

Time-Consistent Management of a Liquidity Trap: Monetary and Fiscal policy with Debt*

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Abstract

I study optimal monetary and fiscal policy in a New Keynesian model with an occasionally binding zero lower bound that leads to liquidity trap episodes. I analyze the use of government spending and labor income tax as components of the discretionary fiscal stimulus package at the liquidity trap. Reliance on either of these instruments depends on whether the government budget is relaxed or has to be balanced. If the government has to balance its budget period by period, it relies more on the spending instrument. Varying the debt burden across time makes the government rely more on the use of labor taxes because discretionary incentives introduced by debt help to reduce the time-inconsistency problem of the tax rate response at the liquidity trap. Moreover, I show that the risk of falling into the liquidity trap leads to the accumulation of the optimal long run government debt buffer that reduces the frequency of reaching the zero lower bound.

JEL Codes: E52, E62, E63, H21, H63

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1 Introduction

A shortfall of demand in an economy calls for a firm policy response. The conventional wisdom until the Great Recession was for monetary policy to bear the burden of stabilizing the economy.¹ In December 2008, in the midst of the Great Recession, the Federal Reserve lowered the federal funds rate almost down to zero. Situations in which the zero lower bound prevents the monetary authority from providing enough stimuli are often referred to as liquidity traps. The European Central Bank, the Bank of Japan, and the central banks of other smaller advanced countries are in liquidity traps today. The constraints imposed on the conventional monetary policy have led academics to consider a number of alternative policy tools to stabilize business cycles, such as fiscal policy and unconventional monetary policy.

Renewed academic interest in fiscal policy was coupled with the implementation of policy during the Great Recession. In February 2009, an unprecedented stimulus package was enacted in the United States, the “American Recovery and Reinvestment Act.” Based on Keynesian ideas, the majority of the package and the subsequent fiscal adjustment came in the form of government spending increases and payroll tax changes. Following the onset of the Great Recession, other countries with policy rates trapped near the minimum feasible levels turned to fiscal adjustments of government spending and taxation in various magnitudes and directions. Figure 1 illustrates this fact by depicting the changes in government spending and taxation since 2007 for a subset of the OECD countries.

[Figure 1 about here.]

The binding zero lower bound is known to increase the time-inconsistency of the optimal monetary policy and to severely impair stabilization under discretion, as demonstrated by [Adam and Billi \(2007\)](#).² Moreover, existing institutional arrangements typically support only a limited horizon of accountability for the treasuries. This motivates me to focus on the challenging case of monetary discretion and to examine the scope of solutions that fiscal policy can offer to improve stabilization while remaining discretionary.

This paper contributes to the line of research that studies optimal time-consistent monetary and fiscal policy with an occasionally binding zero lower bound constraint on the nominal interest rates. The discretionary use of government spending for stabilization purposes has been shown to be optimal in a number of studies with New Keynesian models by [Nakata \(2013\)](#), [Schmidt \(2013\)](#), and [Werning \(2011\)](#).³ The assumptions of lump-sum taxes and Ricardian equivalence are frequently made in such models. The modeling choice of using lump-sum taxes restricts the study of the various ways that taxes can influence the margins of private decisions and how they may be used to partially offset the effect of the zero lower bound. In fact, a diverse set of tax instruments can be used to circumvent the relevance of the zero lower bound along the lines of unconventional fiscal policy, as discussed in [Correia et al. \(2013\)](#). Although their work set an important benchmark in the study of the optimal fiscal policy at the liquidity trap, the application of unconventional fiscal policy requires the kind of consumption tax which is applied after prices are set by firms; in the United States, it is imposed at the state or local level, and it is absent in many other countries. It is therefore instructive to consider the case of the joint use of government spending and a

¹See [Kirsanova et al. \(2009\)](#) for a discussion of the consensus assignment of monetary and fiscal policy.

²See also [Levine et al. \(2008\)](#) and [Nakov \(2008\)](#) for assessment of welfare gain from monetary commitment.

³Government spending is neutral under joint monetary and fiscal discretion and no zero lower bound; see, e.g., [Gnocchi \(2013\)](#).

restricted set of tax instruments to determine whether there are reasons to rely on one over the other.

I frame the analysis in a standard New Keynesian model with monopolistic competition and costly price adjustment augmented with a fiscal sector. The government is benevolent and chooses a discretionary short-term nominal interest rate, a level of government spending that has an intrinsic value, and the government debt supply. I assume that the only tax available to the government is a distortionary flat-rate labor income tax.⁴ In doing so, I follow the tradition of the literature on optimal fiscal policy built upon [Lucas and Stokey \(1983\)](#). The model economy is subject to uncertain demand stemming from *ad hoc* variation in the time preference of households. A sufficiently strong preference shock may bring demand for output low enough to call for a negative real interest rate and, at moderate inflation rates, for full stabilization to require a negative nominal interest rate, which would effectively make the zero lower bound binding and bring the economy into the liquidity trap.⁵ Failure to offset falling demand leads to downward pressure on prices and output. In order to focus on the dynamic demand stabilization problem, I abstract from the static distortions that would otherwise lead to the usual average inflation bias due to policy discretion. The model is set up in such a way that the deterministic steady state features zero government debt. By establishing zero debt as the *status quo* in the setting without uncertainty, I ensure that any long run level of debt that appears in the stochastic version of the model is due to the risk of binding zero lower bound and any debt dynamics attached to this risk are for stabilization purposes. To solve a stochastic version of the model, I use a global nonlinear solution method.

In my model economy, with two fiscal instruments, the government can improve stabilization when the economy is subject to demand fluctuations that make the zero lower bound effectively binding. For small demand fluctuations that call for positive real rates, it is optimal for the government to rely on nominal interest rate variation in order to stabilize the economy. However, variations in both government spending and labor taxes have a stabilizing role upon entering the liquidity trap. When the nominal interest rate hits the zero lower bound, it is optimal to respond by increasing government spending and raising the labor tax rate. Initial government spending and the labor tax responses monotonically decline over time, and both instruments revert to pre-crisis levels by the time the liquidity trap is over. The magnitude of both responses depends crucially on whether the government budget is balanced or relaxed.

The government with a balanced budget relies heavily on the use of government spending. Increasing government spending is helpful in cushioning the fall of aggregate demand, but its use requires deviation from the constant efficient level. The efficacy of the labor tax as a stabilization instrument is improved when it is used in conjunction with debt so that the government shifts reliance away from the spending instrument. Relaxing the government budget makes the increase of spending one order of magnitude smaller and the labor tax change roughly three times larger than in the case with the balanced budget. Raising the labor tax rate, when the nominal interest rate is at the zero lower bound, is inflationary because of the positive cost push effect. Higher taxes can offset deflationary pressure at the liquidity trap and reduce real interest rates. The discretionary government in a given period of time at the liquidity trap, however, does not internalize the effect of the tax increase on demand stabilization in preceding periods. Presence of debt is necessary to create a discre-

⁴Assuming, instead, a VAT tax does not affect my considerations.

⁵To build richer quantitative models, one has to incorporate risk premium shocks, as discussed by [Amano and Shukayev \(2012\)](#) and [Coibion et al. \(2012\)](#).

tionary incentive for the government to reduce its debt burden via the inflationary effect of higher taxes. Households, anticipating this incentive, are less willing to hold government bonds with a lower return and run down their bond holdings. Households substitute savings with current consumption and improve aggregate demand. Over the course of the liquidity trap the government with a flexible budget runs a surplus and every period rolls over only a fraction of the inherited debt to accommodate smoothing of consumption by households.

In normal times, when the zero lower bound is not binding, optimal discretionary policy dictates that the government accumulate or reduce debt until it reaches a positive optimal stock exceeding the efficient deterministic level of zero. Such an outcome is an example of how responses around the deterministic steady state are sub-optimal and how short run dynamics drive the long run of the economy away from its counterpart without uncertainty. Keeping moderate positive debt in the long run requires a higher labor tax, which pushes marginal costs up. To counteract the pass-through to prices, the nominal interest rate policy has to be tighter, which in the long run increases the magnitude of the shock necessary to make the zero lower bound binding. The optimal long run level of debt balances the cost associated with extra taxes against the benefit of reaching the zero lower bound at a lower frequency.

My work is closely related to [Eggertsson \(2006\)](#), who studied optimal discretionary policy in an economy with a binding zero lower bound and taxes of the costly lump-sum type *à la* [Barro \(1979\)](#).⁶ In his analysis, Eggertsson shows that, when taxes do not affect relative prices in the economy, it is optimal to cut taxes and accumulate debt during the liquidity trap. Such a response is effective only because it provides a way to support credibility of the forward guidance on nominal interest rates. When the liquidity trap is over, policy discretion does not allow to sustain the accumulated debt. As a result, a great deal of the debt adjustment is done fiscally irresponsible not just by running budget surpluses but rather by keeping the nominal interest policy loose to reduce real interest rates. My analysis suggests that, when raising taxes pushes marginal costs of the firm up, gains from abandoning fiscal responsibility in order to support the credibility of the monetary forward guidance can be outweighed by the gains from maintaining fiscal responsibility and using taxation as a stabilizing instrument during the liquidity trap.

The set of my results for the case of a balanced budget suggests that inability of households to smooth their consumption by running down their holdings of government bonds, could be one of the reasons explaining the active use of government spending in the economies that are currently in the liquidity trap.⁷ The results in this paper also show that the absence of commitment on its own does not overturn the optimal prescription of raising taxes and reducing debt at the liquidity trap—first described by [Eggertsson and Woodford \(2006\)](#)—which is at odds with conventional wisdom. It is important to note that the fiscal part of the optimal commitment policy mix features a forward guidance component, which is not sustained under discretion. [Nakata \(2011\)](#) showed that for a government currently in the liquidity trap, it is optimal to commit to expansionary lower taxes and the reversal of government spending expansion following the liquidity trap.⁸ Such policy is not part of the optimal time-consistent policy considered in my paper. Therefore, while the debt level as a natural state variable can largely insulate fiscal policy from the time-consistency problem during the liquidity trap, it may not be capable of making fiscal forward guidance credible.

⁶[Burgert and Schmidt \(2013\)](#) study optimal discretionary monetary and fiscal policy with distortionary taxes, but keep taxes constant in their baseline model.

⁷See [Mckay et al. \(2014\)](#) studying effects of borrowing constraints on the monetary forward guidance.

⁸[Belgibayeva and Horvath \(2014\)](#) showed that this result is robust to wages being sticky.

The paper is organized as follows. Section 2 contains the description of the model. Section 3 defines equilibria under two policy regimes: (1) Ramsey equilibrium with commitment policy and (2) Markov-perfect equilibrium with time-consistent policy. I then discuss the design of a labor subsidy that eliminates static distortions and allows to focus on the role of policy for stabilizing demand. Section 4 presents numerical results of the optimal time-consistent policy in the Markov-perfect equilibrium with occasionally binding zero lower bound. Section 5 concludes.

2 The Model

I consider a standard dynamic stochastic general equilibrium New Keynesian model of a closed economy with monopolistically competitive intermediate goods market and costly price adjustment. This section describes the economy, defines the competitive equilibrium and derives the first-best allocation as a reference point for the policy problem defined in the next section.

2.1 Households

The representative household consumes a composite good, which is produced from a continuum of differentiated products indexed by $i \in [0, 1]$ using constant-elasticity-of-substitution production technology. Total supply of the aggregate good Y_t is given by

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where $Y_{i,t}$ is the input of differentiated good i , and $\theta > 1$ is the elasticity of substitution. This composite good is used either for private or public consumption. Private consumption of the aggregated consumption good is denoted by C_t , and G_t denotes spending on public good provision by the government.

The representative household values private consumption and public goods, and dislikes labor. Preferences of the representative household are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) \left[u(C_t) + g(G_t) - \int_0^1 v(h_{i,t}) di \right], \quad (2.1)$$

where $0 < \beta < 1$ is a discount factor, and $h_{i,t}$ denotes the quantity (hours) of labor supplied to the firm producing intermediate good of type i . I assume that function v is increasing and convex in labor, function g is increasing and concave in consumption of the public good, and function u is increasing and concave in consumption of the private good. Exogenous stochastic disturbance ξ_t is a preference shock that affects the marginal rate of substitution between consumption at time t and consumption at time $t+1$.⁹ I normalize $\xi_{-1} = 1$ and assume it follows an AR(1) process

$$\ln(\xi_t) = \rho \ln(\xi_{t-1}) + \varepsilon_t,$$

where $0 \leq \rho < 1$, and $\varepsilon \sim N(0, \sigma^2)$. The preference shock can be interpreted as a shock to a natural real interest rate. The natural rate of interest is the real interest rate associated

⁹Preference shock enters preferences analogously to Braun et al. (2013), Nakata (2013) and Ngo (2013).

with the optimal allocation in the flexible price economy. Given the specification of the preference shock, the natural real interest rate is equal to $\frac{1}{\beta\xi_t}$.

