# The Credit Scoring Channel in the Subprime Conduit Mortgage Market *

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**Abstract**

The Great Recession was preceded by an unprecedented expansion of subprime mortgage lending. We provide a theory that rationalizes how new credit scoring models - like the ones that became available in the early 1990s to subprime lenders - make remote subprime conduit lenders become a desirable option for consumers - the *credit scoring channel*. Also, we show that when the level of securitization increases, equilibrium subprime lending expands but conduit lenders find optimal to rely less on credit-relevant soft information. This can bring the subprime conduit mortgage market to the edge of the collapse region. House prices, mortgage rates, loan amounts, consumers' tenure choice, and the structure of the subprime mortgage market are all endogenous in our model.

**Key words**: subprime lending; credit scoring technology; portfolio lenders; conduit lenders; general equilibrium; endogenous mortgage market segmentation.

**JEL Classification numbers**: R21, R3, R52, D4, D5, D53.

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# 1 Introduction

One of the developments preceding the Great Recession was the initial segmentation of the subprime mortgage market - traditional portfolio lenders versus remote conduit lenders - and the huge expansion of subprime conduit lending and subsequent collapse. We build a theory based on informational and liquidity differences that can generate different subprime mortgage market structures, similar to the ones observed in the decades of 1990s and 2000s. We then use this framework to illustrate the *credit scoring channel*: access to new hard credit scoring technology in the early 1990s can explain the emergence of the subprime conduit mortgage market. However, this expansion of the subprime mortgage market also has a negative aspect, as reported by Rajan, Seru and Vig (2015): conduit lenders only rely on this hard information to originate mortgages, ignoring other credit-relevant soft information, and this lack of perfect screening generates a substantial amount of mortgages that end up defaulting. Moreover, as the level of securitization increases, lenders have an incentive to originate loans that rate high based on characteristics that are reported to investors, even if other unreported variables imply a lower borrower quality. Our model rationalizes these important facts. In addition, we also show how other factors, such as security investors' liquidity, affect the boom-bust of subprime mortgage lending and the housing market.

Our theory relies on a general equilibrium model that incorporates the following important elements that are characteristic of a subprime economy: (1) limited recourse mortgages, (2) asymmetric borrower credit quality information (high v. low default risk borrowers), and (3) two funding sources for consumers, portfolio lenders (originate-to-hold business model) and conduit lenders (originate-to-distribute business model). The loan amounts, the mortgage rates, the house prices, and the household's tenure choice (owning versus renting) are all endogenous determined in equilibrium. In addition, because consumers can choose between portfolio loans and conduit loans, the subprime mortgage market structure is also endogenous in our model.

Explicitly recognizing the possibility of two different funding sources for consumers is important to understand the subprime mortgage market structure. In our baseline model *portfolio lenders* have access to soft information and this allows them to perfectly screen between borrower types; however, their business model is such that they keep all originated mortgages in their portfolios. *Conduit lenders* distribute a fraction of their originated mortgages and thus have access to security investors' liquidity; however, conduit lenders lack soft information, so they rely on an imperfect hard credit scoring technology. These differences imply that mortgage rates and loan amounts are different between the two types of lenders. While portfolio lenders incorporate soft information into the determination of a (borrower specific) risk-based subprime loan rate, conduit lenders recognize that their borrower-lending clientele is lower credit quality on average. Thus, the conduit mortgage rate contains an adverse selection component on the part of borrowers (also referred here as "borrower adverse selection"), captured by the lack of soft information, but also a liquidity component coming from the conduit lender's access to the securitized investment market. These two components move the conduit loan rate in opposite directions. On the one hand, securitization allows customization (conduit loans are priced using the investors' time discount rate), which lowers the cost of capital in the conduit loan market. On the other hand, borrower adverse selection increases the cost of capital in the conduit loan market.

We show that an equilibrium with an endogenous subprime mortgage market structure exists for our economy, and derive mortgage pricing implications that reflect the trade-off between borrower adverse selection and security investors' liquidity. Because in our model the bad type consumers always default, we can focus on a pooling equilibrium (alternative specifications on minimum house size can also rule out a separating equilibrium). After this initial descriptive analysis, we consider a simple version of our model with linear utility functions for consumers to illustrate how changes in the credit scoring technology and other parameters of our economy can modify the equilibrium subprime mortgage market structure. Our first result establishes a relationship between the intensity of the borrower adverse selection problem and the subprime mortgage market structure. We identify three regimes. Regime 1 has only portfolio lenders active and borrower adverse selection is high; Regime 2 has both portfolio lenders and conduit lenders active and adverse selection is intermediate; and Regime 3 has conduit lenders dominating the subprime mortgage market and adverse selection is low.

In our model the intensity of the borrower adverse selection problem is analogous to the conduit lender's belief on the proportion of good type borrowers in its pool of mortgages. Using Bayes' rule we write this belief as a function of the hard credit scoring technology (CST) and the proportion of good type consumers that attempt to borrow from conduit lenders. The latter is an endogenous object that captures the aggregate loan market choice of good type consumers, which must be consistent with the conduit lender's belief and the CST parameter (we show that this consistency property holds in equilibrium). A change in CST is thought as a technology shock and can bring the economy to a different equilibrium mortgage market structure regime. In particular, we show that sufficient improvements in the CST can move the economy from Regime 1 to Regime 2 and then to Regime 3, i.e., the conduit loan market emerges and then becomes dominant. We refer to this sequence as *the credit scoring channel*. In addition, we show how a higher security investors' liquidity can fuel the boom of the subprime conduit market.

Our model also examines the role of other parameters in bringing the economy from Regime 3 to Regimes 2 or 1 (the bust). In particular, we exploit an increase in the mortgage distribution rate and a worsening in the fundamental proportion of good type subprime consumers (e.g., unemployment increases) to generate this transition. For this result we extend our baseline model to one with endogenous soft information acquisition and consider two steps. In the first step we demonstrate that when the distribution of mortgages to investors increases, conduit lenders have less incentives to acquire soft information. This worsens the quality of the pool of conduit loans. Yet conduit loans are still the preferred option for good type consumers. The second step just requires any change that further worsens the quality of this pool of mortgages and makes the conduit loan market the least preferred choice for subprime consumers. For example, a negative shock to the fundamental proportion of good type consumers in the economy has this effect. Depending on the severity of the shock, the economy can go back to either Region 2 or Region 1.

**The credit scoring channel**

The aforementioned framework rationalizes the following changes in the subprime mortgage market: the emergence of the subprime conduit market in early mid-1990s and its subsequent dominance over the traditional relationship lending model in the early 2000s.

Regime 1 with a high borrower adverse selection is similar to the period before early mid-1990s where the hard credit scoring technology was crude or even non-existent. In this equilibrium regime, traditional relationship lenders - also referred to as "portfolio lenders" -, whose business model is to "originate-to-own", are the only ones that operate in the subprime mortgage market. These lenders can acquire at no cost soft credit risk information, and this allows them to screen between subprime borrowers of different default risk type. However, portfolio lenders are capacity constrained, and this leaves many potential high quality subprime borrowers without a mortgage - these leftovers preferred to rent than borrowing at a prohibitive high mortgage rate from other potential lenders who only relied on poor hard credit information.

When the hard credit information improves in the early mid 1990s, conduit lenders, who only rely on hard information and whose business model is primarily "originate-to-distribute", are able to attract low risk consumers (good type) by offering them a better mortgage rate than before, but still at worse terms than portfolio lenders. This is similar to a transition from Region 1 to Region 2 in our model, where the distinct feature is the adoption of a new and better hard CST.[1] The equilibrium market structure in Regime 2 has both portfolio lenders and conduit lenders actively lending to different pools of borrowers at different mortgage rates.

Afterwards, in the early 2000s, the conduit lender's "originate-to-distribute" business model became predominant: all higher quality borrowers preferred to "migrate" to the subprime conduit mortgage market leaving traditional portfolio lenders with a small market share of leftovers. In our model, this is equivalent to a transition from Regime 2 to Regime 3, and can be rationalized by a wider usage of the new hard credit scoring technology and a higher liquidity from the secondary securities market, as mentioned above. In Regime 3, there is a lot of credit in the subprime economy because conduit lenders, who become the preferred choice for good type consumers, can accommodate any "number" of borrowers as long as the hard credit scoring technology identifies them as good borrowers. This boom of subprime credit is accompanied in our model by a jump in house prices and subprime home ownership rates.

**Further insights: Housing affordability and the collapse of subprime lending**
The bust of the subprime mortgage market in early 2007 is equivalent to a transition from Regime 3 to Regime 1 in our model. In this environment we can show that conduit mortgages become an expensive and less attractive option for good type consumers compared to renting - the subsidy paid by the higher quality borrowers to support a pooling loan rate becomes so high that discourages home ownership. When high credit quality consumers run away from the conduit loan market, conduit lenders' pool of borrowers is only composed of risky bad type consumers and the market collapses. Importantly, the drop in available subprime credit makes the equilibrium house price plummet.

Finally, our theory is also relevant to understand home affordability problems and their connections with real estate finance and housing policies. In particular, we establish a relationship between the subprime mortgage market and the urban configuration. Minimum

[1] FICO scores and consumer's credit history became available.

house size policies prevent subprime borrowers with small loans from buying houses with lot size above a minimum threshold. This lower bound on house size rules out a mortgage market for high risk (bad type) consumers, and forces subprime consumers without a mortgage to go to the rental market. This result illustrates how housing regulations prevent the least well-endowed subprime consumers who cannot afford from purchasing a house with a minimum lot size. Thus, the structural details underlying mortgage contract design and market organization consequently feed back to affect the rent versus own decision in our model.

**Relationship with the literature**

The subprime crisis that started in 2007 and its aftermath has been coined as the Great Recession. Much of its discussion has focused on the problems around the secondary mortgage market, see e.g. Gennaioli, Shleifer, and Vishny (2012) and Gorton and Ordonez (2014). This paper focuses instead on the changes that occurred in the supply side of the primary mortgage market (underwriting standards and growth of mortgage securitization), as pointed out by Mian and Sufi (2009, 2015) and Rajan, Seru and Vig (2015), to understand the role of the hard credit scoring technology and securities market liquidity in the boom-bust episode of the subprime mortgage market.

The literature on collateralized lending with asymmetric information is vast and has captured attention in recent years in light of the subprime mortgage lending and financial crisis. For recent work that focuses on how different lenders' information sets affect mortgage loan outcomes, borrowers' default, and market unraveling, see, e.g., Karlan and Zinman (2009), Adams et al. (2009), Rajan, Seru and Vig (2010), and Einav et al. (2013). See Miller (2015) for a related analysis of the importance of information provision to subprime lender screening. More generally, see Stein (2002) for a description of how private information includes soft information, and how difficult is to communicate soft information to other agents at a distance. In brief, and at a high level, this paper contributes to this literature by providing a general equilibrium model that shows how credit scoring and mortgage securitization possibilities affect subprime mortgage originations, securitization, and house prices.

Our equilibrium analysis of the subprime mortgage market also contributes to the recent empirical literature that attempts to identify the pricing determinants of differences between portfolio loans and conduit loans (see Keys, Mukherjee, Seru, and Vig (2010) and Krainer and Laderman (2014) and references therein). Agarwal, Amromin, Ben-David, Chomsisengphet and Evano (2011) recognized the lack of a theoretical model. To this extent, our paper provides a framework that enables to decompose the conduit mortgage spread into a credit information component, a foreclosure recovery rate component, and a component that captures the access to liquidity in the securitized investment market. We then show how these different pricing components relate to the rise and fall of the subprime conduit mortgage market.

Our paper is also related to the literature of shadow banking and subprime lending - see Ashcraft and Schuermann (2008) for overview of the subprime mortgage securitization process, and Geanakoplos (2010) on how to manage the leverage cycle. As in Gennaioli, Shleifer and Vishny (2012), our model can also illustrate that investors' wealth drives up securitization, but in addition our model is able to generate the result that adverse selection in the loan origination market can be the only reason why the conduit loan market shuts down, even when there is investors' appetite for mortgage-backed securities. This provides a different angle to the role of adverse selection on the rise and fall of subprime mortgage lending, which so far has focused on adverse selection in the secondary mortgage market - see e.g., Gorton and Ordonez (2014) leading paper. Our paper also departs from Mayer, Piskorski and Tchistyi (2013), Makarov and Plantin (2013), and Piskorski and Tchistyi (2011) by distinguishing between shadow bank and formal bank funding models, and relating their change in market share to different equilibrium subprime mortgage configuration regimes that result from changes in the credit scoring technology and securitization. Importantly, house prices in our model are endogenous.[2]

Our equilibrium mechanism links subprime mortgage lending standards to the run-up and eventually collapse in home-prices, and thus fills a gap in the literature that studies mortgage leverage and the foreclosure crisis - see e.g. Corbae and Quintin (2015) and Guler (2015) work on foreclosure dynamics with exogenous house prices. In our model house prices are endogenously determined by the intersection between demand and supply, and reflect parameters such as the credit scoring technology and the securities market liquidity. Consumers with a mortgage take these prices as given, and thus accommodate their borrowing to any change in house prices. Because housing supply is inelastic in our model, any debt growth leads inevitably to a high increase in house prices. Mian and Sufi (2011) provide evidence of this.

On a different front, Brueckner, Calem and Nakamura (2012) provide a model that links house price expectations of mortgage lenders and the extent of subprime lending. They show that expectations of a high house price growth increase the consumer's FICO score through current prices and spur subprime lending. In our paper the hard credit scoring model isolates the probability of a consumer being of good type from house prices. This is important because in our model bad type consumers only have a subsistence rent in the second period and the nature of the mortgage contract for subprime consumers is limited recourse, so bad type borrowers always choose to default, no matter what the house price level is. It is the predictive power of the hard credit scoring technology to differentiate between consumer's rents what determines the collapse region of the subprime conduit mortgage market in our model.

Our model captures the ebbs and flows of shadow bank activity, often peaking just prior to a downturn. The peak corresponds with poor access to soft information acquisition by conduit lenders and high liquidity flowing from security investors to conduit lenders (which is their largest if not exclusive source of funds).[3] This is consistent with Purnanandam's (2010) evidence that lack of screening incentives coupled with leverage-induced risk-taking behavior significantly contributed to the current subprime mortgage crisis. We also rationalized Dell' Ariccia, Igan and Laeven (2012) finding that when subprime mortgage

[2] As in any theory of competitive general equilibrium, prices are taken as given by the agents of our economy, who optimize subject to some constraints, and then prices emerge endogenously in equilibrium when supply equals demand. This is a well known difference from partial equilibrium models, where prices enter as parameters and do not reflect fundamentals.

[3] As Ashcraft, Adrian, Boesky and Pozsar (2012) point out, at the eve of the financial crisis, the volume of credit intermediated by the shadow banking system was close to $20 trillion, or nearly twice as large as the volume of credit intermediated by the traditional banking system at roughly $11 trillion.

securitization increases, lenders are more encouraged to make riskier loans (for this result, we extend our baseline model to accommodate for soft information acquisition). Our model also differs from Ordonez's (2014) theory that crisis appear when mortgage-backed security investors neglect systemic risks by focusing instead on the information problems that are specific to the primary conduit loan origination market.

Our model interprets the hard credit scoring as an information technology that assigns a good or bad rating to subprime consumers. This technology is imperfect and this imperfection is the source of borrower's adverse selection in the conduit mortgage market in our model - see Mian and Sufi (2015) for empirical evidence of borrower income misrepresentation as an important indicator of mortgage fraud. Our treatment of the credit scoring technology is different than Chatterjee, Corbae, and Rios-Rull (2011) and Guler (2015) in that they do not distinguish between hard information and soft information, nor between portfolio lender versus conduit lender, and also assume the same technology for all lenders. Importantly, the equilibrium structure of the subprime mortgage market is endogenous in our model. Finally, our result that an improvement in hard credit scoring technology leads to increases in the quantity of lending and also more lending to relatively opaque risky borrowers is similar to the effects of the small business credit scoring on commercial bank lending, as empirically documented by Berger, Frame and Miller (2005).

Finally, we highlight that this paper abstracts from any lender's adverse selection problems (asymmetric information between lenders and investors), and focus instead on borrower's adverse selection problems to examine the credit scoring channel in the subprime conduit mortgage market. Interestingly, even without any lender's adverse selection, we are able to generate a boom-bust episode similar to the recent subprime crisis. This is different from previous works that considered lender's adverse selection as the main reason of the expansion and collapse of lending (see e.g. Frankel and Jin (2015) and Gorton and Ordonez (2014)).

The rest of this paper is as follows. In Section 2 we present the baseline model. Section 3 gives the equilibrium definition, states the equilibrium existence result, and discusses the pricing implications on mortgage rates. Section 4 identifies different equilibrium regimes when the intensity of the conduit lender's adverse selection problem changes. Section 5 examines the role of the credit scoring technology channel in fostering the boom of the subprime conduit mortgage market. Section 6 provides some additional results that our model can generate. In particular, we analyze the impact that an expansion in the securitization industry has on the relaxation of lending standards, and how a negative shock to the fundamental proportion of good type consumers (e.g., drop in employment) can trigger the collapse of the conduit mortgage market in our model. We also add some remarks on the implications of lender's adverse selection for our model. Section 8 concludes.

