Inflation, default and sovereign debt: The role of denomination and ownership

Laura Sunder-Plassmann*
University of Copenhagen

October 2016

Abstract

Emerging market governments borrow in a mix of nominal and real terms both externally and domestically. We propose a theory to study the effects of debt portfolio composition along the dimensions of ownership and denomination on sovereign default and inflation risk. In the model, a benevolent government issues bonds and prints money to smooth domestic household consumption, but lacks commitment and thus faces incentives to default outright or inflate away the debt. We show how these incentives and equilibrium outcomes depend on the portfolio structure of the debt, and present three main findings in a calibrated version of the model: First, Borrowing domestically, and especially in real terms, raises inflation. Second, ownership is the more important determinant of macroeconomic outcomes than denomination for portfolios typically observed in the data. Third, there are welfare gains from nominal and domestic borrowing because it facilitates the use of inflation as a countercyclical policy tool to adjust debt burdens. We document empirical evidence supporting the testable predictions of the model.

*Email: Lsunderplassmann@gmail.com

This is based on a chapter of my dissertation previously circulated under the title "Inflation, default and the denomination of sovereign debt". I would like to thank without implicating Cristina Arellano, Tim Kehoe, Fabrizio Perri, Manuel Amador, Illenin Kondo, Andrea Waddle, David Wiczer, seminar participants at the Minneapolis Fed, the Minnesota Trade Workshop, the Federal Reserve Board, University of Bonn, SUNY Stony Brook, Florida State University, Stockholm School of Economics, Paris School of Economics, Ryerson and Copenhagen University. All errors are mine.

1 Introduction

Sovereign debt crises in emerging market economies are often blamed on governments borrowing from abroad in real terms. With the debt burden fixed and foreigners holding the bonds, governments find it easy to default to reduce financing pressures in downturns. Having nominal debt at their disposal on the other hand is thought to be preferable in principle, but unaffordable in practice due to high inflation premia. The goal of this paper is to evaluate this reasoning. The question of what kind of debt structure is well-suited for crisis prevention is an important - and open - one for both national and supranational policymakers,[1] and we contribute to the answer by sharpening our understanding of the role that debt ownership and denomination plays in determining macroeconomic outcomes including default and inflation risk.

The framework of our analysis is a dynamic general equilibrium theory of optimal monetary and fiscal policy with incomplete markets. The government borrows in a mix of nominal, real, external and domestic debt. Taking the fixed portfolio structure as given, it issues short term debt prints money and raises taxes in order to smooth resident consumption against persistent productivity shocks. We assume that the government lacks commitment to its policies which creates incentives to borrow, inflate and default outright on outstanding debt.

Inflation and outright default are substitutes as both enable the government to relax its budget constraint: Printing money generates seigniorage revenue and - to the extent that bonds are nominal - reduces the real value of the debt burden; outright default directly reduces the outstanding debt. But both are costly which limits the extent to which the government optimally uses them: Domestic residents hold money because of a cash-in-advance constraint on consumption so increases in the price level lower their real purchasing power; outright default results in temporary productivity drops and bond market exclusion for the government.

Equilibrium inflation and default rates are determined as a result of the interaction with optimal debt accumulation. On the one hand, inflation and default incentives are stronger when debt is high or productivity is low because lower real debt burdens require the government to raise less revenue via distortionary labor taxes on households. On the other hand, high inflation and default risk reduce debt accumulation, since the government takes into account that its borrowing costs rise with inflation and default risk. Bond portfolio characteristics that imply stronger incentives to inflate, for example, can thus in equilibrium lead to higher or lower inflation, depending on how much the government optimally borrows.

We calibrate the model to Mexico, using government debt data to match the observed debt portfolio composition, and show that simulated data from the numerical solution of the model successfully reproduce key features of the Mexican economy, both targeted (including average inflation and default rates) and untargeted (including average levels of seigniorage and tax rates, as well as countercyclical inflation and net exports). The calibrated model is then used to analyze the effects of shifts in portfolio composition. We present three main results.

First, borrowing domestically, and in real terms, drives up average inflation and default rates, and implies higher debt to GDP ratios in the long run. When debt is owed to domestic households, the government faces weaker incentives to default and inflate since expropriation of residents entails negative wealth effects. This in turn makes borrowing cheaper for the government, and encourages debt accumulation - which in equilibrium is strong enough for average inflation and default rates to increase in the domestic debt share.

Inflation also increases the higher the real share of domestic debt. Despite the fact that, when debt is real, inflation is less useful as a tool to reduce the debt burden, the absence of inflation premia in bond prices

[1] See for example Borensztein et al. (2005)

means that the government faces stronger incentives to borrow. In equilibrium this leads to high inflation in the model even without substantially higher debt levels, as the tax base for inflation is lower with real debt, and so the government needs to inflate more to generate the same amount of revenue. Higher shares of real debt held by external investors, on the other hand, lower equilibrium inflation. The interaction of reduced inflation premia with own residents' consumption risk is thus crucial to encourage sufficient debt accumulation to drive up inflation.

The second result we present is that, when comparing portfolios within the range typically observed in the data, ownership is the more important determinant of equilibrium debt and inflation than denomination. For default rates, denomination is equally important. Ownership has a large, and positive, effect on inflation, default rates and debt to GDP ratios, while conditional on ownership, the effects of denomination are much smaller. Specifically, the denomination of external debt has relatively minor effects on equilibrium outcomes, and even though high real shares of domestic debt do increase inflation and debt substantially above a certain threshold, economies typically stay well below that. We compare model predictions for four types of average portfolios observed in the data - largely external/nominal, external/real, domestic/nominal or domestic/real portfolios - and show that the two domestic debt economies are more similar to one another than the two nominal debt economies.

Both of these findings are testable predictions of the model, and we show that both are borne out in the data: In a panel of 24 emerging market countries for which government debt portfolio breakdowns are available, we show that inflation and debt to GDP ratios are positively associated with the share of real domestic debt, but not the share of real external debt, as implied by the model. We further show that inflation and debt to GDP ratios are positively related to internal debt shares, but not the overall nominal debt share.

The third, normative result that we highlight is that portfolios that facilitate the use of inflation are welfare improving. We compare economies across the entire spectrum of possible portfolio configurations, and show that the ones with high domestic debt shares and low external real shares dominate in terms of ex-ante consumption-equivalent welfare gains. In those models, the government faces relatively low incentives to inflate or default ex-ante, and so in equilibrium can afford to use inflation as a countercyclical policy tool to adjust the debt burden in downturns. External, real debt portfolios are detrimental for welfare.

The rest of this paper proceeds as follows. We discuss the context and literature, present an empirical overview of government debt portfolios, introduce the model with a discussion of the forces and mechanisms, followed by the calibration, and numerical results on the role of the portfolio composition in driving positive and normative model outcomes.

2 Literature

2.1 Theoretical literature

This paper draws on two main strands of literature. First, it builds on the quantitative sovereign default literature following Arellano (2008) and Aguiar and Gopinath (2006) who study sovereign default and borrowing in small open endowment economies with incomplete markets and lack of commitment. Second, it draws on the literature of time-consistent public policy following Klein et al. (2008), Klein and Rios-Rull (2003) and Paul et al. (2005), which focuses likewise on commitment problems, but largely in closed economies and pertaining to fiscal policies other than default.

Within the latter set of studies, this paper is most closely related to Martin (2009) and Diaz-Gimenez et al. (2008). Martin (2009) studies debt accumulation and monetary policy when the government cannot commit to inflation and borrowing. He does not consider outright default and his economy is a closed one so all debt is held domestically. He highlights the role that lack of commitment plays in driving debt accumulation and equilibrium inflation - a mechanism that is also at work in this paper. Diaz-Gimenez et al. (2008) analyze monetary and fiscal policy with lack of commitment in a setting as in Nicolini (1998), focusing on the role of debt denomination. Their framework like Martin (2009)'s is a closed economy with no option to default. They characterize analytically equilibria for economies with either nominal or real debt, and show that the denomination implies different paths for debt accumulation. A key message of their paper is to show that welfare in nominal debt economies can be higher or lower than in real debt economies, depending on parameters and initial conditions. We build on their results and find conditions under which nominal debt is welfare improving quantitatively in a more general framework that considers the interaction of denomination with ownership and outright default.

Within the sovereign default literature, attention has been focused on external, real debt and default as the relevant policy instruments available to the government. There are a few recent contributions studying aspects of nominal or local currency sovereign borrowing: Na et al. (2014) study optimal default and exchange rate policy in an open economy sovereign default model with downward nominal wage rigidities. They find that defaults are accompanied by large devaluations, and that equilibrium external debt levels are lower in fixed-exchange rate economies, mirroring results in this paper. We contributes to their analysis by relaxing the assumption that public debt is external and studying the interaction of default and inflation with domestic debt. Du et al. (2016) and Engel and Park (2016) model the choice between nominal and real debt, the former without endogenous default, the latter without the distinction between domestic and foreign creditors. Du et al. (2016)'s theoretical result that countercyclical inflation emerges with nominal debt only if investors are risk averse is consistent wit the results in this paper. Du and Schreger (2015) propose a model that explains outright default on nominal debt as a result of the corporate sector being exposed to exchange rate risk, which makes economy-wide inflation relatively more costly for the benevolent government than outright default on external creditors. Roettger (2014) studies the interaction of fiscal and monetary policy in a closed economy model with no external or real debt. He focuses on the role of default and shows that the option to default deteriorates welfare, reduces debt and lowers equilibrium inflation. Arellano et al. (2015) study sovereign crisis risk and default incentives in a monetary union with both domestic and external debt, but focus on selective default between the two types of debt in a stylized two period setting. Gumus (2013) shows in a two-sector sovereign default model with real (tradable) and nominal (non-tradable) bonds that if shocks are such that inflation (real depreciation) is countercyclical, then on average real debt leads to lower default rates. Cuadra et al. (2010) introduce endogenous fiscal policy into a sovereign default model and show that it can generate public expenditures and tax rates that are optimally procyclical, as is often observed in emerging market countries.

None of these papers differentiate between debt ownership and denomination, and to the best of our knowledge this is the first paper to highlight their role and interaction in shaping equilibrium default, debt and inflation outcomes.

2.1.1 Other related contributions

There are a number of other papers focusing on the interaction between monetary and fiscal policy that are related to the present paper but take quite different approaches. Alfaro and Kanczuk (2010) highlight one trade-off involved in issuing nominal debt - tax smoothing versus distorting inflation costs - and find that the optimal share of nominal debt is zero in a closed economy money-in-the-utility function model that keeps the level of debt fixed exogenously. They focus on monetary policy and labor taxes as the only policy instruments, and do not explore the interplay of borrowing, outright default, openness and inflation, that is studied in this paper. They also show that conditional on carrying some nominal debt, it is optimal to have large amounts of it in order to reduce the distortionary inflation needed to smooth taxes - a result that hinges on the exogeneity of the debt, and that this paper shows is reversed once borrowing is endogenized.

Niemann et al. (2013) analyze inflation dynamics in a New Keynesian model with nominal debt and find, similar to this paper, that inflation is correlated with debt if the government pursues policy under discretion due to incentives to monetize nominal liabilities. Arellano and Heathcote (2010) study how a country's exchange rate regime affects a country's ability to borrow externally, comparing full dollarization with free floats in a model of limited enforcement and endogenous borrowing limits. They find that dollarization increases incentives to maintain debt market access and thus debt in equilibrium. Durdu (2009) analyzes the effects of GDP-indexed debt on consumption volatility in a small open economy with endogenous sudden stops. Indexation introduces state contingency in debt payments akin to inflation and nominal debt in this paper. In her setting, an intermediate degree of indexation minimizes consumption volatility: On the one hand it provides a hedge, but on the other it introduces interest rate volatility. Indexation in her framework is fully exogenous while we consider endogenous, optimal inflation.

Nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Da-Rocha et al. (2013), Araujo et al. (2013) and Corsetti and Dedola (2016) among others. These contributions focus on expectations-driven debt crises as in Calvo (1988) and Cole and Kehoe (2000) whereas we consider default driven by weak fundamentals, and study economies without any role for domestic monetary policy - a currency union or dollarized economy. They highlight the idea that there is a trade off involved in choosing to inflate - the benefits of flexibility versus the costs of distortion - which are also present in this paper.[2]

2.2 Empirical literature

On the empirical side, there are a number of papers that study and document the composition of sovereign debt portfolios that this paper is related to: The "original sin" literature beginning with Eichengreen and Hausmann (1999) pointed out the relative lack of external borrowing in local currency. Reinhart and Rogoff (2011) emphasize the prevalence of domestic relative to external public debt as well as explicit default on domestic debt. Lane and Shambaugh (2010) study the currency composition of external liabilities. A series of recent papers document increasing foreign participation in local domestic currency sovereign bond markets which this paper studies, including Burger and Warnock (2007), Arslanalp and Tsuda (2014), Du and Schreger (2015) and Claessens et al. (2007).

[2] Several studies have explored the costs and benefits of indexed debt instruments in the context of public finance, see for instance Fischer (1975), Bohn (1990), Missale (1997) and Barro (1997), among others.

Table 1: Sovereign debt structure

	Nominal	Real
Internal	$\delta(1-\alpha)$	$\delta\alpha$
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$

3 Empirical motivation

The key observation motivating this paper is that emerging market governments hold mixed debt portfolios both in terms of the residence of the investor base and the denomination of the debt. In this section we summarize these empirical patterns of government debt portfolio composition that will be incorporated in the theoretical model to study their implications.

The two features of a government's debt portfolio that are the focus of this paper are ownership and denomination. Ownership is defined as whether the immediate holder of the bond is resident in the borrowing country or abroad. It is unrelated to the market of issuance as investors can purchase bonds listed on foreign exchanges. For denomination we distinguish between bonds that promise payments in nominal terms in the borrowers own currency versus those that do not, which includes both foreign currency denominated bonds and local currency indexed bonds. The aspect of denomination that we are interested in is the degree to which the government has control over the real value of the promised payment stream via inflation. For the remainder of the paper, we will use local currency and nominal interchangeably, and similarly for foreign currency, indexed, and real. Historically, the overlap between the sets of bonds that are held abroad and those that are real has been high, but this relationship is weakening as international investors increasingly purchase nominal government bonds. Table (1) contains a stylized matrix representation of the four types of debt that the paper focuses on.

The focus of the analysis implies that we abstract from other dimensions of sovereign debt portfolios, including investor exposure to exchange rate risk and the role that the jurisdiction of the market of issuance plays. In the data, we cannot distinguish between bonds that can be inflated and those that can be only devalued. The countries of interest to this analysis - relatively advanced emerging market countries - maintain floating exchange rates so it seems more appropriate to focus on inflation as the choice of the government in adjusting payment streams. See Na et al. (2014) for a study of default and devaluation. The jurisdiction under which a bond is issued determines the legal terms of a bond that determine how hard it is to restructure, and thus affects primarily the restructuring process rather than normal borrowing, and is studied for example in Zettelmeyer et al. (2011), Pitchford and Wright (2012) and Schumacher et al. (2015).

The empirical regularities documented here use the database compiled by Arslanalp and Tsuda (2014).[3] The database contains quarterly general government debt stock series by denomination and ownership for 24 emerging market countries from 2004Q1 to 2015Q4. The countries are all large emerging market borrowers included in JP Morgan's emerging market bond index and its foreign currency bond index. The underlying data sources are wherever possible cross-country comparable data sources, in particular the IMF's quarterly external and public debt statistics databases (QEDS and QPDS), supplemented with national sources.

