NBER WORKING PAPER SERIES

THE COLLATERAL TRAP

Frédéric Boissay
Russell Cooper

Working Paper 20703
http://www.nber.org/papers/w20703

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2014

We thank seminar participants at CREI, Sciences-Po Paris, Pennsylvania State University, BIS, CSEF (Capri), and Zurich University for their comments. A special thanks to Elena Carletti, Ed Green, and Antoine Martin for comments and suggestions. A previous version of this paper circulated under the title "Inside and Outside Collateral: Implications for Financial Market Instability" as NBER Working Paper #20703.The views expressed in this paper are those of the authors and do not necessarily reflect those of the ECB The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. E44,E51,G21,G23

ABSTRACT

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Frédéric Boissay
European Central Bank
Kaiserstrasse 29, 60311 Frankfurt am main
Germany
frederic.boissay@ecb.europa.eu

Russell Cooper
Department of Economics
The Pennsylvania State University
611 Kern
State College, PA 16802
and NBER
russellcoop@gmail.com

The Collateral Trap

Frédéric Boissay* and Russell Cooper† ‡

This Version: July 15, 2015
First Version: November 17, 2014

Abstract

This paper studies the fragility of interbank markets. Due to moral hazard and asymmetric information problems, collateral and borrowing constraints are needed to provide appropriate incentives for banks. A key element of the analysis is the distinction between collateral based upon assets created outside of the banking system (*e.g.* government debt) and assets created through the lending process (*e.g.* cash flows). While active interbank markets help reallocate deposits across heterogeneous banks, because of incentive problems these flows are constrained and the markets are fragile, *i.e.* there are multiple equilibria. This paper explores the mechanism for the multiplicity, emphasizing its dependence upon inside and outside collateral, the complementarity between these two types of collateral, and how the endogenous creation of inside collateral gives rise to a fragile "collateral pyramid". It also relates a crisis, termed a "collateral trap" because of the paucity of safe assets, in the model the recent financial crisis in the US economy. Finally, the paper considers policy interventions to avoid interbank market fragility and promote the efficient flow of funds between banks.

1 Introduction

This paper studies the fragility of interbank markets. These markets facilitate the reallocation of funds: banks with relatively profitable opportunities borrow from other banks. These exchanges may include various market–financed transfers of assets between banks and take a variety of forms, ranging from the overnight funds market to bank equity.

Our analysis highlights the significance of collateral, both its volume and its composition, on the functioning of the interbank markets. Reflecting incentive problems, a bank's collateral determines its

*European Central Bank, frederic.boissay@ecb.int

†Department of Economics, The Pennsylvania State University; Research Associate, NBER, russellcoop@gmail.com

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ability to borrow and thus the reallocation of savings to productive projects. In equilibrium, the value of the collateral itself depends, through adverse selection, on the quality of borrowers in the interbank market. The resulting complementarity may generate multiple equilibria including a "collateral trap" characterized by (i) a reduction in the creation of safe assets by banks, (ii) reduced interbank activity, (iii) resource mis–allocation, (iv) a recession.

Models of fragility provide a framework for understanding the power of expectations in driving outcomes in financial markets. Our model of strategic uncertainty in financial markets focuses on loans between financial entities, and on the characteristics of the assets used to secure those loans. It purposefully downplays the role of financial instability stemming from bank runs along the lines of Diamond and Dybvig (1983).[1] Indeed, capital flow reversals and "sudden stops" are not key elements in our analysis.[2]

In our setting, there are two frictions in the interbank market: (i) the profitability of borrowing banks is hidden information and (ii) banks are able to abscond with borrowed funds. This interaction of adverse selection and moral hazard limits the ability of the interbank market to reallocate funds and provides a basis for multiple equilibria.[3] In particular, there may be an active interbank market in which relatively high productivity banks borrow from less productive banks. Lenders believe that borrowers are high quality and quantity constraints on the size of loans are relatively lax. Funds are thereby channeled to productive projects, yielding high output and, across production sites, a relatively low standard deviation of productivity.

Yet, for the same parameter values there may exist a collateral trap in which lenders are pessimistic about the quality of borrowers. In our model, the beliefs about the quality of borrowers play an important role in determining the degree of pledgeability of borrowers' cash flows. In a collateral trap, pledgeability is low and the total volume of safe assets that borrowers can engineer out of those cash flow is endogenously reduced. In this equilibrium, borrowing constraints are tight, and the flow of funds is reduced. Since the sorting of banks is less efficient, relatively low productivity projects are funded, and counter–party fears arise on the interbank market. Output is low and the standard deviation of profitability across active production sites is much larger. Thus, a collateral trap is associated with a financial crisis and a recession. In our model, switches from optimism to pessimism look similar to the observed collapse in the interbank market during the Great Recession.

The paper provides a crisp characterization of the conditions on the economy's fundamentals for this type of multiplicity. Key elements in the analysis are the volume and the value of assets in the economy, which banks can use as collateral in interbank transactions. Those assets can either be outside assets, such

[1]See, for example, Krishnamurthy, Nagel, and Orlov (2014), Gorton and Metrick (2012), Covitz, Liang, and Suarez (2009), Copeland, Martin, and Walker (2011), Acharya and Schnabl (2010), and Fleming, Hrung, and Keane (2010) for arguments supporting this emphasis. Moreover, the runs observed on asset–backed commercial paper (ABCP) markets in 2007-8 were different from the traditional deposit–based bank runs, which Gorton (1988) showed to be mainly related to economic fundamentals. In particular, Covitz, Liang, and Suarez (2009) find that investors in ACBP markets ran from all types of programs, even from those with apparently solid fundamentals.

[2]As emphasized by Caballero (2009), the financial crisis in the US was not marked by a sudden withdrawal of external funds.

[3]The paper builds on the multiplicity discussed in Boissay (2011).

as the pledging of treasuries in a repo arrangement, or inside assets, such as the pledging of cash flows in the asset backed commercial paper market.[4] We emphasize the specific role of treasuries in avoiding the collateral trap. As we shall see, the use of inside assets creates a type of "collateral pyramid" in that cash flows from one loan can be pledged to secure another. Treasuries serve as the foundation of this pyramid. The larger is this pyramid, the more efficient is the operation of the interbank market yet, if pessimism occurs, the larger too is its fall.

As in Kiyotaki and Moore (1997) and the literature that follows, collateral is central to our analysis as a means of coping with incentive problems. This literature focuses on the effects of endogenous variations in collateral value. In contrast, we emphasize how endogenous variations in the composition of collateral, *i.e.* the relative magnitudes of inside and outside collateral, determine the ability of banks to borrow against their assets.

The complementarity between the two types of collateral in our model stands in contrast with Krishnamurthy and Vissing-Jorgensen (2015). In their model, banks provide safe assets to households and the rise in the supply of treasuries crowds out banks deposits via effects on the equilibrium price of treasuries. In contrast, we focus on the holdings of treasuries by collateral constrained banks and on the role of such securities in securing banks' wholesale funding. As Krishnamurthy and Vissing-Jorgensen (2015), we too predict that following an increase in treasury supply the yield spread between risky loans and safe assets should go down (see also Krishnamurthy and Vissing-Jorgensen (2012)). But in our case the reduced spread reflects the lower shadow collateral value of safe assets for banks, not a deterioration of the non–pecuniary services of safe assets for households.

In Benmelech and Bergman (2012) a higher value of collateral leads to more lending. Further, they assume that the value of collateral depends positively on the liquidity constraints of firms who would be the natural purchasers of the collateral in the event of default. This generates a complementarity between collateral values and lending. Benmelech and Bergman (2012) discuss the types of equilibria that can arise, including one they term a "credit trap" in which collateral values and lending are low and injections of liquidity do not increase lending. In the collateral trap banks too are led to invest deposits in a relatively low return activity because the return on the interbank market is driven to a low value through adverse selection. In this sense, liquidity is hoarded rather than lent to more productive activities, as in a credit trap. A distinguishing feature of our model is the collapse of interbank market transactions, particularly those backed by inside collateral. Indeed, consistent with evidence we present, our analysis highlights that the interbank loans backed by inside assets are much less resilient than those backed by outside assets.

The collateral trap is also distinct from the traditional Keynesian liquidity trap and from Caballero and Fahri (2015)'s "safety trap". In those traps, the excess demand for liquidity or safe assets leads to a recession when the nominal interest rate hits the zero lower bound. Also, like Benmelech and Bergman (2012), the focus is on the anemic effects of monetary policy post crisis. In our model, in contrast, a

[4]While the terminology is reminiscent of the distinction between private and public liquidity in Holmstrom and Tirole (1998), our focus is on different forms of collateral as pledgeable assets rather than liquidity *per se*. In our model, as in theirs, a complementarity emerges between inside and outside collateral.

recession results from the collapse of the interbank market and is associated with an inefficient allocation of resources. Moreover, the economy falls into the collateral trap when interest rates are low, not necessarily when they are at the zero lower bound.

The predictions of the model relate to both financial flows and real activity and are consistent with observations during the recent financial crisis. For example, our model predicts the excess volatility of the Asset Backed Commercial Paper (ABCP) relative to repo markets, particularly those secured by treasuries. Our model creates comparable variations in the use of inside and outside collateral. The implications of the model for output and capital reallocation also relate to the evidence presented in Foster, Grim, and Haltiwanger (2013) on factor reallocation. They argue that, in contrast to previous periods of low economic activity, the Great Recession did not correspond to a period of increased reallocation of factors of production.

Though the paper is not intended as a basis for assessing the full costs and benefits of policy analysis, it is possible to consider some crude forms of interventions within our framework, for example restrictions on bank portfolios, the setup of a central bank deposit facility, bank subsidies, or collateral swaps. A key finding is that asset portfolio restrictions are costly insofar as they limit reallocation through the interbank market. This restriction may eliminate the fragility of these markets but they do so by supporting a unique equilibrium with low interbank flows. We also find that bank subsidies, the setup of a central bank deposit facility, and collateral swaps, contribute to the stability of the financial sector.

The paper proceeds as follows. Section 2 documents the evolution of secured funding markets in the US around the recent financial crisis. Section 3 describes our theoretical framework and Section 4 describes the conditions under which financial markets may collapse. The implications of those events are analysed in Section 5. Section 6 explores to the link between our model and the facts, and Section 7 discusses the potential policy interventions that influence both the multiplicity of equilibria as well as the efficiency of the interbank markets. The last section concludes.

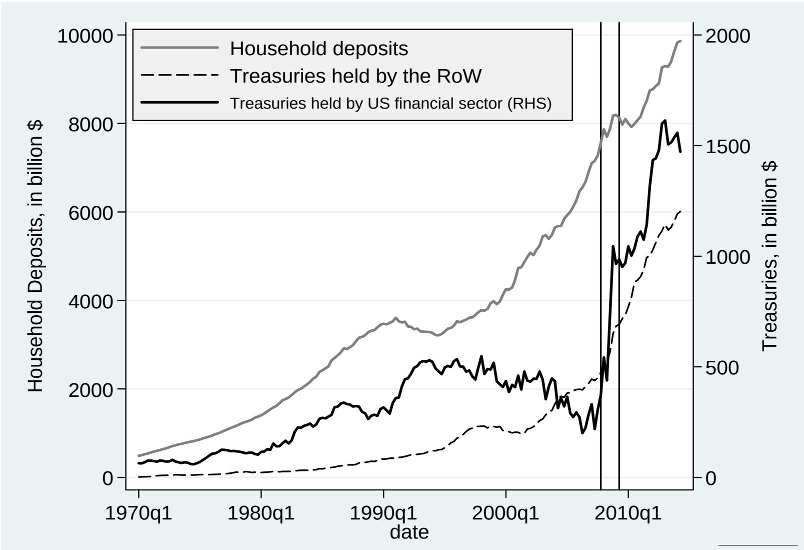
2 Facts

This section presents some pertinent facts related to the flows of funds and rates in interbank markets in the US before and during the 2007-8 financial crisis. These observations are used initially as motivation. Later, the discussion returns to them to evaluate our model.

