Resolution of Collateral Crises*

P. Sebastián Fanelli

MIT[†]

Martín Gonzalez-Eiras

University of Copenhagen[‡]

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Abstract

Allowing for strategic default eliminates the dynamic amplification effects in collateral-driven credit constraint models by decoupling the net worth of borrowers from decreases in asset prices. However, when there are costs associated with the liquidation of a firm and the setting up of a new one, borrowers will sometimes prefer not to default and keep assets used as collateral even if their market value is below the outstanding debt. This creates a wedge between the inside and outside value of assets setting the stage for bargaining over debt repayments. We study how these technological factors, together with institutions that determine the bargaining power of borrowers and lenders, affect the amplification and persistence of aggregate shocks. We find strong amplification effects for moderate shocks, while for larger shocks debt renegotiation dampens amplification. There is more amplification, and shocks are more persistent, the higher the start-up costs of new firms. Furthermore, the presence of asymmetric information on default costs leads to "V-shaped" recoveries. These results are consistent with features of observed behavior in some macroeconomic crises.

KEYWORDS: Credit cycles; balance sheet recessions; default; renegotiation.

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[†]50 Memorial Dr., Cambridge, MA 02142, USA. E-mail: sfanelli@mit.edu

[‡]Øster Farimagsgade 5, 1353 Copenhagen, Denmark. E-mail: mge@alum.mit.edu

1 Introduction

Financial mechanisms play a large role in the propagation and amplification of macroeconomic impulses, particularly in states of crisis.[1] Among the various potentially relevant effects, the strengthening of collateral restrictions as a result of falling asset prices is one of the more salient. After a negative shock, credit-constrained entrepreneurs find themselves with too much debt, which may force them to sell assets at fire- sale prices in an effort to reduce their leverage. This depresses the access to credit of agents who are likely to have comparatively high productivity in the use of resources. The consequent reallocation of assets impinges on aggregate productivity, and causes further contractions in the value of the collateral of prospective borrowers. This deviation-amplifying feedback effect was formalized with a stylized model in a celebrated paper by Kiyotaki and Moore (1997) (hereafter KM). A question arises, both for analytical and practical concerns, about the conditions that generate the large amplification effects found in that work and, more generally, about the circumstances that may determine the intensity of financial multipliers.

The aim of this paper is to study the ex post resolution of crises when the repayment of large masses of private business debts is put into question. In particular, we focus on the relative bargaining power of borrowers and lenders in case of default, determined by institutional and technological factors, and its consequences for the amplification and persistence of aggregate shocks. Economies where the business sector has accumulated excessive debts can adapt in several ways. Leaving aside government interventions (like bailouts or legislated write-offs), the process of adjustment to an overhang may take two polar forms: a gradual deleveraging or a sudden burst of defaults and debt restructurings. In the first instance, firms go through a prolonged period where they use current income for the purpose of servicing debts; consequently, their access to productive resources is diminished. In the other extreme, after bankruptcy procedures have been executed, the restoration of solvency would bring about a relief of the financial constraints on production and investment. We build a model based on KM which, in contrast with that basic framework, allows for the renegotiation of pre- existing debts. In that way, we can capture in broad terms those two scenarios and study conditions that would make an economy go one way or another.

The institutional features of bankruptcy legislation can influence substantially the outcome of a macroeconomic crisis through their impact on the allocation of rights of control over assets after a debtor proves incapable to repay obligations in full. There is significant heterogeneity in the distribution of bargaining power between borrowers and lenders across countries, and even at the subnational level (e.g. in the U.S. there is considerable variation across States in the homestead exemption in the personal bankruptcy procedures under Chapter 7). When considering whether to go for bankruptcy, a borrower faces costs which are affected by the legal environment. On the other hand, when bankruptcy occurs, if the borrower contemplates starting a new firm after the liquidation of the failed one, fixed setup costs have typically to be incurred, and it may take some time of, say, learning by operating the relevant resources, until the recently initiated project reaches its potential level of productivity. These effects indicate disadvantages of giving

[1]See for example the empirical findings of Aikman et al. (2010), and Jordà et al. (2011).

up the economies generated in established firms by "asset specificity" (we use the term in a broader sense that the usual meaning in the literature, which relates closely to the market liquidity of assets). At the same time, the significance of that specificity may vary considerably according to the type of firm and the configuration of the economy, with potentially macroeconomic implications.

Our model suggests that the degree of asset specificity may be a key determinant of the evolution of a given economy after a negative macroeconomic shock. When assets are very specific, so that their productivity depends strongly on the identity of their users, "incumbents" would be reluctant to allow the liquidation of existing firms; that would reinforce the bargaining power of creditors in potential repayment negotiations, by strengthening the weight of their threat to capture the assets. Thus, a shock would create a tension between the potential non-appropriability of output by creditors and their ability to extract resources from debtors who try to maintain access to the assets that they have learned to operate effectively. This feature implies that for small shocks, the desire to keep their assets induces productive entrepreneurs to use a larger proportion of their resources in debt servicing (which tends to depress asset prices), while for larger shocks, the threat of default by the debtor is credible and induces lenders to renegotiate debt repayments. When asset specificity is stronger, the larger would be the threshold size of the shock for which lenders start finding incentives to offer debt relief. We also find that more asset specificity leads to more persistence of the output effects of a one-time shock, since a lower productivity of new assets forces borrowers to rebuild their working capital at a slower pace.

The association found in the model between the degree of asset specificity and the form and intensity of the financial propagators of debt- related economic downturns recalls features of observed behaviors in macroeconomic crises. A long-drawn process of corporate debt reductions in a stagnant economy with, at the same time, few breakdowns or restructurings of large business firms or groups, like that experienced in the 1990's in Japan (e.g. Koo, 2003; Chang, 2006a) would seem to correspond to a case with highly specific productive assets and institutions providing strong protection of creditor rights, which motivates strong efforts to repay debts on the parts of owner/managers of assets and, as a counterpart, accentuates the effect of financial multipliers.[2] By contrast, large-scale transfers of property and control took place in some South East countries in the late 1990's, with a macroeconomic crisis characterized by a very agitated period with numerous renegotiations of private debts and widespread shifts in asset holdings. This pattern would roughly correspond to a low- specificity scenario and with institutions that give borrowers significant bargaining power.[3]

[2]Ahmadjian (2006) has found, through an analysis of ownership ties within business groups in Japan, that although some peripheral relationships were disrupted, the links between core firms remained robust. The image is that of a collection of physical and relational capital that incumbents find costly to take apart, and prefer to go through the painful process of debt reduction to preserve.

[3]Thus, for example, Gomez (2006) finds that in the Malaysian crisis some major capitalists lost control of their corporate assets since they were burdened with enormous debts and depended too much on the will of government leaders, while business groups with better political connections thrived. It has been suggested (e.g. Chang, 2006b; Hanani, 2006) that the changes in the property and management of assets which happened in the Indonesian case were often linked to political influences: political contacts are

In the real world, it may be hard for a creditor to correctly assess the specificity of the relationship between a debtor and her assets. It seems plausible that specificity is harder to ascertain when it is based on the operation of more portable or intangible assets like business networks or contacts. The model can be extended to allow for asymmetric information about default costs; this makes renegotiating more problematic. In that setting, there is more equilibrium default the stronger then informational asymmetry, as lenders will offer debt reductions which are attractive only to borrowers with particularly high asset specificity. This scenario, which replicates the observed variety of responses in crises, with both negotiated agreements on new repayment schedules and terminal defaults leading to the liquidation of firms, would entail more output costs in the initial periods due to the lower productivity of new assets of the entrepreneurs who lose their old businesses while, at the same time, default implies that more capital remains in the hand of entrepreneurs, which moderates the fall in production. This ambiguity notwithstanding, the model implies that the weight of the costs of default dominates at first, but over time, the effect of an allocation of capital more favorable to entrepreneurs becomes stronger; economies with a higher degree of asymmetric information would experience sharper post-shock recessions and more pronounced "V-shaped" recoveries.

An important caveat is in order. Given our interest in the ex post response of the economy to large negative aggregate shocks, the usefulness of the KM environment follows naturally. At the same time, the simplifications that give the basic model its tractability carry out their limitations, particularly the assumption that the shock hitting the economy is a perfectly unanticipated "rare event". This implies that we do not consider the ex ante behavior of lenders and entrepreneurs, and particularly how it is affected by expectations of potential debt renegotiation. However, our analysis would capture qualitatively relevant patterns of macroeconomic and sectoral performance after severe low-probability shocks: economies characterized by high default costs, both for technological and institutional reasons, should exhibit more deleveraging than those where entrepreneurs have more bargaining power and can easily reallocate resources to new firms and incur in small productivity costs. In future work we intend to study ways to identify empirically observable features of the transmission mechanisms analyzed in the model, and particularly the link between bankruptcy procedures, asset specificity, the magnitude of debt renegotiations in the event of large shocks and the behavior of credit constrained firms over the business cycle.

Our model is related to the literature on the macroeconomic implications of financial imperfections (See e.g. Bernanke and Gertler, 1989; Bernanke et al., 1999; Carlstrom and Fuerst, 1997), and the consequences of the limited enforceability of debt contracts (see Cooley et al. (2004)); for a survey, see Brunnermeier et al. (2013), Freixas and Rochet (2008), or Bhattacharya et al. (2004). The large multiplier effects found in KM have been discussed in various contexts. Kocherlakota (2000) builds an economy with a representative agent who has an incentive to smooth consumption, making it optimal to hold liquid assets to self- insure against negative shocks. In that setting, the quantitative significance of the amplification effects generated by collateral constraints hinges on the

more "portable" across business activities than technological or managerial expertise.

parameters of the economy, in particular on factor shares. In the same spirit, Cordoba and Ripoll (2004) suggest, on the basis of a calibrated model of an economy with heterogeneous agents, that the amplification and propagation generated by credit constraints need not be quantitatively large if preferences allow for risk aversion and the capital share in production has values similar to those in actual economies. Moreover, they find a trade-off between amplification and persistence. In contrast, Liu et al. (2013) indicates that when economic shocks impact directly upon the price of collateral assets, amplification effects may still be quantitatively strong.

Brunnermeier and Sannikov (2014) have built a continuous time two- sector model which is solved without using a log-linear approximation. The resulting dynamics feature highly non-linear amplification effects and, unlike in KM, multipliers are asymmetric, stronger for negative impulses. Entrepreneurs are aware that they might go through adverse times, and choose to hold liquidity buffers. This mitigates the effects of moderate shocks, dampening their amplification. However, in response to more severe shocks, entrepreneurs reduce their asset demand, affecting their price and triggering amplification loops. The analogs of balance sheet crises thus generated have a high degree of persistence.

