Breaking the Spell with Credit-Easing*

Self-Confirming Credit Crises in Competitive Search Economies

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Abstract

We introduce Self-Confirming Equilibria in a competitive credit market to explain market freezes. Lenders offer loans at a fixed interest rate and borrowers apply to them, as in a directed search model. In a self-confirming crises, lenders only offer high interest rates and borrowers only invest in risky projects; in particular, lenders are pessimist about the possibility that lower interest rates could attract riskless projects instead. Lenders could assess their beliefs experimenting with low interest rates, but competition dries any eventual benefit of individual experimentation. On the other hand, a policy maker can induce any interest rate as a market equilibrium rate implementing a targeted subsidy policy. We show that, for a policy maker with the same pessimistic beliefs than private lenders, it is socially efficient to target a low interest rate. The social experiment produces evidence that can eventually refutes the pessimistic beliefs and restores the optimality of the decentralized market. Using new micro data on the ABS auto loans in the US, we show that the 2009 TALF intervention by the Fed took the form of our optimal policy and induced a market recovery, as predicted by our theory.

Keywords: unconventional policies, learning, credit crisis, social experimentation, self-confirming equilibrium, directed search.

JEL Classification: D53, D83, D84, D92, E44, E61, G01, G20, J64.

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1 Introduction

Economic crisis are usually characterized by high uncertainty – not just risk – regarding the state of the economy. Such uncertainty shows up in credit markets in the form of perceived counterpart risk, resulting in high lending rates, and in some cases, complete market freezes. The common wisdom is that disruptions of liquidity markets are collateral effects of recessions, so that, conventional (and unconventional) policies aiming at lowering the cost of money or providing more liquidity are appropriate reactions to both. Unfortunately, there are two problems with these policies. One is empirical: they seldom translated into a substantial improvement of the credit conditions for private firms (the crises in the Euro area is an example). The other is theoretical: if there is high uncertainty regarding the state of the economy, how can a policy maker, who faces the same uncertainty, design an optimal policy? In other words, are these widespread policies optimal given the uncertain state of the economy?

Both problems are related. For example, robust decision theory is often applied to address the problem of designing policies in situation of high uncertainty. However, this approach leads to recommend prudence, which does not seem to be a characteristic of recent unconventional policies. An instance of this risk-taking behavior are the *credit-easing* interventions implemented by the Federal Reserve Bank during the 2009 recession in the US. These policies were specifically designed to partly ensure private lenders from the *perceived* counterpart risk in important credit markets under pressure. One of these policies was the TALF whose introduction in the AAA-rated ABS market (completely frozen in late 2008) coincided with a permanent recovery of transactions without that any subsidy was actually dispensed: a proof, in retrospect, that in that case the perceived counterpart risk in that market was excessive. Should we conclude that the Fed was less risk averse, better informed, or just lucky? More broadly, how should a central bank, which does not know more than the private sector, react when lower policy rates are not effective in improving credit market conditions? Should a central bank take risks the private sector is not taking, as the FRB did with the TALF?

These, and similar, questions bring us back to the theoretical problem: the need to develop a suitable theory for situations of high economic uncertainty, which can encompass policy experiences like the TALF, without being specific to them. This is what this paper does providing three original contributions: first, we develop the theory of self-confirming equilibrium in a competitive environment to characterize credit-freezes determined by excessive - but subjectively rational - perceived counterpart risk; second, we show that in this environment a *credit easing policy* of subsidizing lenders' losses can be an optimal policy for a monetary (and fiscal) authority sharing the same subjective beliefs than the private sector, and third, we provide new micro-evidence about the effect of TALF policy in the US automotive AAA-rated ABS in 2009, which is consistent with our theory.

Section 2 introduces a competitive credit market consisting of a continuum of borrowers (e.g. entrepreneurs with projects) and lenders (e.g. banks). The latter, are intermediaries that borrow money from the interbank market, and post credit offers at a fixed interest rate; the former apply for these contracts and choose the projects to be financed. Their expected profits depend on the probability that their loan application is accepted and, if so, on the expected net return of their project. When a risky project fails borrowers only repay the principal of the loan (the *pledgeable* part) and, therefore, the lender also bears part of the risk, which is compensated by the loan's interest risk-premium. However, borrowers also have the choice of implementing a riskless project paying a fixed cost[1]. Safe projects are implemented if the interest rate is low enough to compensate the fixed cost.

Borrowers observe the menu of debt contracts posted by lenders, and choose where to send an application under complete information. On the other hand, lenders do not observe the choice of borrowers and, therefore, have to anticipate the borrower's reaction to their credit offers in order to maximize the value of their contract. According to the logic of a Self-Confirming Equilibrium (SCE), we require that lenders hold beliefs that only need to be correct in equilibrium. Thus, rational lenders can entertain missperceptions about borrowers' reaction to out-of-equilibrium offers that they never offer and they never experience. In this sense equilibrium beliefs are self-confirming.

A key feature of SCE, as originally introduced in Game Theory by Fudenberg and Levine (1993), is that individual actions can potentially produce the observables that correct these missperceptions[2]. In Macroeconomics, Sargent (2001) and Primiceri (2006) used the concept of SCE by modeling the learning problem of a major actor, who actually has the power to affect observables and hence can be trapped in a SCE.[3] In this paper, we instead characterize a SCE in a search and matching competitive environment. Whereas atomistic lenders have no power to affect general equilibrium outcomes, they can have misspecified beliefs about the borrowers' incentives that, within a match, could be experimented on an individual basis. Nevertheless, agents do not have incentive to experiment as competition will dry any eventual gain. In fact, in a competitive equilibrium, lenders believe that there are no expected gains from individual deviations (by definition), and no expected gains in any other equilibrium (zero profit condition). In particular, they place a sufficiently low probability on an individual deviation being profitable; furthermore, even in the case of a prof-

[1]Different interpretations are possible. For example, in the case of the Auto loans, the return from buying a car can be the utility of having that car; the pledgeable principal is the car itself; the fix cost is the utility lost when buying a less fancy car which, nevertheless, has cheaper operating costs that allow the buyer to repay the loan for sure.

[2]It should be noticed that in the macro literature sometimes the term SCE is (mis-)used without this key feature.

[3]In their case, the Fed was taking actions based on a wrong theory of the economy, which could not be confuted by the outcomes that the policy itself was determining, at least up to the point where enough experimentation finally revealed the actual working of the economy.

itable deviation, they anticipate that the new information revealed by the experiment will trigger competition at the new equilibrium and restores the zero profit condition anyway. In sum, the *private expected value* of experimentation is negative. In this sense, our competitive version of the SCE strengthens the resilience of this equilibrium concept in contrast to the original Game Theory formulation, which is instead fragile to the presence of patient experimenters.[4]

In section, 3, we study the problem of a Ramsey planner who takes into account the directed search equilibrium constraints, and has no more information or less uncertainty than lenders have. As it is well known, competitive directed search is efficient, in the sense that the decentralized market fulfills the constrained first best allocation.[5] This implies that a Ramsey planner – with the same (or more pessimistic) beliefs than lenders, and no instrument that can affect the distribution of matches – will choose to implement the same allocation achieved by the decentralized market.[6]

On the contrary, the availability of a subsidy (which requires fiscal backing) allows the Ramsey planner to directly affect the payoffs of lenders and borrowers and determine the split of the surplus independently from the distribution of matches. We characterize *credit easing* as the optimal policy that takes the form of a fixed subsidy to eventual lenders' losses, lump-sum financed by borrowers. Varying the amount of the subsidy the policy maker can induce different interest rates in the market. In particular, with the subsidy lenders stop pricing the perceived risk and compete for attracting more borrowers. Hence, the subsidy acts as an implicit tax to lenders that offer interest rates different from the targeted interest rate. The implicit tax takes the form of lower matching rates for offers at non-targeted interest rates. Furthermore, the subsidy has the property of restoring the Hosios condition at the targeted interest rate.

An optimal target is a sufficiently low interest rate that could unveil the actual incentives of lenders. In fact, no subsidy will be actually given in the favorable case of safe project adoption! Thus, the social value of one-shot[7] experimentation can be indefinitely large: at no cost the policy can both reveal and implement a low interest

[4]In contrast, to rational expectations models of market malfunctioning, here lenders do not offer the optimal contract, not because is not available, but because they fail to recognize its optimality. In fact, an optimal contract is such when it optimally provides out-of-equilibrium incentives which enforce the preferred contract. In the SCE logic, lenders have missbeliefs about out-of-equilibrium borrowers' incentives, so they cannot engineer an optimal contract as 'rational expectations mechanism design theory' would vindicate.

[5]As we show, any SCE in our environment satisfies the Hosios' efficient matching condition (Moen, 1997), although, as we emphasize, this only ensures local – rather than global – efficiency.

[6]For example, in our model, a central bank conventional policy instruments, such as interest rate policies is not effective because typically is a 'local instrument' and therefore in a locally efficient equilibrium should not be used (unless the economy parameters are just at the margin), furthermore, not being an instrument target to specific markets may not be optimal to consider large deviations. Similarly, a less conventional policy, such as QE liquidity provision to banks, is not effective since more liquidity does not change the pessimistic beliefs of the banks.

[7]In a stationary environment, the policy only needs to be implemented once, since this is what it takes for misbeliefs to vanish.

no risk REE that will persist in a stationary environment. If instead the experiment reveals that such low risk REE does not exists (e.g. that in fact borrowers do not have low risk projects to finance), then it bears a temporary finite cost. In other words, the *social subjective expected value* of such experiment it is likely to be positive.

In section 4, we explore the explanatory power of our theory using new micro data in the US automotive credit market which benefited from the Term Asset Backed Securities Lending Facility (TALF). The TALF has the features of our optimal credit easing policy. The program provides non-recourse loans at low interest rates collateralized by AAA-rated Asset Backed Securities (ABS) with a 15% haircut: in practice, it constitutes a subsidy contingent to realized losses to the ABS owner.[8]

We collected data on interest rates, amounts and montly losses relative to Auto loans constituting AAA-rated ABS issued between January 2007 to December 2012 from nine companies. The dataset is particularly suitable to test our theory: it is a representative sample (52% coverage) of the second most important highly rated AAA-rated ABS sector participating in TALF, after credit cards; it is a fixed interest rage contract (independent of size); it is a secured loan (car as a pledge[9]), and, most importantly, this market does not exhibit externalities in repayment likelihood, that the credit card market for example has.[10] This last feature rules out alternative explanations based on strong complementarities as with models of multiple equilibria (for example, (Bebchuk and Goldstein, 2011)).

Our theory would say that, before TALF, the market was in a SCE with high interest rates and high realized losses, where increasing interest rates was an optimal choice to minimize losses given the available information. This is what we find in our first econometric exercise. After controlling for the business cycle and company fixed effects, we find that higher interest rates[11] are associated with lower losses in the subsample of ABS generated before the TALF.

The introduction of the TALF made the interest rates on the newly generated Auto ABS, and consequently on the underlying Auto loans, to dramatically fall. Nevertheless, even after the end of the TALF, interest rates stay low. Our theory would say that this is the effect of learning that, in contrast to that which Auto companies believed, sufficiently low interest rates generate lower losses. This is indeed what we find looking at the subsample of ABS generated during and after TALF. Using the same econometric model, now we find that lower interest rates are associated with lower losses. We interpret these two regressions as capturing the beliefs of Auto companies before and after TALF which were justified by their experience in the market.

[8] Krugman's explanation can be found here: http://krugman.blogs.nytimes.com/2009/03/23/geithner-plan-arithmetic/

[9] Credit cards, the largest sector participating in TALF, provides unsecured loans.

[10] The fact that costumers have access to more convenient loans, does not increase the probability that old costumers repay more easily their debt.

[11] Here we mean interest rates spreads, i.e. interest rate minus Libor, which is the actual variable controlled by companies.

We finally run a regression discontinuity analysis that uses the whole dataset and find evidence of a switch in the effect of interest rates on losses after TALF. This last exercise demonstrates that the riskness of Auto loans was indeed affected in a non-linear way by interest rates, which is a condition for the emergence of a self-confirming crises.

2 Self-Confirming Crises in Competitive Markets

This section introduces Self-Confirming Equilibria in a model of competitive search for credit. Competition strengths the resilience of this equilibrium concept in contrast to its original game-theoretic formulation. We then describe how the economy can slide in a Self-Confirming market freeze.

2.1 A simple game of the credit market

We start by describing the credit relationship between a single lender and a single borrower. For the sake of clarity, we introduce a minimal pay-off structure and focus attention on a one period economy; it will be clear in due course that none of the main insights of the paper hinge on these simplifications. We use this structure to briefly discuss a fragile aspect of the game theoretic formulation of a Self-Confirming Equilibrium.

A borrower

A borrower can obtain liquidity to invest in a project from a lender. A lending contract specifies an interest rate R that the borrower pays to the lender at the end of the period. Given credit conditions characterized by an interest rate R, a borrower chooses the type of the project.

The borrower can invest one unit of capital in one between two types of projects, namely a safe and a risky one, which differ for the likelihood of success and implementation costs. The table below summarizes the payoff projects that finally determine the incentives of the borrower. Both types have the same conditional per-unit return: in case of success is $1 + y$, whereas 1 in case of failure. Safe projects do not fail, but their implementation requires a fix per unit cost of k. Risky projects do not have any fix per-unit additional cost, but they can fail with a probability of $1 - \alpha$. Finally, only in case of success the borrower needs to pay $1 + R$ to the lender, otherwise the borrower repays just the capital 1.

A set of options available to the borrower is characterized by $\tilde{\omega} \equiv (\tilde{\alpha}, \tilde{k})$ belonging to $\Omega \equiv \{[0, 1], R_+\}$, that is, the couple of random parameters $\tilde{\alpha}$ and $\tilde{k}$ that characterize the return of the risky and the safe option, respectively.[12] In order to save on notation,

[12] We use a tilde to denote a random variable, $\tilde{x}$, in contrast to one of its particular realizations, x.

we will denote the choice of the borrower by $\rho \in \tilde{\omega}$, so that $\rho = \tilde{\alpha}$ or $\rho = \tilde{k}$ indicate that the borrower adopts the risky or the safe project, respectively.