The database contains sufficient information to infer all debt shares from Table (1) that we are interested in. It contains direct information on the share of debt that is owned externally, δ. It also contains a measure of the share of nominal debt that is held abroad, $\frac{(1-\delta)(1-\kappa)}{\delta(1-\alpha)+(1-\delta)(1-\kappa)}$, and of the nominal share in total debt,

[3] Available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip

$\delta(1-\alpha)+(1-\delta)(1-\kappa)$. Using these three pieces of information, we can back out estimates of δ, α, κ, the portfolio shares we are interested in [4]

Figures (1) and (2) plot the resulting time series shares for the share of internal debt δ, the real share of internal debt α and the real share of external debt κ for all countries, grouped by geographical region. They show that (i) domestic debt shares are large, (ii) that the majority of domestic debt is nominal, and (iii) that a relatively smaller but increasing share of external debt is nominal.

The top half of Figure (1) shows the first point: The share of domestic debt ranges from at least 40% (with very few exceptions) to 100%. The simple pooled average is 67%. Asian countries lead in terms of domestic debt shares, with both India and China never dropping below 95% domestic debt, and Malaysia, the Philippines and Thailand with domestic shares above 75% throughout the sample. Indonesia displays a relatively strong downwards trend over time, dropping from near 100% domestically held deb to around 40% between 2004 and end the end of 2015. Latin American countries in the Figure (1) show high cross-country heterogeneity in domestic debt shares. Brazil's are relatively high throughout the sample above 80%, while Uruguay remains below 50%. Mexico, the country we will choose as the calibration target for the theoretical model later on, is close to the average with its share varying between 50 and 75% domestically held debt over time. For the Ukraine we can see how the crisis has affected its debt composition, with external investor shares dropping drastically in 2008/09 bringing the domestic share from 20% to over 60% in just two years. Egypt stands out as a country with a high share of domestic investors, while Russia has increased its share of domestic investors steadily over the sample to around 80% now. European countries have fairly low domestic debt shares, all remaining below 80%, especially Lithuania with the lowest average throughout the sample across countries of just 23% domestically held debt.

The share of externally held debt that is real, κ, tends to be large across the sample with a pooled mean of 61%, but falling over time. The bottom half of Figure (1) shows this most dramatically for Latin American countries. The average real external debt share at the beginning of the sample was over 90%, while at the end of the sample it has dropped to around 60%. There are differences across countries, however. Argentina is one notable exception that has maintained a largely real external debt stock, perhaps unsurprisingly given its ongoing default renegotiations throughout the sample and questions regarding the reliability of its inflation statistics and general governance. Brazil on the other end of the extreme has seen the largest shift in its external debt composition, with its real share dropping below 20% at the end of the sample. In Asian countries, there are several countries that have either fully or nearly exclusively nominal external debt, namely China and India, as well as more recently Thailand and Malaysia. These countries overall then borrow largely from their own residents, and in nominal terms. Indonesia bucks the trend of increasingly nominal external debt shares as the only exception. Russia has shifted towards borrowing from abroad in nominal terms in the course of the sample, dropping from 100% real external debt to only 50%. South Africa issues much of its externally held debt in nominal terms, while the Ukraine remains almost exclusively real.

Finally, Figure (2) shows that the majority of domestically held debt is nominal. Argentina and Brazil are notable exceptions with real domestic debt shares of around 55% and 30%, respectively, but the average is just 18%.

Since there is relatively less variation within countries over time in the portfolio breakdown than across countries, we summarize the portfolio composition by taking time-averages of the portfolio shares for each country and plotting the resulting distribution of countries by portfolio characteristics in the internal-real

[4] See Section (B) in the appendix for details.

Figure 1: Debt portfolio structure

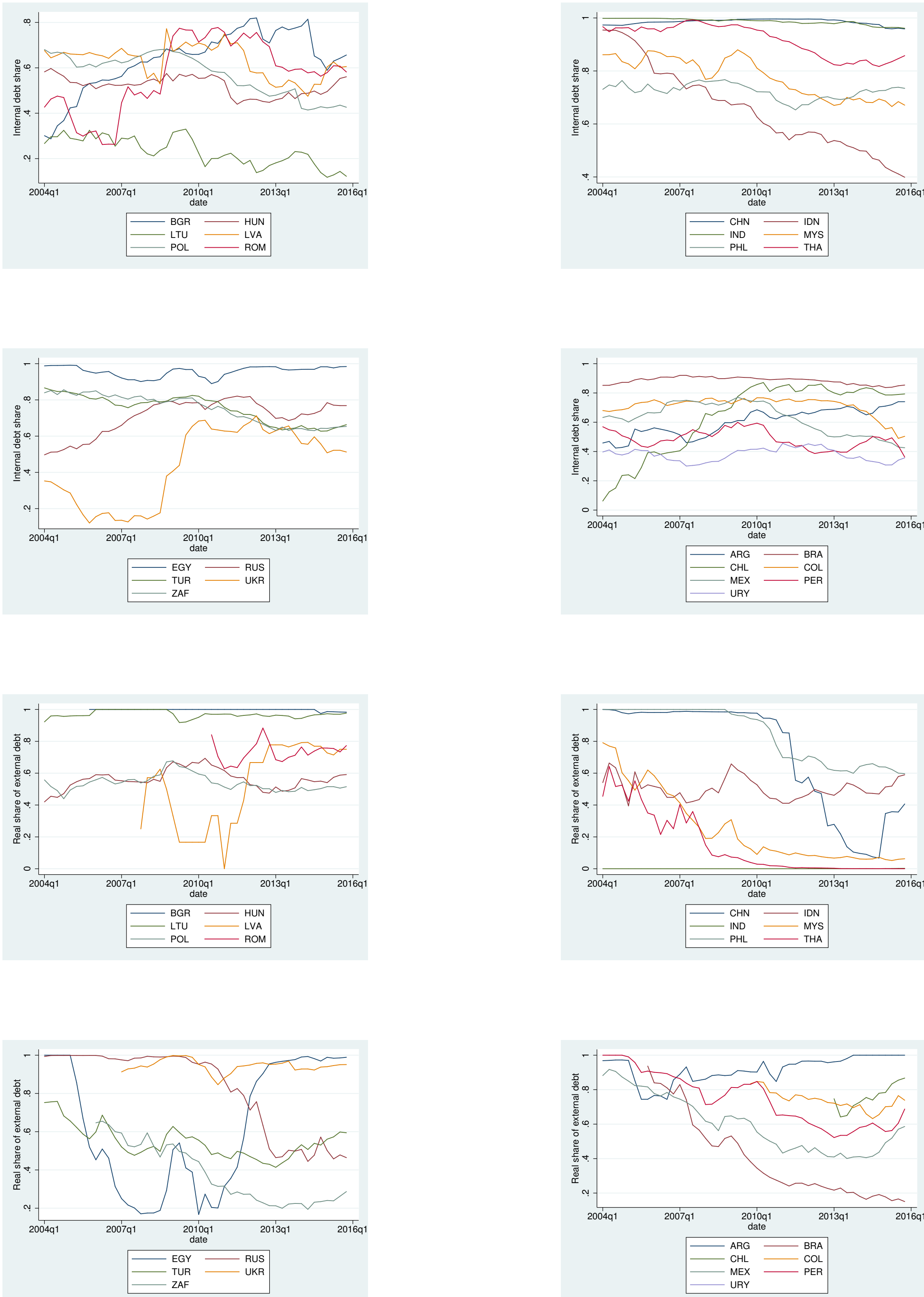


Figure 2: Debt portfolio structure ctd.

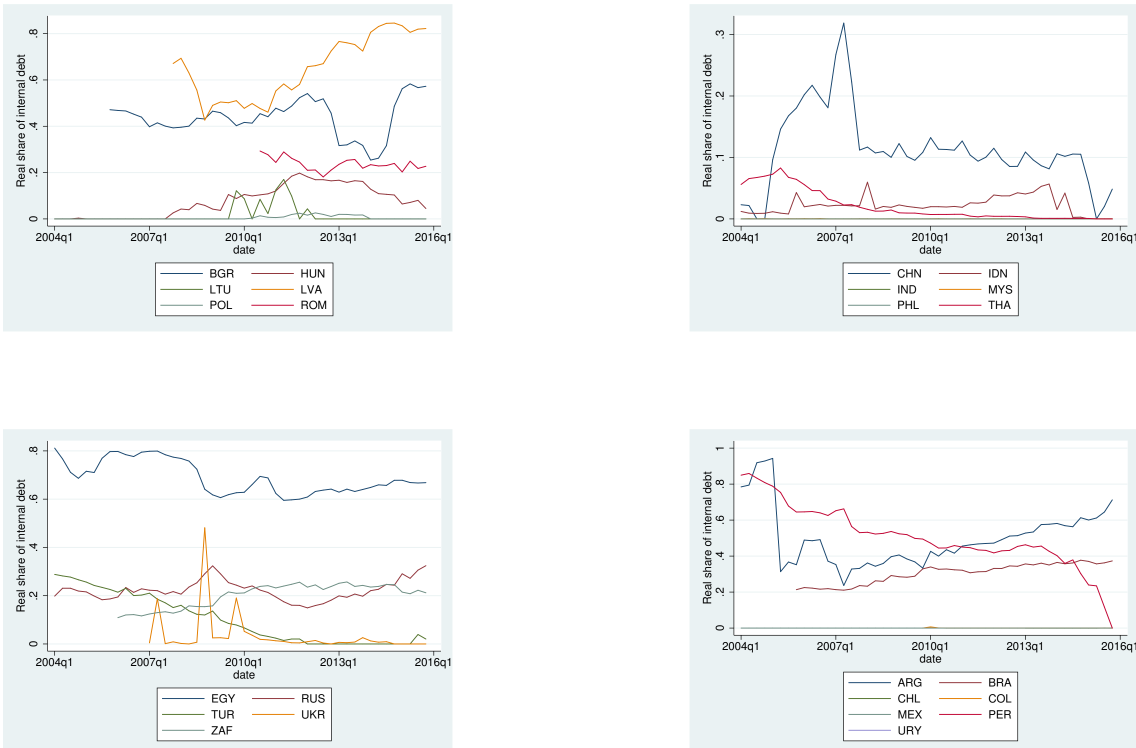
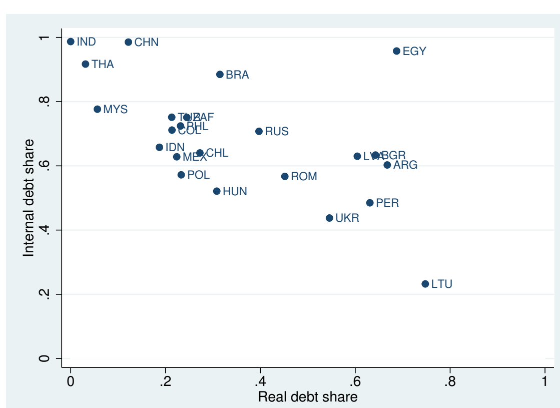


Figure 3: Portfolio composition in the cross-section



share plane in Figure (3). The real share is computed as the weighted average of domestic and external real debt. The Figure shows that in terms of the quadrants from Table (1), there are relatively few countries in the top right or bottom left quadrants. The majority of sovereigns borrows largely from foreigners in real terms, or from their own citizens in nominal terms. Asia tends to occupy the top left domestic/nominal part of the graph, while Argentina is close to the bottom right external/ real corner.

Having documented that governments borrow in a mix of nominal and real debt from both foreigners and abroad, we next develop a model that incorporates these portfolios and allows us to characterize and quantify their effects.

4 Model

This section presents a model of sovereign borrowing, inflation and default with bonds that differ along two key dimensions: Denomination (nominal or real) and ownership (external or internal). It is a small open economy in discrete and infinite time with three types of agents: Households, the government and international investors. Households work and save in money and government bonds to smooth consumption over time. Part of household consumption must be purchased with money. Labor productivity is stochastic and the only source of uncertainty in the model. The benevolent government levies labor income taxes and issues non-contingent, partially indexed bonds to both domestic households and international investors to finance exogenous and fixed government spending. The government lacks commitment to its policies, so that it has an incentive to borrow to finance expenditures, and to reduce its real debt burden through costly explicit default or distortionary price inflation. We discuss each element of the model in turn before stating the optimal policy problem of the government and defining the equilibrium.

4.1 Debt structure

We assume the following exogenous debt structure. The government issues one-period claims that can be bought by foreign or domestic bondholders. For every unit bond issuance, we assume that fraction δ is bought by domestic residents and fraction $1-\delta$ is bought by international investors. We further assume that fraction α of domestic holdings and fraction κ of external holdings are indexed to the domestic price level, while the remaining fractions are nominal.[5] The law of one price holds in this economy, the nominal exchange rate is unity and there is no distinction between indexed and foreign currency bonds.

For concreteness, assume that the government issues b bonds today. If the entire issuance is bought by international investors ($\delta = 0$) and is indexed to the price level ($\kappa = 1$), then b represents a promise to repay b units of consumption tomorrow to the foreign investors, just like in a standard sovereign default model with external debt only. If the bonds are nominal ($\kappa = 0$), issuing b represents a claim to b pesos tomorrow. This claim is worth b/P_t units of consumption today, but only b/P_{t+1} when repaid tomorrow, where P_t and P_{t+1} are the domestic price levels in the two periods. When inflation $\pi_{t+1} = P_{t+1}/P_t - 1$ is positive, the real value of this nominal debt will be eroded. In general, b is a promise to pay $(\kappa+\alpha)b$ units of consumption pus $((1-\kappa)+(1-\alpha))b$ pesos, split between foreigners and locals according to δ.

[5] Instead of each creditor group purchasing a mix of real and nominal debt, we can also imagine separate groups of investors who only invest in one type, with shares κ and $1-\kappa$ of the total mass of external investors respectively, and similarly for domestic investors, each with no pricing power within their group.

The government takes this structure as given, and we only need to consider optimal total debt issuance b when solving its optimal policy problem. It is reasonable to assume a fixed portfolio structure like this since in the data the cross-section heterogeneity of sovereign debt portfolios is much larger than the within country variation over time (see the data section). It has the advantage of allowing us to study the implications for sovereign crises of departing from the standard assumption of real and external only. We can readily use this framework to study the effects on macroeconomic outcomes of shifts in the composition of sovereign debt portfolios for policy analyses, as we will further below.[6]

4.2 Household problem

The representative agent maximizes the discounted expected lifetime utility from cash good consumption, credit good consumption and labor:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \tag{1}$$

where β is the subjective discount factor and

$$u(c_1, c_2, n) = \gamma \frac{c_1^{1-\sigma}}{1-\sigma} + \log c_2 - \phi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

Preferences are separable and logarithmic in credit good consumption, and thus consistent with balanced growth (King et al., 1988). The Frisch elasticity of labor supply is assumed to be constant and equal to ν. The curvature σ and weight on cash good consumption γ will determine the extent to which money and inflation are important in the economy.

The within-period timing of events follows Svensson (1985). Households enter period t with nominal money balances $\bar{m}_t$ and sovereign bonds δb_t if the government is in good credit standing. The productivity shock realizes, and households observe all prices: w_t, the real wage rate, τ_t the labor income tax rate, q_t the unit bond price denominated in units of consumption, and $\bar{p}_t$, the nominal price level. Next, bond, labor and goods markets open. Households decide how much labor n_t to supply and receive labor income net of taxes in exchange. In the bond market, if the government is in good credit standing, they receive payments from the government for outstanding debt and they choose how much new government bonds δb_{t+1} to buy. Repayments on outstanding debt are made in cash for fraction $1-\alpha$ of the debt, and in consumption units for the remainder. If the government is in bad credit standing because it has defaulted in the past and has not yet regained access to credit markets, the bond market remains closed. In the goods market, households make consumption purchases: Cash goods can only be purchased with existing money balances, that is they are subject to the following cash-in-advance (CIA) constraint:

$$\bar{p}_t c_{1t} \leq \bar{m}_t, \forall t \tag{2}$$

This makes expected inflation costly if it constrains and thus distorts optimal cash good consumption, and makes unexpected inflation costly because the real value of existing money balances is lower than households budgeted for at the end of the previous period. Credit good purchases $\bar{p}_t c_{2t}$ can be financed with all income. Next, the money market opens where households choose new money holdings $\bar{m}_{t+1}$ to carry over to tomorrow. Finally, production and consumption take place.