Fact # 1: *The 2007–8 crisis occurred after a historical increase in bank deposits and a historical fall in US treasuries holdings by the US financial sector.*

Figure 1 shows the evolution of the levels of treasuries held by the financial sector and the rest of the world as well as household deposits from a long term perspective. As is clear from the figure, treasuries and deposits both increased from 1970 until 1995. After 1995, the growth of deposits and of the demand for treasuries by the rest of the world accelerated, whereas banks reduced their holding of treasuries, until the start of the crisis. At that time, banks deposits were at a historical high, and treasuries held by the financial sector were at a historical low, reflecting a "crowding out effect" and the shortage of safe assets in the US financial sector (Caballero (2009)).

Figure 1: US treasuries held by the financial sector and household deposits



Note: Total outstanding amount of US treasuries securities held by the US financial sector. *Source:* US Flows of Funds Accounts (Table L.209, series FL763061100.Q + FL753061103.Q + FL743061103.Q + FL633061105.Q + FL653061105.Q + FL663061105.Q). Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Treasuries held by the Rest of the World: *Source:* US Flows of Funds Accounts (Table L.209, series FL263061105.Q). Vertical lines: Great Recession's troughs and peaks (NBER dates).

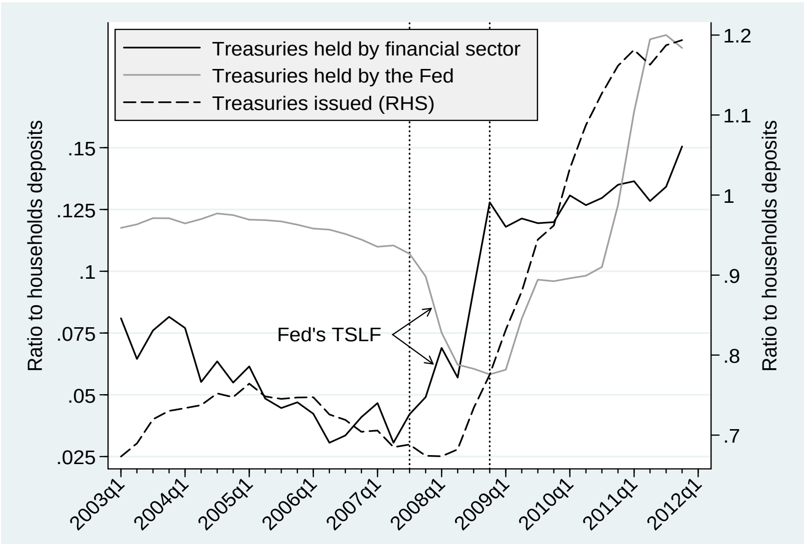
Fact # 2: *The US financial sector gradually reduced its holding of US treasury securities relative to other assets in the run–up to the crisis, and significantly increased it during the crisis.*

Figure 2 illustrates Fact #2. It reports the evolution of the outstanding amount of US treasuries issued by the US federal government (on the right scale) and held by the US financial sector and the Federal Reserve System (on the left scale) from 2003q1 until 2011q4, as a fraction of households' deposits. The figure shows that the crisis arose at a time when both the supply of US treasury securities and the US financial intermediaries' holding of those assets were low by historical standards: in 2007q2, notably, only 2.5% of the funds deposited were invested into treasuries. This suggests that, at that time, the rate of transformation of risky assets into safe deposits by US banks was unusually high. As the repo market was becoming illiquid in the last quarter of 2007 (see Figure 3), and following Bear Stearns' near–bankruptcy in the first quarter of 2008, the Federal Reserve Bank launched the Term Securities Lending Facility program (TSLF). This program offered primary dealers the possibility to swap less liquid collateral for more liquid Treasury collateral held by the federal reserve bank system.[5] Following the TSLF, financial intermediaries significantly and rapidly increased their holdings of treasuries (plain black line), which more than tripled

[5]The Fed also put in place other new liquidity facilities during the early stage of the crisis, like the Term Auction Facility (TAF) or the Primary Dealer Credit Facility (PDCF). One important difference with the TAF is that the TSLF was available to Primary Dealers only (whereas the TAF was intended for depository institutions) and addressed conditions in the secured

within one year, while treasury holdings by the federal reserve system concomitantly decreased (gray line). The overall supply of treasuries by the US government increased too, but later, with a three quarter lag. Fleming, Hrung, and Keane (2010) provide empirical evidence that this provision of treasury collateral through the TSLF mitigated a more general shortage of collateral on the repo market, which may explain —at least in part— why this market did not collapse as much as other secured loan markets at that time (see Figure 3).

Figure 2: US treasuries, as a ratio of Households' Bank Deposits



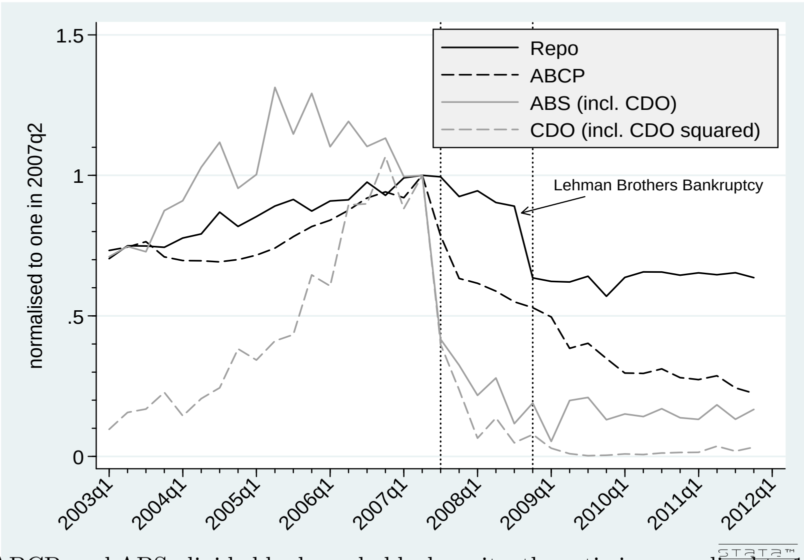
Note: Total outstanding amount of US treasuries securities and Total outstanding amount of US treasuries securities held by the US financial sector and by the monetary authorities, divided by households' total deposits; *Source:* US Flows of Funds Accounts (Table L.209, series FL313161505.Q, FL713061100.Q, and FL763061100.Q + FL753061103.Q + FL743061103.Q + FL633061105.Q + FL653061105.Q + FL663061105.Q, and Table L.100, series FL154000025.Q). Vertical lines: Great Recession's troughs and peaks (NBER dates).

Fact # 3: *During the 2007-8 crisis, wholesale funding markets secured by privately issued collateral collapsed, whereas those secured by treasuries or government guaranteed assets remained stable.*

Fact #3 has been documented in a number of papers. For example, Copeland, Martin, and Walker (2011) show that the segments of the US repo market, such as tri-party repos and the bilateral treasury repos, which represent more than two thirds of the US repo market, remained strikingly stable during the crisis. By contrast, Gorton and Metrick (2012) show that the bilateral private asset repo markets, such as corporate debt or ABS repos, did exhibit instability. Those markets were small with respect to the tri-party repo market, though. Overall, the reduction in repo transactions was rather limited. Krishnamurthy, Nagel, and Orlov (2014) show that it was dwarfed by the reduction in Asset Backed Commercial Paper (ABCP) transactions. Covitz, Liang, and Suarez (2009) provide evidence that the collapse of the ABCP market resembled a run, as it was pervasive and affected all ABCP market segments, irrespective of the

funding markets. One important difference with the PDCF is that the TSLF was an auction facility (whereas the PDCF was a standing facility) and therefore less subject to stigma effects. See Fleming, Hrung, and Keane (2010) for a detailed discussion.

Figure 3: Volume of Secured Loans, as a ratio to Households' Deposits



Note: Volumes of repos, ABCP, and ABS, divided by households deposits; the ratio is normalised to 1 in 2007q2. Repo and Fed Funds volumes: outstanding amounts borrowed by the financial sector (excluding foreign banks offices); *Source:* US Flows of Funds (Table L.207, series FL892150005.Q). ABCP: outstanding amounts; *Source:* Federal Reserve Bank of St Louis (FRED) database (series DTBSPCKAM). ABS and CDO: quarterly issuances; *Source:* Asset Backed Alert and Dealogic. Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Vertical lines: Great Recession's troughs and peaks (NBER dates).

fundamental quality of the underlying private assets. Altogether, this literature seems to indicate that one key determinant of the stability of secured wholesale funding had to do with the type of the assets used as collateral, and more specifically with whether those assets were publicly or privately issued.

Figure 3 reports the volumes of loans on several wholesale funding markets (repos, ABCP, and Asset Backed Securities —ABS) as a ratio to households' total deposits before and during the recent financial crisis.[6] To ease the comparison, the ratio is normalized to one in 2007q2, one quarter before the start of the crisis. In the run–up to the crisis, the three markets essentially evolved together, and their size increased. When in Summer 2007 the crisis hit, the volume of transactions fell by 60% in the ABS market, by 40% in the ABCP market, and by a bit less than 10% in the repo market, which confirms the resilience of the latter documented in earlier work. In effect, the repo market remained stable until the fourth quarter of

[6] ABSs are securities whose income payments is derived from and "backed" by a specified pool of assets. The pool of assets is typically a group of small and illiquid assets such as mortgage, credit card loans etc, which cannot be sold individually. Collateralized Debt Obligations (CDO) are structured financial products that pools together cash flow–generating assets and repackage those assets into discrete tranches that can be sold to investors. CDOs can be viewed as complex –because more structured— types of ABSs. The maturity of those claims is usually longer (above eighteen months) than that of repos and ABCPs, which does not exceed a quarter (overnight for the repos and less than six weeks for ABCPs). In the latter cases, there is therefore little difference between the outstanding volume of loans and the volume of loan issuances. So, to compare the evolution of those markets during the crisis, we report the quarterly issuances of ABS together with the outstanding volumes (end–of–quarter) of repos and ABCPs.

2008, when Lehman Brothers went bankrupt.[7] Interestingly, Figure 3 also points to some heterogeneity within the private collateral market segment. For example, the ABCP market was more resilient than the ABS market and, within the ABS market, the CDO segment completely dried up. This latter observation suggests that whether or not the collateralized assets were publicly or privately issued is probably not enough to explain the various degrees of resilience of secured loan markets.

Fact # 4: *During the crisis the Fed fund rate and the yields on treasury repos and three–month treasuries fell sharply but the spreads between those yields remained close to zero. In contrast, the spread between corporate loan rates rose abruptly.*

Figure 4 shows that the treasury repo rate is essentially the same as the risk–free Federal Fund and three month treasury rates. Those rates increased together in the run–up to the crisis, and then plummeted from 5.25% in 2007q2 to about 0.25% in 2009q1. The figure also shows how the average corporate loan rate (dashed black line) and the ratio of the Federal Fund rate to this average rate (plain black line) evolved during those times. The latter ratio —which is the inverse of the spread on corporate loans— will play an important role in our analysis. We will argue that its sudden fall during the crisis is indicative of the impairment of the financial intermediation process at that time.

3 Environment

We consider an economy populated by a government, a representative household, and banks. All agents live one period.[8] We present their choice problems and constraints in turn.

3.1 Government

The government finances expenditures $\mathbf{b}$ by borrowing from banks at rate r^b, and repays debt by raising taxes of $\mathbf{b}r^b$ on households at the end of the period. This budget constraint is built into the consumption of the representative household in (1). Though public expenditures have no direct benefit, the debt will have indirect social value through its use as collateral by banks.