Labor of type i is used to produce differentiated good i . Private consumption of a bundle of differentiated goods is indexed using a constant-elasticity-of-substitution aggregator:

$$C_t = \left(\int_0^1 C_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

The index of public goods G_t is defined analogously. Given individual goods prices $P_{i,t}$, the price index corresponds to the minimum cost of a unit of the aggregated good:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

The household enters period t holding assets in the form of nominal non-contingent (risk free, zero coupon) one-period bonds B_{t-1} . The supply of bonds in equilibrium is determined by government policies, which will be discussed later. Under described market arrangement, household's flow budget constraint is of the following form:

$$P_t C_t + R_t^{-1} B_t = (1 - \tau_t) \int_0^1 W_{i,t} h_{i,t} di + B_{t-1} + \int_0^1 \Pi_{i,t} di - T_t, \quad (2.2)$$

where $W_{i,t}$ is the nominal wage of labor of type i , $\Pi_{i,t}$ is the nominal profit from sales of differentiated good of type i distributed in a lump-sum way, and T_t is the lump-sum government transfer.¹⁰ Labor income of the household is taxed at a linear tax rate τ_t . To avoid complicating the notation I do not explicitly describe the market for private claims. Regardless, this setup is isomorphic to the model with a complete set of private state-contingent securities, as these would not be traded in equilibrium under the assumption of representative household. Also, note that I consider a "cashless" limit of the monetary economy in the spirit of Woodford (2003) and therefore I abstract from money holdings.

To have a well-defined intertemporal budget constraint and rule out "Ponzi schemes" I impose an additional constraint on household behavior that has to hold at each contingency:

$$\lim_{T \rightarrow \infty} \mathbb{E}_t \left[\left(\prod_{k=t}^T R_k^{-1} \right) B_T \right] \geq 0. \quad (2.3)$$

The household maximizes (2.1) by choosing consumption, industry-specific labor and bond purchases $\{C_t, h_t(i), B_t\}$ subject to budget constraint (2.2) and no-Ponzi condition (2.3), taking as given prices, policies and firms' profits $\{P_t, W_t, R_t, \tau_t, G_t, T_t, \Pi_t(i)\}_{t=0}^{\infty}$, the exogenous stochastic process of preference shocks $\{\xi_t\}_{t=0}^{\infty}$, and initial bond holdings B_{-1} .

The optimal plan of the household has to satisfy (2.2) and (2.3) with equality (the latter is then referred to as the transversality condition), and the following first order conditions:

$$w_{i,t} = \frac{1}{(1 - \tau_t)} \left(\frac{v'(h_{i,t})}{u'(C_t)} \right), \quad (2.4)$$

$$R_t^{-1} = \beta \xi_t \mathbb{E}_t \left(\frac{u'(C_{t+1})}{u'(C_t) \pi_{t+1}} \right), \quad (2.5)$$

¹⁰The lump-sum transfer is used for the sole purpose of financing labor (employment) subsidy at the steady state. See discussion below.

where $\pi_{t+1} \equiv P_{t+1}/P_t$ is the gross one period inflation rate, and $w_{i,t} \equiv W_{i,t}/P_t$ is the real wage. Equation (2.4) describes intratemporal trade-off between consumption and leisure. Equation (2.5) is an Euler equation describing the intertemporal allocation of consumption and savings.

2.2 Intermediate goods producers

There is a continuum of firms of unit mass producing imperfectly substitutable differentiated goods with a technology that is linear in labor $Y_{i,t} = h_{i,t}$. The firm producing good i sets the price $P_{i,t}$ and hires the quantity labor of type i necessary to satisfy realized demand in a perfectly competitive labor market. I assume that the government allocates its spending on the good varieties identically to the household, so that the resulting demand for good i is then

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t. \quad (2.6)$$

The firm chooses the price $P_{i,t}$ so as to maximize its present discounted real value of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{P_{i,t}}{P_t} Y_{i,t} - (1-s)w_{i,t}Y_{i,t} - \kappa_{i,t}Y_t \right],$$

subject to demand function (2.6), where λ_t is the marginal utility of real income for the representative household, and s is the time-invariant rate of a labor (employment) subsidy.¹¹ Following Rotemberg (1982), I assume a quadratic cost of price adjustment:

$$\kappa_{i,t} \equiv \frac{\varphi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2.$$

The choice of this specification for sticky prices over Calvo (1983) is commonly used in the literature when solving optimal policy problems without using the linear-quadratic approach, which would require an enlargement of the state-space due to the need to track price dispersion.¹²

In a symmetric equilibrium where all the firms charge identical prices or, equivalently, if there exists a single representative firm, the first order condition for the firm's problem is

$$\theta \left((1-s)w_t - \frac{(\theta-1)}{\theta} \right) = \varphi \left((\pi_t - 1) \pi_t - \beta \xi_t \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right). \quad (2.7)$$

Condition (2.7) is a New Keynesian Phillips curve in its nonlinear form, stating that current inflation depends on the marginal cost of production and expected inflation.

Goods market clearing requires the following aggregate resource constraint:

$$(1 - \kappa_t)Y_t = C_t + G_t, \quad (2.8)$$

As in Braun et al. (2013), I define GDP as the gross output net of the resource cost of price adjustment incurred by producers of the differentiated goods, $(1 - \kappa_t)Y_t$. This definition assumes that the resource costs of the price adjustment is the input of producers of the differentiated goods.

¹¹This subsidy can be used by the government to eliminate the deterministic steady state distortions associated with monopolistic competition and a distortionary labor income tax. See discussion below. The specification follows Leith and Wren-Lewis (2013). One can alternatively design a labor income subsidy, which works equivalently when the labor market is competitive.

¹²See exceptions Anderson et al. (2010), Ngo (2013) for fully nonlinear solutions with Calvo (1983) pricing.

2.3 Government

The government consists of a central bank and a treasury. The treasury decides on the amount of public goods G_t to provide to the household in the form of the aggregate consumption good. To cover spending on public goods, it levies a labor income tax, τ_t , and participates in the bond market. The central bank controls the short-term interest rate by means of open-market operations that vary the level of the real money balances held by the household. In equilibrium, the policy rate controlled by the central bank by no arbitrage condition has to be equal to the interest rate on government bonds, R_t .

In the “cashless” economy that I consider, the government supplies nominal claims (also known as “money”) that do not provide nonpecuniary return and thus only impose a zero lower bound on gross nominal interest rates:

$$R_t \geq 1. \quad (2.9)$$

Substituting firms’ profit, I can map household’s flow budget constraint (2.2) and resource constraint (2.8) into the nominal flow budget constraint of the government:

$$R_t^{-1}B_t = P_t(G_t - (\tau_t - s)w_tY_t) + B_{t-1} - T_t.$$

As I leave money out of consideration I automatically forgo seigniorage revenues obtained by the government. I assume that lump sum transfers T_t are used for the sole purpose of transferring resources corresponding to the labor subsidy. Furthermore, since the purpose of the subsidy is to address only permanent distortions in the economy, I set the real value of the lump-sum transfers over time equal to the steady-state value of the subsidy level. The flow budget constraint of the government in real terms is then given by:

$$R_t^{-1}b_t = \frac{b_{t-1}}{\pi_t} + (G_t + \varsigma_t - \tau_t w_t Y_t), \quad (2.10)$$

where $b_t \equiv B_t/P_t$ is the real value of government bonds, and $\varsigma_t \equiv s w_t Y_t - s w^{ss} Y^{ss}$ is the real deviation of the subsidy from its steady state level.

Following [Chari and Kehoe \(1993\)](#) and [Lustig et al. \(2008\)](#), I assume that households participate at the bond market anonymously, so that bonds issued by households are unenforceable and the government would not buy them. This assumption restricts the range of the government portfolio positions. I capture this assumption with the following inequality constraint that has to hold at all times and in all states

$$b_t \geq 0. \quad (2.11)$$

Under this constraint the government can borrow from households but does not provide loans to households.¹³ Note that under (2.11), the no-Ponzi condition (2.3) is automatically satisfied.

Therefore the government, subject to the zero lower bound constraint (2.9), chooses $\{R_t, G_t, \tau_t\}$ that, at the equilibrium prices, uniquely pin down $\{b_t\}$ as satisfying (2.10) and (2.11). The government’s problem will be introduced and discussed in the next section.

¹³No government lending constraint is also imposed in [Faraglia et al. \(2013\)](#).

2.4 Competitive equilibrium

I focus on the symmetric equilibria where producers of intermediate goods charge equivalent prices, $P_{i,t} = P_t$ for all $i \in [0, 1]$, and therefore face the same demand $Y_{i,t} = Y_t$, hire the same amount of labor $h_{i,t} = h_t$, and pay the same competitive wage $w_{i,t} = w_t$. The production function for intermediate goods then implies that $Y_t = h_t$. Taking these conditions into account, I now define a competitive equilibrium

Definition 1 (Competitive equilibrium). *Given exogenous process for household's preference shocks $\{\xi_t\}_{t=0}^\infty$ and initial outstanding government debt $b_{-1} \geq 0$, a rational expectations symmetric equilibrium is a sequence of stochastic processes $\{C_t, Y_t, \pi_t, w_t, b_t, R_t, G_t, \tau_t\}_{t=0}^\infty$ satisfying*

$$u'(C_t) = \beta \xi_t R_t \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{\pi_{t+1}} \right\}, \quad (2.12)$$

$$v'(Y_t) = (1 - \tau_t) w_t u'(C_t), \quad (2.13)$$

$$\theta(1 - s) w_t = (\theta - 1) + \varphi \left((\pi_t - 1) \pi_t - \beta \xi_t \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right\} \right), \quad (2.14)$$

$$0 = C_t + G_t - \left(1 - \frac{\varphi}{2} (\pi_t - 1)^2 \right) Y_t, \quad (2.15)$$

$$R_t^{-1} b_t = \frac{b_{t-1}}{\pi_t} + (G_t + \varsigma_t - \tau_t w_t Y_t), \quad (2.16)$$

$$R_t \geq 1, \quad (2.17)$$

$$b_t \geq 0, \quad (2.18)$$

and the transversality condition, for all $t \geq 0$; where the Euler equation (2.12) and equation (2.13) are first-order conditions of the household problem, the New-Keynesian Phillips curve equation (2.14) is the first-order condition of the intermediate goods producer, equation (2.15) is the aggregate resource constraint, equation (2.16) is the government budget constraint, (2.17) represents the zero lower bound constraint on nominal interest rates, and (2.18) is the government no-lending constraint.

2.5 First-best allocation

The first-best allocation is the one maximizing the household's utility (2.1) subject to technology constraints while abstracting from economic distortions in the private markets. Such an allocation is efficient and coincides with the solution of the fictitious Social Planner's problem described in appendix A. Efficiency dictates, for all $t \geq 0$, that the marginal utilities of private and public consumption be set equal to the marginal disutility of labor

$$\begin{aligned} u'(C_t) &= v'(Y_t), \\ g'(G_t) &= v'(Y_t). \end{aligned}$$

I use these efficiency conditions as a benchmark when discussing the problem of the government in the next section.

3 Policy regimes

In this section I formulate the optimal policy problem. Throughout the paper I assume full cooperation between fiscal and monetary authorities. The government is assumed to be benevolent and, hence, have the maximization of the household's utility as an objective. The first-best allocation is in general not attainable even in the absence of any shocks due to a number of economic distortions in the model. First, the intermediate goods producers charge a mark-up over the marginal cost. Second, government spending has to be financed with the distortionary labor tax that introduces a wedge into a private leisure decision. Further distortions include the price-adjustment cost as well as the bounds on bond holdings and nominal interest rate.

I start by establishing a second-best policy problem as a part of the Ramsey equilibrium that takes the described economic distortions into account. The Ramsey policymaker is assumed to fully commit to the policy it announces. Then I proceed by dropping the commitment assumption and describing the optimal Markov-perfect allocation under discretionary policymaking. The last part of this section is devoted to the analysis of the deterministic steady states of the two policy regimes. It discusses the design of an employment subsidy capable of eliminating static differences between the two equilibria that are not related to the demand stabilization problem, which is the focus of this paper. The stochastic version of the Markov equilibrium is then analyzed in the next section.

3.1 Ramsey equilibrium

Assume that the government can commit to follow through the policies it announces at the beginning of time. The government then announces policy plan for all future contingencies in order to implement the best competitive equilibrium which I refer to as the Ramsey equilibrium. I formally capture this idea in the following definition.

