Inattention to Rare Events*

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Abstract

The world recently experienced several rare events with disastrous consequences: the global financial crisis, the European sovereign debt crisis, and the Fukushima nuclear accident. These events have in common that key decision-makers were unprepared for them, which aggravated these events. We develop a model in which agents make state-contingent plans – prepare to act in different contingencies – subject to the constraint that agents can process only a finite amount of information. We identify the forces that make agents prepare little for some contingencies. We study whether a social planner would want agents to prepare more for rare events.

Keywords: rare events, disasters, rational inattention, efficiency. (*JEL*: D83, E58, E60).

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1 Introduction

The world recently experienced several rare events with disastrous consequences: the global financial crisis, the European sovereign debt crisis, and the Fukushima nuclear accident. These events have in common that key decision-makers were unprepared for them, which aggravated these events. Should decision-makers think more about optimal actions in unusual times, even if this means that they will think less about optimal actions in normal times?

To address this question formally, we study a model where agents make state-contingent plans, subject to an information processing constraint. That is, agents prepare for different contingencies but can process only a finite amount of information. The different contingencies have different probabilities, mistakes may be more costly in some contingencies than in others, agents may face limited liability, and actions may be strategic complements or strategic substitutes. We identify the forces that make agents prepare little for rare events. We then study whether a social planner would want agents to be more prepared for rare events. We find that under reasonable assumptions this is the case.

In the model, agents can process only a finite amount of information. Agents therefore cannot prepare perfectly for all contingencies. The expected benefit of thinking about the optimal action in a contingency is higher when the contingency is more likely. Thus, the extent to which agents think about a contingency is increasing in the probability of the contingency. The first-order condition for an optimal allocation of attention says: agents allocate attention so as to equate the *probability-weighted* expected loss due to suboptimal actions across contingencies. As a result, the expected loss due to suboptimal action in a contingency is inversely related to the probability of the contingency. For example, if the probability of one contingency is one thousand times smaller than the probability of another contingency (think of the first contingency as a rare event and of the second contingency as normal times), the expected loss due to suboptimal action is one thousand times larger in the first contingency than in the second contingency. Hence, the observation that agents take good actions in normal times does not imply that agents will take good actions in unusual times.

This result still holds when mistakes are more costly in some contingencies than in other contingencies. The optimal allocation of attention is still to equate the *probability-weighted* expected loss due to suboptimal actions across contingencies.

Since limited liability is a feature of many real world situations and limited liability kicks in more frequently in unusual times than in normal times, we allow for limited liability in the model. We begin by introducing limited liability symmetrically across contingencies, that is, the extent of limited liability protection is the same in all contingencies. We find that this form of limited liability makes agents prepare *even less* for rare events. The intuition is the following. Since agents think less about the optimal actions in unusual times than about the optimal actions in normal times, agents take on average worse actions in unusual times than in normal times. Limited liability therefore is more relevant in unusual times than in normal times. Hence, limited liability reduces more strongly the incentive to think about unusual times than the incentive to think about normal times.

We also allow for strategic interactions in the model. Actions may be strategic complements or strategic substitutes. We begin by assuming that the degree of strategic complementarity in actions is the same for all contingencies. We obtain the following result. Start in a situation in which there are no strategic interactions and agents think less about the optimal actions in unusual times than about the optimal actions in normal times. Suppose that actions become strategic complements. Then agents think *even less* about rare events. Strategic complementarity reduces more strongly the incentive to think about unusual times than the incentive to think about normal times. This is true even though the degree of strategic complementarity is the same in all contingencies.

The model helps us understand what we think was a critical feature of the recent events: In each case an adverse shock occurred, key decision-makers were unprepared to take action in response to that shock, and catastrophic consequences followed. In Fukushima, the adverse shock was the earthquake and tsunami that cut the power supply and disabled the cooling system of a nuclear power plant. In order to prevent an explosion, the staff on duty had to vent a nuclear reactor. They opened the emergency manual and discovered that it contained no instructions on how to vent the reactor in the absence of electricity. The staff had to improvise the venting and failed to prevent an explosion. The model suggests why the staff were unprepared: The adverse shock they had to respond to was a low probability event and the management of the company owning the plant faced limited liability. Possibly, there was also strategic complementarity in actions. In the paper we also relate the model to the global financial crisis and the European sovereign debt crisis. The story of each crisis fits the Fukushima parable: “something bad happens, you have to take action, you open an emergency manual and discover that it tells you nothing about what to do." Furthermore, the interaction of low probability, limited liability, and possibly strategic complementarity helps us understand why the emergency manual was empty in each case.

Would a planner want people to be more prepared for rare events? To answer this question, we study the following planner problem. The planner chooses the agents' attention allocation, subject to the agents' information processing constraint. The planner maximizes ex-ante welfare. We find that under reasonable assumptions limited liability creates an inefficiency: The planner wants agents to think more about optimal actions in unusual times, even if this means that they think less about optimal actions in normal times.

This paper makes contact with several recent strands of literature. It is related to the literature on rational inattention building on Sims (2003).[1] In contrast to the existing literature on rational inattention, this paper studies how agents make state-contingent plans, subject to an information processing constraint.[2]

Our work is also related to the literature on rare disasters. See for example Barro (2006), Gabaix (2012), Gourio (2012), and Barro, Nakamura, Steinsson, and Ursua (2013). This literature investigates the implications of rare disasters for asset prices and business cycles when agents act perfectly in a rare event. By contrast, we model agents as acting imperfectly in a rare event and we investigate how much incentive agents have to prepare for a rare event. If people had been prepared to take good action in historical rare adverse events, these events would have unfolded less dramatically and perhaps would not be called "disasters" today.

The part of the paper in which we compare the equilibrium allocation of attention with the effi-

[1] For theoretical papers, see e.g. Sims (2003, 2006, 2010), Luo (2008), Maćkowiak and Wiederholt (2009, 2014), Van Nieuwerburgh and Veldkamp (2009, 2010), Woodford (2009), Mondria (2010), Matejka (2012), Paciello (2012), Tutino (2013), Matejka and McKay (2014), Paciello and Wiederholt (2014), and Yang (2014). For empirical papers, see Maćkowiak, Moench, and Wiederholt (2009), Coibion and Gorodnichenko (2012), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014), and Melosi (2014). For an alternative approach to modeling attention choices, see Reis (2006a,b) where agents incur a fixed cost and receive perfect information when they pay attention and agents choose the duration of inattentiveness.

[2] In a somewhat related paper, Bolton and Faure-Grimaud (2008) propose a model of costly decision-making based on time-costs of deliberating current and future decisions. An agent can invest and choose between a risky action and a safe action. The agent is uncertain about the return of the risky action in different states of nature. The agent can choose to think ahead, think on the spot, or not think at all. The cost of thinking is that it delays the project.

cient allocation of attention is related to two recent papers on efficiency of information acquisition. Colombo et al. (2014) consider a model in which agents with a quadratic payoff function take an action based on a noisy public and private signal. Agents' actions may be strategic complements or substitutes. Before acting agents choose the precision of the private signal. Colombo et al. (2014) characterize the conditions under which agents' information choice is efficient and study the social value of the public signal. Llosa and Venkateswaran (2013) consider a simple business cycle model in which various agents take actions, e.g., firms set prices, based on a noisy private signal. Before acting agents choose the precision of the private signal. Llosa and Venkateswaran show that typically agents' information choices are inefficient in that business cycle model.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the equilibrium allocation of attention. Section 4 looks at recent events from the perspective of the model. Section 5 studies the efficient allocation of attention. Section 6 concludes.

2 Model

There are two periods, called today and tomorrow. Tomorrow the economy is in one of two regimes indexed by $j \in \{r, n\}$. Let $p_j > 0$ denote the probability of regime j with $p_r < p_n$. We refer to regime r as "unusual times" (or "the rare event") and regime n as "normal times."[3]

There is a continuum of agents indexed by $i \in [0, 1]$. Today each agent commits to a state-contingent plan for tomorrow. The contingent plan of agent i specifies an action for each contingency

$$a_i = \begin{pmatrix} a_{i,r} \\ a_{i,n} \end{pmatrix} \in \mathbb{R}^2.$$

Tomorrow nature draws the regime and the contingent plan of each agent is implemented. The assumption that agents cannot process additional information after nature has drawn the regime and before the plan is implemented captures the idea that once the regime realizes agents have to act quickly. Therefore, agents have to plan ahead. As an extension, we will allow agents to process a finite amount of additional information after nature has drawn the regime and before the plan is implemented.

[3] In the baseline model presented here, we assume that agents know the true values of p_r and p_n. As an extension, we will consider many periods and Bayesian learning about the probabilities of the two regimes.

The payoff of agent i in regime $j = r, n$ is given by the payoff function $V^j(a_{i,j}, a_j, z_j)$, where $a_{i,j}$ is the own action in regime j, z_j is a fundamental in regime j, and a_j is the mean action in the population in regime j. We will assume that the fundamental in the regime affects the optimal action in the regime, and since agents are uncertain about the fundamental, they are uncertain about the optimal action. The superscript j on the payoff function indicates that the payoff function may differ across regimes.

Since limited liability is a feature of many real world situations, we allow for limited liability in the model

$$V^j(a_{i,j}, a_j, z_j) = \max\left\{U^j(a_{i,j}, a_j, z_j), \omega_j\right\},$$

where $U^j(a_{i,j}, a_j, z_j)$ is the payoff function in regime j in the absence of limited liability and ω_j is the lowest possible payoff in regime j in the presence of limited liability. A higher value of ω_j means more limited liability protection.

For tractability, we assume that the payoff function with unlimited liability is quadratic and strictly concave in its first argument. The first assumption can be viewed as a second-order approximation of any twice differentiable function with the same three arguments. Then,

$$U^j(a_{i,j}, a_j, z_j) = U^j\left(a^*_{i,j}, a_j, z_j\right) - \frac{\left|U^j_{a_i a_i}\right|}{2}\left(a_{i,j} - a^*_{i,j}\right)^2, \tag{1}$$

where $a^*_{i,j}$ denotes the optimal action in regime j, which is given by

$$a^*_{i,j} = \arg\max_{a_{i,j}\in\mathbb{R}} U^j(a_{i,j}, a_j, z_j) = \frac{U^j_{a_i}}{\left|U^j_{a_i a_i}\right|} + \frac{U^j_{a_i a}}{\left|U^j_{a_i a_i}\right|} a_j + \frac{U^j_{a_i z}}{\left|U^j_{a_i a_i}\right|} z_j. \tag{2}$$

The coefficient $U^j_{a_i a_i}$ denotes the second derivative of the function $U^j(a_{i,j}, a_j, z_j)$ with respect to $a_{i,j}$, the coefficients $U^j_{a_i a}$ and $U^j_{a_i z}$ denote the cross derivatives of the function $U^j(a_{i,j}, a_j, z_j)$ involving $a_{i,j}$, and the coefficient $U^j_{a_i}$ denotes the first derivative of the function $U^j(a_{i,j}, a_j, z_j)$ with respect to $a_{i,j}$ evaluated at the origin. We assume that the fundamental in regime j affects the optimal action in regime j ($U^j_{a_i z} \neq 0$) and we assume without loss of generality that the coefficients on a_j and z_j in equation (2) sum to one.[4] Defining $\delta_j \equiv \left|U^j_{a_i a_i}\right|/2$, $\varphi_j \equiv U^j_{a_i}/\left|U^j_{a_i a_i}\right|$, and $\gamma_j \equiv U^j_{a_i a}/\left|U^j_{a_i a_i}\right|$, the last two equations become

$$U^j(a_{i,j}, a_j, z_j) = U^j\left(a^*_{i,j}, a_j, z_j\right) - \delta_j\left(a_{i,j} - a^*_{i,j}\right)^2, \tag{3}$$

[4] If this assumption is not satisfied, one can always redefine the fundamental z_j by multiplying it with a constant to ensure that this assumption is satisfied.

and

$$a_{i,j}^{*}=\varphi_{j}+\gamma_{j}a_{j}+\left(1-\gamma_{j}\right)z_{j}. \tag{4}$$

The coefficient δ_j governs the cost of a mistake in regime j, while the coefficient γ_j governs the degree of strategic complementarity in actions in regime j. If $\gamma_j = 0$, the optimal action in a regime does not depend on the mean action in the population. For most of the paper, we will focus on the case $\gamma_j = 0$. In this case, one can simply think of the fundamental in a regime as the optimal action in the regime.[5] In general, one can think of the fundamental in a regime as some exogenous variable that affects the optimal action in the regime.

Agents have some prior knowledge of the optimal actions in the two regimes. In particular, agents have the common prior belief that the vector of fundamentals is normally distributed with mean μ and covariance matrix Σ

$$z=\begin{pmatrix} z_r \\ z_n \end{pmatrix}\sim N\left(\mu,\Sigma\right).$$

In the baseline model, we assume that Σ is a diagonal matrix with strictly positive diagonal entries Σ_{rr} and Σ_{nn}. As an extension, we will relax the assumption that Σ is diagonal. The larger Σ_{rr} and Σ_{nn}, the more uncertain agents are about the optimal actions in the two regimes.

Agents can process information about the optimal actions before committing to a plan. Processing information about the optimal actions in the two regimes is modeled as receiving noisy signals about the fundamentals in the two regimes

$$s_i=\begin{pmatrix} z_r \\ z_n \end{pmatrix}+\begin{pmatrix} \varepsilon_{i,r} \\ \varepsilon_{i,n} \end{pmatrix},$$

where the vector of noise $(\varepsilon_{i,r},\varepsilon_{i,n})'$ is assumed to be independent of z, independent across agents, and normally distributed with mean zero and covariance matrix Λ. Let $\Omega=\Sigma-\Sigma\left(\Sigma+\Lambda\right)^{-1}\Sigma$ denote the posterior covariance matrix of z after receiving s_i.

Following Sims (2003), we assume that agents can process only a finite amount of information, and we model agents' limited ability to process information as a constraint on uncertainty reduction,

[5] In Section 3, we set the intercept φ_j to zero because the value of φ_j has no effect on the equilibrium allocation of attention. In Section 5, we take into account that the value of the intercept φ_j may affect the efficient allocation of attention.

where uncertainty is measured by entropy. Formally, agents face the constraint

$$H(z) - H(z|s_i) \leq \kappa.$$

Here $H(z)$ denotes the prior uncertainty about the vector of fundamentals and $H(z|s_i)$ denotes the posterior uncertainty after processing information. Since the entropy of a bivariate normal distribution with covariance matrix Σ equals $(1/2)\log_2\left[(2\pi e)^2|\Sigma|\right]$, where $|\Sigma|$ denotes the determinant of the covariance matrix, the information processing constraint reduces to

$$\frac{1}{2}\log_2\left(\frac{|\Sigma|}{|\Omega|}\right) \leq \kappa.$$

The parameter $\kappa > 0$ indexes the ability of an agent to process information, where a larger κ means that an agent can process more information and therefore reduce uncertainty by more.

