Stagnation Traps

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Abstract

We provide a Keynesian growth theory in which pessimistic expectations can lead to very persistent, or even permanent, slumps characterized by unemployment and weak growth. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap. In a stagnation trap, the central bank is unable to restore full employment because weak growth depresses aggregate demand and pushes the interest rate against the zero lower bound, while growth is weak because low aggregate demand results in low profits, limiting firms' investment in innovation. Policies aiming at restoring growth can successfully lead the economy out of a stagnation trap, thus rationalizing the notion of job creating growth.

JEL Codes: E32, E43, E52, O42.

Keywords: Secular Stagnation, Liquidity Traps, Growth Traps, Endogenous Growth, Multiple Equilibria.

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1 Introduction

Can insufficient aggregate demand lead to economic stagnation, i.e. a protracted period of low growth and high unemployment? Economists have been concerned with this question at least since the Great Depression,[1] but recently interest in this topic has reemerged motivated by the two decades-long slump affecting Japan since the early 1990s, as well as by the slow recoveries experienced by the US and the Euro area in the aftermath of the 2008 financial crisis. Indeed, as shown by table 1, all these episodes have been characterized by long-lasting slumps in the context of policy rates at, or close to, their zero lower bound, leaving little room for conventional monetary policy to stimulate demand. Moreover, during these episodes potential output growth has been weak, resulting in large deviations of output from pre-slump trends (figure 1).[2]

In this paper we present a theory in which very persistent, or even permanent, slumps characterized by unemployment and weak growth are possible. Our idea is that the connection between depressed demand, low interest rates and weak growth, far from being casual, might be the result of a two-way interaction. On the one hand, unemployment and weak aggregate demand might have a negative impact on firms' investment in innovation, and result in low growth. On the other hand, low growth might depress aggregate demand, pushing real interest rates down and nominal rates close to their zero lower bound, thus undermining the central bank's ability to maintain full employment by cutting policy rates.

To formalize this insight, and explore its policy implications, we propose a *Keynesian growth* framework that sheds lights on the interactions between endogenous growth and liquidity traps. The backbone of our framework is a standard model of vertical innovation, in the spirit of Aghion and Howitt (1992) and Grossman and Helpman (1991). We modify this classic endogenous growth framework in two directions. First, we introduce nominal wage rigidities, which create the possibility of involuntary unemployment, and give rise to a channel through which monetary policy can affect the real economy. Second, we take into account the zero lower bound on the nominal interest rate, which limits the central bank's ability to stabilize the economy with conventional monetary policy. Our theory thus combines the Keynesian insight that unemployment might arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that productivity growth is the result of investment in innovation by profit-maximizing agents. We show that the interaction between these two forces can give rise to prolonged periods of low growth and high unemployment. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap.

[1] See Hansen (1939) for an early discussion of the relationship between aggregate demand, unemployment and technical progress.

[2] Ball (2014) estimates the long-run consequences of the 2008 global financial crisis in several countries and documents significant losses in terms of potential output. Christiano et al. (2015) find that the US Great Recession has been characterized by a very persistent fall in total factor productivity below its pre-recession trend. Cerra and Saxena (2008) analyse the long-run impact of deep crises, and find, using a large sample of countries, that crises are often followed by permanent negative deviations from pre-crisis trends. A similar conclusion is reached by Blanchard et al. (2015), who also find that recessions are in many cases followed by a slowdown in the growth rate of the economy.

Table 1: Japan, United States, Euro area: before/during slump

	Japan		United States		Euro area	
	1981-1990	1991-2014	1998-2007	2008-2014	1999-2007	2008-2014
Policy rate	4.34	0.86	3.68	0.35	2.91	1.15
Unemployment rate	2.50	4.02	4.90	7.88	8.67	10.37
Labor productivity growth	4.13	1.63	2.23	1.18	1.20	0.76

Notes: All the values are subsample averages expressed in percentage points. Labor productivity is real GDP/hours worked. Data from IMF International Financial Statistics and OECD.

In our economy there are two types of agents: firms and households. Firms' investment in innovation determines endogenously the growth rate of productivity and potential output of our economy. As in the standard models of vertical innovation, firms invest in innovation to gain a monopoly position, and so their investment in innovation is positively related to profits. Through this channel, a slowdown in aggregate demand that leads to a fall in profits, also reduces investment in innovation and the growth rate of the economy. Households supply labor and consume, and their intertemporal consumption pattern is characterized by the traditional Euler equation. The key aspect is that households' current demand for consumption is affected by the growth rate of potential output, because productivity growth is one of the determinants of households' future income. Hence, a low growth rate of potential output is associated with lower future income and a reduction in current aggregate demand.

This two-way interaction between productivity growth and aggregate demand results in two steady states. First, there is a full employment steady state, in which the economy operates at potential and productivity growth is robust. However, our economy can also find itself in an unemployment steady state. In the unemployment steady state aggregate demand and firms' profits are low, resulting in low investment in innovation and weak productivity growth. Moreover, monetary policy is not able to bring the economy at full employment, because the low growth of potential output pushes the interest rate against its zero lower bound. Hence, the unemployment steady state can be thought of as a stagnation trap.

Expectations, or animal spirits, are crucial in determining which equilibrium will be selected. For instance, when agents expect growth to be low, expectations of low future income reduce aggregate demand, lowering firms' profits and their investment, thus validating the low growth expectations. Through this mechanism, pessimistic expectations can generate a permanent liquidity trap with involuntary unemployment and stagnation. We also show that, aside from permanent liquidity traps, pessimistic expectations can give rise to liquidity traps of finite, but arbitrarily long, expected duration.

We then examine the policy implications of our framework. First we study optimal interest rate policy. We show that a central bank operating under commitment can design interest rate rules that eliminate the possibility of expectations-driven stagnation traps. However, we also show that if the central bank lacks the ability to commit to its future actions stagnation traps are possible even

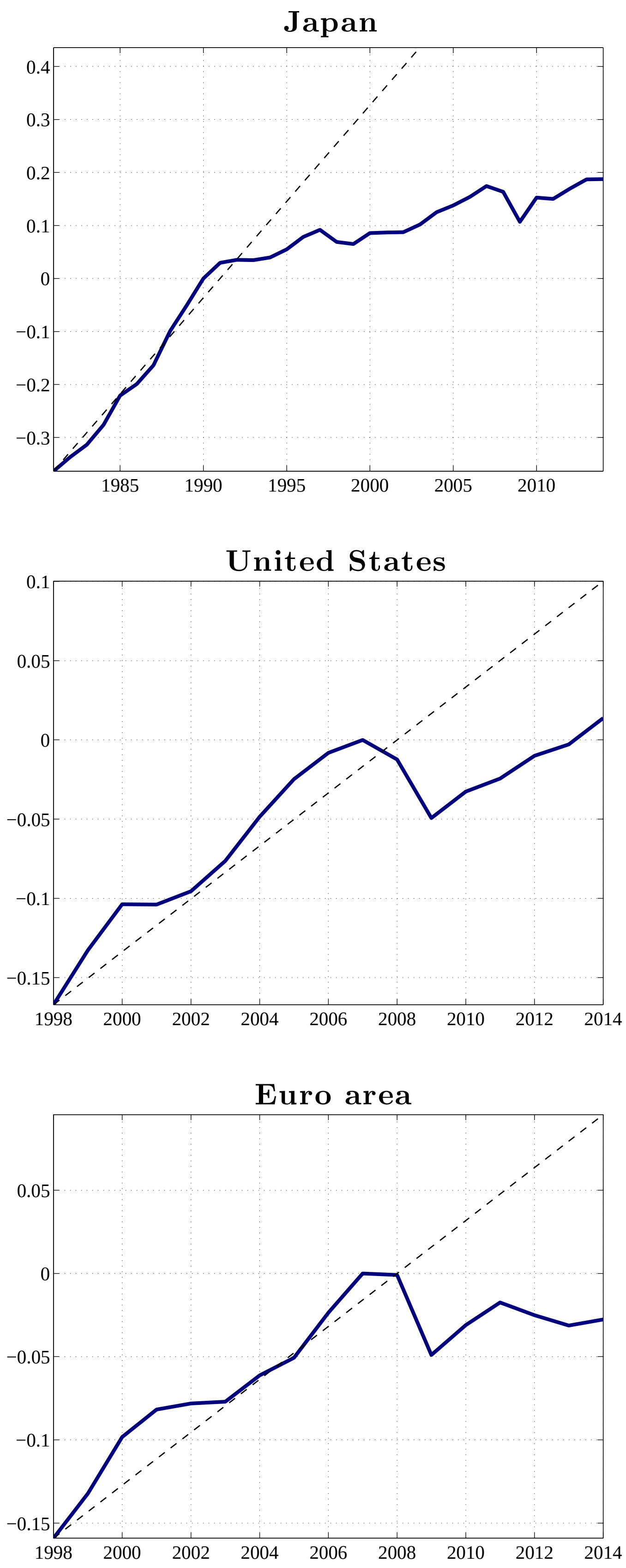

Figure 1: Real GDP per capita. Notes: Series shown in logs, undetrended, centered around 1990 for Japan, and 2007 for United States and Euro area. Gross domestic product, constant prices, from IMF World Economic Outlook, divided by total population from World Bank World Development Indicators. The linear trend is computed over the period 1981-1990 for Japan, and 1998-2007 for United States and Euro area.

when interest rates are set optimally. We then turn to policies aiming at sustaining the growth rate of potential output, by subsidizing investment in productivity-enhancing activities. While these policies have been studied extensively in the context of the endogenous growth literature, here we show that they operate not only through the supply side of the economy, but also by stimulating aggregate demand during a liquidity trap. In fact, we show that an appropriately designed subsidy to innovation can push the economy out of a stagnation trap and restore full employment, thus capturing the notion of job creating growth. However, our framework suggests that, in order to be effective, the subsidy to innovation has to be sufficiently aggressive, so as to provide a "big push" to the economy.

This paper is related to several strands of the literature. First, the paper is related to the literature studying liquidity traps. Traditionally, this literature has focused on slumps generated by ad-hoc preference shocks, as in Krugman (1998), Eggertsson and Woodford (2003), Eggertsson (2008) and Werning (2011), or by financial shocks leading to tighter access to credit, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), or by periods of disinvestment, as in Rognlie et al. (2014). In all these frameworks liquidity traps are driven by a temporary fall in the natural interest rate, and permanent liquidity traps are not possible. Instead, our paper builds on Hansen's secular stagnation hypothesis (Hansen, 1939), that is the idea that a drop in the natural interest rate might push the economy in a long-lasting liquidity trap, characterized by the absence of any self-correcting force to restore full employment.[3] Hansen formulated this concept inspired by the US Great Depression, but recently some commentators, most notably Summers (2013) and Krugman (2013), have revived the idea of secular stagnation to rationalize the long duration of the Japanese liquidity trap and the slow recoveries characterizing the US and the Euro area after the 2008 financial crisis. To the best of our knowledge, the only existing frameworks in which permanent liquidity traps are possible have been provided by Benhabib et al. (2001) and Eggertsson and Mehrotra (2014). However, the source of their liquidity traps is very different from ours. In their influential paper, Benhabib et al. (2001) show that in the standard New Keynesian model, when monetary policy is conducted through a Taylor rule, self-fulfilling expectations about future deflation can give rise to permanent liquidity traps.[4] Instead, in Eggertsson and Mehrotra (2014) permanent liquidity traps are the outcome of shocks that alter households' lifecycle saving decisions. Different from these important contributions, in our framework the drop in the real natural interest rate that generates a permanent liquidity trap originates from an endogenous drop in investment in innovation and productivity growth.

Second, our paper is related to the research on poverty and growth traps. This literature, surveyed by Azariadis and Stachurski (2005), discusses several mechanisms through which a country can find itself stuck with inefficiently low growth. We contribute to this literature by showing that a liquidity trap can be the driver of a growth trap. Indeed, the intimate connection between the

[3] Interestingly, Hansen (1939) lists a slowdown in technical progress as one of the possible causes of an episode of secular stagnation.

[4] See Schmitt-Grohé and Uribe (2012) and Mertens and Ravn (2014) for examples of recent contributions building on Benhabib et al. (2001).

two traps lead us to put forward the notion of stagnation traps. Interestingly, the result that large policy interventions are needed to lead the economy out of a stagnation trap is in line with the notion of "big push", formalized by Murphy et al. (1989) in the context of poverty traps.

As in the seminal frameworks developed by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990), long-run growth in our model is the result of investment in innovation by profit-maximizing agents. A small, but growing, literature has considered the interactions between short-run fluctuation and long run growth in this class of models (Fatas, 2000; Barlevy, 2004; Comin and Gertler, 2006; Aghion et al., 2010; Nuño, 2011; Queraltó, 2013; Aghion et al., 2009, 2014). In particular, two recent independent contributions by Anzoategui et al. (2015) and Bianchi and Kung (2015) explore the role of innovation-based growth in quantitative business cycle models. However, to the best of our knowledge, we are the first ones to study monetary policy in an endogenous growth model featuring a zero lower bound constraint on the policy rate, and to show that the interaction between endogenous growth and liquidity traps creates the possibility of long periods of stagnation.

Finally, our paper is linked to the literature on fluctuations driven by confidence shocks and sunspots. Some examples of this vast literature are Kiyotaki (1988), Benhabib and Farmer (1994, 1996), Francois and Lloyd-Ellis (2003), Farmer (2012) and Bacchetta and Van Wincoop (2013). We contribute to this literature by describing a new channel through which pessimistic expectations can give rise to economic stagnation.

The rest of the paper is composed of four sections. Section 2 describes the baseline model. Section 3 shows that pessimistic expectations can generate arbitrarily long-lasting stagnation traps. Section 4 extends the baseline model in several directions. Section 5 discusses some policy implications. Section 6 concludes.

2 Baseline Model

In this section we lay down our *Keynesian growth* framework. The economy has two key elements. First, the rate of productivity growth is endogenous, and it is the outcome of firms' investment in research. Second, the presence of nominal rigidities imply that output can deviate from its potential level, and that monetary policy can affect real variables. As we will see, the combination of these two factors opens the door to fluctuations driven by shocks to agents' expectations. To emphasize this striking feature of the economy, in what follows we will abstract from any fundamental shock.[5] Moreover, in order to deliver transparently our key results, in this section we will make some simplifying assumptions that enhance the tractability of the model. These assumptions will be relaxed in section 4.

We consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a central bank that sets monetary policy.

[5] To be clear, we believe that the study of fluctuations driven by fundamental shocks in the context of our model is a promising research area, that we plan to pursue in the future.

2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}\right)\right], \tag{1}$$

where C_t denotes consumption, $0<\beta<1$ is the subjective discount factor, σ is the inverse of the elasticity of intertemporal substitution, and $E_t[\cdot]$ is the expectation operator conditional on information available at time t.

Each household is endowed with one unit of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t<1$ units of labor on the market. Hence, when $L_t=1$ the economy operates at full employment, while when $L_t<1$ there is involuntary unemployment, and the economy operates below capacity.