The state $\tilde{\omega}$ is distributed on Ω according to a density function $\phi(\tilde{\omega})$. The borrower chooses the quality of the project before $\tilde{\omega}$ realizes, based on $\phi(\tilde{\omega})$. In our leading example, we will focus on the simplest case in which $\phi(\tilde{\omega})$ is degenerate with mass one on a particular value $\omega = (\alpha, k)$, which is therefore known to the borrower. However, in the rest of the paper, we will maintain the notation $\phi(\tilde{\omega})$ whenever the analysis is is not restricted to the specific pay-off specification that we introduce in this section.

Projects $(\alpha,\ k)$	Safe	Risky
Success	cost: $1 + R + k$ return: $1 + y$ probability: 1	cost: $1 + R$ return: $1 + y$ probability: α
Failure	cost: 1 return: 1 probability: 0	cost: 1 return: 1 probability: $1 - \alpha$

Table 1. Borrower's payoffs.

The optimal investment policy is then

$$\rho^* (R, \omega) \equiv \underset{\{\rho \in \omega\}}{\arg\max} \{\pi^b (\rho; R, \omega)\}, \tag{1}$$

with

$$\pi^b (\alpha; R, \omega) \equiv (y - R)\, \alpha, \tag{2a}$$
$$\pi^b (k; R, \omega) \equiv y - R - k, \tag{2b}$$

where $\pi^b(\rho; R, \omega)$ is the expected net return associated with the project $\rho \in \omega$ of a borrower, given a finance interest rate R and a set of project types ω available to a borrower. The participation of the borrower to the market requires that $R \leqslant y$.

A lender

A lender is an agent that has access to the money market, but cannot implement projects. A lender borrows money at a rate, δ, and chooses the interest rate R at which she makes a take or leave lending offer to a borrower. The lender observes the choice of the borrower only ex-post, so contracts are restricted to a fixed R.

The expected net return of a loan depends on the interest rate R, the cost of money δ, and the choice of the borrower ρ. In particular, the latter determines the probability that the project succeeds and hence a loan can be repaid. In particular, if the lender offers a contract at an interest R and the borrower chooses $\rho^*(R, \omega)$ the expected net return will be

$$\pi^l(R; \rho^*(R, \omega), \delta), \tag{3}$$

with

$$\pi^l(R; \alpha, \delta) \equiv \alpha R - \delta, \tag{4a}$$

$$\pi^l(R; k, \delta) \equiv R - \delta. \tag{4b}$$

Implicitly we assume that the money market is a secured market; on the contrary in the credit market liability is limited as only the project income is pledgeable. Therefore, the lender bears the cost of an eventual failure of the borrower.

Modeling counterpart risk

The surplus generated by a credit relationship, i.e. the sum of agents' payoffs, is independent from the level of the interest rate R. In the case of a *risky project* the sum of the interim surplus is $\alpha y - \delta$ whereas in the case of the *safe project* is $y - k - \delta$. Therefore as soon as $y - k/(1-\alpha)$ becomes positive the surplus generated adopting the safe project is larger.

However, the borrower has no interest to adopt the safe option when offered a too high interest rate. Specifically, for a given R, the borrower will choose to implement a safe project if and only if $\alpha(y - R) \leqslant y - k - R$ or

$$R \leqslant \bar{R} \equiv y - \frac{k}{1-\alpha}.$$

Whenever $\bar{R}$ is negative – which occurs when $k > (1-\alpha)y$, that is, the fixed cost associated with the safe project is sufficiently high – borrowers will never adopt the safe technology no matter which interest rate R is offered.

Therefore $\bar{R}$ is an important parameter in the choice of the lender. The lender needs to understand to which extent a lower R can induce the borrower to implement a safe project, which is not subject to default risk[13]; $\bar{R}$ represents the highest interest rate compatible with safe project adoption.

Nevertheless, computing $\bar{R}$ requires knowing the payoff structure of the borrower, i.e. the values y, k, and α. Whereas it is natural that an agent observes her own payoffs, it is less obvious that she can directly observe all the underlying incentives of other players. This is the key idea motivating the formulation of a Self-Confirming equilibrium, which in our case takes the following form:

Assumption (A): *A lender does not know the payoff structure of a borrower.*

[13]We refer to default risk as the risk that interest rates are not repayed by the borrower at the end of the contract.

As a consequence, the lender is a-priori uncertain about the actual behavior of the borrower. Such uncertainty generates counterpart risk in the lending contract as the borrower's choice ρ affects the returns of the lender. Let us denote $\beta(\tilde{\omega})$ the *subjective* density function of a lender, describing her beliefs about the probability that a borrower has access to a set of choices $\omega \subset \Omega$. In particular, for a given R and δ,

$$E^{\beta}\left[\pi^{l}(R;\rho^{*}(R,\tilde{\omega}),\delta)\right] \equiv \int \pi^{l}(R;\rho^{*}(R,\tilde{\omega}),\delta)\beta(\tilde{\omega})\, d\tilde{\omega}, \tag{5}$$

denotes the expected lender's profit evaluated with the probability distribution induced by β. Note that we allow for subjective density function $\beta(\tilde{\omega})$ to possibly - but not necessarily - differ from the objective density function, $\phi(\tilde{\omega})$. Nevertheless, we assume that $\mathrm{supp}(\phi(\tilde{\omega})) \in \mathrm{supp}(\beta(\tilde{\omega}))$ and, moreover, lenders' beliefs comply with Bayesian updating.

Fragility to patient experimentation in non-competitive environments

The setting introduced above can be used to briefly discuss the game-theoretical notion of self-confirming equilibrium and help to illuminate one of its limit.[14] To do so, let us assume for a moment that a lender could only choose between two interest rates, namely R_h and R_l, such that $R_h > \bar{R} > R_l$ and $R_h\alpha < R_l$.

If the lender knows the payoff structure of the borrower, she would understand that the dominant strategy is to offer R_l, anticipating that the borrower has interest to implement the safe project. Nevertheless, under the assumption (A), a lender can well entertain beliefs about $\bar{R}$, namely $\bar{R}^e$, such that $R_h > R_l > \bar{R}^e$. In this case a lender will never offer R_l.

A Self-Confirming equilibrium is one in which the lender offers R_h and the borrower implements risky projects. In this situation, even ex-post the lender will not observe the counterfactual in which the borrower adopts a safe project in response to an offer R_l (i.e. that indeed $R_h > \bar{R} > R_l$). Therefore, the equilibrium itslef do not produce any observable that can confute lender's beliefs, i.e. lender's beliefs are self-confirmed. In a Self-Confirming equilibrium the strategy of the lender is not a violation of Bayesian rationality in any respect, neither ex-ante nor ex-post.

Of course, one can wonder to which extent such an equilibrium is robust in the context of a repeated game. After any repetition, the lender will observe either the value of α if the last project of the borrower was risky, or k otherwise. Therefore, the lender could play R_l to assess the reaction of the borrower and then exploit this piece of information for future repetitions: i.e. the lender can experiment. The choice to experiment involves a trade-off. On the one hand, a one-period deviation from the believed best action R_h generates an expected opportunity cost. On the other hand, if the lender discovers unexploited opportunities, the information yield a rent for future

[14]For a formal and exhaustive discussion refer to **?**.

repetition of the game whose value can be potentially unbounded. Thus, a lender – who is patient enough and can secure the eventual gains of the experimentation – will always choose to play the perceived lottery offering R_l, at least once. In a dynamic context a Self-Confirming equilibrium is fated to break down when the lender gives high value to future payoffs and can secure rents from experiments. In this sense, the game-theoretic formulation of the Self-Confirming equilibrium (Fudenberg and Levine, 1993) is fragile to patient experimentation.

This kind of fragility is common to applications of the Self-Confirming equilibrium in Macroeconomics that hinge on the learning friction of a major actor who is large enough to influence observables (Sargent, 2001; Primiceri, 2006).[15] Deviations from the typical large player framework appears problematic given the difficulty to define settings in which atomistic agents can still retain the power to affect what they observe. In the following, we will overcome this difficulty. We will provide a static characterization of Self-Confirming equilibrium in competitive environments. Then, we will argue that competition makes Self-Confirming equilibria robust to patient experimentation, so that the main insights from the static model naturally extend to dynamic frameworks.

2.2 From games to competitive markets

Below, we present a competitive search a matching environment where credit relationships are formed randomly. In this environment, we characterize Self-Confirming equilibria. Our analysis is general in the class of linear economies characterized by the generic payoff functions $\pi^b(\rho; R, \tilde{\omega})$ and $\pi^l(R; \rho, \tilde{\omega})$, where $\pi^b_R(\rho; R, \tilde{\omega}) = -\pi^l_R(R; \rho, \tilde{\omega})$ for a given ρ, and a generic distribution $\phi(\tilde{\omega})$.

We focus on a static economy and then discuss how the insights naturally extend into a dynamic version. In contrast to non-competitive environments, even if lenders discount little the future, competition dries out the private incentives of experimentation, exactly as in a models of investment in R&D where discoveries are public goods.

Matching in the credit market

Atomistic lenders and borrowers match to form a credit relationship in the context of a competitive direct search framework, as intoduced by by Moen (1997) along the simplified variant described by Shi (2006). We normalize the mass of borrowers to one, whereas we allow free entry on the side of lenders.

Each lender can send an application for funds replying to an offer of credit posted by a borrower. The search is *directed*, meaning that at a certain interest rate R there is a subset of applications $a(R)$ and offers $o(R)$ looking for a match at that specific R.

[15] Such a major actor is typically a policy maker that, implementing policies based on a misspecified theory, prevents available data from revealing the misspecification.

The per-period flow of new lender-borrower matches in a submarket R is determined by a standard Cobb Douglas matching function

$$x\left(a(R), o(R)\right) = A a(R)^{\gamma} o(R)^{1-\gamma} \tag{6}$$

with $\gamma \in (0,1)$. This assumption, which is standard in the literature, ensures a constant elasticity of matches to the fraction of vacancies and applicants, for each submarket R.[16] . The probability that an application for a unit of credit at interest rate R is considered is $p\left(R\right) = x\left(a(R), o(R)\right)/a(R)$ and the probability that a unit of credit offered R is used is $q\left(R\right) = x\left(a(R), o(R)\right)/o(R)$. Once the match is formed the lender lends one unit to the borrower at a rate R. We will say that a submarket is active if there is at least a contract posted.

Borrowers send applications once lenders have posted their offers. A borrower sends an application to one posted contract R among the set of posted contracts H to maximize

$$J(R) \equiv p\left(R\right) E^{\phi}[\pi^{b}\left(\rho^{*}\left(R, \tilde{\omega}\right)\right)], \tag{7}$$

where $\pi^{b}\left(\rho^{*}\left(R, \tilde{\omega}\right)\right)$ is a shorter notation for $\pi^{b}\left(\rho^{*}\left(R, \tilde{\omega}\right); R, \tilde{\omega}\right)$. In (7) we assume that borrowers apply for credit without knowing the realization of their individual state ω, which ensures $p(R)$ being independent of $\tilde{\omega}$. A relaxation of this assumption is quite innocuous for the purposes of this section, although it introduces technical caveats for the analysis in section 3 (see footnote **??**). The competitive behavior of borrowers implies that the mass of applicants to a submarket $R' \in H$, namely $a(R')$ increases (resp. decreases), whenever $J(R') > J(R'')$ for each $R'' \in H$ (resp. $J(R') < J(R'')$ for at least a $R'' \in H$). Competition among borrowers implies that $J(R)$ is equalized across the posted contracts, i.e. more profitable contracts are associated with lower probabilities of matching.

Lenders are first movers in the search: they choose whether or not to pay an entry cost c and, once in the market, at which interest rate R they post a contract. A posted R is a solution to **the lender's problem**:

$$\max_{R} E^{\beta}[q\left(R\right) E^{\beta}[\pi^{l}\left(R; \rho^{*}(R, \tilde{\omega}), \delta\right)] - c], \tag{8}$$

subject to

$$p\left(R\right) E^{\beta}[\pi^{b}\left(\rho^{*}(R, \tilde{\omega})\right)] = \bar{J}, \tag{9}$$

and

$$q\left(R\right) = A^{\frac{1}{1-\gamma}} p\left(R\right)^{-\frac{\gamma}{1-\gamma}}, \tag{10}$$

where $\bar{J}$ is an arbitrary constant[17], and (10) is a direct implication of (6). The con-

[16] In particular, the ratio $\theta\left(R\right) = a\left(R\right)/o\left(R\right)$ denote the tightness of the submarket R. The tightness is a ratio representing the number of borrowers looking for a credit line *per-unit of vacancies.* Notice that the tightness is independent of the absolute number of vacancies open in a certain market.

[17] Here, we emphasize that the lender does not need to know the equilibrium value of $J(R)$, but just

straints (9) and (10) make sure that the individual lender takes the probability of matching in a submarket as given. Such probabilities are evaluated according to the subjective probability distribution $\beta(\tilde{\omega})$. Lenders cannot individually affect the distribution of offers and applications, and in particular, the expected utility granted to borrowers. Thus, the competitive behavior of borrowers implies that (9) holds, which together with (10), defines $q(R)$.

On the side of the lenders, free entry guarantees competition, so that the mass of lenders posting a contract in the submarket R, namely $o(R)$, increases (resp. decreases) whenever $E^{\beta}[V(R)] > 0$ (resp. $E^{\beta}[V(R)] < 0$), where

$$V(R) \equiv q(R)\, E^{\beta}[\pi^{l}(R; \rho^{*}(R, \tilde{\omega}), \delta)] - c, \tag{11}$$

is the value of posting a vacancy. Competition among lenders implies that $E^{\beta}[V(R)] = 0$, i.e. lenders run at zero profits.

