How Excessive Is Banks' Maturity Transformation?*

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Abstract

We quantify the gains from regulating banks' maturity transformation in an infinite horizon model of banks which finance long-term assets with non-tradable debt. Banks choose the amount and maturity of their debt trading off investors' preference for short maturities with the risk of systemic crises. As in Stein (2012), pecuniary externalities make unregulated debt maturities inefficiently short. The assessment is based on the calibration of the model to Eurozone banking data for 2006. Lengthening the average maturity of wholesale debt from its 2.8 months to 3.3 months would produce welfare gains with a present value of euro 105 billion.

JEL Classification: G01, G21, G28

Keywords: liquidity risk, maturity regulation, pecuniary externalities, systemic crises

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1 Introduction

One of banks' core functions is maturity transformation: allowing the financing of long-term assets while accommodating investors' preferences for shorter investment horizons. Such function is played by commercial banks, investment banks and many shadow banking entities which finance a significant part of their assets with liabilities that are either callable or short term (including retail demand deposits, term deposits, commercial paper, repos, etc.). The value of maturity transformation and the vulnerability associated with it have long been recognized by the banking literature (Diamond and Dybvig, 1983), which initially devoted most of its attention to banks financed with demand deposits and the problem of bank runs (see Allen and Gale, 2007). The Global Financial Crisis turned the attention to the inefficiencies associated to banks' maturity transformation activities. Indeed, the severity of the refinancing problems at a vast array of institutions and their role in amplifying the subprime crisis (Brunnermeier, 2009; Gorton, 2009) led regulators to the view that maturity transformation in the years leading to the crisis was excessive (see, for example, Tarullo, 2009).

Since then, several papers have addressed the rationale for regulating banks' exposure to funding liquidity risk. They generally share the idea that banks' refinancing needs during a crisis produce negative pecuniary and non-pecuniary externalities (see, for example, Perotti and Suarez, 2011). This may happen because refinancing needs force banks to undertake fire sales whose impact on asset prices contributes to tightening financial constraints (Stein, 2012). As pointed by earlier references, it can also happen through contagion, because of direct losses coming from interbank positions (Rochet and Tirole, 1996; Allen and Gale, 2000), or through the damage that the disruption of the financial system inflicts on the rest of the economy (Kroszner, Laeven, and Klingebiel, 2007).

While conceptually very valuable, existing papers do not quantify the inefficiency associated with excessive maturity mismatches. In fact, the stylized time dimension of the underlying models (typically with two or three dates) is unsuitable for calibration. Yet, measuring the social costs and benefits of banks' maturity transformation is essential to inform policy makers in the task of designing and calibrating new regulatory tools such as the Net Stable Funding Ratio (NSFR) of Basel III (see BCBS, 2014).

Our paper is a first attempt in such direction. We develop and calibrate a tractable infinite horizon model focused on banks' maturity transformation function. Banks choose the amount and maturity of the debt issued against their long-term assets taking into account two forces pushing in opposite directions: first, investors' preference for liquidity (which calls for issuing debt with short maturities) and, second, the existence of systemic liquidity crises in which refinancing the maturing debt is especially costly (which calls for borrowing at long maturities). As in Stein (2012), pecuniary externalities make unregulated debt maturity decisions inefficiently short. After calibrating the model to Eurozone banking data for 2006, we quantify the extent to which banks' average debt maturities were excessively short and the size of the welfare gains that would have been associated with regulating liquidity risk in such an environment.

In our recursive model, banks place non-tradable debt among unsophisticated investors who are initially patient but may suddenly turn impatient. Short maturities reduce the expected time the savers have to wait before recovering their funds if they become impatient. Without systemic liquidity crises, banks might satisfy investors' preferences by issuing debt of the shortest maturity (or, equivalently, demandable debt) that would be repeatedly rolled over among (subsequent cohorts of) patient investors. However, we assume that in systemic liquidity crises banks are unable to place debt among unsophisticated investors and have to rely on the more expensive funding provided by some *crisis financiers*. Such financiers are experts whose heterogenous outside investment opportunities effectively produce an upward sloping aggregate supply of funds during crises.[1]

At an initial non-crisis period, banks decide their capital structure by trading off the lower interest cost of shorter debt maturities with their impact on the cost of refinancing during crises. Individual banks choose longer debt maturities (implying smaller refinancing needs) if they anticipate crisis financing to be more costly. The intersection between crisis financiers' upward sloping supply of funds and banks' downward sloping refinancing needs produces a unique equilibrium cost of crisis financing, and some unique bank capital structure decisions associated with it.

For brevity, the core of our analysis focuses on the simple case in which our representative bank finds it optimal to choose debt structures that prevent it from going bankrupt (which

[1] This upward sloping supply of funds during crises works like a generalized version of a cash-in-the-market constraint *à la* Allen and Gale (1998).

implies being liquidated) during crises.[2] Debt structures guarantee the bank to survive a crisis if they satisfy what we call the bank's *crisis financing constraint*: The bank must keep sufficient equity value in normal times so as to be able to absorb the excess cost of refinancing its maturing debt in a crisis. This constraint imposes an upper limit on the amount and immediacy of the bank's debt.

The model allows us to decompose the private and social value of banks into four intuitive present value terms. The first three are the same in both values. The first (positive) term is the unlevered value of bank assets, that is, the asset cash flows discounted using impatient bankers' discount rate. The second (positive) term captures the value added by maturity transformation in the absence of liquidity crises, which in turn comes from financing the banks with debt held by savers who are initially more patient than bankers. The third (negative) term reflects that in crises banks' maturing debt has to be temporarily financed by impatient experts rather than more patient savers.

The fourth (negative) term in the expression for banks' private value discounts the costs of refinancing maturing debt at an excess cost during crises. Instead, in the expression for banks' social value, such term discounts the value of investment opportunities that crisis financiers pass up when financing the banks during crises. Crucially, banks' maximize their private value taking the (anticipated) equilibrium excess cost of crisis financing as given, while a social planner would maximize social value internalizing the impact of banks' aggregate refinancing needs on such excess cost.

In fact, the pecuniary externality that banks neglect when making their capital structure decisions affects efficiency because rises in the excess cost of crisis financing tighten banks' crisis financing constraints. As a result, in the unregulated equilibrium, debt maturities are excessively short and crisis financing is excessively costly, which eventually reduces the aggregate amount of leverage that the banking industry can sustain. We find that, by inducing banks to lower the intensive margin of their maturity transformation activities (i.e. to choose longer maturities), a regulator can increase the use of the extensive margin (i.e. facilitate the issuance of more debt) and increase the social surplus.

To assess the quantitative importance of the inefficiencies coming from this externality and the potential gains from regulation, we calibrate the model to Eurozone data for 2006.

[2] In subsection 7.1 we show that banks find it optimal to avoid going bankrupt during crises for a range of liquidation values of bank assets that encompasses the most empirically plausible values of such parameter.

We combine information about banks' liability structure and the average maturities of the various debt categories to estimate the refinancing needs of a representative Eurozone bank in a crisis. The calibrated model matches the average maturity of banks' wholesale debt, which is of 2.8 months.[3] Reaching social efficiency would require lengthening that maturity to 3.3 months. Although this increase may look modest, it would allow banks to remain solvent in a crisis with an equity ratio of 4.0% rather than 5.2%, and would generate a net welfare gain with a present value of euro 105 billion or 0.8% of the unlevered value of bank assets. These gains can be broken down into a rise by euro 424 billion (or 3.4%) in the total market value of banks and a fall in 319 billion (or 21%) in the present value of the rents appropriated by crisis financiers. All in all, this points to a substantial welfare impact associated with the optimal regulation of banks' debt maturity.[4]

Optimal regulation under our calibration implies an increase in the average maturity of banks' wholesale debt of only 0.5 months. This raises some concern about the possibility that the limitation of banks' maturity transformation envisaged by regulatory proposals such as the NSFR of Basel III is excessive. Roughly speaking, requiring banks to hold a NSFR higher than one implies that liabilities with maturity longer than one year ("stable funding") should exceed assets with maturity longer than one year ("illiquid assets"). This points to reducing maturity transformation much more drastically than what our model prescribes.

In the final part of the paper we analyze the sensitivity of our quantitative results to key aspects of the calibration strategy and we discuss several possible extensions of the model. The extensions deal with the case in which systemic crises may lead some banks to default and being liquidated, the case in which crises not only cause an increase in refinancing costs but also some asset-side losses, and the case in which bailout expectations push banks into the violation of their crisis financing constraints, justifying the social desirability of regulating not only their debt maturity but also their leverage.

The paper is organized as follows. Section 2 places the contribution of the paper in the context of the literature. Section 3 presents the model. Section 4 defines and characterizes its equilibrium. Section 5 derives its efficiency and regulatory implications. The calibration and

[3] The calibrated model is a straightforward extension of the baseline model that separately accounts for the availability, stability and lower cost of funding coming from insured retail deposits.

[4] To put the above numbers in perspective, if the 424 billion gain in banks' market value were appropriated by bank equityholders, it would imply a windfall gain equivalent to 36.6% of equity value in the unregulated equilibrium.

key quantitative results of the paper appear in Section 6. Section 7 discusses some extensions. Section 8 concludes. The appendices at the end contain details on the calibration, proofs, and other technical derivations.[5]

2 Related literature

Our paper addresses the important task of quantifying the value of banks' maturity transformation and the involved inefficiencies by combining ingredients from the recent normative analysis of externalities associated with banks' funding decisions (Stein, 2012) and the previous literature on the microfoundations of banks' liquidity provision role (Diamond and Dybvig, 1983).

Conceptually, the closest paper to ours is Stein (2012), where the inefficiency in banks' debt maturity choices also comes from the combination of pecuniary externalities and financial constraints.[6] The mechanism that in Stein works through fire sale prices in our paper works through the cost of refinancing during crises. The main differential contribution of our paper is at the richer timeframe and the quantitative exercise.

In addition to pecuniary externalities, the literature has found other theoretical mechanisms that may justify debt maturity regulation. In Perotti and Suarez (2011), banks neglect their contribution to generating systemic risk (modeled as a technological externality) when they expand their credit activity using short-term funding. In Farhi and Tirole (2012), public support to distressed institutions during crises (e.g. via central bank lending) makes bank leverage decisions strategic complements, also producing excessive short-term borrowing. Finally, in Brunnermeier and Oehmke (2013), lack of enforceability of debt covenants creates a conflict of interest between long-term and short-term creditors, pushing banks to choose inefficiently short debt maturities.

Of course, the rationale for short-term debt financing has been extensively analyzed in

[5] Further technical details on the extensions appear in an Online Appendix that can be found in the authors' personal webpages.

[6] Pecuniary externalities are a common source of inefficiency in models with financial constraints (e.g. Lorenzoni, 2008) and more generally in economies with incomplete markets (Geanakoplos and Polemarchakis, 1986, Greenwald and Stiglitz, 1986). Most of the recent papers (including Bianchi and Mendoza, 2011, Korinek, 2011, and Gersbach and Rochet, 2012) emphasize them as a potential cause of excessive fluctuations in credit and/or excessive credit. Bengui (2011) presents a model *à la* Kiyotaki and Moore (1997) where firms' choice between short-term and long-term debt is inefficient because firms' neglect part of the net worth and asset price stabilization effects of long-term debt.

the corporate finance and banking literatures, typically using models with highly stylized timeframes. In contributions following Bryant (1980) and Diamond and Dybvig (1983), demand deposits help satisfy investors' idiosyncratic liquidity needs coming from preference shocks but create a maturity mismatch that makes banks vulnerable to runs. Flannery (1994), in a corporate finance context, and Calomiris and Kahn (1991), Diamond and Rajan (2001), and Huberman and Repullo (2010), in a banking context, attribute a disciplinary role to short-term debt and the possibility of runs. Quite differently, in Flannery (1986) and Diamond (1991), short-term debt allows firms with private information to profit from future rating upgrades, while in Diamond and He (2012) short maturities have a non-trivial impact on a classical debt overhang problem. The rationale for short-term debt in our model is close to the first of these literature streams but, instead of offering demandable debt, banks in our model find it optimal to offer debt with an interior debt maturity that trades off investors' higher valuation of short maturities with banks' concerns about refinancing costs during crises.

Rochet and Vives (2004), Goldstein and Pauzner (2005), and Martin, Skeie, and von Thadden (2014a), among others, model the emergence of roll-over risk as the combined result of doubts about the solvency of banks and a coordination problem between short-term creditors. Various papers, including Allen and Gale (1998), Acharya and Viswanathan (2011) and Acharya, Gale, and Yorulmazer (2011), study the implications of roll-over risk and runs for issues such as risk-sharing, risk-shifting, fire sales, and the collateral value of risky securities. In our paper we also study the implications of roll-over risk but we abstract from endogenizing the risk of runs or the emergence of liquidity crises. Instead, we model crises as an exogenous "sudden stop" of the type introduced by Calvo (1998) in the emerging markets literature.[7]

From a technical perspective, our work is related to the literature that incorporates debt refinancing risk in infinite-horizon capital structure models. Leland and Toft (1996) study the connection between credit risk and refinancing risk, and show that short debt maturities increase the threshold of a firm's fundamental value below which its costly bankruptcy occurs. He and Xiong (2012a) extend the analysis to a setup with shocks to market liquidity. He and Milbradt (2014) introduce a secondary market for corporate debt subject to search frictions

[7]See Bianchi, Hatchondo, and Martinez (2013) for a recent application.

and explore the interactions between credit risk and the endogenous liquidity of such market. While these papers mainly focus on asset pricing implications and the determinants of credit spreads and market liquidity from a strictly positive standpoint, ours focuses on banks' debt maturity decisions, the assessment of inefficiencies due to pecuniary externalities, and their potential correction through regulation.[8]

3 The model

We consider an infinite horizon economy in which time is discrete $t = 0, 1, 2,...$ and a special class of *expert agents* own and manage a continuum of measure one of *banks*, which are describable as an exogenous pool of long-term assets. The economy alternates between normal states ($s_t = N$) in which banks can roll over their debt among *unsophisticated savers*, and crisis states ($s_t = C$) in which they cannot. The crisis states represent systemic liquidity crises in a reduced-form manner. For tractability, we assume $\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon$ and $\Pr[s_{t+1} = C \mid s_t = C] = 0$, so that crises are short-lived episodes with a constant probability of following any normal state. Since crises last for just one period, for calibration purposes one must think of a period as the standard duration of a crisis. Finally, we assume that the economy starts up in a normal state ($s_0 = N$).

3.1 Agents

Both expert agents and unsophisticated savers are long-lived risk-neutral agents who enter the economy in a steady flow of sufficiently large measure per period and exit it whenever their investment and consumption activities are completed.[9] Each entering agent is endowed with a unit of funds.

3.1.1 Experts

Experts are relatively impatient. They discount future consumption at rate ρ_H. When entering the economy, each expert has the opportunity to invest his endowment either in bank

[8]Other infinite horizon analyses with a positive focus include He and Xiong (2012b), which shows that "dynamic runs" may occur when lenders fear that future lenders will stop rolling over maturing debt before the currently offered debt matures, and Cheng and Milbradt (2012), which shows that this type of runs may have a beneficial effect on an asset substitution problem.

[9]Specifically, the entering agents are assumed to be sufficient to cover banks' refinancing needs, while exit ensures that the measure of active agents remains bounded.

claims or in an indivisible private investment project with a net present value z which is heterogeneously distributed over the entrants.[10] The distribution of z has support $[0, \overline{\phi}]$ and the measure of agents with $z \leq \phi$ is described by a differentiable and strictly increasing function $F(\phi)$, with $F(0) = 0$ and $F(\overline{\phi}) = \overline{F}$. These assumptions imply that the access to experts' funding (which banks will need in crises states as specified below) will have a cost that increases in the overall amount of funding required from them.

3.1.2 Savers

Entering savers are initially *patient*. They start discounting next period utility from consumption at rate $\rho_L < \rho_H$. However, in every period they face an idiosyncratic probability γ of turning irreversibly *impatient* and starting to discount the utility of future consumption at rate ρ_H from that point onwards.

Unsophisticated savers have no other investment opportunity than bank debt. So, in the normal state the entering savers decide between buying bank debt or consuming their endowment, while in the crisis state they simply consume their endowment. Preexisting savers with maturing debt face an analogous choice on the use of the recovered funds.

