delta^d L_b^i(X_t - \mu), \tag{9}$$

with $\lambda = (d, b, \mu, \alpha, \beta, \Gamma_i)'$. It is shown in Johansen and Nielsen (2012) and Dolatabadi, Nielsen and Xu (2016) that, for fixed (d, b), the estimation of model (6) is carried out as in Johansen (1995). In this way the parameters $(\mu, \alpha, \beta, \Gamma_i)'$ can be concentrated out of the likelihood function. Then we only need to optimize the profile likelihood function over the two fractional parameters, d and b. Through our analysis, we show the results implied by estimation which allows for different estimates of d and b. We also comment below on the estimates implied by a model where d is forced to be equal to b. As explained by Johansen and Nielsen (2018), the likelihood ratio test of the usual CVAR is asymptotically $\chi^2(2)$ and the likelihood ratio test of the hypothesis that $d = b$ in the fractional model is asymptotically $\chi^2(1)$. Hence these tests are very easy to implement and can be calculated using the software package of Nielsen and Popiel (2016)

4 Empirical Results

In this section we show and discuss the first empirical results of the paper. We first show the empirical estimates of the FCVAR model and relate the implied model rates with monetary policy management. We then extract the FCVAR-term premium, compare it with alternative I(0) and I(1) counterparts and provide an interpretation of its underlying economic sources. We then discuss the implied term structure of term premia, provide subsample analysis and obtain analogous results for two additional yield curves: UK and Germany. Finally, we examine the dynamic effects of the recent quantitative easing policies on term premium dynamics.

4.1 Baseline Estimates

The dataset in Gürkaynak, Sack and Wright (2007) provides daily data of sovereign rates from maturities 1-year to 30 years. To capture some relevant maturities at the short, medium and long end of the yield curve, we work with the 1-year ($i_t^{(12)}$), 3-year ($i_t^{(36)}$), 5-year ($i_t^{(60)}$) and 10-year ($i_t^{(120)}$) US sovereign rates. We work with the monthly frequency, as results can then be related to key macro variables, such as unemployment, consumer inflation or industrial production. We use end-of-the-month interest rate observations over each month to construct the monthly dataset, which spans the August 1971-April 2018 sample period.

When we run the FCVAR system with the four interest rates, we obtain the following estimated model:

$$\Delta^{0.756}(X_t - \mu) = \alpha\beta' L_{1.184}\Delta^{0.756-1.184}(X_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^{0.756} L^i_{1.184}(X_t - \mu) + \varepsilon_t. \quad (10)$$

Results are based on a VAR(1) for short-run dynamics ($k = 1$), as selected by the Hannan-Quinn criterion.[4] Table 1 reports the cointegrating rank test –analogous to a cointegration test– and identifies a single long-run cointegration relation for interest rates. In turn, the alternative of not having a cointegrating relationship is clearly rejected. Hence, the FCVAR model is validated. The estimated common order of integration of the four interest rates is 0.756 (with standard deviation 0.035), with a 95% confidence interval including the set (0.688, 0.824). This value turns out to be statistically higher than 0.5 and different from 0 and 1. Table 2 also shows the results of an LR test and reveals that the CVAR is rejected in favor of the FCVAR. The parameter b is estimated to be 1.184 (with standard deviation 0.087).[5] This implies that the error term displays anti-persistence, being therefore stationary and with the shocks reverting more often than those expected from a random series. In turn, the level parameter is μ estimated at $[5.264, \quad 5.771, \quad 5.993, \quad 6.188]'$.

[Insert Tables 1 and 2: Results of the Cointegrating Rank and LR Tests]

[4] Other likelihood criterion like AIC and BIC produced the same result. This is a relevant issue noting that the FCVAR model can suffer from identification issues when the number of lags is unknown (see Carlini and Santucci de Magistris, 2017).

[5] This large number can be a consequence of the conditioning on few initial values, although the conclusions of our study seem not to be affected by this large number.

The estimated long-run fractional cointegration vector is:

$$\hat{\beta}' = [1, \quad -2.598, \quad 2.347, \quad -0.760]',$$

where the elements of this vector are associated with the 1, 3, 5 and 10-year bond rates, respectively. Thus, while loadings on the medium end of the yield curve are more than twice higher than those in the short and long ends, the sum of the four loadings is close to zero. Economically, the finding of this single long-run cointegration vector –together with the values of this vector– is consistent with the existence of long-run parallel shifts in the yield curve. This is implicitly consistent with the existence of (at least) one long-run level factor shifting the entire yield curve in the long-run. In contrast, changes in one (or a subset) of yields may be temporary if they imply a departure with respect to the cointegration vector.

The corresponding estimated speed of adjustment vector is estimated at:

$$\hat{\alpha}' = [0.016, \quad 0.042, \quad 0.053, \quad 0.064]'.$$

As a result, the implied speed of adjustment with respect to deviations from the long-run relationship is fastest (and statistically significant, given that its standard deviation is 0.021) for the 10-year rate. In contrast, the 1-year rate adjustment to deviations from this fractional cointegration is very sticky, almost null (and statistically non-significant, given that its standard deviation is 0.020). The short-rate thus tends to be less driven by the long-run relation among rates and more influenced by its own short-run dynamics, at least at high frequencies. So, shocks affecting specifically the medium and long-end of the yield curve –and which generate deviations from the long-run relationship– are transmitted to the short-rate very slowly, while specific shocks affecting the short and medium ends of the yield curve –and, again, to the extent that they generate deviations from the long-run relationship– are transmitted to the 10-year rate relatively fast.

One metric to evaluate the fit of the model is to check whether the implied risk-premium dynamics of the model capture the deviations from the expectations hypothesis. Several authors have documented this fact and in particular, Dai and Singleton (2002) have developed the LPY(i)/LPY(ii) criteria to compare the sample v/s model-implied deviations from the expectations hypothesis. In particular, the LPY(i) criterion is based on a regression of the change in the long-rate on the scaled long-short yield spread. If the expectations hypothesis were true, the slope coefficient should be unity, yet typical sample regressions yield negative coefficients –and increasingly so as a function of the long-rate maturity–. We do confirm this in our data and with our model fitted values, where the regression slopes are -0.298 and -11.649 for the 3 and 10-year yield regressions, respectively. In contrast, in the LPY(ii) regressions, where risk-adjusted (with the one period excess holding period return) long-run yield returns are regressed on the scaled yield slope, the sample coefficients should be close to unity. This is also what we obtain across the fitted yields of our FCVAR model. Thus our model is largely consistent with the standard deviations of the expectations hypothesis.

To understand the impact of monetary policy in our fractionally cointegrated yield curve, Figure 2 plots the monetary policy rate (Federal Funds Rate, FFR), together with the 1, 3, 5 and 10-year bond rates implied by the long-run equilibrium relation vector β'. Figure 2 shows that the FFR is very similar to the 1-year and 3-year rates implied by the long-run relation. So our estimated fractional cointegration relation captures the fact that monetary policy is the driving factor behind the short-end of the yield curve at both high and low frequencies (see Moreno (2004)). When comparing the FFR with the medium and long ends of the yield curve (5 and 10-year rates), some differences arise. In particular, while the long-run trend of both rates is similar, the strong counter-cyclical dynamics of the FFR are not replicated by the implied 10-year-rate, which exhibits a less volatile pattern and more nuanced changes. In the next subsection, we turn to further understand the dynamics of the 10-year rate by decomposing it into the risk-neutral rate and the term premium.

[Insert Figure 2: Federal Funds Rate and FCVAR-implied Rates]

By construction, one restriction of the FCVAR model is that it constrains all the sovereign rates to have the exact same integration order. One may argue that sovereign rates could have different integration orders and thus should not be cointegrated. We entertain this possibility and estimate an alternative Fractional VAR model, which allows for different integration orders across yields. In particular, we estimate a VARFIMA(1,D,0), i.e. allowing for a first order VAR without an MA part, and fractional dynamics (D is a 4×4 diagonal matrix where each entry includes the (potentially) fractional order of integration of each yield). Our estimates imply that the orders of integration are 0.771, 0.783, 0.799 and 0.823 for the 1, 3, 5 and 10-year yields, respectively. We can statistically reject that they are different from each other, thus lending support to the FCVAR specification. Moreover, neither of these values is significantly different from the FCVAR estimate of the common integration order, which is 0.756.