Definition 2 (Ramsey equilibrium). *Ramsey equilibrium consists of a state-contingent plan $\{C(\xi^t), Y(\xi^t), \pi(\xi^t), w(\xi^t), b(\xi^t), R(\xi^t), G(\xi^t), \tau(\xi^t)\}_{t=0}^{\infty}$ chosen at the initial time period $t = 0$ and for all possible histories $\xi^t \equiv (\xi_0, \dots, \xi_t)$ of preference shocks that, given initial outstanding government debt $b_{-1} \geq 0$, maximizes expected discounted sum of future utilities*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) [u(C_t) + g(G_t) - v(Y_t)]$$

subject to equations (2.12) - (2.18) characterizing competitive equilibrium.

The Lagrangian of the Ramsey problem and the first-order conditions characterizing the Ramsey allocation are described in Appendix B.

3.2 Markov-perfect equilibrium

Assume that the government is benevolent and credibly repays its debt in the future, but the use of the rest of its fiscal and monetary instruments is credible only within a period of announcement.¹⁴ The government has incentives to renege on its past promises of taxes,

¹⁴Credible debt repayment can be supported by a high implicit cost of the outright debt default.

government spending, and the nominal interest rate. This gives rise to a time-inconsistency problem of the Ramsey policy. Policy without commitment is discretionary. I model optimal discretionary policy as an outcome of a game between the private sector and the government represented by the treasury and the central bank playing cooperatively to maximize expected discounted utility (2.1) of the representative household. More precisely, I look for policy that is a part of the stationary Markov-perfect equilibrium of this game.

The timing of the game is as follows. Each period, the government acts first as a Stackelberg leader and announces current period policies. When doing so, it internalizes the effect of its decision on the actions of the private sector, acting as a Stackelberg follower and taking future policy as given. The government chooses the optimal policy sequentially based on the minimal payoff-relevant state of the economy. Such an aggregate state of the economy at the time t is described by realizations of the preference shock ξ and inherited government liabilities from the previous period b_{-1} . Although the government at a given period can not credibly commit to its future policy and, hence, does not directly influence future actions of the private sector, it can indirectly influence both through the current debt issuance policy that determines the future inherited state of the economy. In the Markov-perfect equilibrium, sequentially optimal choices are time-consistent and recursively determined by stationary rules.

I proceed by using a recursive formulation to formally describe the concept of the optimal discretionary policy. I drop time indices and use b_{-1} and b to denote outstanding and newly issued government bonds in a given period. For all the remaining variables I denote the next period value of a given variable x by x' . The government, at a given period in time correctly anticipates the future policy as well as the corresponding equilibrium allocation and prices that I denote as governed by the stationary functions $\mathcal{C}, \mathcal{Y}, \Pi, \mathcal{W}, \mathcal{R}, \mathcal{B}, \mathcal{T}, \mathcal{G}$. The future value function is denoted by the stationary function \mathcal{V} . When announcing policy for an ongoing period the government is free to deviate from the anticipated rules by implementing the best possible competitive equilibrium. As it is common in the optimal taxation literature, one can think of the government as choosing simultaneously its policy, prices and allocation, provided that they satisfy equilibrium conditions from definition 1.¹⁵ I capture this with the following Markov optimization problem of the discretionary government:

$$\max_{C, Y, \pi, w, R, b, \tau, G} u(C) + g(G) - v(Y) + \beta \xi \mathbb{E} \{ \mathcal{V}(b; \xi') \}$$

subject to the constraints

¹⁵Strictly speaking, I also assume that the primitives of the model permit correspondence between sequential formulation in definition 1 and recursive formulation employed here to be valid.

$$\begin{aligned}
0 &= [\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi] Y u_c \\
&\quad + \varphi\beta\xi\mathbb{E}\{\mathcal{V}(\cdot)(\Pi(\cdot)-1)\Pi(\cdot)u_c(\mathcal{C}(\cdot))\}, \\
0 &= \left(1 - \frac{\varphi}{2}(\pi-1)^2\right) Y - C - G, \\
0 &= R^{-1}b - \pi^{-1}b_{-1} - (G + \varsigma - \tau w Y), \\
0 &= u_c - R\beta\xi\mathbb{E}\left\{\frac{u_c(\mathcal{C}(\cdot))}{\Pi(\cdot)}\right\}, \\
0 &= w(1-\tau)u_c - v_y, \\
b &\geq 0, \\
R &\geq 1.
\end{aligned}$$

For optimal policy to be time-consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov-perfect equilibrium.

Definition 3 (Markov-perfect equilibrium). *A Markov-perfect equilibrium is a function $\mathcal{V}(b_{-1}; \xi)$ and a tuple of rules $\{\mathcal{C}, \mathcal{Y}, \Pi, \mathcal{W}, \mathcal{R}, \mathcal{B}, \mathcal{T}, \mathcal{G}\}$ each being a function of b_{-1} and ξ , such that:*

1. *Given $\mathcal{V}(\cdot)$, tuple of rules solves Markov problem of the government,*
2. *$\mathcal{V}(\cdot)$ is the value function of the government*

$$\mathcal{V}(b_{-1}; \xi) = u(\mathcal{C}(\cdot)) + g(\mathcal{G}(\cdot)) - v(\mathcal{Y}(\cdot)) + \beta\xi\mathbb{E}\{\mathcal{V}(\mathcal{B}(\cdot); \xi')\}.$$

I restrict attention to equilibria with differentiable value and policy functions. Under this assumption, equilibria are characterized by the first-order conditions of the government problem.¹⁶ The derivation of these conditions is delegated to Appendix C.

3.3 Deterministic steady state analysis

Standard in this class of models, there is in fact a continuum of the Ramsey steady states indexed with an initial level of government debt. The Markov steady state in general is different from the Ramsey steady states. Both the Ramsey and the Markov steady states are in general different from the first-best allocation. These differences are discussed in detail in Appendix D. The focus of this study is on the discretionary policy that stabilizes demand and I want to study it in isolation from the long-term effects caused by the absence of the commitment. To do so I assume that a labor subsidy is used by the government to offset permanent distortions in the economy as described in the following proposition.

Proposition 1 (Employment subsidy). *Given debt level b , there exists a unique employment subsidy rate $s^e(b)$ such that*

- *corresponding Ramsey steady state is efficient,*

¹⁶I refrain from a formal general proof of equilibrium existence or uniqueness. For a parametrized model numeric results demonstrate existence. Multiplicity is known to exist in a standard neoclassical economy with capital, see [Ortigueira et al. \(2012\)](#).

- *there is a Markov steady state with debt level b that is also efficient.*

Proof. See Appendix D.1. □

According to this proposition, in the deterministic setting without uncertainty one can design the subsidy to support any debt-to-GDP ratio with efficient allocation under both policy regimes. In what follows I pick a particular subsidy rate under which efficient steady states of both policy regimes features zero government debt.

4 Results

This section presents results for the stochastic Markov-perfect equilibrium of the model with an occasionally binding zero lower bound. I apply numerical methods because the equilibrium can not be solved for in the closed form. First I lay out the parametrization strategy. Then I discuss the optimal long run debt policy and after that proceed with discussion of the optimal policy responses to variations in the preference shock when the government budget constraint is relaxed. The final part of this section considers the case when the government budget is balanced.

4.1 Parametrization and Solution Method

Parameter values are summarized in table 1. Each period in the model represents one quarter of a year. The steady state time discount rate, β , is set to 0.995, corresponding to the annual real interest rate of 2 per cent.

Preferences of the households for consumption of private and public goods are described by $u(c_t) \equiv \frac{c_t^{1-\gamma_c}}{1-\gamma_c}$, $g(G_t) \equiv \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g}$, and the disutility from work is specified as $v(h_t) \equiv \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h}$. Curvature parameters of the utility functions, γ_c , γ_g and γ_h , are set to 2, 1, and 1 correspondingly. I choose values of the utility weights ν_h and ν_g equal to 100 and 1.25 correspondingly so that in the deterministic steady state households spend one quarter of their unitary time endowment working and government spending amounts to 20 per cent of the value added.

The monopoly power of the firms is described by the elasticity of substitution between intermediate goods, θ , which I set equal to 11 in order to match markup of the price over the marginal cost of 10 per cent. The parameter of price adjustment cost, φ , is set to 116.505, which is consistent with a Calvo (1983) price-setting specification where one quarter of the firms reoptimize their prices every period.

The parameter governing persistence of the demand shock, ρ , is set equal to 0.8. This value is a common choice in the literature studying zero lower bound and, as Fernández-Villaverde et al. (2012) discuss, it implies that the demand shock process has a half life of about three quarters. The standard deviation of the preference shock innovations is used to target unconditional probability of the model economy being at the zero lower bound. Coibion et al. (2012) calibrate the duration of the binding zero lower bound at roughly five per cent of the time, which is consistent with the post-WWII to 2011 period in the United States. Reifschneider and Williams (2000) estimate this probability to be in the range of five to six per cent based on the data for the Great Moderation period in the United States. Taking into account that the zero lower bound in the United States is expected to be binding well into the 2015 implies that the zero lower bound will have been binding for

Parameter	Description	Value
β	Discount rate	0.995
γ_c	Interpemoral elasticity for C	2
γ_g	Intertemporal elasticity for G	1
γ_h	Inverse Frisch elasticity	1
ν_g	Utility weight on G	1.25
ν_h	Utility weight on labor	100
φ	Price adjustment cost	116.505
θ	Elasticity of substitution among goods	11
ρ	Demand shock persistence AR(1)	0.8
σ_ε	Demand shock innovation s.d.	0.002

Table 1: Baseline parametrization

almost a quarter of time since the beginning of the Great Moderation period, which by far exceeds the five per cent probability estimate. Even if one considers current events to be the extreme tail event, [Chung et al. \(2012\)](#) included first two years of the Great Recession into the sample and employed a variety of estimation techniques to conclude that five per cent frequency is an underestimation. Also, recently revived hypothesis of the secular stagnation (see [Summers \(2013\)](#)) and [IMF \(2014\)](#) forecast of downward trend in the natural real interest rate would suggest that the probability of hitting the zero lower bound could be higher in the medium run. I set the standard deviation of the preference shock innovations equal to 0.002. This value results in having the unconditional probability of being at the zero lower bound around eight per cent of the time.

I solve the model numerically by means of a global nonlinear approximation method. [Appendix E](#) contains the details.

4.2 Optimal long run debt

In this section I show that the risk of falling into the liquidity trap permanently drives the economy away from its efficient deterministic steady state. Compared to the zero debt in the deterministic steady state, long run debt level is positive in the stochastic equilibrium with preference shocks. The optimal long run level of government debt, b^* , amounts to ~5 per cent of the annual GDP. [Figure 2](#) shows the optimal government debt change policy when the preference shock is equal to its average value ($\xi = 1$). It plots the absolute change of the outstanding debt across two consecutive periods as a function of the outstanding debt in the first out of these two periods, scaled by the constant annual efficient GDP. Let the economy start without outstanding government debt, $b_{-1} = 0$. Conditional on $\xi = 1$, the optimal policy prescription is to accumulate government debt until the optimal level b^* is reached. Conversely, an economy starting with a debt level above the optimal level b^* , conditional on $\xi = 1$, should experience debt stock decline until the optimal level b^* is reached.

The allocation corresponding to the optimal long run level of debt, b^* , and the average preference shock, $\xi = 1$, is a *risky* steady state in the terminology of [Coourdacier et al. \(2011\)](#). Keeping positive debt in the long run is an instrument to permanently change the point of the state space around which the economy fluctuates. The optimal positive long run debt level b^* creates an incentive to reduce the real interest rate that balances out

the increase of the real interest rate due to the discretionary response to the binding zero lower bound. Both of these effects on the real interest rate stem from the failure of the discretionary government to provide optimal (history-dependent) dynamic policy control.¹⁷ This general failure of the discretionary policy is often referred to as the stabilization bias.¹⁸ The remainder of this section explains in more detail the two counteracting forms of the stabilization bias in my model that lead to the accumulation of the positive government debt buffer in the long run.

[Figure 2 about here.]

The first form of the stabilization bias present in this model is the debt stabilization bias due to labor income taxation.¹⁹ The government in an economy with an outstanding debt exceeding the efficient level of zero has to collect additional revenue via distortionary taxation to support its liabilities. Under discretion the government can not enjoy the benefits of spreading costs of the distortionary taxation over time and has to stabilize debt stock around its efficient level.²⁰ At the beginning of each period discretionary government will be tempted to manipulate the real interest rate in order to increase the price of its bonds.²¹ Without the second form of the stabilization bias introduced by the binding zero lower bound and discussed below, current discretionary government would have to discipline future self by adjusting the debt level toward the efficient level where the efficient allocation is implemented and the discretionary incentive is eliminated.