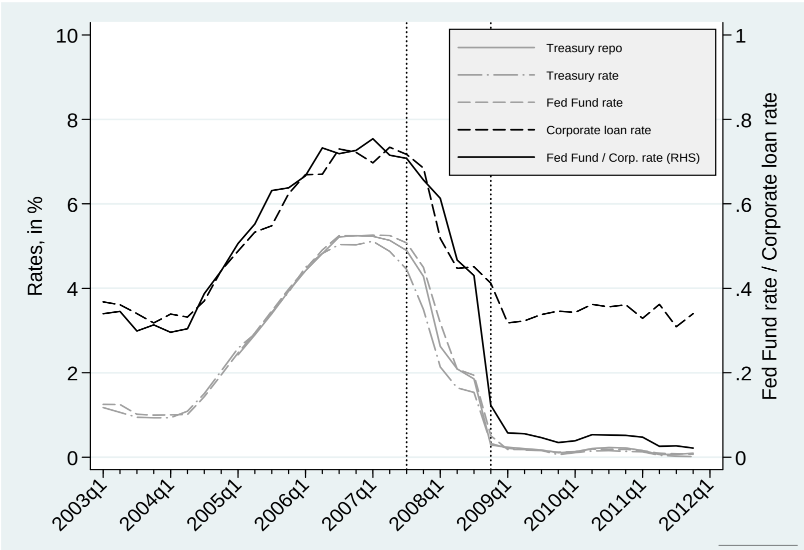
3.2 Household

The representative household is endowed with savings, which take the form of s units of the single good deposited into the banks at the beginning of the period. At the end of the period the household consumes

[7] In its Primary Dealer statistics, the Federal Reserve Bank of New York reports a $1.5 trillion fall in outstanding repos in that quarter. This suggests that most of the collapse in the repo market in 2008q4 was imputable to primary dealers, and especially to Lehman's bankruptcy. Copeland et al. (2012) point that, in anticipation of Lehman Brothers upcoming demise, hedge funds and other Lehman Brothers clients moved their business to other broker–dealers, thus depriving Lehman from repo–able assets. The Flows of Fund statistics for the whole broker–dealer sector indicate that only about half of the $1.5 trillion slack was picked up by other broker–dealers, hence the sudden fall in repo activities in 2008q4.

[8] The model is static as the interactions take place just once. Note though that there are two stages to these interactions.

Figure 4: Nominal Interest Rates by Asset Classes



Note: Quarterly averages. Repo rates are the DTCC GCF Repo Index of the interest rates paid for the most-traded GCF Repo contracts on US treasury securities, federal agency securities, and mortgage-backed securities (MBS) issued by Fannie Mae and Freddie Mac; *Source:* DTCC. The treasury rate is the interest rate on the 3–month treasury securities (constant maturity); *Source:* Federal Reserve Bank of St Louis FRED database. The corporate loan rate is the average of commercial and industrial loan rates ; *Source:* Federal Reserve Board Survey of Terms of Business Lending. Vertical lines: Great Recession's troughs and peaks (NBER dates).

the return on deposits, sr^d, minus the taxes $\mathbf{b}r^b$.[9] For simplicity, assume linear utility over consumption, c:

$$u(c) = c = sr^d - \mathbf{b}r^b. \tag{1}$$

3.3 Banks

There is a continuum of mass one of banks collecting deposits s from the household, endowed with projects as well as a storage technology. There are two stages of bank investments. In the first stage, banks are identical and use their deposits to purchase government bonds, b, and set the remainder, $s - b$, aside for second stage investments. In the second stage, banks learn about the productivity of their projects, ε, and become heterogenous. They can then invest into their projects, or lend to more productive banks on an interbank market. The interbank market arises endogenously to allow banks to reallocate their funds once they have learned their productivity. In the second stage, banks may also store goods.

We first study the choices of the bank in the second stage. Using this, we formulate the optimization problem of the bank in the first stage.

[9]In the model, there will be no friction between banks and households. Therefore, there will be no material distinction between bank deposits and bank equity, and r^d may be equally interpreted as the return on deposits plus dividends.

3.3.1 Stage 2 Choices: Interbank Market Participation

In the second stage banks, after making a portfolio choice in the first stage, learn their productivity ε. They choose to borrow or lend on the interbank market, to invest into their projects or store goods. The choice between these options is made at the start of stage 2.

A project financed by bank ε yields gross unit return εa. The εs are distributed over $[0, 1]$ with a continuously differentiable cumulative distribution function $G(\varepsilon)$, $\forall \varepsilon \in [0, 1]$. Banks can also store goods, generating a return γa, with $\gamma < 1$. In the discussion that follows, we proceed under the assumption that the storage technology is (weakly) inefficient, *i.e.* banks (weakly) prefer to either initiate projects or lend to other banks rather than undertake storage (see later Assumption 2). This potential to store goods nonetheless matters for the equilibrium outcome through its effects on incentives.

Once the ε's are drawn, banks allocate their available funds, $s - b$, into interbank loans or their own projects in order to maximize their profit.

Let r be the interbank loan rate. Then there is a critical bank intermediation skill, denoted $\bar{\varepsilon}$, with

$$\bar{\varepsilon} = \frac{r}{a}, \tag{2}$$

such that a bank with $\varepsilon = \bar{\varepsilon}$ is indifferent between undertaking her project and lending on the interbank market. In fact, banks with $\varepsilon \geq \bar{\varepsilon}$ will want to borrow on the interbank market as the return on their project generates a net profit; likewise, those with $\varepsilon < \bar{\varepsilon}$ will lend. To fix ideas, we will refer to those banks as "borrowers" and " lenders", respectively.

As it allows funds to be channelled to the most productive projects, the interbank market improves the efficiency of the financial sector. The higher $\bar{\varepsilon}$, the more productive the banks operating projects, and thus the more efficient the banking sector. Accordingly, interpret $\bar{\varepsilon}$ as a measure of financial efficiency. Clearly, in a frictionless world, only the most productive bank would operate its project: $\bar{\varepsilon}$ would be equal to one. However, there is a limit, determined in equilibrium, on how much productive banks (those with $\varepsilon \geqslant \bar{\varepsilon}$) can borrow on the interbank market. This limit takes into account the presence of asymmetric information and moral hazard.

Asymmetric Information Asymmetric information arises from the assumption that ε is known to bank ε only, and not verifiable *ex post*. Thus, in equilibrium lenders will be able to compute the expected value of ε for borrowers but contracts cannot be conditioned on bank specific variables.

Moral Hazard A bank that borrows on the interbank market has a choice to invest the funds either in its project or in storage. We assume that the cash flows from storage cannot be seized by creditors, so that the bank has the option to abscond and default on its interbank loans. This creates a moral hazard problem, which will be important for determining an equilibrium restriction on the amount borrowed.

Valued collateral and borrowing restrictions are jointly needed to deal with banks' incentive issues. As banks cannot easily divert the cash flows generated by government bonds and their projects, they can pledge those assets as collateral to secure interbank funding. Banks issue several types of securities, depending on which type of asset they pledge as collateral, government bonds or cash flows. In effect, the pledging of those assets implies a transfer of ownership *ex ante* from the borrower to the lenders, who will return the assets at the end of the period only if the loan is paid back. Throughout, government bonds are considered *outside collateral* while cash flows from the projects are termed *inside collateral.*

While both types of collateral can be used to secure funding, there are two important differences between them that make clear the costs of using inside collateral. Suppose that a security backed by government bonds is defaulted upon. In this event, the lender simply assumes ownership of the bonds.

In contrast, suppose that a security backed by cash flows is defaulted upon. There are two types of costs involved that jointly capture the notion that, in contrast to government bonds, collecting the promised cash flows is costly to a lender who assumes ownership of the assets post default. There is a fixed cost, f, that is incurred whenever a security backed by cash flows is defaulted upon. This fixed cost may reflect, *inter alia*, the transaction costs related to the transfer of ownership of the projects backing the security, or the cost related to the lenders having to operate the projects themselves. Further, the lender can only seize up to a fraction θ of the cash flows from existing projects. The parameter θ affects the degree of pledgeability of cash flows. As the analysis proceeds, we study how these two costs influence the flow of funds between participants in the interbank market.

To understand the operation of the interbank market, it is convenient to present the process by which banks raise interbank funds in several stages. Let $\phi^b s$ be the funds secured by the government bonds, $\phi^c s$ those secured by cash flows, and ϕs the total amount of interbank loans, with $\phi \equiv \phi^b + \phi^c$.

Initially, banks only have government bonds to pledge based upon their stage 1 portfolio decision. They borrow in the interbank market with the loans secured by these bonds. The incentive constraint that prevents a bank ε from borrowing funds and absconding is:

$$\gamma a \left(s - b + \phi^b s\right) \leqslant br^b + \varepsilon a \left(s - b + \phi^b s\right) - \phi^b sr, \tag{3}$$

A borrower who runs away does so with all of her funds and gets $\gamma a(s - b + \phi^b s)$. But the returns on the pledged government bonds, br^b, are lost. By contrast, a borrower ε who does not run away earns the return on government bonds plus the return on the projects, $\varepsilon a \left(s - b + \phi^b s\right)$, net of the interbank loan repayment, $\phi^b sr$. This is indicated by the right side of (3).

This incentive constraint must hold for all banks who borrow, *i.e.* for all $\varepsilon \geq \bar{\varepsilon}$. For bank $\bar{\varepsilon}$ the net return from not running away is independent of ϕ^b, whereas the net return of running away increases with ϕ^b. It follows that, in equilibrium, (3) binds at $\varepsilon = \bar{\varepsilon}$, which yields (using (2)):

$$\phi^b = \frac{\frac{b}{s}(r^b - r) + \bar{\varepsilon} a}{\gamma a} - 1 + \frac{b}{s}. \tag{4}$$

Building upon those first projects, bank ε can raise additional funds on the interbank market. It can issue a new security by pledging the future cash flows of those projects.[10] At this stage, the future cash flows are the only assets the bank can pledged as collateral, insofar as government bonds have already been pledged to the first stage lenders. So, no asset is being pledged multiple times.

Let $\phi^{c(1)}s$ be the new funding that banks can raise against the cash flow of the existing projects. The incentive constraint that prevents bank ε from absconding is:

$$br^b + \gamma a\phi^{c(1)}s + (1-\theta)\varepsilon a\left(s - b + \phi^b s\right) - \phi^b sr \leqslant br^b + \varepsilon a\left(s - b + \phi^b s\right) - \phi^b sr + \phi^{c(1)}s(\varepsilon a - r) \quad (5)$$

The left side is what the bank retains if it stores the new funds and runs away from the new creditors. In this case, those creditors recover the assets pledged to them, *i.e.* a fraction θ of the cash flow coming from the projects financed by the pledging of government bonds. The right side is what the bank earns if instead it invests $\phi^{c(1)}$ into new projects, and repays the new creditors *ex post*. To be clear, this condition ensures that the bank will repay, assuming, given (3), that it will also repay its other creditors whose interbank loans are backed by government debt. Thus (5) is an incentive constraint for the first round of interbank loans backed by cash flows.

Simplifying, (5) becomes

$$\gamma a\phi^{c(1)} - \theta\varepsilon a\left(1 - \frac{b}{s} + \phi^b\right) \leqslant \phi^{c(1)}(\varepsilon a - r) \quad (6)$$

Again, this condition holds with equality for the marginal bank, *i.e.* $\varepsilon = \bar{\varepsilon}$. Using (2), (6) becomes

$$\phi^{c(1)} = \frac{\theta\bar{\varepsilon}}{\gamma}\left(1 - \frac{b}{s} + \phi^b\right). \quad (7)$$

Under Assumption 1 (see below), the term $\frac{\theta\bar{\varepsilon}}{\gamma}$ is smaller than one, and is the fraction of the project that can be pledged as collateral in subsequent wholesale funding transactions. Indeed, once $\phi^{c(1)}s$ is obtained, the bank invests $\phi^{c(1)}s$ into its projects, and may then raise again additional funds $\phi^{c(2)}s$ on the interbank market by issuing another security backed by projected cash flows. In this way, the use of cash flows as collateral gives rise to a sequence of new securities and collateral creation inside the banking sector. This is why we refer to cash flows as "inside" collateral.