We finally note in this subsection that when we estimate the FCVAR imposing that $d = b$, we also obtain a unique fractional cointegration relation with d=0.765 –very similar to our benchmark 0.756– and a standard deviation of 0.050. Table 3 shows the results of a likelihood ratio testing the benchmark model ($d \neq b$) v/s the restricted model ($d = b$). The table shows a statistical rejection of the restricted model. From an economic perspective, subsequent results turn out quite similar under both specifications, although the term premium implied by our more flexible benchmark –a topic to which we now turn– is even more stable.

[Insert Table 3: LR Test, $d = b$ v/s $d \neq b$]

4.2 Term Premium Analysis

Once we have determined that sovereign rates are fractionally cointegrated and its relation with monetary policy management, we can examine the implied term premium. Following Wright (2011), we compute the term premium as the model-implied five-to-ten-year forward rate minus the average expected one-year interest rate from five to ten years hence:

$$tp_t = f_t^{(120-60)} - \frac{1}{5} E_t \sum_{j=5}^{9} i_{t+12j}^{(12)}. \tag{11}$$

Based on the estimates of our FCVAR model, we can identify the implied baseline term premium (tp_t). This is plotted in Figure 3. As the figure shows, the implied term premium is markedly counter-cyclical and no clear trend emerges. While the term premium is positive during most of the sample period, it also displays low negative values at the end of the 70s and beginning of the 80s (reaching values around -0.5%). During the recent 2008 financial crisis, the term premium also increased to values higher than 3%, but it has declined since then, with term premium levels below 1% by the end of the sample.

[Insert Figure 3: Term Premium Implied by the FCVAR System]

Table 4 shows the mean and standard deviation of the term premia and risk-neutral rates implied by the I(0)-VAR, I(1)-CVAR and FCVAR models, respectively. The I(0)-VAR model generates the least variable risk neutral rate, due to the fast mean reversion of forward-looking expectations. The opposite is the case for the CVAR model, where expectations are the most volatile. The FCVAR model is the one which clearly delivers the most stable term premium in terms of standard deviation (one third lower than CVAR and I(0)-VAR counterparts). Its mean is also the lowest, 20 and 40 basis points lower than the CVAR and I(0)-VAR models, respectively. Table 5 shows the correlation of the term premia and risk-neutral rates with four macro variables: Federal Funds rate, unemployment, industrial production growth and the term premium itself. While the FCVAR and the CVAR term premia display a negative correlation with the Federal Funds rate, this correlation is positive for the I(0)-VAR. Also, the risk-neutral rates implied by the FCVAR and the CVAR have a negative correlation with their respective term premia, whereas the opposite is the case for the I(0)-VAR. All term premia have a positive correlation with unemployment, a theme we revisit below.

[Insert Tables 4 and 5: Term Premium Descriptive Statistics]

The top graph in Figure 4 plots the term premia implied by the three models. It shows how the I(0)-implied term premium is substantially higher during the early 80s and becomes increasingly negative since 2015. The bottom graph in Figure 4 shows the differences between the term premium implied by both the I(0) and CVAR models, respectively, and that implied by our FCVAR. The differences are quite sizable during some periods. The I(0)-implied term premium is higher than the FCVAR-term premium from 1976 to 1995 (reaching almost 3% in the early 80s). This gap exhibits a downward trend, revealing the downward trend in the I(0) implied term premium during the first part of the sample. The downward trend in the I(0)-implied term premium thus reveals the challenge that I(0) models face when describing the true counter-cyclical nature of the term premium (see Bauer, Rudebusch and Wu (2012) small-sample analysis of I(0)-type models). In contrast to the I(0)-implied model, the CVAR-implied term premium is lower than the FCVAR-implied one during most of the first 15 years of the sample. This difference reaches its maximum value in the last years of the 70s and the first years of the 80s (reaching above -3%), when sovereign rates were especially volatile due to monetary policy tightening in an era of high inflation rates.

By the last years of the sample –at the time of policy rates close to the zero lower bound–, important differences remain and they have different signs depending on the model at hand: around 0.5% higher in the CVAR and around 2% lower in the I(0) VAR. The first column of Figure 5 examines the patterns in the 1-year rate expectations for the three models during the post-2006 period for three alternative horizons (1-year, 5-year and 10-year). Differences are striking. While the I(0) model produces long-run (10-year) expectations above 4% (close to the full sample average) –implying a negative term premium by the end of the sample (see top graph in the second column of Figure 5)–, the opposite is the case for the I(1)-CVAR, where implied expectations are very close to zero (in fact they are negative during almost three years!). The FCVAR-implied long-run expectations are between 1 and 2%, showing a realistic slow mean reversion in the context of a slow economic recovery. In sum, our FCVAR-identified term premium is less volatile than its I(0) and CVAR counterparts. Our analysis shows that this is due to the fact that the I(0) model impinges too little volatility to the risk-neutral rate, whereas the CVAR imparts too much volatility.

[Insert Figures 4 and 5: Term Premium and Risk-Neutral Differences:
FCVAR v/s CVAR and VAR]

Theoretical and empirical research identifies two main reasons behind an increase in term premia: On the one hand, an increase in inflation uncertainty (see, e.g., Wright, 2011) and on the other an increase in economic risk (see e.g. Bauer, Rudebush and Wu, 2012). Thus, a correct identification of the term premium is crucial for an understanding of the economic forces behind term premium dynamics as well as for the appropriate policy response. In fact, these two risk factors call for opposite monetary policy response: Central banks should increase interest rates if increasing risk premia reflect inflation uncertainty, while they should reduce them when a spike in term premia reflects economic and financial risk (see related comments in Bernanke, 2006). It is therefore quite important to understand which one is likely to be the dominating factor behind eventual term premium increases.

To shed some light on this issue, we follow, e.g, Backus and Wright (2007), Gagnon, Raskin, Remache and Sack (2011), and Wright (2011), which introduce the following ordinary least squares regression model so as to explain historical time variation in the term premium:

$$tp_t = \alpha + \beta x_t + \eta_t, \tag{12}$$

where tp_t is a measure of the term premium –I(0)-VAR, FCVAR or CVAR–, x_t denotes a vector of regressors and η_t is the error term. In practice, we will consider two models. In the first model, which is very similar to the ones in Backus and Wright (2007), Wright (2011) and Bauer, Rudebusch and Wu (2012), we regress the term premium on measures of inflation uncertainty and real economic activity. Specifically, we measure inflation uncertainty with the long-run inflation disagreement series measured by the Michigan Survey of Consumers, which captures the interquartile range of five-to ten-year-ahead inflation expectations. Business cycle uncertainty is captured with the unemployment rate, and an NBER recession dummy.

We compare in Table 6 the results obtained with the I(0)-VAR TP, the CVAR and the FCVAR term premium. The dimension of the full sample, which starts in April 1990 and finishes in April 2018, is constrained by the availability of the long-run inflation disagreement series. We find that the correct identification of the persistence of the term premium has a strong influence in their interpretation. While the stationary I(0) term premium is strongly positively related to inflation uncertainty, the opposite is the case for the unit root-I(1) term premium, as the conditional correlation with inflation uncertainty is negative. In contrast, the results of the FCVAR model show no evidence of correlation with our measure of inflation uncertainty.

[Insert Table 6: Term Premium Sources, Simple Regression Model]

The differences between the results obtained with the stationary I(0) model and the ones of the I(1) and FCVAR alternatives are due to the implied volatility of the risk neutral rate. As explained by Bauer, Rudebush and Wu (2012), the stationary I(0) model implies too fast mean reversion of expected interest rates and too little volatility of the risk neutral rate. As a consequence, the term premium identified with the I(0) model inherits the trend of nominal interest rates, which is in turn related to the downward inflation trend. In the I(1) and FCVAR specification, instead, the risk neutral rates respond much more to changes in the short rate, and the implied term premium does not inherit the downward trend of nominal interest rates and inflation. Interestingly, the recession dummy is significant in the I(1) model but not in the FCVAR model. This happens because, in the FCVAR model, the increase in term premium actually precedes a few months the actually recession dates, providing an early warning of recession risk.