The reminder of the paper is organized as follows. Section 2 presents the basic framework, which is a variant of KM's model with endogenous default costs. In this setting, the strategic decisions on default and renegotiation imply a connection between the degree of asset specificity and the form and intensity of the financial multiplier. Section 3 develops an extension with asymmetric information about default costs. In section 4 we show that stronger asset specificity leads to more persistence of the effects of shocks. Section 5 concludes.

2 Amplification and asset specificity

2.1 Basic setup

We outline our model that builds on KM's setup. There are two types of producers, entrepreneurs and lenders, each with measure one. The entrepreneurs will turn out to be the (constrained) borrowers in equilibrium. Both sets of agents are risk neutral and maximize their expected utility given by

$$E_t \sum_{s=0}^{\infty} \beta^s x_{t+s} \qquad \text{and} \qquad E_t \sum_{s=0}^{\infty} \prod_{j=0}^{s} \beta'_j x'_{t+s},$$

where $0 < \beta < \beta'_t < 1$ are their respective discount factors, and x_{t+s} and x'_{t+s} are their consumptions in period $t+s$ of a perishable good.

There is a fixed aggregate endowment of a productive asset or capital, $\overline{K}$. Capital is the only factor of production. Entrepreneurs have access to a linear production technology. Their output is divided into two parts. A subset of it (with productivity a) is "tradable", meaning that it can be used for market transactions; by contrast, "non-tradable" output (c per unit of input) can only be consumed by the entrepreneur. This assumption makes consumption positive in every period even though entrepreneurs choose to apply all the tradable output to acquire capital. Lenders have access to a standard production technology with decreasing returns:

$$y'_{t+1} = G(k'_t), \text{ where } G' > 0,\, G'' < 0,\, G''' \geq 0,\, G'(0) > aR_t > G'\left(\overline{K}\right),$$

where R_t is the gross interest rate (equal in equilibrium to the inverse of β'_{t+1}).We further assume $G''' \geq 0$ to limit the number of equilibria in the model to at most 3.[4]

The model's key assumption is a constraint that limits entrepreneurs' ability to borrow to the collateral they can provide, given by the anticipated value of their capital holdings. Specifically, if at date t an entrepreneur has assets k_t, then she can borrow b_t as long as the promised repayment does not exceed the market value of her assets at date $t+1$:

$$R_t b_t \leq q_{t+1} k_t,$$

where q is the price of capital.

This credit constraint is rationalized through the impossibility of the borrower to precommit to making productive use of the firm's assets. KM make the assumption that an enforcement technology exists which precludes default *after* the productive process has started (see footnote 13 in KM). By construction, debt contracts are specified unconditionally in terms of quantities of output. If, now, an entirely unpredicted negative aggregate shock disturbs for one period levered firms, their promised debt repayments exceed the post-shock value of their collateral. And lenders can collect debt services against both the assets and the tradable output of the borrower. Consequently, while the potential of a default restricts ex ante the supply of credit below the value of the capital stock, once the shock is realized, debtors honor their commitments in full even if ex post their collateralized assets prove insufficient for that purpose.

The previous assumption is crucial for the existence of large multiplier effects. These emerge as a lower demand for capital by levered entrepreneurs reduces asset prices, and forces them to allocate more output to debt servicing, which causes further drops in prices: a *transitory* reduction in borrowing firms' net worth brings about a fall of the same order of magnitude in asset prices and aggregate output, and the economy takes a substantial time to return to the steady state. In contrast, if repayments are actually bounded by the collateral of the debtor firm, so that the whole volume of current output remains available to borrowers, a transitory decline in productivity only has second order effects on asset prices and total production. Figure 1 shows the impulse response functions of output after a temporary negative productivity shock for the cases of full repayment and costless default.

We extend the basic KM model in three directions. First, we allow borrowers to default ex post at a cost. We assume this cost is proportional to the value of assets and that it affects entrepreneurs' net worth. Hence, if a borrower defaults, she must

[4] It is well know that, in models that feature a demand for assets that is increasing in their price, multiple equilibria might exist. Kiyotaki and Moore (1997) limited their attention to one of these, the one that features the highest asset level in the hands of entrepreneurs. Default and renegotiation have implications that affect the other expectational equilibria. Studying them is particularly useful when shocks are so severe that the original KM equilibrium ceases to exist.

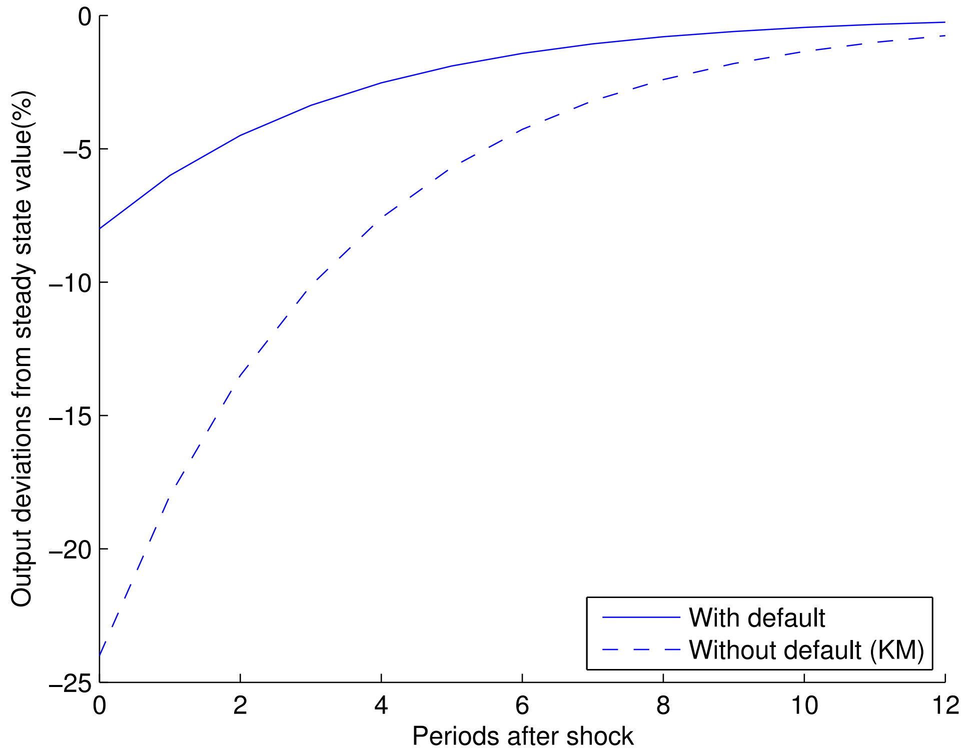

Figure 1: Strategic default dampens the amplification mechanism.

suffer a loss of tradable output, which is larger the higher her default costs. This implies that the entrepreneur has incentives to avoid outright default. Second, we allow lenders to negotiate a haircut $\varphi \geq 0$ in the contractual value of debt to prevent entrepreneurs from defaulting. We assume lenders and entrepreneurs split the surplus (given prices) according to Nash bargaining. Third, we will consider two types of shocks: a temporary productivity shock for entrepreneurs, and a temporary preference shock for lenders that reduces asset prices. Henceforth, we follow the convention that upper-case letters refer to aggregate quantities while lower-case letters denote individual quantities.

We assume entrepreneurs' operate a linear technology, such that given our assumptions on default, output is given by,[5]

$$y_{t+1} = F(k_t) = \begin{cases} (a_t + c)k_t, & \text{if there is no default in period } t+1, \\ (a_t + c)k_t - \alpha q_t k_t, & \text{if there is default in period } t+1, \end{cases}$$

where $0 \leq \alpha \leq 1$ is a measure of default costs.

We are interested in studying the response of the economy to unexpected transitory shocks that hit the economy at $t = 0$. As said, we consider two types of shocks. First, a

[5]Note that we assume default costs are proportional to assets valued at their purchase price, instead of the current market price. This assumption simplifies the derivation of our main results. We provide intuition for changes in setup and outcomes if default costs were given by $-\alpha q_{t+1} k_t$.

productivity shock that makes tradable production, a_t, shift from a to $a(1-\delta)$ with $\delta > 0$. Second, a preference shock that makes lenders more impatient for one period, reducing $\beta_1' = \tilde{\beta}$ to $\beta_1' = \tilde{\beta}(1-\epsilon)$.[6] We assume there is perfect foresight from period 1 onwards.

To ensure that the economy converges to the steady state, we need the following assumption,[7]

Assumption 1.

$$c > \left(\frac{1}{\beta} - 1\right) a.$$

Suppose lenders offer borrowers a haircut of φ. Then, the flow-of-funds-constraint of a borrower in t would be:

$$q_t k_t + I_t^{ND}\left((1-\varphi)R_{t-1}b_{t-1} - q_t k_{t-1}\right) + x_t - ck_{t-1} = (a_t - \alpha(1 - I_t^{ND})q_{t-1})k_{t-1} + b_t\,, \quad (1)$$

where I_t^{ND} is an indicator variable that takes the value of 1 whenever $t \neq 0$, and for $t = 0$ if there is no default.

We denote steady-state quantities by $*$. We guess, and later verify, that in equilibrium $\{K_0\}$ is an increasing sequence that converges to K^*. Let $R = \tilde{\beta}^{-1}$ denote the steady state interest rate. Thus, $R_0 = \frac{R}{1-\epsilon}$ and, since lenders are unconstrained, their Euler equation yields $q_t = \frac{1}{R_t} G'\left(\bar{K} - K_0\right) + \frac{1}{R_t} q_{t+1}$. Iterating forward and imposing a no-bubble condition yields

$$q_0 = (1-\epsilon)\left\{u(K_0) + \sum_{s=1}^{\infty} \frac{1}{R^s} u\left(K_s\right)\right\}, \quad (2)$$

where $u(K_0) \equiv \frac{1}{R} G'(\bar{K} - K_0)$ is the equilibrium down-payment when $R_t = R$. Under our conjecture, $\{q_t\}$ is an increasing sequence that converges to $q^* = \left(\frac{R}{R-1}\right) a$. This, together with assumption 1, ensures that investing as much as possible ($x_t = ck_{t-1}$, $R_t b_t = q_{t+1} k_t$) is an optimal strategy for entrepreneurs. In the period of the shock,[8]

$$\begin{aligned} 1 + \hat{k}_0^R(\varphi) &= \frac{a}{u(K_0)(1-\epsilon)} \left(1 - \delta + \frac{R}{R-1}(\hat{q}_0 + \varphi)\right) \\ 1 + \hat{k}_0^D &= \frac{a}{u(K_0)(1-\epsilon)} \left(1 - \delta - \frac{R}{R-1}\alpha\right), \end{aligned} \quad (3)$$

where $\hat{k}_0^i = \frac{k_0^i - k^*}{k^*}$, $\hat{q}_0 = \frac{q_0 - q^*}{q^*}$. Since there is perfect foresight from $t = 1$ onwards,

$$1 + \hat{k}_t^i = \frac{a}{u(K_t)}\left(1 + \hat{k}_{t-1}^i\right). \quad (4)$$

Since all entrepreneurs are identical, default entails output losses, and there is no information asymmetry, in equilibrium there is no default, i.e. $K_0 = k_t^R$.[9] Thus, we can

[6] Note that $\epsilon < \bar{\epsilon}$, with $\bar{\epsilon}$ given by $\tilde{\beta}(1-\bar{\epsilon}) = \beta$. Otherwise lenders would not have an incentive to lend.

