Size-dependent Regulation and Factor Income Distribution

Preliminary and Incomplete

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Abstract

In many countries, labor regulation is more binding for large firms than for small firms. By inducing a reallocation of productive resources away from large firms, such policy designs are likely to generate a loss of aggregate output. Moreover, as small firms are less capital intensive that large firms, size-dependent regulation might also induce a reallocation of income from capital to labor. The objective of this paper is to quantify these macroeconomic implications in the case of the 50 employees threshold in France. In order to achieve this, I first estimate the distortion induced by the regulation at the micro-level using French firm-level data. Then I use my estimates to calibrate policy experiments in a general equilibrium framework. Estimation is based on a structural model in which firms produce by combining capital and labor through a CES technology. Capital productivity and labor productivity both differ across firms. At the threshold, the model predicts (i) a break in the distribution of firm sizes, (ii) a break in capital intensity and (iii) the presence of an excessive mass of firms. These predictions are empirically verified and identify my parameters. I find that regulation raises the unit cost of labor by 7.5% and total costs of firms at the threshold by 3%. Firms above 50 raise their capital-labor ratio by 5% as a response to the regulation, which corresponds to a capital-labor elasticity of substitution of 0.6. Full deregulation raises aggregate welfare be 0.2% and decreases labor share of income by 0.14%.

Keywords: Size-dependent Regulation, Firm size distribution, Capital-labor elasticity of substitution, Factor income distribution

JEL Classification: L11, E25, J08, L25, D33

1 Introduction

In many countries, labor regulation is more stringent for large firms than for small firms. An illustration is the obama care which constrains employers with more than 50 employees to purchase health insurance for their workers. Such regulation designs as suspected of impacting aggregate outcomes because (i) they induce a reallocation of market shares toward small firms and (ii) small firms are different from large firms. Most of the literature so far has explained the impact of size dependent regulations through the lens of productivity differences. By contrast, I focus on firm-level heterogeneity in factor intensity.

In particular, I consider the implications for the aggregate productivity and factor income distribution. Micro differences in factor intensity matter for two reasons. Firstly, the extend of misallocation resulting from regulating the use a factor depends on the share of this factor in the total costs of *regulated firms*. Ignoring factor heterogeneity therefore leads to mis-estimate the extend of misallocation by inferring it from the aggregate (wrong) factor share. Secondly, because large firms tend to be more capital intensive, size dependent regulations induce a reallocation of market shares towards labor intensive firms and thence potentiallydistort the aggregate distribution of income across factors. [1]

I carry out my analysis in the context of the 50 employees threshold in the employees rights legislation which applies to French firms. Drawing upon a census of French manufacturing firms above 20 employees over the period 1995-2007, I structurally estimate the implicit cost of the regulation above 50 employees using observed distortions in firms factor demands in the neighborhood of the threshold. Using my estimates to calibrate a general equilibrium model, I get preliminary results suggesting that the regulation shifts the distribution of income towards labor, even if regulated firms substitute capital for labor. The rationale for this result being that the regulation generates a reallocation of market shares from large capital intensive firms to small labor intensive firms.

There are three main contributions to this paper. First I document new facts on firms behavior around a workforce threshold in the labor regulation. Namely, I show that regulated firms significantly increase there capital labor ratio in response to the regulation. Second, I propose a simple way to structurally estimate the implicit cost of the regulation. Incidentally, my approach delivers an estimate of the elasticity of substitution at the firm level. Since my structural model is stationary, I interpret it as long-run elasticity of substitution. Third, I extend the closed version of Melitz (2003) to allow firms to produce out of both capital and labor and to have non-hick neutral productivity differences. The calibration of this model allows me to analyze the distributional implication of the regulation.

The analysis developed in this paper proceed in several steps. First, I develop a structural model from which I identify the parameters of interest. It is a partial equilibrium model with heterogeneous firms producing a differentiated good through a production function which is

1. See Raval (2011) and Forslid & Okubo (2011) for examples of papers documenting a positive relationship between size and capital intensity.

CES in capital and labor. Firms face a regulation discontinuity which is modeled as an increase in the payroll tax rate above a given number of employees. Productivity differences across firms are driven by a single parameter and can be factor-biased. Based on this parameter, firms self-select into size so that high productivity firms are above the threshold, low productivity firms are below the threshold and there is a range of intermediate productivity firms which choose to bunch at the threshold. The model predicts a break in the capital-labor ratio and in the firm size distribution at the threshold respectively because (i) firms at the left and the right of the threshold have different technologies and (ii) because labor is more costly above the threshold.

Then I describe the French regulation along with some descriptive facts on firms behavior around the 50 threshold. The French case offers a perfect set-up to empirically assess the effect of size-dependent labor regulations. In fact, when a French firm exceeds the 50 threshold, it has to provide its employees with a handful of important additional rights (e.g. set-up a works council, agree on a profit sharing rule with its employees, etc.). I verify that main qualitative predictions of the model are empirically verified. In particular, I get reduced form evidence suggesting that firms substitute capital for labor at the threshold as well as above the threshold.

Using the model, I estimate the tax equivalent of the regulation and the capital-labor elasticity of substitution. My approach consists in estimating the individual factor demands for a given technology parameter under two counterfactual situations: one where regulation binds all firms irrespective of size and another one where there is no regulation. The estimated parameters are those which equate estimated counterfactual factor demands to their structural counterparts. I estimate the vector of factor demands for a firm which would locate exactly at the 50 threshold in the absence of regulation. Its counterfactual labor demand when regulation applies to all firms is identified from the excessive mass of firms at the threshold. The insight for why the excess mass is related to the size of a firm in the counterfactual situation when regulation binds all firms is simply that both are a function of the cost of the regulation: the model predicts that a larger cost of the regulation leads more firms to bunch and would also lead bunching firms to be smaller, should they be regulated. Counterfactual capital demands are identified from the relationship between the number of employees and the average relative factor expenditures in a neigboorhood of the threshold.

I find that the regulation raises the unit cost of labor by 7.5% and total costs by 3%. Moreover, firms above 50 raise their capital-labor ratio by 5%. Estimated capital-labor elasticity of substitution is 0.6 and consistent with the literature[2].

Finally, I use estimated parameters to implement counterfactual exercises. I am especially interested in the response of the aggregate production and of the share of labor in national income to a variation of the micro cost of the regulation. To this end, I superimpose a general equilibrium layer to the problem of the firm. The general structure builds upon essentially follow Melitz (2003). I assume that upon entry, firms pay a sunk cost and randomly draw a

2. See Chirinko et al. (2004) for a review.

capital and labor productivity. Once on the market, firms pay a fixed operating cost. The joint entry distribution of capital and labor productivity is calibrated to replicate the empirical size distribution and the empirical size-factor cost ratio below the threshold. I am still working on the results from this section.

My work relates to several strands of literature. It contributes to the literature on the impact of (workforce) size-dependent regulations on firm's policy. These papers differ in the source of identification. Some have an exogenous source of variation in the regulation. This is the case of Bauernschuster (2009) and Kugler & Pica (2008) who use time-variations in the size threshold for dismissal protection legislation to identify an impact on the employment dynamics at the firm-level. Given their source of identification, they estimate a short-run to medium-run effect of the change in regulation. Some other papers like Boeri & Jimeno (2005) document the impact of the threshold by regressing firm level variables on a dummy variable for whether the firm is above the threshold or not. However, the obtained coefficient can not have a causal interpretation as firms are susceptible of choosing their position to the threshold based on unobservables. Ceci-Renaud & Chevalier (2010) and Schivardi & Torrini (2004) do not have an instrument variable either and so identify threshold effects from the distortion in firms'size dynamics around the discontinuity under the assumption that firms'dynamics is a smooth function of size in the absence of a threshold. I depart from these different contributions by adopting a structural approach while they rely on reduced forms.

Closer to what I do, Gourio & Roys (2012) and Garicano et al. (2013) estimate a structural model. Both are able to estimate the micro and macro cost of the regulation. However, since labor is the only production factor in both papers, no analysis of the substitution effects across factors is possible, whether at the micro or macro level. The present paper, by contrast, considers both capital and labor as production factors.

Finally, size-dependent labor regulation belongs to the broader family of heterogeneous firm-level regulation. As pointed out in a series of recent paper (e.g. Guner et al. 2008, Restuccia & Rogerson 2008, Hsieh & Klenow 2009), the heterogeneity of policy distortions at the firm-level can generate large aggregate inefficiencies by inducing a reallocation of resources across firms. I add by letting firms capital intensity vary within industries. This makes it possible to investigate the macro-level substitution effect induced by heterogeneous policies.

The remainder of the paper is organized as follows. Next section introduces the dataset, the content of the French regulation as well as some evidences of its effect on firms' behavior. Section 3 describes the structural model. Section 4 presents the estimation strategy. Section 5 reports the results. section 6 provides the results from preliminary policy experiments.

2 Regulatory Background and Motivating Evidence

In this section, I first describe the French Regulation and my dataset. Then I provide reduced form evidences that labor and capital demand respond to the regulation at the firm-level. These evidences motivate the structural model.

2.1 The French Regulation

French regulation has important thresholds at 10,20 and 50 employees. In the present study, I will focus on the 50 threshold. There are two reasons why the 50 threshold is arguably the most interesting one. First, the 50 threshold is generally recognized as the most binding to firms and as such, identifying its effects is easier from a statistical point of view and has a greater interest for economic policy. Second, existing works on the thresholds in France have mostly focused on the 50 threshold so that my results will be comparable to the largest possible set of existing results. [3]

When a firm exceeds 50 employees in France, it has to deal with a long list of additional rules on labor and accounting (see Ceci-Renaud & Chevalier (2010) for the exhaustive list). For instance, above 50, a firm must:

— organize the election of a works council upon the request of at least one employee. In France, a works council is in charge of providing services to employees and to represent them with respect to executive management.

— set-up a committee for health, safety and working conditions.

— produce a *plan social* when more than 9 employees are fired simultaneously. A plan social is a legal process through which, among other things, firms must prove to some legal institutions that they have tried to find a new job to dismissed employees.

— share profits with their employees. The sharing rule must be jointly established by the managers and workers' representatives at the firm level.

When, exactly, does the regulation binds to a firm? The definition of the number of employees in the regulation is the monthly full-time equivalent workforce. This measure takes into account part-time workers and temporary workers but not trainees nor subsidized employment. The 50 threshold triggers a set of laws and not all of them have the exact same criterion as to how long a firm should stay above 50 before it applies. Most of the rules, however, become binding as soon a firm has spent more than 12 months, over the three past years, above 50. As I will explain at the time of presenting the data, my measure of workforce is the monthly full-time equivalent averaged over the 12 months of an accounting year. As such it is a good, although imperfect, measure of whether a firm is regulated or not. Moreover, some elements of the regulation are based on the firm exceeding a given number of employees *or* a given turnover (around 4 million Euros). [4]. This is another factor which makes my measure of workforce a slightly noisy proxy for whether a firm is regulated or not [5]

3. Note that even if I focus on the 50 employees threshold, my estimation strategy virtually applies to any threshold of the regulation.

4. The turnover criterion is only on accounting rules, not on labor rules.

5. note, however, that following the insight from the ordinary least squares literature, this noise should play against me by biasing estimated parameters toward zero.