To control for the correlation between unemployment and inflation, we correct the baseline regression by substituting the unemployment variable for the residual of the unemployment regression on a constant and long-run inflation disagreement. Table 7 shows the associated results. In this instance, long-run inflation disagreement enters also significantly in the FCVAR-term premium regressions for the FCVAR and CVAR models, but this relationship is weaker and less robust, and appears to be dominated by the strong relationship between term premium and unemployment rates. Overall, our results are in line with the findings of Bauer, Rudebush and Wu (2012), who claim that a more precise estimation of the persistence of interest rates is crucial to avoid too fast mean reversion of expected interest rates and thus an underestimation of risk neutral rates. While these regression specifications are admittedly very simple, their insights carry over to regression models with additional controls considered by Gagnon, Raskin, Remache and Sack (2011) (core PCE inflation, interest rate volatility and the Economic Policy Uncertainty index of Baker, Bloom and Davis (2016)).

[Insert Table 7: Term Premium Sources, Alternative Regression Model]

We now turn to analyze the term structure of the term premia. For our baseline analysis, we have chosen a standard term premium (the difference between the five-to-ten-year forward rate and the average of the five expected 1-year rates 5 years hence), but we can alternatively compute the three following term premia based on a standard decomposition:

$$tp_t^{(n)} = i_t^{(12n)} - \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+12j}^{(12)}, \tag{13}$$

where n can be equal to 3, 5 or 10 years. Figure 6 shows these three term premia together with the baseline one. Results are quite sensible and reveal three key features. First, the term premia are highly correlated and clearly counter-cyclical. Second, term premia associated with longer maturities are higher than shorter maturities. Third, differences across term premium are clearly non-linear: They become larger at high values across term premia.

We shed further light on the term structure of term premia by graphing the 4 term premia for the 3 models (FCVAR, I(1)-CVAR, I(0) VAR), together with the associated risk-neutral rates. Results are shown in Figure 7. Overall, the term premia estimated by the FCVAR display less volatility than the other 2 sets of counterparts. The I(0) VAR-implied term premia all display a clear downward trend, with very large differences across premia during some periods, such as the 1980s. The I(1) CVAR-implied term premia display counter-cyclical dynamics but take unrealistic negative values in the 1970s.

[Insert Figures 6 and 7: Term Structure of Term Premia and Comparison across Models]

As a last exercise in this subsection, we derive the term premium for alternative subsamples. One potential limitation of term premium computations is that they may be quite sensitive to sample selection. We now elaborate on this point and compare our results with those of the two alternative term premia (I(1)-CVAR and I(0) VAR). Figure 8 shows the baseline term premia implied by our FCVAR model for alternative subsamples. The top graph compares the full-sample term premium with an alternative sample ending in 2008, right before the financial crisis. The two term premia turn out to be very similar across these two subsamples. The graphs below show the differences of term premia across two other subsamples (post-1979 and post-1990). In this latter subsample differences seem more noticeable, but overall dynamics are quite similar. We thus conclude that the FCVAR term premium is quite robust across subsamples despite relevant macro and monetary policy changes during our baseline sample.

Figure 9 shows the subsample term premia across models. It shows that the I(0) model -the most commonly used in term premium analysis- is the most unstable one, with some important discrepancies across subsamples. In contrast, the I(1) CVAR-implied term premia are very stable. This should come as no surprise since the I(1) model by construction depends almost entirely on the previous period interest rate, independently of the subsample taken.

[Insert Figures 8 and 9: Term Premium for Alternative Subsamples and Comparison across Models]

4.3 International Term Premia

The previous analysis suggests that the FCVAR model provides a realistic representation of the yield curve dynamics in the United States. An interesting question is whether these findings can be extended to other countries. To address this issue, in this section we estimate the FCVAR model for two other advanced economies, the UK and Germany. The data for the UK is from the Bank of England, and covers the sample from December 1972 to April 2018. The data for Germany is from the Bundesbank and covers the sample from September 1972 to April 2018. As for the US, results are based on a VAR(1) for short-run dynamics.

Table 8 shows the estimated values for the parameters d and b. Our tests allow us to reject the hypothesis that interest rates are stationary I(0). While we do not report them for brevity, tests also reject the I(1) cointegration in favor of fractional cointegration. In the case of Germany, however, we cannot reject the hypothesis that $d = b$.

[Insert Table 8: FCVAR Orders of Integration, US, UK and Germany]

We also test for the presence of fractional cointegration relationships between interest rates. As in the case of the US, we find the presence of a single fractional cointegrating relationship for the UK and Germany. The implied cointegrating vectors for the UK and Germany are, respectively:

$$\hat{\beta}'(UK) = [1, \quad -6.333, \quad 7.343, \quad -1.982]',$$

$$\hat{\beta}'(GER) = [1, \quad -3.969, \quad 4.033, \quad -1.100]'.$$

Interestingly, the interpretation of the two estimated cointegrating vectors is essentially the same as in the US case. They imply long-run parallel shifts in the yield curve, and so this finding is consistent with level factor/s shifting the yield curve in the long-run.

Figure 10 shows the risk neutral rates and term premia implied by the VAR, CVAR and FCVAR models for the UK and Germany. As was the case for the US, an accurate estimation of the persistence of interest rates is crucial for the identification and interpretation of term premium dynamics. For both countries, the I(0) term premium is relatively stable and presents a downward trend inherited by the interest rates. This happens because, by implying too fast mean-reversion of interest rate changes, the VAR model tend to underestimate the variability of risk neutral rates. On the contrary, the CVAR and FCVAR imply more volatile risk neutral rates and countercyclical term premia -probably too volatile in the case of the CVAR–. Importantly, the different models imply starkly different term premia dynamics. For example, in both countries term premia implied by the stationary VAR are around 4 percent higher than the ones implied by the FCVAR in the early 1980s and 1990s. The differences with the CVAR are smaller but still non-negligible.

As a final exercise, Figure 11 shows the implied term premia of the three countries -the US, the UK and Germany- with the three models. Three facts stand out. First, in all cases the FCVAR model implies quite different term premia dynamics from the ones implied by the I(0) VAR. This is likely to be important for economic interpretation and policy-making. For example, in the US in 2018 the term premium is around -2 percent according to the VAR model while slightly positive according to the FCVAR model. Second, the US term premium is the least volatile of the three countries according to the FCVAR model, while the most volatile according to the VAR model. Finally, the FCVAR model implies a strong increase in synchronization and correlation starting with the financial crisis, while the term premia implied by the VAR model significantly depart after 2010.

[Insert Figures 10 and 11: Term Premia, UK and Germany]

4.4 Quantitative Easing and the Term Premium

In this final subsection, we provide a further illustration of the importance of correct term premium identification for economic analysis and policy. It is based on a topic that has captured a lot of attention in the recent years: The effects of large assets purchase programs (also called "quantitative easing") macro-finance variables, such as the term premium. To shed light on this issue, we compute impulse responses to large shocks to the Central Banks balance sheet by means of local projections (LPs). The local projections methodology, developed by Jordà (2005), consists of running sequential predictive regressions of the endogenous variables on a structural shock and a set of controls for different prediction horizons. Specifically, we estimate local projections of the following form:

$$y_{t+h} = \alpha_h + \beta_{0(h)}\epsilon_t + \sum_{i=1}^{p} \gamma_{i(h)} w_{i,t} + u_{(h),t+h}, \tag{14}$$

where y_{t+h} is the projection of the endogenous variable at the horizon h, ϵ_t is the shock of interest and $w_{i,t}$ is a vector of control variables. We consider four endogenous variables: The term premium, the risk neutral rate, core inflation and the year-on-year industrial production growth. The vector of control variables include a constant, the term premium, industrial production, inflation, the Gilchrist and Zakrajšek (2012) credit spread and the nominal Federal Funds rate. We also control for the lags 1 to 6, 9 and 12 of each of these variables and of the endogenous variable of interest. The model is estimated by simple OLS, and confidence intervals can be computed using Newey-West corrected standard errors.

As discussed in Jordà (2005), LPs present several advantages with respect to a standard VAR. LPs do not require the specification and the estimation of the unknown data generating process and are therefore more robust to specification, are less affected by the curse of dimensionality and can easily accommodate non-linearities. Moreover, it seems to be unaffected by the level of persistence of the dependent variable. Even though LPs estimates of impulse responses are less efficient than VAR-based estimates, when the VAR is correctly specified and it is the true model, these efficiency losses are usually not large.

