Density Forecast Evaluation in Unstable Environments

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Abstract

We propose a density forecast evaluation method in the presence of instabilities, which are defined as breaks in any conditional moment of interest and/or in the functional form of the conditional density of the process. Following the autocontour-based tests proposed in González-Rivera *et al.* (2011, 2014), we construct Sup- and Ave-type tests calculated over a collection of subsamples in the evaluation period. These tests enjoy asymptotic distributions that are nuisance-parameter free, they are correctly sized and very powerful on detecting breaks in the parameters of the conditional mean and conditional variance. We also provide an accurate procedure to detect the location of the breaks and the unstable periods. We analyze the stability of a dynamic Phillips curve and find that the best one-step-ahead density forecast of changes in inflation is generated by a Markov switching model where the unemployment coefficient is state-dependent.

Key Words: Autocontour testing, Structural breaks, Phillips Curve

JEL Classification: C01, C22, C53.

1 Introduction

Generally, instability is understood as changes in the parameters of a proposed forecasting model over the forecasting horizon. For clarification purposes, consider a simple model $y_{t+1} = \beta' x_t + \sigma \varepsilon_{t+1}$ with $\varepsilon_t \sim i.i.d.N(0,1)$. The model is unstable over time if the slope coefficients β can change over the forecasting sample, either smoothly or abruptly to contain one or multiple breaks. We may also entertain a time varying variance such that σ may be also subject to breaks, and we may have different conditional probability density functions, e.g. more or less thick tails, over different periods of time. This definition is general enough to account for most types of instability discussed in the current applied econometric literature. Up to today, the most comprehensive survey in the subject is Rossi (2014) in the *Handbook of Economic Forecasting* that reports extensive empirical evidence of instabilities in macroeconomic and financial data. Some examples follow.

The instability of predictive regressions, in which the significance of predictive regressors varies over different subsamples, has been documented in studies of predictability of stock returns (see Goyal and Welch, 2003; Paye and Timmermann, 2006; Rapach and Zhou, 2014), in exchange rate predictions (see Rossi, 2006; Rogoff and Stavrakeva, 2008) and in output growth and inflation forecasts (see Stock and Watson, 2003; Rossi and Sekhposyan, 2010). Naturally linked to this evidence is the econometric question on testing for parameter stability and structural breaks in the data, which has an illustrious history. From Chow (1960) test to most recent works such as Andrews (1993), Andrews and Ploberger (1994) , Pesaran and Timmermann (2002), among others, testing for breaks has mainly focused on the behavior of the conditional mean. Our contribution aims to extend testing for instabilities to the *full* conditional density forecast that includes not only any conditional measure of interest, e.g. mean, variance, duration, etc. but also the functional form of the assumed conditional density function. To our knowledge, this is the first contribution that explores breaks in the functional form of the conditional density. Our analysis is framed more closely within the approach of Giacomini and Rossi (2009, 2010) and Rossi and Sekhposyan (2011) on the evaluation of out-of-sample forecasts in the presence of instability. However, differently from these works, we do not choose a particular loss function as we do not deal just with a point

forecast but with the full density model. We ask: is the density forecast stable over time? and if it is not, where are the breaks? and to some extent, where are they coming from? [1]

The testing methodology that we propose is based on the AutoContouR (ACR) device introduced by González-Rivera *et al.* (2011, 2012) and generalized later on in González-Rivera and Sun (2014). The null hypothesis of our tests is a correctly specified density forecast (joint hypothesis of correct dynamics in the moments of interest and correct functional form of the density). Following Diebold's (1998) seminal work, we work with Rosenblatt's probability integral transforms (PIT) associated with the point forecasts. Under the null, the PITs must be i.i.d uniformly distributed U[0,1]. The Generalized AutoContouR (G-ACR) is a device (set of points) that is very sensitive to departures from the null in either direction and consequently, it provides the basis for very powerful tests. More specifically, for a time series of PITs, the G-ACRs are squares (in the univariate case) or hyper-cubes (in the multivariate case) of different (probability) areas or volumes within the maximum square or hyper-cube formed by a multidimensional uniform density $[0,1]^n$ for $n \geq 2$. By statistically comparing the location of the empirical PITs and the volume of the empirical G-ACRs with the location and volume of the population G-ACRs, we are able to construct a variety of tests for correct density forecast. Since the shapes of the G-ACRs can be visualized, we can extract information about where and how the rejection of the null hypothesis comes from. A great advantage of our approach is that it can also be applied quite easily to multivariate random processes of any dimension.

In a potential unstable data environment, we will form rolling subsamples within the forecasting sample. For every subsample, we apply a battery of G-ACR tests, and to detect instabilities, we construct a Sup- and an Ave-type statistics. Though the limiting distribution of these tests is a function of Brownian motions, the tests are nuisance-parameter free and their distribution can be tabulated. If the null hypothesis is rejected, we will be able to detect the timing of the break(s) rather accurately and the unstable periods. Though precise break detection is difficult, it is very important (Elliott and Muller, 2007) because it will help to improve the model forecast. Decisions

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[1]Though we focus on out-of-sample density forecasts, the methodology proposed in this paper can also be applied to in-sample specification testing.

regarding the choice of the estimation and evaluation windows and the estimation methods (e.g. recursive/rolling or time-varying parameter estimation) will be affected by the location of the break (Pesaran and Timmermann, 2007). In some instances, the break process can be modeled (Pesaran, Pettenuzzo, and Timmermann, 2006) and added to the model forecast.

The paper is organized as follows. In section 2, we review the G-ACR approach and introduce the new statistics with their asymptotic distributions. In section 3, we assess the finite sample properties (size and power) of the tests, and show how to detect potential breaks. We offer an extensive assessment by considering fixed, rolling, and recursive estimation schemes. In section 4, we apply the tests to assess the stability of the Phillips curve from 1958 on by evaluating the models proposed in Amisano and Giacomini (2007). We conclude in section 5. The appendix contains mathematical proofs.

2   Statistics and Asymptotic Distributions

2.1   Construction of the Statistics

The test statistics are based on the autocontour (ACR) and generalized autocontour (G-ACR) methodology proposed by González-Rivera *et al.* (2011, 2012, 2014) that provides powerful tests for dynamic specification of the conditional density model either in-sample or out-of-sample environments. In the present context, we adapt these tests to instances where instabilities may be present in the data so that, beyond the evaluation of the density model, we will also be able to detect unstable periods.

Let Y_t denote the random process of interest with conditional density function $f(y_t|\Omega_{t-1})$, where Ω_{t-1} is the information set available up to time $t-1$. Observe that the random process Y_t could enjoy very general statistical properties, e.g. heterogeneous, dependence, etc. The researcher will construct the conditional model by specifying a conditional mean, conditional variance or other conditional moments of interest, and making distributional assumption on the functional form of $f(.)$. Based on the conditional model, she will proceed to construct a density forecast. If the

proposed predictive density model for Y_t, i.e. $\{f_t^*(y_t|\Omega_{t-1})\}_{t=1}^T$ coincides with the true conditional density $\{f_t(y_t|\Omega_{t-1})\}_{t=1}^T$, then the sequence of probability integral transforms (PIT) of $\{Y_t\}_{t=1}^T$ w.r.t $\{f_t^*(y_t|\Omega_{t-1})\}_{t=1}^T$ i.e. $\{u_t\}_{t=1}^T$ must be $i.i.d\ U(0,1)$ where $u_t = \int_{-\infty}^{y_t} f_t^*(v_t|\Omega_{t-1})dv_t$. Thus, the null hypothesis $H_0 : f_t^*(y_t|\Omega_{t-1}) = f_t(y_t|\Omega_{t-1})$ is equivalent to the null hypothesis $H_0' : \{u_t\}_{t=1}^T$ is $i.i.d\ U(0,1)$ (see Diebold $et\ al.$, 1998). Following González-Rivera and Sun (2014), we construct the generalized autocontours (G-ACR) under i.i.d. uniformity of predictive densities. Under $H_0' : \{u_t\}_{t=1}^T\ i.i.d\ U(0,1)$, the G-ACR$_{\alpha_i,k}$ is defined as the set of points in the plane (u_t, u_{t-k}) such that the square with $\sqrt{\alpha_i}$-side contains $\alpha_i\%$ of observations, i.e.,

$$\text{G-ACR}_{\alpha_i,k} = \{B(u_t, u_{t-k}) \subset \Re^2 \| \ 0 \leq u_t \leq \sqrt{\alpha_i} \text{ and } 0 \leq u_{t-k} \leq \sqrt{\alpha_i}, \ s.t. : u_t \times u_{t-k} \leq \alpha_i\}$$

We construct an indicator series I_t^{k,α_i} as follows

$$I_t^{k,\alpha_i} = \mathbf{1}((u_t, u_{t-k}) \subset \text{G-ACR}_{\alpha_i,k}) = \mathbf{1}(0 \leq u_t \leq \sqrt{\alpha_i}, 0 \leq u_{t-k} \leq \sqrt{\alpha_i})$$

Based on this indicator, González-Rivera and Sun (2014) proposed the following t-tests and chi-square statistics to test the null hypothesis $H_0' : \{u_t\}_{t=1}^T\ i.i.d\ U(0,1)$, which is equivalent to testing for correct specification of the full conditional density model.

(1) t-ratio testing

$$\frac{\sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)}{\sigma_{k,i}} \xrightarrow{d} N(0,1)$$

where $\widehat{\alpha}_i = \frac{\sum_{t=k+1}^T I_t^{k,\alpha_i}}{T-k}$, and $\sigma_{k,i}^2$ is the asymptotic variance of $\widehat{\alpha}_i$.

(2) chi-squared testing

(2.1) For a fixed autocontour α_i, $L_{\alpha_i}' \Lambda_{\alpha_i}^{-1} L_{\alpha_i} \xrightarrow{d} \chi_K^2$ where $L_{\alpha_i} = (\ell_{1,\alpha_i}, ... \ell_{K,\alpha_i})'$ is a $K \times 1$ stacked vector with element $\ell_{k,\alpha_i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$, and Λ_{α_i} is the asymptotic variance-covariance matrix of the vector L_{α_i}.

(2.2) For a fixed lag k, $\mathbf{C}_k' \Omega_k^{-1} \mathbf{C}_k \xrightarrow{d} \chi_C^2$ where $\mathbf{C}_k = (c_{k,1}, ... c_{k,C})'$ is a $C \times 1$ stacked vector with

element $c_{k,i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$, and Ω_k the asymptotic variance-covariance matrix of the vector $\mathbf{C}_k$.

In a potential unstable environment, we will construct the tests within the following rolling sample scheme. The total sample size T is divided into two parts: in-sample observations (R) and out-of-sample observations (P). We form subsamples of size r from $t-r+1$ up to t, where $t = R+r, \cdots, T$. In each subsample, we evaluate the proposed predictive density by calculating three different statistics (t, C and L). As a result, we obtain three sets of $n \equiv T - r - R + 1$ tests each i.e., $\{t_j\}_{j=1}^n$, $\{C_j\}_{j=1}^n$ and $\{L_j\}_{j=1}^n$. Finally, to detect instabilities, we construct Sup-type and Avg-type statistics by taking the supremum (S) and the average (A) respectively over each set $\{|t_j|\}_{j=1}^n$, $\{C_j\}_{j=1}^n$ and $\{L_j\}_{j=1}^n$ so that we obtain the following six statistics: $S_{|t|}$, S_C, S_L and $A_{|t|}$, A_C, A_L.

2.2 Asymptotic Properties of the Statistics

Under the following set of assumptions, we provide three propositions, which proofs are provided in the Appendix.

- A1: For $T \to \infty$, $R \to \infty$, $P \to \infty$, $\lim_{T\to\infty} \frac{P}{R} = 0$ and $\lim_{T\to\infty} \frac{r-k}{P} = m$, as $r, P \to \infty$, where r is the size of the rolling subsample in the evaluation set, $m \in (0,1)$ and k is the lag in the indicator I_t^{k,α_i} [2].