[6] See Engel and Park (2016) for a model of endogenous denomination choice.

The budget constraint of the household is then given by

$$c_{1t} + c_{2t} + \frac{\bar{m}_{t+1}}{\bar{p}_t} + (1 - d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right) q_t \delta b_{t+1} = (1 - \tau_t) w_t n_t + \frac{\bar{m}_t}{\bar{p}_t} + (1 - d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right) \delta b_t, \forall t$$

where d_t is a binary indicator equal to 1 if the government chooses to default and 0 otherwise.[7] In order to make this problem stationary, we divide the nominal variables $\bar{p}_t$ and $\bar{m}_t$ as well as the nominal portion of bonds, $(1-\alpha)\delta$ by the aggregate money supply, $\bar{M}_t$, following Cooley and Hansen (1991).[8] Defining the money growth rate between tomorrow and today as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$, we can then write the normalized household budget constraint as

$$c_{1t}+c_{2t}+\frac{1+\mu_t}{p_t} m_{t+1}+\left(\frac{(1-\alpha)(1+\mu_t)}{p_t} + \alpha\right)(1-d_t) q_t \delta b_{t+1} = (1-\tau_t) w_t n_t + \frac{m_t}{p_t} + \left(\frac{1-\alpha}{p_t} + \alpha\right)(1-d_t)\delta b_t, \forall t \tag{3}$$

4.3 Solution to the household problem and prices

Since we will be solving the optimal policy problem of the government when taking household choices into account, it is convenient to characterize the solution to the household problem first and find implied expressions for prices that have to hold in a competitive equilibrium. A solution to the household problem are sequences $\{c_{1t}, c_{2t}, n_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ that maximize (1) subject to (2) and (3), taking as given price sequences $\{\mu_t, q_t, w_t, \tau_t, p_t\}_{t=0}^{\infty}$, a sequence of credit standings $\{d_t\}_{t=0}^{\infty}$ and initial money and bond holdings m_0 and b_0. The associated first order conditions that have to hold in every period are given by

$$\frac{-u_{3t}}{u_{2t}} = (1-\tau_t) w_t \tag{4}$$

$$u_{2t}\frac{1+\mu_t}{p_t} = \beta E_t\left[\frac{u_{1t+1}}{p_{t+1}}\right] \tag{5}$$

$$u_{2t}\left(\frac{(1-\alpha)(1+\mu_t)}{p_t} + \alpha\right) q_t = \beta E_t\left[(1-d_{t+1}) u_{2t+1}\left(\frac{1-\alpha}{p_{t+1}} + \alpha\right)\right] \tag{6}$$

$$u_{1t} - u_{2t} \geq 0 \tag{7}$$

where expectations are taken over the productivity shock tomorrow, conditional on the realization today and u_i denotes the partial derivative of the utility functions with respect to the i^{th} argument.

The first equation is the standard intratemporal first order condition that characterizes the optimal trade-off between credit good consumption and leisure. The last condition arises from the CIA constraint. If the constraint is not binding, the condition holds with equality and consumption is not distorted - marginal utilities are equalized. Equation (6) is an Euler condition for the marginal utility of credit good consumption in periods t and $t+1$. We can rearrange it as a standard asset pricing equation to see that the bond price

[7] Here d is equal to 1 both of the government has chosen to default in the current period, and if he has chosen to default in a prior period and has not yet regained market access, so his credit standing is bad. We will distinguish between the default decision and the credit standing in the recursive formulation of the government problem below.

[8] We should write bond service as $\frac{1-\alpha}{\bar{p}_t}\delta\bar{b}_t + \alpha\delta b_t$ (and analogously bond revenue), but after normalizing $\frac{\bar{b}}{\bar{M}} = b$ because of market clearing so we go straight to b for notational simplicity.

compensates domestic bondholders for consumption, default and inflation risk:

$$q_t = \beta E_t \left[\underbrace{\frac{u_{2t+1}}{u_{2t}}}_{\substack{\text{Consumption}\\\text{risk}}} \underbrace{(1-d_{t+1})}_{\text{Default risk}} \underbrace{\frac{\frac{1-\alpha}{p_{t+1}}+\alpha}{\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha}}_{\text{Inflation risk}} \right] \tag{8}$$

Everything else equal, a high stochastic discount factor increases household savings demand and thus the bond price. A high probability of default in the next period lowers the price and raises the interest rates that households demand in order to hold government bonds. When all bonds are nominal ($\alpha = 0$), expression (8) reduces to

$$q_t = \beta E_t \left[\frac{u_{2t+1}}{u_{2t}} (1-d_{t+1}) \frac{1}{1+\pi_{t+1}} \right]$$

where

$$1+\pi_{t+1} \equiv \frac{p_{t+1}(1+\mu_t)}{p_t} \tag{9}$$

is consumer price inflation.[9] Households need to be compensated to hold bonds whose real payout is expected to be eroded through price inflation.

First order condition (5), finally, can shed light on how the cash in advance constraint distorts the economy. Rearranging we can write

$$1 = \beta E_t \left[\frac{u_{1t+1}}{u_{2t}} \frac{1}{1+\pi_{t+1}} \right]$$

On the left hand side, we have the price of money. In a steady state when the cash in advance constraint does not bind we have $u_1 = u_2$ from (7), and note that the gross risk free long run real interest rate is just $\frac{1}{\beta}$ from (8). Thus in an undistorted steady state, the Friedman rule holds in this economy: Inflation is minus the real interest rate (gross inflation equals β), and the nominal interest rate is zero. If the cash in advance constraint binds in steady state, or outside a steady state if marginal utility of cash consumption is expected to be sufficiently high, (expected) price inflation may be positive. Households are willing to pay a positive nominal interest rate to hold money balances if they expect to be cash constrained.

4.4 External bond pricing

External debt is assumed to be bought by competitive, risk neutral, international investors as is standard in the sovereign default literature (see among many others Chatterjee and Eyigungor 2012). The investors' opportunity cost is the international real risk free rate r, and they choose government bond purchases, b_{t+1}, in order to maximize expected profits. Their static maximization problem in each period t is

$$\max_{b_{t+1}} \Pi_t = q_{et} \left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa \right) b_{t+1} - \frac{1}{1+r} E_t \left[(1-d_{t+1}) \left(\frac{1-\kappa}{p_{t+1}} + \kappa \right) \right] b_{t+1}$$

which pins downs the price of external debt as

$$q_{et} = \frac{1}{1+r} E_t \left[(1-d_{t+1}) \frac{\frac{1-\kappa}{p_{t+1}}+\kappa}{\frac{(1-\kappa)(1+\mu_t)}{p_t}+\kappa} \right] \tag{10}$$

[9] Recall that p is the nominal price level scaled by the aggregate money stock, that is $p \equiv \frac{\bar{p}}{\bar{M}}$, and that $1+\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t}$. Then $1+\pi_t \equiv \frac{\bar{p}_t}{\bar{p}_{t-1}} = \frac{p_t}{p_{t-1}} \frac{M_t}{M_{t-1}} = \frac{p_t(1+\mu_{t-1})}{p_{t-1}}$

Notice how this is analogous to the pricing of domestic debt. The price compensates bond holders for default risk and, to the extent that $\kappa < 1$, inflation risk. Since international investors are assumed to be risk neutral, there are no risk premia in the external bond price expression.

4.5 Government budget constraint and default

The government finances exogenous expenditures g using labor income taxes, seigniorage and net bond revenues, so its budget constraint in period t is given by

$$\begin{aligned} g = &\tau_t w_t n_t + \frac{(1+\mu_t)}{p_t} - \frac{1}{p_t} \\ &+ (1-d_t)\left\{\left[\delta q_t\left(\frac{(1-\alpha)(1+\mu_t)}{p_t} + \alpha\right) + (1-\delta)q_{et}\left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa\right)\right] B_{t+1} \right. \\ &\left. - \left[\delta\left(\frac{1-\alpha}{p_t} + \alpha\right) + (1-\delta)\left(\frac{1-\kappa}{p_t} + \kappa\right)\right] B_t \right\} \end{aligned} \tag{11}$$

If the government's credit standing is bad or it chooses to default in the current period, it no longer services outstanding debt, but also cannot borrow. Its credit standing will remain bad for a random period of time, as discussed below in the recursive formulation of the government problem.

Notice that we assume that the government defaults simultaneously on domestic residents and foreigners. This assumption has support in the data. Empirically, sovereign bonds are frequently structured such that default in one triggers default in another via cross-default and acceleration clauses. Choi et al. (2011) for example find that 85% of Brady issuers included cross-default and acceleration clauses in their international bond issues between 1982 and 2000, and 63% of other sovereigns (excluding very safe borrowers like the US and Germany). Trebesch et al. (2012) document that collective action clauses (CACs), including cross-default and acceleration clauses, have become well-established market practice for bonds issued under international law. They show that Mexico in particular, which we will calibrate the model to later, has since 2003 issued more than 90% of its sovereign bonds under New York law which typically includes CACs.[10]

Note also that we assume that a default decision entails a complete write down of outstanding debt. In the data, debt is typically renegotiated following sovereign defaults and bondholders recover part of their investment. We abstract from debt renegotiations for tractability. Allowing for non-zero debt recovery rates would, everything else equal, reduce default incentives uniformly across bond types.[11]

In addition to an exclusion cost of default, we assume that default entails direct output costs in the form of drops in productivity. This assumption is standard in the sovereign default literature, and used to capture in a reduced form way real costs that are frequently associated with sovereign debt crises. It is quantitatively important here, as well as in these other studies, that these costs are asymmetric and higher in times of high productivity. The asymmetry implies that the sovereign accumulates significant amounts of debt in good times, when he has strong disincentives to default.

[10] Other support for the prevalence of simultaneous default on domestic and foreign bondholders includes Erce and Diaz-Cassou (2010) who show that seven out of eleven external restructurings in a sample of recent debt crises were preceded or followed by domestic debt restructurings.

[11] In particular, there is no systematic evidence of discrimination against groups of bondholders in renegotiations. Trebesch et al. (2012) find in a comprehensive review of sovereign defaults globally that foreign bondholders have not received systematically unfavorable terms relative to domestic bondholders.

4.6 Production and market clearing

Output is produced using a linear technology,

$$y_t = z_t n_t \tag{12}$$

where z_t is an exogenous productivity shock, y_t is output and n_t labor supply. Labor will be paid its marginal product, so that in equilibrium

$$w_t = z_t \tag{13}$$

In equilibrium, markets clear and so money balances held by domestic households must equal money supplied by the government: $\bar{m}_t = \bar{M}_t, \forall t$, and thus normalized money supply $m_t = M_t = 1, \forall t$. Similarly for bond markets, bonds held must be equal bonds supplied, $B_t = \delta b_t + (1-\delta) b_t = b_t, \forall t$. The cash in advance constraint will hold with equality in equilibrium (but not necessarily bind) since households at the margin prefer to consume rather than carry over money balances that at best have a zero return, so

$$c_{1t} = \frac{1}{p_t}, \forall t \tag{14}$$

Finally the resource constraint of the small open economy, derived by combining the household and government budget constraints after imposing money and bond market clearing, will have to hold at all points in time:

$$c_{1t} + c_{2t} + g + x_t \quad = \quad y_t, \forall t \tag{15}$$

where x_t is equal to net external savings:

$$x_t \quad \equiv \quad (1-\delta)\left[\left(\frac{(1-\kappa)}{p_t} + \kappa\right) b_t - \left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa\right) q_{et} b_{t+1}\right] \tag{16}$$

5 Recursive optimal policy problem

We will next state the optimal policy problem of the government recursively and define the equilibrium we solve for. Denote from now on all current period variables x_t x and future variables x_{t+1} by x'.

The government is benevolent and will choose new bond issuances and how much money to print in order to maximize household utility, while taking into account resource constraints and that the implied allocations must be a competitive equilibrium consistent with household optimization. We assume that the government cannot commit to its policies. This introduces a time inconsistency problem: Since debt issuance today is non-distortionary, it faces incentives to accumulate debt in favor of other distortionary sources of finance at its disposal, like labor income taxes and the inflation tax. Given an outstanding stock of debt, however, it faces incentives to erode its real value through inflation or explicit default in order to lower the debt burden. Due to lack of commitment, the government is unable to take into account that its past actions have lead to its current, indebted state.

We analyze throughout the time consistent Markov perfect equilibrium of the economy, in line with much of the literature on sovereign default and public policy under lack of commitment (e.g. Klein et al. 2008 and Arellano 2008). In this equilibrium, the government's optimal policy rules are functions of the current state of the economy only: the productivity shock z and its debt position b. This means that, for example, the optimal borrowing and default decisions will be given by some rule $b' = \boldsymbol{h}(z,b)$ and $d = \boldsymbol{d}(z,b)$, and similarly credit good consumption will be given by, say, $c_2 = \boldsymbol{c_2^1}(z,b)$ if the government is in good credit standing and $c_2 = \boldsymbol{c_2^0}(z)$ otherwise.[12]

When solving its problem, the government takes as given the rules that future governments follow, and so it internalizes how its choice of borrowing affects the future state of the economy and future policies. This implies in particular that it internalizes how additional bond issuances and seigniorage raise future default and inflation risk, and thus reduce the current revenues they can generate from selling additional bonds or printing money today. In equilibrium the optimal rules of the current government must coincide with the ones of future governments.

To define the government problem and the equilibrium, it will be helpful to rewrite the government budget constraint (11) in terms of allocations only as an implementability constraint. We can substitute out the the money growth rate, the tax rate and the domestic bond price using the competitive equilibrium expressions (4) through (6), and substitute out the price level using (14) to get:[13]

$$u_2(c_2)\left((1-d)\left(c_1(1-\alpha)+\alpha\right)\delta b - c_2\right) - u_3(n)n = (1-d)\left(\zeta_1^1(z,b') + \zeta_2^1(z,b')\delta b'\right) + d\zeta_1^0(z) \tag{17}$$

where

$$\zeta_1^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))\boldsymbol{c_1^1}(z',b')u_1\left(\boldsymbol{c_1^1}(z',b')\right) + \boldsymbol{d}(z',b')\boldsymbol{c_1^0}(z')u_1\left(\boldsymbol{c_1^0}(z')\right)\right] \tag{18}$$

$$\zeta_2^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b')u_2\left(\boldsymbol{c_2^1}(z',b')\right)\left(\boldsymbol{c_1^1}(z',b')(1-\alpha)+\alpha\right)\right] \tag{19}$$

$$\zeta_1^0(z) \equiv \beta\eta\zeta_1^1(z',0) + \beta(1-\eta)E_{z'|z}\left[\boldsymbol{c_1^0}(z')u_1\left(\boldsymbol{c_1^0}(z')\right)\right] \tag{20}$$

Since utility is separable, we include with a slight abuse of notation only the dependence on the respective argument in the $u_i(\cdot)$ expressions. On the right hand side of (17), $\zeta_1^1(z,b')$ and $\zeta_1^0(z)$ are revenues from printing money in repayment and default, respectively, and $\zeta_2^1(z,b')\delta b'$ is revenue from selling domestic bonds. Notice that written in this way, the right hand side of (17) is independent of outstanding debt and solely a function of borrowing b'. It describes how the government generates revenue from selling new bonds, and how this revenue is affected by future policies. We will discuss these effect further below.

The expression for money revenues in default, (20), captures our assumption that all outstanding debt is written off and the government cannot issue new bonds in default, but has a η chance each period of re-entering capital markets with zero debt. The assumptions of a complete debt writedown implies that the functions in bad credit standing do not depend on debt or borrowing.