We identify a "quantitative easing" (QE) shock with two different strategies:

- First, we identify the shock as the residual of the regression of the first difference of the St. Louis Adjusted Monetary Base, Δm_t, on a set of controls: $\epsilon_t = \Delta m_t - E(\Delta m_t | w_{1,t}, \ldots, w_{p,t})$. The set of controls includes industrial production growth, the inflation rate, the Gilchrist and Zakrajšek (2012) credit spread, the term premium and the short term interest rate. We also control for 12 lags of each of these variables and for 12 lags of Δm_t.

- Second, we identify the QE shock using the actual timing of the FED announcements. Specifically, we build a shock variable defined as the product of a dummy D taking value 1 when a new round of quantitative easing is implemented, and the growth rate in monetary base, Δm_t, which proxies for the importance of the policy change: $\epsilon_t = D \cdot \Delta m_t$. In particular, the variable D takes value 1 in the following dates:

1. December 2008 (QE1): The FOMC approves the purchase of agency mortgage-based securities (MBS) and agency debt for up to 600 billion dollars.

2. March 2009: The FOMC expands its asset purchase program to a total of 1.25 trillion in purchases of agency MBS, 200 billion in government-sponsored-enterprises (GSE) obligations, and up to 300 billion of longer term securities.

3. November 2011 (QE2). The FOMC announces the intention of purchasing 600 billion of longer-term securities. Since in our data the actual monetary base started to increase the following month (in December 2012), the QE dummy takes a value 1 in December. Notice, however, that this timing assumption does not affect the results.

4. September 2012 (QE3): The FOMC announces an open-ended commitment to purchase 40 billion agency MBS per month. Since in our data the actual monetary base started to increase the following month, the QE dummy takes a value 1 in October 2012. Again, this timing assumption does not affect the results.

Figure 12 shows the impulse responses of the macro-finance variables –including the FCVAR implied baseline term premium and risk-neutral rate– to the QE shock following the second identification strategy. Results with the first strategy are quite similar, and we do not report them for brevity. The figure shows a protracted decline of the term premium together with an increase in the risk-neutral rate. This increase in the risk-neutral rate seems to be driven by the expansionary effects in inflation and economic activity which are shown in the two bottom panels of the Figure. Thus, the QE shocks had a clear effective expansionary effect in the real economy together with a considerable dynamic reduction in the risk aversion of investors toward long-term bonds. Because of this expected increase in real activity and inflation, investors –in a zero-lower bound environment– may have priced a future increase in the future monetary policy (short-term) interest rate. While this increase may have only materialized at a later date (see Bauer and Rudebusch (2016)), the risk neutral rate clearly reflects these expectations.

As a final illustration of the differences among the term premia identified through the alternative modelling strategies, Figure 13 compares the impulse responses of three term premia and associated risk-neutral rates (I(0)-VAR, I(1)-CVAR and FCVAR) to the same QE shock. Qualitatively, the responses of the FCVAR and the I(1)-VAR are similar –decline of the term premium and increases in the risk-neutral rate– but the responses of the I(1)-VAR are quantitatively larger. This is in contrast to the I(0)-VAR model, where the term premium response to the QE shock is basically null and not even statistically significant. Thus, policy analysis based on purely I(0) VAR models can be especially misleading when focusing on the monetary policy effects on term premium dynamics. This is especially relevant at this point in time, when policy makers are measuring the effects of future tapering of the monetary stimulus on term premium dynamics. While the current withdrawal of monetary stimulus is asymmetric in timing and size with respect to the QE injections (see Yellen (2017)), our results support the view that term premia will tend to increase over time.

[Insert Figures 12 and 13: Term Premium and Risk-Neutral Responses to QE Shock]

Several papers have analyzed the impact of the QE policies on yields and term premium dynamics. Gagnon, Raskin, Remache and Sack (2011) perform an event study with daily data where they show a baseline effect on the 10-year term premium of 71 accumulated basis points across 8 announcements of large scale asset purchases (2008-2010). Their methodology is quite different from ours, as they consider 1-day windows for each of these 8 announcements and aggregate these 8 one-day impacts to construct this figure. In our case where we obtain an accumulated significant impact of 25 basis points, we directly work with monthly data and thus aggregation occurs at the monthly frequency. We thus may miss some of the daily frequency effects that they report. They also work with a different term premium, as they base their results on the Kim and Wright (2005) term premium, which is based on an I(0) model. In a related exercise, Hamilton and Wu (2012) use a model of risk-averse arbitrageurs to estimate the change in maturity structure of the Fed's balance sheet (away from the short end and into the long) on yields. Their results imply a decrease of the term premium above 12 basis points. Krisnamurthy and Vissing-Jorgensen (2011) essentially find no impact of QE on duration risk premium (the one associated with the term premium). They however find effects of QE on yields through a signaling channel (interest rate changes going forward). This study is also quite different from ours since they perform daily and intra-daily event studies and they mostly focus on yields, rather than the term premium.

5 Conclusions

This paper presents a very natural model of the yield curve capturing a long run equilibrium relationship among the U.S. sovereign rates. Our estimates of the flexible FCVAR model confirmed the existence of this pattern and characterized it as a fractionally integrated mean-reverting process. Our analysis also rejects some of the standard stationary I(0) and unit-root alternatives to joint modelling of interest rates. The estimates implied by our general FCVAR model are thus able to capture both the low-frequency movements in bond yields and the mean reversion commonly assumed in many financial models. We show that the implied term premium is quite robust to alternative subsamples and also derive the term premia for the UK and Germany.

As an important outcome of our exercise, this term structure model affords the identification of a credible term premium which can be readily used by both academics and policy makers. We also shed light on the sources of the term premium, which are mainly real, i.e. while economic growth lowers the term premium, economic slack and recessions increase the risk priced by investors in long-term bonds. In a final exercise, we investigate the role of quantitative easing policies on term premium dynamics. In contrast to the I(0) stationary-implied term premium, our estimates of the FCVAR imply a significant term premium decline following the quantitative easing episodes.

This article can be extended in several directions. Firstly, the term premium here has been specified in terms of a long memory property, characterized by a spectral density function which has a pole or singularity at the smallest (zero) frequency. However, it might be the case that the spectrum of the term premium contains peaks at other frequencies, referring, for example, to the business cycles. In this context, Gegenbauer (fractionally integrated) processes can be employed as an alternative to the standard I(d) approach used in this work. In addition, the FCVAR can be extended by adding other macro-finance variables potentially cointegrated with interest rates. Work in these directions is now in progress.

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Table 1: Cointegrating Rank Test, Sovereign Yields

Rank	Log-Likelihood	LR statistic
0	1343.603	60.193
1	**1361.136**	**25.127**
2	1371.076	5.248
3	1372.619	2.162
4	1373.700	—-

This table shows the results of the cointegrating rank test for the FCVAR model. In bold, the selected cointegration rank.

Table 2: LR Test, Sovereign Yields, CVAR v/s FCVAR

Unrestricted log-like:	1361.136
Restricted log-like:	1345.663
LR statistic:	30.947
p-value:	0.000

This table shows the results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model vis à vis the I(1) CVAR model.

Table 3: LR Test, Sovereign Yields, $d = b$ v/s $d \neq b$

Unrestricted log-like:	1361.136
Restricted log-like:	1357.200
LR statistic:	7.873
p-value:	0.014

This table shows the results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model with d different from b and the FCVAR model with the restricted model where $d = b$.

Table 4: Table 4: Term Premium, Risk Neutral Rate: Descriptive Statistics

	Variable	Mean	St.dev.
I(0)-VAR	Risk neutral rate	5.0113	1.3513
	Term Premium	2.0030	1.7407
I(1)-CVAR	Risk neutral rate	5.2122	3.6278
	Term Premium	1.8020	1.7498
FCVAR	Risk neutral rate	5.4074	2.7848
	Term Premium	1.6069	1.1265

This table shows the first and second moments of the term premium and risk neutral rates implied by the three alternative term structure models.