[7] This is just KM's Assumption 2.

[8] If default cost were proportional to the current value of assets, the last term in (3) would be $-\frac{R}{R-1}\alpha(1 + \hat{q}_t)$.

[9] This follows from Coase's theorem.

find K_{t+i} as a function of K_{t+i-1} from (4). Iterating backwards we obtain $K_{t+i} = f_i(K_t)$ with $f_0(K) \equiv K$. Using this relation, it can be shown that provided $K_t \leq K^*$, $\{K_t\}$ is, as conjectured, an increasing sequence that converges to K^*.[10]

Next, we compute the implied entrepreneurs' utilities of default and renegotiation given the shocks, aggregate capital K_t, and the proposed haircut φ,

$$U^i(\varphi; K_0) = cK^* + \beta c k_1^i + \beta^2 c k_2^i + ... + \lim_{t\to\infty} \beta^t c k_{t-1}^i.$$

Using our previous results, we obtain[11]

$$\frac{U^R(\varphi; K_0) - U^D(K_0)}{\beta c K^*} = \frac{a}{u(K_0)} \frac{1}{(1-\epsilon)} \frac{R}{R-1} \left[(\hat{q}_0 + \varphi) + \alpha\right] \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=1}^{t} \frac{a}{u\left(f_s(K_0)\right)} \right). \tag{5}$$

By renegotiating, a borrower saves on the default costs, α and, in exchange, accepts to keep a share of the (negative) capital gains, $\hat{q}_0 + \varphi \leq 0$, which translates in a uniformly lower level of capital, both initially and in subsequent periods.[12] From (5), it follows that when $\varphi \geq -\hat{q}_0 - \alpha$ the entrepreneur does not default. Inspecting equation (5) we see that, conditional on a haircut level φ, K_0 has a negative effect on the utility difference, $U^R - U^D$. This effect follows from the fact that a higher K_0 implies higher current and future capital prices, leading to a lower impact of a given change in period t's net worth on capital demand, and a slower convergence to steady state (see (3)). Thus, the difference $\widehat{k}_0^R - \widehat{k}_0^D$ is reduced when K_0 increases. Note that beyond their impact on K_0 and $\hat{q}_0$, the only direct effect of the shocks is that a larger preference shock amplifies the utility difference.

Renegotiation gives an entrepreneur surplus $U^R(\varphi; K_0) - U^D(K_0)$ while a lender gets surplus $(1-\varphi)\, q^* K^* - q_0 K^* = -\left(\hat{q}_0 + \varphi\right) q^* K^*$. We assume these surpluses are divided according to a Nash-bargaining rule. In other words, φ solves

$$\max_{\varphi \in [0,1]} \left(-\hat{q}_0 - \varphi\right)^{\theta} \left(U^R(\varphi; K_0) - U^D(K_0)\right)^{1-\theta}$$

where $\theta \in [0,1]$ is the bargaining power of lenders. The objective function is concave so we can use the FOC to characterize the solution. Taking logarithms, this gives[13]

$$\varphi = \max\left[-\hat{q}_0 - \frac{\theta}{1-\theta} \frac{U^R(\varphi; K_0) - U^D(K_0)}{\frac{d(U^R(\varphi;K_0) - U^D(K_0))}{d\varphi}}, 0 \right] = \max\left[-\hat{q}_0 - \theta\alpha, 0\right].$$

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[10]To see this result, first note that since $u(K)K$ is increasing in K, so $u(K_{t+1})K_{t+1} < aK_t < aK^*$. Thus, $K_{t+1} < K^*$ and, by induction, $K_{t+i} < K^*\ \forall i$. This, in turn, implies that $u(K_{t+i}) < a\ \forall i$ and, since $u(K_{t+i})K_{t+i} = aK_{t+1-1}$, $K_{t+i} > K_{t+i-1}$.

[11]This follows since $U^i(\varphi; K_t) = cK^* + (1 + \widehat{k}_t^i)\beta c K^* \left[\sum_{s=0}^{\infty} \beta^s \left(\prod_{i=0}^{s} \frac{a}{u(f_i(K_t))}\right)\right]$.

[12]If $\frac{\beta a}{u(0)} > 1$, the last term does not converge when $K = 0$. In that case the only offer the entrepreneur will accept is $\varphi = -\hat{q}_t$

[13]If default cost were proportional to the current value of assets, then we would get $\varphi = \max\left[-\hat{q}_t - \theta\alpha(1 + \hat{q}_t), 0\right]$.

The equilibrium haircut depends on the proportional effect that φ has on the entrepreneurs' surplus as captured by the term $\frac{U^R(\varphi;K_0)-U^D(K_0)}{\frac{d(U^R(\varphi;K_0)-U^D(K_0))}{d\varphi}}$. This term is independent of the shocks. Let $\bar{q} \equiv -\theta\alpha$. When $\hat{q}_0 \geq \bar{q}$, the price of capital is sufficiently high that the threat of default is noncredible even if $\varphi = 0$. Hence, entrepreneurs bear all the capital losses in this region. In contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs are able to bargain a positive haircut. Finally, note that when entrepreneurs are very powerful or default is not very costly, $\theta\alpha \approx 0$, lenders bear most of the losses, $\varphi \approx -\hat{q}_0$. In contrast, when lenders have more bargaining power or default is more costly, they manage to extract some surplus from the entrepreneurs and $\varphi < -\hat{q}_0$. Note, however, that there is a limit to doing so since they can never increase the burden of debt. Hence, once $\varphi = 0$ increasing θ further has no effect.

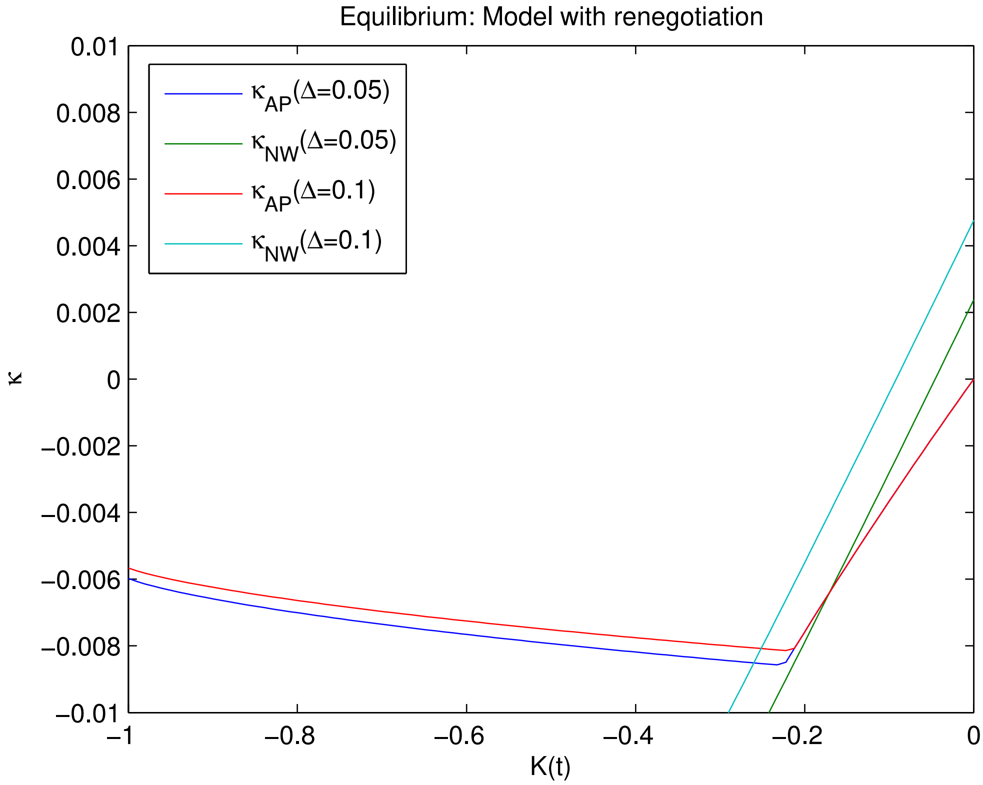

Figure 2: Renegotiation.

Using our previous results it follows that the equilibrium is characterized by the following equations,[14]

$$\text{[net worth]} \quad : \quad \frac{u(K_0)K_0}{K^*} = \frac{a}{1-\epsilon}\left(1-\delta+\frac{R}{R-1}\max\left[-\theta\alpha, \hat{q}_0\right]\right) \tag{6}$$

$$\text{[asset pricing]} \quad : \quad \hat{q}_0 = \hat{q}(K_0,\epsilon) = \frac{R-1}{R}\frac{(1-\epsilon)}{a}\left\{u(K_0)+\sum_{t=1}^{\infty}\frac{1}{R^t}u\left(K_s\right)\right\}-1 \tag{7}$$

The first equation is the "net worth" relation, which links the size of the capital losses faced by the entrepreneur with the amount of capital she can retain the first period. When $\hat{q}_0 \geq \bar{q}$, the threat of default is not credible and the entrepreneur bears all the capital losses. Since the entrepreneur is borrowing constrained, her demand of capital is increasing in its price in this region. In contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs' capital losses are independent of the shock (lenders absorb any further increases in capital losses). The second equation is the standard "asset pricing" relation, which states that the price of capital is the discounted sum of future dividends.

The next proposition characterizes equilibria.

