Ambiguous Policy Announcements

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Abstract

We study the effects of monetary policy announcements in a New Keynesian model, where ambiguity averse households with heterogenous net financial wealth use a worst-case criterion to evaluate the credibility of announcements. An announcement of a future loosening in monetary policy leads to rebalancing in financial asset positions, can cause credit crunches, and might be contractionary in the interim period before implementation. This is because households with positive net financial wealth (creditors) are the most likely to believe the announcement, due to the wealth losses that future monetary policy can cause to them. And when creditors believe the announcement more than debtors do, the wealth losses that creditors expect to incur are larger than the wealth gains that debtors expect to realize. So aggregate net wealth is perceived to fall, and the economy can contract due to lack of aggregate demand, which is more likely when wealth inequality is large. We evaluate the importance of this mechanism by focusing on the start of the Forward Guidance practice by the ECB in July 2013. We show that the inflation expectations of households have responded in accordance with the theory. After matching the entire distribution of European households' net financial wealth, we find that the ECB announcement is contractionary in our model. Generally, redistributing expected future wealth might have unintended perverse effects when agents are ambiguity averse.

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1 Introduction

Policy makers often rely on announcements about future reforms of economic institutions or future changes in fiscal or monetary policy to stimulate the economy in the short run. Once implemented, these policies typically have important redistributive implications. For example during the recent Great Recession and since short term nominal interest rates had hit the zero lower bound, Central Banks have relied extensively on announcements about future changes in monetary policy to raise today inflation and stimulate the economy, a practice which is generally known as *Forward Guidance*. And it is well known that higher inflation tends to redistribute wealth from creditors to debtors (Fisher 1933, Doepke and Schneider 2006, and Adam and Zhu 2015). In this paper we show that, when agents are ambiguity averse, policy announcements might have unintended perverse effects in the interim period before the policy is actually implemented. Generally the effect of the announcement depends on (i) the amount of redistribution induced by the policy if implemented, (ii) the concentration of future hypothetical wealth losses, and (iii) the (endogenous) correlation between agents' wealth and their changes in expectations at the time of the announcement.

To show these points, we consider the effects of monetary policy announcements in a New Keynesian model, where ambiguity averse households with heterogenous net financial wealth use a worst-case criterion to evaluate the credibility of announcements, according to the Maximin preferences specification proposed by Gilboa and Schmeidler (1989). Ambiguity aversion is a natural paradigm when characterizing the behavior of households who do not know the probability distribution of outcomes, which is likely to happen when households have to deal with news about unfamiliar contingencies, as for example in the case of announcements about future unconventional policies in an unusual economic environment.

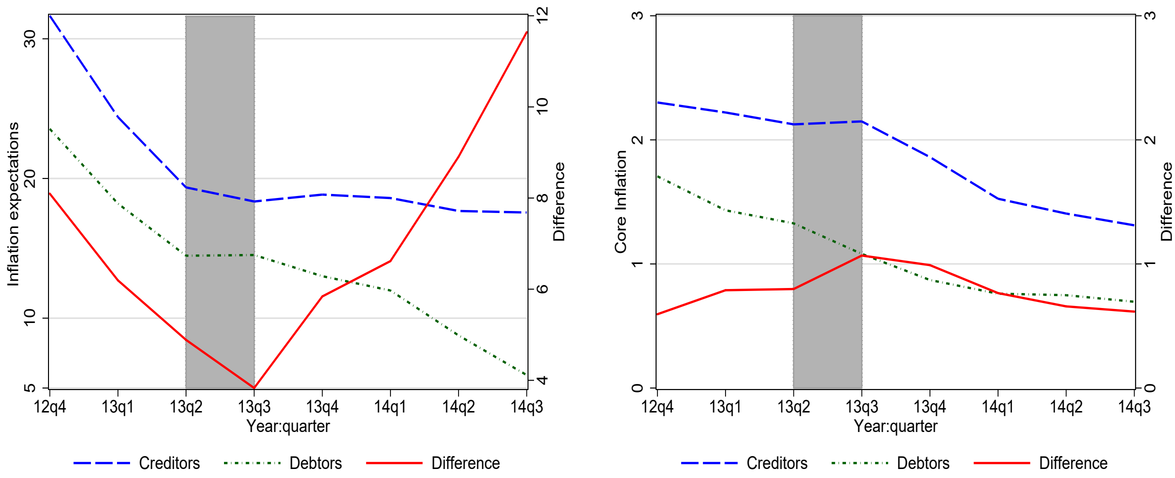
We focus on the effects of announcements of future changes in real rates due to changes in inflation and/or nominal rates, in an economy which is initially in a steady state equilibrium. If implemented, a reduction in real rates tends to stimulate the consumption of agents with negative net financial wealth (debtors) through both a substitution and an income effect. However, for agents with positive net financial wealth (creditors), a change in real rates involves a substitution and an income effect on consumption which tends to have opposite effects on consumption. This is because a fall in real returns reduces the capital income that pertains to creditors. As a result, creditors are more likely to believe the announcement of a future loosening in monetary policy than debtors are, because the worst-case scenario for creditors—at least for those with sufficiently large net financial wealth—is that real rates and thereby their financial income will actually fall. However, if creditors believe the announcement more than debtors do, the wealth losses that creditors expect to incur are larger than the gains that debtors expect to realize and as a result aggregate net wealth—equal to the sum of the expected wealth of creditors and debtors—is perceived to fall. This is what we call the *forward misguidance* effect, which can be strong enough to dominate the conventional substitution effect and thereby to lead to a contraction in aggregate activity due to lack of aggregate demand. This fall in demand is more likely when wealth inequality is large so that the capital losses induced by future monetary policy are concentrated among a small group of relatively wealthy households. Generally, as a result of the announcement of a future loosening in monetary policy, the real rate expected by creditors is smaller than the real rate expected by debtors, which leads to a rebalancing in the financial asset positions of agents and can even cause credit crunches, which happen because agents stop trading in financial markets so as to get fully insured against future changes in monetary policy.

An announcement of a future tightening in monetary policy with an associated future increase in real rates is unambiguously contractionary in our model. In this case debtors are the most likely to believe the announcement and for them the future increase in real rates leads to a reduction in their consumption through a substitution and an income effect which both work in the direction of reducing consumption. So aggregate consumption and thereby output unambiguously fall. The fall in output is larger than the one that would arise in an hypothetical equilibrium where the announcement is fully believed by all agents in the economy. This again emphasizes that, under ambiguity aversion, redistributing future expected wealth is a negative-sum game.

We evaluate the importance of this mechanism by focusing on the start of the Forward Guidance practice by the ECB on 4 July 2013.[1] After the announcement, long-term government bonds yields and EONIA swap rates have fallen by around 10-15 basis points at maturities between 2 and 4 years, see Coeuré (2013), ECB (2014), and Section 5. There is evidence that the inflation expectations of households have responded to the announcement as predicted by the misguidance effect. Figure 1 shows the evolution of households' expected inflation, in panel (a), and realized inflation in panel (b) in the Euro 11 countries, separately for the group of creditor countries (as a blue dashed line) and of debtor countries (as a green dotted line). Creditor countries are defined as those that at the time of the ECB announcement had a positive Net Foreign Asset position, which include Austria, Finland, Germany, Luxembourg, and Netherlands. Debtor countries are all the others. The difference between the two lines corresponds to the red solid line in the figure.

[1]On that date the ECB Governing Council has started its Forward Guidance practice by announcing that "it expected the key ECB interest rates to remain at present or lower levels for an extended period of time."

Figure 1: Forward Guidance in Euro 11: Expected and realized inflation



(a) Expected change in trend inflation (b) Actual Inflation

Notes: Core Inflation is yearly log differences in consumer prices excluding energy and seasonal food multiplied by 100. Expectations are calculated in terms of balances—differences between people providing positive and negative answer. Price expectations come from the following question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall. Probabilities are calculated in terms of balances: differences between people saying that he answer is very likely versus people saying the answer it is unlikely. Price expectations are calculated as equal to $(f_i + 1/2f_{ii} - 1/2f_{iv} - f_v) \times 10$, where f_j is the fraction of individuals who opted for option j = i, ii, iii, iv, v in the survey. Creditor countries are Austria, Finland, Germany, Luxembourg, and Netherlands. Others countries include all the remaining countries in the group of Euro 11 countries. Source of data: ECB, Joint Harmonized Programme of Business and Consumer Surveys by European Commission and External Wealth of Nation Mark II (EWN).

The area in grey identifies the quarter of the announcement. Inflation has kept falling after the announcement, which is consistent with the relatively muted effects of Forward Guidance estimated in the US (Del Negro, Giannoni, and Patterson 2015). Interestingly, after the announcement, inflation has kept falling at a higher speed in creditor countries than in debtor countries, see panel (b). Still, and relative to trend, inflation expectations have been revised upwards only in creditor countries, whereas they have remained stable in debtor countries, see panel (a). As a result the difference in inflation expectations between creditor and debtor countries has increased, which, together with the evidence for actual inflation above, implies that the wedge between expected future inflation and future realized inflation has widened in creditor countries relative to debtor countries.

To provide further evidence in favor of the misguidance effect, we rely on a Difference-in-Differences identification strategy using Italian provinces, which differ substantially in their net financial wealth and are subject to the same country specific shocks. We exploit a unique quarterly-frequency dataset with information on both expected and realized inflation at a very disaggregated level. In each province we construct a measure of the inflation expectation bias of households by calculating the difference between today expected future inflation and future realized inflation. We classify provinces according to the fraction of households in the province who have positive net financial wealth (creditor households). We show that, in response to the ECB announcement, provinces with a larger share of creditor households experience a relative increase in their inflation expectation bias: roughly, a one standard deviation increase in the share of creditor households in the province is associated with an after-the-announcement relative increase in the expectation bias of 9 basis points.

To study the quantitative relevance of the effect, we calibrate our heterogenous agents new Keynesian model to match data from the Household Finance and Consumption Survey (HFCS) on the entire distribution of European households' net financial wealth. We calibrate the ECB announcement to match the response on 4 July 2013 of the yield curve of interest rates at all maturities between 2 and 10 years. We find that in our model the output effect of the ECB announcement is substantially muted relative to the effect that would arise in the alternative standard benchmark where all households fully believe the announcement. Under our preferred parametrization, the announcement is actually contractionary, with a cumulated output loss of 1.6% in the fifteen months before implementation, which should be compared with the cumulated output gain of almost 6% under the alternative benchmark.

Relation to the Literature. Forward Guidance has become a central tool of monetary policy during the current recession because conventional expansionary monetary policies were no longer available as short term interest rates hit the zero lower bound. And there is a growing literature studying optimal monetary policy in a liquidity trap (Eggertsson and Woodford 2003) as well as the effects of Forward Guidance (Del Negro, Giannoni, and Patterson 2015, Swanson 2016). From the point of view of conventional New Keynesian sticky price models it is a puzzle why Forward Guidance announcements have been little effective in stimulating the economy and moving it out of a liquidity a trap, and some papers have proposed explanations for this puzzle, see Andrade, Gaballo, Mengus, and Mojon (2015), Caballero and Farhi (2014), Kaplan, Moll, and Violante (2016b), McKay, Nakamura, and Steinsson (2015) and Wiederholt (2014). In this paper we abstract away from the reason why the monetary authority relies on announcements to stimulate the economy. We just emphasize, that, under ambiguity aversion, announcements of a future loosening in monetary policy are more likely to affect the expectations of agents with large net financial wealth than those of agents with negative financial wealth, and because of this the economy can even contract in the presence of large financial imbalances.

It has been known at least since Fisher (1933) that expansionary monetary policy redistributes wealth from creditors to debtors. It has also been observed that this redistribution could be expansionary on aggregate demand because agents differ either in their marginal propensity to consume out of wealth, as first emphasized by Tobin (1982), or in the liquidity and term structure of their portfolios, as in Kaplan, Moll, and Violante (2016a) and Auclert (2015) respectively. However, see Doepke, Schneider, and Selezneva (2015) for an overlapping generation model where this redistribution makes aggregate consumption fall. Here we focus on redistributing future expected wealth, which under ambiguity aversion is typically a negative-sum game, because net losers out of the redistribution tend to believe any news about the future more than net winners do.

Other papers have emphasized that ambiguity aversion matters for business cycle analysis. Ilut and Schneider (2014) show that shocks to the amount of ambiguity can be an important source of business cycle volatility. Ilut, Valchev, and Vincent (2016) study the implication of ambiguity aversion for sticky prices. Backus, Ferriere, and Zin (2015) focus on asset pricing, while Ilut, Krivenko, and Schneider (2015) provide methods to study dynamic economies where ambiguity averse agents differ in their perception of exogenous shocks and study the implications for precautionary savings, asset premia and insurance gains. Here we focus on the effects of announcements and more generally of news about the future and how they interact with wealth inequality and redistribution.

Recent research has emphasized that there is substantial heterogeneity in the inflation expectations of agents, see for example Coibion, Gorodnichenko, and Kumar (2015) for evidence. Here we emphasize that, when agents are ambiguity averse, changes in inflation expectations of agents are related to their wealth position.

Section 2 provides some evidence. Section 3 characterizes the economy. Section 4 studies the effect of monetary policy announcements in a simple case. Section 5 evaluates quantitatively the effect of the ECB announcement on July 2103. Section 6 studies robustness. Section 7 concludes. The Appendix contains details on data and model computation.

2 Some more evidence on the misguidance effect

Figure 1 indicates that, in response to the Forward Guidance announcement by the ECB, the inflation expectations of households have responded more in creditor than in debtor countries, which is evidence consistent with a misguidance effect. However, aggregate country-level data do not provide information on the within-country distribution of households' net financial assets. Moreover, country-specific asymmetric shocks have played an important role in the current crisis. To address these concerns, we rely on a Difference-in-Differences identification strategy which exploits quarterly data on realized and expected inflation at the Italian province level. Italian provinces exhibit substantial variation in their households' net financial wealth and, by construction, are subject to the same country specific shocks. We construct a measure of the (average) inflation expectation bias of households within a province by calculating the difference between the expected future inflation of agents in the province and the realized future inflation in the province. We test whether, in response to the ECB announcement, the inflation expectation bias of households has increased more in provinces with a larger fraction of creditor households. We start with a brief discussion of the construction of the dataset, for full details see the Appendix.

2.1 Data

Our province level data come from three different sources: realized inflation is from the official Italian statistical institute (ISTAT); expected inflation is from confidential data collected in the Bank-of-Italy-Sole24Ore Survey on Expectations; Net Financial Assets (NFA) of households are calculated using the Survey of Household Income of Wealth (SHIW) by the Bank of Italy. Data are quarterly and our regressions cover the sample period 2012:I-2014:II. The last observation of the sample is dictated by the start of the Quantitative Easing (QE) program by the ECB which has started in 2015:I. Realized inflation in the province corresponds to the yearly log-difference of the general price index in the province, which is consistent the ECB practice of monitoring price stability at the annual frequency. Expected inflation measures 2-quarters-ahead expected inflation, by averaging the reported estimates of all individuals in the province in the survey. The NFA of a household is calculated as equal to the difference between the sum of the household's holdings of money, postal deposits, saving certificates, pension funds, government securities, and other securities minus the sum of all household's financial liabilities to banks or other financial institutions. A creditor household is a household with positive NFA. For each province we calculate the pre-announcement fraction of creditor households, by using the last two (to increase sample size) available waves of SHIW before the ECB announcement—which refer to 2010 and 2012. Table 1 describes our sample. The average NFA of an Italian household is equal to 14,589 Euros (at constant 2010 prices). The fraction of households with positive NFA is 67%, but this fraction varies considerably across provinces—from 26% to 97%. In each province i and quarter t, we calculate the difference between expected inflation, $E_{it}[\pi_{it+2}]$, and future realized inflation, π_{it+2},

$$\hat{\pi}_{it} \equiv E_{it}[\pi_{it+2}] - \pi_{it+2}, \tag{1}$$

Table 1: Descriptive statistics

VARIABLES	(1) mean	(2) sd	(3) N	(4) min	(5) max
Pre-announcement fraction of creditor households	0.67	0.13	1078	0.26	0.97
Pre-announcement fraction of creditor households divided by SD, F_i	5.47	1	1078	2.56	7.75
Inflation rate in province π_{it}	1.77	1.24	1078	-0.47	4.76
Two quarters ahead expected inflation, $E_{it}[\pi_{it+2}]$	2.02	1.23	1078	-10	8.72
Two quarters ahead realized inflation, π_{it+2}	1.15	1.16	1078	-9.62	4.53
Inflation expectation bias, $\hat{\pi}_{it}$	0.86	0.74	1078	-3.61	6.79
Year	2012.80	0.75	1078	2012	2014

Notes: Quarterly data over the sample period 2012:I-2014:II. Realized inflation comes from ISTAT. Data on expected inflation are based on confidential data from the Bank of Italy Sole24Ore survey on expectations. The Net Financial Asset position of households is calculated using the 2010 and 2012 waves of the Survey of Household Income of Wealth (SHIW).

which is a measure of the (average) inflation expectation bias of agents in province i at time t. Interestingly, $\hat{\pi}_{it}$ is positive on average in our sample.