- A2: $E|I_t^{k,\alpha_i}|^q < \Delta < \infty$ for some $q \geq 2$. This assumption is trivial as the second moments of the indicator are well defined as we will see next.

- A3: The data $\{y_t\}$ comes from a stationary and ergodic process [3].

Proposition 1 Let J be the index that identifies a particular subsample in the evaluation period, i.e. $J = [Ps]$, $s \in [m,1]$, $[Pm] = r - k$, and let $\widehat{\alpha}_i(J) = \frac{\sum_{t=R+1+J-r+k}^{R+J} I_t^{k,\alpha_i}}{r-k}$ be the corresponding

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[2] When this assumption is not satisfied, e.g., in a rolling forecasting scheme when the estimation sample R tends to be small and $\lim_{T\to\infty} \frac{P}{R} \neq 0$ we, in practice, implement a parametric bootstrapping procedure to assess the distribution of the statistics.

[3] This assumption can be relaxed to include more general mixing processes because the relevant conditions to invoke limiting theorems as the FCLT are those of the indicator process.

subsample proportion based on the indicator. The Sup- and Avg-tests are

$$
S_{|t|} = \sup_{J} \left| \frac{\sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,\alpha_i}} \right|
$$

$$
A_{|t|} = \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \left| \frac{\sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,\alpha_i}} \right|
$$

where $\sigma_{k,\alpha_i}^2 = \alpha_i(1-\alpha_i) + 2\alpha_i^{3/2}(1-\alpha_i^{1/2})$

Given assumptions A1-A3, and under the null hypothesis of $i.i.d\ U(0,1)$ PITs, the asymptotic distribution of the tests, for $P \to \infty$, are as follows

$$
S_{|t|} \xrightarrow{d} \sup_{s \in [m,1]} \frac{|W(s) - W(s-m)|}{\sqrt{m}}
$$

$$
A_{|t|} \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{|W(s) - W(s-m)|}{\sqrt{m}} ds
$$

where $W(.)$ is a standard univariate Brownian motion.

Proposition 2 For a given lag k, write $c_{k,i}(J) = \sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)$ and stack $c_{k,i}(J)$ for different autocontours levels $i = 1, 2, ...C$ such that $\mathbf{C}_k(J) = (c_{k,1}(J), ...c_{k,C}(J))'$ is the $C \times 1$ stacked vector. The Sup- and Avg-tests are

$$
S_C = \sup_{J} \mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J)
$$

$$
A_C = \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \mathbf{C}_k(J)' \Omega_k^{-1} \mathbf{C}_k(J)
$$

where Ω_k is the asymptotic variance and covariance matrix of the random vector $\mathbf{C}_k(J)$. Given assumptions A1-A3, and under the null hypothesis of $i.i.d\ U(0,1)$ PITs, the asymptotic distribution of the tests, for $P \to \infty$, are as follows,

$$
S_C \xrightarrow{d} \sup_{s \in [m,1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m}
$$

$$
A_C \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m} ds
$$

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where $\mathbf{W}(.)$ is a standard C-variate Brownian motion.

Proposition 3 For a given contour α_i, write $\ell_{k,\alpha_i}(J) = \sqrt{r-k}(\widehat{\alpha}_i(J) - \alpha_i)$ and stack ℓ_{k,α_i} for $k = 1,....K$. Let $\mathbf{L}_{\alpha_i}(J) = (\ell_{1,\alpha_i}(J),...\ell_{K,\alpha_i}(J))'$ be the $K \times 1$ stacked vector. The Sup- and Avg-tests are

$$S_L = \sup_{J} \mathbf{L}_{\alpha_i}(J)'\Lambda_{\alpha_i}^{-1}\mathbf{L}_{\alpha_i}(J)$$

$$A_L = \frac{1}{P - r + 1}\sum_{\underline{J}}^{\bar{J}} |\mathbf{L}_{\alpha_i}(J)'\Lambda_{\alpha_i}^{-1}\mathbf{L}_{\alpha_i}(J)|$$

where Λ_{α_i} is the asymptotic variance and covariance matrix for the random vector $\mathbf{L}_{\alpha_i}$. Given assumptions A1-A3, and under the null hypothesis of $i.i.d\ U(0,1)$ PITs, the asymptotic distribution of the tests, for $P \to \infty$, are as follows,

$$S_L \xrightarrow{d} \sup_{s\in[m,1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m}$$

$$A_L \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m}ds$$

where $\mathbf{W}(.)$ is a standard L-variate Brownian motion.

We tabulate the percentiles of the asymptotic distributions of the tests provided in Propositions 1 to 3. Since the distributions depend on m, which is proportion of the rolling sample to the total evaluation sample, we consider the following values of $m \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$. In Table 1, we report the percentiles of the distribution of the $S_{|t|}$ and $A_{|t|}$ statistics; in Table 2, those for the S_C and A_C considering the 13 autocontour $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$; and in Table 3 those for the S_L and A_L tests considering 5 lags.

[TABLES 1-3 ABOUT HERE]

Since a model can be considered misspecified whenever does not take into account breaks present in the data, the proposed stability tests can also be viewed as statistics assessing jointly the specification and the time stability of the model. However, in practice, the tests will be most useful

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when we have models that have passed in-sample specification tests and we wish to understand whether the model is stable over different time periods, which may not be necessarily included in the estimation sample. In the following sections, Monte Carlo simulations and the evaluation of the Phillips curve, we show how to use the stability tests.

3 Monte Carlos Simulations

We perform extensive Monte Carlos simulations to assess the finite sample properties (size and power) of the proposed statistics. For $\sqrt{R}$-consistent estimators of the parameters of the model, i.e. $(\widehat{\theta} - \theta_0) = O_p(R^{-1/2})$ with a well-defined asymptotic distribution, and under assumption A1, $R \to \infty$, $P \to \infty$, and $P/R \to 0$ as $T \to \infty$, parameter uncertainty is asymptotically negligible. In this case, the critical values tabulated in Tables 1 to 3 can be used directly. In cases where the condition $P/R \to 0$ is violated, we can either bootstrap the variance of the tests (and use the tabulated critical values) or the tests themselves to approximate their asymptotic distribution. In the following simulations, we keep the ratio P/R constant and we implement a parametric bootstrap for the distribution of the statistics. This is a sensible approach because the null hypothesis fully specifies the parametric data generating process. We consider fixed, rolling, and recursive forecasting schemes.

3.1 Size of the Tests

Under the null hypothesis of a stable density model, we consider the following data generating process: $y_t = \alpha_1 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$ where $x_t = \phi_1 + \phi_2 x_{t-1} + \varepsilon_t$, and $\varepsilon_t \sim N(0, 1)$, $\phi_1 = 1.38$, $\phi_2 = 0.77$, $\alpha_1 = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$. We consider sample sizes of $T = 150$ (with evaluation sample $P{=}60$), $T = 375$ (with $P{=}150$), and $T = 750$ (with $P{=}300$) observations, and for each sample size, we consider the proportion $m = r/P$ equal to $m = 1/3$, $m = 1/2$ and $m = 2/3$. We maintain the ratio P/R constant and equal to 2/3. In total, we run nine experiments, of which we present here the two most extremes: small sample size with small subsample window

$(T = 150, m = 1/3)$ and large sample size with large subsample window $(T = 750, m = 2/3)$. The size results of these two cases are presented in Tables 4 and 5 respectively. The remaining seven cases are available in a "supplementary material" file. We work with 13 autocontour coverage levels $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$. The number of Monte Carlo replications is 1,000, and the number of bootstrap samples is 500.

[TABLES 4-5 ABOUT HERE]

The overall size of the tests is very good in the nine experiments considered. There are not substantial differences among the fixed, rolling, and recursive estimation schemes. We find some small size distortions (under-sized) when the sample is small $(T = 150)$ and the autoncotour levels are extreme (1% and 99%) but as the sample size increases, the distortion disappears. For the individual tests $S_{|t|}^{k,\alpha_i}$ and $A_{|t|}^{k,\alpha_i}$ (k and α_i fixed), the Ave-test tends to have better size than the Sup-test. The umbrella tests S_C, A_C, S_L, and A_L have very good size even in small samples.

3.2   Power of the Tests

To assess the power of the tests, we generate data from four different processes, all of them containing a break point. The model that we maintain under the null hypothesis is the same as the one considered in the study of the size properties: $y_t = \alpha_1 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$ with $x_t = \phi_1 + \phi_2 x_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$. The total sample size (T) is 650, $R = 350$, $P = 300$, and $m = 1/3$. The break point happens at at $R + \tau P$, where $\tau = 1/3$. In the following experiments, the number of Monte Carlo replications is 1000 and bootstrapped samples 500. We maintain a nominal test size of 5%.

The four data generating mechanisms are the following:

DGP1: Break in the intercept of $y_t = \alpha_t + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0,1)$:

$$\alpha_t = \begin{cases} \alpha_1 = 1.5 & \text{if } t < \text{break} \\ \alpha_2 = 2 & \text{otherwise} \end{cases}$$

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with $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$.

DGP2: Break in the variance of $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma_t \epsilon_t$, $\epsilon_t \sim N(0,1)$:

$$\sigma_t = \begin{cases} \sigma_1 = 1.5 & \text{if } t < \text{break} \\ \sigma_2 = 1.8 & \text{otherwise} \end{cases}$$

with $\alpha = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$.

DGP3: Breaks in the slope coefficients of $y_t = \alpha + \beta_{1,t} y_{t-1} + \beta_{2,t} x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0,1)$:

$$\beta_{1,t} = \begin{cases} \beta_{1,1} = 0.5 & \text{if } t < \text{break} \\ \beta_{1,2} = 0.3 & \text{otherwise} \end{cases}$$

$$\beta_{2,t} = \begin{cases} \beta_{2,1} = 0.6 & \text{if } t < \text{break} \\ \beta_{2,2} = 0.4 & \text{otherwise} \end{cases}$$

with $\alpha = 1.5$, $\sigma = 1$.

DGP4: Breaks in the intercept, variance and slope coefficients of $y_t = \alpha_t + \beta_{1,t} y_{t-1} + \beta_{2,t} x_{t-1} + \sigma_t \epsilon_t$, $\epsilon_t \sim N(0,1)$:

$$\alpha_t = \begin{cases} \alpha_1 = 1.5 & \text{if } t < \text{break} \\ \alpha_2 = 2 & \text{otherwise} \end{cases}$$

$$\sigma_t = \begin{cases} \sigma_1 = 1.5 & \text{if } t < \text{break} \\ \sigma_2 = 1.8 & \text{otherwise} \end{cases}$$

$$\beta_{1,t} = \begin{cases} \beta_{1,1} = 0.5 & \text{if } t < \text{break} \\ \beta_{1,2} = 0.3 & \text{otherwise} \end{cases}$$

$$\beta_{2,t} = \begin{cases} \beta_{2,1} = 0.6 & \text{if } t < \text{break} \\ \beta_{2,2} = 0.4 & \text{otherwise} \end{cases}$$

Note that the breaks considered are not very extreme. We perform all the simulations under fixed, rolling, and recursive estimation schemes. We report the power results for the fixed and rolling

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schemes in Tables 6 to 9, and those for the recursive scheme in the supplementary material.

Under the fixed scheme, the tests are most powerful (power of about 90%) to detect breaks in intercept and slope coefficients (DGP1, DGP3, and DGP4). Both Ave- and Sup- tests enjoy similar performance, either for single hypothesis ($S_{|t|}$ and $A_{|t|}$) or for joint hypothesis (S_C, A_C, S_L, A_L). The power drops when testing for breaks in the variance (DGP2) with values of 10-60% for the single hypothesis tests and of 60% for the joint hypothesis tests. Under the rolling scheme, as we expected, the tests are less powerful overall because by rolling the estimation sample, the model adjusts slowly to the new parameters values. Nevertheless, the power of S_C, A_C, S_L and A_L is still very high (around 50-80%) for DGP1, DGP3, and DGP4, and around 40% for DGP2. For all DGPs, the Ave-test is more powerful that the Sup. Under the recursive scheme, the tests performance is slightly worse than in the fixed scheme but slightly better than in the rolling scheme.