Proposition 1. (i) An equilibrium exists.
(ii) There exists a threshold $\bar{\Delta}(\epsilon)$ (with $\frac{d\bar{\Delta}}{d\epsilon} \leq 0$) such that for $\delta < \Delta(\epsilon)$ there exists an equilibrium $\{K(\delta,\epsilon), \hat{q}(\delta,\epsilon)\}$ with no renegotiation, i.e. $\varphi = 0$. The equilibrium capital and prices are continuous in ϵ and δ and strictly decreasing in both arguments.
(iii) There exists a threshold $\underline{\Delta}(\epsilon) \leq \bar{\Delta}(\epsilon)$ (with $\frac{d\underline{\Delta}}{d\epsilon} \leq 0$) such that for $\delta > \underline{\Delta}(\epsilon)$ there exists an equilibrium $\{K(\delta,\epsilon), \hat{q}(\delta,\epsilon)\}$ with renegotiation, i.e. $\varphi > 0$. The equilibrium capital and prices are continuous in ϵ and δ, and strictly decreasing in δ. Equilibrium capital is strictly increasing, an prices ambiguous, in ϵ. The haircut φ is increasing in δ and ambiguous with respect to ϵ.
(iv) When $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$, the equilibrium is unique and $\frac{d\Delta^*}{d\epsilon} \leq 0$.

Proof. See Appendix 5.1. □

Figure XX illustrates equilibria graphically. The net-worth curve has a kink at the point in which haircuts start to be positive. When $\hat{q}_0$ is above this point, defaulting is sufficiently unattractive that the threat of default is not credible. Hence, in this region, entrepreneurs bear all the losses and capital demand increases with its price. When $\hat{q}_0$ is below the threshold, default is credible, there is renegotiation and creditors share the burden of capital losses with debtors. The figure shows the case in which there are three equilibria.[15] In this case equilibrium (ii) is the one with the highest level of capital while equilibrium (iii) is the one with the lowest level. As δ increases the net-worth curve moves up and to the left. for both equilibria of type (ii) and (iii) capital and prices decrease. Since for equilibrium (iii) entrepreneurs' capital gains are independent of δ (i.e. $\varphi + \hat{q}_0$ is constant), this implies φ has to increase substantially to compensate the change in $\hat{q}_0$.

[14] If $K_t = 0$ and $\frac{\beta a}{u(0)} > 1$, then $\hat{q}_t = (1-\epsilon)\frac{u(0)}{a} - 1$ is the solution to the second equation.

[15] There are no more than three because we assumed $G''' > 0$, which guarantees that the non-vertical branch of the NW and the AP intersect at most twice.

In contrast, preference shocks have two opposite effects on entrepreneurs' capital demand. First, there is a positive effect since the shock reduces the required downpayment and thus allows an increase in leverage. Second, there is a negative effect since lower asset prices imply capital losses, and therefore a reduction in demand. Lemma A0 in Appendix 5.1 shows that preferences shocks shift both curves downwards (and the vertical part of the net worth to the right), but the effect is stronger on the asset pricing curve. Thus preference shocks reduce capital demand and asset prices for equilibrium (ii). In contrast, for equilibrium (iii) capital demand increases with preferences shocks. This happens because renegotiation puts a lower bound on capital losses, which eliminates the negative effect of a preference shock on demand, leaving only the positive effect from an increase in leverage. The effect of preference shocks is negative for equilibrium (i) and ambiguous for equilibrium (iii). Since for equilibrium (iii) $\varphi + \hat{q}_0$ is constant, preference shocks have also ambiguous effects on haircuts.

For the equilibrium to be unique, the NW curve has to be steeper than the AP curve at the point in which it has the kink. In this case $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$, and the threshold is such that the first, and only, intersection between the asset-pricing and net-worth curves is exactly at the kink of the latter. Then, the economy behaves like an equilibrium of type (ii) for $\delta < \Delta^*(\epsilon)$ and like an equilibrium of type (iii) for $\delta > \Delta^*(\epsilon)$.

The following proposition characterizes how the equilibrium with renegotiation is affected by default costs and bargaining power.

Proposition 2. For the equilibrium characterized in Proposition 1 (iii)
(i) The threshold $\Delta^*(\epsilon)$ is increasing in θ and α
(ii) When $\delta > \Delta^*(\epsilon)$, equilibrium capital and prices are strictly decreasing in θ and α. The effect on the haircut is ambiguous.

Proof. See Appendix 5.1 □

Figure XX illustrates changes in default costs or bargaining power (their effects enter symmetrically in equilibrium equations (6) and (7)). Note that the only curve that moves in this case is the vertical renegotiation branch of the net-worth curve. More specifically, when lenders' bargaining power increases, or entrepreneurs' default cost is higher, $\bar{q}$ decreases and the kink in the net-worth curve shifts to the left. As long as we stay within the region where $\varphi > 0$, this implies that both K_0 and $\hat{q}_0$ decrease with θ and α. In other words, higher bargaining power by lenders and higher default costs reduce the stabilizing effect of renegotiation and move us closer to the KM world, with significant amplification. Since $\hat{q}_0$ is decreasing the effect on φ is ambiguous. To understand this result, first note that keeping K_0 constant at the original equilibrium value, the shift in the renegotiation branch of the net-worth curve measures the size of the "partial equilibrium" effect. For example, if α decreases, then entrepreneurs' threat of default is more credible and they are able to extract a larger surplus from creditors. This partial equilibrium effect triggers a general equilibrium force in the opposite direction: As entrepreneurs become wealthier (higher $\hat{q}_0 + \varphi$), they are able to buy more capital, raising prices and lowering the threat of default. The size of this general equilibrium effect is related to the steepness of the

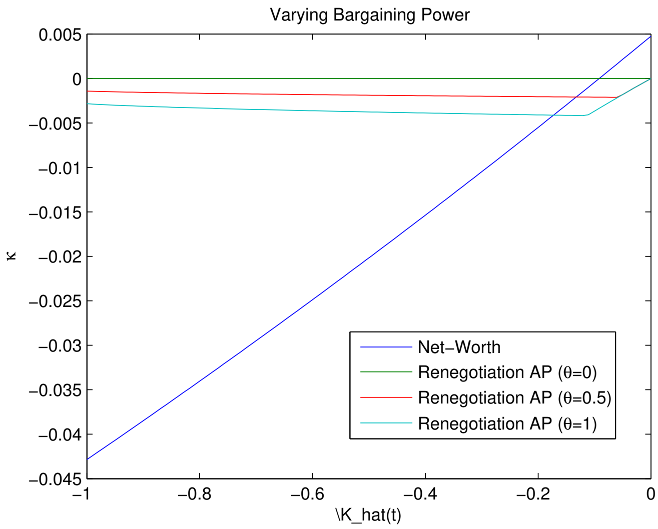

Figure 3: theta alpha.

asset-pricing function. If it is very steep, a small additional amount of capital in the hands of entrepreneurs may raise prices enough such that the initial partial equilibrium effect is overturned and haircuts decrease.[16]

Finally, note that an economy with $\theta = 1$ generates the same behavior of capital and prices as an economy in which renegotiation is not allowed and entrepreneurs default. The reason is that lenders extract all the surplus when $\theta = 1$ and, hence, entrepreneurs' net-worth is unaffected by renegotiation. The only difference between both scenarios is that, in an economy with default, output will decrease further due to the default costs.

3 Asymmetric Information about Default Costs

The aftermath of a financial crisis is often characterized not only by debt restructuring negotiations but also by outright default and bankrupcies. In this section, we extend our model to allow for default in equilibrium. In order to do so, we postulate that the entrepreneurs' default costs are private information.[17] More specifically, we allow for heterogeneity in the size of the default cost α_i faced by each entrepreneur i, which is known by the entrepreneur but unknown to the lender, who only knows the cumulative distribution function $F(\alpha) \in C^2$.

Since lenders ignore the type of entrepreneurs they have lent to, they face a tradeoff in the event of an unforeseen negative shock: a higher level of debt relief makes more borrowers willing to accept, but the rent extracted from each entrepreneur gets smaller. Lenders will balance the two effects, recognizing that the willingness of borrowers to

[16] In numerical simulations we found that the effect on φ in either direction is quantitatively very small.

[17] It may be noted that, since all agents are assumed to treat ex ante the likelihood of a shock as strictly negligible, informational asymmetries about features that become relevant only in the event of a disturbance have no influence on the contracts outstanding when the shock arrives.

accept a certain deal will be weaker for those with low default costs. For simplicity we now assume that lenders have all the ex post bargaining power, i.e. $\theta = 1$. We keep the timing of the previous section: First lenders make an offer and then entrepreneurs decide whether to accept it or not.[18]

We solve the problem by backward induction. First, an entrepreneur must decide whether to accept or decline the debt reduction φ, taking as given the dynamics of aggregate capital and prices. From equation (5), we know that entrepreneurs will only accept a haircut offer if $\alpha_i \geq -(\hat{q}_0 + \varphi)$. Taking this into account lenders minimize expected losses. For a given debt offer φ, a lender incurs in a cost (in percentage terms) given by $-\hat{q}_0$ on the fraction $F(-(\hat{q}_0 + \varphi))$ of the borrowers who default and deliver their collateral, whereas he loses φ (in percentage terms) on the complementary fraction $1 - F(-(\hat{q}_0 + \varphi))$ of credits that are renegotiated. Since individual lenders take prices, $\hat{q}_0$, as given, we can write their problem as,

$$\min_{\varphi \geq 0} \; (\hat{q}_0 + \varphi)\,(1 - F(-(\hat{q}_0 + \varphi)))\,.$$

The first order condition yields,

$$1 - F\left(-\left(\hat{q}_0 + \varphi\right)\right) + f\left(-\left(\hat{q}_0 + \varphi\right)\right)\left(\hat{q}_0 + \varphi\right) \geq 0, \quad \text{with equality if } \varphi > 0. \tag{8}$$

We make assumptions to guarantee a unique solution for the lenders problems. Either $-\alpha(1 - F(\alpha))$ is strictly convex, or the Mills ratio, $\frac{1-F}{f}$, is weakly decreasing with $\lim_{\alpha \to 1} \frac{1-F}{f} < 1$.[19]

Lemma 1. Let μ denote the share of defaulting entrepreneurs. If $K_0 < K^*$, then $\hat{q}_0 + \varphi < 0$ and $\mu > 0$. In other words, there is default in equilibrium.

Proof. Note that $\hat{q}_t + \varphi = 0$ would imply the first term of (8) is positive so $\hat{q}_t = \hat{q}_t + \varphi = 0$. This is a contradiction since $K_t < K^*$ implies $\hat{q}_t < 0$. Thus $\hat{q}_t + \varphi < 0$. From (5), it follows that entrepreneurs with $\alpha_i < -(\hat{q}_t + \varphi)$ would default, and thus $\mu = F(-(\hat{q}_t + \varphi)) > 0$. □

Let $\bar{\alpha}$ denote the solution of (8) ignoring the non-negativity constraint, which is constant. The solution to lenders' problem can then be written as

$$\varphi^{AI} = \max[-\hat{q}_0 - \bar{\alpha}, 0],$$

where $\hat{q}_0$ is still given by (2). Note the symmetry with the derivation of the equilibrium haircut in the previous section, with $-\bar{q}$ replaced by $\bar{\alpha}$. This simplifies the derivation of most of our results.