Households can trade in one-period, non-state contingent bonds b_t. Bonds are denominated in units of currency and pay the nominal interest rate i_t. Moreover, households own all the firms and each period they receive dividends d_t from them.[6]

The intertemporal problem of the representative household consists in choosing C_t and b_{t+1} to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint:

$$P_tC_t+\frac{b_{t+1}}{1+i_t}=W_tL_t+b_t+d_t,$$

where P_t is the nominal price of the final good, b_{t+1} is the stock of bonds purchased by the household in period t, and b_t is the payment received from its past investment in bonds. W_t denotes the nominal wage, so that W_tL_t is the household's labor income.

The optimality conditions are:

$$\lambda_t=\frac{C_t^{-\sigma}}{P_t} \tag{2}$$

$$\lambda_t=\beta(1+i_t)E_t\left[\lambda_{t+1}\right], \tag{3}$$

where λ_t denotes the Lagrange multiplier on the budget constraint, and the transversality condition $\lim_{s\to\infty}E_t\left[\frac{b_{t+s}}{(1+i_t)...(1+i_{t+s})}\right]=0$.

2.2 Final good production

The final good is produced by competitive firms using labor and a continuum of measure one of intermediate inputs x_j, indexed by $j\in[0,1]$. Denoting by Y_t the output of final good, the production function is:

$$Y_t=L_t^{1-\alpha}\int_0^1 A_{jt}^{1-\alpha}x_{jt}^{\alpha}dj, \tag{4}$$

[6] To streamline the exposition, in the main text we consider a cashless economy. In appendix B we show that introducing money does not affect our results.

where $0 < \alpha < 1$, and A_{jt} is the productivity, or quality, of input j.[7]

Profit maximization implies the demand functions:

$$P_t (1-\alpha) L_t^{-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{jt}^{\alpha} dj = W_t \tag{5}$$

$$P_t \alpha L_t^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha-1} = P_{jt}, \tag{6}$$

where P_{jt} is the nominal price of intermediate input j. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

2.3 Intermediate goods production and profits

In every industry j producers compete as price-setting oligopolists. One unit of final output is needed to manufacture one unit of intermediate good, regardless of quality, and hence every producer faces the same marginal cost P_t. Our assumptions about the innovation process will ensure that in every industry there is a single leader able to produce good j of quality A_{jt}, and a fringe of competitors which are able to produce a version of good j of quality A_{jt}/γ. The parameter $\gamma > 1$ captures the distance in quality between the leader and the followers. Given this market structure, it is optimal for the leader to capture the whole market for good j by charging the price:[8]

$$P_{jt} = \xi P_t \quad \text{where} \quad \xi \equiv \min\left(\gamma^{\frac{\alpha}{1-\alpha}}, \frac{1}{\alpha}\right) > 1. \tag{7}$$

This expression implies that the leader charges a constant markup ξ over its marginal cost. Intuitively, $1/\alpha$ is the markup that the leader would choose in absence of the threat of entry from the fringe of competitors. Instead, $\gamma^{\alpha/(1-\alpha)}$ is the highest markup that the leader can charge without losing the market to its competitors. It follows that if $1/\alpha \leq \gamma^{\alpha/(1-\alpha)}$ then the leader will charge the unconstrained markup $1/\alpha$, otherwise it will set a markup equal to $\gamma^{\alpha/(1-\alpha)}$ to deter entry. In any case, the leader ends up satisfying all the demand for good j from final good producers.

Expressions (6) and (7) imply that the quantity produced of a generic intermediate good j is:

$$x_{jt} = \left(\frac{\alpha}{\xi}\right)^{\frac{1}{1-\alpha}} A_{jt} L_t. \tag{8}$$

Combining expressions (4) and (8) gives:

$$Y_t = \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t, \tag{9}$$

where $A_t \equiv \int_0^1 A_{jt} dj$ is an index of average productivity of the intermediate inputs. Hence, pro-

[7] More precisely, for every good j, A_{jt} represents the highest quality available. In principle, firms could produce using a lower quality of good j. However, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

[8] For a detailed derivation see, for instance, the appendix to chapter 7 of Barro and Sala-i Martin (2004).

duction of the final good is increasing in the average productivity of intermediate goods and in aggregate employment. Moreover, the profits earned by the leader in sector j are given by:

$$P_{jt}x_{jt} - P_t x_{jt} = P_t \varpi A_{jt} L_t,$$

where $\varpi \equiv (\xi - 1)(\alpha/\xi)^{1/(1-\alpha)}$. According to this expression, a leader's profits are increasing in the productivity of its intermediate input and on aggregate employment. The dependence of profits from aggregate employment is due to the presence of a market size effect. Intuitively, high employment is associated with high production of the final good and high demand for intermediate inputs, leading to high profits in the intermediate sector.

2.4 Research and innovation

There is a large number of entrepreneurs that can attempt to innovate upon the existing products. A successful entrepreneur researching in sector j discovers a new version of good j of quality γ times greater than the best existing version, and becomes the leader in the production of good j.[9]

Entrepreneurs can freely target their research efforts at any of the continuum of intermediate goods. An entrepreneur that invests I_{jt} units of the final good to discover an improved version of product j innovates with probability:

$$\mu_{jt} = \min\left(\frac{\chi I_{jt}}{A_{jt}}, 1\right),$$

where the parameter $\chi > 0$ determines the productivity of research.[10] The presence of the term A_{jt} captures the idea that innovating upon more advanced and complex products requires a higher investment, and ensures stationarity in the growth process. We consider time periods small enough so that the probability that two or more entrepreneurs discover contemporaneously an improved version of the same product is negligible. This assumption implies, mimicking the structure of equilibrium in continuous-time models of vertical innovation such as Aghion and Howitt (1992) and Grossman and Helpman (1991), that the probability that a product is improved is the sum of the success probabilities of all the entrepreneurs targeting that product.[11] With a slight abuse of notation, we then denote by μ_{jt} the probability that an improved version of good j is discovered at time t.

We now turn to the reward from research. A successful entrepreneur obtains a patent and becomes the monopolist during the following period. For simplicity, in our baseline model we

[9] As in Aghion and Howitt (1992) and Grossman and Helpman (1991), all the research activities are conducted by entrants. Incumbents do not perform any research because the value of improving over their own product is smaller than the profits that they would earn from developing a leadership position in a second market.

[10] Our formulation of the innovation process follows closely chapter 7 of Barro and Sala-i Martin (2004) and Howitt and Aghion (1998). An alternative is to assume, as in Grossman and Helpman (1991), that labor is used as input into research. This alternative assumption would lead to identical results, since ultimately output in our model is fully determined by the stock of knowledge and aggregate labor.

[11] Following Aghion and Howitt (2009), we could have assumed that every period only a single entrepreneur can invest in research in a given sector. This alternative assumption would lead to identical equilibrium conditions.

assume that the monopoly position of an innovator lasts a single period, after which the patent is allocated randomly to another entrepreneur.[12] The value $V_t(\gamma A_{jt})$ of becoming a leader in sector j and attaining productivity γA_{jt} is given by:

$$V_t(\gamma A_{jt}) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \varpi \gamma A_{jt} L_{t+1} \right]. \tag{10}$$

$V_t(\gamma A_{jt})$ is equal to the expected profits to be gained in period $t+1$, $P_{t+1}\varpi\gamma A_{jt}L_{t+1}$, discounted using the households' discount factor $\beta\lambda_{t+1}/\lambda_t$. Profits are discounted using the households' discount factor because entrepreneurs finance their investment in innovation by selling equity claims on their future profits to the households. Competition for households' funds leads entrepreneurs to maximize the value to the households of their expected profits.

Free entry into research implies that expected profits from researching cannot be positive, so that for every good j:[13]

$$P_t \geq \frac{\chi}{A_{jt}} V_t(\gamma A_{jt}),$$

holding with equality if some research is conducted aiming at improving product j.[14] Combining this condition with expression (10) gives:

$$\frac{P_t}{\chi} \geq \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \gamma \varpi L_{t+1} \right].$$

Notice that this condition does not depend on any variable specific to sector j, because the higher profits associated with more advanced sectors are exactly offset by the higher research costs. As is standard in the literature, we then focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt} = \chi I_{jt}/A_{jt} = \mu_t$ for every j. We can then summarize the equilibrium in the research sector with the complementary slackness condition:

$$\mu_t \left(\frac{P_t}{\chi} - \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \gamma \varpi L_{t+1} \right] \right) = 0. \tag{11}$$

Intuitively, either some research is conducted, so that $\mu_t > 0$, and free entry drives expected profits in the research sector to zero, or the expected profits from researching are negative and no research is conducted, so that $\mu_t = 0$.

[12] This assumption, which is drawn from Aghion and Howitt (2009) and Acemoglu et al. (2012), simplifies considerably the analysis. In section 4.3 we show that our results extend to a setting in which, more conventionally, the innovator's monopoly position is terminated when a new version of the product is discovered.

[13] To derive this condition, consider that an entrepreneur that invests I_{jt} in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt} V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the free entry condition in the research sector implies:

$$P_t I_{jt} \geq \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

[14] It is customary in the endogenous growth literature to restrict attention to equilibria in which in every period a positive amount of research is targeted toward every intermediate good. We take a slightly more general approach, and allow for cases in which expected profits from research are too low to induce entrepreneurs to invest in innovation. This degree of generality will prove important when we will discuss the policy implications of the framework.

2.5 Aggregation and market clearing

Market clearing for the final good implies:[15]

$$Y_t - \int_0^1 x_{jt} dj = C_t + \int_0^1 I_{jt} dj, \tag{12}$$

where the left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that all the GDP has to be consumed or invested in research. Using equations (8) and (9) we can write GDP as:

$$Y_t - \int_0^1 x_{jt} dj = \Psi A_t L_t, \tag{13}$$

where $\Psi \equiv (\alpha/\xi)^{\alpha/(1-\alpha)} (1 - \alpha/\xi)$.

The assumption of a unitary labor endowment implies that $L_t \leq 1$. Since labor is supplied inelastically by the households, $1 - L_t$ can be interpreted as the unemployment rate. For future reference, when $L_t = 1$ we say that the economy is operating at full employment, while when $L_t < 1$ the economy operates below capacity and there is a negative output gap.

Long run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index A_t. By the law of large numbers, a fraction μ_t of intermediate products is improved every period. Hence, A_t evolves according to:

$$A_{t+1} = \mu_t \gamma A_t + (1 - \mu_t) A_t,$$

while the (gross) rate of productivity growth is:

$$g_{t+1} \equiv \frac{A_{t+1}}{A_t} = \mu_t (\gamma - 1) + 1. \tag{14}$$

Recalling that $\mu_t = \chi I_{jt}/A_{jt}$, this expression implies that higher investment in research in period t is associated with faster productivity growth between periods t and $t + 1$.

[15] The goods market clearing condition can be derived combining the households' budget constraint with the expression for firms' profits:

$$d_t = \underbrace{P_t Y_t - W_t L_t - \xi P_t \int_0^1 x_{jt} dj}_{\text{profits from final sector}} + \underbrace{(\xi - 1) P_t \int_0^1 x_{jt} dj - P_t \int_0^1 I_{jt} dj}_{\text{profits from intermediate sector}},$$

where profits are net of research expenditure, and the equilibrium condition $b_{t+1} = 0$, deriving from the assumption of identical households.

2.6 Wages, prices and monetary policy

We consider an economy with frictions in the adjustment of nominal wages.[16] The presence of nominal wage rigidities plays two roles in our analysis. First, it creates the possibility of involuntary unemployment, by ensuring that nominal wages remain positive even in presence of unemployment. Second, it opens the door to a stabilization role for monetary policy. Indeed, as we will see, prices inherit part of wage stickiness, so that the central bank can affect the real interest rate of the economy through movements in the nominal interest rate.

In our baseline model, we consider the simplest possible form of nominal wage rigidities and assume that wages evolve according to:

$$W_t = \bar{\pi}^w W_{t-1}. \tag{15}$$

This expression implies that nominal wage inflation is constant and equal to $\bar{\pi}^w$, and could be derived from the presence of large menu costs from deviating from the constant wage inflation path.

To be clear, our results do not rely at all on this extreme form of wage stickiness. Indeed, in section 4.2 we generalize our results to an economy in which wages are allowed to respond to fluctuations in employment, giving rise to a wage Phillips curve. However, considering an economy with constant wage inflation simplifies considerably the analysis, and allows us to characterize transparently the key economic forces at the heart of the model.

Turning to prices, combining equations (5) and (8) gives:

$$P_t = \frac{1}{1-\alpha}\left(\frac{\xi}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \frac{W_t}{A_t}.$$

Intuitively, prices are increasing in the marginal cost of firms producing the final good. An increase in wages puts upward pressure on marginal costs and leads to a rise in prices, while a rise in productivity reduces marginal costs and prices. This expression, combined with the law of motion for wages, can be used to derive an equation for price inflation:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{\bar{\pi}^w}{g_t}, \tag{16}$$

which implies that price inflation is increasing in wage inflation and decreasing in productivity growth.

The central bank implements its monetary policy stance by setting the nominal interest rate

[16] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Christiano et al. (2005) using an estimated medium-scale DSGE model of the US economy, and by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Fabiani et al. (2010).

according to the truncated interest rate rule:

$$1 + i_t = \max\left((1 + \bar{i})\, L_t^{\phi}, 1\right),$$

where $\bar{i} \geq 0$ and $\phi > 0$. Under this rule the central bank aims at stabilizing output around its potential level by cutting the interest rate in response to falls in employment.[17] The nominal interest rate is subject to a zero lower bound constraint, which, as we show in appendix B, can be derived from standard arbitrage between money and bonds.

2.7 Equilibrium

The equilibrium of our economy can be described by four simple equations. The first one is the *Euler* equation, which captures households' consumption decisions. Combining households' optimality conditions (2) and (3) gives:

$$C_t^{-\sigma} = \beta(1 + i_t)E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right].$$

According to this standard Euler equation, demand for consumption is increasing in expected future consumption and decreasing in the real interest rate, $(1 + i_t)/\pi_{t+1}$.