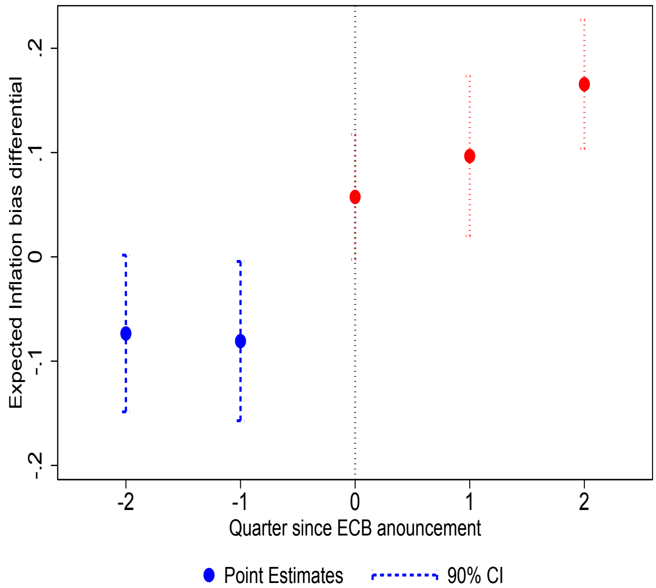
2.2 Evidence

To evaluate whether, in response to the ECB announcement, the inflation expectation bias has changed more in provinces with a larger fraction of creditor households, we run the following regression:

$$\hat{\pi}_{it} = \sum_{n=-6}^{5} \phi_n \left(F_i \times \mathbb{I}_{n=t-t_0}\right) + \beta X_{it} + \varepsilon_{it} \ , \tag{2}$$

where F_i is equal to the pre-announcement fraction of creditor households in the province, divided by its standard deviation, which is equal to 0.13. $\mathbb{I}_{n=t-t_0}$ is a dummy which is equal to one just at quarter n since the announcement which occurred at time t_0=2013:III. The set of controls X_{it} include a full set of time dummies. The coefficient ϕ_n, whose profile is reported in Figure 2, measures the difference in the inflation expectation bias of two provinces that differ in a one-standard-deviation amount of creditor households at quarter n since the ECB announcement. The point estimates of ϕ_n for $n < 0$ correspond to the blue bullet point in Figure 2; those for $n \geq 0$ correspond to the red bullet point. Vertical bars denote 90% confidence interval using Robust Standard Errors. The vertical dotted black line indicates the quarter of the announcement, $n = 0$. There is clear indication that the coefficient ϕ_n's have increased at the time of the announcement and in the quarters thereafter, which means that the inflation expectation bias of agents has increased more in

Figure 2: Expected inflation bias of creditor households before and after FG



Notes: Profile of the ϕ_n's coefficient of regression (2). The dependent variable is the difference between expected future inflation and future realized inflation, $\hat{\pi}_{it}$. The independent variable is the pre-announcement fraction of creditor households in the province as obtained from SHIW divided by its standard deviation, F_i. The vertical dotted black line indicates the quarter of the announcement, $n = 0$. Vertical bars denote 90% confidence interval using Robust Standard Errors.

provinces with a larger share of creditor households.

To evaluate the effect of the ECB announcement on the expectation bias $\hat{\pi}_{it}$, one can calculate the difference between the average value of the coefficients ϕ_n's in the quarters after the shock and their average value in the quarters before the shock. As it is common in the Dif-in-Dif literature, we measure this difference by running the following regression:

$$\hat{\pi}_{it} = \overline{\phi} F_i + \phi F_i \times \mathbb{I}_{t \geq t_0} + \beta X_{it} + \varepsilon_{it} \quad . \tag{3}$$

Again F_i is equal to the (standardized) fraction of creditor households in the province. The set of controls X_{it} include a full set of time dummies and province dummies. $\mathbb{I}_{t \geq t_0}$ is a dummy which is equal to one in the quarter of the announcement and in all quarters thereafter, while it is equal to zero otherwise. The coefficient $\overline{\phi}$ measures the average effect of F_i on the expectation bias in the province. The Difference-in-Differences coefficient ϕ measures how the effect of F_i on the inflation expectation bias has increased in the quarters after the announcement. Roughly ϕ measures the difference between the average value of the red and the blue bullet points in Figure 2. The results from estimating (3) are reported in Table 2. Column 1 refers to the specification with no province fixed effects, column 2 reports the results after controlling for a full set of province level dummies. The estimates indicate that, after the ECB announcement, provinces with a one-standard-deviation higher share of creditor households have experienced a relative increase in their inflation expectation bias of around 9 basis points.

Table 2: Effects of FG on the expected inflation bias of creditor households

VARIABLES	(1)	(2)
Fraction of creditors, F_i, (coefficient $\overline{\phi}$)	-0.02	-0.23
	(0.03)	(0.15)
Announcement-dummy × F_i, (coefficient ϕ)	0.09***	0.09***
	(0.03)	(0.03)
R^2	0.35	0.49
N. of observations	1078	1078
N. of provinces	108	108
Year FE	Y	Y
Province FE	N	Y

Notes: Results from running regression (3). All regressions include year fixed effect. The dependent variable is $\hat{\pi}_{it}$ as defined in (3). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA in the province. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

3 The model

We start considering a simple economy in discrete time, which we extend in Section 5 for a quantitative analysis. The economy is populated by a unit mass of households, indexed by $x \in [0, 1]$, who are ambiguity averse and just differ in their net financial wealth, $a_{xt} \in [\underline{a}_t, \bar{a}_t]$, invested in one-period bonds. There is a unit mass of firms that demand labor to produce intermediate goods sold under a regime of monopolistic competition and prices are sticky. The nominal interest rate is continuously adjusted to achieve the inflation target set by a monetary authority with an unambiguous mandate of price stability. The monetary authority has always complied with its mandate by fully stabilizing prices over the history of the economy. We focus the analysis on the short run response of the economy, after the monetary authority, suddenly and unexpectedly, announces a future change in the inflation target, which makes households doubt whether the authority will actually deviate from its historical mandate as announced. Hereafter we adopt the convention that, unless otherwise specified, variables are real—measured in units of the final consumption good.

Households Household $x \in [0,1]$ is infinitely lived, with a subjective discount factor $\beta < 1$ and per period preferences over consumption c_{xt} and labor l_{xt} given by

$$U(c_{xt}, l_{xt}) = \frac{\left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma}, \tag{4}$$

with $\psi_0, \psi > 0$ and $\sigma \geq 1$. When all households share the same beliefs, these preferences (Greenwood, Hercowitz, and Huffman 1988) guarantee that the economy features a representative household, which is a canonical benchmark in the New Keynesian literature. Financial markets are incomplete, in that households can invest just in a one-period bond, which, at time t, pays (gross) return r_t per unit invested.[2] Households can freely borrow by going short in the asset. The labor market is perfectly competitive, so households take the wage w_t as given. At each point in time t, household-x chooses the triple $\{c_{xt}, l_{xt}, a_{xt+1}\}$ subject to the budget constraint

$$c_{xt} + a_{xt+1} \leq w_t\, l_{xt} + r_t\, a_{xt} + \lambda_t \ , \tag{5}$$

where a_{xt+1} measures the units invested in bonds at time t which will yield return r_{t+1} at time $t+1$, while λ_t denotes (lump sum) government transfers (specified below).

Monetary policy rule The monetary authority sets the (gross) nominal interest rate R_{t+1} between time t and $t+1$ (paid in $t+1$) according to

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{\Pi_t}{\Pi_t^*}\right)^{(1-\rho_r)\,\phi} \left(\frac{R_t}{\bar{R}}\right)^{\rho_r} \ , \tag{6}$$

where $\phi > 1$ and $\rho_r \in [0,1)$, $\bar{R}$ is the steady state interest rate, $\Pi_t \equiv p_t/p_{t-1}$ is gross inflation, and Π_t^* is the time-t *inflation target*, which is equal to one in steady state, $\bar{\Pi}^* = 1$.

Firms The final consumption good is produced by a (representative) competitive firm, which uses a continuum of varieties $i \in [0,1]$ as inputs according to

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}}, \tag{7}$$

[2] In Appendix C.1, we consider an extension with long-term bonds.

where y_{it} is the amount of variety i used in production. The variety i is produced only by a firm i which uses a linear-in-labor production function, so that $y_{it} = \ell_{it}$, where ℓ_{it} denotes firm-i labor demand, whose unitary cost is w_t. Firm $i \in [0, 1]$ sets the nominal price for its variety p_{it} to maximize the beginning of period expected profits, $d_{it} \equiv y_{it}\,(p_{it}/p_t - w_t)$, taking as given the demand schedule by the competitive firm, the aggregate nominal price, p_t, and the wage rate, w_t. We assume firm i chooses its nominal price at time t, p_{it}, after the monetary authority has set the inflation target for that period Π_t^* (but before any time-t policy announcement). Finally, we start assuming that the government owns all firms in the economy and rebates all profits back to households in a lump-sum fashion, so that $\lambda_t = \int_0^1 d_{it}\,di$. We relax this assumption in Section 5.

Markets clearing In equilibrium, output Y_t is equal to aggregate consumption $C_t \equiv \int_0^1 c_{xt}\,dx$, so that $Y_t = C_t$, and labor demand is equal to labor supply, $\int_0^1 \ell_{it} di = \int_0^1 l_{xt} dx$. Since bonds are in zero net supply, clearing of the financial market requires that $\int_0^1 a_{x,t} = 0$ at the return $r_t = R_t/\Pi_t$, where the nominal interest rate R_t satisfies (6).

Steady state At $t = 0$, the economy is initially in a steady state, where a monetary authority with an unambiguous mandate of price stability has always set $\Pi_t^* = 1$, and households expect Π_t^* to remain equal to one also in the future $\forall t$, implying $\bar{r} = \bar{R} = 1/\beta$ and $\bar{\Pi} = 1$, where an upper-bar denotes the steady state value of the corresponding quantity.

Policy announcement At $t = 0$, (and after firms have set their nominal price), the monetary authority announces that in period $T > 0$, and only at T, the inflation target will deviate from full price stability, implying that $\Pi_T^* = \varepsilon$, and $\Pi_t^* = 1$ for all $t \neq T$. If $\varepsilon > 1$, the announcement is *inflationary*; if $\varepsilon < 1$, it is *deflationary*. Given the announcement, household $x \in [0, 1]$ makes her consumption, labor supply and saving decisions, while firms $i \in [0, 1]$ supplies any amount demanded at their (previously) set price.

Ambiguity aversion There is ambiguity about whether the monetary authority will actually deviate from its mandate of price stability and households are ambiguity averse. Households doubt whether the monetary authority can commit to implement the announced inflation target at time T, Π_T^*, and do not know how to calculate the probability distribution of Π_T^*.[3] Thus, $\forall t < T$, household-x perceives Π_T^* as a random variable with a probability distribution about which the household has to form her own subjective beliefs. To model ambiguity aversion, we rely on the multiple priors utility model by Gilboa and Schmeidler (1989), whose axiomatic foundations are provided by Epstein and Schneider (2003). The utility of household-x is then given by the sum of the felicity from time-t consumption

[3] For simplicity, we assume that households face ambiguity just about the intensity of the implementation of the announcement, ε, but not about the time horizon of the implementation, T.

and labor plus the expected continuation utility, which is evaluated under the household-x worst-case scenario about the future realizations of the inflation target. Formally, we assume that preferences at time t order future streams of consumption, $\mathbf{C_t} = \{c_s(h^s)\}_{s=t}^{\infty}$, and labor supply, $\mathbf{L_t} = \{l_s(h^s)\}_{s=t}^{\infty}$, so that utility is defined recursively as follows:

$$V_t(\mathbf{C_t}, \mathbf{L_t}) = U(c_t, l_t) + \beta \min_{\Omega \subseteq \mathcal{S}_t,\ G \in \mathcal{P}(\Omega)} \int_{\Omega} V_{t+1}(\mathbf{C_{t+1}}, \mathbf{L_{t+1}})\, G(d\Pi^*_{t+1}) \ , \tag{8}$$

where $h^t = \{\Pi^*_{-\infty}, ..., \Pi^*_{t-1}, \Pi^*_t\}$ denotes the history up to time t, Ω is the support of the probability distribution G that household x associates to the realizations of the inflation target one period ahead, Π^*_{t+1}. Expected utility arises when the household is forced to take Ω and the associated probability distribution G as given. Under ambiguity aversion, to rank the utility from future streams of consumption and labor, the household chooses a support Ω and an associated probability distribution G so as to minimize the continuation utility V_{t+1} (worst case criterion). The support Ω is chosen among the possible realizations of the inflation target in $t+1$, which is denoted by $\mathcal{S}_t$. A non-degenerate set of beliefs captures the household's lack of confidence in probability assessments, with a larger set implying greater ambiguity about the future. The probability distribution G is chosen in the set of all probability distributions $\mathcal{P}(\Omega)$ that assign positive probability to all values in the support Ω. We assume that, $\forall t$, household $x \in [0,1]$ can condition her choices to the entire history up to time t, h^t, which is fully characterized by the observed realizations of Π^*_t up to t. Household x chooses her consumption plans, $c_t(h^t)$, her labor supply $l_t(h^t)$ and her savings $a_{t+1}(h^t)$ so as to maximize (8). Notice that, if the realizations of the inflation target affect the consumption and labor streams of different households differently, these preferences will give rise to actions that are taken under heterogeneous beliefs.

In our experiment, the household faces ambiguity about the realization of Π^*_T only, so the set $\mathcal{S}_t$ is non-degenerate just at $t = T-1$. In particular, we assume that $\mathcal{S}_{T-1} = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}]$: when the announcement is inflationary, $\varepsilon > 1$, we have $\mathcal{S}_{T-1} = [1, \varepsilon]$; when it is deflationary, $\varepsilon < 1$, we have $\mathcal{S}_{T-1} = [\varepsilon, 1]$. The motivation for this specification is that a monetary authority with a mandate of price stability, which has been maintained successfully over a long past history, makes household-x doubt whether the central bank will actually deviate from its mandate as announced and, if so, by how much.[4] There is no ambiguity about the inflation target pursued by the monetary authority at $t < T-1$ or at $t \geq T$. So we have $\mathcal{S}_t = 1\ \forall t \neq T-1$. Finally, notice that, since the set $\mathcal{S}_t$ is common to

[4] As it will become clear from the analysis below, this is a conservative assumption because any larger support implies more heterogeneity in equilibrium households' beliefs, which would generally strengthen our results.

all households $x \in [0, 1]$, they all face the same amount of ambiguity.[5] We can now define an equilibrium as follows:

Equilibrium An equilibrium is a set of beliefs, quantities, and prices such that, $\forall t$,

1. Each *household* $x \in [0, 1]$ chooses c_{xt}, l_{xt}, and a_{xt+1} to maximize the utility in (8), which also determines her beliefs about the support for the next period realizations of the inflation target, $\Omega_x \subseteq \mathcal{S}_t$, and the associated probability distribution $G_x \in \mathcal{P}(\mathcal{S}_t)$;

2. The *monetary authority* sets the nominal interest rate R_t as in (6);

3. Each *firm* $i \in [0, 1]$ sets the price $p_{it} = p_t$ optimally, after the inflation target for the period has been determined (but before any policy announcement);

4. The labor market, the goods markets, and the financial market *all clear* at the wage w_t, the inflation Π_t, and the return r_t.

4 Model solution

We start assuming that the policy announcement at $t = 0$ is about the next-period inflation target Π_1^*, so that $T = 1$. We also assume that there are only two types of households who differ just in their initial financial wealth.[6] A fraction one half of households is a *creditor*, $j = c$, with wealth equal to $a_{x0} = a_{c0} = B > 0$ $\forall x \in [0, 1/2]$, while the remaining fraction is a *debtor*, $j = d$, with financial wealth $a_{x0} = a_{d0} = -B < 0$, $\forall x \in [1/2, 1]$. Here B denotes the amount of initial *financial imbalances* in the economy. We start proving two simple results that clarify the functioning of the model. Then we solve for the equilibrium by proceeding in three steps: we first determine the allocation of the economy for given households' degenerate beliefs about Π_1^*; we endogenize beliefs by using (8); and finally we fully characterize the equilibrium.

[5]There is empirical evidence suggesting that more educated individuals and those with greater financial literacy are characterized by smaller ambiguity when investing in financial markets and dealing with financial institutions, see Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016). Here we do not allow for exogenous differences in ambiguity to better isolate the effects of wealth inequality on the formation of households' expectations, which endogenously generate heterogeneity in beliefs.

[6]Both assumptions are relaxed in the quantitative model of Section 5. To keep the notation consistent throughout the paper, we have described the economy for general T and for an arbitrary distribution of households' assets a_{xt}. In this simple model the assumption $T = 1$ is with minor loss of generality, because firms adjust prices in every period so output can respond just at $t = 0$. The time horizon of the announcement will matter in the quantitative model because prices are slowly adjusted.

4.1 Two preliminary results

Figure 3 describes the time line of the experiment. At the time of the announcement, $t = 0$, prices are predetermined at a value normalized to one, $p_0 = 1$. The focus of the analysis is in characterizing output at time zero, Y_0 which, given sticky prices, is determined by the saving decisions of creditors, a_{c1}, and debtors, a_{d1}. Clearing of the financial market implies that $a_{c1} = -a_{d1} = B'$, where B' denotes the amount of financial imbalances at the end of period zero. In the following periods, $t \geq 1$, firm $i \in [0, 1]$ sets its price p_{it} to maximize expected profits at the beginning of period, $d_{it} \equiv y_{it}\,(p_{it}/p_t - w_t)$, after taking as given the demand for the variety of the competitive firm, which has the conventional form:

$$y_{it} = Y_t \left(\frac{p_{it}}{p_t}\right)^{-\theta}.$$

The resulting optimal nominal price is a markup over firm-i expected nominal wage:

$$p_{it} = \frac{\theta}{\theta - 1}\, E_{it}[w_t\, p_t] \quad \forall i \in [0, 1], \tag{9}$$

which immediately implies $p_{it} = p_t\ \forall i$. Also, since firms set their price after observing Π_t^*, firms choices are taken under perfect information $\forall t \geq 1$, which allow us to conclude that

$$w_t = \frac{\theta - 1}{\theta}, \quad \forall t \geq 1. \tag{10}$$

The utility in (4) together with the preferences in (8) also imply that the labor supply of a household of type $j = c, d$ solves a simple static maximization problem, which yields the familiar condition

$$\psi_0\, l_{jt}^{\psi} = w_t. \tag{11}$$

This implies that all households supply the same labor (independently of their wealth and their beliefs): $l_{jt} = Y_t$, $\forall j$, which uses the fact that, in equilibrium, labor supply equals output. The latter, together with (10) and (11), immediately implies that:

Lemma 1 *Output Y_t converges back to steady state at $t = 1$, so that $Y_t = \bar{Y}$ for all $t \geq 1$.*

Moreover Lemma 1 together with the interest rate rule in (6) allows us to prove that

Lemma 2 *At any point in time $t \geq 0$, the inflation is equal to the inflation target, $\Pi_t = \Pi_t^*$, and the nominal interest remains unchanged at its steady state value, $R_t = \bar{R}$.*

Proof of Lemma 2. $R_0 = \bar{R}$ because the economy is initially in a steady state. Given the timing of the monetary announcement, prices do not respond at $t = 0$ so $\Pi_0 = \Pi_0^* = 1$, which after using (6) yields $R_1 = \bar{R}$. Lemma 1 implies that the economy is back to steady state starting from $t = 1$ so it must be that $r_t = \bar{R}\ \forall t \geq 2$ (which obviously follows from the Euler equation of consumption below). By assumption we also have $\Pi_t = \Pi_t^* = 1$, $\forall t \geq 2$ so we have $R_t = r_t = \bar{R}\ \forall t \geq 2$, which immediately allows us to conclude that $R_t = \bar{R}\ \forall t$. But this together with (6) also implies that $\Pi_t = \Pi_t^*\ \forall t \geq 0$. ■

In Section 5 we consider an extension of the model where changes in the inflation target also affect nominal interest rates.[7]

Figure 3: Timing

4.2 Output and the financial market for given beliefs

We now assume that a household $j = c, d$ has given degenerate beliefs about the realization of Π_1^*, which are represented by a point $\varepsilon^{\tau_j} \in \mathcal{S}_0$ with $\tau_j \in [0, 1]$. In practice it is useful to define $\bar{\tau} \equiv (\tau_c + \tau_d)/2$ and $\rho \equiv (\tau_c - \tau_d)/(2\bar{\tau}) \in [-1, 1]$, which are related to τ_c and τ_d as follows: $\tau_c \equiv \bar{\tau}\,(1 + \rho)$ and $\tau_d \equiv \bar{\tau}\,(1 - \rho)$. $\bar{\tau}$ measures the average credibility of the announcement among all households in the economy; while ρ measures the correlation between the financial asset position of households and their perceived credibility of the announcement. When $\rho > 0$, creditors believe the announcement more than debtors do, while the opposite is true when $\rho < 0$; $\rho = 0$ means that all households share the same beliefs. The problem of a household of type $j = c, d$ at $t = 0$ is then given by

$$\max_{\{c_{js}, l_{js}, a_{js+1}\}_{s \geq 0}} \quad E_{j0}\left[\sum_{s=0}^{\infty} \beta^s\, U(c_{js}, l_{js})\right] \ ,$$

subject to the budget constraint in (5). The expectation operator is indexed by j, since households of different types (might) have different beliefs. The first order condition for

[7]But the result that nominal interest rates do not move mimics a situation where nominal rates cannot move (say because they have hit the zero lower bound) and the monetary authority tries to stimulate the economy in the short run through promises of higher future inflation, by announcing that current nominal rates will remain low for an extended period of time.

the consumption choices of household j yields the Euler condition

$$\left(c_{jt} - \psi_0 \frac{l_{jt}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta\, E_{jt}\left[r_{t+1}\left(c_{jt+1} - \psi_0 \frac{l_{jt+1}^{1+\psi}}{1+\psi}\right)^{-\sigma}\right], \tag{12}$$

which together with the budget constraint in (5) and the labor supply condition (11), fully determine household-j's consumption, c_{jt}, savings, a_{jt+1}, and labor supply, l_{jt}, $\forall t$.