3.2 Banks

At the initial period ($t = 0$), each of the banks possesses a pool of long-term assets that, if not liquidated, yields a constant cash flow $\mu > 0$ per period. If liquidated, bank assets produce a terminal payoff L. For brevity, the experts who own and manage the banks at any given point in time are called *bankers*.

Bankers can profit from the lower discount rates of the patient savers by issuing among them debt claims against the return of the long-term assets.[11] Debt is issued at par in the form of (infinitesimal) contracts with a principal normalized to one. Importantly, bank debt is assumed to be non-tradeable.[12] At the initial period ($t = 0$), bankers choose a triple

[10] The experts who opt for their own projects rather than bank claims exit the economy immediately.

[11] To keep the model tractable, we abstract from the possibility that existing or new bankers create new bank assets. In a fuller model in which banks' role as financiers of long-term projects were explicitly formalized, the gains from maturity transformation might be passed through to the owners of those projects in the form of better financing conditions.

[12] The lack of tradability might be structurally thought as the result of savers' geographical dispersion and the lack of access to centralized trading. We discuss the importance of this assumption and its connection with the literature in subsection 7.5.

(r, δ, D), where r is the per-period interest rate, δ is the constant probability with which each contract matures in each period, and D is the overall principal of the debt. So debt maturity is random, which helps for tractability, and has the property that the expected time to maturity of any non-matured contract is equal to $1/\delta$.[13] We also assume that contract maturity arrives independently across contracts so that there is constant flow δD of maturing debt in every period. Failure to pay interest or repay the maturing debt in any period leads the bank to be liquidated at value L.

In normal periods, the refinancing of maturing debt δD is done by replacing the maturing contracts with identical contracts placed among patient savers. So the bank generates a free cash flow of $\mu - rD$ that is paid to bankers as a dividend.[14]

In crisis periods, financing the repayment of the maturing debt requires bankers to turn to other experts. With the sole purpose of simplifying the algebra, we assume that bankers learn about their banks' refinancing problems after having consumed the normal dividends.[15] Thus, they require δD units of funds. Otherwise, the bank fails and its assets are liquidated at value L, which would be distributed among debt and equity folders according to standard bankruptcy procedures. To obtain the funds, the bankers are assumed to offer a fraction α of the residual continuation value of their bank (i.e. of its future free cash flows) to some of the entering experts.[16] The arrangement reduces the bank's debt in hands of savers to $(1 - \delta)D$ during the crisis in the understanding that an extra amount δD of debt will be optimally reissued among savers, at the same terms as the remaining debt, once the crisis is over. The proceeds from such placement are part of the residual continuation value of the

[13]With $\delta = 1$, the debt issued by banks could be interpreted as demand deposits. However, as it will become clear below, if the probability and cost of systemic crises are large enough, choosing $\delta = 1$ is neither privately nor socially optimal.

[14]We have considered an extension in which banks (or bankers) can use their free cash flow to build a buffer of liquidity with which to partially cover refinancing needs in a crisis. We have checked that if the probability of suffering a systemic crisis and/or the cost of liquidity in a crisis are not too large, then holding liquidity is strictly suboptimal. This is the case under our calibration of the model; with parameterizations not satisfying this property, analytical tractability is lost.

[15]Otherwise, they may find it optimal to cancel the dividends and reduce the bank's funding needs to $\delta D - (\mu - rD)$. The algebra in this case is more tedious but the results are barely affected because under realistic parameterizations (e.g. our calibration below) the dividends $\mu - rD$ are very small relative to the refinancing needs δD.

[16]In some related papers of bank runs, institutions can satisfy the repayment of their non-rolled over debt during crises either via asset sales (Stein, 2012, and Martin, Skeie, and von Thadden, 2014a) or by reducing investment (Martin, Skeie, and von Thadden, 2014b). For tractability, we consider a similarly costly way to accommodate the disappearing funding that keeps asset size constant.

bank that counts towards the compensation of the financing experts. In practical terms, one can interpret experts' financing as the provision of a short-term bridge loan in exchange for a fraction of the equity of the bank once its original debt structure gets restablished.

For tractability, the core of our analysis focuses on the case in which the liquidation value L is low enough for bankers to find it optimal to choose initial debt structures (r, δ, D) that guarantee their refinancing during crises. This gives rise to the notion of *equilibrium with crisis financing* defined below. The corresponding formal condition on L (which is easily satisfied under the calibration of the model) is discussed in subsection 7.1, while subsection 7.2 discusses how the analysis could be extended to cover parameterizations leading (some) banks to default on the equilibrium path.

3.3 The cost of crisis financing

By virtue of competition, the fraction α of the residual continuation value offered to the funding experts in a crisis must be just enough to compensate the marginal entering expert for the opportunity cost of her funds, say ϕ. Given the heterogeneity in experts' private investment opportunities and the size of the aggregate refinancing needs, clearing the refinancing market in a crisis requires $F(\phi) = \delta D$. So the market-clearing *excess cost of crisis financing* can be found as $\phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$. Under our prior assumptions on $F(\phi)$, the *inverse supply* of crisis financing $\Phi(x)$ is strictly increasing and differentiable, with $\Phi(0) = 0$ and $\Phi(\overline{F}) = \overline{\phi}$. Thus, the excess cost of crisis financing ϕ is increasing in banks' aggregate refinancing needs.

4 Equilibrium analysis

We use the following definition of equilibrium:

Definition 1 *An* equilibrium with crisis financing *is a tuple* $(\phi^e, (r^e, \delta^e, D^e))$ *describing an excess cost of crisis financing* ϕ^e *and a debt structure for banks* (r^e, δ^e, D^e) *such that:*

1. *Patient savers accept the debt contracts involved in* (r^e, δ^e, D^e)*.*
2. *Among the class of debt structures that allow banks to be refinanced during crises,* (r^e, δ^e, D^e) *maximizes the value of each bank to its initial owners.*

3. The market for crisis financing clears in a way compatible with the refinancing of all banks, i.e. $\phi^e = \Phi(\delta^e D^e)$.

In the next subsections we undertake the steps necessary to prove the existence and uniqueness of this equilibrium, and establish its properties.

4.1 Savers' required maturity premia

Let us first analyze the conditions upon which the debt contracts associated with some debt structure (r, δ, D) are acceptable to savers in the normal state. Since the bank will fully pay back its maturing debt even in crisis periods, a saver's valuation of a contract does not depend on the aggregate state of the economy but only on whether the saver is patient ($i = L$) or impatient ($i = H$). The ex-coupon values of the contract in each of these individual states, U_L and U_H, must satisfy the following system of equations:

$$\begin{aligned} U_L &= \frac{1}{1+\rho_L}\{r + \delta + (1-\delta)[(1-\gamma)U_L + \gamma U_H]\}, \\ U_H &= \frac{1}{1+\rho_H}\left[r + \delta + (1-\delta)U_H\right]. \end{aligned} \tag{1}$$

These recursive formulas express U_L and U_H in terms of the discount factors, payoffs, and continuation values relevant in each state. A non-matured debt contract pays r with probability one in each next period. Additionally it matures with probability δ, in which case it pays its face value of one and loses its continuation value. With probability $1 - \delta$, it does not mature and then its continuation value is U_L or U_H depending on the saver's individual state in the next period. The terms multiplying these continuation values in the right hand side of the equations reflect the probability of each individual state next period.

When banks place debt among savers, patient savers are abundant enough to acquire all the issue, so the acceptability of the terms (r, δ) requires

$$U_L(r,\delta) = \frac{r+\delta}{\rho_H+\delta}\frac{\rho_H+\delta+(1-\delta)\gamma}{\rho_L+\delta+(1-\delta)\gamma} \geq 1, \tag{2}$$

where $U_L(r, \delta)$ is the solution for U_L arising from (1). Obviously, for any given maturity choice δ, bankers' value is maximized by issuing contracts with the minimal r that satisfies $U_L(r, \delta) = 1$.

Proposition 1 *The minimal interest rate acceptable to patient savers for each maturity choice δ is given by the function*

$$r(\delta) = \frac{\rho_H \rho_L + \delta \rho_L + (1-\delta)\gamma \rho_H}{\rho_H + \delta + (1-\delta)\gamma}, \tag{3}$$

which is strictly decreasing and convex, with $r(0) = \rho_H \frac{\rho_L+\gamma}{\rho_H+\gamma} \in (\rho_L, \rho_H)$ and $r(1) = \rho_L$.

The proofs of all propositions are in Appendix B. Having $r'(\delta) < 0$ evidences the advantage of offering short debt maturities to a patient saver. For any expected maturity $1/\delta$ longer than one, the saver bears the risk of turning impatient and having to postpone his consumption until his contract matures. Compensating the cost of waiting generates a maturity premium $r(\delta) - \rho_L > 0$, which is increasing in $1/\delta$. Figure 1 illustrates the behavior of $r(\delta)$ under the calibration described in Section 6.[17]

4.2 Banks' optimal debt structures

From now on, we will set $r = r(\delta)$ and refer to banks' debt structures as (δ, D). And, to further save on notation, we will generally refer to $r(\delta)$ as simply r.

4.2.1 Value of bank equity in normal times

The continuation value of bank equity in the normal state depends on both the bank's debt structure (δ, D) and the fraction α of its residual continuation value which is relinquished to crisis financiers in subsequent crises.

The continuation value of equity in a normal period that follows another normal period, $E(\delta, D; \alpha)$, can be found as the solution to the following recursive equation:

$$\begin{aligned} E(\delta, D; \alpha) \;\; = \;\; & \frac{1}{1+\rho_H} \Big\{ (\mu - rD) + (1-\varepsilon)E(\delta, D; \alpha) + \\ & \varepsilon(1-\alpha)\frac{1}{1+\rho_H}[\mu - (1-\delta)rD + \delta D + E(\delta, D; \alpha)] \Big\}. \end{aligned} \tag{4}$$

To explain this formula, recall that bankers' discount rate is ρ_H and next period they receive a dividend $\mu - rD$. With probability $1 - \varepsilon$, the next period is a normal period too and the continuation value of equity is $E(\delta, D; \alpha)$ once again. With probability ε, a systemic crisis

[17] As explained in Section 6, we calibrate an extended version of the model that allows for insured retail deposits. All our figures would look qualitatively the same if insured deposits were made equal to zero.

arrives and a fraction α of the residual continuation value of the bank is relinquished to the crisis financiers.

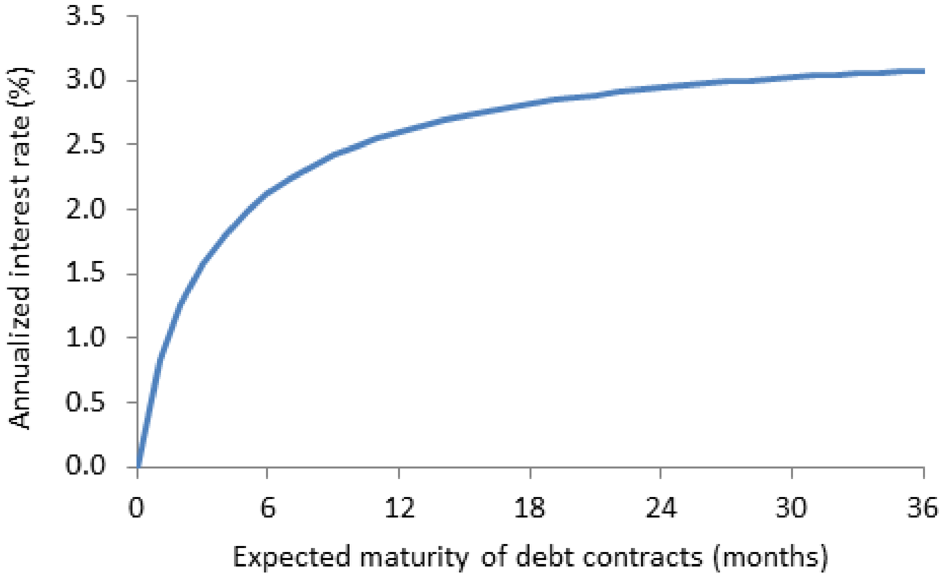

Figure 1 Annualized interest rate as a function of $1/\delta$

The term $\frac{1}{1+\rho_H}[\mu-(1-\delta)rD+\delta D+E(\delta, D; \alpha)]$ represents the present value of the payoffs that the bank will make to its residual claimants (crisis financiers in proportion α and prior equityholders in proportion $1-\alpha$) after getting refinanced in the crisis.[18] It is expressed in terms of free cash flows available once the crisis is over. So $\mu-(1-\delta)rD$ are the asset returns net of interest payments to unsophisticated savers (whose debt is reduced to $(1-\delta)D$ during the crisis) in the period right after the crisis, δD is the revenue from reissuing the debt financed by the experts during the crisis (which is paid out to the residual claimants), and the last term reflects that, once the initial debt structure is fully restored, the present value of subsequent free cash flows is $E(\delta, D; \alpha)$ again.

Competition between entering experts implies that bankers will obtain δD in exchange for the minimal α that satisfies

$$\alpha\frac{1}{1+\rho_H}[\mu-(1-\delta)rD+\delta D+E(\delta, D; \alpha)] \geq (1+\phi)\delta D, \tag{5}$$

[18] The exact form of the claims that split in proportions α and $1-\alpha$ the residual continuation value of the bank between crisis financiers and prior equityholders, respectively, is irrelevant due to a Modigliani-Miller type of result.

which simply says that the residual continuation value appropriated by crisis financiers must compensate them for the opportunity cost $(1+\phi)\,\delta D$ of the provided funding. Under the implied α, (5) holds with equality and can be used to substitute for α in (4) and obtain the following Gordon-type formula for equity value:

$$E(\delta, D; \phi) = \frac{\mu}{\rho_H} - \frac{r(\delta)}{\rho_H}D - \frac{1}{\rho_H}\frac{\varepsilon\{[(1+\rho_H)\phi + \rho_H] - r(\delta)\}}{1+\rho_H+\varepsilon}\delta D. \tag{6}$$

The interpretation of this expression is very intuitive: Equity resembles a perpetuity in which the relevant payoffs are discounted at the impatient rate ρ_H; μ is the unlevered cash flow of the bank; $r(\delta)$ is the interest rate paid on the debt placed among savers; and $\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\}$ is the term reflecting the (discounted) differential cost of refinancing each unit of maturing debt each time a crisis arrives.

Finally, taking into account that (5) holds with equality and α cannot be larger than one, the feasibility of refinancing the bank during crises requires:

$$\mu - (1-\delta)rD + \delta D + E(\delta, D; \phi) \geq (1+\rho_H)(1+\phi)\delta D, \tag{7}$$

which we will call the *crisis financing constraint* (CF). It establishes that the free cash flow plus the continuation value of equity in the period after the crisis must be no lower than the amount the bank needs to compensate, at the rate ρ_H, the cost $1+\phi$ of each unit of refinancing during the crisis.

4.2.2 Optimal debt structure problem

Bankers' goal when choosing the bank's initial debt structure is to maximize the total market value of the bank, $V(\delta, D; \phi) = D + E(\delta, D; \phi)$, which using (6) can be written as:

$$V(\delta, D; \phi) = \frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H}D - \frac{1}{\rho_H}\frac{\varepsilon(\rho_H - r(\delta))}{1+\rho_H+\varepsilon}\delta D - \frac{1}{\rho_H}\frac{\varepsilon(1+\rho_H)\phi}{1+\rho_H+\varepsilon}\delta D. \tag{8}$$

The first term in this expression is the value of the unlevered bank. The second term is the value obtained by financing the bank with debt claims held by savers' initially more patient than the bankers (notice that $r(\delta) < \rho_H$, by Proposition 1). The third term reflects that crisis financing is made by experts (whose discount rate is ρ_H) instead of by patient savers (who require a yield $r(\delta) < \rho_H$). The last term accounts for the excess cost coming from having to compensate all crisis financing according to the excess opportunity cost of funds ϕ

of the marginal crisis financier. Importantly, by the logic of perfect competition, the owners of each individual bank take ϕ as given when making their decisions on (δ, D).