3.3 Detection of the break point

An advantage of these tests is that they can be informative regarding the approximated location of the break in the data. Though we do not provide an exact test for a break, we will show how to spot the periods of time where the instability happens. We consider two cases: a break in the intercept of the model (DGP1B) and a break in the variance (DGP2B). We generate a time series of $T = 600$ observations from each of the following processes:

DGP1B: $y_t = \alpha_t + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0,1)$

$$\alpha_t = \begin{cases} \alpha_1 = 1.5 & \text{if } t < \text{break} \\ \alpha_2 = 0.1 & \text{otherwise} \end{cases}$$

with $\beta_1 = 0.5$, $\beta_2 = 0.6$, $\sigma = 1$, and the break occuring at the 480 observation, i.e. break=480.

DGP2B: $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma_t \epsilon_t$, $\epsilon_t \sim N(0,1)$

$$\sigma_t = \begin{cases} \sigma_1 = 2 & \text{if } t < \text{break} \\ \sigma_2 = 0.5 & \text{otherwise} \end{cases}$$

with $\alpha = 1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, and break=480 observation.

We proceed by estimating the model $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim N(0,1)$ and choosing the following samples R=350, P=250, and $m = r/P = 100/250 = 0.4$. We implement the three estimation schemes (fixed, recursive, and rolling). For each subsample ($r = 100$) in the evaluation window P we calculate the t-ratio and C statistics for a total of 151 tests ($T - R + 1 - r$). Next, from the asymptotic distribution tables, we choose the 5% critical values corresponding to $S_{|t|}$ and S_C as the thresholds that will help to locate the first significant subsample that should contain the break point. Given the sample windows that we have chosen, i.e. $P/R = 0.71$, we recommend bootstrapping the tests $S_{|t|}$ and S_C in order to find the 5% critical value. In Figures 1 and 2, we plot the 151 t-tests and C-tests in a sequential fashion together with the bootstrapped 5% critical values corresponding to the distribution of $S_{|t|}$ and S_C under the three estimation schemes and for both DGP1B and DGP2B.

Regarding DGP1B, the C-tests react faster to the break than the t-tests for all estimation schemes. For instance, under the fixed scheme, it is the 80th t-statistics that hits the threshold for the first time (from that point on all t-tests are above the threshold) and it marks the first subsample that contains the break, which corresponds to the interval of observations [430, 529]. For the C-statistics, it is the 36th test hitting the threshold line for the first time, which corresponds to the observations in the subsample [386, 485]. The break must be contained in the intersection of these two subsamples, i.e. [430, 485]. The actual break happens at the 480th observation. We observe that the maximum values of the t-tests and C-tests are those of the 130th statistic, which corresponds to the subsample [480, 579]. From this point on, the values of the tests start decreasing, which means that the break point is already passed. As expected, under the recursive and rolling schemes, the tests react slower to the break but nevertheless they are also accurate on pointing out the subsample where the break occurs.

Regarding DGP2B, both t-tests and C-tests react at about the same time. For instance, under the recursive scheme, it is the 73th statistic (for both t and C tests) that hits the threshold for the first time. This corresponds to the subsample [423, 522]. The maximum values of the t-tests and C-tests are those of the 130th statistic, which corresponds to the subsample [480, 579], and putting these two pieces of information together, we locate the break within the observations [480, 522].
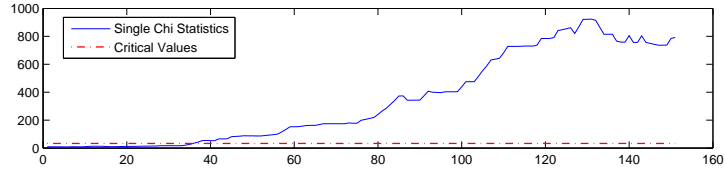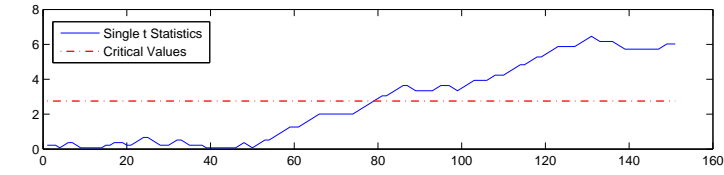
Fixed Scheme
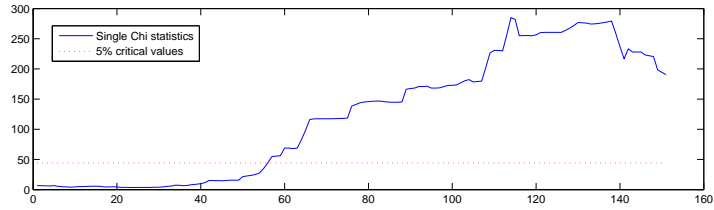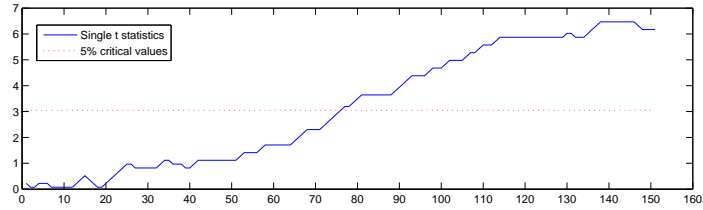
Recursive Scheme
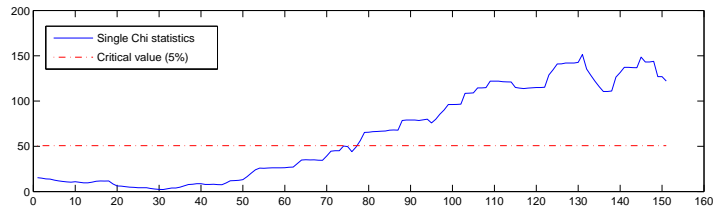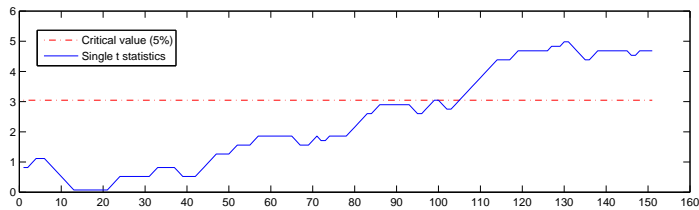
Rolling Scheme

Figure 1: **Plots of t and C Statistics for DGP1B: Break in the intercept**
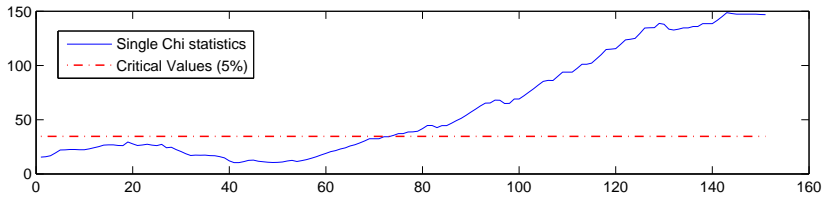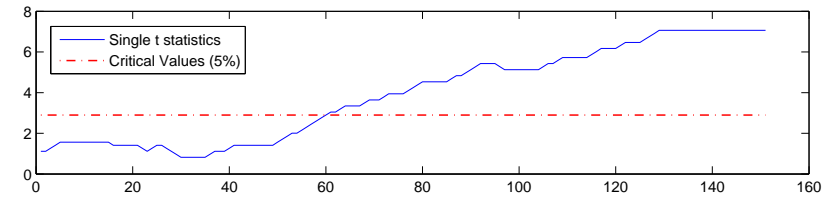
Fixed Scheme
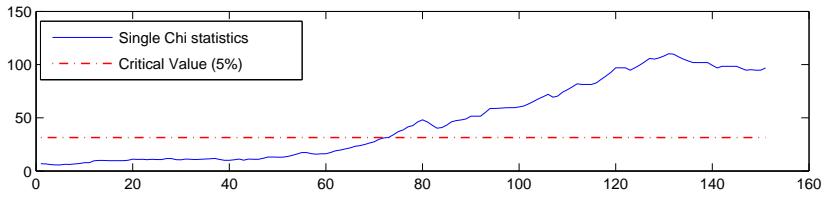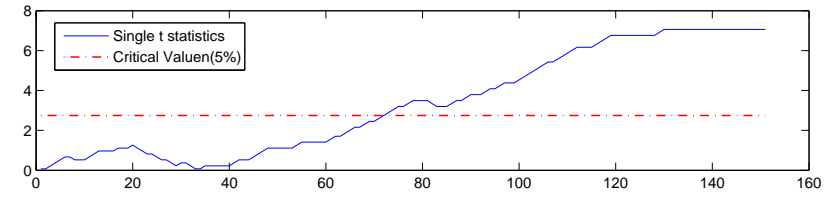
Recursive Scheme
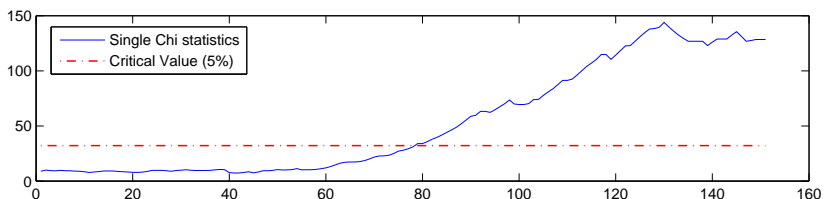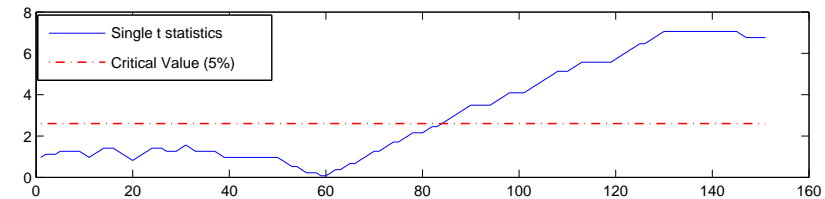
Rolling Scheme

Figure 2: **Plots of t and C Statistics for DGP2B: Break in the variance**

4 Density Forecast Evaluation of the Phillips Curve

We apply the proposed tests to analyze the stability of the Phillips curve. Stock and Watson (1999) found some empirical evidence in favor of the Phillips curve as a forecasting tool, they showed that inflation forecasts produced by the Phillips curve were more accurate than forecasts based on simple autoregressive or multivariate models but they also found parameter instabilities across different subsamples. Rossi and Skehposyan (2010) showed that the predictive power of the Phillips curve disappeared around the time of the Great Moderation. Based on scoring rules, Amisano annd Giacomini (2007) compared the density forecast accuracy of several models of the Phillips curve and concluded that the best density forecast is produced by a Markov-switching model. Since their comparisons are based on the *average* forecasting performance of competing models over time, they cannot directly address the instabilities widely documented in the literature. In this section, we consider the models in Amisano annd Giacomini (2007) and we focus on their absolute density forecast performance in the presence of instabilities. Our starting model is a linear Phillips curve (Stock and Watson, 1999), in which changes of the inflation rate depend on their lags and on lags of the unemployment rate i.e.,

$$\Delta \pi_t = \alpha_1 + \beta_1 \Delta \pi_{t-1} + \beta_2 \Delta \pi_{t-2} + \beta_{12} \Delta \pi_{t-12} + \gamma u_{t-1} + \sigma \epsilon_t$$

where $\pi_t = 100 \times \ln(CPI_t/CPI_{t-12})$; CPI_t is the consumer price index for all urban consumers and all items; u_{t-1} is the civilian unemployment rate; and $\epsilon_t \sim N(0,1)$. The dats is collected from the FRED database; monthly CPI and unemployment series are both seasonally adjusted. The time series run from 1958M01 to 2012M01 (updated sample from 1958M01-2004M07 in Amisano and Giacomini). Standard tests on $\Delta \pi_t$ and u_t show that they do not have a unit root. On implementing our tests, we consider the same estimation sample as in Amisano and Giacomini, from 1958M01 to 1987M12 (360 observations). The evaluation sample runs from 1988M01 to 2012M01 (289 observations) with subsamples of size $r = 200$.