Table 5: Table 5: Term Premium, Risk Neutral Rate: Descriptive Statistics

	Variable	Corr w/FFR	Corr w/tp	Corr w/unempl	Corr w/Δ Y
I(0)-VAR	Risk neutral rate	0.9297	0.5034	-0.0294	0.1937
	Term Premium	0.5767	1	0.5694	-0.0839
I(1)-CVAR	Risk neutral rate	0.9772	-0.7088	0.0855	0.1115
	Term Premium	-0.7344	1	0.3665	-0.1650
FCVAR	Risk neutral rate	0.9642	-0.2869	0.1184	0.0625
	Term Premium	-0.3774	1	0.5518	-0.0520

This table shows the correlations of the three alternative term premia and risk neutral rates with several macro-finance variables: Federal Funds Rate, term premium, unemployment and industrial production growth, respectively.

Table 6: Term Premium Drivers: Simple Model

1990m4-2017m12	TP I(0)	TP I(1)	TP FCVAR
Constant	-5.82***	-0.41	-0.91**
	(0.49)	(0.44)	(0.38)
Long-run Inflation Disagreement	1.88***	-0.59***	-0.19
	(0.18)	(0.16)	(0.14)
Unemployment Rate	0.24***	0.77***	0.54***
	(0.04)	(0.04)	(0.03)
Recession dummy	0.33	0.87***	0.13
	(0.21)	(0.19)	(0.16)
Adj. R2	0.45	0.59	0.50

This table shows the result of the simple OLS regressions of the alternative term premium on macro-finance variables. Newey-West-corrected standard errors appear in parentheses.

Table 7: Term Premium Drivers: Altermative Model

1990m4-2017m12	TP I(0)	TP I(1)	TP FCVAR
Constant	-5.47***	0.71	-0.12
	(0.48)	(0.44)	(0.38)
Long-run Inflation Disagreement	2.24***	0.56***	0.62***
	(0.16)	(0.15)	(0.13)
Unemployment Residual	0.24***	0.77***	0.54***
	(0.04)	(0.04)	(0.03)
Recession dummy	0.33	0.87***	0.13
	(0.21)	(0.19)	(0.16)
Adj. R2	0.45	0.59	0.50

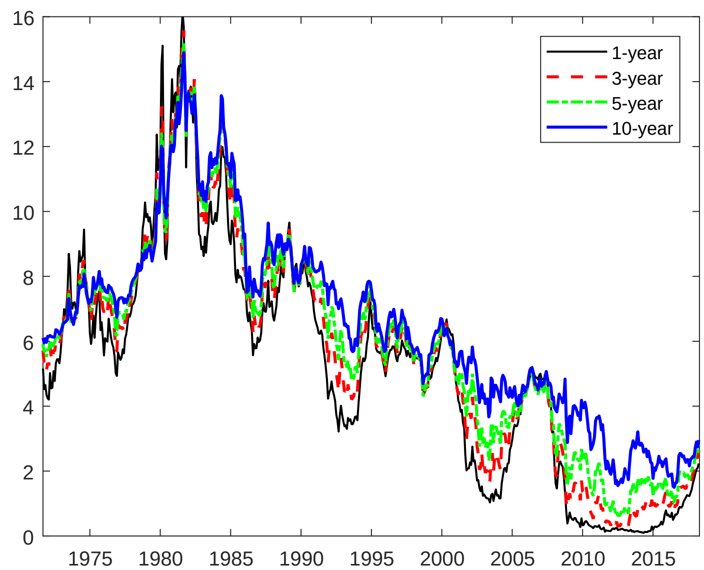
This table shows the result of the OLS regressions of the alternative term premium on macro-finance variables. The Unemployment Residual variable is the residual of the unemployment rate regression on a constant and long-run inflation disagreement. Newey-West-corrected standard errors appear in parentheses.

Table 8: FCVAR Orders of Integration, US, UK and Germany

	US	UK	GER
d	0.756	0.919	0.871
	(0.034)	(0.052)	(0.041)
b	1.184	0.457	0.844
	(0.087)	(0.192)	(0.094)

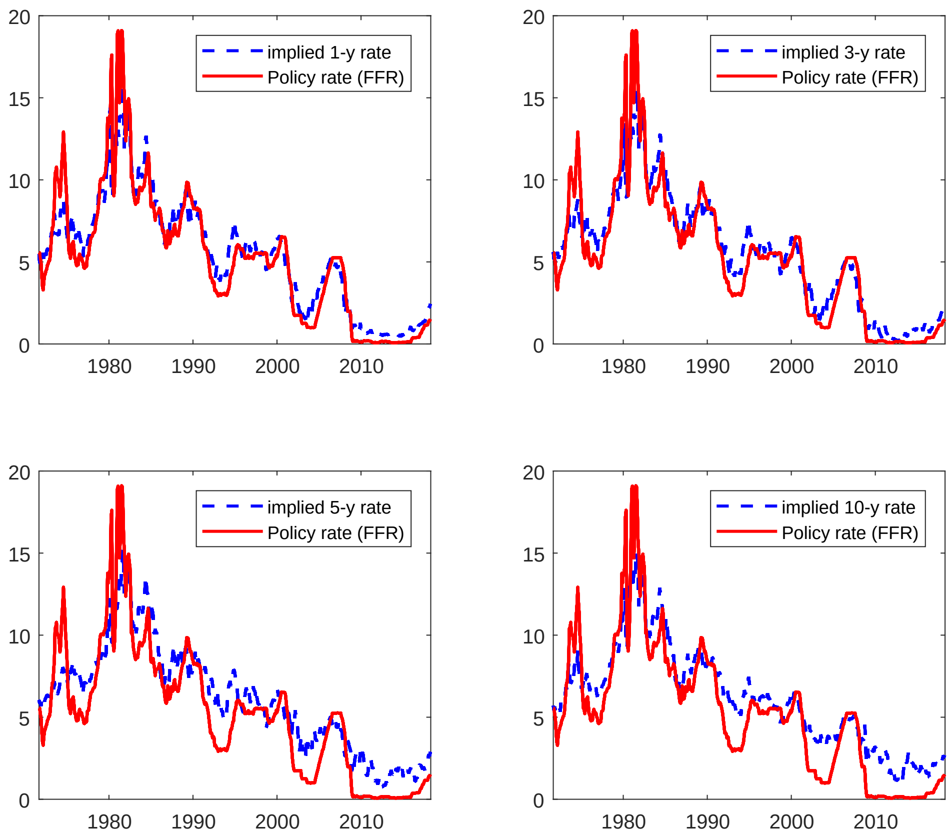
This table shows the estimates of the orders of integration of the FCVAR models for the three countries analyized in this study: United States (US), United Kingdom (UK) and Germany (GER). Standard errors appear in parentheses.

Figure 1: US Sovereign Rates



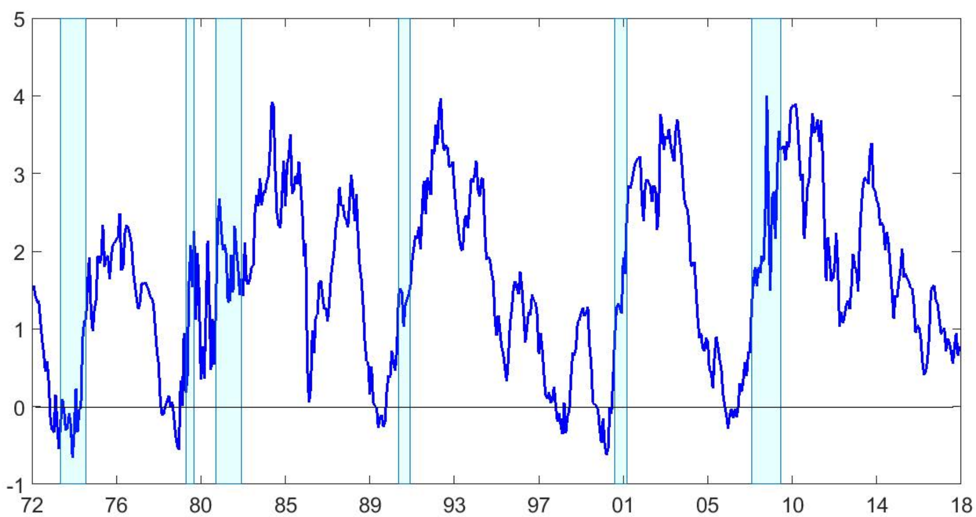
This figure plots the historical monthly series of zero-coupon US sovereign rates (1-year, 3-year, 5-year and 10-year).