# 2 Baseline model

Our baseline model consists of a two-periods (periods 1 and 2) deterministic economy with asymmetric information between borrowers and lenders. We focus on the market of subprime mortgages, leaving aside the market of prime mortgages. In the Supplementary Material we provide further details that characterize subprime consumers, subprime housing markets and subprime mortgage markets, and compare these to their prime counterparts. Our model has the agents: subprime households ($h$), subprime portfolio lenders ($pl$), subprime conduit lenders ($cl$), and security investors ($i$). We also refer to portfolio lenders and a conduit lenders as PLs and CLs, respectively, and use the terms household and consumer indistinctly.

## 2.1 Main assumptions

The general equilibrium model we are about to describe has the following main assumptions:

*Two types of subprime households:* In our economy all subprime households fall below some subsistence poverty line and have a subsistence income in period 1 equal to $\omega^{SR} > 0$ units of the numeraire good. We think of $\omega^{SR}$ as a government subsidy that is fungible and can be used to either rent a house or to fund a down payment on a owner-occupied house should the borrower qualify for a sub-prime mortgage. When the subprime consumer uses $\omega^{SR}$ to rent a house, this is equivalent as getting access to one of the housing affordable units provided by local governments. In the second period some of the subprime consumers experience a positive income shock (e.g., get a better job) $\omega^{+} > \omega^{SR}$, while the rest of the pool remains at their current (poverty) income level $\omega^{SR}$. Label the consumers that experience an increase in their second period endowment as a G-type (or good type) and those who don't as a B-type (or bad type). Consumers know their type in period 1, but G-type consumers are unable to verifiably convey their unrealized increase in income level to outside parties. This is an important aspect of our model with subprime consumers - as discussed below, the lenders' credit scoring technology that screens borrower types is coarse in absence of soft information, and, in general, considerably worse that the credit scoring technology in the prime lending market. The measures of types G and B households in the economy are $\lambda_G \equiv \lambda(A(G))$ and $\lambda_B \equiv \lambda(A(B))$, respectively. We refer to the ratio $\lambda_G/(\lambda_G + \lambda_B)$ as the *fundamental proportion* of G-type consumers in the economy.

*Limited recourse mortgages*: Recourse mortgages are specific to the subprime market in the US and Europe, except few special cases such as purchase money mortgages in California and 1-4 family residences in North Dakota. Most of these recourse mortgages are subject to some limited liability. This is especially true for subprime borrowers that have few resources (wealth). The intuition is that lenders cannot take everything and leave a consumer homeless when he defaults and becomes bankrupt. In fact, bankruptcy is designed to shield consumers from too much recourse on mortgage loans. With this motivation in mind, we consider a model with recourse mortgage contracts that are subject to some ungarnished minimum subsistence consumption ($\omega^{SR}$) by the borrower ("limited recourse"). This limited liability nature of the contract is similar to a mortgage exemption that protects the subprime borrower from consuming less than a subsistence rent (see Davila (2015) for an analysis of bankruptcy exemptions from a welfare point of view).[4] The implications for our model are as follows. Under this limited recourse contract, G-type consumers (with no default risk) can credibly commit to pay back the loan even

[4]See Poblete-Cazenave and Torres-Martinez's (2013) for a general equilibrium model with limited-recourse collateralized loans and securitization of debts, where equilibrium is shown to exist for any continuous garnishment rule and multiple types of reimbursement mechanisms.

if the loan repayment is higher than the house value, but a B-type consumer cannot. Adverse selection in the primary subprime mortgage market is a result of this limited recourse nature of the mortgage contract. Also, this type of contract implies that bad type borrowers, who by assumption are only endowed with a subsistence rent at time of mortgage repayment, end up defaulting and giving their housing asset to the lenders. Hence, the limited recourse mortgage is effectively a non-recourse mortgage for the B-type borrowers. In the Supplementary Material we elaborate on the details of limited recourse mortgage contracts, their implications for adverse selection, and also explain the differences if we were to consider non-recourse mortgages instead - *a la* Geanakoplos and Zame (2014) - where adverse selection would be absent in our baseline model.

*Two funding sources for consumers*: Portfolio lenders (PLs) originate mortgages to be held in the entity's asset portfolio ("originate-for-ownership"). In contrast, conduit lenders (CL) are transactional, specializing only in originating mortgages for sale to a third party ("originate-for-distribution"). This access to secondary mortgage markets can possibly reduce the cost of capital when secondary subprime mortgage markets are liquid and competitive. Another difference is that PLs and CLs have different credit scoring technologies. CL generally work out of a small office with computers, with no established presence in a community. In the baseline model below we assume that CLs have access to hard credit information (e.g., credit history and FICO scores), which is always accurate, but it does not necessarily lead to a perfect assessment of consumer type. PLs have soft information as a supplement to the available hard credit information, and by assumption this is enough to fully reveal the borrower's type (PLs know their borrowers and their communities and borrowers maintain checking and other personal accounts with them).[5] As such CLs are not capable of resolving asymmetric information over and above what is available with hard information and their credit scoring technology. The lack of soft information by CLs introduces asymmetric information in the primary CL mortgage market.[6] Later, in Section 6, we will depart from the baseline model and allow lenders to choose their optimal amount of soft information and show that the assumed differences in soft information acquisition between lender types in the baseline model do not speak against optimality. Exhibit 1 below illustrates the flow of borrowing with these two potential funding sources for consumers.

[5] Soft information may include listening to and analyzing the borrower's explanation for past difficulties in making credit payments and determining whether the hard numbers for the borrower or property make sense given what a loan agent can perceive about them. For a discussion of how securitization discourages lenders from engaging in "soft" mortgage underwriting, see "Comments to the Federal Deposit Insurance Corporation" by the National Association of Consumer Advocates on February 22, 2010.

[6] Later in the paper (see Section 6) we will allow lenders to choose their optimal amount of soft information and show that the assumed differences in soft information acquisition between lender types do not speak against optimality.

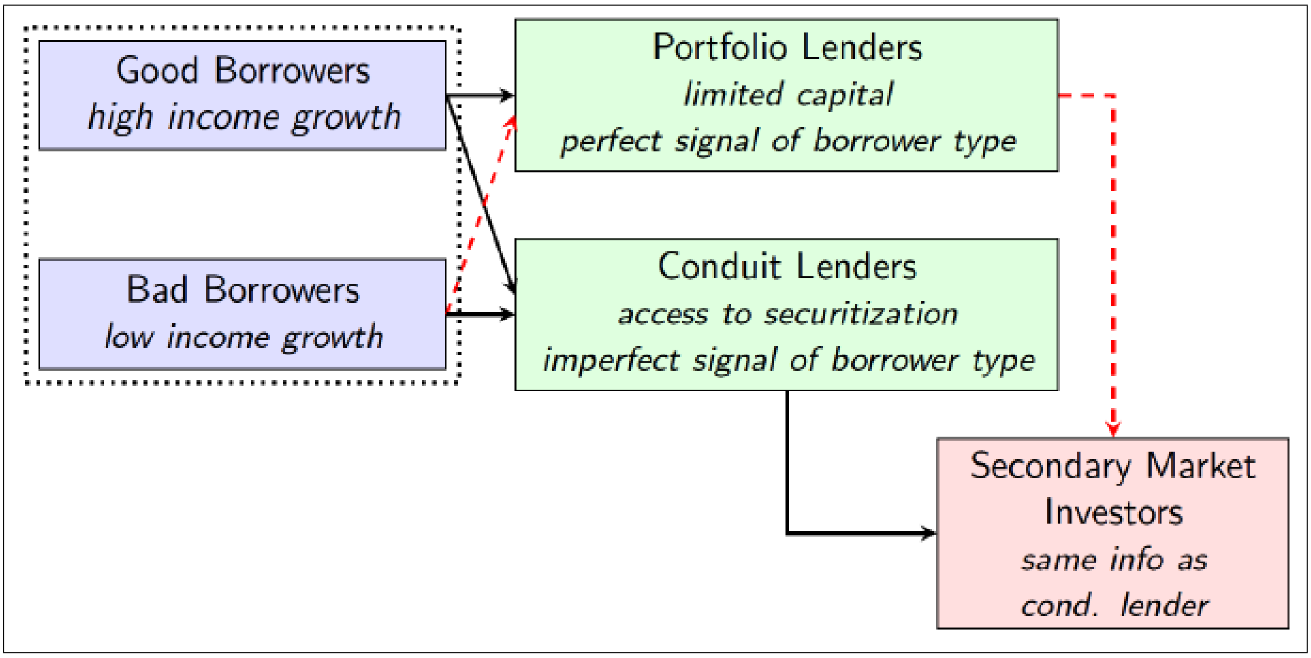


Exhibit 1. The flow of borrowing

*Capacity constrained portfolio lenders*: Another assumption is that PLs are capacity constrained, i.e., they cannot lend to more than $v(PL)$ consumers. In particular, we assume $\lambda_G > v(PL)$ (portfolio lenders can only lend to some but not all good type consumers). This assumption is motivated by additional constrains faced by portfolio lenders - in the context of the 1980s and 1990s-, such as the time constraint to originate loans that require face-to-face contact between borrower and lender (one important source of soft information). In addition, other considerations may also apply here, such as the inability of portfolio lenders in the short run to raise equity to finance new mortgages. The assumption of capacity constrained PLs then implies that when portfolio loans are the first choice among consumers, the rest of good type consumers who did not get a portfolio loan have no other option but to go to the conduit loan market in order to get a mortgage. Also, bad type consumers, who are identified as such by the portfolio lender's additional soft credit information, only can get a mortgage if misrepresenting their type in the conduit mortgage market. Exhibit 2 summarizes the main distinctions between traditional portfolio lenders and conduit lenders, which can be seen as representative of the subprime mortgage market in the 1980s and early 1990s.[7]

| | Soft information | Originate-to-distribute | Capacity constrained |
|---|---|---|---|
| (traditional) PL | YES | NO | YES |
| CL | NO | YES | NO |

Exhibit 2

[7]The pre-1990s US depository model was of thousands of small portfolio lenders that generally operated over very narrow geographic areas. Thrifts were a particular type of depository that were designed to make residential mortgage loans – the subject of this paper. The banking crisis in the 1980s, coupled with the the relaxation of many banking laws involving geographic- and product-market expansion, led to fewer and larger depositories. This consolidation and expansion was further propelled by the IT revolution.

*Adverse selection in the primary conduit mortgage market*: CLs cannot perfectly screen the type of borrowers using hard information only; only with additional soft information lenders can identify the type of subprime borrower.[8] In our baseline model investors rely on the same credit scoring technology than CLs (both without soft information), thus leaving aside the possibility of adverse selection in the secondary market of mortgage backed securities. Later, in Section 6, we examine the implications of dropping this assumption.

*Inelastic owner-occupied housing supply*: The owner-occupied housing consumption space is $[0, \bar{H}]$ where $\bar{H}$ denotes the aggregate amount of owner-occupied housing in the economy. For simplicity, we take the aggregate supply of owner-occupied housing in the first period and the aggregate demand of owner-occupied housing in the second period as exogenously given and equal to $\bar{H} = 1$. Later, in Section 4, we will explain how the baseline model naturally extends to an overlapping generations economy under specific assumptions on the consumer's utility function.

Before presenting the formal model, we introduce some useful notation. We write $l$ to denote a lender independently of his type (PL or CL), and denote an agent type by $a = h, pl, cl, i$ with respective sets $A(\mathcal{H})$, $A(PL)$, $A(CL)$, and $A(I)$. We define $\mathbf{A} = A(\mathcal{H}) \cup A(PL) \cup A(CL) \cup A(I)$, and represent the non-atomic measure space of agents in this economy by $(\mathbf{A}, \mathcal{A}, \lambda)$, where $\mathcal{A}$ is a $\sigma$-algebra of subsets of the set of agents in $\mathbf{A}$, and $\lambda$ is the associated Lebesgue measure. For simplicity, we set $\lambda(A(PL)) = \lambda(A(CL)) = \lambda(A(I)) = 1$.

## 2.2 Subprime households

Consumption can take two forms: owner-occupied housing ($H_\tau$) and rental housing ($R_\tau$), where $\tau = 1, 2$ denotes the corresponding time period. In period 1 a household can buy a house of size $H_1$ at price $p_1$ per house size unit or rent a house of size $R_1$ at per unit price 1 (numeraire good). House purchases are long term contracts that once signed the house can be "consumed" in both periods (if the consumer buys a house in period 1, the same house enters in period 2 budget constraint as an asset endowment evaluated at market price $p_2$). On the other hand, buying good $R$ can be seen as a one-period contract: this good can only be consumed for one period.

The rental housing market for subprime consumers usually requires of a government voucher. We assume that these rental housing units (e.g., shelter) are provided by the government in exchange of a voucher. To streamline our analysis, we assume that the voucher is fungible and can also be used to fund a down payment on a owner-occupied house should the borrower qualify for a sub-prime mortgage. This is similar to a situation where the government is subsidizing the home equity part of a mortgage. Once the second period starts, households expect to die at the end of the period. Thus, we refer to households in period 2 as old households, and households in period 1 as young households. When households

[8] Chatterjee, Corbae, and Rios-Rull (2011) allow consumers to borrow multiple times to study the role of reputation acquisition where the individual's type score is updated every period according to some exogenous rule. These are characteristics of prime borrowers who build some credit reputation over time by borrowing in multiple occasions. In our paper we study subprime consumers whose access to credit is rather limited and in general can borrow only once. Thus, there is no reputation acquisition in our model, nor a need to update the individual's type score.

are old, they choose to consume owner-occupied housing $H_2$ or the numeraire good $R_2$ or a combinations of the two. Household $h$'s preferences are represented by an utility function $u^h(R_1, H_1, R_2, H_2)$ that is continuous, strictly increasing and strictly quasiconcave.

In period 1 (impatient) households can increase their consumption by borrowing from either a *portfolio lender* ($PL$) or a *conduit lender* ($CL$). Both types of lenders originate mortgages in a competitive environment, although they differ in the terms of their contracts. The matching between consumers and lenders is endogenous in our model and will be addressed later. Consider a consumer that borrows from primary mortgage market $l = PL, CL, \emptyset$ (we write $l = \emptyset$ if the consumer does not borrow from a PL or a CL). Denote the consumer's loan amount in the subprime mortgage market $l$ by $q^l\psi^l \geq 0$, where $q^l$ and $\psi^l \geq 0$ denote the $l$-type mortgage discount price and loan repayment due at the beginning of the second period (when $l = \emptyset$, we write $\psi^{\emptyset} = 0$). For example, if the discount price is $q^l = 0.8$ and the promise is $\psi^l = \$100$, the loan amount of this consumer is \$80. For simplicity, we normalize the loan interest rate to 0.

Equilibrium existence requires an upper bound $B > 0$ on $\psi^l$, but this bound can be chosen arbitrarily (in our characterization of equilibrium below we will choose this bound such that this short sale constraint is non-binding):

$$\psi^l \leq B \tag{1}$$

The period 1 budget constraint of a consumer with access to primary mortgage market $l$ is:

$$p_1 H_1 + R_1 \leq q^l \psi^l + \omega^{SR} \tag{2}$$

Observe that the consumer's mortgage down payment is endogenous in our model; for example, if $R_1 = 0$, then the maximum down payment is equal to $\omega^{SR}/p_1 H_1$.

Sub-prime loans are subject to a *limited recourse mortgage* contract that stipulates that a borrower is allowed to consume his subsistence income $\omega^{SR}$ if default occurs. Accordingly, we write the second period budget constraint as follows:

$$p_2 H_2 + R_2 \leq \max\{\omega^{SR}, \omega_2^t + p_2 H_1 - \psi^l\} \tag{3}$$

where $\omega_2^t$ denotes the period 2 endowment of a consumer of type $t = G, B$ and is such that $\omega_2^G = \omega^+$ and $\omega_2^B = \omega^{SR}$. The term $p_2 H_1$ in the right hand side of the inequality (3) captures the value of the house purchased in the previous period and is interpreted as a sale at market price $p_2$ per house size unit. The consumer can then use the proceeds of this sale for consumption after repaying his mortgage.[9] The maximum operator in (3) allows the household to strategically default and consume at least the minimum subsistence income $\omega^{SR}$.[10] There is no default if $p_2$, $H_1$, and $\psi^l$ are such that $\omega^{SR} \leq \omega_2^t + p_2 H_1 - \psi^l$. Loan payment is (partially) enforced by the nature of the limited recourse loan in our model.

[9] A consumer with an owner-occupied house at the beginning of period 2 decides whether to sell it at market price, or to consume it. The latter is equivalent to the joint transactions of selling the house the consumer owns at the beginning of period 2 and then buying immediately after a house with same size.