We will rewrite the resource constraint (15) in a similar way as the government budget constraint. Substituting out the external bond price (10) (and again using (14)), the resource constraint becomes

$$c_1 + c_2 + g + (1-d)(1-\delta)\left[\left(c_1(1-\kappa)+\kappa\right)b - \zeta_3^1(z,b')b'\right] = zn \tag{21}$$

where

$$\zeta_3^1(z,b') \equiv \frac{1}{1+r}E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))\left(\boldsymbol{c_1^1}(z',b')(1-\kappa)+\kappa\right)\right] \tag{22}$$

[12] One could also include the credit standing as one of the states, but the notation then becomes more burdensome, so we instead define different functions conditional on good and bad credit standing. It will become clear below why rules in bad credit standing do not depend on b.

[13] See (A.1) in the appendix for the derivation.

Here, $\zeta_3^1(z,b')(1-\delta)b'$ is the revenue from issuing external bonds. In the case of only real bonds, $\kappa = 1$, this reduces to the standard expression present in many models of sovereign default, where the bond price compensates for default risk only.

We are now ready to state the government problem recursively as follows.

Problem 1 (Government Problem). The state of the economy is $z \in \mathbb{R}_{++}$ and $b \in \mathbb{R}$. If the government is in good credit standing, it has the choice of whether to remain current or default. Its option value of default is given by

$$V(z,b) = \max_{d\in\{0,1\}} (1-d)V^1(z,b) + dV^0(z) \tag{23}$$

In case of repayment, it chooses optimal new bond issuances b', cash good consumption c_1, credit good consumption c_2 and labor n subject to the implementability constraint, resource constraint and the household FOC for the cash in advance constraint. Its value of repayment is given by

$$\begin{aligned} V^1(z,b) = &\max_{b',c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[V(z',b')] \\ &\text{subject to (7),(17) and (21)} \\ &b' \in \mathbb{R}, c_1, c_2 \in \mathbb{R}_+, n \in [0,1] \end{aligned} \tag{24}$$

In case of default, it chooses optimal cash good consumption c_1, credit good consumption c_2 and labor n subject to the same constraints, and re-enters capital markets with no debt in the next period with probability η, so the value of default is

$$\begin{aligned} V^0(z) = &\max_{c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[\eta V(z',0) + (1-\eta)V^0(z')] \\ &\text{subject to (7),(17) and (21)} \\ &c_1, c_2 \in \mathbb{R}_+, n \in [0,1] \end{aligned} \tag{25}$$

With the statement of the government problem in hand, we can define the equilibrium that we solve for.

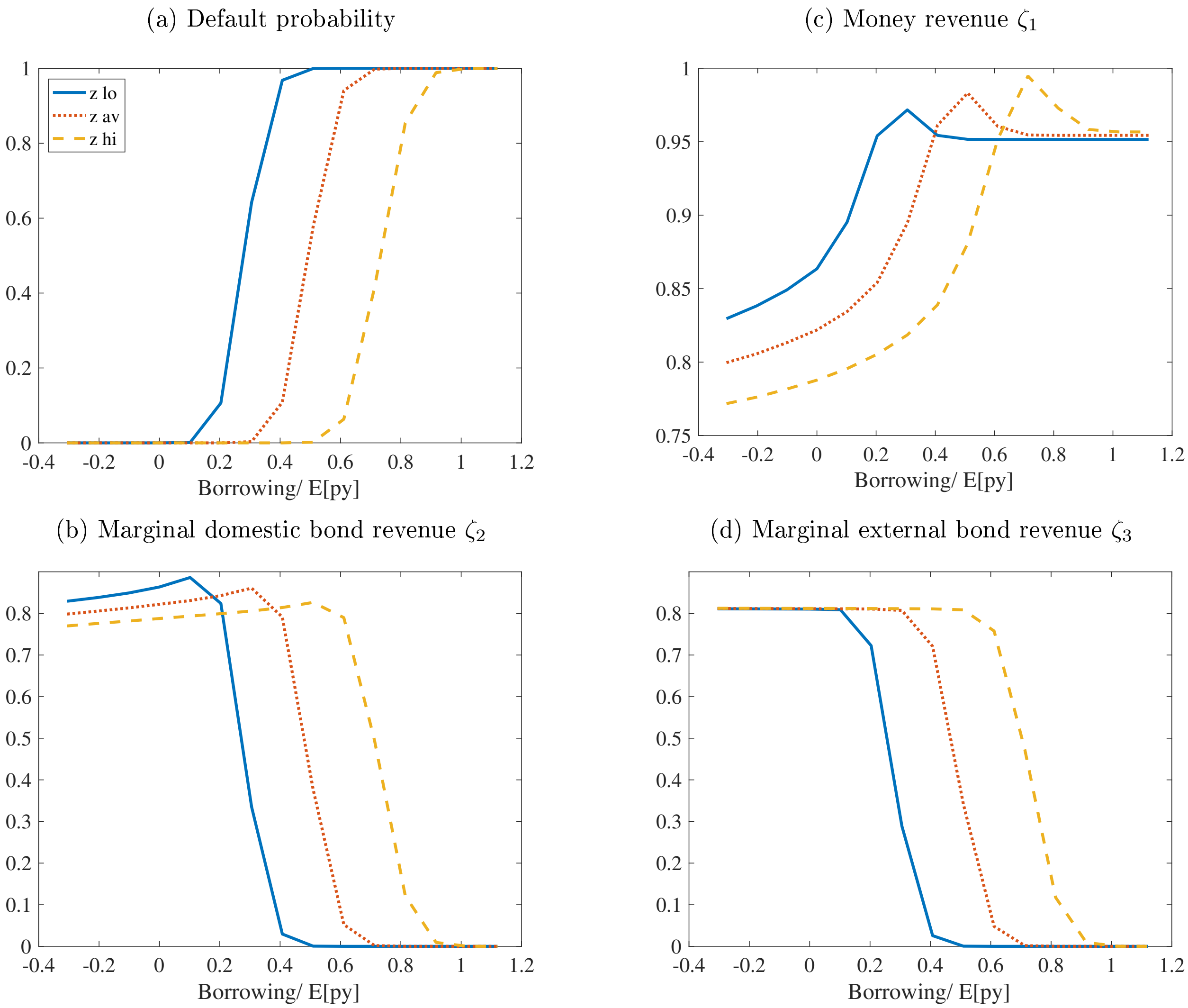
Definition 1 (Markov Perfect Equilibrium). *A Markov perfect equilibrium of the economy are value function $V(z,b)$ with associated policy function $\boldsymbol{d}(b,z)$, value function in repayment $V^1(z,b)$ with associated policy functions $\boldsymbol{h}(z,b)$, $\boldsymbol{c_1^1}(z,b)$, $\boldsymbol{c_2^1}(b,z)$, $\boldsymbol{n^1}(b,z)$, and value function in default $V^0(z)$ with associated policy functions $\boldsymbol{c_1^o}(z)$, $\boldsymbol{c_2^0}(z)$, and $\boldsymbol{n^0}(z)$ that solve (23), (24) and (25), with $b' = \boldsymbol{h}(z,b)$, $d = \boldsymbol{d}(z,b)$, and for (24) in good credit standing $c_1 = \boldsymbol{c_1^1}(z,b)$, $c_2 = \boldsymbol{c_2^1}(z,b,)$, $n = \boldsymbol{n^1}(z,b)$, while for (25) in bad credit standing $c_1 = \boldsymbol{c_1^0}(z)$, $c_2 = \boldsymbol{c_2^0}(z)$, $n = \boldsymbol{n^0}(z)$.*

Equilibrium prices can then be read off the competitive equilibrium conditions (4) through (6), (10) and (14).

5.1 Government incentives

We now turn to a discussion of the revenue that the government generates from bond issuance to understand the incentives that it faces when deciding on optimal borrowing. We will make use of the numerical solution of the calibrated model here to develop the intuition, and Figure (4) plots the equilibrium expressions of equations (18), (19) and (22) as well as the default probability as a function of borrowing b' and productivity

Figure 4: Revenues from borrowing and default risk



z. Suppose also, as will turn out to be the case, that the the government faces stronger incentives to default and to raise the price level the higher its debt and the lower productivity, so that the default rule $\boldsymbol{d}(z,b)$ and price level, the inverse of cash consumption $\boldsymbol{c_1^1}(z,b)$, are both increasing in b and decreasing in z, while credit good consumption $\boldsymbol{c_2^1}(z,b,)$ is decreasing in b and increasing in z.

Consider first the marginal revenue generated from issuing bonds, (19). The expression shows that higher borrowing today b' increases the risk of default and higher prices tomorrow via $\boldsymbol{d}(z',b')$ and $\boldsymbol{c_1^1}(z',b')$. The revenue that the government is able to generate from selling b' today is thus reduced to the extent that the issuance raises future default and inflation risk. Notice too that when domestic debt is real so $\alpha = 1$, only default risk continues to reduce the revenue from bond issuance. Consumption risk works the opposite way as default and inflation risk and tends to offset some of their effects: If credit good consumption $\boldsymbol{c_2^1}(z',b')$ is decreasing in b', then household demand for bonds increases as their expected marginal utility of consumption tomorrow increases, and so they are willing to pay more for government bonds today.

The net effect of these three forces varies with parameters as well as the state. We plot the equilibrium solution in panel (b) of Figure (4). It shows that for sufficiently low borrowing, the consumption risk factor dominates and increased household demand for savings raises the marginal revenue from issuing bonds. But for high borrowing, default and inflation risk become increasingly important factors and reduce $\zeta_2^1(z,b')$.[14]

Turning to the revenue generated from printing money, equation (18) shows that borrowing affects it through similar channels as in the case of bond revenue. On the one hand, more borrowing will erode the real value of money balances tomorrow, and thus reduce households' willingness to hold money today. This is the direct effect of $\boldsymbol{c_1^1}(z',b')$ in the expression, analogous to its appearance in the (19) where it was multiplied by $1-\alpha$. On the other hand, this increases the marginal utility of cash consumption tomorrow, $u_1\left(\boldsymbol{c_1^1}(z',b')\right)$, and households that expect to be cash-strapped increase their demand for money. This is parallel to the effect via the marginal utility of credit consumption in (19). Whether borrowing raises or lowers revenues from money creation depends on the relative size of these effects, and in particular the elasticity of substitution for cash consumption. For our calibration, the net effect is positive, as shown in panel (c) of Figure (4): Borrowing raises revenue from money creation, conditional on repayment. Intuitively, high borrowing implies that the expected future price level is high. Agents thus expect their real money balances to be worth less and to be cash strapped, and so increase their demand for money balances to hold today and carry over to tomorrow. To meet this money demand, the government can print money today, a source of revenue, and relax its budget constraint. The Figure shows that revenues from this money creation peak for borrowing such that default risk is very high but not quite certain. If borrowing is so high that default next period is certain, ζ_1^1 is below its peak since in default the government is not expected to raise prices as much - after all, it has no debt to service.

Revenue from borrowing from foreigners, finally, is shown in panel (d) of the Figure and is similar to bond price functions in standard sovereign default models. If borrowing is sufficiently low for default risk to be zero, ζ_3^1 is below the international risk free rate only to the extent that external debt is nominal and subject to inflation risk. We can see that numerically this effect is small, the function is barely decreasing at these low levels of b'. Beyond a certain point, default risk dominates the shape of this revenue function, much like its domestic counterpart ζ_2^1.

What can we say about the effect of borrowing on inflation risk? Borrowing turns out to increase inflation risk, but only up to a point. Inflation between tomorrow and today is given by (use equation (5) and recall equation (9)):

$$1+\pi(z,z',b') = \frac{c_1(1+\mu(z,b'))}{\boldsymbol{c_1^1}(z',b')} = \frac{1}{u_2}\frac{\zeta_1^1(z,b')}{\boldsymbol{c_1^1}(z',b')} \tag{26}$$

Note that in equilibrium c_1 and u_2 will depend on z and b, but we suppress this dependence in the expression and focus only on the effect of borrowing, b'. Cash consumption $\boldsymbol{c_1}(z',b')$ in the denominator is decreasing in b', which tends to increase inflation. $\zeta_1^1(z,b')$ is increasing in b' up to a point when default risk kicks in, as we have seen. So as long as default risk plays no large role, borrowing drives up inflation. However, increasing borrowing beyond a certain point will raise default risk and thus may, depending on the relative changes in the ratio $\frac{\zeta_1^1(z,b')}{\boldsymbol{c_1^1}(z',b')}$, actually lower inflation. Intuitively, in default the government faces weaker inflation incentives since it has no debt to service, and so when default risk becomes sufficiently high, it becomes increasingly likely that the government will not need to inflate tomorrow because it defaults instead.

[14]Note that this is not inconsistent with the household's stochastic discount factor being negatively correlated with productivity, thus implying positive risk premia. This will in fact turn out to be a feature of the model in equilibrium. But ζ_2 (as well as ζ_1 and ζ_3) do not depend on b, and thus do not depend on current period credit consumption, $\boldsymbol{c_2^1}(z,b)$. The government's marginal revenue from issuing bonds expressed in this way does not depend on the stochastic discount factor, but only on the marginal utility of consumption tomorrow, effectively.

The preceding discussion centered on the effects of borrowing on revenues, default and inflation risk. What then shapes the government's optimal borrowing decision, how does it decide how much to borrow in equilibrium? The optimal policy problem implies an intertemporal Euler equation that characterizes the trade-offs that it faces. The problem is in general non differentiable because of the presence of default, but conditional on repaying tomorrow, we can write down the first order condition for an interior borrowing choice (as in Arellano and Ramanarayanan 2012, for example). Denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (17) and the resource constraint (21) respectively, let $\mathcal{Z}(b)$ be the set of z for which the government does not default given b, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b') = E_{z'|z}[\hat{\zeta}_i^1(z',b')], i = 1,2,3$. Then the government's Euler equation is given by:[15]

$$\lambda_1\left(\delta\zeta_2^1(z,b') + \delta b'\frac{\partial\zeta_2^1(z,b')}{\partial b'} + \frac{\partial\zeta_1^1(z,b')}{\partial b'}\right) + \lambda_2(1-\delta)\left(\zeta_3^1(z,b') + b'\frac{\partial\zeta_3^1(z,b')}{\partial b'}\right) = \beta E_{z'|z}\left[\left(\lambda_1'\delta\hat{\zeta}_2^1(z',b') + \lambda_2'(1-\delta)\hat{\zeta}_3^1(z',b')\right)|z' \in \mathcal{Z}(b')\right] \quad (27)$$

This equation describes how the government trades off costs and benefits of additional borrowing. On the right hand side, we have the marginal costs of an additional unit of bonds issued. Borrowing more today implies that debt is high tomorrow, and so both the government budget constraint and the resource constraint are tighter as the government faces higher debt service to both households and foreigners. This is captured by their respective future Lagrange multipliers λ_1' and λ_2', weighted by the size of the repayment that has to be made for each unit of debt. These costs are balanced against the marginal benefits of borrowing on the left hand side of the equation. Issuing an additional unit of bonds yields direct revenue of $\delta\zeta_2^1$ and $(1-\delta)\zeta_3^1$, but it also raises future default, inflation and consumption risk and thus changes the revenue that government can generate from b' today, as shown by the presence of the derivatives of ζ_1^1, ζ_2^1 and ζ_3^1.

The sign of the derivatives in the Euler equation shapes the government's incentives to borrow. We saw in Figure (4), that for sufficiently high borrowing the slopes of the functions with respect to borrowing become weakly negative. In other words, the marginal benefit of borrowing falls and the government has less of an incentive to do so. Total revenues, $\zeta_1^1(z,b') + \zeta_2^1(z,b')\delta b'$ and $(1-\delta)\zeta_3^1(z,b')b'$ trace out a Laffer curve.

Note that ζ_1 remains positive throughout since even in default the government continues to have control over the money stock. The presence of money in the economy gives the government an additional reason to borrow. Similarly, domestic ownership increases incentives to borrow due to consumption risk - recall the positive slope of ζ_2^1. Whether stronger incentives to borrow in the presence of nominal and domestic debt also translate into higher inflation and default rates depends of course on how much the government chooses to borrow in equilibrium. This is what we turn to next - the calibration and simulated model results.