2.2 Data

My empirical analysis is ran on the EAE (Annual Business Survey) dataset. This dataset is a panel of the universe of French manufacturing firms above 20 employees over 1995-2007. The data is collected by INSEE (French National Statistical institute) out of about 20000 firms each year. The dataset also contains a randomly drawn sample of firms below 20 which I never use in my analysis. I also limit my analysis to firms which main activity is in the manufacturing sector. The main elements of firms' balance sheet are present in the database (e.g., turnover, value added, investment, materials, etc.). There is no variable at the worker level (e.g., individual workers' wage, occupation or education). The dataset has no information either on whether the main elements of the regulation are implemented by a firm (works council, committee for health and safety, etc.). Only is there information on profit shared by the firm with its employees.

I essentially use three variables: the workforce size, the wage bill and the gross stock of fixed assets. I have separate information on tangible and intangible fixed asset, at the beginning of the year and at the end of the year. Hereafter, the variable I name "capital" is the sum of these four measures of fixed assets divided by two (I average the beginning and the end of year stocks). For now I have not implemented the usual treatments to the stock of capital (i.e., deflating each vintage of capital by the price index at the time of purchase and implementing perpetual inventory methods). Since capital is one of the most important variable in my analysis, it is clear that these treatments will be present in the final version of the paper.

At the core of my analysis is the factor cost ratio variable which I define as the ratio of the user cost of capital and wage bill. I obtain the user cost of capital very simply by multiplying the stock of capital, defined above, by a unit user cost of capital $r = 0.1$. Here I follow the value proposed by Hsieh & Klenow (2009). It corresponds to a rental rate of 5% and a depreciation rate of 5%. The purpose of this calibration is just to present factor cost ratios with sensible levels. My estimation, however, is independent of this calibration as only do relative factor cost ratios across firms serve my identification.

In order to compute TFP, I also construct the value added and materials variables. I define materials as the sum of the cost of goods and the cost of materials. I compute TFP using Levinsohn & Petrin (2003) semi-parametric method. Once again, value added and materials are not deflated although they will be in future versions of the paper.

Table 1 reports correlations for main variables over the whole sample. Not surprisingly, value added, capital and employees are positively correlated. More interestingly, these different measures of size are positively and significantly correlated with the factor cost ratio. This is a finding consistent with previous evidences in the literature (see Raval 2011). It is suggestive of a model where productivity differences across firms are factor biased. In appendix A, I present additional descriptive statistics on the main variable of interest.

Table 1 – Correlation Matrix for Main Variables

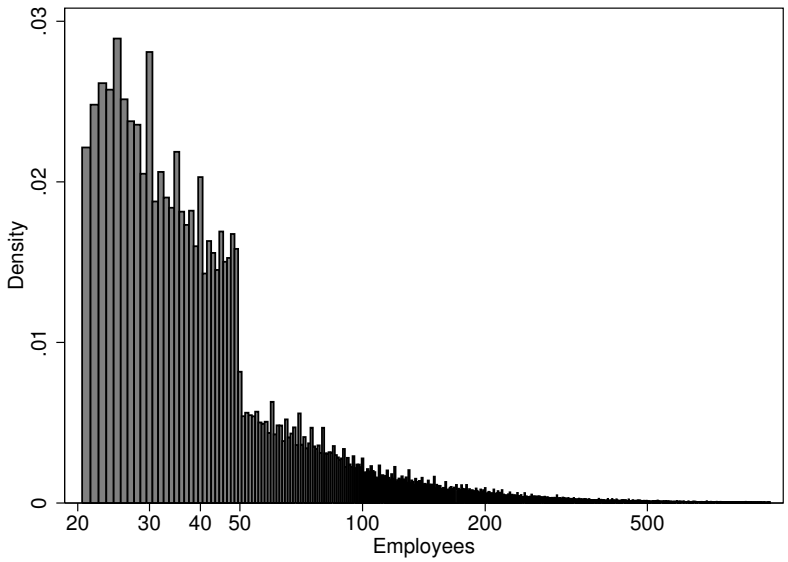
	Value Added	Fixed Assets	Employees	Factor Cost Ratio
Value Added	1.00			
Fixed Assets	0.88	1.00		
	(0.00)			
Employees	0.88	0.81	1.00	
	(0.00)	(0.00)		
Factor Cost Ratio	0.03	0.04	0.01	1.00
	(0.00)	(0.00)	(0.00)	

Note: P-values in parentheses. Correlations are computed over the whole sample.

2.3 Motivating evidence

The present paper proposes to identify the implicit cost of the regulation along with the capital-labor elasticity of substitution. To this end, I will structurally estimate firm-level response of capital and labor demands. This sub-section gives suggestive evidence that there are such responses.

Figure 1 – Firm Size Distribution between 20 and 1000 Employees

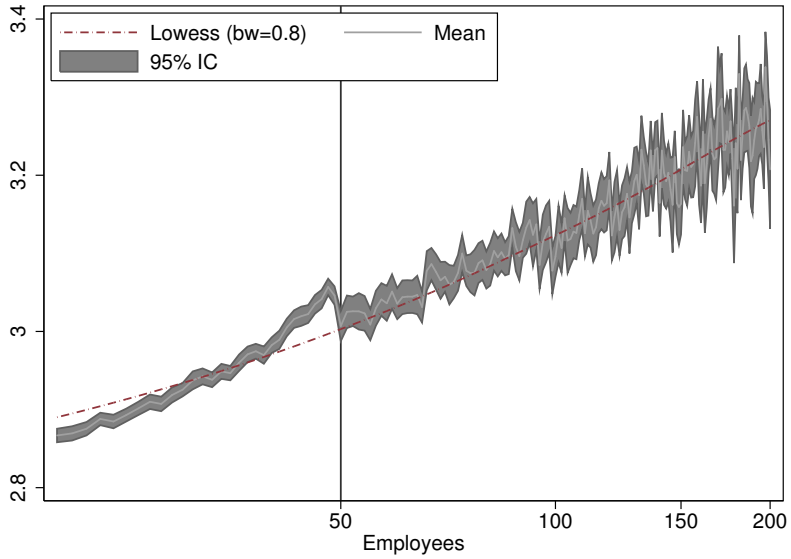


In figure 1, I plot the workforce size distribution of French firms on a log x-axis. This distribution exhibits features which are common in the literature: it is highly skewed and its upper tail is akin to a pareto distribution. [6]. Yet, visual inspection reveals two departures from the traditional pattern of firm size distribution. First, there is a break at 50 employees: the number of firms is approximately twice larger at 49 than at 50. Second, while the distribution

6. Influential papers documenting the pareto shape of firm size distribution include Simon & Bonini (1958) and Luttmer (2007)

follows a negative trend everywhere, it seems to be increasing just below the threshold. This excessive mass supports the existence of firms bunching at the threshold so as not to get regulated. In my estimation, this excessive mass identifies the labor demand response to the regulation.

Figure 2 – log TFP Versus Size

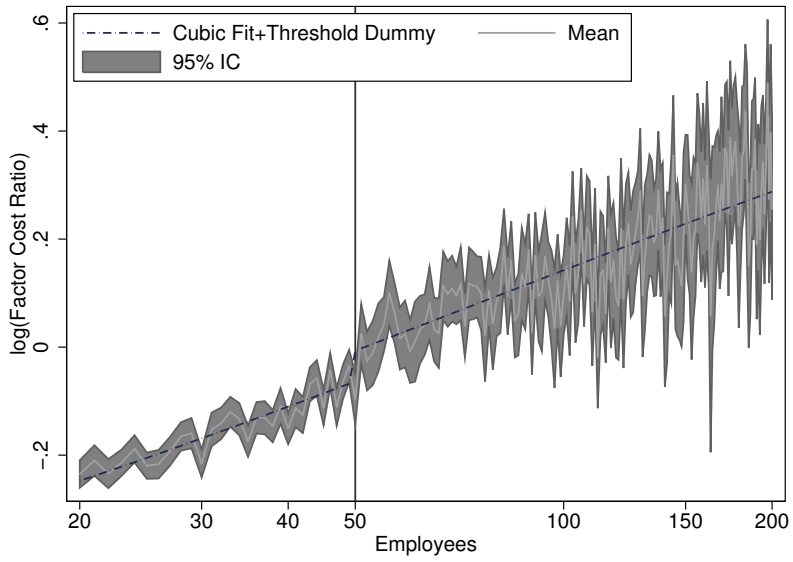


Note: TFP is estimated using Levinsohn and Petrin (2003) methodology with intermediate input as a control for unobserved productivity.

As an additional evidence of bunching, in figure 2 I plot workforce against log TFP. The x-axis is log-scaled. Figure 2 features a positive relationship between TFP and size: it seems that more productive firms self-select into larger sizes. However, one can observe that the average productivity below the threshold, i.e. between 40 and 49, significantly differs from the value predicted from non-parametrically regressing productivity on size. Put differently, some firms below the threshold are too small given their productivity, which indicates that some firms shrink their labor demand to stay below 50.

Let us now look at the response of the factor cost ratio as it informs on the substitution behavior of firms. I first describe the relationship between employees and the factor cost ratio. In figure 3, I report the firm-level relationship between employees and the average factor cost ratio. Prior to computing the average, I demean the factor cost ratio at the year-2-digit industry level (36 industries) so that relationship in figure 3 is within year-industry. This relationship is upward sloping consistently with the correlation matrix presented in previous subsection. More interestingly, there is a positive break in the relationship at the threshold. This means that firms tend to be more capital intensive above 50. In order to check the statistical magnitude of this break, I regress the factor cost ratio on a third order polynomial and a dummy equal to one if a firm has more than 50 employees. The results are in table 3.

Figure 3 – Factor Cost Ratio Versus Size



Note: 'Cubic Fit+Threshold Dummy' is the prediction of a regression of log Factor Cost Ratio over a dummy (*Employees* $\geq$50), *Employees*, *Employees*^{2} and *Employees*^{3}, a set of year-2-digit industry dummies and a constant term.

Table 2 – Factor Cost Ratio VS. Size

	log Factor Cost Ratio		
	(1)	(2)	(3)
"Above 50 Employees" Dummy	0.082^a	0.057^a	0.010
	(0.000)	(0.000)	(0.104)
Employees	-0.286^a	-0.139^b	0.302^a
	(0.000)	(0.036)	(0.000)
Employees^2	0.113^a	0.083^a	-0.084^a
	(0.000)	(0.000)	(0.000)
Employees^3	-0.008^a	-0.006^a	0.005^a
	(0.000)	(0.000)	(0.000)
Fixed Effect	No	2-digits Industry×year	Firm
Model	OLS	OLS	OLS
Observations	242437	242437	242437
R^2	0.047	0.043	0.005
rho		0.150	0.859

p-values in parentheses

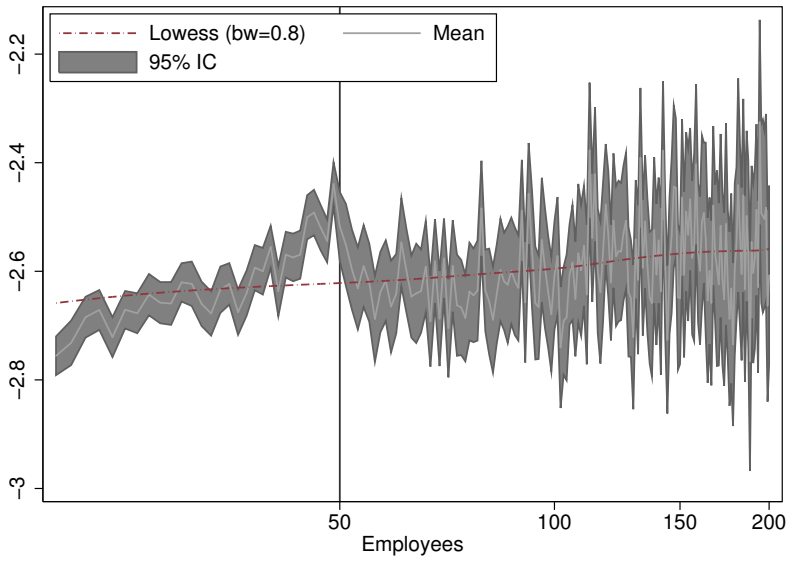
^{c} $p < 0.1$, ^{b} $p < 0.05$, ^{a} $p < 0.01$

Coefficients from column (1) were obtained without controls. The coefficient associated to the dummy variable is 0.08 which means that a firm at 50 employees is on average 8% more capital intensive. Adding industry fixed effects (Column (2)) lowers the coefficient to 5% but significance remains. The effect of adding industry fixed effects on the coefficient suggests that more capitalistic industries also have a larger proportion of firms above 50, on average. Specification in column (3) has firm fixed effects and the "above 50 employees" dummy is close to be significant at 10%.