Notice that, in order to solve (8), a lender needs to anticipate the reaction of the borrower $\rho^{*}(R, \tilde{\omega})$ to an offer R, to determine both the probability $q(R)$ that an offer R is accepted and the default risk associated with it. Hence a lender bears the risk associated with the probability that a contract is not filled and the uncertainty, or possible missperception, concerning the payoff incentives of lenders.

Equilibria: SSCE and REE

We present the definition of Strong Self-Confirming Equilibrium (SSCE) and we contrast it to the notion of Self-Confirming Equilibrium (SCE) and the one of Rational Expectation Equilibrium (REE).

Definition 1 (SSCE)**.** *Given an objective density function $\phi(\omega)$, a Strong Self-Confirming equilibrium (SSCE) is a set of posted contracts H^{*} and beliefs $\beta(\omega)$ such that:*

sc1) for each R^{*}, the maximizing value for the borrower $J(R^{*}) = \bar{J}$;

sc2) each R^{*} solves the lender's problem (8)-(10);

sc3) there is an open neighborhood of R^{*}, namely $\mathfrak{I}(R^{*})$, such that for any $R \in \mathfrak{I}(R^{*})$ it is

$$E^{\beta}[V(R)] = E^{\phi}[V(R)], \tag{12}$$

that is, borrowers correctly anticipates lenders' reaction only locally around the realized equilibrium contracts.

The third condition (sc3) restricts lenders' beliefs $\beta(\tilde{\omega})$ about borrowers' actions to be correct in a neighborhood of an equilibrium R^{*}. This is also a stronger beliefs' restriction than the one usually assumed in the notion of Self-Confirming Equilibrium

that she cannot affect it.

(SCE), which instead does not contemplate any belief restriction out of equilibrium. In fact, at a SCE, condition (sc3) holds punctually for any R^* rather than for any $R \in \mathfrak{I}(R^*)$.

Crucially, the definition of SSCE does not require lenders to have correct beliefs about non-realized out-of-equilibrium behavior. This leaves open the possibility that, in a SSCE, better contracts out of a neighborhood of the equilibrium could be wrongly believed by lenders to be strictly dominated by existing ones. In particular, lenders may missperceive the actions of the borrowers would take – and the resulting risks – when offered lower interest rates. Since such contracts will not be posted, then in equilibrium there do not exist counterfactual realizations that can confute wrong beliefs, neither ex-ante nor ex-post.

A REE is a stronger notion than a SSCE requiring that no agent holds wrong out-of-equilibrium beliefs. In the present model this equals to impose that lenders' unbiased beliefs about borrowers' payoffs. In such a case the equilibrium contract is the one which objectively yields the highest reward with respect to every possible feasible contract.

Definition 2 (REE)**.** *A rational expectation equilibrium (REE) is a Self-Confirming equilibrium such that, at any $R \in \mathfrak{R}$, (12) holds, that is, lenders correctly anticipates borrowers' reaction for any possible contract.*

A REE obtains from a tightening of condition (sc3) in the definition of a SSCE. This implies that every $R^\star \in H^\star$ is such that lenders can exactly forecast the out-of-equilibrium value of posting a credit line as they can correctly anticipate borrowers' responses. Therefore, posting in the submarket $R^\star$ is a globally dominant strategy both from an objective and a subjective point of view.

Equilibrium Characterization

We provide now a characterization of an equilibrium. We develop a non-marginal technique that can be generally used to identify the equilibrium of search and matching economies with non convex payoff structures and potentially multiple local maxima. As we will show the Hosios condition obtains from a linear local approximation of the global criterion.

Plugging constraints into the objective we can derive the condition for an equilibrium contract as:

$$R^* = \arg\max \left(A^{\frac{1}{1-\gamma}} \bar{J}^{-\frac{\gamma}{1-\gamma}} E^\beta[\pi^b\left(\rho^*(R,\tilde{\omega})\right)]^{\frac{\gamma}{1-\gamma}} E^\beta[\pi^l\left(R;\rho^*(R,\tilde{\omega}),\tilde{\omega}\right)] - c] \right),$$

so that, after defining

$$\mu(R) \equiv E^\beta[\pi^b\left(\rho^*(R,\tilde{\omega})\right)]^{\frac{\gamma}{1-\gamma}} E^\beta[\pi^l\left(R;\rho^*(R,\tilde{\omega}),\tilde{\omega}\right)], \tag{13}$$

we have the following lemma as a direct consequence.

Lemma 1. *Consider two contracts posted respectively at* R_1 *and* R_2*. From the point of view of a single atomistic lender*

$$E^{\beta}\left[V\left(R_1\right)\right] \geqslant E^{\beta}\left[V\left(R_2\right)\right] \Leftrightarrow \mu\left(R_1\right) \geqslant \mu\left(R_2\right), \tag{14}$$

for any profile of contracts offered by other lenders.

Note that the evaluation of R does not depend on $\bar{J}$, i.e. it does not depend on the level of utility granted to the other side of the market, which a single lender cannot affect. However, lenders partly internalize the welfare of borrowers as contracts that provides better conditions for borrowers are more likely to be signed. In particular, with $\gamma = 0$, when all the surplus is extracted by lenders, (14) becomes $E^{\beta}\left[\pi^{l}(R_{;}\rho^{*}(R_1,\tilde{\omega}),\tilde{\omega})\right] \geqslant E^{\beta}\left[\pi^{l}(R_2,\rho^{*}(R_2,\tilde{\omega}),\tilde{\omega})\right]$, that is at the equilibrium only the interim payoff of lenders is maximized as borrowers will always earn zero. With $\gamma = 1$ instead, when the whole surplus is extracted by borrowers, (14) becomes $E^{\beta}\left[\pi^{b}(\rho^{*}(R_1,\tilde{\omega}))\right] \geqslant E^{\beta}\left[\pi^{b}(\rho^{*}(R_2,\tilde{\omega}))\right]$, that is only the interim payoff of borrowers is maximized as lenders will always earn nothing.

Let us introduce here a definition of local maxima of μ evaluated by the system of beliefs β and ϕ, respectively.

Definition 3. *A contract* R' *is a* β*-local maximum for the lender if there exists a neighborhood of* R'*, namely* $\mathcal{I}(R')$*, such that*

$$\mu(R') = \sup_{R\in\mathcal{I}(R')} \mu(R), \tag{15}$$

with $\mathcal{M}^{\beta}$ *denoting the set of* β*-local maxima. An interior* β*-local maximum is a contract* R *such that*

$$\mu\left(R\right)\left(\frac{\gamma}{1-\gamma}\frac{1}{E^{\beta}[\pi^{b}\left(\rho^{*}(R,\tilde{\omega})\right)]} - \frac{1}{E^{\beta}[\pi^{l}\left(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega}\right)]}\right) = 0, \tag{16}$$

with $\hat{\mathcal{M}}^{\beta} \subseteq \mathcal{M}^{\beta}$ *denoting the set of interior* β*-local maxima. The corresponding sets of local* ϕ*-maxima, namely* $\hat{\mathcal{M}}^{\phi}$ *and* $\mathcal{M}^{\phi}$*, obtain for* $\beta = \phi$*.*

The definition above allows a simple characterization of the equilibria as follows.

Lemma 2. *For a given* $\phi(\tilde{\omega})$ *and* $\beta(\tilde{\omega})$*, a set of contracts* H^* *is a SSCE but not REE if any* $R^* \in H^*$ *is such that: i)* $R^* = \sup \mathcal{M}^{\beta}$*, ii)* $R^* \in \mathcal{M}^{\phi}$ *but* $R^* \neq \sup \mathcal{M}^{\phi}$*; whereas it is a REE if any* $R^* \in H^*$ *is such that:* $R^* = \sup \mathcal{M}^{\beta} = \sup \mathcal{M}^{\phi}$*.*

The requirement $R^* \in \mathcal{M}^{\phi}$ is a direct consequence of having $\beta = \phi$ locally around the equilibrium. Of course, (16) is satisfied locally by any interior SSCE (or REE), i.e. an equilibrium where neither incentive-compatibility nor participation constraints are binding. In such a case we can obtain a marginal condition on the elasticity of the $\mu(R)$ function which identifies the local maximum.

The criterion, (16) reduces to the famous Hosios condition, that is, R^* is such that $\pi^{b} = \gamma(\pi^{b} + \pi^{l})$ and $\pi^{l} = (1-\gamma)(\pi^{b} + \pi^{l})$.

Robustness to Patient Experimentation

In contrast to the non-competitive versions of the self-confirming equilibrium, our characterization is robust to experimentation by highly patient agents. To sketch our argument we shall extend our static economy to an infinite horizon economy where each period is a repetition of our one-period version. A SSCE in this environment is a sequence of static SSCE. The only key difference is that now beliefs are updated dynamically in light of the equilibrium realizations of the period before. Of course, given that lender's beliefs are correct at the equilibrium, what a lender will observe along the equilibrium path is of no help to refine their beliefs.

We ask ourselves whether, when the competitive economy is on a SSCE, a patient lender, who is aware of her ignorance, has any incentive to experiment lower interest rates. The answer is "no" and follows as a consequence of two very general points.

First, any individual deviation from the equilibrium is perceived to be costly. This is of course true by definition of any equilibrium. Notice that, in this respect, the restriction to one-period contract is innocuous, we could extend the model including richer contract specifications, without changing the conclusion that on a SSCE each lender perceives that deviating entails a loss (big or small does not matter) with respect to the status quo.

Second, all the information generated by the market is public, i.e. there are no informational rents. This condition is a pre-requisite for the decentralized economy to achieve the competitive outcome. The implication is that the outcome of any eventual deviation from the equilibrium will be observed by all competitors and used to update beliefs. This argument does not require to specify which information is produced or observed, but only that *if* a deviation produces information which is sufficiently informative for a single lender to update beliefs, then all lenders will update beliefs in the same way. As a result, the zero-profit condition will necessarily hold after any individual deviation, whatever is the outcome.

In the end, competition dries out lenders' profits so that they always expect to earn zero profits at *any* equilibrium: there are no informational rents that the individual lender could eventually exploit in this environment. Individual gains are only possible out of equilibrium. Nevertheless, deviations from an equilibrium, are, by definition, expected to yield negative profits. Therefore, the typical fragility of self-confirming equilibrium to patient experimentation in non-competitive environments does not apply in competitive ones. For this reason, the insights of our simple static formulation are not less general that the ones we could obtain in any dynamic extension of the model.

2.3 Self-Confirming credit crises

We use now our simple specification – with pay-offs (2) and (4), and a given ω – to describe how an economy can slide into a Self-Confirming crises.

The first step to compute equilibria is to work out the set $\mathcal{M}^{\phi}$ to which a SSCE belongs. We can use (16) to compute the interior local maximum relative to safe and risky project choice, respectively. Then, we check whether such contracts are within the bounds imposed by incentive-compatibility and participation constraints. Note that, conditionally to a particular project choice, the problems are nicely concave, so that a unique maximum typically arise.

Proposition 1. *For a given ω, the set of ϕ-local maxima is $\mathcal{M}^{\phi} = \{R_s^*, R_r^*\}$ where $R_s^* = \min(\bar{R}, \hat{R}_s)$ with*

$$\hat{R}_s = (1-\gamma)(y-k) + \gamma\delta, \tag{17}$$

provided that $R_s^* > \delta$, and $R_r^* = \min(y, \hat{R}_r)$ with

$$\hat{R}_r = (1-\gamma)y + \frac{\gamma}{\alpha}\delta \tag{18}$$

and it exists if $R_r^* > \bar{R}$.

Proof. Postponed to Appendix A.1 ∎

$\hat{R}_s$ and $\hat{R}_r$ represent interior local maxima, namely the contracts which locally maximize lenders' profits when no constraints are binding; R_s^* and R_r^* instead account for the possibility that constraints bind. In particular, $R_r^* > R_s^*$, that is, ceteris paribus, risky projects imply higher interest rates. Nevertheless, the expected profit of both a borrower and a lender can be higher when a risky project is implemented depending on parameters (for example, when $\alpha = 1$).

Let us now characterize the set of REE, i.e. $\sup \mathcal{M}^{\phi}$.

Proposition 2. *For a given ω, there exists a threshold value $\hat{\alpha}(k) \in (\underline{\alpha}(k), \overline{\alpha}(k))$, where*

$$\underline{\alpha}(k) = \frac{y - \hat{R}_s - k}{y - \hat{R}_s} \quad \textit{and} \quad \overline{\alpha}(k) = \frac{y - \delta - k}{y - \delta}$$

which is decreasing in k, such that:

(i) *if $\alpha < \hat{\alpha}(k)$ then $\sup \mathcal{M}^{\phi} = \{R_s^*\}$,*

(ii) *if $\alpha > \hat{\alpha}(k)$ then $\sup \mathcal{M}^{\phi} = \{R_r^*\}$,*

(iii) *only for $\alpha = \hat{\alpha}(k)$ then $\sup \mathcal{M}^{\phi} = \{R_r^*, R_s^*\}$.*

Proof. Postponed to Appendix A.2 ∎

The proposition establishes that for a sufficiently high level of riskiness ($\alpha < \hat{\alpha}$) then the safe equilibrium is the unique REE, otherwise the risky equilibrium is the unique REE. The threshold $\hat{\alpha}$ is the only value of α where two REE exist in this model. This threshold lies in $(\underline{\alpha}, \overline{\alpha})$, that is the interval for which $R^s = \bar{R}$ that is a safe equilibrium arises as a corner contract.

We state now the existence of a unique equilibrium contract which is SSCE but not REE.