Let $\phi^{c(n)}s$ be the funds that bank ε can raise against the pledgeable fraction of the cash flows of the projects financed at the $(n-1)^{\text{th}}$ round. Then, the corresponding n^{th} incentive constraint is (for $n \geqslant 2$)

$$\gamma a\phi^{c(n)} - \theta\varepsilon a\phi^{c(n-1)} \leqslant \phi^{c(n)}(\varepsilon a - r), \quad (8)$$

and must again hold for all $\varepsilon \geqslant \bar{\varepsilon}$.[11] In equilibrium, using the fact that (8) binds at $\varepsilon = \bar{\varepsilon}$, one gets (for all

[10]To obtain this next stage of funding, the borrowing bank indicates, in a verifiable way, that it has already obtained the first stage of funding. Notice that, as long as (3) holds, all potential lenders know that these funds have been invested into the projects.

[11]Because they depend on ε, borrowers' cash flows, $\varepsilon a\phi^{c(n-1)}s$, are information sensitive, subject to the asymmetric infor-

$n \geqslant 2$)

$$\phi^{c(n)} = \frac{\theta\bar{\varepsilon}}{\gamma}\phi^{c(n-1)}. \tag{9}$$

This process of loan creation is limited by the fixed cost f. Lenders must pay this cost whenever an interbank security (here the n^{th} security) is defaulted upon, in order to assume the ownership of, and collect, the cash flows that back the security. The lenders' threat to take over those assets, which is crucial in preventing the borrower from absconding, is credible only if the cash flows recovered by the lenders cover the fixed cost, though. It follows that a borrower can issue the n^{th} interbank security only if:

$$\theta\bar{\varepsilon}a\phi^{c(n-1)}s \geqslant f. \tag{10}$$

The left side of (10) is the cash flows of the marginal borrower, which the new creditors can seize if this borrower default on the new security. The constraint thus means that lenders must be in the position to claim the cash flows of all borrowers, even the least productive of them, $\bar{\varepsilon}$.

Let N be the total number of interbank debt issuances. It is determined by the relation (using (7) and (9)):

$$\left(\frac{\theta\bar{\varepsilon}}{\gamma}\right)^N (1 - \frac{b}{s} + \phi^b) = \frac{f}{\gamma as}, \tag{11}$$

which implies that the total amount of interbank loans secured by inside collateral is given by:

$$\phi^c = \sum_{n=1}^{n=N}\phi^{c(n)} = \frac{\theta\bar{\varepsilon}}{\gamma}\cdot\frac{\frac{b}{s}(r^b - r) + \bar{\varepsilon}a - \frac{f}{s}}{\gamma a - \theta\bar{\varepsilon}a}. \tag{12}$$

In order for inside collateral to be created and yet maintain a moral hazard problem, we assume:

Assumption 1. ***(Liquidation Cost, Seizable Cash Flow)*** *(i)* $\gamma > \theta$ *and (ii)* $\gamma a > f/s$.

The first part of Assumption 1 means what borrowers get when they run away, γ, more that compensates their loss of the pledged assets, θ, so that banks may gain from absconding. The second part of the assumption means that the fixed liquidation cost is not too large compared with the size of the economy, so that a sophisticated financial sector can emerge (*i.e.* $N > 0$). Ultimately, the total outstanding amount of interbank loans that banks can raise is (using (4), (9) and (11)):

$$\phi = \phi^b + \phi^c = \frac{\gamma\frac{b}{s}(r^b - r) + (\gamma - \frac{\theta f}{as})\bar{\varepsilon}a}{\gamma(\gamma a - \theta\bar{\varepsilon}a)} - 1 + \frac{b}{s} \tag{13}$$

One pending issue is, whether a lender at stage $n-1$ would still agree to lend, knowing that the cash flow generated from this loan will be used as collateral in a stage n. Lemma 1 shows that it would, and

mation problem, and cannot be pledged as collateral (see Gorton and Ordonez (2014)). Only a fraction of it, $\theta\bar{\varepsilon}a\phi^{c(n-1)}s$ can be repackaged into information insensitive assets. This information insensitive fraction of the cash flows can be seen as the "senior" tranche of the bank's cash flows.

illustrates how the incentive constraints interact across the stages of the loan process.

Lemma 1. *The pledging of the cash flow from stage $n-1$ loans to stage n lenders does not impair the repayment of these loans. Banks borrowing in the interbank market generate sufficient cash flow to repay their loans.*

Proof. Using backwards induction, start in the final stage. By lending $\phi^{c(N)}$, the lenders of round N make sure that their borrowers do not run away from them. Since they are the last lenders in the process, they know that the cash flows from the projects will not be pledged as collateral afterward. So, for those lenders, the N^{th} incentive compatibility constraint is met. In turn, the lenders of $\phi^{c(N-1)}$ can contract on the premise that borrowers will not run away at round N. Using this argument, the incentive compatibility conditions imply that loans will be paid-off *ex post*. □

3.3.2 Stage 1 Choices: Portfolio Choice

Ex ante banks maximize the expected gross return on assets, denoted π^b, with respect to b:

$$\max_b \pi^b \equiv \left(r + \frac{b}{s}(r^b - r) + (1 - \frac{b}{s} + \phi) \int_{\bar{\varepsilon}}^{1} (\varepsilon a - r) dG(\varepsilon) \right) s, \tag{14}$$

given (13). The return on assets is the sum of returns on government bonds, interbank loans, and projects, weighted by the share of those assets in banks' assets. The second and third terms reflect the excess return on governments bonds and projects over interbank loans. This optimization problem assumes that, *ex post*, banks with $\varepsilon < \bar{\varepsilon}$ will be net lenders in the interbank market and banks with $\varepsilon \geqslant \bar{\varepsilon}$ will be net borrowers on the interbank market and invest into projects. Only the latter banks enjoy excess returns. Using (13), the first order condition yields a no-arbitrage condition:

$$r^b = r, \tag{15}$$

which implies that banks are indifferent between investing in government bonds and lending to other banks. Using this, relations (4), (12) and (13) simplify to:

$$\phi^b = \frac{\bar{\varepsilon}}{\gamma} - 1 + \frac{b}{s}, \tag{16}$$

$$\phi^c = \frac{\theta\bar{\varepsilon}}{\gamma} \cdot \frac{\bar{\varepsilon} - \frac{f}{as}}{\gamma - \bar{\varepsilon}\theta}, \tag{17}$$

$$\phi = \frac{\bar{\varepsilon}}{\gamma} \cdot \frac{\gamma - \theta\frac{f}{as}}{\gamma - \theta\bar{\varepsilon}} - 1 + \frac{b}{s}, \tag{18}$$

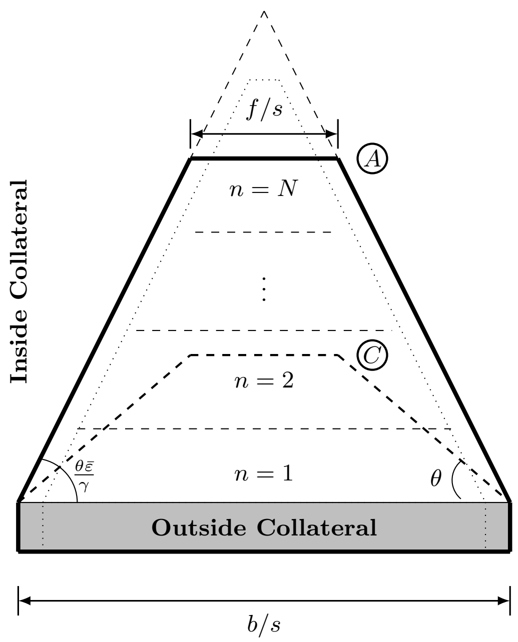
and, using (11), the total number of rounds becomes:

$$N = \frac{\ln(\bar{\varepsilon}) - \ln\left(\frac{f}{as}\right)}{\ln(\gamma) - \ln(\theta\bar{\varepsilon})}. \tag{19}$$

3.3.3 Collateral Pyramid

The multiple rounds of lending described above form a "collateral pyramid"; see Figure 5. The foundation of this pyramid (in gray in the figure) is made of assets created outside the financial sector, *i.e.* government bonds. The rest of the pyramid is made of collateral created inside the financial sector along the various lending rounds. The size of the pyramid corresponds to the total volume of safe assets that can be pledged as collateral in interbank transactions. It is limited by two main factors. First, it depends on the volume of government bonds held by banks relative to their deposits, b/s: all things equal, the larger this collateral base, the bigger the pyramid. Thus, inside collateral builds upon outside collateral. Second, it also depends on the fixed liquidation cost, f/s, and on the capacity of the banking sector to create new assets, *i.e.*, on the degree of pledgeability of the cash flows, $\theta\bar{\varepsilon}/\gamma$. The latter plays a critical role in our analysis, to the extent that (i) it determines how much inside collateral can be generated out of cash flows (*i.e.* a "collateral multiplier") and (ii) it is endogenous and varies with $\bar{\varepsilon}$. As we shall see later, this is the reason why the volume of inside collateral is much more unstable than that of outside collateral, and collapses during crises.

Figure 5: Collateral Pyramid



The dependence of inside collateral on outside collateral stands in stark contrast with Krishnamurthy and Vissing-Jorgensen (2015), who view the two types of safe assets as substitutes. They too make a distinction between outside safe assets (treasuries) and inside safe assets (in their case, bank deposits). In their model, however, both types of safe assets are demanded by households for the non-pecuniary services those assets provide. When the supply and the yield of treasuries go down, households increase their demand for bank deposits. In contrast, in our case safe assets are engineered and supplied by banks as a way to alleviate financing constraints on the interbank market. When the supply of treasuries goes down, banks have less outside collateral to secure wholesale funding, and counter–party fears on the interbank market rise, which reduces the pledgeability of cash flows and the creation of inside collateral. In our case, the two types of assets are therefore complements.

Our model of the interbank market, which emphasizes the stages of the lending process, can be seen as a stylized representation of how the US shadow banking industry manufactured new financial products in the run–up to the 2008 financial crisis (see Coval, Jurek, and Stafford (2009); Pozard, Adrian, Ashcraft, and Boesky (2012)). To recap, the first step is to form a diversified pool of loans. The next step is to slice the pool and to issue a safe security backed by the information insensitive slice of the pool. This funding then gives rise to new projects and cash flows, which will be later used as collateral to raise new funding, and so on.

While for simplicity the model is strictly about a single asset, notice that the interbank claims issued at each stage could potentially be rather complex and that richer models would include alternative forms of securitization, such as CDO, CDO^2, or CDO^N.[12] Accordingly, in the discussion that follows we think of N as capturing the complexity of financial markets and securities. These securities would build upon each other as in our multiple lending rounds. The closer to the top of the collateral pyramid, the more complex the securities pledged as collateral. Of course, in our model, in equilibrium the sequencing in the collateral creation process is instantaneous.

4 Equilibrium Conditions

This section describes the set of equilibria. A sunspot variable is included in the construction of an equilibrium as there may be multiple equilibria in the interbank market. The sunspot only serves as an indicator of expectations; no significant economic decisions are taken prior to its realization.[13]

[12] To see this point, notice that $\phi^{c(n)}s$ is the *net* volume of interbank borrowing in stage n. Without loss of generality, one could alternatively focus on *gross* flows, by assuming that banks with $\varepsilon \geqslant \bar{\varepsilon}$ both lend, say δs and borrow $(\phi^{c(n)} + \delta)s$ on the interbank market, with δ being an arbitrary positive quantity. Assume further that, like government bonds, interbank loans can be pledged at no cost. It is easy to see that under those assumptions the marginal borrower's incentive compatibility constraints, and therefore net interbank borrowing, are unchanged. In this case, however, at a given stage n borrowers would pledge the cash flows from both the projects and the interbank loans they made at stage $n-1$. The stage $n-1$ interbank loans being themselves backed by the cash flow from projects and interbank loans made in the previous stages, the securities issued at stage n would be backed by a non–trivial mix of interbank securities and projects. It follows that, implicitly, the collateral backing interbank loans in our model can be complex and collateral complexity increases with n.