Output at $t = 0$, Y_0, can be obtained by using the market clearing condition for final consumption

$$Y_0 = \frac{c_{c0} + c_{d0}}{2},$$

where c_{j0} and c_{j1} $\forall j \in \{c, d\}$ satisfy the equilibrium budget constraint of type j household at $t = 0$ and $t = 1$, which after using Lemma 1 yields

$$c_{j0} = Y_0 + \bar{R}\, a_{j0} - a_{j1} \quad \text{and} \quad c_{j1} = \bar{Y} + (\bar{R} - 1)\varepsilon^{-\tau_j}\, a_{j1}. \tag{13}$$

Notice that, under degenerate beliefs ad given Lemma 2, $\varepsilon^{-\tau_j}\, a_{j1}$ can be interpreted as the expected real wealth of household j after the inflation target in period one has been realized, which will yield real return $\bar{R}$ in all remaining periods. We now use equation (13) to substitute for c_{j0} and c_{j1} into the Euler equation (12) evaluated at $t = 0$ for both $j = c$ and $j = d$. In the resulting expression, we use the conditions for market clearing in the financial market $a_{c0} = -a_{d0} = B$ and $a_{c1} = -a_{d1} = B'$. After using the fact that, under degenerate beliefs, we have that $E_{j0}[r_1] = \bar{R}\,\varepsilon^{-\tau_j}$ and after remembering that $l_{jt} = Y_t$, we finally obtain the following two conditions:

$$\frac{\bar{N} + (\bar{R} - 1)\varepsilon^{-\bar{\tau}\,(1+\rho)}\, B'}{N_0 + \bar{R}B - B'} = \varepsilon^{-\bar{\tau}\,\frac{1+\rho}{\sigma}}\ , \tag{DA}$$

$$\frac{\bar{N} - (\bar{R} - 1)\varepsilon^{-\bar{\tau}\,(1-\rho)}\, B'}{N_0 - \bar{R}B + B'} = \varepsilon^{-\bar{\tau}\,\frac{1-\rho}{\sigma}}\ , \tag{SA}$$

where $\bar{N} \equiv N(\bar{Y})$ and $N_0 \equiv N(Y_0)$, with

$$N(Y) \equiv Y - \psi_0 \frac{Y^{1+\psi}}{1+\psi}$$

representing output net of the effort cost of working, which in equilibrium, is just a monotonically increasing transformation of output Y.[8] Equation (DA) can be interpreted as a

[8]Notice that $N'(Y) > 0$ when $w < 1$, which is implied by (10).

demand for assets by creditors: by the consumption smoothing motive, the demand for assets of creditors is increasing in time-zero net output, N_0, because households want to save more when output temporarily increases. This corresponds to the positively sloped straight blue line in Figure 4. By the same logic, equation (SA) can be interpreted as characterizing the supply of assets by debtors: the supply of assets in the economy is decreasing in the level of output at $t = 0$ as debtors want to borrow less (save more) when time-zero output is higher. This corresponds to the negatively sloped straight red line in Figure 4. The financial market clears at the point where the two schedules cross, which is unique and corresponds to point A in the Figure. The associated value of time-zero net output N_0, is obtained by combining (DA) with (SA) to obtain

$$N_0 = N_0(\varepsilon, \bar{\tau}, \rho) \equiv \bar{N} \left(\omega \, \tilde{\varepsilon}^{\,\frac{1+\rho}{\sigma}} + (1-\omega) \, \tilde{\varepsilon}^{\,\frac{1-\rho}{\sigma}} \right) + B \, \kappa \, \left(\tilde{\varepsilon}^{\,(1+\rho) \left(\frac{1}{\sigma} - 1 \right)} - \tilde{\varepsilon}^{\,(1-\rho) \left(\frac{1}{\sigma} - 1 \right)} \right), \quad (14)$$

where $\tilde{\varepsilon} \equiv \varepsilon^{\,\bar{\tau}}$ measures the announcement rescaled by its average credibility while

$$\omega \equiv \frac{1 + (\bar{R} - 1) \, \tilde{\varepsilon}^{\,(1-\rho) \left(\frac{1}{\sigma} - 1 \right)}}{2 + \frac{\bar{R}-1}{2} \left(\tilde{\varepsilon}^{\,(1+\rho) \left(\frac{1}{\sigma} - 1 \right)} + \tilde{\varepsilon}^{\,(1-\rho) \left(\frac{1}{\sigma} - 1 \right)} \right)} \quad \in [0,1] \ ,$$

$$\kappa \equiv \frac{\bar{R} \, (\bar{R} - 1)}{2 + (\bar{R} - 1) \, \left[\tilde{\varepsilon}^{\,(1+\rho) \left(\frac{1}{\sigma} - 1 \right)} + \tilde{\varepsilon}^{\,(1-\rho) \left(\frac{1}{\sigma} - 1 \right)} \right]} > 0.$$

We can summarize this discussion by stating the following Lemma:

Lemma 3 *When the beliefs of households are given and degenerate, as characterized by* $\bar{\tau} \in [0,1]$ *and* $\rho \in [-1,1]$*, net output* N_0 *and financial imbalances* B' *are determined by the point where the demand for assets* (DA) *and the supply of assets* (SA) *cross. The intersection is unique and the resulting* N_0 *is given by the function* $N_0(\varepsilon, \bar{\tau}, \rho)$ *in* (14).

The first term in the right hand side of (14) is always positive and characterizes how aggregate consumption is affected by the substitution effect, implied by temporary changes in the expected real rate. The second term in the right hand side of (14) can be positive or negative and characterizes the effects on consumption of redistributing future expected wealth from one household type to the other. This term is zero, when $B = 0$ because there is no wealth redistribution. It also zero when $\rho = 0$, because in this case the wealth losses expected by the type of household who looses (creditors when $\varepsilon > 1$, debtors when $\varepsilon < 1$) are identical to the wealth gains expected by the other type. And zero-sum transfers of wealth from one household's type to the other have no effects on consumption because all households share the same marginal propensity to consume—due to both the utility function in (4) and the absence of financial constraints. So when $B = 0$ or when $\rho = 0$, net output is equal to $N_0 = \tilde{\varepsilon}^{\frac{1}{\sigma}} \bar{N}$, as in a standard New Keynesian model with a representative household where the announcement is just scaled down by its average credibility $\tilde{\varepsilon} = \varepsilon^{\bar{\tau}}$. The canonical New Keynesian benchmark model, where all households in the economy fully believe the announcement, arises when $\bar{\tau} = 1$ and $\rho = 0$. By substituting these values into (14), we immediately obtain that $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$, which can be substituted back into (DA) to solve for B' so as to obtain

$$B' = \frac{\bar{R}\, B}{1 + (\bar{R} - 1)\, \varepsilon^{\frac{1}{\sigma} - 1}},$$

which also implies that the new steady state imbalances that result once the announcement is implemented are equal to

$$\frac{B'}{\varepsilon} = \frac{\bar{R}}{\bar{R}\varepsilon^{\frac{1}{\sigma}} + \varepsilon - \varepsilon^{\frac{1}{\sigma}}} B.$$

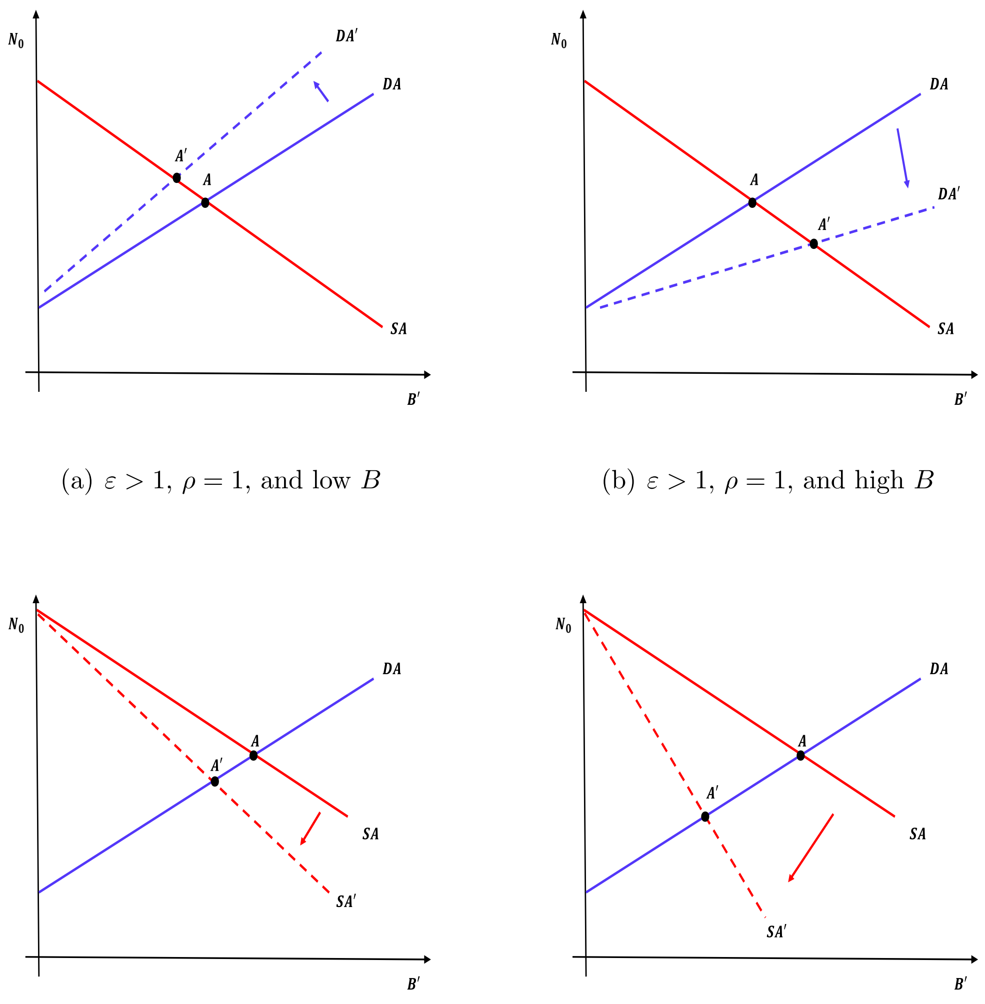
Under $\sigma > 1$ we immediately see that $B'/\varepsilon - B < 0$ if $\varepsilon > 1$, while $B'/\varepsilon - B > 0$ if $\varepsilon < 1$, which leads to the following Proposition:

Proposition 1 (The New Keynesian benchmark) *When all households in the economy fully believe the announcement, $\bar{\tau} = 1$ and $\rho = 0$, output Y_0 is a strictly increasing function of ε and $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$. The new steady state financial imbalances after the announcement is implemented, B'/ε, fall if the announcement is inflationary, $\varepsilon > 1$, while they increase if the announcement is deflationary, $\varepsilon < 1$.*

To clarify how (initial) financial imbalances, B, and the correlation between a household's wealth and its beliefs, ρ, affect time-zero output, we use the diagram representation in Figure (4). Start noticing that ε affects the demand of assets (DA) and the supply of assets (SA) both through a substitution and an income effect. Importantly, the substitution and the income effect affect the consumption decisions of debtors $j = d$ in the same direction, while they have opposite effects on the consumption choices of creditors $j = c$. To see this notice that the substitution effect is represented by the term in the right hand side of equation (DA), for households of type $j = c$, and of equation (SA), for households of type $j = d$: in response to an inflationary announcement $\varepsilon > 1$, these terms tend to stimulate consumption and thereby output by shifting upward both the (DA) and the (SA) schedule. But $\varepsilon > 1$ also redistributes expected future wealth from creditors to debtors, which explains why ε enters negatively in the numerator of the left hand side of (DA), while it enters positively in the numerator of the left hand side of (SA). This income effect has a greater negative impact on the consumption of creditors when the initial financial imbalances B, are larger because of the implied larger expected wealth losses. And importantly, when $\rho \neq 0$, these losses are no longer necessarily equal to the wealth gains expected by debtors. So redistributing future expected wealth i sno longer a zero-sum game.

To better see these points, let's assume that the announcement is inflationary, $\varepsilon > 1$, and

Figure 4: Clearing of the financial market for different values of ε, ρ, and B



(a) $\varepsilon > 1$, $\rho = 1$, and low B

(b) $\varepsilon > 1$, $\rho = 1$, and high B

(c) $\varepsilon < 1$, $\rho = -1$, and low B

(d) $\varepsilon < 1$, $\rho = -1$, and high B

that only creditors believe it, $\rho = 1$. When so the supply of assets (SA) remains unchanged. The demand for assets (DA) can shift up or down: if B is small, the substitution effect prevails and (DA) shifts up, as in panel (a) of Figure 4;if B is large, the income effect prevails and (DA) shifts down as in panel (b). Case (b) arises because the lower return on assets perceived by creditors implies that creditors feel poorer, so creditors consume less, which leads to a contraction in today aggregate income. This fall in income, due to consumption smoothing, make creditor saves more which increase their demand for assets and allows the financial market to clear even if the real rate on assets expected by debtors has remained unchanged.

Things are different when focusing on debtors, because for them the income and substitution effect work in the same direction. Panel (c) and (d) of Figure 4 illustrate this point by focusing on a deflationary announcement $\varepsilon < 1$ that is believed just by debtors, $rho = -1$. In this case, (DA) remains unchanged, while the supply of assets SA always move down, the more so the larger is B. And as debtors borrow less, their consumption and thereby aggregate output fall which allow to restore equilibrium in the financial market by reducing the demand for assets of creditors, at their given expected return on assets. The next proposition summarizes this discussion:

Proposition 2 (Output with heterogeneous beliefs) *For given households beliefs, as characterized by $\bar{\tau} \in [0, 1]$ and $\rho \in [-1, 1]$, net output N_0 is given by the function $N_0(\varepsilon, \bar{\tau}, \rho)$ in* (14)*, which implies that*

1. *If $\rho > 0$, greater initial imbalances, B, reduce the response of time-zero output Y_0 to inflationary announcement, $\varepsilon > 1$. And if they are large enough, Y_0 contracts;*

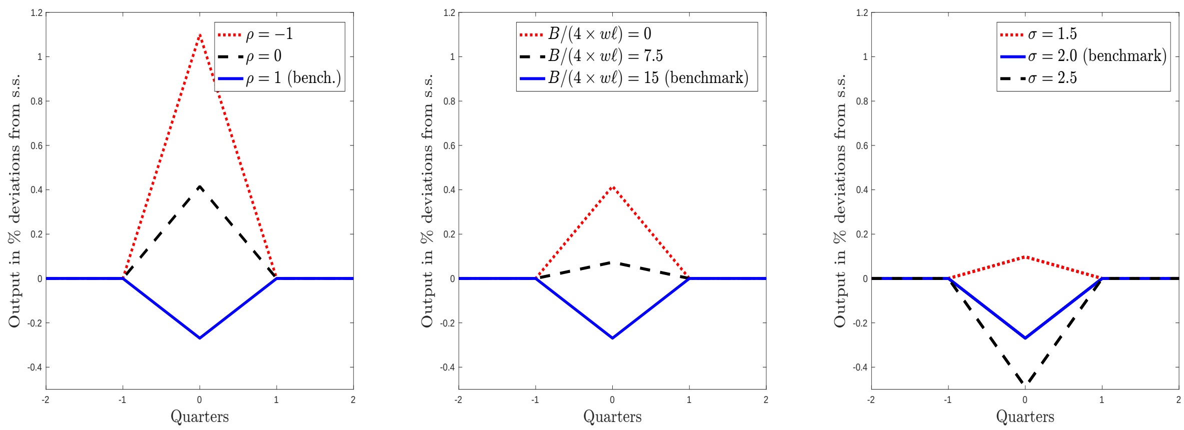
2. *When $\rho < 0$, Y_0 always falls in response to a deflationary announcement, $\varepsilon < 1$. And more so when initial imbalances, B, are larger.*

Numerical illustration In Figure 5 we plot the response of output to a 1% inflationary announcement for different parameter values. The blue solid line in each plot uses the parameter values of Table 3 when $\rho = 1$ and $\bar{\tau} = 1/2$—which will turn out to be the equilibrium beliefs. B is set equal to the standard deviation of the ratio of wealth to yearly labor income in the Euro Area. We discuss the effects of changing ρ in panel (a); of changing B, in panel (b); and of changing σ in panel (c). The lower the correlation between the wealth of households and their perceived credibility of the inflationary announcement, the larger is the output response: it increases from -0.3% of steady state under $\rho = 1$ to a 110% increase of steady state when $\rho = -1$. Panel (b) shows how smaller imbalances, leads to larger increases in output: when $B = 0$ output increase by 0.4% of steady state output as compared to the the fall of 0.3% under the baseline specification. Panel (c) shows how a higher Elasticity of Intertemporal Substitution (smaller σ) leads to a a larger increase in output.