Proposition 3. *Given $\phi(\omega)$, only R^*_r can be SSCE without being REE. In particular, for $R^*_r \in \mathcal{M}^\phi$ it is $\sup \mathcal{M}^\beta = \{R^*_r\}$ without being $\sup \mathcal{M}^\phi = \{R^*_r\}$ if : $\alpha < \hat{\alpha}$ and $\mathbb{E}^\beta[k]$ is sufficiently high.*

Proof. Postponed to Appendix A.3 ∎

The two conditions for the existence of a risky SSCE that is not a REE are intuitive. First, the safe equilibrium must be globally a strictly dominant contract when evaluated with the objective distribution. Second, lenders must believe that it is sufficiently unlikely that borrowers will adopt the safe project if they lower the interest rate; i.e. they expect to be in a risky REE. A sufficiently high level of k is, for example, $\mathbb{E}^\beta[k] > (1-\alpha)r$.

There are two feature of a SSCE that are worth to notice. First, the model has the important feature of generating a unique determinate SSCE that is not a REE which that for excessive credit tightening and risk taking. Hence, missbeliefs can only sustain credit crises. Second, the beliefs that sustain a Self-Confirming crisis should satisfy a threshold condition, which is compatible with belief heterogeneity. Moreover, considering risk aversion or ambiguity would enlarge the set of beliefs that sustain a Self-Confirming crisis.

Figure 1 illustrates our findings in the space (α, R). Our baseline configuration is $r = 0.03, \delta = 0.008, k = 0.005, \gamma = 0.4, c = 0.001, A = 0.1$. The feasible range of equilibrium interest rates compatible with the adoption of a safe (resp. risky) technology is the region below (resp. above) the dotted thin curve representing the adoption frontier of borrowers. For any α, R_r and R_s are denoted by respectively the upper and lower thick lines. In particular, the red solid line denotes the unique REE for a given $\alpha > \hat{\alpha}$. For $\alpha < \hat{\alpha}$ the unique SSCE which is not REE is plotted in solid blue. The red dotted line represent the REE safe equilibrium for $\alpha < \hat{\alpha}$.

We are ready now to describe how a transition from a REE period to a Self-Confirming crisis. We can think about a credit crises as determined by an exogenous fall in α. When the risk is low ($\alpha_{high} = 0.9$) then the unique REE is the risky equilibrium where borrowers only adopt risky projects, which are indeed not that risky. This is optimal. When risk increases up to a sufficiently high level ($\alpha_{low} = 0.55$), rational expectation would predict that the lenders switch to the low interest rate regime. In the logic of Self-Confirming equilibria whether or not the “jump" occurs at the right point crucially depends on lenders’ expectations about the counterfactual reaction of borrowers. In particular, the actual borrowers’ reaction to low interest rates could not materialize as long as lenders keep posting high interest rates. Therefore, the trap

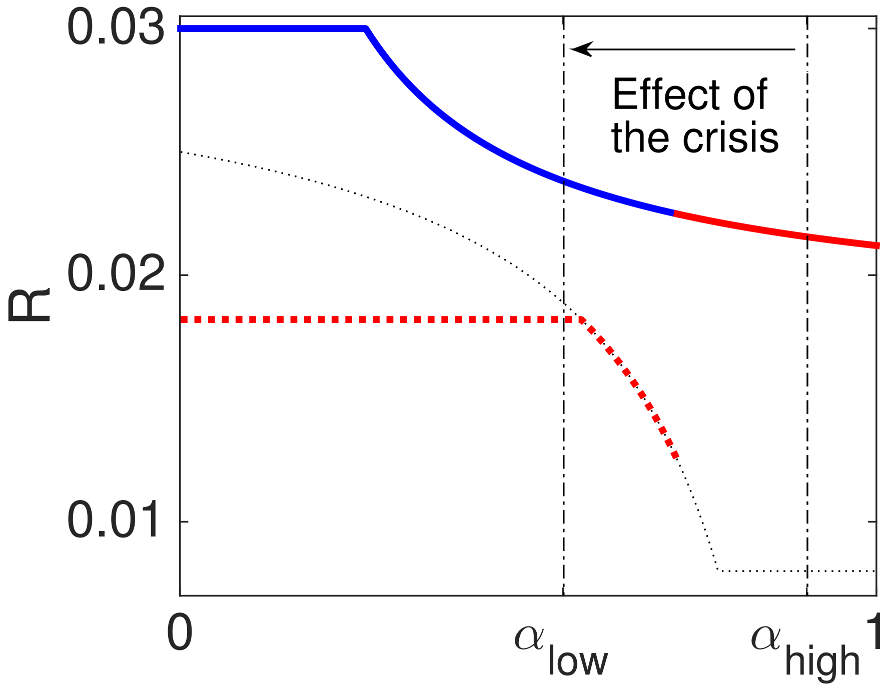


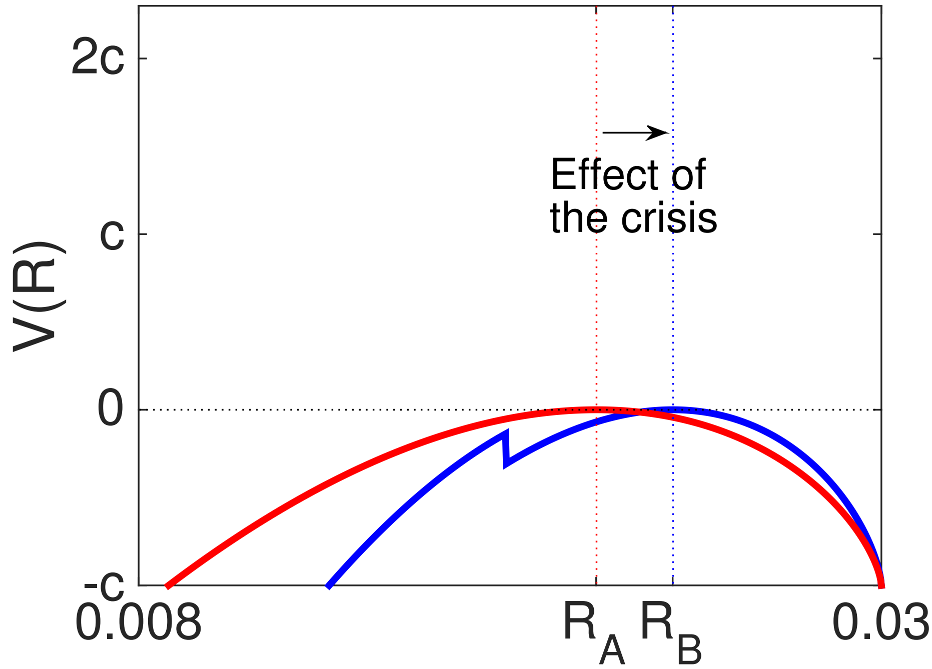
Figure 1: Weaker fundamentals create room for a Self-Confirming crisis.

is self-fulfilling as high interest rates, justified by perceived counterpart risk, induces risky choices by borrowers.

Figure 2 illustrates the individual maximization problem corresponding to the situation plotted in previous figure. On the x-axes we represent R, i.e. the individual choice of a lender. On the y-axis we measure the expected pay-off of the individual lender $\mathbb{E}^{\beta}[V(R)]$ *when all the other lenders post at the equilibrium* R. Thus, the figure shows the individual incentive to deviate from an equilibrium prescription. When risk is low ($\alpha = \alpha_{low}$ as in figure 1) the maximization problem of the lender is represented by the convex solid red curve, whose maximum at R_A yields zero, as implied by the zero profit condition. After an exogenous increase in risk ($\alpha = \alpha_{high}$ as in figure 1) the maximization problem becomes the solid blue line. The new curve picks at $R_B > R_A$, accounting for a larger risk premia factored in interest rates. Moreover, a discontinuity arises for a sufficiently low interest rate, as a result of the uncertainty of a lender about the counterfactual reaction of a borrower. In particular, in this example, we consider the case $\beta(k = 0.005) = 0.07$, whereas the probability that the state is $\beta(k = 0.015) = 0.93$ where, notice $0.015 > r(1 - \alpha)$ that is, $k = 0.015$ is too high to induce a safe adoption for any feasible level of R.

Figure 3 illustrates the perceived lottery of the lender. The lower dotted blue line denotes the lender's payoff in the case $k = 0.015$, whereas the higher dotted blue line represents the case $k = 0.007$. In the latter, an *individual* deviation from R_B to R_C, with all other lenders posting at R_B, would yield a large profit. Nevertheless, this possibil-

Figure 2: In a Self-Confirming crises, an increase in risk implies higher interest rates.



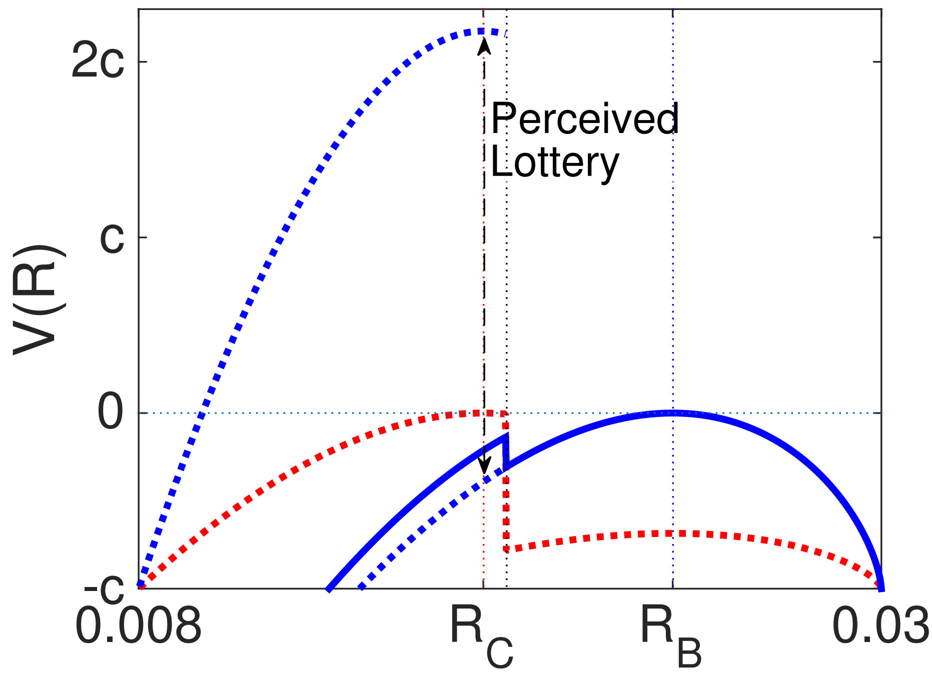


Figure 3: At a Strong Self-Confirming Equilibrium, a lender perceives a out-of-equilibrium lottery with negative expected value.

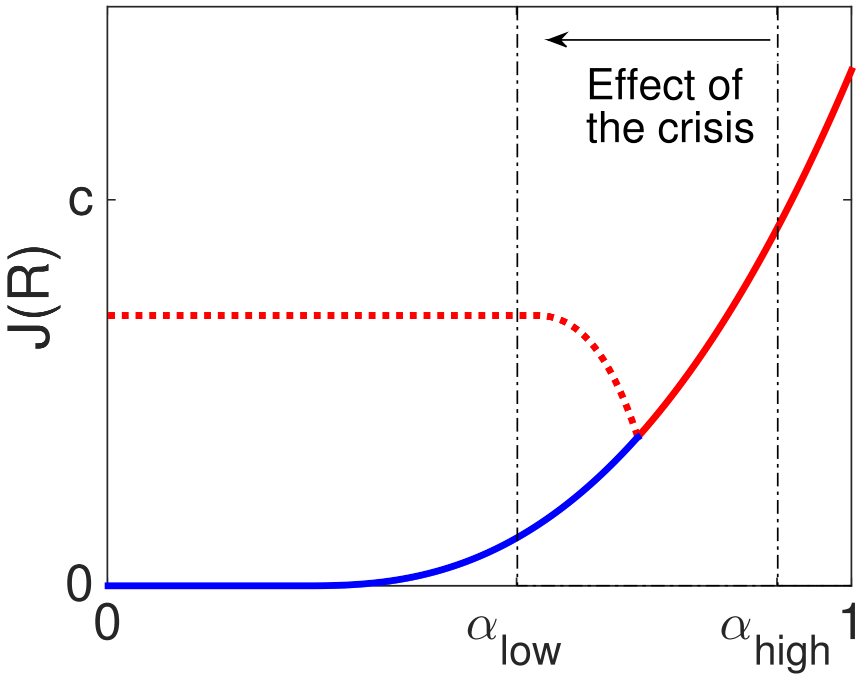


Figure 4: Social welfare in a self-confirming crises.

ity is believed with a too small probability to induce an individual deviation. This is a necessary condition for R_B to be an equilibrium. Hence, a set of misspecified beliefs can sustain a SSCE that is not a REE when, as in our example, an existing globally optimal state for the lender is not believed sufficiently likely. Such missbelief cannot even be confuted ex-post because, given that only high interest rates are offered, no borrower adopts the safe option in equilibrium.

The maximization problem of a lender when all other lenders post at the unique REE interest rate R_C (unique conditionally to $\alpha = 0.55$ and $k = 0.007$) is represented by the dotted red curve in figure 3. The local and global maximum is at R_C and yields zero profits. This is useful to illustrate how competition dries out private incentives of experimentation in a dynamic extension of the model. Suppose for a moment that (despite pessimism) a lender discovers that posting R_C grants profits, i.e. $k = 0.007$ is known with certainty now. This outcome will be observed by other competitors that in turn will choose to post R_C, fulfilling the zero-profit condition anyway.

Finally, figure 4 plots the equilibrium social welfare, measured in terms of cost-per-vacancy c, as a function α. Colors and traits are used to distinguish the REE from the other SSCE, as in figure 1. Notice that since lenders run at zero expected profits, the social welfare coincides with the expected profits of borrowers. Social welfare is increasing in α (and so decreasing in R_r) when the economy is on a risky equilibrium, whereas it is insensitive to risk at a REE where borrowers adopt safe projects.[18] The

[18]Social welfare is decreasing in α whenever $\alpha \in (\underline{\alpha}, \bar{\alpha})$ for which the safe equilibrium arises as a

effect of a Self-Confirming crises triggered by an increase in fundamental risk is a dramatic decrease of social welfare. The drop would have been much lower in the REE instead. How a policy maker, who shares the same beliefs of lenders, would (should) act in a Self-Confirming crises?