4.1 Evaluation of the Linear Phillips Curve

We present the evaluation results for the linear Phillips Curve in Table 10 under fixed and rolling estimation schemes. The recursive case is in the supplementary material.

[TABLE 10 ABOUT HERE]

The umbrella tests S_C, A_C, S_L, and A_L indicate a clear rejection of the linear model. On examining the individual tests $S_{|t|}$ and $A_{|t|}$, the rejection comes from the middle autocontours, between 40% and 70% coverage, and from the 95-99% levels (large changes in inflation). In Figure 3, we plot the t- and C-statistics over the evaluation period in a sequential fashion. The tests break through their corresponding critical values around the 60th statistic. From this point on, the values of the tests keep on increasing reaching two local maxima, which points to two potential breaks: the first in the 64th statistic for the t-tests (1993M03 to 2009M11) and in the 71th statistic for the C tests (1993M11 to 2010M06), and the second local maximum in the 78th statistic that corresponds to the period 1994M06 to 2011M01. All these periods include the years 1993-2007 in the so-called Great Moderation and the years after the deep financial crisis of 2008.

4.2 Evaluation of Non-Linear Phillips Curve

Given the rejection of the linear Phillips curve, we proceed with a flexible specification by assuming that the coefficients in the linear model vary according to a Markov switching mechanism. We consider a two-state Markov switching model, i.e.,

$$\Delta \pi_t = \alpha^{s_t} + \beta_1^{s_t} \Delta \pi_{t-1} + \beta_2^{s_t} \Delta \pi_{t-2} + \beta_{12}^{s_t} \Delta \pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma^{s_t} \epsilon_t$$

where the unobserved state variable s_t switches between two states,1 or 2, with transition probabilities $Pr(s_t = j | s_{t-1} = i) = p_{ij}$ for $i, j = 1, 2$; and ϵ_t is assumed to be a standard normal variate. Thus, this model allows for non-Gaussian density forecasts. Since all the coefficients depend on the state variable (Model 1), the model is extremely flexible and it will adapt to potential breaks or instabilities that may occur over time. We have run our test statistics and we fail to reject the null
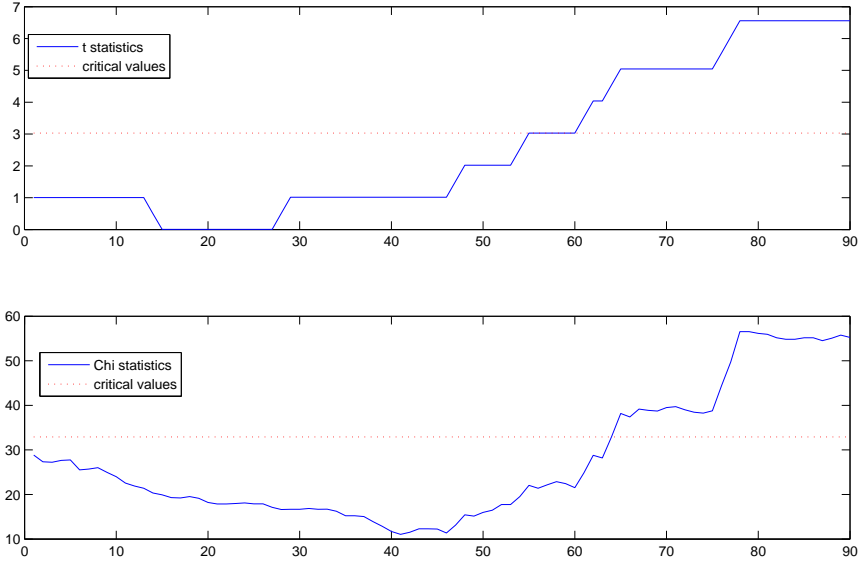
Figure 3: **Plots of t-ratio (99% autocontour) and C Statistics for Linear Phillips Curve (fixed scheme)**

hypothesis of correct specification. However, we would like to investigate what coefficients are key to understand where the nonlinear behavior comes from. We consider two additional specifications,

Model 2: $\Delta \pi_t = \alpha + \beta_1^{s_t} \Delta \pi_{t-1} + \beta_2^{s_t} \Delta \pi_{t-2} + \beta_{12}^{s_t} \Delta \pi_{t-12} + \gamma u_{t-1} + \sigma \epsilon_t$, (intercept and unemployment coefficient do not depend on s_t).

Model 3: $\Delta \pi_t = \alpha^{s_t} + \beta_1 \Delta \pi_{t-1} + \beta_2 \Delta \pi_{t-2} + \beta_{12} \Delta \pi_{t-12} + \gamma^{s_t} u_{t-1} + \sigma_{s_t} \epsilon_t$, (inflation coefficients do not depend on s_t).

We present the test results in Table 11.

[TABLE 11 ABOUT HERE]

For Model 2, we still fail to reject the overall model as the umbrella tests S_C, A_C, S_L, and A_L have p-values larger than 5%. However, on a close examination of the individual tests $S_{|t|}$ and $A_{|t|}$, we observe a rejection of the model in the autocontours 90 to 99%. In Model 3, we allow the intercept, the unemployment coefficient and the variance of the error to be state dependent while the rest of the parameters are constant. In this case, all statistics (joint and individual)

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enjoy p-values larger than 5% and consequently we fail to reject the model even in the presence of instabilities. A nonlinear Phillips curve has better predictive performance than a linear model, which is unstable over time. The advantage comes from letting the coefficient that links inflation and unemployment to be state-dependent.

5 Conclusion

We have provided a battery of tests to assess the stability of the density forecast over time, which offer important advantages for the empirical researcher. These tests are nuisance-parameter free and their asymptotic distributions can be tabulated. If the tests reject the null hypothesis of a stable density forecast, the shapes of the empirical generalized autocontours can be visualized to extract information regarding the direction of the rejection. Regardless of the estimation scheme (fixed, rolling, or recursive), their finite sample properties are superior. In some instances, the Ave-tests tend to have a slightly better size than the Sup-tests. Both types are more powerful on detecting breaks in the intercept and slope coefficients than on detecting breaks in the variance. In addition, we have proposed a rather accurate procedure to find the location of the breaks and the unstable periods. The tests can also be easily applied to multivariate random processes of any dimension. As an application of the tests, we have analyzed the stability of the Phillips curve. A linear model is strongly rejected in favor of a non-linear specification that allows the coefficient linking inflation changes and unemployment to be state-dependent. The break of the linear model occurs during the Great Moderation years.

Appendix

Proof of Proposition 1:

The indicator function I_t^{k,α_i} is a Bernoulli random variable with the following moments: $E(I_t^{k,\alpha_i}) = \alpha_i$, $Var(I_t^{k,\alpha_i}) = \alpha_i(1-\alpha_i)$ and covariance

$$r_h^{\alpha_i} \equiv cov(I_t^{k,\alpha_i}, I_{t-h}^{k,\alpha_i}) = \begin{cases} 0 & \text{if } h \neq k \\ \alpha_i^{3/2}(1-\alpha_i^{1/2}) & \text{if } h = k \end{cases}$$

Since the indicator process is stationary and ergodic, $\widehat{\alpha}_i(J)$ satisfies the condition of global covariance stationarity required for the FLCT to apply (Theorem 7.17) in White (2001). Since $J = [Ps]$, $s \in [m,1]$, and $r - k = [Pm]$, we write as $P \to \infty$

$$\begin{aligned} W_P(s) &\equiv \frac{\sqrt{(r-k)}(\widehat{\alpha}_i(J) - \alpha_i)}{\sigma_{k,i}} = \frac{\sum_{t=R+1+J-r+k}^{R+J}(I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{r-k}\sigma_{k,i}} \\ &= \frac{\sum_{t=R+1}^{R+J}(I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{(r-k)}\sigma_{k,i}} - \frac{\sum_{t=R+1}^{R+J-(r-k)}(I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{(r-k)}\sigma_{k,i}} \\ &= \frac{\sqrt{P}}{\sqrt{Pm}}\frac{\sum_{t=R+1}^{R+[Ps]}(I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{P}\sigma_{k,i}} - \frac{\sqrt{P}}{\sqrt{Pm}}\frac{\sum_{t=R+1}^{R+[P(s-m)]}(I_t^{k,\alpha_i} - \alpha_i)}{\sqrt{P}\sigma_{k,i}} \\ &\xrightarrow{d} \frac{1}{\sqrt{m}}(W(s) - W(s-m)) \end{aligned}$$

where $W(.)$ is the standard Brownian motion and the limiting distribution, in the last line, is a direct consequence of the FCLT. Finally, by the Continuous Mapping Theorem, we have:

$$\begin{aligned} S_{|t|} &= \sup_J |\frac{\sqrt{r-k}\widehat{\alpha}_i(J)}{\sigma_{k,\alpha_i}}| \xrightarrow{d} \sup \frac{|W(s) - W(s-m)|}{\sqrt{m}} \\ A_{|t|} &= \frac{1}{P-r+1}\sum_{\underline{J}}^{\bar{J}} |\frac{\sqrt{r-k}\widehat{\alpha}_i(J)}{\sigma_{k,\alpha_i}}| \xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{|W(s) - W(s-m)|}{\sqrt{m}} \end{aligned}$$

where $\underline{J} = [P\underline{s}]$ and $\bar{J} = [P\bar{s}]$.

Proof of Proposition 2

Let Ω_k be the variance-covariance matrix of $\mathbf{C_k}(J)$ whose typical element $\omega_{i,j}$ is calculated as follows

$$cov(c_{k,i}, c_{k,j}) = cov(I_t^{k,\alpha_i}, I_t^{k,\alpha_j}) + cov(I_t^{k,\alpha_i}, I_{t-k}^{k,\alpha_j}) + cov(I_{t-k}^{k,\alpha_i}, I_t^{k,\alpha_j}) + o(1)$$

If $i = j$, by Proposition 1, $\omega_{i,i} = var(\sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)) = \alpha_i(1 - \alpha_i) + 2\alpha_i^{3/2}(1 - \alpha_i^{1/2})$. If $i < j$, $\alpha_i < \alpha_j$, and we have

$$cov(I_t^{k,\alpha_i}, I_t^{k,\alpha_j}) = E(I_t^{k,\alpha_i} \times I_t^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i(1 - \alpha_j)$$

$$cov(I_t^{k,\alpha_i}, I_{t-k}^{k,\alpha_j}) = E(I_t^{k,\alpha_i} \times I_{t-k}^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i \times \alpha_j^{1/2} - \alpha_i \times \alpha_j$$

$$cov(I_{t-k}^{k,\alpha_i}, I_t^{k,\alpha_j}) = E(I_{t-k}^{k,\alpha_i} \times I_t^{k,\alpha_j}) - \alpha_i \times \alpha_j = \alpha_i \times \alpha_j^{1/2} - \alpha_i \times \alpha_j$$

If $i > j$, the above expressions hold by just switching the subindexes i and j.