Bankers solve the following problem:

$$\begin{aligned} \max_{\delta\in[0,1],\ D\geq 0} \quad & V(\delta, D;\phi) = D + E(\delta, D;\phi) && (9) \\ \text{s.t.} \quad & E(\delta, D;\phi) \geq 0 && \text{(LL)} \\ & \mu - (1-\delta)rD + \delta D + E(\delta, D;\phi) \geq (1+\rho_H)(1+\phi)\delta D && \text{(CF)} \end{aligned}$$

The first constraint imposes the non-negativity of equity value in the normal state and can be thought of as bankers' limited liability constraint (LL) in such state.[19] The second constraint is the crisis financing constraint (7) (or bankers' limited liability in the crisis state). It can be shown that the two constraints boil down to the same constraint on D for $\delta = 0$, but (CF) is tighter than (LL) for $\delta > 0$.[20] Thus (LL) can be ignored.

The following technical assumptions help us prove the existence and uniqueness of the solution to the bank's optimization problem:[21]

A1. $\overline{\phi} < 2\frac{1+\rho_L}{1+\rho_H} - 1$.

A2. $\gamma < \frac{1-\rho_H}{2}$.

Proposition 2 *For each given excess cost of crisis financing ϕ, the bank's maximization problem has a unique solution (δ^*, D^*). In the solution: (1) (CF) is binding, that is, in a crisis the financiers appropriate 100% of the bank's residual continuation value. (2) Optimal debt maturity $1/\delta^*$ is increasing in ϕ and the optimal amount of maturing debt per period $\delta^* D^*$ is decreasing in ϕ. In fact, if $\delta^* \in (0,1)$, both δ^* and $\delta^* D^*$ are strictly decreasing in ϕ.*

The intuition for these results is the following. First, the bank is always interested in maximizing its leverage, so its (CF) constraint is always binding, which in turn means that bankers get fully diluted ($\alpha = 1$) in each crisis.[22] One can interpret the crisis financing

[19] Notice that satisfying (LL) requires bankers' dividends, $\mu - r(\delta)D$, to be non-negative.

[20] See the proof of Proposition 2 in Appendix B.

[21] A1 and A2 are sufficient conditions that impose rather mild restrictions on the parameters. For instance, for the discount rates ρ_L, ρ_H, used in the calibration of the model (see Section 6), A1 and A2 impose $\overline{\phi} < 0.9957$ and $\gamma < 0.4986$.

[22] Full dilution is an implication of the simplifying assumption that all crises have the same severity. With heterogeneity in this dimension (for example, due to random shifts in $\Phi(x)$), the corresponding crisis financing constraint might only be binding (or even not satisfied, inducing bankruptcy) in the most severe crises, and perhaps leave some residual continuation value in hands of the prior bankers in the mildest crises.

arrangement as a one period loan with a principal of δD which the crisis financiers grant to the bank. The principal of such loan is repaid right after the crisis out of the reissuance of debt with face value δD among savers. Additionally, the crisis financiers get also compensated with 100% of the bank's equity after the crisis.

Second, other things equal, as the excess cost of crisis financing ϕ increases, the value of maturity transformation diminishes and all banks choose a longer expected maturity (a lower δ^*). The implied tightening of (CF) also induces banks to reduce the amount of funding $\delta^* D^*$ demanded to crisis financiers.

The bank's optimal debt structure decisions (δ^*, D^*) determine, as a residual, its *equity ratio*, E/V. As shown in Figure 2, this ratio is strictly increasing in the excess cost of crisis financing ϕ. Intuitively, by (CF), each bank needs a larger value of equity in the normal state in order to be able to pay its crisis financiers for the larger cost of financing during crises.[23] Under the calibration described in Section 6, equity ratios fall in a realistic 0%–6% interval for a wide range of values of ϕ.

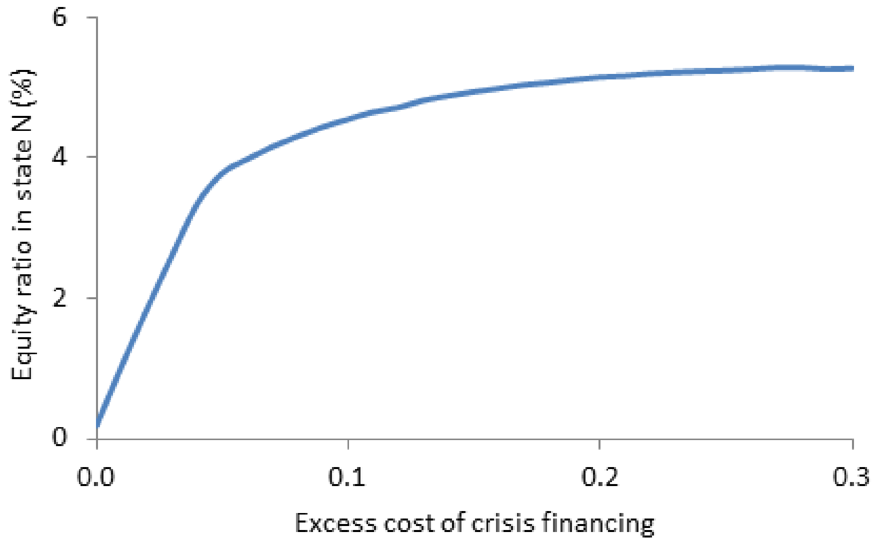

Figure 2 Banks' optimal equity ratio E/V in state N

[23]Even with $\phi = 0$ banks need some (tiny) positive equity because crisis financiers demand a return ρ_H larger than r for their funds.

4.3 Equilibrium

Banks' optimization problem for any given excess cost of crisis financing ϕ already embeds savers' participation constraint so the only condition for equilibrium that remains to be imposed is the clearing of the market for crisis financing. The continuity and monotonicity in ϕ of the function that describes excess demand in such market guarantees that there exists a unique excess cost of crisis financing ϕ^e for which the market clears:

Proposition 3 *(1) The equilibrium* $(\phi^e, (r^e, \delta^e, D^e))$ *exists and is unique. (2) If the inverse supply of crisis financing* $\Phi(x)$ *shifts upwards, (i) expected debt maturity* $1/\delta^e$ *increases, (ii) total refinancing needs* $\delta^e D^e$ *fall, (iii) bank debt yields* r^e *increase, and (iv) the excess cost of crisis financing* ϕ^e *increases. (3) If initially* $\delta^e \in (0,1)$*, all these variations are strict.*

The proposition also states the pretty intuitive effects associated with a shift in the inverse supply of crisis financing. Other comparative statics results are omitted for brevity.[24]

5 Efficiency and regulatory implications

In this section we solve the problem of a social planner who has the ability to control banks' funding decisions subject to the same constraints that banks face when solving their private value maximization problems. We show that debt maturity in the unregulated competitive equilibrium is inefficiently short because of a pecuniary externality that operates through the cost of crisis financing and its impact on banks' frontier of maturity transformation possibilities.

Suppose that a social planner can regulate both the amount D and the maturity parameter δ of banks' debt. Since in our economy only existing bankers and future crisis financiers obtain a surplus, a natural objective for the social planner is to maximize the sum of the present value of such surpluses. Crisis financiers appropriate the difference between the equilibrium excess cost of crisis financing, $\phi = \Phi(\delta D)$, and the net present value of their alternative investment opportunity, $z = \Phi(x) < \Phi(\delta D)$ for all $x < \delta D$. Hence, their surplus in a crisis is:

$$u(\delta, D) = \int_0^{\delta D} \left(\Phi(\delta D) - \Phi(x)\right) dx = \delta D \Phi(\delta D) - \int_0^{\delta D} \Phi(x) dx. \tag{10}$$

[24] The interested reader may find them in a working paper predecessor of the current paper (see Segura and Suarez, 2013).

Evaluated at the normal state, the present value of their expected future surpluses can be written as:[25]

$$U(\delta, D) = \frac{1}{\rho_H}\frac{\varepsilon(1+\rho_H)}{1+\rho_H+\varepsilon}u(\delta, D). \tag{11}$$

From here, using (8), the objective function of the social planner can be expressed as:

$$\begin{aligned} W(\delta, D) &= V(\delta, D; \Phi(\delta D)) + U(\delta, D) \\ &= \frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H}D - \frac{1}{\rho_H}\frac{\varepsilon(\rho_H - r(\delta))}{1+\rho_H+\varepsilon}\delta D - \frac{1}{\rho_H}\frac{\varepsilon(1+\rho_H)}{1+\rho_H+\varepsilon}\int_0^{\delta D}\Phi(x)dx, \end{aligned} \tag{12}$$

which contains four terms: the value of an unlevered bank, the value added by maturity transformation in the absence of systemic crises, the value loss due to financing the bank with impatient experts during liquidity crises, and the value loss due to the sacrifice of the NPV of the investment projects given up by the experts who act as banks' crisis financiers.

Importantly, and differently from bankers when looking at (8), the social planner does not evaluate $W(\delta, D)$ at a taken-as-given excess cost of crisis financing ϕ, but internalizes the impact of δD on the market clearing ϕ (and its impact on crisis financiers' surplus). This will be key to understanding the differences between the competitive and the socially optimal allocations.

With this key ingredient, the social planner's problem can be written as:

$$\begin{aligned} \max_{\delta\in[0,1],\ D\geq 0} \quad & W(\delta, D) \qquad \qquad (13) \\ \text{s.t.} \quad & \mu - (1-\delta)rD + \delta D + E(\delta, D; \Phi(\delta D)) \geq (1+\rho_H)(1+\Phi(\delta D))\delta D \quad \text{(CF')} \end{aligned}$$

This problem differs from banks' optimization problem (9) in two dimensions. First, the objective function includes the surplus of the crisis financiers. Second, the social planner internalizes the effect of D and δ on the market-clearing excess cost of crisis financing, so (CF') contains $\Phi(\delta D)$ in the place occupied by ϕ in the (CF) constraint.[26]

[25] $U(\delta, D)$ satisfies the following recursive equation:

$$U(\delta, D) = \frac{1}{1+\rho_H}\left[(1-\varepsilon)U(\delta, D) + \varepsilon\left(u(\delta, D) + \frac{1}{1+\rho_H}U(\delta, D)\right)\right].$$

The first term in square brackets takes into account that, with probability $1-\varepsilon$, next period is also a normal period and crisis financiers' continuation surplus remains equal to $U(\delta, D)$. The second term captures that with probability ε there is a crisis, in which case crisis financiers obtain $u(\delta, D)$ plus the continuation surplus that, one more period later, is again $U(\delta, D)$.

[26] The constraint called (LL) in (9) can be ignored in the planner's problem as well because it is implied by (CF').

Comparing the solution of the planner's problem with the unregulated equilibrium, we obtain the following result:

Proposition 4 *If the competitive equilibrium features $\delta^e \in (0,1)$ then a social planner can increase social welfare by choosing a longer expected debt maturity than in the competitive equilibrium, i.e. some $1/\delta^s > 1/\delta^e$, which then allows banks to issue more debt, i.e. some $D^s > D^e$.*

The root of the discrepancy between the competitive and the socially optimal allocations is at the way individual banks and the social planner perceive the frontier of the set of maturity transformation possibilities. As illustrated in Figure 3, banks choose their individually optimal (δ, D) along the (CF) constraint (where ϕ^e is taken as given) while the social planner does it along the (CF') constraint (where $\phi = \Phi(\delta D)$).

At the equilibrium allocation (δ^e, D^e) both the social planner's and the initial bankers' indifference curves are tangent to (CF). Moreover, (CF) and (CF') intersect at (δ^e, D^e) since the competitive equilibrium obviously satisfies $\phi^e = \Phi(\delta^e D^e)$. However, the social planner's indifference curve is not tangent to (CF') at (δ^e, D^e), implying that this allocation does not maximize welfare. In the neighborhood of (δ^e, D^e), (CF') allows for a larger increase in D, by reducing δ, than what seems implied by (CF), where ϕ remains constant. It turns out that maturity transformation can produce a larger surplus with a lower use of its intensive margin (short maturities) and a larger use of its extensive margin (leverage), like at (δ^s, D^s). Implementing this allocation would simply require imposing δ^s as a regulatory upper limit to banks' choice of δ, and then allowing banks to decide how much debt to issue (as they would choose as much as compatible with their (CF) constraint).

These results offer a new perspective on regulatory proposals emerged in the aftermath of the recent crisis that defend reducing *both* banks' leverage *and* their reliance on short-term funding. In the context of the current model, once debt maturity (δ) is regulated, limiting banks' leverage (D) would be counterproductive. Our results also indicate that simply limiting banks' leverage (e.g., through higher capital requirements) would not correct the inefficiencies identified above. In fact, as one can see in Figure 3, forcing banks to choose debt lower than D^e without intervening on the choice of δ, would induce them, in the new regulated equilibrium, to move along (CF') in the direction that implies a shorter expected debt maturity (larger δ), thus lowering welfare even further.

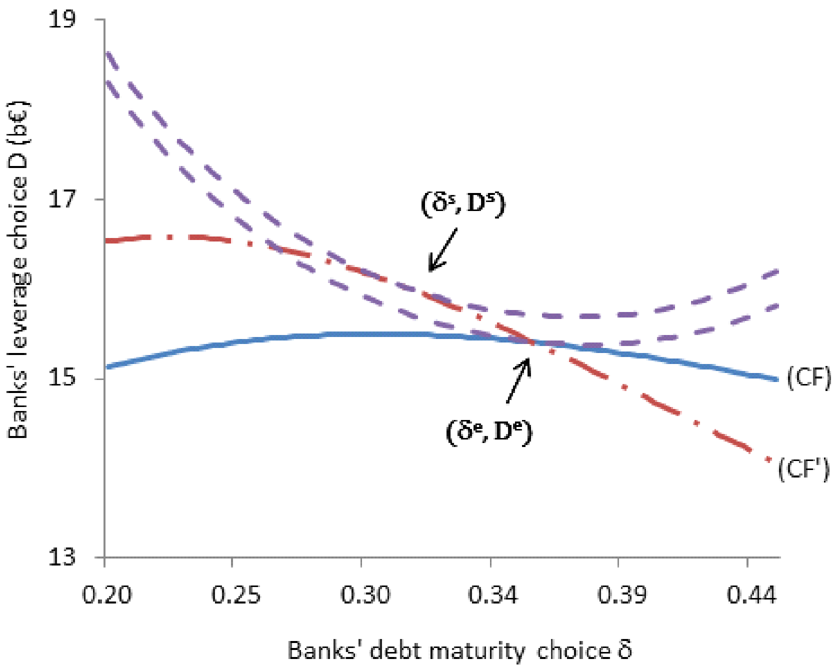

Figure 3 Equilibrium vs. socially optimal debt structures. The solid curve is the private (CF) constraint in the unregulated equilibrium, the dot-and-dashed curve is the social (CF') constraint. The two dashed curves are indifference curves of the social planner.

Of course, the regulation of bank leverage might be desirable for reasons not captured in the model that have been extensively discussed elsewhere (e.g. Santos, 2001). These notably include the presence of asset side risk and the existence of costs of bank failure that banks do not fully internalize or distortions due to explicit or implicit government guarantees. In subsection 7.4 we outline an extension of the model in which banks that violate their (CF) constraint may expect the government to bail them out (e.g. by subsidizing their access to crisis financing). We argue that regulatory limits to bank leverage would be socially desirable in such case.

6 Quantitative results

In this section we calibrate the model in order to assess the potential quantitative importance of its implications. In order to render the exercise more realistic we first extend the model to allow bank debt to incorporate an exogenous base of stable retail deposits together with possibly unstable wholesale debt.

6.1 The model with stable retail funding

In the baseline model we assume that banks in normal periods face a perfectly elastic demand for their debt from investors who "run" (do not refinance such debt) in crisis periods. We think these features are a good description of banks' wholesale debt funding but do not capture well the greater stability of retail deposits, which arguably comes from the existence of deposit insurance (see Gorton, 2009).[27]

To account for the possibility of stable retail debt funding, we extend the model as follows. We assume that on top of the debt investors considered so far, hereinafter called *wholesale investors*, each bank has the opportunity to raise funding among a (captive) population of *retail investors* that contains a measure $\overline{D}_R \geq 0$ of patient agents in each period. Exactly like wholesale investors, retail investors are born with one unit of funds each and suffer idiosyncratic shocks to their discount rate as specified in the baseline model. But differently from them, retail investors are attached to each specific bank, their funding is limited and, crucially, they do not run in crisis periods.[28]

Under this extension, the problem of the bank consists in optimally choosing the pair $((\delta, D),\ (\delta_R, D_R))$, of debt structures for wholesale and retail funding, respectively. The stability of retail funding implies trivially that banks will choose $\delta_R = 1$ (the lowest maturity), thus guaranteeing that retail debt is only held by the retail investors who are patient in each period. Retail funding can consequently be interpreted as demand deposits that, according to (3), pay the lowest possible interest rate $r(1) = \rho_L$, and are issued in amount $D_R = \overline{D}_R$ by each bank.