The second form of the stabilization bias present in this model is the deflation bias due to an occasionally binding zero lower bound.²² Following high enough preference shock, the binding zero lower bound prevents the nominal interest rate policy from offsetting the fall of private demand. Abstracting from fiscal policy, falling demand pulls down prices and output contracts. Deflationary pressure is larger under the discretionary monetary policy because it can not credibly compromise stabilization upon exiting the liquidity trap and generate output boom, which would effectively reduce desire of households to save into the future and improve stabilization at the liquidity trap. Matters get worse when the zero lower bound is recurring. The deflationary pressure and output contraction associated with the liquidity trap spill into the rational expectations of households even when the nominal interest rate is positive and the economy is away from the liquidity trap. As a result, contemporaneous inflation and output in the times when the zero lower bound is not binding are adjusted downward. The adjustment of the former makes the real interest rate higher (compared to commitment).

[Figure 3 about here.]

¹⁷See [Currie and Levine \(1987\)](#) for a discussion of the time-inconsistency of the optimal dynamic control of a system under commitment.

¹⁸See [Clarida et al. \(1999\)](#) for a discussion of the stabilization bias in application to dynamic inflation control in a New Keynesian model.

¹⁹See [Leith and Wren-Lewis \(2013\)](#) and [Vines and Stehn \(2007\)](#) for a discussion of the debt stabilization bias in New Keynesian models without the risk of binding zero lower bound.

²⁰Given the incomplete markets structure, it would be optimal for the government to commit and spread associated costs of distortionary taxation over time. Under commitment, debt then would follow a random walk type of behavior described in [Aiyagari et al. \(2002\)](#) and [Schmitt-Grohé and Uribe \(2004\)](#).

²¹See [Debortoli and Nunes \(2012\)](#) for a discussion of the discretionary fiscal policy with debt in a real economy.

²²See [Adam and Billi \(2007\)](#) and [Eggertsson \(2006\)](#) for a discussion of the deflation bias in New Keynesian models without labor income taxation.

The two forms of the stabilization bias influence the conduct of nominal interest rate policy in the normal times when the zero lower bound is not binding. The deflation bias makes the central bank to conduct more accommodative interest rate policy, which leads to the lower bound being reached at a higher frequency. The debt stabilization bias at the low debt levels, comparable to the optimal long run level b^* , makes the central bank to conduct tighter interest rate policy. Figure 3 shows how the optimal nominal interest rate response to preference shocks on impact changes with the outstanding debt in the beginning of period. An outstanding debt level above b^* makes the debt stabilization bias prevail and requires larger preference shock for the zero lower bound to be reached. Such a debt level in the long run is, however, unsustainable because it would require higher tax rates that can not be smoothed over time under discretion. On the other hand, reducing level of debt below b^* makes the deflation bias prevail, which leads to more frequent liquidity trap episodes. Optimal long run level of debt, b^* , balances the benefit from less frequently binding zero lower bound against the cost of distortionary taxation induced by this level.

4.3 Optimal responses

I proceed with an examination of the optimal policy and allocation in the Markov-perfect equilibrium by considering the impulse responses to varying demand of households. I consider the case when the government budget constraint is relaxed and the level of outstanding government debt prior to the shock is equal to the optimal long run level b^* . The variation of demand is a result of the exogenous time preference changes. Temporary increase of the preference shock ξ makes the household more patient and reduces their contemporaneous demand.

Figure 4 shows the impulse responses under the optimal discretionary policy following a positive preference shock equal to the one unconditional standard deviation. I report consumption, government spending, labor supply (hours), the GDP, and the real wage in percentage deviation from the deterministic steady state. Inflation and the (net) nominal interest rate are reported in annualized percentage points. The labor tax rate is reported in percentage points. Primary balance is the difference between tax revenue and government spending, reported as a fraction of the quarterly efficient GDP. Real value of debt is reported as a fraction of the annual efficient GDP.

The conventional short-term nominal interest rate policy is fully capable on its own to entirely insulate the economy from moderate negative demand shocks. Lowering the policy rate reduces excess savings desire of the household and keeps the allocation of aggregate economic activity intact. Labor supply and both private and public consumption are stabilized at the values that are slightly below their deterministic steady state counterparts because the risky steady state level of government debt is positive compared to the debt level of zero in the deterministic steady state. Temporary reduction of the government debt stock reflects the reduction in excess savings of the private sector. Keeping allocation constant also requires keeping government spending, the tax rate and, hence, the primary surplus unaltered. Despite adjusting debt, the government can keep the primary balance constant because lower nominal interest rate raises government bond prices and adjustment is self-financed.

[Figure 4 about here.]

If the preference shock deviation from its average generates large enough fall in demand, the zero lower bound starts binding. Figure 5 shows the impulse responses under the optimal

discretionary policy following a positive preference shock equal to the three unconditional standard deviations; the magnitude of the shock is the only difference compared to Figure 4. When the zero lower bound is binding, the nominal interest rate fails to offset the fall of private demand. Falling private demand manifests itself into downward pressure on both prices and hours worked. Resulting deflationary pressure makes households to adjust their expectations of future inflation downward because of the shock persistence and the nominal rigidity in the price setting decision of the firms. Reduction in the expectations of inflation increases the real interest rate, which reinforces savings desire and endogenously aggravates the fall of private demand. The larger and more persistent is the preference shock the more pronounced are the hours worked decline and deflationary pressure. The nominal interest policy is ineffective at the liquidity trap, yet the government has two fiscal instruments remaining at its disposal. The available set of fiscal instruments does not allow to entirely stabilize the economy but is still used to improve stabilization of the economy at the liquidity trap. The remainder of this section discusses the response of fiscal instruments at the liquidity trap.

[Figure 5 about here.]

In models without the fiscal sector where the private sector is the only source of demand, changing private demand is the only force driving changes in aggregate demand. Differently, in the class of models with the endogenous government spending, the government actively uses the spending instrument, which endogenously responds to shocks, so as to stabilize aggregate demand. The optimal policy mix at the liquidity trap features increase of government spending in the impact period. The initial government spending expansion dies out until reaching the pre-crisis level when the zero lower bound stops binding. The response of government spending cushions decline of aggregate demand due to the fall of private demand. The higher aggregate demand improves hiring incentives of the firms. As a result, the fall of the hours worked is mitigated and improving labor demand supports wages and therefore prices from falling. Thus, government spending is optimally used for stabilization purposes when the zero lower bound is binding, which is not the case when demand fluctuations are not strong enough to put the economy into the liquidity trap.²³

Using government spending for cushioning the fall of aggregate demand is, however, costly because it opens a gap by driving public consumption away from its efficient level. Like government spending that takes on a stabilizing role during the liquidity trap, so does the labor tax respond to the binding zero lower bound when its variation is otherwise avoided. As a part of the optimal fiscal policy response to the shock that puts economy into the liquidity trap, it is optimal to temporarily raise taxes returning them back to the pre-crisis level once the zero lower bound stops binding. The higher labor tax is optimal due to its supply-side effect on prices. Increasing taxes in a given period makes agents less willing to work. Firms are then forced to increase wages to produce a given amount of output and pass some degree of the marginal cost increase on prices. Keeping taxes elevated in the future remaining periods of the liquidity trap creates inflationary expectations that offset expectations of deflation due to the binding zero lower bound. It reduces the real interest rate in the current period and breaks the feedback loop between high real interest rates and the falling private demand.

The discretionary government does not internalize the benefit of the contemporaneous tax increase in a given period for stabilizing the fall of private demand in the preceding

²³Nakata (2013) and Schmidt (2013) make the case for using government spending as stabilization instrument at the liquidity trap when taxes are lump-sum and the Ricardian equivalence holds.

periods. Under discretionary policy, tax hikes can only be sustained to the extent they are time-consistent. The contemporaneous tax increase comes at a cost of driving labor supply down. The benefit of the contemporaneous tax increase comes from its inflationary effect on the real value of the outstanding government bonds. If the government inherits a positive level of debt, it can benefit from reducing the debt burden via inflation. Importantly, this discretionary incentive to manipulate the price of government debt is not time-inconsistent at the liquidity trap because higher inflation reduces the real interest rate in the previous period and, therefore, improves stabilization.²⁴ Amount of debt issued by the current government regulates the trade-off of tax setting by the government in the next period. Bequeathing a certain amount of debt, the current government influences the dynamic tax response to reduce the real interest rate for stabilization purposes.

Reduction of the real interest rate, resulting from the anticipation by households of the discretionary incentive to manipulate the price of government bonds, offsets the fall of private demand so that households are willing to run down their assets. For households to smooth their consumption by running down their assets, the government has to reduce the amount of the outstanding debt. The price of government bonds is, however, relatively too low because the real interest rate falls short of the level that would entirely offset the fall of private demand. Therefore, debt reduction is not self-financed as in response to moderate demand shocks that do not lead to the binding zero lower bound. The government runs extra surplus to implement debt reduction.

In sum, flexibility of the government budget creates a discretionary incentive that reduces the time-inconsistency problem of the dynamic tax response. As a result, the government can credibly exploit the intertemporal effect of tax hikes on the real interest rate to improve consumption smoothing of households. Flexibility of the government budget is crucial for the efficacy of taxes in stabilizing the fall of private demand. Next section demonstrates that, when the government budget is balanced period by period, the composition of the fiscal response is reversed.

4.4 Comparison with the balanced budget case

I contrast previous analysis to the case where the government is restrained to keep flow budget balanced period by period. When doing so, I do not change the nature of the tax instrument. Government spending has to be financed with revenue collected via the distortionary tax on labor income. The balanced budget assumption eliminates the discretionary incentive to manipulate the real interest rate and prevents households from using variation in government debt holdings to smooth their consumption because bonds are in zero net supply.

Figure 6 shows the impulse responses under the optimal discretionary policy to a positive preference shock equal to the three unconditional standard deviations when the government budget is balanced in every period (solid black lines) compared to the responses from the previous section when the government budget is relaxed (dotted red lines, primary balance and debt dynamics are not reported). As in the relaxed budget case, following a large negative demand shock the short-term nominal interest rate is brought down to the zero lower bound in an attempt to offset increased savings desire of the household. The fiscal responses are qualitatively similar to the case with government debt dynamics. Both govern-

²⁴This result stands in contrast to the time-inconsistency problem that arises in the models of discretionary policy where the zero lower bound is not binding, see e.g., [Debortoli and Nunes \(2012\)](#) and [Leith and Wren-Lewis \(2013\)](#).

ment spending and the labor income tax rate temporarily increase. When the government budget is balanced, however, tax increase becomes much more costly instrument from the perspective of the government in a given period of time at the liquidity trap. Therefore, using taxes to push inflation expectations is not a credible stabilization strategy as it is in the case with the relaxed government budget. When the government budget is balanced, the real rate stays high and the economy experiences stronger fall of private demand. In response to falling demand wages plummet and the economy experiences larger private consumption gap and falling prices. As a result, the government is pushed toward considerably more pronounced response of government spending in order to cushion the fall of aggregate demand.

When increasing government spending in order to cushion the fall of aggregate demand, the government goes much further to the point where its response is one order of magnitude larger than in the case with government debt dynamics. Not surprisingly, there is less reliance on the tax instrument: the rate set on impact in response to the binding zero lower bound is around one percentage point lower than in the case with government debt dynamics. This change of the roles leads to a different response of hours worked. Stronger use of the government spending and milder use of the labor tax do not depress labor supply as much as when taxes is the prime stabilizing instrument. Under baseline parametrization I find positive hours worked gap at the liquidity trap with a balanced government budget. Really important, however, is the qualitative fact that the this gap is smaller in absolute value than in the case with government debt dynamics. Smaller gap of hours worked comes at a cost of having larger gaps in public and private consumption accompanied by deflation.

[Figure 6 about here.]

Absence of government debt not only affects fiscal policy and the allocation during the liquidity trap, but it also affects policy and the allocation when the zero lower bound is not binding. Without government debt the economy fluctuates around its efficient deterministic steady state, but moderate fluctuations in demand are not precisely offset with variations in the nominal interest rate. Figure 7 shows the impulse responses under the optimal discretionary policy following a positive preference shock equal to the one unconditional standard deviation magnitude.

[Figure 7 about here.]