[13] One extension of the model would allow the strategic uncertainty to influence households' demand for bank deposits.

4.1 Timing

The timing of the interactions between households, banks, and the government:

1. The household deposits s.
2. The sunspot is revealed.
3. The banks allocate deposits across government bonds (b) and future investment ($s - b$).
4. Banks learn about the profitability of their projects, ε.
5. Banks decide to make either interbank loans, to invest into their project, or store goods.

4.2 Market Clearing Conditions

There are three markets: government bonds, interbank loans, and bank deposits. We solve for the equilibria, given savings s and a stock of bonds $\mathbf{b}$.

The interbank market equilibrium will determine $\bar{\varepsilon}$. Once this is known, the other components of the equilibria can be determined. The key to fragility comes from the multiple values of $\bar{\varepsilon}$ solving the interbank market clearing condition.

Government Bond Market Since the government issues $\mathbf{b}$ bonds per bank, and each bank demands b, the market clears when:

$$b = \mathbf{b}. \tag{20}$$

Here b reflects the holding of government bonds by the banks. By our timing, this decision is made after the realization of the sunspot and hence given the rate of return on interbank loans and government debt.

Bank Deposit Market Banks are identical *ex ante*, compete for depositors, and break even *ex post.* In equilibrium, the return on bank deposits is given by (using (14), (15) and (18)):

$$r^d \equiv \frac{\pi^b}{s} = r\left(1 + \frac{1 - \frac{\theta}{\gamma a} \cdot \frac{f}{s}}{\gamma - \theta\bar{\varepsilon}} \int_{\bar{\varepsilon}}^{1} (\varepsilon - \bar{\varepsilon}) dG(\varepsilon)\right). \tag{21}$$

Under Assumption 1, the deposit rate is larger than the interbank loan rate and the return on government bonds. The spread reflects the shadow value of government bonds as collateral. Despite the low return on government bonds, banks are willing to hold such assets *ex ante* because they can pledge them as collateral to relax their financing constraints.

Interbank Market Banks with $\varepsilon \geqslant \bar{\varepsilon}$ demand funds ϕs; so demand is $(1-G(\bar{\varepsilon}))\phi s$. Banks with $\varepsilon < \bar{\varepsilon}$ supply $s-\mathbf{b}$, provided that $\bar{\varepsilon} > \gamma$; in this case, those banks strictly prefer to lend than to store goods, and supply is $G(\bar{\varepsilon})(s-\mathbf{b})$. In the special case when $\bar{\varepsilon} = \gamma$, banks are indifferent between those two options, and so supply up to $G(\bar{\varepsilon})(s-\mathbf{b})$ on the interbank loan market. It follows that the market clearing condition is:

$$(1-G(\bar{\varepsilon}))\phi \begin{cases} = G(\bar{\varepsilon})(1-\frac{\mathbf{b}}{s}) \text{ if } \bar{\varepsilon} > \gamma & (a) \\ \\ \in \left[0, G\left(\bar{\varepsilon}\right)\left(1-\frac{\mathbf{b}}{s}\right)\right] \text{ if } \bar{\varepsilon} = \gamma & (b). \end{cases} \tag{22}$$

In (22a), banks with $\varepsilon < \bar{\varepsilon}$ lend all of their assets in the interbank market. Importantly, these banks do not store goods. It is possible that the interbank market also clears with some of the suppliers using their storage technology. By arbitrage, this arises only when $\bar{\varepsilon} = \gamma$, as (22b) suggests. Using (18) and (20), relation (22) can be re–written as:

$$H(\bar{\varepsilon}) \equiv \frac{(1-G(\bar{\varepsilon}))\bar{\varepsilon}}{\frac{\gamma}{\theta}-\bar{\varepsilon}} \begin{cases} = \frac{\theta\left(1-\frac{\mathbf{b}}{s}\right)}{1-\frac{\theta}{\gamma a}.\frac{f}{s}} \text{ if } \bar{\varepsilon} > \gamma & (a) \\ \\ \in \left[0, \frac{\theta\left(1-\frac{\mathbf{b}}{s}\right)}{1-\frac{\theta}{\gamma a}.\frac{f}{s}}\right] \text{ if } \bar{\varepsilon} = \gamma & (b). \end{cases} \tag{23}$$

Banks with $\varepsilon \geqslant \bar{\varepsilon}$ finance those projects with own funds $s-\mathbf{b}$ and by interbank loans $\phi^b s$ and $\phi^c s$ secured by outside collateral (government bonds) and inside collateral (cash flow from projects), respectively.

Relation (23) is a convenient representation of the market clearing condition, as only its left side depends on $\bar{\varepsilon}$. The richness of the model will come from the non–monotonicity — *i.e.* the hump–shape— of $H(\bar{\varepsilon})$.

As $\bar{\varepsilon}$ increases, the fraction of banks who borrow falls, and their average quality goes up. Since higher quality banks have less incentives to abscond, the amount they can borrow increases. That is, from (18), ϕ is increasing in $\bar{\varepsilon}$. The multiplicity will come from the interaction of these extensive and intensive margins.

The term on the right side of (23a) decreases with aggregate productivity (a), or as the fixed liquidation (f) goes down. One can also show that, under Assumption 1, $H(\bar{\varepsilon})$ goes up faster with the fraction of seizable cash flows (θ) than the term on the right hand side does, which means that the net demand of interbank loans increases with seizable cash flows. The return on storage, γ, has an opposite effect on net demand.

4.3 Equilibria

We discuss the types of equilibria that might arise and then give conditions for their co-existence. We will show that there are potentially multiple equilibria in this model economy due to the asymmetric information and moral hazard of banks. For this discussion, we consider restrictions on $G(\cdot)$ and γ so that those two frictions are meaningful:

Assumption 2. ***(Initial Resource Mis–allocation)*** *(i)* $G''(\bar{\varepsilon}) \geqslant 0 \ \forall \bar{\varepsilon} \in [0,1]$, *and (ii)* $\gamma < \bar{\varepsilon}^{max} \equiv$

$\arg\max_{\bar{\varepsilon}\in[0,1]} H(\bar{\varepsilon})$.

Point *(i)* means that there is a sufficient mass of high ε–banks in the economy. Point *(ii)* means that the storage technology is (weakly) inefficient. Altogether, Assumption 2 means that deposits are initially mis–allocated across banks, and that there is scope for re–allocation through an interbank market. Under this assumption, $H(\bar{\varepsilon})$ is hump–shaped and reaches an interior maximum at $\bar{\varepsilon} = \bar{\varepsilon}^{\max} \in (\gamma, 1)$.[14] The hump shape of $H(\bar{\varepsilon})$ reflects the ambiguous effect of $\bar{\varepsilon}$ on loan demand, which decreases with $\bar{\varepsilon}$ on the extensive margin but increases with $\bar{\varepsilon}$ on the intensive margin. This latter unusual effect is due to the fact that borrowers' average quality goes up with $\bar{\varepsilon}$, as this works to relax banks' borrowing constraint. The negative intensive margin effect dominates when $1 - G(\bar{\varepsilon})$ is large enough, that is when $\bar{\varepsilon}$ is low; in this case, the loan demand curve bends backward.

Definition 1. *An equilibrium is a vector of returns (r, r^d, r^b), a critical level of ε denoted $\bar{\varepsilon}$, government bond holdings b, and borrowing limits ϕ^b, ϕ^c, such that*

- *Ex ante, banks choose b to maximize their expected return on deposits;*
- *Ex post, banks with*
  - *$\varepsilon \geqslant \bar{\varepsilon}$ borrow in the interbank market up to the limit $\phi = \phi^b + \phi^c$;*
  - *$\varepsilon < \bar{\varepsilon}$ either lend in the interbank market or store goods;*
- *ϕ^b, ϕ^c are determined so that banks with $\varepsilon \geqslant \bar{\varepsilon}$ invest rather than abscond;*
- *beliefs about the distribution of types of banks borrowing in the interbank market are consistent with the equilibrium outcome;*
- *markets for interbank loans, deposits and government bonds clear.*

The flow of funds between banks is illustrated in Figure 6. As is clear, the determination of $\bar{\varepsilon}$ is fundamental to characterizing an equilibrium and determining the reallocation of funds within the banking sector through the interbank market.

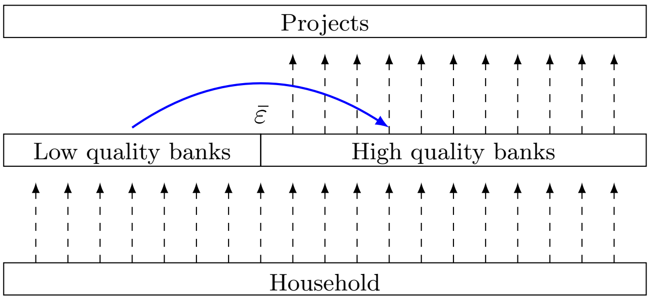
4.3.1 Normal Times

We first study outcomes which are solutions to (23a). These solutions become the basis for a subset of the equilibria. The right side of (23a) is predetermined and constant. Under Assumption 2, the left side is hump shaped —the exact shape will depend on the assumed form for $G(\cdot)$ and the parameters of the model. As a consequence, a solution $\bar{\varepsilon} > \gamma$ to (23a) does not always exist.

Equilibria in the interbank loan market are shown in Figure 7. Since banks always have the outside option to get a return γa by storing goods, only the values $\bar{\varepsilon} \geqslant \gamma$ are relevant. The figure illustrates one

[14] For a proof of this, see the Appendix.

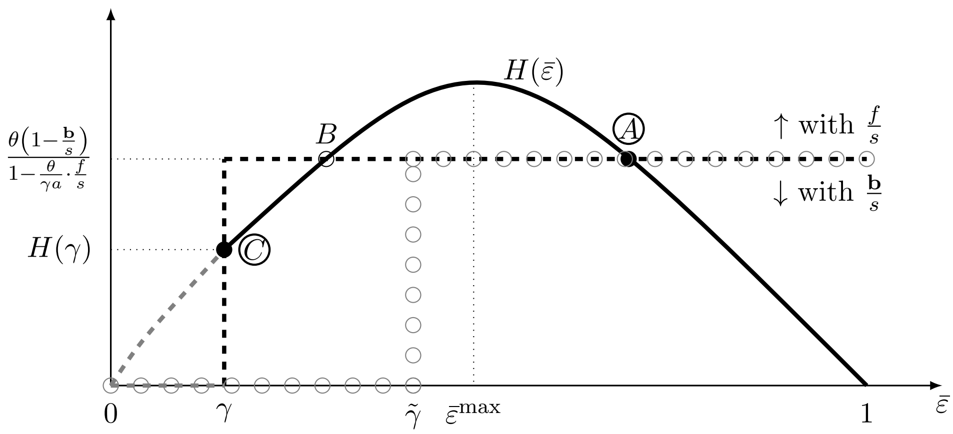
Figure 6: Interbank Reallocation and Financial Intermediation



solution to (23) at point A (see also point A in Figure 5). This equilibrium determines a value of $\bar{\varepsilon} > \gamma$ such that banks with $\varepsilon \geqslant \bar{\varepsilon}$ borrow on the interbank market, and will be referred to as "normal times". In this type of equilibrium, there banks do not use the storage technology, and all deposits are invested into projects.

From this value of $\bar{\varepsilon}$, we can construct an equilibrium as follows. Use (2) to determine the interbank rate: $r = \bar{\varepsilon}a$. From the no-arbitrage condition, (15), $r^b = r$. Relations (16) and (17) yield ϕ^b and ϕ^c given $\bar{\varepsilon}$. Finally, the equilibrium number of "rounds" in the interbank market that underlies ϕ^c is given by (19). Under Assumption 1, ϕ^c and N are strictly positive.