The net-worth relation now needs to take into account that there is default in equilibrium. Recall that entrepreneurs with $\alpha < \min[\bar{\alpha}, -\hat{q}_0]$ default, while agents with

[18] Given that lenders are risk neutral, we proceed as if each one of them faces a continuum of entrepreneurs. This makes the number of borrowers who default for a given debt reduction offer a deterministic quantity from the point of view of a single lender, and not only at an aggregate level.

[19] For example, a uniform distribution would satisfy either requirement.

$\alpha \geq \min[\bar{\alpha}, -\hat{q}_0]$ renegotiate. Then, the net-worth relationship for an entrepreneur i yields

$$\frac{u(K_0)k_t(\alpha_i)}{aK^*} = \frac{1}{1-\epsilon}(1-\delta-\frac{R}{R-1}\min[\alpha_i,\min[\bar{\alpha},-\hat{q}_0]])$$

Integrating individual capital holdings yields

$$\frac{u(K_0^{AI})K_0^{AI}}{aK^*} = \frac{1}{1-\epsilon}\left(1-\delta-\frac{R}{R-1}\min[\bar{\alpha},-\hat{q}_0]-\frac{R}{R-1}\int_0^{\min[\bar{\alpha},-\hat{q}_0]}(\alpha-\min[\bar{\alpha},-\hat{q}_0])dF\left(\alpha\right)\right) \tag{9}$$

Note that the new net-worth relationship still describes an upward relationship between K_0 and $\hat{q}_0$ for $\hat{q}_0 \geq -\bar{\alpha}$ and a vertical line when $\hat{q}_0 < -\bar{\alpha}$. The model is closed by the same asset-pricing relationship as before.

With asymmetric information, after a preference shock entrepreneurs with low default costs take advantage of the positive leverage effect of a decrease in asset prices, with minor effects on their net worth since they default. Depending on parameters this effect might dominate the overall negative effect that the preference shock has on non-defaulting entrepreneurs. We make the following assumption to rule out this case.[20]

Assumption 2.

$$\frac{R-1}{R} < 1 - F(1-\beta R).$$

The following proposition characterizes the equilibrium,

Proposition 3. Under assumption 2, the following holds:
(i) There exists a threshold $\bar{\Delta}^{AI}(\epsilon)$ such that for $\delta < \bar{\Delta}^{AI}(\epsilon)$ there exists an equilibrium $\left\{K^{AI}\left(\delta,\epsilon\right),\hat{q}^{AI}\left(\delta,\epsilon\right)\right\}$ with no renegotiation, i.e. $\varphi^{AI}=0$. The equilibrium capital and prices are continuous in ϵ and δ and strictly decreasing in both arguments. The share of defaulting entrepreneurs $F(\alpha)$ increases with δ and ϵ.
(ii) There exists a threshold $\underline{\Delta}^{AI}(\epsilon) \leq \bar{\Delta}^{AI}(\epsilon)$ such that for $\delta > \underline{\Delta}^{AI}(\epsilon)$ there exists an equilibrium $\left\{K^{AI}\left(\delta,\epsilon\right),\hat{q}^{AI}\left(\delta,\epsilon\right)\right\}$ with renegotiation, i.e. $\varphi^{AI} > 0$. The equilibrium capital and prices are continuous in ϵ and δ, strictly increasing in ϵ and strictly decreasing in δ. The haircut φ^{AI} is increasing in δ and ambiguous with respect to ϵ. The share of defaulting entrepreneurs is constant at $F\left(\bar{\alpha}\right)$.
(iii) When a set of conditions specified in Appendix 1 are satisfied, $\Delta^{AI*}(\epsilon) \equiv \bar{\Delta}^{AI}(\epsilon) = \underline{\Delta}^{AI}(\epsilon)$. The equilibrium is unique and $\frac{d\Delta^{AI*}}{d\epsilon} < 0$.

Proof. See Appendix 5.2. ☐

Proposition XX shows that the features we analyzed in the previous section are robust to introducing asymmetric information. Furthermore, it states that the share of defaulting entrepreneurs increases while renegotiation is redundant (i.e. $\varphi^{AI}=0$) and stays constant

[20]Under assumption 2 we have $\frac{\partial\hat{q}^{NW,AI}(K^*,\delta,\epsilon)}{\partial\epsilon} = \frac{\frac{\partial\hat{q}^{NW}(K^*,\delta,\epsilon)}{\partial\epsilon}}{1-F(\hat{q}^{NW,AI})} = -\frac{R-1}{R}\frac{\epsilon}{1-F(\hat{q}^{NW,AI})} \geq -\epsilon = \frac{\partial\hat{q}(K^*,\epsilon)}{\partial\epsilon}$. Where the inequality follows since $\epsilon < \bar{\epsilon} = 1-\beta R$. Thus K_0 cannot be above K^*.

when renegotiation is triggered.[21] To understand the differential effects of asymmetric information on the equilibrium, we compare the solution to the case of perfect information. In that case, lenders can tailor the offered haircut to each entrepreneur, offering $\varphi_i^{PI} = \max[-\hat{q}_0 - \alpha_i, 0]$. Thus, the individual net-worth relation yields

$$\frac{u(K)k_0(\alpha_i)}{aK^*} = \frac{1}{1-\epsilon}(1 - \delta - \frac{R}{R-1}\min[\alpha_i, -\hat{q}_0]),$$

which integrating across individual capital holdings yields

$$\frac{u(K_0^{PI})K_0^{PI}}{aK^*} = \frac{1}{1-\epsilon}\left(1 - \delta + \frac{R}{R-1}\hat{q}_0 - \frac{R}{R-1}\int_0^{-\hat{q}_0}(\alpha + \hat{q}_0)dF(\alpha)\right) \tag{10}$$

Comparing (9) and (10), we see that asymmetric information transfers wealth from creditors to debtors when shocks are large enough to trigger haircuts. Since lenders are unable to discriminate, they are forced to give large haircuts to high-default cost agents, which in turn boosts their wealth and dampens the response of asset prices. On the other hand, asymmetric information leads to a positive share of defaulting entrepreneurs, which generates output losses associated with default costs. Because there are no costs associated to default in subsequent periods, output will recover faster when there is asymmetric information. We collect these observations in the following proposition,

Proposition 4. In an economy with asymmetric information, a shock has the following effects, relative to an equivalent economy with perfect information:
(i) When $\hat{q}_0 \geq -\bar{\alpha}$, $\varphi^{AI} = 0$, $K_0^{AI} = K_0^{PI}$, and output is lower.
(ii) When $\hat{q}_0 < -\bar{\alpha}$, $\varphi^{AI} > 0$, $K_0^{AI} > K_0^{PI}$, and the effect on output is ambiguous.
(iii) For both (i) and (ii) output recovers faster after the initial shock.

Proof. See Appendix 5.2. □

4 Conclusions

A representation of deep macroeconomic crises requires attention to the ways in which parties in financial contracts, and the legal system itself, process situations of widespread broken promises. We extend the existing literature (see for example Kiyotaki and Moore (1997), Bernanke et al. (1999), or Brunnermeier and Sannikov (2014) among others) by assuming that, when the value of assets posted as collateral is below the value of debt, the parties in financial contracts may engage in negotiations to re-define payments. This corresponds to the existence of bankruptcy procedures that limit the ability of creditors to collect debts, and the observed fact that renegotiations are common in moments of large macroeconomic disturbances. The resulting analysis highlights the connection between the repercussions of a shock and the ex post bargaining power of borrowers and lenders.

[21]The fact that it is exactly constant relies on the assumption that there is no feedback between asset prices and default costs. Depending on other details of the model, renegotiation may slow down the pace (or even reverse) at which the share of defaulting entrepreneurs increases with the size of the shock.

The outcome of debt renegotiations, and thus the macroeconomic consequences of the shock, are seen to be influenced by institutional factors and by structural and technological features which operate on the degree of asset specificity, understood as the differential in productivity of an ongoing firm and that of a new venture started after an entrepreneur defaulted and gave up the assets pledged as collateral.

In order to study the implications of the costs of strategic default on the ex post resolution of crises, we introduced a number of changes to the Kiyotaki and Moore (1997)'s model. We found that there is always a threshold level for the initial shock above which the amplification effects through the reallocation of capital to lower productivity uses and through lower asset prices is dampened as lenders are led to accept some debt reductions in order to avoid default; that threshold is larger for firms with higher default costs. When these default costs are assumed to be private information, the model suggests that economies with more severe asymmetries would experience sharper "V-shaped" recoveries after a crisis.

The model suggests that credit- related recessions may be of two types, according to the institutional and technological costs of default. In some instances, either through government intervention or bankruptcy procedures that preserve financially troubled firms, constraints on the access to resources for production would get somehow relieved (even if this happens after a period of turmoil). Other "balance sheet crises" would show long periods of deleveraging where firms hold to their capital but investment and production are financially restricted and asset prices stay depressed, as argued by Koo (2003) for Japan's "lost decade" and the Great Depression.

Our work produces some implications that can be empirically tested. First, amplification is higher when default costs are high, and when institutions give a higher ex post bargaining power to lenders. Second, we should observe sharper "V-shaped" recoveries in economies with more asymmetric information on default costs. We leave for future research the design of an identification strategy to test for these implications.

The basic model that we have considered is apt to be extended in several ways. The treatment of the shock as a zero probability bolt from the blue can be a useful analytical device, and suitable to our purpose of studying the ex post resolution of large aggregate disturbances. But it certainly leaves aside very relevant analytical and practical issues. If, in the context described by the model, the parties of debt contracts completely disregard the possibility of an aggregate shock, it would be fully indifferent to them to specify repayments in terms of quantities of output or in terms of the value of collateral (as in US mortgages); the amplification effects of unforeseen disturbances that would emerge in the first case would happen to be absent in the ex ante identical second case.

Since most debt obligations are defined as non- contingent amounts of goods, various kinds of behavior can modify the results of the model if agents contemplate the potential of disturbances, especially by moderating the impacts of small shocks if, for instance, debtors find it convenient to leave a slack in the collateral constraint in normal times for precautionary reasons according to the specificity of their assets (Brunnermeier et al. (2013) find that firms with higher default costs borrow less, and thus have lower financial risk), or the supply of credit incorporates the likelihood of debt reductions, depending on the characteristics of the institutions that deal with defaulted debts (as in the evidence presented by Berger et al. (2011), that in US states with high exemption levels in personal bankruptcy procedures under Chapter 7, small unlimited liability firms are able to borrow smaller amounts than similar firms in states with low debtor protection). Then, the macroeconomic responses may show two types of non-linearities: one, an increase in the financial multipliers with the size of the shock (evocative of a corridor effect; see Leijonhufvud (1973)) as the financial buffer stocks are exhausted and, in the other extreme, a moderation of impacts as debts are renegotiated in the event of a very strong shock.