Each agent i decides how carefully to think about the optimal action in the rare event and the optimal action in normal times so as to maximize the expected payoff subject to the information processing constraint:

$$\max_{\Lambda} \sum_{j=r,n} p_j E\left[V^j(a_{i,j}, a_j, z_j)\right], \tag{5}$$

subject to

$$\forall s_i \in \mathbb{R}^2 : a_{i,j} = \arg\max_{x\in\mathbb{R}} E\left[V^j(x, a_j, z_j)|s_i\right], \tag{6}$$

$$s_i = \begin{pmatrix} z_r \\ z_n \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,r} \\ \varepsilon_{i,n} \end{pmatrix}, \tag{7}$$

and

$$\frac{1}{2}\log_2\left(\frac{|\Sigma|}{|\Omega|}\right) \leq \kappa, \tag{8}$$

and the restriction that Λ is a positive semidefinite matrix. The agent chooses the allocation of attention so as to maximize the expected payoff. Recall that p_j is the probability of regime j and $V^j(a_{i,j}, a_j, z_j)$ is the payoff in regime j. The expectation operator in (5) is the expectation under the prior. The agent anticipates that for each signal realization he or she will commit to the best contingent plan given his or her posterior. See equation (6). The agent faces the information processing constraint (8).[6]

[6] The covariance matrix of noise Λ and the posterior covariance matrix of the fundamentals Ω have no subscript i because the solution to the attention choice problem (5)-(8) is the same for all agents.

In problem (5)-(8) the informational constraint depends only on the prior covariance matrix of the fundamentals, Σ, and the posterior covariance matrix of the fundamentals, Ω. This setup formalizes the idea that learning is the mental process of absorbing available information. All information required for the agent to take the optimal actions in both regimes is in principle available. The agent, due to limited cognitive ability, cannot attend to all this information and therefore cannot prepare a perfect action plan for each contingency. Furthermore, once the agent has formed a conditional expectation of the optimal action, there is no physical cost of implementing the action. We think that this setup captures the critical feature of the recent events: people had failed to think through what action to take in certain contingencies, while information about what action to take was available and the physical cost of implementing good action was negligible.

3 The equilibrium allocation of attention

In this section, we derive the equilibrium allocation of attention. We begin by abstracting from limited liability and strategic interactions (Section 3.1). Afterwards, we study the effects of limited liability (Section 3.2) and strategic interactions (Section 3.3) on the equilibrium allocation of attention. Finally, we consider three extensions of the baseline model (Sections 3.4-3.6).

3.1 The role of probabilities

In this subsection, we characterize the equilibrium allocation of attention in the special case of unlimited liability and no strategic interactions. We show that the odds of the rare event determine the equilibrium ratio of the expected loss due to suboptimal action in the rare event to the expected loss due to suboptimal action in normal times.

When there is unlimited liability ($V^j\left(a_{i,j}, a_j, z_j\right) = U^j\left(a_{i,j}, a_j, z_j\right)$) and no strategic complementarity or strategic substitutability in actions ($\gamma_r = \gamma_n = 0$), the optimal action in each regime equals the fundamental in the regime, $a^*_{i,j} = z_j$. The best contingent plan of an agent given his or her posterior equals the conditional expectation of the optimal actions, $a_{i,j} = E\left[z_j | s_i\right]$, and the expected payoff in regime j is given by

$$\begin{aligned} E\left[U^j\left(a_{i,j}, a_j, z_j\right)\right] &= E\left[U^j\left(a_{i,j}^*, a_j, z_j\right)\right] - \delta_j E\left[\left(a_{i,j} - a_{i,j}^*\right)^2\right] \\ &= E\left[U^j\left(a_{i,j}^*, a_j, z_j\right)\right] - \delta_j E\left[\left(E\left[z_j|s_i\right] - z_j\right)^2\right] \\ &= E\left[U^j\left(a_{i,j}^*, a_j, z_j\right)\right] - \delta_j \Omega_{jj}. \end{aligned} \tag{9}$$

Equation (9) implies that the expected loss in payoff in regime j due to suboptimal action equals $\delta_j \Omega_{jj}$, where δ_j is the coefficient that governs the cost of a mistake in regime j and Ω_{jj} is the posterior variance of the fundamental in regime j.

Since the fundamentals are independent across regimes (Σ is diagonal), it is optimal to think independently about the optimal action in the rare event and the optimal action in normal times. This result is proved in Section 3.3. Formally, the optimal covariance matrix of noise in the signal Λ is diagonal. As a result, the posterior covariance matrix of the fundamentals Ω is diagonal and the information processing constraint (8) reduces to

$$\frac{1}{2}\log_2\left(\frac{\Sigma_{rr}}{\Omega_{rr}}\right) + \frac{1}{2}\log_2\left(\frac{\Sigma_{nn}}{\Omega_{nn}}\right) \leq \kappa. \tag{10}$$

Let κ_j denote the attention devoted to regime j

$$\kappa_j \equiv \frac{1}{2}\log_2\left(\frac{\Sigma_{jj}}{\Omega_{jj}}\right). \tag{11}$$

This definition implies the following simple relationship between the posterior and prior variance of the fundamental in regime j: $\Omega_{jj} = \Sigma_{jj} 2^{-2\kappa_j}$. When no attention is devoted to a regime ($\kappa_j = 0$), the posterior variance equals the prior variance. When attention is devoted to a regime ($\kappa_j > 0$), the posterior is less diffuse than the prior.

Agents decide how carefully to think about the optimal actions in the different regimes. Using equations (9)-(11), the attention choice problem (5)-(8) can be expressed as

$$\max_{(\kappa_r, \kappa_n) \in \mathbb{R}_+^2} \left(-\sum_{j=r,n} p_j \delta_j \Omega_{jj}\right), \tag{12}$$

subject to

$$\Omega_{jj} = \Sigma_{jj} 2^{-2\kappa_j}, \tag{13}$$

and

$$\kappa_r + \kappa_n \leq \kappa. \tag{14}$$

The unique solution to this problem is

$$\kappa_r = \begin{cases} 0 & \text{if } \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \leq 2^{-\kappa} \\ \frac{1}{2}\left[\kappa + \log_2\left(\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}}\right)\right] & \text{if } \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \in \left[2^{-\kappa}, 2^{\kappa}\right] \\ \kappa & \text{if } \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \geq 2^{\kappa} \end{cases}. \tag{15}$$

If the ratio of $p_r \delta_r \Sigma_{rr}$ to $p_n \delta_n \Sigma_{nn}$ is equal to one, the attention allocation is fifty-fifty. Starting from this situation, reduce the probability of the rare event, p_r. Agents decide to think less about the optimal action in the rare event and more about the optimal action in normal times. Since the expected benefit of thinking about a contingency is higher when the contingency is more likely, the extent to which agents think about a contingency is increasing in the probability of the contingency. Note that a corner solution is possible. If the rare event is sufficiently unlikely, agents decide to not think at all about the optimal action in the rare event. Finally, for agents to think more about the optimal action in unusual times than about the optimal action in normal times, the cost of a mistake has to be sufficiently larger in unusual times than in normal times ($\delta_r > \delta_n$) or agents have to be sufficiently more uncertain about the optimal action in unusual times than about the optimal action in normal times ($\Sigma_{rr} > \Sigma_{nn}$).

It is clear from equation (15) that the attention devoted to the rare event depends on several factors. By contrast, the expected loss due to suboptimal action in the rare event divided by the expected loss due to suboptimal action in normal times depends *only* on the odds of the rare event. At an interior solution ($0 < \kappa_r < \kappa$) the first-order condition for an optimal allocation of attention reads

$$p_r \delta_r \Omega_{rr} = p_n \delta_n \Omega_{nn}.$$

Agents equate the *probability-weighted* expected loss due to suboptimal action across contingencies. Thus

$$\frac{\delta_r \Omega_{rr}}{\delta_n \Omega_{nn}} = \frac{1}{\frac{p_r}{pn}}. \tag{16}$$

The expected loss in the rare event divided by the expected loss in normal times (left-hand side of the last equation) is equal to one over the odds of the rare event (right-hand side of the last equation). This equation holds for any parameter values, so long as the attention problem has an interior solution. Consider the following example. If the rare event has a relative probability of 0.1 percent, the expected loss due to suboptimal action is one thousand times larger in the rare event than in normal times. Hence, the observation that agents take good actions in normal times does not imply that agents will take good actions in unusual times.

3.2 Limited liability

In this subsection, we study the effects of limited liability on the equilibrium allocation of attention. We continue to assume that there are no strategic interactions ($\gamma_r = \gamma_n = 0$).

For tractability, we assume that the payoff at the payoff-maximizing action is independent of the fundamental and the mean action of others, i.e., whenever the agent takes the payoff-maximizing action in regime j he or she will get the payoff $\bar{u}_j$. The payoff function with unlimited liability (3) then reduces to

$$U^j\left(a_{i,j}, z_j\right) = \bar{u}_j - \delta_j \left(a_{i,j} - z_j\right)^2.$$

This assumption will allow us to derive analytical results about the optimal allocation of attention with limited liability.

Recall that the payoff function with limited liability is

$$V^j\left(a_{i,j}, z_j\right) = \max\left\{U^j\left(a_{i,j}, z_j\right), \omega_j\right\}.$$

The expected payoff in regime j when the agent commits to action $a_{i,j}$ after receiving signal s_i equals

$$E\left[V^j\left(a_{i,j}, z_j\right)|s_i\right] = \int_{-\infty}^{\infty} \max\left\{\bar{u}_j - \delta_j\left(a_{i,j} - z\right)^2, \omega_j\right\} f\left(z|s_i\right) dz,$$

where $f\left(z|s_i\right)$ denotes the conditional density of the optimal action in regime j given the signal. Since limited liability kicks in if and only if the absolute distance between the action and the optimal action exceeds $\Delta_j \equiv \sqrt{\frac{\bar{u}_j - \omega_j}{\delta_j}}$, this expected payoff in regime j can be written as

$$E\left[V^j\left(a_{i,j}, z_j\right)|s_i\right] = \int_{a_{i,j}-\Delta_j}^{a_{i,j}+\Delta_j} \left[\bar{u}_j - \delta_j\left(a_{i,j} - z\right)^2 - \omega_j\right] f\left(z|s_i\right) dz + \omega_j.$$

The action that maximizes this expression is the conditional mean of the optimal action given the signal, $a_{i,j} = E\left[z_j|s_i\right]$. Furthermore, in Appendix A we show that the maximized expected payoff is independent of the value of this conditional mean. Therefore, we can consider without loss in generality the special case where the density $f(z|s_i)$ has the property $E[z_j|s_i] = 0$. We arrive at the following expression for the maximized expected payoff in regime j

$$\max_{a_{i,j}\in\mathbb{R}} E\left[V^j(a_{i,j}, z_j)\,|s_i\right] = \int_{-\Delta_j}^{\Delta_j} \left[\bar{u}_j - \delta_j z^2 - \omega_j\right] f(z|s_i)\,dz + \omega_j.$$

Finally, one can write the maximized expected payoff in regime j as the sum of the expected payoff with unlimited liability and the expected benefit from limited liability

$$\begin{aligned}\max_{a_{i,j}\in\mathbb{R}} E\left[V^j(a_{i,j}, z_j)\,|s_i\right] &= \bar{u}_j - \delta_j\Omega_{jj} \\ &\quad + \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] f(z|s_i)\,dz \\ &\quad + \int_{\Delta_j}^{\infty} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] f(z|s_i)\,dz.\end{aligned}$$

The first term on the right-hand side is the expected payoff with unlimited liability. The second term plus the third term is the expected benefit from limited liability. In the following, we denote the expected benefit from limited liability in regime j by $B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)$, recognizing that the expected benefit from limited liability depends only on Ω_{jj}, $\bar{u}_j - \omega_j$ and δ_j, which is shown in Appendix A. The following lemma summarizes properties of the expected benefit from limited liability.

Lemma 1 *The expected benefit from limited liability in regime j equals*

$$B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j) = \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] f(z|s_i)\,dz + \int_{\Delta_j}^{\infty} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] f(z|s_i)\,dz,$$

where $\Delta_j = \sqrt{\frac{\bar{u}_j - \omega_j}{\delta_j}} > 0$. The expected benefit from limited liability has the following properties:

- $\frac{\partial B(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j)}{\partial\Omega_{jj}} \in (0, \delta_j)$,
- $\frac{\partial^2 B(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j)}{\partial\Omega_{jj}\partial\omega_j} > 0$,
- $\frac{\partial^2 B(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j)}{\partial^2\Omega_{jj}} > 0$ *if* $\Delta_j \geq 1.732\sqrt{\Omega_{jj}}$,
- $\frac{\partial^3 B(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j)}{\partial^2\Omega_{jj}\partial\omega_j} > 0$ *if* $\Delta_j > 1.732\sqrt{\Omega_{jj}}$ *and* $\frac{\partial^3 B(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j)}{\partial^2\Omega_{jj}\partial\omega_j} < 0$ *if* $\Delta_j \in \left(0, 1.732\sqrt{\Omega_{jj}}\right)$.

Proof. See Appendix A. ■

We use these results about the partial derivatives of the expected benefit from limited liability when we study how limited liability affects the equilibrium allocation of attention.[7]

Let us turn to the equilibrium allocation of attention. The attention choice problem with limited liability can be expressed as

$$\max_{(\kappa_r,\kappa_n)\in\mathbb{R}^2_+}\left(\sum_{j=r,n} p_j\left[\bar{u}_j-\delta_j\Omega_{jj}+B\left(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j\right)\right]\right), \tag{17}$$

subject to

$$\Omega_{jj}=\Sigma_{jj}2^{-2\kappa_j}, \tag{18}$$

and

$$\kappa_r+\kappa_n\leq\kappa. \tag{19}$$

The difference to problem (12)-(14) is the presence of the benefit from limited liability in objective (17). The following proposition describes how limited liability affects the equilibrium allocation of attention.

Proposition 1 *(The effect of limited liability) Let κ_r^{LL} and κ_r^{UL} denote the attention allocated to the rare event in the case of limited liability and in the case of unlimited liability, respectively. Furthermore, let $\Omega_{rr}^{UL}=\Sigma_{rr}2^{-2\kappa_r^{UL}}$ denote the optimal posterior variance under unlimited liability. If $\bar{u}_r-\omega_r=\bar{u}_n-\omega_n$, $\delta_r=\delta_n$, $\Sigma_{rr}\geq\Sigma_{nn}$, and $\Delta_r\geq 1.732\sqrt{\Omega_{rr}^{UL}}$, limited liability reduces the attention allocated to the rare event:*

$$\kappa_r^{UL}\in(0,\kappa)\Rightarrow\kappa_r^{LL}<\kappa_r^{UL}.$$

Proof. See Appendix B. ■

To understand the effect of limited liability, Proposition 1 considers a symmetric situation. The difference between the highest possible payoff and the lowest possible payoff is the same in the two regimes ($\bar{u}_r-\omega_r=\bar{u}_n-\omega_n$) and the cost of a mistake is the same in the two regimes ($\delta_r=\delta_n$). Furthermore, the extent of limited liability protection is not too large relative to the

[7] In Maćkowiak and Wiederholt (2012) we prove part of the first bullet point of Lemma 1 (the sign of $\frac{\partial B\left(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j\right)}{\partial\Omega_{jj}}$). For the following proposition, one needs the other part of the first bullet point of Lemma 1 and the third bullet point of Lemma 1.

posterior variance under unlimited liability ($\Delta_r \geq 1.732\sqrt{\Omega_{rr}^{UL}}$).[8] Then limited liability *reduces* the attention allocated to the rare event and *increases* the attention allocated to normal times. The reason is the following. In the absence of limited liability, agents choose to be more uncertain about the optimal action in the rare event than about the optimal action in normal times, implying that in expectation agents take worse actions in the rare event than in normal times. Hence, limited liability is more likely to matter in the rare event than in normal times. As a result, the introduction of limited liability makes agents pay even less attention to the rare event and even more attention to normal times.