How to interpret these results? Clearly, the dummy in both first specifications has no causal interpretation. This is because, as the analysis of the TFP and the firm distribution make clear, firms self-select with respect to the threshold . Therefore, firms at 49 *are not* comparable to firms at 50, a priori. There could be a positive break in capital intensity at 50 employees even if firms do not substitute capital for labor as a response to the regulation. It will for instance be the case if (i) more productive firms self-select above 50 and (ii) more-productive firms are also more capital intensive (for reasons exogenous to the regulation). For now, all one can conclude from the positive break is that it is consistent with firms substituting capital for labor because of the regulation.

I see three possible reasons why the dummy coefficient is not significant in the within-firm specification. First, firms do not substitute capital for labor at all. Second, within-firm capital-labor substitution takes time so that it can not be observed in the time horizon of the panel. My structural estimation of the capital-labor substitution will be in the cross-section of firms. It will have a long-run interpretation so that it will be consistent with the absence of substitution obtained in the time series. Third, firms might anticipate that they will exceed 50 in the future and therefore increase their capital-labor ratio before to get regulated.

Figure 4 – Investment over Fixed Assets Versus Size



As a way to test these rationales for the low explanatory power of the "above 50" dummy variable, I look at the investment behavior of firms. In figure 4 is plotted the mean log ratio of investment to fixed assets (hereafter the investment rate), conditional on workforce. The investment rate exhibits a clear bulge below the threshold. Interestingly, there is no equivalent distortion visible on the factor cost ratio below 50, in figure 3. Combining these features suggests that firms do not bunch long enough so their investment effort translates into an excess factor cost ratio below the threshold. This analysis supports the idea that firms invest more intensively below the threshold in order to have a higher capital labor ratio once above the threshold.

With these empirical facts in hand, I turn to the description of the structural model. The structure will guide the rest of our empirical analysis and will allow me to identify a causal effect of the regulation on factor demands.

3 Theory: Heterogeneous Firms and Legal Threshold

This section first lays down the structure of the model. At this stage, I limit my presentation to the problem of the firm in a partial equilibrium set up as it suffices for my estimation (I postpone the description of the rest of the model to the counterfactual section). Then, I derive the empirical predictions that will be used to identify the parameters.

3.1 The Set-up

Technology and Demand I consider a closed and static economy populated with heterogeneous firms producing a differentiated good out of capital and labor. The technology of a firm is CES and is summarized by a scalar parameter α which determines both the efficiency of capital $A(\alpha)$ and the efficiency of labor $B(\alpha)$. The expression of the production function is

$$y(k,l;\alpha) = \left[(A(\alpha_i)k_i)^{\frac{\sigma-1}{\sigma}} + (B(\alpha_i)n_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with n_i the number of employees , k_i the stock of capital and $\sigma \geq 0$ the capital-labor elasticity of substitution. I do not postulate the pattern of complementarity across factors: σ can be smaller or larger than unity. Through following assumption, I ensure that the production cost of the firm are decreasing with α

Assumption 1. *Functions $A(\bullet)$ and $B(\bullet)$ are strictly increasing*

As indicated in assumption 1, I assume both labor efficiency and capital efficiency to increase with α. Parameter α is therefore a positive measure of a firm's TFP and I refer to it as "efficiency" in the rest of the paper. α is distributed across firms according to a PDF function $g(\alpha)$. This function is subject to following assumption:

Assumption 2. *$g(\alpha)$ is smooth.*

Assumption 2 will be useful in the estimation presented in section 4. Intuitively, under assumption 2 and in the absence of a regulation threshold, the model delivers a smooth distribution of firm size. So the impact of the regulation can be identified from deviations to a smooth distribution in the neighborhood of the threshold .

All firms face an identical isoelastic demand function

$$y_i = p_i^{-\varepsilon} Z, \tag{2}$$

with ε the price elasticity of demand, p_i the price charged by firm i and Z a demand shifter common to all firms. As is standard, we impose $\varepsilon > 1$ to ensure that firms charge finite prices.

The Regulation I model regulation as a tax rate τ on labor which applies to firms above $\bar{n}$ ($\bar{n} = 50$ in my application to the French regulation). [7]

The expression of total labor costs of a firm with n employees is:

$$(1 + \tau \mathbb{1}\{l \geq \bar{l}\}) wn.$$

Here, I implicitly assume that the regulation does not impact the cost of capital. This is obviously a simplifying assumption which will not be verified in the general case of a size-dependent regulation. However, most rules binding above 50 employees in France are relative to EPL. In the French case, it seems therefore reasonable to assume that the productivity of other production factors is not affected by the regulation.

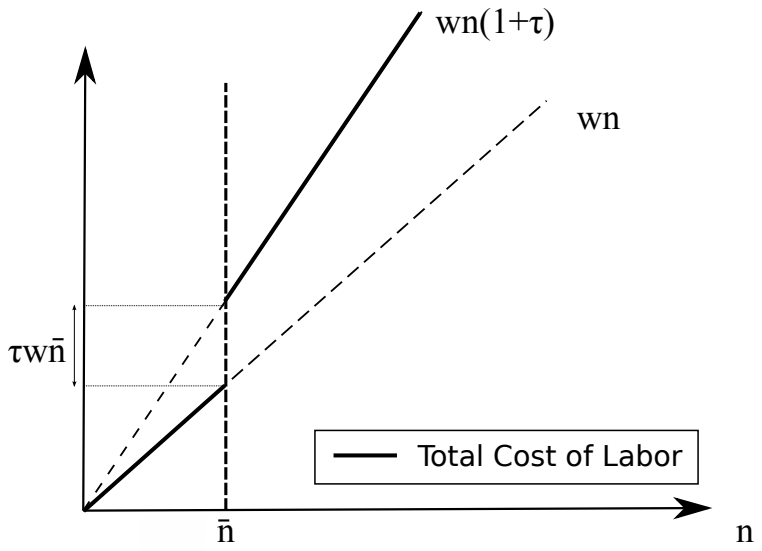
The regulation could also have dynamic components that we omit here. For instance, the French regulation contains elements relative to redundancy rules which potentially impact the firing costs of the firm. However, as proved by Bentolila & Bertola (1990), when financial markets are complete and firms can fully hedge, firing costs are equivalent to a per period cost on labor. The regulation could also factor in firms'dynamics by taking the form of a sunk cost. Gourio & Roys (2012) investigate that possibility by assuming that firms incur an adjustement cost to activate the regulation and that once activated, the regulation can be maintained at no cost. [8]

In figure 5, I plot the cost function of a firm. Observe that at $\bar{n}$, the cost of labor features both a positive discontinuity of magnitude τwl and a change of slope from w to $w(1+\tau)$. The discontinuity results from the fact that when regulation binds, it applies to every workers and not only to the $\bar{n}$-th worker and beyond. This discontinuity in firms' production set distorts firms' decisions. Next subsection solves the problem of the firm and analyzes these distortions.

7. I do not include a fixed cost component to the regulation. This choice is motivated by Garicano et al. (2013) who estimate a small and insignificant fixed component of the 50 employees threshold in France.

8. Although Gourio & Roys (2012) get results suggestive of the presence of a sunk cost, they are unable to estimate this jointly with a per period cost of the regulation. Moreover, the break in the size distribution at the threshold found in the data is strongly supportive of a per period-per unit cost of the regulation.

Figure 5 – Total Cost of Labor



3.2 The problem of the firm

In this subsection, I derive the expression of firms optimal output and factor demand. I start by investigating the counterfactual cases where regulation binds all firms ($\bar{n} = 0$) and where regulation binds no firms ($\bar{n} = +\infty$). The fact that in these cases firms do not have to choose to be regulated or not facilitates the analysis. Then I present the actual case: $0 < \bar{n} < +\infty$.

3.2.1 Firm's Decisions in Absence of a Threshold

Let $c_0(\alpha)$ and $c_1(\alpha)$ be the unit production cost function of a firm with efficiency α respectively in the absence of regulation and when regulation applies to all firms . The expression of these functions is standard under CES technology

$$\begin{cases} c_0(\alpha) &= \left[\left(\frac{r}{A(\alpha)}\right)^{1-\sigma} + \left(\frac{w}{B(\alpha)}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ c_1(\alpha) &= \left[\left(\frac{r}{A(\alpha)}\right)^{1-\sigma} + \left(\frac{(1+\tau)w}{B(\alpha)}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} . \end{cases} \tag{3}$$

The production cost is a CES index of the cost per efficiency unit of labor and capital. Given assumption 1, $c_i(\alpha)$ is strictly decreasing with α. Due to the regulation cost, $c_1(\alpha)$ is larger than $c_0(\alpha)$. The optimal price and output are

$$\begin{aligned} p_i(\alpha) &= \frac{\varepsilon}{\varepsilon - 1} c_i \\ y_i(\alpha) &= \left(\frac{\varepsilon}{\varepsilon - 1} c_i\right)^{-\varepsilon} Z, \end{aligned}$$

Because a firm's production costs are larger in presence of regulation, $p_1(\alpha) > p_0(\alpha)$ and, as a result, output is $y_1(\alpha) > y_0(\alpha)$.

Labor demand $n_i(\alpha)$ involved in the production of output $y_i(\alpha, \tau), \quad i = 0, 1$ has usual CES expression:

$$\begin{cases} n_0(\alpha) &= \left(\frac{w}{c_0(\alpha)}\right)^{-\sigma} B(\alpha)^{\sigma-1} y_0(\alpha) \\ n_1(\alpha) &= \left(\frac{(1+\tau)w}{c_1(\alpha)}\right)^{-\sigma} B(\alpha)^{\sigma-1} y_1(\alpha). \end{cases} \tag{4}$$

Regulation reduces labor demand through two effects. First, it leads firms to substitute capital for labor as it shifts up the relative cost of labor. Second, it causes a scale effect as the production costs go up which shrinks the output of the firm. It follows that $n_1(\alpha)$ is smaller than $n_0(\alpha)$.