4.3 Endogenous beliefs

We now characterize how households form their beliefs about the likelihood and magnitude of the future change in the inflation target Π_1^*. Since by Lemma 1 the economy is back to

Figure 5: Effects of a 1% inflationary announcement on time-zero output Y_0



(a) The role of beliefs, ρ (b) The role of wealth, B (c) The role of EIS, σ

Notes: The blue solid line is the impulse response to an announcement of $\varepsilon = 1.01$ when $\rho = 1$, and $\bar{\tau} = 1/2$ while the other paramaters are equal to $B/(4 \times w\ell) = 15$, $\bar{R} = 1.015$, $\sigma = 2$, $\theta = 3$, $\psi = 1/2$, and $\psi_0 = 2/3$, which are the same as in Table 3. The dashed and dotted lines are the analogous impulse responses after changing one parameter at a time relative to the baseline.

steady state at $t = 1$, we can express the problem of a household at time $t = 0$ with initial asset position a as equal to

$$\max_{a',c,l} \left\{ U(c,l) + \beta \min_{\Omega \subseteq \mathcal{S}_0,\ G \in \mathcal{P}(\mathcal{S}_0)} \left[\int_{\Omega} \bar{V}\left(\frac{a'}{\Pi_1^*}\right) G(d\Pi_1^*) \right] \right\} \tag{15}$$

$$\text{s.t.} \quad c + a' \leq w_0\, l + \bar{R}\, a + D_0 \ , \tag{16}$$

where $\mathcal{S}_0 = [\min(1, \varepsilon); \max(1, \varepsilon)]$, and the continuation utility is given by the function

$$\bar{V}\,(s) = \frac{\left[\bar{N} + (\bar{R} - 1)s\right]^{1-\sigma}}{(1-\sigma)(1-\beta)}, \tag{17}$$

which is obviously increasing in the wealth that the household has at the beginning of period one. When the announcement is inflationary $\varepsilon > 1$, we have that $\mathcal{S}_0 = [1; \varepsilon]$. In this case higher $\Pi_1^* \in \mathcal{S}_0$ yields lower utility when $a' > 0$, while it increases utility when $a' < 0$. If $a' = 0$, households' utility is unaffected by the value of $\Pi_1^* \in \mathcal{S}_0$. These consideration immediately imply that the equilibrium beliefs of households depend on the financial imbalances chosen by the household at the end of period zero (B'), and on whether the announcement is inflationary $\varepsilon > 1$ or deflationary $\varepsilon < 1$. In brief we have that:

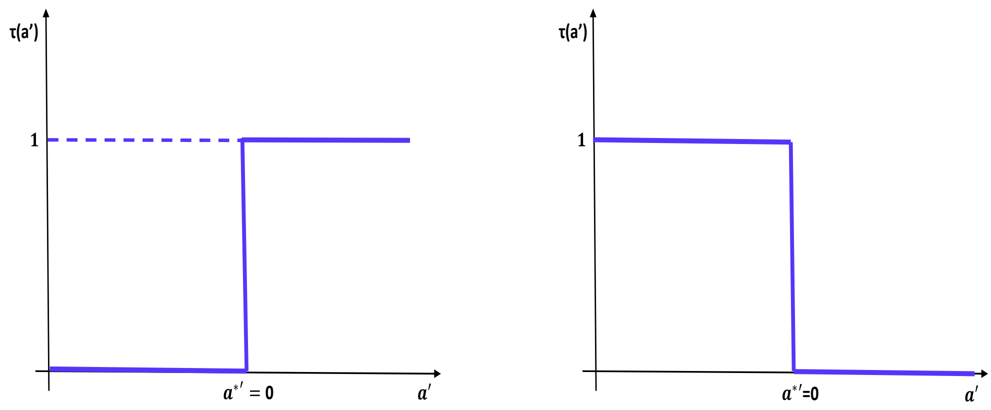
Proposition 3 (Individual beliefs) *A household's beliefs depend of the announcement, ε, and of the household's end-of period savings, a'. When $a' = 0$ beliefs are undeterminate. If $a' \neq 0$ they are always degenerate and equal to $\varepsilon^{\tau(a',\varepsilon)}$ where*

$$\tau(a', \varepsilon) = \mathbb{I}(\varepsilon > 1) \times \mathbb{I}(a' > 0) + \mathbb{I}(\varepsilon < 1) \times \mathbb{I}(a' < 0), \tag{18}$$

where $\mathbb{I}$ denotes the indicator function.

If households have no savings at the end period $t = 0$, $a' = 0$, the beliefs of the household are undeterminate because the household is indifferent about any future choices of the monetary authority. Figure 6 characterizes the beliefs of households as a function of the end-of-period financial imbalances chosen by the household, $a' > 0$, and of the nature of the announcement ε. The function $\tau(a', \varepsilon)$ measures the percentage amount of the announcement ε that the household thinks the monetary authority will implement in period one. If the announcement is inflationary, $\varepsilon > 1$, $\tau(a', \varepsilon) = 1$ if $a' > 0$ and zero otherwise, which corresponds to panel (a) of Figure 6. If the announcement is deflationary, $\varepsilon < 1$, $\tau(a', \varepsilon) = 1$ if $a' < 0$ while it is zero otherwise, which corresponds to panel (b).

Figure 6: Endogenous determination of beliefs for $\varepsilon < 1$ and $\varepsilon > 1$



(a) Case $\varepsilon > 1$

(b) Case $\varepsilon < 1$

4.4 Equilibrium

We now characterize the equilibrium beliefs, time-zero output and the new steady state financial imbalances that emerges at $t \geq 1$ when the announcement is actually implemented. The next proposition characterizes the mapping from financial imbalances at the beginning of $t = 0$, B, with imbalances at the end of the period, B'.

Proposition 4 (No reversal in households' NFA) *In equilibrium, creditors and debtors never switch their net financial asset position, i.e. if $B > 0$, then $B' \geq 0$.*

Proof of Proposition 4. Assume that $\varepsilon > 1$ (similar logic applies when $\varepsilon < 1$) and argue by contradiction. Suppose that $B > 0$ leads to $B' < 0$. Then, (18) implies that households of type $j = c$ do not believe the announcement and think that the expected real rate is unchanged at its steady state level $\bar{R} = 1/\beta$. Then households of type $j = c$ find optimal to dissave (reduce wealth from B to $B' < 0$) only if we have $Y_0 < \bar{Y}$. But in an equilibrium with $B' < 0$, it also follows from (18) that households of type $j = d$ do believe the announcement and thereby expect a low return. Then households of type $j = d$ would find optimal to save (move from a debtor position of B to a creditor position of $-B' > 0$) only if $Y_0 > \bar{Y}$, which immediately leads to a contradiction. ■

By combining Proposition 3 and 4 we immediately obtain a characterization of the aggregate equilibrium beliefs in trems of $\bar{\tau}$ and ρ:

Proposition 5 (Aggregate beliefs) *In a credit crunch equilibrium, $B' = 0$, households' beliefs are undeterminate. Otherwise, $B' > 0$, only one household's type believes the announcement, $\bar{\tau} = 1/2$: if the announcement is inflationary, $\varepsilon > 1$, creditors believe it, $\rho = 1$; if it is deflationary, $\varepsilon < 1$, debtors believe it, $\rho = -1$. So we generally have*

$$\rho = \rho(\varepsilon) \equiv 1 - 2\mathbb{I}(\varepsilon < 1), \tag{19}$$

where $\mathbb{I}$ denotes the indicator function.

If there is a credit crunch, $B' = 0$, the beliefs of all households in the economy are undeterminate because all households have reached their "peace of mind" and are indifferent about future choices of the monetary authority. If $B' > 0$, instead, equilibrium beliefs are determinate and degenerate.

We are now able to fully characterize equilibrium output. We calculate the intercept on the y-axis of the supply of asset by debtors, $j = d$, and of the demand for assets by creditors, $j = c$, both evaluated at the equilibrium beliefs of Proposition 5, so $\bar{\tau} = 1/2$ and

$\rho = \rho(\varepsilon)$ as given in (19). The intercept of (SA) is given by

$$N_0^A = \min\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} + \bar{R}B, \tag{20}$$

while the intercept of (DA) is given by

$$N_0^B = \max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}B. \tag{21}$$

Clearly $N_0^B < N_0^A$ is equivalent to

$$B > \frac{|\varepsilon^{\frac{1}{\sigma}} - 1|\bar{N}}{2\bar{R}}, \tag{22}$$

which is a necessary and sufficient condition to guarantee that (SA) and (DA) cross at $B' > 0$. If this happens we have an equilibrium. For example, the condition (22) is satisfied both in panel (a) or (b) of Figure 4, which deals with an inflationary announcement $\varepsilon > 1$, and in panels (c) or (d), which deals with a deflationary announcement $\varepsilon < 1$. But when (22) fails, we have that $N_0^A < N_0^B$, which implies that (SA) and (DA) would cross at $B' < 0$, as in panel (b) of Figure 7. Given Proposition 4, in this case we have a credit crunch equilibrium, $B' = 0$, where both households types $j = c, d$ stop trading in financial markets. This equilibrium is sustained because, the households' beliefs in (18) imply that the real rates expected by households exhibit a discontinuity at $a' = 0$, as shown in panel (a) of 7 for the case $\varepsilon > 1$. In a credit crunch equilibrium, the Euler equation of households will hold as an inequality. For each household type $j = c, d$, it has to be that: (i) the cost of investing in financial markets today (in terms of lower utility from consumption) is greater than tomorrow gains (again in terms of consumption utility) from financial returns; (ii) the today gain from borrowing are smaller than the tomorrow cost from debt repayments. After writing down these conditions we obtain that:

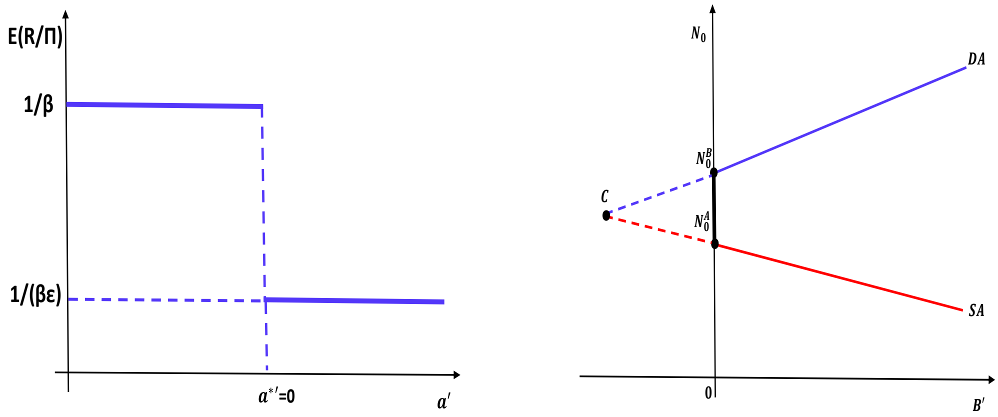
Lemma 4 *If (22) fails, $N_0^A < N_0^B$ and the equilibrium features a credit crunch $B' = 0$.*

Proof of Lemma 4. In a credit crunch equilibrium, a household of type $j = d$ should be such that

$$\frac{1}{\max(1, \varepsilon^{\frac{1}{\sigma}})\bar{N}} < \frac{1}{N_0 - \bar{R}B} < \frac{1}{\min(1, \varepsilon^{\frac{1}{\sigma}})\bar{N}} \tag{23}$$

The first inequality implies that a household of type $j = d$ does not want to lend—which involves lower consumption today in exchange of higher consumption tomorrow—-, while the second inequality implies that household $j = d$ does not want to borrow—which involves higher consumption today in exchange of lower consumption tomorrow. The analogous

Figure 7: Credit crunches with $\varepsilon > 1$: the Zen effect



(a) Discontinuity in expected real rates (b) Credit crunch equilibrium

inequalities for a household of type $j = c$ read as follows

$$\frac{1}{\max(1, \varepsilon^{\frac{1}{\sigma}})\bar{N}} < \frac{1}{N_0 + \bar{R}B} < \frac{1}{\min(1, \varepsilon^{\frac{1}{\sigma}})\bar{N}} \tag{24}$$

After some algebra, one can check that these inequalities are equivalent to the condition $N_0^A < N_0^B$, or alternatively that the inequality in (22) fails. ■

Lemma 4 together with the previous considerations imply that

Proposition 6 (Equilibrium output) *An equilibrium always exists. If* (22) *holds, then* $B' > 0$ *and net output,* N_0*, is given by* (14) *evaluated at the equilibrium beliefs of Proposition 5, so that* $N_0 = N_0(\varepsilon, 1/2, \rho(\varepsilon))$*. If* (22) *fails, households' beliefs are indeterminate and the equilibrium features a credit crunch,* $B' = 0$*, where net output,* N_0*, can be any value in the range* $[N_0^A, N_0^B]$*, where* N_0^A *and* N_0^B *are given by* (20) *and* (21)*, respectively.*

Notice that, in a credit crunch equilibrium caused by an inflationary announcement $\varepsilon > 1$, output never falls because it is always true that $N_0^A > \bar{N}$. However net output is always smaller than the output level that would arise in the conventional New Keynesian model characterized in Proposition 1, because we always have that $N_0^B < \varepsilon^{\frac{1}{\sigma}}\bar{N}$.

After combining Proposition 6 with Propositions 1 and 2 we immediately obtain the following corollary:

Corollary 1 *Under ambiguity, after an inflationary announcement* $\varepsilon > 1$*, output* Y_0 *in-*

creases less than in the New Keynesian benchmark and it can even fall if B is large enough. In response to a deflationary announcement, $\varepsilon < 1$, Y_0 always falls and more so than in the New Keynesian benchmark.

Finally we can compare the amount of rebalancing that emerge in our model whence the announcement is implemented with the corresponding amount of rebalancing that would emerge in the canonical New Keynesian model where the announcement is fully believed by all households in the economy:

Proposition 7 (Rebalancing under ambiguity) *After an inflationary announcement $\varepsilon > 1$, the new steady state financial imbalances once the announcement is implemented, B'/ε, always fall ($B'/\varepsilon < B$), and more so than in the New-Keynesian benchmark. After a deflationary announcement $\varepsilon < 1$, B'/ε also falls if*

$$B < \frac{(1-\varepsilon^{\frac{1}{\sigma}})\bar{N}}{2\bar{R} - (\bar{R}-1)(\varepsilon^{\frac{1}{\sigma}} - \varepsilon)}, \tag{25}$$

while it always increases in the New-Keynesian benchmark (with $\varepsilon < 1$). Generally, after any announcement, B'/ε is smaller than the corresponding value in the New-Keynesian benchmark.

Announcements always generate more rebalancing than in the New Keynesian benchmark In responses to an inflationary announcement $\varepsilon > 1$, debtors have strong incentives to deleverage while creditors have little incentive to save because the real rate expected by creditors is smaller than the real rate expected by debtors, see panel (a) in Figure 7. This leads to a rebalancing in the financial asset positions of agents and can even cause credit crunches, which happen because agents stop trading in financial markets so as to get fully insured against future changes in monetary policy. As a result financial imbalances naturally fall. Agents naturally reach a level of financial assets that makes them indifferent about the future choices of monetary policy. This is different from precautionary savings. Agents naturally tend to obtain a financial position where they reach the peace of mind, in that their future welfare becomes independent of future monetary policy. This is what we call the *Zen effect*. Due to this effect, credit crunches might arise where $B' = 0$ even if $B = 0$.

5 A quantitative analysis of the forward Guidance announcement by the ECB

In this section we use our framework to study the implications of the Forward Guidance announcement by the European Central Bank in July 2013 for the Euro area economy.

The Euro area is a particularly interesting case to study both because, as we document later, there was substantial heterogeneity in household wealth at the time of the Forward Guidance announcement, and because of the evidence we provided in Section 2 about the relationship between household financial wealth and inflation expectation responses to the announcement. We model the Euro area as a closed economy populated by households that differ in net wealth as in Section 3. For simplicity, our analysis abstracts from other sources of household heterogeneity, such as heterogeneity in production and labor income, and focuses only on heterogeneity arising from capital income. In order to match key features of the Euro area economy at the time of the Forward Guidance announcement by the ECB, we extend the model of Section 3 to allow for persistent responses of output and inflation to the monetary policy announcement, and for a non zero net supply of financial assets. We next discuss these extensions in detail.

Sticky prices We assume that firms can adjust prices after the monetary announcement at $t = 0$ but face price adjustment costs as in Rotemberg (1982). In particular, we assume that each intermediate producer chooses its price in each period to maximize profits subject to price adjustment costs expressed in units of firm output. These adjustment costs are quadratic in the rate of price change and scaled by aggregate output, Y_t:

$$\Theta_t\left(\pi_{it}\right) = \frac{\kappa}{2}\left(\pi_{it}\right)^2 Y_t \ , \tag{26}$$

where $\pi_{it} = (p_{it} - p_{it-1})/p_{it-1}$ is the firm level inflation rate and $\kappa > 0$.

Monetary policy experiment As in Section 4, we assume the economy is in steady state at $t = 0$, when an announcement is made that the inflation target will stay at its steady state value, $\Pi^*_t = 1$, for all $t < T$, will be equal to $\Pi^*_T = \varepsilon$ at $t = T$. Differently from before, we allow here for persistence in the deviation of the inflation target from steady state. In particular, we assume that the inflation target converges back to to its steady state value according to an AR(1) process, $\log(\Pi^*_t) = \rho^* \ \log(\Pi^*_{t-1})$, for $t > T$. For simplicity, as in in Section 4, we assume that household face ambiguity with respect to the implementation of the policy, but not with respect to its horizon, T, or its persistence, ρ^*. We consider two cases for the path of the nominal interest rate in the interim periods $t \in [0, T)$. In our baseline case, we assume that the nominal interest rate will stay constant at its steady state value, $\bar{R}$, for all $t \in [0, T)$, and then be given by equation (6) afterwards. This assumption mimics the inaction of the policy rate around the forward guidance announcement by the ECB. In the other case we will instead assume that the nominal interest rate will be given by equation (6) for all $t \geq 0$.