[10] Strategic default is simply an optimality condition in which the borrower, subject to the relevant recourse requirements, decides whether mortgage loan payoff to retain ownership of the house or default with house forfeiture generates greater utility. See Davila (2015) for an exhaustive analysis of exemptions in recourse mortgages.

Denote the household $h$'s consumption bundle by $x^h = (H_1^h, R_1^h, H_2^h, R_2^h) \in \mathbb{R}_+^4$. We say that the pair $(x^h, \psi^h)$ is feasible if it satisfies constraints (1), (2) and (3). The households' optimization problem is as follows: each household maximizes his utility function subject to constraints (1), (2) and (3).

## 2.3 Lenders

We require that in order to receive a mortgage, the consumer must be identified as a G-type, i.e., rating=G.[11] Denote by $CST_G^l$ the lender $l$'s credit scoring technology. It is formally defined as the probability that a lender of type $l$ assigns a good rating to a G-type type borrower, i.e., $CST_G^l \equiv \Pr^l(\text{rating=G}|\text{G})$. Similarly, we write $CST_B^l \equiv \Pr^l(\text{rating=G}|\text{B})$. For simplicity, we assume $\Pr^l(\text{rating=G}|\text{G}) = \Pr^l(\text{rating=B}|\text{B})$ and $\Pr^l(\text{rating=G}|\text{B}) = \Pr^l(\text{rating=B}|\text{G})$, which imply $CST_G^l = 1 - CST_B^l$. By assumption, CLs only rely on hard information and thus $CST_G^{CL} < 1$. Portfolio lenders have access to soft information on top of the hard credit information, and thus, by assumption, always assign a good signal to G-type consumers, i.e., $CST_G^{PL} = 1$. The measure of consumers that receive a loan from CLs is equal to

$$\mu^{CL}(\text{rating=G}) = CST_G^l \cdot \mu_G^{CL} + CST_B^l \cdot \mu_B^{CL}$$

where $\mu_G^{CL}$ and $\mu_B^{CL}$ denote the measure of G-type and B-type consumers that attempt to borrow from CL. $\mu_G^{CL}$ and $\mu_B^{CL}$ are endogenous in our model, and so is $\mu^{CL}(\text{rating=G})$.

We use Bayes' rule to write the expected probability of lending to a G-type consumer, given that the lender $l$'s CST assigns to that consumer a good rating, as follows:

$$\Pr{}^l(\text{G}|\text{rating=G}) = \frac{CST_G^l \cdot \hat{\pi}_G^l}{CST_G^l \cdot \hat{\pi}_G^l + CST_B^l \cdot \hat{\pi}_B^l} \tag{4}$$

where $\hat{\pi}_G^l$ denotes the proportion of G-type consumers among all consumers that attempt to borrow from lender $l$. For example, if a CL's pool contains 60 B-type borrowers and 40 G-type borrowers, then $\hat{\pi}_G^{CL} = 0.4$. In general, if the PLs are the first choice, CLs are the second choice and a conduit mortgage is preferred to renting, then only a mass $\lambda_G - v(PL)$ of good type consumers attempt to borrow from CLs. And then, $\hat{\pi}_G^{CL} = (\lambda_G - v(PL))/(\lambda_G - v(PL) + \lambda_B)$. If, on the other hand, CLs are the first loan market choice for G-type consumers and a conduit mortgage is preferred to renting, then $\hat{\pi}_G^{CL} = (\lambda_G)/(\lambda_G + \lambda_B)$. Thus, in our model, $\hat{\pi}_G^{CL}$ will be an endogenous object determined by the consumer choices on which mortgage market they prefer to borrow from and the fundamental proportion of G-type consumers $\lambda_G/(\lambda_G + \lambda_B)$. As we will see below, this is a central element in our theory of emergence and collapse of the subprime conduit mortgage market.

To simplify our notation, we shall write the lender $l$'s *belief* on the proportion of G-type consumers in its pool of borrowers as

$$\pi^l \equiv \Pr{}^l(\text{G}|\text{rating=G}).$$

[11] As explained in Section 3, we can rule out a market for B-type consumers by appealing to minimum house size regulations, or to common practice where lender don't want to lend to a consumer that is known to default. Also, observe that the adverse selection problem in the mortgage market would not disappear if we were to allow for a market of "bad ratings", since B-type consumers would still prefer to misrepresent their type and borrow a large loan amount as G-type consumers do.

Then, by assumption, we can write $\pi^{PL} = 1$ and $\pi^{CL} < 1$.

Lenders are subject to an "originate-to-distribute" type constraint, which says that a lender $l$ cannot distribute more than a fraction $d^l$ of its mortgages originated;[12] in particular,

$$z^l \leq d^l \varphi^l \tag{5}$$

where $\varphi^l \geq 0$ denotes the total amount of mortgages originated by lender $l$, $z^l \geq 0$ is the amount of mortgages the lender sells to the investors, and $d^l$ is the fraction of mortgages that are originated for distribution.[13] $\varphi^l$ and $z^l$ are choice variables, and $d^l$ is a parameter that takes value 0 if the lender is a portfolio lender ($l = PL$), and $d^l \in (0, 1]$ if the lender is a conduit lender ($l = CL$). In practice, $d^{CL}$ is typically close to 1 for CLs. A distribution rate smaller than 1 can be the result of a regulation or a self-imposed constraint due to reputation concerns (not modelled here). We then say that a pair $(\varphi^l, z^l)$ is feasible if it satisfies (5).

Given the nature of the limited recourse mortgage contract, when there is borrower default, the lender garnishes all borrower's income above the subsistence rent $\omega^{SR}$. This includes repossessing the house and reselling it if the borrower happened to buy a house in the first period. However, the foreclosure process is costly for the lender: foreclosure cost and other indirect costs associated with foreclosure delays result in a loss $(1 - \delta)p_2 H_1$ to the lender, where $\delta \in [0, 1]$ denotes the foreclosure recovery rate.

Lenders are risk neutral with time discount factor $\theta^l > \theta^h$ and belief $\pi^l$ on the fraction of G-type borrowers in the pool.[14] In particular, we consider the following linear separable profit function for a lender $l$:

$$\Phi^l(\varphi^l, z^l) = (\omega_1^l - q^l \varphi^l + \tau z^l) + \theta^l (1 - d^l)(\pi^l \varphi^l + (1 - \pi^l)\delta p_2 H_1^G), \tag{6}$$

where $\tau$ denotes the sale price of a mortgage in the secondary market. The lender's first period endowment is positive, $\omega_1^l > 0$ (for simplicity, we assumed $\omega_2^l = 0$). Notice that only a fraction $1 - d^l$ of mortgages affect the lender's profit function in the second period because it distributes a fraction $d^l$ of the mortgage payment proceeds to investors. Also notice that the interaction between the originate-to-distribute constraint (5) and the profit function (6) determines the two possible loan origination models. On the one hand, CLs can distribute a fraction $d^{CL} > 0$ of the originated mortgages, but lack soft information (so $\pi^{CL} < 1$). PL, on the other hand, have soft information ($\pi^{PL} = 1$) but don't sell their mortgages to the investors ($d^{PL} = 0$).

The lenders optimization problem is as follows. Each lender $l$ chooses $\varphi^l$ and $z^l$ to maximize $\Phi^l(\varphi^l, z^l)$ subject to the originate-to-distribute constraint (5). We denote the lender $l$'s choice set by $\mathbf{C}^l \subseteq \mathbb{R}_+^2$, which is composed of all pairs $(\varphi^l, z^l)$ that are feasible.

[12] For previous work in general equilibrium that incorporates security possession, see Bottazzi, Luque, and Pascoa (2012) and Faias and Luque (2016).

[13] In our model, homogeneous loans are pooled and securitized into one asset - see Aksoy and Basso (2014) for a model with tranching. We also ignore agency issues regarding securitization and its implications on distressed loans.

[14] Risk-neutrality implies that the lender's first order condition determines the competitive mortgage price $q^l$, $l = PL, CL$ (see Section 3). The assumption of lender's risk neutrality is common in the literature. See e.g. Arslan, Guler, and Taskin (2015), Chatterjee, Corbae and Rios-Rull (2011), Guler (2015), and Fishman and Parker (2015).

Choices $\varphi^l$ and $z^l$ determine the lender $l$'s consumption vector $x^l \equiv (\omega_1^l - q^l\varphi^l + \tau z^l, (1-d^l)(\pi^l\varphi^l + (1-\pi^l)\delta p_2 H_1^G)) \in \mathbb{R}_+^2$.

## 2.4 Investors

Investors assign a smaller weight to period 1 consumption than lenders do, i.e., $\theta^l < \theta^i$. We also assume that both CLs and investors only rely on hard credit information $CST^{CL} = CST^i$ and their beliefs are such that $\pi^{CL} = \pi^i < 1$. This assumption is convenient as it allows us to focus on the adverse selection problem in the primary mortgage market, leaving aside potential information problems that may arise between conduit lenders and secondary mortgage investors (later, in Section 6 we will discuss the implications of dropping this assumption).

The investor $i$'s optimization problem consists of choosing $z^i$ to maximize the following profit function:

$$\Lambda^i(z^i) \equiv \omega_1^i - \tau z^i + \theta^i(\pi^i z^i + (1-\pi^i)d^l\delta p_2 H_1^G) \tag{7}$$

where $\omega_1^i > 0$ denotes the investor's endowment in period 1 (for simplicity, we assume $\omega_2^i = 0$). The term $\pi^i z^i + (1-\pi^i)d^l\delta p_2 H_1^G$ captures the investor's second period revenue from buying mortgages in the first period. The first term, $\pi^i z^i$, corresponds to the payment from the fraction $\pi^i$ of G-type borrowers. The second term, $(1-\pi^i)d^l\delta p_2 H_1^G$ , corresponds to the income from lending to a fraction $(1-\pi^i)$ of B-type borrowers. The term $d^l$ stands for the percentage of mortgages that lenders sell and hence investors are entitled to that revenue. Because B-type consumers are not able to honor the loan payment corresponding to a G-type loan contract, the investor only receives the depreciated value of the foreclosed house, $\delta p_2 H_1^G$, from these defaulted mortgages. Finally, define the investor $i$'s consumption bundle by $x^i \equiv (\omega_1^i - \tau z^i, \pi^i z^i + (1-\pi^i)d^l\delta p_2 H_1^G) \in \mathbb{R}_+^2$.

# 3 Equilibrium and Mortgage Pricing

In this section we propose an equilibrium notion of a competitive economy with endogenous segmented markets, assert that an equilibrium exists, and examine its pricing implications.

## 3.1 Equilibrium definition and the existence result

A consumer can choose among three possibilities: (1) borrow in the PL market , (2) borrow in the CL market, or (3) not borrow. We denote these possibilities by $m^{PL}$, $m^{CL}$, and $m^{\emptyset}$, respectively. Thus, the set of consumer's "market choice" possibilities is $\mathbf{M} = \{m^{PL}, m^{CL}, m^{\emptyset}\}$. The consumer's market choice is consumer-type ($t(h) = G, B$) and market-type ($l \in \mathbf{L} \equiv \{PL, CL, \emptyset\}$) specific, and is denoted by $m_{t(h)}^l \equiv (t(h), l)$. A *list* is a function $\iota : \mathbf{M} \to \{0,1\}$, where $\iota(t(h), l)$ denotes the number of market choices of type $(t(h), l)$. We assume that a consumer $h$ can only belong to one market in $\mathbf{M}$ (i.e., $\sum_{l=PL,CL,\emptyset} \iota(t(h), l) = 1$). We write $\mathbf{Lists} = \{\iota : \iota \text{ is a list}\}$, define the consumer's market choice function by $\mu : \mathbf{A} \to \mathbf{Lists}$, and denote the aggregate of type $(t(h), l)$-choices by $\hat{\mu}(t(h), l) \equiv \int_{A(t(h))} \mu^h(t(h), l)d\mu$. We find convenient to rewrite the consumer $h$'s utility function as a function of his consumption and market choice, e.g., $u^h(x^h, \mu^h(m))$. We assume that the utility mapping $(h, x, \mu) \to u^h(x, \mu)$ is a jointly measurable function of all its arguments. The consumer $h$'s choice set $\mathbf{X}^h \subset \mathbb{R}^5_+ \times \mathbf{Lists}$ consists of the feasible set of elements $(x^h, \psi^h, \mu^h)$ that this consumer can choose. The consumption set correspondence $h \to \mathbf{X}^h$ is a measurable correspondence.

Below we formally define our notion of equilibrium, which is similar to the standard concept of a competitive equilibrium in general equilibrium with the additional condition that lenders' beliefs $\pi \equiv (\pi^{PL}, \pi^{CL})$ must be consistent with the distribution of consumers into markets given by the market aggregate choice function $\hat{\mu}$ through function (4). In particular, notice that the consumer $h$'s market choice $\mu^h(m^l_{t(h)})$ is a function of the lender's belief $\pi^{CL}$, in turn a function of the proportion $\hat{\pi}^{CL}_G$ of G-type consumers that attempts to borrow from $l$-type lenders and the CST as dictated by function (4). Then, the aggregate consumers' market choice $(\hat{\mu})$ for the $l$-type loan market is also a function of $\hat{\pi}^{CL}$ and the CST. Let $f^l$ be a continuous function that brings $\hat{\mu}^l \equiv \hat{\mu}(m^l)$ into a proportion of G-type consumers in the $l$-type loan market. Also let a continuous function $g$ that brings $f(\hat{\mu}^l)$ and $CST^l_G$ into a real number in interval $[0, 1]$ as in (4). Then, given $CST^l_G$, we say that the aggregate market choice vector $\hat{\mu}^l \in \mathbb{R}^{\mathbf{M}}$ is *consistent* with $\pi^l$ if $\pi^l = g(f(\hat{\mu}^l(CST^l_G, \pi^l), CST^l_G)$. In addition, the market choice function $\mu$ must be such that the PL's capacity constraint holds, i.e.,

$$\int_{A(G)} \mu^h(G, PL) d\mu \leq v(PL) \tag{8}$$

**Definition 1:** *Given the triplet $(CST^{PL}, CST^{CL}, CST^i)$, an equilibrium for this economy consists of a vector of market choices $\mu$, prices $(p_1, p_2, q^{PL}, q^{CL}, \tau)$ and allocations $((x^h, \psi^h)_{h \in A(G) \cup A(B)}, (x^l, \varphi^l, z^l)_{l \in A(l), l=PL,CL}, (x^i, z^i)_{i \in A(I)})$ such that:*

*(1.1) Each consumer $h$ chooses $(x^h, \psi^h, \mu^h) \in \mathbf{X}^h$ that maximizes $u^h(x^h, \mu^h(m))$.*
*(1.2) Each lender $l$ chooses $(\varphi^l, z^l) \in \mathbf{C}^l$ that maximizes $\Phi^l(\varphi^l, z^l)$.*
*(1.3) Each investor $i$ chooses $z^i \in \mathbb{R}_+$ that maximizes $\Lambda^i(\varphi^i, z^i)$.*
*(1.4) $\hat{\mu}$ is consistent with $\pi$.*
*(1.5) Market clearing:*
(MC.1) $\int_{A(\mathcal{H})} \psi^{h,PL} \mu^h(t(h), PL) dh = \int_{A(PL)} \varphi^{PL} dr$
(MC.2) $\int_{A(\mathcal{H})} \psi^{h,CL} \mu^h(t(h), CL) dh = \int_{A(CL)} \varphi^{CL} dk$
(MC.3) $\int_{A(CL)} z^l dl = \int_{A(I)} z^i di$
(MC.4) $\sum_{l \in \mathbf{L}} \int_{A(\mathcal{H})} R^h_\tau \mu^h(t(h), l) dh + \int_{A(CL) \cup A(PL)} x^l_\tau dl + \int_{A(I)} x^i_\tau di = \int_{\mathbf{A}} \omega^a_\tau da,\ \tau = 1, 2$
(MC.5) $\sum_{l \in \mathbf{L}} \int_{A(\mathcal{H})} H^h_1 \mu^h(t(h), l) dh = \sum_{l \in \{\mathbf{L}} \int_{A(\mathcal{H})} H^h_2 \mu^h(t(h), l) dh = \bar{H}$

**Theorem 1 (Existence):** *There exists an equilibrium as defined in Definition 1.*

Proving existence of equilibrium for our subprime economy is not straightforward. The size of the portfolio and conduit mortgage markets is endogenous - it depends from the consumers' preferred mortgage market choices. In addition, there are non-convexities in our model, e.g., the maximum operator in the consumer's second period budget constraint, which captures the limited recourse nature of most subprime mortgage contracts in the US, and the consumers' discrete choice of mortgage market. Our large economy allows us to deal with these subtleties. Our approach is as follows. We construct a generalized game and show that there is a mixed strategies equilibrium. Then claim that because the auctioneers' payoff functions depend on a profile of mixed strategies only through finitely many indicators, there is a degenerate equilibrium profile of the generalized game. And finally, we show that the equilibrium of the generalized game is in fact an equilibrium in the sense of Definition 1. We leave the details of the existence proof for the Appendix A.1.