The way labor demand varies with α is more ambiguous. To see this, let us write down the elasticity of labor demand to α

$$\frac{\partial \log n_i(\alpha)}{\partial \alpha} = \overbrace{-\frac{\partial \log B(\alpha)}{\partial \alpha}}^{\text{Labor Saving Effect}<0} \underbrace{+\sigma \frac{\partial \log \frac{B(\alpha)}{c_i(\alpha)}}{\partial \alpha}}_{\text{Substitution Effect}} \overbrace{+\varepsilon \frac{\partial \log c_i(\alpha)}{\partial \alpha}}^{\text{Scale Effect}>0}. \tag{5}$$

Efficiency α has three effects on labor demand. First effect is what we call a "Labor saving effect". When α goes up, so does labor productivity so that a firm needs fewer physical units of workers for a given number of efficiency units of labor. This effect is unambiguously negative on labor demand. The second effect is a scale effect: production costs decrease with α which allows a firm to charge lower prices and sell more. This effect is unambiguously positive. The last effect is a substitution effect. It comes from the fact that I did not impose α to be a hicks neutral technology parameter. Therefore, a change in α may induce a substitution effect by shifting the relative efficiency $\frac{A}{B}(\alpha)$.

Assumption 3. $\frac{\partial \log B}{\partial \alpha} < \sigma(\frac{\partial \log B}{\partial \alpha} - \frac{\partial \log A}{\partial \alpha}) + \varepsilon \frac{\partial \log A}{\partial \alpha}$

3 guarantees that labor demand is strictly increasing with α which will prove useful in the estimation. Intuitively, it constraints the labor saving effect the be more than compensated by both other effect. If $\varepsilon > 1 > \sigma$, as we find in the data, this is amounts to assuming the labor productivity B do not increase "too much" with α relative to capital productivity A.

I now define $\rho_i(\alpha) \equiv \frac{rk_i}{wn_i}(\alpha)$ as the factor cost ratio. This function will show useful in my estimation as it will allow me to use the observed substitution behavior of firms to back out structural parameters. $\rho_i(\alpha)$ verifies

$$\begin{cases} \rho_0(\alpha) &= \left(\frac{r}{w}\right)^{1-\sigma} \left(\frac{A}{B}(\alpha)\right)^{\sigma-1} \\ \rho_1(\alpha) &= (1+\tau)^{\sigma} \left(\frac{r}{w}\right)^{1-\sigma} \left(\frac{A}{B}(\alpha)\right)^{\sigma-1}. \end{cases} \tag{6}$$

ρ_1 is larger than ρ_0 and $\rho_1(\alpha)$ is increasing with τ.[9]

9. The ranking of ρ_1 and ρ_0 would be more ambiguous if I had included the cost of the regulation in the

The factor cost ratio depends on α through $\frac{A}{B}(\alpha)$. Actually, relative factor efficiency is the only source of variation in the factor cost ratio across firms (conditional on the regulation status). We could have made the structure of the model more complex in order to incorporate alternative sources of variation like adjustment costs, heterogeneous factors costs, heterogeneous credit constraints could come as complements. However, as will appear clearly below, the parsimony of my model pays off through the simplicity of the estimation. In now describe firms' decisions in the presence of a regulation threshold.

The General Case

I now consider the case $\infty > \bar{n} > 0$. Unlike counterfactual cases studied above, firms now have to choose whether to be regulated or not, consistently with the French context. Let α_c be such that

$$n_0(\alpha_c) = \bar{n}.$$

Firms below α_c are unaffected with the regulation since their optimal demand, $n_0(\alpha)$, is inferior to $\bar{n}$. Firms above α_c trade-off between (i) bunching at the threshold and thence giving up revenues generated by a larger workforce or (ii) expanding beyond the threshold and paying a higher unit cost per worker. The profit from bunching is

$$\pi_b(\alpha) = \max_k \left\{ R(k, \bar{n}, \alpha) - w\bar{n} - r.k \right\},$$

with $R(k, n; \alpha)$ the revenue production function of a firm α obtained by combining equations (1) and (2) :

$$R(k, l; \alpha) = Z^{\frac{1}{\varepsilon}} \left[(A(\alpha)k)^{\frac{\sigma-1}{\sigma}} + (B(\alpha)l)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}}. \tag{7}$$

It results that the profit loss from bunching instead of being regulated is

$$\Delta(\alpha) \equiv \pi_1(\alpha) - \pi_b(\alpha),$$

, with $\pi_1(\alpha)$ the optimal profit of a regulated firm:

$$\pi_i(\alpha) = Z^{\frac{1}{\varepsilon}} y_i(\alpha)^{\frac{\varepsilon-1}{\varepsilon}}. \tag{8}$$

A firm bunches at the threshold if it is affected by the regulation ($\alpha \geq \alpha_u$) and its profit loss from bunching is negative ($\Delta \leq 0$). Respectively, a firm chooses to exceed $\bar{n}$ if $\alpha \geq \alpha_u$ and $\Delta > 0$. Simple analysis of $\Delta(\alpha)$ delivers following proposition:

cost of labor used to compute the factor cost ratio (i.e. if I had considered an alternative factor cost ratio $\tilde{\rho}_1(\alpha) \equiv \frac{rk_1}{(1+\tau)wn_1}(\alpha)$). In that case, there would be a negative price effect of the regulation on the factor cost ratio. This effect would even dominate the substitution effect when $\sigma < 1$. In terms of the identification, this point implies that we should make sure that what we observe in the data above 50 employees is actually ρ_1 and not $\tilde{\rho}_1$ because it matters for the way my structural model rationalizes the discontinuity in the factor cost ratio at the threshold. As a matter of fact, in the French case, regulation triggers monetary transfers which are not recorded as wages. Therefore, I am confident that the factor cost ratio plotted in figure 3 is indeed the empirical counterpart of ρ_i.

Proposition 1. *Let α_u be such that $\Delta(\alpha_u) = 0$ and $\alpha_u > \alpha_c$ then:*

1. *α_u is unique*
2. *$\Delta'(\alpha_u) > 0$*
3. *$n_1(\alpha_u) > \bar{n}$*

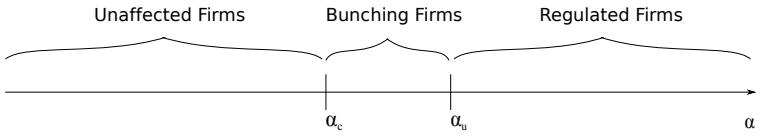
Proof. See Appendix B □

Point 1 of proposition 1 imply that there is a cutoff firms α_c indifferent between bunching or exceeding the threshold:

$$\Delta(\alpha_c) = 0. \tag{9}$$

From point 2 of proposition 1 it follows that firms between α_c and α_u bunch at $\bar{n}$ and firms above α_u exceed $\bar{n}$. I represent the sorting of efficiencies in figure 6. The rationale for this sorting is that returns to capital are decreasing and optimal output is increasing with α. Since the only way of increasing its output for a bunching firm is to consume more capital, the profit loss from bunching, $\Delta(\alpha)$, is increasing. As established in third bullet of proposition 1, no firm finds it optimal to have a workforce size in a range $[\bar{n}, n_1(\alpha_u)]$. This is because regulation shifts up the average cost of labor: by crossing $\bar{n}$, a firm is hit by a finite cost $\tau w \bar{n}$ which can not be compensated for by a marginal increase of n.

Figure 6 – Sorting of Efficiencies



To summarize, labor demand $n(\alpha)$ and factor cost ratio $\rho(\alpha)$ verify:

$$\{n(\alpha), \rho(\alpha)\} = \begin{cases} \{n_0(\alpha), \rho_0(\alpha)\} & \text{if} \quad \alpha < \alpha_c \\ \left\{\bar{n}, \frac{rk_b(\alpha)}{w\bar{n}}\right\} & \text{if} \quad \alpha_c \leq \alpha < \alpha_u \\ \{n_1(\alpha), \rho_1(\alpha)\} & \text{if} \quad \alpha \geq \alpha_u, \end{cases} \tag{10}$$

With $k_b(\alpha)$ the optimal capital of a bunching firm. $k_b(\alpha)$ equalizes the marginal revenue from capital at $\bar{n}$ with the rental rate:

$$\frac{\partial R}{\partial k}(k_b(\alpha), \bar{n}; \alpha) = r. \tag{11}$$

In figure 7 and 8, I give a sketch of $n(\alpha)$ and $\rho(\alpha)$. Labor demand grows with α below α_c. It is flat over $[\alpha_c, \alpha_u]$ as firms bunch at $\bar{n}$. There is a positive discontinuity in $n(\alpha)$ at α_u, then $n(\alpha)$ increases again over $[\alpha_u, \alpha_{max}]$, although more slowly than below α_c, due to the per unit cost of the regulation. As to $\rho(\alpha)$, the model does not determine whether it is increasing, decreasing or even monotonic. Consistently with figure 3 where factor cost ratio is increasing, I plotted figure 8 assuming $\rho(\alpha)$ is increasing. Over $[\alpha_u, \alpha_{max}]$, bunching firms substitute capital for labor in order to mitigate the cost of bunching so that $\rho(\alpha)$ diverges positively from $\rho(\alpha, 0)$ beyond α_c. So far I have expressed relationships between observables and α, which is unobservable. Next subsection derives empirical predictions from the model.

Figure 7 – Labor Demand and Efficiency

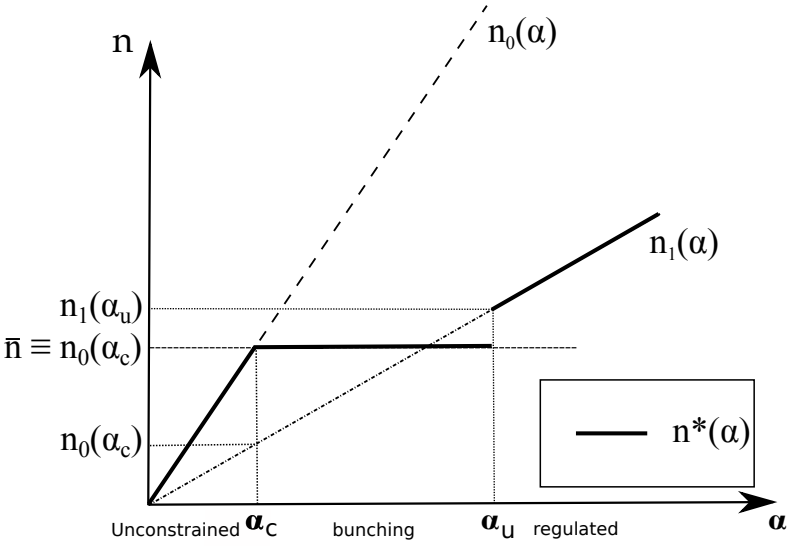
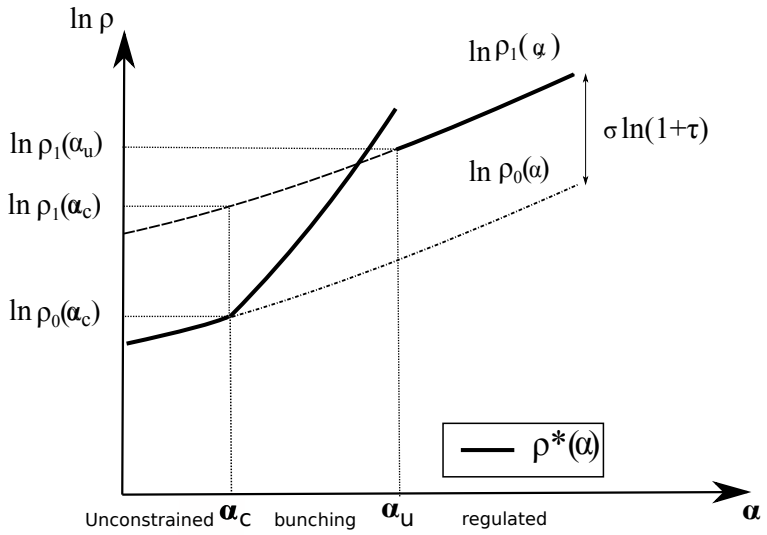


Figure 8 – Factor Cost Ratio and Efficiency



3.3 Empirical Predictions

The purpose of this subsection is twofold. First, I show that the simple model presented above is capable of replicating the features observed in the data around the threshold. Second, I lay the ground for the estimation section below. I order to achieve these ends, I derive the relationship between the factor cost ratio and workforce as well as the firm size distribution. I verify that, consistently with the data, the model predicts (i) an excessive mass of firms below the threshold, (ii) a break in the size distribution and (iii) a break in the factor cost ratio at the threshold.