Furthermore, one may conjecture that the ex ante anticipation of the ex post behavior in the event of shocks could strengthen the connection between default costs and the size of multipliers, although that remains to be checked with a specific analysis, using setups such as those in Lorenzoni (2008) or Brunnermeier and Sannikov (2014).

References

[1]

Ahmadjian, C. (2006), 'Japanese Business Groups: Continuity in the Face of Change', in S-J Chang, ed.: *Business Groups in East Asia: Financial Crisis, Restructuring, and New Growth*, Oxford University Press, New York.

Aikman, D., Haldane, A. G. and Nelson, B. (2013), 'Curbing the Credit Cycle', *The Economic Journal* forthcoming.

Berger, A. N., Cerqueiro, G. and Penas, M. F. (2011), 'Does debtor protection really protect debtors? Evidence from the small business credit market', *Journal of Banking & Finance* **35**(7), 1843–1857.

Bernanke, B. and Gertler, M. (1989), 'Agency Costs, Net Worth and Business Fluctuations', *American Economic Review* **79**(1), 14–31.

Bernanke, B., Gertler, M. and Gilchrist, S. (1999), 'The financial accelerator in a quantitative business cycle framework', *Handbook of Macroeconomics* **1**(C), 1341–1393.

Bhattacharya, S., Boot, A. WA. and Thakor, A. V. (2004), *Credit, Intermediation, and the Macroeconomy: Models and Perspectives*, Oxford University Press, New York.

Brunnermeier, M. K., Eisenbach, T. M. and Sannikov, Y. (2013), 'Macroeconomics with Financial Frictions: A Survey', in *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society*, Cambridge University Press.

Brunnermeier, M. K. and Sannikov, Y. (2014), 'A Macroeconomic Model with a Financial Sector', *American Economic Review* forthcoming.

Carlstrom, C. T. and Fuerst, T. S. (1997), 'Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis', *American Economic Review* **87**(5), 893–910.

Chang, S. J. (2006a), *Business Groups in East Asia: Financial Crisis, Restructuring, and New Growth*, Oxford University Press, New York.

Chang, S. J. (2006b), 'Introduction: Business Groups in East Asia', in S-J Chang, ed.: *Business Groups in East Asia: Financial Crisis, Restructuring, and New Growth*, Oxford University Press, New York.

Cooley, T., Marimon, R. and Quadrini, V. (2004), 'Aggregate consequences of limited contract enforceability', *Journal of Political Economy* **112**(4), 817–847.

Cordoba, J. C. and Ripoll, M. (2004), 'Credit Cycles Redux', *International Economic Review* **45**(4), 1011–1046.

Freixas, X. and Rochet, J. C. (2008), *Microeconomics of Banking*, MIT Press, Cambridge.

Gomez, E. (2006), 'Malaysian Business Groups: The State and Capital Development in the Post Currency Crisis Period', in S-J Chang, ed.: *Business Groups in East Asia: Financial Crisis, Restructuring, and New Growth*, Oxford University Press, New York.

Hanani, A. (2006), 'Indonesian Business Groups: The Crisis in Progress', in S-J Chang, ed.: *Business Groups in East Asia: Financial Crisis, Restructuring, and New Growth*, Oxford University Press, New York.

Hynes, R. M., Malani, A. and Posner, E. A. (2004), 'The political economy of property exemption laws', *Journal of Law and Economics* **47**(1), 19–43.

Jordà, O., Schularick, M. HP. and Taylor, A. M. (2011), 'When Credit Bites Back: Leverage, Business Cycles, and Crises' NBER Working Paper 17621.

Kiyotaki, N. and Moore, J. (1997), 'Credit Cycles', *Journal of Political Economy* **105**(2), 211–248.

Kocherlakota, N. (2000), 'Creating Business Cycles Through Credit Constraints', *Federal Reserve Bank of Minneapolis Quarterly Review* **24**(3), 2–10.

Koo, R. (2003), *Balance Sheet Recession: Japan's Struggle with Uncharted Economics and its Global Implications*, John Wiley & Sons, Singapore.

Leijonhufvud, A. (1973), 'Effective Demand Failures', *The Scandinavian Journal of Economics* **75**(1), 27–48.

Liu, Z., Wang, P. and Zha, T. (2013), 'Land-Price Dynamics and Macroeconomic Fluctuations', *Econometrica* **81**(3) 1147–1184.

Lorenzoni, G. (2008), 'Inefficient Credit Booms', *Review of Economic Studies* **75**(3), 809–833.

Olver, F., Lozier, D., Boisvert, R. and Clark, C. (2010), *NIST Handbook of Mathematical Functions*, Cambridge University Press.

5 Appendix

5.1 Proof of proposition 1

We start with the following lemmas

Lemma A0. We can rewrite the net worth condition, (6), as $\hat{q}^{NW}(K_0, \delta, \epsilon)$ given by

$$\hat{q}^{NW}(K_0, \delta, \epsilon) = \max\left[\frac{R-1}{R}\left(\frac{u(K_0)K_0}{aK^*}(1-\epsilon) - (1-\delta)\right), -\theta\alpha\right].$$

It follows that if K_0 is an equilibrium such that $\hat{q}^{NW}(K_0, \delta, \epsilon) = \hat{q}(K_0, \epsilon)$, then $\frac{\partial \hat{q}(K_0,\epsilon)}{\partial\ \epsilon} \leq \frac{\partial \hat{q}^{NW}(K_0,\delta,\epsilon)}{\partial\ \epsilon} \leq 0$.

Proof. When $\hat{q}^{NW}(K_0, \delta, \epsilon) = -\theta\alpha$, $\frac{\partial \hat{q}^{NW}}{\partial\ \epsilon} = 0$ (note that in this case $\frac{dK_0}{d\epsilon} > 0$). Else $\frac{\partial \hat{q}^{NW}}{\partial\ \epsilon} = -\frac{\hat{q}^{NW} + \frac{R-1}{R}(1-\delta)}{1-\epsilon}$, and $\frac{\partial \hat{q}}{\partial\ \epsilon} = -\frac{\hat{q}+1}{1-\epsilon}$. The result then follows when $\hat{q}^{NW} = \hat{q}$ since $\frac{R-1}{R}(1-\delta) \leq 1$. QED

Lemma A1. At the steady state, and when there is no renegotiation, $\hat{q}^{NW}(K_0, \delta, \epsilon)$ is steeper than $\hat{q}(K_0, \epsilon)$.

Proof. The slopes of $\hat{q}^{NW}(K_0, \delta, \epsilon)$ and $\hat{q}(K_0, \epsilon)$ when $K_0 = K^*$ are given by[22]

$$\begin{aligned}
\frac{d\hat{q}^{NW}}{dK_0}|_{K_0=K^*} &= (1-\epsilon)\frac{R-1}{R}\frac{1}{K^*}\left(\frac{1}{\eta}+1\right), \\
\frac{d\hat{q}}{dK_0}|_{K_0=K^*} &= (1-\epsilon)\frac{R-1}{R}\frac{1}{K^*}\left(\frac{1}{\eta}+\frac{1}{\eta}\frac{\frac{\eta}{R(\eta+1)}}{1-\frac{1}{R}\frac{\eta}{\eta+1}}\right).
\end{aligned}$$

Since $R > 1$, it is always the case that the former is steeper. QED

Lemma A2. Let $K^\varphi(\delta, \epsilon)$ and $\hat{q}^\varphi(\delta, \epsilon)$ be the implicit solutions to

$$\begin{aligned}
-\theta\alpha - \hat{q}^{NW}\left(K^\varphi(\delta,\epsilon), \delta, \epsilon\right) &= 0 \\
\hat{q}^\varphi(\delta,\epsilon) - \hat{q}\left(K^\varphi(\delta,\epsilon), \epsilon\right) &= 0.
\end{aligned}$$

Then, $K^\varphi(\delta, \epsilon)$ and $\hat{q}^\varphi(\delta, \epsilon)$ are well-defined and continuous functions, with $\frac{dK^\varphi}{d\delta} < 0$, $\frac{d\hat{q}^\varphi}{d\delta} < 0$, and $\frac{dK^\varphi}{d\epsilon} > 0$.

Proof. By definition, the pair $(K^\varphi, \hat{q}^\varphi)$ denotes the intersection between the asset pricing relation (7) with a vertical line at capital demand consistent with the onset of renegotiation according to (6). This would be an equilibrium of the model if $-\theta\alpha \geq \hat{q}(K^\varphi, \epsilon)$ (and provided $K^\varphi \geq 0$). Note that since $-\theta\alpha$ is unaffected by δ while $\hat{q}^{NW}$ is strictly increasing in δ, there is a unique solution to this system of equations. Furthermore, since both curves are continuous, by the implicit function theorem, K^φ and $\hat{q}^\varphi$ are continuous functions. Applying again the implicit function theorem, we can get the derivative of K^φ with respect to the productivity shock,

$$\frac{dK^\varphi}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K}} < 0.$$

[22]Here $\frac{1}{\eta}$ is, as in KM, $\frac{d \log u(K)}{d \log K_0}|_{K=K^*}$.

Furthermore, it is immediate that $\frac{d\hat{q}^{\varphi}}{d\delta} < 0$. A similar analysis for preference shocks shows that $\frac{dK^{\varphi}}{d\epsilon} > 0$ (since $\frac{\partial \hat{q}^{NW}}{\partial \epsilon} < 0$). The sign of $\frac{d\hat{q}^{\varphi}}{d\epsilon}$ is ambiguous. QED

Lemma A3. Consider the model without renegotiation. Then, for any $K < K^*$ and ϵ, there exists at most one δ such that $\hat{q}^{NW}(K, \delta, \epsilon) = \hat{q}(K, \epsilon)$.