6 Calibration

The model is calibrated to Mexico, both for reasons of data availability and because it has a relatively mixed government debt portfolio along the dimensions of interest to this paper. We pick the parameters governing the debt portfolio to replicate its public debt composition. Between 2004Q1 and 2015Q4, the years for which data is available, its debt was on average 65% domestically owned, 78% of which was in real terms, while all of the domestic debt was nominal. We thus set $\delta = 0.65, \kappa = 0.78$ and $\alpha = 0$.

[15] See (A.2) in the appendix for the derivation.

Table 2: Parameters

Parameter			Description/ target
Internal debt share	δ	0.65	Mexico debt portfolio
Real external debt share	κ	0.78	Mexico debt portfolio
Real internal debt share	α	0.00	Mexico debt portfolio
International real risk free rate	r	0.0034	90-day US Treasury bill yield 1.37% annually
Subjective discount factor	β	0.98	Mexico real risk free rate 8.4% annually
Re-entry probability after default	η	0.25	Average exclusion duration 1 year
Frisch elasticity of labor supply	ν	2.00	Standard macro estimate
Government spending	g	0.0363	Average government spending 11% of GDP
Persistence productivity process	ρ	0.95	Neumeyer and Perri (2005)

Parameter			Description/ target	Data	Model
Volatility productivity process	σ_ϵ	0.015	Mexico GDP volatility	0.018	0.017
Labor weight in utility	ϕ	10.00	Fraction of time working	0.33	0.33
Cash good weight in utility	γ	0.025	Cash/credit good ratio	0.82	0.88
Curvature cash good	σ	2.80	Inflation rate	0.042	0.040
Default cost threshold	χ	-0.10	Default rate	0.016	0.018

We assume that log productivity follows a stationary AR(1) process

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \epsilon \sim N(0, \sigma_\epsilon^2)$$

In choosing the persistence and volatility of the productivity process, we follow Neumeyer and Perri (2005) and set $\rho = 0.95$ while calibrating σ_ϵ to match the volatility of quarterly Mexican HP-filtered log real GDP from 1997Q1, the end of its last debt crisis, through 2015Q4.

For the default cost, we follow Arellano (2008) and assume that in default, productivity is given by

$$z_{def} = \begin{cases} z & \text{if } z < \chi \\ \chi & \text{otherwise} \end{cases}$$

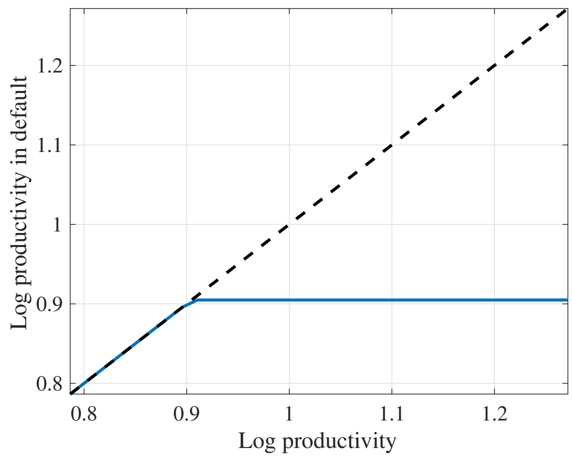
and set the parameter χ to match an annual default rate of 1.6%. Mexico did not default since 2004, when our debt portfolio data starts, so we choose a default rate based on its postwar (1945-2014) experience. Going back much further would include times with very different macroeconomic conditions, especially the public debt portfolio which is the center of this analysis. Figure (5) plots the resulting effective productivity.

The probability of market re-access following a default is set to $\eta = 0.25$ implying an average exclusion period of 1 year. This is in line with estimates by Asonuma and Trebesch (2016) according to which Mexico renegotiated its debt within around 1 year of defaulting in the 1980s. The international risk free rate is set to $r = 0.0034$, which corresponds to an average annualized interest rate of 1.37% on 90-day US T-bills between 1997Q1 and 2015Q4. We choose the disutility of labor ϕ to match a fraction of time spent working of 33%, which then determines $g = 0.0363$ to give average government spending to GDP of 11% between 1997Q1 and 2015Q4. The Frisch elasticity of labor supply is set to $\nu = 2$, within the standard range of macro estimates for the elasticity of aggregate hours,[16] and we set $\beta = 0.98$ for an annual domestic real risk free rate of 8.4%.

The weight of cash goods in utility γ is set to match the cash to credit good ratio observed in the data in

[16] Chetty et al. (2011) for example report a value of 2.84 from a meta-analysis.

Figure 5: Default cost function



order to capture the extent to which households are exposed to monetary distortions. Let ϕ be the long term share of cash consumption in production net of national savings, $c_1 = \phi(y - g - x)$, so the targeted cash credit ratio is given by

$$\frac{c_1}{c_2} = \frac{\phi}{1 - \phi}$$

If $c_1 = M/p$, as is the case in the model in equilibrium, we can express ϕ as a function of observables: $\phi = \frac{M}{P(y-g-x)}$. Using quarterly data on nominal GDP, government spending, net exports and the M1 money stock, this implies an average cash/credit good ratio of 0.82 in Mexico between 2004Q1 and 2015Q4.

The parameter governing the elasticity of cash good consumption, σ, is chosen to match average consumer price inflation of 4.2% annually between 2004Q1 and 2015Q4. Inflation is computed using quarterly CPI data from the IMF. Table (2) presents the calibrated parameter values, details on data sources are in the appendix.

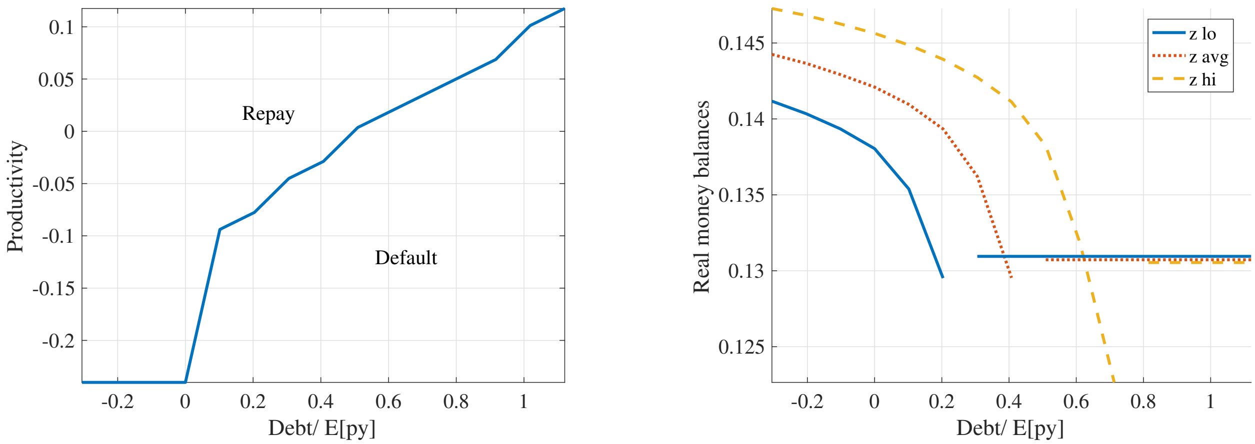
7 Numerical results

7.1 Equilibrium policies

In this section, we discuss the optimal policies for inflation, default, interest and tax rates as a function of the state. Since these equilibrium functions underlie the simulation results, it will be useful to discuss them first. The main take-away will be that inflation and seigniorage move together with default risk, and rise as debt increases or productivity falls: The government resorts to the printing press whenever borrowing becomes expensive because of default risk, but outright default is still too unappealing.

Figure (6) shows how equilibrium real money balances and the default decision depend on debt and output in the numerical solution of the model. The left panel shows that, as in standard sovereign default models, default incentives are high when debt is high and output is low. The right panel shows that a similar logic applies to the price level, as alluded to earlier in section (5.1): The Figure plots equilibrium real money balances as a function of debt relative to average GDP for three levels of productivity, average and one standard deviation above and below the mean. We see that in repayment, when debt is high or productivity is low, the government chooses a relatively high price level, equivalently low real money balances, to reduce

Figure 6: Default and real money balances as functions of the state



the real debt burden. For sufficiently high debt or low productivity, default becomes optimal which in the Figure corresponds to the range where the functions no longer depend on debt.[17]

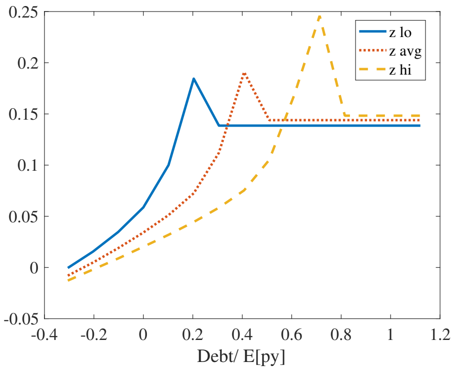
We next turn to equilibrium interest and tax rate policies. The top two rows of Figure (7) plot labor income tax rates, the domestic bond yield (equivalently, the nominal interest rate since the share of real domestic debt is zero in the calibration), expected inflation and the money growth rate. Note that these are all functions of the state as well as the optimal borrowing choice $b' = \boldsymbol{h}(z, b)$. Their general shapes reflect the same government incentives to erode the real value of debt: Since these incentives higher when productivity is low or debt is high, we see that, in repayment, both tax rates and domestic bond yields fall with productivity and rise with debt, up to the point of default. Expected inflation follows a similar pattern but instead of increasing right up until the point of default, it drops at levels of debt just shy of default. The reason is that in default inflation is low as there is no outstanding debt to finance, so as default risk rises, agents incorporate that expectation of lower inflation. This pattern is also reflected in money growth rates, which in general rise with debt, but fall just prior to default: Household money demand falls as they expect inflation to be low in default, and thus their real money balances to be worth more.

How does the government optimally choose among sources of public finance for its expenditures in different regions of the state space? The bottom two rows of Figure (7) illustrate the contributions of labor income taxes, seigniorage and net borrowing to expenditures as a function of the state, assuming that borrowing follows the optimal rule $b' = \boldsymbol{h}(z, b)$. As debt rises or output falls, the government finances itself increasingly with taxes and seigniorage. Bond finance becomes too expensive since interest rates rise with inflation and default risk, as we have seen. Quantitatively, seigniorage makes a relatively minor contribution to public finance of at most 10% of expenditures at average output levels. Labor income taxes make up the bulk of the revenue throughout. At low levels of debt, bond revenue contributes more than 50%, but this quickly falls as default and inflation risks rise. Note that this is not a statement about the equilibrium observed contributions but conditional on the level of debt. We will investigate in the next section whether the model

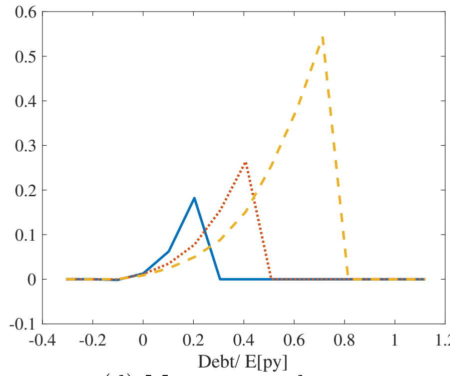
[17]Note that in default the price level is slightly higher when productivity is high than when it is low. This means that cash good consumption is lower in good times than in bad. The reason for this is that optimal money issuance is forward-looking. When choosing money balances for tomorrow, the government takes into account expectations of what its real value will be. These expectations vary with the state, since there is a probability that he will regain market access (see equation (20)). In particular, when productivity is high, since it is mean reverting, future productivity in the case of re-entry is expected to be lower, which implies lower revenues from money creation today than if productivity is expected to be higher. This revenue shortfall in turn implies lower consumption.

Figure 7: Equilibrium rates and revenues as a function of the state

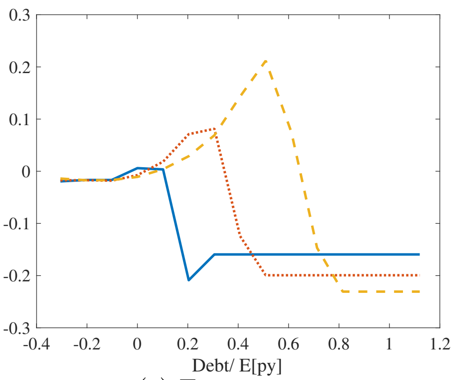
(a) Tax rate



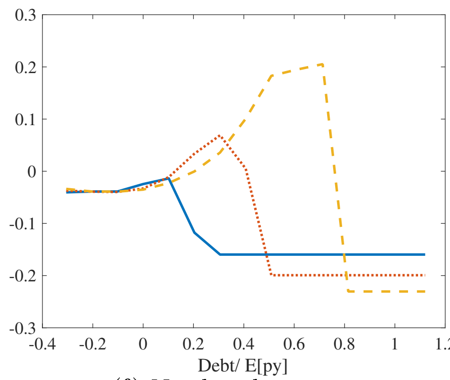
(b) Domestic bond yield



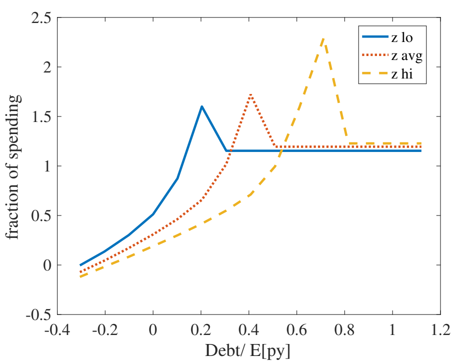
(c) Expected inflation rate



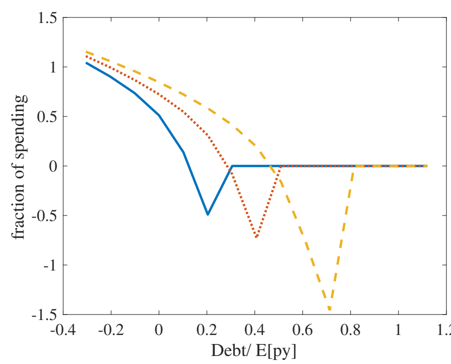
(d) Money growth rate



(e) Tax revenue



(f) Net bond revenue



(g) Seigniorage revenue

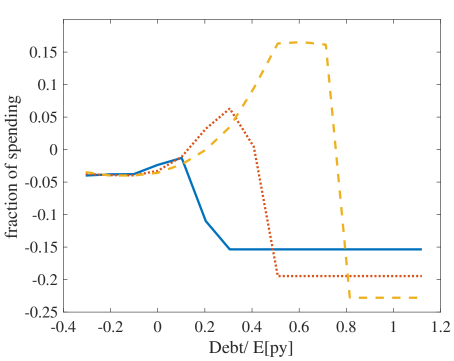
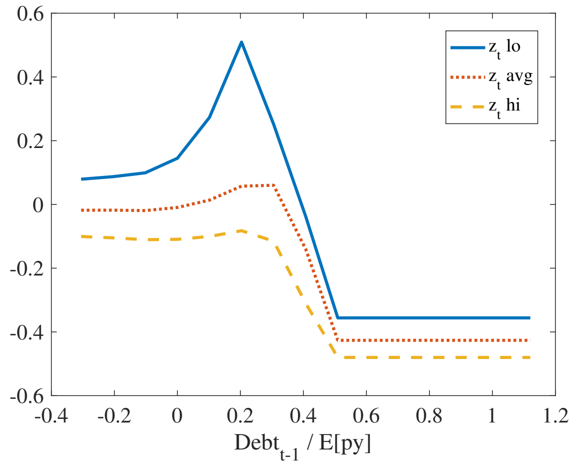


Figure 8: Equilibrium inflation $\pi_t = \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$ as a function of debt outstanding b_{t-1} for a range of productivity realizations z_t, conditional on productivity being at its long run mean the previous period, $z_{t-1} = E[z]$



also replicates reasonable financing shares in equilibrium in simulations.