Size Distribution While I assume workforce size is a continuous decision variable of the firm, I model *observed* labor as a discrete variable. In other words, firms pick their workforce size n in $\mathbb{R}^+$ but the econometrician only observes the integer part $\lfloor n \rfloor \in \mathbb{N}$. Let $\alpha_i(n)$ be the inverse of function $n_i(\alpha)$. For $n \notin [\bar{n}, n_1(\alpha_u)]$, i.e for n not in the region above the threshold 'deserted' by firms, observed size distribution $g^n(n)$ verifies:

$$g^n(n) = \begin{cases} g_0^n(n) & \text{if} \quad n < \bar{n} \\ g_1^n(n) & \text{if} \quad n \geq \lfloor n(\alpha_u, \tau) \rfloor + 1, \end{cases} \tag{12}$$

with

$$g_i^n(n) = \int_{\alpha_i(n+1)}^{\alpha_i(n)} g(\alpha) d\alpha$$

$g_1^n(n)$ (respectively $g_0^n(n)$) can be interpreted as the size distribution in a counterfactual economy where the regulation applies to all firms (respectively applies to no firm). $g_1^n(n)$ is

parametrized by τ because τ drives the relationship between efficiency and size $\alpha_1(n)$: the larger τ, the larger the efficiency α associated to a given size. Consequently, and as shown in equation (12), regulation distorts the size distribution of firms not only in a neighborhood of the threshold but for any size above $\bar{n}$.

True size of bunching firms is $\bar{n}$ minus some epsilon so that their observed size is $\bar{n}-1$. This mass of firms comes on top of firms which are unaffected by the regulation and which optimal size is in $[\bar{n}-1, \bar{n}]$:

$$g^n(\bar{n}-1) = g_0^n(\bar{n}-1) + \int_{\alpha_c}^{\alpha_u} g(\alpha)d\alpha. \tag{13}$$

So the model predicts an excessive mass of firms $\delta^g \equiv \int_{\alpha_c}^{\alpha_u} g(\alpha)d\alpha$ at $\bar{n}-1$. All else equal, this mass is increasing with τ as α_u, the efficiency level making a firm indifferent to bunch or the exceed $\bar{n}$, is obviously increasing with τ itself. To conclude on the size distribution, no firm is observed over the segment $[\bar{n}, \lfloor n_1(\alpha_u, \tau)\rfloor - 1]$. To sum up, the general expression of firm distribution is:

$$g^n(0) = \begin{cases} g_0^n(n) & \text{if} \quad n < \bar{n} \\ g_0^n(\bar{n}) + \delta^g & \text{if} \quad n = \bar{n} \\ 0 & \text{if} \quad n \in [\bar{n}, \lfloor n_1(\alpha_u)\rfloor - 1] \\ \int_{n_1(\alpha_u)}^{\lfloor n_1(\alpha_u)\rfloor+1} g(\alpha)d\alpha & \text{if} \quad n = \lfloor n_1(\alpha_u)\rfloor \\ g_1^n(n) & \text{if} \quad n \geq \lfloor n_1(\alpha_u)\rfloor + 1, \end{cases}$$

Figure 9 – Firm Size Density

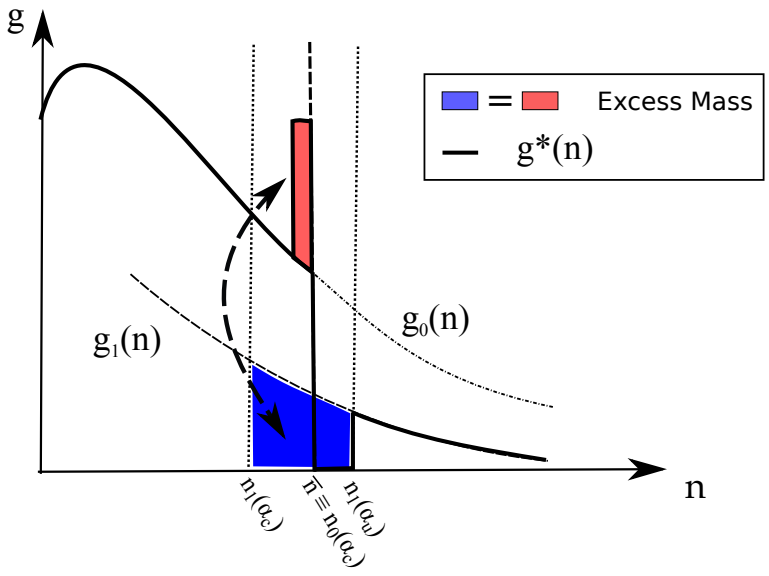
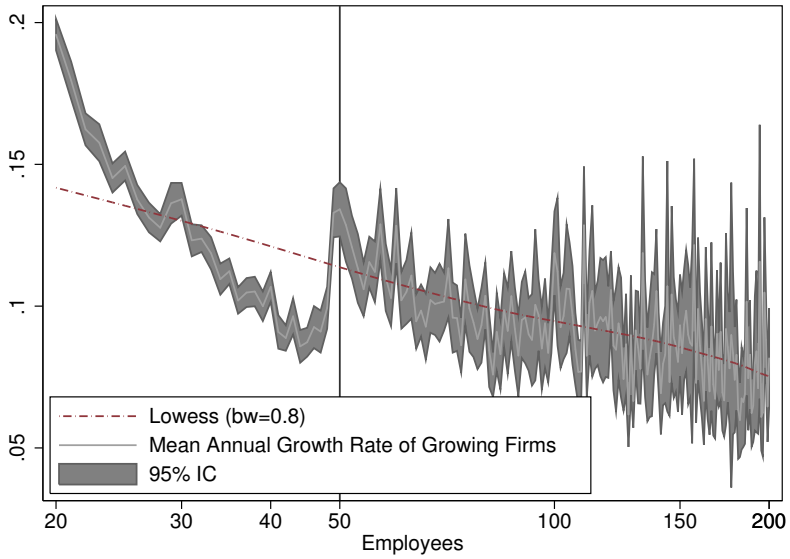


Figure 9 gives a representation of the firm size density. Consistently with the data, it features a break at 50 due to the fact that the labor demand function is distorted above the threshold, and hence the size distribution. The model also predicts an excessive mass below 50 and the absence of firms over a size interval above 50. This is not verified in the data. There are, at least, two possible explanations to that. First, the size distribution is measured with error. As explained when presenting the data, this might be due to the fact that my measure of workforce does not exactly match the one from the regulation. Second, unlike what I assume in the model, the regulation tax might induce a fixed cost on top of a marginal cost. If this fixed cost is negative, this might compensate the finite cost $w\bar{n}\tau$ which hits firms at the threshold so that firms do not need to exceed the threshold by a finite amount of workers to find it profitable to be regulated. In order to test the latter explanation, in figure 10 I plot the expected annual growth of the number of employees conditional on growing. This variable peaks at the threshold and decreases quickly above it. It seems that firms "jump" when they cross the threshold. This is consistent with the idea that regulation induces a positive discontinuity in the cost function of firms, and that the discontinuity in the support of the distribution above the threshold is masked by some measurement error.

Figure 10 – Do firms "jump" when crossing the 50 threshold ?



Notes: This figure displays the average growth of the number of employees conditional on (i) the current number of employees and on (ii) the current number of employees being strictly larger than the number of employees in the following year. Lowess line is a locally weighted scatterplot smoothing.

Capital-Labor ratio VS. Size I now derive the model's prediction regarding the relationship between the factor cost ratio and size, which the second relationship I use in my

estimation. Mean factor cost ratio given a size $n \notin [\bar{n}, n_1(\alpha_u)]$ equals

$$\rho^n(n) \equiv \mathbb{E}\left[\frac{rk}{wn}\middle| n\right] = \begin{cases} \rho_0^n(n) & \text{if} \quad n < \bar{n} \\ \rho_1^n(n) & \text{if} \quad n \geq \lfloor n_1(\alpha_u) \rfloor + 1, \end{cases}$$

with

$$\rho_i^n(n) = \frac{\int_{\alpha_i(n)}^{\alpha_i(n+1)} \rho_i(\alpha) g(\alpha) d\alpha}{\int_{\alpha_i(n)}^{\alpha_i(n)} g(\alpha) d\alpha}, \tag{14}$$

the average factor cost ratio of firms with size n when regulation applies to all firms (i=1) and to no firms (i=0). Hereafter, I make the approximation

$$\rho_i^n(n) = \rho_i(\alpha_i(n)), \tag{15}$$

i.e. I do as if firm' size was not observed with a rounding error. Similarly to what happens on the size distribution, regulation distorts the size-capital intensity relationship above $\bar{n}$. To better understand this distortion, one can write the expression of the distortion $\delta^\rho(n) \equiv \rho_1^n(n) - \rho_0^n(n)$ for τ small:

$$\delta^\rho(n) = \overbrace{\frac{\partial \rho_1(\alpha_1(n))}{\partial \alpha} \times \frac{\partial \alpha_1(n)}{\partial \tau}}^{\text{Selection Effect}} + \overbrace{\frac{\partial \rho_1(\alpha_1(n))}{\partial \tau}}^{\text{Substitution Effect}} .$$

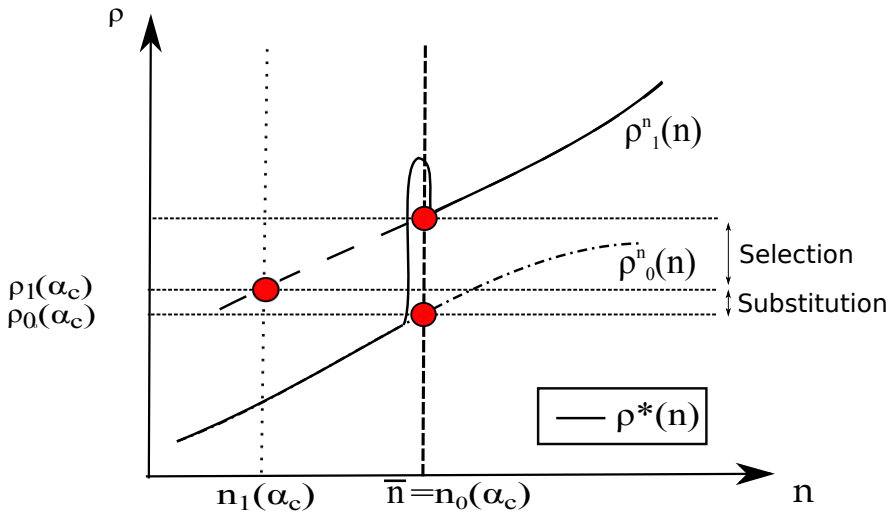
$\delta^\rho(n)$ has two parts: a selection effect and a substitution effect. Selection effect comes from the fact that regulated firms are more efficient for a given size and that higher efficiency is potentially associated with a different capital intensity. Selection effect differs from 0 insofar as α causes factor biased technological differences ($\frac{\partial \rho_1(\alpha_1(n))}{\partial \alpha} \neq 0$). Substitution effect reflects the within-firm capital-labor substitution that occurs as a response to the upward shift in the relative cost of labor to capital created by the regulation. It is of the same sign as σ. Remark that if σ tends to 0 (and so substitution effect is nil) and technological differences are biased, regulation still creates a distortion in the factor cost ratio-size relationship, just because of the selection effect. A regression discontinuity approach would take $\delta^\rho(\bar{n})$ as an estimate of the substitution effect induced by the regulation. It would thus be biased upward if more productive firms are also more capitalistic (as suggested by my data). This analysis points out the importance of allowing for factor biased technological differences in my structural estimation.