**Remarks about the notion of equilibrium:**

1. In our model adverse selection is present only in the conduit mortgage market and we use the concept of pooling equilibrium. There are two reasons why we can rule out a separating equilibrium for this market without compromising the robustness of equilibrium.[15] First, it is common sense that CLs do not want to write a mortgage contract that has high default probability (default probability is 1 in our model with limited recourse mortgages since the B-type borrower's income in the second period equals his subsistence rent). So we can rule out this type of mortgage contracts as a matter of common practice. Second, B-type consumers with a small conduit loan are not able to buy a house if there is a minimum house size larger than

$$H_{CL}^{\min} \equiv \frac{\bar{H}(\omega^{SR} + \bar{L})}{2\omega^{SR} + \bar{L} + L^G} \tag{9}$$

where $\bar{L} = \theta^i \delta \omega^{SR}/(1 - \theta^i \delta)$ is the maximum loan amount that a CL would give to a B-type consumer being compatible with non-negative profits for the lender, and $L^G = \bar{\theta}(\omega^{SR} + \bar{L})/(1 - \bar{\theta})$ is the loan amount that a G-type consumer would obtain from a CL when mortgage markets are segmented (using the CL's first order condition and G-type consumer's first period budget constraint). Minimum house size regulations have applied in the U.S. and other countries for decades, and it is well known that consumption standards such as minimum lot sizes can exclude low-income groups if they are set too high - see e.g. Malpezzi and Green (1996).[16] These two reasons together (common practice in mortgage lending and minimum house size regulation) make it clear that separation will not happen, and therefore we can ignore it without compromising the robustness of the model.

2. In our model default risk is the result of the CL's inability to perfectly screen between borrower types, and thus it can be attributed to the endogenous behavior of consumers with whom they are matched in equilibrium. We treat this risk as idiosyncratic in the sense that the matching between lenders and consumers is independent and uniform, and the law of large numbers applies.

3. Our notion of equilibrium assumes that lenders and investors form beliefs about the composition of the lenders' pool of borrowers. These beliefs are common, degenerate and governed by the lender's CST. Lenders and investors take their beliefs as given and optimize without taking into account the consumers' choice of mortgage market.[17] Then,

[15] A separating equilibrium can exist even when B-types always default in the second period because the CL would gets income $\delta p_2 H_1$ by foreclosing the house. This is profitable for the CL when the loan amount is small because by foreclosing the house the CL is able to seize the B-type borrower's house equity ($\omega^{SR}$). Formally, the CL gets $\delta p_2 H_1$ in $t = 2$ and gives $q^B \varphi^B$ to the borrower in $t = 1$.

[16] See also NAHB Research Center (2007) and the Wharton Housing Regulation Index for measures of housing regulation and Duraton, Henderson and Strange (2015).

[17] This is similar to general equilibrium models of firm formation where agents optimize without taking the supply of jobs into account.

equilibrium condition (1.4) requires that lenders' beliefs are consistent with the distribution of consumers into mortgage markets.

4. Given the PL's capacity constraint and the CL's imperfect CST, consumers of the same type may end up with different loan amounts, and thus different realized housing consumption and ex-post utility (e.g., there will be an equilibrium configuration where some G-type consumers are lucky and obtain a portfolio loan, some G-type consumers obtain a conduit loan, and the remaining G-type consumers cannot borrow and must rent). Our approach to equilibrium existence does not speak against this possibility.

Next, we derive asset pricing conditions that any equilibrium in this economy must satisfy using the lender and investor's optimality conditions.

## 3.2 Mortgage Discount Prices

Using the lender and investor's first order conditions we obtain the following discount price for conduit loans:

$$q^{CL} = \frac{\bar{\pi}\bar{\theta}}{1-\delta(1-\bar{\pi})\bar{\theta}} \tag{10}$$

where $\bar{\pi} \equiv \pi^{CL} = \pi^i$ and $\bar{\theta} \equiv d^l\theta^i + (1-d^l)\theta^l$. Since $\theta^i > \theta^l$, a higher mortgage distribution rate $d^l$ implies a higher $q^{CL}$. Adverse selection is captured by belief $\bar{\pi} < 1$ and decreases the CL's discount price. The term $1-\delta(1-\bar{\pi})\bar{\theta}$ in (10) is the "default loss" that the CL incurs when its pool contains an expected fraction $1-\bar{\pi}$ of B-type borrowers: the higher the default loss, the lower is the discount price that the CL offers to its borrowers. The CL's mortgage rate (or cost of capital) is $1/q^{CL}$.

The inability of CLs to fully resolve information asymmetries with their the hard information-based screening technology ($\bar{\pi} < 1$) implies that some borrowers in their pool are of bad type. Since bad type borrowers (endogenously) fail to comply with mortgage payment contract terms and conditions, with the net post-foreclosure sales proceeds less than the promised payment, the CL incurs in a "default loss". As a result, based on observables and expectations at the time of mortgage loan origination, the lender finds it optimal to tack on a pooling rate premium to the base loan rate to account for adverse selection risk. However, the loan rate may move indirectly with the credit risk of its borrowers if the lender's access to liquidity in the secondary market is sufficiently high (i.e., high $\bar{\theta}$). Roughly speaking, securitization lowers the cost of capital ($1/q^{CL}$) in the conduit loan market where lemons are present.

The discount price that investors pay for the subprime mortgages is $\tau = \bar{\pi}\theta^i/(1-\delta(1-\bar{\pi})\bar{\theta})$. The PLs, who by assumption has $d^{PL}=0$ and $\pi^{PL}=1$, find optimal to set their mortgage discount price equal to its discount factor $\theta^l$, i.e., $q^{PL}=\theta^l$. Prices $q^{CL}$ and $q^{PL}$ can be compared as follows:

$$q^{CL} < q^{PL} \text{ if } \pi^{CL} < \pi_2 \equiv \frac{\theta^l(1-\delta\bar{\theta})}{\bar{\theta}(1-\delta\theta^l)}.$$

Threshold $\pi_2$ will appear again in the next section when we characterize the different equilibrium regimes. Interesting, we see that as the distribution rate $d^{CL}$ increases, threshold $\pi_2$ decreases, and hence more information is needed to sustain an environment where the conduit mortgage rate is below the PL's rate.

By excess premium (EP) we mean the difference between the rate of return of conduit loans and the risk free rate of portfolio loans, i.e.,

$$EP \equiv (1/q^{CL}) - (1/q^{PL}) \tag{11}$$

**Proposition 1:** *The EP increases with default losses and decreases with the predictive power of the CL's CST, a higher mortgage distribution rate, and a higher risk free rate.*

Notice that our pricing results have some analogies with Sato's (2015) analysis of transparent versus opaque assets. Sato shows that transparent firms own transparent assets and opaque firms own opaque assets in equilibrium. This is analogous to us showing PLs hold only higher quality loans and CLs own a mix. The reasons for such holdings are different in the two models, however. In our model, CLs are intermediaries that originate and sell opaque subprime MBS. Sato also shows that opaque assets trade at a premium to transparent assets. This is primarily due to agency distortions in the opaque firm. For us a premium in opaque asset prices comes through the investors' demand for subprime MBS.

# 4 Equilibrium regimes

To streamline our analysis, we focus on a more analytically tractable setting where owner-occupied housing ($H$) and rental housing ($R$) are perfect substitutes and consider the following linear separable utility function:

$$u^h(R_1, H_1, R_2, H_2) = R_1 + \eta H_1 + \theta^h(R_2 + H_2),$$

where $\theta^h < 1$ denotes the consumer's discount factor and $\eta > 1$ denotes a preference parameter that captures that, all else equal, in the first period young households prefer to consume owner-occupied housing over rental housing (this can be possibly due to a better access to schools, for example; see Corbae and Quintin (2015) for a model with also an "ownership premium" in preferences). When households are old, the utility from consumption of owner-occupied housing $H_2$ and the utility from consumption of rental housing $R_2$ are the same. To get simple closed form solutions, we assume $\omega_2^+ = 1$, $\omega^{SR} = 1/2$, $v(PL) = 1$, and $\lambda_G = 1.5$.

## 4.1 House prices

This subsection discusses the effect of the owner-occupied housing price on consumers' housing choices. First, recall that the aggregate demand for owner-occupied housing in the *first* period and the aggregate supply of owner-occupied housing in the *second* period are inelastic, both equal to $\bar{H} = 1$. A constant stock of owner-occupied housing is convenient to get simple closed form equilibrium solutions because the market clearing house prices are such that $p_1 = p_2 = p$.[18] Defaults occur in our model due to the imperfect screening of

[18] The owner-occupied market clearing equations in periods 1 and 2 and the households' optimal choice $H_2^h = 0$ (shown in the Appendix) imply that $p_1 = p_2 = p$.

the hard CST, and not due to house price movements.[19] Secondly, in equilibrium $p > 1$, which implies that old households with a mortgage will sell their house in the second period and move to rental housing, as the benefits to owning go away as the younger household transitions to older age.[20] In the first period, however, young consumers with a mortgage will find optimal to buy a house, provided that the credit scoring technology parameter $\pi^{CL}$ exceeds a certain threshold, as argued below.

Thirdly, notice that our model can be conceived as an overlapping generations (OLG) economy, where in each period there are new lenders and investors (alternatively, we could assume instead that lenders and investors cannot share risk across time among different generations of households).[21] In that case, our baseline two periods economy becomes similar to an OLG economy where households in the second period choose to sell their houses to a new generation of younger households (the stock of owner-occupied housing changes hands from old households to young households).

## 4.2 Minimum house size

This subsection examines the role of a minimum house size on the exclusion of subprime consumers from mortgage markets. First, notice that PLs can in general lend to G-type consumers or to B-type consumers. Similarly to our discussion on the effect of $H_{CL}^{\min}$ on a conduit mortgage market specific for B-type consumers, we can also find a threshold $H_{PL}^{\min}$ that rules out a portfolio mortgage market specific for B-type consumers. In particular, we find that G-type consumers crowd out B-type consumers from the portfolio mortgage market if there is a local policy that requires a minimum house (lot) size equal to

$$H_{PL}^{\min} \equiv \omega^{SR}/p(1 - \delta\theta^{l}) \tag{12}$$

This housing policy implies that subprime consumers with a small portfolio loan (or no loan) have no other option but to rent in the first period, because when $p > 1$ these consumers can only afford buying a house of size $\omega^{SR}/p$, which is certainly below $H_{PL}^{\min}$.

Now, define $H^{\min} \equiv \max\{H_{PL}^{\min}, H_{CL}^{\min}\}$, which is the minimum house size that rules out both a portfolio mortgage market and a conduit mortgage market specific to B-type consumers. Then, threshold $H^{\min}$ captures how a local minimum house size regulation affects the bottom of the housing market by excluding subprime borrowers of bad type from the mortgage market.

[19] For a model where default is triggered by a fall in house prices, see e.g. Arslan, Guler and Taskin (2015) where mortgages are non-recourse.

[20] Also, as households get older, their needs may change and may prefer independent living, assistance living, or even nursing care than living by their own in a big owner-occupied house. See Hochguertel and van Soest (2001) for evidence that young households buy a house to accommodate the new family members and possibly to get access to better schools, but when they are old and the family size decreases, these households sell their houses and move to smaller rental houses.

[21] Extending the OLG model to a more general setting with infinitely lived agents and more than one good is subtle because the presence of such agents may preclude equilibrium existence due to the possibility of Ponzi schemes (see Seghir (2006)).

### 4.3 Mortgage market collapses

This section identifies three thresholds, $\pi_0$, $\pi_1$ and $\pi_2$, for the CL's belief $\pi^{CL}$. These thresholds determine different subprime mortgage market configurations, and all can be expressed as a function of the parameters of our economy, including key parameters such as $\theta^l$, $\theta^i$, $d^l$, and $CST_G^{CL}$.

1. *In presence of a minimum house size constraint, the conduit market can collapse if belief $\pi^{CL}$ sufficiently deteriorates*: there is a threshold $\pi_0$ that solves the following equation:
$$H_1^{G,CL}(\pi_0) = H^{\min} \tag{13}$$
such that when $\pi^{CL} < \pi_0$ conduit loans are so small that borrowers cannot afford to buy a house with size above $H^{\min}$.

2. *There is a conduit mortgage market as long as G-type consumers prefer to borrow from CLs than renting in the first period*: When $\pi^{CL}$ decreases below a given threshold $\pi_1$, the implicit conduit mortgage rate is so high that G-type consumers prefer to rent in both periods ($R_1 = \omega^{SR}$ and $R_2 = \omega_2^+$) than borrowing from CL and buying a house in the first period. Threshold $\pi_1$, at which indifference between buying a house with a conduit loan and renting in both periods occurs, solves the following equation:[22]
$$\eta H_1^{G,CL}(\pi_1) + \theta^h \omega^{SR} = \omega^{SR} + \theta^h \omega^+ \tag{14}$$
When $\pi^{CL} < \pi_1$, conduit loans are so small that G-type consumers prefer to rent in both periods.

**Lemma 1:** *The conduit mortgage market collapses when $\pi^{CL} < \max\{\pi_0, \pi_1\}$.*

3. *Consumers may prefer to borrow from CLs if the conduit loan is larger than the portfolio loan*: There is a threshold $\pi_2$ at which the G-type consumer is indifferent between a conduit loan and a portfolio loan. This threshold solves the following expression:[23]
$$\eta H_1^{G,CL}(\pi_2) + \theta^h \omega^{SR} = \eta H_1^{G,PL} + \theta^h \omega^{SR} \tag{15}$$
Observe that when $\pi^{CL} > \pi_2$, consumers prefer conduit loans even when conduit lenders risk-price the presence of lemons and their subsequent default into the mortgage discount price. In this case, the proportion $\hat{\pi}_G^{CL}$ of G-type consumers that attempts to borrow from CLs improves as now conduit loans are the first best option for G-type consumers. Also interestingly, when the mortgage distribution rate $d^{CL}$ increases, threshold $\pi_2$ decreases and the conduit mortgage market expands.

[22] In the left hand side term of equation (14) both portfolio loan and conduit loan markets are active and the market clearing house price is computed accordingly.

[23] The left hand side term in equation (15) represents the G-type consumer's utility from buying a house in the first period with a conduit loan and then renting (in a setting where only the conduit loan market is active). The right hand side term in equation (15) represents the G-type consumer's utility from buying a house in the first period with a portfolio loan and then renting (in a setting where both portfolio loans and conduit loans markets are active).

**Lemma 2:** *The portfolio mortgage market becomes the first choice for G-type consumers when $\pi^{CL} > \pi_2$.*

Below we summarize the different possible market configurations in terms of the CL's belief $\pi^{CL}$ and indicate the size of the portfolio and conduit mortgage markets for each of these configuration. For simplicity, we assumed that CLs are not capacity constrained,[24] so whenever a G-type is not able to borrow from a PL, he can always try to borrow from a CL. However, not all G-type consumers that attempt to borrow from a CL end up with a loan. This is because the CL's CST identifies a G-type consumer as B-type with positive probability.

**Proposition 2** (Subprime mortgage market configurations)**:**

- *If $\pi^{CL} < \max\{\pi_0, \pi_1\}$, the conduit mortgage market collapses and only a mass $v(PL)$ of G-type consumers can borrow to buy a house. The rest of consumers, with mass $\lambda_G - v(PL) + \lambda_B$, rent in both periods.*

- *If $\pi^{CL} > \pi_2$, G-type consumers prefer the conduit mortgage market. A mass $CST_G^{CL}\lambda_G$ $+CST_B^{CL}\lambda_B$ of consumers receive a good rating are able to borrow at the conduit loan rate and buy a house. A mass $\min[(1 - CST_G^{CL})\lambda_G, 1]$ of G-type consumers without a conduit loan will borrow from their second best option, the portfolio loan market. The rest of consumers will rent in both periods.*

- *When $\pi^{CL} \in [\max\{\pi_0, \pi_1\}, \pi_2]$, portfolio lenders lend to a mass $v(PL)$ of G-type consumers. A mass $(1 - CST_G^{CL})(\lambda_G - v(PL)) + (1 - CST_B^{CL})\lambda_B$ of consumers receive a bad rating in the conduit loan market have no option but to rent in both periods.*

The proof follows immediately from our previous analysis and is thus omitted. Next, we explain the effect of changes of key parameters on $\pi_0$, $\pi_1$ and $\pi_2$. First, when the predictive power of the hard credit scoring technology worsens, $\pi^{CL}$ decreases, and there is more asymmetric information between borrowers and CLs, and all else equal, the conduit market is closer to its collapse (or enters in the collapse region). Second, when the consumer's discount factor $\theta^h$ increases and the owner-occupied preference parameter $\eta$ decreases, consumers find renting in the first period relatively more attractive than borrowing-to-own, and thus the conduit loan market shrinks as $\pi_1$ increases. Third, when the investor's discount factor $\theta^i$ and/or the distribution rate $d^{CL}$ increase, all else equal, the conduit loan market expands (as threshold values $\pi_0$, $\pi_1$ and $\pi_2$ decrease). This is because conduit mortgages become more attractive due to the higher investors' willingness to pay for mortgages. Fourth, a higher foreclosure cost expands the region where both portfolio and conduit loan markets are active, as a lower $\delta$ decreases the value of thresholds $\pi_0$, $\pi_1$ and increases the value of $\pi_2$.