Figure 7: Multiple Equilibria in the Interbank Market



Since $H(\bar{\varepsilon})$ is hump–shaped, another interior solution (point B) can exist. Relative to A, the value of $\bar{\varepsilon}$ is lower, there are more banks borrowing but their borrowing constraint is tighter, loan quality is lower, and the intermediation process is less efficient. In the analysis that follows, we focus on the equilibrium at A as it is locally stable and thus has "reasonable" comparative statics. But, as we shall see, there is another equilibrium outcome other than B that is of interest.

4.3.2 Collateral Trap

Another equilibrium, which we term a "collateral trap", can arise as a solution to (23b) with $\bar{\varepsilon} = \gamma$. In this case, there is an excess supply of loans on the interbank market, and some banks store goods. Hence, aggregate productivity and output are low.

In Figure 7, this equilibrium is represented by point C (see also point C in Figure 5). At this point, the banks with $\varepsilon \leqslant \gamma$ are indifferent between lending to other banks and storing goods, and they are the only banks that supply interbank loans. The supply of interbank loans is infinitely elastic, as illustrated by the vertical dashed line. In a collateral trap, banks engineer little inside collateral, and the borrowing constraint holds tight: productive banks, who cannot distinguish themselves from other banks, are unable to commit themselves to lending to projects, and therefore cannot raise more funding. Despite the low interbank rate, the demand for loans remains below supply, there is an excess supply of interbank funding, and the interbank market clears by having low productivity banks use the storage technology. This equilibrium exists if and only if (using (23b)):

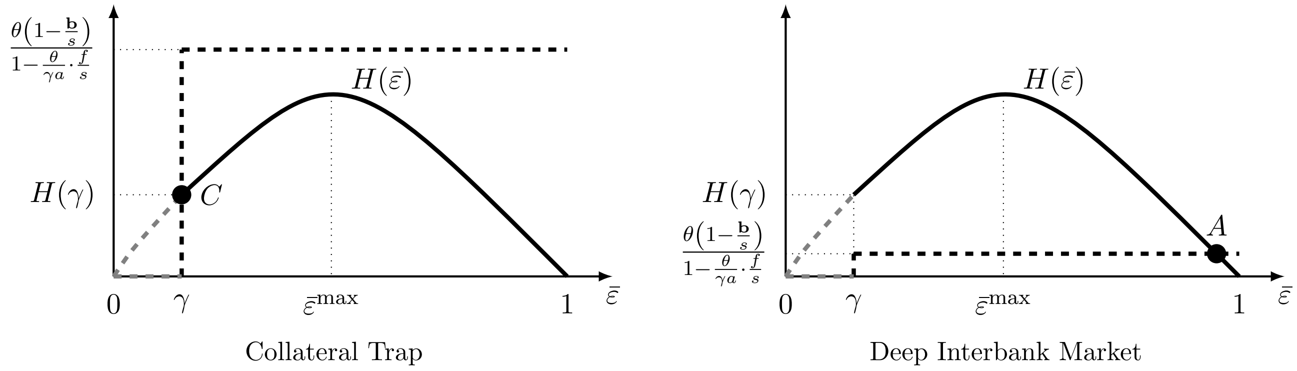
$$H(\gamma) < \frac{\theta\left(1 - \frac{\mathbf{b}}{s}\right)}{1 - \frac{\theta}{\gamma a} \cdot \frac{f}{s}}. \tag{24}$$

Though some banks store goods, interbank borrowing is still possible, *i.e.* $\phi > 0$, because banks can pledge as collateral both their government bonds as well as the cash flows of the projects that are operated.[15] However, the volume of collateral is reduced compared to normal times, and so is the level of interbank activity. On the one hand, banks are limited in the creation of safe assets, because counter–party fears on the interbank market reduce the degree of pledgeability of the cash flows. On the other hand, counter–party fears are high because banks have little safe assets to pledge as collateral. Hence the collateral trap.

In Figure 7, the collateral trap equilibrium in C co-exists with the normal times equilibrium in A. In this sense, the interbank market is susceptible to swings in confidence. It is natural to associate outcomes like A with optimism about the quality of borrowing banks, with high returns on interbank loans and relaxed borrowing restrictions. It is also natural to think of crises, like C, as the outcome of pessimism. A and C are self–fulfilling equilibria. Assume for example that banks are pessimistic: they believe that unproductive banks demand interbank funding. Since unproductive banks are prone to absconding, lenders require borrowers to put more "skin in the game", and reduce ϕ. As every borrower demands less funding, the equilibrium interbank loan rate goes down, and the net present value of the unproductive banks' projects increases. Hence, unproductive banks do indeed demand interbank funding.

[15] If $\mathbf{b} = 0$ (no outside collateral) and $\theta = 0$ (no inside collateral) then in a collateral trap $\phi = 0$ and there is no demand for interbank loans.

Figure 8: Unique Equilibrium Configurations



4.3.3 Conditions for a Collateral Trap

Not all economies possess multiple equilibria. Other possible outcomes are illustrated in Figure 8, which shows cases of unique equilibria. In panel (a), there are no solutions to (23a), and there is a collateral trap equilibrium. In panel (b), by contrast, there is a solution to (23a) but not to (23b); in this case the normal times equilibrium is the only possible outcome. To describe the conditions of existence of a collateral trap and normal times equilibria, it is useful to define two critical levels of government debt, $\bar{b}$ and $\underline{b}$, as:

$$\bar{b} = \frac{f}{\gamma a} H(\gamma) + \left(1 - \frac{H(\gamma)}{\theta}\right) s \tag{25}$$

and

$$\underline{b} = \frac{f}{\gamma a} H(\bar{\varepsilon}^{\max}) + \left(1 - \frac{H(\bar{\varepsilon}^{\max})}{\theta}\right) s \tag{26}$$

with $\underline{b} < \bar{b}$. There are three regions for the level of public debt. In the upper dominance region, with $\mathbf{b} > \bar{b}$, banks can use so much public debt as collateral that crises are ruled out, irrespective of banks' beliefs about borrowers' quality in the interbank market. Then there is a lower dominance region, with $\mathbf{b} < \underline{b}$, where instead there is a shortfall of pledgable assets in the economy. The government did not issue enough debt to avoid the collateral trap. Finally, there is an intermediate region, where beliefs matter and sunspot equilibria arise; in this case, crises are self–fulfilling. Proposition 2 formalizes this characterization.

Proposition 2. ***(Equilibrium)***

- *When $\mathbf{b} < \underline{b}$, there is a unique collateral trap equilibrium, with $\bar{\varepsilon} = \gamma$ as solution to (23b);*
- *When $\mathbf{b} > \bar{b}$, there is a unique normal times equilibrium, with $\bar{\varepsilon} > \bar{\varepsilon}^{max}$ as solution to (23a);*
- *When $\mathbf{b} \in [\underline{b}, \bar{b}]$, there are sunspot equilibria, with either $\bar{\varepsilon} > \bar{\varepsilon}^{max}$ or $\bar{\varepsilon} = \gamma$.*

Proof. The results follow directly from the definition of $\bar{b}$ and $\underline{b}$ in (25) and (26). □

5 Equilibrium Analysis

This section looks at some properties of the equilibria. It contains comparative static results on the effects of savings on the operation of the interbank market. It also compares collateral traps and normal times.

5.1 Aggregate Output and Consumption

Aggregate output, y, is the sum of the outputs of banks' projects and storage:

$$y = \underbrace{\int_{\bar{\varepsilon}}^{1} \varepsilon a \left((1+\phi)s - \mathbf{b}\right) \mathrm{d}G(\varepsilon)}_{\text{projects}} + \gamma a \underbrace{\left((s-\mathbf{b})G(\bar{\varepsilon}) - (1-G(\bar{\varepsilon}))\phi s\right)}_{\text{storage}},$$

which using relation (18) and Proposition 2 yields:

$$y = \begin{cases} \frac{s-\mathbf{b}}{1-G(\bar{\varepsilon})} \int_{\bar{\varepsilon}}^{1} \varepsilon a \mathrm{d}G(\varepsilon) \text{ in normal times} \\ \\ \frac{1-\frac{\theta}{\gamma a}\cdot\frac{f}{s}}{1-\theta} s \int_{\gamma}^{1} (\varepsilon-\gamma) a \mathrm{d}G(\varepsilon) + \gamma a (s-\mathbf{b}) \text{ in a collateral trap.} \end{cases} \tag{27}$$

The above relation shows that aggregate output is determined both by the size of the economy s and by the degree of efficiency, $\bar{\varepsilon}$, with which the banking sector allocates resources across projects. Since the household consumes aggregate output (*i.e.* $c = y$), it too is determined by the level of $\bar{\varepsilon}$. Thus, output, consumption, and welfare increase with $\bar{\varepsilon}$.

5.2 Stability, Efficiency, Complexity

The rapid increase in bank deposits and the large demand for safe assets addressed to the US banking sector by the rest of the world (see Figure 1) has been put forward as one of the key structural factors behind the 2007–8 financial crisis (see, *e.g.*, Caballero and Krishnamurthy (2009)). The aim of this section is to shed light on the relationship between savings and financial stability, efficiency, and complexity.[16]

Proposition 3. ***(Savings and Stability)*** *An increase in savings s widens the range of government debt levels, $\bar{b} - \underline{b}$, for which there are multiple equilibria, and thus is conducive to interbank market instability. Moreover, following the increase, the level of government debt required to avoid the collateral trap ($\bar{b}$) goes down if and only if $\theta > G(\gamma)$.*

Proof. From Appendix 9, $\bar{\varepsilon}^{\max}$ is independent of s. Using (25) and (26), it is easy to see that $(\bar{b} - \underline{b})/s$ is proportional to $(1 - \theta f/\gamma a s)$, which increases monotonically with s; hence $\bar{b} - \underline{b}$ increases with s. From

[16] In the context of our model, we interpret the size of the region of multiplicity as indicative of financial instability: the larger $\bar{b} - \underline{b}$, the higher the degree of financial instability. There is a distinction between financial stability and financial efficiency. The financial sector can be efficient and yet unstable, as in Point A on Figure 7; or it can be stable and yet inefficient, as in Point C on Figure 8; or it can be both stable and efficient, as in Point A in Figure 8.

relation (25) it is clear that $\bar{b}$ decreases with s if and only if $\theta < H(\gamma)$, which is equivalent to (using the definition of $H(\bar{\varepsilon})$ in (23)) $\theta > G(\gamma)$. □

Besides influencing the range of multiplicity, the level of savings also has an effect on the degree of efficiency and complexity of financial markets during normal times.

Proposition 4. ***(Savings and Efficiency)*** *Following an increase in savings, financial efficiency ($\bar{\varepsilon}$) improves in normal times if and only if $\boldsymbol{b} < \frac{\theta f}{\gamma a}$.*

Proof. From Figure 7 it is easy to see that $\bar{\varepsilon}$ increases with s if and only if the term on the right hand side of (23) decreases with s, which yields $\mathbf{b} < \frac{\theta f}{\gamma a}$. □

Proposition 4 describes the conditions, under which an increase in savings enhances financial efficiency. On the one hand, an increase in s reduces the fixed liquidation cost relative to the size of the financial sector, f/s, which stimulates the creation of new structured financial products and private collateral. The production of inside collateral contributes to the functioning of the interbank market, whose efficiency improves in the sense that $\bar{\varepsilon}$ goes up. This effect is all the larger when, all things being equal, f is high or the degree of pledgeability of the cash flows (θ) is high. On the other hand, an increase in s reduces the available amount of public collateral relative to the size of the financial sector, $\mathbf{b}/s$, which limits the collateral base in the first place; this effect is all the larger than $\mathbf{b}$ is high. Ultimately, which effect dominates depends on the relative size of f and $\mathbf{b}$. When $\mathbf{b} < \theta f/\gamma a$, then the net effect is positive and financial efficiency is enhanced.