Financial market We allow for the supply of financial assets in the economy to be positive through a competitive mutual fund that owns the firms and is a net debtor with respect to the households, to which it supplies bonds. The flow budget constraint of the mutual fund is such that aggregate net payments, $r_t \int_0^1 a_{xt}\, dx$, are balanced by dividends D_t and new net issuance of bonds, $\int_0^1 a_{xt+1}\, dx$, in each period:

$$r_t \int_0^1 a_{xt}\, dx + \Upsilon = D_t + \int_0^1 a_{xt+1}\, dx \ . \tag{27}$$

We assume that a value Υ of the overall inflows to the mutual fund is not paid to households, and interpret it as a payment to hypothetical foreign agents for debt held abroad. We will use Υ to calibrate the net supply of financial assets in the economy.

5.1 Characterization of the equilibrium

In this section we derive the conditions that determine the equilibrium and discuss its key properties. Each firm i sets the price p_{it} in period t to solve the following problem

$$\max_{\{p_{is}\}_{s=t}^{\infty}} E_{it}\left[\sum_{s=t}^{\infty} q_{t,s}\left(\left(\frac{p_{is}}{p_s} - w_s\right) Y_t \left(\frac{p_{is}}{p_s}\right)^{-\theta} - \Theta_s\left(\pi_{is}\right)\right)\right] \ , \tag{28}$$

where $q_{t,t+j}$ is the real discount factor between periods t and $t+j$. The expectation operator is indexed by i to denote the beliefs of firm i about the path of future output, inflation and real rate. In our baseline specification, we assume that firms fully trust the policy announcement of the monetary authority and forms beliefs with rational expectations. We will later consider the alternative assumption where firms don't believe the announcement and show that our quantitative results are even stronger. The solution to the firm problem implies symmetric pricing, $p_{it} = p_t$ and $\pi_{it} = \pi_t$ for all i, and an equation determining the dynamics of inflation,

$$1 - \kappa\left(\Pi_t - 1\right)\Pi_t + \kappa\, E_{it}\left[q_{t,t+1}\left(\Pi_{t+1} - 1\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = \theta\left(1 - w_t\right) \ . \tag{29}$$

The demand of each variety is equal to aggregate output, Y_t. Hence, the aggregate dividend of firms is given by

$$D_t = \left(1 - w_t - \frac{\kappa}{2}\pi_t^2\right) Y_t \ . \tag{30}$$

For given path of output Y_t, the nominal interest rate R_t, the real wage w_t and inflation Π_t are jointly determined by equations (6), (11) and (28), and the real rate is given by $r_t = R_t/\Pi_t$. Aggregate output is given by the condition for equilibrium in the goods market,

$$Y_t = \frac{C_t + \Upsilon}{1 - \frac{\kappa}{2}\pi_t^2} \ , \tag{31}$$

where $C_t = \int_0^1 c_{xt}\, dx$. Thus, to close the model, we are left to determine aggregate consumption C_t, which we obtain from the solution to the household problem which we next discuss. We distinguish between the household problem after the resolution of the ambiguity, i.e. for $t \geq T$, and before it, i.e. for $t < T$.

Aggregate demand at $t \geq T$ The problem of the household at any $t \geq T$ is to maximize the present discounted value of all future utility, $\sum_{s=0}^{\infty} \beta^s U(c_{xt+s}, l_{xt+s})$, subject to the intertemporal budget constraint, $\sum_{s=0}^{\infty} q_{t,t+s}\, (c_{xt+s} - w_{t+s}\, l_{xt+s}) \leq r_t\, a_{x,t}$, under perfect foresight. Given the perfect foresight, the optimal choice of $\{c_{xt}, l_{xt}, a_{xt+1}\}_{t \geq T}$ is given by the first order conditions in equations (11)-(12) together with the budget constraint.

Using these equations we next characterize the solution to the household problem for a given path of $\{w_s, \pi_s, r_s\}_{s \geq T}$. Consider the economy at $t \geq T$, after ambiguity is resolved. For a given sequence of real wages $\{w_s\}_{s \geq T}$, inflation $\{\pi_s\}_{s \geq T}$, and real interest rates $\{r_s\}_{s \geq T}$, the aggregate consumption is independent of the distribution of assets and is given by

$$C_t = \frac{w_t}{1+\psi} \left(\frac{w_t}{\psi_0}\right)^{\frac{1}{\psi_0}} + \alpha_t \sum_{j=0}^{\infty} q_{t,t+j} \left(\psi\, \frac{w_{t+j}}{1+\psi} \left(\frac{w_{t+j}}{\psi_0}\right)^{\frac{1}{\psi_0}} + D_{t+j} - \Upsilon\right) \ , \tag{32}$$

where $\alpha_t = \left[\sum_{j=0}^{\infty} q_{t,t+j} \left(\prod_{s=1}^{j} (\beta\, r_{t+s})^{\frac{1}{\sigma}}\right)\right]^{-1}$. The consumption of household x with wealth a_{xt} is given by

$$c_{xt} = C_t + \alpha_t\, [r_t\, a_{xt} - S_t] \ , \tag{33}$$

where $S_t \equiv D_t + r_{t+1}^{-1}\, S_{t+1}$ is the value of the market portfolio.

Equation (32) shows that aggregate consumption is independent of the distribution of assets a at $t \geq T$. This is because, differently from $t < T$, there is no relationship between the wealth and the beliefs, which are now homogeneous. Moreover, there is no wealth effect on labor supply, so that the distribution of financial assets does not affect labor supply. Thus, the aggregate economy behaves as a standard representative household new-Keynesian economy at $t \geq T$.

Equation (33) fully characterizes the optimal household behavior of each household. At $t \geq T$, ambiguity on the behavior of the monetary authority is resolved, the nominal interest rate is determined by equation (6) together with the realized path of Π_t^*, and all agents have perfect foresight and identical beliefs about the path of economic variables. The deviation of the consumption of a household from the average consumption in the economy depends from the difference between the value of her financial assets, $r_t\, a_{xt}$, and the value of the market portfolio, S_t.

The next proposition describes some useful properties of the equilibrium at $t \geq T$ when the nominal interest rate is not a state variable of the economy.

Proposition 8 *Assume $\rho_r = 0$, or alternatively $R_t = \bar{R}$ for all $t \leq T$. Then the equilibrium dynamics at $t < T$ do not affect the equilibrium dynamics of aggregate variables at $t \geq T$, e.g. $\{Y_t\}_{t\geq T}$ is independent of $\{Y_t\}_{t<T}$. Moreover, if $\Pi^*_T = 1$, the economy is in steady state at $t = T$.*

Proposition (8) characterizes the equilibrium of the economy at $t \geq T$ when there is no inertia in the interest rate rule ($\rho_r = 0$), or when the interest rate has been at its steady state value at $t < T$ (as we will assume later). In these cases, past economic dynamics do not affect the equilibrium of the economy at $t \geq T$ as the distribution of financial assets is the only state of the economy, which is however is irrelevant for aggregate dynamics as shown in Proposition 1. An implication of the corollary is that if the policy announcement is not implemented, $\Pi^*_T = 1$, then the economy is in steady state at any $t \geq T$.

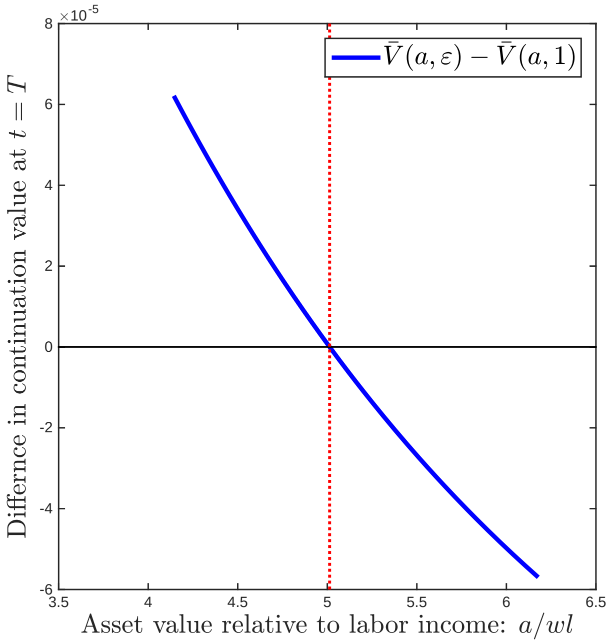
Aggregate demand at $t < T$ We now characterize aggregate demand at $t < T$, when households are ambiguous about the implementation of the policy announcement. Let's focus for simplicity on the case of an expansionary announcement, i.e. $\varepsilon > 1$. Given the preferences defined in equation (8) the problem of a household with wealth $a_{x,0}$ at the time of the policy announcement is

$$\begin{aligned} \max_{\{c_{xt},l_{xt}\}_{t=0}^{T-1}} \quad & \sum_{t=0}^{T-1} \beta^t\, U(c_{xt}, l_{xt}) \;+ \min_{\Omega\subseteq[1,\varepsilon],\; G\in\mathcal{P}(\Omega)} \beta^{T-1} \int_{\Omega} \bar{V}(a_{xT}, \Pi^*_T)\, G(d\Pi^*_T) \;, \qquad (34) \\ & \text{s.t.} \\ & \sum_{t=0}^{T-1} q_{0,t}\; (c_{xt} - w_t\, l_{xt}) + q_{0,T-1}\, a_{xT} \leq\; r_0\, a_{x,0} \;, \end{aligned}$$

where $\bar{V}(a_{xT}, \Pi^*_T) = \sum_{t=T}^{\infty} \beta^{t-T}\, U(c_{xt}, Y_t)$ is the continuation value at T that depends both on the value of assets accumulated and on the realization of the inflation target, determining labor supply equal to output Y_t, and household consumption as of equations (32)-(33).

We next discuss how households with different wealth at the time of the (eventual) implementation of the announcement are affected by an increase in inflation. To help our discussion, in Figure (8) we plot the difference in the continuation values $\bar{V}(a_{xT}, \Pi^*_T)$ between the case when the policy is implemented, $\Pi^*_T = \varepsilon > 1$, and is not, $\Pi^*_T = 1$, as a function of the asset position of the household at $t = T$, at our baseline calibration. At low

Figure 8: Determination of beliefs



or negative values of wealth at $t = T$, the implementation of the policy announcement is beneficial as it reduces the capital payments for debtors and increases the share of output to labor, which is inefficiently low due to monopolistic competition. The worst case scenario for these households is that the announcement is not implemented, so they will handle the ambiguity by using a degenerate probability that has all the mass at $\Pi_T^* = 1$. We call these households *believers*. We notice that, differently from the simple case of Section 4, here also households with positive wealth at $t = T$ may benefit from inflation because of the higher output at $t \geq T$. As wealth increases so that capital income becomes a more and more important share of overall income of the individual, an inflationary announcement is less and less good news, to the point that if a_{xT} is larger than a threshold a^* (depicted by the vertical dashed line in the plot) then higher inflation is bad for the continuation value as it reduces capital income more than it increases labor income. The worst case scenario for these households is that the announcement is implemented, so they will handle the ambiguity by using a degenerate probability that has all the mass at $\Pi_T^* = \varepsilon$. We call these households *unbelievers*. Households that end up with a level of net wealth $a_{xT} = a^*$ will be indifferent with respect to the implementation of the policy. We call these households *indifferent*.

Thus, aggregate consumption C_t at $t < T$ will be determined by aggregating over the demand of believers, unbelievers and indifferent households. As both believers and unbelievers have degenerate beliefs about the equilibrium at $t \geq T$, their demand at $t < T$ will be given by equations (32)-(33), where each type of household will have different (degenerate) beliefs about the path of w_t, r_t and π_t for $t \geq T$. The demand of indifferent households is instead given by the first order conditions in equation (12) together with the inter-temporal budget constraint under the constraint that $a_{xT} = a^*$.

Numerical solution of the model We conjecture two paths of output Y_t for $t \geq 0$, one for the case where the policy is implemented at $t = T$ and one for the case where the policy is not implemented at $t = T$. We notice that the two paths necessarily coincide at $t < T$. We also notice that when $R_t = \bar{R}$ for $t < T$, Proposition 8 applies and the equilibrium output in the case the policy is not implemented coincides with the steady state output. Given the path of output, we can solve for the implied path of real wages, inflation and interest rates from equations (6), (11) and (29). Given the path of prices $\{w_s, \pi_s, r_s\}_{s\geq 0}$, we can solve for the implied aggregate demand, C_t, using equation (32) for $t \geq T$ and the solution to the household problem in equation (34) together with $C_t = \int_0^1 c_{tx}\, dx$ for $t < T$. Finally, we obtain the implied value of output that satisfies the demand so obtained from equation (31). We update our conjecture about $\{Y_t\}$ until convergence.

5.2 Calibration

The calibration exercise can be described as consisting of three different parts. The first part deals with the choice of parameter values that govern household preferences and the firm price adjustment technology. The second part deals with the calibration of the distribution of households net financial assets. The third part deals with the calibration of the monetary policy experiment.

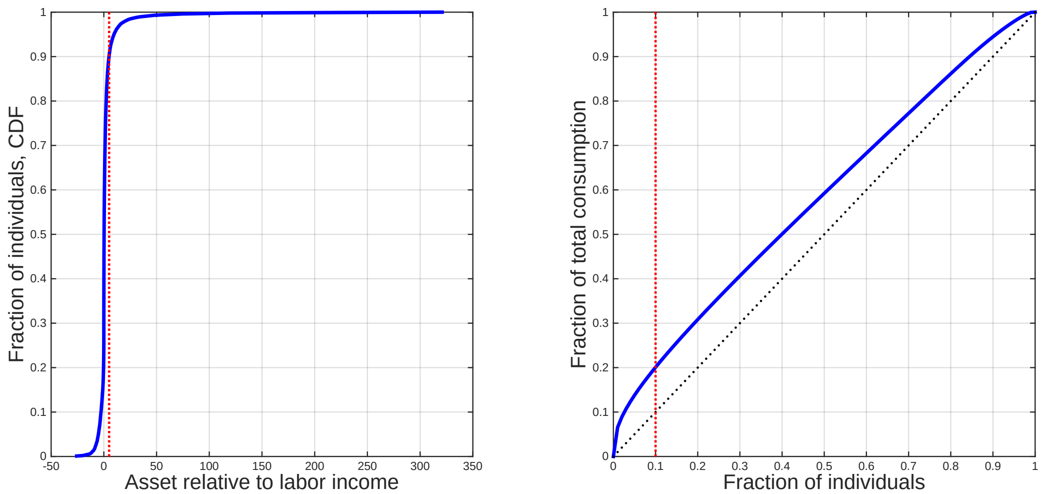
Table 3 describes the choice of the parameters. We set the time period to be a quarter and $\beta = 0.985$ so that the real interest rate is 6% in steady state (reference?).[9] We set $\sigma = 2$ so that the elasticity of interetempboral substitution (EIS) is 0.5, and $\psi = 0.5$ so that the Frisch elasticity of labor supply is 2. They are both in the range of values commonly used in the literature.[10] The elasticity of substitution across different varieties is set to $\theta = 3$ to match a steady state labor share of output equal to 2/3. This value of θ is at the lower end of the range of values typically used in macro models when estimating the price

[9]This choice of β implies that aggregate capital income is 12% as large as labor income in our model. It is a conservative choice from the perspective of matching the capital income share of output. For instance, Elsby, Hobijn, and Şahin (2013) report estimates according to which the profit share is 20% of the payroll share.

[10]See Guvenen (2006) for a review on the estimates of the EIS. See Keane and Rogerson (2012) for a discussion of the Frisch elasticity.

elasticity of demand. For instance Midrigan (2011) assumes an elasticity of 3, as we do, whereas Golosov and Lucas Jr (2007) an elasticity of 7. It is however an upper bound from the perspective of estimating the labor share which has been falling below 2/3 in the last decade.[11] The parameter governing the cost of price adjustment is set to $\kappa = 30$ so that the slope of the Phillips curve (the elasticity of inflation to current marginal cost) implied by our model is $\theta/\kappa = 0.1$. This value is in the middle of the range commonly used in the literature.[12] The value of transfers abroad is set to $\Upsilon = 0.25$ to match the ratio of mean net financial assets to yearly labor income equal to 2 in the Euro area. Below we describe the data used to compute such statistic. Finally, the parameter governing the disutility of labor ψ_0 is set, without loss of generality, to normalize labor supply in steady state to 1, whereas the tax rate on profits, δ, is assumed to be zero.

Figure 9: The distribution of net financial assets: excluding government debt



(a) CDF of households net financial assets

(b) Consumption Lorenz Curve

Notes: The left panel plots the CDF of Euro are households net financial assets scaled by Euro area average labor income; the right panel plots the fraction of aggregate demand that is accounted for by a given fraction of individuals, ordered from the richest to the poorest in our model calibrated as in Table 3.

Next, we describe the calibration of the cross-households distribution of financial assets, which in steady state we denote by $\bar{G}(a)$ with support on $[\underline{a}, \bar{a}]$, to represent the fraction of households having financial asset position smaller than a. We proceed as follows. We discretize the interval $[\underline{a}, \bar{a}]$ into 1,000 grid points, $B_1, B_2, \ldots B_{1000}$, such that these points

[11]See Karabarbounis and Neiman (2014).

[12]See Schorfheide (2008) for a review.