Figure 2: Federal Funds Rate and FCVAR-implied Rates



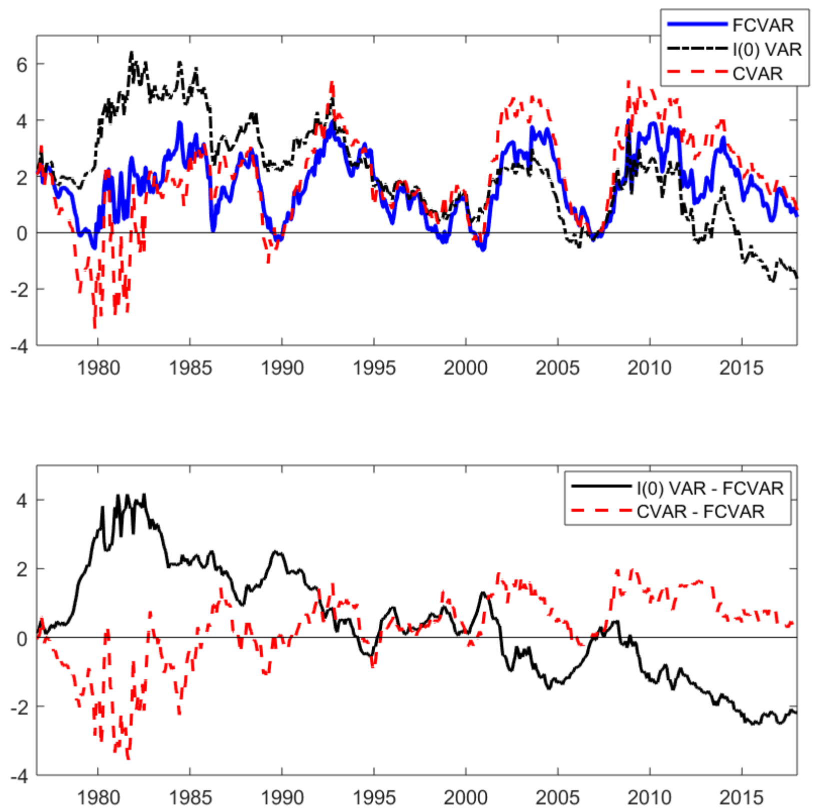
This figure plots the historical monthly series of the Federal Funds Rate (FFR) together with the FCVAR-long-run-implied US sovereign rates (1-year, 3-year, 5-year and 10-year).

Figure 3: Term Premium FCVAR



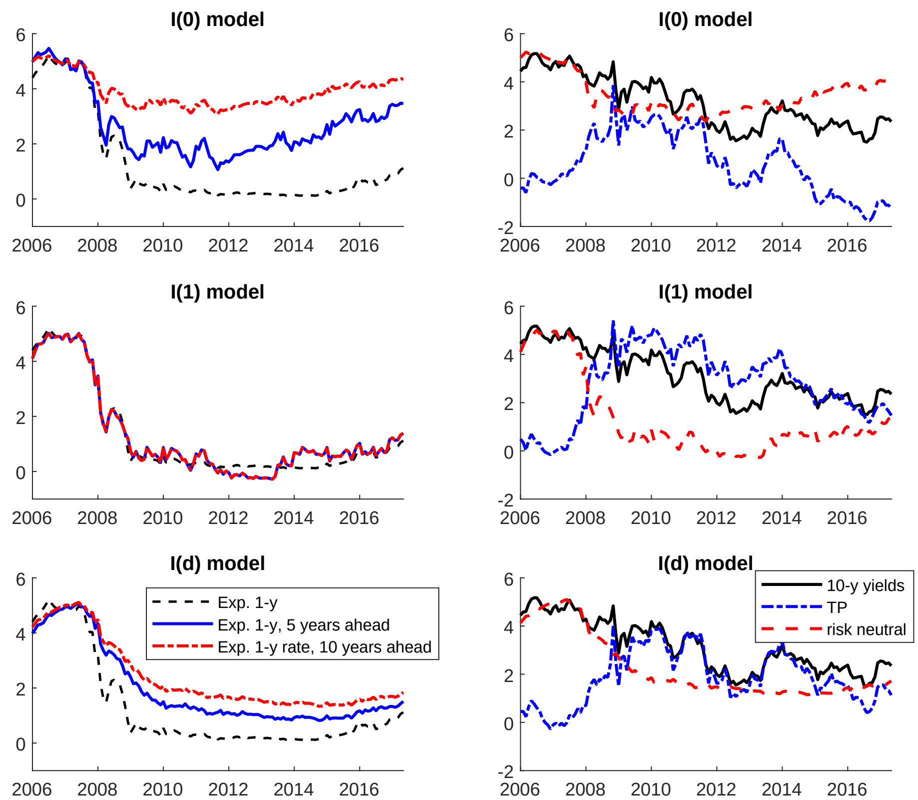
This figure plots the monthly term premium implied by the FCVAR. Shaded areas reflect NBER recession periods.

Figure 4: Differences in Term Premium: I(0) and CVAR v/s FCVAR



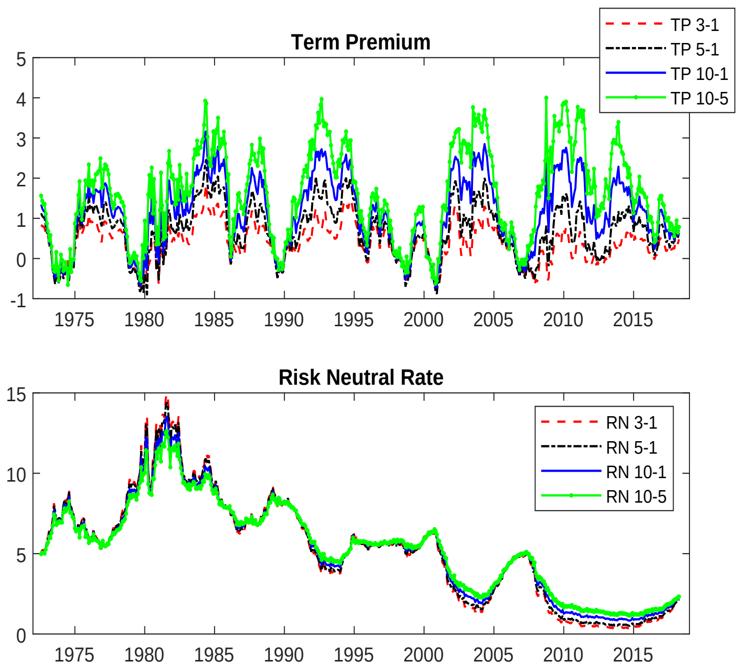
This figure plots the three monthly term premia (CVAR, FCVAR and I(0)-VAR), as well as the differences between the FCVAR-implied term premium and the other two.