Next, we illustrate how the excess premium (EP) and equilibrium loan amounts ($q^{PL}\psi^{PL}$ and $q^{CL}\psi^{CL}$) change when we vary the CL's belief $\pi^{CL}$. We represent these functions in Figures 1 and 2, respectively. For that, we assume that $d^{CL} = 0.8$, $\theta^h = 0.4$, $\theta^l = 0.7$,

[24] Alternatively, $v(CL) > \Pr(\text{rating=G}|\text{G})\lambda_G + \Pr(\text{rating=G}|\text{B})\lambda_B$.

$\theta^i = 0.9$, $\eta = 4$, $\delta = 0.5$, $\lambda_G = 1.5$, $\lambda_B = 1$ and $v(PL) = 1$. The $\pi$-thresholds for these parameters are $\pi_0 = 0.15$, $\pi_1 = 0.63$ and $\pi_2 = 0.71$.[25] In Figure 1 we observe two lines. The line $d^{CL} = 0$ in Figure 1 computes $EP$ when CLs cannot distribute mortgages to investors. In this case, the G-type consumers always prefer portfolio loans over conduit loans and the conduit mortgage rate is always above $1/q^{PL}$ (so $EP > 0$). When belief $\pi^{CL}$, the $EP$ decreases. The second line in Figure 1 illustrates the EP when $d^{CL} = 0.8$. It changes from positive to negative at $\pi^{CL} = \pi_2 \equiv 0.71$. At this point the CL's gains from intermediation exactly offset its loss from bad type (defaulted) loans, and the EP coincides with the risk-free rate $(1/q^{PL})$. When $\pi^{CL} > 0.71$, the CL's mortgage rate is smaller than the PL's rate, and G-types consumers prefer conduit loans to portfolio loans in equilibrium. In Figure 2 we can see that it is exactly at $\pi^{CL} = \pi_2 \equiv 0.71$ when the conduit loan amount coincides with the portfolio loan amount, and when $\pi^{CL} > 0.71$, CLs offer a higher loan amount than PLs.

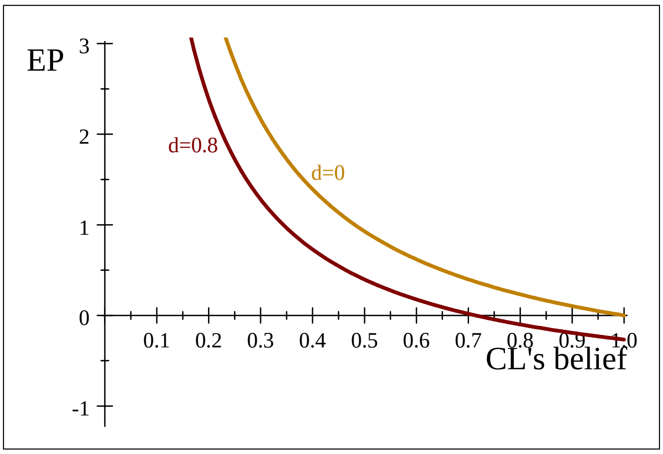

Figure 1

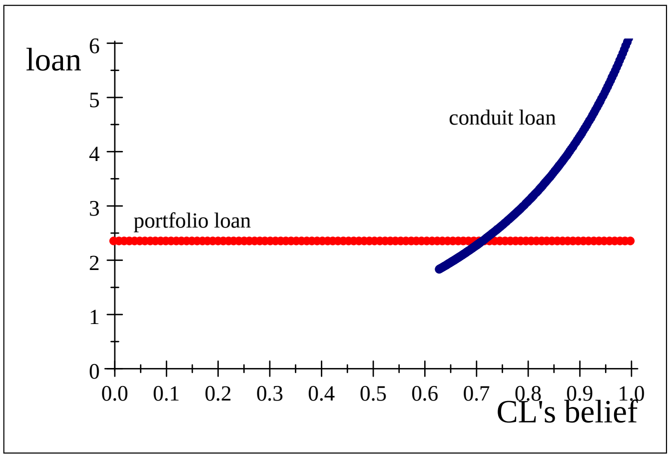

Figure 2

# 5 The credit scoring channel

To motivate the analysis in this section, let us start with a brief discussion of the evolution of the credit scoring technology in the subprime mortgage market. In particular, let us go back to the 1980s and early 1990s, where the subprime loan credit scoring technology was crude and there did not exist powerful summary statistics on consumer credit quality (FICO score). In that state of the world, it was very difficult for subprime loan originators to reliably distinguish between good and bad credit borrowers based on hard information. If transaction-based lending were to occur based on hard information only, the high likelihood to confusing good and bad types in underwriting decisions would increase loan rates substantially due to adverse selection concerns, thus potentially pricing all borrowers out of the conduit mortgage market. But relationship (portfolio) lenders, such as local depository

[25] Observe that threshold $\pi_2$ that solves equation (15) exactly coincides with the threshold that solves equation $q^* = q^{CL}$ (or equivalently, $EP = 0$) and also equation $q^{PL}\psi^{PL} = q^{CL}\psi^{CL}$.

financial institutions, were capable to soliciting soft information to improve their underwriting decision outcomes. Potentially based on regulatory requirement (e.g., credit rating agency), localized relationship lenders were the only available source of subprime loans, but were subject to capacity constraints that resulted in the rationing of credit (to good types) in subprime neighborhoods. Now consider the evolved period from the middle 1990s to early 2000s in which credit information became available to improve credit scoring decisions (FICO is introduced and provides accurate assessments of borrower credit quality), and where credit scoring models ($CST_G^{CL}$) themselves improved. This created a foundation for more credibly distributing subprime loans into a secondary market. A reduction in the pooling rate on subprime loans due to better (perceived if not actual) sorting of good and bad types made it feasible for low-cost transaction-based lenders (brokers and other conduit lenders) to set up shop to apply automated underwriting based on hard information only.

## 5.1 A credit scoring technology shock triggers the "boom"

With the above events in mind, let us examine the role of the CL's credit scoring technology $CST_G^{CL}$ in triggering changes in the equilibrium structure of the subprime mortgage market. We attempt to show that a big enough improvement in $CST_G^{CL}$ can trigger the emergence of the conduit mortgage market first, and then trigger a regime change from $\hat{\pi}_G^{CL}$ low to $\hat{\pi}_G^{CL}$ high (i.e., G-type consumers preferred mortgage market changes from PL to CL, or in other words a change in $\pi^{CL}$ from below $\pi_2$ to above $\pi_2$). For this, we identify three different economies, for different periods (e.g., early 1990s, mid 1990s, and early 2000s), as we change $CST_G^{CL}$ from low to moderate and then from moderate to high.

**Proposition 3**: *For this economy, there are three possible equilibrium regimes:*

- *Regime 1 is characterized by an inactive conduit mortgage market, and occurs if a low $\hat{\pi}_G^{CL}$ and a low $CST_G^{CL}$ are such that $\pi^{CL} < \pi_1$.*
- *Regime 2 is characterized by an emergent conduit mortgage market that coexists with the portfolio mortgage market (still the preferred option for G-type consumers), and occurs if a low $\hat{\pi}_G^{CL}$ and a moderate $CST_G^{CL}$ are such that $\pi^{CL} \in (\pi_1, \pi_1)$*
- *Regime 3 is characterized by a dominant conduit mortgage market and a relatively small portfolio mortgage market, and occurs if a high $\hat{\pi}_G^{CL}$ and high $CST_G^{CL}$ are such that $\pi^{CL} \geq \pi_2$.*

In Regime 1 there are only PLs in the subprime mortgage market, whose loan amount is independent of the CL's CST, and therefore the house price is low, and the size of the owner-occupied house is large precisely because house price is low.

In Regime 2 the conduit mortgage market emerges because CLs offer loan amounts that are sufficiently attractive to G-type households without a portfolio loan than renting. In this regime there are new consumers with a mortgage relative to Regime 1. The higher credit supply increases the demand for housing and in turn increases the house price. Also, because housing supply is inelastic, more credit coming from the conduit loan market decreases the equilibrium house size that consumers with portfolio loans can buy. On the other hand, consumers with conduit loans can buy a larger house size as $\pi^{CL}$ keeps increasing.

In Regime 3 G-type consumers' preferred option is the conduit loan market, so the proportion of G-type consumers that attempts to borrow from CLs ($\hat{\pi}_G^{CL}$) is high. Transition from Regime 2 (with $\hat{\pi}_G^{CL}$ low) to Regime 3 (with $\hat{\pi}_G^{CL}$ high ) is similar to a "boom" of the subprime mortgage market, where mortgage credit and house prices increase and home affordability problems decrease.

Next, we illustrate the different regimes in Proposition 3 and their corresponding equilibrium values of mortgage lending and house price for different values of $CST_G^{CL}$ in Figures 3 and 4, respectively.[26]

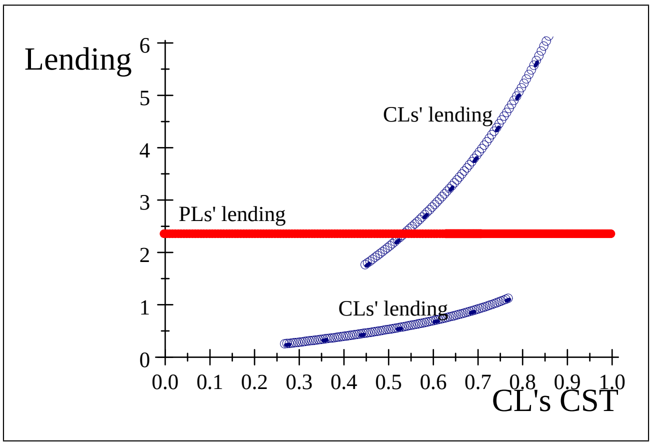


Figure 3: This figure portraits the total amount of PL and CL lending as a function $CST_G^{CL}$.

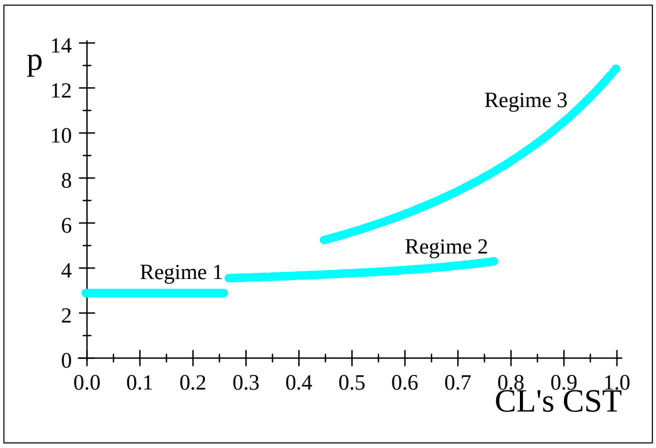


Figure 4: This figure illustrates the equilibrium house price $p$ as a function of $CST_G^{CL}$.

Figures 3 and 4 capture the different equilibrium regimes identified above. The portfolio mortgage market is the preferred choice for G-type consumers ($\hat{\pi}_G^{CL} = 0.33$) when the predictive power of the CL's credit scoring technology is such that $CST_G^{CL} \leq CST_{G,1}^{CL} \equiv 0.77$. This is the case of both Regime 1 and Regime 2, with respective upper bound thresholds for $CST_G^{CL}$ equal to $CST_{G,0}^{CL} \equiv 0.27$ and $CST_{G,1}^{CL} \equiv 0.77$, respectively.[27]

[26] The total amount of PL lending and CL lending is given by expressions $q^{PL}\psi^{PL}(\min\{\lambda_G - CST_G^{CL}\mu_G^{CL}, 1\})$ and $q^{CL}\psi^{CL}\mu^{CL}(\text{rating=G})$, respectively. See the Appendix A.3 for the corresponding closed form solutions.

[27] In Figures 3 and 4, the thresholds for $CST_G^{CL}$ follow from expression (4) and are equal to $CST_{G,0}^{CL} = 0.27$, $CST_{G,1}^{CL} = 0.77$ and $CST_{G,2}^{CL} = 0.45$. In particular, we use the following expression derived from expression (4):

$$CST_G^{CL} = \frac{1 - \hat{\pi}_G^{CL}}{\hat{\pi}_G^{CL}} \frac{1}{\frac{1}{\pi^{CL}} + \frac{1}{\hat{\pi}_G^{CL}} - 2} \tag{16}$$

and then replace $\pi^{CL}$ by $\pi_0 = 0.15$, $\pi_1 = 0.63$ and $\pi_2 = 0.71$, and use the corresponding $\hat{\pi}_G^{CL}$ of each regime.

When $CST_G^{CL}$ hits $CST_{G,1}^{CL} \equiv 0.77$, $\hat{\pi}_G^{CL} = 0.33$ is no longer consistent with $\pi^{CL} \geq \pi_2$, and G-type consumers migrate to the conduit loan market ($\hat{\pi}_G^{CL} = 0.75$).[28] A fraction $CST_G^{CL}$ of these G-type consumers will be able to get a conduit loan and the remainder will go to their second best option, the portfolio loan market.

It is important to notice that inequality $CST_{G,1}^{CL} > CST_{G,2}^{CL}$ holds because $CST_{G,1}^{CL}$ and $CST_{G,2}^{CL}$ are computed using $\hat{\pi}_G^{CL} = 0.33$ (low) and $\hat{\pi}_G^{CL} = 0.75$ (high), respectively (using (16)). In other words, to maintain a given level of $\pi^{CL}$, a high $\hat{\pi}_G^{CL}$ tolerates a lower $CST_G^{CL}$ than a low $\hat{\pi}_G^{CL}$.

In the Appendix we provide additional simulations that capture the changes in the equilibrium values of house size and rental market size for different values of $CST_G^{CL}$. As explained there, the size of the rental market is the largest in Regime 1, and then decreases as we move towards Regime 3 where subprime mortgage lending attains the highest amount. This point illustrates the tight relationship between home affordability and the financial sector in our model.

# 6 Additional insights from the model

In this section we discuss, under the lens of our model, how the equilibrium variables and mortgage market structure of economies with different investor's appetite, mortgage distribution rate, fundamental proportions of good type consumers, and lender's adverse selection may change. We start with a brief description of important events of the boom-bust episode to motivate our analysis.[29]

## 6.1 Motivation

In the early 1990s, in addition to adverse selection concerns as related to loan pricing with transaction-based lending, there was also little demand for subprime loans packaged as securities. However, in the late 1990s and early 2000s, things looked quite differently in the securities market. Concurrent with the new available credit information and the better credit scoring models was the introduction of capital reserve regulation (Basel II) that increased the attractiveness of owning low credit risk (AAA-rated) securities. There were also shocks (the Asian and Russian financial crises) that shifted foreign capital flows towards dollar-denominated U.S. Treasuries and close substitutes. This shift in demand decreased yields of riskless and near riskless bonds, causing fixed-income investors to move further out the credit risk curve in search for higher yields. The search for higher yields and favorable capital treatment caused demand for AAA-rated securities to skyrocket. But these securities were not in sufficient supply to meet all of the demand. The subprime mortgage market represented a vast untapped market, where the pooling of such loans could then be converted (in part, but large part) into AAA-rated securities in large quantities to help satisfy the demand.

[28] i.e., increases from $(\lambda_G - v(PL))/(\lambda_G - v(PL) + \lambda_B) = 0.33$ to $(\lambda_G)/(\lambda_G + \lambda_B) = 0.75$.

[29] For the sake of brevety, we kept this narrative short. We refer to Mian and Sufi's (2014b) book "House of Debt" for an in-depth look at the boom and subsequent bust of the subprime mortgage market.

In 2006 the US economy was hit by a sustained increase in unemployment. There were also concerns about the performance of subprime mortgages due to lenders' potential lax screening, and confidence in the credit scoring based conduit loan business model was shaken. Also, demand for credit-risky MBS fell (e.g., investors became more impatient), and consequently investors increased the required pooling loan rate. All these events brought the conduit loan market into the collapse region. Subprime home ownership rates stalled and the housing boom ended.

## 6.2 Investors' appetite, securitization, and fundamentals

In this section we explain how investors' appetite, a growth in securitization, and a shock to fundamentals fueled the boom, relaxed lending standards, and triggered the bust, respectively. Results are summarized in the following proposition.