Proof. First, note that $\frac{d\hat{q}^{NW}}{dK}$ is independent of δ. By the fundamental theorem of calculus, we can always write

$$\hat{q}^{NW}(K, \delta, \epsilon) = \hat{q}^{NW}(K^*, \delta, \epsilon) - \int_K^{K^*} \frac{d\hat{q}^{NW}}{dK}(s)\, ds.$$

Since - given K and ϵ - $\hat{q}(K, \epsilon)$ is a constant and $\hat{q}^{NW}(K^*, \delta, \epsilon)$ is strictly increasing in δ, there is at most one δ that solves $\hat{q}^{NW}(K, \delta, \epsilon) = \hat{q}(K, \epsilon)$. QED

(i) First note that $\hat{q}^{NW}(K^*, \delta, \epsilon) = \max[\frac{R-1}{R}(\delta - \epsilon), \bar{q}]$, and that $\hat{q}(K^*, \epsilon) = -\epsilon$, such that $\hat{q}^{NW}(K^*, \delta, \epsilon) \geq \hat{q}(K^*, \epsilon)$. Next, consider two cases. First if $\hat{q}(K^{\varphi}(\delta, \epsilon), \epsilon) \leq \max[-\frac{R-1}{R}(1-\delta), \bar{q}]$, then $(K^{\varphi}(\delta, \epsilon), \hat{q}(K^{\varphi}(\delta, \epsilon), \epsilon))$ is an equilibrium. If $\hat{q}(K^{\varphi}(\delta, \epsilon), \epsilon) > \max[-\frac{R-1}{R}(1-\delta), \bar{q}]$, then $\hat{q}(K^{\varphi}(\delta, \epsilon), \epsilon) \geq \hat{q}^{NW}(K^{\varphi}(\delta, \epsilon), \delta)$. Since both $\hat{q}$ and $\hat{q}^{NW}$ are continuous in K,[23] it follows that there exists $0 < K^{eq} \leq K^*$ such that $\hat{q}(K^{eq}, \epsilon) = \hat{q}^{NW}(K^{eq}, \delta, \epsilon)$.

(ii) When $\delta = \epsilon = 0$, $\hat{q}^{NW}(K^*, 0, 0) = \hat{q}(K^*, 0) = 0$. Since $\hat{q}^{NW}$ is continuous in K, δ, and ϵ, and $\hat{q}$ is continuous in K and ϵ, we can define the function $K^{KM}(\delta, \epsilon)$ implicitly from the equation $\hat{q}^{NW}(K, \delta, \epsilon) - \hat{q}(K, \epsilon) = 0$ in a neighborhood of $\delta = \epsilon = 0$. By the implicit function theorem $K^{KM}(\delta, \epsilon)$ is continuous in both δ and ϵ. Clearly, $\hat{q}^{KM}(\delta, \epsilon) = \hat{q}^{NW}\left(K^{KM}(\delta, \epsilon), \delta\right)$ is also continuous in δ and ϵ. For the productivity shock

$$\frac{dK^{KM}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K} - \frac{\partial \hat{q}}{\partial K}} < 0,$$

where the inequality follows since $G''' \geq 0$ and lemma A1 guarantee that $\frac{\partial \hat{q}^{NW}}{\partial K} > \frac{\partial \hat{q}}{\partial K} > 0$. A similar analysis yields $\frac{dK^{KM}}{d\epsilon} < 0$, since from lemma A0, $\frac{\partial \hat{q}^{NW}}{\partial \epsilon} - \frac{\partial \hat{q}}{\partial \epsilon} \geq 0$. Since productivity shocks do not affect the asset pricing curve, $\frac{d\hat{q}^{KM}}{d\delta} < 0$. The sign of $\frac{d\hat{q}^{KM}}{d\epsilon}$ is ambiguous.

The threshold $\bar{\Delta}(\epsilon)$ is defined as the $\min[\delta^{KM}(\epsilon), \delta^{REN}(\epsilon)]$,where $\delta^{KM}(\epsilon)$ is the highest productivity shock for which the solution of $\hat{q}^{NW}(K, \delta, \epsilon) - \hat{q}(K, \epsilon) = 0$ in a neighborhood of $\delta = \epsilon = 0$ is continuous in δ for given ϵ. And $\delta^{REN}(\epsilon)$ is determined by $\hat{q}^{KM}(\delta^{DEF}(\epsilon), \epsilon) = -\theta\alpha$, i.e. when the first intersection between the net worth and asset pricing curves occurs at the kink of the former. By the implicit function theorem, $\frac{d\bar{\Delta}(\epsilon)}{d\epsilon} \leq 0$.

(iii) The threshold $\underline{\Delta}(\epsilon)$ is determined by $\hat{q}^{NW}(K^{\varphi}(\underline{\Delta}(\epsilon), \epsilon), \underline{\Delta}(\epsilon), \epsilon) = \hat{q}(K^{\varphi}(\underline{\Delta}(\epsilon), \epsilon), \epsilon) = -\theta\alpha$, i.e. it is the productivity shock at which the net worth and asset pricing curves intersect at the kink of the former. Since the asset pricing is not affected by the productivity

[23] More rigorously, $\hat{q}^{NW}$ is upperhemicontinuous. Henceforth, we ignore this technicality which is inconsequential for our analysis.

shock, while this shifts the net worth upwards, an equilibrium with renegotiation, as characterized in lemma A2 exists for all $\delta \geq \underline{\Delta}(\epsilon)$ (as long as $K^{\varphi}(\delta,\epsilon) \geq 0$). From lemma A0 we have $\frac{d\underline{\Delta}(\epsilon)}{d\epsilon} \leq 0$. Comparative statics for the equilibrium with renegotiation with respect to the shocks follow from lemma A2. Since when there is renegotiation $\varphi + \hat{q}_0$ is constant, the effect of shocks on the haircut is the opposite of the effect on prices. Thus $\frac{d\varphi}{d\delta} > 0$, while the sign of $\frac{d\varphi}{d\epsilon}$ is ambiguous. But, if at the original equilibrium the $\hat{q}^{NW}$ curve is steeper than the $\hat{q}_t$ curve, then φ is increasing in ϵ.

(iv) For $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$ it must be that $\bar{\Delta}(\epsilon) = \delta^{REN}(\epsilon)$, i.e. the first intersection between the net worth and asset pricing curves occurs at the kink of the former. This implies that $K^{KM}(\Delta^*(\epsilon),\epsilon) = K^{\varphi}(\Delta^*(\epsilon),\epsilon)$. Since $\frac{dK^{KM}}{d\epsilon} < 0$, $\frac{dK^{\varphi}}{d\epsilon} > 0$, and

$$\frac{dK^{KM}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K} - \frac{\partial \hat{q}}{\partial K}} > -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K}} = \frac{dK^{\varphi}}{d\delta},$$

it must be the case that $\frac{d\Delta^*(\epsilon)}{d\epsilon} \leq 0$.

We now want to prove equilibrium uniqueness. We start considering the case $\delta < \Delta^*(\epsilon)$. Clearly, there can be no equilibrium with renegotiation. $G''' \geq 0$ and Lemma A3 guarantee that $K^{KM}(\delta,\epsilon)$ is the unique equilibrium. Next, suppose $\delta \geq \Delta^*(\epsilon)$. For $K > K^{\varphi}(\Delta^*(\epsilon),\epsilon)$ there can be no other equilibrium from Lemma A3. For $K < K^{\varphi}(\Delta^*(\epsilon),\epsilon)$, we know that $-\theta\alpha > \hat{q}(K,\epsilon)$. Hence, the renegotiation equilibrium is unique.

5.1.1 Proposition 3

(i) This follows from the fact that $\Delta^*(\epsilon)$ is defined by the intersection of the $\hat{q}_t^{NW}$ and $\hat{q}_t$ curves when the first one has a kink, i.e. when $\hat{q}_t^{NW} = -\theta\alpha$. Since the upward sloping part of the former and the latter are unaffected by change in θ or in α while the vertical branch of the former shifts to the left with an increase in α or θ, $\Delta^*(\epsilon)$ is an increasing function of α or θ (at the kink, since $G''' \geq 0$, the net worth curve is steeper than the asset pricing curve).

(ii) A decrease in α or θ shifts the vertical branch of the net worth curve to the right, thus increasing the equilibrium K^{φ} and prices. The effect on haircuts is ambiguous. This depends on the slope of the $\hat{q}_t$ and $\hat{q}_t^{NW}$ curves at the original equilibrium K^{φ}. If the former is steeper (which since $G''' \geq 0$ happens when $\delta > \Delta(\epsilon)$) then the decrease in α or θ will result in lower haircuts.

5.2 Proofs asymmetric information

5.2.1 Proposition 4

(i) The proof parallels that of proposition 1 (i). Assumption 2 guarantees that $\hat{q}^{NW,AI}(K^*,\delta,\epsilon) \geq \hat{q}(K^*,\epsilon)$. We need to define capital holdings if every entrepreneur defaults, $\tilde{K}^{AI}(\delta,\epsilon)$, as $(1-\epsilon)u(\tilde{K}^{AI}(\delta,\epsilon))\tilde{K}^{AI}(\delta,\epsilon) = \left(1 - \delta - \frac{R}{R-1}E[\alpha]\right) aK^*$ (note that if every entrepreneur is defaulting, from (9) it follows that $-\hat{q}^{NW,AI} \geq 1$). If $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \leq \underline{\kappa} = -1$, then $(\max[\tilde{K}^{AI}(\delta,\epsilon),0], \hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon)$ is an equilibrium. If $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) >$

$\underline{\kappa}$, then $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \geq \hat{q}^{NW,AI}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\delta,\epsilon)$. Since both $\hat{q}$ and $\hat{q}^{NW,AI}$ are continuous, then it follows that there exists $\max[\tilde{K}^{AI}(\delta,\epsilon),0] < K^{eq} \leq K^*$ such that $\hat{q}(K^{eq},\epsilon) = \hat{q}^{NW,AI}(K^{eq},\delta,\epsilon)$.

(ii) This follows the proof of Proposition 1 (ii). When $\delta = \epsilon = 0$, $\hat{q}^{NW,AI}(K^*,0,0) = \hat{q}(K^*,0) = 0$. Since $\hat{q}^{NW,AI}$ is continuous in K, δ, and ϵ, and $\hat{q}$ is continuous in K and ϵ, we can define the function $K^{KM,AI}(\delta,\epsilon)$ implicitly from the equation $\hat{q}^{NW,AI}(K,\delta,\epsilon) - \hat{q}(K,\epsilon) = 0$ in a neighborhood of $\delta = \epsilon = 0$. By the implicit function theorem $K^{KM}(\delta,\epsilon)$ is continuous and decreasing in δ, since

$$\frac{dK^{KM,AI}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW,AI}}{\partial \delta}}{\frac{\partial \hat{q}^{NW,AI}}{\partial K} - \frac{\partial \hat{q}}{\partial K}} < 0,$$

where the inequality follows since $G''' \geq 0$, assumption 2, and lemma A1 guarantee that $\frac{\partial \hat{q}^{NW,AI}}{\partial K} > \frac{\partial \hat{q}}{\partial K} > 0$ for all $\delta < \Delta^{AI}(\epsilon)$, and $\frac{\partial \hat{q}^{NW,AI}}{\partial \delta} > 0$. A similar analysis yields $\frac{dK^{KM,AI}}{d\epsilon} < 0$, since from assumption 2, $\frac{\partial \hat{q}^{NW,AI}}{\partial \epsilon} - \frac{\partial \hat{q}}{\partial \epsilon} \geq 0$.