Taking this into account, the formal analysis can be conducted by minimally modifying the equations of the baseline model: the asset cash flow μ must be replaced by $\mu - \rho_L \overline{D}_R$ (subtracting the interest payments on retail deposits) and the market value of the bank as defined in (8) must incorporate $\overline{D}_R$ as an additional term .

[27] Differences in the propensity to run of different classes of debt are explicitly recognized in regulations regarding the liquidity coverage ratio (LCR) in Basel III (BCBS, 2013). Specifically, when establishing rules for the estimation of banks' potential refinancing needs during a crisis, they set lower minimum run-off rates for insured retail deposits than for other debt categories.

[28] Since a measure $\gamma \overline{D}_R$ of each bank's retail investors become impatient in each period, we implicit assume that a measure $\gamma \overline{D}_R$ of new patient retail investors become accessible to each bank in each period.

6.2 Calibration

We calibrate our extended model to the Eurozone banking sector. The general objective of the calibration is to match the debt structure (outstanding amounts, average maturities, and interest rates of retail and wholesale debt) of a representative Eurozone bank in 2016. We choose that year so as to describe the situation just prior to the first signs of arrival of the 2007-2009 financial crisis. Given the absence of relevant liquidity risk regulations in 2006, we will interpret the liability structure observed in that year as corresponding to the unregulated equilibrium of previous sections.

6.2.1 Duration of crises

We assume that liquidity crises last for one month. Hence, a model period will represent one month. This is consistent with the duration of the "liquidity stress scenarios" that the new liquidity coverage ratio (LCR) of Basel III mandates banks to cover (see BCBS, 2013).

6.2.2 Eurozone banks' debt structure in 2006

Data on the liability structure of Eurozone banks is published regularly in the Monthly Bulletin and the Monetary Statistics of the ECB (see Appendix A for details). We define retail funding as the deposits held by euro area households and non-financial corporations (NFCs) in euro area banks. The remaining debt liabilities of Eurozone banks (see Table 1) are considered wholesale funding. As previously discussed, retail deposits are attributed a maturity parameter $\delta_R = 1$ and assumed stable during crises. In turn, wholesale funding is treated as a homogeneous debt category with an associated flow of refinancing needs which is the source of trouble during crises. Estimating such flow is the most data-intensive part of our calibration exercise.

To do it, we first attribute an average δ_i to each of the five wholesale debt categories $i = 1, 2, 3, 4, 5$ that appear in Table 1. Existing data only provides some coarse partitions of debt categories by maturity ranges. To estimate each average δ_i, we assume that banks' debt is uniformly distributed over the maturities contained in each of the maturity ranges. Finally, we estimate the overall δ of wholesale debt (0.359) as a weighted average of each component δ_i. Such δ corresponds to an average debt maturity of 2.8 months.

Table 1
Structure of Eurozone banks' outstanding debt in 2006

Debt category	Amount (b€)	Fraction (%)	Weight in overall δ	Assigned δ_i	Implied $1/\delta_i$
Retail deposits	$5,821$	27.4			
Wholesale debt	$15,404$	72.6	1.000	0.359	2.8
- Deposits & repos from banks	$7,340$	34.6	0.476	0.560	1.8
- Commercial paper & bonds	$4,463$	21.0	0.290	0.027	37.0
- Other deposits	$2,906$	13.7	0.189	0.336	3.0
- Other repos	245	1.2	0.016	0.693	1.4
- Eurosystem lending	451	2.1	0.000	–	–
Total outstanding debt	$21,225$	100.0			

This table describes the structure of Eurozone banks' outstanding debt in 2006 and assigns a maturity parameter δ_i to each of the wholesale debt categories based on existing breakdowns by maturity ranges. For details on the underlying data and the assigment of δ_i, see Appendix A. The value of δ_i assigned to Wholesale debt ("the overall δ") is a weighted average of the δ_i assigned to its components (excluding Eurosystem lending for which no maturity data is publicly available). One model period is one month.

6.2.3 Choice of functional forms and parameter values

Following Greenwood, Landier and Thesmar (2015), we assume a simple linear functional form for the inverse supply of crisis financing:

$$\Phi(x) = ax, \tag{14}$$

where the parameter $a \geq 0$ can be interpreted as the impact on the cost of funds during crises of an increase in banks' aggregate refinancing needs.[29] In the context of aggregate liquidity crises, direct empirical evidence regarding a does not exist, so we will calibrate this parameter within the model.

Our overall calibration strategy is as follows. Our extended model has seven parameters: ε, $\overline{D}_R$, ρ_L, ρ_H, γ, μ and a. Parameters ε and $\overline{D}_R$ can be calibrated to match empirical counterparts directly available in the existing evidence or data. The rest are set to make an equal number of model equilibrium outcomes match calibration targets based on existing data (exact identification). In fact the structure of the model allows us to split the remaining parameters in two blocks. The block made by ρ_L, ρ_H, and γ can be calibrated without having

[29]Greenwood, Landier and Thesmar (2015) adopt this linear specification in a model of fire sales and calibrate a using prior studies on the price impact of secondary market sales.

fixed μ and a, so as to make the model produce a yield curve that fits the average interest rate paid by deposits of three different maturity ranges (defined based on data availability). The block made by μ and a can subsequently be set in order to exactly match the overall amount and the average maturity of Eurozone banks' wholesale debt in 2006.[30] The calibration targets appear on Table 2. The resulting parameter values appear in Table 3.

Table 2
Calibration targets

Description	Model variable	Matched value	Interpretation
Frequency of crises	ε	0.0076	Every 11.4 years
Aggregate banks' retail funding	$\overline{D}_R$	5,821	5,821 b€
Average interest rates			
- Overnight deposits	$r(1)$	0.000685	0.8% yearly
- Deposits with maturity $\leq$ 2 years	$\frac{1}{24-1}\int_1^{24} r(\frac{1}{t})dt$	0.002012	2.4% yearly
- Deposits with maturity $>$ 2 years	$\frac{1}{60-24}\int_{24}^{60} r(\frac{1}{t})dt$	0.002552	3.1% yearly
Banks' aggregate wholesale debt	D^e	14,953	14,953 b€
Maturing wholesale debt per month	δ^e	0.359	Every 2.8 months

This table describes the targets for the baseline calibration of the model. The parameter values that appear in Table 3 below are found so as to exactly match the values of the moments described in columns 1 and 2 of this table with their data counterparts. Columns 3 and 4 show the matched values and their interpretation in terms of meaningful units of measurement. One model period is one month. See the main text and Appendix A for further details on the calibration strategy.

Frequency of systemic crises (ε) To obtain an estimate of ε, we use the subsample of advanced economies in the systemic banking crises database of Laeven and Valencia (2012), which covers the period 1970-2011 and is the largest in terms of the number of countries that it covers. We compute the yearly frequency of systemic crises ε_Y by dividing the number of systemic crises in the subsample by the maximum number of potential occurrences of a crisis (the number of country-year observations).[31] This is translated into a monthly probability of a crisis using the equation $1-\varepsilon_Y = (1-\varepsilon)^{12}$, which presumes that not registering a crisis

[30] Most calibrated macroeconomic models do not have this recursive structure and hence fix most of the internally calibrated parameters simultaneously in order to match or approximate a selected equal or larger number of empirical moments. Given the recursivity of our model, we think that splitting its calibration in two stages renders the identification of the parameters more transparent.

[31] For most countries in advanced economies, the number of possible occurrences is (2011-1970)+1=42, but we take into account that some countries were created in the 1990s, after the collapse of the communist block in Eastern Europe.

in one year requires not registering it in any of its months. The calibrated ε corresponds to suffering a crisis every 11.4 years on average.

Table 3
Parameter values

Description	Parameter	Value	Interpretation
Frequency of crises	ε	0.0076	Every 11.4 years
Aggregate banks' retail funding	$\overline{D}_R$	5,821	5,821 b€
Patient agents' discount rate	ρ_L	0.000685	0.8% yearly
Impatient agents' discount rate	ρ_H	0.002816	3.4% yearly
Frequency of idiosyncratic preference shocks	γ	0.204	Every 4.9 months
Per period asset cash flow	μ	35.3	35.3 b€/month
Impact of needs on cost of crisis funds	a	0.000038	0.38 bps/b€

This table describes the parameters values emerging from the baseline calibration of the model. These values are set so as to exactly match the calibration targets specified in Table 2 following a two-stage procedure described in the text. One model period is one month. The last column in the table provides an interpretation of the parameter values in terms of meaningful units of measurement.

Aggregate retail debt funding ($\overline{D}_R$) We set the aggregate amount of banks' retail funding $\overline{D}_R$ equal to its empirical counterpart in Table 1.

Preference parameters (ρ_L, ρ_H, and γ) The discount rates ρ_L and ρ_H, and the frequency of idiosyncratic preference shocks γ completely determine the interest rate curve $r(\delta)$ (see (3)). We calibrate these parameters to match the average interest rates paid by Eurozone banks in 2006 on household deposits of three different maturities: overnight deposits (that we match to the rate that corresponds to the shortest maturity in the model, $r(1) = \rho_L$), term deposits with maturity up to two years (that we match to the model implied average $\frac{1}{24-1}\int_1^{24} r(1/t)dt$), and term deposits with maturity above two years (that we match to the model implied average $\frac{1}{60-24}\int_{24}^{60} r(1/t)dt$).[32] See Appendix A for further details on the data.

Asset cash flow (μ) The per period asset cash flow μ is mostly a scale parameter that determines banks' debt capacity. We set it to make the equilibrium amount of wholesale

[32] We use household deposits because of the availability of interest rate data by maturity range. However, the distribution of deposits within each maturity range is not provided, so we just assume a uniform distribution over the stated ranges. We cap the over two years range at five years based on the casual observation that deposits of longer maturities are very rare.

debt D equal to its empirical counterpart in Table 1. The resulting μ implies attributing to Eurozone banks monthly aggregate asset returns of 35.3 b€.

Impact of refinancing needs on cost of funds (a) Parameter a has a direct effect on the equilibrium excess cost of funds during crises, which in turn determines banks' equilibrium choice of δ. Hence, we calibrate this parameter to exactly match the average δ in the data. The calibrated a implies that an increase of 1 b€ per month in Eurozone banks' refinancing needs increases the excess cost of crisis financing, ϕ, in 0.38 bps. This estimate is of the same order of magnitude as the 1.00 bps/b€ used in the calculations of Greenwood, Landier and Thesmar (2015) and the 0.62 bps/b€ implied by the overall 10 bps impact of the 16 b€ bond issue by Deutsche Telekom reported by Newman and Rierson (2004).[33]

6.2.4 Other model implied variables

In the calibrated economy the equilibrium excess cost of crisis funds is $\phi^e = 0.210$, which means that crisis financiers obtain a return from refinancing banks during crises of 21.0% in excess of their required annual discount rate of 3.4%. This model variable might be assimilated to the "underpricing gains" that experts make when acquiring banks' equity in periods of distress. Unfortunately we are not aware of studies reporting these gains.

In the unregulated equilibrium of our calibrated economy, banks operate with an equity ratio of 5.2%, and a return on assets, ROA, of 0.7%.[34] Before comparing these numbers with their empirical counterparts, it is worth recalling that our model abstracts from asset risk while, of course, in reality banks use their capital also as a buffer to absorb potential asset-side losses. As further explained in subsection 7.3, if asset risk is potentially concurrent with refinancing risk, banks may want to have enough capital to cover both types of risks at the same time. What this means is that the equity ratio produced by the model should be interpreted as an indication of how much capital would be needed to cover refinacing risk alone. Therefore, the model generated equity ratio should be expected to be lower than the one observed in the data. Indeed, according to the Saint Louis Fed's FRED database,

[33] See Ellul, Jotikasthira, and Lundblad (2011) and Feldhutter (2012) for additional references estimating price impacts in the context of fire sales.

[34] ROA is conventionally defined as net income over market value of assets. Its monthly counterpart in our model is $(\mu - \rho_L \overline{D}_R - r(\delta)D)/V$.

Eurozone banks' equity ratio in 2006 was 6.6%, i.e. 1.4 percentage points higher than the equity ratio produced by the model.

Relatedly, abstracting from asset risk and investors' risk aversion, the ROA generated by the model should also be expected to be lower than its data counterpart. And, in fact, the ROA of Eurozone banks in 2006 as reported in FRED was 1.0%, 34 bps higher than the figure generated by the model.

6.3 Value of maturity transformation

Table 4 shows the value generated by maturity transformation in the unregulated equilibrium obtained under the parameters described in Table 3. It reports the unlevered value of bank assets and the sources of value or social surplus gains and losses captured in the expressions for V and W in (8) and (12), respectively.[35]

Table 4
Value of maturity transformation and gains from regulation

	Unregulated equilibrium		Gains from regulation	
Components	Value (b€)	% of μ/ρ_H	ΔValue (b€)	% of μ/ρ_H
Unlevered value of bank assets (μ/ρ_H)	$12,543$	100.0	0	0.0
Gains from maturity transformation if $\varepsilon = 0$	$12,971$	103.4	-215	-1.2
Losses from refinancing risk ($\varepsilon > 0$) if $\phi = 0$	-23	-0.2	4	0.0
Losses from excess cost of crisis funding ($\phi > 0$)	$-3,107$	-24.8	636	5.1
Total market value of banks (V)	$22,383$	178.5	424	3.4
Market value of bank debt ($D + \overline{D}_R$)	$21,225$	169.2	660	5.3
Market value of bank equity (E)	$1,158$	9.2	-235	-1.9
Present value of experts' rents (U)	$1,554$	12.4	-319	-2.5
Social surplus ($W = V + U$)	$23,937$	190.9	105	0.8

This table describes the value generated by maturity transformation in the unregulated equilibrium and the gains from optimally regulating debt maturity. For a comparison of key model variables across the unregulated and regulated economies, see Table 5. The breakdown of the sources of value is based on trivially extended versions of the expressions for V and W in (8) and (12), respectively. ε is the probability of a suffering a crisis. ϕ is the excess cost of crisis financing.

Maturity transformation allows the representative bank to increase its value by a net amount equivalent to 78.5% of the unlevered value of its assets, μ/ρ_H. Indeed, before sub-

[35] Recall that in the extended model, the expression for V is as in (8) plus the term $\frac{\rho_H-\rho_L}{\rho_H}\overline{D}_R$ that captures the value of the maturity transformation associated with retail funding.

tracting the costs associated with refinancing risk, maturity transformation produces a gross extra value of 103.4% of μ/ρ_H. However, the anticipated discounted costs of all future crises (almost entirely due to having $\phi > 0$) subtract value equivalent to 24.8% of μ/ρ_H. Having enough capacity to pay for the excess costs incurred in each crisis requires the bank to operate with equity worth about 9.2% of the unlevered value of its assets.

The social surplus W generated by banks exceeds banks' total market value V because it also includes the intramarginal rents appropriated by the experts who finance the banks in each crisis, which represent about 12.4% of μ/ρ_H.

6.4 Regulating the externality

Internalizing the pecuniary externality associated with the cost of crisis refinancing pushes the social planner to impose a longer expected maturity to wholesale debt (3.3 months) than the one chosen by banks in the unregulated economy (2.8 months). Table 5 compares the value of several equilibrium variables across the unregulated and the optimally regulated economies. The reduction in the intensive margin of maturity transformation allows banks to expand their wholesale debt by 4.3% (660 b€), while still reducing their aggregate refinancing needs in a crisis by 10.8% (600 b€)

This in turn leads to a reduction of 2.3 percentage points in the excess cost of crisis financing. Regulation also reduces the value of the investment opportunities given up by crisis financiers in each crisis (by 20.4% or 118 b€) and the equity ratio that banks need in order to assure their refinancing during crises (4.0% rather than 5.2%).[36]

The implications of regulation for the various components of banks' market value and social surplus are described in the last two columns of prior Table 4. The bulk of the net gains are due to the reduction in the excess cost of crisis financing. The market value of the bank absorbs about one third of the savings from such reduction, while the remaining two thirds are offset by the lower gains from maturity transformation. Interestingly, while banks' market value increases by an amount equivalent to 3.4% of unlevered asset value (424 b€), the final increase in social surplus is equivalent to 0.8% of μ/ρ_H (105 b€). The remaining 2.6% (319 b€) represents the reduction in the value of the rents appropriated by experts during crises. To put these numbers in perspective, if the 424 billion gain in banks' market

[36] As shown in (12), the NPV of investment opportunities sacrificed by experts in each crisis is $\int_0^{\delta D} \Phi(x)dx$.

value that can be achieved through regulation were appropriated by bank equityholders, it would imply a windfall gain equivalent to 36.6% of the banks' equity value in the unregulated equilibrium.