We can finally also look at the model predictions for realized inflation as a function of current and past fundamentals. Recall from equation (26) that realized inflation between t and $t-1$ depends not just on the state in $t-1$, but also the output realization in t. Figure (8) therefore plots equilibrium inflation $\pi_t \equiv \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$ as a function of debt outstanding b_{t-1} for a range of output realizations z_t, conditional on productivity being at its long run mean the previous period, $z_{t-1} = E[z]$, and with borrowing following the equilibrium rule $b_t = \boldsymbol{h}(z_{t-1}, b_{t-1})$.

The Figure shows that the government optimally inflates when productivity turns out to be worse than in the previous period (in this case, below average) and debt is sufficiently high but not high enough for outright default. If debt is high but productivity improved relative to yesterday and is now above trend, the government avoids very high inflation rates. The reason for this is again that its ability to raise revenue from bonds varies with productivity. In good times, bond finance is relatively cheap as well as non-distortionary, so the government uses that instead of inflation to balance its budget. In bad times it cannot raise as much revenue from borrowing due to inflation and default risk premia. As a result, it shifts its financing to inflation, as long as debt is not so high that this becomes too onerous and he prefers outright default. The Figure shows that we expect inflation in the model to be negatively correlated with productivity.

7.2 Simulations: Model vs data

To evaluate model performance we compare simulated model statistics to their empirical counterparts. We simulate the model for 5000 periods, discarding the first 100 to eliminate the effects of initial conditions, and calculate statistics from the simulated model data. Table (3) summarizes key model and data statistics that were not targeted in the calibration. Model and actual data are treated symmetrically, that is both are filtered using an HP-filter, and output, consumption and hours are in logs. The data sources are mostly the OECD or Banco de Mexico, and are discussed in detail in the appendix in section (B.5).

Table 3: Model and data: Targeted and untargeted moments

		Data	Model
Means	Domestic nominal bond yield	0.075	0.144
	External spread	0.019	0.034
	Tax revenue/GDP	0.120	0.110
	Seigniorage/GDP	0.004	0.003
	External public debt service/GDP (annualized)	0.024	0.052
Volatilities	Domestic nominal interest rate	0.0057	0.0443
	External spread	0.0066	0.0245
	Tax revenue/GDP	0.0118	0.0315
	Seigniorage/GDP	0.0002	0.0051
	External public debt service/GDP (annualized)	0.0056	0.0100
	Inflation	0.0067	0.1318
Correlations with GDP	Domestic nominal interest rate	-0.29	-0.24
	External spread	-0.65	-0.57
	Tax rate	-0.17	-0.67
	Seigniorage/GDP	-0.73	0.71
	External debt service/GDP (annualized)	0.25	0.48
	Inflation	-0.40	-0.03
	Volatility of aggregate private consumption relative to GDP	1.30	2.25
	Volatility of net exports/GDP relative to GDP	0.31	1.23

The Table shows that the model is successful at matching aspects of public finance in Mexico and its macroeconomy more generally. Overall, it does better at capturing first moments of the data, as well as its cyclical properties, than at second moments, generally overpredicting volatility. The first section of the table compares average interest rates and spreads, tax and seigniorage revenues and as well as debt service in the model and the data. The model predicts that the average yield on nominal, domestically held bonds 14.4%, compared to 7.5% in the data, and that the spread on external bonds is 2.8% compared to 1.9% in the data. In the data, the external spreads are from JP Morgan's EMBI while in the model it is the spread over the international risk free rate of externally held bonds. Note that in the benchmark calibration these are denominated in a mix of nominal and real terms. In the data, the EMBI likewise includes both real (foreign currency denominated) and nominal bonds.

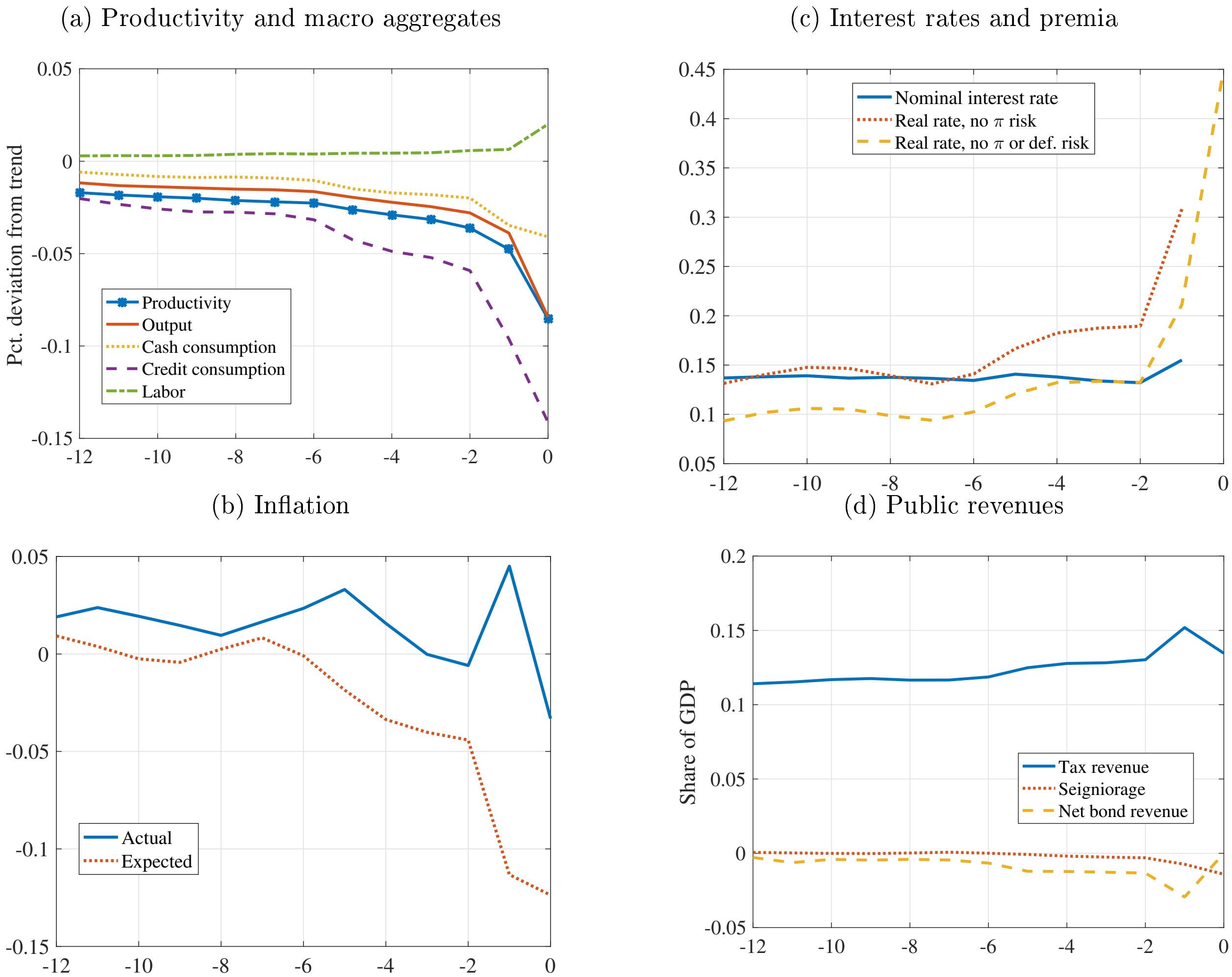
In terms of public finance, the model predicts that the majority of government revenue comes from labor taxation - 11% of GDP compared to 12% in the data - and very little from seigniorage at less than half a percent of GDP in both the data and the model. In the data, we include personal income taxes less consumption taxes. We do not include oil related tax revenues or corporate revenues, in line with the model abstracting from oil sectors and a fully-fledged corporate sector with capital accumulation.[18] The model overpredicts public debt service compared to the data, at just over 5% of GDP compared to 2.4%.[19]

The second section of Table (3) compares volatilities of the same variables in the model and the data. For the calibrated output volatility, the model variables are generally more volatile than their data counterparts.

[18]We still choose to calibrate to the overall government consumption to GDP ratio of the economy, since extractive industries and capital increase both public revenues and investment expenditures. We effectively assume that they are revenue neutral.

[19]As is well known in the literature, net borrowing flows and debt stocks are too closely related in any 1 period debt model compared to the data where borrowers do not roll over all their debt stocks every period. There is thus a tension whether to evaluate model predictions of stocks or flows. We choose to compare flows (debt service) rather than stocks, as for example in Arellano (2008), given that in this model flow revenues of the government and their relative contributions to its budget are important. The reason we focus on external public debt service is that total public debt service, or more generally net bond revenues are more difficult to measure in the data.

Figure 9: Event study: The run-up to a default in the model, average over simulations



This reflects that hours in the model are not volatile enough, so we need a relatively volatile shock to match output volatility, whereas in the data hours contribute more to the volatility of GDP.[20]

The model does well at capturing the cyclical aspects of the data. It successfully predicts countercyclical domestic interest rates, external spreads, tax rates and inflation. It underpredicts the countercyclicality of inflation, and predicts procyclical rather than countercyclical seigniorage, which we will return to below.

7.2.1 A typical default episode

To better understand the role of outright default in the model, Figure (9) plots the behavior of key variables in the four years prior to a default episode. The figure is based on 100 default samples from the simulated data. It shows how defaults are preceded by productivity, output and consumption downturns. Hours move very little. Output in the period of default is on average around 10% below trend. Actual inflation rises prior to a default as productivity continues to disappoint and the government has an incentive to raise the price level. This is in contrast with expected inflation which falls in the Figure. The reason for this is twofold.

[20] In the model, the volatility of log hours relative to the volatility of log GDP is 0.24. Neumeyer and Perri (2005) report a value of 0.43 for Mexico in the data.

Table 4: The role of denomination and ownership

	Benchmark	(1) Internal debt		(2) Real external debt		(3) Real internal debt	
		$\delta = 0.90$	$\delta = 0.00$	$\kappa = 1.00$	$\kappa = 0.00$	$\alpha = 0.50$	$\alpha = 0.25$
Debt/GDP	0.068	0.098	0.055	0.067	0.047	0.095	0.080
Default rate	0.018	0.025	0.016	0.018	0.007	0.030	0.025
Inflation	0.040	0.385	-0.069	0.020	0.128	0.190	0.101
ρ(Inflation,GDP)	-0.03	-0.36	0.16	0.01	-0.25	-0.41	-0.24
Tax rate	0.11	0.08	0.12	0.11	0.10	0.10	0.11
ρ(Tax rate,GDP)	-0.67	-0.82	-0.44	-0.64	-0.59	-0.84	-0.79

Benchmark: $\delta = 0.65, \kappa = 0.78, \alpha = 0$.

First, productivity is mean reverting so agents continue to expect a recovery with lower inflation. Second, because inflation is low in default, expected inflation falls as default risk rises. This points to why in this benchmark calibration inflation is only mildly countercyclical, and seigniorage procyclical.

The third panel of the Figure shows that the main factor that contributes to high nominal interest rates prior to a default is not default or inflation, but primarily consumption risk: Default risk increases moderately, but the real rate increases much more than the nominal rate as expected inflation falls. The real, default- and inflation-risk-free rate on the other hand, which only moves because of consumption risk (see Equation (8)), increases the most. This reflects household desire to dissave in order to smooth consumption during the downturn.

A limitation of the model as a complete representation of an actual economy is the lack of detail of what happens in a default episode. In reality, debt crises are frequently associated with high inflation. Nonetheless, whatever the reason that drives high inflation during debt crises, it is not related to servicing the debt if the country stops making payments, and thus beyond the scope of the current paper. We restrict attention to inflation as a result of access to bonds and limited commitment only.

7.3 The role of debt denomination and ownership

We now answer the key question of the paper: What role do portfolio characteristics play in determining equilibrium model outcomes? The key conclusion from this section will be that domestic ownership, especially when the debt is real, and nominal external debt are important drivers of inflation.

With the calibrated model as the benchmark, we vary one of the parameters determining the portfolio characteristics at a time and report the resulting statistics from the simulated models in Table (4).[21] The Table shows that higher shares of internal debt (column 1) raise equilibrium debt, inflation and default rates. For our calibration, if all debt is external, the government implements inflation of -6.9% annually. This rises to nearly 40% when 90% of the debt are held internally. The increase in inflation is accompanied by an increase in default rates to 2.5% and higher debt - the reason for higher inflate and default rates - of almost 10% rather than just over 5% with no domestic debt.

Turning to the role of denomination, higher shares of real debt raise debt levels and default rates, regardless

[21] One concern is that these experiments conflate the effects of denomination and ownership. For example, starting from the calibrated benchmark model and moving towards a higher domestic share at the same time increases the overall nominal share of the debt, since more of the domestic than the external debt is nominal in the calibration. The conclusions from this table carry through to an alternative experiment where we keep denomination constant when varying ownership, and vice versa. The results are available on request.

Table 5: Four portfolio types

	(1) Internal real	(2) Internal nominal	(3) External real	(4) External nominal
Debt/GDP	0.106	0.104	0.074	0.070
Default rate	0.031	0.022	0.026	0.020
Inflation	0.355	0.452	0.036	0.044
ρ(Inflation,GDP)	-0.46	-0.46	-0.14	-0.08
Tax rate	0.09	0.08	0.11	0.11
ρ(Tax rate,GDP)	-0.85	-0.83	-0.73	-0.71

$\{(\delta_i, \kappa_i, \alpha_i)_{i=1,4}\} = \{(0.79, 0.63, 0.24), (0.85, 0.36, 0.04), (0.49, 0.85, 0.36), (0.59, 0.64, 0.06)\}$

of whether the share of real external (column 2) or real internal debt (column 3) is increased. The effect of denomination on inflation however depends on whether the debt is owned domestically or abroad. If a large share of *external* debt is real, then this reduces inflation, whereas if a large share of *internal* debt is real, then this increases inflation. In both cases there are two incentives at play. On the one hand, the incentive to inflate is lower with real debt because inflation is less effective at reducing the debt burden. On the other, inflation incentives are stronger the higher the stock of debt, and real debt in the model raises equilibrium debt levels. In the case of external debt, the former dominates and the government reduces inflation optimally, while in the case of internal debt the latter dominates and the government inflates more.

These effects of portfolio characteristics on inflation and borrowing highlight that it is debt denomination and ownership *jointly* that matter for equilibrium crisis risk and inflation outcomes. It cautions against looking at denomination in isolation when considering portfolio management policies, and suggests, for example, that pursuing a strategy of borrowing abroad in nominal terms, like Mexico, or domestically in real terms, like Brazil, may not be conducive towards a low inflation environment.

7.4 Four portfolio types

In the empirical motivation, we documented to what extent countries differ in their average portfolio structures (see for example Figure (3)), and we can now use the model to get a better of sense of how much macroeconomic outcomes differ depending on the types of debt portfolios that we typically observe in the world. For example, how do the predictions compare for a typical external, real debt economy relative to a typical external, nominal debt economy?

To this end, we assign each country in the data to one of four portfolio types: (i) internal real (IR), (ii) internal nominal (IN), (iii) external real (ER) or (iv) external nominal (EN). Countries above the median internal debt share are categorized as E countries, otherwise as I. Within each of those two categories, countries above the median share of real debt are then categorized as R, and N otherwise. The real debt share here is defined as $\omega \equiv \delta\alpha + (1-\delta)\kappa$.[22] We solve the model four times, once for each type of portfolio, where the parameters governing the portfolio shares are given by the averages within each type in the data. The resulting model statistics and portfolio compositions are reported in Table (5).