To understand how productivity growth relates to demand for consumption, it is useful to combine the previous expression with $A_{t+1}/A_t = g_{t+1}$ and $\pi_{t+1} = \bar{\pi}^w/g_{t+1}$ to obtain:

$$c_t^{\sigma} = \frac{g_{t+1}^{\sigma-1}\bar{\pi}^w}{\beta(1 + i_t)E_t\left[c_{t+1}^{-\sigma}\right]}, \tag{17}$$

where we have defined $c_t \equiv C_t/A_t$ as consumption normalized by the productivity index. This equation shows that the relationship between productivity growth and present demand for consumption can be positive or negative, depending on the elasticity of intertemporal substitution, $1/\sigma$. In fact, there are two contrasting effects. On the one hand, faster productivity growth is associated with higher future consumption. This income effect leads households to increase their demand for current consumption in response to a rise in productivity growth. On the other hand, faster productivity growth is associated with a fall in expected inflation. Given i_t, lower expected inflation increases the real interest rate inducing households to postpone consumption. This substitution effect points toward a negative relationship between productivity growth and current demand for consumption. For low levels of intertemporal substitution, i.e. for $\sigma > 1$, the income effect dominates and the relationship between productivity growth and demand for consumption is positive. Instead, for high levels of intertemporal substitution, i.e. for $\sigma < 1$, the substitution effect dominates and the relationship between productivity growth and demand for consumption is neg-

[17] To clarify, this particular form of interest rate rule is by no means essential for the results of the paper. For instance, following the work of Erceg et al. (2000), it is often assumed that in presence of flexible prices and rigid wages the central bank aims at stabilizing wage inflation. We consider this possibility in section 4.2. Instead, in section 5.1 we derive the optimal interest rate policy.

ative. Finally, for the special case of log utility, $\sigma = 1$, the two effects cancel out and productivity growth does not affect present demand for consumption.

Empirical estimates point toward an elasticity of intertemporal substitution smaller than one.[18] Hence, in the main text we will focus attention on the case $\sigma > 1$, while we provide an analysis of the cases $\sigma < 1$ and $\sigma = 1$ in the appendix.

Assumption 1 *The parameter σ satisfies:*

$$\sigma > 1.$$

Under this assumption, the Euler equation implies a positive relationship between the pace of innovation and demand for present consumption.

The second key relationship in our model is the *growth* equation, which is obtained by combining equation (2) with the optimality condition for investment in research (11):

$$(g_{t+1} - 1)\left(1 - \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{-\sigma} \chi\gamma\varpi L_{t+1}\right]\right) = 0. \tag{18}$$

This equation captures the optimal investment in research by entrepreneurs. For values of profits sufficiently high so that some research is conducted in equilibrium and $g_{t+1} > 1$, this equation implies a positive relationship between growth and expected future employment. Intuitively, a rise in employment, and consequently in aggregate demand, is associated with higher monopoly profits. In turn, higher expected profits induce entrepreneurs to invest more in research, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature.

The third equation combines the goods market clearing condition (12), the GDP equation (13) and the fact that $\int_0^1 I_{jt} dj = A_t(g_{t+1} - 1)/(\chi(\gamma - 1))$:[19]

[18] Havránek (2015) performs a meta-analysis of the literature and finds that, though substantial uncertainty about the exact value of the elasticity of intertemporal substitution exists, most estimates lie well below one. Examples of papers estimating an elasticity smaller than one are Hall (1988), Ogaki and Reinhart (1998), Basu and Kimball (2002), who use macro data, and Vissing-Jørgensen (2002) and Best et al. (2015), who use micro data.

That said, different assumptions about wage or price setting can lead to a positive relationship between productivity growth and present demand for consumption even in presence of an elasticity of intertemporal substitution larger than one. For instance, consider a case in which wages are indexed to productivity growth, so that:

$$W_t = \bar{\pi}^w g_t^{\omega} W_{t-1},$$

where $\omega > 0$. In this case, the Euler equation becomes:

$$c_t^{\sigma} = \frac{g_{t+1}^{\sigma-1+\omega}\bar{\pi}^w}{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]},$$

so that a positive relationship between demand for consumption and productivity growth arises as long as $\sigma > 1 - \omega$.

[19] To derive this condition, consider that:

$$\int_0^1 I_{jt} dj = \int_0^1 A_{jt} I_{jt}/A_{jt} dj = A_t \int_0^1 I_{jt}/A_{jt} dj = A_t I_{jt}/A_{jt} = A_t \mu_t/\chi = A_t(g_{t+1} - 1)/(\chi(\gamma - 1)),$$

where we have used the fact that I_{jt}/A_{jt} is the same across all the j sectors.

$$c_t = \Psi L_t - \frac{g_{t+1} - 1}{\chi(\gamma - 1)}. \tag{19}$$

Keeping output constant, this equation implies a negative relationship between productivity-adjusted consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output to innovation activities, reducing the resources available for consumption.

Finally, the fourth equation is the monetary policy rule:

$$1 + i_t = \max\left((1 + \bar{i})\, L_t^{\phi}, 1\right). \tag{20}$$

We are now ready to define an equilibrium as a set of processes $\{g_{t+1}, L_t, c_t, i_t\}_{t=0}^{+\infty}$ satisfying equations (17) – (20) and $L_t \leq 1$ for all $t \geq 0$.

3 Stagnation traps

In this section we show that the interaction between aggregate demand and productivity growth can give rise to prolonged periods of low growth, low interest rates and high unemployment, which we call *stagnation traps*. We start by considering non-stochastic steady states, and we derive conditions on the parameters under which two steady states coexist, one of which is a stagnation trap. We then show that stagnation traps of finite expected duration are also possible.

3.1 Non-stochastic steady states

Non-stochastic steady state equilibria are characterized by constant values for productivity growth g, employment L, normalized consumption c and the nominal interest rate i satisfying:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}^w} \tag{21}$$

$$g^{\sigma} = \max\left(\beta\chi\gamma\varpi L, 1\right) \tag{22}$$

$$c = \Psi L - \frac{g - 1}{\chi(\gamma - 1)} \tag{23}$$

$$1 + i = \max\left((1 + \bar{i})\, L^{\phi}, 1\right), \tag{24}$$

where the absence of a time subscript denotes the value of a variable in a non-stochastic steady state. We now show that two steady state equilibria can coexist: one characterized by full employment, and one by involuntary unemployment.

Full employment steady state. Let us start by describing the full employment steady state, which we denote by the superscripts f. In the full employment steady state the economy operates at full capacity, and hence $L^f = 1$. The growth rate associated with the full employment steady state g^f can then be found by setting $L = 1$ in equation (22), while the nominal interest rate that supports this steady state can be obtained by setting $g = g^f$ in equation (21).

We summarize our results about the full employment steady state in a proposition.

Assumption 2 *The parameters $\bar{i}$, $\bar{\pi}^w$ and ϕ satisfy:*

$$\bar{i} = \bar{\pi}^w \beta^{-\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{\sigma-1}{\sigma}} - 1 \tag{25}$$

$$\bar{\pi}^w > \beta^{\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{1-\sigma}{\sigma}} \tag{26}$$

$$\phi > 1 - \frac{1}{\sigma}. \tag{27}$$

Proposition 1 *Suppose assumptions 1 and 2 hold and that*

$$(\beta\chi\gamma\varpi)^{\frac{1}{\sigma}} < 1 + \Psi\chi(\gamma-1) + \min\left(0, \left(1-\frac{1}{\sigma}\right)\Psi\chi(\gamma-1) - \frac{1}{\sigma}\right) \tag{28}$$

$$1 < (\beta\chi\gamma\varpi)^{\frac{1}{\sigma}} < \gamma \tag{29}$$

Then, there exists a unique full employment steady state with $L^f = 1$. The full employment steady state is characterized by positive growth ($g^f > 1$) and by a positive nominal interest rate ($i^f > 0$). Moreover, the full employment steady state is locally determinate.[20]

Intuitively, assumption 2 guarantees that monetary policy and wage inflation are consistent with the existence of a full employment steady state. Condition (25) ensures that the intercept of the interest rate rule is consistent with existence of a full employment steady state, while condition (26) implies that inflation and productivity growth in the full employment steady state are sufficiently high so that the zero lower bound constraint on the nominal interest rate is not binding. Instead, condition (27), which requires the central bank to respond sufficiently strongly to fluctuations in employment, ensures, in conjunction with condition (28), that the full employment steady state is locally determinate.[21]

Turning to condition (28), its role is to ensure that consumption in the full employment steady state is positive. Instead, condition (29) implies that in the full employment steady state the innovation probability lies between zero and one ($0 < \mu^f < 1$), an assumption often made in the endogenous growth literature.

Summing up, the full employment steady state can be thought as the normal state of affairs of the economy. In fact, in this steady state, which closely resembles the steady state commonly considered both in New Keynesian and endogenous growth models, the economy operates at its full potential, growth is robust, and monetary policy is not constrained by the zero lower bound.

Unemployment steady state. Aside from the full employment steady state, the economy can find itself in a permanent liquidity trap with low growth and involuntary unemployment. We

[20] All the proofs are collected in appendix A.

[21] Similar assumptions are commonly made in the literature studying monetary policy in New Keynesian models (Galí, 2009). In fact, analyses based on the New Keynesian framework typically focus on fluctuations around a steady state in which output is equal to its natural level, that is the value that would prevail in absence of nominal rigidities, and the nominal interest rate is positive. Moreover, local determinacy is typically ensured by assuming that the central bank follows an interest rate rule that reacts sufficiently strongly to fluctuations in inflation or output.

denote this unemployment steady state with superscripts u. To derive the unemployment steady state, consider that with $i = 0$ equation (21) implies:

$$g^u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}.$$

Since $\bar{i} > 0$ it follows immediately from equation (21) that $g^u < g^f$. Moreover, notice that equation (21) can be written as $(1+i)/\pi = g^\sigma/\beta$. Hence, $g^u < g^f$ implies that the real interest rate $(1+i)/\pi$ in the unemployment steady state is lower than in the full employment steady state. To see that the liquidity trap steady state is characterized by unemployment, consider that by equation (22) $(g^u)^\sigma = \max(\beta\chi\gamma\varpi L^u, 1)$. Now use $\beta\chi\gamma\varpi = g^f$ to rewrite this expression as:

$$L^u \leq \left(\frac{g^u}{g^f}\right)^\sigma < 1,$$

where the second inequality derives from $g^u < g^f$.

The following proposition summarizes our results about the unemployment steady state.

Proposition 2 *Suppose assumptions 1, 2 and condition (29) hold and that*

$$1 < \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} \tag{30}$$

$$\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} < 1 + \frac{\frac{\xi}{\alpha} - 1}{\xi - 1}\left(\frac{\beta}{(\bar{\pi}^w)^\sigma}\right)^{\frac{1}{\sigma-1}}\frac{\gamma - 1}{\gamma}. \tag{31}$$

Then, there exists a unique unemployment steady state. At the unemployment steady state the economy is in a liquidity trap ($i^u = 0$), there is involuntary unemployment ($L^u < 1$), and both growth and the real interest rate are lower than in the full employment steady state ($g^u < g^f$ and $1/\pi_u < (1 + i^f)/\pi^f$). Moreover, the unemployment steady state is locally indeterminate.

Condition (30) implies that $g^u > 1$, and its role is to ensure existence and uniqueness of the unemployment steady state. This condition is needed because there is no depreciation in the quality of intermediate inputs, implying that a steady state with negative productivity growth cannot exist.[22] Moreover, condition (31) makes sure that $c^u > 0$. Uniqueness is ensured by the fact that by equation (22) there exists a unique value of L consistent with $g = g^u > 1$.[23] Instead, assumption (27) guarantees that the zero lower bound on the nominal interest rate binds in the

[22]Since $\beta < 1$ and $g^u > 1$, in our baseline model an unemployment steady state exists only if $\bar{\pi}^w < 1$, that is if wage inflation is negative. This happens because in a representative agent economy with positive productivity growth the steady state real interest rate must be positive. In turn, when the nominal interest rate is equal to zero, deflation is needed to ensure that the real interest rate is positive. However, this is not a deep feature of our framework, and it is not hard to modify the model to allow for positive wage inflation and a negative real interest rate in the unemployment steady state. For instance, in section 4.2 we show that the presence of precautionary savings due to idiosyncratic shocks creates the conditions for an unemployment steady state with positive inflation and negative real rate to exist.

[23]Notice that this assumption rules out the case $g^u = 1$. Under this knife-edged case an unemployment steady state might exist, but it will not be unique, since by equation (22) multiple values of L are consistent with $g = 1$.

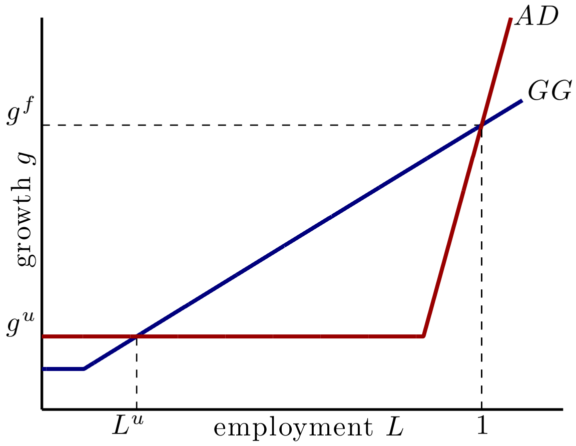

Figure 2: Non-stochastic steady states.

unemployment steady state.

The proposition states that the unemployment steady state is locally indeterminate, so that animal spirits and sunspots can generate local fluctuations around its neighborhood. This result is not surprising, given that in the unemployment steady state the central bank is constrained by the zero lower bound, and hence monetary policy cannot respond to changes in aggregate demand driven by self-fulfilling expectations.

We think of this second steady state as a stagnation trap, that is the combination of a liquidity and a growth trap. In a liquidity trap the economy operates below capacity because the central bank is constrained by the zero lower bound on the nominal interest rate. In a growth trap, lack of demand for firms' products depresses investment in innovation and prevents the economy from developing its full growth potential. In a stagnation trap these two events are tightly connected. We illustrate this point with the help of a diagram.

Figure 2 depicts the two key relationships that characterize the steady states of our model in the $L-g$ space. The first one is the growth equation (22), which corresponds to the GG schedule. For sufficiently high L, the GG schedule is upward sloped. The positive relationship between L and G can be explained with the fact that, for L high enough, an increase in employment and production is associated with a rise in firms' profits, while higher profits generate an increase in investment in innovation and productivity growth. Instead, for low values of L the GG schedule is horizontal. These are the values of employment for which investing in research is not profitable, and hence they are associated with zero growth.

The second key relationship combines the Euler equation (21) and the policy rule (24):

$$g^{\sigma-1} = \frac{\beta}{\bar{\pi}^w} \max\left((1+\bar{\imath})L^{\phi}, 1\right).$$

Graphically, this relationship is captured by the AD, i.e. aggregate demand, curve. The upward-sloped portion of the AD curve corresponds to cases in which the zero lower bound constraint on the nominal interest rate is not binding.[24] In this part of the state space, the central bank responds to a

[24]Precisely, the zero lower bound constraint does not bind when $L \geq (1+\bar{\imath})^{-1/\phi}$.