Table 5
Effects of regulation on key equilibrium variables

Description	Unregulated economy	Regulated economy	Change
Expected maturity (months)	2.8	3.3	0.5m
Aggregate wholesale debt (b€)	15,404	16,064	4.3%
Aggregate refinancing needs (b€)	5,530	4,930	−10.8%
Excess cost of crisis funds (%)	21.0	18.7	−2.3pp
Value sacrificed by financiers in each crisis (b€)	579	461	−20.4%
Equity ratio (%)	5.2	4.0	−1.2pp

This table compares key model variables across the unregulated equilibrium and the equilibrium emerging under the optimal regulation of banks' debt maturity decisions. In both economies the underlying parameter values are those of the baseline calibration of the model (Table 3). The last column reports variations associated with moving from the unregulated to the regulated economy which are measured in months (m), per cents (%) or percentage points (pp).

All in all, these results show the quantitative importance of the pecuniary externalities captured by the model and the substantial implications of their optimal regulation. Yet these results do not point to the need for a drastic lengthening of the maturity of banks' debt (the difference between the unregulated and optimally regulated expected maturity of wholesale debt is of just 0.5 months). Hence, they constitute a call for caution against regulations such as the NSFR of Basel III (see BCBS, 2014), which might effectively impede banks to finance illiquid assets with potentially unstable funds.[37]

6.5 Sensitivity analysis

We now analyze the robustness of our results to changes in some critical calibration choices. We examine each one of those choices at a time, recalculating, if needed, all the parameters that depend on it according to the calibration strategy described above. Table 6 summarizes the results.

[37] Roughly speaking, NSFR regulation would call banks to operate with a ratio of *stable funding* (liabilities with maturity longer than one year) to *illiquid assets* (assets with maturity longer than one year) larger than one. So most bank debt ought to have maturities of more than twelve months.

Table 6
Sensitivity analysis

Description	Baseline	Higher crisis frequency	Instability of retail funding	Higher cost impact
A.Recalibrated parameters				
Cost impact of refinancing needs (bps/b€)	0.38	0.32	0.20	0.62
Per period asset cash flow (b€)	35.3	35.0	40.5	35.3
B. Non-targeted model implied moments				
Equity ratio (%)	5.2	4.3	6.7	5.2
Return on assets (%)	0.65	0.64	0.85	0.65
C. Normative implications				
Expected regulated maturity (months)	3.26	3.24	2.84	3.46
Welfare gain (% of unlevered assets)	0.84	0.88	1.25	2.75

This table reports relevant parameters, some non-targeted moments generated by the model, and key variables for the normative analysis across three variations of the calibration exercise. Column 2 revisits the baseline calibration of the model. The recalibrations explored in columns 3 to 5 are explained in the text. Parameters ρ_L, ρ_H, and γ remain unchanged across all exercises. When exploring the instability of retail funding, the unregulated expected debt maturity matched in the calibration shifts to 2.4 months and corresponds to banks' overall debt instead of wholesale debt only.

Higher frequency of crises If instead of looking at the whole subsample of advanced economies in Laeven and Valencia (2012), we only consider banking crises affecting the countries that were part of the Eurozone in 2006, we obtain a frequency of crises that would correspond to having $\varepsilon = 0.0094$. This implies an average interval between crises of about 8.9 years (2.5 years shorter than in the baseline calibration). Facing a higher frequency of crises, banks would increase (all else equal) the maturity of their debt. Therefore, in the recalibrated economy (which still targets to match the average maturity of banks' wholesale debt in 2006), the implied excess cost of crisis funding ϕ^e falls to 0.17, leading the parameter a that determines the cost impact of each unit of refinancing needs in a crisis to decline to 0.32 bps/b€. Since each crisis becomes effectively less costly, banks can operate with a lower equity ratio (4.3%). In the regulated economy the expected maturity of wholesale debt becomes 3.2 months and social surplus increases by an amount equivalent to about 0.9% of the unlevered value of bank assets.

Instability of retail funding What would happen if opposite to the assumption made above (and the wisdom of Basel III) retail funding is not stable during crisis? To answer this question, we add the debt previously assigned to $\overline{D}_R$ to the wholesale debt previously contained in D^e and recalculate the target for δ^e after taking into account the maturity structure of retail deposits and the new composition of D^e. This implies setting $D^e = 21,244$ and $\delta^e = 0.415$ (or an average maturity of 2.4 months for banks' entire debt).

Since the implied refinancing needs during crises increase significantly, the alternative calibration yields a higher value for μ and a lower value for a. It also raises the equity ratio with which banks need to operate (6.7%). As for the regulated economy, controlling the externality leads to an increase of 0.4 months in expected debt maturity (somewhat lower than the one seen in the baseline calibration) and an increase in welfare equal to 1.25% of the unlevered value of bank assets (higher than in the baseline calibration).[38]

Higher cost impact of crisis refinancing needs As already noted above, our calibrated cost impact parameter a takes a value lower than the price impact of fire sales estimated in Newman and Rierson (2004). To examine the normative implications of facing a higher marginal cost impact, we can alternatively assume that the inverse supply of funds curve $\Phi(x)$ is not necessarily linear but can be locally approximated by $a_0 + ax$, and we can fix its parameter a equal to 0.62 bps/b€ which is the marginal price impact estimated in Newman and Rierson (2004). This strategy leaves a_0 as an additional parameter that can be calibrated to target δ^e. We obtain $a_0 = -0.134$ and an unchanged value for μ. Moreover, this strategy also leaves the implied unregulated excess cost of crisis financing, the equity ratio, and the return on assets unaffected. The increase in the cost impact of crisis financing raises the importance of the pecuniary externality, pushing up the regulated expected maturity to 3.5 months and the welfare gains from regulation to 2.7% of the unlevered value of bank assets.

Overall, the different alternative calibration scenarios explored in this subsection yield plausible values for both the recalibrated parameters and the model-implied moments. Besides, the conclusions on the quantitative importance of the inefficiency (and the order of magnitude of the required regulatory intervention) remain fairly robust.

[38] The required lengthening of maturity is lower because a is lower, while the welfare gains are larger because the pecuniary externality affects a broader base of bank liabilities.

7 Discussion and possible extensions

In this section we discuss the importance of some of the assumptions of the model and outline some of its potential extensions. Formal details on the extensions described in subsections 7.2, 7.3 and 7.4 are provided in an Online Appendix available at the authors' personal webpages.

7.1 Optimality of not defaulting during crises

We have so far assumed that the liquidation value of banks in case of default, L, is small enough for banks to find it optimal to rely on funding structures that satisfy the (CF) constraint. How small L has to be (and what happens if it is not) is discussed next.

If a bank were not able to refinance its maturing debt, it would default, and we assume that this would precipitate its liquidation. As we explain in Appendix C, we assume that the bank in this case is put into resolution, retail depositors are paid first, out of L, and the wholesale debtholders share the remains, if positive. Assuming $L \geq \overline{D}_R$ (or alternatively the existence of full deposit insurance), retail deposits would remain riskless even if the bank is expected to default in a crisis. In contrast, the interest rate paid on wholesale debt would have to include compensation for credit risk. After undertaking the necessary adjustments, Appendix C derives the maximum liquidation value $L^{\max}$ for which, if all other banks opt for debt structures that prevent default during crises, an individual bank prefers preventing default during crises. Thus for $L \leq L^{\max}$, the equilibrium without default is sustainable as a laissez-faire equilibrium, i.e. without the need to impose that banks must prevent default during crises.

In the calibrated economy, $L^{\max}$ equals $18,507$ b€, a value that exceeds by 47.5% the unlevered value of bank assets (μ/ρ_H) and represents 82.6% of the total market value of the bank in a normal state (V). Therefore, our focus on situations in which the representative bank chooses debt structures that allow it to survive a crisis is consistent with assuming that its liquidation value is below 82.6% of its total market value in normal times. In light of existing evidence on bank resolutions, we consider this assumption plausible.[39]

[39] For instance, Bennett and Unal (2014), using data from bank holding companies resolved in the US by the FDIC from 1986 to 2007, estimate an average discounted total resolution cost to asset ratio of 33.2%, a number compatible with our assumption. See Hardy (2013) for related evidence.

7.2 Crises that lead to default

The model could nevertheless be extended to situations in which the liquidation value of banks in case of default is so large that an equilibrium with no default during crises is not sustainable. Following the terminology of the previous subsection, this would happen when $L > L^{\max}$. In this case, some banks will necessarily default in equilibrium. To keep the aggregate size of the banking system constant, we will assume that each defaulting bank gets replaced, right after the crisis, by an identical bank that pays an entry cost c.

In such setup, it is possible to prove the existence of an equilibrium in mixed strategies in which a fraction x^e of the banks (safe banks) choose debt structures that satisfy the (CF) constraint, and the remaining banks (risky banks) choose debt structures that expose them to default during crises. Specifically, let the triple $(\phi^e, (\delta^e, D^e))$ denote the excess cost of funds faced by safe banks during crises and their debt structure in normal times, respectively, and let V^d denote the total market value of a risky bank under its optimal debt structure in normal times. Then, having a mixed strategies equilibrium requires

$$V(\delta^e, D^e; \phi^e) = V^d, \text{ with}$$

$$\phi^e = \Phi(x^e \delta^e D^e).$$

The first condition says that banks must be indifferent between being safe and risky, while the second one is the market clearing condition for crisis funds when only a fraction x^e of banks (the safe ones) relies on experts' funding.

Importantly, the need to regulate the debt maturity choices of safe banks is preserved under this extension.[40] In addition, a bank that decides to be safe relies on experts' funds during crises and, hence, creates a negative pecuniary externality on the remaining safe banks, so a social planner may find it optimal to regulate how many banks are allowed to be safe. Indeed, denoting by (δ^s, D^s) and x^s the socially optimal debt structure and measure of safe banks, respectively, it is possible to prove that:

$$V(\delta^s, D^s; \Phi(x^s \delta^s D^s)) > V^d,$$

which implies the need to regulate the measure x^s of banks choosing (δ^s, D^s). A way to do so would be to make safe banks pay a fee $V(\delta^s, D^s; \Phi(x^s \delta^s D^s)) - V^d$ (in addition to fixing

[40] The normative analysis gets further simplified when $V^d = c$, which prevents having to take into account surpluses $(V^d - c)$ appropriated by the entrants that replace defaulting banks.

their expected debt maturity at $1/\delta^s$).[41] This would lead to an intuitive split of the banking sector between safe regulated banks and risky unregulated banks (a sort of shadow banks).

The main point of this discussion is to show that the essential insights of the model also apply when allowing for the possibility that some banks default during crises. Having said that, taking full account of the repercussions of bank default might require several other modifications in the model, e.g. regarding the value at which banks can be liquidated, which might negatively depend on the measure of failing banks, as in papers focused on fire-sale externalities.[42] However, exploring such modifications exceeds the scope of the current paper.

7.3 Asset risk

Our model focuses on the refinancing risk associated with banks' wholesale debt and provides a novel theory of bank equity as a buffer to cover the losses due to such risk. This view is not a substitute but a complement to the view that bank equity serves as a buffer to absorb potential asset-side losses. We have abstracted from asset risk to focus on the truly novel aspects of our contribution. Here we sketch how our model could be extended to include asset risk.

Suppose that crises, additional to interrupting access to usual wholesale refinancing channels, destroy a fraction $\chi \in (0,1)$ of bank assets.[43] To keep the simple recursive structure of the model, assume that in the normal state following a crisis, banks can replenish their damaged asset base by acquiring at a cost $c > 0$ the assets χ lost in the crisis (where we can assume $\chi\frac{\mu}{\rho_H+\varepsilon\chi} > c$ so that such investment has positive NPV even if not accompanied by any gains from maturity transformation). Banks can pay for c with the proceeds from reestablishing their pre-crisis debt structure, δD, or with direct contributions from the experts.[44] Finally, in order to abstract from a potential debt overhang problem that may lead to inefficient asset replenishment decisions, assume that each bank operates under a

[41] This reasoning does not necessarily imply having $x^s < x^e$.

[42] Assuming that the same experts that refinance banks during crises can also buy the assets of defaulting banks might produce an interesting nexus between the excess refinancing cost of safe banks and the liquidation value of defaulting banks.

[43] This would capture in reduced form the intertwined fundamental and panic aspects of banking crises typically captured in the literature on bank runs (e.g. Goldstein and Pauzner, 2005).

[44] In the Online Appendix, we simplify the modeling of this last possibility by assuming that the upward sloping supply of funds among experts only operates during crisis periods, while in normal periods experts' funds have a constant opportunity cost ρ_H. This is similar to Bolton, Chen, and Yang (2011), who assume time variation in the conditions at which banks can tap equity markets.

covenant that forces it to reestablish its damaged asset base after crises.[45]

Conditional on having access to crisis financing, banks in this extended setup reestablish their original asset size after each crisis and the recursivity of the problem leads to expressions for equity value, total market value, and the crisis financing constraint very similar to those in the baseline model.[46] In an equilibrium with no default, the (CF) constraint is binding, and the entire model logic remains essentially the same as in the baseline model: unregulated debt maturities are too short and regulated banks would be able to operate with lower equity ratios because of the lower need to cover excess refinancing costs during crises. Importantly, banks' equity in this extension acts as a buffer to accommodate not only excess refinancing costs but also asset-side losses suffered during crises.

7.4 Bailout expectations and the regulation of leverage

Our normative results imply that the optimal regulation of debt maturity would allow banks to increase their leverage. This implication is in contrast to policy makers' view that banks' leverage is excessive because banks expect to obtain public support when suffering financial distress. In this extension we show that introducing the possibility of government bailouts in our model would lead to the need for regulating banks' leverage rather than just debt maturity.

Suppose that there is a government that can subsidize the refinancing of banks by experts during crises. Recall that in the baseline model, if for a given excess cost of crisis funds ϕ, a bank does not satisfy the (CF) constraint, then experts will not refinance its maturing debt during crises. This would trigger default and the liquidation of the bank. Suppose that liquidation produces some external social costs $C > 0$ so that the government may have an ex post motive to avoid the failure of the bank. Suppose further that the government can

[45] See Dangl and Zechner (2007) for a model of debt maturity decisions in which there is no such covenant and shorter maturities can serve to commit equityholders to reducing leverage after poor performance.

[46] Specifically, similarly to (6) and (7), equity value is

$$E^{AR}(\delta, D; \phi) = \frac{\mu}{\rho_H} - \frac{r(\delta)}{\rho_H} D - \frac{1}{\rho_H} \frac{\varepsilon\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\}}{1+\rho_H+\varepsilon}\delta D - \frac{\varepsilon\chi}{\rho_H(1+\rho_H+\varepsilon)}\mu - \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}c,$$

while the new crisis financing constraint imposes

$$(1-\chi)\mu - (1-\delta)rD + \delta D - c + E^{AR}(\delta, D; \phi) \geq (1+\rho_H)(1+\phi)\delta D.$$

promise potential financing experts some transfer τ just after the crisis, financed by taxing savers at that point.[47] Assume finally that the bailout process involves some intervention cost $\lambda > 0$. Clearly, for a large enough subsidy τ, the bank will be able to obtain experts' funding and its liquidation will be avoided.

When the social cost of default C is sufficiently large relative to the government intervention cost λ, the government will find ex-post optimal to bail out the bank. But banks, anticipating this, will ignore their (CF) constraint and debtholders will not demand any compensation for default risk. In this polar situation, the "moral hazard" problem is extreme: the limited liability constraint during normal times (LL) is the only relevant constraint.[48] Not surprisingly, banks will choose debt with the shortest maturity ($\delta^e = 1$) and the maximum possible leverage ($D^e = \mu/\rho_L$) so that their equity ratio will be zero.