Under a balanced budget assumption, the government decreases nominal rate more aggressively than it does in the economy without balanced budget in response to the preference shock of the same magnitude. Different dynamics during liquidity trap episodes without government debt spill over into dynamics when the zero lower bound is not binding through expectations. Forward-looking agents and firms take into account higher real rates and lower wages corresponding to the states with the binding zero lower bound and adjust correspondingly their consumption and prices downward even when zero lower bound is not binding. The higher is conditional probability of reaching the liquidity trap, the stronger is such pass-through. Monetary policy lowering nominal interest rate more aggressively counteracts this effect. In the equilibrium, more accommodative monetary policy leads to the GDP boom and positive private consumption gap. More accommodative monetary policy also leads to the zero lower bound being hit more often, which increases the overall time the economy spends at the liquidity trap.

5 Conclusion

The global economic downturn put into a liquidity trap many advanced economies. Large fiscal adjustments were implemented in an attempt to stabilize these economies.

In this paper I characterize optimal monetary and fiscal policy under discretion in a New Keynesian model with recurring episodes of a liquidity trap. I focus the analysis on the use of discretionary government spending and labor income taxation jointly as stabilizing instruments at the liquidity trap. A government which must keep a balanced budget relies more on the spending instrument than a government which is allowed to borrow and can temporarily run down its debt during the liquidity trap. On the other hand, a government which is allowed to borrow and can temporarily run down its debt during the liquidity trap places more weight on the use of the labor income tax. My results suggest that, given lack of fiscal commitment, inability of households to smooth their consumption by running down their holdings of government bonds, could be one of the reasons explaining the active use of the spending instrument in the economies that are currently in the liquidity trap. I plan to investigate effects of market incompleteness and borrowing constraints on the conduct of fiscal policy at the liquidity trap.

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A First-best allocation characterization

Lagrangian corresponding to the Planner's problem is

$$L \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) [u(C_t) + g(G_t) - v(Y_t) + \gamma_t (Y_t - C_t - G_t)].$$

First-order conditions are as follows

$$\begin{aligned} u'(C_t) &= \gamma_t, \\ g'(G_t) &= \gamma_t, \\ v'(Y_t) &= \gamma_t. \end{aligned}$$

Eliminating Lagrange multiplier γ_t leaves the system with two equations

$$\begin{aligned} u'(C_t) &= v'(Y_t), \\ g'(G_t) &= v'(Y_t), \end{aligned}$$

that together with resource constraint $Y_t = C_t + G_t$ characterize first-best allocation as a solution of the Planner's problem.

B Ramsey equilibrium characterization

Lagrangian corresponding to the Ramsey problem is given by

$$\begin{aligned} \mathcal{L} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) & \left[u(C_t) + g(G_t) - v(Y_t) \right. \\ & - \lambda_{1,t} \left(u_{c,t} - R_t \xi_t \beta \mathbb{E}_t \left\{ \frac{u_{c,t+1}}{\pi_{t+1}} \right\} \right) \\ & - \frac{\lambda_{1,t-1}}{\xi_{t-1} \beta} \left(u_{c,t-1} - R_{t-1} \xi_{t-1} \beta \frac{u_{c,t}}{\pi_t} \right) \\ & - \lambda_{2,t} \left(\begin{aligned} & [\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t-1)\pi_t] Y_t u_{c,t} \\ & + \varphi \xi_t \beta \mathbb{E}_t \{ (\pi_{t+1}-1)\pi_{t+1} Y_{t+1} u_{c,t+1} \} \end{aligned} \right) \\ & - \frac{\lambda_{2,t-1}}{\xi_{t-1} \beta} \left(\begin{aligned} & [\theta(1-s)w_{t-1} - (\theta-1) - \varphi(\pi_{t-1}-1)\pi_{t-1}] Y_{t-1} u_{c,t-1} \\ & + \varphi \xi_{t-1} \beta (\pi_t-1)\pi_t Y_t u_{c,t} \end{aligned} \right) \\ & - \lambda_{3,t} (w_t(1-\tau_t)u_{c,t} - v_{y,t}) \\ & - \lambda_{4,t} \left(\left(1 - \frac{\varphi}{2}(\pi-1)^2 \right) Y_t - C_t - G_t \right) \\ & - \lambda_{5,t} \left(R_t^{-1} b_t - \frac{b_{t-1}}{\pi_t} - (G_t + \varsigma_t - \tau_t w_t Y_t) \right) \\ & - \mathbb{E}_t \left\{ \lambda_{5,t+1} \xi_{t+1} \beta \left(R_{t+1}^{-1} b_{t+1} - \frac{b_t}{\pi_{t+1}} - (G_{t+1} - \tau_{t+1} w_{t+1} Y_{t+1}) \right) \right\} \\ & \left. + \nu_t (R_t - 1) + \eta_t b_t \right], \end{aligned}$$

First-order conditions with respect to the decision variables $C_t, Y_t, \pi_t, w_t, \tau_t, G_t, R_t, b_t$ are

$$u_{c,t} = \lambda_{1,t}u_{cc,t} + \lambda_{2,t} [\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t-1)\pi_t] Y_t u_{cc,t} \quad (\text{B.1})$$

$$+ \lambda_{3,t}w_t(1-\tau_t)u_{cc,t} - \lambda_{4,t} \\ - \lambda_{1,t-1}R_{t-1}\frac{u_{cc,t}}{\pi_t} - \lambda_{2,t-1}\varphi(\pi_t-1)\pi_t Y_t u_{cc,t} \\ v_{y,t} = -\lambda_{2,t} [\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t-1)\pi_t] u_{c,t} \quad (\text{B.2})$$

$$+ \lambda_{3,t}v_{yy,t} - \lambda_{4,t} \left(1 - \frac{\varphi}{2}(\pi-1)^2\right) - \lambda_{5,t}((\tau_t-s)w_t) \\ - \lambda_{2,t-1}\varphi(\pi_t-1)\pi_t u_{c,t} \\ 0 = \lambda_{2,t}\varphi(2\pi_t-1)Y_t u_{c,t} + \lambda_{4,t}\varphi(\pi_t-1)Y_t - \lambda_{5,t}\frac{b_{t-1}}{\pi_t^2} \quad (\text{B.3})$$

$$- \lambda_{1,t-1}R_{t-1}\frac{u_{c,t}}{\pi_t^2} - \lambda_{2,t-1}\varphi(2\pi_t-1)Y_t u_{c,t} \\ 0 = \lambda_{2,t}\theta(1-s)Y_t u_{c,t} + \lambda_{3,t}(1-\tau_t)u_{c,t} + \lambda_{5,t}((\tau_t-s)Y_t) \\ 0 = \lambda_{3,t}u_{c,t} - \lambda_{5,t}Y_t \quad (\text{B.4})$$

$$g_{G,t} = -\lambda_{4,t} - \lambda_{5,t} \quad (\text{B.5})$$

$$0 = \lambda_{1,t}\xi_t\beta\mathbb{E}_t \left\{ \frac{u_{c,t+1}}{\pi_{t+1}} \right\} + \lambda_{5,t}\frac{b_t}{R_t^2} + \nu_t \quad (\text{B.6})$$

$$0 = \beta\mathbb{E}_t \left\{ \xi_{t+1}\frac{\lambda_{5,t+1}}{\pi_{t+1}} \right\} - \frac{\lambda_{5,t}}{R_t} + \eta_t \quad (\text{B.7})$$

Multipliers $\nu_t \geq 0, \eta_t \geq 0$ and have to satisfy Kuhn-Tucker complementary slackness conditions

$$0 = \nu_t (R_t - 1), \\ 0 = \eta_t b_t.$$

C Markov equilibrium characterization

Lagrangian for the Markov problem of the government is as follows

$$\mathcal{L} \equiv u(C) + g(G) - v(Y) + \beta\xi\mathbb{E}\{\mathcal{V}(b; \xi')\} \\ - \lambda_1 [u_c - R\beta\xi\mathbb{E}\{S(b; \xi')\}] \\ - \lambda_2 [[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi] Y u_c + \varphi\beta\xi\mathbb{E}\{Z(b; \xi')\}] \\ - \lambda_3 [w(1-\tau)u_c - v_y] \\ - \lambda_4 \left[\left(1 - \frac{\varphi}{2}(\pi-1)^2\right) Y - C - G \right] \\ - \lambda_5 \left[R^{-1}b - \frac{b-1}{\pi} - (G + \varsigma - \tau w Y) \right] \\ + \nu [R - 1] + \eta b,$$

where

$$S(b; \xi') \equiv \frac{u_c(\mathcal{C}(b; \xi'))}{\Pi(b; \xi')},$$

$$Z(b; \xi') \equiv \mathcal{Y}(b; \xi') \cdot (\Pi(b; \xi') - 1) \cdot \Pi(b; \xi') \cdot u_c(\mathcal{C}(b; \xi')).$$

Multipliers $\nu \geq 0$, $\eta \geq 0$ and have to satisfy Kuhn-Tucker conditions

$$0 = \nu(R - 1),$$

$$0 = \eta b.$$

Ignoring functions' arguments, the corresponding first-order conditions, apart from the competitive equilibrium constraints, include

$$[C]: \quad u_c = (\lambda_1 + \lambda_2[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi])Y + \lambda_3w(1-\tau)u_{cc} - \lambda_4 \quad (C.1)$$

$$[Y]: \quad v_y = -\lambda_2[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi]u_c + \lambda_3v_{yy} - \lambda_4\left(1 - \frac{\varphi}{2}(\pi-1)^2\right) - \lambda_5(\tau-s)w \quad (C.2)$$

$$[\pi]: \quad 0 = \varphi(\lambda_2(2\pi-1)u_c + \lambda_4(\pi-1))Y - \lambda_5\frac{b_{-1}}{\pi^2} \quad (C.3)$$

$$[w]: \quad 0 = (\lambda_2\theta(1-s)Y + \lambda_3(1-\tau))u_c + \lambda_5(\tau-s)Y \quad (C.4)$$

$$[\tau]: \quad 0 = \lambda_3u_c - \lambda_5Y \quad (C.5)$$

$$[G]: \quad g_G = -\lambda_4 - \lambda_5 \quad (C.6)$$

$$[R]: \quad 0 = \lambda_1\beta\xi\mathbb{E}\{S'\} + \lambda_5bR^{-2} + \nu \quad (C.7)$$

$$[b]: \quad 0 = \beta\xi\mathbb{E}\{V'_b\} + \beta\xi\lambda_1R\mathbb{E}\{S'_b\} - \lambda_2\beta\xi\varphi\mathbb{E}\{Z'_b\} - \lambda_5R^{-1} + \eta \quad (C.8)$$

where subscripts denote partial derivatives. Envelope theorem implies

$$V'_b = \frac{\lambda'_5}{\Pi'} \quad (C.9)$$

Simplifying the system

One can work around with the system of first order conditions and simplify it. Using resource constraint (2.8) I solve for government expenditures

$$G = \hat{G}(Y, C, \pi) \equiv \left(1 - \frac{\varphi}{2}(\pi-1)^2\right)Y - C.$$

Analogously I use Phillips curve (2.7) and consumption leisure trade-off (2.4) to express wage and labor tax

$$w = \hat{w}(Y, C, \pi, b) \equiv \frac{(\theta-1)}{\theta(1-s)} + \frac{\varphi}{\theta(1-s)} \left((\pi-1)\pi - \beta \frac{\mathbb{E}\{Z'\}}{Y u_c} \right),$$

$$\tau = \hat{\tau}(Y, C, \pi, b) \equiv 1 - \frac{1}{\hat{w}} \frac{v_y}{u_c}.$$

Private Euler equation (2.5) along with zero-lower bound condition (2.9) deliver

$$R = \hat{R}(C, b) \equiv \max \left\{ 1, \frac{u_c}{\beta \xi \mathbb{E} \{ S' \}} \right\},$$

I proceed by explicitly solving for Lagrange multipliers $\lambda_2, \lambda_3, \lambda_4$ and λ_5 from equations (C.3)-(C.6).

$$\lambda_2 = \hat{\lambda}_2(Y, C, \pi, b) \equiv \Omega \frac{g_G}{u_c} \left[\frac{\hat{\tau}}{\theta} + \frac{Y}{(1-\hat{\tau})u_c} \right], \quad (\text{C.10})$$

$$\lambda_3 = \hat{\lambda}_3(Y, C, \pi, b) \equiv -\Omega \frac{g_G}{u_c} Y, \quad (\text{C.11})$$

$$\lambda_4 = \hat{\lambda}_4(Y, C, \pi, b) \equiv (\Omega - 1) g_G, \quad (\text{C.12})$$

$$\lambda_5 = \hat{\lambda}_5(Y, C, \pi, b) \equiv -\Omega g_G, \quad (\text{C.13})$$

where $\Omega < 1$ is defined as follows

$$\Omega(Y, C, \pi, b) \equiv \frac{\varphi \pi^2 (\pi - 1) Y}{\varphi \pi^2 (\pi - 1) Y + \varphi \pi^2 (2\pi - 1) Y \left(\frac{\hat{\tau}}{\theta} + \frac{Y}{(1-\hat{\tau})u_c} \right) + b_{-1}}.$$