Since the vector $\mathbf{C_k}(J)$ is globally stationary, we can invoke a multivariate FLCT (see Theorem 7.29 and 7.30 in White (2001)). By following the same arguments as in Proposition 1, we have as $P \to \infty$,

$$\begin{aligned} \mathbf{W}_P(s) &\equiv \Omega_k^{-1/2}\mathbf{C_k}(J) \\ &\xrightarrow{d} \frac{1}{\sqrt{m}}(\mathbf{W}(s) - \mathbf{W}(s-m)) \end{aligned}$$

where $\mathbf{W}(s)$ is a C-variate Brownian process. By the Continuous Mapping Theorem, we have

$$\begin{aligned} S_C &= \sup_J \mathbf{C}_k(J)'\Omega_k^{-1}\mathbf{C}_k(J) \\ &\xrightarrow{d} \sup_{s\in[m,1]} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m} \end{aligned}$$

and

$$A_C = \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} |\mathbf{C}_k(J)'\Omega_k^{-1}\mathbf{C}_k(J)|$$

$$\xrightarrow{d} \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(r)-\mathbf{W}(r-m))'(\mathbf{W}(r)-\mathbf{W}(r-m))}{m}$$

Proof of Proposition 3: Let Λ_{α_i} be the variance-covariance matrix of $\mathbf{L}_{\alpha_i}(J)$ whose typical element $\lambda_{j,k}$ is calculated as

$$\lambda_{j,k} = \begin{cases} \alpha_i(1-\alpha_i) + 2\alpha_i^{3/2}(1-\alpha_i^{1/2}) & \text{if } j=k \\ 4\alpha_i^{3/2}(1-\alpha_i^{1/2}) & \text{if } j \neq k \end{cases}$$

Therefore, the vector $\mathbf{L}_{\alpha_i}(J)$ is globally stationary, and we can invoke a multivariate FLCT (see Theorems 7.29 and 7.30 in White (2001)). By following the same arguments as in Proposition 1, we have as $P \to \infty$,

$$\mathbf{W}_P(s) \equiv \Lambda_{\alpha_i}^{-1/2}\mathbf{L}_{\alpha_i}(J)$$

$$\xrightarrow{d} \frac{1}{\sqrt{m}}(\mathbf{W}(s)-\mathbf{W}(s-m))$$

where $\mathbf{W}(s)$ is a L-variate Brownian process. By the Continuous Mapping Theorem, we have

$$S_L = \sup_{J} \mathbf{L}_{\alpha_i}(J)'\Lambda_{\alpha_i}^{-1}\mathbf{L}_{\alpha_i}(J)$$

$$\xrightarrow{d} \sup_{s\in[m,1]} \frac{(\mathbf{W}(s)-\mathbf{W}(s-m))'(\mathbf{W}(s)-\mathbf{W}(s-m))}{m}$$

$$
\begin{aligned}
A_L \quad &= \quad \frac{1}{P-r+1} \sum_{\underline{J}}^{\bar{J}} \mathbf{L}_{\alpha_{\mathsf{i}}}(J)' \Lambda_{\alpha_i}^{-1} \mathbf{L}_{\alpha_{\mathsf{i}}}(J) \\
&\xrightarrow{\;d\;} \quad \int_{\underline{s}}^{\bar{s}} \frac{(\mathbf{W}(s) - \mathbf{W}(s-m))'(\mathbf{W}(s) - \mathbf{W}(s-m))}{m}
\end{aligned}
$$

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Tables: Asymptotic Distributions

In the following six tables, the percentiles are obtained from 2000 replications with a sample size of 20,000 observations in each replication.

Asymptotic Distribution of $S_{|t|}$ Statistic									
	m								
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	3.950	3.713	3.510	3.396	3.428	3.213	3.089	3.112	2.951
95%	3.502	3.267	3.066	2.926	2.866	2.679	2.537	2.439	2.406
90%	3.292	2.987	2.845	2.642	2.571	2.361	2.240	2.121	2.015
80%	3.020	2.684	2.522	2.372	2.217	2.053	1.935	1.849	1.655
70%	2.843	2.489	2.315	2.156	1.999	1.834	1.697	1.582	1.392
60%	2.702	2.343	2.145	1.983	1.819	1.653	1.529	1.383	1.197
50%	2.569	2.203	2.015	1.830	1.664	1.515	1.365	1.221	1.021
40%	2.457	2.090	1.884	1.684	1.515	1.373	1.212	1.066	0.867
30%	2.339	1.975	1.747	1.540	1.379	1.228	1.071	0.915	0.730
20%	2.201	1.822	1.586	1.384	1.226	1.074	0.923	0.791	0.606
10%	2.033	1.629	1.420	1.199	1.055	0.901	0.761	0.631	0.479
5%	1.922	1.492	1.290	1.065	0.943	0.813	0.663	0.535	0.394
1%	1.730	1.265	1.072	0.829	0.739	0.631	0.498	0.409	0.287

Asymptotic Distribution of $A_{|t|}$ Statistic									
	m								
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	1.241	1.435	1.656	1.824	2.204	2.181	2.360	2.542	2.583
95%	1.078	1.199	1.355	1.487	1.697	1.694	1.774	1.854	1.998
90%	1.004	1.088	1.206	1.300	1.418	1.406	1.490	1.586	1.666
80%	0.918	0.970	1.035	1.101	1.115	1.128	1.183	1.264	1.256
70%	0.870	0.891	0.922	0.948	0.939	0.928	0.966	1.017	1.019
60%	0.825	0.825	0.827	0.837	0.818	0.796	0.807	0.824	0.825
50%	0.789	0.760	0.760	0.741	0.710	0.685	0.682	0.675	0.661
40%	0.752	0.706	0.690	0.653	0.615	0.581	0.551	0.534	0.507
30%	0.711	0.649	0.617	0.568	0.524	0.487	0.453	0.417	0.378
20%	0.673	0.589	0.544	0.497	0.451	0.409	0.358	0.320	0.274
10%	0.608	0.517	0.465	0.409	0.359	0.319	0.269	0.234	0.186
5%	0.567	0.458	0.413	0.342	0.310	0.260	0.219	0.188	0.140
1%	0.478	0.381	0.325	0.251	0.235	0.200	0.152	0.135	0.096

Table 1: Asymptotic Distributions of $S_{|t|}$ and $A_{|t|}$ Statistics

Asymptotic Distribution of S_C Statistic									
	m								
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	42.488	39.592	38.534	37.694	37.361	35.925	36.051	34.591	31.804
95%	37.159	34.956	32.983	32.277	30.902	29.597	29.008	27.773	25.962
90%	35.155	32.562	30.722	29.378	28.069	27.176	26.181	24.685	23.356
80%	32.624	29.964	28.071	26.673	25.329	24.182	22.935	21.621	20.175
70%	30.932	28.330	26.359	24.906	23.421	22.132	20.932	19.423	18.125
60%	29.689	26.991	24.908	23.416	21.858	20.579	19.179	17.901	16.550
50%	28.540	25.798	23.648	22.111	20.626	19.136	17.708	16.501	15.241
40%	27.482	24.522	22.468	20.900	19.254	17.672	16.440	15.111	13.829
30%	26.508	23.385	21.197	19.510	17.873	16.413	15.058	13.685	12.477
20%	25.351	22.174	19.836	18.193	16.514	14.927	13.582	12.250	10.898
10%	23.731	20.400	18.067	16.333	14.568	13.061	11.741	10.480	8.987
5%	22.376	18.902	16.821	14.879	13.338	11.876	10.383	9.037	7.852
1%	20.394	16.833	14.737	12.807	11.208	9.884	8.275	6.929	5.905

Asymptotic Distribution of A_C Statistic									
	m								
Percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	16.342	18.610	20.265	22.301	24.658	26.534	27.422	28.202	27.987
95%	15.298	16.576	17.794	18.927	19.924	20.838	21.866	22.084	22.533
90%	14.762	15.734	16.456	17.287	18.022	18.629	18.996	19.612	19.965
80%	14.099	14.648	15.143	15.477	16.014	16.449	16.761	16.975	17.096
70%	13.672	14.029	14.209	14.439	14.705	14.937	15.107	15.111	15.290
60%	13.295	13.393	13.478	13.566	13.597	13.552	13.629	13.790	13.772
50%	12.955	12.873	12.847	12.820	12.666	12.591	12.534	12.514	12.547
40%	12.623	12.388	12.178	12.014	11.722	11.471	11.290	11.362	11.311
30%	12.262	11.851	11.535	11.227	10.909	10.576	10.364	10.166	10.073
20%	11.828	11.264	10.837	10.423	9.991	9.555	9.288	8.965	8.786
10%	11.298	10.483	9.871	9.330	8.746	8.297	7.860	7.558	7.143
5%	10.840	9.895	9.120	8.436	7.849	7.358	6.750	6.380	5.967
1%	10.030	8.791	7.993	7.279	6.363	5.659	5.069	4.629	4.382

Table 2: Asymptotic Distribution of S_C and A_C Statistics (13 autocontours)

					m				
percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	26.825	25.069	23.492	23.010	22.057	20.621	19.801	19.370	17.756
95%	22.719	21.121	19.941	18.372	17.199	16.042	15.645	14.470	13.631
90%	20.867	19.109	17.489	16.318	15.203	14.241	13.301	12.552	11.636
80%	18.859	16.979	15.273	14.045	12.859	11.973	11.213	10.267	9.325
70%	17.452	15.505	13.917	12.671	11.665	10.585	9.708	8.788	7.949
60%	16.389	14.279	12.920	11.587	10.492	9.537	8.610	7.765	6.927
50%	14.810	12.425	11.052	9.796	8.827	7.916	6.830	6.071	5.192
40%	15.534	13.300	11.910	10.616	9.576	8.678	7.698	6.917	6.068
30%	13.970	11.675	10.164	8.891	7.999	6.948	6.048	5.213	4.423
20%	13.066	10.697	9.249	8.023	7.039	6.054	5.113	4.388	3.646
10%	11.896	9.535	8.158	7.011	5.863	4.966	4.213	3.484	2.743
5%	11.145	8.544	7.070	6.093	5.099	4.286	3.492	2.786	2.187
1%	9.583	7.267	5.857	4.821	3.950	3.173	2.585	1.907	1.510

Asymptotic Distribution of A_L Statistic

					m				
percentile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
99%	7.260	8.419	9.620	10.837	12.021	12.943	13.509	14.458	14.629
95%	6.507	7.336	7.948	8.396	9.045	9.636	10.116	10.569	10.913
90%	6.122	6.673	7.034	7.503	7.961	8.313	8.562	8.929	9.220
80%	5.705	6.038	6.259	6.452	6.643	6.822	6.995	7.104	7.309
70%	5.383	5.583	5.669	5.733	5.820	5.868	5.929	5.977	6.040
60%	5.164	5.208	5.200	5.198	5.212	5.158	5.191	5.122	5.128
50%	4.742	4.604	4.468	4.361	4.224	4.021	3.885	3.817	3.767
40%	4.951	4.890	4.831	4.770	4.663	4.570	4.506	4.452	4.438
30%	4.535	4.316	4.123	3.922	3.717	3.476	3.324	3.174	3.088
20%	4.294	3.988	3.712	3.418	3.174	2.973	2.715	2.584	2.480
10%	4.017	3.539	3.189	2.922	2.602	2.320	2.086	1.927	1.746
5%	3.752	3.237	2.867	2.508	2.246	1.921	1.695	1.459	1.292
1%	3.337	2.681	2.222	1.875	1.621	1.388	1.150	0.960	0.777

Table 3: Asymptotic Distribution of S_L and A_L Statistics (five lags)

Tables: Size and Power of the Tests

I. Small Sample (T) and Small Subsample Window (m)

T=150, R=90,P=60	$S^{1,1}_{|t|}$	$S^{1,2}_{|t|}$	$S^{1,3}_{|t|}$	$S^{1,4}_{|t|}$	$S^{1,5}_{|t|}$	$S^{1,6}_{|t|}$	$S^{1,7}_{|t|}$	$S^{1,8}_{|t|}$	$S^{1,9}_{|t|}$	$S^{1,10}_{|t|}$	$S^{1,11}_{|t|}$	$S^{1,12}_{|t|}$	$S^{1,13}_{|t|}$
fixed	0.042	0.037	0.033	0.046	0.049	0.037	0.04	0.051	0.045	0.045	0.048	0.036	0.039
rolling	0.028	0.04	0.041	0.041	0.038	0.042	0.04	0.04	0.041	0.046	0.044	0.036	0.038
recursive	0.029	0.041	0.042	0.04	0.039	0.039	0.038	0.039	0.044	0.042	0.039	0.035	0.032