In this setup, banks ignore the costs associated with government bailouts. To the extent that these costs are sufficiently important, the social planner may want to implement the optimal debt structure (δ^s, D^s) of the baseline model, which satisfies (CF) and, thus, does not lead to bailouts. Two important differences with respect to the baseline model arise. First, leverage under the optimal regulated debt structure, D^s, is lower than the unregulated one, D^e. Second, regulating (average) debt maturity only is not enough, since bailout expectations would lead banks to choose a leverage level $D' > D^s$ and the government to bail out the banks during crises.[49]

7.5 Tradability of debt

The non-tradability of banks' debt plays a key role in the model. The holders of non-mature debt who turn impatient suffer disutility from delaying consumption until their debt matures because there is no secondary market where to sell the debt (or where to sell it at a sufficiently good price). If bank debt could be traded without frictions, debtholders would sell their debts to patient investors as soon as they become impatient. Banks could issue perpetual debt ($\delta = 0$) at some initial period and get rid of refinancing concerns. In practice a lot of bank debt instruments apart from retail deposits, including certificates of

[47] We do not consider the possibility of taxing savers during crises because this would provide an arguably artificial means to avoid paying the excess cost of crisis financing.

[48] Banks unable to satisfy (LL) are assumed not to be allowed to operate.

[49] If the government could commit not to bail out banks then, as in the baseline model, the regulation of $1/\delta$ only would suffice to achieve the socially optimal debt structure.

deposit, interbank deposits, repos, and commercial paper, are commonly issued over the counter (OTC) and have no liquid secondary market.

Our model does not contain an explicit justification for the lack of tradability. Arguably, it might stem from administrative, legal compliance, and operational costs associated with the trading (specially using centralized trade) of heterogenous debt instruments issued in small amounts, with a short life or among a dispersed mass of investors. In fact, if some investors possess better information about banks than other, then costs associated with asymmetric information (e.g. exposure to a winners' curse problem in the acquisition of bank debt) may make the secondary market for bank debt unattractive to investors in the first place (Gorton and Pennacchi, 1990).

Additionally, the literature in the Diamond and Dybvig (1983) tradition has demonstrated that having markets for the secondary trading of bank claims might damage the insurance role of bank debt.[50] Yet, Diamond (1997) makes the case for the complementarity between banks and markets when some agents' access to markets is not guaranteed.

Our model could be extended to describe situations in which debt is tradable but in a decentralized secondary market characterized by search frictions (like in Duffie, Gârleanu, and Pedersen, 2005, Vayanos and Weill, 2008, and Lagos and Rocheteau, 2009). In such setting, shortening the maturity of debt would have the effect of increasing the outside option of an impatient saver who is trying to find a buyer for his non-matured debt.[51] This could allow sellers to obtain better prices in the secondary market, making them willing to pay more for the debt in the first place and encouraging banks to issue short-term debt.[52]

8 Conclusion

In this paper, we have assessed the value of maturity transformation, the inefficiencies caused by underlying pecuniary externalities, and the gains from regulating banks' debt maturity decisions. The assessment is based on the calibrated version of an infinite horizon equi-

[50] The result refers explicitly to bank deposits. See von Thadden (1999) for a review of the results obtained in this tradition.

[51] He and Milbradt (2014) and Bruche and Segura (2013) explicitly model the secondary market for corporate debt as a market with search frictions.

[52] In Bruche and Segura (2013), these trade-offs imply a privately optimal maturity for bank debt. The empirical evidence in Mahanti et al. (2008) and Bao, Pan, and Wang (2011), among others, shows that short-term bonds are indeed more "liquid" (as measured by the narrowness of the bid-ask spread) than long-term bonds.

librium model in which banks with long-lived assets decide the overall principal, interest rate payments, and maturity of their debt. Savers' preference for short maturities comes from their exposure to idiosyncratic preference shocks and the lack of tradability of bank debt. Banks' incentive not to set debt maturities as short as savers might, ceteris paribus, prefer comes from the fact that there are episodes (systemic liquidity crises) in which their access to savers' funding fails and their refinancing becomes more expensive. Unregulated debt maturities are inefficiently short because banks do not internalize the impact of their decentralized capital structure decisions on the equilibrium cost of the funds needed for their financing during crises, which tightens the frontier of maturity transformation possibilities faced by all banks.

The calibration of the model to Eurozone banking data for 2006 yields the result that the welfare gains from the optimal regulation of banks' debt maturity decisions are substantial (with an aggregate present value of about euro 105 billion). Yet, the required lengthening in the average maturity of banks' wholesale debt is quite moderate: from its estimated 2.8 months in the unregulated equilibrium to 3.3 months in the optimally regulated equilibrium. This introduces a call for caution regarding the desirability of more drastic reductions in maturity transformation such as those envisaged by the NSFR regulation of Basel III.

We have sketched a number of extensions of the baseline model that establish possible avenues for further research, including the integrated analysis of asset risk and refinancing risk, allowing for default and asset liquidations to occur in equilibrium, and the calibration of an extension of the model in which bailout expectations might justify the need for regulating banks' leverage. Additionally, future research might address other issues that we have left out of the current paper, including the potentially endogenous determination of the probability of systemic crises (which for tractability we have treated as an exogenous parameter, possibly leading to understate the importance of maturity risk regulation) and the quantitative analysis of the role of private or public liquidity insurance arrangements.[53]

[53] Such role was theoretically analyzed in a working paper predecessor of the current paper (see Segura and Suarez, 2013).

Appendix

A Data description and robustness

In this appendix we first describe the data on the structure of Eurozone banks' liabilities in 2006. Then we provide details on the steps followed in the calibration of the model. We conclude with some robustness analysis.

A.1 Debt categories and outstanding amounts

The data on the outstanding debt liabilities of the aggregate Eurozone banking sector at the end of 2006 comes from the Monthly Bulletin and the Monetary Statistics published by the ECB. We assume that retail funding consists of the deposits held by euro area households and non-financial corporations (NFCs) in euro area banks whose figures are reported in Section 2.5 of the Monthly Bulletin. We adjust the original figures to exclude repurchase agreements, that are not covered by deposit insurance, and include them in one of the wholesale funding debt categories.

The remaining debt liabilities are considered wholesale funding. The breakdown shown in the first column of Table 1 is chosen to provide a convenient match with the sources of data that allow us to impute an average maturity to each debt category.

Deposits and repos from banks is a category created by adding and subtracting several items. It is the result of adding (i) deposits issued by euro area banks with euro area monetary and financial institutions (MFIs) (Monthly Bulletin, Section 2.1, Aggregate balance sheet of euro area MFIs) and (ii) deposits issued by euro area banks with non-euro area banks (Monthly Bulletin, Section 2.5), and subtracting (iii) lending from the Eurosystem to euro area banks (Monthly Bulletin, Section 1.1) and (iv) loans from euro area MMFs to euro area MFIs (Monetary Statistics, Aggregate balance sheet of euro area MMFs).

Commercial paper and bonds includes the outstanding principal of tradable debt securities issued by euro area banks (Monthly Bulletin, Section 2.7).

Other deposits is a category created from Section 2.5 of the Monthly Bulletin that includes deposits held by insurance corporations and pension funds, other financial intermediaries, general government, non-bank non-euro area residents, and money market funds (MMFs). We adjust the original figures to exclude, whenever feasible, repurchase agreements, that we group in the next category.[54]

Other repos includes the repurchase agreements from households, NFCs, insurance corporations and pension funds, and other financial intermediaries.

[54]This can be done for the first two sectors. For the last three sectors (whose deposits account for 5.9% of total bank debt) it is not possible to distinguish between unsecured deposits and secured deposits (repos).

Eurosystem lending is the lending made by the ECB and the national central banks of the euro area (Monthly Bulletin, Section 1.1).

A.2 Average δ for each wholesale debt category

To impute an average δ to *Other deposits* we use the data on the maturity profile of the corresponding deposits (Monthly Bulletin, Section 2.5). The data distinguishes between the following maturities at issuance: (a) overnight, (b) up to two years, (c) more than two years, (d) redeemable at notice of up to three months, and (e) redeemable at notice of more than three months. We assign an average maturity δ_j to each of these maturity intervals (j =a,b,c,d,e) and then compute a weighted average to obtain the average δ of the corresponding debt category.[55] For the maturity interval (a), the probability that the deposit matures in a one month crisis is $\delta_{\mathrm{a}} = 1$. For the maturity interval (b), we assume that the maturity at issuance of the corresponding debt is uniformly distributed in the interval from 0 to 24 months and that the issuance of this debt occurs in a perfectly staggered manner over time, which implies assigning $\delta_{\mathrm{b}} = 0.18$.[56] Similar assumptions allow us to impute $\delta_{\mathrm{c}} = 0.025$, $\delta_{\mathrm{d}} = 0.33$ and $\delta_{\mathrm{e}} = 0$ to the remaining intervals.

To impute an average δ to *Deposits and repos from banks*, we use the Euro Money Market Survey of the ECB. This yearly survey reports the average daily volumes of euro denominated interbank borrowing and lending (secured and unsecured) transactions of a sample of European Union (EU) banks.[57] The data is broken down in the following intervals of maturity at issuance: one day, two days to one week, one week to one month, one to

[55] For three of the sectors in the category (general government, non-bank non euro-area residents, and MMFs), there is no data on the maturity profile. To the deposits from the general goverment (11.3% of this category), we assign an average δ equal to that of non-financial corporations which can be computed using similar data from Monthly Bulletin, Section 2.5. To those from non-bank non euro-area residents (29.9%), which are mostly non-bank financial intermediaries, and MMFs (1.9%) we impute an average δ equal to that of the deposits from other financial intermediaries (which are also part of this category).

[56] Under the stated assumptions, the probability that an outstanding debt with maturity at issuance of t months matures during a crisis that lasts one month is one if $t \leq 1$, and $1/t$ if $t > 1$. Integrating these probabilities over $t \sim U[0, 24]$ we obtain:

$$\delta_{\mathrm{b}} = \frac{1}{24}\left[1 + \int_{1}^{24} \frac{1}{t} dt\right] = 0.18.$$

Similarly, for a maturity interval $[T_{1j}, T_{2j}]$ with $T_{1j} > 1$, we can use the formula

$$\delta_j = \frac{1}{T_{2j} - T_{1j}} \int_{T_{1j}}^{T_{2j}} \frac{1}{t} dt.$$

[57] For this and other categories described below, the data source refers to (samples of banks from) the whole EU rather than the Eurozone, and constitutes the best proxy to the reality of Eurozone banks in 2006 available to us.

three months, three to six months, six to twelve months, more than twelve months. We set $\delta_j = 1$ for all debt issued with maturity of less than one month and use uniformity assumptions to impute values of δ_j to the remaining intervals in the same way we described for the *Other deposits* category. For the more than twelve months interval, we assume a maximum maturity of twenty-four months.

To impute an average δ to *Commercial paper and bonds*, we use data from the Risk Dashboard, a report published quarterly by the European Systemic Risk Board. Section 4.6 of the Risk Dashboard provides the outstanding amounts of debt securities issued by EU banks and their breakdown in a number of time-to-maturity intervals: less than one year, one to two years, two to three years, three to four years, four to five years, five to ten years, and more than ten years. Taking into account that the reported maturities are times-to-maturity instead of maturities-at-issuance, we assign an average $\delta_j = 1/12$ to debt in the first interval and $\delta_j = 0$ to the remaining ones.

To impute an average δ to *Other repos*, we rely on Survey on the European Repo Market conducted by the International Capital Market Association (ICMA). This yearly survey reports the outstanding repo transactions of a sample of European financial groups, mainly banks. The survey distinguishes essentially the same maturity intervals as the Euro Money Market Survey.[58] The reported maturities are times-to-maturity instead of maturities-at-issuance, which implies assigning $\delta_j = 1$ to all the intervals with time to maturity of less than one month and $\delta_j = 0$ to residual maturities of more than one month.

Finally, in the absence of precise published data on the maturity profile of *Eurosystem lending* and given that it accounts for only 2.1% of Eurozone bank debt in 2006, we exclude this category from the computation of the overall average δ set as a target in our calibration.

A.3 Average interest rates by maturity range

For the calibration of the preference parameters ρ_L, ρ_H and γ, we use data on the average interest rates paid on outstanding deposits issued by Eurozone banks and held by domestic households. Table 4 in Section 4.5 of the ECB's Monthly Bulletin contain the average rates r_{jt} paid in every month $t = 1, 2, ..., 12$ of 2006 on deposits of various maturity categories j. We consider three categories that correspond to specific maturity ranges: overnight deposits ($j = 1$), maturity of up to 2 years ($j = 2$), and maturity over 2 years ($j = 3$).[59] We understand that, in the case of households, "overnight deposits" are mostly made of demand deposits. We set the target empirical moment for each category equal the simple average of

[58] In fact, it includes an additonal category to account for open-ended repos. These contracts can be terminated on demand and thus we assign $\delta_j = 1$ to them.

[59] Other categories include deposits redeemable at notice of less than 3 months and deposits redeemable at notice of more than 3 months.

its monthly observations and match it to the model implied moments described in the main text.

B Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Proposition 1 Using (3) it is a matter of algebra to obtain that:

$$r'(\delta) = \frac{-\gamma(1+\rho_H)(\rho_H-\rho_L)}{[\rho_H+\delta+(1-\delta)\gamma]^2} < 0,$$

$$r''(\delta) = \frac{2\gamma(1-\gamma)(1+\rho_H)(\rho_H-\rho_L)}{[\rho_H+\delta+(1-\delta)\gamma]^3} > 0.$$

The other properties stated in the proposition are immediate.■

Proof of Proposition 2 The proof is organized in a sequence of steps.

1. If (CF) is satisfied then (LL) is strictly satisfied. Using equation (6), (LL) can be written as:

$$0 \le E(\delta, D; \phi) = \frac{1}{\rho_H}(\mu - rD) - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D,$$

while, using (7), (CF) can be written as

$$\begin{aligned} 0 &\le \frac{1}{1+\rho_H}[\mu - r(1-\delta)D + \delta D + E(\delta, D; \phi)] - (1+\phi)\delta D = \\ &= \frac{1}{\rho_H}(\mu - rD) - \left(1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\right)\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D. \end{aligned}$$

Now, since $1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon} > \frac{(1+\rho_H)\varepsilon}{\rho_H(1+\rho_H+\varepsilon)}$ we conclude that whenever (CF) is satisfied, (LL) is strictly satisfied.

2. Notation and useful bounds. Using equation (8) we can write:

$$V(\delta, D; \phi) = \frac{\mu}{\rho_H} + D\Pi(\delta; \phi), \tag{15}$$

where

$$\Pi(\delta, \phi) = 1 - \frac{1}{\rho_H}\left[\left(1-\frac{\varepsilon}{1+\rho_H+\varepsilon}\delta\right)r + \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\delta\left(\phi+\frac{\rho_H}{1+\rho_H}\right)\right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function $\Pi(\delta, \phi)$ is concave in δ.

(CF) in equation (7) can be rewritten as:

$$\mu + V(\delta, D; \phi) \geq [(1+\rho_H)(1+\phi)\delta + (1+r)(1-\delta)]D,$$

and if we define $C(\delta, \phi) = (1+\rho_H)(1+\phi)\delta + (1+r)(1-\delta)$, (CF) can be written in a more compact form that will be used from now onwards:

$$\frac{1+\rho_H}{\rho_H}\mu + [\Pi(\delta, \phi) - C(\delta, \phi)]D \geq 0. \tag{16}$$

Using Proposition 1 we can see that the function $C(\delta, \phi)$ is convex in δ.

We have the following relationship:

$$\Pi(\delta, \phi) = 1 - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[r(\delta) + \frac{\varepsilon}{1+\rho_H}(C(\delta, \phi) - 1)\right]. \tag{17}$$

Assumption A1 implies $(1+\rho_H)(1+\phi) \leq 2(1+\rho_L) \leq 2(1+r(\delta))$ for all δ, and we can check that the following bounds (that are independent from ϕ) hold:

$$\begin{aligned} C(\delta, \phi) &\geq 1 + r(\delta), \\ \frac{\partial C(\delta, \phi)}{\partial \delta} &\leq 2(1+r(\delta)) - (1+r(\delta)) = 1 + r(\delta). \end{aligned} \tag{18}$$

Using assumption A2, it is a matter of algebra to check that, for all δ,

$$\frac{d^2 r}{d\delta^2} + \frac{dr}{d\delta} \geq 0.$$

And, from this inequality, $\frac{dr}{d\delta} < 0$, and $r < \rho_H$, it is possible to check that:

$$\frac{\partial^2 \Pi(\delta, \phi)}{\partial \delta^2} + \frac{\partial \Pi(\delta, \phi)}{\partial \delta} < -\frac{1}{\rho_H}\left(1 - \frac{\varepsilon}{1+\rho_H+\varepsilon}\delta\right)\left(\frac{dr}{d\delta} + \frac{d^2 r}{d\delta^2}\right) \leq 0. \tag{19}$$

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

3. $D^* = 0$ is not optimal It suffices to realize that $\frac{\partial V(D,0;\phi)}{\partial D} = \Pi(0, \phi) = 1 - \frac{r(0)}{\rho_H} > 0$.