Non-binding ZLB

If zero-lower bound is not binding, then from (C.7)

$$\lambda_1 = \hat{\lambda}_1(Y, C, \pi, b) \equiv \Omega \frac{g_G}{(u_c)^2} b \beta \xi \mathbb{E} \{ S' \}, \quad \text{if } \eta = 0, \quad (\text{C.14})$$

and I am left with 4 unknowns (Y, C, π, b) and the same number of equations

$$0 = u_c - \left(\hat{\lambda}_1 + \hat{\lambda}_2 [\theta \hat{w} - (\theta - 1) - \varphi(\pi - 1)\pi] Y + \hat{\lambda}_3 \hat{w} (1 - \hat{\tau}) \right) u_{cc} + \hat{\lambda}_4, \quad (\text{C.15})$$

$$0 = v_y + \hat{\lambda}_2 [\theta(1-s)\hat{w} - (\theta - 1) - \varphi(\pi - 1)\pi] u_c - \hat{\lambda}_3 v_{yy} + \hat{\lambda}_4 \left(1 - \frac{\varphi}{2} (\pi - 1)^2 \right) + \hat{\lambda}_5 \hat{\tau} \hat{w}, \quad (\text{C.16})$$

$$0 = \hat{R}^{-1} b - \frac{b_{-1}}{\pi} - \left(\hat{G} + \hat{\zeta} - \hat{\tau} \hat{w} Y \right), \quad (\text{C.17})$$

$$0 = \beta \xi \mathbb{E} \left\{ \frac{\hat{\lambda}'_5}{H'} \right\} + \beta \xi \hat{\lambda}_1 \hat{R} \mathbb{E} \{ S'_b \} - \hat{\lambda}_2 \beta \xi \varphi \mathbb{E} \{ Z'_b \} - \hat{\lambda}_5 \hat{R}^{-1}, \quad (\text{C.18})$$

where first two correspond to first-order conditions (C.1) and (C.2), equation (C.17) is the government budget constraint, and (C.18) is the so-called generalized Euler equation (GEE) characterizing optimal bond purchases choice. The GEE equates the discounted expected utility loss resulting from a tighter budget constraint in the future with the current direct and indirect gains resulting from a marginal relaxation of the budget constraint today and the other competitive equilibrium constraints. There is also ad-hoc lending constraints (2.11), which means that GEE has to hold only for the interior solution $b > 0$.

Binding ZLB

For the case of binding zero-lower bound I can not use (C.7) to express λ_1 . Instead, the very fact of binding zero-lower bound delivers extra equation of the form

$$\xi\beta\frac{\mathbb{E}\{S(b;\xi')\}}{u_c} = 1. \quad (\text{C.19})$$

One can then use equation (C.1) to solve for λ_1 as follows

$$\lambda_1 = \hat{\lambda}_1(Y, C, \pi, b) \equiv \frac{u_c - g_g}{u_{cc}} + \Omega Y \frac{g_G}{u_c} \left(\frac{1}{Y} \frac{u_c}{u_{cc}} + \hat{w}(1 - \hat{\tau}) - \left[\frac{\hat{\tau}}{\theta} + \frac{Y}{(1 - \hat{\tau})u_c} \right] [\theta(1 - s)\hat{w} - (\theta - 1) - \varphi(\pi - 1)\pi] \right)$$

and use system of equations (C.16)-(C.18) along with (C.19) to solve for unknown (Y, C, π, b) .

Solution

Described system of equations has to be satisfied for each $(b_{-1}; \xi)$, hence it is a system of functional equations with solution described by a tuple of functions $\{\mathcal{Y}, \mathcal{C}, \Pi, \mathcal{B}\}$. As it is common, the system characterizing Markov-perfect equilibria appears to contain yet unknown policy functions and their derivatives.

D Deterministic steady state analysis

To analyze time-invariant deterministic long run steady states of both Ramsey and Markov-perfect equilibria set $\sigma = 0$, $\xi_t = 1$ for all $t \geq 0$. I restrict attention by considering interior steady states where inequality constraints on nominal interest rate R and government debt holdings b do not bind, i.e. $\nu = 0$ and $\eta = 0$. Until further notice, consider the case when there is no employment subsidy, $s = 0$.

Start with the Ramsey policy regime. Standard to this class of models, there is in fact a continuum of Ramsey steady states indexed with initial level of government debt. To put it the other way, a continuous range of debt levels can be supported in equilibrium, hence the model exhibits indeterminacy of degree one. I characterize this continuum of steady states with the following proposition.

Proposition 2 (Ramsey Steady State). *Level of debt in the Ramsey steady state is indeterminate. For a given debt level b , Ramsey steady state is characterized by*

$$\pi = 1 \text{ and } R = \beta^{-1},$$

and marginal utility conditions

$$\left(\frac{\theta - 1}{\theta} \frac{1}{(1 - s)} - \frac{G + (1 - \beta)b}{Y} \right) u'(C) = v'(Y), \quad (\text{D.1})$$

$$g'(G) \geq v'(Y). \quad (\text{D.2})$$

Proof. Euler equation at the steady state implies

$$R\beta = \pi.$$

The same condition is implied by the first-order condition w.r.t. b_t , (B.7), at the steady state. For this reason, the system of equations determining equilibrium at the steady state exhibits indeterminacy of dimension one. There is a continuum of steady states that can be indexed with the level of debt. One can freely choose steady state level of debt knowing there exist initial condition that would support it. First-order condition w.r.t. nominal interest rate R_t , (B.6), at the steady state becomes $\lambda_5 b R^{-2} = -\lambda_1 \beta \pi^{-1} u_c$. Combining the two and using it together with first-order condition w.r.t. inflation π_t , (B.3), one can derive the following steady state condition

$$\lambda_4 \varphi(\pi - 1) = 0.$$

Largange multiplier λ_4 corresponds to resource constraint and using envelope theorem argument is equal to the marginal value of relaxing resource constraint. Clearly $\lambda_4 \neq 0$ and it then follows that at the steady state

$$\pi = 1 \text{ and } R = \beta^{-1}.$$

I take into account previous result in all the following derivations. Government budget constraint, Phillips curve, and consumption-leisure optimality condition of the private sector at the steady state can be correspondingly rewritten as

$$\begin{aligned} \tau w &= \frac{G + (1 - \beta)b}{Y}, \\ w &= \frac{(\theta - 1)}{\theta}, \\ \frac{v_y}{u_c} &= w(1 - \tau). \end{aligned}$$

First two equations can be used to eliminate real wage w and tax rate τ from the third one in order to get

$$\underbrace{\left(\frac{\theta - 1}{\theta} - \frac{G + (1 - \beta)b}{Y} \right)}_{\in(0,1)} u_c = v_y,$$

and the implied inequality

$$u_c > v_y \tag{D.3}$$

First-order conditions w.r.t government spending G_t , (B.5), at the steady state is

$$g_G = -\lambda_4 - \lambda_5. \tag{D.4}$$

Combining it with first-order condition w.r.t. labor supply Y_t , (B.2), to get the following steady state condition $g_G - v_y = -\lambda_3 (v_{yy} + (1 - \tau w))$. By assumption $v_{yy} \leq 0$. For any plausible calibrations $(1 - \tau w) \geq 0$. Therefore necessary and sufficient condition for the last part of the proposition is

$$g_G \geq v_y \text{ iff } \lambda_3 \leq 0, \tag{D.5}$$

where λ_3 is the Lagrange multiplier attached to consumption-leisure optimality condition of the private sector.

The remaining part is proved by contradiction. Using first-order conditions w.r.t. tax rate τ_t , (B.4), and inflation π_t , (B.3), another first-order condition w.r.t. consumption C_t , (B.1), can be rearranged into the following steady state condition

$$\underbrace{u_c}_{\geq 0} = \lambda_3 \underbrace{\left(\frac{(1-\beta)b}{Y} + \frac{v_y}{u_c}(1-\tau) \right)}_{\geq 0} \underbrace{u_{cc}}_{\leq 0} - \lambda_4.$$

Assume $\lambda_3 > 0$, then from the last condition it follows that

$$-\lambda_4 \geq u_c \geq 0.$$

Recall that $u_c > v_y$ by (D.3). Moreover, if $\lambda_3 > 0$ then $v_y > g_G$ by (D.5). The chain of inequalities then becomes

$$-\lambda_4 \geq u_c > v_y > g_G \geq 0. \quad (\text{D.6})$$

Recall that Lagrange multiplier λ_4 corresponds to the resource constraint. Resource constraint can be relaxed in a number of ways adjusting labor supply, public and private consumption. It therefore has to be the case that the marginal gain from relaxing budget constraint has to be larger or equal than the marginal cost of reducing public consumption, i.e. $\lambda_4 \geq g_G$, which contradicts (D.6). Therefore $\lambda_3 \leq 0$ and $g_G \geq v_y$. \square

This result is similar to Adam (2011) and Motta and Rossi (2013). All Ramsey steady states share the same zero inflation condition and identical positive nominal interest rate.

Compare marginal utility conditions for Ramsey steady state with analogous conditions for the first-best allocation. Conditions (D.1) and (D.2) show that in general there are wedges between marginal utilities of private and public consumption and marginal disutility from labor. Wedge in equation (D.1) implies $u'(C) \geq v'(Y)$ so that private consumption in Ramsey steady state falls below first-best optimum. This wedge appears due to firms charging monopolistic mark-up and distortionary nature of taxes required to finance steady state government spending and debt interest payments (for positive levels of debt). The fact that government spending requires collection of distortionary tax provides incentive for the planner to reduce government spending below first-best optimum as described by condition (D.2). Clearly the output also appears to be below efficient level.

If no-lending constraint (2.18) was not in place, one could show that there exists a unique negative level of government debt for which Ramsey steady state is in fact efficient. In this steady state government uses interest receipts from holding assets in order to finance government spending and offset monopolistic distortion without resorting to distortionary labor tax. The magnitude of such efficient asset level is generally a large fraction of the GDP.²⁵

Now turning to the Markov-perfect equilibrium. Steady-state of the Markov-perfect equilibrium in general is hard to characterize, because first-order conditions of the government problem contain derivatives of unknown functions. One can still devise a limited analytical insight into properties of the Markov steady state allocation. To do so, I use Ramsey steady state as a reference and see what happens if policymaker were to achieve price stability

²⁵See for instance Aiyagari et al. (2002) for a neoclassical model without monopolistic competition and Gnocchi and Lambertini (2014) for a New Keynesian model.

$\pi = 1$ in Markov steady state. Under price stability, a subset of first-order conditions imply the reaction function

$$\begin{aligned} u'(C) &= v'(Y), \\ g'(G) &= v'(Y), \end{aligned}$$

under which the marginal utilities of private and public consumption are equated to marginal disutility of labor just like in the first-best allocation. Such behavior is counterproductive in the presence of economic distortions and cannot be sustained in equilibrium, which leads to deviation of Markov steady state from Ramsey steady state. In particular the following results regarding inflation holds.

Proposition 3 (Markov steady state). *Given discount factor β close enough to 1, Markov-perfect steady state exhibits positive trend inflation*

$$\pi > 1.$$

Proof. Like in the Ramsey steady state, set of the competitive equilibrium constraints imply

$$R\beta = \pi, \tag{D.7}$$

$$u_c > v_y. \tag{D.8}$$

Condition (D.7) together with zero lower bound on interest rate $R \geq 1$ imply $\pi \geq \beta$. Moreover, $\pi = 1$ is not a equilibrium. To see this first note that equations (C.10)-(C.13) and (C.14) for $\pi = 1$ would imply $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_5 = 0$ and $\lambda_4 = -g_G$. This fact together with equations (C.1) and (C.2) would imply $u_c = v_y$, which contradicts (D.8). Therefore for β close to one it must be that $\pi > 1$. \square

It is then straightforward to show that government spending in Markov steady state is in general different from its counterpart in Ramsey steady state. This result shows that lack of credibility has consequences for the long run properties of optimal policies. Discretionary policymaker in attempt to boost inefficiently low output in the long run leads to biases in inflation and government spending. The former is a reminiscence of the debated outcome due to credibility problem from [Kydland and Prescott \(1977\)](#), and the latter is an accompaniment that arises due to distortionary financing of endogenous government spending akin [Adam and Billi \(2014\)](#) where compared to my setup there is no coordination between monetary and fiscal government authorities.²⁶

D.1 Efficient subsidy rate

Here I relax the assumption of zero subsidy and provide proof of the proposition 1.

