

Domestic Debt and Sovereign Defaults

Enrico Mallucci*

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Abstract

This paper examines how domestic holdings of government debt affect sovereign default risk and government debt management. I develop a dynamic stochastic general equilibrium model with endogenous default risk which includes both external debt and domestic debt. In this setup I show that domestic debt endogenously explains output contraction observed upon default. Defaults weaken investors balance sheet and cause a contraction of the credit supply to the private sector. Ultimately output falls. Consequently, domestic holdings of government debt act as a disciplinary device and reduce government incentives to default. I calibrate the model to the Argentinean economy and I show that the model improves the quantitative performance of existing endogenous default models generating default rates and debt to GDP ratios that are close to the empirics. The introduction of domestic debt also has important normative implications. I show that the socially efficient domestic vs. external composition of debt cannot be achieved without government intervention. Atomistic domestic investors fail to internalize the consequences of their purchases on the sustainability of public balances. Pigouvian subsidies sustaining domestic bond purchases can restore efficiency.

JEL classification: F34, G21, E32.

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*London School of Economics and Centre for Macroeconomics. Houghton Street, WC2A 2AE London. E-mail: e.mallucci@lse.ac.uk. I would like to thank Gianluca Benigno and Keyu Jin for their support and advice, and Ethan Ilezki, Daniel Osorio, Guido Sandleris, Kevin Sheedy, Roman Šustek and Silvana Tenreyro and participants to the LSE brown bag seminar, 2014 EEA meeting, II Macro Banking and Finance Workshop and European Financial Crisis Workshop for comments and suggestions.

1 Introduction

Understanding why countries default is central to studies about the sustainability of public finances. The scholarly literature has focused on two factors to explain default episodes: the size of external debt and the fluctuation of the economic cycle. While these two dimensions are certainly important, another aspect is equally important: the internal versus external composition of debt. Reinhart and Rogoff (2008) highlight the importance of domestic debt. They study the evolution of domestic debt from 1914 to 2007 for a sample of 64 countries and conclude that there is a “forgotten history of domestic debt”. While domestic debt dynamics are relevant to understand sovereign default episodes, very little research has been devoted to it. My paper fills in the gap by incorporating domestic debt in the sovereign default literature.

Four key empirical regularities demonstrate the importance of domestic debt. (i) Domestic debt is large and constitutes the largest fraction of government debt. (ii) Contrary to conventional wisdom, domestic investors are not strictly junior to external investors. Outright defaults on domestic debt happen and happen frequently. About 45% of the default episodes between 1980 and 2007 also involved domestic debt. (iii) Output contracts more around domestic default episodes rather than around external default episodes (iv) as does credit. Based on these regularities pertaining to domestic debt, I construct a dynamic stochastic general equilibrium model with endogenous default risk á la Eaton and Gersovitz (1981) that incorporates domestic debt and thereby rationalizes the four empirical regularities.

The theoretical model is composed of four sectors: a benevolent government, households (domestic investors), firms, and international investors. Households purchase domestic bonds and use them to store liquidity. As in Holmstrom and Tirole (1998) and in Gennaioli et al. (2014) liquidity from the maturing bonds is transferred from households (investors) to firms that are subject to a working capital constraint and require credit to finance a fraction of their wage payments. Finally, the price of government debt is determined by international investors that have access to both government debt and a risk-free asset. Within this framework, I show that the introduction of domestic debt has important positive and normative implications.

My main contributions are the following. First, while standard sovereign default models (i.e. Arellano, 2008 and Aguiar and Gopinath, 2006) assume exogenous output costs to default, the study of domestic debt allows me to illustrate an endogenous mechanism linking defaults

and output contractions through the credit market which is consistent with the empirics.¹ The mechanism works as follows: Sovereign defaults weaken domestic investor's balance sheets causing a contraction of credit and a fall in output which is the stronger the larger the exposure towards domestic debt. Consequently, domestic debt disciplines governments and reduces incentives to default. The bigger the domestic holdings of government debt, the lower the probability of default and the larger the quantity of government debt that is sustainable in equilibrium.

Second, I calibrate the model to Argentina. Endogenous sovereign default models have difficulties matching simultaneously the high default rates and the high debt to GDP ratios. High default rates reduce debt to GDP ratios in equilibrium and vice-versa. The disciplinary power exerted by domestic debt reduces the tension between the size of government debt and the incidence of defaults. Thus, I show that accounting for domestic debt helps to explain high debt to GDP ratios given the observed probability of default.