It may be useful to give a sketch of the proof of Proposition 1. The first bullet point in Lemma 1 implies that the constraint on uncertainty reduction (19) is always binding. Substituting constraint (18) and the binding constraint (19) into the objective (17) yields an objective function that depends only on the choice variable κ_r and parameters. The partial derivative of this objective function with respect to the choice variable κ_r equals

$$p_r \delta_r \Omega_{rr} 2\ln(2)\left[1 - \frac{\frac{\partial B(\Omega_{rr}, \bar{u}_r - \omega_r, \delta_r)}{\partial \Omega_{rr}}}{\delta_r}\right] - p_n \delta_n \Omega_{nn} 2\ln(2)\left[1 - \frac{\frac{\partial B(\Omega_{nn}, \bar{u}_n - \omega_n, \delta_n)}{\partial \Omega_{nn}}}{\delta_n}\right]. \tag{20}$$

This expression is strictly negative at the point $\kappa_r = \kappa_r^{UL}$ if (i) $\Omega_{nn}^{UL} < \Omega_{rr}^{UL}$, (ii) the B function is strictly convex in its first argument on $\left[\Omega_{nn}^{UL}, \Omega_{rr}^{UL}\right]$, and (iii) the second and third argument of the B function are the same across contingencies. The assumptions in Proposition 1 imply that (i)-(iii) hold. See equation (16) and Lemma 1. As a result, if $\kappa_r^{UL} \in (0, \kappa)$, any solution to the attention choice problem with limited liability has to satisfy $\kappa_r \neq \kappa_r^{UL}$. Proving the stronger statement in Proposition 1 that any solution to the attention choice problem with limited liability has to satisfy $\kappa_r < \kappa_r^{UL}$ is more work, because the objective function that depends only on the choice variable κ_r and parameters is not necessarily concave in κ_r, but it can be done. See Appendix B.

Starting from the symmetric situation covered in Proposition 1, one can introduce asymmetry across regimes (in addition to the asymmetry that the rare event is less likely than normal times). For example, if $\bar{u}_r - \omega_r < \bar{u}_n - \omega_n$, because the rare event is a bad event or limited liability protection is higher in the rare event than in normal times, then limited liability reduces the attention devoted

[8] Note that less limited liability protection in regime j means smaller ω_j and thus larger Δ_j (a larger absolute distance between the action and the optimal action is necessary for limited liability to kick in). The case of no limited liability can be viewed as the limit as Δ_j goes to infinity.

to the rare event even more compared to the case of $\bar{u}_r - \omega_r = \bar{u}_n - \omega_n$. To see this, note that lowering $\bar{u}_r$ or increasing ω_r reduces the term in the first square bracket in expression (20). See Lemma 1.

Next, suppose that mistakes are more costly in the rare event than in normal times ($\delta_r > \delta_n$). This assumption has subtle effects on the equilibrium allocation of attention because δ_r and δ_n affect both the equilibrium in the case of unlimited liability and the expected benefit from limited liability. However, one can show that the term $\frac{\partial B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)}{\partial \Omega_{jj}} / \delta_j$ appearing in expression (20) depends only on Ω_{jj} and Δ_j. Recall that $\Delta_j = \sqrt{\frac{\bar{u}_j - \omega_j}{\delta_j}}$. Furthermore, the term $\frac{\partial B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)}{\partial \Omega_{jj}} / \delta_j$ is strictly increasing in Ω_{jj} and strictly decreasing in Δ_j so long as $\Delta_j \geq 1.732\sqrt{\Omega_{jj}}$. Hence, if $\Omega_{rr}^{UL} > \Omega_{nn}^{UL}$, $\Delta_r \leq \Delta_n$, and $\Delta_r \geq 1.732\sqrt{\Omega_{rr}^{UL}}$, then limited liability reduces the attention allocated to the rare event.[9]

3.3 Strategic complementarity in actions

In this subsection, we study the effects of strategic interactions on the equilibrium allocation of the attention. The takeaway is as follows. Start in a situation in which there are no strategic interactions and agents pay less attention to the rare event than to normal times. Suppose that actions become strategic complements. Then agents will pay *even less* attention to the rare event. This is true even though the degree of strategic complementarity is *the same* in the two regimes.

Formally, we relax the assumption that $\gamma_r = \gamma_n = 0$. For ease of exposition, we assume that there is unlimited liability, and in the main text, we assume that the degree of strategic complementarity in actions is the same in the rare event and in normal times, $\gamma_r = \gamma_n \equiv \gamma$.[10] When $\gamma > 0$ actions are strategic complements. An individual agent wants to do what other agents do. When $\gamma < 0$ actions are strategic substitutes. An individual agent wants to do the opposite of

[9] We have assumed that the signals are drawn from a Gaussian distribution. Under unlimited liability the payoff function is quadratic and, given a quadratic objective and a Gaussian fundamental, one can prove that optimal signals are Gaussian. See, for example, Maćkowiak and Wiederholt (2009). Under limited liability the assumption that the signals are Gaussian allows us to derive analytical results, in particular Proposition 1. We studied numerically the attention allocation problem under limited liability without the assumption that the signals are Gaussian. We allowed the agent solving the problem to choose directly the optimal joint distribution of the action and the fundamental. We found that the effect of limited liability stated in Proposition 1 continues to hold, i.e., limited liability reduces the attention allocated to the rare event also when one does not assume that the signals are Gaussian.

[10] In Appendix C we cover the case when the degree of strategic complementarity in actions differs across regimes.

what other agents do.

The first part of the following proposition states that it is optimal to think independently about the optimal action in unusual times and the optimal action in normal times. The second part of the proposition characterizes the equilibrium allocation of attention for any value of $\gamma \in (-1,1)$.

Proposition 2 *Consider equilibria of the form $a_j = \phi_j z_j$, where $\phi_j \in \mathbb{R}$ is a coefficient. Since the fundamentals are independent across regimes (Σ is diagonal), each agent decides to receive independent signals about the fundamental in unusual times and the fundamental in normal times (the equilibrium Λ is diagonal). The information-processing constraint (8) then reduces to*

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{rr}}{\Omega_{rr}}\right)}_{\kappa_r} + \underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{nn}}{\Omega_{nn}}\right)}_{\kappa_n} \leq \kappa.$$

Furthermore, if $\gamma \in (-1,1)$ and $2^\kappa > \gamma/(1-\gamma)$, the equilibrium is unique and

$$\kappa_r = \begin{cases} 0 & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}} \leq \frac{1}{(1-\gamma)2^\kappa+\gamma 2^{-\kappa}} \\ \frac{1}{2}\left[\kappa + \log_2(x)\right] & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}} \in \left[\frac{1}{(1-\gamma)2^\kappa+\gamma 2^{-\kappa}}, (1-\gamma)2^\kappa + \gamma 2^{-\kappa}\right] \\ \kappa & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}} \geq (1-\gamma)2^\kappa + \gamma 2^{-\kappa} \end{cases}, \tag{21}$$

where

$$x = \frac{\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}} - \frac{\gamma}{1-\gamma}2^{-\kappa}}{1 - \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma}{1-\gamma}2^{-\kappa}}. \tag{22}$$

The set of equilibria when $\gamma \in (-1,1)$ and $2^\kappa \leq \gamma/(1-\gamma)$ is given in Appendix C.

Proof. See Appendix C. ■

Proposition 2 shows that raising the degree of strategic complementarity in both regimes makes the equilibrium attention allocation more extreme (if possible, i.e., if the attention allocation in the absence of strategic interactions is not already a corner solution). Figure 1 illustrates this result by depicting equilibrium attention to the rare event, κ_r, as a function of the square root of $(p_r\delta_r\Sigma_{rr}/p_n\delta_n\Sigma_{nn})$.[11] In the figure, $\gamma = 0$ denotes the case of no strategic interactions, $\gamma >> 0$ denotes a value of γ close to the value at which $2^\kappa = \gamma/(1-\gamma)$, and $\gamma > 0$ denotes a value of γ between these two extremes. Pick any point on the horizontal axis to the left of 1. In the absence of strategic interactions (i.e., $\gamma = 0$) agents think less about the optimal action in unusual times

[11] Figure 1 assumes that the parameters κ and γ satisfy $\gamma \in (-1,1)$ and $2^\kappa > \gamma/(1-\gamma)$.

than about the optimal action in normal times (i.e., $\kappa_r < 0.5\kappa$). Raising the degree of strategic complementarity (from $\gamma = 0$ to $\gamma > 0$ and further to $\gamma >> 0$) makes agents think even less about the optimal action in unusual times (i.e., κ_r falls).

When actions are strategic complements, the fact that other agents do not think carefully about the optimal action in a regime reduces the incentive for an individual agent to think about the optimal action in that regime. This effect is known in the literature.[12] We find that this effect is stronger for the regime that agents think less about than for the regime that agents think more about. Therefore, raising the degree of strategic complementarity makes the attention allocation more extreme. This is true although the degree of strategic complementarity is the same in the two regimes.

As the degree of strategic complementarity rises, corner solutions occur more easily. See Figure 1. In fact, for a high degree of strategic complementarity, a small change in parameters (e.g., a small change in the probability of the rare event) can have a large effect on the equilibrium allocation of attention. In particular, as γ approaches the value at which $2^{\kappa} = \gamma / (1-\gamma)$, the parameter region in which the equilibrium allocation of attention is an interior solution collapses to a single point.[13]

Strategic substitutability has the opposite effect. As one can see from equations (21)-(22), strategic substitutability in actions (i.e., $\gamma < 0$) makes the equilibrium attention allocation less extreme.[14]

3.4 Extension: Correlated optimal actions

In the rest of this section, we consider three extensions of the baseline model studied so far. In this subsection, we relax the assumption that optimal actions are independent across regimes. The upshot is as follows. Start in a situation in which the optimal actions are independent across regimes, the odds of the rare event are small, and the expected loss in the rare event is therefore larger than the expected loss in normal times. Suppose that the optimal actions become correlated.

[12] See Hellwig and Veldkamp (2009) and Maćkowiak and Wiederholt (2009).

[13] For a sufficiently high degree of strategic complementarity in actions (namely, $\gamma / (1-\gamma) \geq 2^{\kappa}$), there exist multiple equilibria. See Appendix C for the details.

[14] In Appendix C we also cover the case when the degree of strategic complementarity differs across regimes. The upshot is that raising the degree of strategic complementarity in a single regime makes agents allocate less attention to that regime and more attention to the other regime.

Then the expected loss in the rare event falls, because from thinking about the optimal action in normal times agents learn about the optimal action in unusual times. However, the expected loss in the rare event falls little so long as the optimal actions are not strongly correlated.

Formally, the decision problem of an individual agent is still given by expressions (5)-(8), except that the prior covariance matrix of the fundamentals Σ is nondiagonal.[15] We solve problem (5)-(8) numerically assuming different values of the covariance between the optimal actions in the two regimes, Σ_{rn}. For simplicity, we consider the case of no strategic interactions, i.e., $\gamma_r = \gamma_n = 0$. We always find that the expected loss in the rare event initially falls little as the correlation between the optimal actions in the two regimes increases. Furthermore, we always find that the agent chooses a nondiagonal Λ.

Consider a numerical example. We set $\delta_r = \delta_n$, $\Sigma_{rr} = \Sigma_{nn} = 1$, $p_r = 0.01$, and we choose a value of κ such that the posterior variance of the optimal action in normal times, Ω_{nn}, is equal to 0.01 in the case when the optimal actions are independent across regimes ($\Sigma_{rn} = 0$).[16] Figure 2 shows how the solution of the model changes as we raise Σ_{rn} from zero (no prior correlation of the optimal actions) towards one (perfect prior correlation of the optimal actions), holding the other parameters constant. We note the following results: (1) As the prior correlation of the optimal actions becomes stronger, the expected loss in the rare event Ω_{rr} falls. The stronger the prior correlation of the optimal actions, the more agents learn about what to do in unusual times by thinking about what to do in normal times, and therefore the better agents do on average in the rare event.[17] (2) This effect is nonlinear and sets in slowly, i.e., Ω_{rr} is concave in Σ_{rn}. So long as the optimal actions are not strongly correlated, the expected loss in the rare event falls little compared with the case when the optimal actions are independent. What is the source of this nonlinearity? As the prior correlation of the optimal actions increases, agents learn about the optimal action in unusual times mainly from thinking about the optimal action in normal times. Furthermore, if a fundamental (here z_r) and a signal (here $z_n + \varepsilon_{i,n}$) have a bivariate normal distribution, then the conditional variance of this fundamental given that signal equals the prior variance multiplied by one *minus* the *squared* correlation coefficient. For this reason, Ω_{rr} is concave in Σ_{rn}. In other

[15] We assume that Σ is nonsingular, i.e., the fundamentals are not perfectly correlated across regimes.

[16] In other words, this value of κ means that agents choose to reduce the variance of the optimal action in normal times by a factor of 100.

[17] The expected loss in normal times Ω_{nn} also decreases with Σ_{rn}.

words, Ω_{rr} initially falls little as Σ_{rn} rises.

3.5 Extension: Learning the probability of a rare event

The baseline model assumes that agents know the true probability of the economy being in a particular regime next period. In this subsection, we study a version of the model in which the probability of the economy being in a particular regime next period is a random variable. The following insights emerge. Start in a situation in which agents know the true probabilities and agents pay less attention to the rare event than to unusual times. Suppose that agents are Bayesians who must infer the true probabilities over time. When the rare event fails to occur, agents pay *even less* attention to the rare event because the rational estimate of the probability of the rare event falls. Furthermore, once the rare event takes place, agents pay *a lot more* attention to the rare event because the rational estimate of the probability of the rare event occuring again jumps up.

Consider a random variable X that has a Bernoulli distribution with an unknown parameter p, i.e., X can take only the values 0 and 1, the probabilities are

$$\Pr(X=1)=p \qquad \text{and} \qquad \Pr(X=0)=1-p,$$

and p itself is a random variable. We think of $X=1$ as unusual times and we think of $X=0$ as normal times. Suppose that: (i) agents observe sequentially random variables $X_1,...,X_s,...$ that are i.i.d. over time and each has this Bernoulli distribution; (ii) in period 0, the agents' prior distribution of p is a beta distribution with parameters $\alpha>0$ and $\beta>0$; and (iii) in every period $t=1,2,...$, agents observe whether $X=1$ or $X=0$ and agents update their prior distribution of p. Then the agents' posterior distribution of p given that $X_t=x_t$, $t=1,...,s$, is a beta distribution with parameters $\alpha+y$ and $\beta+s-y$, where $y=\sum_{t=1}^{s}x_t$. Furthermore, agents still solve the attention problem (5)-(8), except that in objective (5) the probability of the economy being in a particular regime next period has been replaced by the agents' posterior expectation of that probability.[18]

[18] To evaluate the agents' objective when agents are uncertain about p, in general one must keep track of the agents' posterior distribution of p and perform integration with respect to p. However, if the agents' prior distribution of p and the stochastic process $\{X_t\}$ are independent of the stochastic process $\{z_t,\varepsilon_{i,t}\}$, the agents' objective reduces to expression (5) except that the probability of the economy being in a particular regime next period must be replaced by the agents' posterior expectation of that probability.