Proposition 5. ***(Savings and Complexity)*** *There exists $\Delta > 0$ such that, in normal times, an increase in savings s raises financial complexity N if and only if $\boldsymbol{b} < \frac{\theta f}{\gamma a} + \Delta$.*

Proof. From relation (19), N is a continuous, increasing function of both s and $\bar{\varepsilon}$. The direct effect of s is independent of $\mathbf{b}$. The indirect effect of s goes through $\bar{\varepsilon}$ and —from Proposition 4— is negative if $\mathbf{b} > \theta f/\gamma a$, and positive otherwise. It follows that there exists $\Delta > 0$ such that those two effects exactly offset each others when $\mathbf{b} = \frac{\theta f}{\gamma a} + \Delta$. Hence, the overall effect of s on N is positive if and only if $\mathbf{b} < \frac{\theta f}{\gamma a} + \Delta$. □

The results in Propositions 4 and 5 reflect the ambiguous effects of an increase in savings on collateral availability. To see this, compare in Figure 5 the collateral pyramid before the increase in savings, delineated by the thick black line, with that after, delineated by the dotted line. When s goes up, the volume of outside collateral relative to savings (b/s) diminishes, and the base of the pyramid shrinks. However, as projects grow larger, economies of scale make the fixed liquidation cost and the financial engineering of new assets affordable (*i.e.* f/s decreases). Hence, the height of the pyramid goes up, as well as financial complexity (Proposition 5). When $\mathbf{b} = \theta f/\gamma a$, then the two effects exactly offset each other, which means that the economy creates endogenously exactly the same amount of inside collateral as the amount of outside collateral it lost (relative to savings). In this case, financial efficiency does not change, but the financial sector is more complex and less stable (as $\bar{b} - \underline{b}$ is larger). In other cases, the total volume of collateral can go either up (if $\mathbf{b} < \theta f/\gamma a$) or down (if $\mathbf{b} > \theta f/\gamma a$).

5.3 Comparing Outcomes: Collateral Traps versus Normal Times

The key feature of collateral traps is the reduction in $\bar{\varepsilon}$, as in Figure 7; this has important implications.

First, at a lower $\bar{\varepsilon}$, a larger fraction of banks borrow in the interbank market. This increase on the extensive margin means that lower productivity banks that would lend to higher productivity banks during normal times actually borrow in a collateral trap. The fact that low productivity banks demand funding raises counter-party fears, and therefore reduces the degree of pledgeability of cash flows. Hence, in a collateral trap, the collateral multiplier is low, as illustrated by the smaller collateral pyramid C in Figure 5.

Countering this change on the extensive margin is also a reduction in borrowing limits. With lower quality banks borrowing, these limits fall to avoid moral hazard. In addition to the reduction in size of the interbank loans, there is a composition effect. Specifically, the volume of inside collateral collapses and the share of outside collateral in total collateral goes up. Essentially, the limit on the borrowing backed by outside collateral (ϕ^b) is less sensitive that on the borrowing backed by the cash flow from projects (ϕ^c), which is driven by variations in the collateral multiplier and in the complexity of loans based on inside collateral, *i.e.* N (see (19)).

In a collateral trap there is less activity in the interbank market, and some banks store goods, which reduces aggregate productivity. This fall in productivity has not only an overall negative real effect on the *levels* of output, consumption, and welfare, but also a positive effect on the cross-sectional *dispersion* of banks' returns. The output and productivity implications are consistent with the evidence in Foster, Grim, and Haltiwanger (2013). To the extent the Great Recession contained more of a financial collapse than other recessions, the implications of our model, which attributes recessions to the collapse of the interbank market, are relevant. A decline of economic activity is due to a reduction in the reallocation of deposits in the interbank market. This decline in activity also corresponds to a period with an increased cross-sectional dispersion in productivity.

Proposition 6. ***(Features of a Collateral Trap)*** *Relative to normal times and all other things being equal, in a collateral trap:*

1. *the cut-off bank productivity, $\bar{\varepsilon}$, is lower;*
2. *the interbank loan rate, r, is lower;*
3. *the deposit rate is lower;*
4. *the spread between the deposit rate and the interbank loan rate (and therefore the shadow value of government bonds as collateral) is higher;*

5. *the spread between the government bond yields and the interbank loan rate remains null;*
6. *the volume of interbank transactions secured by outside collateral, ϕ^b, is lower;*
7. *the volume of interbank transactions secured by inside collateral, ϕ^c, is lower;*
8. *the ratio $\frac{\phi^b}{\phi^c}$ is higher;*
9. *the complexity of interbank loans, N is reduced;*
10. *banks store goods, and not all deposits (net of government bonds) are used to finance productive projects;*
11. *output, consumption and welfare fall;*
12. *the cross–sectional dispersion of bank productivity (the εs) is higher.*

Proof. The proposition compares multiple equilibria A and C in Figure 7. The effects of the reduction in $\bar{\varepsilon}$ (item 1) so that banks use the storage technology (item 10) defines a collateral trap, as in Figure 7. As $\bar{\varepsilon} = \frac{r}{a}$, the interbank rate fall with $\bar{\varepsilon}$ (item 2), and from (2) and (21), it is easy to see that the spread between r^d and r go up (item 4). Item 5 results from (15). The effects of a fall in $\bar{\varepsilon}$ on outside and inside collateral (items 6 and 7) come directly from (16) and (17). The effects of a fall in the ratio of outside to inside collateral (item 8) can be seen from relation (see (16) and (18)) $\left(\phi^b + 1 - \mathbf{b}/s\right) / \left(\phi^c + \phi^b + 1 - \mathbf{b}/s\right) = (\gamma - \theta\bar{\varepsilon}) / (\gamma - \theta f/(as))$: the left hand side increases with ϕ^b/ϕ^c, and the right hand side decreases with $\bar{\varepsilon}$ and hence rises in a collateral trap. Complexity (item 9) is given by (19) and this is increasing in $\bar{\varepsilon}$ and so falls. Since aggregate output increases monotonically with $\bar{\varepsilon}$ (see (27)), output, consumption, and welfare are lower (item 11). Item (12) follows from $\bar{\varepsilon}$ being reduced in a collateral trap. □

6 Back to the Facts

We use our model to understand the link between the observed increase in savings and shortage of outside collateral (Facts #1 and #2) and the evolution in interbank market flows and interest rates before and during the 2007–8 crisis (Facts #3 and #4). The model is structured around "normal" and "crisis" times. With this in mind, the pre-crisis period, roughly from 2003 to mid-2007, is viewed as "normal" times. The crisis begins at that point.

6.1 Pre-Crisis

Figure 1 shows that the increase in the level of household deposits and holding of US treasuries by the rest of the world accelerated from 1995 to the onset of the crisis and that, over the same period, the volume of US treasuries held by the US financial sector fell like never before. As a consequence, in 2008 the treasuries–to–deposit ratio of the US financial sector was less than half of what it was in 2003 (Figure

2). At the same time the volume of complex financial assets, such as repos, ABCP, or ABS-CDO rose (see Figure 3). One interpretation of those events is that the US financial industry created complex "safe assets" as substitutes to treasuries (Caballero (2009)), as a way to quench the rest of the world's thirst for US safe assets.

From the perspective of our model, the increase in bank deposits is seen as an increase in s and the fall in treasuries as a decrease in $\mathbf{b}$ (Fact #1). The sophistication of asset markets is seen as N going up (Fact #3). Our interpretation of those events is that, by reducing US banks' collateral base, the crowding of US banks out of treasuries by the rest of the world made the US financial system more fragile. On the other hand, the concomitant increase in deposits enabled banks to generate more cash flows and to create more inside collateral, which made it up for the fall in treasury holdings. In an environment of relatively scarce treasuries ($\mathbf{b} < \theta f/\gamma a + \Delta)$), as was the case in the pre–crisis phase, the increase in deposits also gave rise to more financial complexity (Proposition 5).

These dynamics resulted in a change in the composition of banks' collateral. From a financial stability perspective, this change is important, as the wholesale funds secured by cash flows are less stable than those secured by treasuries; they are subject to changes in beliefs. Based on our model, we argue that the reduction in the volume of US treasuries held US banks drove the economy from the upper dominance region (with $\mathbf{b} > \bar{b}$) into the multiple equilibrium or the collateral trap region (with $\mathbf{b} \leqslant \bar{b}$); hence the crisis.

6.2 Crisis: Falling into the Collateral Trap

We interpret events post-2008 as a fall in confidence of lenders in the interbank market. The crisis is defined by a dramatic fall in cut-off productivity determining the borrowers and lenders in the interbank market ($\bar{\varepsilon}$). The implications of this switch from optimistic to pessimism are summarized in Proposition 6. During the crisis, the repo rate (r) and the ratio between the repo and the corporate loan rates ($\bar{\varepsilon}$) fell, as as seen in Figure 4, and so does the repo, ABCP and ABS-CDO activity (relative to deposits), as seen in Figure 3. The latter figure also shows that repo activity fell the least and stabilized in 2009 at its 2003 level. In contrast, the ABS-CDO activity fell by over 80% from its peak in 2007 to the 2009. The ABCP volume fell more than repo but less than the ABC-CDO volumes. These relative flows (Fact #3) are in accord with the model's predictions, particularly items 8–9 in Proposition 6. We interpret the ABCP flow as indicative of $\phi^{c(1)}$ and the more dramatic reduction in ABS-CDO as a variation in the sophistication of lending, *i.e.* a reduction in N. We interpret the resilience of the repo market as due to Fed swapping inside collateral for inside collateral in the context of its TSLF program, and by the implied widening on collateral base. In this way, we argue, repo activity was supported through government intervention (Fact #2). This is our collateral trap.

7 Policy Responses

Our focus is on analyzing policies intended to increase household consumption. To discipline our discussion, we assume that, absent any policy interventions, there initially exist multiple equilibria (*i.e.* $\mathbf{b} \in [\underline{b}, \bar{b}]$). In this context, one question that we ask is, whether policy interventions can guide the economy out of multiplicity toward the normal times equilibrium. Formally, this amounts to discussing which policy lowers the upper dominance region threshold, $\bar{b}$. As expression (25) shows, this threshold depends on two different types of parameters, namely: the parameters governing the collateral creation process, θ and f; and those governing the banks' returns, γ and a. We discuss the policies related to those parameters in turn.

7.1 Narrow Banking

The first type of intervention that we consider is a restriction on the assets that can be pledged as collateral in interbank trades. We refer to such a restriction as "narrow banking". In the context of our model, narrow banking takes the form of an interdiction to pledge as collateral the future cash flows from projects. This corresponds to a reduction in the recovery rate θ down to zero. Gorton and Metrick (2010) recently proposed a similar intervention, with the creation of regulated "narrow funding banks", whose regulator would decide which classes of assets are eligible as collateral.[17] In our case, only the interbank trades secured by government debt are feasible, interbank loans are backed by outside collateral only ($\phi^n = 0$ and $\phi = \phi^b$), and financial complexity is eliminated ($N = 0$).

The following proposition compares the equilibrium without inside collateral to the outcome with inside collateral.