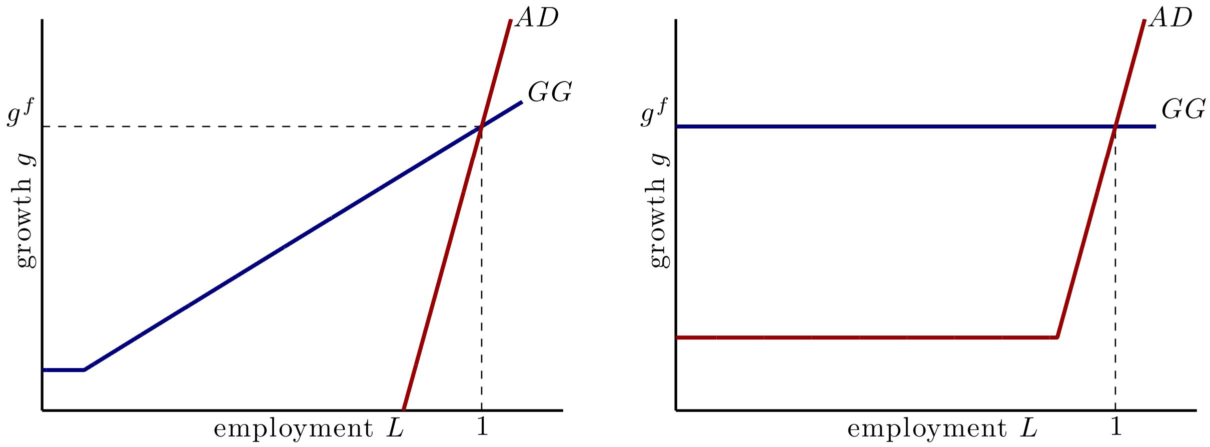

Figure 3: Understanding stagnation traps. Left panel: economy without zero lower bound. Right panel: economy with exogenous growth.

rise in employment by increasing the nominal rate. In turn, to be consistent with households' Euler equation, a higher interest rate must be coupled with faster productivity growth.[25] Hence, when monetary policy is not constrained by the zero lower bound the AD curve generates a positive relationship between L and g. Instead, the horizontal portion of the AD curve corresponds to values of L for which the zero lower bound constraint binds. In this case, the central bank sets $i = 0$ and steady state growth is independent of L and equal to $(\beta/\bar{\pi}^w)^{1/(\sigma-1)}$. As long as the conditions specified in propositions 1 and 2 hold, the two curves cross twice and two steady states are possible.

Importantly, both the presence of the zero lower bound and the procyclicality of investment in innovation are needed to generate steady state multiplicity. Suppose that the central bank is not constrained by the zero lower bound, and hence that liquidity traps are not possible. As illustrated by the left panel of figure 3, in this case the AD curve reduces to an upward sloped curve, steeper than the GG curve, and the unemployment steady state disappears. Intuitively, the assumptions about monetary policy ensure that, in absence of the zero lower bound, the central bank's reaction to unemployment is always sufficiently strong to ensure that the only possible steady state is the full employment one.[26]

Now suppose instead that productivity growth is constant and equal to g^f. In this case, as shown by the right panel of figure 3, the GG curve reduces to a horizontal line at $g = g^f$, and again the full employment steady state is the only possible one. Indeed, if growth is not affected by variations in employment, then condition (26) guarantees that aggregate demand and inflation are sufficiently high so that in steady state the zero lower bound constraint on the nominal interest

[25] Recall that we are focusing on the case $\sigma > 1$.

[26] To see this point, consider that ignoring the zero lower bound and using $1+\bar{i} = \bar{\pi}^w \beta^{-\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{\sigma-1}{\sigma}}$ the AD curve can be written as

$$g = (\beta\chi\gamma\varpi)^{\frac{1}{\sigma}} L^{\frac{\phi}{\sigma-1}}.$$

Recalling that the GG curve is $g = \max\left((\beta\chi\gamma\varpi L)^{1/\sigma}, 1\right)$, we have that the assumption $\phi > 1 - 1/\sigma$ implies that for any $0 < L < 1$ the AD curve lies below the GG curve. The result is that the two curves cannot cross at any $0 < L < 1$, unless the zero lower bound binds.

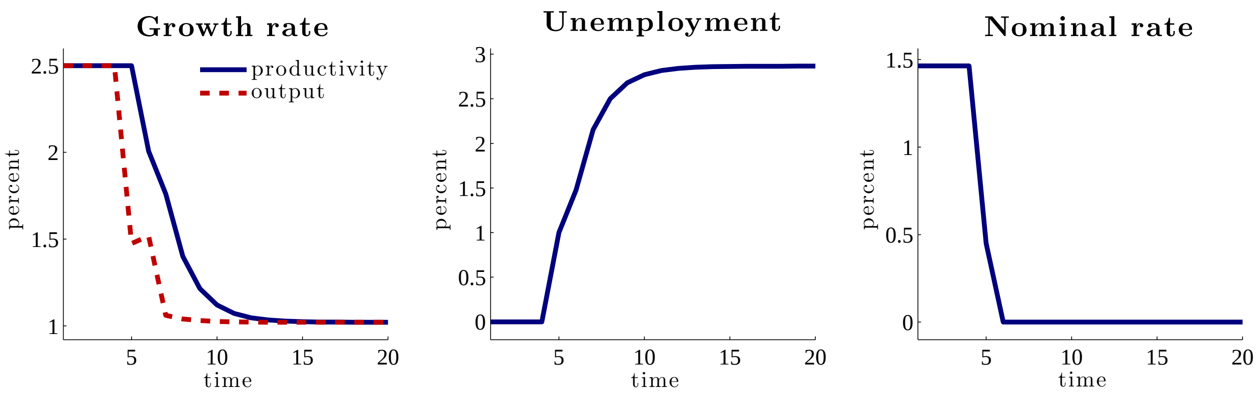

Figure 4: An example of transition toward the unemployment steady state.

rate does not bind, ensuring that the economy operates at full employment. We refer to the unemployment steady state as a stagnation trap to capture the tight link between liquidity and growth traps suggested by our model.

We are left with determining what makes the economy settle in one of the two steady states. This role is fulfilled by expectations. Suppose that agents expect that the economy will permanently fluctuate around the full employment steady state. Then, their expectations of high future growth sustain aggregate demand, so that a positive nominal interest rate is consistent with full employment. In turn, if the economy operates at full employment then firms' profits are high, inducing high investment in innovation and productivity growth. Conversely, suppose that agents expect that the economy will permanently remain in a liquidity trap. In this case, low expectations about growth and future income depress aggregate demand, making it impossible for the central bank to sustain full employment due to the zero lower bound constraint on the interest rate. As a result the economy operates below capacity and firms' profits are low, so that investment in innovation is also low, justifying the initial expectations of weak growth. Hence, in our model expectations can be self-fulfilling, and sunspots, that is confidence shocks unrelated to fundamentals, can determine real outcomes.

Interestingly, the transition from one steady state to the other need not take place in a single period. In fact, there are multiple perfect foresight paths, on which agents' expectations can coordinate, that lead the economy to the unemployment steady state. Figure 4 shows one of these paths.[27] The economy starts in the full employment steady state. In period 5 the economy is hit by a previously unexpected shock to expectations, which leads agents to revise downward their expectations of future productivity growth. From then on, the economy embarks in a perfect foresight transition toward the unemployment steady state. Initially, pessimism about future productivity triggers a fall in aggregate demand, leading to a rise in unemployment, to which the central bank responds by lowering the policy rate. In period 6 there is a further drop in expected productivity growth, causing a further rise in unemployment which pushes the economy

[27] Appendix D provides the numerical algorithm used to find perfect foresight paths that lead to the unemployment steady state. The parameters used to construct the figure are $\beta = .99$, $\sigma = 2$, $\gamma = 1.05$, $\chi = 89.14$, $\alpha = .5$, $\bar{\pi}^w = .98$ and $\phi = 1$. This parameterization is purely illustrative. Section 4.3 presents a calibration exercise.

in a liquidity trap. This initiates a long-lasting liquidity trap, during which the economy converges smoothly to the unemployment steady state.

Summarizing, the combination of growth driven by investment in innovation from profit-maximizing firms and the zero lower bound constraint on monetary policy can produce stagnation traps, that is permanent, or very long lasting, liquidity traps characterized by unemployment and low growth. All it takes is a sunspot that coordinates agents' expectations on a path that leads to the unemployment steady state.

Before moving on, it is useful to compare our notion of stagnation traps with the permanent liquidity traps that can arise in the New Keynesian model. In the standard New Keynesian model productivity growth is exogenous, and there is a unique real interest rate consistent with a steady state. As shown by Benhabib et al. (2001), permanent liquidity traps can occur in these frameworks if agents coordinate their expectations on an inflation rate equal to the inverse of the steady state real interest rate. Because of this, the New Keynesian model typically feature two steady states, one of which is a permanent liquidity trap. These two steady states are characterized by the same real interest rate, but by different inflation and nominal interest rates, with the liquidity trap steady state being associated with inflation below the central bank's target.

In contrast, in our framework endogenous growth is key in opening the door to steady state multiplicity and permanent liquidity traps. Crucially, in our model the two steady states feature different growth and real interest rates, with the liquidity trap steady state being associated with low growth and low real interest rate. Instead, inflation expectations do not play a major role. In fact, once a wage Phillips curve is introduced in the model, it might very well be the case that inflation in the unemployment steady state is the same, or even higher, than in the full employment one. We will go back to this point in section 4.2.

3.2 Temporary stagnation traps

Though our model can allow for economies which are permanently in a liquidity trap, it is not difficult to construct equilibria in which the expected duration of a trap is finite.

To construct an equilibrium featuring a temporary liquidity trap we have to put some structure on the sunspot process. Let us start by denoting a sunspot by ξ_t. In a sunspot equilibrium agents form their expectations about the future after observing ξ, so that the sunspot acts as a coordination device for agents' expectations. To be concrete, let us consider a two-state discrete Markov process, $\xi_t \in (\xi^o, \xi^p)$, with transition probabilities $Pr\left(\xi_{t+1} = \xi^o | \xi_t = \xi^o\right) = 1$ and $Pr\left(\xi_{t+1} = \xi^p | \xi_t = \xi^p\right) = q < 1$. The first state is an absorbing optimistic equilibrium, in which agents expect to remain forever around the full employment steady state. Hence, once $\xi_t = \xi^o$ the economy settles on the full employment steady state, characterized by $L = 1$ and $g = g^f$. The second state ξ^p is a pessimistic equilibrium with finite expected duration $1/(1-q)$. In this state the economy is in a liquidity trap with unemployment. We consider an economy that starts in the pessimistic equilibrium.

Under these assumptions, as long as the pessimistic sunspot shock persists the equilibrium is

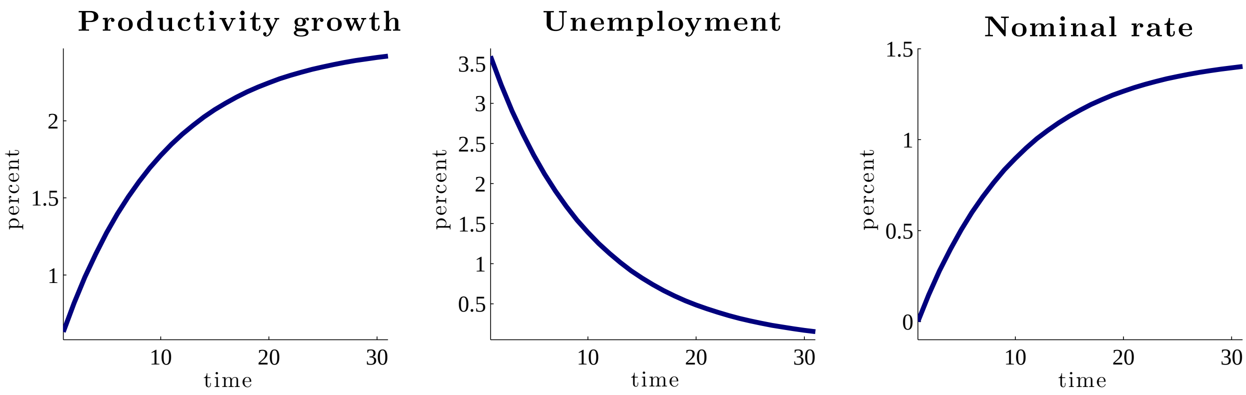

Figure 5: A temporary stagnation trap: expected dynamics.

described by equations (17), (18) and (19), which, using the fact that in the pessimistic state $i = 0$, can be written as:

$$(g^p)^{\sigma-1} = \frac{\beta}{\bar{\pi}^w}\left(q + (1-q)\left(\frac{c^p}{c^f}\right)^{\sigma}\right) \tag{32}$$

$$(g^p - 1)\left((g^p)^{\sigma} - \beta\chi\gamma\varpi\left(qL^p + (1-q)\left(\frac{c^p}{c^f}\right)^{\sigma}\right)\right) = 0. \tag{33}$$

$$c^p = \Psi L^p - \frac{g^p - 1}{\chi(\gamma - 1)}, \tag{34}$$

where the superscripts p denote the equilibrium while pessimistic expectations prevail. Similar to the case of the unemployment steady state, in the pessimistic equilibrium the zero lower bound constraint on the interest rate binds, there is involuntary unemployment and growth is lower than in the optimistic state.

Characterizing analytically the equilibrium described by equations (32) − (34) is challenging, but some results can be obtained by using $g^u = (\beta/\bar{\pi}^w)^{1/(\sigma-1)}$ to write equation (32) as:

$$(g^p)^{\sigma-1} = (g^u)^{\sigma-1}\left(q^p + (1-q^p)\left(\frac{c^p}{c^f}\right)^{\sigma}\right).$$

It can be shown that c^p/c^f is smaller than one, i.e. switching to the optimistic steady state entails an increase in productivity-adjusted consumption. Hence, the equation above implies that temporary liquidity traps feature slower growth compared to permanent ones. Indeed, for reasonable parameterizations, growth and employment are both increasing in the expected duration of the trap, so that traps of shorter expected duration are characterized by sharper contractions.

Figure 5 displays the expected path of productivity growth, unemployment and the nominal interest rate during a temporary liquidity trap.[28] The economy starts in the pessimistic equilibrium, characterized by low growth, high unemployment and a nominal interest rate equal to zero. From the second period on, each period agents expect that the economy will leave the trap and go back to the full employment steady state with a constant probability. Hence, the probability that the economy remains in the trap decreases with time, explaining the upward path for expected pro-

[28]To construct this figure we have used the same parameters used to construct figure 4, and set $1/(1-q) = 10$.

ductivity growth, employment and the nominal interest rate. However, even though the economy eventually goes back to the full employment steady state, the post-trap increase in the growth rate is not sufficiently strong to make up for the low growth during the trap, so that the trap generates a permanent loss in output.

This example shows that pessimistic expectations can plunge the economy into a temporary liquidity trap with unemployment and low growth. Eventually the economy will recover, but the liquidity trap lasts as long as pessimistic beliefs persist. Hence, long lasting liquidity trap driven by pessimistic expectations can coexist with the possibility of a future recovery.

4 Extensions and Numerical Exercise

In this section we extend the model in three directions. We first show that the introduction of precautionary savings can give rise to stagnation traps characterized by positive inflation and a negative real interest rate. We then show that our key results do not rely on the assumption of a constant wage inflation rate. Lastly, we perform a simple calibration exercise to examine a setting in which, consistent with standard models of vertical innovation, the duration of innovators' monopoly rents is endogenous.