Table 3: Baseline Calibration

Model		Data	
Parameter	Value	Moment	Value
β	0.985	Yearly real return	0.06
σ	2	Elasticity of intertemporal substitution	0.5
ψ	0.5	Frisch elasticity of labor supply	2
ψ_0	0.66	Labor supply normalization	1
θ	3	Labor share	0.66
κ	30	Slope of the Phillips Curve	0.1
Υ	0.25	Mean net financial asset to yearly labor income	2
ϕ	1.5	Taylor rule response to inflation	1.5
ρ_r	0.8	Taylor rule inertia in nominal rate	0.8
T	6	Max fit to forward rates response at 2-10 years maturity	-
ρ^*	0.9	Max fit to forward rates response at 2-10 years maturity	-
ε	1.0016	Max fit to forward rates response at 2-10 years maturity	-

represent the permilles of the distribution, i.e. $\bar{G}(B_1) = \bar{G}(B_{i+1}) - \bar{G}(B_i) = 0.001$ for all $i < 1000$. We then identify the grid points B_i, for $i = 1, 2, ...1000$ from the distribution of net financial assets obtained from the Euro area Household Finance Consumption Survey (HFCS). The data refers to 2010 except for Austria where the data refers to 2012. The data cover all countries in the group of Euro 11 with the exclusion of Ireland, which is not included in HFCS. Net Financial assets are calculated as the the difference between total financial assets and total financial liabilities.[13] In our benchmark calibration we do not in-

[13]Financial assets include Deposits (da2101); Mutual Funds (da2102); Bonds (da2103); non self-employment private business (da2104); value of self-employment business (da1140); shares of publicly traded companies (ds2105); managed accounts (da2106); money owed to households (da2107); Other Assets (da2108); Voluntary pensions plus whole life insurance (da2109). Financial liabilities is calculated as the sum of outstanding balance of HMR mortgages (dl1110); outstanding balance of mortgages on other

clude government debt among the liabilities of the household. In an alternative calibration we add government debt to the liabilities of the household.[14] The left panel of Figure 9 plots the cumulated probability distribution of net financial assets, scaled by the average yearly labor income of the Euro area, in the case where government debt is excluded by household liabilities. The distribution displays substantial standard deviation and skewness, about 23 and 116 respectively, and is characterized by a mean approximately equal to 2. The distribution of net financial assets including government debt (not displayed) is characterized by similar standard deviation and skewness, but lower mean which drops to 0.35 of yearly labor income. The right panel of Figure 9 plots the Lorenz curve for the distribution of consumption predicted by our model at the baseline calibration. It gives on the vertical axis the fraction of total consumption accounted by the top $x\%$ of households on the horizontal axis ordered by their financial wealth. As a result, the Lorenz curve is always above the 45 degree line. The vertical dashed red line denotes the fraction of households that believe the inflationary announcement. These correspond to the top 10% of the distribution of financial wealth and account for 20% of aggregate demand in our model.

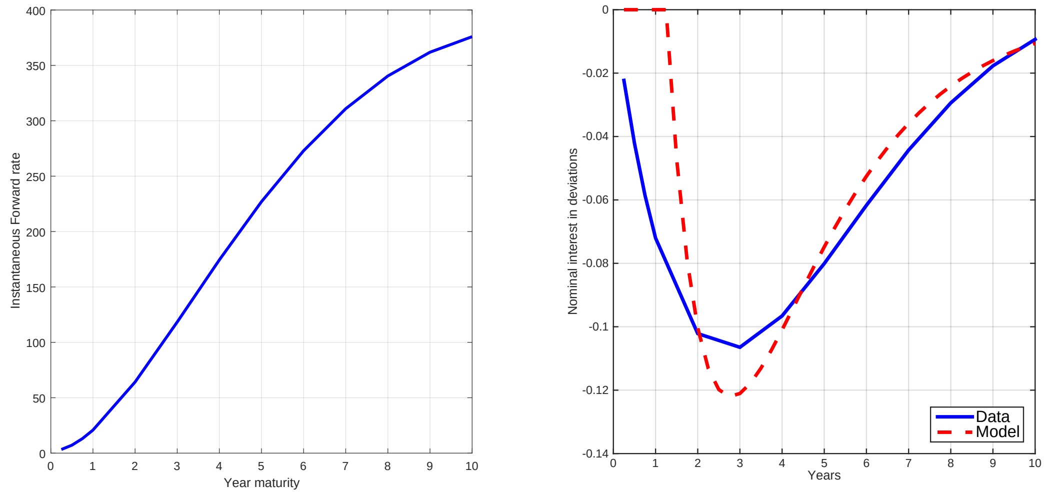
Finally, we calibrate the monetary policy parameters. We set the parameters of the interest rate rule in equation (6) to standard values, i.e. $\phi = 1.5$ and $\rho_r = 0.8$. We choose the horizon of the announcement T, its intensity ε, together with the persistence of the inflation target ρ^*, to maximize the fit of the path of the nominal interest rate predicted by our model with the path of nominal interest rates in the Euro area predicted at the time of the announcement by Mario Draghi that started the Forwad Guidance Policy by the ECB on July 4^{th} 2013. In particular, we choose these parameters to match the difference in the path of instantaneous forward rates associated to the yield curves of Triple A government bonds in the Euro area between July 4^{th} and July 2^{nd} for maturities up to 10 years . We plot such difference in the right panel of Figure 10 for maturities up to 10 years. In the left panel of Figure 10 we plot instead the forward rates on July 2^{nd}.[15] We notice that forward guidance announcement has had a substantial impact on the path of interest rates.

properties (dl1120); and outstanding balance of other non mortgage debt (dl1200). Statistics are weighted using weights provided by HFCS (hw0010) and then aggregated by taking the simple mean of the values obtained using the five different implicates reported by HFCS (im0100).

[14]Government debt of the household is the product of the country specific level of net government debt per capita as reported in Table 1 of Adam and Zhu (2015) multiplied by the number of people in the household who are older than 16 years of age as obtained by HFCS (dh0006).

[15]Effects are similar when focusing on the yield curve of the EOANIA Swap interest rate. The instantaneous Forward rate is calculated using the methodology by Svensson (1995), "Estimating Forward Interest Rates with the Extended Nelson and Siegel Method," Sveriges Riksbank Quarterly Review 1995:3, pp 13-26, as calculated by the ECB. We focus on government bonds rather than on the EONIA swap rate because the yield curve of government bonds is characterized also at maturity greater than 2 years, while the interest rate swap of the Eonia market is available only up to maturities of 2 years. The data are obtained at the datawarehouse of the ECB.

Figure 10: The effects of forward guidance on the Yield curve



(a) Instantaneous forward rates, July 2^{nd} 2013 (b) Shift in forward rates, July 4^{th}

Notes: The path at date t of instantaneous forward rates at maturities $u \geq t$, is such that the price at t of a zero coupon bond maturing at T is given by $\exp(-\int_t^T f(t,u)du)$. The left panel plots the instantaneous forward rates extrapolated from the yield curve associated to Triple A Government bonds on July 2^{th} 2013. The right panel (blue solid line) plots the difference between the same object evaluated on July 4^{th} and July 2^{nd} 2013, and the equivalent object in our model equal to the change in the path of short term interest rates given by equation (6). Basis points.

The differential between the post-announcement and pre-announcement forward curves is negative, with a peak at a horizon of 3 years, and very persistent, corroborating the thesis that such policy announcement has the largest impact on interest rates in the medium run. The red dashed line in the left panel of Figure 10 displays the path of the short term nominal interest rate in deviation from steady state in our model, i.e. $R_t - \bar{R}$ that best fits the data. In particular, given the parameters of Table 3, we set T, ε and ρ to minimize the square distance between the data (blue line in the right panel of Figure 10) and the model, at maturities larger than 2 years. Our methodology implies $T = 6$, which corresponds to a horizon of the announcement of 6 quarters, $T = 6$, an intensity of the announcement equal to 16 basis points, i.e. $\varepsilon = 1.0016$, and a persistence $\rho^* = 0.9$ implying an half life of about a year and a half.

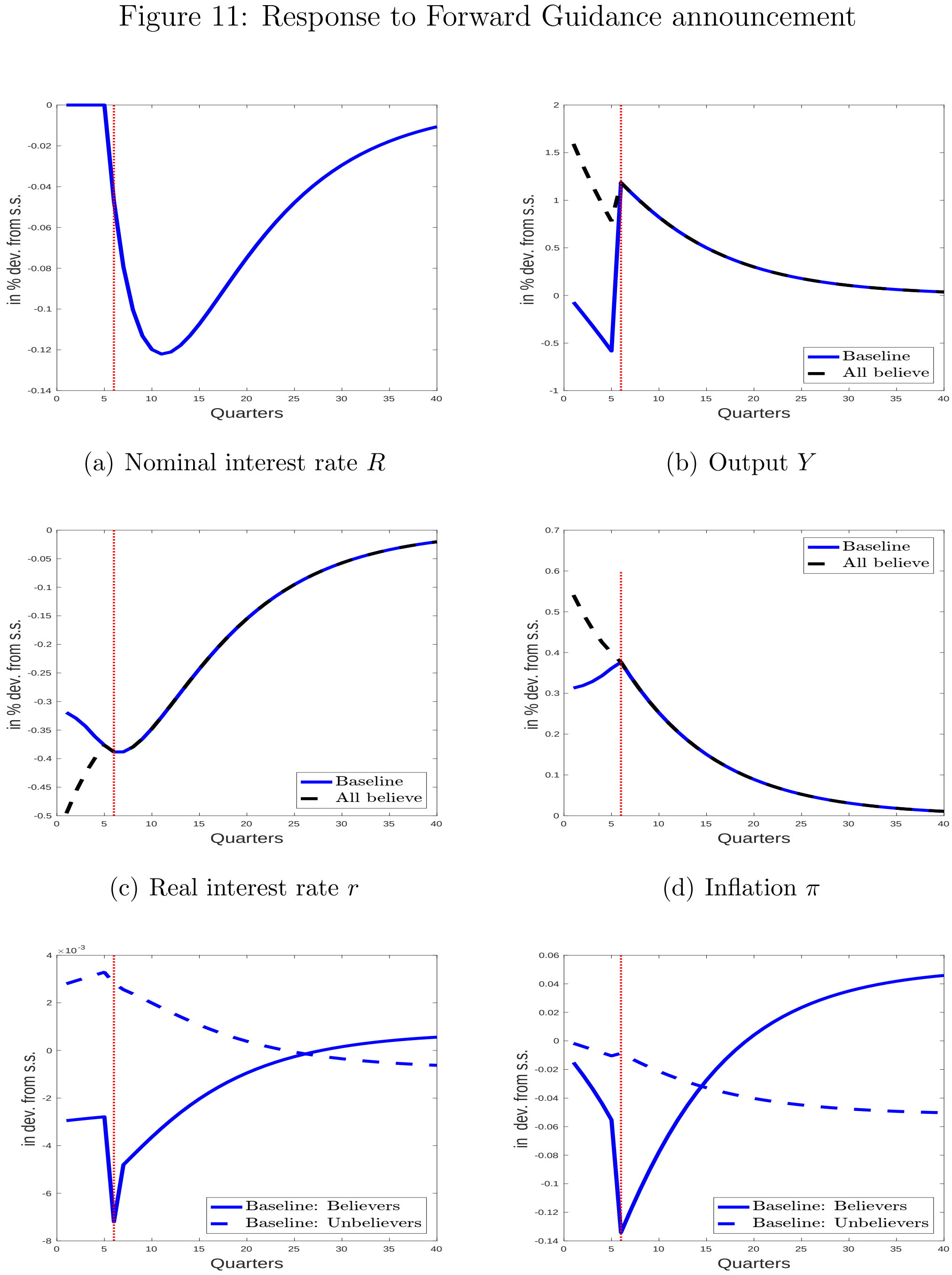
5.3 Impulse responses

Figure (11) plots the impulse responses of selected economic variables to the Forward Guidance policy announcement in our model where parameters are set as in Table 3, and where we assume that the announcement is actually implemented. We emphasize that, while the implementation of the monetary policy announcement will affect equilibrium outcomes at $t \geq T$, it has no effect on the equilibrium outcomes at $t < T$, as only beliefs about the implementation matter for the equilibrium. The blue lines plot the responses of our economy, the black dashed lines plot the responses that would be predicted in a counterfactual economy where all households are assumed to believe the announcement. The vertical red line denotes the time of the implementation of the announcement.

As a consequence of the application of Proposition 8, the economy behaves as a standard representative agent new-keynesian economy after the implementation of the announcement at $t \geq T$, independently of the dynamics at $t < T$: output and inflation increase as a consequence of the fall in nominal and real interest rates. Thus the monetary policy is indeed expansionary once implemented. The economy behaves very differently from the counterfactual (all believe) economy in the interim period between the announcement and the implementation: output falls on impact of the announcement and keeps falling until the implementation, whereas in the counterfatual economy the announcement would have caused an output boom. The fall in output in our economy is driven by the fall in demand of believers, who are households with larger financial asset position and are hurt by the (eventual) implementation of the expansionary policy. Notice that also in the counterfactual (all-believe) economy these households would reduce their demand. What differentiates our economy from the counterfactual one is the behavior of households with financial asset position low enough that they behave under the belief that the monetary announcement will not be implemented at $t = T$. As these are the household that would benefit the most from the fall in real rates, and would increase the most their demand if they were to believe, their relative inaction matters substantially for the overall response of aggregate demand. Intuitively, in the counterfactual (all believe) economy the higher increase in demand of households with low financial asset positions exactly offsets the fall in demand of households with high financial asset position, so that the distribution of wealth is irrelevant. In our economy, the wealth distribution instead matters a lot.

We notice that household with low financial asset position (unbelievers) increase nevertheless their demand at $t < T$. This is explained by the fall in the real interest rate that takes place at $t < T$. Because of the substitution effect and the lower interest payment on debt at $t < T$, these households will increase their consumption, but the increase in consumption would have been much larger if these households were to believe that the real rate will keep being low also at $t \geq T$. As t approaches T, the positive impact on demand of the substitution effect becomes smaller, so that household reduce their overall demand, explaining the further fall in output. Finally, the fall in the real interest rate at $t < T$ is explained by the increase in inflation of forward-looking firms who we have assumed to

Figure 11: Response to Forward Guidance announcement



(a) Nominal interest rate R

(b) Output Y

(c) Real interest rate r

(d) Inflation π

(e) Consumption over Labor Income

(f) Assets over Labor Income

believe the implementation of the policy announcement. The fall in real rates is however smaller than in the all-believe economy as inflation increases less due to the fall in output induced by the announcement at $t < T$.

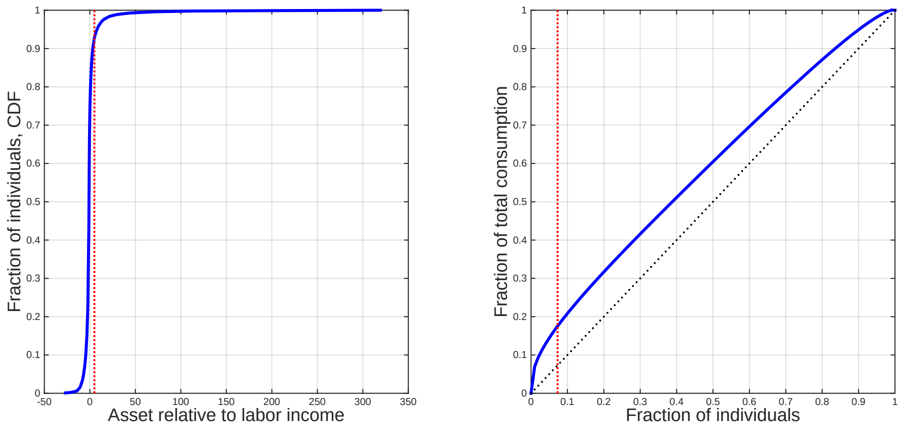
6 Robustness

In this section we perform a series of robustness exercises to assess the relevance of the assumptions of our economic model. In each exercise, we change one assumption at the time. When needed, we recalibrate the parameters governing the monetary policy experiment, T, ρ^* and ε so that each economy we consider is consistent and observationally equivalent with respect to the path of nominal interest rates described in Figure 10. The other parameters are unchanged with respect to Table 3, unless a parameter is the object of the robustness exercise.

Government debt into households liabilities. In this section we calibrate the distribution of net financial assets including government debt, of which we plot the cumulative distribution function in the left panel of Figure 12. As a result of the higher liabilities, the mean value of net financial asset relative to labor income drops from 2 in the benchmark case to 0.38 in this case. The right panel of Figure 12 plots the consumption Lorenz curve in our model at new distribution of financial assets. Now, the fraction of individuals that have enough financial assets drops from 10% in the benchmark case to 8% in this case, accounting for 18% of aggregate consumption. The first row of Figure 13 plots the impulse response of output (left panel) to the monetary policy announcement of a change in the path of the inflation target (right panel), that is then actually implemented, in the baseline model (blue solid line) and in the counterfactual with government debt (black dashed line). As a result of the different parametrization of the distribution of financial assets, the estimated path of the inflation target is different from the baseline, displaying a larger drop at the time of implementation, $T = 6$. This explains the larger response of output at $T = 6$. In the interim period, $t < T$, the output response with government debt is larger than in the baseline model, but the economy nevertheless experiences a recession a year after the announcement. Moreover, the cumulated output gain in the five quarters after the announcement is 0.6%, much smaller than the 5.8% output gain in the counterfactual all believe economy displayed in Figure 11. Thus we conclude that, even if households were to incorporate government debt as part of their liabilities, the Forward Guidance announcement would have been far less effective in our model than in the standard homogeneous beliefs economy.

Firms do not believe the announcement. In this section we assume that firms don't believe that the announcement will be implemented and assign a probability zero to this happening. We notice that this assumption does not affect the path of the nominal interest rates which are constant by assumption at $t < T$ and independent of past beliefs at $t \geq T$.

Figure 12: The distribution of net financial assets: including government debt



(a) CDF of households NFA (b) Consumption Lorenz Curve

Notes: The left panel plots the CDF of Euro are households net financial assets scaled by Euro area average labor income, net of government debt; the right panel plots the fraction of aggregate demand that is accounted for by a given fraction of individuals, ordered from the richest to the poorest the model.

The output response is identical to our baseline case at $\geq T$, but differs from it before at $t < T$. In particular, the output drops more at the time of the announcement and then is almost invariant until the implementation. The reason is that firms are not front-loading the future inflation in the current prices as they don't believe the announcement and, as a result, the real rate is almost constant in the interim period, so that the drop in output is all driven by the fall in demand of wealthier households.

Taylor rule at $t < T$. In this section we assume that the nominal interest rate is not constant at $t < T$ but is instead given by equation (6) at all t. We reestimate the path of the inflation target, displayed in the third row of Figure 13, and plot the output response in the left panel. We notice that the output drops on impact of the announcement, and then changes little until the time of the implementation relatively to the dynamics of the output in the baseline economy. This is due to the reaction of the nominal interest rate to inflation in the interim period that reduces the dynamics of the real rate, which instead falls much more when the nominal rate is constant as in the baseline model, and inflation rises.

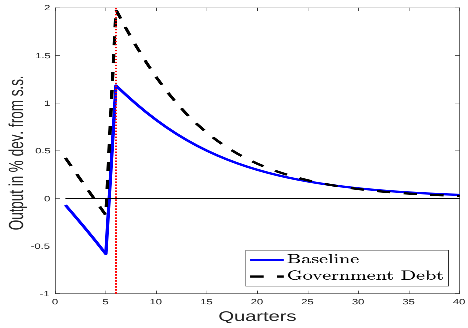
Announcement of monetary policy tightening. In this section we study the output effect of an announcement of a deflationary in our baseline economy. All parameter are as in Table 3, but for the inflation target at $t = T$ which we instead assume equal to $\varepsilon =$

1/1.0016. The fourth row of Figure 13 shows that the output falls more in the interim period after the announcement than in the counterfactual economy where all households believe the announcement. The reason is that the households that would benefit from the lower inflation and higher real rates, the wealthy households, do not believe the announcement and thus have a lower consumption demand than in the baseline economy. Therefore, we conclude that ambiguous announcements of monetary tightenings are more recessionary than unambiguous ones.