I conclude with describing the behavior of the factor cost ratio at the threshold. There is a range of efficiencies $[\alpha_0(\bar{n}-1), \alpha_u]$ such that firms bunch at $\bar{n}-1$. Firms with $\alpha \in [\alpha_c, \alpha_u]$ demand $k_b(\alpha)$ units of capital. So capital intensity at $\bar{n}-1$ is

$$\rho^n(\bar{n}-1) = \frac{\int_{\alpha_0(\bar{n}-1)}^{\alpha_c} \rho_0(\alpha) g(\alpha) d\alpha + \int_{\alpha_c}^{\alpha_u} \frac{r k_b(\alpha)}{w\bar{l}} g(\alpha) d\alpha}{\int_{\alpha_0(\bar{n}-1)}^{\alpha_u} g(\alpha) d\alpha}.$$

Bunching firms mitigate the cost of being small by imbalancing their capital-labor ratio, this suggests that the factor cost ratio should be abnormally high in a neighborhood below the threshold.

Figure 11 – Theoretical Relationship Between Factor Cost Ratio and Size



The main patterns of the relationship between factor cost ratio and size are illustrated in figure 11. As in the data, this relationship is broken at the threshold. However, the excessive factor cost ratio below the threshold is not visible in the data. As explained when presenting the motivating evidence, it might be that firms only transitorily bunch at the threshold so that they do not have time to adjust there capital stock. In other words, my model fail at replicating the investment behavior of firms below the threshold because it it a zone where firms transit from being not regulated to being regulated. And as a static model, my model is obviously not equipped to predict a transition. Nevertheless, as firms move away from the threshold, they converge back to a long run investment behavior for which my static model is more adapted. In the estimation, we address that limitation of the model by not using the information on capital around the threshold as a source of identification.

Next section describes how I use these predictions to identify the tax equivalent τ and the elasticity of substitution σ.

4 Estimation Strategy

My estimation strategy consists in two steps. In a first step, I estimate the factor demands of firm α_c (i.e. the lowest efficiency firm bunching at 50) in two counterfactual situations: a situation where regulation binds all firms (situation 1) and a situation where regulation binds no firm (situation 0). A this stage, I do not make use of the CES assumption on technology.

Then, in a second stage, I use the structural expression of the factor demands (equations (4) and (6)). The estimated tax equivalent and elasticity of substitution equalize the structural factor demands to factor demands estimated in the first step.

4.1 Estimation of Counterfactual Factor Demands

The first step of the estimation has two substeps which I explain now. First I identify the counterfactual labor demands from the firm size distribution. Intuitively, the excess mass of firms below the threshold will identify by how much firms would shrink their workforce by switching from full regulation to no regulation. Then, I use the counterfactual labor demands and the relationship between the factor cost ratio and size to estimate the counterfactual factor cost ratios.

Estimation of Labour Response In this paragraph, we propose an estimation strategy for $\{n_0(\alpha_c), n_1(\alpha_c)\}$. One of these two labor demands is known by definition since $n_0(\alpha_c) \equiv \bar{n}$, with $\bar{n} = 50$ in the French case. As for the counterpart $n_1(\alpha_c)$, it is identified from the distortion in the size distribution at the threshold. I will first give a graphical intuition of the estimation.

Let us consider figure 9 again. In red is the mass of bunching firms. How would this mass of firms distributes itself in a counterfactual situation where regulation applies to all firms? First of all, in that situation there would be no incentive to bunch at 50 since regulation applies anyway, whether a firm is at 49 or at 51. Therefore bunching firms would spread over an interval of sizes. The boundaries of this interval would correspond to the counterfactual labor demand $n_1(\alpha)$ of, respectively, the least efficient and the most efficient bunching firms, namely α_c and α_u. This is precisely what the blue area is, namely the density of firms located between $n_1(\alpha_c)$ and $n_1(\alpha_u)$ in the full regulation counterfactual. It results that *the blue area is equal to the red area.* I can now write this mathematically:

$$\delta^g = \int_{n_1(\alpha_c)}^{n_1(\alpha_u)} g_1^n(n')\, dn'. \tag{16}$$

The left hand side of equation(16) is the excess mass at 49, it corresponds to the red area in figure 9. The right hand side corresponds to the blue area. Based on this insight and up to a few technical details which I cover below, the way I estimate $n_1(\alpha_c)$ is the following:

1. I estimate the excessive mass at the threshold (the red area)
2. I look for the value of $n_1(\alpha_c)$ which equates the blue area to the excessive mass.

In order to implement this estimation, I must be able to evaluate the blue area. Put differently, I must know the value of function $h(x) \equiv \int_x^{n_1(\alpha_u)} g_1^n(n')\, dn'$ for any x in a neighborhood of 50. I identify function $g^n(n)$ by regressing firm size distribution above 50 over a polynomial of n. In order to account for the fact that the counterfactual distribution, $g_1^n(n)$, departs from the actual distribution over $[50, n_1(\alpha_u)]$, I add to the regression size dummies over the interval

$[50, n_B]$ where n_B is a size calibrated to be arguably larger than $n_1(\alpha_u)$. Another issue with evaluating $h(x)$ is that the upper bound of the integral, $n_1(\alpha_u)$, is unknown. However, after a few lines of algebra, it is possible to get rid of $n_1(\alpha_u)$ in $h(x)$:

$$\begin{aligned} h(x) &= \int_x^{50} g_1^n(n')\,dn' + \int_{50}^{n_1(\alpha_u)} g_1^n(n')\,dn' \\ &= \int_x^{50} g_1^n(n')\,dn' + \int_{50}^{n_1(\alpha_u)} g_1^n(n') - \underbrace{g^n(n')}_{=0}\,dn' + \int_{n_1(\alpha_u)}^{n_B} \underbrace{g_1^n(n') - g^n(n')}_{=0}\,dn' \\ &= \int_x^{50} g_1^n(n')\,dn' + \int_{50}^{n_B} g_1^n(n') - g^n(n')\,dn' \end{aligned}$$

$\int_x^{50} g_1^n(n')$ is picked up by extrapolating the estimate of function $g_1^n(n)$ (obtained from size distribution above n_B) to the interval $[x, 50]$. $\int_{50}^{n_B} g_1^n(n', \tau) - g^n(n)\,dn'$ is estimated as the sum of the dummies over $[50, n_B]$, in the regression fitting the size distribution.

Since I estimate $n_1(\alpha_c)$ as a solution to equation (16), I also need to estimate the excessive mass δ^g. The way I proceed is similar to the literature on bunching (e.g. Saez (2010), Chetty et al. (2009)). I regress the observed firm size distribution below the threshold over a polynomial of n and a set of size dummies over an interval $[n_A, 50]$, with n_A calibrated to fall in a region in which firm size distribution is arguably not distorted by the presence of bunching firms at the threshold. These dummies capture the excessive mass.

To summarize and formalize, to estimate $n_1(\alpha_c)$, I first fit the firm size distribution above the threshold and below the threshold by running following regression:

$$g^n(n) = \sum_{p=0}^{P_0} b_{p,0} n^p + \sum_{p=0}^{P_1} b_{p,1} n^p \times \mathbb{1}\{n \geq 50\} + \sum_{i=n_A}^{n_B} d_i \mathbb{1}\{n = i\} + u_g \tag{17}$$

With $g^n(n)$ the firm size density, observed in the data, P_0 and P_1 the order of the polynomial used to fit the size distribution respectively below and above 50, and u_g the error term. Coefficients $\{b_{p,0}\}$ are identified from the size distribution below n_A. Coefficients $\{b_{p,1}\}$ are identified from the size distribution above n_B. Dummies $\{d_i\}$ are identified from the distortion in the distribution between n_A and n_B.

From equation (17), excess mass is identified by the sum of dummies below the threshold

$$\hat{\delta^g} = \sum_{i=n_A}^{49} \hat{d_i}$$

while $h(x)$ is obtained as the sum over $[x, 49]$ of the distribution fitted above 50, minus the size dummies $\{\hat{d_i}\}_{i=50,..,n_B}$:

$$\hat{h}(x) = \sum_{n=x}^{49} \sum_{p=0}^{P_1} \hat{b}_{p,1} n^p - \sum_{i=50}^{n_B} \hat{d_i}$$

Then, the estimator of $\hat{n_1}(\alpha_c)$ is a solution to $\hat{\delta^g} = \hat{h}(x)$:

$$\hat{n_1}(\alpha_c) : \hat{\delta^g} = \hat{h}\left(\hat{n_1}(\alpha_c)\right). \tag{18}$$

4.2 Estimation of Capital Intensity Response

By definition of ρ_i^n (see equation (15)), $\{\rho_0(\alpha), \rho_\tau(\alpha)\}$ verifies

$$\rho_0(\alpha_c) = \rho_0^n(\bar{n})$$
$$\rho_1(\alpha_c) = \rho_1^n(n_1(\alpha_c))$$

Since $\bar{n}$ is known and $n_1(\alpha_c)$ has been estimated in a first step, the estimation of counterfactual capital intensities amounts to estimating functions $\rho_i^n(n)$. Analogously to the labour response estimation, I assume that functions ρ_0^n and ρ_1^n can be approximated by finite order polynomials which I estimate be regressing factor cost ratio over size through following regression:

$$\rho^n(n) = \sum_{p=0}^{P_0'} b'_{p,0} n^p + \sum_{p=0}^{P_1'} b'_{p,1} n^p \times \mathbb{1}\{n \geq \bar{n}\} + \sum_{i=n_A}^{n_B} d'_i \mathbb{1}\{n = i\} + u'_n \tag{19}$$