	$A^{1,1}_{|t|}$	$A^{1,2}_{|t|}$	$A^{1,3}_{|t|}$	$A^{1,4}_{|t|}$	$A^{1,5}_{|t|}$	$A^{1,6}_{|t|}$	$A^{1,7}_{|t|}$	$A^{1,8}_{|t|}$	$A^{1,9}_{|t|}$	$A^{1,10}_{|t|}$	$A^{1,11}_{|t|}$	$A^{1,12}_{|t|}$	$A^{1,13}_{|t|}$
fixed	0.047	0.048	0.049	0.056	0.055	0.058	0.049	0.059	0.049	0.057	0.052	0.055	0.057
rolling	0.032	0.038	0.048	0.043	0.041	0.046	0.046	0.045	0.04	0.046	0.048	0.046	0.031
recursive	0.024	0.037	0.037	0.043	0.043	0.038	0.043	0.052	0.043	0.044	0.042	0.035	0.033

	$S^{1,13}_{C}$	$A^{1,13}_{C}$	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$
fixed	0.059	0.051	0.06	0.06	0.052	0.045	0.063	0.063	0.058	0.055
rolling	0.038	0.043	0.045	0.052	0.042	0.042	0.051	0.05	0.045	0.045
recursive	0.033	0.043	0.044	0.055	0.053	0.052	0.052	0.058	0.039	0.033

	$S^{1,7}_{|t|}$	$S^{2,7}_{|t|}$	$S^{3,7}_{|t|}$	$S^{4,7}_{|t|}$	$S^{5,7}_{|t|}$	$A^{1,7}_{|t|}$	$A^{2,7}_{|t|}$	$A^{3,7}_{|t|}$	$A^{4,7}_{|t|}$	$A^{5,7}_{|t|}$
fixed	0.04	0.038	0.042	0.04	0.038	0.049	0.053	0.054	0.051	0.039
rolling	0.04	0.033	0.032	0.033	0.034	0.036	0.038	0.039	0.04	0.032
recursive	0.038	0.037	0.031	0.035	0.034	0.043	0.039	0.04	0.05	0.037

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
In $S^{k,\alpha_i}_{|t|}$ and $A^{k,\alpha_i}_{|t|}$, lag k and autocontour α_i are fixed.
In $S^{k,\alpha}_{C}$ and $A^{k,\alpha}_{C}$, k is fixed and α is the total number of autocontours considered.
In S^{k,α_i}_{L} and A^{k,α_i}_{L}, up to k lags are considered and α_i is a fixed autocontour.

Table 4: **Size of the statistics:T=150 R=90 P=T-R=60 $m = 1/3$ (nominal size 5%)**

II. Large Sample (T) and Large Subsample Window (m)

T=750, R=450,P=300	$S^{1,1}_{|t|}$	$S^{1,2}_{|t|}$	$S^{1,3}_{|t|}$	$S^{1,4}_{|t|}$	$S^{1,5}_{|t|}$	$S^{1,6}_{|t|}$	$S^{1,7}_{|t|}$	$S^{1,8}_{|t|}$	$S^{1,9}_{|t|}$	$S^{1,10}_{|t|}$	$S^{1,11}_{|t|}$	$S^{1,12}_{|t|}$	$S^{1,13}_{|t|}$
fixed	0.048	0.040	0.044	0.049	0.046	0.051	0.052	0.048	0.052	0.04	0.042	0.05	0.032
rolling	0.049	0.041	0.056	0.051	0.048	0.041	0.041	0.045	0.057	0.056	0.049	0.038	0.039
recursive	0.046	0.048	0.045	0.044	0.051	0.048	0.044	0.045	0.047	0.057	0.058	0.039	0.04

	$A^{1,1}_{|t|}$	$A^{1,2}_{|t|}$	$A^{1,3}_{|t|}$	$A^{1,4}_{|t|}$	$A^{1,5}_{|t|}$	$A^{1,6}_{|t|}$	$A^{1,7}_{|t|}$	$A^{1,8}_{|t|}$	$A^{1,9}_{|t|}$	$A^{1,10}_{|t|}$	$A^{1,11}_{|t|}$	$A^{1,12}_{|t|}$	$A^{1,13}_{|t|}$
fixed	0.049	0.045	0.049	0.051	0.046	0.049	0.051	0.043	0.049	0.052	0.043	0.047	0.049
rolling	0.048	0.042	0.046	0.055	0.044	0.047	0.043	0.044	0.044	0.043	0.049	0.044	0.048
recursive	0.04	0.043	0.041	0.063	0.065	0.045	0.053	0.045	0.044	0.051	0.042	0.044	0.043

	$S^{1,13}_{C}$	$A^{1,13}_{C}$	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$
fixed	0.045	0.049	0.047	0.044	0.053	0.05	0.049	0.042	0.046	0.051
rolling	0.045	0.044	0.043	0.064	0.063	0.044	0.048	0.041	0.044	0.05
recursive	0.043	0.047	0.045	0.057	0.052	0.048	0.048	0.048	0.051	0.048

	$S^{1,7}_{|t|}$	$S^{2,7}_{|t|}$	$S^{3,7}_{|t|}$	$S^{4,7}_{|t|}$	$S^{5,7}_{|t|}$	$A^{1,7}_{|t|}$	$A^{2,7}_{|t|}$	$A^{3,7}_{|t|}$	$A^{4,7}_{|t|}$	$A^{5,7}_{|t|}$
fixed	0.052	0.053	0.042	0.042	0.043	0.051	0.048	0.042	0.054	0.049
rolling	0.041	0.048	0.043	0.039	0.04	0.043	0.043	0.047	0.049	0.043
recursive	0.044	0.051	0.041	0.038	0.041	0.053	0.054	0.043	0.045	0.048

Notation: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
In $S^{k,\alpha_i}_{|t|}$ and $A^{k,\alpha_i}_{|t|}$, lag k and autocontour α_i are fixed.
In $S^{k,\alpha}_{C}$ and $A^{k,\alpha}_{C}$, k is fixed and α is the total number of autocontours considered.
In S^{k,α_i}_{L} and A^{k,α_i}_{L}, up to k lags are considered and α_i is a fixed autocontour.

Table 5: **Size of the statistics:T=750 R=450 P=T-R=300 $m = 2/3$ (nominal size 5%)**

Fixed Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.14	0.45	0.765	0.956	0.979	0.987	0.995	0.992	0.987	0.976	0.938	0.867	0.589
$l=2$	0.14	0.464	0.789	0.955	0.981	0.99	0.993	0.993	0.988	0.974	0.943	0.864	0.593
$l=3$	0.14	0.419	0.792	0.965	0.985	0.997	0.997	0.994	0.985	0.971	0.937	0.871	0.594
$l=4$	0.11	0.439	0.797	0.953	0.986	0.993	0.994	0.994	0.99	0.979	0.938	0.866	0.595
$l=5$	0.12	0.453	0.796	0.952	0.984	0.992	0.994	0.993	0.989	0.971	0.938	0.867	0.598
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.08	0.244	0.698	0.898	0.948	0.969	0.981	0.978	0.968	0.945	0.904	0.812	0.588
$l=2$	0.09	0.277	0.741	0.896	0.952	0.965	0.979	0.983	0.972	0.947	0.898	0.815	0.585
$l=3$	0.07	0.276	0.72	0.893	0.941	0.969	0.98	0.984	0.967	0.943	0.906	0.815	0.588
$l=4$	0.07	0.319	0.702	0.887	0.957	0.971	0.979	0.981	0.971	0.944	0.904	0.817	0.586
$l=5$	0.08	0.339	0.722	0.906	0.951	0.968	0.985	0.976	0.972	0.944	0.903	0.816	0.584
	$S^{l,13}_C$	$A^{l,13}_C$											
$l=1$	0.961	0.921											
$l=2$	0.965	0.924											
$l=3$	0.967	0.923											
$l=4$	0.963	0.921											
$l=5$	0.965	0.921											
	$S^{2,7}_L$	$S^{3,7}_L$	$S^{4,7}_L$	$S^{5,7}_L$	$A^{2,7}_L$	$A^{3,7}_L$	$A^{4,7}_L$	$A^{5,7}_L$					
$C=7$	0.989	0.985	0.976	0.964	0.98	0.969	0.956	0.942					
Rolling Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.18	0.12	0.308	0.416	0.459	0.394	0.42	0.264	0.21	0.08	0.119	0.092	0.08
$l=2$	0.11	0.11	0.295	0.324	0.323	0.431	0.433	0.333	0.193	0.084	0.123	0.093	0.08
$l=3$	0.11	0.11	0.258	0.402	0.386	0.39	0.472	0.325	0.207	0.096	0.123	0.094	0.09
$l=4$	0.13	0.11	0.289	0.439	0.41	0.413	0.378	0.325	0.173	0.093	0.121	0.092	0.082
$l=5$	0.16	0.11	0.262	0.445	0.418	0.423	0.438	0.318	0.171	0.091	0.121	0.092	0.083
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.25	0.318	0.559	0.783	0.738	0.710	0.534	0.634	0.418	0.18	0.21	0.102	0.091
$l=2$	0.22	0.292	0.527	0.733	0.71	0.714	0.56	0.621	0.438	0.17	0.216	0.100	0.092
$l=3$	0.24	0.275	0.521	0.768	0.756	0.744	0.499	0.638	0.442	0.155	0.224	0.103	0.092
$l=4$	0.25	0.28	0.519	0.806	0.775	0.759	0.611	0.638	0.421	0.133	0.21	0.101	0.092
$l=5$	0.25	0.305	0.516	0.797	0.781	0.769	0.533	0.626	0.392	0.121	0.24	0.101	0.093
	$S^{l,13,}_C$	$A^{l,13}_C$											
$l=1$	0.638	0.749											
$l=2$	0.521	0.733											
$l=3$	0.619	0.801											
$l=4$	0.641	0.822											
$l=5$	0.637	0.803											
	$S^{2,7}_L$	$S^{3,7}_L$	$S^{4,7}_L$	$S^{5,7}_L$	$A^{2,7}_L$	$A^{3,7}_L$	$A^{4,7}_L$	$A^{5,7}_L$					
$C=7$	0.414	0.419	0.408	0.432	0.563	0.59	0.578	0.532					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S^{l,7}_{|t|}$,$A^{l,7}_{|t|}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S^{l,7}_L, A^{l,7}_L$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S^{l,13}_C$ and $A^{l,13}_C$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
1000 Monte Carlo replications and 500 bootstrap samples.
T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 6: **Power for DGP1 under Fixed and Rolling Schemes**

Fixed Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.292	0.271	0.191	0.084	0.073	0.109	0.281	0.278	0.391	0.512	0.638	0.638	0.518
$l=2$	0.26	0.239	0.171	0.079	0.075	0.106	0.282	0.292	0.418	0.518	0.637	0.639	0.52
$l=3$	0.285	0.265	0.175	0.089	0.075	0.12	0.283	0.29	0.404	0.53	0.635	0.639	0.523
$l=4$	0.311	0.26	0.162	0.088	0.063	0.096	0.282	0.284	0.399	0.512	0.624	0.636	0.526
$l=5$	0.273	0.253	0.158	0.068	0.068	0.108	0.282	0.288	0.408	0.539	0.645	0.638	0.531
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.345	0.242	0.162	0.09	0.078	0.127	0.275	0.271	0.361	0.472	0.561	0.604	0.527
$l=2$	0.327	0.225	0.16	0.081	0.077	0.12	0.269	0.277	0.377	0.467	0.56	0.606	0.522
$l=3$	0.347	0.239	0.167	0.089	0.087	0.123	0.272	0.286	0.374	0.468	0.558	0.6	0.527
$l=4$	0.368	0.246	0.159	0.09	0.069	0.111	0.272	0.277	0.345	0.455	0.55	0.592	0.523
$l=5$	0.317	0.222	0.146	0.086	0.072	0.12	0.273	0.288	0.37	0.469	0.564	0.6	0.523
	$S^{l,13}_{C}$	$A^{l,13}_{C}$											
$l=1$	0.659	0.595											
$l=2$	0.651	0.604											
$l=3$	0.666	0.625											
$l=4$	0.644	0.625											
$l=5$	0.652	0.611											
	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$					
$C=7$	0.286	0.285	0.281	0.285	0.271	0.273	0.276	0.275					