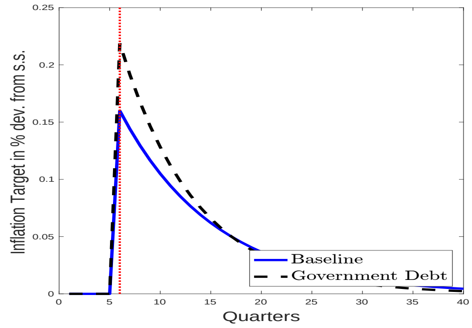
7 Conclusions

We have studied the effects of monetary policy announcements in a New Keynesian model, where ambiguity averse households with heterogenous net financial wealth use a worst-case criterion to evaluate the credibility of announcements. An announcement of a future tightening in monetary policy is always contractionary. An announcement of a future loosening in monetary policy leads to rebalancing in the financial asset positions of households, can cause credit crunches, and might be contractionary in the interim period before implementation. This is because households with large positive net financial wealth (creditors) are the most likely to believe the announcement, because the future reduction in real interest rates might cause large wealth losses to them. But when creditors believe the announcement more than debtors do, the wealth losses that creditors expect to incur are larger than the gains that debtors expect to realize. So aggregate net wealth is perceived to fall, and the announcement can misguide the economy towards a contractionary period caused by lack of aggregate demand, which is more likely when wealth inequality is large. We have evaluated the importance of this mechanism by focusing on the start of the Forward Guidance practice by the ECB in July 2013. By using micro data for Italian provinces and aggregate data for the Euro area, we find that the inflation expectation of households have responded in accordance with the theory. After matching the entire distribution of European households' net financial wealth, we have found that the ECB announcement is contractionary in our model, and particularly so when households do not feel liable for their country's government debt. Generally this analysis shows that, under ambiguity aversion, announcement of future policies with important redistributive implications might have unintended effects in the interim period before the policy is implemented.

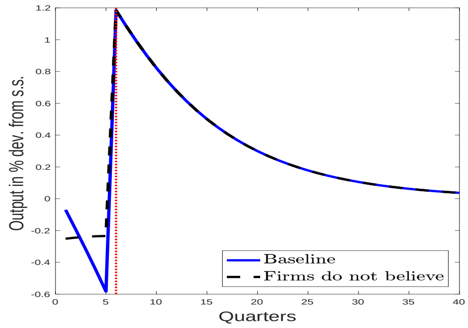
Figure 13: Output Responses to Forward Guidance: Robustness Analysis and Extensions



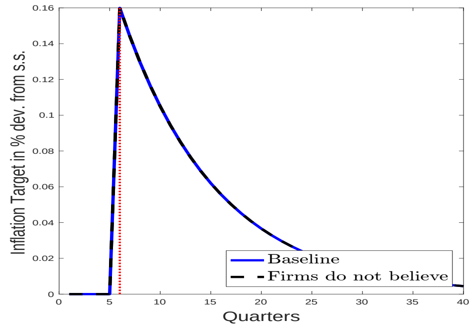
(a) Y: Govt. Debt in net wealth



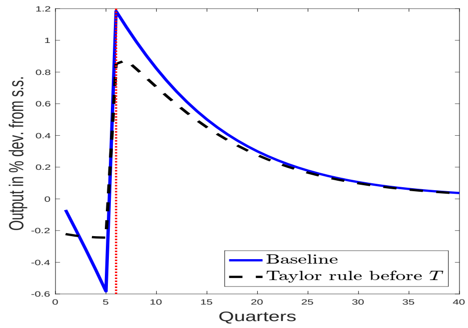
(b) Π^*: Govt. Debt in net wealth



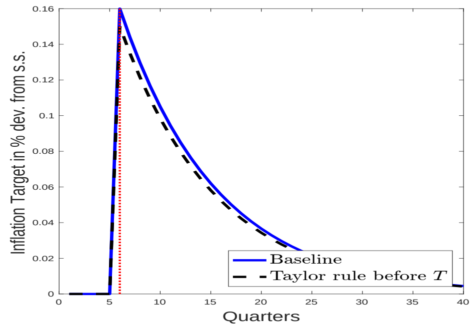
(c) Y: firms don't believe annoucement



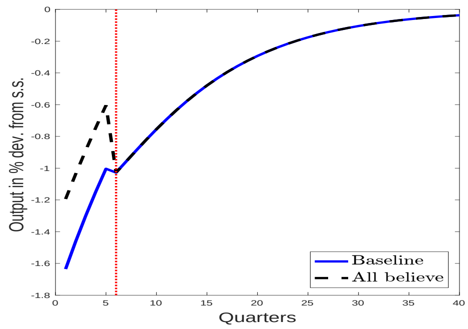
(d) Π^*: firms don't believe announcement



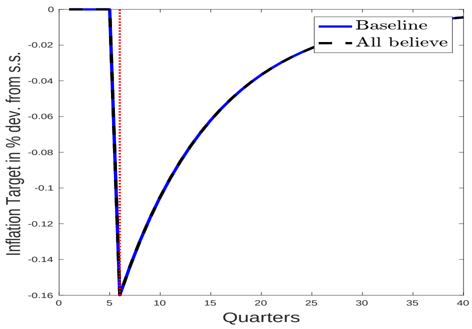
(e) Y: Taylor rule at $t < T$



(f) Π^*: Taylor rule at $t < T$



(g) Y: deflationary announcement



(h) Π^*: deflationary announcement

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A Data appendix

We describe the source of data for (i) realized and expected inflation in the Euro Area, (ii) Italian provinces, and (iii) the Net Financial Assets of European households.

A.1 Euro Area Data on realized and expected inflation

We focus on the group of Euro 11 countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Portugal, Spain, and Netherlands.

Core Inflation is equal to the yearly log differences in the Harmonised Index of Consumer Prices (HICP) energy and unprocessed food multiplied by 100, which comes from the Eurostat data warehouse available at "http://ec.europa.eu/eurostat/".

Price expectations data come from the Joint Harmonized Programme of Business and Consumer Surveys run by the European Commission. These surveys are administered at a monthly frequency. The key advantage of the Consumer Survey is that it directly asks European households for their expectations about future inflation, which makes the survey very different from the commonly used Survey of Professional Forecasters, which just focuses on professionals. The sample size for each survey varies across countries. Price expectations are obtained from the following question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall. Probabilities are calculated in terms of balances (differences between people saying that the answer is very likely versus people saying the answer it is unlikely) so that Price expectations are calculated as equal to $(f_i + 1/2 f_{ii} - 1/2 f_{iv} - f_v) \times 10$, where $f_j =$ i, ii, iii, iv, v is the fraction of individuals who opted for option j in the survey. The series are seasonally adjusted by the Commission.

Creditor vs debtor countries Creditor countries are Austria, Finland, Germany, Luxembourg, and Netherlands. Others countries include all the remaining countries in the group of Euro 11 countries. Countries are classified using their Net Foreign Asset Position as obtained from the External Wealth of Nation Mark II (EWN), see Lane and Milesi-Ferretti (2007). The only exception is Austria that according to the last observation in the EWN dataset, which pertains to 2011, had a Net Foreign Asset position over GDP ratio of -4.5%. Since Austria has accumulated large current account surpluses which average 2 percent of GDP in all the following years since 2011, we include it in the group of creditor countries. Results are robust to whether Austria is treated as a creditor or a debtor country.

A.2 Italian data

Our Italian data come from the Italian statistical institute (ISTAT), from the Survey of Inflation Expectations ran by the Bank of Italy and Sole24Ore (which is the main Italian national daily business newspaper), and from the Survey of Households Income and Wealth, administered by the Bank of Italy.

Realized inflation at the province level is obtained directly from ISTAT, as extracted from the I.Stat online archive. We focus on the general price index, denoted as pgen in the

ISTAT database. Realized inflation in the province corresponds to the yearly log-difference of the general price index in the province. We take yearly log-differences, because the ECB monitors price stability on the basis of the annual rate of change of the Harmonised Index of Consumer Prices (HICP) and because of the phrasing of the inflation expectations data, see below.

Expected inflation measures 2-quarters-ahead expected inflation, by averaging the reported estimates of all observations in the province in the Survey of Inflation Expectations ran by the Bank of Italy and Sole24Ore. The disaggregated province level data are confidential data kindly made available to us by the Bank of the Italy. The Survey of Inflation Expectations is ran at a quarterly frequency since 1999. Data is gathered in March, June, September and December. The sample comprises about 800 companies, operating in all industries including construction. Entrepreneurs in the company are asked to predict the Consumer price inflation in Italy 6 months ahead, by answering the following question: "[If the survey is ran in June 2013] What do you think consumer price inflation in Italy, measured by the 12-month change in the Harmonized Index of Consumer Prices (HICP), will be in December 2013?". Notice that entrepreneurs are asked to predict the evolution of the same index (HICP at the national level) . In practice, here we are assuming that the replies of entrepreneurs in a province reflect the average beliefs of all agents in the province.

Net Financial Assets (NFA) Our data on the NFA of Italian households come from the Survey of Household Income and Wealth (SHIW), which is administered by the Bank of Italy to a representative sample of Italian households. The survey, which is currently ran at a biannual frequency, collects detailed data on households' balance sheets. Each wave surveys about 8,000 households, which, after using the weights provided by SHIW (mnemonic Pesofit in SHIW), are fully representative of the Italian resident population. To increase sample size, we rely on both the 2010 and the 2012 waves of SHIW. NFA are calculated as the difference between the sum of households' holdings of postal deposits, saving certificates and CDs (mnemonic shiwaf1 in SHIW), government securities (mnemonic shiwaf2 in SHIW) and other securities (mnemonic shiwaf3 in SHIW) minus the sum of households financial liabilities to banks and other financial companies (mnemonic shiwpf1 in SHIW) plus trade debt (mnemonic shiwpf2 in SHIW) plus liabilities to other households (mnemonic shiwpf3 in SHIW).

Creditor household is a household with positive NFA, see the construction of the variable NFA for details.

Fraction of creditor households For each province we calculate the pre-announcement fraction of creditor households, by using the 2010 and the 2012 waves of SHIW. To calculate the fraction, we weight each household according to the weights provided by SHIW (mnemonic Pesofit).

Inflation expectation bias In each province i and quarter t, we calculate the difference between expected inflation, and future realized inflation, which corresponds to equation (1) in the main text.

A.3 European households' Net Financial Assets in HFCS

The Eurosystem Household Finance and Consumption Survey (HFCS) collects fully harmonised information on the asset portfolio allocation of households and their consumption expenditures in all euro area 11 countries (other than Ireland). Wealthy individuals are over-sampled to characterize well the right tail of the income and wealth distribution of European households. Within each country, the sum of the estimation weights equals the total number of households in the country, so that the sum of weights in the whole dataset equals the total number of households in the 10 countries of Euro 11 we consider. The structure of HFCS resembles that of the Survey of Consumer Finances in the US. To account for measurement error and missing observations, HFCS reports five separate imputation replicates (implicates) for each record. All statistics are calculated following the procedure suggested by HFCS: for each implicate we calculate the desired statistic using HFCS weights (mnemonic hw0010) and then average across the five implicates (mnemonic im0100). The surveys in each country were carried out in 2010 except in Finland and Netherlands where it was carried out in 2009 and in Spain where it was carried out in 2008. All statistics are deflated at 2010 prices.

Net Financial Assets (NFA) are calculated as the the difference between total financial assets and total financial liabilities. Financial assets include (i) deposits (mnemonic da2101); (ii) mutual funds (mnemonic da2102); (iii) bonds (mnemonic da2103); (iv) non self-employment private business (mnemonic da2104); (v) value of self-employment business (mnemonic da1140); (vi) shares of publicly traded companies (mnemonic ds2105); (vii) managed accounts (mnemonic da2106); (viii) money owed to households (mnemonic da2107); (ix) other assets (mnemonic da2108); and (x) voluntary pensions plus whole life insurance (mnemonic da2109). Financial liabilities are the sum of (i) outstanding balance of mortgages on household main residence (mnemonic dl1110); (ii) outstanding balance of mortgages on other properties (mnemonic dl1120); and (iii) outstanding balance of other non mortgage debt (mnemonic dl1200).

Net Financial Assets (NFA) including government debt is obtained by subtracting to NFA, the per household government debt of the country where the household resides. The household's country of residence is obtained from mnemonic sa0100. Per household government debt is the product of the country specific level of net government debt per capita as reported in Table 1 of Adam and Zhu (2015) multiplied by the country's average number of people in the household who are older than 16 years of age as obtained by HFCS (mnemonic dh0006).

Consumption expenditures are the sum of the consumption expenditures during the last 12 months on food and beverages at home (mnemonic hi0100) and on food and beverages outside home (mnemonic hi0200).

Average labor income in the Euro Area is calculated by averaging the employee income of all household members (mnemonic di1100) for all households whose household head has between 20 and 65 years of age. The mnemonic for the age of the household head is ra0300. The resulting average labor income is 21631 Euros.

B Proofs

Proof of Proposition ?? We start by proving point 2). At $t \geq T$ uncertainty about the path of nominal interest rates and inflation is resolved. From the first order condition to the household problem in equation (12) and its budget constraint in (5) we obtain the optimal path of consumption:

$$c_{xt+j} = \psi_0 \frac{l_{t+j}^{1+\psi}}{1+\psi} + \prod_{s=1}^{j} (\beta\, r_{t+s})^{\frac{1}{\sigma}} \left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi} \right) ,$$

$$c_{xt} + \sum_{j=1}^{\infty} \prod_{s=1}^{j} r_{t+s}^{-1}\, c_{xt+j} = a_{xt}\, r_t + w_t l_{xt} + s_t + \sum_{j=1}^{\infty} \prod_{s=1}^{j} r_{t+s}^{-1}\, (w_{t+j} l_{t+j} + s_{t+j}) \; .$$

Combining the two expressions and using $\psi_0\, l_{xt}^{1+\psi} = w_t l_{xt}$ we have:

$$c_{xt} + \left(c_{xt} - \frac{w_t l_{xt}}{1+\psi} \right) \alpha_{xt} = a_{xt}\, r_t + w_t l_{xt} + s_t + \sum_{j=1}^{\infty} \left(\prod_{s=1}^{j} r_{t+s}^{-1} \right) \frac{\psi}{1+\psi} (w_{t+j} l_{t+j} + s_{t+j}) \; ,$$

implying equation(33). The path of real assets is given by

$$a_{xt+1} = a_{xt}\, r_t + w_t\, l_{xt} + s_t - c_{xt} \; . \tag{35}$$

We next prove point 1). We notice that from the equilibrium conditions in the financial markets in (27) we have $\int a_{xt}\, r_t dx = \int a_{xt+1} dx + D_t - \Upsilon$. By iterating forward we have $\int a_{xt}\, r_t dx = \sum_{j=0}^{\infty} q_{t,t+j}\; (D_t - \Upsilon)$. Using this result into equation (33) we obtain equation (32).

Proof of Proposition 7 Using $\rho = 0$ and $\bar{\tau} = 1$ in the AD and AS equations and assuming $\Pi_1^* = \varepsilon$ we have

$$a_{j1} - a_{j0} = \frac{\varepsilon^{-\frac{1}{\sigma}}\, N_0 - \bar{N} + a_{j0}(\bar{R}-1)(\varepsilon^{-\frac{1}{\sigma}} - \varepsilon^{-1})}{(\bar{R}-1)\, \varepsilon^{-1} + \varepsilon^{-\frac{1}{\sigma}}} \; , \tag{36}$$

$$a_{j2} - a_{j0} = \frac{\varepsilon^{-\frac{1}{\sigma}}\, N_0 - \bar{N} + a_{j0}(\bar{R}-1) \left(\varepsilon^{-\frac{1}{\sigma}} \frac{\bar{R}-\varepsilon}{\bar{R}-1} - 1 \right)}{\bar{R} - 1 + \varepsilon^{1-\frac{1}{\sigma}}} \; , \tag{37}$$

where $j = \{c, d\}$ rand recall that $a_{c2} = B' \varepsilon^{-1}$, $a_{c1} = B'$, $a_{c0} = B$, $a_{d2} = -B' \varepsilon^{-1}$, $a_{d1} = -B'$ and $a_{d0} = -B$. Using equation (14) we have $N_0 = \bar{N}\, \varepsilon^{\frac{1}{\sigma}}$, which together with the last equations implies $a_{c1} - a_{c0} > 0$ and $a_{c2} - a_{c0} < 0$ if $\varepsilon > 1$, and $a_{c1} - a_{c0} > 0$ and $a_{c2} - a_{c0} > 0$ if $\varepsilon < 1$.

Rebalancing with ambiguity: $\varepsilon > 1$. Using $\rho = 1$ and $\varepsilon > 1$ in the AD and AS equations, we have that $a_{c1} - a_{c0}$ and $a_{c2} - a_{c0}$ are given by equations (36) and (37) respectively, while

$$a_{d1} - a_{d0} = \frac{N_0 - \bar{N}}{\bar{R}}, \quad \text{and} \quad a_{d2} - a_{d0} = \varepsilon^{-1} \frac{N_0 - \bar{N}}{\bar{R}} + (\varepsilon^{-1} - 1)\, a_{d0}$$

When $\varepsilon > 1$ and $\rho = 1$, using equation (14) we have:

$$N_0 = \bar{N}\,(\omega\, \varepsilon^{\frac{1}{\sigma}} + 1 - \omega) - B\,\kappa\,(1 - \varepsilon^{\frac{1}{\sigma}-1}) < \bar{N}\, \varepsilon^{\frac{1}{\sigma}}$$

Using the last equation and the definitions of ω and κ we have:

$$a_{d2} - a_{d0} = \varepsilon^{-1} \frac{\bar{N}}{\bar{R}}\, \omega\, (\varepsilon^{\frac{1}{\sigma}} - 1) - \, a_{d0} \left[1 - \frac{2\varepsilon^{-1}\, \bar{R}}{\bar{R} - 1 + (\bar{R}+1)\, \varepsilon^{1-\frac{1}{\sigma}}} \right] > 0 \; .$$

Moreover, comparing equation (37) at $j = c$ in the case with ambiguity to the case when all believe we have that $a_{c2} - a_{c0}$ is smaller (and $a_{d2} - a_{d0}$ is smaller) in the case of ambiguity than when all believe because equilibrium output N_0 is smaller.

Rebalancing with ambiguity: $\varepsilon < 1$.