Here is a numerical example. Suppose that the true value of p is 0.01. In period 0, the agents' prior distribution of p is a beta distribution with parameters $\alpha = 1$ and $\beta = 99$. Note that the agents' prior expectation of p equals the truth, because the prior expectation of p equals $\alpha / (\alpha + \beta) = 0.01$. Let $X_t = 0$ for $t = 1, ..., s-1$, $X_t = 1$ for $t = s$, and $s = 101$. In words, the regime turns out to be normal times one hundred periods in a row and in period 101 the regime turns out to be unusual times.[19] The agents' posterior expectation of p evolves over time as shown in Figure 3. Note that between period 1 and period 100, the agents' posterior expectation of p falls slowly. Just before the rare events occurs, the agents' posterior expectation of p is equal to 0.005. Agents underestimate the probability of the rare event by fifty percent. Consequently, agents think *even less* about the optimal action in the rare event. Next, observe that just after the rare event occurs the agents' posterior expectation of p changes by a large amount. The agents' posterior expectation of p doubles to 0.01. Consequently, the occurrence of the rare event causes a large reallocation of attention toward thinking about what to do in the rare event, because agents now find it much more likely that the rare event will occur again.

3.6 Extension: Information processing after regime realizes

So far we have assumed that agents cannot process additional information about the optimal action in a regime after the regime realizes and before agents take actions. This assumption captures the idea that once the regime realizes agents have to act quickly. Therefore, agents have to plan ahead. We now replace this assumption by a weaker assumption. Suppose agents can process a finite amount of additional information about the optimal action in a regime after the regime realizes. Let $\hat{\kappa} \geq 0$ denote the additional uncertainty reduction that agents can achieve about the optimal action in a regime after the regime realizes. Before taking an action, the posterior variance of the optimal action in the rare event then equals

$$\Omega_{rr}^{\text{tomorrow}} = \Omega_{rr}^{\text{today}} 2^{-2\hat{\kappa}} = \Sigma_{rr} 2^{-2\kappa_r} 2^{-2\hat{\kappa}} = \left(\Sigma_{rr} 2^{-2\hat{\kappa}}\right) 2^{-2\kappa_r},$$

and the posterior variance of the optimal action in normal times equals

$$\Omega_{nn}^{\text{tomorrow}} = \Omega_{nn}^{\text{today}} 2^{-2\hat{\kappa}} = \Sigma_{nn} 2^{-2\kappa_n} 2^{-2\hat{\kappa}} = \left(\Sigma_{nn} 2^{-2\hat{\kappa}}\right) 2^{-2\kappa_n}.$$

[19] The probability that unusual times fail to occur in one hundred Bernoulli trials with $p = 0.01$ equals about 0.36.

Assuming that agents can process additional information about the optimal action in a regime after the regime realizes is isomorphic to reducing the prior variances of the optimal actions by a factor $2^{-2\hat{\kappa}}$. Since the only assumptions we made about the prior variances are: (i) $\Sigma_{rr} > 0$ and $\Sigma_{nn} > 0$, and (ii) $\Sigma_{rr} \geq \Sigma_{nn}$ in Proposition 1, this setup is nested in our earlier analysis.

4 Applications

In this section we use the model to understand the recent events: the Fukushima nuclear accident, the global financial crisis, and the European sovereign debt crisis.

4.1 Fukushima nuclear accident

We define "unusual times" as the regime in which an earthquake and tsunami disable the cooling system of a nuclear power plant.

The 9.0-magnitude earthquake that struck off the coast of Japan on March 11, 2011 cut all off-site power supply to the Fukushima Dai-ichi nuclear power plant, owned and operated by Tokyo Electric Power Company (Tepco). The ensuing tsunami waves knocked out all of the plant's emergency diesel generators apart from one. After the earthquake and tsunami had cut power supply and thereby disabled the cooling system, workers at the plant tried to avoid a catastrophe. The most severe problem was that the fuel rods inside the reactors were overheating, causing a buildup of steam and hydrogen inside the reactor buildings, which meant a possible explosion. After communicating with Tepco officials in Tokyo and the prime minister of Japan, the workers on site decided to vent reactor Unit 1 to reduce pressure. The workers opened the emergency manual and discovered that it did not contain *any* instructions on how to vent the reactor in the absence of electricity. Throughout the night, the workers tried to figure out ad hoc ways to vent the reactor in the absence of electricity.[20] At about 2:30pm on March 12, the operators confirmed a decrease in pressure inside the reactor, providing some indication that the improvised venting was starting to work. Unfortunately, this good news came too late. Shortly thereafter, a hydrogen explosion destroyed the Unit 1 reactor building.[21]

[20] See the program "One year later, inside Japan's nuclear meltdown" that National Public Radio broadcast on February 28, 2012. The program is available at www.npr.org.

[21] See the report by the International Atomic Energy Agency international fact finding expert mission after the

What happened was *not* that the Tepco staff had thought carefully what action to take if the cooling system were disabled and then judged that action to be too costly to implement. Instead, the Tepco staff had not thought about what to do if the cooling system were disabled: The emergency manual contained *no* instructions on how to vent a nuclear reactor in the absence of electricity and therefore the workers on site had to improvise corrective measures. This corner allocation of attention ($\kappa_r = 0$) is consistent with the model if p_r is sufficiently close to zero.

The model provides the following explanation for why Tepco staff were unprepared for the regime "an earthquake and tsunami disable the cooling system of a nuclear reactor." First, this regime was a low-probability event. Second, Tepco officials face limited liability. Third, optimal day-to-day actions in a nuclear power plant are at most weakly correlated with optimal emergency actions. Thinking carefully about how to run a nuclear power plant optimally in normal times fails to improve actions in times when an earthquake and tsunami have disabled the plant's cooling system.[22]

4.2 Global financial crisis

Let us focus on the defining moment of the global financial crisis which came when Lehman Brothers filed for bankruptcy on September 15, 2008. We think of "unusual times" as the regime in which an investment bank like Lehman Brothers has a sizable negative net present value. We add the word "sizable" to make it clear that in this regime Lehman Brothers cannot be rescued with a small amount of public support; a large amount of public support is necessary. "Normal times" is the regime in which Lehman Brothers has a positive net present value, or at worst a negative net present value close to zero.

We consider the actions of U.S. policy-makers. The policy-makers were uncertain about their optimal action in each regime. The possible actions in normal times were "don't intervene" and "orchestrate a sale of Lehman to another financial institution, possibly with a small amount of public support." To take the optimal action in normal times the policy-makers needed to process information about a few potential buyers of Lehman, the price at which a sale would occur, and

Fukushima nuclear accident. The report is available at www.iaea.org.

[22] *Financial Times* in its May 7-8, 2011 issue quotes Goshi Hosono, a senior aide to Japan's prime minister, saying "Tepco's job is to deliver a constant supply of electricity – extremely routine work. It is a company for stable times."

the details of any public support. On the other hand, the possible actions in unusual times were "don't intervene" and "offer a large amount of public support of some form." Crucially, assessing the consequences of the "don't intervene" action is a very different thought process in unusual times than in normal times. In normal times the shutdown of Lehman will not trigger bankruptcies of other financial institutions. By contrast, in unusual times when Lehman has a sizable negative net present value, its bankruptcy is likely to trigger other bankruptcies. Hence to prepare for unusual times the policy-makers had to think about an entire network of closely connected financial institutions. There was uncertainty about the structure of the network ("who owns whom how much") and about how much more losses financial institutions could still absorb.[23]

The policy-makers reduced their uncertainty as they processed information about what to do in each regime. Our reading of the events is that the policy-makers thought carefully about what to do in normal times. In particular, the policy-makers prepared to orchestrate a sale of Lehman Brothers to another financial institution (like Bank of America or Barclays) with a small amount of public support. Importantly, almost the entire meeting of the policy-makers and bankers at the Federal Reserve Bank of New York on the weekend of September 13-14, 2008, was devoted to planning a sale of Lehman Brothers. By contrast, little time during the meeting was spent thinking about what to do if the hole in Lehman's balance sheet was too deep for Lehman to be sold with a small amount of public support. Timothy Geithner, then president of the Federal Reserve Bank of New York, asked one of the working groups formed at the meeting to "put foam on the runway," in case Lehman's sale could not be orchestrated, and "be prepared to do *something*."[24]

As the weekend drew to a close, it turned out that unusual times had occurred: Lehman Brothers had a sizable negative net present value and therefore could not be sold with only a small amount of public support. The policy-makers had thought little about what to do in that regime. They had to decide quickly and chose to take the action "don't intervene." Lehman filed for bankruptcy. Within days, the policy-makers reversed themselves as they offered a large amount of public support to American International Group, money market funds, and so on. We take this policy reversal as an indication that the optimal action on the weekend of September 13-14, 2008, would have been a

[23] For recent models featuring uncertainty about a financial network, see Caballero and Simsek (2013) and Alvarez and Barlevy (2014).

[24] We quote Geithner after Wessel (2009, p.17), with emphasis added. See Wessel (2009) for a description of the meeting at the Federal Reserve Bank of New York on September 13-14, 2008.

different one.

The model proposes the following explanation for why the policy-makers were unprepared for the regime "Lehman Brothers has a sizable negative net present value." First, this regime was a low-probability event (p_r close to zero). Second, the policy-makers faced limited liability because their losses would be bounded in the event of failure ($\omega > 0$). Third, the optimal actions were at most weakly correlated across the two regimes (Σ_{rn} close to zero).[25]

Of course, assessing the quality of policy-makers' actions is difficult. Even today we cannot be certain what the optimal action on the weekend of September 13-14, 2008, was. However, the fact that the policy-makers reversed themselves so dramatically indicates that their initial action was far from optimal.

4.3 European sovereign debt crisis

We focus on what we see as the defining moment of the European sovereign debt crisis which came in April 2010 when the prime minister of Greece asked other euro-area member states for help in resolving Greece's fiscal crisis.[26] We think of "normal times" as the regime in which the government of a euro-area country is solvent but may be illiquid (i.e., may be unable to roll over its maturing debt.) "Unusual times" is the regime in which the government of a euro-area country is insolvent.[27]

Greece entered the post-Lehman era with a large amount of government debt. In October 2009, a new Greek government announced that the fiscal situation was a lot worse than had previously been understood. The immediate problem was that a sizable amount of public debt was due to mature in May 2010. There was uncertainty whether by that time Greece would turn out to be in "normal times" or "unusual times." The possible actions of the euro-area policy-makers in normal times were "don't intervene" and "make Greece a loan." Possible actions in unusual times were likewise "don't intervene" and "make Greece a loan," but possible actions in unusual times also included "give Greece a transfer," "guarantee Greek government debt," and "help Greece organize

[25] Possibly, actions of different policy-makers were also strategic complements ($\gamma > 0$). Any policy decision had to be made by a committee, and therefore each individual policy-maker had an incentive to propose a policy action acceptable to other policy-makers.

[26] See Bastasin (2012), IMF (2013), and Irwin (2013) for a chronology of the fiscal crisis in Greece.

[27] By the government being insolvent we mean that government debt exceeds the present value of primary budget surpluses in the absence of reform and under any politically feasible reform.

an orderly default."

Importantly, preparing for unusual times is a very different activity than preparing for normal times. Preparation for normal times involves figuring out the size and conditions of a loan. On the other hand, figuring out how to make government default orderly is a very different task from designing the conditions of a loan. Figuring out the modalities of an inter-country transfer or an inter-country debt guarantee is also a very different thought process because it would change the way EMU works.

The euro-area policy-makers processed information about what to do in each regime. Far-reaching options such as different forms of a public debt guarantee were on the table. However, our interpretation of the events is that the policy-makers spent most of the time between October 2009 and April 2010 thinking about the size and conditions of any loan to Greece. In particular, much attention was given to figuring out the interest rate on the loan and designing the reform measures Greece would have to promise in order to get the loan. By contrast, we are not aware of any planning for an orderly default by the government of a euro-area country during that period.

In the end, the news coming from Greece between October 2009 and April 2010 turned out to be bad. Greece found itself in unusual times. Nevertheless, in response to the prime minister's request in April 2010, the euro-area policy-makers together with the International Monetary Fund made the Greek government a loan on May 2, 2010. By October 2010, the chancellor of Germany and the president of France decided that government default would have to be an option in the euro area. Preparation for an orderly default by Greece began. In October 2011, a new assistance package for Greece was announced, this time including provisions for default. We take this policy reversal as an indication that the optimal action in the spring of 2010 would have been a different one.

The model proposes the following explanation for why the policy-makers were unprepared for the regime "the government of a euro-area country is insolvent." This regime was a low-probability event. Limited liability and possibly strategic complementarity amplified the asymmetry in the allocation of attention. Once Greece turned out to be insolvent, all the thinking that went into designing the optimal conditionality for a loan to Greece was of little use.

4.4 Low-probability events, *not* unthinkable events

It is sometimes argued that the recent events were unthinkable, zero-probability events. Another popular view is that the probability of each of the recent events was impossible to estimate. The evidence suggests otherwise.

The probability of default by Lehman Brothers and the probability of default by Greece were simple to estimate, at least crudely, based on publicly available data. In both cases, publicly available data suggested that the probability of default was small but strictly greater than zero. Figure 4 plots the probability of default on one-year senior debt of Lehman, based on credit default swap (CDS) premia.[28] Prior to August 9, 2007, the day on which the interbank market froze up, the probability of default by Lehman was 0.002 on average. The probability of Lehman's default between August 9, 2007, and the last day on which the Lehman CDS was traded, September 12, 2008, was 0.03 on average. An event with a probability of 0.03 is a low-probability event but it is not unthinkable. Figure 5 plots the probability of default on one-year government debt of Greece, likewise based on CDS premia.[29] Prior to September 15, 2008, the day of Lehman's bankruptcy, the probability of default by the Greek government was 0.002 on average. The probability of Greek default between September 15, 2008, and the day on which the first assistance package for Greece was agreed, May 2, 2010, was 0.03 on average. The similarity between Figure 5 and Figure 4 is striking.

The probability of the combination of a 9.0-magnitude earthquake and tsunami near Fukushima could not be estimated based on financial market data. However, this earthquake-tsunami combination had a well-known precedent. The so-called Jogan earthquake of 869 knocked down a castle

[28] To produce Figure 4, we took from Bloomberg CDS premia on one-year senior debt of Lehman Brothers, at daily frequency, from the beginning of July 2003 to the last trading day, September 12, 2008. We computed the probability of default, plotted in Figure 4, from this data assuming risk neutrality and a recovery rate equal to 8.625 percent, the actual recovery rate reported in Singh and Spackman (2009). The dataset had occasional missing observations which accounts for the missing values in Figure 4.