Proposition. *Given debt level b , there exists a unique employment subsidy rate $s^e(b)$ such that*

- *corresponding Ramsey steady state is efficient,*
- *there is a Markov steady state with debt level b that is also efficient.*

²⁶The idea that discretionary conduct of the monetary policy will lead to a (higher) average inflation bias was criticized for instance in [Blinder \(1997\)](#).

Proof. When $s \neq 0$, implimentability constraints of the competitive equilibrium at the steady state imply

$$\left(\frac{\theta - 1}{\theta} \frac{1}{(1 - s)} - \frac{G + (1 - \beta)b}{Y} \right) u_c = v_y.$$

Given efficiency conditions derived from the Planner's problem in Appendix A, necessary condition for competitive equilibrium to be efficient is

$$\left(\frac{\theta - 1}{\theta} \frac{1}{(1 - s)} - \frac{G + (1 - \beta)b}{Y} \right) = 1.$$

Subsidy rate $s^e(b)$ is a solution to this equation. One can then verify that with $s^e(b)$ remaining first-order conditions in Ramsey and Markov equilibria coincide and remaining efficiency conditions are satisfied. \square

E Solution method

I use value function iteration procedure to search for a fixed point of value function and policy functions. Starting with a guess of the next period value function and future policy functions, on a discretized state space, I solve Markov optimization problem of the discretionary government. New solution is used to update guesses of the value and policy functions, and repeat the procedure until convergence when value and policy functions from two consecutive iterations become arbitrarily close.

Off the grid points I use cubic splines to interpolate value and policy functions. Expectations are computed with Gauss-Hermite quadrature. As for the grid, I start with equidistant points and then augment them with a set of adaptive points. Borrowing idea from [Brumm and Grill \(2014\)](#), I choose adaptive part of the grid to better capture the kink due to occasionally binding zero lower bound.

I implement solution method in Matlab using open source nonlinear optimization solver IPOPT. Interface of IPOPT solver for Matlab environment is implemented in a freeware third-party OPTI toolbox. To improve computation speed, solution of the Markov optimization problem over the grid is done via parallel computing.

Figure 1: Core government spending and taxation since 2007 in Eurozone countries, Czech Republic, Denmark, Japan, Sweden, Switzerland, the United Kingdom, and the United States.

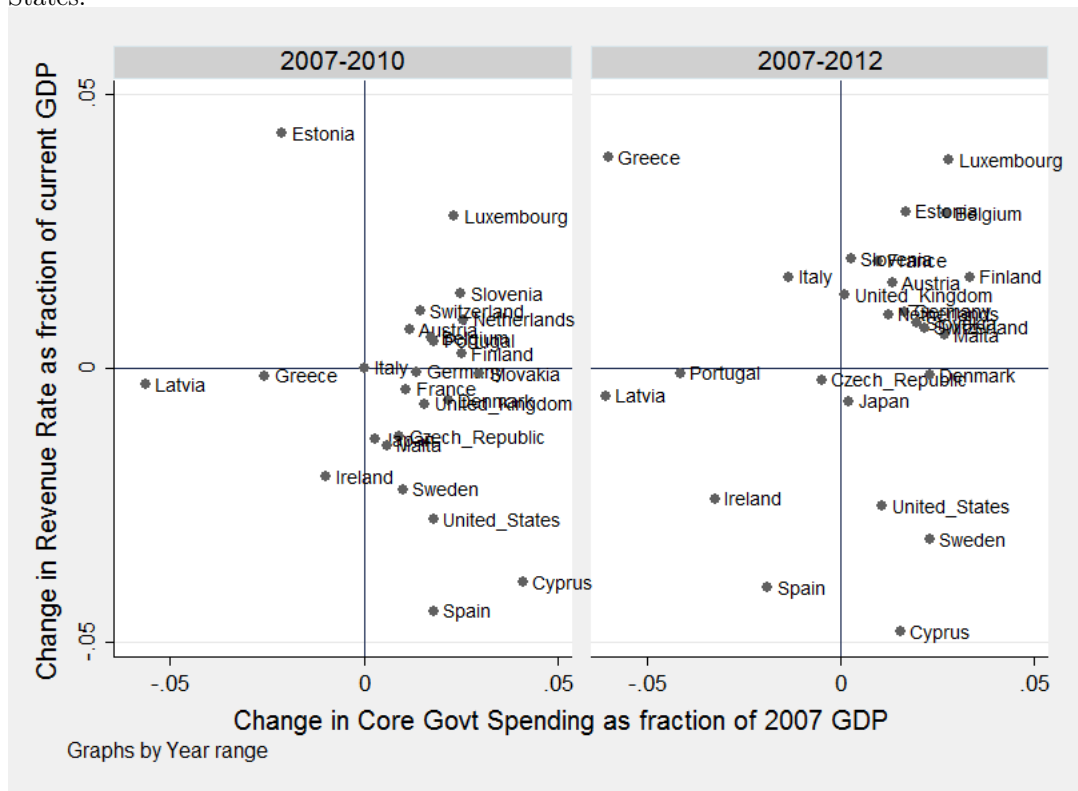
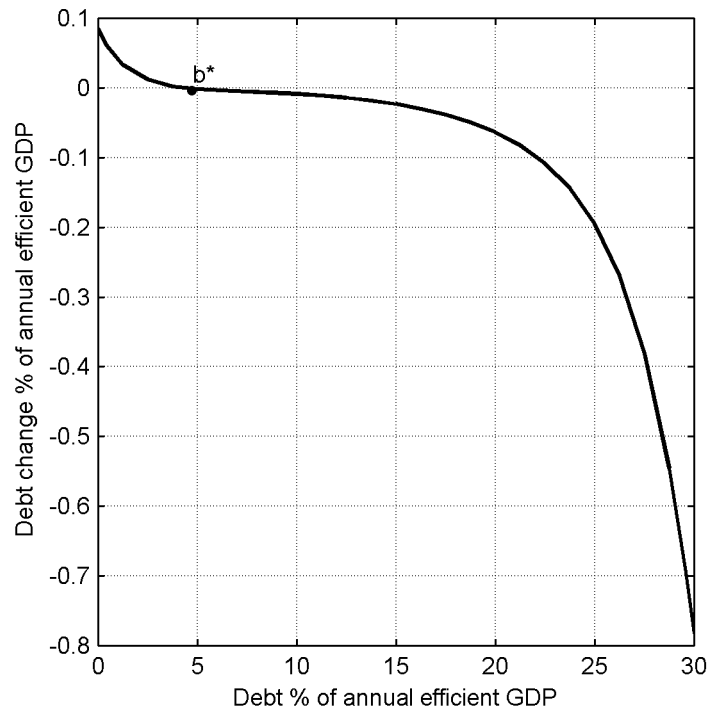
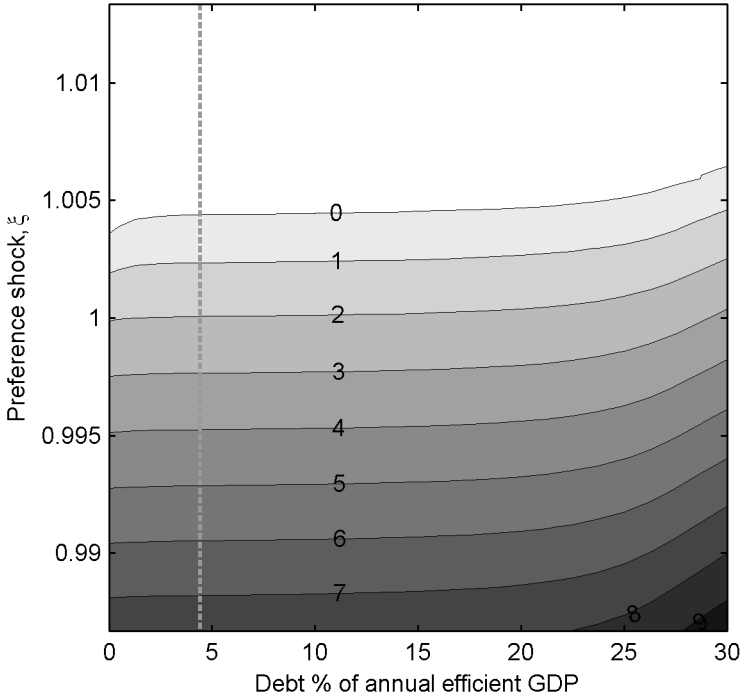


Figure 2: Optimal debt change policy, $(\mathcal{B}(b_{-1}, \xi) - b_{-1})$, conditional on $\xi = 1$



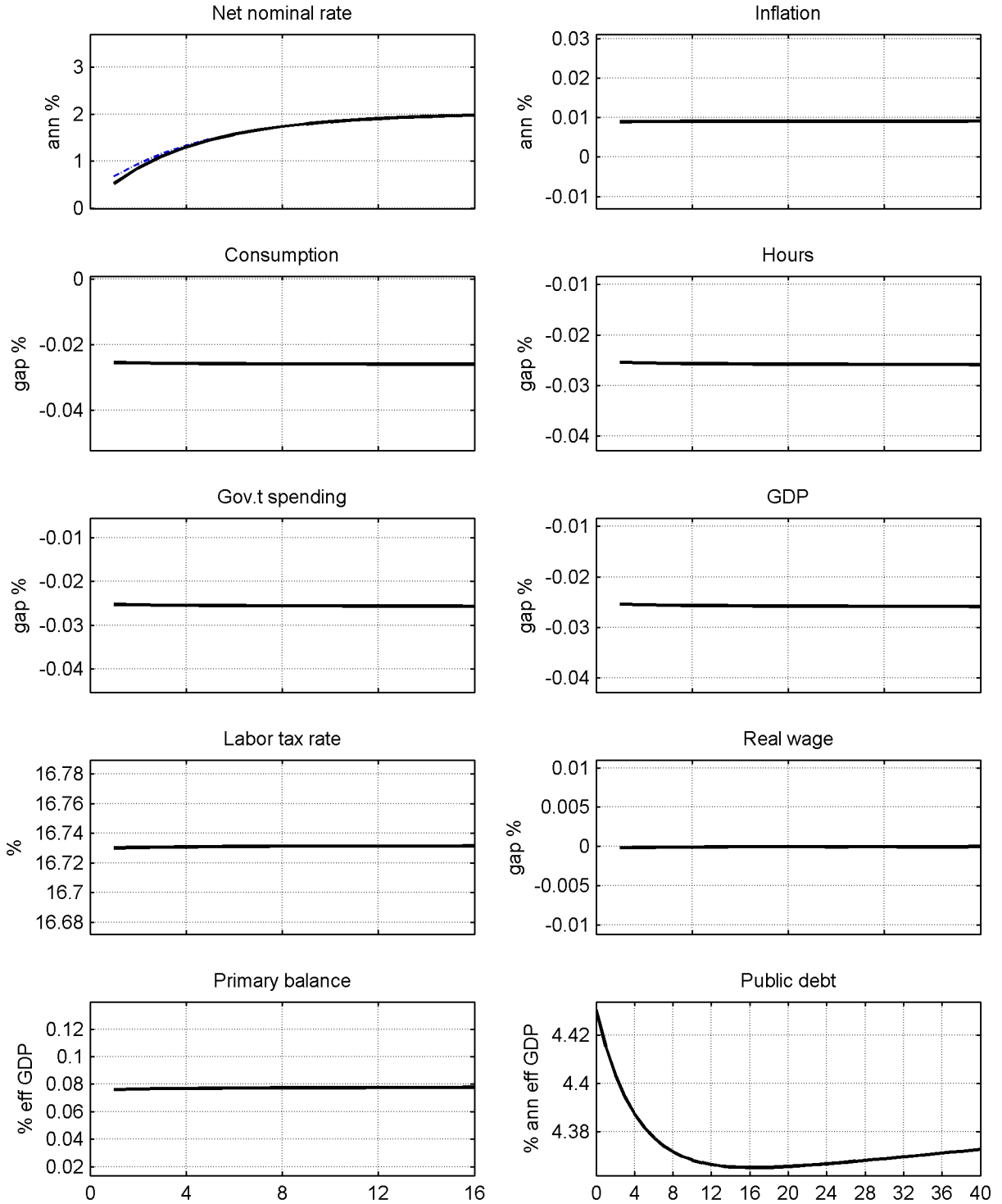
Notes: b^* is the optimal long run debt level at the risky steady state.