Rolling Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.116	0.104	0.088	0.063	0.05	0.056	0.112	0.11	0.171	0.341	0.302	0.264	0.261
$l=2$	0.114	0.09	0.086	0.063	0.054	0.051	0.118	0.113	0.172	0.318	0.296	0.264	0.262
$l=3$	0.107	0.094	0.076	0.065	0.06	0.053	0.108	0.126	0.189	0.335	0.28	0.273	0.266
$l=4$	0.113	0.104	0.072	0.054	0.058	0.049	0.175	0.114	0.167	0.329	0.287	0.266	0.267
$l=5$	0.109	0.093	0.08	0.053	0.05	0.053	0.16	0.121	0.184	0.337	0.274	0.265	0.267
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.179	0.129	0.099	0.078	0.066	0.075	0.19	0.151	0.204	0.36	0.344	0.35	0.378
$l=2$	0.169	0.112	0.082	0.075	0.066	0.076	0.18	0.145	0.205	0.386	0.345	0.343	0.379
$l=3$	0.175	0.139	0.092	0.07	0.067	0.077	0.196	0.159	0.228	0.378	0.352	0.348	0.378
$l=4$	0.183	0.143	0.083	0.063	0.066	0.077	0.192	0.144	0.201	0.358	0.342	0.352	0.375
$l=5$	0.159	0.129	0.09	0.063	0.066	0.065	0.191	0.151	0.208	0.371	0.347	0.35	0.377
	$S^{l,13}_{C}$	$A^{l,13}_{C}$											
$l=1$	0.409	0.414											
$l=2$	0.363	0.394											
$l=3$	0.381	0.427											
$l=4$	0.398	0.431											
$l=5$	0.306	0.318											
	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$					
$C=7$	0.168	0.115	0.113	0.112	0.129	0.189	0.191	0.182					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S^{l,7}_{|t|}$, $A^{l,7}_{|t|}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S^{l,7}_{L}$, $A^{l,7}_{L}$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S^{l,13}_{C}$ and $A^{l,13}_{C}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
1000 Monte Carlo replications and 500 bootstrap samples.
T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 7: **Power for DGP2 under Fixed and Rolling Schemes**

Fixed Scheme	$S_{|t|}^{l,1}$	$S_{|t|}^{l,2}$	$S_{|t|}^{l,3}$	$S_{|t|}^{l,4}$	$S_{|t|}^{l,5}$	$S_{|t|}^{l,6}$	$S_{|t|}^{l,7}$	$S_{|t|}^{l,8}$	$S_{|t|}^{l,9}$	$S_{|t|}^{l,10}$	$S_{|t|}^{l,11}$	$S_{|t|}^{l,12}$	$S_{|t|}^{l,13}$
$l=1$	0.95	0.998	0.999	0.999	0.999	1	0.97	0.985	0.961	0.849	0.449	0.33	0.29
$l=2$	0.96	0.998	0.999	1	0.999	0.997	0.982	0.981	0.96	0.859	0.444	0.16	0.19
$l=3$	0.97	0.997	0.998	1	0.999	0.998	0.979	0.982	0.956	0.85	0.436	0.35	0.31
$l=4$	0.96	0.997	0.998	1	0.998	0.998	0.98	0.984	0.965	0.853	0.446	0.36	0.13
$l=5$	0.96	0.997	1	0.999	1	0.999	0.982	0.983	0.96	0.854	0.447	0.2	0.23
	$A_{|t|}^{l,1}$	$A_{|t|}^{l,2}$	$A_{|t|}^{l,3}$	$A_{|t|}^{l,4}$	$A_{|t|}^{l,5}$	$A_{|t|}^{l,6}$	$A_{|t|}^{l,7}$	$A_{|t|}^{l,8}$	$A_{|t|}^{l,9}$	$A_{|t|}^{l,10}$	$A_{|t|}^{l,11}$	$A_{|t|}^{l,12}$	$A_{|t|}^{l,13}$
$l=1$	0.93	0.951	1	0.999	0.999	0.995	0.95	0.973	0.914	0.475	0.38	0.25	0.16
$l=2$	0.931	0.967	0.998	1	0.999	0.994	0.952	0.974	0.896	0.518	0.34	0.22	0.17
$l=3$	0.937	0.98	0.998	0.999	0.997	0.995	0.95	0.973	0.883	0.555	0.36	0.24	0.19
$l=4$	0.935	0.96	1	0.998	0.997	0.994	0.95	0.968	0.908	0.487	0.31	0.32	0.22
$l=5$	0.939	0.95	0.999	0.998	0.997	0.995	0.95	0.962	0.875	0.545	0.31	0.12	0.23
	$S_C^{l,13}$	$A_C^{l,13}$											
$l=1$	1	0.97											
$l=2$	1	0.98											
$l=3$	0.999	0.97											
$l=4$	1	0.999											
$l=5$	1	1											
	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
$C=7$	0.98	0.98	0.98	0.98	0.97	0.969	0.97	0.97					
Rolling Scheme	$S_{|t|}^{l,1}$	$S_{|t|}^{l,2}$	$S_{|t|}^{l,3}$	$S_{|t|}^{l,4}$	$S_{|t|}^{l,5}$	$S_{|t|}^{l,6}$	$S_{|t|}^{l,7}$	$S_{|t|}^{l,8}$	$S_{|t|}^{l,9}$	$S_{|t|}^{l,10}$	$S_{|t|}^{l,11}$	$S_{|t|}^{l,12}$	$S_{|t|}^{l,13}$
$l=1$	0.46	0.461	0.395	0.184	0.26	0.329	0.396	0.457	0.401	0.134	0.212	0.092	0.08
$l=2$	0.358	0.32	0.338	0.381	0.421	0.475	0.511	0.508	0.39	0.121	0.241	0.093	0.08
$l=3$	0.437	0.398	0.389	0.442	0.465	0.524	0.524	0.518	0.385	0.099	0.223	0.094	0.09
$l=4$	0.445	0.425	0.444	0.481	0.502	0.544	0.54	0.512	0.345	0.095	0.232	0.092	0.082
$l=5$	0.448	0.409	0.425	0.468	0.511	0.54	0.546	0.506	0.361	0.075	0.232	0.092	0.083
	$A_{|t|}^{l,1}$	$A_{|t|}^{l,2}$	$A_{|t|}^{l,3}$	$A_{|t|}^{l,4}$	$A_{|t|}^{l,5}$	$A_{|t|}^{l,6}$	$A_{|t|}^{l,7}$	$A_{|t|}^{l,8}$	$A_{|t|}^{l,9}$	$A_{|t|}^{l,10}$	$A_{|t|}^{l,11}$	$A_{|t|}^{l,12}$	$A_{|t|}^{l,13}$
$l=1$	0.189	0.219	0.332	0.473	0.584	0.678	0.726	0.739	0.706	0.395	0.163	0.082	0.06
$l=2$	0.585	0.628	0.686	0.788	0.808	0.834	0.826	0.815	0.716	0.356	0.171	0.081	0.06
$l=3$	0.68	0.711	0.728	0.796	0.832	0.839	0.832	0.811	0.7	0.334	0.163	0.081	0.07
$l=4$	0.684	0.733	0.765	0.832	0.851	0.861	0.846	0.815	0.698	0.308	0.162	0.081	0.062
$l=5$	0.674	0.717	0.762	0.82	0.846	0.86	0.848	0.814	0.669	0.291	0.161	0.081	0.063
	$S_C^{l,13}$	$A_C^{l,13}$											
$l=1$	0.57	0.817											
$l=2$	0.539	0.845											
$l=3$	0.53	0.814											
$l=4$	0.532	0.78											
$l=5$	0.534	0.782											
	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
	0.477	0.496	0.48	0.421	0.728	0.75	0.749	0.703					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S_{|t|}^{l,7}$, $A_{|t|}^{l,7}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S_L^{l,7}$, $A_L^{l,7}$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
1000 Monte Carlo replications and 500 bootstrap samples.
T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 8: **Power for DGP3 under Fixed and Rolling Schemes**

Fixed Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	1	1	0.999	1	1	1	0.999	0.999	0.989	0.984	0.969	0.93	0.79
$l=2$	0.999	1	0.999	0.996	1	1	1	1	0.98	0.983	0.967	0.901	0.78
$l=3$	0.998	1	1	1	1	0.999	1	1	0.996	0.98	0.965	0.912	0.781
$l=4$	1	0.999	1	1	0.998	1	0.999	1	0.99	0.98	0.962	0.923	0.744
$l=5$	1	0.996	1	1	1	1	1	0.998	1	0.98	0.966	0.917	0.775
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.989	1	0.995	1	1	1	0.98	0.998	0.989	0.96	0.914	0.812	0.639
$l=2$	0.989	1	0.995	0.995	0.997	1	0.988	0.989	0.986	0.967	0.934	0.815	0.633
$l=3$	0.99	1	1	1	1	0.997	0.985	0.99	0.986	0.966	0.914	0.815	0.622
$l=4$	1	0.996	1	1	0.996	0..	0.989	0.991	0.986	0.967	0.954	0.817	0.644
$l=5$	1	0.996	1	1	1	1	0.989	0.996	0.986	0.97	0.914	0.816	0.675
	$S^{l,13}_C$	$A^{l,13}_C$											
$l=1$	1	1											
$l=2$	1	1											
$l=3$	1	1											
$l=4$	1	1											
$l=5$	1	1											
	$S^{2,7}_L$	$S^{3,7}_L$	$S^{4,7}_L$	$S^{5,7}_L$	$A^{2,7}_L$	$A^{3,7}_L$	$A^{4,7}_L$	$A^{5,7}_L$					
$C=7$	1	1	1	1	1	1	1	1					
Rolling Scheme	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.569	0.461	0.486	0.589	0.696	0.746	0.76	0.729	0.69	0.643	0.481	0.331	0.123
$l=2$	0.44	0.387	0.358	0.577	0.664	0.73	0.744	0.733	0.703	0.629	0.464	0.332	0.12
$l=3$	0.541	0.465	0.421	0.528	0.649	0.695	0.705	0.71	0.691	0.624	0.445	0.332	0.127
$l=4$	0.586	0.514	0.48	0.525	0.639	0.698	0.703	0.694	0.663	0.618	0.447	0.319	0.131
$l=5$	0.571	0.495	0.484	0.525	0.643	0.724	0.722	0.723	0.678	0.627	0.448	0.318	0.135
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.25	0.318	0.559	0.761	0.831	0.861	0.858	0.839	0.819	0.763	0.606	0.461	0.236
$l=2$	0.22	0.292	0.527	0.739	0.794	0.835	0.857	0.852	0.82	0.767	0.601	0.46	0.242
$l=3$	0.24	0.275	0.521	0.705	0.789	0.835	0.85	0.854	0.819	0.767	0.609	0.45	0.237
$l=4$	0.25	0.28	0.519	0.699	0.795	0.833	0.86	0.842	0.8	0.736	0.606	0.443	0.24
$l=5$	0.25	0.305	0.516	0.704	0.789	0.837	0.846	0.848	0.823	0.75	0.588	0.446	0.242
	$S^{l,13}_C$	$A^{l,13}_C$											
$l=1$	0.75	0.85											
$l=2$	0.74	0.85											
$l=3$	0.76	0.87											
$l=4$	0.76	0.83											
$l=5$	0.771	0.81											
	$S^{2,7}_L$	$S^{3,7}_L$	$S^{4,7}_L$	$S^{5,7}_L$	$A^{2,7}_L$	$A^{3,7}_L$	$A^{4,7}_L$	$A^{5,7}_L$					
$C=7$	0.656	0.558	0.558	0.56	0.801	0.724	0.656	0.605					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S^{l,7}_{|t|}$,$A^{l,7}_{|t|}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S^{l,7}_L$,$A^{l,7}_L$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S^{l,13}_C$ and $A^{l,13}_C$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
1000 Monte Carlo replications and 500 bootstrap samples.
T=650, R=350, P=300, $m = 1/3$, and break point at $R + \tau P$ for $\tau = 1/3$