[29] To produce Figure 5, we took from Datastream CDS premia on one-year government debt of Greece, at daily frequency, from the beginning of 2004 to May 2, 2010, the day on which the first assistance package for Greece was agreed. We computed the probability of default, plotted in Figure 5, from this data assuming risk neutrality and a recovery rate equal to 21.5 percent, the actual recovery rate in the case of Greece reported by *Financial Times* in its March 20, 2012 issue.

and sent a tsunami wave more than two miles inland in the same region. This fact was brought up in a meeting of a commission evaluating the safety of the Fukushima Dai-ichi nuclear power plant in June 2009. Several Tepco officials attended this meeting.[30] Thus the earthquake-tsunami combination of March 11, 2011, was a low-probability but not an unthinkable event and this was known inside Tepco.

5 The efficient allocation of attention

Should decision-makers think more about optimal actions in unusual times, even if this means that they will think less about optimal actions in normal times? More formally, would society be better off from an ex-ante perspective if agents allocated attention differently than in equilibrium?

To answer this question, we consider the following planner problem. The planner can tell agents how to allocate attention. The planner has to respect the agents' information processing constraint (8).

5.1 Benchmark

As a benchmark, assume that the planner maximizes ex-ante utility of the agents, agents have unlimited liability, and each agent's payoff does not depend on the mean action in the population. Then the planner's problem equals the agents' attention choice problem (12)-(14), and the equilibrium allocation of attention equals the efficient allocation of attention.

5.2 Limited liability

Next, let us introduce limited liability. It seems reasonable to assume that when limited liability kicks in, society still incurs a loss. The planner's problem now differs from the agents' attention choice problem. The planner solves problem (12)-(14), while the agents solve problem (17)-(19).

If (i) the extent of limited liability protection is at least as large in the rare event as in normal times ($\Delta_r \leq \Delta_n$), (ii) agents with unlimited liability would decide to be more uncertain about the optimal action in unusual times than about the optimal action in normal times ($\Omega_{rr}^{UL} > \Omega_{nn}^{UL}$), and (iii) limited liability protection is not too large ($\Delta_r \geq 1.732\sqrt{\Omega_{rr}^{UL}}$), then limited liability reduces

[30] See, for example, the March 23, 2011 issue of *The Washington Post*.

the attention allocated to the rare event:

$$\kappa_r^{UL} \in (0,\kappa) \Rightarrow \kappa_r^{LL} < \kappa_r^{UL}.$$

Hence, if $\kappa_r^{UL} \in (0,\kappa)$, the planner wants agents to think more carefully about the optimal action in the rare event than they do in equilibrium.

5.3 Strategic interactions

In the working paper version of this paper, we also study in detail whether strategic complementarity in actions creates inefficiencies in the allocation of attention. See Maćkowiak and Wiederholt (2011). Here we only summarize these results, because we believe that strategic complementarity in actions was of lesser importance in the examples given in Section 4.

Consider the following setup. Suppose that the equilibrium actions under perfect information equal the welfare-maximizing actions (i.e., inefficiencies, if any, arise due to the agents' information processing constraint). Furthermore, suppose that the conditions of Proposition 2 are satisfied (unlimited liability, Σ diagonal, $\gamma_r = \gamma_n \equiv \gamma$, and $2^\kappa > \frac{\gamma}{1-\gamma}$) and the equilibrium allocation of attention satisfies $0 < \kappa_r < \kappa_n$. In addition, suppose that the planner problem is convex. Under these conditions, the equilibrium allocation of attention equals the efficient allocation of attention if and only if the agents' payoff function satisfies

$$U_{a_i a} + U_{aa} = 0. \tag{23}$$

If the left-hand side is strictly negative, agents think too little about the optimal action in the rare event from an ex-ante perspective. If the left-hand side is strictly positive, the opposite is true. Here U_{aa} denotes the second derivative of the payoff function with respect to the mean action in the population.

The reason for the ambiguous direction of the inefficiency is the following. There are two externalities – a positive and a negative externality. When agents think more carefully about the optimal action in a regime, the mean action in the regime moves more with the fundamental in that regime, which directly increases ex-ante utility. This positive externality is present for both regimes and is stronger for the regime that agents are paying less attention to. Hence, if this positive externality were the only externality, the planner would want agents to think *more* about unusual times. On the other hand, when agents think more carefully about the optimal action in a regime and therefore the mean action in the regime moves more with the fundamental in that regime, the problem of other agents becomes more complicated. This negative externality is present for both regimes and is stronger for the regime that agents are paying less attention to. Hence, if this negative externality were the only externality, the planner would want agents to think *less* about unusual times. When condition (23) holds, the positive externality and the negative externality exactly cancel and the equilibrium allocation of attention equals the efficient allocation of attention.

The condition (23) also appears in Angeletos and Pavan (2007). Angeletos and Pavan (2007) study an economy with a continuum of agents in which each agent observes a noisy private and public signal. The precision of the two signals is exogenous. Due to the quadratic Gaussian structure of the economy, actions are a linear function of the two signals and Angeletos and Pavan (2007) refer to the coefficients on the two signals as the "use of information." They then compare the equilibrium use of information to the efficient use of information, where the latter is defined as the one that maximizes ex-ante utility. For economies that are efficient under perfect information, it turns out that the equilibrium use of information equals the efficient use of information if and only if condition (23) is satisfied. We thus arrive at the following conclusion. The same condition that governs the relationship between the equilibrium use of information and the efficient use of information in the model in Angeletos and Pavan (2007) also governs the relationship between the equilibrium allocation of attention and the efficient allocation of attention in our model with an endogenous signal precision. This result is slightly different from the result in Colombo et al. (2014), because they assume that ex-ante welfare depends also directly negatively on dispersion in actions.

6 Conclusions

We develop a model of state-contingent planning under rational inattention. The key feature of the model is that agents make avoidable mistakes. In other words, when a rare event occurs our agents think: "We wish we had allocated attention differently and prepared more for the rare event. We are going to make mistakes that could have been avoided if we had prepared more." We believe that the same feature was critical in several recent rare events with disastrous consequences.

Furthermore, that agents make avoidable mistakes is a feature specific to a model with information choice like rational inattention; it does not arise in models with perfect information or in models with exogenous imperfect information.

The model helps us understand why people can be unprepared in the real world: the interaction of low probability, limited liability, and strategic complementarity in actions is critical. Furthermore, the model helps us evaluate in which real-world situations the degree of preparation will be inefficient. The model's results can be applied in many contexts. For example, one can think of managers of a financial institution as agents who must allocate attention and one can think of the institution's owners as the social planner. The model suggests that, due to limited liability of the managers, it is a good idea for the owners to mandate that the managers prepare a plan to preserve shareholder value at a time of severe, unlikely stress. Of course, owners of a financial institution themselves face limited liability: they don't care about the consequences of winding down the institution for others. Therefore, as another example one can think of owners of financial institutions as agents who must allocate attention and one can think of the government as the social planner. In this context, the model suggests that it is a good idea for regulators to require that each systemically important financial institution prepare a "living will," a plan for an orderly resolution of that institution in the event of its failure. The Dodd-Frank Act introduced a living will mandate of this kind in the United States.[31]

The model is simple in some dimensions. For instance, in future research one could relax the assumption that the probability of unusual times is independent of actions taken by agents. We think of this assumption as a reasonable approximation because, no matter what humans do, failure of a systemically important financial institution, severe fiscal stress, and a nuclear emergency will probably remain low-but-non-zero-probability events.

[31] Public summaries of the living wills, or resolution plans, can be accessed at the website of the Federal Deposit Insurance Corporation, http://www.fdic.gov/regulations/reform/resplans. Interestingly, the summary by The Goldman Sachs Group, Inc., from July 29, 2012, discusses both a recovery plan (our first example in this paragraph) and a resolution plan (our second example in this paragraph): "Recovery plans focus on the steps that management would take to reduce risk, divest non-core businesses and conserve capital in times of severe stress. In contrast to a recovery plan, a resolution plan is premised on failure. The objective of a resolution plan is to identify and mitigate obstacles to an orderly resolution (...)."

A Proof of Lemma 1

Step 1: The expected benefit from limited liability in regime j equals

$$
\begin{aligned}
B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right) &= \int_{-\infty}^{E[z_j|s_i]-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j \left(E\left[z_j|s_i\right] - z\right)^2\right)\right] f\left(z|s_i\right) dz \\
&\quad + \int_{E[z_j|s_i]+\Delta_j}^{\infty} \left[\omega_j - \left(\bar{u}_j - \delta_j \left(E\left[z_j|s_i\right] - z\right)^2\right)\right] f\left(z|s_i\right) dz \\
&= 2 \int_{-\infty}^{E[z_j|s_i]-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j \left(E\left[z_j|s_i\right] - z\right)^2\right)\right] f\left(z|s_i\right) dz. \qquad (24)
\end{aligned}
$$

The second equality is due to the fact that the density is symmetric around its mean. Standard formulas for the moments of a truncated normal distribution yield

$$
\begin{aligned}
&\int_{-\infty}^{E[z_j|s_i]-\Delta_j} \left[\omega_j - \bar{u}_j + \delta_j \left(E\left[z_j|s_i\right] - z\right)^2\right] f\left(z|s_i\right) dz \\
&= \Phi\left(-\frac{\Delta_j}{\Omega_{jj}}\right) \left[\omega_j - \bar{u}_j + \delta_j \Omega_{jj} \left(1 + \frac{\Delta_j}{\Omega_{jj}} \frac{\phi\left(-\frac{\Delta_j}{\Omega_{jj}}\right)}{\Phi\left(-\frac{\Delta_j}{\Omega_{jj}}\right)}\right)\right],
\end{aligned}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the pdf and cdf of the standard normal distribution. It follows that the expected benefit from limited liability depends only on Ω_{jj}, $\bar{u}_j - \omega_j$, and δ_j. The expected benefit from limited liability does not depend on the conditional mean of the payoff-maximizing action. Hence, without loss in generality, we can set $E\left[z_j|s_i\right] = 0$ when we study the expected benefit from limited liability.

Step 2: We now compute and sign four derivatives of the expected benefit from limited liability in regime j. The first derivative with respect to Ω_{jj} equals

$$
\begin{aligned}
\frac{\partial B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj}} &= 2 \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] \frac{\partial f\left(z|s_i\right)}{\partial \Omega_{jj}} dz \\
&= 2 \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] \left[f\left(z|s_i\right) \left(\frac{z^2}{2\Omega_{jj}^2} - \frac{1}{2\Omega_{jj}}\right)\right] dz. \qquad (25)
\end{aligned}
$$

The cross derivative with respect to Ω_{jj} and ω_j equals

$$\frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj} \partial \omega_j} = 2 \int_{-\infty}^{-\Delta_j} \left[f\left(z|s_i\right) \left(\frac{z^2}{2\Omega_{jj}^2} - \frac{1}{2\Omega_{jj}} \right) \right] dz. \tag{26}$$

The second derivative with respect to Ω_{jj} equals

$$\begin{aligned} \frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial^2 \Omega_{jj}} &= 2 \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] \frac{\partial^2 f\left(z|s_i\right)}{\partial^2 \Omega_{jj}} dz \\ &= 2 \int_{-\infty}^{-\Delta_j} \left[\omega_j - \left(\bar{u}_j - \delta_j z^2\right)\right] \frac{f\left(z|s_i\right)}{\Omega_{jj}^2} \left[\frac{1}{4} \left(\frac{z^2}{\Omega_{jj}} - 1 \right)^2 - \left(\frac{z^2}{\Omega_{jj}} - \frac{1}{2} \right) \right] dz \end{aligned} \tag{27}$$

The derivative with respect to Ω_{jj}, Ω_{jj}, and ω_j equals

$$\frac{\partial^3 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial^2 \Omega_{jj} \partial \omega_j} = 2 \int_{-\infty}^{-\Delta_j} \frac{f\left(z|s_i\right)}{\Omega_{jj}^2} \left[\frac{1}{4} \left(\frac{z^2}{\Omega_{jj}} - 1 \right)^2 - \left(\frac{z^2}{\Omega_{jj}} - \frac{1}{2} \right) \right] dz. \tag{28}$$

Let us start with the cross derivative (26). If $\Delta_j \geq \sqrt{\Omega_{jj}}$, the integral on the right-hand side of equation (26) is strictly positive because the integrand is strictly positive for all $z \in (-\infty, -\Delta_j)$. If $\Delta_j \in \left[0, \sqrt{\Omega_{jj}}\right)$, then

$$\begin{aligned} \frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj} \partial \omega_j} &= 2 \int_{-\infty}^{-\Delta_j} \left[f\left(z|s_i\right) \left(\frac{z^2}{2\Omega_{jj}^2} - \frac{1}{2\Omega_{jj}} \right) \right] dz \\ &\geq 2 \int_{-\infty}^{0} \left[f\left(z|s_i\right) \left(\frac{z^2}{2\Omega_{jj}^2} - \frac{1}{2\Omega_{jj}} \right) \right] dz \\ &= \int_{-\infty}^{\infty} \left[f\left(z|s_i\right) \left(\frac{z^2}{2\Omega_{jj}^2} - \frac{1}{2\Omega_{jj}} \right) \right] dz \\ &= 0. \end{aligned}$$

The weak inequality is a strict inequality if $\Delta_j > 0$ and the weak inequality is an equality if $\Delta_j = 0$. Collecting results yields

$$\frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj} \partial \omega_j} > 0 \text{ for all } \Delta_j > 0, \tag{29}$$

and

$$\frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj} \partial \omega_j} = 0 \text{ if } \Delta_j = 0. \tag{30}$$

Let us turn to the first derivative (25). If $\Delta_j \geq \sqrt{\Omega_{jj}}$, the integral on the right-hand side of equation (25) is strictly positive because the integrand is strictly positive for all $z \in (-\infty, -\Delta_j)$. Furthermore, take any value for Δ_j satisfying $\Delta_j \geq \sqrt{\Omega_{jj}}$ and increase ω_j so as to reduce Δ_j to any value $\Delta_j \geq 0$ (recall that $\Delta_j = \sqrt{\frac{\bar{u}_j - \omega_j}{\delta_j}}$). The results about the cross derivative $\frac{\partial^2 B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)}{\partial \Omega_{jj} \partial \omega_j}$ imply that this increase in ω_j raises the first derivative $\frac{\partial B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)}{\partial \Omega_{jj}}$. It follows that

$$\frac{\partial B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj}} > 0 \text{ for all } \Delta_j \geq 0.$$

It also follows that, for any Ω_{jj} and δ_j, the derivative $\frac{\partial B(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j)}{\partial \Omega_{jj}}$ is maximized at $\bar{u}_j - \omega_j = 0$. Furthermore, equation (25) implies

$$\frac{\partial B\left(\Omega_{jj}, 0, \delta_j\right)}{\partial \Omega_{jj}} = \delta_j.$$

Collecting results yields

$$\frac{\partial B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj}} \in (0, \delta_j) \text{ for all } \Delta_j > 0, \tag{31}$$

and

$$\frac{\partial B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial \Omega_{jj}} = \delta_j \text{ if } \Delta_j = 0. \tag{32}$$