Proposition 7. ***(Narrow Banking)*** *The introduction of narrow banking does not rule out the collateral trap, may eliminate the normal times equilibrium ($\bar{b}$ goes up), and impairs the functioning of the interbank market in normal times ($\bar{\varepsilon}$ goes down), if such an equilibrium still exists. Whatever the initial level of $\boldsymbol{b} \in [\underline{b}, \bar{b}]$, narrow banking reduces consumption both in normal times and in the collateral trap.*

Proof. It is easy to see from (23) that both $H(\bar{\varepsilon})$ and $\bar{\varepsilon}^{\max}$ increase monotonically with θ, and therefore that $\bar{b}$ and $\underline{b}$ decrease with θ (from (25) and (26)). It follows that, under narrow banking (*i.e.*, for $\theta = 0$) the normal times equilibrium may be eliminated ($\underline{b}$ goes up), the collateral trap is not eliminated ($\bar{b}$ does not go down), and the banking sector is less efficient ($\bar{\varepsilon}$ is lower) in the normal time equilibrium, if such an equilibrium still exists. Since the banking sector is less efficient in normal times, output is lower, and so is consumption in normal times (from (27)). Finally, since no inside collateral is created, there is less reallocation in a collateral trap, and both output and consumption go down. □

The implications of this intervention are clear. First, the normal times equilibrium has a lower level of $\bar{\varepsilon}$ than in the comparable equilibrium with valued inside collateral. This reduction in $\bar{\varepsilon}$ means that ϕ is

[17] For a discussion of similar narrow banking proposals, see *e.g.* Chow and Surti (2011).

lower in the economy without inside collateral. Accordingly, the reallocation through the interbank market is reduced.

Second, the collateral trap remains and is associated with even less funding of projects. For this economy, collateral restrictions not only do not eliminate the moral hazard and asymmetric information frictions but also reduce the ability of the banking system to respond to these sources of inefficiency. This makes clear that a collateral trap does not arise because of the presence of inside collateral: multiple equilibria can exist even if all loans are backed by outside collateral.

Also, notice that it is possible that collateral restrictions eliminate the multiplicity of equilibria. But this occurs by supporting a unique equilibrium with low interbank flows. This is exactly the same outcome as in the collateral trap, so that the intervention does not improve welfare.

7.2 Bank Subsidy

The second policy intervention is a government subsidy that raises the return on projects. The subsidy is announced at the beginning of the period, and is financed by a lump sum tax on the household at the end of the period. Therefore it is conditional on the bank operating its projects. Such a subsidy could be seen as a conditional bank capital injection by a government, akin to the US Treasury's 2009 "Capital Assistance Program". Indeed, under this program, banks received capital from the Treasury conditional on them lending to the real economy.[18]

In the context of our model we assume that banks are given *ex post* a subsidy that raise the unit return on their projects from a to $\tilde{a}$, with $a > \tilde{a}$. The following proposition compares the equilibrium without the subsidy to the outcome with the subsidy.

Proposition 8. ***(Bank Subsidy)*** *A subsidy on projects may eliminate the collateral trap equilibrium, and always improves the functioning of the interbank market, be it in normal times or in the collateral trap. The effect of such a subsidy on consumption is positive in both cases.*

Proof. Following the rise in a, the term on the right hand side of (23) decreases. Hence, the banking sector is more efficient in normal times ($\bar{\varepsilon}$ goes up). From (25) it is easy to see that $\underline{b}$ goes down, which means that the collateral trap equilibrium may be avoided. Since in normal times $\bar{\varepsilon}$ is higher, so are output and consumption (see (27)). Finally, since the subsidy stimulates the creation of inside collateral, there is also more reallocation in a collateral trap, hence higher output and consumption in this case too. □

By giving banks more incentives to operate their projects, the subsidy mitigates the moral hazard problem on the interbank market. For a given volume of collateral, the demand for interbank loans goes up, and so does the equilibrium interbank rate. As a consequence, $\bar{\varepsilon}$ goes up: some unproductive banks that were borrowing on the interbank market now lend, and funds are allocated to the best projects. As

[18]More precisely, as part of the application process, US banks were required to submit a plan for how they intended to use the capital injected to preserve and strengthen their lending capacity. The access to the program was also conditional on restrictions on dividend payments, executive compensation requirements, etc.

the quality of borrowers goes up and their heterogeneity diminishes, the asymmetric information problem recedes, and a larger fraction of cash flows become pledgeable, which further relaxes the borrowers' collateral constraint. Hence the positive effect of a subsidy on the economy.

7.3 Deposit Facility

The next intervention that we consider is the opening of a deposit facility by a central bank, through which the latter offers banks interest–bearing deposits. One recent example of such intervention is the "Term Deposit Facility", which the US Federal Reserve Bank established in 2008.

To analyse this policy we assume that banks now have the possibility to deposit their excess cash with a central bank at a rate of return $\tilde{\gamma}a$, with $\tilde{\gamma} > \gamma$. Hence, banks have two outside options: storage and central bank deposits. Moreover, we assume that banks cannot abscond with funds deposited at the central bank, and that the central bank stores those funds. Hence, central bank deposits are not being used productively, and the government raises lump sum taxes from the household at the end of the period to make it up for the central bank's losses (*i.e.* for $\tilde{\gamma} - \gamma$).

The following proposition compares the equilibrium without the deposit facility to the outcome with the deposit facility.

Proposition 9. ***(Deposit Facility)*** *The existence of a deposit facility eliminates the collateral trap equilibrium if and only if the offered deposit rate, $\tilde{\gamma}a$, is above a certain threshold. In the normal times equilibrium, the deposit facility is not used, and consumption is the same as in a normal times equilibrium without deposit facility. In the collateral trap (if this equilibrium still exists), the deposit facility is used and consumption is higher than without deposit facility.*

Proof. Let $\bar{\varepsilon}^B$ be the smallest solution to equation (23a); this solution corresponds to point B in Figure 7, with $\bar{\varepsilon}^B > \gamma$. Then, by construction, the collateral trap is still a possible outcome if and only if aggregate demand intersects aggregate supply for $\bar{\varepsilon} = \tilde{\gamma}$, that is, if and only if $\bar{\varepsilon}^B \leqslant \tilde{\gamma}$. It follows that the threshold for $\tilde{\gamma}$, above the collateral trap is avoided, is $\bar{\varepsilon}^B$. In the collateral trap (if this equilibrium still exists), the banks that borrow funds on the interbank market are on average more productive, and output and consumption are therefore higher, than without the deposit facility. □

One important result in Proposition 9 is that a central bank can eliminate coordination failures on the interbank market by setting its deposit facility rate high enough. Indeed, as the central bank raises $\tilde{\gamma}$, it also reduces banks' supply of funds on the interbank market, and the equilibrium interbank increases. This, in turn, gives unproductive banks more incentives to lend, which by resorbing the asymmetry of information about borrowers' quality helps avoid the collateral trap. The use of central bank deposit as coordination device is illustrated in Figure 7. The gray circles represent the supply curve after the central bank opens the deposit facility. For a rate of return below $\tilde{\gamma}a$, there is no supply, and point C is ruled out as an equilibrium. Banks then coordinate on the normal time equilibrium in A, where the interbank rate is above $\tilde{\gamma}a$; in this equilibrium, banks do not use the central bank's deposit facility.

7.4 Collateral Swap

In March 2008, the Federal Reserve set up the TSLF program, which gave banks (in effect, primary dealers) the possibility to bid a fee to borrow Treasury securities from the Fed for one month, while agreeing to provide other (less liquid) securities as collateral in exchange. Banks could then use the borrowed Treasury securities as collateral to obtain cash in the private markets. Fleming, Hrung, and Keane (2010) show that such a measure precipitated a significant narrowing of repurchase agreement (repo) spreads between Treasury collateral and less liquid collateral and that, overall, the provision of Treasury collateral through the TSLF mitigated a more general shortage of liquid collateral.

The last intervention that we consider is similar to the TSLF. We assume that the government issues new government debt, and swaps every unit of the new debt against $1/\tilde{\theta}$ units of cash flow from projects at the beginning of the period, with $\tilde{\theta} \in (0, 1)$. At the end of the period, all swaps are reversed, and banks get their corporate claims back.[19] Following those swaps, banks then have the possibility to pledge the new government bonds as collateral in lieu of cash flows.

Proposition 10. ***(Collateral Swap)*** *A collateral swap may rule out the collateral trap only if $\tilde{\theta} > \theta$. In any case, the policy always improves the functioning of the interbank market, and the effect on consumption is always positive.*

Proof. The results follow from the fact that the collateral swap has the same effect as an increase in θ up to $\tilde{\theta}$. □

The collateral swap policy enhances the pledgeability of banks' cash flows, so that banks have more skin in the game, and lose more if they abscond. The effects of this policy are therefore similar to those of a bank subsidy (Section 7.2).

8 Conclusions

In this paper we explore how banks' holdings of outside collateral (like treasuries) affect banks' production of inside collateral, and how this collateral creation process affects the efficiency and stability of wholesale financial markets. We highlight sudden swings in both collateral value and collateral creation as a source of fragility in interbank markets leading to a collateral trap.

Due to the presence of moral hazard and adverse selection, loans between heterogeneous banks have two main features. First, these loans are collateralized, either by treasuries (outside collateral) and/or cash flows (inside collateral). Second, limits emerge endogenously on the amount banks can borrow in this market. Confidence is central to the functioning of the interbank market. When lenders are optimistic

[19] In effect, those swaps can be seen as the government lending government debt securities to the banks against the corporate cash flows as collateral. The securities lending transactions unravel at the end of the period, and banks collect cash flows; in this way, we make sure that even the banks with a high ε are willing to enter the swap, and that the productivity of bank ε cannot be revealed based on whether or not the bank enters the swap.

about the quality (productivity) of borrowing banks, quantity restrictions are relatively lax and interbank loan rates are high. In equilibrium, only high productivity banks borrow in the market and the reallocation of deposits across banks is relatively efficient.

But, for the same parameters, the economy may be stuck in a collateral trap. In this case, lenders are pessimistic about bank quality. The volume of inside collateral is low, borrowing restrictions are tight, and lending rates are low. Relatively low productivity banks borrow so that the interbank market is much less efficient in the reallocation of deposits. The reduction in interbank flows is more pronounced in loans backed by inside collateral than in loans backed by outside collateral.

The disruption of financial reallocation in a collateral trap has real effects as well. The most profitable banks are unable to attract the flow of funds they would receive in normal times. The aggregate economy is less productive: output, consumption and welfare are lower.

Though the model is purposefully simple, structured to capture the interactions of inside and outside collateral, it is capable of matching some key features of the recent financial crisis. This includes the reduction in the interbank loan rate, the fall in the flow of funds financed by inside collateral relative to outside collateral and the consequent fall in output.

Finally, we use the model to discuss the effects of four policies on financial efficiency and stability. A key finding is that asset portfolio restrictions are costly in terms of welfare; indeed, such restrictions may eliminate the fragility of these markets but they do so by supporting a unique equilibrium with low interbank flows. We also find that bank subsidies, a central bank deposit facility, and collateral swaps contribute to the stability of the financial sector.

9 Appendix

We want to prove that $H(\bar{\varepsilon})$ is humped–shaped over interval $[0,1]$. From the definition of $H(\bar{\varepsilon})$ in (23), one gets:

$$H'(\bar{\varepsilon}) > 0 \Leftrightarrow \left(\frac{\gamma}{\theta} - \bar{\varepsilon}\right) \frac{\bar{\varepsilon} G'(\bar{\varepsilon})}{1 - G(\bar{\varepsilon})} < \frac{\gamma}{\theta}. \tag{28}$$

Given Assumption 1, $\gamma > \theta$ and it is easy to see that the left hand side of relation (28) is equal to zero when $\bar{\varepsilon} = 0$ and goes to infinity when $\bar{\varepsilon} \nearrow 1$. Therefore, $H(\bar{\varepsilon})$ reaches at least one maximum over $[0,1]$. It remains to show that this maximum is unique. To do so, we re–write (28) as:

$$\underbrace{\frac{1}{\bar{\varepsilon}} \left(1 - \frac{1 - G(\bar{\varepsilon})}{\bar{\varepsilon} G'(\bar{\varepsilon})}\right)}_{\chi(\bar{\varepsilon})} < \frac{\theta}{\gamma}, \tag{29}$$

and show that, under Assumption 2, $\chi'(\bar{\varepsilon}) > 0$: $\chi'(\bar{\varepsilon}) > 0 \Leftrightarrow 2G'(\bar{\varepsilon}) + \bar{\varepsilon} G''(\bar{\varepsilon}) > 0$.

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