3 Credit Easing as an optimal policy

In this section, we demonstrate how a targeted subsidy from the central bank to the private lenders, financed by borrowers, can be a powerful tool to break a socially inefficient equilibrium and implement, if it exists, a welfare maximizing equilibrium. Under fairly general conditions, we show that if the objective of the CB is to maximize *ex-ante* social welfare, this is equivalent to maximize the *interim* total surplus of a match between a borrower and a lender, when an optimal subsidy is implemented. Moreover, the CB will implement the policy whenever it assigns positive probability to the existence of a REE associated with strictly higher *ex-ante* surplus.

On the other hand, the lenders' private evaluation of the policy does not align with the social evaluation because, as the subsidy moves the whole distribution of matches, the lenders will always expect to earn zero profit.

3.1 Welfare evaluation of *laissez-faire* economies

Let us first analyze the problem of a benevolent social planner who maximizes social welfare, by choosing R as a policy instrument and subject to the directed search competitive restrictions; in particular, the free entry lender's expected zero profit condition, which means that the planner maximizes $J(R)$. Importantly, we will provide the planner with the same subjective beliefs of the lenders, and no fiscal capacity, i.e. without power to operate transfers between lenders and borrowers.

The problem of a planner *without* fiscal capacity is:

$$\max_{R} E^{\beta}[p(R)\, E^{\beta}[\pi^{b}(\rho^{*}(R,\tilde{\omega}))]], \tag{19}$$

subject to

$$c = q(R)\, E^{\beta}[\pi^{l}(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega})]$$

and

$$q(R) = A^{\frac{1}{1-\gamma}} p(R)^{-\frac{\gamma}{1-\gamma}};$$

i.e, the social planner maximizes social welfare taking the zero profit condition and the market tighthenss as a constraint. Notice that in (19) the subjective beliefs are those of the planner. Here we assume that they share the same beliefs, β, but in general there may be different. We will refer to a *laissez-faire* economy as one in

corner solution constrained by the borrowers' adoption.

which a planner has no other instrument than R to affect the terms of trade (and in particular has no fiscal power).

As before, plugging constraints into the objective we can derive the constrained first-best contract as[19]

$$R^{\star} = \arg_R \max \left(A^{\frac{1}{\gamma}} c^{-\frac{1-\gamma}{\gamma}} E^{\beta}[\pi^l(R; \rho^*(R, \tilde{\omega}), \tilde{\omega})]^{\frac{1-\gamma}{\gamma}} E^{\beta}[\pi^b(\rho^*(R, \tilde{\omega}))] \right),$$

so that after defining

$$\bar{\mu}(R) \equiv E^{\beta}[\pi^l(R; \rho^*(R, \tilde{\omega}), \tilde{\omega})]^{\frac{1-\gamma}{\gamma}} E^{\beta}[\pi^b(\rho^*(R, \tilde{\omega}))], \tag{20}$$

we have a criterion to rank the welfare generated by different contracts.

Lemma 3. *Consider two alternative laissez-faire economies trading at interest rate* R_1 *and* R_2*,respectively. From a the point of view of a planner:*

$$E^{\beta}[J(R_1)] \geqslant E^{\beta}[J(R_2)]$$

if and only if

$$\bar{\mu}(R_1) \geqslant \bar{\mu}(R_2), \tag{21}$$

for any profile of contracts offered by other lenders.

Comparing (21) and (14) we can easily see that the two criteria are maximized for the same equilibrium contract, i.e. $R^* = R^{\star}$. We therefore obtain the following proposition, which is a version of the well known result on the efficiency of the directed search competitive equilibrium.

Lemma 4. *In a laissez-faire economy where lenders and the planner have the same subjective beliefs, the competitive allocation is a solution to the planner's problem.*

The proposition states that in an economy in which the social planner has no other instrument than R to alter the terms of trade, the socially preferred allocation coincides with the one determined by the decentralized market. It reproduces the standard result on the constrained efficiency of directed search equilibria, except that in our context it is a local equilibrium result when the planner and the competitive agents (private lenders) share the same subjective beliefs.

3.2 Welfare evaluation with a subsidy as policy instrument

In this subsection we will introduce the possibility that the social planner can implement linear transfers between borrowers and lenders. The subsidy has three important features:

[19] From here onward, we sill use a $\star$ to denote an outcome determined by the planner, as opposed to to $*$, which denotes the outcome determined by private agents.

i) the subsidy that an individual lender *expect* to receive in a match is independent of her offer;

ii) the tax that an individual borrower *expect* to pay in a match is independent of her project choice;

iii) the subsidy is financed by a lump sum tax on matched borrowers;

The subsidy is a powerful instrument, which the social planner can use to implement the competitive equilibrium that maximizes social welfare. As said, the TALF in the ABS auto-loan market intervention by the FED in 2009 is a particular example of this subsidy.

Optimal subsidy

To recover the optimal subsidy let us first work out the problem of the authority that, for a given R, chooses an optimal subsidy $s^\star(R)$ in order to maximize social welfare. **The problem of a planner *with* fiscal capacity** is:

$$\max_{R,s} E^{\beta}[p\,(R)\,E^{\beta}[\pi^{b}(\rho^{*}(R,\tilde{\omega}))-s]], \tag{22}$$

subject to

$$c = q\,(R)\,E^{\beta}[(\pi^{l}\,(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega})+s)],$$

and

$$q\,(R) = A^{-\frac{1}{1-\gamma}}p\,(R)^{-\frac{\gamma}{1-\gamma}},$$

where s denotes a subsidy to lenders financed by taxing borrowers so that in expectation there is no distortion on the individual project choice. Plugging constraints into the objective we can derive the optimal subsidy for a given R as:

$$s^\star(R) = \arg_s \max\left(A^{\frac{1}{\gamma}}c^{-\frac{1-\gamma}{\gamma}}E^{\beta}[\pi^{l}\,(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega})+s]^{\frac{1-\gamma}{\gamma}}E^{\beta}[\pi^{b}(\rho^{*}(R,\tilde{\omega}))-s]\right),$$

so that after defining

$$\hat{\mu}(R,s^\star(R)) \equiv E^{\beta}[\pi^{l}\,(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega})+s^\star(R)]^{\frac{1-\gamma}{\gamma}}E^{\beta}[\pi^{b}(\rho^{*}(R,\tilde{\omega}))-s^\star(R)], \tag{23}$$

we have a criterion to rank the welfare generated by different contracts provided the authority implements the optimal subsidy. In the case of linear economies (i.e. when $\pi^{l}_{R} = -\pi^{b}_{R}$) the optimal subsidy $s^\star$ targeting a contract R satisfies the first-order condition:

$$\hat{\mu}(R,s^\star(R)\left(\frac{1-\gamma}{\gamma}\frac{1}{E^{\beta}[\pi^{l}(R;\rho^{*}(R,\tilde{\omega}),\tilde{\omega})]+s^\star(R)}+\frac{1}{E^{\beta}[\pi^{b}(\rho^{*}(R,\tilde{\omega}))]-s^\star(R)}\right)=0. \tag{24}$$

Therefore, the optimal subsidy targeting a R satisfies:

$$s^\star(R) = (1-\gamma)E^\beta[\pi^b(\rho^*(R,\tilde{\omega}))] - \gamma E^\beta[\pi^l(R;\rho^*(R,\tilde{\omega}),\tilde{\omega})]. \tag{25}$$

Notice that the optimal subsidy implies a split of the total expected interim surplus determined by the relative elasticities of the matching function to the mass of applicants and offers:

$$\begin{aligned} E^\beta[\pi^b(\rho^*(R,\tilde{\omega})) - s^\star(R)] &= \gamma E^\beta[\mathcal{S}(R,\rho^*(R,\tilde{\omega}))], && (26)\\ E^\beta[\pi^l(R;\rho^*(R,\tilde{\omega}),\tilde{\omega}) + s^\star(R)] &= (1-\gamma)E^\beta\left[\mathcal{S}(R,\rho^*(R,\tilde{\omega}))\right]. && (27)\end{aligned}$$

where $\mathcal{S}(R,\rho^*(R,\tilde{\omega})) \equiv \pi^b(\rho^*(R,\tilde{\omega})) + \pi^l(R;\rho^*(R,\tilde{\omega}),\tilde{\omega})$ is the total interim surplus generated by the project choice of the borrower as an optimal reaction to an offer R. Finally, plugging (25) back into (23) gives

$$\hat{\mu}(R,s^\star(R)) = \gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}} E^\beta[\mathcal{S}(R,\rho^*(R,\tilde{\omega}))]^{\frac{1}{\gamma}},$$

which, reduces the social evaluation to a simple total expected surplus criterion, given by $\log\left(\hat{\mu}(R,s^\star(R))\right)$. In particular,

Lemma 5. *Consider two alternative subsidized economies trading at interest rate* R_1 *with subsidy* $s^\star(R_1)$ *and* R_2 *with* $s^\star(R_2)$*, respectively. From a the point of view of a planner:*

$$E^\beta\left[J\left(R_1,s^\star(R_1)\right)\right] \geqslant E^\beta\left[J\left(R_2,s^\star(R_2)\right)\right]$$

if and only if

$$E^\beta[\mathcal{S}(R_1,\rho^*(R_1,\tilde{\omega}))] \geqslant E^\beta\mathcal{S}(R_2,\rho^*(R_2,\tilde{\omega}))]. \tag{28}$$

In general the criteria (21) and (28) do not necessarily coincide. The reason is that, without the subsidy, the interest rate R determines at the same time both: i) the incentives of the borrower in the project choice and ii) the incentive of the lender to post an offer, which depends on the split of the expected surplus. The presence of the subsidy makes possible to disentangle these two dimensions. In particular, the optimal subsidy achieves an efficient share of the (subjectively expected interim) surplus for a given R, so that now R can be targeted to induce lenders to select the type of project which maximizes the surplus.

Credit Easing as a Social Experiment

Since the criteria (21) and (28) may not coincide, there will be situations in which the planner would like to implement the subsidy and change the decentralized allocation in order to achieve a higher *expected* welfare. Moreover, in the case of a SSCE that is not REE, the policy constitutes a social experiment as it is based on subjective beliefs β, which can be misspecified. In such a case the policy has the effect of producing evidence that can correct beliefs and clear uncertainty. A temporary policy intervention could be necessary to break the spell of missbeliefs about counterpart risk.[20]

As we have seen, the design of the optimal subsidy is an exercise on optimal implementation. As a result, the authority can induce whatever equilibrium contract she prefers by means of the subsidy. To see this, suppose the authority targets a certain $R^\star$ and implements the subsidy $s^\star(R^\star)$. By our definition of SSCE, subjective and objective beliefs coincide *locally*, therefore it is easy to see that the lenders' evaluation becomes

$$\mu(R, s^\star(R)) = \hat{\mu}(R, s^\star(R))^{\frac{\gamma}{1-\gamma}}, \tag{29}$$

where $\mu(R, 0)$ is nothing else than (13). Following our notation for local equilibria, let us denote by $\mathcal{M}^{\beta}_{s^\star(R)}$ (resp. $\hat{\mathcal{M}}^{\beta}_{s^\star(R)}$) the local (resp. interior) maxima of the function $\mu(R, s^\star(R^\star))$. Therefore, the first order condition of $\mu(R, s^\star(R))$ with respect to R (the analogous to (16)), satisfies

$$\mu(R, s^\star(R^\star)) \left(\frac{\gamma}{1-\gamma} \frac{1}{E^{\beta}[\pi^b(\rho^*(R,\tilde{\omega})) - s^\star(R^\star)]} - \frac{1}{E^{\beta}[\pi^l(R; \rho^*(R,\tilde{\omega}), \tilde{\omega}) + s^\star(R^\star)]} \right) = 0, \tag{30}$$

exactly at $R^\star$ because of (25). The conditions (26), (27) and (30) makes clear that that the optimal subsidy restores *locally* optimality at *any* targeted contract in the sense of the Hosios condition (see Hosios (1990)). We can finally state the following.

Proposition 4. *Suppose the authority targets a contract $R^\star$ fixing a targeted subsidy $s^\star(R^\star)$, then the lenders' best reply to this policy is to offer:*

$$R^\star = \sup \mathcal{M}^{\beta}_{s^\star(R^\star)} = \sup \hat{\mathcal{M}}^{\beta}_{s^\star(R^\star)}$$

We have showed that, in the case of linear economies, for the decentralized market to sustain a certain $R^\star$ it is sufficient that the authority commits to $s^\star(R^\star)$. Let us now clarify under which conditions the authority will decide to implement a subsidy.

Proposition 5. *Consider a decentralized SSCE equilibrium $R^* \in \mathcal{M}^{\Phi}$ delivering an expected total surplus $\mathcal{S}(R^*, \rho^*(R^*, \tilde{\omega}))$. The authority will implement a contingent subsidy $s^\star(R^\star)$ targeting a contract $R^\star$ whenever $E^{\beta}[\mathcal{S}(R^\star, \rho^*(R^\star, \tilde{\omega}))] \geqslant E^{\beta}[\mathcal{S}(R^*, \rho^*(R^*, \tilde{\omega}))]$.*

Furthermore, by the definition of SSCE, $E^{\beta}[\mathcal{S}(R^*, \rho^*(R^*, \tilde{\omega}))] = E^{\Phi}[\mathcal{S}(R^*, \rho^*(R^*, \tilde{\omega}))]$. In sum, the authority will implement the subsidy no matter how small is the subjective probability that total surplus could improve. That is, the implementation of the subsidy is, in general, the *ex-ante* the right decision for the authority, irrespective of

[20]It should be noticed that this mechanism can also work in a case where there is a structural externality that prevents the decentralized market to potentially achieve the efficient equilibrium, as in models of Self-Fulfilling credit crisis.

what agents can eventually learn after exploring new submarkets (notably, that the status quo was not a REE). In other words, although the CB has in principle the power to unveil the true state experimenting on few matches, she finds worth implementing the policy on the whole distribution to maximize the expected benefit across the population. On the other hand, the fact that all the distribution of posted contracts moves after the subsidy, leaves the lenders at zero expected profits in any case. In this sense the private and social value of experimentation diverge.