**Proposition 4:** *Our model identifies three additional channels:*

1. *When the investor's appetite for mortgage backed securities ($\theta^i$) increases, a lower threshold $CST_{G,2}^{CL}$ is necessary to trigger the transition from Regime 2 to Regime 3.*

2. *Assume that soft information acquisition is costly. Then, CLs find optimal to decrease $CST_G^{CL}$ when the mortgage securitization rate $d^{CL}$ increases, yet the equilibrium regime does not change.*

3. *For a given $CST_G^{CL}$, a shock $\varepsilon > 0$ to the fundamental proportion of G-type consumers ($\lambda_G' = \lambda_G - \varepsilon$ and $\lambda_B' = \lambda_B + \varepsilon$) is sufficient to bring the economy from Region 3 to Regions 2 or 1.*

The hypothesis in Proposition 4.1 captures what happened during the period 2000-2004, when demand for AAA-rated bonds and related securities intensified. When the investor's time discount factor $\theta^i$ increases, the CL's mortgage discount price and loan amount increase because a fraction $d^{CL}$ of the conduit loans is now priced at a higher price.[30] The thesis part of Proposition 4.1 follows because a higher $\theta^i$ decreases threshold $CST_{G,2}^{CL}$ (as well as thresholds $CST_{G,0}^{CL}$ and $CST_{G,1}^{CL}$). For example, using the specified parameters in our simulations above, when $\theta^i$ goes from 0.9 to 0.95, $\pi_2$ falls from 0.71 to 0.66, and $CST_{G,2}^{CL}$ falls from 0.45 to 0.39.

Proposition 4.2 has two hypothesis. One is that $d^{CL}$ increases, e.g., from moderate to high, as it occurred during period 2001-2005. The second one is that soft information is costly. This can be accommodated in our model by modifying the CL's profit function $\Phi^{CL}$ as follows:

$$\Phi^{CL} = (\omega_1^{CL} - s - q^{CL}\varphi^{CL} + \tau z^{CL}) + \theta^l(1 - d^{CL})(\pi^{CL}(s)\varphi^{CL} + (1 - \pi^{CL})\delta p_2 H_1^G), \quad (17)$$

where $s$ denotes the cost of acquiring soft information in the first period, and $\pi^l(s)$ is an increasing and concave function of $s$. This additional hypothesis extends our baseline model to one with *endogenous soft information acquisition*. In the Appendix we prove

[30] See the pricing equation (10) and the equilibrium loan amount expression in the Appendix A.3.

Proposition 4.2 using the specific functional form $CST_G^{CL} = h + \sqrt{s}$, where the first and second components correspond to the hard and soft information components, respectively. Also, in the Appendix, we provide a numerical example that illustrates Proposition 4.2.

Proposition 4.2 is according to Rajan, Seru and Vig's (2015) empirical evidence that when the level of securitization increases, lenders have an incentive to originate loans that rate high based on characteristics that are reported to investors, even if other unreported variables imply a lower borrower quality. See also Dell' Ariccia, Igan and Laeven (2012) who find that when subprime mortgage securitization increases, lenders are more encouraged to make riskier loans. Also notice that this proposition rationalizes our simplifying assumption in the baseline model that PL with $d^{PL} = 0$ have access to soft information, whereas CLs with $d^{CL} > 0$ don't. Finally, Proposition 4.2 implies that when the mortgage securitization rate increases, the securitized-portfolio spread increases (this follows because when $CST_G^{CL}$ decreases, the securitized-portfolio equilibrium spread increases - see Section 4.3).

Proposition 4.3 considers a negative shock to the fundamental proportion $\lambda_G/(\lambda_G + \lambda_B)$. This shock can be thought of a result of a deterioration in household's net worth, as documented by Mian and Sufi (2014a). The proof of this proposition is left for the Appendix. The intuition is that this shock to the fundamental proportion of G-types does not affect the $\pi$-thresholds identified above, but changes $\hat{\pi}_G^{CL}$.

The channels identified in Proposition 4.3 provide additional insights on the boom-bust episode of the subprime mortgage market. Other channels not examined in this paper might be also relevant. Also, more than one channel identified here might be contemporaneous, and thus our results should be seen with some perspective. For example, Propositions 4.1 reinforces Proposition 4.3 if thinking in terms of a decrease in investors' liquidity during the bust (lower $\theta^i$).

## 6.3 Adverse selection in the secondary market

Information problems in secondary mortgage markets have now been widely studied. For instance, Fishman and Parker (2015) consider a setting where investors may acquire more information than intermediaries (CLs in terms of our model, as CLs only rely on hard information only). In their model valuation by sophisticated investors creates an adverse selection problem. This is because investors who do valuation fund only good assets, leaving bad ones to approach unsophisticated investors. This worsens the pool of assets purchased by unsophisticated investors who do not do valuation, in turn lowering the price, in turn making valuation even more profitable. In this model, a move from an equilibrium with valuation to an equilibrium without valuation has the features of a credit crunch: lower prices, lower levels of investment, and profitable valuation. Some of these features also appear in Gorton and Ordonez (2014) theory of short-term collateralized debt. In their setting, when the economy relies on informationaly insensitive debt, firms with low quality collateral can borrow, generating a credit boom and an increase in output. A crisis occurs when a ( possibly small) shock causes agents to suddenly have incentives to produce information, leading to a decline in output.

Another strand of the literature has focused instead on the strategic considerations that lenders have when securitizing their mortgages for distribution to security investors. For instance, Frankel and Jin (2015) show that under securitization ignorance is bliss: a remote bank can compete successfully for applicants with strong observables because investors will not suspect the remote lender of choosing only bad loans to sell.

Our baseline model rules out the possibility of adverse selection in the secondary mortgage markets because (1) CLs and investors rely on the same (hard) information to screen between borrower types (i.e., $\pi^i = \pi^{CL}$), and (2) PLs, who have access to soft information in the baseline model, are not allowed to distribute mortgages to investors. Adverse selection in secondary markets would arise in our model if investors - who only rely on hard information - buy mortgage-backed securities from lenders that have superior (soft) information. In the Supplementary Material we explore this possibility and its implications on the equilibrium regime, mortgage spreads and realized defaults. There we consider "sophisticated portfolio lenders" - a mixed formed between PL and CL - that are able to securitize mortgages and distribute them to the investors. We consider two situations: one where sophisticated PLs only distribute mortgages that the hard credit scoring technology assigns a good rating; and another where sophisticated PLs behave strategically and sell bad mortgages to naive investors whose CST identifies as good mortgages. In the first case, we show that "sophisticated PLs" become the first choice for G-type consumers as they can distribute mortgages as CLs do, but also have better information than CLs, who by assumption only rely on hard information. In the second case, we show how investors are selected against by informed sophisticated portfolio mortgage originators and, as a result, investor's default expectations are lower than their realized default.

# 7 Conclusions

This paper provided a general equilibrium model of a subprime economy with endogenous market segmentation, tenure choice, house prices, mortgage rates and loan amounts. The distinction between the two different sources of funding for consumers (portfolio vs. conduit lenders) was important to capture the trade-off between access to soft information and access to the liquidity from the secondary securities market, as well as illustrating the consumers migration from one subprime mortgage market to another and their respective market sizes. Another important element of our theory was the limited recourse nature of the subprime mortgages, which brings adverse selection on the part of borrowers to the model. Despite the presence of several non-convexities, an equilibrium exists in our large economy.

With this setting, we then examine the impact of a new available hard credit scoring technology to subprime lenders. Borrowing from subprime conduit lenders was less preferred than renting in absence of the hard credit scoring technology because adverse selection made mortgage terms prohibitive for subprime consumers. When the new hard credit scoring technology was introduced, adverse selection diminished, and subprime conduit lending emerged. We call this the *credit scoring channel.* In addition, our model also identifies three additional channels. First, subprime lending grows when investors pay a higher price for mortgage securities. Second, an increase in securitization expands housing affordability but reduces lending standards. With lax screening, the economy moves closer to the collapse region. Third, a shock to the fundamental proportion of G-type consumers can trigger the bust.

There are several other interesting theoretical extensions of our model that we have not explored in this paper. First, we think that it would be interesting to examine whether pre-payment penalties and mortgage refinancing have any role in implementing a Pareto superior equilibrium when adverse selection is present. Secondly, it also seems interesting to allow for a house price bubble and study its implications on the rise and fall of the subprime mortgage market. This possibility, although it has already been widely studied in the literature, could bring new insights on how the bubble relates to innovations in the credit scoring technology and subsequent beliefs. In this respect, one could try to incorporate accurate predictions of home prices as part of scoring, and test the failure of the housing collateral assessment module as a reason of a surge in default rates nationwide. However, this extension seems to speak against the general equilibrium spirit of our model, where house prices are endogenous and agents are rational. Finally, our model can be enriched by incorporating agency issues regarding securitization and examining its implications on distressed loans. Any of the aforementioned extensions would enrich our model and provide additional perspectives on new channels.

Our model and results provide new insights for empirical work. First, and most importantly, empirical work that test the channels identified in this paper seems interesting to us. Also, one would like to compare the severity of the adverse selection problem in the subprime mortgage market between non-recourse US states and limited recourse US states following our discussion in the Supplementary Material. For that, Ghent and Kudlyak's (2011) table 1 serves as an excellent summary of the different state recourse laws. It would be also interesting to examine how the severity of the adverse selection problem changed during the different securitization regimes, or when differences along time in foreclosure costs, banks' lending capacities, or the credit scoring technology are observed. Last, but not least, one could examine the economic and statistical significance of the components that we identify in the mortgage credit spread.

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# A Appendix

## A.1 Equilibrium existence

**Proof of Theorem 1:** We investigate the problem of equilibrium existence by transforming it first into a problem of existence of a social system equilibrium. Our approach is by simultaneous optimization. There, a player's payoff function and constraint set are parameterized by the other players' actions. This second dependence does not occur in games. The extension is a mathematical object referred to as a generalized game by Debreu (1952).

We carry out this analysis in the continuum of agents framework. Most of our extensions follow by application of Hildenbrand's (1974) results.[31]

*The generalized game*: In the generalized game a player $a$ chooses his strategy $\varkappa^a$ parameterized by the other players' strategies $\bar{\varkappa}^{-a}$. For our economy this game is played by the consumers, the lenders, the investors, and five fictitious auctioneers. To incorporate consumers' market choice decisions into the generalized game, we divide the consumers' optimization problem in two stages.

*Stage 1 (Non-convex generalized game with given market choices)*: Consumer $h$ chooses his most preferred consumption for a given mortgage market choice $m^l_{t(h)} \equiv (t(h), l)$, i.e., taking $\bar{\mu}^h(m^l_{t(h)}) = 1$ as given. The consumer $h$'s consumption and loan demand when market choice is $m^l_{t(h)}$ is given by

$$(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)})) \in \arg\max\{\ u^{h_\theta}\left(\cdot, \bar{\mu}^h(m^l_{t(h)})\right) : \bar{p}_1 H^h_1(m^l_{t(h)}) + R^h_1(m^l_{t(h)}) \leq \bar{q}\psi^h(m^l_{t(h)}) + \omega^{SR},$$
$$\psi^h(m^l_{t(h)}) \leq B,\ \ \bar{p}_2 H^h_2(m^l_{t(h)}) + R^h_2(m^l_{t(h)}) \leq \max\{\omega^{SR}, \omega^t_2 + \bar{p}_2 H^h_1(m^l_{t(h)}) - \psi^h(m^l_{t(h)})\}\}$$

Observe that the choice variables in the constrained optimization problem should all be multiplied by $\bar{\mu}^h(m^l_{t(h)})$, but we chose to omit it as we are already assuming that $\bar{\mu}^h(m^l_{t(h)}) = 1$ (the consumer is evaluating his utility at specific market choice $m^l_{t(h)}$) - e.g., when writing $H^h_1(m^l_{t(h)})$ we mean $H^h_1(m^l_{t(h)})\bar{\mu}^h(t(h), l)$ with $\bar{\mu}^h(t(h), l) = 1$.

Let us show that $(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$ has nonempty compact values and is continuous. First, notice that $h \to (x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$ has a measurable graph (see Hildenbrand (1974, p. 59, Proposition 1.b)). Non-emptiness follows from the positive endowment assumption. Compactness follows because $H^h(m^l_{t(h)}) \leq \bar{H} < \infty$ , $R^h(m^l_{t(h)}) \leq \int_{\mathbf{A}} \omega^a da < \infty$, and $\psi^h(m^l_{t(h)}) \leq B$ if prices $p_1$ and $p_2$ are uniformly bounded away from 0 (i.e., $p_1$,$p_2 \geq \alpha$, $\alpha > 0$).[32] Continuity of $(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$ follows if consumer's demand is both upper and lower hemi-continous. Since the consumer's consumption set and utility function are both continuos in $(x, \psi)$ and endowments are desirable, we can apply Berge's Maximum theorem to show that $(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$ is upper hemi-continuous. Next, we prove lower hemi-continuity of $(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$. Denote by $B^h = \{(x^h, \psi^h) : p_1 H^h_1(m^l_{t(h)}) + R^h_1(m^l_{t(h)}) \leq q\psi^h(m^l_{t(h)}) + \omega^{SR},\ \psi^h(m^l_{t(h)}) \leq B,\ p_2 H^h_2(m^l_{t(h)}) + R^h_2(m^l_{t(h)}) \leq \max\{\omega^{SR}, \omega^t_2 + p_2 H^h_1(m^l_{t(h)}) - \psi^h(m^l_{t(h)})\}\}$ the set of consumer $h$'s consumption and borrowing amounts that are budget feasible.

**Claim 1:** *$(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}))$ is lower hemi-continuous.*

**Proof:** Fix $\bar{\mu}^h(m^l_{t(h)}) = 1$ and consider consumer $h$'s correspondence $\dot{B}^h$ that associates to each vector $(p_1, p_2, q)$ the collection of plans $(x^h, \psi^h, \bar{\mu}^h(m^l_{t(h)})) \in \mathbf{X}^h$ that satisfies consumer's budget constraints of $B^h$ as strict inequalities. $\dot{B}^h$ has non-empty endowments

[31] See Luque (2013) for a similar approach in a local public goods non-atomic economy, and Luque (2014) for a review of different approaches to the presence of equilibrium in a continuum of agents framework.

[32] One can show that prices are indeed positive with a strictly monotonic utility. The argument is standard and thus ommitted for the sake of brevety (one should consider a sequence of truncated generalized games by relaxing $\alpha$ and apply the multidimensional Fatou's lemma (see Hildenbrand 1974, p. 69) to obtain a cluster point of this sequence; see Poblete-Cazenave and Torres-Martinez (2013)).

as consumer's endowments are strictly positive. Also, since the constraints that define $\dot{B}^h$ are given by inequalities that only include continuous functions, the correspondence $\dot{B}^h$ has an open graph. Therefore, for any consumer $h$, $\dot{B}^h$ is lower hemi-continuous (see Hildebrand (1974, Prop. 7, p. 27)). Moreover, the correspondence that associates any vector $(p_1, p_2, q)$ to the closure of the set $\dot{B}^h(p_1, p_2, q)$ is also lower hemi-continuous (see Hildebrand (1974, Prop. 7, p. 26)). Now define the closure of $\dot{B}^h$ by $\overline{\dot{B}^h}$. We affirm that $\overline{\dot{B}^h} = B^h$. Since for any $(p_1, p_2, q)$ we have $\overline{\dot{B}^h}(p_1, p_2, q) \subset B^h(p_1, p_2, q)$, it is sufficient to show that $B^h(p_1, p_2, q) \subset \overline{\dot{B}^h}(p_1, p_2, q)$.

Given $(x^h, \psi^h) \in B^h(p_1, p_2, q)$ and $(\varepsilon, \delta_1, \delta_2) \in [0,1]^3$, let $\psi^h(\varepsilon, \delta_1) = (1-\delta_1)\psi^h + \varepsilon$. We first want to prove that $((1-\delta_1)x_1^h, (1-\delta_2)x_2^h, \psi^h(\varepsilon, \delta_1)) \in \dot{B}^h$, where $x_1^h = (H_1^h, R_1^h)$ and $x_2^h = (H_2^h, R_2^h)$. It is not difficult to see that this last property holds if $\delta_1\omega^{SR} > \delta_1\psi^h - \varepsilon > 0$ (C1) and $\delta_2 = \delta_1(\omega^{SR} + p_2H_1^h)/(\psi^h + p_2H_1^h)$ (C2). In fact, when $(x^h, \psi^h)$ is changed to $((1-\delta_1)x_1^h, \psi^h(\varepsilon, \delta_1))$, a quantity $\delta_1\omega^{SR} + \varepsilon$ becomes available at the first period. Thus, if (C1), the possible lower revenue from modified debt (if $\delta_1\psi^h - \varepsilon > 0$) is covered by a portion $\delta_1$ of period 1 endowment.

It remains to show that a consumer can buy $(1-\delta_2)x_2^h$ after deciding whether to strategically default or not. This follows by (C2). To see this, notice that the new resources that become available in the second period are $\max\{\delta_2\omega^{SR}, \delta_2\omega_2^t + p_2\delta_2H_1^h - p_2\delta_1H_1 - \delta_2\psi^h + \delta_1\psi^h - \varepsilon\}$. New resources must be greater than $\delta_2\omega^{SR}$ in the event of no-default, i.e., $\delta_2\omega_2^t + p_2\delta_2H_1^h - p_2\delta_1H_1 - \delta_2\psi^h + \delta_1\psi^h - \varepsilon \geq \delta_2\omega^{SR}$. We know that $\omega_2^G > \omega^{SR}$, so by choosing $\omega_2^t = \omega^{SR}$ we immediately see that sufficient condition (C2) follows.