Next, consider the second derivative (27). The term

$$\frac{\partial^2 f\left(z|s_i\right)}{\partial^2 \Omega_{jj}} = \frac{f\left(z|s_i\right)}{\Omega_{jj}^2}\left[\frac{1}{4}\left(\frac{z^2}{\Omega_{jj}} - 1\right)^2 - \left(\frac{z^2}{\Omega_{jj}} - \frac{1}{2}\right)\right] \tag{33}$$

has the following properties. The term equals zero for two values of $\left(z^2/\Omega_{jj}\right)$: $3+\sqrt{6}$ and $3-\sqrt{6}$. Furthermore, if $\left(z^2/\Omega_{jj}\right) \notin \left[3-\sqrt{6}, 3+\sqrt{6}\right]$, the term (33) is strictly positive. If $\left(z^2/\Omega_{jj}\right) \in \left(3-\sqrt{6}, 3+\sqrt{6}\right)$, the term (33) is strictly negative. We arrive at the following conclusion. If $\Delta_j \geq \sqrt{3+\sqrt{6}}\sqrt{\Omega_{jj}}$, the integral on the right-hand side of equation (27) is strictly positive because the integrand is strictly positive for all $z \in (-\infty, -\Delta_j)$. Thus

$$\frac{\partial^2 B\left(\Omega_{jj}, \bar{u}_j - \omega_j, \delta_j\right)}{\partial^2 \Omega_{jj}} > 0 \text{ for all } \Delta_j \geq \sqrt{3+\sqrt{6}}\sqrt{\Omega_{jj}}. \tag{34}$$

Finally, let us turn to the cross derivative (28). We already showed that the integrand on the right-hand side of equation (28) has the following properties: it equals zero if $\left(z^2/\Omega_{jj}\right) \in \left\{3-\sqrt{6}, 3+\sqrt{6}\right\}$, it is strictly positive if $\left(z^2/\Omega_{jj}\right) \notin \left[3-\sqrt{6}, 3+\sqrt{6}\right]$, and it is strictly negative if $\left(z^2/\Omega_{jj}\right) \in$

$\left(3-\sqrt{6},3+\sqrt{6}\right)$. Furthermore, the integral (28) equals zero if $\Delta_j = 0$, because the fourth central moment of a normal distribution equals three times the squared variance. We arrive at the following conclusion. There exists a unique threshold value $\bar{\Delta} > 0$ with the property

$$\left.\frac{\partial^3 B\left(\Omega_{jj},\bar{u}_j-\omega_j,\delta_j\right)}{\partial^2\Omega_{jj}\partial\omega_j}\right|_{\Delta_j=\bar{\Delta}} = 2\int_{-\infty}^{-\bar{\Delta}}\frac{f\left(z|s_i\right)}{\Omega_{jj}^2}\left[\frac{1}{4}\left(\frac{z^2}{\Omega_{jj}}-1\right)^2-\left(\frac{z^2}{\Omega_{jj}}-\frac{1}{2}\right)\right]dz = 0. \tag{35}$$

Furthermore, $\bar{\Delta} \in \left(\sqrt{3-\sqrt{6}}\sqrt{\Omega_{jj}},\sqrt{3+\sqrt{6}}\sqrt{\Omega_{jj}}\right)$ and the integral (28) is strictly positive if $\Delta_j > \bar{\Delta}$ while the integral (28) is strictly negative if $\Delta_j \in \left(0,\bar{\Delta}\right)$. This threshold value $\bar{\Delta}$ is linear in $\sqrt{\Omega_{jj}}$ and numerical integration yields that $\bar{\Delta} = 1.732\sqrt{\Omega_{jj}}$.

B Proof of Proposition 1

Step 1: The attention choice problem with limited liability reads

$$\max_{\kappa_r\in[0,\kappa]} g\left(\kappa_r,\theta\right), \tag{36}$$

where κ_r is the choice variable, θ is a vector of parameters, and $g\left(\kappa_r,\theta\right)$ is the objective function:

$$\begin{aligned} g\left(\kappa_r,\theta\right) &= p_r\left[\bar{u}_r-\delta_r\Sigma_{rr}2^{-2\kappa_r}+B\left(\Sigma_{rr}2^{-2\kappa_r},\bar{u}_r-\omega_r,\delta_r\right)\right] \\ &\quad +p_n\left[\bar{u}_n-\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}+B\left(\Sigma_{nn}2^{-2(\kappa-\kappa_r)},\bar{u}_n-\omega_n,\delta_n\right)\right]. \end{aligned}$$

For comparison, the attention choice problem with unlimited liability reads

$$\max_{\kappa_r\in[0,\kappa]} h\left(\kappa_r,\theta\right), \tag{37}$$

where

$$h\left(\kappa_r,\theta\right) = p_r\left[\bar{u}_r-\delta_r\Sigma_{rr}2^{-2\kappa_r}\right]+p_n\left[\bar{u}_n-\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}\right].$$

The only difference is that the expected benefit from limited liability in the two regimes (i.e., the term $B\left(\Sigma_{jj}2^{-2\kappa_j},\bar{u}_j-\omega_j,\delta_j\right)$ with $j=r,n$) only appears in the first objective function.

Step 2: Let us first study the attention choice problem with unlimited liability. The objective function $h:\mathbb{R}\times\mathbb{R}^{11}\to\mathbb{R}$ is twice continuously differentiable and strictly concave in its first argument. Furthermore, the set $[0,\kappa]$ is compact. Hence, the maximization problem has a unique solution and the solution is given by

$$\begin{array}{ll} \kappa_r^{UL}=0 & \text{if } \left.\frac{\partial h(\kappa_r,\theta)}{\partial\kappa_r}\right|_{\kappa_r=0}\leq 0, \\ \kappa_r^{UL}=\kappa & \text{if } \left.\frac{\partial h(\kappa_r,\theta)}{\partial\kappa_r}\right|_{\kappa_r=\kappa}\geq 0, \\ \left.\frac{\partial h(\kappa_r,\theta)}{\partial\kappa_r}\right|_{\kappa_r=\kappa_r^{UL}}=0 & \text{otherwise.} \end{array}$$

The partial derivative of the objective function with respect to κ_r equals

$$\frac{\partial h\left(\kappa_r,\theta\right)}{\partial\kappa_r}=p_r\delta_r\Sigma_{rr}2^{-2\kappa_r}2\ln\left(2\right)-p_n\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}2\ln\left(2\right).$$

Combining results yields

$$\kappa_r^{UL}=\begin{cases} 0 & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\leq 2^{-\kappa} \\ \frac{1}{2}\kappa+\frac{1}{2}\log_2\left(\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\right) & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\in\left(2^{-\kappa},2^{\kappa}\right) \\ \kappa & \text{if } \sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\geq 2^{\kappa} \end{cases}.$$

Step 3: Let us turn to the attention choice problem with limited liability. The objective function $g:\mathbb{R}\times\mathbb{R}^{11}\rightarrow\mathbb{R}$ is twice continuously differentiable in its first argument and the set $[0,\kappa]$ is compact. Hence, the maximization problem has a solution. The partial derivative of the objective function with respect to κ_r equals

$$\begin{aligned} \frac{\partial g\left(\kappa_r,\theta\right)}{\partial\kappa_r} &= p_r\delta_r\Sigma_{rr}2^{-2\kappa_r}2\ln\left(2\right)\left[1-\frac{\frac{\partial B\left(\Sigma_{rr}2^{-2\kappa_r},\bar{u}_r-\omega_r,\delta_r\right)}{\partial\Sigma_{rr}2^{-2\kappa_r}}}{\delta_r}\right] \\ &\quad -p_n\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}2\ln\left(2\right)\left[1-\frac{\frac{\partial B\left(\Sigma_{nn}2^{-2(\kappa-\kappa_r)},\bar{u}_n-\omega_n,\delta_n\right)}{\partial\Sigma_{nn}2^{-2(\kappa-\kappa_r)}}}{\delta_n}\right]. \end{aligned} \tag{38}$$

First, consider the case $\kappa_r^{UL}\in(0,\kappa)$. In this case, we have

$$\begin{array}{l} \frac{\partial h(\kappa_r,\theta)}{\partial\kappa_r}=0 \quad \text{if } \kappa_r=\kappa_r^{UL} \\ \frac{\partial h(\kappa_r,\theta)}{\partial\kappa_r}<0 \quad \text{if } \kappa_r>\kappa_r^{UL} \end{array},$$

which implies

$$\begin{array}{ll} p_r\delta_r\Sigma_{rr}2^{-2\kappa_r}2\ln\left(2\right)=p_n\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}2\ln\left(2\right) & \text{if } \kappa_r=\kappa_r^{UL} \\ p_r\delta_r\Sigma_{rr}2^{-2\kappa_r}2\ln\left(2\right)<p_n\delta_n\Sigma_{nn}2^{-2(\kappa-\kappa_r)}2\ln\left(2\right) & \text{if } \kappa_r>\kappa_r^{UL} \end{array}. \tag{39}$$

Furthermore, let $\kappa_r^{equality}\in\mathbb{R}_+$ denote the attention allocation at which the posterior uncertainty about the optimal action in the rare event equals the posterior uncertainty about the optimal action in normal times, that is,

$$\Sigma_{rr}2^{-2\kappa_r^{equality}}=\Sigma_{nn}2^{-2\left(\kappa-\kappa_r^{equality}\right)},$$

or equivalently

$$\kappa_r^{equality} = \frac{1}{2}\left[\kappa + \log_2\left(\sqrt{\frac{\Sigma_{rr}}{\Sigma_{nn}}}\right)\right].$$

The assumption $\delta_r = \delta_n$ implies $\kappa_r^{UL} < \kappa_r^{equality}$. The assumptions $\bar{u}_r - \omega_r = \bar{u}_n - \omega_n$, $\delta_r = \delta_n$, and $\Delta_r \geq 1.732\sqrt{\Omega_{rr}^{UL}}$ imply that

$$\begin{array}{lll} \frac{\frac{\partial B\left(\Sigma_{rr}2^{-2\kappa_r},\bar{u}_r-\omega_r,\delta_r\right)}{\partial \Sigma_{rr}2^{-2\kappa_r}}}{\delta_r} > \frac{\frac{\partial B\left(\Sigma_{nn}2^{-2(\kappa-\kappa_r)},\bar{u}_n-\omega_n,\delta_n\right)}{\partial \Sigma_{nn}2^{-2(\kappa-\kappa_r)}}}{\delta_n} & \text{if } \kappa_r \in \left[\kappa_r^{UL}, \kappa_r^{equality}\right) \\ \frac{\frac{\partial B\left(\Sigma_{rr}2^{-2\kappa_r},\bar{u}_r-\omega_r,\delta_r\right)}{\partial \Sigma_{rr}2^{-2\kappa_r}}}{\delta_r} = \frac{\frac{\partial B\left(\Sigma_{nn}2^{-2(\kappa-\kappa_r)},\bar{u}_n-\omega_n,\delta_n\right)}{\partial \Sigma_{nn}2^{-2(\kappa-\kappa_r)}}}{\delta_n} & \text{if } \kappa_r = \kappa_r^{equality} \end{array}. \tag{40}$$

Namely, the assumption $\Delta_r \geq 1.732\sqrt{\Omega_{rr}^{UL}}$ implies that the function B is strictly convex in its first argument on $\left(0, \Omega_{rr}^{UL}\right]$, and we have $\Sigma_{nn}2^{-2(\kappa-\kappa_r)} \leq \Sigma_{rr}2^{-2\kappa_r} \leq \Omega_{rr}^{UL}$ for all $\kappa_r \in \left[\kappa_r^{UL}, \kappa_r^{equality}\right]$. Combining results (38)-(40) yields

$$\forall \kappa_r \in \left[\kappa_r^{UL}, \kappa_r^{equality}\right] : \frac{\partial g\left(\kappa_r, \theta\right)}{\partial \kappa_r} < 0. \tag{41}$$

Hence, any $\kappa_r \in \left[\kappa_r^{UL}, \kappa_r^{equality}\right]$ cannot be a solution to the attention choice problem (36). Next, we show that any $\kappa_r > \kappa_r^{equality}$ cannot be a solution to the attention choice problem with limited liability. If $\kappa_r^{equality} \geq \kappa$, this result follows from the fact that κ_r cannot exceed κ. If $\kappa_r^{equality} < \kappa$, this result follows from the following argument. Let Ω denote the posterior uncertainty about the optimal action in the two regimes at $\kappa_r = \kappa_r^{equality}$

$$\Omega = \Sigma_{rr}2^{-2\kappa_r^{equality}} = \Sigma_{nn}2^{-2\left(\kappa-\kappa_r^{equality}\right)}.$$

One can express the posterior uncertainty in the two regimes as

$$\Omega_{rr} = \Omega 2^{-2\left(\kappa_r-\kappa_r^{equality}\right)},$$

and

$$\Omega_{nn} = \Omega 2^{2\left(\kappa_r-\kappa_r^{equality}\right)}.$$

For all $\kappa_r > \kappa_r^{equality}$, we have $\Omega_{rr} < \Omega_{nn}$ and one can swap the value of Ω_{rr} and the value of Ω_{nn} by changing the sign of $\kappa_r - \kappa_r^{equality}$. Changing the sign of $\kappa_r - \kappa_r^{equality}$ without violating $\kappa_r \in [0, \kappa]$ is always feasible because the assumption $\Sigma_{rr} \geq \Sigma_{nn}$ implies $\kappa_r^{equality} \geq \frac{1}{2}\kappa$. Furthermore, the objective function under limited liability can be written as

$$\begin{aligned} g\left(\kappa_r, \theta\right) \quad = \quad & p_r\bar{u}_r - p_r\left[\delta_r\Omega_{rr} - B\left(\Omega_{rr}, \bar{u}_r - \omega_r, \delta_r\right)\right] \\ & + p_n\bar{u}_n - p_n\left[\delta_n\Omega_{nn} - B\left(\Omega_{nn}, \bar{u}_n - \omega_n, \delta_n\right)\right]. \end{aligned}$$

The first square bracket on the right-hand side is the expected loss due to suboptimal action in the rare event. This expected loss is strictly positive and strictly increasing in Ω_{rr}. See Lemma 1. The second square bracket on the right-hand side is the expected loss due to suboptimal action in normal times. This expected loss is strictly positive and strictly increasing in Ω_{nn}. Recall that for all $\kappa_r > \kappa_r^{equality}$ we have $\Omega_{rr} < \Omega_{nn}$ and swaping the values of Ω_{rr} and Ω_{nn} is feasible. Note that swaping yields a higher value of the objective because $p_r < p_n$, $\bar{u}_r - \omega_r = \bar{u}_n - \omega_n$, and $\delta_r = \delta_n$. Hence, any $\kappa_r > \kappa_r^{equality}$ cannot be a solution to the attention choice problem (36). Combining results we arrive at the conclusion stated in Proposition 1: If $\kappa_r^{UL} \in (0, \kappa)$, every solution to the attention choice problem with limited liability satisfies $\kappa_r < \kappa_r^{UL}$.