Third, the inclusion of domestic debt has important normative implications. Debt composition as well as debt size and cycle fluctuation matter to determining the default risk. In this paper, I evaluate whether markets can autonomously allocate domestic and external debt efficiently. I find that the competitive equilibrium is not efficient, whenever domestic investors are small and take government debt prices as given. Domestic investors consume too much and lend too little to the government: i.e. domestic holdings of government debt are sub-optimally low. The sub-optimality of debt composition introduces a pecuniary externality in the economy through the price of government debt. As this price determines the terms of borrowing for government debt, the resulting management of the aggregate sovereign debt is also inefficient. Efficiency can be restored if the set of available policy instruments is extended to include a Pigouvian subsidy that incentives domestic purchases of government bonds.

My work is closely related to quantitative models of sovereign default that extend the seminal work of [Eaton and Gersovitz \(1981\)](#) (i.e. [Arellano, 2008](#) and [Aguiar and Gopinath, 2006](#)). My paper also relates to the literature that studies the interplay between sovereign defaults, financial intermediaries and the credit market. [Gennaioli et al. \(2014\)](#), [Bolton and Jeanne](#)

¹Theoretical sovereign default models have typically associated output losses to exogenous external forces (i.e. [Arellano, 2008](#) and [Mendoza and Yue, 2012](#)): in retaliation to a default episode, lenders exclude countries from international financial markets and international trade causing output contraction. This view, however, has been challenged by several empirical works (i.e. [De Paoli et al., 2009](#); [Sandleris, 2012](#) and [Borensztein and Panizza, 2009](#)) that have shown that the bulk of output contraction around defaults is actually explained by domestic credit contraction.

(2011), [Brutti \(2011\)](#) and [Sosa Padilla \(2014\)](#) build endogenous default models about the relation between domestic government debt and the banking sector.² My paper builds on this literature as it also relates sovereign default costs to the contraction of the credit market induced by losses incurred by domestic investors. I extend this literature studying domestic and external debt simultaneously.

My paper also relates to the work of [Broner et al. \(2010\)](#). This influential paper argues that the presence of complete secondary markets for government debt paired with the ability of the government to distinguish between domestic and foreign lenders can rationalize the existence of government debt even in the absence of default costs.³ In my work I also emphasize the crucial role of domestic debt, in that domestic debt quantities determine default risk and default costs. However, my paper departs from [Broner et al. \(2010\)](#) in that it does not aim to rationalize the existence of government debt but it aims to study the quantitative and normative implications of the existence of domestic debt.

Finally, my paper also relates to the literature about externalities in small open economies. [Bianchi and Mendoza \(2011\)](#) show that sudden stop episodes are explained by excessive borrowing in the wake of crises. Agents fail to internalize the consequences of their own actions on borrowing costs and borrow too much. In this paper, I show that a similar mechanism applies to the sovereign debt market, but with different implications. Whenever domestic investors are too small to internalize the impact of their purchases on the price of government debt, domestic investors consume too much and lend too little to the government. Governments, thus, borrow too little and default too often. Efficiency can be restored with the introduction of subsidies that promote domestic holdings of government bonds.

The rest of the paper is organized as follows: Section 2 presents some stylized facts and narrative evidence about the relationship between sovereign defaults and internal debt. Section 3 introduces the quantitative model that I employ to study debt composition and its implication for default risk. Section 4 introduces a simplified version of the model and derives the

²[Gennaioli et al. \(2014\)](#) propose a stylized model relating default costs to the development of the domestic financial sector. More developed financial sectors sustain higher quantities of government debt. As sovereign defaults weaken the balance sheet of banks, greater exposure reduces the incentives to default. Similarly, [Brutti \(2011\)](#) proposes a stylized model where government debt is used to store liquidity and sovereign default causes a liquidity shortage. Finally [Bolton and Jeanne \(2011\)](#), [Sosa Padilla \(2014\)](#) also develop models that link default costs to banks holdings of government debt. In a recent paper [Bocola \(2014\)](#) also studies the relation between sovereign defaults and banking crises in a quantitative model where sovereign defaults are exogenous

³The argument is very intuitive. Efficient secondary markets can be used to transfer government bonds holdings from foreign lenders to domestic ones if the government decides to default. If this is the case, an altruistic government never defaults and the existence of government debt is justified.

main conclusions of the paper in an intuitive and analytical form. Section 5 formally defines the Pareto Optimal Equilibrium and the Competitive Equilibrium for the economy and discusses under what circumstances the two equilibria do not coincide. Section 6 calibrates the full quantitative model. Section 7 investigates the positive implications of introducing domestic debt in the framework, while Section 8 discusses the normative implications. Section 9 concludes the paper.