4. The solution (D^*, δ^*) of the maximization problem in equation (9) exists, is unique, and satisfies (CF) with equality, i.e. $\frac{1+\rho_H}{\rho_H}\mu + (\Pi(\delta^*, \phi) - C(\delta^*, \phi))D^* = 0$. We are going to prove existence and uniqueness in the particular case that there exist $\delta_\Pi, \delta_C \in [0, 1]$ such that $\frac{\partial \Pi(\delta_\Pi, \phi)}{\partial \delta} = \frac{\partial C(\delta_C, \phi)}{\partial \delta} = 0$. This will ensure that the solution of the maximization problem is interior in δ. The other cases are treated in an analogous way but might give rise to corner solutions in δ.[60]

[60] More precisely, if for all $\delta \in [0, 1]$ $\frac{\partial C(\delta, \phi)}{\partial \delta} > 0$ we might have $\delta^* = 0$ and if for all $\delta \in [0, 1]$, $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} > 0$ we might have $\delta^* = 1$.

First, since $\Pi(\delta, \phi)$ is concave in δ we have that $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \leq \delta_\Pi$. Since $C(\delta, \phi)$ is convex in δ we have that $\frac{\partial C(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \geq \delta_C$. It is easy to prove from equation (17) that $\delta_C < \delta_\Pi$.

Now, let (δ^*, D^*) be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution (δ^*, D^*) are:

$$\begin{aligned} (1+\theta)\Pi - \theta C &= 0, \\ (1+\theta)\frac{\partial \Pi}{\partial \delta} - \theta \frac{\partial C}{\partial \delta} &= 0, \\ \theta \left[\frac{1+\rho_H}{\rho_H}\mu + (\Pi - C)D^* \right] &\geq 0, \\ \theta &\geq 0, \end{aligned} \tag{20}$$

where θ is the Lagrange multiplier associated with (CF) and we have used that $D^* > 0$ in order to eliminate it from the second equation.

If $\theta = 0$ then the second equation implies $\delta^* = \delta_\Pi$ and thus $\Pi(\delta^*, \phi) \geq \Pi(0, \phi) > 0$ and the first equation is not satisfied. Therefore we must have $\theta > 0$ so that (CF) is binding at the optimum. Now we can eliminate θ from the previous system of equations, which gets reduced to:

$$\frac{\partial \Pi(\delta^*, \phi)}{\partial \delta} C(\delta^*, \phi) = \frac{\partial C(\delta^*, \phi)}{\partial \delta} \Pi(\delta^*, \phi), \tag{21}$$

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta^*, \phi) - \Pi(\delta^*, \phi) \right] D^*. \tag{22}$$

We are going to show that equation (21) has a unique solution in δ. For $\delta \leq \delta_C < \delta_\Pi$, we have $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$ and thus the left hand side (LHS) of (21) is strictly bigger than the RHS. For $\delta \geq \delta_\Pi > \delta_C$, we have $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$ and thus RHS of (21) is strictly bigger.

Now, the function $\frac{\partial C(\delta,\phi)}{\partial \delta}\Pi(\delta, \phi)$ is strictly increasing in the interval (δ_C, δ_Π) since both terms are positive and increasing. Thus, it suffices to prove that for $\delta \in (\delta_C, \delta_\Pi)$ the function $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} C(\delta, \phi)$ is decreasing.[61] Using the the bounds in (18), inequality (19) and $\frac{\partial^2 \Pi}{\partial \delta^2} < 0, \frac{\partial \Pi}{\partial \delta} > 0$ for $\delta \in (\delta_C, \delta_\Pi)$, we have:

$$\frac{\partial}{\partial \delta}\left(\frac{\partial \Pi}{\partial \delta} C \right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta}\frac{\partial C}{\partial \delta} \leq (1+r)\left(\frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta} \right) \leq 0.$$

This concludes the proof on the existence and uniqueness of a δ^* that satisfies the necessary FOC in (21).

Now, for given δ^*, the other necessary FOC (22) determines D^* uniquely.[62]

[61] This is not trivial since $C(\delta, \phi)$ is increasing.

[62] Let us observe that for all δ, $C(\delta, \phi) \geq 1 > \Pi(\delta, \phi)$.

5. δ^* **is independent from** μ **and** D^* **is strictly increasing in** μ. Equation (21) determines δ^* and is independent from μ. Then (22) shows that D^* is increasing in μ.

6. δ^* **is decreasing in** ϕ **and, if** $\delta^* \in (0,1)$**, it is strictly decreasing.** Let $\delta(\phi)$ be the solution of the maximization problem of the bank for given ϕ. Let us assume that $\delta(\phi)$ satisfies the FOC (21). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function $\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi$ is decreasing in δ around $\delta(\phi)$. In order to show that $\delta(\phi)$ is decreasing, it suffices to show that the derivative of this function w.r.t. ϕ is negative. Using the definitions of $C(\delta, \phi), \Pi(\delta, \phi)$ after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi\right] = -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[(1+\rho_H)\left(\frac{dr}{d\delta}\delta - r\right) + \varepsilon\right].$$

Now we have $\frac{d}{d\delta}\left(\frac{dr}{d\delta}\delta - r\right) = \frac{d^2 r}{d\delta^2}\delta \geq 0$ and thus $\frac{dr}{d\delta}\delta - r \geq \frac{dr}{d\delta}\delta - r\big|_{\delta=0} = -r(0)$, and finally:

$$\begin{aligned}\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi\right] &\leq -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[-(1+\rho_H) r(0) + \varepsilon\right] \\ &< -(1+\rho_H) + \frac{1}{\rho_H}(1+\rho_H) r(0) = -(1+\rho_H)\left(1 - \frac{r(0)}{\rho_H}\right) < 0.\end{aligned}$$

This concludes the proof that $\frac{d\delta}{d\phi} < 0$.[63]

7. $\delta^* D^*$ **is decreasing with** ϕ. **If** $\delta^* > 0$ **it is strictly decreasing.** Let $\delta(\phi), D(\phi)$ be the solution of the maximization problem of the bank for given ϕ. We have:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta(\phi), \phi) - \Pi(\delta(\phi), \phi)\right] D(\phi).$$

Let $\phi_1 < \phi_2$. In Step 6 we showed that $\delta(\phi_1) \geq \delta(\phi_2)$. If $\delta(\phi_2) = 0$ then trivially $\delta(\phi_1) D(\phi_1) \geq \delta(\phi_2) D(\phi_2) = 0$. Let us suppose that $\delta(\phi_2) > 0$. Since trivially $\Pi(\delta(\phi_1), \phi_1) D(\phi_1) \geq \Pi(\delta(\phi_2); \phi_2) D(\phi_2)$, we must have $C(\delta(\phi_1), \phi_1) D(\phi_1) \geq C(\delta(\phi_2), \phi_2) D(\phi_2)$. Now, suppose that $\delta(\phi_1) D(\phi_1) \leq \delta(\phi_2) D(\phi_2)$, then we have the following two inequalities:

$$\begin{aligned}(1+\rho_H)(1+\phi_1)\delta(\phi_1) D(\phi_1) &< (1+\rho_H)(1+\phi_2)\delta(\phi_2) D(\phi_2), \\ (1+r(\delta(\phi_1)))(1-\delta(\phi_1)) &\leq (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),\end{aligned}$$

that imply $C(\delta(\phi_1), \phi_1) D(\phi_1) < C(\delta(\phi_2), \phi_2) D(\phi_2)$, but this contradicts our assumption. Thus, $\delta(\phi_1) D(\phi_1) > \delta(\phi_2) D(\phi_2)$.■

[63] In the case of corner solution $\delta^*(\phi) = 1$, we might have $\frac{d\delta^*}{d\phi} = 0$ and obviously for $\delta^*(\phi) = 0$, $\frac{d\delta^*}{d\phi} = 0$.

Proof of Proposition 3 This proof has two parts:

1. Existence and uniqueness of equilibrium. Let us denote $(\delta(\phi), D(\phi))$ the solution of the bank's optimization problem for every excess cost of crisis financing $\phi \geq 0$. Proposition 2 states that $\delta(\phi)D(\phi)$ is decreasing in ϕ. For $\phi \in [0, \overline{\phi}]$ let us define $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$. This function represents the difference between the excess cost of financing during a crisis by banks' decisions and banks' expectation on such variable. Since Φ is an increasing function on the aggregate demand of funds during a crisis the function $\Sigma(\phi)$ is strictly decreasing. Because of the uniqueness of the solution to the problem that defines $(\delta(\phi), D(\phi))$, the function is also continuous. Moreover, we trivially have $\Sigma(0) \geq 0$ and $\Sigma(\overline{\phi}) \leq 0$. Therefore there exists a unique $\phi^e \in \mathbb{R}^+$ such that $\Sigma(\phi^e) = 0$. By construction $(\phi^e, (\delta(\phi^e), D(\phi^e)))$ is the unique equilibrium of the economy.

2. Comparative statics with respect to a shift in $\Phi(x)$. We are going to follow the notation used in the proof of Proposition 3. Let Φ_1, Φ_2 be two curves describing the inverse supply of financing during a crisis and assume they satisfy $\Phi_1(x) > \Phi_2(x)$ for all $x > 0$. Let us denote $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$ for $i = 1, 2$. By construction we have $\Sigma_1(\phi_1^e) = 0$. Let us suppose that $\phi_1^e < \phi_2^e$. Then we would have:

$$\Sigma_2(\phi_2^e) = \Phi_2(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e \leq \Phi_1(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e < \Phi_1(\delta(\phi_1^e)D(\phi_1^e)) - \phi_1^e = \Sigma_1(\phi_1^e) = 0, \tag{23}$$

where in the first inequality we use the assumption $\Phi_2(x) \leq \Phi_1(x)$ for $x \geq 0$, and in the second inequality we use that if $\phi_1^e < \phi_2^e$ then $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$ (Proposition 2), and that $\Phi_1(\cdot)$ is increasing. Notice that the sequence of inequalities in (23) implies $\Sigma_2(\phi_2^e) < 0$, which contradicts the definition of ϕ_2^e. We must therefore have $\phi_1^e \geq \phi_2^e$. And Proposition 2 implies that $\delta_1^e \leq \delta_2^e$, $\delta_1^e D_1^e \leq \delta_2^e D_2^e$, and $r_1^e \geq r_2^e$, proving all the results in weak terms.

Finally, let us suppose that $\delta_2^e \in (0, 1)$. Then the first inequality in (23) is strict, since $\delta_2^e D_2^e > 0$, and we can straightforwardly check that the previous argument implies $\phi_1^e > \phi_2^e$. In this case, since $\delta_2^e \in (0, 1)$, Proposition 2 implies that $\delta_1^e < \delta_2^e$, $\delta_1^e D_1^e < \delta_2^e D_2^e$, and $r_1^e > r_2^e$.$\blacksquare$

Proof of Proposition 4 We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries From first principles, using equations (8) and (12), we can obtain

$$\frac{\partial W(\delta, D)}{\partial \delta} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial \delta} = D \frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}, \tag{24}$$

where the last equality follows from (15). Similarly we can obtain

$$\frac{\partial W(\delta, D)}{\partial D} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial D} = \Pi(\delta, \Phi(\delta D)). \tag{25}$$

2. (CF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 4. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that ϕ is not taken as given but as the function $\Phi(\delta D)$ in D and δ.

3. Definition of function $D^c(\delta)$ and its properties Let $(\phi^e, (\delta^e, D^e))$ be the competitive equilibrium. Let us assume that $\delta^e < 1$. By definition of equilibrium we have $\phi^e = \Phi(\delta^e D^e)$. For every δ let $D^c(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right] D^c(\delta). \tag{26}$$

Differentiating w.r.t. δ:

$$\left[\frac{\partial C(\delta,\phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta,\phi^e)}{\partial \delta}\right] D^c(\delta) + \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right]\frac{dD^c(\delta)}{d\delta} = 0. \tag{27}$$

Using the characterization of δ^e in equation (21), the inequalities $C(\delta,\phi^e) \geq 1 > \Pi(\delta,\phi^e)$ imply $\frac{\partial C(\delta^e,\phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta} > 0$ and, then, we can deduce from the equation above that $\frac{dD^c(\delta^e)}{d\delta} < 0$. Since (CF) is binding at the optimal debt structure we can think of the bank problem as maximizing the univariate function $V(\delta, D^c(\delta);\phi^e)$. Hence δ^e must satisfy the necessary FOC for an interior solution to the maximization of $V(\delta, D^c(\delta);\phi^e)$:

$$\frac{dV(\delta^e, D^c(\delta);\phi^e)}{d\delta} = 0 \Leftrightarrow D^c(\delta^e)\frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta} + \Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0, \tag{28}$$

which multiplying by δ^e can be written as

$$D^c(\delta^e)\frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta^e}\delta^e = \Pi(\delta^e,\phi^e)\left(-\frac{dD^c(\delta^e)}{d\delta}\delta^e\right).$$

Since $\frac{\partial\left(\Pi - \frac{\partial \Pi}{\partial \delta}\delta\right)}{\partial \delta} = -\frac{\partial^2 \Pi}{\partial \delta^2}\delta \geq 0$ and $\Pi(0,\phi) - \frac{\partial \Pi(0,\phi)}{\partial \delta}0 > 0$, we have $\Pi(\delta,\phi) > \frac{\partial \Pi(\delta,\phi)}{\partial \delta}\delta$ for all $\delta \in [0,1]$ and the previous equation implies

$$D^c(\delta^e) > -\frac{dD^c(\delta^e)}{d\delta}\delta^e \Leftrightarrow \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0.$$

4. Evaluation of $\left.\frac{d(D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ and $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ For every δ, let $D^s(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta,\Phi(\delta D^s(\delta))) - \Pi(\delta,\Phi(\delta D^s(\delta)))\right] D^s(\delta).$$

Differentiating w.r.t. δ, we obtain

$$\left[\frac{\partial C(\delta,\Phi)}{\partial\delta}-\frac{\partial\Pi(\delta,\Phi)}{\partial\delta}\right]D^s(\delta)+\left[C(\delta,\Phi(\delta D^s(\delta)))-\Pi(\delta,\Phi(\delta D^s(\delta)))\right]\frac{dD^s(\delta)}{d\delta}+$$
$$+\left[\frac{\partial C(\delta,\Phi)}{\partial\phi}-\frac{\partial\Pi(\delta,\Phi)}{\partial\phi}\right]\Phi'(\delta D^s(\delta)))\frac{d\left(\delta D^s(\delta)\right)}{d\delta}=0. \tag{29}$$

By construction, $D^s(\delta^e)=D^c(\delta^e)=D^e$. Now, subtracting equation (27) from equation (29) at the point $\delta=\delta^e$ we obtain

$$\left[C(\delta^e,\phi^e)-\Pi(\delta^e,\phi^e)\right]\left(\frac{dD^s(\delta^e)}{d\delta}-\frac{dD^c(\delta^e)}{d\delta}\right)+\left[\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}\right.$$
$$\left.-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\phi}\right]\Phi'(\delta^e D^e)\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=0. \tag{30}$$

Suppose that $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}\leq 0$, then we would have $\frac{dD^s(\delta^e)}{d\delta}\geq\frac{dD^c(\delta^e)}{d\delta}$, since trivially $\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}-\frac{\partial\Pi(\delta,\phi^e)}{\partial\phi}>0$. But then

$$\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=D^s(\delta^e)+\frac{dD^s(\delta^e)}{d\delta}\delta^e>D^c(\delta^e)+\frac{dD^c(\delta^e)}{d\delta}\delta^e=\left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}>0,$$

which contradicts the hypothesis. We must thus have $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0$, in which case equation (30) implies $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}<0$.