Using $\rho = -1$ and $\varepsilon < 1$ in the AD and AS equations, we have that $a_{d1} - a_{d0}$ and $a_{d2} - a_{d0}$ are given by equations (36) and (37) respectively, while

$$a_{c1} - a_{c0} = \frac{N_0 - \bar{N}}{\bar{R}} \; , \quad \text{and} \quad a_{c2} - a_{c0} = \varepsilon^{-1} \frac{N_0 - \bar{N}}{\bar{R}} + (\varepsilon^{-1} - 1)\, a_{c0}$$

When $\varepsilon < 1$ and $\rho = -1$, using equation (14) we have:

$$N_0 = \bar{N}\,(\omega + (1 - \omega)\, \varepsilon^{\frac{1}{\sigma}}) - B\,\kappa\,(\varepsilon^{\frac{1}{\sigma}-1} - 1) < \bar{N}\, \varepsilon^{\frac{1}{\sigma}}$$

Using the last equation and the definitions of ω and κ we have:

$$a_{c2} - a_{c0} = \frac{\kappa}{\bar{R} - 1} \left[\varepsilon^{-1} \bar{N}\, (\varepsilon^{\frac{1}{\sigma}} - 1) - B \, \Big((\bar{R} - 1)\, \varepsilon^{\frac{1}{\sigma}-1} + \bar{R}\,(-2\varepsilon^{-1} + 1) + 1 \Big) \right] \; ,$$

implying $a_{c2} - a_{c0} < 0$ if and only if

$$B < \frac{\bar{N}\, \varepsilon^{-1}\, (1 - \varepsilon^{\frac{1}{\sigma}})}{\bar{R}\,(2\varepsilon^{-1} - 1 - \varepsilon^{\frac{1}{\sigma}-1}) + \varepsilon^{\frac{1}{\sigma}-1} - 1} \; .$$

Moreover, comparing equation (37) at $j = d$ in the case with ambiguity to the case when all believe we have that $a_{d2} - a_{d0}$ is smaller (and $a_{c2} - a_{c0}$ is larger) in the case of ambiguity than when all believe because equilibrium output N_0 is smaller.

Derivations:

$$\begin{aligned}
\bar{R}(a_{d2}-a_{d0}) &= \varepsilon^{-1}\bar{N}\,\omega\,(\varepsilon^{\frac{1}{\sigma}}-1) - \varepsilon^{-1}\,a_{d0}\,\kappa\,(\varepsilon^{\frac{1}{\sigma}-1}-1) + \bar{R}\,a_{d0}\,(\varepsilon^{-1}-1) \\
&= -\varepsilon^{-1}\bar{N}\,\omega\,(1-\varepsilon^{\frac{1}{\sigma}}) - a_{d0}\kappa\left[\varepsilon^{-1}\,(\varepsilon^{\frac{1}{\sigma}-1}-1) + \frac{2}{\bar{R}-1}(1-\varepsilon^{-1}) + (\varepsilon^{\frac{1}{\sigma}-1}+1)(1-\varepsilon^{-1})\right] \\
&= -\varepsilon^{-1}\bar{N}\,\omega\,(1-\varepsilon^{\frac{1}{\sigma}}) - \frac{\bar{R}\,a_{d0}}{\bar{R}+1+(\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}-1}}\left[(\bar{R}-1)\,(-2\varepsilon^{-1}+1+\varepsilon^{\frac{1}{\sigma}-1}) + 2\,(1-\varepsilon^{-1})\right] \\
&= -\varepsilon^{-1}\bar{N}\,\omega\,(1-\varepsilon^{\frac{1}{\sigma}}) - \frac{\bar{R}\,a_{d0}\varepsilon^{1-\frac{1}{\sigma}}}{\bar{R}-1+(\bar{R}+1)\,\varepsilon^{1-\frac{1}{\sigma}}}\left[(\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}-1} - \bar{R}\,(2\varepsilon^{-1}-1)+1\right] \\
&= -\varepsilon^{-1}\bar{N}\,\omega\,(1-\varepsilon^{\frac{1}{\sigma}}) - \bar{R}\,a_{d0}\left[1 - \frac{2\varepsilon^{-1}\,\bar{R}}{\bar{R}-1+(\bar{R}+1)\,\varepsilon^{1-\frac{1}{\sigma}}}\right]
\end{aligned}$$

$$\begin{aligned}
\bar{R}(a_{c2}-a_{c0}) &= \varepsilon^{-1}\bar{N}\,(1-\omega)\,(\varepsilon^{\frac{1}{\sigma}}-1) - \varepsilon^{-1}\,a_{c0}\,\kappa\,(\varepsilon^{\frac{1}{\sigma}-1}-1) + \bar{R}\,a_{c0}\,(\varepsilon^{-1}-1) \\
&= \frac{\kappa\,\varepsilon^{-1}\bar{N}\,(\varepsilon^{\frac{1}{\sigma}}-1)}{\bar{R}-1} - B\,\kappa\left[\varepsilon^{-1}\,(\varepsilon^{\frac{1}{\sigma}-1}-1) + \frac{2}{\bar{R}-1}(1-\varepsilon^{-1}) + (\varepsilon^{\frac{1}{\sigma}-1}+1)(1-\varepsilon^{-1})\right] \\
&= \frac{\kappa\,\varepsilon^{-1}\bar{N}\,(\varepsilon^{\frac{1}{\sigma}}-1)}{\bar{R}-1} - \frac{B\,\kappa}{\bar{R}-1}\left[(\bar{R}-1)\,(-2\varepsilon^{-1}+1+\varepsilon^{\frac{1}{\sigma}-1}) + 2\,(1-\varepsilon^{-1})\right] \\
&= \frac{\kappa}{\bar{R}-1}\left[\varepsilon^{-1}\bar{N}\,(\varepsilon^{\frac{1}{\sigma}}-1) - B\,\Big((\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}-1} + \bar{R}\,(-2\varepsilon^{-1}+1)+1\Big)\right]
\end{aligned}$$

so that $a_{c2}-a_{c0}<0$ if and only if

$$B < \frac{\bar{N}\,\varepsilon^{-1}\,(1-\varepsilon^{\frac{1}{\sigma}})}{\bar{R}\,(2\varepsilon^{-1}-1-\varepsilon^{\frac{1}{\sigma}-1}) + \varepsilon^{\frac{1}{\sigma}-1}-1}$$

C Extensions of the simple model of Section 4

C.1 Long term nominal bonds

In this section we extend our model to allow for long term nominal bonds and show that the equilibrium of the economy is unaffected. In particular, we assume that households can now trade two different assets a one period real bond as before (whose units are denoted by a_x), that for each unit bought in period t pays r_{t+1} units at $t+1$, and a perpetual bond (whose units are denoted by z_x) that pays a constant nominal return $\bar{i}$ each period, and has a price μ_t in period t. Let $B_t = a_{ct} + \mu_t\,z_{ct}$ be the value of financial assets held by creditor households, so that $-B_t = a_{dt} + \mu_t\,z_{dt}$ is the value owed by debtors. Assume, as before, that creditors are half of the population. Equilibrium in the financial market now requires $z_{ct} + z_{dt} = 0$ and $a_{ct} + a_{dt} = 0$.

For given beliefs, the first order conditions of the household x problem with respect to

a_{xt+1} and z_{xt+1} are given by equation (12) and by

$$\mu_t \left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi} \right)^{-\sigma} = \beta\, E_{xt} \left[\left(c_{xt+1} - \psi_0 \frac{l_{xt+1}^{1+\psi}}{1+\psi} \right)^{-\sigma} \frac{\mu_{t+1} + \bar{i}}{\Pi_{t+1}} \right] .$$

We now conjecture and verify that the equilibrium obtained in this economy with long-term nominal bonds is identical to the one obtained in Section 4, when only short term bonds are present. The proof is immediate once we show that the long-term nominal bond and the short term real assets are equivalent in our model, so that the household is indifferent between holding one or the other, and the portfolio choice is irrelevant. Under the conjecture that the equilibrium is invariant, we have that $R_t = \bar{R}$ for all t, implying $r_t = \bar{R}/\Pi_t$ for all t. Moreover, under the conjectured equilibrium households have degenerate beliefs so that the first order conditions to the household problem imply: $\frac{\mu_{t+1}+\bar{i}}{\mu_t} = \bar{R}$, so that the evolution of the price of the nominal bond has to be such that the nominal return per period is $\bar{R}$. Therefore, the short term real bond and the long-term nominal bond are equivalent as their return only varies as a result of variation in the realization of inflation.

D Solution of the quantitative model

We set a date $\bar{T}$ large arbitrarily large at which the economy is back to steady state where $\Pi_t = 1$, $Y_t = \bar{Y}$ and the real wage $w_t = \psi_0 \bar{Y}^{\psi}$ for all $t > \bar{T}$. We recall that $\delta = 0$ so that there are no subsidies to the household. We then conjecture a path of real wages, $\{w_t\}_{t=0}^{\bar{T}}$. Given the path of real wages, we use the equilibrium condition in the labor market to obtain the path of labor supply and output, $Y_t = l_{xt} = (w_t/\psi_0)^{\frac{1}{\psi}}$. Given the path of output and real wages, we use the sequence of equations (29) for each t and equations (6) to jointly determine the path of inflation Π_t, of nominal interest rates R_t and therefore of real rates, $r_t = R_t/\Pi_t - 1$. Given the path of inflation and the equilibrium condition in the goods market we obtain aggregate consumption as $c_{xt} = Y_t \left(1 - \frac{\kappa}{2} \left(\Pi_t - 1\right)^2\right) - \phi$. Furthermore, from equation (6) we obtain the path of nominal interest rates R_t and therefore real rates, $r_t = R_t/\Pi_t - 1$. Given the dynamics of aggregate variables we are ready to solve for the household problem. We do this in two steps. We first solve the household problem after the implementation of the announcement, i.e. for $t \geq T$, and conditional on a given implementation of the announcement; in the second step we obtain household consumption and saving decisions for $t < T$ for given beliefs of the household about the implementation of the announcement at $t = T$.

The solution to the household problem at $t \geq T$ is given by Proposition **??**. In the case of the announcement not being implemented we have $w_t^1 = \psi_0 \bar{Y}^{\psi}$ for $t \geq T$. In the case of the announcement being implemented instead, we update the guess for $\{w_t^0\}_{t=T}^{\bar{T}}$ until $Y_t = (c_{xt} + \Upsilon)/\left(1 - \frac{\kappa}{2}\pi_t^2\right)$ is satisfied. Once we have determined the equilibrium path of $\{w_t^m\}_{t=T}^{\infty}$, and associated equilibrium aggregate variables, we are able to determine the value function of an household j with real asset value a_{xt} at the beginning of period T, as a function of the implementation of the monetary announcement, denoted by $V_T^m(a_{xt})$. Such value function is given by $\sum_{j=0}^{\infty} \beta^j \left(c_{xt+j} - \psi_0 \frac{l_{t+j}^{1+\psi}}{1+\psi} \right)^{1-\sigma} /1 - \sigma$, where c_{xt+j} and l_{t+j}

are obtained from equations (33)-(35) under the given path real wages $\{w_t^m\}_{t=T}^{\infty}$. The value function $V_T^m(a_{xt})$ is strictly increasing in a_{xt}. We conjecture and verify an equilibrium where there exists a unique value $\hat{a} \geq 0$ such that $V_T^0(\hat{a}) = V_T^1(\hat{a})$, $V_T^0(a) < V_T^1(a)$ for $a > \hat{a}$ and $V_T^0(a) > V_T^1(a)$ for $a < \hat{a}$.

Step 2: solving the household problem for $t < T$ We take as given the solution for the path of aggregate variables at $t \geq T$ obtained in the first step. We conjecture a path of real wages and inflation, $\{w_t^n\}_{t=0}^{T-1}$ where n indexes the conjecture. Let $\{\hat{w}_s^m\}_{s=t}^{\infty} = [w_t^n, w_{t+1}^n, ..w_{T-1}^n, w_T^m, w_{T+1}^m, ...]$ be the beliefs about the path of real wages of a household as a function of whether she thinks the announcement will be implemented ($m = 0$) or not ($m = 1$) at period t. For a given path of real wages we are able to determine all other aggregate variables as describe above.

We distinguish thees possible cases for each household j. In the first two cases, the household has degenerate beliefs about the implementation of the announcement and in particular the path of her consumption is given by equation (33) and the path of her real asset is given by equation (35) under the given set of beliefs about the path of aggregate variables indexed by $\{\hat{w}_s^m\}_{s=t}^{\infty}$, with $m = 0$ or $m = 1$. The path of beliefs associated to not trusting the implementation of the announcement is admissible if and only if the implied level of real asset at $t = T$ is such that $a_{xt} < \hat{a}$. Equivalently, the path of beliefs associated to trusting the implementation of the announcement is admissible if and only if the implied level of real asset at $t = T$ is such that $a_{xt} > \hat{a}$.

In the third case, the household does not have degenerate beliefs about the implementation of the policy, but her beliefs are indeterminate. This case is possible only when the path of consumption and saving of the household is such that $a_{xt} = \hat{a}$. We can then use the final condition $a_{xt} = \hat{a}$, together with the first order condition for consumption at $t < T$,

$$c_{xt} = \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi} + \left(\beta^{\frac{j}{\sigma}} \prod_{s=1}^{t} (1+r_s)^{\frac{1}{\sigma}} \right) \left(c_{x0} - \psi_0 \frac{l_0^{1+\psi}}{1+\psi} \right), \quad \text{for} \quad t \in [1, T-1]$$

to determine the path of consumption of the household until $t = T - 1$. Each household j evaluates the three possible path of consumption and labor according to

$$\sum_{j=0}^{T-1} \beta^j \frac{\left(c_{xt+j} - \psi_0 \frac{l_{t+j}^{1+\psi}}{1+\psi} \right)^{1-\sigma}}{1-\sigma} + \beta^{T-1} V_T^m(a_{xt}),$$

and choose the highest value.

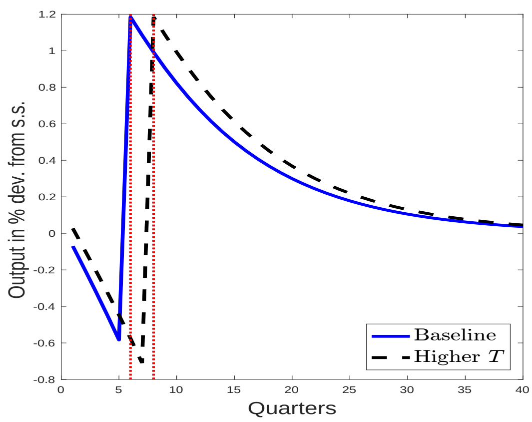
Once we have determine the path of consumption for each household j for $t \in [0, T-1]$, we evaluate the equilibrium condition in (31). We update the guess for $\{w_t^n\}_{t=0}^{T-1}$ until such condition is satisfied.

E Robustness

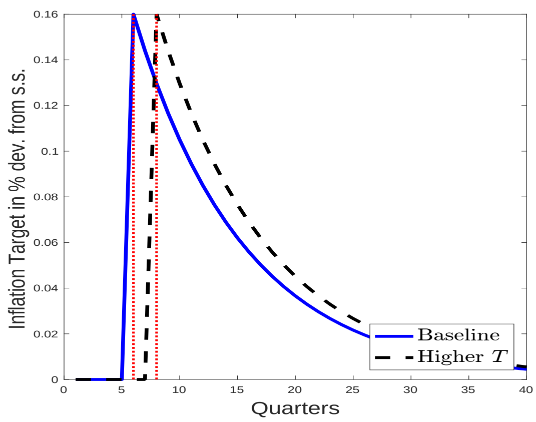
E.1 Horizon of the announcement

In this section we consider a longer horizon for the date of the implementation of the announcement. In particular, we consider an announcement with an horizon of 2 years corresponding to $T = 8$. As shown in Figure 14, output drops less on impact of the announcement than in the baseline case of $T = 6$ because the expected cumulated drop of the real rate before the implementation date is mechanically larger due to the longer time period where inflation is higher (due to firms front-loading the future drop in the inflation target) and the nominal interest rate is constant. Thus, the substitution effect on demand is relatively stronger, and output is higher on impact. As we approach the date of the implementation, unbelievers expect the real rate to go back to steady state sooner and drop their demand, and the output drops at a level even smaller that the one of the baseline economy.

Figure 14: Response to Forward Guidance announcement with $T = 8$



(a) Output Y

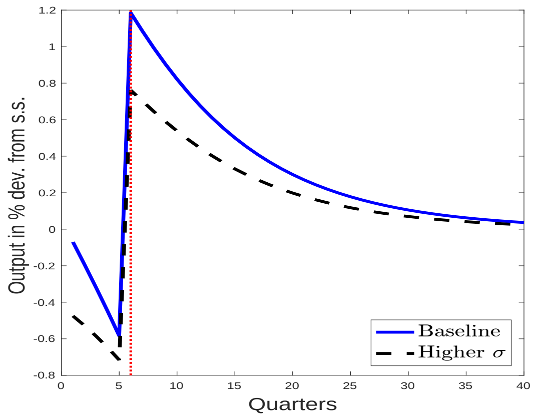


(b) Inflation Target Π^*

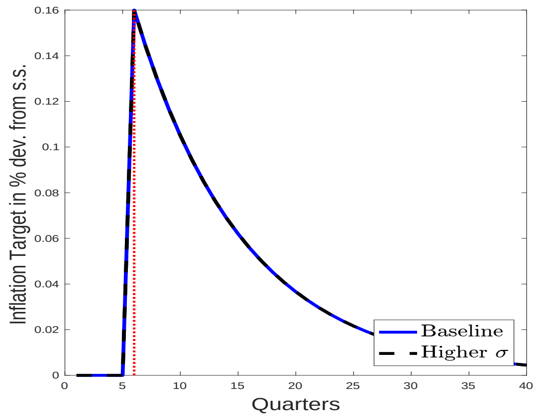
E.2 Inter-temporal elasticity of substitution

In this Section we consider the role of the elasticity of intertemporal substitution in the propagation of the forward guidance announcement. We consider an increase in σ from 2 to 2.5, corresponding to a drop in the EIS from 0.5 to 0.4. As Figure 15 shows the output response of the economy to the announcement is substantially more recessionary at $t < T$, and less expansionary once implemented at $t \geq T$. Therefore, the forward guidance announcement is more recessionary when the EIS is lower because the positive substitution effect of lower real rates on output is smaller.

Figure 15: Response to Forward Guidance announcement with $\sigma = 2.5$



(a) Output Y



(b) Inflation Target Π^*