Implementation of the subsidy

To gain more intuition on how *social experimentation*, with the use of an *optimal subsidy* works, it is useful to go back to our baseline economy with pay-offs (2) and (4), and a given ω. There, a large enough interest rate, for example R' such that $R' > y - k/(1-\alpha)$, will induce borrowers to always choose the risky project. In this case the surplus is $\mathcal{S}(R', \alpha) = \alpha y - \delta$. The matching instead yields a surplus of $\mathcal{S}(R'', k) = y - k - \delta$ in the case the borrowers are offered a positive R'' such that $R'' < y - k/(1-\alpha)$. Notice that whenever such a positive R'' exists we also have $\mathcal{S}(R'', k) > \mathcal{S}(R', \alpha)$. Therefore,

$$k < (1-\alpha)y \tag{31}$$

identifies the condition for which a planner, with the same beliefs of lenders, would like to target an interest rate satisfying $0 < R < y - k/(1-\alpha)$, if the decentralize equilibrium does not belong to this range.

In the case of figure 5, we considered an example where lenders believe $k = k^L = 0.005$ with probability $p^L = 0.07$, whereas $k = k^H = 0.015$, with probability $p^H = 1 - p^L$, otherwise. Therefore, p^L is also the probability that $\mathcal{S}(R, k) = y - k - \delta$ at any $R < y - k/(1-\alpha)$. In particular, for any positive $R < y - k/(1-\alpha)$, the following inequality

$$E^{\beta}[\mathcal{S}(R,k)] > E^{\beta}\left[\mathcal{S}(R,\alpha)\right], \tag{32}$$

holds. This implies that the authority would like to implement a contingent subsidy at any $R < y - k/(1-\alpha)$. Let us focus on the case when the authority targets a contract $R^{\star} = R_C$ (note that $R_C < y - k/(1-\alpha)$), which is the best contract conditional to the realization of the good state k^L (as shown in Figure 5). In particular, the optimal targeted subsidy is given by

$$\begin{aligned} s^{\star}(R_C^*) &= p^L\left[(1-\gamma)[y - R_C^* - k^L] - \gamma[R_C^* - \delta]\right] \\ &\quad + p^H\left[(1-\gamma)\alpha(y - R_C^*) - \gamma(\alpha R_C^* - \delta)\right] \end{aligned}$$

The subsidy makes (30) being satisfied, so that lenders - *all of them* - strictly prefer to post offers at R_C. This is illustrated by the curve green in figure 5, which represents the expected payoff when the the subsidy $s^{\star}(R_s^*)$ is implemented. The pick of the

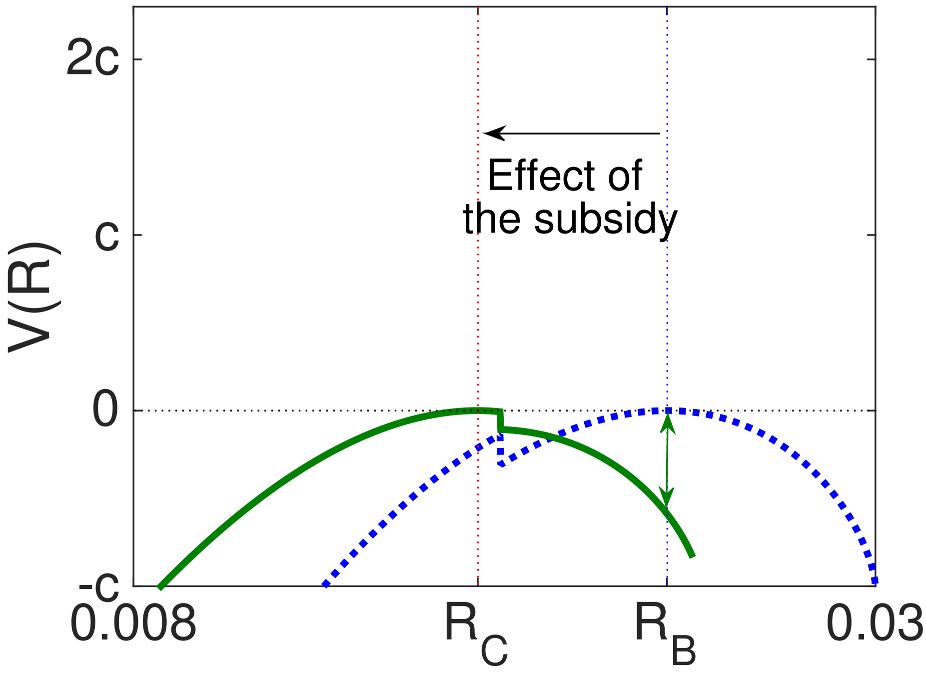


Figure 5: The effect of the optimal subsidy.

green line is exactly at R_C, where the zero profit condition is satisfied. Notice that in this case lenders reply to the *credit easing* policy by offering loans at the R_C interest and, at such low interest, borrowers choose the safe technology. As a result, the effect of implementing a subsidy to the lenders at the cost of taxing borrowers results in no effective transfer in equilibrium, but in an implicit tax outside equilibrium. Lenders with beliefs β, as described, perceive that they are being taxed at R_B. In practice, once R_C is being offered, borrowers prefer the new choice to the old R_B; i.e. the effect is through a sharp decrease of $q(R_B)$. Therefore, a policy of subsidization of lenders at the expense of borrowers results in an implicit tax to lenders if they do not offer the planner's desired equilibrium interest rate. *Credit easing* is a powerful instrument!

4 The case of TALF in the Automotive ABS market

A brief history of the TALF

Asset Backed Securities (ABS) are assets through which private companies liquidate credit that they have agreed to their customers. The value of an ABS is the value of claims over future receivables originated from an underlying pool of credit contracts. ABS cover important sector of the US economy like home equity, automotive, students loans, credit cards, to quote the four most important.

In the second half of 2007 the ABS market experiences a sudden contraction after a constant increase in volumes since early 2000 (see figure 6).[21] The crash was mostly driven by lower-than-expected returns in the housing markets which depressed the value of subprime home equities. The dramatic increase in perceived risk and the lack of confidence in rating agencies resulted in an abrupt freeze of the AAA-rated ABS segment whose interest-rate rose at exceptionally high levels reflecting unusually high risk premiums (see figure **??**). Private liquidity collapsed rapidly, and investors directed available resources to quality assets like treasury bills which almost doubled their daily volumes of trade from 40 to 80 USD billions during 2008-2009.

Within this context, the Fed stepped in with the lunch of the Term Asset Backed Securities Lending Facility (TALF) which supplied about 71 billions of non-recourse loans at lower interest rates, to any U.S. company provided of highly rated (AAA and AAA-) collateral. The TALF was set in such a way that, if the ABS given as collateral fall sharply in value, an investor can put the collateral that secures its TALF loan back on the hands of the Fed, only losing a collateral haircut of 15%. Thus, TALF constitutes a subsidy contingent on credit losses on the underlying ABS security. This intervention was made primarily to sustain the credit market in a period of high perceived counterpart risk. More precisely, the Fed acted as a borrower of last resort taking the risk of experimenting contractual conditions which were perceived as too risky by the private sector.

Nevertheless, despite malign prophecies welcoming the birth of the programs, on the 30th of September 2010, the Fed announced that more than 60% of the TALF loans have been repaid in full, with interest, ahead of their legal maturity dates. In other words, the more favorable conditions offered by the Fed, instead of triggering an adverse selection process, have been the prelude of a remarkable business performance. The NYFed, which was in charge of the operations, finally announced that "as of May 2011, there has not been a single credit loss. Also, as of May 2011, TALF loans have earned billion in interest income for the US taxpayer".[22]

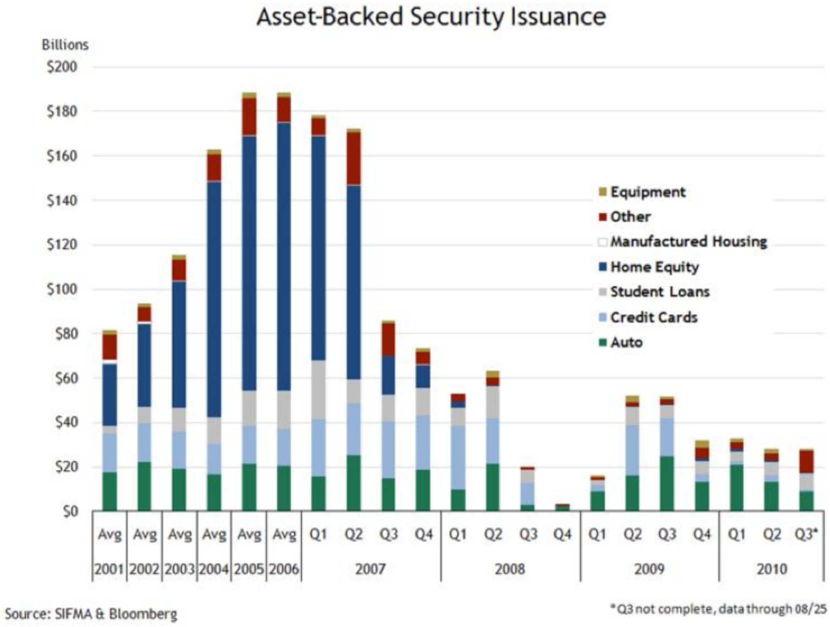
How can our theory explain the effect of TALF?

The aim of this section is to demonstrate that our theory can be used to rationalize the TALF intervention and its success. We already saw that the TALF intervention increased the value of ABS, providing insurance against perceived counterpart risk. In particular, by providing a non-recourse loan at very low interest rates against the 85% of the value of an AAA-rated ABS, the Fed was offering an implicit subsidy contingent to the loss on the ABS being higher than 15% of its market value. However, this fact alone does not imply yet the key mechanism of our theory, which relates to the effect of the TALF in the underlying market for the ABS security. In particular, our theory would predict that:

[21] New issuances of consumer ABS plunged from $50 billion per quarter of new originations in 2007 to only $4 in the last quarter of 2008.

[22] Source: http://www.newyorkfed.org/education/talf101.html

Figure 6: Amounts issued in the US ABS market for different categories.



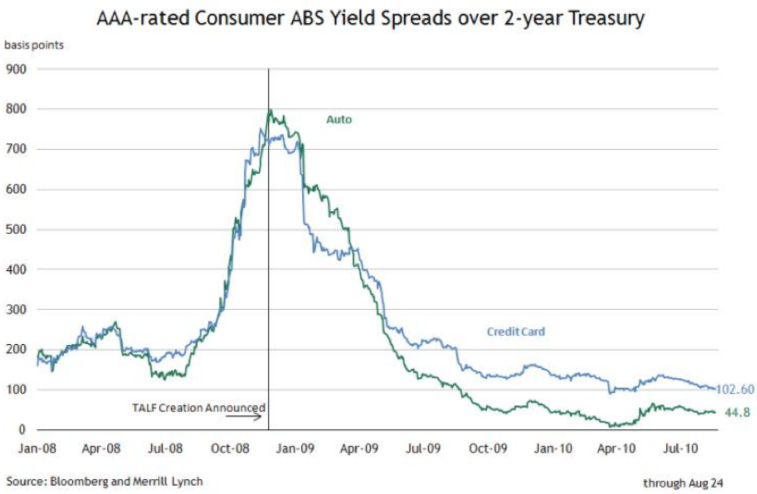


Figure 7: Interest rates spreads in the AAA-rated Auto and Credit Card ABS market.

1. during the crises, losses and interest rates were raising together;
2. the introduction of TALF translated in a decrease of the interest rates applied by financing companies to their costumers;
3. after TALF, loans become less risky than that which could have been predicted by using all information available in the market before TALF intervention;
4. after TALF interest rates remained low as a consequence of firms' learning that low interest rates generates low losses.

In other words, through the lens of our theory, the crises of the ABS market is the result of an increase in the underlying riskness of the economy that triggered an increase in interest rates and an increase in losses, evolving in a spiral of self-confirming pessimism. In such a context the TALF insured companies against the perceived counterpart risk, companies stopped charging risk premia and competition mechanically drove their interest rates low. This way the policy maker produced the counterfactual which eventually corrected firms' missbeliefs: lower interest rates actually yielded lower rather than higher losses. As a consequence, the TALF subsidy was never implemented, although the recovery was permanent.

Of course, at the same time there was a general recovery in the US economy. Therefore, to test our theory we need to insulate the effect of TALF from other business cycle externalities. In particular, to argue that our mechanism is in play we need to exclude that the recovery of the market could be explained by models of multiple REE as the one of Bebchuk and Goldstein (2011). We designed the following empirical strategy to overcome all these difficulties.

4.1 The ABS automotive market

Why looking at the Automotive Market?

First of all, let us explain why the automotive market constitutes an excellent case for our analysis. The automotive ABS is the second most important category of ABS, after credit card, in terms of amount supported by the TALF, about 3 millions US. We choose to look at this market because the peculiar characteristics of the underlying contracts make our analysis particularly informative. Below there is a list of the advantages that this market offers for the purposes of our research inquiry, in contrast to natural alternatives, namely credit cards.

There are no cross-customer externalities. It is hard to argue that, in this market, lower interest rates to new costumers may affect the likelihood of repayment of old costumers, which instead may happen for credit cards (even controlling for the business cycle). Importantly, the absence of this characteristic rules out potential explanations that requires multiple REEs.

The contract is only based on a fixed interest rate. In an automotive loan contract there is no particular structure of delay fees or renegotiation procedure, which instead may differentiate across credit card contracts.