Finally, making $\delta_1 \to 0$ (so $\varepsilon$ and $\delta_2$ vanish too), we conclude that $(x^h, \psi^h) \in \overline{\dot{B}^h}(p_1, p_2, q)$, as long as consumers can consume their resources. Thus, correspondence $\dot{B}^h$ is lower hemi-continuous for each consumer. □

Now, let $\int_{A(G)\cup A(B):t(h)=t}(x^h(m_{t(h)}^l), \psi^h(m_{t(h)}^l))d\lambda$ represent the measurable demand of goods and loan payments by the continuum of type $t$ consumers in market $m_t^l$. Because the aggregate consumer demands function

$$\int_{A(G)\cup A(B):t(h)=t}(x^h(m_{t(h)}^l; \bar{p}, \bar{q}), \psi^h(m_{t(h)}^l; \bar{p}, \bar{q}))d\lambda$$

is the integral of upper semi-continuous demands with respect to a nonatomic measure, we have that $\int_{A(G)\cup A(B):t(h)=t}(x^h(m_{t(h)}^l), \psi^h(m_{t(h)}^l))d\lambda$ is upper hemi-continuous. The compact-valued function $h \to (x^h(m_{t(h)}^l), \psi^h(m_{t(h)}^l))$ is bounded above and below by $(\int_{\mathbf{A}} \omega(a)da, \bar{H}, \int_{\mathbf{A}} Bda)$ and 0, respectively. According to Hildenbrand (1974, p. 62, Theorem 2), the aggregate consumer demands function is nonempty. And according to Hildenbrand (1974, p. 73, Proposition 7) this set, which is bounded below by 0, is also compact. Therefore, $\int_{A(G)\cup A(B):t(h)=t}(x^h(m_{t(h)}^l; \bar{p}, \bar{q}), \psi^h(m_{t(h)}^l; \bar{p}, \bar{q}))d\lambda$ is compact and has nonempty values. Using a similar reasoning, we can show that the measurable aggregate demand $\int_{A(CL)\cup A(PL):t(l)=t}(R^l, H^l, \varphi^l)d\lambda$ is compact and has nonempty values.

Observe that the consumer's consumption budget set does not have convex values due to the maximum operator in the second period budget constraint, and therefore, we cannot claim that $(x^h(m_{t(h)}^l), \psi^h(m_{t(h)}^l))$ has convex values.[33] However, Lyapounov's convexity

[33]If the budget set had convex values, then we could have used quasiconcavity of $u^h$ to demonstrate that

theorem of an atomless finite dimensional vector measure (see Hildenbrand (1974, p. 62, Theorem 3)) implies that the aggregate consumer demand is convex-valued.

*Stage 2 (Non-convex generalized game with endogenous mortgage market choices)*: Given the consumers' optimal consumptions in each mortgage market, consumers choose their most preferred mortgage market (recall that $l = \emptyset$ is a possibility). Let $U^h(m^l_{t(h)}) \equiv u^h(x^h(m^l_{t(h)}), \psi^h(m^l_{t(h)}), \mu^h(m^l_{t(h)}))$. Then, $\mu^h(m^l_{t(h)}) = 1$ if $l \in \arg\max U^h(l)$ and 0 otherwise (as $M(h) = 1$). We represent the pure strategy of consumer $h$ by a basis vector of dimension $M$. The vector $\mu^h(m^l_{t(h)})$ is the vector in $\mathbb{R}^M$ with 1 as $(m)^{\text{th}}$ coordinate and zero otherwise. By a parallel argument as above, there is a measurable selection $h \to \mu^h(m^l_{t(h)})$ with an associated aggregate demand vector $\int_{A(G)\cup A(B)} \mu^h(m^l_{t(h)})d\lambda$, which is the integral of upper hemi-continuous demands with respect to a non-atomic measure. Thus, $\int_{A(G)\cup A(B)} \mu^h(m^l_{t(h)})d\lambda$ is upper hemi-continuous, with compact (by the assumption $M(h) = 1$, for a.e. $h$), convex (by Lyapounov's convexity theorem) and nonempty values.

Lenders and investors objective functions are linear and their choice variables belong to non-empty closed compact sets. Thus, their respective first order conditions pin down prices $q^{PL}$, $q^{CL}$ and $\tau$.

*Auctioneer 1* chooses $p_1$ to minimize $\left(\sum_{l\in\mathbf{G}} \int_{A(G)\cup A(B)} H^h_1(m^l_{t(h)})\bar{\mu}^h(m^l_{t(h)})d\lambda - \bar{H}\right)^2$, where $\bar{H}$ stands for the exogenous supply of housing from an old previous generation. *Auctioneer 2* chooses $p_2$ to minimize $\left(\sum_{l\in\mathbf{G}} \int_{A(G)\cup A(B)} H^h_2(m^l_{t(h)})\bar{\mu}^h(m^l_{t(h)})d\lambda - \bar{H}\right)^2$, where $\bar{H}$ stands for the exogenous demand of housing from a young future generation. *Auctioneer 3* chooses $\varphi^{PL}$ and $\varphi^{CL}$ to minimize $\sum_{l=PL,CL} \int_{A(G)\cup A(B)}(\bar{\psi}^h(m^l_{t(h)})\bar{\mu}^h(m^l_{t(h)})d\lambda - \int_{A(l)} \varphi^l(m^l_{t(h)})\bar{\mu}^h(m^l_{t(h)})d\lambda)^2$. *Auctioneer 4* chooses $z^{CL}$ to minimize $\left(d^{CL}\bar{\varphi}^{CL} - z^{CL}\right)^2$. *Auctioneer 5* chooses $z^i$ to minimize $(z^i - \bar{z}^{CL})^2$. Finally, to guarantee the consistency condition (1.4), we introduce Auctioneer 6, whose optimization problem is choosing $\pi^l \in [0,1]$ to minimize $(\pi^l - g(f(\hat{\mu}^l(CST^l_G, \pi^l), CST^l_G))^2$, for $l = PL, CL$, where functions $f$ and $g$ are as defined in Section 3.

All Auctioneers' strategy sets are nonempty, convex, and compact. An equilibrium for the constructed generalized game consists of a vector $(\bar{x}, \bar{\mu}, \bar{\psi}, \bar{\varphi}, \bar{z}, \bar{p}_1, \bar{p}_1, \bar{q}, \bar{\tau})$ such that each player $a$ chooses a strategy $\bar{\varkappa}^a$ to solve his respective optimization problem parameterized in the other players' actions $\bar{\varkappa}^{-a}$.

**Claim 2:** *There exists an equilibrium in mixed strategies for the constructed generalized game.*

**Proof:** Note that the consumer's strategy set for choosing his most preferred mortgage market in stage 2 has a finite and discrete space domain $\mathbf{M}$. In order to circumvent this problem, we extend our generalized game to allow for consumers' mixed strategies in the set of group types $\mathbf{M}$. Let $\Sigma(\mathbf{M}) = \{\sigma = (\sigma(m))_{m\in\mathbf{M}} : \sigma(m) \geq 0, \sum_{m\in\mathbf{M}} \sigma(m) = 1\}$. Then, $\Sigma(\mathbf{M})$ stands for the convex hull of $\{PL, CL, \emptyset\}$, which is the set of mixed strategies for each consumer. A profile of strategies $\rho : A(G) \cup A(B) \rightarrow \Sigma(\mathbf{M})$ brings the continuum of consumers into strategies (pure or mixed). Consumer $h$'s stage 2 optimization problem extended to mixed strategies is such that this consumer randomizes over

$x(h,s)$ has convex values.

the possible consumptions in the set of different market choices. We write $U^h(\sigma) \equiv u^h\left(\sum_{m\in\mathbf{M}}\sigma(m)(x^h(m),\psi^h(m)),\sigma\right)$. That is, *consumer randomizes in* $\mathbf{M}$*, but not directly in consumption.* Then, consumer $h$'s stage 2 maximization problem is $\max_{\sigma\in\Sigma(\mathbf{\Omega})} U^h(\sigma)$. Utility function $u^h\left(\sum_{m\in\mathbf{M}}\sigma(m)(x^h(m),\psi^h(m)),\sigma\right)$ is a continuous bounded real valued function on $\sum_{m\in\mathbf{M}}\sigma(m)(x^h(m),\psi^h(m))$, and the mixed strategy $\sigma$ belongs to the convex compact set $\Sigma(\mathbf{M})$. $\mathbf{K}(h) = \{\sigma \in \Sigma(\mathbf{M}) : \sigma \in \arg\max U^h(\sigma)\}$ denotes the set of mixed strategies that solve consumer $h$'s second stage maximization problem.

We must extent the fictitious auctioneers' problems to allow for consumers' mixed strategies. Given a mixed strategy profile $\rho : A(G)\cup A(B) \to \Sigma(\mathbf{G})$, we can rewrite the auctioneers 1, 2 and 3's objective functions extended to mixed strategies as follows: *Auctioneer 1* chooses $p_1$ to minimize $\left(\sum_{m\in\mathbf{M}}\int_{A(G)\cup A(B)} H_1^h(m)\rho^h(m)d\lambda - \bar{H}\right)^2$; *Auctioneer 2* chooses $p_2$ to minimize $\left(\sum_{m\in\mathbf{M}}\int_{A(G)\cup A(B)} H_2^h(m)\rho^h(m)d\lambda - \bar{H}\right)^2$; *Auctioneer 3* chooses $\varphi^{PL}$ and $\varphi^{CL}$ to minimize $\sum_{m\in\mathbf{M}}\left(\int_{A(G)\cup A(B)} \bar{\psi}^h(m^l_{t(h)})\rho^h(m^l_{t(h)})d\lambda - \int_{A(l)}\varphi^l(m^l_{t(h)})d\lambda\right)^2$, for $l = PL, CL$. All the conditions of Debreu's (1952) theorem hold. Thus, we can assert that the extended generalized game has an equilibrium, possibly in mixed strategies.

At this point it remains to observe that auctioneers 1-3' new (extended) objective functions do not depend only on the average of the consumers' profile, as consumers' demands for commodities may be different among consumers of the same type as they can have different access to the mortgage market and, therefore, we cannot apply Schmeidler (1973) to show that a degenerate equilibrium of the extended generalized game is, in fact, an equilibrium of the original game. Instead, we apply a particular result of Pascoa (1998), used by Araujo and Páscoa (2002, Lemma 2) in an incomplete markets economy, which says that purification can be possible if in the extended generalized game, players' mixed strategies depend only on finitely many indicators, one for each type (a statistical indicator).

In particular, auctioneers 1's extended payoff functions depend on the profile of mixed strategies $\rho$ only through finitely many indicators, one for each consumer type $t = G, B$ in $m \in \mathbf{M}$, of the form $\int_{A(G)\cup A(B)}\int_{\mathbf{M}} H_1^h(m;\bar{p}_1)d\rho^h(m)d\lambda$.[34] Given a mixed strategies equilibrium profile $\rho$, there exists a profile $(h, m^l_{t(h)})_{h\in A(G)\cup A(B);t(h)=G,B;m\in\mathbf{M}}$ such that the Dirac measure $\hat{\rho}^h$ at $m^l_{t(h)}$ is an extreme point of the set $\mathbf{K}(h)$, which is the consumer $h$'s best response to the price chosen by Auctioneer 1 in the previous equilibrium in mixed strategies. And moreover, $\int_{A(G)\cup A(B)}\int_{\mathbf{M}} H_1^h(m^l_{t(h)};\bar{p}_1)d\rho^h(m^l_{t(h)})d\lambda$ is the same as $\int_{A(G)\cup A(B)}\int_{\mathbf{G}} H_1^h(m^l_{t(h)};\bar{p}_1)d\bar{\rho}^h(m^l_{t(h)})d\lambda$. Hence, we can replace $(h, m^l_{t(h)})$ by $(h, \hat{\rho}^h(m^l_{t(h)}))$, for all $h \in A(G)\cup A(B)$, $t(h) = G, B$ and $l \in \mathbf{G}$, and keep all the equilibrium conditions satisfied. The indicators that the atomic auctioneer takes as given evaluated at $\hat{\rho}$ are still the same as when evaluated at $\rho$. The proofs for auctioneers 2 and 3's payoff functions follow the same lines. Therefore, we conclude that $\hat{\rho}$ is a degenerate equilibrium profile. $\square$

**Claim 3:** *An equilibrium for our generalized game (in pure strategies) is an equilibrium as defined in Definition 1.*

**Proof:** Let $(x, \psi, \varphi, z, \mu, p_1, p_1, q, \tau)$ be an equilibrium in pure strategies of the generalized game introduced above. Our construction of consumer's optimization in stage 1 and

[34] Observe that we could have written $\sum_{m\in\mathbf{M}} H_1^h(m)\rho^h(m)$ instead of $\int_{\mathbf{M}} H_1^h(m)d\rho^h(m)$.

stage 2 of the above generalized game imply that equilibrium condition (1.1) is satisfied - otherwise, we could find a smaller consumption bundle and use continuity to get into a contradiction with the proposed optimum. Equilibrium conditions (1.2) and (1.3) follow from the solutions of lender $l$ and investor $i$' linear optimization problems, respectively. Equilibrium condition (1.4) follows from the auctioneer 6's optimization problem. Market clearing conditions are satisfied due to the following reasons: (MC.1) and (MC.2) result from the solutions to auctioneer 3's optimization problem. (MC.2) results from the solutions to auctioneer 4 and 5's optimization problems. (MC.5) follows from the solutions to auctioneers 1 and 2 optimization problems. (MC.4) follow by Walras' law in periods 1 and 2. In particular, we can aggregate all agents' resources in period 1, including the exogenous supply of owner-occupied housing in period 1 from a previous old generation of consumers, and obtain:

$$\zeta_1 \equiv \sum_{m\in\mathbf{M}} \int_{A(G)\cup A(B)} \left(p_1 H_1^h(m_{t(h)}^l) + R_1^h(m_{t(h)}^l) - q^l\psi^h(m_{t(h)}^l) - \omega^{SR}\right)\mu^h(m_{t(h)}^l)d\lambda - p_1\bar{H} +$$
$$\sum_{l=PL,CL} \int_{A(l)} (\omega_1^l - q^l\varphi^l + \tau z^l)d\lambda + \int_{A(i)} (\omega_1^i - \tau z^i)d\lambda \leq 0$$

It is easy to see that, when market clearing conditions (MC.1), (MC.2), (MC.3) and (MC.5) hold, there is no excess demand of the numeraire good consumption in period 1 ($\zeta_1 \leq 0$). Otherwise, we would contradict the above aggregation of budget constraints. In fact, the previous inequality holds with equality (i.e., the market of the numeraire good in period 1 clears). Suppose, by contradiction, that $\zeta < 0$. Then, there is a nonnull set of agents with non-binding budget constraints, a contradiction with optimization. Thus, $\zeta_1 = 0$. By a similar argument, we can also prove that $\zeta_2 = 0$. $\square$

## A.2 Minimum house size

### A.2.1 Conduit mortgage market specific to B-type consumers

We focus on the existence of a pooling equilibrium. This is because, as we argued in Section 3, there are two reasons why we can rule out the existence of a mortgage market for B-type consumers. The first reason is that common practice (and common sense) seems against lending to consumers where default always occurs. The second reason is the existence of a minimum house size $H_{CL}^{\min}$ that prevents B-type consumers with a small conduit loan to buy a house with a lot size larger than $H_{CL}^{\min}$.

We now identify threshold $H_{CL}^{\min}$ as a function of the parameters of our economy. First, notice that the CL would get positive profits by lending to a B-type consumer if $q^B\varphi^B \leq \theta^i\delta p H_1^B$ (here we are assuming $d^{CL} = 1$ as this gives the largest loan amount to the consumer since pricing uses the investor's discount factor $\theta^i$). Then, using $pH_1^B = \omega^{SR} + q^B\varphi^B$ from the first period budget constraint (assuming $\bar{H}$ constant in both periods), we get

$$q^B\varphi^B \leq \frac{\theta^i\delta\omega^{SR}}{1-\theta^i\delta} \equiv \bar{L}$$

that is, $\bar{L}$ is the maximum loan amount that a CL would give to a B-type consumer being compatible with non-negative profits for the lender. Now, going back to the minimum house size regulation argument, we can rule out a mortgage market for B-type consumers if $H^B < H_{CL}^{\min}$, i.e., if $(\omega^{SR} + \bar{L})/p < H_{CL}^{\min}$. The market clearing price for owner-occupied housing is $p = (2\omega^{SR} + \bar{L} + L^G)/\bar{H}$, where $L^G$ is the loan amount that a G-type consumer would obtain from a CL when mortgage markets are segmented (using the CL's first order condition and that G-type consumer's first period budget constraint we get $L^G = \bar{\theta}(\omega^{SR} + \bar{L})/(1 - \bar{\theta})$). Then, back to the inequality for $H_{CL}^{\min}$ we can write

$$H_{CL}^{\min} > \frac{\bar{H}(\omega^{SR} + \bar{L})}{2\omega^{SR} + \bar{L} + L^G}$$

Hence, we conclude that a minimum house size policy can rule out the possibility of a separating equilibrium if inequality (9) holds.