Second, consider the case $\kappa_r^{UL} = 0$. In this case, the unique solution to the attention choice problem with limited liability is $\kappa_r = 0$. The arguments are almost identical to the arguments in the case of $\kappa_r^{UL} \in (0, \kappa)$. There are two differences. The first difference is that result (39) is replaced by

$$\begin{array}{ll} p_r \delta_r \Sigma_{rr} 2^{-2\kappa_r} 2 \ln(2) \leq p_n \delta_n \Sigma_{nn} 2^{-2(\kappa - \kappa_r)} 2 \ln(2) & \text{if } \kappa_r = \kappa_r^{UL} \\ p_r \delta_r \Sigma_{rr} 2^{-2\kappa_r} 2 \ln(2) < p_n \delta_n \Sigma_{nn} 2^{-2(\kappa - \kappa_r)} 2 \ln(2) & \text{if } \kappa_r > \kappa_r^{UL} \end{array}.$$

The second difference is that result (41) and the result that any $\kappa_r > \kappa_r^{equality}$ cannot be a solution to the attention choice problem with limited liability now imply that the unique solution to the attention choice problem with limited liability is $\kappa_r = 0$.

Third, consider the case $\kappa_r^{UL} = \kappa$. In this case, the fact that $\kappa_r \in [0, \kappa]$ implies that every solution to the attention choice problem with limited liability satisfies $\kappa_r \leq \kappa_r^{UL}$.

C Proof of Proposition 2

Step 1: We consider equilibria where the mean action in the population in a regime is a linear function of the fundamental in the regime:

$$a_j = \phi_j z_j, \tag{42}$$

where $\phi_r \in \mathbb{R}$ and $\phi_n \in \mathbb{R}$ are undetermined coefficients that we need to solve for.

Step 2: The attention choice problem (5)-(8) can now be stated as follows. Substituting $V^j(a_{i,j}, a_j, z_j) = U^j(a_{i,j}, a_j, z_j)$ as well as equations (3), (4), and (6) into objective (5), using equation (42) to substitute for a_j in the objective, and deducting from the objective a constant that the agent cannot affect yields

$$\max_{\Lambda}\left(-\sum_{j=r,n}p_j\delta_j\left(\gamma_j\phi_j+1-\gamma_j\right)^2\Omega_{jj}\right), \tag{43}$$

subject to

$$\Omega=\Sigma-\Sigma\left(\Sigma+\Lambda\right)^{-1}\Sigma, \tag{44}$$

$$\frac{1}{2}\log_2\left(\frac{|\Sigma|}{|\Omega|}\right)\leq\kappa, \tag{45}$$

and the restriction that Λ is a positive semidefinite matrix. Furthermore, using the formula for the determinant of a two-by-two matrix, the information flow constraint (45) can be written as

$$\frac{1}{2}\log_2\left(\frac{\Sigma_{rr}\Sigma_{nn}-\Sigma_{rn}^2}{\Omega_{rr}\Omega_{nn}-\Omega_{rn}^2}\right)\leq\kappa. \tag{46}$$

Step 3: When the optimal action in unusual times and the optimal action in normal times are independent ($\Sigma_{rn}=0$), it is optimal to receive independent signals about the optimal action in unusual times and the optimal action in normal times ($\Lambda_{rn}=0$). The proof is as follows. First, the information flow constraint (46) is always binding. Second, increasing Ω_{rn}^2 for a given Ω_{rr} and Ω_{nn} raises the information flow on the left-hand side of constraint (46) without improving objective (43). Third, when $\Sigma_{rn}=0$ then $\Omega_{rn}=0$ if and only if $\Lambda_{rn}=0$. Next, using $\Sigma_{rn}=\Lambda_{rn}=\Omega_{rn}=0$ and the definition $\kappa_j\equiv\frac{1}{2}\log_2\left(\frac{\Sigma_{jj}}{\Omega_{jj}}\right)$ the attention choice problem (43)-(45) can be stated as

$$\max_{(\kappa_r,\kappa_n)\in\mathbb{R}_+^2}\left(-\sum_{j=r,n}p_j\delta_j\left(\gamma_j\phi_j+1-\gamma_j\right)^2\Omega_{jj}\right),$$

subject to

$$\Omega_{jj}=\Sigma_{jj}2^{-2\kappa_j},$$

and

$$\kappa_r+\kappa_n\leq\kappa.$$

The unique solution to this decision problem is

$$\kappa_r=\begin{cases}0 & \text{if } x\leq 2^{-\kappa}\\ \frac{1}{2}\left[\kappa+\log_2\left(x\right)\right] & \text{if } x\in\left[2^{-\kappa},2^{\kappa}\right]\\ \kappa & \text{if } x\geq 2^{\kappa}\end{cases}, \tag{47}$$

with

$$x \equiv \sqrt{\frac{p_r \delta_r \left(\gamma_r \phi_r + 1 - \gamma_r\right)^2 \Sigma_{rr}}{p_n \delta_n \left(\gamma_n \phi_n + 1 - \gamma_n\right)^2 \Sigma_{nn}}}; \tag{48}$$

and

$$\kappa_n = \kappa - \kappa_r. \tag{49}$$

Step 4: Equations (47)-(49) give the optimal allocation of attention as a function of the parameters of the model and the undetermined coefficients ϕ_r and ϕ_n. The next step is to solve for the undetermined coefficients ϕ_r and ϕ_n as a function of the optimal allocation of attention. Combining results one then obtains the equilibrium of the model. The actions by agent i are given by equation (6). Substituting $V^j(a_{i,j}, a_j, z_j) = U^j(a_{i,j}, a_j, z_j)$, equations (3)-(4), and the guess (42) into equation (6) yields

$$a_{i,j} = \left(\gamma_j \phi_j + 1 - \gamma_j\right) E\left[z_j | s_i\right].$$

Calculating first the conditional expectation in the last equation and then the mean action in the population yields

$$a_j = \left(\gamma_j \phi_j + 1 - \gamma_j\right)\left(1 - 2^{-2\kappa_j}\right) z_j.$$

Assume $\gamma_j \in (-1, 1)$ for $j = r, n$. The last equation implies that for a given allocation of attention (i.e., for a given $(\kappa_r, \kappa_n) \in \mathbb{R}_+^2$) the guess $a_j = \phi_j z_j$ is correct if and only if

$$\phi_j = \frac{\left(1 - \gamma_j\right)\left(1 - 2^{-2\kappa_j}\right)}{1 - \gamma_j\left(1 - 2^{-2\kappa_j}\right)}. \tag{50}$$

The last equation gives the undetermined coefficients $(\phi_r, \phi_n) \in \mathbb{R}_+^2$ as a function of the allocation of attention $(\kappa_r, \kappa_n) \in \mathbb{R}_+^2$ and the parameters γ_r and γ_n.

Step 5: An equilibrium allocation of attention is a $(\kappa_r, \kappa_n) \in \mathbb{R}_+^2$ satisfying equations (47)-(49), where $(\phi_r, \phi_n) \in \mathbb{R}_+^2$ is given by equation (50). Using equation (50) to substitute for ϕ_r and ϕ_n in equation (48) yields

$$x = \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{\frac{1-\gamma_r}{1-\gamma_r\left(1-2^{-2\kappa_r}\right)}}{\frac{1-\gamma_n}{1-\gamma_n\left(1-2^{-2\kappa_n}\right)}}. \tag{51}$$

An equilibrium allocation of attention is a $(\kappa_r, \kappa_n) \in \mathbb{R}_+^2$ satisfying equations (47), (49), and (51). It is useful to distinguish three types of equilibria: (i) the equilibrium allocation of attention has the property $\kappa_r = 0$, (ii) the equilibrium allocation of attention has the property $\kappa_r = \kappa$, and (iii) the equilibrium allocation of attention has the property $\kappa_r = \frac{1}{2}\left[\kappa + \log_2(x)\right]$.

First, consider an equilibrium with the property $\kappa_r = 0$. Substituting $\kappa_r = 0$ and $\kappa_n = \kappa$ into equation (51) yields

$$x = \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{1-\gamma_r}{1-\gamma_n} \left[1 - \gamma_n \left(1 - 2^{-2\kappa}\right)\right].$$

It follows from the last equation and equation (47) that $\kappa_r = 0$ is an equilibrium if and only if

$$\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{1-\gamma_r}{1-\gamma_n} \left[1 - \gamma_n \left(1 - 2^{-2\kappa}\right)\right] \leq 2^{-\kappa}.$$

This condition can be stated as

$$\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \leq \frac{1}{(1-\gamma_r)\left[2^{\kappa} + \frac{\gamma_n}{1-\gamma_n} 2^{-\kappa}\right]}. \tag{52}$$

Second, consider an equilibrium with the property $\kappa_r = \kappa$. Substituting $\kappa_r = \kappa$ and $\kappa_n = 0$ into equation (51) yields

$$x = \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{1-\gamma_r}{1-\gamma_n} \frac{1}{1-\gamma_r\left(1-2^{-2\kappa}\right)}.$$

It follows from the last equation and equation (47) that $\kappa_r = \kappa$ is an equilibrium if and only if

$$\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{1-\gamma_r}{1-\gamma_n} \frac{1}{1-\gamma_r\left(1-2^{-2\kappa}\right)} \geq 2^{\kappa}.$$

This condition can be stated as

$$\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \geq (1-\gamma_n)\left[2^{\kappa} + \frac{\gamma_r}{1-\gamma_r} 2^{-\kappa}\right]. \tag{53}$$

Third, turn to an equilibrium with the property $\kappa_r = \frac{1}{2}\left[\kappa + \log_2(x)\right]$. Substituting $\kappa_r = \frac{1}{2}\left[\kappa + \log_2(x)\right]$ and $\kappa_n = \kappa - \kappa_r$ into equation (51) yields

$$x = \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{\frac{1-\gamma_r}{1-\gamma_r\left(1-2^{-\kappa}\frac{1}{x}\right)}}{\frac{1-\gamma_n}{1-\gamma_n\left(1-2^{-\kappa}x\right)}}.$$

Rearranging the last equation yields

$$\left[1 - \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{\gamma_n}{1-\gamma_n} 2^{-\kappa}\right] x = \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} - \frac{\gamma_r}{1-\gamma_r} 2^{-\kappa}. \tag{54}$$

If $\left[1 - \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{\gamma_n}{1-\gamma_n} 2^{-\kappa}\right] \neq 0$, the unique solution to the last equation is

$$x = \frac{\sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} - \frac{\gamma_r}{1-\gamma_r} 2^{-\kappa}}{1 - \sqrt{\frac{p_r \delta_r \Sigma_{rr}}{p_n \delta_n \Sigma_{nn}}} \frac{\gamma_n}{1-\gamma_n} 2^{-\kappa}}. \tag{55}$$

Thus, when $\left[1-\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}\right]\neq 0$, it follows from the last equation and equation (47) that $\kappa_r=\frac{1}{2}\left[\kappa+\log_2(x)\right]$ is an equilibrium if and only if

$$\frac{\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}-\frac{\gamma_r}{1-\gamma_r}2^{-\kappa}}{1-\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}}\in\left[2^{-\kappa},2^{\kappa}\right]. \tag{56}$$

Furthermore, when

$$\left[1-\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}\right]>0, \tag{57}$$

condition (56) is equivalent to

$$\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\in\left[\frac{\frac{1}{1-\gamma_r}}{2^{\kappa}+\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}},\frac{2^{\kappa}+\frac{\gamma_r}{1-\gamma_r}2^{-\kappa}}{\frac{1}{1-\gamma_n}}\right]. \tag{58}$$

When

$$\left[1-\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}\right]<0, \tag{59}$$

condition (56) is equivalent to

$$\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\in\left[\frac{2^{\kappa}+\frac{\gamma_r}{1-\gamma_r}2^{-\kappa}}{\frac{1}{1-\gamma_n}},\frac{\frac{1}{1-\gamma_r}}{2^{\kappa}+\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}}\right]. \tag{60}$$

Finally, if

$$\left[1-\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}\frac{\gamma_n}{1-\gamma_n}2^{-\kappa}\right]=0, \tag{61}$$

equation (54) reduces to

$$\sqrt{\frac{p_r\delta_r\Sigma_{rr}}{p_n\delta_n\Sigma_{nn}}}=\frac{\gamma_r}{1-\gamma_r}2^{-\kappa}. \tag{62}$$

In summary, if conditions (57)-(58) or conditions (59)-(60) hold, a unique equilibrium with the property $\kappa_r=\frac{1}{2}\left[\kappa+\log_2(x)\right]$ exists and in this equilibrium x is given by equation (55). If conditions (61)-(62) hold, a continuum of equilibria with the property $\kappa_r=\frac{1}{2}\left[\kappa+\log_2(x)\right]$ exist; namely any $\kappa_r\in[0,\kappa]$ is such an equilibrium.

This completes the characterization of equilibria of the form $a_j=\phi_j z_j$. In the special case of $\gamma_r=\gamma_n\equiv\gamma$, conditions (52), (53), (57)-(58), (59)-(60), and (61)-(62) and equation (55) reduce to the conditions and equation given in Proposition 2.

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Figure 1: Equilibrium attention to the rare event