We can control for the duration and amount of the loan. In an automotive loan contract, once the amount of the loan and the interest rate is fixed, montly payments will follow according to a pre-determined schedule. In contrast, with a credit card loan the amount of the loan depends on the current propensity to consume, and the debt can in principle be indefinitely rolled-over.

We can control for the collateral. Automotive loan contracts are secured loans; the collateral is typically the car which originate the debt. Credit cards are typically unsecured credit.

We can control for the wealth of costumers. The company issuing the automotive loan is typically the same company selling the car. Thus, the company is an indicator of the wealth of the costumer (i.e. the one who buys a BMW is typically richer than the one who buys a Honda), which instead it is not the case with credit card companies (almost everyone can have a Visa).

Taking advantage of all these nice features comes at the mild cost of extending a bit the interpretation of our simple payoff structure. In the case of the automotive market, a project must be interpret as a car, so that the project return is a flow of utility given by possession of a car. The fixed cost k instead shall be interpreted as the effort (in terms of portfolio management) needed to avoid a stochastic liquidity shock that when hits forces the agent to miss her payment and, as a consequence of the delinquency, lose the car. In the end, this interpretation is not crucial. Our payoff structure aims to simply reproduce a demand for credit whose quality (i.e. probability of repayment) increases as the agreed interest rates decreases.

Dataset on Automotive AAA-rated ABS

We collected data on Asset Backed Securities (ABS) in the US automotive sector for 9 different issuers in the automotive sector who appear in the balance sheet of the New York Fed as being accepted as part of the Term Asset-Backed Securities Loan Facility (TALF) program starting in March 2009. In particular, we collected all the free online available information on Trusts issued from 2007 till 2012, a time span which includes when TALF has been introduced.[23]

Table 1 reports the tranches sorted by issuer for which we have found information. All the issuing entities listed above have benefited from the TALF programme

[23]Every year each of these companies delivers in the market a variable number of Trusts (or tranches). For instance, World Omni in 2008 extended loans in two tranches (2008-A, 2008-B), while the same company extended only one tranche in 2009 (2009-A).

Company (i) → Year of issuance	**BMW**	**Carmax**	**Ford**	**Harley**	**Honda**	**Hyundai**	**Nissan Lease**	**Nissan Owner**	**World Omni**
2007	2007-1	2007-1 2007-2 2007-3	2007-A 2007-B	2007-1 2007-2 2007-3	2007-1 2007-2 2007-3	2007-A	2007-A	2007-A 2007-B	2007-A 2007-B
2008		2008-1 2008-A 2008-2	2008-B 2008-B 2008-C	2008-1	2008-1 2008-2	2008-A	2008-A	2008-A 2008-B 2008-C	2008-A 2008-B
2009	**2009-1**	**2009-1** **2009-A** 2009-2	**2009-A** **2009-B** **2009-C** **2009-D** 2009-E	**2009-1** **2009-2** 2009-3 2009-4	**2009-2** **2009-3**	**2009-A**	**2009-A** 2009-B	2009-1 **2009-A**	**2009-A**
2010	2010-1	2010-1 2010-2 2010-3	2010-A 2010-B	2010-1	2010-1 2010-2 2010-3	2010-A 2010-B	2010-A 2010-B	2010-A	2010-A
2011	2011-1	2011-1 2011-2 2011-3	2011-A 2011-B	2011-1 2011-2	2011-1 2011-2	2011-A 2011-B 2011-C	2011-A 2011-B	2011-A 2011-B	2011-A 2011-B
2012	2012-1	2012-1 2012-2 2012-3 2012-3	2012-A 2012-B 2012-C 2012-D	2012-1		2012-A 2012-B 2012-C	2012-A 2012-B	2012-A 2012-B	2012-A 2012-B

Table 1: List of tranches for every issuing company (In **bold** the Trusts eligible collateral under the Federal Reserve Bank's Term Asset-Backed Securities Loan Facility (TALF)).

implemented by the Federal Reserve (those reported in **bold** in Table 1). The number of tranches that have been eligible varies by issuer. The programme started in March 2009; however, the loans covered by TALF have been extended by each issuer at different points in time within year 2009.[24] The sum of these loans' amounts mentioned above represents around 46,5% of all ABS covered by TALF.

The data[25] includes information on the following three dimensions:[26]

1. Principal amounts in US dollars. For each of the trust we report the total value of the the credit pool gathered by the asset. Moreover, we have the breakdown for different "Class" (or Asset Backed Notes) of riskiness, going from the more secure A1 to the riskiest A4. Solely the Asset Backed Notes A1, A2, A3 and A4 are the ones classified as AAA by rating agencies (i.e. they are above the minimal level of FICO points credit scores to get the AAA label). The evolution of the total principal amounts in our sample is plotted in figure 8 where a different solid line denotes a different riskiness category. Note the collapse of issuance at the end of 2008 and the following recovery in 2009 during the TALF period. Contrast this with the course of

[24]For instance, for BMW Vehicle Lease Trust 2009-1 the prospectus has been made in May 2009, while the correspondent prospectus for Hyundai Auto Receivables Trust 2009 A is September 2009.

[25]The data have been collected one information at a time from prospectuses publicly available online. The major source utilized is *https://www.bamsec.com/companies/6189/208* where the majority of observations are available. The other sources are the issuers' websites which sometimes contain Trusts' prospectuses. Official TALF transaction data available at: http://www.federalreserve.gov/newsevents/reform_talf.htm#data

[26]Further details on the composition of the dataset, the sources and the procedure through which the data have been collected are presented in Appendix 4.1.

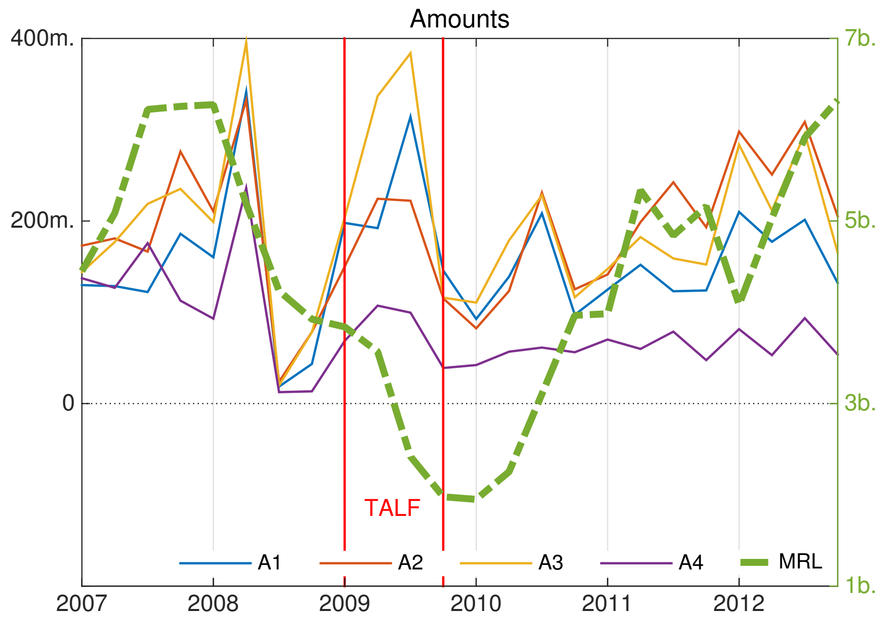


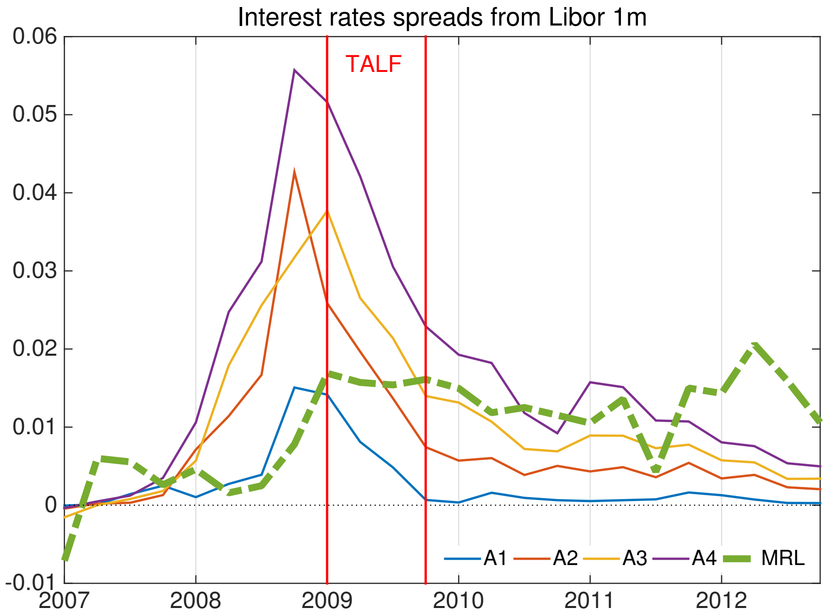
Figure 8: Quarterly total principal amounts issued in our sample for different categories of riskiness; the dashed line denotes the total amount of minimal risk loans agreed during the same period by US banks (3-quarter rolling window; scale on the right axis).

the green dashed line, which denotes the total value (y-axis on the left) of minimal risk loan issued by US bank in the same period (source: ST. Louis FRED dataset). The contrast highlightens that the recovery in the automotive market during the TALF intervention coincides with the deepest depression of the safest credit market in the US economy.

2. *Interest Rates.* Each tranche for each company is characterized by a fixed interest rate for each category at which the underlying pool of credit has been signed.[27] The evolution of the weighted average of the interest rates spreads – i.e. interest rate minus 1 month Libor – fixed at the time of the issuance is plotted in figure 9. A different solid line denotes a different riskiness category. Note the sharp increase of interest rates at the end of 2008 and the following decrease in 2009 during the TALF period. Contrast this with the course of the green dashed line, which denotes the interest rate on minimal risk loan issued by US bank in the same period (source: ST. Louis FRED dataset). From the comparison we note that the decrease of interest rates in the automotive market during the TALF intervention is in stark contrast with a permanent increase in interest rates in the safest credit market in the US economy at the beginning of 2009.

[27]Few trusts included a fraction of credit subject to variable interest rates. See Appendix 4.1 for an explanation on how we treat these cases.

Figure 9: Quarterly weighted average (each company is weighted by its relative issued amount) of interest rate differential from one month Libor for different categories of riskiness; the dashed line denotes the interest rate differential from the one month Libor on minimal risk loans agreed during the same period by US banks.



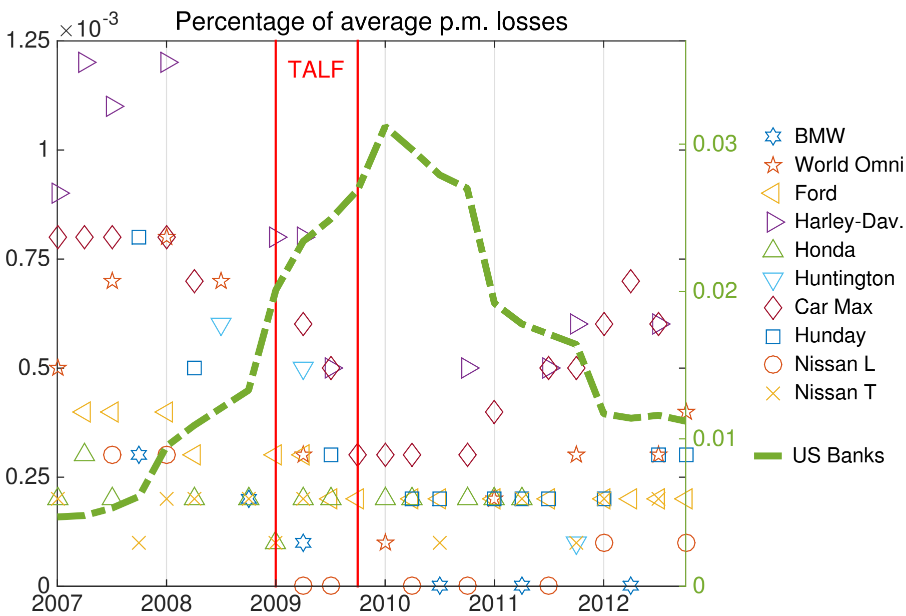


Figure 10: Average monthly loss for each tranche in the sample plotted in its quarter of issuance; the dashed line denotes losses experienced by US banks in the same period by US banks (scale on the right axis).

3. Losses with respect to the original pool balance. On the 15th day of each month following the date issuance, the tranche will pay the payment received from the pool of contractors of the originating loans. This amount constitutes the coupon of the tranche.[28] Thus, the investor in the ABS bears the default risk on the underlying loan contracts. For each tranche, we collected the series of cumulative losses on this receivables as a percentage of the original pool balance, which is published ex-post by the company issuing the ABS.

For each trust, the first difference of this series gives the per-month flow of losses. In figure 10, each point represent the average per month loss relative to a specific trust identified by a company and a time of issuance (on the x-axis). Notice that the TALF period coincides with a drastic drop of average losses for each company. After TALF losses never reached the pre-crises level. The picture also plots with a green dashed line the evolution of credit losses reported by all US bank in the same period (source: FRED st. Louis Fed). One can notice that while in this specific market losses diminished, in the US banks were experiencing a rapid increase of delinquencies.

The contrast of the automotive market in our sample with the evolution of macro benchmarks - as the ones represented by dashed lines in each picture - is suggestive of the specific effect of the introduction of the TALF in the ABS markets. At a first glance, we can distinguish between two periods: a pre-TALF period where interest rate differential were high, volumes became low and losses high, and a post-TALF where interest rates were low, volumes recovered and losses were low. This evolution is consistent with our interpretation that TALF was effective in giving the opportunity to learn that lower interest rates were associated with lower, rather than higher, losses: a conterfactual never observed in that market before!

In what follows, we are going to make this statement precise by means of an econometric analysis that will allow to distinguish between cyclical and market specific components of losses.