The Table shows that a typical internal debt economy has more debt and higher inflation than one with

[22]The implied portfolio characteristics of each group change either not at all or only slightly if we (i) first group by denomination and then by ownership, (ii) define the real debt share based on the direct measure x (see the appendix) rather than the weighted average, (iii) do any of this in the pooled data set rather than the cross-section. Section (B.3) in the appendix lists the member countries of each category. For example, Argentina is classified as ER, India as IN, Mexico as EN, and Russia as IR.

Table 6: Portfolio features: The correlation with inflation and debt in the data

	(1) Log Inflation	(2) Log Debt/GDP	(3) Log Inflation	(4) Log Debt/GDP
Log real share of internal	0.001***	0.002**		
	(0.000)	(0.001)		
Log real share of external	-0.002***	-0.006*		
	(0.001)	(0.003)		
Log internal share of total			0.021***	0.066*
			(0.006)	(0.037)
Log real share of total			-0.000	-0.002
			(0.001)	(0.003)
Observations	886	970	886	970

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

largely external debt. Moreover, while ownership is clearly related to debt and inflation, denomination is not. Economies that both finance themselves largely internally, but differ in the denomination of their bonds, are much more similar in terms of inflation and debt accumulation than two economies that borrow in similar denomination, but one domestically and one from abroad.

Note finally that in these illustrative cases portfolio composition is such that the positive effect on inflation of real internal debt is not obvious. This happens because economies that have relatively high real shares of internal debt also tend to have relatively high real shares of external debt in the data. While the former drives up inflation, the latter lowers it, as discussed. For the illustrative portfolio cases here, we see that the representative IR economy has less inflation than the representative IN economy.

7.5 Testing the model predictions

We find an important interaction effect between denomination and ownership in shaping debt, inflation and default outcomes. Domestic real debt raises inflation, while external real debt does not. The importance of ownership in determining the effect of denomination on inflation is a testable implication of the theory so we check whether it is borne out in the data. Specifically, we estimate

$$\log \pi_{it} = \beta_1 \log \alpha_{it} + \beta_2 \log \kappa_{it} + \epsilon_{it}$$

where the model predicts that we should find $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 < 0$. We use our panel data set of 24 countries from 2004Q1 through 2015Q4, and use panel-feasible generalized least squares (FGLS) to estimate the coefficients, allowing for heteroskedastic error structures and panel-specific autocorrelation, with both dependent and independent variables in logs. The results are reported in column (1) of Table 6: The model prediction is consistent with the data, with the coefficients of the expected sign and significant at the 1% level. Real internal debt shares increase inflation in the data, real external shares lower it.

To further check whether there is also support for the mechanism that the model suggests for this relationship, namely higher debt when internal debt is real but not when external debt is real, we also estimate

$$\log\left(\frac{b_{it}}{y_{it}}\right) = \beta_1 \log \alpha_{it} + \beta_2 \log \kappa_{it} + \epsilon_{it}$$

The results from this specification are in column (2) of Table 6, and likewise corroborate the theory: $\hat{\beta}_1$ is significantly positive, $\hat{\beta}_2$ negative.

A second testable prediction of the model is the relative importance of ownership over denomination in determining debt levels and inflation rates for portfolio compositions that we typically observe in the data. We therefore estimate, again using FGLS with heteroskedastic errors and panel-specific autocorrelation structure:

$$\log\left(\frac{b_{it}}{y_{it}}\right) = \beta_1 \log \delta_{it} + \beta_2 \log \omega_{it} + \epsilon_{it}$$

and

$$\log \pi_{it} = \beta_1 \log \delta_{it} + \beta_2 \log \omega_{it} + \epsilon_{it}$$

Columns 3 and 4 of Table 6 report these results. Consistent with the theory, they show that internal debt shares δ are positively correlated with both debt and inflation ($\hat{\beta}_1$ is significantly positive in both regressions), while real shares are not ($\hat{\beta}_2$ is not significant).

7.6 Welfare

We are not only interested in the positive predictions of the model, but also the welfare implications of different portfolio structures. What composition is optimal not from a crisis prevention standpoint, but in terms of utility? To answer this, we compute the welfare gain of alternative economies, defined as the percentage of permanent credit good consumption that households in the benchmark economy would be willing to give up to live in an alternative economy instead.[23] We report the ex-ante welfare gain, that is the unconditional expectation of the gain with zero assets:

$$\Delta_1 = \exp\left((1-\beta)\left(E_z\left[V_A(z,0)\right] - E_z\left[V_B(z,0)\right]\right)\right) - 1$$

Table (7) reports the resulting gains associated with the variants of the calibrated benchmark model from section (7.3). It shows that borrowing internally is desirable. Increasing the nominal debt share from the benchmark level of two third to 90% yields welfare gains of nearly 0.1% of consumption. Moreover, doing so in real terms is preferable. Raising the real domestic debt share to 50% from 0 implies gains of 0.16% of consumption. Finally, if you have to borrow abroad, doing so in nominal terms is better: Lowering the real external debt share to 0 from the benchmark value of 78% gives gains of 0.18%.

Figure (10) presents welfare gains for economies for a wider range portfolio structures, with domestic debt shares $\delta \in [0, 0.9]$, real external debt shares $\kappa \in [0, 1]$, and real domestic debt shares $\alpha \in [0, 0.5]$. A similar picture emerges here: Higher domestic debt shares δ and domestic real shares α tend to yield welfare gains,

[23] Specifically, the welfare gain of economy A relative to benchmark economy B is defined as the percentage increase Δ in credit good consumption in the benchmark economy c_2^B that equalizes the expected lifetime utility from the two economies:

$$E_0 \sum_t \beta^t u(c_{1A}, c_{2A}, n_A) = E_0 \sum_t \beta^t u(c_{1B}, c_{2B}(1+\Delta), n_B)$$

Note that $E_0 \sum_t \beta^t u(c_{1X}, c_{2X}, n_X) = V_X(z,b), x = \{A, B\}$ and, due to our functional form assumption,

$$E_0 \sum_t \beta^t u(c_{1B}, c_{2B}(1+\Delta), n_B) = \frac{\log(1+\Delta)}{1-\beta} + E_0 \sum_t \beta^t u(c_{1B}, c_{2B}, n_B)$$

so that

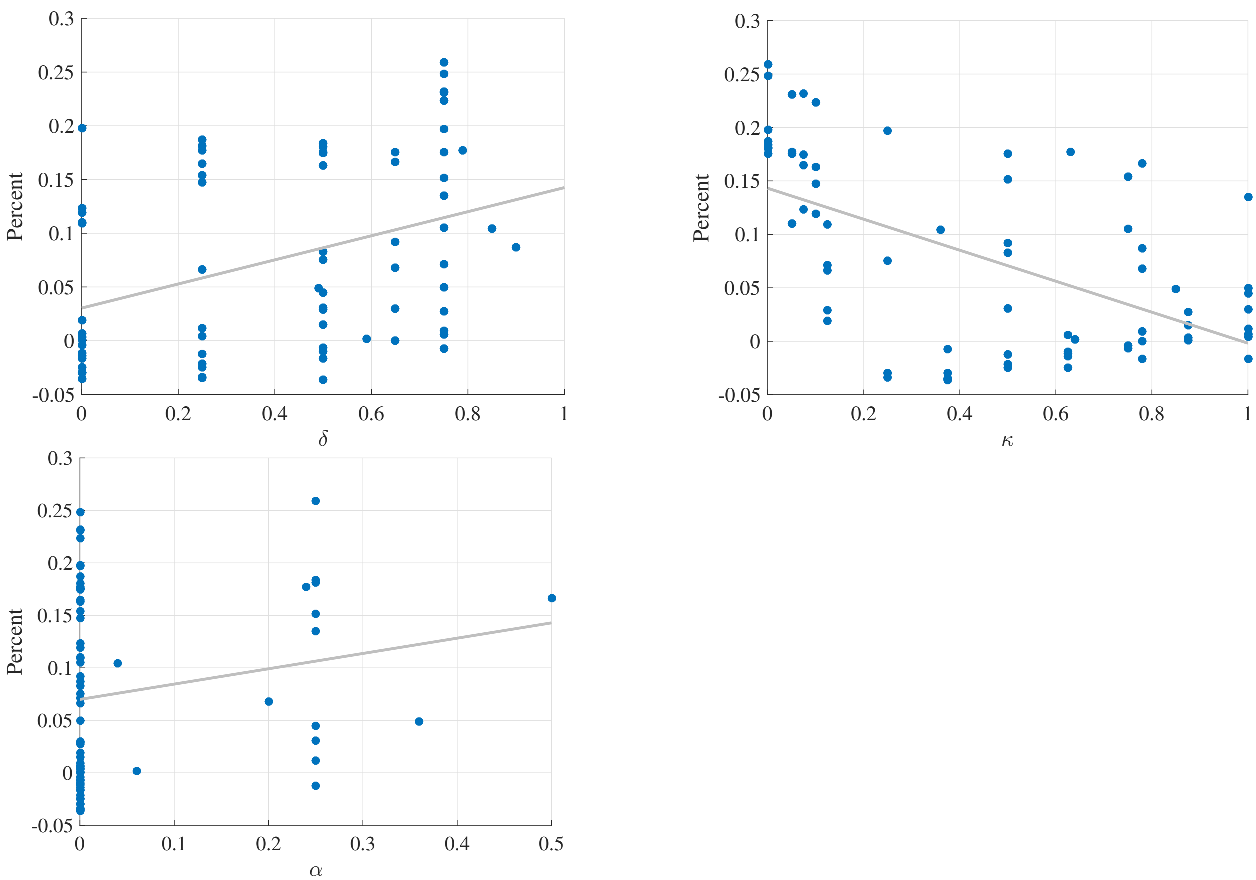
$$\Delta(z,b) = \exp\left((1-\beta)\left(V_A(z,b) - V_B(z,b)\right)\right) - 1$$

Table 7: Welfare gains (% of expected lifetime credit good consumption)

	(1) Internal debt		(2) Real external debt		(3) Real internal debt	
	$\delta = 0.90$	$\delta = 0.00$	$\kappa = 1.00$	$\kappa = 0.00$	$\alpha = 0.50$	$\alpha = 0.25$
Gain relative to benchmark	0.09	-0.02	-0.01	0.18	0.16	0.06

Benchmark: $\delta = 0.65, \kappa = 0.78, \alpha = 0$.

Figure 10: Ex-ante welfare gains as a function of internal debt shares (left) and external real debt shares (right)



while higher real external shares κ are detrimental for welfare from an ex-ante perspective.

Underlying these welfare results are two main reasons. First, there is a level effect. It is welfare improving to be able to implement a relatively high level of consumption overall. Disincentives to distort result in lowering borrowing costs, higher debt levels, and thus lower labor income tax rates, higher hours and overall higher consumption. These disincentives are stronger the higher domestic debt shares, or external nominal debt shares. The government also lowers the cash to credit goods ratio in these economies, and since credit goods carry a higher weight in the utility, increasing their consumption is at the margin better for welfare than consuming more cash goods. Despite higher inflation with more nominal portfolios, as well as higher hours, the reduced distortions from labor income taxes and resulting higher levels of total consumption are worth the monetary distortions and disutility from work for our calibration.

Second, there is a volatility effect. Higher real debt shares exacerbate volatility of consumption as inflation is used less readily to adjust debt burdens and smooth consumption. This is reflected in inflation becoming increasingly volatile and less countercyclical the higher the real share of external debt. The government lacks the commitment not to use inflation at all and so it will in "emergencies", but it is too costly to use efficiently when it would be beneficial, namely in downturns.

In the case of real domestic debt, the levels effect dominates. Despite relatively costlier inflation, the disincentives to expropriate your own residents is strong enough to lower borrowing costs and raise sustainable debt levels to allow the government to implement relatively high overall consumption levels, which more than offsets any welfare losses from increased volatility.

Overall, the model thus suggests that the possibility to inflate is valuable in a setting of borrowing without commitment. Countries are better off with market arrangements in which inflation is relatively high, but in which it can and is being used as an effective alternative to outright default to adjust debt burdens in downturns. Borrowing externally and in real terms like much of Latin America in the 1980s prevents exactly that and thus according to the model is detrimental for welfare.

8 Conclusion

Who holds emerging market government debt and whether it pays in nominal or real terms matters for crisis risk. In this paper we have characterized and quantified the role that debt denomination and ownership play for default and inflation outcomes by incorporating realistic debt portfolio structures into a dynamic general equilibrium model of sovereign borrowing without commitment. We have used the calibrated model to sharpen our understanding of the effects of shifts in the portfolio structure. Our key conclusions, which find support in the data, are that domestic debt ownership, and especially real domestic debt, is an important driver of inflation; that ownership is the more important factor affecting inflation and debt levels while denomination matters equally for default rates; and that portfolios that lead to high inflation and default rates are not detrimental for welfare, on the contrary. In terms of policy, the model thus suggests that developing domestic debt markets, particularly in real terms, and nominal external debt markets, like Mexico, can be a useful strategy for emerging market governments. It cautions against relying on purely external, real financing.

References

Aguiar, M., Amador, M., Farhi, E. and Gopinath, G. (2013), Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises, mimeo. 5

Aguiar, M. and Gopinath, G. (2006), Defaultable debt, interest rates and the current account, *Journal of International Economics* 69(1), 64–83. 3

Alfaro, L. and Kanczuk, F. (2010), Nominal versus indexed debt: A quantitative horse race, *Journal of International Money and Finance* 29(8), 1706–1726. 5

Araujo, A., Leon, M. and Santos, R. (2013), Welfare analysis of currency regimes with defaultable debts, *Journal of International Economics* 89(1), 143–153. 5

Arellano, C. (2008), Default risk and income fluctuations in emerging economies, *American Economic Review* 98(3), 690–712. 3, 15, 21, 26

Arellano, C., Atkeson, A. and Wright, M. (2015), External and Public Debt Crises, *in* 'NBER Macroeconomics Annual 2015, Volume 30', NBER Chapters, National Bureau of Economic Research, Inc, pp. 191–244. 4

Arellano, C. and Heathcote, J. (2010), Dollarization and financial integration, *Journal of Economic Theory* 145(3), 944–973. 5

Arellano, C. and Ramanarayanan, A. (2012), Default and the maturity structure in sovereign bonds, *Journal of Political Economy* . 20

Arslanalp, S. and Tsuda, T. (2014), Tracking Global Demand for Emerging Market Sovereign Debt, IMF Working Papers 14/39, International Monetary Fund. 5, 6, 38

Asonuma, T. and Trebesch, C. (2016), Sovereign Debt Restructurings: Preemptive Or Post-Default, *Journal of the European Economic Association* 14(1), 175–214. 21

Barro, R. J. (1997), Optimal management of indexed and nominal debt, NBER Working Papers 6197, National Bureau of Economic Research, Inc. 5

Bohn, H. (1990), Tax smoothing with financial instruments, *American Economic Review* 80(5), 1217–30. 5

Borensztein, E., Jeanne, O. D., Mauro, P., Zettelmeyer, J. and d Chamon, M. (2005), Sovereign Debt Structure for Crisis Prevention, IMF Occasional Papers 237, International Monetary Fund. 2

Burger, J. D. and Warnock, F. E. (2007), Foreign participation in local currency bond markets, *Review of Financial Economics* 16(3), 291–304. 5

Calvo, G. A. (1988), Servicing the public debt: The role of expectations, *American Economic Review* 78(4), 647–61. 5

Chatterjee, S. and Eyigungor, B. (2012), Maturity, indebtedness, and default risk, *American Economic Review* 102(6), 2674–99. 13

Chetty, R., Guren, A., Manoli, D. and Weber, A. (2011), Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins, *American Economic Review* 101(3), 471–75. 21

Choi, S. J., Gulati, G. M. and Posner, E. A. (2011), Political risk and sovereign debt contracts, *University of Chicago Institute for Law & Economics Olin Research Paper* (583). 14