4.1 Precautionary savings and negative real rates

In our baseline framework positive growth and positive inflation cannot coexist during a permanent liquidity trap. Intuitively, if the economy is at the zero lower bound with positive inflation, then the real interest rate must be negative. But then, to satisfy households' Euler equation, the steady state growth rate of the economy must also be negative. Conversely, to be consistent with positive steady state growth the real interest rate must be positive, and when the nominal interest rate is equal to zero this requires deflation.

However, it is not hard to think about mechanisms that could make positive growth and positive steady state inflation coexist in an unemployment steady state. One possibility is to introduce precautionary savings. In appendix E, we lay down a simple model in which every period a household faces a probability p of becoming unemployed. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income of an employed household. Unemployment benefits are financed with taxes on the employed households. We also assume that unemployed households cannot borrow and that trade in firms' share is not possible.

As showed in the appendix, under these assumptions the equilibrium is described by the same equations of the baseline model except for the Euler equation (17), which is replaced by:

$$c_t^{\sigma} = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]},$$

where:

$$\rho \equiv 1 - p + p/b^{\sigma} > 1.$$

The unemployment steady state is now characterized by:

$$g^u = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}.$$

Since $\rho > 1$, an unemployment steady state in which both inflation and growth are positive is now possible.

The key intuition behind this result is that the presence of uninsurable idiosyncratic risk depresses the real interest rate.[29] Indeed, the presence of uninsurable idiosyncratic risk drives up the demand for precautionary savings. Since the supply of saving instruments is fixed, higher demand for precautionary savings leads to a lower equilibrium interest rate. This is the reason why an economy with uninsurable unemployment risk can reconcile positive steady state growth with a negative real interest rate. Hence, once the possibility of uninsurable unemployment risk is taken into account, it is not hard to imagine a permanent liquidity trap with positive growth, positive inflation and negative real interest rate.

4.2 Introducing a wage Phillips curve

Our basic model features a constant wage inflation rate. Here we introduce a wage Phillips curve, and discuss the implications of our model for inflation and the role of wage flexibility.

To make things simple, let us assume that nominal wages are downwardly rigid:

$$W_t \geq \psi\left(L_t\right) W_{t-1},$$

with $\psi' > 0$, $\psi(1) = \bar{\pi}^w$. This formulation, in the spirit of Schmitt-Grohé and Uribe (2012), allows wages to fall at a rate which depends on unemployment. Capturing some nonmonetary costs from adjusting wages downward, here wages are more downwardly flexible the more employment is below potential. This formulation gives rise to a nonlinear wage Phillips curve. For levels of wage inflation greater than $\bar{\pi}^w$ output is at potential. Instead, if wage inflation is less than $\bar{\pi}^w$ there is a positive relationship between inflation and the output gap.

Similar to the baseline model, monetary policy follows a truncated interest rate rule in which the nominal interest rate responds to deviations of wage inflation from a target π^*:

$$1 + i_t = \max\left(\left(1+\bar{i}\right)\left(\frac{\pi_t^w}{\pi^*}\right)^{\phi}, 1\right). \tag{35}$$

We assume that $\pi^* \geq \bar{\pi}$, so that when wage inflation is on target the economy operates at full employment. We also assume that $1 + \bar{i} = \pi^*\left(\beta\chi\gamma\varpi\right)^{1/\sigma}$ and that ϕ is sufficiently large so that $\pi^* L^{\phi\sigma/(\sigma-1)} < \psi(L)$ for any $0 \leq L \leq 1$. This assumption, similar to assumption (27) of the baseline

[29] See Huggett (1993).

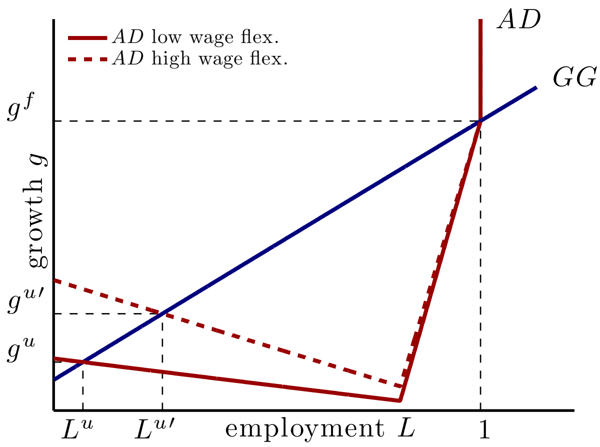

Figure 6: Steady states with variable inflation.

model, ensures local real determinacy of the full employment steady state and that, in the absence of the zero lower bound, there are no steady states other than the full employment one.

A steady state of the economy is now described by (22), (23), (35) and:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\pi^w} \tag{36}$$

$$\pi^w \geq \psi(L). \tag{37}$$

It is easy to check that there exists a unique full employment steady state with $L = 1$, $g = (\beta\chi\gamma\varpi)^{1/\sigma}$ and $\pi^w = \pi^*$. Hence, the presence of the wage Phillips curve does not affect employment and growth in the full employment steady state.

Let us now turn to the unemployment steady state. Combining equations (35) − (37) and using $i = 0$, gives:

$$g^u = \left(\frac{\beta}{\psi(L^u)}\right)^{\frac{1}{\sigma-1}}. \tag{38}$$

This expression implies a negative relationship between growth and employment. To understand this relationship, consider that in a liquidity trap the real interest rate is just the inverse of expected inflation. Due to the wage Phillips curve, as employment increases wage inflation rises generating higher price inflation. Hence, in a liquidity trap a higher employment is associated with a lower real interest rate. The consequence is that during a permanent liquidity trap a rise in employment must be associated with lower productivity growth, to be consistent with the lower real interest rate. As illustrated by figure 6, graphically this is captured by the fact that the AD curve, obtained by combining equations (35) − (37), is downward sloped for values of L low enough so that the zero lower bound constraint binds.[30]

To solve for the equilibrium unemployment steady state, combine equations (22) and (38) to

[30] Moreover, the AD curve has now a vertical portion to capture the fact that the wage Phillips curve becomes vertical at $L = 1$.

obtain:

$$L^u = \frac{1}{\beta\chi\gamma\varpi}\left(\frac{\beta}{\psi(L^u)}\right)^{\frac{\sigma}{\sigma-1}}. \tag{39}$$

Since the left-hand side of this expression is increasing in L^u, while the right hand-side is decreasing in L^u, there is a unique L^u that characterizes the unemployment steady state. Moreover, since $L^u < 1$, the presence of a Phillips curve implies that the unemployment steady state is now characterized by lower wage inflation than the full employment steady state. In sum, the presence of a wage Phillips curve does not alter the key properties of the unemployment steady state, while adding the realistic feature that in the unemployment steady state the central bank undershoots its wage inflation target.

Turning to price inflation, recalling that $\pi_t = \pi_t^w / g_{t+1}$, we have that:

$$\frac{\pi^u}{\pi^f} = \frac{\psi(L^u)}{\pi^*}\frac{g^f}{g^u}.$$

Since $\psi(L^u) < \pi^*$ and $g^f > g^u$, depending on parameter values price inflation in the unemployment steady state can be above, below or even equal to price inflation in the full employment steady state. This result is due to the fact that in the unemployment steady state the depressive impact on price inflation originating from low wage inflation is counteracted by the upward pressure exerted by low productivity growth.

To conclude this section, we note that higher wage flexibility, captured by a steeper wage Phillips curve, is associated with better outcomes in the unemployment steady state. For instance, this result can be seen by considering that expression (39) implies that the endogenous fall in wage inflation, captured by the term $\psi(L^u)$, sustains employment in the unemployment steady state. Figure 6 illustrates graphically the impact of higher wage flexibility on the determination of the unemployment steady state. The figure shows that higher wage flexibility steepens the downward portion of the AD curve, leading to higher growth and employment in the unemployment steady state. This is an interesting result, in light of the fact that analyses based on the standard New Keynesian framework suggest that higher price or wage flexibility typically lead to worse outcomes in terms of output during liquidity traps (Eggertsson, 2010).

4.3 Numerical exercise

In this section, we explore further the properties of the model by performing a simple calibration exercise. To be clear, the objective of this exercise is not to provide a careful quantitative evaluation of the framework or to replicate any particular historical event. In fact, both of these tasks would require a much richer model. Rather, our aim is to show that the magnitudes implied by the model are quantitatively relevant and reasonable.

For our numerical exercise we enrich the baseline model in two dimensions. First, along the lines of section 4.2, we consider an economy with downward wage rigidities. Second, we relax the assumption of one period monopoly rents for innovators in favor of a, more conventional, setting in which the duration of innovators' rents is endogenous.

Endogenous duration of rents from innovation. In our baseline model we assume that the rents from innovation last a single period, after which the innovator's patent expires. Here we consider a setting in which every period the innovator retains its patent with probability $1 - \eta$.[31] Under these assumptions, the value of becoming a leader in sector j is:

$$V_t(\gamma A_{jt}) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(P_{t+1} \varpi \gamma A_{jt} L_{t+1} + (1 - \mu_{jt+1} - \eta) V_{t+1}(\gamma A_{jt}) \right) \right]. \tag{40}$$

The first term inside the parenthesis on the right-hand side, also present in the baseline model, captures the expected profits to be gained in period $t+1$. In addition, the value of a successful innovation includes the value of being a leader in period $t+1$, $V_{t+1}(\gamma A_{jt})$, times the probability that the entrepreneur remains the leader in period $t+1$, $1 - \mu_{jt+1} - \eta$. Notice that the probability of maintaining the leadership is decreasing in μ_{jt+1}, capturing the fact that the discovery of a better version of product j terminates the monopoly rents for the incumbent. As in the baseline model, future payoffs are discounted using the households' discount factor $\beta \lambda_{t+1}/\lambda_t$.

To streamline the exposition, in this section we restrict attention to equilibria in which in every period a positive amount of research is targeted toward every intermediate good.[32] In this case, free entry into research implies that expected profits from researching are zero for every product. This zero profit condition implies that:[33]

$$P_t = \frac{\chi}{A_{jt}} V_t(\gamma A_{jt}),$$

for every good j. Moreover, we focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt} = \chi I_{jt}/A_{jt} = \mu_t$ for every j. In this case, $V_t(\gamma A_{jt}) = \bar{V}_t \gamma A_{jt}$ for every j, while free entry into the research sector in period $t+1$ implies $\bar{V}_{t+1} = P_{t+1}/(\gamma\chi)$. Combining these conditions with expression (40) gives:

$$\frac{P_t}{\chi} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(P_{t+1} \varpi \gamma L_{t+1} + (1 - \mu_{t+1} - \eta) \frac{P_{t+1}}{\chi} \right) \right].$$

This expression summarizes the equilibrium in the research sector. Combining this expression with equation (2) and using $\mu_t = (g_{t+1} - 1)/(\chi(\gamma - 1))$ gives:

$$g_{t+1}^{\sigma} = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \left(\chi \gamma \varpi L_{t+1} + 1 - \frac{g_{t+2} - 1}{\gamma - 1} - \eta \right) \right],$$

[31] Standard models of vertical innovation typically assume that $\eta = 0$. We follow Acemoglu and Akcigit (2012) and consider the case $\eta > 0$ to be able to match a realistic value for the spending in R&D-to-GDP ratio.

[32] We verify that this condition holds in all the simulations presented below.

[33] To derive this condition, consider that an entrepreneur that invests I_{jt} in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt} V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the zero expected profits condition in the research sector implies:

$$P_t I_{jt} = \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

Table 2: **Parameters**

	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Discount factor	$\beta = 0.99$	$(1 + i^f)/\pi^f = 1.04$
Wage inflation at full emp.	$\pi^* = 1.035$	$\pi^f = 1.02$
Intercept of wage Phillips curve	$\bar{\pi}^w = 1.02$	Schmitt-Grohé and Uribe (2012)
Slope of wage Phillips curve	$\tilde{\psi} = 1.07$	$\bar{\pi}^w 0.95^{\tilde{\psi}} = 0.98$
Productivity of research	$\chi = 42.08$	$g^f = 1.015$
Innovation step	$\gamma = 1.05$	Acemoglu and Akcigit (2012)
Share of labor in gross output	$1 - \alpha = 0.488$	Profits/GDP = 5%
Prob. patent expires	$\eta = 0.19$	$i^f/GDP^f = 3\%$

which replaces equation (18) of the baseline model.

Parameters. We choose the length of a period to correspond to a year. For many parameters we follow Schmitt-Grohé and Uribe (2012). Hence, the discount factor is set to $\beta = 0.99$ to target a real interest rate in the full employment steady state of 4 percent, and the inverse of the elasticity of intertemporal substitution is set to $\sigma = 2$, a standard value in the business-cycle literature. Moreover, we choose the value of χ, the parameter determining the productivity of research, so that growth in the full employment steady state is equal to 1.5 percent, the average growth rate of per capita output in the postwar United States. We set the central bank's wage inflation target so that price inflation in the full employment steady state is 2 percent. Recalling that wage inflation is the product of price inflation and productivity growth, this implies $\pi^* = 1.02 \cdot 1.015 = 1.035$.

The step size of innovations is set to $\gamma = 1.05$, as in Acemoglu and Akcigit (2012). We set the labor share in gross output to $1 - \alpha = 0.488$ to target a share of profits in GDP of 5 percent, and the probability that a patent expires to $\eta = 0.19$ to match a ratio of spending in R&D-to-GDP of 3 percent.[34] These targets correspond to the profit share and spending in R&D-to-GDP ratio implied by the benchmark calibration of the model in Acemoglu and Akcigit (2012).

We specify a functional form for the wage Phillips curve similar to the one in Schmitt-Grohé and Uribe (2012). We thus assume that:

$$\psi(L) = \bar{\pi}^w L^{\tilde{\psi}}.$$

To set $\bar{\pi}^w$ and $\tilde{\psi}$ we follow Schmitt-Grohé and Uribe (2012). Hence, we set $\bar{\pi}^w = 1.02$, so that at full employment nominal wages are indexed at a rate at least as large as the price inflation target. However, note that, since wage inflation is the product of price inflation and productivity growth, the downward wage rigidity constraint is not binding at the full employment steady state. Moreover, we calibrate the parameter $\tilde{\psi}$, which governs the elasticity of wage inflation to employment,

[34] Here we follow a large part of the endogenous growth literature that uses spending in R&D as the data counterpart of the model's investment in innovation. It is also possible, and in our judgement plausible, to take a broader interpretation of investment in innovation that includes other types of investment, often difficult to measure, that contribute to firms' productivity growth. For instance, Comin and Gertler (2006) emphasize the importance of investment in the implementation of existing technologies as a key driver of productivity growth in the medium term.

so that at an unemployment rate of 5 percent nominal wages fall by 2 percent per year. That is we impose the restriction $0.98 = \bar{\pi}^w 0.95^{\tilde{\psi}}$.