### A.2.2 Portfolio mortgage market specific to B-type consumers

The portfolio mortgage contract $(q^{B,r}, \psi^{B,r})$ specific for B-type consumers must satisfy budget constraints $pH_1^{B,r} = \omega^{SR} + q^{B,r}\psi^{B,r}$ and $\omega^{SR} = \omega^{SR} - \psi^{B,r} + pH_1^{B,r}$ (the latter coming from the limited recourse requirement), which implies $\psi^{B,r} = pH_1^{B,r}$ and $\psi^{B,r} = \omega^{SR}/(1 - q^{B,r})$. PL's optimization implies that $q^{B,r} = \theta^l\delta$. Thus, $\psi^{B,r} = \omega^{SR}/(1 - \theta^l\delta)$ and using again equation $\psi^{B,r} = pH_1^{B,r}$ we get $H_1^{B,r} = \omega^{SR}/p(1 - \theta^l\delta)$. Then, set $H_{PL}^{\min} = \omega^{SR}/p(1 - \theta^l\delta)$.

## A.3 Equilibrium amounts

Here we characterize the equilibrium house prices and loan amounts. We refer to the pairs $(q^{PL}, \varphi^{PL})$ and $(q^{CL}, \varphi^{CL})$ as the pooling contracts offered by portfolio lenders and conduit lenders, respectively. First, given the portfolio loan discount price $q^{PL}$, G-type consumers will borrow against all their second period revenue, provided they consume exactly the subsistence rent $\omega^{SR}$. The equilibrium portfolio loan amount is an increasing function of the PL's discount factor. In particular, it is given by the following expression

$$q^{PL}\psi^{PL} = \frac{\theta^l}{1 - \theta^l} \tag{18}$$

Similarly, a G-type consumer with a conduit loan takes as given the conduit loan discount price $q^{CL}$ and borrows against his future income, provided that he consumes exactly the subsistence rent $\omega^{SR}$. B-type consumers that receive a good rating by the CL are lucky to misrepresent their type and will borrow under the same terms and conditions than G-type consumers. The equilibrium conduit loan amount increases with the predictive power of the CST (and thus with $\pi^{CL}$), the foreclosure recovery rate $\delta$, and the $d^l$-weighted discount factor $\bar{\theta}$, which in turn increases with the distribution rate $d^{CL}$ and the investor's discount factor $\theta^i$ and decreases with the lender's discount factor $\theta^l$. In particular, the equilibrium conduit loan amount is given by the following expression:

$$q^{CL}\psi^{CL} = \frac{\pi^{CL}\bar{\theta}}{1 - \bar{\theta}(\pi^{CL}(1 - \delta) + \delta)} \tag{19}$$

The equilibrium value of mortgages distributed to investors is given by the following expression:

$$\tau z^{CL} = \frac{d^{CL}\mu^{CL}(\text{rating=G})}{1-\delta\bar{\theta}(1-\pi^{CL})} \tag{20}$$

where $\mu^{CL}$(rating=G) is the endogenous measure of consumers that borrow from conduit lenders, i.e., $\mu^{CL}(\text{rating=G}) = 0$ if $\pi^{CL} < \max\{\pi_0, \pi_1\}$, $\mu^{CL}(\text{rating=G}) = \pi^{CL}(\lambda_G - v(PL)) + (1-\pi^{CL})\lambda_B$ if $\pi^{CL} \in [\max\{\pi_0, \pi_1\}, \pi_2]$, and $\mu^{CL}(\text{rating=G}) = \pi^{CL}\lambda_G + (1-\pi^{CL})\lambda_B$ if $\pi^{CL} > \pi_2$. If investors had limited wealth, conduit lenders would be constrained by the total amount of credit that can be securitized, i.e., $d^{CL}\varphi^{CL} \leq z^{CL} = z^i$ where the first inequality obeys the originate-to-distribute constraint (5) and the second equality follows from market clearing in the mortgage-backed securities market. One can now see that the equilibrium quantity of mortgages originated by conduit lenders is constrained by the investor's wealth because $\tau z^i \leq \omega_1^i$. Thus, our model is also able to capture Gennaioli, Shleifer, and Vishny (2012) result that investors' wealth may drive up securitization.

Finally, the equilibrium house price depends on the mass of consumers with access to a mortgage. It is given by the following expression:

$$p = \begin{cases} v(PL)\left(\omega^{SR} + \frac{\theta^l\omega^+}{1-\theta^l}\right) \text{ if } \pi^{CL} < \max\{\pi_0, \pi_1\} \\ (v(PL) + \mu_1^{CL}(\text{rating=G}))\omega^{SR} + \frac{\theta^l\omega^+}{1-\theta^l} + \mu_1^{CL}(\text{rating=G})\frac{\omega^+\bar{\theta}\pi^{CL}}{1-\bar{\theta}(\pi^{CL}(1-\delta)+\delta)} \text{ if } \pi^{CL} \in [\max\{\pi_0, \pi_1\}, \pi_2] \\ (v(PL) + \mu_1^{CL}(\text{rating=G}))\omega^{SR} + \frac{\theta^l\omega^+}{1-\theta^l} + \mu_1^{CL}(\text{rating=G})\frac{\omega^+\bar{\theta}\pi^{CL}}{1-\bar{\theta}(\pi^{CL}(1-\delta)+\delta)} \text{ if } \pi^{CL} > \pi_2 \end{cases}$$

where, as pointed out before, $\mu_1^{CL}(\text{rating=G}) = CST_G^{CL}(\lambda_G - v(PL)) + (1 - CST_G^{CL})\lambda_B$ if $\pi^{CL} \in [\max\{\pi_0, \pi_1\}, \pi_2]$, and $\mu_2^{CL}(\text{rating=G}) = CST_G^{CL}\lambda_G + (1 - CST_G^{CL})\lambda_B$ if $\pi^{CL} > \pi_2$.

Figures A1 and A2 illustrate the equilibrium values of house size $(H^{G,PL}, H^{G,CL})$ and size of the rental market, respectively, for different values of $CST_G^{CL}$.

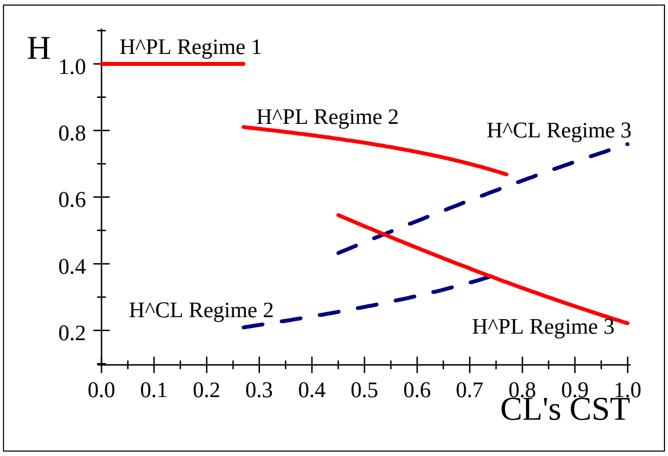


Figure A1: This figure illustrates the equilibrium house sizes $H^{G,PL}$ and $H^{G,CL}$ as a function of $CST_G^{CL}$.

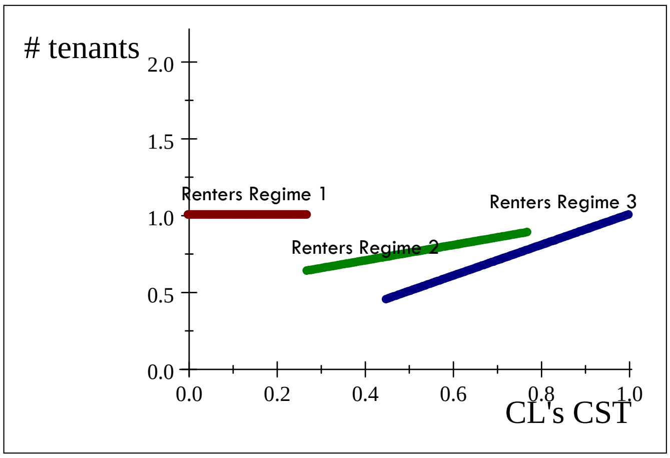


Figure A2: This figure illustrates the size of the subprime rental market as a function of the credit scoring technology $CST_G^{CL}$.

In Figure A1 we see that when the economy enters Regime 3, the house size of borrowers with conduit loans is larger than for borrowers with portfolio loans. This is consistent with the idea that the portfolio mortgage market is not the consumers' first option in Regime 3. We also see that the equilibrium house size of consumers with portfolio loans plummets when the conduit loan size enters in Region 3, as the expansion of the conduit loan market injects more credit in the economy and house price jumps. Also notice that there is a discontinuity in the equilibrium house size purchased with conduit loans when $\hat{\pi}_G^{CL}$ and $CST_G^{CL}$ are such that $\pi^{CL} = \pi_2$ even when the jump in the conduit loan amount is partially offset by the jump in the equilibrium house price at that point.

Figure A2 shows that the size of the rental market is largest in Regime 1 (only the portfolio mortgage market exists). When Regime 2 starts ($\pi^{CL}$ attains $\pi_1$), the rental market shrinks as new consumers get (conduit) mortgages. The rental market shrinks again in Regime 3 ($\pi^{CL}$ attains $\pi_2$), as the conduit mortgage market absorbs a substantial larger fraction of G-type and B-type consumers, while the portfolio mortgage market also absorbs those G-type consumers without a conduit loan. In Regime 3 a mass $(1 - CST_G^{CL})\lambda_B$ of B-type consumers are able to get a conduit loan. However, as $\pi^{CL}$ gets closer to 1, CLs better differentiate between G-type and B-type consumers and reject more B-type consumers, and as a results the size of the rental market converges to the "number" of B-type consumers in the economy ($\lambda_B = 1$). See the Supplementary Material for a more detailed explanation of Figure A2 together with the specific equilibrium expressions of the size of the rental market.

## A.4 Proofs of Propositions 4.2 and 4.3

### A.4.1 Proposition 4.2

How can we rationalize an endogenous transition from $CST_G^{CL}$ high to $CST_G^{CL}$ low, yet keeping the conduit loan market being the preferred option for G-type consumers, i.e., $\hat{\pi}_G^{CL}$

high? Proposition 4.2 answers this question by considering an increment in the mortgage securitization rate $d^{CL}$ similarly to what happened during the booming period 2001-2005. It assumes the new CL's profit function (17) and the CST form $CST_G^{CL} = h + \sqrt{s}$. Notice that from expression (4) we know that $\partial\pi^l/\partial CST_G^{CL} > 0$, and from functional form $CST_G^{CL} = h + \sqrt{s}$ we know that $CST_G^{CL}$ is a continuous, increasing and concave function of $s$. To put this exercise in the context of the boom of the subprime mortgage market, assume that $\hat{\pi}_G^{CL}$ is high (Regime 3).

**Proof of Proposition 4.2:** Taking the partial derivative with respect to $s$, with (5) binding, and writing $D^l = \varphi^l - \delta p_2 H_1^G$ to denote the lender's default loss, we get:

$$[s] : 1 = \theta^l(1 - d^{CL})\frac{\partial\pi^{CL}}{\partial CST_G^{CL}}\frac{\partial CST_G^{CL}}{\partial s}D^l \qquad \text{(FOC[s])}$$

In the FOC[s], we have that the marginal cost (MC) to acquire soft information is constant and equal to 1, while the marginal benefit (MB) is decreasing with slope $-1/(4s^{3/2})$. The intersection between MC and MB pins down the optimal amount of soft information acquired by conduit lenders.[35] Because $\partial CST_G^{CL}/\partial s = 0.5s^{-1/2}$, FOC[s] implies that the CL finds optimal to acquire less soft information the higher is its mortgage distribution rate $d^l$, ceteris paribus.[36] This proves our claim.□

**Numerical example of Proposition 4.2:** Let $CST_G^{CL} = h + \sqrt{s}$ with $h = 0.6$. Take the same parameters used in previous simulations with the exception that $d^{CL}$ can now take two possible values: $d^{CL} = 0.5$ (moderate securitization) and $d^{CL} = 0.8$ (high securitization). In Figure A3 we plot two MB curves corresponding to each of these securitization rates. Exhibit 3 represents the optimal values of soft information acquisition ($s$), the value of $CST_G^{CL}$, the CL's belief $\pi^{CL}$, and the consistency between $\hat{\pi}_G^{CL}$ and $\pi^{CL}$ (when $\pi^{CL} \geq \pi_2$, $\hat{\pi}_G^{CL}$ should equal 0.75; and when $\pi^{CL} < \pi_2$, $\hat{\pi}_G^{CL}$ should equal 0.33). As expected, when $d^{CL}$ increases, the amount of soft information $s$ acquired by a CL decreases. For instance, going from a moderate $d^{CL} = 0.5$ to a high $d^{CL} = 0.8$, decreases $s$, and this change in turn decreases $CST_G^{CL}$ from 0.82 to 0.68. Then, using expression (4) together with these values for $CST_G^{CL}$ and $\hat{\pi}_G^{CL} = 0.75$ (CL is the best option for G-type consumers), we see that $\pi^{CL}$ decreases from 0.93 to 0.83. Notice that when $d^{CL}$ increases, $\pi_2$ decreases, but still we have $\pi^{CL} \geq \pi_2$ for both cases. Thus, the proportion of G-type consumers that attempts to borrow from CLs, $\hat{\pi}_G^{CL} = 0.75$, is consistent with region $\pi^{CL} \geq \pi_2$.

[35] In equilibrium default losses can in turn be expressed as a function of the parameters of our economy as follows: $D^l(\omega_2^+, \delta, d^{CL}, \pi^{CL}, \theta^i, \theta^l) = \frac{\omega_2^+(1-\delta\bar{\theta})}{1-\bar{\theta}(\pi^{CL}(s)(1-\delta)+\delta)} - \frac{\delta\omega^{SR}}{2}$.

[36] Also, from the FOC[s] we find that the optimal amount of soft information is lower the lower is its discount factor $\theta^l$, the lower is the default loss $D(\varphi^l)$, and the weaker is the effect of $s$ on $CST_G^{CL}$ (here given by the square root function).

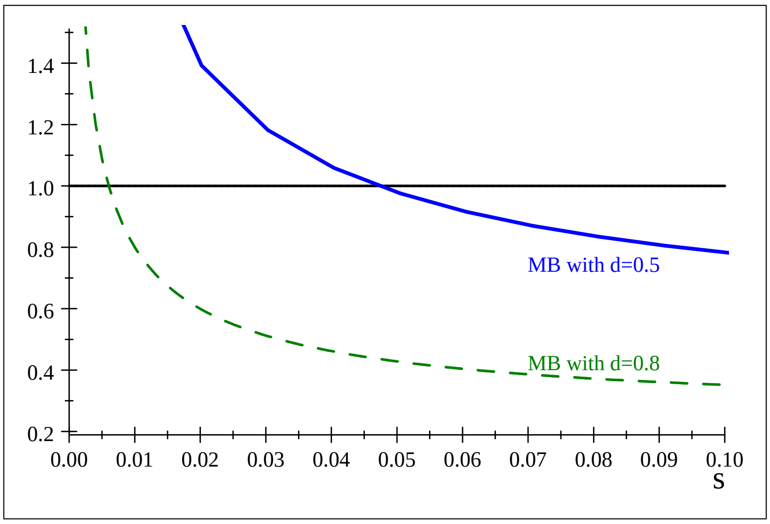


Figure A3

| $d^{CL}$ | $\hat{\pi}_G^{CL}$ | $s$ | $CST_G^{CL}$ | $\pi^{CL}$ | $\pi_2$ | $\hat{\pi}_G^{CL}$ *consistent with* $\pi^{CL}$? |
|---|---|---|---|---|---|---|
| 0.5 | 0.75 | 0.048 | 0.82 | 0.93 | 0.81 | *YES* |
| 0.8 | 0.75 | 0.006 | 0.68 | 0.86 | 0.71 | *YES* |

Exhibit 3

### A.4.2 Proposition 4.3

The proof of Proposition 4.3 is almost immediate. First, notice that a negative shock to $\lambda_G/(\lambda_G+\lambda_B)$ does not change equilibrium threshold $\pi_2$ since

$$\pi_2 = \frac{\theta^l(1-\delta(d^{CL}\theta^i+(1-d^{CL})\theta^l))}{(d^{CL}\theta^i+(1-d^{CL})\theta^l)((1-\theta^l)+\theta^l(1-\delta))}.$$

However, a shock to the fundamental proportion of G-type consumers decreases $\pi^{CL}$ because it depends on $\hat{\pi}_G^{CL}$ (see expression (4), and $\hat{\pi}_G^{CL}$ is a decreasing function of $\lambda_G/(\lambda_G+\lambda_B)$. It stands to reason that a big enough shock to the fundamental proportion can bring $\pi^{CL}$ below threshold $\pi_2$.