Figure 5: Differences in Long-Term Expectations, Term Premium and Risk-Neutral Rates: I(0) and CVAR v/s FCVAR



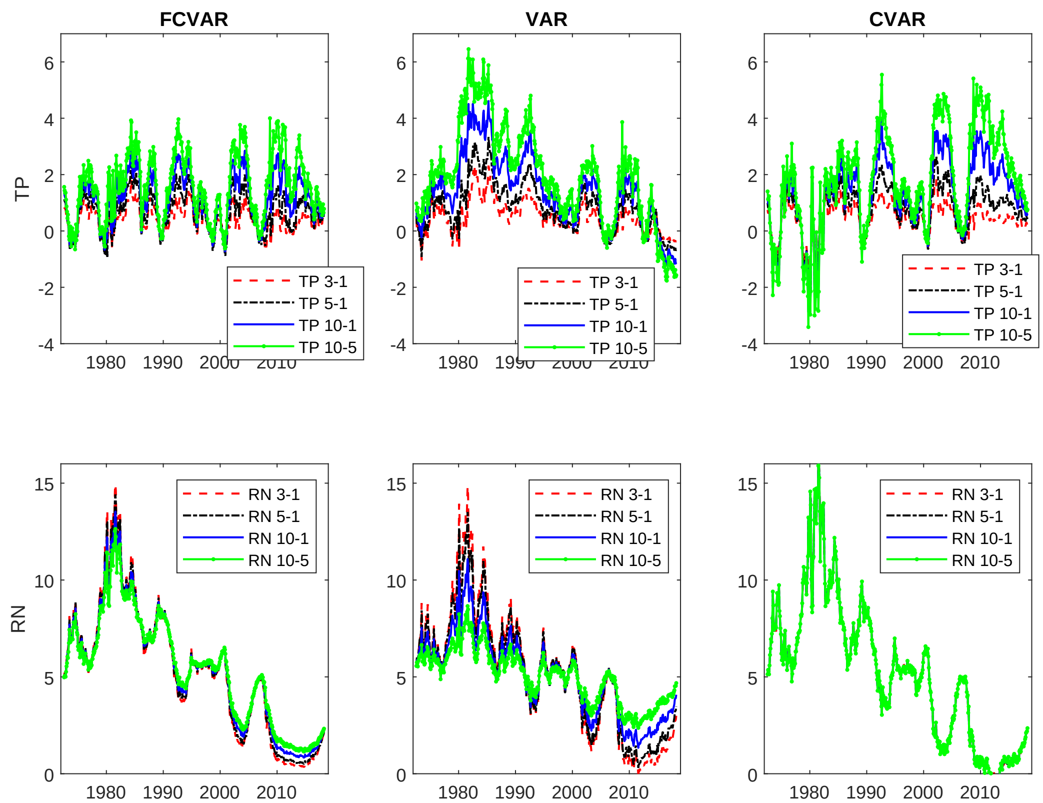
The graphs in the left column show the long-term expectations of the 1-year rate implied by the three models (I(0)-VAR, I(1)-CVAR and FCVAR (I(d))) for different horizons (1-year, 5-years and 10-years ahead), whereas those in the right column plots the implied implied term premium and risk-neutral rates across the three models together with the 10-year yields.

Figure 6: Term Structure of Term Premia: FCVAR



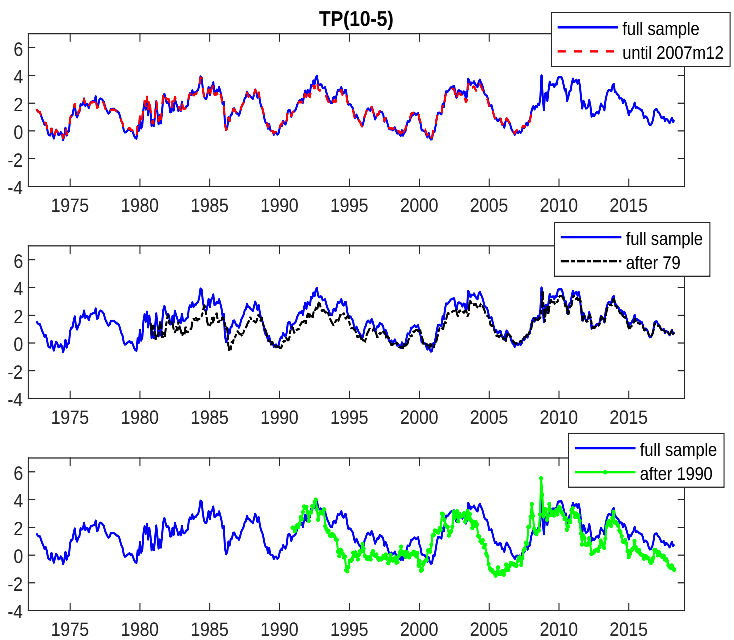
The top figure shows the FCVAR-implied term premia computed at different maturities, with the TP 10-5 being the baseline term premium, and the other three are the 3, 5 and 10-year term premia following equation (13). The bottom figure plots the associated risk-neutral rates.

Figure 7: Term Structure of Term Premia: VAR, FCVAR, CVAR



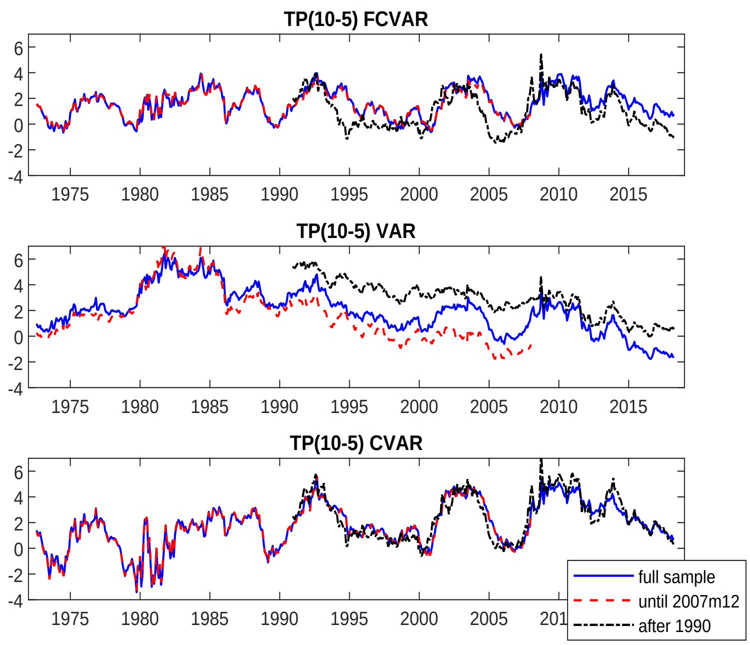
The top figure shows the term structure of term premia for the (from left to right) FCVAR, I(0)VAR and I(1) CVAR models. The term premia are computed at different maturities, with the TP 10-5 being the baseline term premium, and the other three are the 3, 5 and 10-year term premia following equation (13). The bottom figure plots the associated risk-neutral rates across models.

Figure 8: Term Premium Subsample Stability: FCVAR



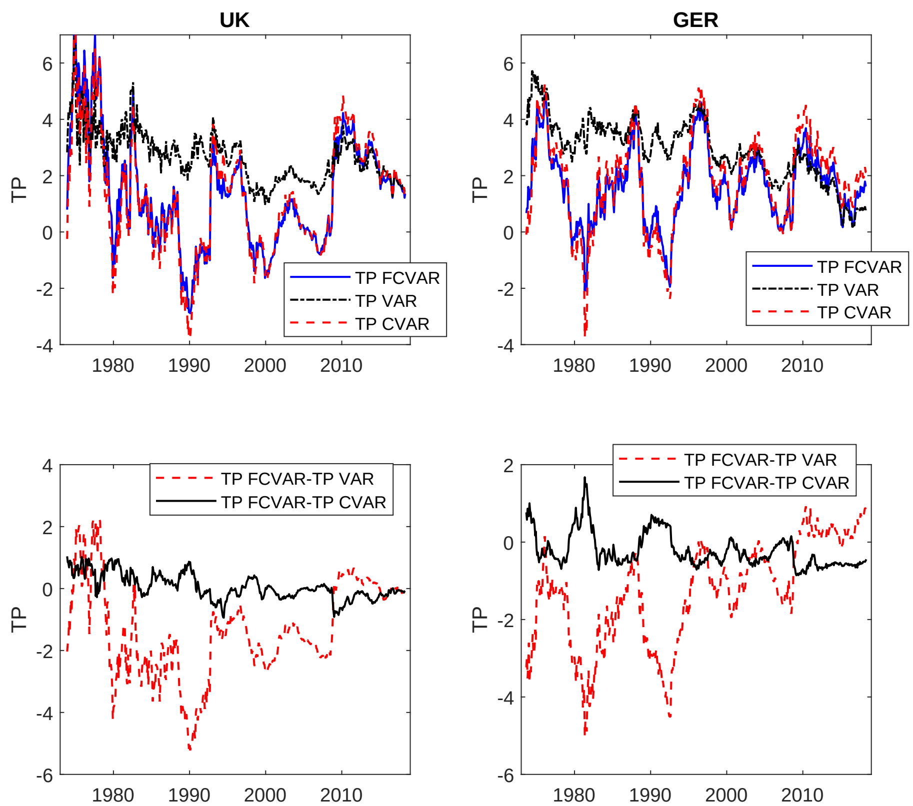
This figure compares the baseline full sample term premium with that associated with other subsamples: Pre-2008 (top graph), post-1979 (medium graph) and post-1990 (bottom graph).