5. Evaluation of $\left.\frac{dW(\delta,D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ Using equations (24) and (25), we have:

$$\begin{aligned}\frac{dW(\delta,D^s(\delta))}{d\delta}&=\frac{\partial W(\delta,D^s(\delta))}{\partial\delta}+\frac{\partial W(\delta,D^s(\delta))}{\partial D}\frac{dD^s(\delta)}{d\delta}\\&=D^s(\delta)\frac{\partial\Pi(\delta,\Phi(\delta D^s(\delta)))}{\partial\delta}+\Pi(\delta,\Phi(\delta D^s(\delta)))\frac{dD^s(\delta)}{d\delta}.\end{aligned}$$

And, using $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}$ and (28), we obtain:

$$\left.\frac{dW(\delta,D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}<D^s(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}+\Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta}=0.$$

Summing up, having

$$\left.\frac{dW(\delta,D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}<0,\quad\left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e}<0,\text{ and }\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0,$$

implies that a social planner can increase welfare by fixing some $\delta^s<\delta^e$, that is associated with debt $D^s>D^e$ and with refinancing needs $\delta^s D^s<\delta^e D^e$.■

C Debt structures that induce default during crises

In this appendix we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt and being liquidated in the crisis state. First, we describe the sequence of events following default. Second, we show how the debt of the bank would be valued by savers who correctly anticipate the possibility of default. Finally, we analyze the bank's capital structure problem when default during crises is an explicit alternative.

Default and liquidation Liquidation following the bank's inability to satisfy its refinancing needs yields a residual value $L \geq 0$. Suppose that partial liquidation is not feasible and in case of liquidation L is first used to repay $\overline{D}_R$ to retail depositors and only if $L > \overline{D}_R$ the residual value is equally distributed among the wholesale debtholders. Assume further, for simplicity, that $L > \overline{D}_R$ so that retail deposits are riskless.[64] It is easy to realize that if the bank exposes itself to default in a crisis (rather than relying on crisis financing), then it will find it optimal to opt for debt contracts that mature in a perfectly correlated manner since this minimizes the probability of defaulting during crises. Hence we assume that the debt issued by the bank when getting rid of the (CF) constraint has perfectly correlated maturities.

Pricing of wholesale debt in the presence of default risk From the perspective of the investor in wholesale debt, there are four states relevant for the valuation of a non-matured debt contract: patience in the normal state ($i = LN$), patience in the crisis state ($i = LC$), impatience in the normal state ($i = HN$), and impatience in the crisis state ($i = HC$).

Let $l = (L - \overline{D}_R)/D < 1$ be the fraction of the principal of wholesale debt which is recovered in case of liquidation and let Q_i be the present value of expected losses due to default as evaluated from each of the states i just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which debtholdes recover the full principal (one unit) at maturity. These values satisfy the following system of recursive relationships:

$$Q_{LN} = \frac{1}{1+\rho_L} \left[\delta\varepsilon(1-l) + (1-\delta)\{(1-\varepsilon)[(1-\gamma)Q_{LN} + \gamma Q_{HN}] + \varepsilon[(1-\gamma)Q_{LC} + \gamma Q_{HC}]\}\right],$$

$$Q_{HN} = \frac{1}{1+\rho_H} \left[\delta\varepsilon(1-l) + (1-\delta)\left[(1-\varepsilon)Q_{HN} + \varepsilon Q_{HC}\right]\right],$$

[64] Under our calibration, this inequality is satisfied by the maximum liquidation value $L^{\max}$ for which banks prefer to avoid defaulting during crises.

$$Q_{LC} = \frac{1}{1+\rho_L}(1-\delta)[(1-\gamma)Q_{LN} + \gamma Q_{HN}],$$
$$Q_{HC} = \frac{1}{1+\rho_H}(1-\delta)Q_{HN}.$$

These expressions essentially account for the principal $1 - l > 0$ which is lost if the bank's wholesale debt matures in the crisis state (pushing the bank into default). The first equation reflects that default as well as any of the four non-default states i may follow state LN. The second and fourth equations reflect that impatience is an absorbing state. The third and fourth equations reflect that after the crisis state the economy reverts deterministically to the normal state. We will denote the solution for Q_{LN} associated with this linear system of equations by $Q_{LN}(\delta, D; L)$ in order to highlight its dependence on δ, D and L.

The value of a debt contract $(r, \delta, 1)$ to a patient investor in the normal state, when default is expected if the bank's debt matures during a crisis, can then be written as

$$U_L^d(r, \delta) = U_L(r, \delta) - Q_{LN}(\delta, D; L),$$

where $U_L(r, \delta)$ is the value of the same contract in the benchmark senario in which banks do not default in crises, given by (2).

The interest rate yield that the bank offers when default may occur, $r^d(\delta)$, satisfies $U_L^d(r^d(\delta), \delta) = 1$, while the non-default yield $r(\delta)$ satisfies $U_L(r(\delta), \delta) = 1$. Thus, we have $U_L^d(r^d(\delta), \delta) = U_L(r(\delta), \delta)$, which allows us to express $r^d(\delta)$ as the sum of $r(\delta)$ and a default-risk premium:

$$r^d(\delta) = r(\delta) + \frac{(\rho_H + \delta)(\rho_L + \delta + (1-\delta)\gamma)}{\rho_H + \delta + (1-\delta)\gamma} Q_{LN}(\delta, D; L).$$

The above equations imply that the default-risk premium $r^d(\delta) - r(\delta)$ is increasing in δ and D, and decreasing in L. Given that δ increases the probability of default, $r^d(\delta)$ is not necessarily decreasing in δ.

Optimal capital structure when debt maturity in crises leads to default If the bank does not satisfy the crisis financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in the normal state $E^d(\delta, D)$ will satisfy

$$E^d(\delta, D) = \frac{1}{1+\rho_H}\left[\mu\text{-}\rho_L\overline{D}_R\text{-}r^d D\text{+}(1\text{-}\varepsilon)E^d(\delta,D)\text{+}\varepsilon\{\delta \cdot 0\text{+}(1\text{-}\delta)\frac{1}{1+\rho_H}[\mu\text{-}r^d D\text{+}E^d(\delta,D)]\}\right],$$

whose solution yields:

$$E^d(\delta, D) = \frac{1+\rho_H+\varepsilon(1-\delta)}{(1+\rho_H)^2 - (1+\rho_H)(1-\varepsilon) - \varepsilon(1-\delta)}(\mu - \rho_L\overline{D}_R - r^d D).$$

And the bank's optimal capital structure decision can be described as

$$\begin{array}{cc} \max\limits_{D \geq 0,\ \delta \in [0,1]} & V^d(\delta, D) = \overline{D}_R + D + E^d(\delta, D), \\ \text{s.t.} & E^d(\delta, D) \geq 0, \quad \text{(LL)} \end{array} \tag{31}$$

where (LL) is trivially equivalent to $\mu - \rho_L \overline{D}_R - r^d D \geq 0$.

To find the maximum value of L which, under our calibration of the model, is consistent with banks' optimizing subject to the (CF) constraint (and, hence, with not getting exposed to default), we proceed in two steps. First, we solve the problem in (31) numerically for an ample grid of values of L. Second, we compare the optimized value of the objective function in (31) with the equilibrium market value of the bank in the scenario with no default. $L^{\max}$ is the maximum L for which the solution of the problem with no default dominates.

References

Acharya, Viral, Douglas Gale, and Tanju Yorulmazer (2011) "Rollover Risk and Market Freezes," *Journal of Finance*, 66, 1177-1209.

Acharya, Viral, and S. Viswanathan (2011) "Leverage, Moral Hazard, and Liquidity," *Journal of Finance*, 66, 99-138.

Allen, Franklin, and Douglas Gale (1998) "Optimal Financial Crises," *Journal of Finance*, 53, 1245-1284.

Allen, Franklin, and Douglas Gale (2000) "Financial Contagion," *Journal of Political Economy*, 108, 1-33.

Allen, Franklin, and Douglas Gale (2007), *Understanding Financial Crises*, Oxford: Oxford University Press.

Bao, Jack, Jun Pan, and Jiang Wang (2010) "The Illiquidity of Corporate Bonds," *Journal of Finance*, 66, 911–946.

Basel Committee on Banking Supervision (2013) "Basel III: The Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools," Bank for International Settlements, January.

Basel Committee on Banking Supervision (2014) "Basel III: The Net Stable Funding Ratio," Bank for International Settlements, October.

Bengui, Julien (2011), "Systemic Risk and Inefficient Debt Maturity," mimeo, University of Maryland.

Bennett, Rosalind, and Haluk Unal (2014) "Understanding the Components of Bank Failure Resolution Costs," FDIC CFR WP 2014-04, July.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez (2013) "International Reserves and Rollover Risk," IMF WP 13/33, January.

Bianchi, Javier, and Enrique Mendoza (2011) "Overborrowing, Financial Crises and 'Macro-Prudential' Policy?," IMF WP 11/24, February.

Patrick Bolton, Patrick, Hui Chen, and Jinqiang Yang (2011), "Market Timing, Investment, and Risk Management," *Journal of Financial Economics*, 109, 40-62.

Bruche, Max, and Anatoli Segura (2013) "Debt Maturity and the Liquidity of Secondary Debt Markets," CEMFI WP 1303, April.

Brunnermeier, Markus (2009) "Deciphering the Liquidity and Credit Crunch 2007-08," *Journal of Economic Perspectives*, 23, 77-100.

Brunnermeier, Markus, and Martin Oehmke (2013) “The Maturity Rat Race,” *Journal of Finance*, 68, 483-521.

Bryant, John (1980) “A Model of Reserves, Bank Runs, and Deposit Insurance,” *Journal of Banking and Finance*, 4, 335-344.

Calomiris, Charles, and Charles Kahn (1991) “The Role of Demandable Debt in Structuring Optimal Banking Arrangements,” *American Economic Review*, 81, 497-513.

Calvo, Guillermo (1998) “Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops,” *Journal of Applied Economics*, 1, 35-54.

Cheng, Ing-Haw, and Konstantin Milbradt (2012) “The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts,” *Review of Financial Studies*, 25, 1070-1110.

Dangl, Thomas, and Josef Zechner (2007), “Debt Maturity and the Dynamics of Leverage,” mimeo, University of Vienna.

Diamond, Douglas (1991) “Debt Maturity Structure and Liquidity Risk,” *Quarterly Journal of Economics,* 106, 709-737.

Diamond, Douglas (1997) “Liquidity, Banks, and Markets,” *Journal of Political Economy*, 105, 928-956.

Diamond, Douglas, and Philip H. Dybvig (1983) “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy,* 91, 401-419.

Diamond, Douglas, and Raghuram G. Rajan (2001) “Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking,” *Journal of Political Economy*, 109, 287-327.

Diamond, Douglas, and Zhiguo He (2012) “A Theory of Debt Maturity: The Long and Short of Debt Overhang,” NBER WP 18160, June.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Pedersen (2005) “Over-the-Counter Markets,” *Econometrica*, 73, 1815-1847.

Ellul, Andrew, Chotibhak Jotikasthira, and Christian Lundblad (2011) “Regulatory Pressure and Fire Sales in the Corporate Bond Market,” *Journal of Financial Economics,* 101, 596-620.

Farhi, Emmanuel, and Jean Tirole (2012) “Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts,” *American Economic Review*, 102, 60-93.

Feldhutter, Peter (2012) “The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressures,” *Review of Financial Studies*, 25, 1155-1206.

Flannery, Mark (1986) "Asymmetric Information and Risky Debt Maturity Choice," *Journal of Finance,* 41, 19-37.

Flannery, Mark (1994) "Debt Maturity and the Deadweight Cost of Leverage: Optimally Financing Banking Firms," *American Economic Review,* 84, 320-331.

Geanakoplos, John, and Herakles Polemarchakis (1986) "Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when Markets Are Incomplete," in: W. Heller, R. Starr and D. Starrett (eds.), *Essays in Honor of Kenneth Arrow*, Vol. 3, Cambridge: Cambridge University Press, 65-95.

Gennaioli, Nicola, Andrei Shleifer, and Robert W. Vishny (2012) "Neglected Risks, Financial Innovation and Financial Fragility," *Journal of Financial Economics*, 104, 452-468.

Gersbach, Hans, and Jean-Charles Rochet (2012) "Aggregate Investment externalities and Macroprudential Regulation," *Journal of Money, Credit and Banking,* 44, 73-109.

Goldstein, Itay, and Ady Pauzner (2005) "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60, 1293-1328.

Gorton, Gary (2009) "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007," paper presented at the Federal Reserve Bank of Atlanta's Financial Markets Conference, May 11-13.

Gorton, Gary, and George Pennacchi (1990) "Financial Intermediaries and Liquidity Creation," *Journal of Finance*, 45, 49-71.

Greenwald, Bruce, and Joseph Stiglitz (1986) "Externalities and Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, 101, 229-264.

Greenwood, Robin, Augustin Landier, and David Thesmar (2015) "Vulnerable Banks," *Journal of Financial Economics,* 115, 471-485.

Hardy, Daniel (2013) "Bank Resolution Costs, Depositor Preference, and Asset Encumbrance," IMF WP 13/172, July.

He, Zhiguo, and Konstantin Milbradt (2014) "Endogenous Liquidity and Defaultable Bonds," *Econometrica*, 82, 1443-1508.

He, Zhiguo, and Wei Xiong (2012a) "Roll over risk and Credit risk," *Journal of Finance*, 67, 391-429.

He, Zhiguo, and Wei Xiong (2012b) "Dynamic Debt Runs," *Review of Financial Studies*, 25, 1799-1843.

Huberman, Gur, and Rafael Repullo (2010) "Moral Hazard and Debt Maturity," mimeo, CEMFI.

Kiyotaki, Nobuhiro, and John Moore (1997) "Credit Cycles," *Journal of Political Economy*, 105, 211-248.

Korinek, Anton (2011) "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses," mimeo, University of Maryland.

Kroszner, Randall, Luc Laeven, and Daniela Klingebiel (2007) "Banking Crises, Financial Dependence, and Growth," *Journal of Financial Economics*, 84, 187-228.

Laeven, Luc, and Fabian Valencia (2012) "Systemic Banking Crises Database: An Update," IMF WP 12/163, June.

Lagos, Ricardo, and Guillaume Rocheteau (2009) "Liquidity in Asset Markets with Search Frictions," *Econometrica*, 77, 403-426.

Leland, Hayne, and Klaus Toft (1996) "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987-1019.

Lorenzoni, Guido (2008) "Inefficient Credit Booms," *Review of Economic Studies*, 75, 809-833.

Mahanti, Sriketan, Amrut Nashikkar, Marti Subrahmanyam, George Chacko, and Gaurav Mallik (2008) "Latent Liquidity: A New Measure of Liquidity, with an Application to Corporate Bonds," *Journal of Financial Economics,* 88, 272-298.

Martin, Antoine, David Skeie, and Ernst-Ludwig von Thadden (2014a) "The Fragility of Short-Term Secured Funding Markets," *Journal of Economic Theory*, 149, 15-42.

Martin, Antoine, David Skeie and Ernst-Ludwig von Thadden (2014b) "Repo Runs," *Review of Financial Studies*, 27, 957-989.

Newman, Yigal, and Michael Rierson (2003) "Illiquidity spillovers: Theory and evidence from European telecom bond issuance," mimeo, Graduate School of Business, Stanford University.

Perotti, Enrico, and Javier Suarez (2011) "A Pigovian Approach to Liquidity Regulation," *International Journal of Central Banking*, 7, 3-41.

Rochet, Jean-Charles, and Xavier Vives (2004) "Coordination Failures and the Lender of Last Resort: Was Bagehot Right after All?," *Journal of the European Economic Association*, 2, 1116-1147.

Santos, Joao (2001) "Bank Capital Regulation in Contemporary Banking Theory: A Review of the Literature," *Financial Markets, Institutions & Instruments*, 10, 41-84.

Segura, Anatoli, and Javier Suarez (2013) "Recursive Maturity Transformation," mimeo, CEMFI.

Stein, Jeremy (2012) "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics*, 127, 57-95.

Tarullo, Daniel (2009) "Regulatory Reform," Testimony before the Committee on Financial Services, U.S. House of Representatives, Washington, D.C., October 29.

Vayanos, Dimitri, and Pierre Olivier Weill (2008) "A Search-Based Theory of the On-the-Run Phenomenon," *Journal of Finance*, 63, 1361-1398.

von Thadden, Ernst-Ludwig (1999) "Liquidity Creation through Banks and Markets: Multiple Insurance and Limited Market Access ," *European Economic Review*, 43, 991-1006.