(iii) Since creditors have all the capital when $K = 0$, $\hat{q}(0,\epsilon) = (1-\epsilon)\frac{R}{R-1}\left(\frac{u(0)-a}{a}\right)$. If $\hat{q}(0,\epsilon) > \underline{\kappa}$, this can only be an equilibrium when $\delta = 1$, i.e. when all entrepreneurs have their net worth wiped out. When $\delta < 1$ default moderates the effect on net worth and it is no longer the case that $K = 0$ is an expectational equilibrium in the neighborhood of $\delta = 1$, since entrepreneurs with very small default costs have positive net worth. An equilibrium with $\mu = 1$ requires that the net worth relation becomes vertical at $\tilde{K}^{AI}(\delta,\epsilon)$. In this case prices are given by $\hat{q}_t(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon)$. If $\hat{q}_t(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \leq \underline{\kappa}$, then indeed every type of entrepreneur is better off defaulting and this is an equilibrium.

5.2.2 Proposition 5

We prove first this preliminary result

Lemma A4. Let $K^{\varphi,AI}(\delta,\epsilon)$ and $\hat{q}^{\varphi,AI}(\delta,\epsilon)$ be the implicit solutions to

$$\begin{aligned} -\bar{\alpha} - \hat{q}^{NW,AI}\left(K^{\varphi,AI}(\delta,\epsilon),\delta,\epsilon\right) &= 0 \\ \hat{q}^{\varphi,AI}(\delta,\epsilon) - \hat{q}\left(K^{\varphi,AI}(\delta,\epsilon),\epsilon\right) &= 0 \end{aligned}$$

Then, $K^{\varphi,AI}(\delta,\epsilon)$ and $\hat{q}^{\varphi,AI}(\delta,\epsilon)$ are well-defined and continuous functions, with $\frac{dK^{\varphi,AI}}{d\delta} < 0$, $\frac{d\hat{q}^{\varphi,AI}}{d\delta} < 0$, and $\frac{dK^{\varphi,AI}}{d\epsilon} > 0$.

Proof. This parallels the proof of Lemma A2. QED

(i) This follows the proof of Proposition 4 (i), considering two cases. First note that if $K^{\varphi,AI}(\delta,\epsilon) < 0$, then $\hat{q}^{NW,AI}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) \leq -1$. If $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) \leq \max[\underline{\kappa},-\bar{\alpha}]$, then $\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right)\right)$ is an equilibrium. If $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) > \max[\underline{\kappa},-\bar{\alpha}]$, then $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) > \hat{q}^{NW,AI}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right)$. Since both $\hat{q}$ and $\hat{q}^{NW,AI}$ are continuous in K, it follows that there exists $\max[K^{\varphi,AI}(\delta,\epsilon),0] < K^{eq} \leq K^*$ such that $\hat{q}(K^{eq},\epsilon) = \hat{q}^{NW,AI}(K^{eq},\delta,\epsilon)$.

(ii) Next, we want to prove that under Assumption 3, there exists some $\bar{\Delta}^{AI}(\epsilon) \in \left[0,\Delta^{AI}(\epsilon)\right)$ such that $\{K^{eq}(\delta,\epsilon),\hat{q}^{eq}(\delta,\epsilon)\}$ defined by

$$\{K^{eq}(\delta,\epsilon), \hat{q}^{eq}(\delta,\epsilon)\} = \left\{ \begin{array}{c} \{K^{KM,AI}(\delta,\epsilon), \hat{q}^{KM,AI}(\delta,\epsilon)\} \text{ if } \delta < \bar{\Delta}^{AI}(\epsilon) \\ \{\max[K^{\varphi,AI}(\delta,\epsilon), 0], \max[\hat{q}^{\varphi,AI}(\delta,\epsilon), \hat{q}(0,\epsilon)]\} \text{ if } \delta \geq \bar{\Delta}^{AI}(\epsilon) \end{array} \right\}$$

is a set of equilibria in which the resulting capital schedule $K^{eq}(\delta,\epsilon)$ and asset prices $\hat{q}^{eq}(\delta,\epsilon)$ are continuous functions of δ and ϵ. Note that, as in proposition 2, large shocks might wipe out entrepreneurs' net worth, i.e. $K^{\varphi,AI}(\delta,\epsilon) < 0$. Since this leads to the trivial equilibrium with $K_0 = 0$, henceforth we will assume that $K^{\varphi,AI}(\delta,\epsilon) \geq 0$.

Assumption 3.

$$\begin{aligned} &\frac{R}{R-1}\left(-\bar{\alpha}(1-F(\bar{\alpha})) - \int_0^{\bar{\alpha}} \alpha dF(\alpha)\right) \\ &> (1-\epsilon)\frac{u\left(K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon)\right)K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon)}{aK^*} - \left(1-\Delta^{AI}(\epsilon)\right). \end{aligned}$$

Under Assumption 3 we know that $-\bar{\alpha} > \hat{q}\left(K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon),\epsilon\right)$. This implies that when $\delta = \Delta^{AI}(\epsilon)$, the equilibrium features renegotiation, $\varphi > 0$, and thus $K^{KM,AI}\left(\Delta^{AI}(\epsilon),\epsilon\right) < K^{\varphi,AI}\left(\Delta^{AI}(\epsilon),\epsilon\right)$. From Lemma A4 we know that as we decrease δ, K_0 increases. Since $K^{KM,AI}(0,\epsilon) > K^{\varphi,AI}(0,\epsilon)$ (where the latter might require a negative haircut), this implies that there exists a $\bar{\Delta}^{AI}(\epsilon) < \Delta^{AI}(\epsilon)$ for which the equilibrium corresponds to $\left(K^{KM,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right), \hat{q}^{KM,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)\right)$, i.e. the net worth and asset pricing curves intersects at the point when the former has a kink.

To prove equilibrium uniqueness, we start considering the case $\delta < \bar{\Delta}^{AI}(\epsilon)$. There can be no equilibrium with renegotiation. $G''' \geq 0$ and Lemma A3 adapted to $q^{NW,AI}$ guarantee that $K^{KM,AI}(\delta,\epsilon)$ is the unique equilibrium. Next, suppose $\delta \geq \bar{\Delta}^{AI}(\epsilon)$. For $K > K^{\varphi,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)$ there can be no other equilibrium from Lemma A3 adapted to $q^{NW,AI}$. For $K < K^{\varphi,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)$, we know that $-\bar{\alpha} > \hat{q}(K,\epsilon)$. Hence, the renegotiation equilibrium is unique. Concerning the response of haircuts, we have seen in the proof of Proposition 4 (ii) that $K^{KM,AI}(\cdot)$ and $\hat{q}^{KM,AI}(\cdot)$ are decreasing in δ and ϵ. We start with productivity shocks. From Lemma A4 we know that $K^{\varphi,AI}(\cdot)$ is decreasing in δ. This implies that prices are also decreasing in δ, and since $\hat{q}_t + \varphi^{AI} = -\bar{\alpha}$, this implies that the haircut, φ^{AI}, is increasing in δ. For preference shocks, Lemma A4 tells us that $K^{\varphi,AI}(\cdot)$ is increasing in ϵ. Since the shock shifts the $\hat{q}_t$ curve downwards the effect of the shock on prices is ambiguous as the direct effect of ϵ is negative but there is a positive effect from the increase in K. As a result the effect on φ^{AI} is ambiguous. But, if at the original equilibrium the $\hat{q}^{NW,AI}$ curve is steeper than the $\hat{q}_t$ curve, then φ^{AI} is increasing in ϵ.

(iii) This parallels the proof of proposition 2 (iii).

5.2.3 Proposition 6

We derive comparative statics results for the following distribution function

$$F(x) = \frac{x^{\gamma}}{x^{\gamma} + (1-x)^{\gamma}},$$

with $\gamma \geq 1$. Its density is given by

$$f(x) = \frac{\gamma x^{\gamma-1}(1-x)^{\gamma-1}}{[x^\gamma + (1-x)^\gamma]^2}.$$

(i) We show first that $\mu = 1 - F(\overline{\alpha}) = 1 - \frac{\overline{\alpha}}{\gamma}$.

From first order condition (8) holding with equality we have

$$\frac{\overline{\alpha}}{\gamma}(\overline{\alpha}^\gamma + (1-\overline{\alpha})\gamma) = (1-\overline{\alpha})^\gamma,$$

using this condition to replace the denominator in the cumulative distribution function

$$F(\overline{\alpha}) = \frac{\overline{\alpha}^{\gamma+1}}{\gamma(1-\overline{\alpha})^\gamma}.$$

First order condition (8) holding with equality can also be rewritten as

$$\overline{\alpha}^{\gamma+1} = (1-\overline{\alpha})^\gamma(\gamma - \overline{\alpha}),$$

Replacing in the cumulative function gives

$$F(\overline{\alpha}) = 1 - \frac{\overline{\alpha}}{\gamma}.$$

(ii) From the previous result it follows that to show that the more severe is the asymmetry of information (indicated by a lower γ), the more default arises in equilibrium requires proving that $\frac{\gamma}{\overline{\alpha}}\frac{d\overline{\alpha}}{d\gamma} < 1$. This follows from the application of the implicit function theorem on the following expression of (8) holding with equality

$$\ln(\gamma - \overline{\alpha}) + \gamma(1-\overline{\alpha}) - (\gamma+1)\overline{\alpha} = 0.$$

The implicit derivative is given by

$$\frac{d\overline{\alpha}}{d\gamma} = -\frac{\ln(\frac{\overline{\alpha}}{1-\overline{\alpha}}) - \frac{1}{\gamma-\overline{\alpha}}}{\frac{\gamma+1}{\overline{\alpha}} + \frac{\gamma}{1-\overline{\alpha}} + \frac{1}{\gamma-\overline{\alpha}}}.$$

Thus the question of whether $\frac{\gamma}{\overline{\alpha}}\frac{d\overline{\alpha}}{d\gamma} < 1$ boils down to whether

$$-\gamma\ln(\frac{\overline{\alpha}}{1-\overline{\alpha}}) + \frac{\gamma}{\gamma-\overline{\alpha}} < \gamma + 1 + \frac{\gamma\overline{\alpha}}{1-\overline{\alpha}} + \frac{\overline{\alpha}}{\gamma-\overline{\alpha}},$$

which can be rewritten as

$$-\gamma\ln\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) < \gamma + \frac{\gamma\overline{\alpha}}{1-\overline{\alpha}}.$$

This condition is satisfied as long as $\overline{\alpha} \geq \frac{1}{2}$, which can be verified from the following expression of (8) holding with equality

$$\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)^{\gamma+1} = 1 + \frac{\gamma-1}{1-\overline{\alpha}}.$$