Results. Table 3 displays several statistics from this calibrated version of the model. The first column refers to the full employment steady state. As targeted in the calibration, productivity growth is 1.5 percent, while, by definition of the full employment steady state, unemployment is equal to zero. The real interest rate is equal to its calibration target of 4 percent, which, coupled with the 2 percent price inflation, implies a nominal interest rate of about 6 percent. Wage inflation is 3.53 percent, approximately equal to the sum of price inflation and productivity growth. Finally, about 3 percent of GDP is spent on research, as targeted in the calibration.

Column two shows the statistics for the unemployment steady state. Productivity growth is 0.13 points lower than in the full employment steady state. This difference might seem small, but, as we will see, it generates sizable welfare losses. Moreover, this number is in line with the response of productivity growth to policy changes typically found in Schumpeterian growth models (Acemoglu and Akcigit, 2012; Aghion et al., 2013). Unemployment, which in this model corresponds to the negative of the output gap, is equal to 5.35 percent. The nominal interest rate is equal to zero, while the real interest rate is 3.74 percent, so 26 basis point lower than in the full employment steady state. Moreover, the unemployment steady state is characterized by deflation and falling nominal wages. Interestingly, the ratio of spending in R&D-to-GDP in the unemployment steady state is 2.86 percent, so only slightly lower than in the full employment steady state. This happens because in the unemployment steady state both spending in R&D and GDP are lower than in the full employment one.

To get a sense of the welfare losses entailed by the unemployment steady state, we computed the consumption equivalent with respect to the full employment steady state. This is defined as the proportional permanent increase in consumption that households living in an economy stuck in the unemployment steady state must receive in order to be indifferent with respect to switching to the full employment steady state.[35] As shown in the last row of table 3, the welfare losses associated with the unemployment steady state are equivalent to an 11.27 percent permanent increase in consumption. This is a large welfare loss compared to the gains from stabilization policies usually obtained in business-cycle models. We will see shortly that about half of these welfare losses are a direct cause of the endogenous drop in productivity growth.[36]

In the third column, we report statistics referring to the unemployment steady state of a coun-

[35] More formally, for any generic expected consumption stream $E_0\{C_t\}_{t=0}^{\infty}$ we compute the consumption equivalent ϵ with respect to the full employment steady state as:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{((1+\epsilon)C_t)^{1-\sigma}-1}{1-\sigma}\right)\right]=\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(C_{f,t})^{1-\sigma}-1}{1-\sigma}\right)\right],$$

where $\{C_{f,t}\}_{t=0}^{\infty}$ is the consumption stream in the full employment steady state. When performing these computations we consider economies that start with the same initial level of productivity A_0.

[36] To be clear, adding some of the features typical of quantitative business cycle models, most notably disutility from working, is likely to reduce these welfare losses. However, following the results of Barlevy (2004), we conjecture that the endogenous drop in productivity will likely lead to significant welfare losses even in more realistic quantitative frameworks. We leave this interesting question for future research.

Table 3: Calibrated examples

	Full employment steady state	Unemployment steady state		Temporary trap	
		Benchmark	Ex. growth	Benchmark	Ex. growth
Productivity growth	1.50	1.37	1.50	1.25	1.50
Unemployment rate	0.00	5.35	5.50	7.50	8.00
Nominal interest rate	6.08	0.00	0.00	0.00	0.00
Real interest rate	4.00	3.74	4.00	5.49	6.20
Price inflation	2.00	−3.60	−3.85	−5.21	−5.84
Wage inflation	3.53	−2.28	−2.40	−4.02	−4.42
R&D/GDP	2.96	2.86	n/a	2.66	n/a
Consumption equivalent	0.00	11.27	5.82	3.59	1.72

Notes: All the values are expressed in percentage points. *Ex. growth* stands for model with productivity growth exogenous and equal to g^f. n/a stands for not applicable.

terfactual economy featuring exogenous productivity growth set equal to g^f.[37] This counterfactual is useful, because it isolates the role played by the endogenous productivity growth in the benchmark economy. The unemployment steady state of the economy with exogenous growth features a higher unemployment rate and lower inflation compared to the benchmark economy. The difference in inflation is partly due to the fact that the endogenous drop in productivity growth featured in the benchmark economy raises firms' marginal cost, inducing firms to charge higher prices and mitigating the fall in inflation,[38] and partly due to the fact that wage inflation is lower in the exogenous growth economy. The full employment and unemployment steady state of the exogenous economy feature the same real interest rate. This highlights the role of the endogenous drop in productivity growth in depressing the real interest rate in the unemployment steady state of the benchmark economy. Turning to welfare, the unemployment steady state of the exogenous growth economy is characterized by large welfare losses, equal to a 5.82 percent permanent decrease in consumption. However, these welfare losses are about half of the ones characterizing the benchmark economy, meaning that the endogenous drop in productivity growth has a large negative impact on welfare.

Columns 4 and 5 show the statistics for a trap with an expected duration of ten years, respectively for the benchmark economy and for the economy with exogenous growth. This experiment is meant to illustrate the impact on the economy of a stagnation trap of long, but finite, expected duration. Starting from the benchmark economy, qualitatively a temporary stagnation trap resembles the unemployment steady state, but the difference with respect to the full employment steady state are quantitatively larger. For instance, during the temporary trap productivity growth is 0.25 percent lower and unemployment is 7.5 percent higher than in the full employment steady state.

[37] Similar to Schmitt-Grohé and Uribe (2012), and contrary to our baseline model, the extended model allows for an unemployment steady state even with exogenous productivity growth because of the presence of a wage Phillips curve. For the economy with exogenous productivity growth we maintained the same parameters of our benchmark economy, and the statistics relative to the full employment steady state reported in column 1 also apply to the economy with exogenous productivity growth. The only exception is that, since in the counterfactual economy productivity growth is fully exogenous, investment in research is set equal to zero.

[38] Christiano et al. (2015) provide evidence on the role of the slowdown in productivity growth during the US Great Recession in sustaining inflation.

The only exception is represented by the real interest rate, which during the temporary stagnation trap rises above its value in the full employment steady state. This can be explained with the large drop in price inflation occurring during the trap. The welfare losses linked to the temporary liquidity trap are smaller than the ones generated by the unemployment steady state, but still quantitatively significant since they are equal to a 3.59 percent permanent drop in consumption. Comparing column 4 and 5 gives broadly the same picture of the unemployment steady state. In particular, when productivity growth is exogenous a temporary liquidity trap generates a sharper rise in unemployment, and a temporary liquidity trap generates welfare losses that are about half the size of the ones of the benchmark economy.

Overall, this simple calibration exercise paints the picture of stagnation traps as persistent periods of weak productivity growth coupled with significantly high unemployment and low output gap. Though the fall in productivity growth associated with a stagnation trap is not dramatic, the welfare losses associated with a prolonged stagnation trap are substantial, due to its permanent effect on long-run output.

5 Policy Implications

We now turn to the policy implications of our model. We start by considering optimal interest rate policy, both under commitment and discretion. We then turn to growth policies aiming at sustaining investment in productivity enhancing activities. For simplicity, we discuss the role of these policies in the context of the baseline model described in section 2.

5.1 Interest rate policy

In section 3 we have shown that stagnation traps can arise if the central bank follows an interest rate rule, in which the interest rate responds monotonically to employment or wage inflation. In this section we examine the robustness of this result to optimal interest rate policy. The key lesson that we derive is that the ability to commit by the central bank is crucial. As we will see, under full commitment a central bank can implement interest rate policies that rule out stagnation traps. Instead, if the central bank operates under discretion stagnation traps are possible, even if interest rates are set optimally.

Commitment. Let us start by examining a central bank that operates under commitment. To build intuition, first consider a central bank that adopts an interest rate peg, by committing to set $i_t = i^f$ in any date and state. Clearly, this policy rules out the unemployment steady state, because the only steady state consistent with $i = i^f$ is the full employment one.[39] Moreover, this policy rules out the persistent liquidity traps described in section 3.2, because they would require the nominal interest rate to be equal to zero. More broadly, pegging the interest rate to i^f rules out stagnation traps, because for these to occur agents should anticipate a protracted period of zero nominal interest rate. However, as it is well known, pegging the interest rate opens the door

[39] See the proof to proposition 1.

to sunspot fluctuations around the full employment steady state.[40] In fact, this is precisely one of the reasons why adopting interest rate rules that respond to employment or inflation might be desirable. Hence, ruling out stagnation traps by pegging the interest rate comes at the risk of self-fulfilling fluctuations around the full employment steady state.

Another option for a central bank under commitment is to adopt a non-linear interest rate rule, in the spirit of the one proposed by Schmitt-Grohé and Uribe (2012). This approach combines a standard interest rate rule, that operates in "normal" times, with an interest rate peg, adopted by the central bank when expectations turn pessimistic. To see how this approach works, define s_t as a binary value that follows:

$$s_t = \begin{cases} 1 & \text{if } i_{t-1} = 0 \\ 0 & \text{if } g_t \geq g^f \\ s_{t-1} & \text{otherwise,} \end{cases}$$

for $t \geq 0$ with $s_{-1} = 0$. Now consider a central bank that follows the rule:

$$i_t = \begin{cases} \max\left(\left(1+\bar{i}\right) L_t^{\phi} - 1, 0\right), & \text{if } s_t = 0 \\ i^f & \text{otherwise.} \end{cases}$$

Under this rule, the central bank switches to an interest rate peg the period after the nominal interest rate hits the zero lower bound. The peg is maintained for one period, after which the central bank returns to the interest rate rule considered in section 3.

This policy eliminates the unemployment steady state and the persistent stagnation traps of section 3.2. In fact, for stagnation traps to occur agents should coordinate their expectations on a protracted period of zero interest rates, a possibility ruled out by the central bank commitment to maintain the interest rate equal to zero for one period at most. At the same time, this policy rule eliminates sunspot fluctuations around the full employment steady state. Hence, a central bank under commitment can rule out the stagnation traps discussed in section 3, while still preserving determinacy of the full employment steady state, by adopting a nonlinear interest rate rule.

Discretion. The picture changes dramatically if the central bank does not have the ability to commit to its future actions. The following proposition characterizes the behavior of a benevolent central bank that operates under discretion.

Proposition 3 *Consider a central bank that operates under discretion and maximizes households' expected utility, subject to* (17)*,* (18)*,* (19)*,* $L_t \leq 1$ *and* $i_t \geq 0$*. The solution to this problem satisfies:*

$$i_t \left(L_t - 1\right) = 0.$$

The intuition behind this result is straightforward. The discretionary central bank seeks to maximize current employment.[41] From the goods market clearing condition, employment is in-

[40] Again, see the proof to proposition 1.

[41] To be precise, the economy is subject to three sources of inefficiency. First, involuntary unemployment is possible.

creasing in consumption and investment in research (both normalized by productivity):

$$\Psi L_t = c_t + \frac{I_t}{A_t}.$$

In turn, equations (17) and (18) imply that, holding expectations about the future constant, both consumption and investment in research are decreasing in the nominal interest rate. In fact, when the nominal interest rate falls also the real interest rate decreases, inducing households to frontload their consumption and entrepreneurs to increase investment in research, thus stimulating output and employment. It follows that, as long as the zero lower bound constraint does not bind, the central bank is able to set the nominal interest rate low enough so that the economy operates at full employment and $L_t = 1$. However, if a negative nominal interest rate is needed to reach full employment then the best that the discretionary central bank can do is to set $i_t = 0$. Hence, the economy can be in one of two regimes. Either the economy operates at full employment and the zero lower bound constraint on the interest rate does not bind, or the economy is in a liquidity trap with unemployment.[42]

We now show that under a discretionary central bank the economy can experience the same kind of stagnation traps described in section 3. Let us take the perspective of a discretionary central bank operating in period $t = 0$. Consider a case in which expectations coordinate on the unemployment steady state, so that $E_0[i_t] = 0$, $E_0[L_{t+1}] = L^u$ and $E_0[c_{t+1}] = c^u$ for every future date $t > 0$ and state. From section 3 we know that if the central bank sets $i_0 = 0$, then $L_0 = L^u$ so the economy will experience unemployment. Can the central bank do better by setting a positive nominal interest rate? The answer is no, because by raising the nominal interest rate above zero the central bank would further depress demand for consumption and investment, thus pushing employment below L^u. Hence, if expectations coordinate on the unemployment steady state the best response of a central bank under discretion is to set $i_0 = 0$, implying that $L_0 = L^u$, $g_1 = g^u$ and $c_0 = c^u$. A similar reasoning holds in any date $t \geq 0$, meaning that the central bank's actions validate agents' expectations and push the economy in the unemployment steady state. Moreover, a similar reasoning implies that if expectations coordinate on the full employment steady state the central bank will set $i = i^f$ and validate them. Hence, under discretionary monetary policy the two steady states analyzed in section 2 are possible equilibria.[43] Moreover, one can show that under discretion the temporary stagnations traps described in section 3.2 are also possible.

These results highlight the key role that the ability to commit plays in avoiding stagnation

Second, due to monopolistic competition production of intermediate goods is inefficiently low. Third, investment in research is subject to the intertemporal spillover and business stealing effects studied by Aghion and Howitt (1992) and Grossman and Helpman (1991), implying that in the laissez faire equilibrium the fraction of output invested in research can be higher or lower than in the social planner allocation. However, as shown in the proof to proposition 3, interest rate policy can only seek to correct the first distortion. The economic intuition for this result is that, since consumption and investment in research are both decreasing in the interest rate, interest rate policy cannot affect the allocation of output between consumption and investment, and hence cannot correct for the inefficiencies due to the intertemporal spillover and business stealing effects.

[42] In fact, the optimal policy under discretion is equivalent to the truncated interest rate rule considered in the baseline model with $\phi \to +\infty$.

[43] This result can also be derived using the graphical approach of figure 2. In fact, the only difference is that in the case of a discretionary central bank the upward portion of the AD curve becomes a vertical line at $L = 1$.

traps through interest rate policy. Under commitment, the central bank can design interest rate policies that make expectations of a prolonged liquidity trap inconsistent with equilibrium, thus ruling out the possibility of long periods of stagnation. Instead, under discretion the central bank inability to commit to its future actions leaves the door open to stagnation episodes.[44]

5.2 Growth Policy

One of the root causes of a stagnation trap is the weak growth performance of the economy, which is in turn due to entrepreneurs' limited incentives to innovate due to weak demand for their products. This suggests that subsidies to investment in innovation might be a helpful tool in the management of stagnation traps. In fact, these policies have been extensively studied in the context of endogenous growth models as a tool to overcome inefficiencies in the innovation process. However, here we show how policies that foster productivity growth can also play a role in stimulating aggregate demand and employment during a liquidity trap.