Figure 9: Term Premium Subsample Stability: FCVAR, VAR, CVAR



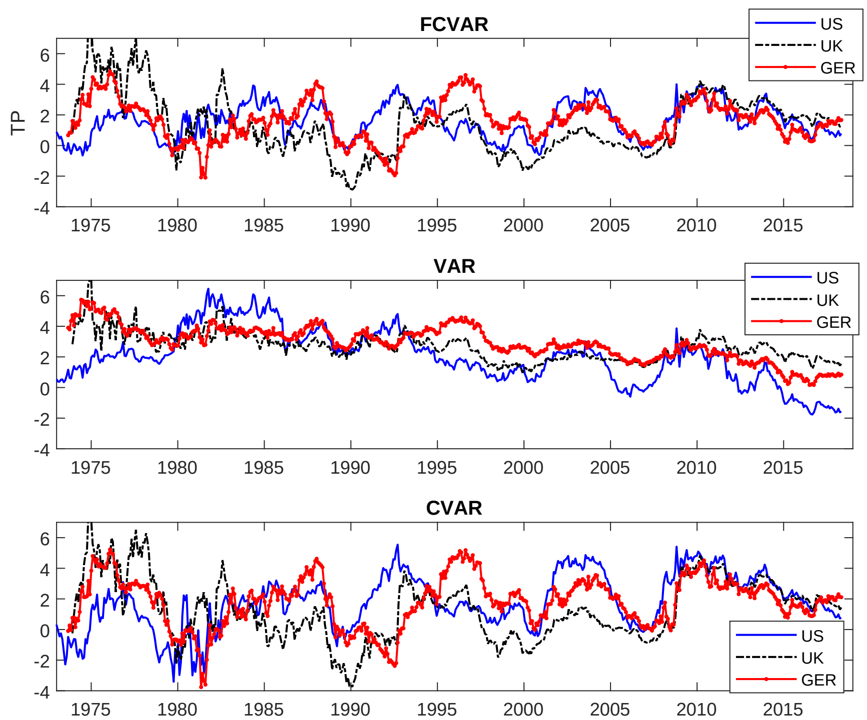
This figure compares the baseline full sample term premium with that associated with other subsamples: FCVAR (top graph), I(0) VAR (medium graph) and I(1) CVAR (bottom graph). Each graph compares the full sample term premium with the pre-2008 and the post-1990 counterparts.

Figure 10: Term Premia: UK, Germany (FCVAR, I(0) VAR, I(1) CVAR)



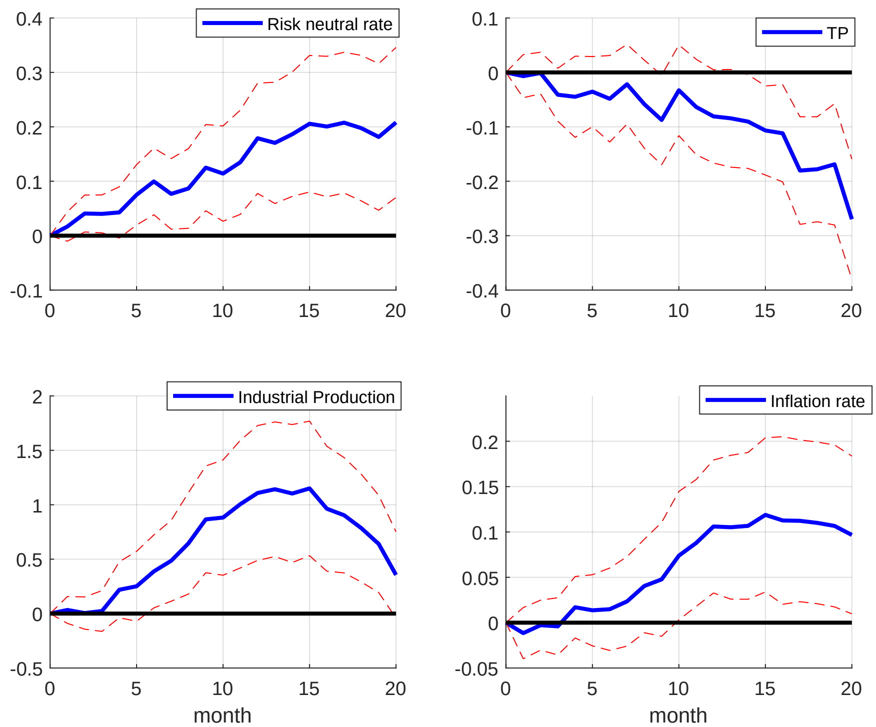
This figure plots the implied term premia across models (FCVAR, I(0) VAR, I(1) CVAR) for the UK and Germany (top graphs). It also plots the differences between the FCVAR and the other two models across countries (bottom graphs).

Figure 11: Term Premia: US, UK, Germany (FCVAR, I(0) VAR, I(1) CVAR)



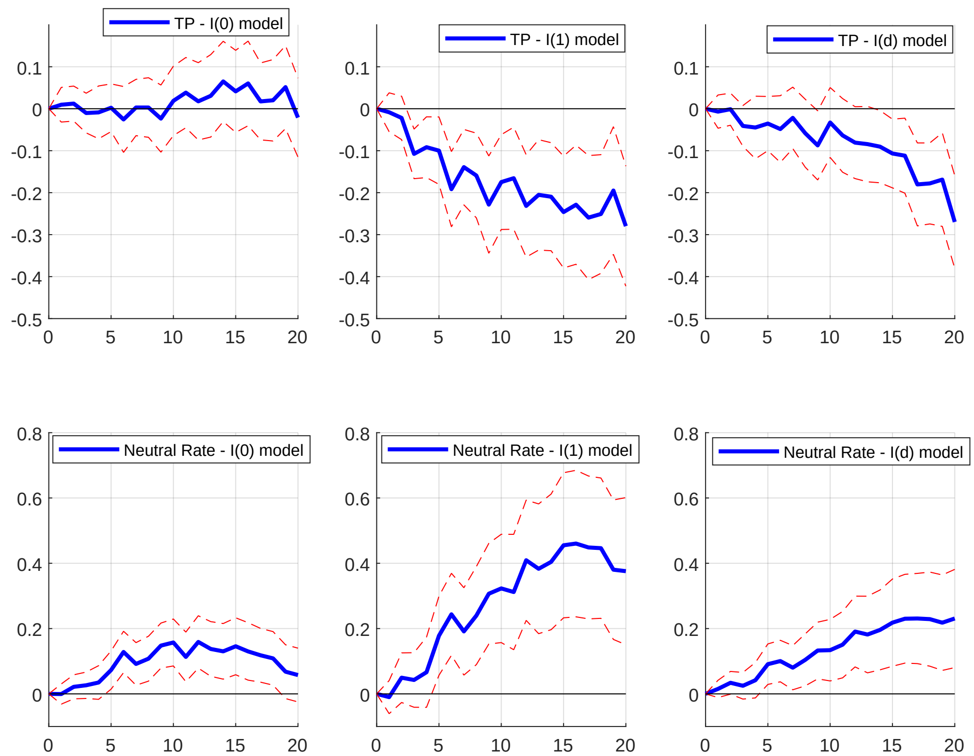
This figure plots the term premia of the three countries (US, UK, Germany) implied by the three models: FCVAR (top graph), I(0) VAR (medium graph) and CVAR (bottom graph).

Figure 12: Macro-Finance Impulse Responses to QE Shock, FCVAR Model



This figure plots the impulse response function of the term premium, the risk-neutral rate, industrial production growth and inflation to a quantitative easing (QE) shock. The responses are identified via local-projection analysis and the term premium and risk-neutral rate through the FCVAR model with sovereign rates.

Figure 13: Term Premium and Risk-Neutral Rate Responses to QE Shock: I(0)-VAR, I(1) CVAR and FCVAR



This figure compares the impulse response function of the term premium and the risk-neutral rate to a quantitative easing (QE) shock (I(0)-VAR, I(1) CVAR and FCVAR (I(d) models)). The responses are identified via local-projection analysis.