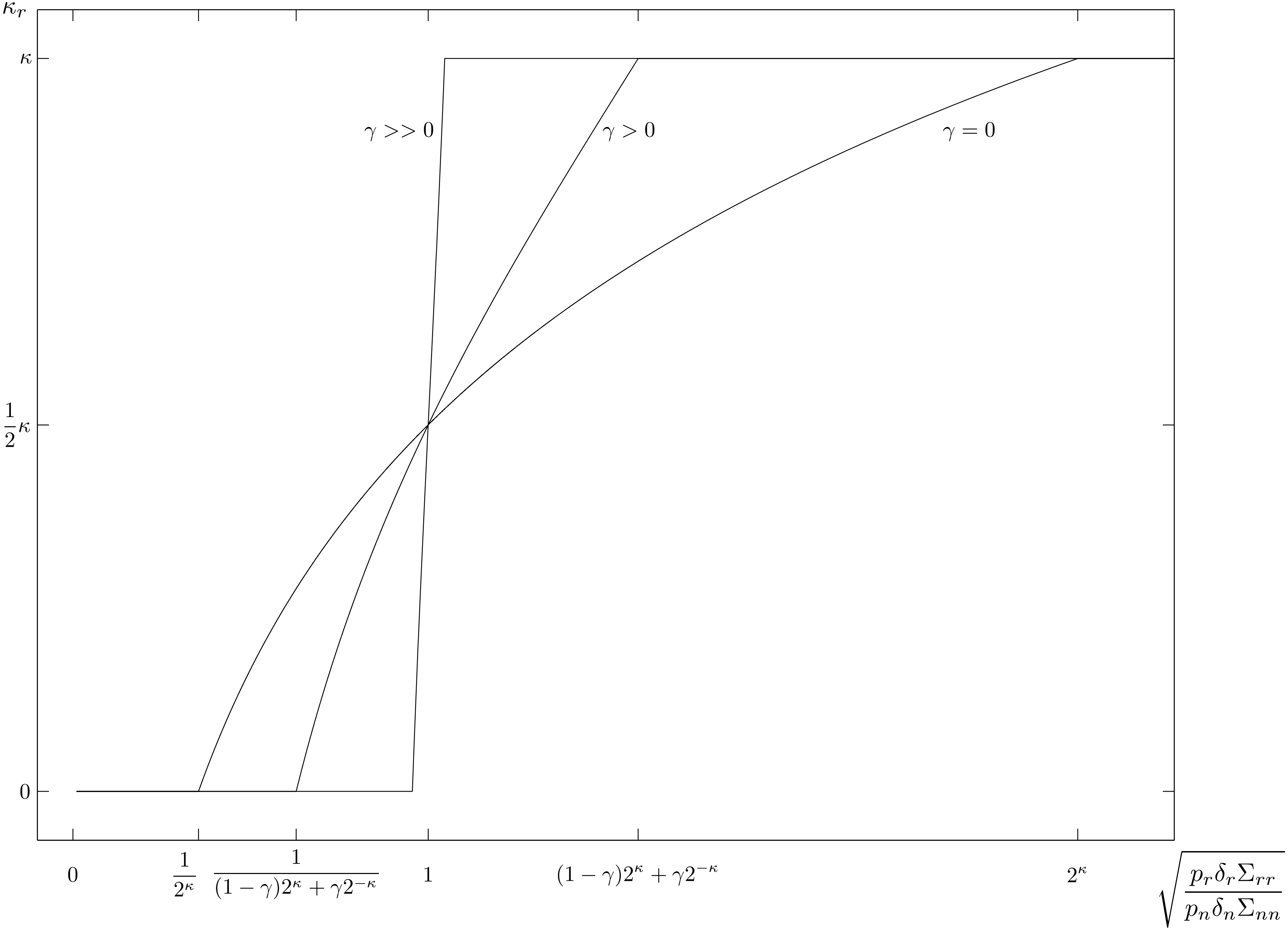
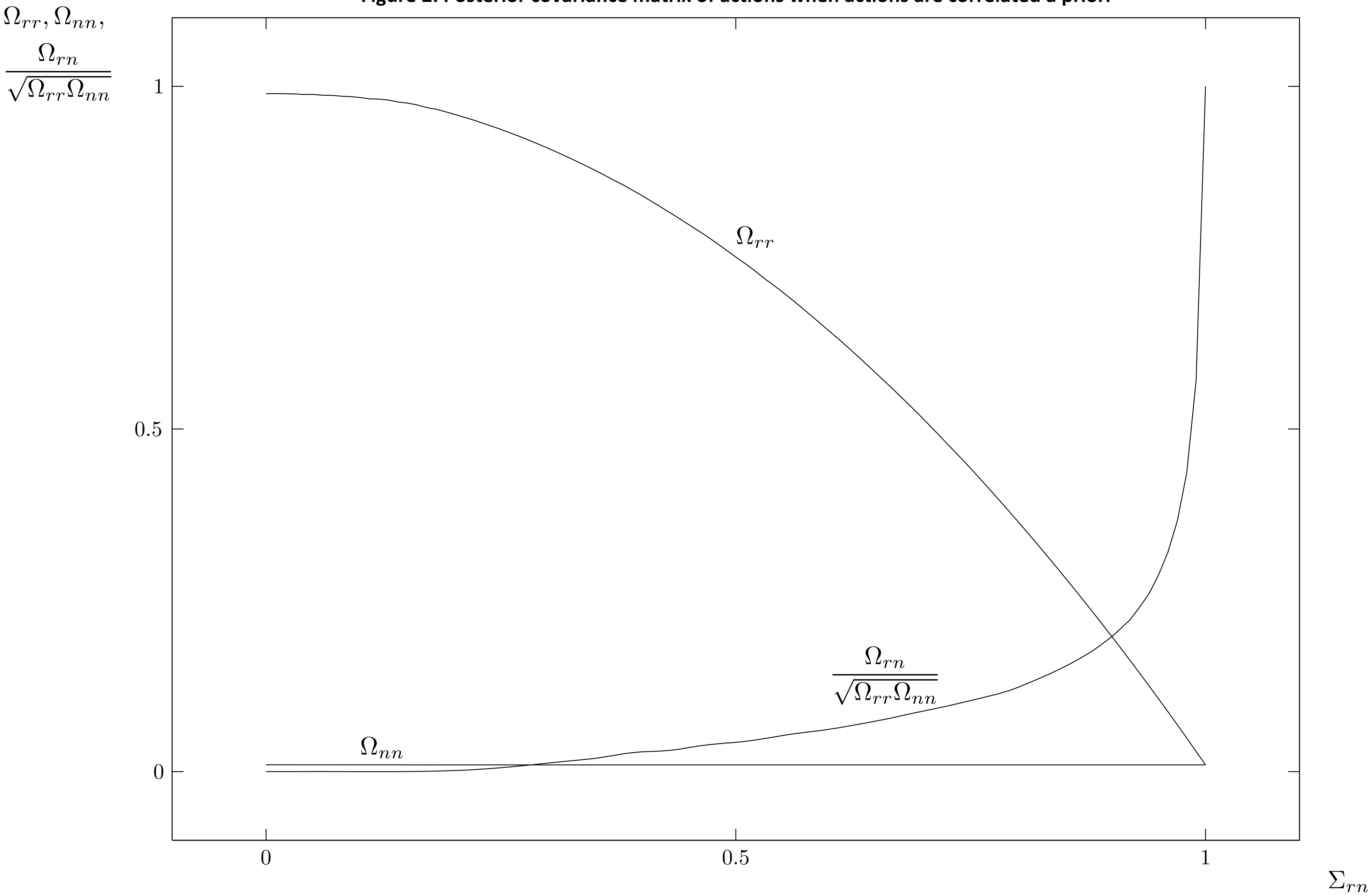


Figure 2: Posterior covariance matrix of actions when actions are correlated a priori



Note: This figure assumes $\Sigma_{rr} = \Sigma_{nn} = 1$ so that Σ_{rn} is the prior correlation of actions.

Figure 3: Posterior expectation of the probability of unusual times

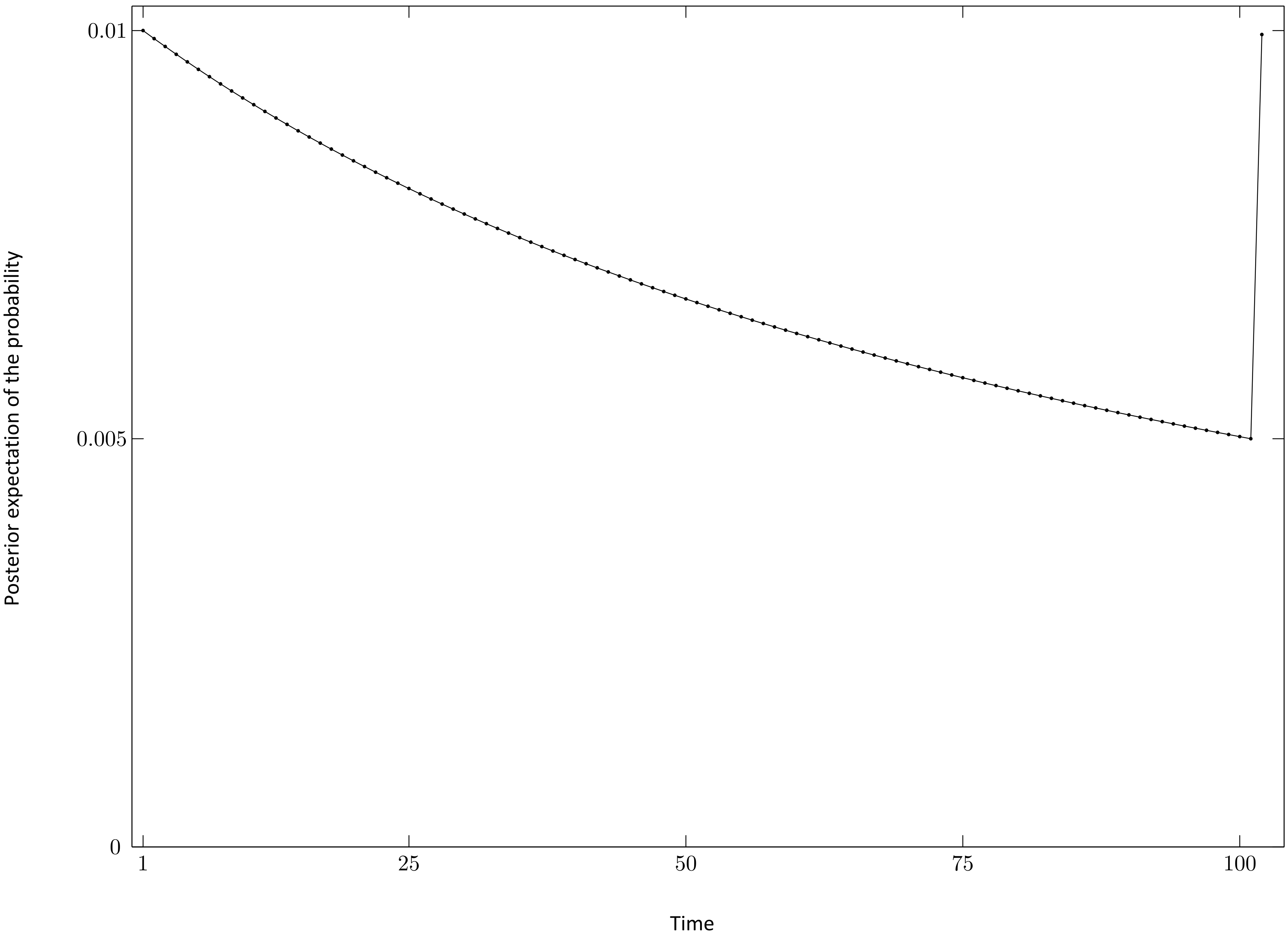


Figure 4: Probability of default, Lehman Brothers

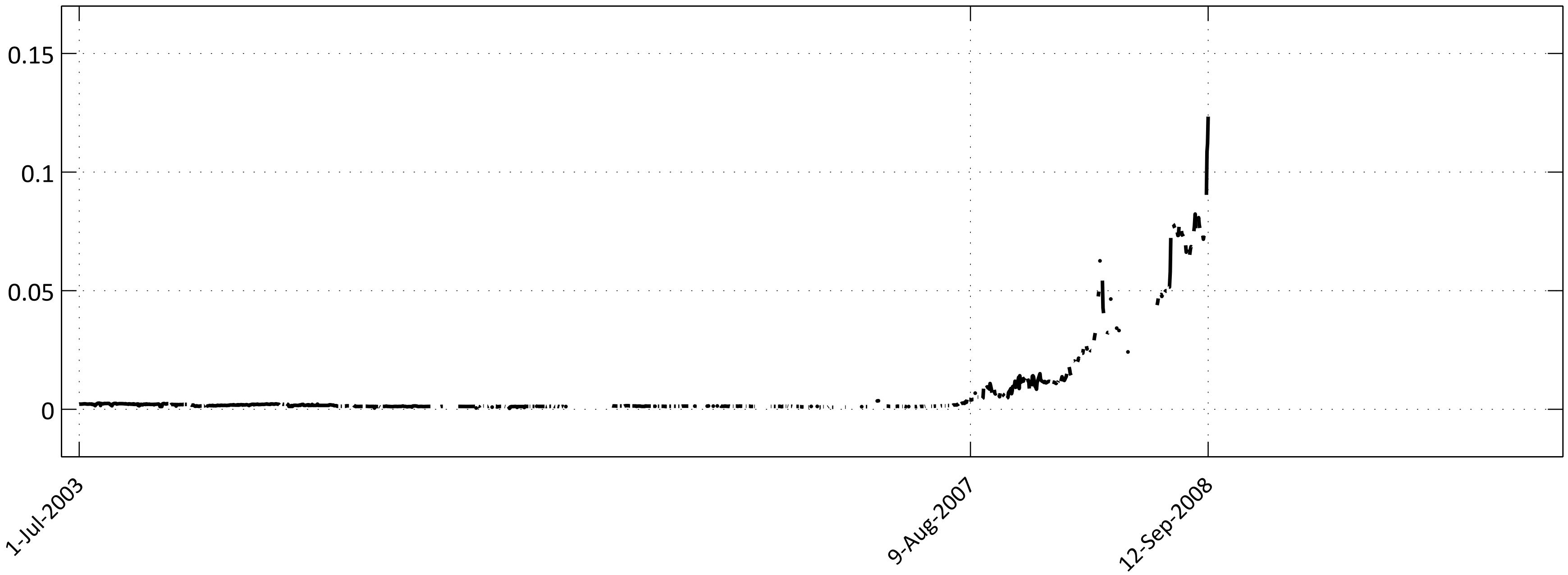
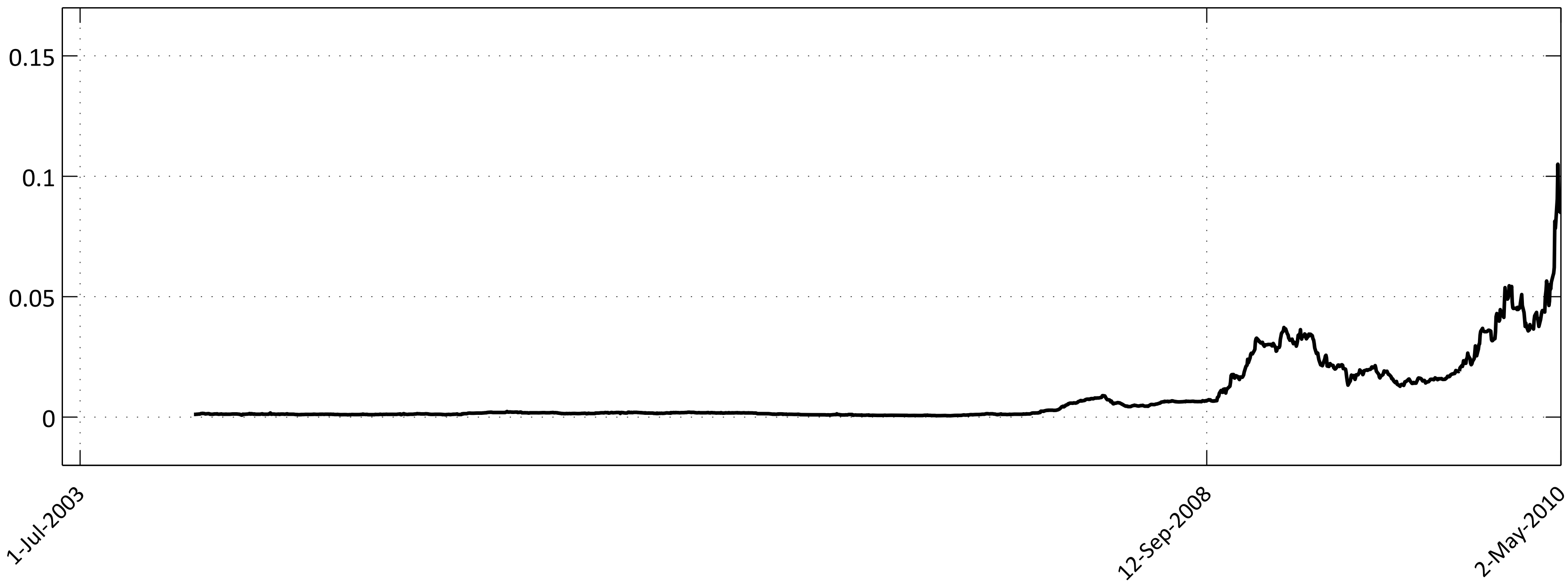


Figure 5: Probability of default, Greece



Note: The probabilities of default in Figures 4-5 are derived from CDS premia. See Section 4.4 for the details.