With $\rho^n(n)$ the factor cost ratio conditional on size observed in the data and u_ρ the error term. In this equation, the set of dummies only controls for the wedge between actual capital intensity and counterfactual capital intensity in a neighborhood of $\bar{n}$. It is of no further use in the estimation. $\{\rho_0(\alpha_c), \rho_1(\alpha_c)\}$ is estimated by using the value of counterfactual capital intensities predicted by the polynomials at the sizes of interest, $\bar{n}$ and $\widehat{n_1(\alpha_c)}$:

$$\widehat{\rho_0(\alpha_c)} = \sum_{p=0}^{P_0'} \hat{b'}_{p,0} \bar{n}^p$$
$$\widehat{\rho_1(\alpha_c)} = \sum_{p=0}^{P_1'} \hat{b'}_{p,1} \widehat{n_1(\alpha_c)}^p$$

4.3 Mapping Counterfactual Factor Demands to Structural Parameters

The system of input demands by α_c in the "no regulation" and the "full regulation" situations is:

$$\begin{cases} \bar{n} & = \left(\frac{w}{c_0(\alpha_c)}\right)^{-\sigma} B(\alpha_c)^{\sigma-1} y_0(\alpha_c) \\ n_1(\alpha_c) & = \left(\frac{(1+\tau)w}{c_1(\alpha_c)}\right)^{-\sigma} B(\alpha_c)^{\sigma-1} y_1(\alpha_c) \\ \rho_0(\alpha_c) & = \left(\frac{r}{w}\right)^{1-\sigma} \left(\frac{A}{B}(\alpha_c)\right)^{\sigma-1} \\ \rho_1(\alpha_c) & = (1+\tau)^\sigma \left(\frac{r}{w}\right)^{1-\sigma} \left(\frac{A}{B}(\alpha_c)\right)^{\sigma-1}. \end{cases}$$

This system can be concentrated over $\bar{n}$, $n_1(\alpha_c)$, $\rho_0(\alpha_c)$, $\rho_1(\alpha_c)$, τ, σ and ε:

$$\begin{cases} \frac{n_1(\alpha_c)}{\bar{n}} & = (1+\tau)^{-\sigma} \left(\frac{(1+\tau)^{1-\sigma} + \rho_0(\alpha_c)}{1+\rho_0(\alpha_c)} \right)^{\varepsilon-\sigma} \\ \frac{\rho_1(\alpha_c)}{\rho_0(\alpha_c)} & = (1+\tau)^{\sigma}. \end{cases} \tag{20}$$

Factor demands were estimated in previous steps. So I can plug their estimates in system (20), calibrate ε and then estimate $\{\hat{\tau}, \hat{\sigma}\}$ as a solution to the system. I estimate standard errors via bootstrap. Namely, I draw with replacements 500 samples of firms of the size of my dataset and I run the estimation for each of them.

Next section presents the results.

5 Results

There is a handful a parameters that I need to calibrate to run the estimation. First parameters are P_0 and P_1. I fit the distribution below and above the threshold with polynomials of order one and two respectively. In practice, choosing too large numbers for P_0 and P_1 generates an over fitting of the distribution and results into a bad estimation of the distortion over $[n_A, n_B]$. Another parameter to calibrate is n_A, namely the lower bound to the interval over which distortions around the threshold are estimated. I choose $n_A = 40$ because it is the size where the bulge of productivities starts in figure 2. I set $n_B = 56$ because it the expected destination size of firms which grow at the threshold (see figure 10). The idea is that the distortion in the size distribution just above 50 comes from firms not wanting to be there. I use the size dynamics to get a sense of where firms 'land' when they jump over this unprofitable range of sizes.

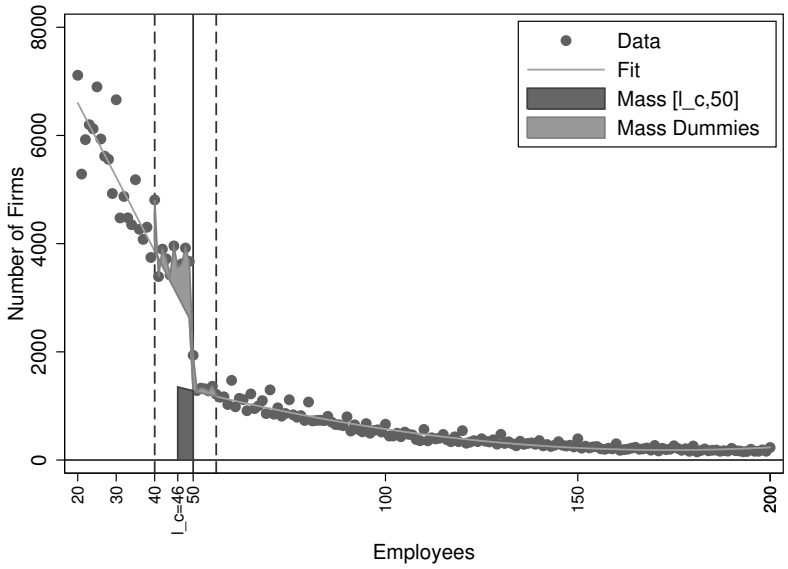
In order to visually assess the quality of the fit, I plot the prediction of the regression of size density (model (17)) in figure 12. The sum of size dummies from 40 to 50, which identifies the mass of bunching firms, represents 3% of firms in my sample. This is a small number, especially if one considers the fact that I do not observe firms below 20 employees. However, this small mass of bunching firms does not imply a small aggregate impact as all firms above 50 are also constrained by the regulation.

I estimate $n_1(\alpha_c)$ by equalizing the sum of size dummies with the mass of firms between $n_1(\alpha_c)$ and n_B. I illustrate the solution to this problem in figure 12. I find $\widehat{n_1(\alpha_c)} = 46$ meaning that the bunching firm with the lowest productivity would be 4 employees smaller if regulation were to bind all firms, all else equal.

In order to estimate counterfactual factor cost ratio, I estimate model (19) and then take the prediction at $n = 50$ and $n = \widehat{n_1(\alpha_c)}$. This procedure is illustrated in figure 13.

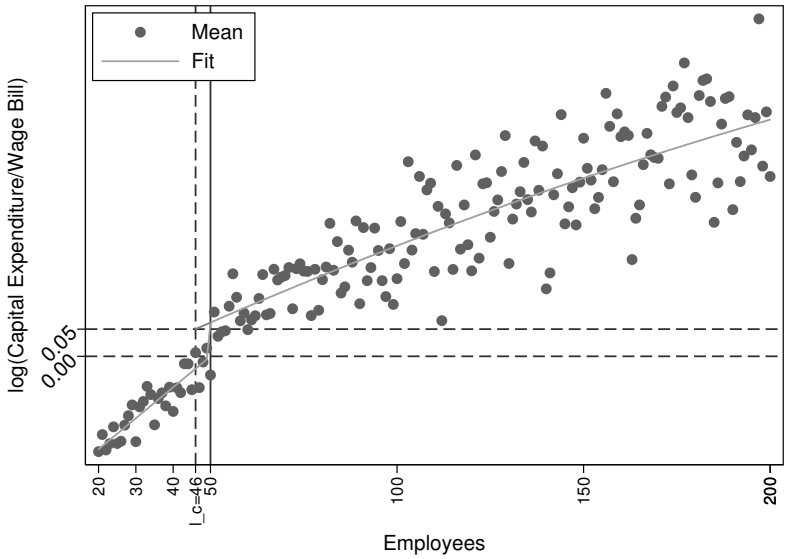
With the counterfactual factor demands estimated, I now invert system (20) over σ and τ. To this end, I need to calibrate ε. The way I proceed is I implement Levinsohn & Petrin (2003) semi-parametric estimation of a cobb-douglas production function with capital and labor. Put simply, the estimation consists in regressing value added on capital and labor and using materials as a control for (unobserved) total factor productivity. I use the sum

Figure 12 – Fitting Firm Size Distribution I



Note: Area 'Mass over $[l_c; 50]$' is the area between the x-axis and the extrapolation of the distribution fit above 50 over the range [46,50]. Area 'Mass over $[l_c; 50]$' and area 'Mass Dummies' are equal.

Figure 13 – Fitting Capital Intensity Versus Size



Note: Capital Expenditure=0.1*Fixed Assets. Fit is the prediction of a regression of log Capital Expenditure on Employees, a dummy (size≥50), a collection of interaction terms $Employees \times dummy(size \geq 50)$ and $Employees^2 \times dummy(size \geq 50)$, a set of dummies for employees $\in [40, 56]$, a set of 2-digit industry dummies and a constant term.

of obtained coefficients on labor and capital as an estimate of the curvature of the revenue function defined in (7). I get $\frac{\varepsilon-1}{\varepsilon} = 0.837$.

Table 3 – Parameter Estimates

Parameter	*Estimate*	95% *Confidence Interval*	
		Lower Bound	Upper Bound
response # employees $(50 - n_1(\alpha_c))$:	4.1	3.4	4.8
response factor cost ratio $(\ln \frac{\rho_1(\alpha_c)}{\rho_0(\alpha_c)})$:	0.05	0.01	0.08
excessive mass at 50 (# Firms)	6.67e+03	5.75e+03	7.59e+03
excessive mass at 50 (% of firms)	3.05	2.63	3.47
τ	0.077	-0.001	0.135
σ	0.58	0.07	4.80

Notes: Estimates of confidence interval are obtained with 500 bootstrap replications.
The confidence interval is at 95% and is estimated from the basic percentile method.

Finally, I find a tax equivalent of 8% and an elasticity of substitution of 0.6. Table 3 summarizes the results. The value of τ is slightly larger than in comparable studies.[10] However, in present paper, the cost of labor is only one part of the total cost of production. In order to make our result more comparable to the relevant benchmark, I compute the increase in percents in the total unit cost of production of a firm at the threshold:

$$\ln \frac{\widehat{C_1(\alpha_c)}}{C_0(\alpha_c)} = \frac{1}{1 + \hat{\rho}_0(\alpha_c)} \tau = 0.035$$

I get that regulation increases the total cost of a firm at the threshold by 3.5% which is smaller (although not significantly) than existing estimates. This is consistent with the idea that controlling for the substitution behavior of firms leads to infer a smaller cost of the regulation from a given dataset. The reason for that is simply that substitution allows firms to bunch at a lower cost so that a smaller regulation cost is necessary to rationalize the excessive mass at the threshold.

As to the elasticity of substitution, I get a value close to recent studies (Chirinko et al. (2004) and Raval (2011) estimate long run capital-labor elasticity of substitution around 0.5). I obtain a standard error too large, however, to reject the null hypothesis that σ is larger or equal to one.

In order to put these results in perspective, next section makes us of our estimates to implements policy experiments.

6 Counterfactual Analysis

In this section, I investigate the impact of variations in τ over the equilibrium. Since regulation potentially affects relative price of labor to capital, I will be especially interested in the factor income redistribution effects of modifying the regulation. To quantify these general equilibrium effects, I first need to specify the structure of the economy.

10. Garicano et al. (2013) and Gourio & Roys (2012) get estimates of τ between 4% and 7%.

6.1 The Structure of the General Equilibrium

My general equilibrium model builds upon the closed version of Melitz (2003) model and embed the partial equilibrium model of the firm developed in section 3. More specifically, I generalize the assumption on technology made by Melitz (2003) by allowing firms to produce out of both capital and labor through a CES function. The economy is at a stationary equilibrium and is populated with a unitary mass of individuals. Each individual is endowed with N_i units of labour and K_i units of capital, with the sum of individual endowments summing to N and K respectively Labor is the numeraire ($w = 1$).