4.2 Econometric analysis

Testing local knowledge

We specify our first econometric exercise in such a way to highlight the learning aspect of our theory. We run two regressions on two different subsamples of our dataset, with a similar number of observations. The first sample - Sample I - consists of all the data available before the 25th. of March 2009, which is the date of implementation of the TALF. The second Sample II consists of all the data relative to tranches issued after the 25th. March 2009 for the following two years, i.e. until the 1st of April 2011.

[28]Each trust provides for about 3 or 4 years of payments.

On each sample we run the following linear regression:

$$\begin{aligned}Y_{i,t,T} = \beta_0 + \beta_1 X_{i,T} + &\beta_2 Val_{i,T} + \beta_3 Lib_T + S_i + \\ &+ \beta_4 Lib_t + \beta_5 u_t + \beta_6 Inf_t + \beta_7 Gdp_t + \beta_8 Vix_t + \epsilon_{i,t,T}\end{aligned}$$

where $Y_{i,t,T}$ denotes the first differences of monthly cumulative losses with respect to initial pool balance occurred at time t relative to the tranche issued by company i at time T; $X_{i,T}$ stands for the differential of the average interest rate relative to the tranche (i, T) with the correspondent one-month libor value at time T; $Val_{i,T}$ is the amount issued, which is relative to the tranche (i, T), valued in value in current US dollars. S_i are company fixed effects that controls for all the time-invariant unobserved company's characteristics. Finally, a set of covariates that control for business cycle are included: u_t is the US monthly unemployment rate; Vix_t is the monthly VIX index; Lib_t is the one-month Libor at time t; Lib_T is the one month-libor at the time of the issuance of the relative tranche T; Inf_t is the monthly inflation rate in US; Gdp_t is the monthly US national GDP's growth rate.

The regression aims at capturing the relation between interest spreads, which is a choice variable of the financing companies, and resulting losses, controlling for a number of tranche specific factors and business cycle variables.[29]

Sample I provides information on the evolution of losses that one could have *expected* as a consequence of lower interest rates at the time of the introduction of the TALF. As Table 2 shows, the estimated coefficient is significant and negative, i.e. lower interest rates were expected to generate higher losses. This finding would explain the tendency of firms to increase interest rates along diverging path documented by figure 9. Based on this beliefs, the mechanical decrease of interest rates due to TALF would be expected to generate higher losses for the policy maker.

Sample II is informative about the *actual* impact that the large decrease in interest rates induced by the TALF had on losses. The relevant coefficient is now significant and positive, meaning that lower interest rates were yielding lower, rather than higher, losses. This result would explain the tendency of firms to further decrease interest rates even after the TALF expired as figure 9 illustrates. This also explains why the TALF subsidy has never been implemented.

In addition, the variables we included to control for the business cycle have all the expected sign when significant. In particular, note that the policy variable Libor impacts on losses always with a negative sign, in line with several studies that identified the risky channel of monetary policy. In Sample I a decrease in the policy rate

[29]Notice that, although we do not deal directly with potential non-linearity of the loss pattern here (but we do that later), the truncation of the samples reduces the eventual misspecification. In fact, restricting the estimation range to values closer to the cutoff point is the basis of the local linear regression approach proposed by Hahn et al. (2001) which provides a reliable solution for the presence of non-linearities of the model.

Local Linear Regressions (dependent variable $Y_{i,t,T}$)

Variable		Coef	se
Sample I: Pre-TALF issuances, payments until March 2009			
$X_{i,T}$		$\boxed{-0.0237^{**}}$	(0.0075)
$Val_{i,T}$		2.07^{-14}	(2.59^{-14})
Lib_t		-0.0113^{**}	(0.0041)
Lib_T		-0.0043	(0.0057)
u_t		-0.0000	(0.0000)
Inf_t		0.0020	(0.0014)
Gdp_t		-0.0039^{*}	(0.0020)
Vix_t		0.0000^{***}	(2.59^{-06})
R^2	0.5445		
Obs.	536		
Sample II: Post-TALF issuances, payments until April 2011			
$X_{i,T}$		$\boxed{0.0200^{**}}$	(0.0072)
$Val_{i,T}$		1.11^{-14}	(4.25^{-14})
Lib_t		-0.0520	(0.0441)
Lib_T		-0.0100^{***}	(0.0029)
u_t		0.0001^{***}	(0.0000)
Inf_t		-0.0010	(0.056)
Gdp_t		0.014	(0.0033)
Vix_t		-0.0000^{**}	(5.48^{-06})
R^2	0.5378		
Obs.	589		

Standard errors clustered by issuing company. The model includes a constant and issuing company fixed effects.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Effect of interest rates on cumulative losses controlled for business cycle variables.

increases losses on simultaneous payments. In Sample II instead, this effect vanishes although the Libor component of the tranche-specific interest rate exhibits now a statistically significant negative coefficient. In fact, in Sample II Libor does not change much as it is constrained by the zero lower bound; unemployment instead becomes significant acting as a shadow policy rate.

Testing global knowledge

Our second econometric exercise is designed to capture the point of view of an external observer that wants to ex-post assess how the introduction of TALF affected the impact of the spread on the losses. We run a linear regression using the whole dataset. On the extended sample an eventual non-linearity of the series of losses can alter the estimate quite significantly. Thus, we step by the problem introducing fixed effects for the number of payments, which are intended to capture the non-linear trend of losses. Our specification is the following:

$$Y_{i,t,T} = \beta_0 + \beta_1 D + \beta_2 X_{i,T} + \beta_3 D X_{i,T} + \beta_4 Val_{i,T} + \beta_5 S_i + \beta_6 S_t + \beta_7 S_p + \epsilon_{i,t,T},$$

where we included the new variables D, S_t and S_p. D, denotes a dummy which is 1 when the payment belong to a tranche issued during or after the TALF period and 0 otherwise. Thus, β_3 measures the differential effect of spreads on losses that the introduction of TALF generated in the newly issued trust. S_t introduces a fixed effect for each date, in such a way to capture all common factors acting at the same date; among them there are the business cycles effects that we included in the linear regression. Finally, S_p includes a fixed effect for the number of the payment which is intended to control for a non-linear deterministic trend that montly losses can have in dependence of the number of the payment (e.g. when on average the fourth payment yields more losses than the tenth).

Global Linear Regressions (dependent variable $Y_{i,t,T}$)

Variable		Coef	se
D		-0.0003^{**}	(0.0001)
$X_{i,T}$		$\boxed{-0.0042^{***}}$	(0.001)
$D * X_{i,T}$		$\boxed{0.0108^{***}}$	(0.003)
$Val_{i,T}$		2.19^{-14}	(3.06^{-14})
R^2	0.6560		
Obs.	4169		

Standard errors clustered by issuing company. The model includes a constant, issuing company fixed effects, time fixed effect and payment number fixed effects.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Effect of interest rates on cumulative losses controlled for time fixed effects.

The results in table 3 confirms the findings of the local linear regressions. The impact of the spread on the losses - namely $X_{i,T}$ - is negative and statistically significant. Nevertheless, the differential effect of the same conditional to the TALF being implemented is positive and statistically significant. Moreover the latter is larger than the former in absolute terms, therefore, the overall effect of spread on losses is positive during and after the introduction of the TALF. This finding confirms that what companies could learn from a partial subsamples is indeed what can be estimated using all the information ex-post available in this market.

5 Conclusions

This paper presents a new approach to monetary policy in situations of high economic uncertainty, where private agents and policy makers may misperceive – and possibly underestimate – the actual strength of the economy. By developing and applying the concept of Self-Confirming Equilibrium to a competitive financial market we can characterize a, previously non-captured, form of credit crisis and, more importantly, we show that *Credit Easing* can be the optimal policy response, breaking the credit freeze. While we present a new theory, the paper also emphasizes that the FRB TALF experience in 2009 can be seen as a frontrunner example of this theory: of its design and its implications, which empirically we test to validate our theory.

A Appendix: Proofs

A.1 Proposition 1

Proof. To find $\hat{R}_s$ and $\hat{R}_r$ we just apply (16) with (1) and (3). Hence, $\hat{R}_s$ and $\hat{R}_r$ result after imposing incentive-compatibility and participation constraints and noting that the two interiors solve well-defined convex problems (so the closer to the interior the higher the payoff). Notice that at the risky equilibrium lenders' participation constraint $\alpha R_r^* - \delta \geqslant 0$ is always satisfied whenever borrowers' participation constraint $y - R_r^* \geqslant 0$ is too: in fact $y \geqslant R_r^*$ implies $y \geqslant \delta/\alpha$ which in turn yields $R_r^* \geqslant \delta/\alpha$. ∎

A.2 Proposition 2

Proof. The two values $\underline{\alpha}(k)$ and $\bar{\alpha}(k)$ correspond respectively to $\bar{R} = \hat{R}_s$ and $\bar{R} = \delta$.

For $\alpha < \underline{\alpha}(k)$ we have

$$\mu(R_r^*) = \pi^b(R_r^*; \alpha, \omega)^{\frac{\gamma}{1-\gamma}} \pi^l(R_r^*; \alpha, \omega) < \mu(\hat{R}_s) = \pi^b(\hat{R}_s; k, \omega)^{\frac{\gamma}{1-\gamma}} \pi^l(\hat{R}_s; k, \omega),$$

that is,

$$\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma) \max\left\{(\alpha y - \delta)^{\frac{\gamma}{1-\gamma}}, 0\right\} < \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)(y-k-\delta)^{\frac{\gamma}{1-\gamma}},$$

whenever $\bar{R} > 0$ which is true for $\alpha < \underline{\alpha}(k)$. We conclude that whenever $R_s^* = \hat{R}_s$ thenR_s^* is a REE.

For $\alpha > \bar{\alpha}(k)$ contracts that induce the safe adoption requires a R lower than the cost of money δ, which violates the participation constraint of the lender; therefore R_r^* will be the unique REE for $\alpha > \bar{\alpha}(k)$.

For $\alpha \in (\underline{\alpha}(k), \bar{\alpha}(k))$ we have that $R_s^* = \bar{R}$. The relevant equation for $R_s^* = \bar{R}$ to be unique REE is

$$\mu(R_r^*) = \pi^b(R_r^*; \alpha, \omega)^{\frac{\gamma}{1-\gamma}} \pi^l(R_r^*; \alpha, \omega) < \mu(\bar{R}|\rho = k) = \pi^b(\bar{R}; k, \omega)^{\frac{\gamma}{1-\gamma}} \pi^l(\bar{R}; k, \omega),$$

that is,

$$\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma) \max\left\{(\alpha y - \delta)^{\frac{\gamma}{1-\gamma}}, 0\right\} < \left(\left(y - \frac{k}{1-\alpha} - \delta\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}}\right).$$

On the one hand, $\mu(R_r^*)$ is always monotonically increasing in α. On the other hand, $\mu(\bar{R})$ is always monotonically decreasing in α, given that:

$$\frac{\partial\left(\left(y - \frac{k}{1-\alpha} - \delta\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}}\right)}{\partial \alpha} = \frac{(1-\alpha)\gamma(y-k-\delta) - 2k\alpha}{\alpha(1-\alpha)^2(1-\gamma)}\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}} < 0,$$

holds for $\alpha \in (\underline{\alpha}, \bar{\alpha})$.[30] Hence, we can conclude that

$$\left(y - \frac{k}{1-\alpha} - \delta\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}} = \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)\max\left\{(\alpha y - \delta)^{\frac{\gamma}{1-\gamma}}, 0\right\},$$

defines a threshold $\hat{\alpha}(k)$, such that for $\alpha < \hat{\alpha}(k)$ $R_s^* = \bar{R}$ is the unique REE, whereas for $\alpha > \hat{\alpha}(k)$, R_r^* is the unique a REE. The hedge case $\alpha = \hat{\alpha}(k)$ is the only one where two REE exist. To conclude, notice that

$$\frac{\partial\left(\left(y - \frac{k}{1-\alpha} - \delta\right)\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}}\right)}{\partial k} = \frac{(1-\alpha)\gamma(y-\delta) - k}{k\,(1-\alpha)\,(1-\gamma)}\left(\frac{\alpha k}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}} < 0$$

holds for $\alpha \in (\underline{\alpha}(k), \bar{\alpha}(k))$.[31] This implies that $\hat{\alpha}(k)$ has to be decreasing in k. ∎

A.3 Proposition 3

Proof. Suppose lenders play R_r^* and that $\alpha < \hat{\alpha}(k)$. By definition of SSCE their expectation about $\rho^*(R_r^*, \omega)$ are correct at the equilibrium, which imply that lenders know α but can have misspecified beliefs about k. In particular, for a $E[k]$ sufficiently high, such that $\hat{\alpha}(E[k])$ is sufficiently low (by proposition A.2), we can have $\alpha > \hat{\alpha}(E[k])$ that implies that lenders wrongly believe that R_r^* is the unique REE (i.e. they global maxima when evaluated by β).

On the other hand, R_s^* cannot be SSCE without being REE. Suppose such an equilibrium exists, then it would arise as a corner solution posted at the frontier $\bar{R}$ because it turns out that interior solutions $\hat{R}_s$ are always REE (i.e. they global maxima when evaluated by ϕ). Nevertheless, by definition of a SSCE, agents would have correct beliefs for marginal deviations from the equilibrium, that at the frontier, are indeed sufficient to induce safe project adoption. Therefore at a SSCE posted along the frontier $\bar{R}$ agents would know the actual α. Hence lenders can correctly forecast $\rho(R, \omega)$ at any R, and so they cannot sustain a safe SSCE that is not a REE. A contradiction arises. ∎

B Appendix: Data

TBA

[30] Notice that $\alpha k/(1-\alpha(k))$ in increasing in α and $k\underline{\alpha}/(1-\underline{\alpha}) = \gamma(y-k-\delta)$.
[31] Notice that $k/(1-\alpha)$ in increasing in α and $k/(1-\underline{\alpha}(k)) = \gamma(y-\delta) + (1-\gamma)k$.

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