Table 9: **Power for DGP4 under Fixed and Rolling Schemes**

Tables: Phillips Curve

Fixed Scheme	$S_{|t|}^{l,1}$	$S_{|t|}^{l,2}$	$S_{|t|}^{l,3}$	$S_{|t|}^{l,4}$	$S_{|t|}^{l,5}$	$S_{|t|}^{l,6}$	$S_{|t|}^{l,7}$	$S_{|t|}^{l,8}$	$S_{|t|}^{l,9}$	$S_{|t|}^{l,10}$	$S_{|t|}^{l,11}$	$S_{|t|}^{l,12}$	$S_{|t|}^{l,13}$
$l=1$	0.017	0.529	0.523	0.471	0.268	0.002	0.008	0.045	0.136	0.091	0.085	0.031	0.0001
$l=2$	0.015	0.285	0.55	0.434	0.481	0.002	0.005	0.041	0.157	0.086	0.09	0.027	0.0001
$l=3$	0.028	0.584	0.614	0.651	0.419	0.006	0.005	0.03	0.049	0.088	0.073	0.013	0.0001
$l=4$	0.013	0.104	0.807	0.285	0.209	0.009	0.003	0.027	0.098	0.088	0.069	0.012	0.0001
$l=5$	0.031	0.221	0.212	0.302	0.206	0.009	0.006	0.034	0.103	0.088	0.057	0.006	0.0001
	$A_{|t|}^{l,1}$	$A_{|t|}^{l,2}$	$A_{|t|}^{l,3}$	$A_{|t|}^{l,4}$	$A_{|t|}^{l,5}$	$A_{|t|}^{l,6}$	$A_{|t|}^{l,7}$	$A_{|t|}^{l,8}$	$A_{|t|}^{l,9}$	$A_{|t|}^{l,10}$	$A_{|t|}^{l,11}$	$A_{|t|}^{l,12}$	$A_{|t|}^{l,13}$
$l=1$	0.342	0.671	0.759	0.405	0.163	0.042	0.013	0.248	0.243	0.243	0.412	0.199	0.042
$l=2$	0.335	0.453	0.621	0.352	0.972	0.043	0.022	0.336	0.297	0.223	0.332	0.25	0.04
$l=3$	0.494	0.579	0.316	0.567	0.309	0.041	0.016	0.216	0.225	0.235	0.359	0.197	0.049
$l=4$	0.338	0.348	0.764	0.319	0.152	0.033	0.024	0.144	0.264	0.223	0.327	0.226	0.042
$l=5$	0.138	0.595	0.637	0.238	0.142	0.032	0.024	0.218	0.273	0.222	0.308	0.184	0.042
	$S_C^{l,13}$	$A_C^{l,13}$											
$l=1$	0.004	0.022											
$l=2$	0.001	0.019											
$l=3$	0.003	0.016											
$l=4$	0.001	0.025											
$l=5$	0.001	0.025											
	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
$C=7$	0.006	0.009	0.012	0.01	0.027	0.018	0.027	0.029					

Rolling Scheme	$S_{|t|}^{l,1}$	$S_{|t|}^{l,2}$	$S_{|t|}^{l,3}$	$S_{|t|}^{l,4}$	$S_{|t|}^{l,5}$	$S_{|t|}^{l,6}$	$S_{|t|}^{l,7}$	$S_{|t|}^{l,8}$	$S_{|t|}^{l,9}$	$S_{|t|}^{l,10}$	$S_{|t|}^{l,11}$	$S_{|t|}^{l,12}$	$S_{|t|}^{l,13}$
$l=1$	0.528	0.46	0.843	0.472	0.142	0.002	0.001	0.007	0.008	0.093	0.281	0.026	0.0002
$l=2$	0.258	0.35	0.58	0.251	0.229	0.007	0.004	0.009	0.002	0.081	0.24	0.024	0.0003
$l=3$	0.559	0.31	0.741	0.437	0.16	0.002	0.001	0.002	0.002	0.066	0.288	0.013	0.0003
$l=4$	0.38	0.31	0.479	0.463	0.17	0.001	0.001	0.003	0.008	0.088	0.275	0.021	0.0002
$l=5$	0.285	0.249	0.297	0.225	0.12	0.007	0.001	0.003	0.003	0.063	0.327	0.019	0.0002
	$A_{|t|}^{l,1}$	$A_{|t|}^{l,2}$	$A_{|t|}^{l,3}$	$A_{|t|}^{l,4}$	$A_{|t|}^{l,5}$	$A_{|t|}^{l,6}$	$A_{|t|}^{l,7}$	$A_{|t|}^{l,8}$	$A_{|t|}^{l,9}$	$A_{|t|}^{l,10}$	$A_{|t|}^{l,11}$	$A_{|t|}^{l,12}$	$A_{|t|}^{l,13}$
$l=1$	0.789	0.801	0.853	0.656	0.143	0.001	0.023	0.033	0.032	0.145	0.297	0.271	0.024
$l=2$	0.267	0.55	0.812	0.244	0.568	0.002	0.018	0.024	0.045	0.127	0.283	0.337	0.037
$l=3$	0.751	0.918	0.923	0.723	0.144	0.007	0.015	0.032	0.047	0.114	0.288	0.24	0.023
$l=4$	0.358	0.476	0.575	0.448	0.108	0.001	0.015	0.031	0.031	0.142	0.313	0.271	0.034
$l=5$	0.305	0.655	0.576	0.251	0.129	0.009	0.018	0.031	0.039	0.13	0.286	0.288	0.033
	$S_C^{l,13}$	$A_C^{l,13}$											
$l=1$	0.001	0.009											
$l=2$	0.0003	0.004											
$l=3$	0.0003	0.01											
$l=4$	0.0003	0.005											
$l=5$	0.002	0.004											
	$S_L^{2,7}$	$S_L^{3,7}$	$S_L^{4,7}$	$S_L^{5,7}$	$A_L^{2,7}$	$A_L^{3,7}$	$A_L^{4,7}$	$A_L^{5,7}$					
$C=7$	0.008	0.008	0.006	0.009	0.038	0.025	0.019	0.031					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S_{|t|}^{l,7}$, $A_{|t|}^{l,7}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S_L^{l,7}$, $A_L^{l,7}$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S_C^{l,13}$ and $A_C^{l,13}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
500 bootstrap samples. T=649, R=360, P=289, $m = 0.69$

Table 10: **Bootstrapped P-values: Linear Phillips Curve (Fixed and Rolling Schemes)**

Model 2	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.542	0.682	0.306	0.254	0.18	0.196	0.178	0.26	0.4	0.188	0.044	0.026	0.036
$l=2$	0.478	0.32	0.226	0.15	0.102	0.118	0.102	0.116	0.228	0.088	0.034	0.024	0.032
$l=3$	0.802	0.394	0.188	0.236	0.172	0.176	0.146	0.234	0.274	0.192	0.038	0.024	0.032
$l=4$	0.496	0.65	0.524	0.296	0.188	0.2	0.19	0.284	0.422	0.21	0.042	0.024	0.034
$l=5$	0.794	0.19	0.286	0.186	0.136	0.148	0.164	0.296	0.386	0.166	0.032	0.024	0.032
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.496	0.628	0.296	0.264	0.176	0.198	0.178	0.288	0.424	0.256	0.046	0.028	0.036
$l=2$	0.438	0.298	0.216	0.152	0.102	0.116	0.104	0.16	0.27	0.128	0.044	0.026	0.032
$l=3$	0.372	0.374	0.298	0.24	0.168	0.17	0.148	0.246	0.288	0.22	0.046	0.026	0.032
$l=4$	0.442	0.416	0.366	0.33	0.21	0.2	0.21	0.31	0.434	0.24	0.046	0.026	0.034
$l=5$	0.362	0.22	0.332	0.192	0.138	0.15	0.17	0.32	0.402	0.232	0.036	0.024	0.032
	$S^{l,13}_{C}$	$A^{l,13}_{C}$											
$l=1$	0.328	0.332											
$l=2$	0.18	0.18											
$l=3$	0.284	0.272											
$l=4$	0.32	0.328											
$l=5$	0.266	0.268											
	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$					
$C=7$	0.112	0.12	0.12	0.116	0.118	0.124	0.12	0.122					

Model 3	$S^{l,1}_{|t|}$	$S^{l,2}_{|t|}$	$S^{l,3}_{|t|}$	$S^{l,4}_{|t|}$	$S^{l,5}_{|t|}$	$S^{l,6}_{|t|}$	$S^{l,7}_{|t|}$	$S^{l,8}_{|t|}$	$S^{l,9}_{|t|}$	$S^{l,10}_{|t|}$	$S^{l,11}_{|t|}$	$S^{l,12}_{|t|}$	$S^{l,13}_{|t|}$
$l=1$	0.452	0.662	0.454	0.406	0.466	0.694	0.702	0.814	0.788	0.57	0.41	0.254	0.296
$l=2$	0.428	0.552	0.218	0.464	0.436	0.678	0.652	0.816	0.684	0.55	0.41	0.276	0.296
$l=3$	0.476	0.784	0.454	0.424	0.466	0.66	0.602	0.874	0.782	0.582	0.43	0.206	0.296
$l=4$	0.656	0.738	0.676	0.55	0.584	0.72	0.807	0.884	0.82	0.584	0.386	0.224	0.296
$l=5$	0.652	0.538	0.422	0.444	0.536	0.78	0.836	0.844	0.842	0.578	0.416	0.244	0.296
	$A^{l,1}_{|t|}$	$A^{l,2}_{|t|}$	$A^{l,3}_{|t|}$	$A^{l,4}_{|t|}$	$A^{l,5}_{|t|}$	$A^{l,6}_{|t|}$	$A^{l,7}_{|t|}$	$A^{l,8}_{|t|}$	$A^{l,9}_{|t|}$	$A^{l,10}_{|t|}$	$A^{l,11}_{|t|}$	$A^{l,12}_{|t|}$	$A^{l,13}_{|t|}$
$l=1$	0.442	0.644	0.43	0.39	0.568	0.682	0.856	0.822	0.818	0.596	0.45	0.278	0.294
$l=2$	0.402	0.446	0.318	0.362	0.326	0.488	0.674	0.834	0.72	0.57	0.446	0.308	0.296
$l=3$	0.606	0.748	0.446	0.388	0.454	0.558	0.804	0.868	0.79	0.606	0.434	0.23	0.294
$l=4$	0.638	0.768	0.666	0.348	0.58	0.722	0.86	0.874	0.824	0.584	0.4	0.226	0.296
$l=5$	0.632	0.514	0.404	0.432	0.522	0.722	0.83	0.852	0.86	0.582	0.412	0.246	0.39
	$S^{l,13}_{C}$	$A^{l,13}_{C}$											
$l=1$	0.614	0.596											
$l=2$	0.48	0.454											
$l=3$	0.494	0.482											
$l=4$	0.51	0.478											
$l=5$	0.488	0.472											
	$S^{2,7}_{L}$	$S^{3,7}_{L}$	$S^{4,7}_{L}$	$S^{5,7}_{L}$	$A^{2,7}_{L}$	$A^{3,7}_{L}$	$A^{4,7}_{L}$	$A^{5,7}_{L}$					
$C=7$	0.518	0.566	0.548	0.598	0.534	0.578	0.564	0.618					

Notes: 13 autocontours $C = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.
$S^{l,7}_{|t|}, A^{l,7}_{|t|}$ for $l = 1, 2, ...5$; 7 refers to the 50% autocontour.
$S^{l,7}_{L}, A^{l,7}_{L}$ stacking lags up to $l = 2, ...5$ and considering the 50% autocontour.
$S^{l,13}_{C}$ and $A^{l,13}_{C}$ stacking all 13 autocontours for one lag $l = 1, 2, 3, 4, 5$.
500 bootstrap samples.
T=649, R=360, P=289, $m = 0.69$

Table 11: **Bootstrapped P-values: Non-Linear Phillips Curve (Fixed Scheme)**