Demand Consumers have CES preferences over a continuum of varieties of a differentiated good. Given the homotheticity of these preference, I rely on a representative consumer approach even if individual consumers have different budget constraints. The demand of the representative consumer for a variety ω is

$$y(\omega) = p(\omega)^{-\varepsilon} Z.$$

Residual demand functions therefore have the same expression as in section 3 although Z is now made endogenous and verifies

$$Z = P^{1-\varepsilon} R,$$

with R the total expenditure of the representative consumer over the differentiated good and P is the CES price index over the varieties. Letting M be the mass of incumbent firms in the economy, P verifies:

$$P = \left(M \int p(\alpha)^{1-\varepsilon} g(\alpha) d\alpha \right)^{\frac{1}{1-\varepsilon}}. \tag{21}$$

Production, entry and exit The technology and the regulatory context are those assumed in section 3. As a consequence the optimal price, output, factor demand and operating profit of incumbent firms extend to present general equilibrium setting.

There is a large pool of ex-ante identical prospective entrants in the industry. To enter, firms pay a sunk cost f_e, labeled in capital and draw an efficiency α in a distribution $\lambda(\alpha)$. Drawn productivity remains constant over time. Upon entry, firms may decide to exit immediately. Given the fact that α is constant, that the equilibrium is stationary, and that the value of exiting the market is zero, endogenous exit will happen if and only if the current profit of a firm is negative. To allow for this possibility, I assume that firms need to pay a fixed cost per period f, labeled in capital, to stay in the market. Since operating profits

are strictly decreasing with α, there exists a cutoff firm α_e, indifferent to exit or stay in the market:

$$\pi(\alpha_e) = rf, \tag{22}$$

and the probability of staying in the market for a new entrant is $1 - \Lambda(\alpha)$, with Λ the CDF associated to λ. The stationary distribution of firms, $g(\alpha)$ is therefore linked to entry distribution as follows

$$g(\alpha) = \begin{cases} \frac{\lambda(\alpha)}{1-\Lambda(\alpha)} & \text{if } \alpha \geq \alpha_e \\ 0 & \text{otherwise.} \end{cases}$$

The value of drawing a productivity to a prospective entrant, v_e, is equal to the probability of staying in the market multiplied by the expectation of the infinite sum of future profits, discounted by the probability of the death shock: $v_e = (1 - \Lambda(\alpha)) \int \sum_{t=0}^{\infty} (1-\delta)^t [\pi(\alpha) - rf] g(\alpha) d\alpha$. To decide whether to enter the market or not, a prospective entrant compares v_e to the cost of entry rf_e. Free entry in the economy equalizes v_e to rf_e.

$$\frac{(1 - \Lambda(\alpha))}{\delta} \int [\pi(\alpha) - rf g(\alpha)] d\alpha = rf_e. \tag{23}$$

Equilibrium At the equilibrium, factor markets clear:

$$\begin{aligned} K &= M \int_{\alpha_c} [k(\alpha) + rf]\, g(\alpha) d\alpha + M_e r f_e \\ N &= M \int_{\alpha_c} n(\alpha) g(\alpha) d\alpha, \end{aligned} \tag{24}$$

with M_e the masse of new entrant firms at every period.

On top of its revenues from factors, the representative consumer owns firms and so gets dividends, and she also gets redistributed the tax on labor. The representative consumer exhaust its income by consuming the differentiated good and creating new firms. Therefore her budget constraint is:

$$rK + L + M \int_{\alpha_c} [\pi(\alpha) - rf] g(\alpha) + \tau \int_{\alpha_u} n(\alpha) g(\alpha) d\alpha = M_e r f_e + R. \tag{25}$$

Finally, stationarity imposes the flow of exiting firms to equal the flow of new firms:

$$(1 - \Lambda(\alpha_c))M_e = \delta M, \tag{26}$$

with M_e the mass of entering firms at each period.

An equilibrium is characterized:

— by a set of cutoffs $\{\alpha_e, \alpha_u, \}$ such that the zero-profit condition (22) and indifference condition on bunching (9) are verified,

— by a set of individual input demands $\{k(\alpha), n(\alpha)\}$ such that profit is maximized, i.e verifying (10) and (11),

— by an aggregate expenditure consistent with the budget constraint (25),

— by a price index P consistent with its definition (21)

— by a mass of incumbent M such that the free entry condition (23) is verified,

— by a flow of new entrants M_e such that the flow balance condition (26) applies

— by an interest rate r insure that capital market clears (equation (24)),

6.2 Results

This subsection is preliminary. In table 4 I describe the calibration which I intend to use.

Table 4 – Calibration

Parameter	**method**	**source/matched moment**
$A(\alpha) = \alpha$	*normalized*	
$B(\alpha) = \alpha^b$	*calibrated*	capital ratio difference below/above 50
$\lambda(\alpha) = \theta\alpha^{-\theta}$	*calibrated*	share workers below 50
f	*normalized*	
f_e	*calibrated*	number of firms in the economy
δ	*normalized*	Burstein & Melitz (2011)
σ	*estimated*	
ε	*estimated*	
τ	*estimated*	
K	*calibrated*	aggregate labor share=0.6
L	*calibrated*	number of workers in the economy

My intention is to compute the effect of **de-regulation** on the real wage $1/P$, the real interest rate r/P, the relative share of capital rK/wL and aggregate welfare R/P. In order to isolate the role of capital intensity heterogeneity, I intend to run this experiment in two cases: one where all firms have the same factor share and one where the relative capital intensity between and above the threshold matches the data.

7 Conclusion

I this paper, I have analyzed the impact of size dependent regulation on factor income distribution. Drawing upon a census of French firms, I find suggestive evidence of the opposite

effects. First, the regulation is costly - as revealed by the excess mass of firms at the threshold - and regulated firms are relatively more capital intensive. This suggests that the regulation, by imposing a cost on large (capital-intensive) firms tend to make small firms expand so that the whole economy is less capital intensive. Second, the pattern on capital labor ratio around the threshold suggest that firms substitute capital for ratio in order to mitigate the cost of the regulation. This second effect tends to make the economy more capital intensive.

To find out which of the between and within firm substitution effect dominates, I write down a general equilibrium model which I intend to calibrate. Among the parameters to be calibrates, two important ones - the implicit cost of the regulation and the capital labor elasticity of substitution - have been structurally estimated.

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APPENDIX

A Appendix Tables

Table 5 – Descriptive Statistics

Year	# Firms	# Employees	Wage Bill	Fixed Assets	Value Added
1995	2.14e+04	2.81e+06	6.69e+10	1.95e+11	1.60e+11
1996	2.10e+04	2.77e+06	7.09e+10	2.28e+11	1.59e+11
1997	2.12e+04	2.75e+06	7.25e+10	2.42e+11	1.67e+11
1998	2.08e+04	2.73e+06	7.37e+10	2.42e+11	1.71e+11
1999	2.07e+04	2.74e+06	7.55e+10	2.54e+11	1.79e+11
2000	2.05e+04	2.77e+06	7.83e+10	2.66e+11	1.88e+11
2001	2.06e+04	2.80e+06	8.08e+10	2.81e+11	1.81e+11
2002	2.03e+04	2.74e+06	8.17e+10	2.90e+11	1.81e+11
2003	1.98e+04	2.66e+06	8.16e+10	3.01e+11	1.81e+11
2004	1.92e+04	2.58e+06	8.09e+10	3.05e+11	1.84e+11
2005	1.88e+04	2.52e+06	8.16e+10	3.17e+11	1.87e+11
2006	1.80e+04	2.47e+06	8.31e+10	3.25e+11	1.87e+11
2007	1.70e+04	2.32e+06	8.07e+10	3.22e+11	1.84e+11

Table 6 – Descriptive Statistics

Variable	Mean	q1	q25	q50	q75	q99
Above 50 Dummy	0.42	0	0	0	1	1
Fixed Assets	1.4e+07	0.00	5.2e+05	1.4e+06	4.3e+06	1.8e+08
Employees	133.7	20	30	44	91	1416
Factor Cost Ratio	0.21	0.00	0.06	0.13	0.24	1.15
Wage Bill	3.9e+06	3.2e+05	6.8e+05	1.1e+06	2.2e+06	4.7e+07
Value Added	8.9e+06	3.3e+05	1.2e+06	2.0e+06	4.3e+06	1.1e+08

B Proof Proposition 1

We first demonstrate that $\Delta(\alpha)$ has a unique root over $[\alpha_c, +\infty)$. Let $\tilde{\alpha}$ be such that

$$n_1(\tilde{\alpha}) = \bar{n}$$

$\frac{\partial n_1(\alpha)}{\partial \alpha} > 0$ guarantees the uniqueness of $\tilde{\alpha}$. We will proceed by showing that:

(1) $\Delta(\alpha_c) < 0$

(2) $\Delta'(\alpha) \leq 0$ over $[\alpha_c, \tilde{\alpha}]$

(3) $\Delta'(\alpha) > 0$ over $[\tilde{\alpha}, +\infty]$

(1) and (2) guarantee that $\Delta(\alpha)$ has no root over $[\alpha_c, \tilde{\alpha}]$. (1), (2) and (3) guarantee that $\Delta(\alpha)$ has a unique root over $[\tilde{\alpha}, +\infty]$.

Regulation does not bind to firms α_c so it is clear that $\Delta(\alpha_c) < 0$. From the envelope theorem we get the general expression of the derivative of $\Delta(\alpha)$:

$$\Delta'(\alpha) = \frac{\partial R}{\partial \alpha}(k_1(\alpha), n_1(\alpha), \alpha) - \frac{\partial R}{\partial \alpha}(k_b(\alpha), \bar{n}, \alpha) \tag{27}$$

With $k_b(\alpha)$ the optimal capital demand of a bunching firm and $k_1(\alpha)$ the optimal capital of a regulated firm:

$$k_b(\alpha) : \frac{\partial R}{\partial k}(k_b(\alpha), \bar{n}; \alpha) = r$$
$$k_1(\alpha) : \frac{\partial R}{\partial k}(k_1(\alpha), n_1(\alpha); \alpha) = r$$

The definition of $\tilde{\alpha}$ and the fact that $\frac{\partial n_1(\alpha)}{\partial \alpha} > 0$ imply that $n_1(\alpha) \leq \bar{n}$ over $[\alpha_c, \tilde{\alpha}]$. $\sigma > 0$ implies that inputs are (imperfect) complements and so the same ranking holds for capital: $\forall\, \alpha \in [\alpha_c, \tilde{\alpha}], \quad k_1(\alpha) \leq k_b(\alpha)$. Since $\frac{\partial R}{\partial \alpha}(k, n; \alpha)$ is increasing in (k,n), it follows from equation (27) that $\Delta'(\alpha) \leq 0$ over $[\alpha_c, \tilde{\alpha}]$. Applying similar reasoning over $[\tilde{\alpha}, \infty]$, one establishes that $\forall\, \alpha > \tilde{\alpha}, \quad \Delta'(\alpha) > 0$. $\Delta(\alpha)$ has therefore a unique root $\alpha_u > \tilde{\alpha}$.

Moreover, α_u verifies $\Delta'(\alpha_u) > 0$. Finally, since $n_1(\tilde{\alpha}) = \bar{n}$ by definition, and $n_1(\alpha)$ is increasing in α, $n_1(\alpha_u) > \bar{n}$.

□