The most promising form of growth policies to exit a stagnation trap are those that loosen the link between profits and investment in innovation. For instance, suppose that the government provides a subsidy to innovation, in the form of a lump-sum transfer s_{jt} given to entrepreneurs in sector j to finance investment in innovation.[45] The subsidy can be state contingent and sector specific, and it is financed with lump-sum taxes on households. Under these assumptions, the zero profit condition for research in sector j becomes:[46]

$$V_t(\gamma A_{jt}) = \frac{P_t A_{jt}}{\chi}\left(1 - \frac{s_{jt}}{I_{jt}}\right),$$

where $V_t(\gamma A_{jt})$ is defined as in (10). The presence of the term s_{jt}/I_{jt} is due to the fact that entrepreneurs have to finance only a fraction $1 - s_{jt}/I_{jt}$ of the investment in research, while the rest is financed by the government. This expression implies that entrepreneurs are willing to invest in innovation even when the value of becoming a leader is zero, since if $V_t(\gamma A_{jt}) = 0$ then $I_{jt} = s_{jt}$. Hence, assuming that the government can ensure that entrepreneurs cannot divert the subsidy away from innovation activities, investment in innovation will be always at least equal to the subsidy s_{jt}, so $I_{jt} \geq s_{jt}$.

Let us now consider the macroeconomic implications of the subsidy. For simplicity, we keep on focusing on symmetric equilibria in which every sector j has the same innovation probability, and hence we consider subsidies of the form $s_{jt} = s_t A_{jt}$. Assuming a positive subsidy $s_t > 0$, the

[44] The result that discretionary monetary policy opens the door to multiple equilibria is reminiscent of the findings of King and Wolman (2004).

[45] More precisely, we assume that in sector j the government devotes an aggregate amount of resources $s_t A_{jt}$ to sustain innovation. These resources are equally divided among all the entrepreneurs operating in innovation in that sector.

[46] With the subsidy, the cost of investing I_{jt} in research is $P_t(I_{jt} - s_{jt})$, which gives an expected gain of $\chi I_{jt} V_{jt}(\gamma A_{jt})/A_{jt}$. The zero expected profits condition for research in sector j then implies:

$$P_t\left(I_{jt} - s_{jt}\right) = \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Rearranging this expression we obtain the expression in the main text.

growth equation (18) is replaced by:

$$\left(1-\frac{s_t\chi(\gamma-1)}{g_{t+1}-1}\right)=\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}g_{t+1}^{-\sigma}\chi\gamma\varpi L_{t+1}\right], \tag{41}$$

where to derive this expression we have followed the same steps taken in section 3 and used $I_{jt}/A_{jt}=\mu_t/\chi=(g_{t+1}-1)/(\gamma-1)$. Notice that the expression above implies that $g_{t+1}>1$, since with the subsidy in place investment in innovation is always positive.

We now show that an appropriately chosen subsidy can eliminate the unemployment steady state. Consider a subsidy of the form $s_t=s(g_{t+1})$ with $s'(\cdot)<0$ and $s(g^f)=0$, where g^f is the productivity growth in the full employment steady state under laissez faire. According to this policy, the government responds to a fall in productivity growth by increasing the subsidy to investment in innovation. With the subsidy in place, in steady state the growth equation becomes:

$$g^{\sigma}\left(1-\frac{s(g)\chi(\gamma-1)}{g-1}\right)=\beta\chi\gamma\varpi L. \tag{42}$$

Notice that the term in round brackets on the left-hand side of expression (42) is smaller than one, because $s(g_{t+1})A_{jt}<I_{jt}$. Hence, given L, steady state growth is increasing in the subsidy.

It is easy to see that, since $s(g^f)=0$, the economy features a full employment steady state identical to the one described in section 3.1. Now turn to the unemployment steady state. In the unemployment steady state productivity growth must be equal to $g^u=(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$, to satisfy households' Euler equation. However, a sufficiently high subsidy can guarantee that investment in innovation will always sufficiently high so that the growth rate of the economy will always be higher than $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. It follows that by setting a sufficiently high subsidy the government can rule out the possibility that the economy might fall in a permanent stagnation trap.

Proposition 4 *Suppose that there is a subsidy to innovation $s(g_{t+1})$ satisfying $s'(\cdot)<0$, $s(g^f)=0$ and:*

$$1+s\left(\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}\right)\chi(\gamma-1)>\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}, \tag{43}$$

and that the conditions stated in proposition 1 hold. Then there exists a unique steady state. The unique steady state is characterized by full employment.

Intuitively, the subsidy to innovation guarantees that even if firms' profits were to fall substantially, investment in innovation would still be relatively high. In turn, a high investment in innovation stimulates growth and aggregate demand, since a high future income is associated with a high present demand for consumption. By implementing a sufficiently high subsidy, the government can eliminate the possibility that aggregate demand will be low enough to make the zero lower bound constraint on the nominal interest rate bind. It is in this sense that growth policies can be thought as a tool to manage aggregate demand in our framework. Importantly, to be effective a subsidy to innovation has to be large enough, otherwise it might not have any positive impact

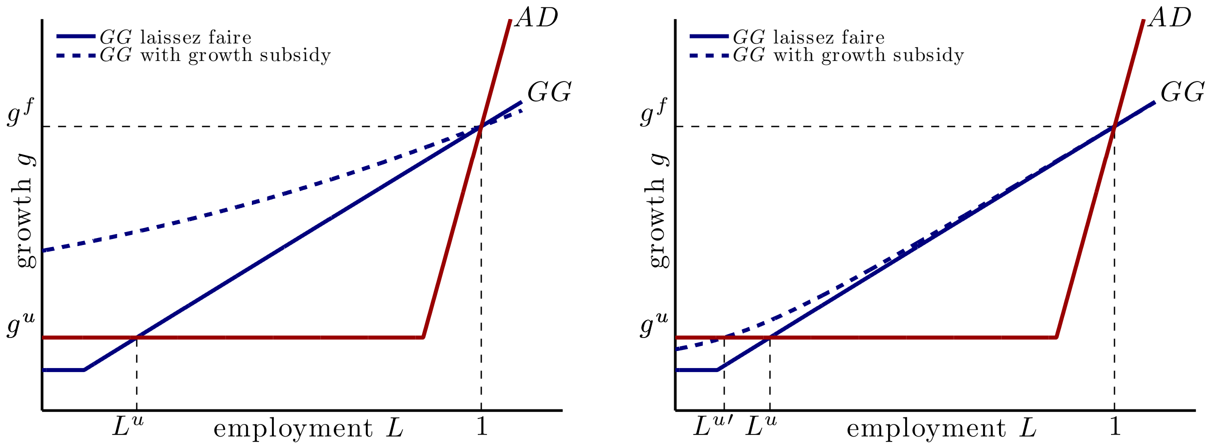

Figure 7: Steady state with growth subsidy. Left panel: large subsidy. Right panel: small subsidy.

on the economy.[47]

Graphically, the impact of the subsidy is illustrated by figure 7. The blue solid line corresponds to the GG curve of the economy without subsidy, while the blue dashed line represents the GG curve of an economy with a subsidy to investment in innovation. The subsidy makes the GG curve rotate up, because for a given level of employment and aggregate demand the subsidy increases the growth rate of the economy. If the subsidy is sufficiently high, as it is the case in the left panel of the figure, the unemployment steady state disappears and the only possible steady state is the full employment one. By contrast, the right panel of figure 7 shows the impact of a small subsidy. A small subsidy makes the GG curve rotate up, but not enough to eliminate the unemployment steady state. In fact, the small subsidy leads to lower employment in the unemployment steady state compared to the laissez faire economy.

Summarizing, there is a role for well-designed subsidies to growth-enhancing investment, a typical supply side policy, in stimulating aggregate demand so as to rule out liquidity traps driven by expectations of weak future growth. In turn, the stimulus to aggregate demand has a positive impact on employment. In this sense, our model helps to rationalize the notion of job creating growth.

[47] One might wonder how important it is that the subsidy is contingent on productivity growth. It turns out that totally analogous results can be obtained with a subsidy contingent on employment. Moreover, even a sufficiently large non-contingent subsidy can rule out the unemployment steady state. However, a non-contingent subsidy will also increase growth, perhaps to an inefficiently high value, in the full employment steady state.

The results summarized in proposition 4 readily extend, under a caveat, to the case of the economy with a wage Phillips curve described in section 4.2. The caveat is that wage inflation should not fall too much as employment approaches zero. Using the notation of section 4.2, the function $\psi(\cdot)$ must be such that:

$$\lim_{L\to 0} \psi(L) \geq \frac{\beta}{(g^f)^{\sigma-1}}.$$

Intuitively, if this condition does not hold, even when the subsidy is sufficiently high so that productivity growth cannot fall below its value in the full employment steady state there still exists an $0 < L < 1$ consistent with a steady state in which $i = 0$. In this case, it is not possible to rule out the unemployment steady state with a subsidy that is a continuous function of productivity growth or employment. However, even when the condition above fails, it is still possible to rule out the unemployment steady state by designing subsidy schemes that are discontinuous functions of productivity growth or employment.

6 Conclusion

In this paper, we have presented a Keynesian growth model in which endogenous growth interacts with the possibility of involuntary unemployment due to weak aggregate demand. The combination of these two factors can give rise to stagnation traps, that is persistent liquidity traps characterized by unemployment and weak growth. All it takes for the economy to fall into a stagnation trap is a wave of pessimism about future growth. We show that large policy interventions to support growth can lead the economy out of a stagnation trap, thus shedding light on the role of growth policies in stimulating aggregate demand and employment.

Our analysis represents a first step toward understanding the interactions between business cycles, growth and stagnation, and it leaves open several exciting avenues for future research. We conclude the paper by mentioning two of them. First, in order to focus the analysis on fluctuations driven by shocks to expectations, in this paper we have abstracted from fundamental shocks. Can fundamental shocks, such as productivity, monetary or news shocks, lead to prolonged periods of stagnation? Does the impact of fundamental shocks on the economy depend on whether the economy is undergoing a period of stagnation? These are examples of questions that our model can help to answer. Second, in this paper we have abstracted from financial frictions. However, it is easy to imagine a channel through which growth and aggregate demand can interact when firms' access to financing is limited. In fact, if access to credit is tight, firms' investment in innovation will be constrained by their internal funds. In turn, a period of low aggregate demand and weak profits will erode firms' internal funds, and thus their ability to invest in productivity-enhancing activities. Through this channel, a period of low aggregate demand will lead to low productivity growth. Indeed, we conjecture that economies undergoing a period of financial stress might be particularly exposed to the risk of expectation-driven stagnation traps. This might contribute to explain why the episodes of stagnation affecting Japan, the US and the Euro area coincided, at least in their beginning, with periods of tight access to credit.

Appendix

A Proofs

A.1 Proof of proposition 1 (existence, uniqueness and local determinacy of full employment steady state)

Proof. We start by proving existence. A steady state is described by the system (21)-(24). Setting $L = 1$ and using the first inequality in condition (28), equation (22) implies:

$$g^f = (\beta\chi\gamma\varpi)^{\frac{1}{\sigma}}, \tag{A.1}$$

so that by assumption (29) $1 < g^f < \gamma$. Then equation (21) implies:

$$1 + i^f = \frac{\bar{\pi}^w \left(g^f\right)^{\sigma-1}}{\beta}. \tag{A.2}$$

Assumptions (25) and (26) guarantee that $i^f > 0$, and that $\bar{i} = i^f$, so that the interest rate rule (24) is compatible with the existence of a full employment steady state. Moreover, combining equations (22) and (23) evaluated at $L = 1$ and $g = g^f$, one can check that the second inequality in condition (28) ensures that $c^f > 0$. Hence, a full employment steady state exists.

To prove uniqueness, consider that equation ($A.1$) implies that there is only one value of g consistent with the full employment steady state, while equation ($A.2$) establishes that there is a unique value of i consistent with $g = g^f$. Hence, the full employment steady state is unique.

We now show that, under the assumption $\phi > 1$, the full employment steady state is locally determinate. A loglinear approximation of equations (17) – (20) around the full employment steady state gives:

$$(\sigma - 1)\hat{g}_{t+1} = \hat{i}_t + \sigma(\hat{c}_t - E_t[\hat{c}_{t+1}]) \tag{A.3}$$

$$\hat{L}_t = \frac{c^f}{y^f}\hat{c}_t + \left(1 - \frac{c^f}{y^f}\right)\frac{g^f}{g^f - 1}\hat{g}_{t+1} \tag{A.4}$$

$$\sigma\hat{g}_{t+1} = \sigma(\hat{c}_t - E_t[\hat{c}_{t+1}]) + E_t\left[\hat{L}_{t+1}\right] \tag{A.5}$$

$$\hat{i}_t = \phi\hat{L}_t, \tag{A.6}$$

where $\hat{x} \equiv \log(x_t) - \log(x^f)$ for every variable x, except for $\hat{g}_t \equiv g_t - g^f$ and $\hat{i} \equiv i_t - \bar{i}$, while $y^f \equiv (\alpha/\xi)^{\alpha/(1-\alpha)}(1 - \alpha/\xi)$ is GDP normalized by the productivity index. This system can be written as:

$$\hat{L}_t = \xi_1 E_t[\hat{L}_{t+1}] + \xi_2 E_t[g_{t+2}] \tag{A.7}$$

$$\hat{g}_{t+1} = \xi_3 E_t[\hat{L}_{t+1}] + \xi_4 E_t[g_{t+2}], \tag{A.8}$$

where

$$\xi_1 \equiv \frac{\frac{y^f}{c^f} - \frac{1}{\sigma} + \left(1 + \frac{y^f - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}{\frac{y^f}{c^f} + \phi\left(1 + \frac{y^f - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}$$

$$\xi_2 \equiv -\frac{\frac{y^f - c^f}{c^f}\frac{g^f}{g^f - 1}}{\frac{y^f}{c^f} + \phi\left(1 + \frac{y^f - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}$$

$$\xi_3 \equiv 1 - \phi\xi_1$$

$$\xi_4 \equiv -\phi\xi_2.$$

The system is determinate if and only if:[48]

$$|\xi_1\xi_4 - \xi_2\xi_3| < 1 \tag{A.9}$$

[48] See Bullard and Mitra (2002).

$$|\xi_1 + \xi_4| < 1 + \xi_1\xi_4 - \xi_2\xi_3. \tag{A.10}$$

Condition (A.9) holds if:

$$\phi > \frac{\frac{y^f - c^f}{y^f}\frac{g^f}{g^f - 1} - 1}{\frac{c^f}{y^f} + \frac{y^f - c^f}{y^f}\frac{g^f}{g^f - 1}},$$

while condition (A.10) holds if:

$$\phi > 1 - \frac{1}{\sigma}.$$

Hence, assumption (27) guarantees that (A.10) holds, while, after some tedious algebra, one can prove that assumptions (27) and (28) imply that (A.9) holds. ■

A.2 Proof of proposition 2 (existence, uniqueness and local indeterminacy of unemployment steady state)

Proof. We start by showing that it is not possible to have an unemployment steady state with a positive nominal interest rate. Suppose that this is not the case, and that there is a steady state with $1 + i = (1 + \bar{i})L^{\phi}$ and $0 \leq L < 1$. Then combining equations (21) and (22), and using $\beta\chi\gamma\varpi = g^f$ and $1 + \bar{i} = \left(g^f\right)^{\sigma - 1}\bar{\pi}^w/\beta$ gives:

$$g^f L^{\frac{\phi}{\sigma-1}} = \left(\max\left(\left(g^f\right)^{\sigma} L, 1\right)\right)^{\frac{1}{\sigma}}. \tag{A.11}$$

Assumption (27) implies that the left-hand side of this expression is smaller than the right-hand side for any $0 \leq L < 1$. Hence, we have found a contradiction and an unemployment steady state with $i > 0$ is not possible.