Figure 3: Contour lines of optimal nominal interest rate policy, $\mathcal{R}(b_{-1}, \xi)$



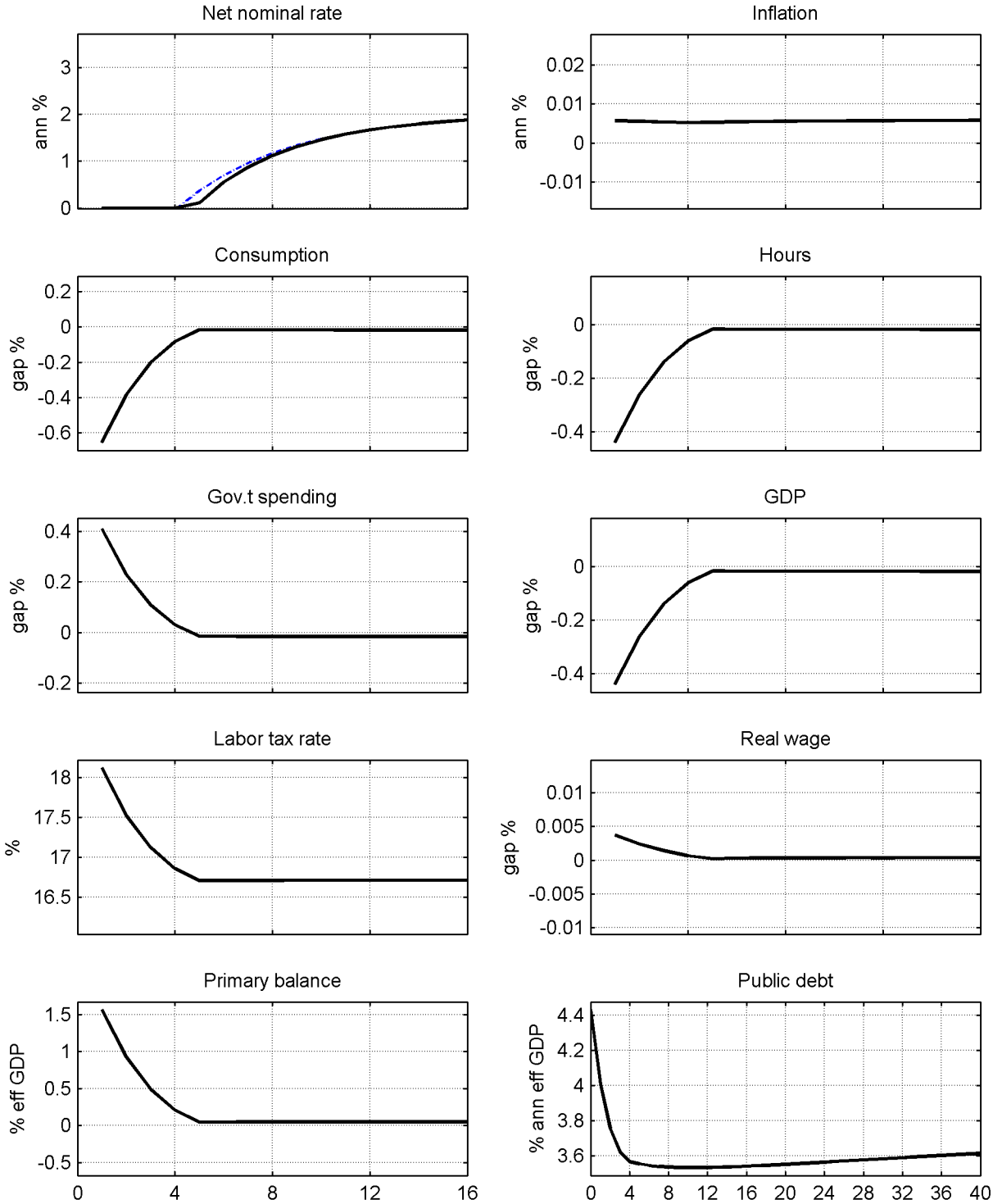
Notes: Numbers on the contour lines are for the net annualized nominal interest rate. White area corresponds to the state space area where the zero lower bound is binding. Dotted line marks optimal long run debt level b^* .

Figure 4: Impulse responses when zero lower bound is not reached



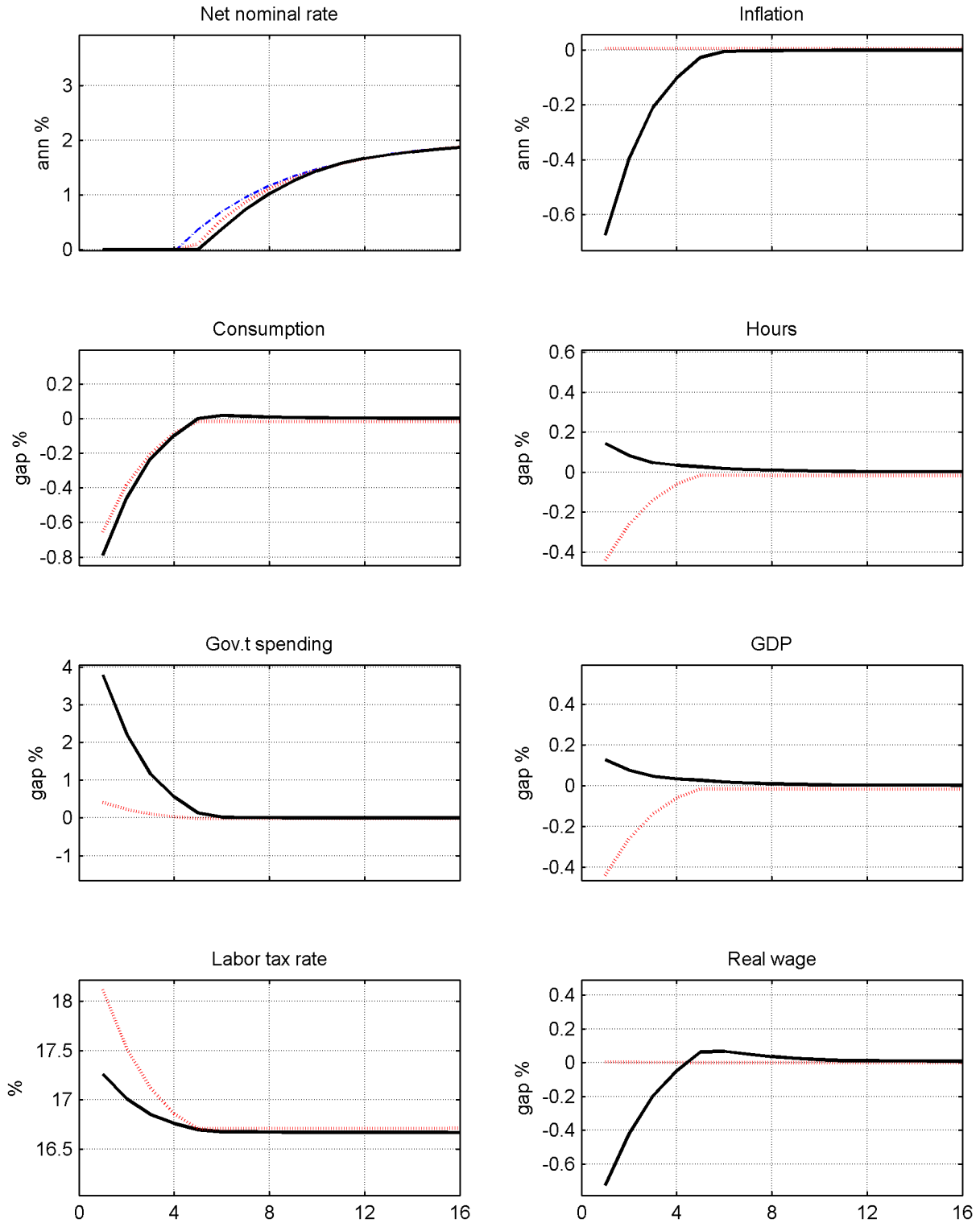
Notes: Impulse responses to a +1 unconditional standard deviation preference shock starting from optimal long run debt level b^* . Dashed blue line in the top left panel shows the (net) natural real interest rate.

Figure 5: Impulse responses when zero lower bound is reached



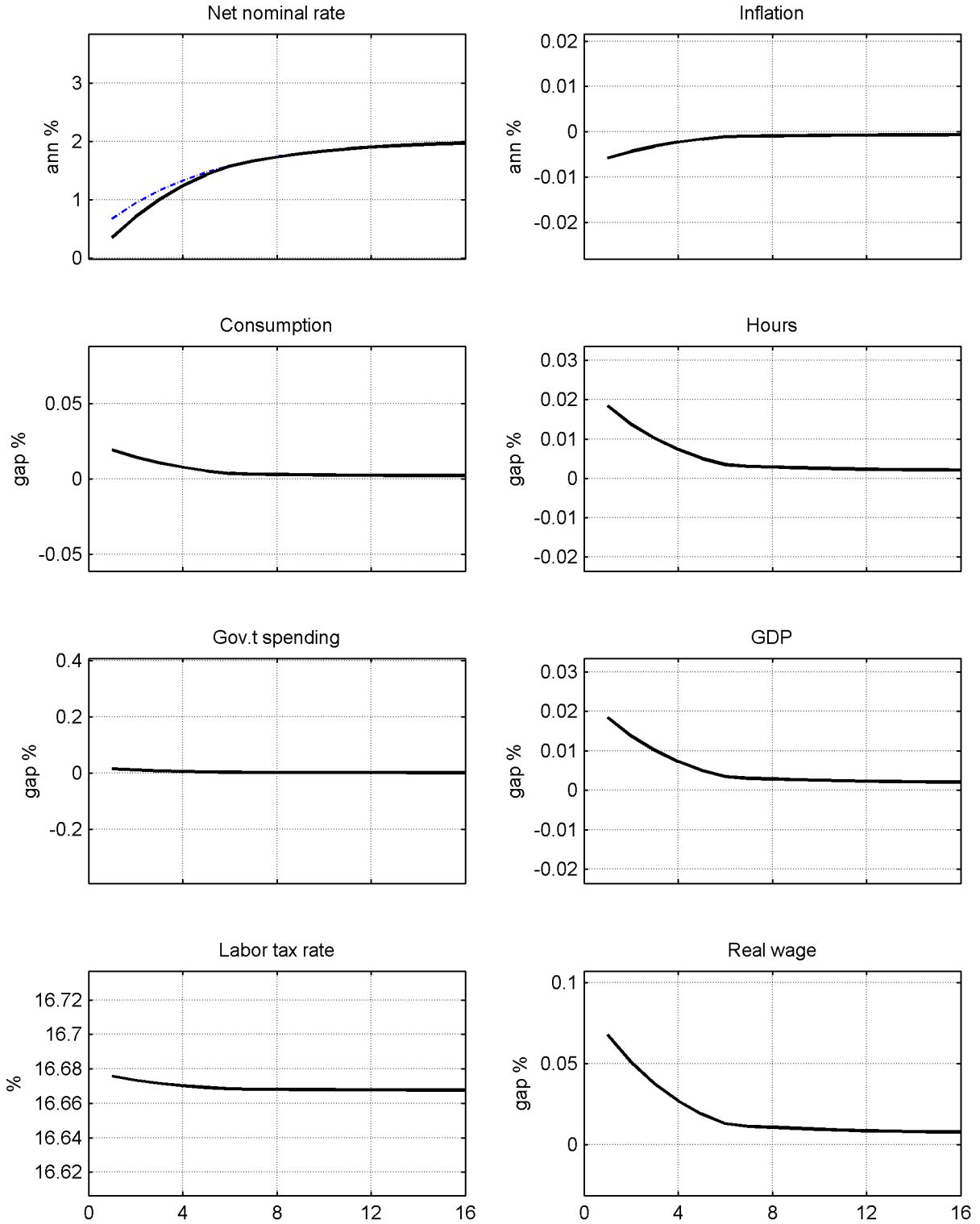
Notes: Impulse responses to a +3 unconditional standard deviations preference shock starting from optimal long run debt level b^* . Dashed blue line in the top left panel shows the natural real interest rate.

Figure 6: Impulse responses when zero lower bound is reached: *balanced vs relaxed budget*



Notes: Impulse responses to a +3 unconditional standard deviations preference shock. Solid black lines are responses under balanced budget. Dotted red lines are responses under relaxed budget. Dashed blue line in the top left panel shows the natural real interest rate.

Figure 7: Impulse responses when zero lower bound is not reached under *balanced budget*



Notes: Impulse responses to a +1 unconditional standard deviation preference shock. Dashed blue line in the top left panel shows the natural real interest rate.