Claessens, S., Klingebiel, D. and Schmukler, S. L. (2007), Government bonds in domestic and foreign currency: the role of institutional and macroeconomic factors, *Review of International Economics* 15(2), 370–413. 5

Cole, H. L. and Kehoe, T. J. (2000), Self-fulfilling debt crises, *Review of Economic Studies* 67(1), 91–116. 5

Cooley, T. F. and Hansen, G. D. (1991), The welfare costs of moderate inflations, *Journal of Money, Credit and Banking* 23(3), 483–503. 12

Corsetti, G. and Dedola, L. (2016), The "Mystery of the Printing Press" Monetary Policy and Self-fulfilling Debt Crises, CEPR Discussion Papers 11089, C.E.P.R. Discussion Papers. 5

Cuadra, G., Sanchez, J. and Sapriza, H. (2010), Fiscal Policy and Default Risk in Emerging Markets, *Review of Economic Dynamics* 13(2), 452–469. 4

Da-Rocha, J.-M., GimÃ©nez, E.-L. and Lores, F.-X. (2013), Self-fulfilling crises with default and devaluation, *Economic Theory* 53(3), 499–535. 5

Diaz-Gimenez, J., Giovannetti, G., Marimon, R. and Teles, P. (2008), Nominal debt as a burden on monetary policy, *Review of Economic Dynamics* 11(3), 493–514. 4

Du, W., Pflueger, C. E. and Schreger, J. (2016), Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy, NBER Working Papers 22592, National Bureau of Economic Research, Inc. 4

Du, W. and Schreger, J. (2015), Sovereign risk, currecny risk, and corporate balance sheets, mimeo. 4, 5, 38

Durdu, C. B. (2009), Quantitative implications of indexed bonds in small open economies, *Journal of Economic Dynamics and Control* 33(4), 883–902. 5

Eichengreen, B. and Hausmann, R. (1999), Exchange rates and financial fragility, NBER Working Papers 7418, National Bureau of Economic Research, Inc. 5

Engel, C. and Park, J. (2016), Debauchery and original sin: The currency composition of sovereign debt, mimeo. 4, 11

Erce, A. and Diaz-Cassou, J. (2010), Creditor discrimination during sovereign debt restructurings, Working Papers 1027, Banco de España;Working Papers Homepage. 14

Fischer, S. (1975), The demand for index bonds, *Journal of Political Economy* 83(3), 509–534. 5

Gumus, I. (2013), Debt denomination and default risk in emerging markets, *Macroeconomic Dynamics* 17(05), 1070–1095. 4

King, R. G., Plosser, C. I. and Rebelo, S. T. (1988), Production, growth and business cycles : I. The basic neoclassical model, *Journal of Monetary Economics* 21(2-3), 195–232. 11

Klein, P., Krusell, P. and Rios-Rull, J.-V. (2008), Time-consistent public policy, *Review of Economic Studies* 75(3), 789–808. 3, 15

Klein, P. and Rios-Rull, J.-V. (2003), Time-consistent optimal fiscal policy, *International Economic Review* 44(4), 1217–1245. 3

Lane, P. R. and Shambaugh, J. C. (2010), Financial Exchange Rates and International Currency Exposures, *American Economic Review* 100(1), 518–40. 5

Martin, F. (2009), A positive theory of government debt, *Review of Economic Dynamics* 12(4), 608–631. 4

Missale, A. (1997), Managing the public debt: The optimal taxation approach, *Journal of Economic Surveys* 11(3), 235–65. 5

Na, S., Schmitt-Grohe, S., Uribe, M. and Yue, V. Z. (2014), A Model of the Twin Ds: Optimal Default and Devaluation, NBER Working Papers 20314, National Bureau of Economic Research, Inc. 4, 6

Neumeyer, P. A. and Perri, F. (2005), Business cycles in emerging economies: the role of interest rates, *Journal of Monetary Economics* 52(2), 345–380. 21, 27

Nicolini, J. P. (1998), More on the time consistency of monetary policy, *Journal of Monetary Economics* 41(2), 333–350. 4

Niemann, S., Pichler, P. and Sorger, G. (2013), Public debt, discretionary policy, and inflation persistence, *Journal of Economic Dynamics and Control* 37(6), 1097–1109. 5

Paul, K., Vincenzo, Q. and Jose-Victor, R.-R. (2005), Optimal Time-Consistent Taxation with International Mobility Of Capital, *The B.E. Journal of Macroeconomics* 5(1), 1–36. 3

Pitchford, R. and Wright, M. L. J. (2012), Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment, *Review of Economic Studies* 79(2), 812–837. 6

Reinhart, C. M. and Rogoff, K. S. (2011), The forgotten history of domestic debt, *Economic Journal* 121(552), 319–350. 5

Roettger, J. (2014), Monetary and fiscal policy with sovereign default, mimeo. 4

Schumacher, J., Chamon, M. and Trebesch, C. (2015), Foreign Law Bonds: Can They Reduce Sovereign Borrowing Costs?, Annual Conference 2015 (Muenster): Economic Development - Theory and Policy 113199, Verein fÃŒr Socialpolitik / German Economic Association. 6

Svensson, L. E. O. (1985), Money and asset prices in a cash-in-advance economy, *Journal of Political Economy* 93(5), 919–44. 11

Trebesch, C., Papaioannou, M. G. and Das, U. S. (2012), Sovereign Debt Restructurings 1950-2010; Literature Survey, Data, and Stylized Facts, IMF Working Papers 12/203, International Monetary Fund. 14

Zettelmeyer, J., d Chamon, M. and Bi, R. (2011), The Problem that Wasn't; Coordination Failures in Sovereign Debt Restructurings, IMF Working Papers 11/265, International Monetary Fund. 6

Appendix

A Model Appendix

A.1 Deriving the implementability constraint

Write the government budget constraint (11) recursively as

$$\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p} + \alpha\right) b + g - \tau w n + x = \frac{1+\mu}{p} + (1-d)\delta q\left(\frac{(1-\alpha)(1+\mu)}{p} + \alpha\right) b'$$

where we have imposed money market clearing and x is defined as in (16). Then substitute for μ and q using (5) and (6), and use the definitions of ζ_1^1 and ζ_2^1 from the main text to get

$$u_2\left[\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p} + \alpha\right) b + g - \tau w n + x\right] = (1-d)\left(\zeta_1^1(z,b') + \delta\zeta_2(z,b')b'\right) + d\zeta_1^0(z)$$

Now substitute for the tax rate using (4)

$$u_2\left[\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p} + \alpha\right) b + g - y + x\right] - u_3 n \quad = \quad (1-d)\left(\zeta_1^1(z,b') + \delta\zeta_2(z,b')b'\right) + d\zeta_1^0(z)$$

ans use market clearing (15) and 14 to arrive at (17). This constraint is the same as in a closed economy (except that the debt stock and borrowing are scaled by δ), external savings only enter explicitly through the resource constraint.

A.2 Deriving the government Euler condition

Recall that we denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (17) and the resource constraint (21), let $\mathcal{Z}(b)$ be the set of z for which the government does not default given b, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b') = E_{z'|z}[\hat{\zeta}_i^1(z',b')], i = 1,2,3.$

FOC with respect to b':

$$\lambda_1\left(\frac{\partial\zeta_1^1(z,b')}{\partial b'} + \delta\zeta_2^1(z,b') + \delta b'\frac{\partial\zeta_2^1(z,b')}{\partial b'}\right) + \lambda_2(1-\delta)\left(\zeta_3^1(z,b') + b'\frac{\partial\zeta_3^1(z,b')}{\partial b'}\right) + \beta E_{z'|z}\left[\frac{\partial V^1(z,b')}{\partial b'}|z' \in \mathcal{Z}(b')\right] = 0$$

Envelope condition:

$$\frac{\partial V^1(z,b)}{\partial b} = -\lambda_1\delta\hat{\zeta}_2^1(z,b) - \lambda_2(1-\delta)\hat{\zeta}_3^1(z,b)$$

Combining these by substituting out the value function yields expressions (27).

B Data Appendix

B.1 Portfolio data

The data set is available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip. We use the following variables with corresponding sheet and row in the data set spreadsheet:

- (GGall) General government debt: Table 1, 1:25
- (ExtGGall) External general government debt: Table 2, 1:25
- (GG) General government debt securities: Table 1, 27:52
- (ExtGG) External general government debt securities: Table 2, 53:78
- (LCCG) Local currency central government debt securities: Table 1, 54:79
- (ExtLCCG) External local currency central government debt securities: Table 2, 107:132
- (GGy) General government debt to GDP: FX, 27:51

Foreign holdings of general government debt exclude foreign official loans. Data on foreign ownership of local-currency central government debt securities are available for most countries in the sample from national data sources. Like Arslanalp and Tsuda (2014), we use this as a proxy for foreign ownership of local-currency general government debt securities, assuming that foreign holdings of local government debt securities are small.

The authors do not discuss how local currency indexed debt is classified. Assuming that it is treated as local currency debt, our estimates for α and κ are lower bounds. Du and Schreger (2015) who construct similar measures of government debt portfolio shares note that, where available, nonresident holdings holdings of indexed debt are very small relative to nonresident holdings of local currency debt, suggesting that our estimate of κ is unlikely to be a substantial underestimate. Similarly for domestic real debt shares, indexed bond markets in general tend to be small relative to non-indexed so α is unlikely to be large and certainly smaller than κ.

B.2 Inferring portfolio shares

Consider the following stylized representation of the sovereign's debt portfolio:

	Nominal	Real	Nom+Real
Internal	$\delta(1-\alpha)$	$\delta\alpha$	δ
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$	$1-\delta$
Int+Ext	x	$1-x$	1

We can obtain estimates of δ, x and the external share of nominal debt $y \equiv \frac{(1-\delta)(1-\kappa)}{x}$ from the database as follows. The internal debt share is computed as $\delta = 1 - \frac{ExtGGall}{GGall}$. The nominal share of debt is approximated as $x = \frac{LCCG}{GG}$. Ideally we would like to know total central government debt securities for the denominator, but these are not available in the database. This is a good approximation as long as local government debt is small relative to general government debt. The external share of nominal debt is calculated as $y = \frac{ExtLCCG}{LCCG}$.

We then use these estimates to infer α and κ: Using the definition of y,

$$\kappa = 1 - \frac{xy}{1-\delta}$$

and since from the table, $1 - \alpha = \frac{x-(1-\delta)(1-\kappa)}{\delta}$,

$$\alpha = 1 - \frac{x(1-y)}{\delta}$$

It is possible that α and also κbecome negative when x is too large relative to y. Given our estimate of x and y this never happens for κ, and we truncate x to ensure it does not happen for α, ie by imposing that

$$x \leq \frac{\delta}{1-y}$$

This is reasonable on the assumption that the measure of local currency debt as a share of the total is noisier and less reliable than the foreign currency share of external debt.

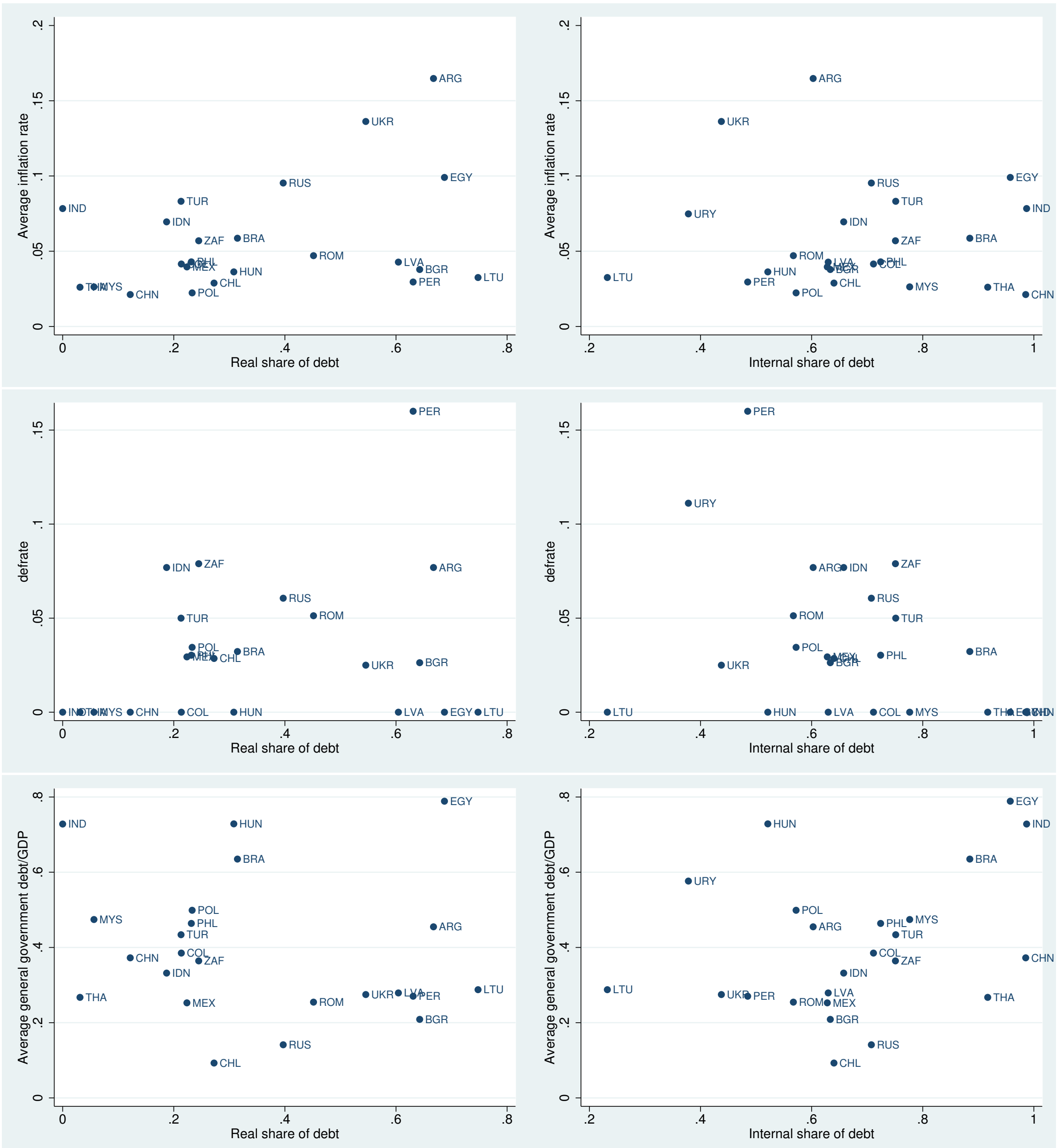
B.3 Country portfolio classification

Countries are assigned to the portfolio quadrants by first splitting along the median internal debt share, and then within that splitting by the median real debt share. The resulting classification is shown below:

	Nominal	Real
Internal	China	Brazil
	Indonesia	Colombia
	India	Egypt
	Malaysia	Philippines
	Thailand	Russia
	Turkey	South Africa
External		Argentina
	Chile	Bulgaria
	Hungary	Lithuania
	Mexico	Latvia
	Poland	Peru
	Romania	Ukraine
		Uruguay

B.4 Portfolio composition and its relationship with inflation, default and debt in the cross-section

Figure 11: Debt, inflation and default by portfolio type in the data



B.5 Data sources for calibration

In the calibration section and to evaluate the performance of the model by comparing it to the data we use the following sources.

Output, consumption, government spending, net exports, CPI, the GDP deflator and M1 money stock are from the OECD, quarterly and seasonally adjusted. 1997Q1 - 2015Q2. The domestic nominal interest rate is the 1 year government bond yield on CETES bonds from the OECD MEI, available from 2001Q3 through

2015Q2. The external spread is the JP Morgan EMBI spread from 1997Q1 through 2012Q3. Tax revenue and the monetary base are from Banco de Mexico, quarterly seasonally adjusted 1997Q1 through 2015Q2, and tax revenue includes all revenue sources except VAT and excise taxes.