We now prove that an unemployment steady state with $i = 0$ exists and is unique. Setting $i = 0$, equation (21) implies that there is a unique value of $g = (\beta/\bar{\pi}^w)^{1/(\sigma-1)} = g^u$ consistent with the unemployment steady state. Moreover, since $i^f > 0$ equation (21) also implies that $g^u < g^f$, while the first inequality in condition (30) implies $g^u > 1$. Evaluating equation (22) at $g = g^u$ we have:

$$L^u = \frac{(g^u)^{\sigma}}{\beta\chi\gamma\varpi},$$

ensuring that there is a unique value of $L = L^u > 0$ consistent with $g = g^u$. Moreover, using $g^u < g^f$ and equation (22) gives $L^u < 1$. Combining equations (22) and (23) evaluated at $L = L^u$ and $g = g^u$, one can check that the second inequality in condition (30) ensures that $c_u > 0$. Hence, the unemployment steady state exists and is unique. Finally, using $i^f > 0$ and $g^u < g^f$ one can see that $1/\pi_u = g^u/\bar{\pi}^w < (1 + i^f)g^f/\bar{\pi}^w = (1 + i^f)/\pi^f$, so that the real interest rate in the unemployment steady state is lower than the one in the full employment steady state.

To prove local indeterminacy one can follow the steps of the proof to proposition one. Since in the neighborhood of the unemployment steady state $\hat{i}_t = 0$, it is easy to show that condition (A.10) cannot be satisfied. ■

A.3 Proof of proposition 3 (optimal discretionary monetary policy)

Proof. Under discretion, every period the central bank maximizes the representative household expected utility subject to (17), (18), (19), $L_t \leq 1$ and $i_t \geq 0$, taking future variables as given. The central bank's problem can be written as:

$$\max_{L_t, c_t, g_{t+1}, i_t} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau} \left(\frac{(C_{\tau}^{1-\sigma} - 1)}{1-\sigma} \right) \right] = E_t \left[\beta^t A_t^{1-\sigma} \left(\frac{c_t^{1-\sigma}}{1-\sigma} + g_{t+1} \nu_t^1 \right) \right] - \frac{1}{(1-\beta)(1-\sigma)},$$

subject to:

$$L_t = \frac{1}{\Psi} \left(c_t + \frac{g_{t+1} - 1}{\chi(\gamma - 1)} \right)$$

$$c_t = \left(\frac{g_{t+1}^{\sigma-1}}{1+i_t} \right)^{\frac{1}{\sigma}} \nu_t^2$$

$$g_{t+1} = \max \left(1, \frac{\nu_t^3}{1+i_t} \right)$$

$$L_t \leq 1$$

$$i_t \geq 0,$$

where the third constraint is obtained by combining (17) and (18), and:

$$\nu_t^1 = E_t \left[\sum_{\tau=t+1}^{\infty} \beta^{\tau} \frac{\left(c_{\tau} \Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}} \right)^{1-\sigma}}{1-\sigma} \right]$$

$$\nu_t^2 = \left(\frac{\bar{\pi}^w}{\beta E_t \left[c_{t+1}^{-\sigma} \right]} \right)^{\frac{1}{\sigma}}$$

$$\nu_t^3 = \bar{\pi}^w \chi \gamma \varpi \frac{E_t[L_{t+1} c_{t+1}^{-\sigma}]}{E_t[c_{t+1}^{-\sigma}]}.$$

ν_t^1, ν_t^2 and ν_t^3 are taken as given by the central bank, because they are function of parameters and expectations about future variables only.

Notice that the objective function is strictly increasing in c_t and g_{t+1}. Also notice that from the second and third constraints we can write $c_t = c(i_t)$ with $c'(i_t) < 0$ and $g_{t+1} = g(i_t)$ with $g'(i_t) \leq 0$. We can then rewrite the problem of a central bank under discretion as

$$\min i_t,$$

subject to:

$$L_t = \frac{1}{\Psi} \left(c(i_t) + \frac{g(i_t) - 1}{\chi(\gamma - 1)} \right)$$

$$L_t \leq 1$$

$$i_t \geq 0.$$

The solution to this problem can be expressed by the complementary slackness condition:

$$i_t\,(L_t - 1) = 0.$$

■

A.4 Proof of proposition 4 (uniqueness of steady state with subsidy to innovation)

The proof that a full employment steady state exists and is unique follows the steps of the proof to proposition 1.

We now prove that there is no steady state with unemployment. Following the proof to proposition 2, one can check that if another steady state exists, it must be characterized by $i = 0$. Equation (17) implies that in this steady state growth must be equal to $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. Suppose that a steady state with $g = (\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$ exists. Then equation (42) implies that there must be an $0 \leq \tilde{L} \leq 1$ such that:

$$\tilde{L} = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{\sigma}{\sigma-1}} \left(1 - \frac{s\left(\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}\right)\chi(\gamma-1)}{\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} - 1}\right)(\beta\chi\gamma\varpi)^{-1}.$$

But condition (43) implies $\tilde{L} < 0$, a contradiction. Hence, an unemployment steady state does not exist. ■

B Model with money

In this appendix we explicitly introduce money in the model. The presence of money rationalizes the zero lower bound constraint on the nominal interest rate, but does not alter the equilibrium conditions of the model.

Following Krugman (1998) and Eggertsson (2008) we assume that households need to hold at least a fraction $\nu > 0$ of production in money balances M:

$$M_t \geq \nu P_t Y_t. \tag{B.1}$$

The household's budget constraint is now:

$$P_t C_t + \frac{b_{t+1}}{1+i_t} + M_t = W_t L_t + b_t + M_{t-1} + d_t - T_t, \tag{B.2}$$

where M_t denotes money holdings at time t to be carried over at time $t+1$, and T are lump-sum taxes paid to the government.

The optimality condition with respect to M_t is:

$$\lambda_t = \beta E_t \left[\lambda_{t+1}\right] + \mu_t, \tag{B.3}$$

where $\mu_t > 0$ is the Lagrange multiplier on constraint ($B.1$). Combining optimality conditions (3) and ($B.3$) it is easy to see that the presence of money implies $i_t \geq 0$. Intuitively, households will always prefer to hold money, rather than an asset which pays a non-contingent negative nominal return. It is also easy to see that constraint ($B.1$) binds if $i_t > 0$, but it is slack if $i_t = 0$. Hence, households' money demand is captured by the complementary slackness condition:

$$i_t \left(M_t - \nu P_t \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t \right) = 0, \tag{B.4}$$

with $i_t \geq 0$ and $M_t \geq \nu P_t \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t$, where we have substituted Y_t using equation (9).

To close the model, we assume that the government runs a balanced budget:

$$T_t = M_t - M_{t-1},$$

so that seignorage revenue is rebated to households via lump sum taxes.

We can now define an equilibrium as a set of processes $\{L_t, c_t, g_{t+1}, i_t, M_t, P_t\}_{t=0}^{+\infty}$ satisfying equations (17) – (20), ($B.4$) and $P_t = \bar{\pi}^w P_{t-1}/g_t$, given initial values P_{-1}, A_0.[49]

Notice that to solve for the path of L_t, c_t, g_{t+1} and i_t only equations (17) – (20) are needed. Given values for L_t, A_t and P_t, the only use of the money demand equation ($B.4$) is to define the money supply M_t consistent with the central bank's interest rate rule. Specifically, when $i_t > 0$ the equilibrium money supply is $M_t = \nu P_t \left(\alpha/\xi\right)^{\frac{\alpha}{1-\alpha}} A_t L_t$, while when $i_t = 0$ any money supply $M_t \geq \nu P_t \left(\alpha/\xi\right)^{\frac{\alpha}{1-\alpha}} A_t L_t$ is consistent with equilibrium.

These results do not rest on the specific source of money demand assumed. For instance, similar results can be derived in a setting in which households derive utility from holding real money balances, as long as a cashless limit is considered (Eggertsson and Woodford, 2003).

C The cases of $\sigma = 1$ and $\sigma < 1$

In the main text we have focused attention on the empirically relevant case of low elasticity of intertemporal substitution, by assuming that $\sigma > 1$. In this appendix we consider the alternative cases $\sigma = 1$ and $\sigma < 1$. The key result is that under these cases the steady state is unique.

We start by analyzing the case of $\sigma = 1$. In steady state, equation (21) can be written as:

$$1 = \frac{\beta(1+i)}{\bar{\pi}^w}.$$

[49] To derive the law of motion for P_t we have used the equilibrium condition $\pi_t = \pi_t^w / g_t$ and the law of motion for W_t.

Intuitively, under this case changes in the growth rate of the economy have no impact on the equilibrium nominal interest rate. Hence, if there exists a full employment equilibrium featuring a positive nominal interest rate, it is easy to check that no unemployment equilibrium can exist.

We now turn to the case $\sigma < 1$. Under this case, equation (21) implies a negative relationship between growth and the nominal interest rate. Supposing that a full employment equilibrium featuring a positive nominal interest rate exists, if a liquidity trap equilibrium exists, it must feature a higher growth rate than the full employment one, i.e. $g^u > g^f$. Since $L^f = 1$, it must be the case that $L^u \leq L^f$. But equation (22) implies a nonnegative steady state relationship between g and L. Then we cannot have a steady state in which $g^u > g^f$ and $L^u \leq L^f$, so that, if the economy features a full employment steady state, an unemployment steady state cannot exist.

D Numerical solution method to compute perfect foresight transitions

The objective is to compute a sequence $\{g_{t+1}, L_t, c_t, i_t\}_{t=0}^{T}$ given an initial condition L_0 for T large such that:

$$c_t^\sigma = \frac{c_{t+1}^\sigma g_{t+1}^{\sigma-1} \bar{\pi}^w}{\beta(1+i_t)} \tag{D.1}$$

$$(g_{t+1} - 1)\left(1 - \beta\left[\left(\frac{c_t}{c_{t+1}}\right)^\sigma g_{t+1}^{-\sigma} \chi\gamma\varpi L_{t+1}\right]\right) = 0 \tag{D.2}$$

$$c_t = \Psi L_t - \frac{g_{t+1} - 1}{\chi(\gamma - 1)} \tag{D.3}$$

$$1 + i_t = \max\left((1+\bar{i}) L_t^\phi, 1\right), \tag{D.4}$$

and $L_t \leq 1$ hold for all $t \in \{0, ..., T\}$. We restrict attention to sequences that converge to the unemployment steady state.

The algorithm follows these steps:

1. Guess a sequence $\{L_t\}_{t=1}^{T+1}$. Set $L_{T+1} = L^u$.

2. Use ($D.1$), ($D.2$), ($D.3$) and ($D.4$) evaluated at $t = 0$, the initial condition L_0 and the guess for L_1 to solve for g_1 and c_0.

3. For any $t \in \{1, ..., T\}$, use equations ($D.1$) and ($D.4$) evaluated at $t - 1$ and values for L_{t-1}, g_t and c_{t-1} to solve for c_t. Use ($D.1$), ($D.2$), ($D.3$) and ($D.4$) evaluated at t, and L_{t+1} to solve for g_{t+1}.

4. Use equations ($D.1$) and ($D.4$) evaluated at T and values for L_T, g_{T+1} and c_T to solve for c_{T+1}.

5. Evaluate convergence by checking that the market clearing condition ($D.3$) holds for any $t \in \{1, ..., T\}$ and that the sequence has converged to the unemployment steady state. If

$\sup ||\Psi L_t - c_t - (g_{t+1} - 1)(\chi(\gamma - 1))||$ for all $t \in \{1, ..., T\}$ and $||c_{T+1} - c^u||$ are sufficiently small we have found a solution. Otherwise compute:

$$\hat{L}_t = L_t - \epsilon \left(\Psi L_t - c_t - \frac{g_{t+1} - 1}{\chi(\gamma - 1)} \right),$$

where ϵ is a small positive number. Update the guess by setting $L_t = \min\left(\hat{L}_t, 1\right)$ for any $t \in \{1, ..., T\}$ and go to step 2.

E Model with unemployment risk

In this appendix, we lay down the model with idiosyncratic unemployment risk described in section 4.2. In this model, each household faces in every period a constant probability p of being unemployed. The employment status is revealed to the household at the start of the period. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income received by employed households. Unemployment benefits are financed with taxes on employed households.

The budget constraint of a household now becomes:

$$P_t C_t + \frac{b_{t+1}}{1 + i_t} = \nu_t W_t L_t + b_t + d_t + T_t.$$

The only change with respect to the benchmark model is the presence of the variables ν and T, which summarize the impact of the employment status on a household's budget. ν is an indicator variable that takes value 1 if the household is employed, and 0 if the household is unemployed. T represents a lump-sum transfer for unemployed households, and a tax for employed households. T is set such that the income of an unemployed household is equal to a fraction b of the income of an employed household.[50]

Moreover, here we assume that households cannot borrow:

$$b_{t+1} \geq 0,$$

and that trade in firms' shares is not possible, so that every household receives the same dividends d.

[50] More precisely, an unemployed household receives a transfer:

$$T = \frac{bW_t L_t + (b-1)d_t}{1 + \frac{bp}{1-p}},$$

while an employed household pays a tax

$$T = -\frac{p}{1-p} \frac{bW_t L_t + (b-1)d_t}{1 + \frac{bp}{1-p}}.$$

The Euler equation is now:

$$c_t^{-\sigma} = \frac{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]}{g_{t+1}^{\sigma-1}\bar{\pi}^w} + \mu_t,$$

where μ_t is the Lagrange multiplier on the borrowing constraint and, as in the main text, $c_t \equiv C_t/A_t$.

We start by showing that the borrowing constraint binds only for unemployed households. Since neither households nor firms can borrow, in equilibrium every period every household consumes her entire income. Denoting, by c_t^e and c_t^{ne} the consumption of respectively an employed and an unemployed household, we have $c_t^{ne} = bc_t^e < c_t^e$. Moreover, due to the assumption of i.i.d. idiosyncratic shocks, $E_t\left[c_{t+1}^{-\sigma}\right]$ is independent of the employment status. Hence, from the Euler equation it follows that $\mu_t > 0$ only for the unemployed, and so the borrowing constraint does not bind for employed households.

The Euler equation of the employed households is:

$$(c_t^e)^\sigma = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[\left(c_{t+1}^e\right)^{-\sigma}\right]},$$

where $\rho \equiv 1-p+p/b^\sigma > 1$, and we have used the fact that the probability of becoming unemployed is independent of aggregate shocks. Moreover, using $c_t = pc_t^{ne} + (1-p)c_t^e = c_t^e(bp+1-p)$, we obtain:

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]}.$$

This equation, which is the equivalent of equation (17) in the baseline model, determines the demand for consumption in the model with idiosyncratic unemployment risk.

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