2 Stylized Facts

The sovereign default literature has concentrated on external debt dynamics while very little has been said about the domestic component of debt. Reinhart and Rogoff (2008) present an interesting overview of domestic debt dynamics and they conclude that there is a “forgotten history of domestic debt”. Domestic debt has been greatly overlooked despite empirical evidence suggests that internal debt dynamics are actually important to assess the sustainability of public balances and the risk of default. In this section I present some stylized facts that confirm the importance of domestic debt dynamics for aggregate debt management.

Data about domestic holdings of government debt were gathered from a number of different data sources. Most of the data were collected from the Quarterly Public Sector Debt database (QPSD) compiled by the World Bank. As some countries with interesting debt dynamics were not surveyed (i.e. Argentina and Japan), I complemented QSPD data with data from the International Debt Statistics database (IDS) also compiled by the World Bank and the Bruegel Sovereign Bond Dataset (BSBD). IDS contains yearly data for the external public debt. Internal debt is estimated as the difference between total debt and external debt. Total public debt figures are taken from Reinhart and Rogoff’s “This time is different” website. Finally BSBD collects quarterly data about public debt and its composition for the major European economies.

Domestic Debt Size

Figure 1 describes the composition of debt at the end of 2013 for a sample of emerging and developed economies. For no country in the sample, the share of domestic debt is negligible. The median internal to total debt ratio is roughly 0.6% making internal debt the most

Figure 1. Internal Debt Component

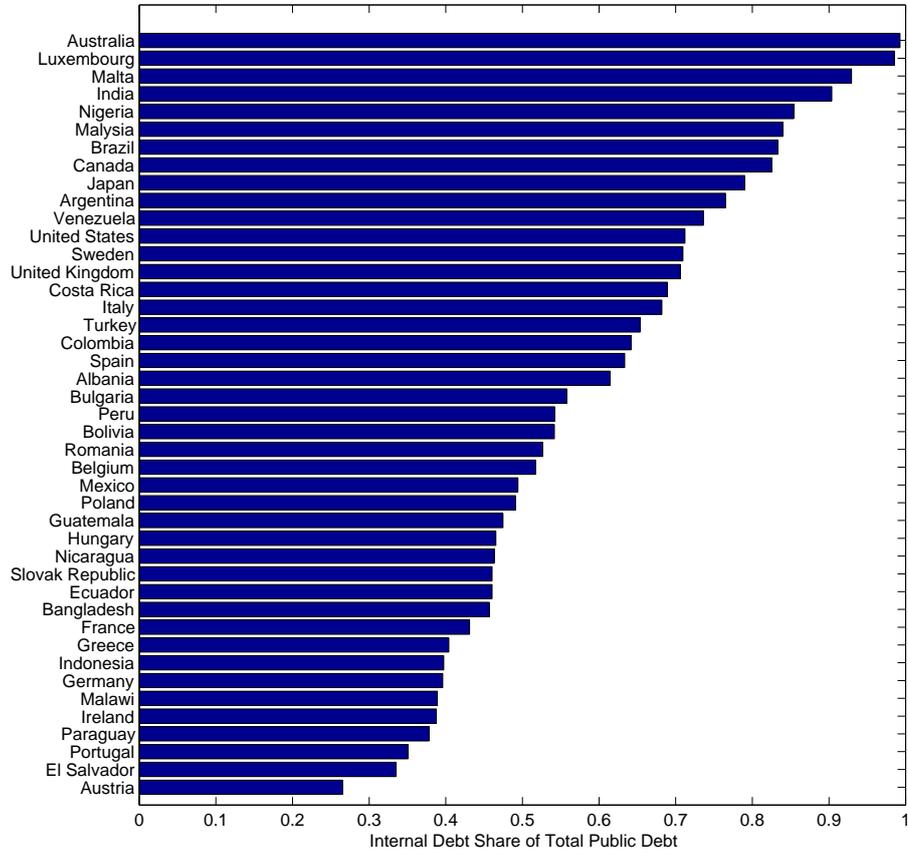


Figure 1 displays the domestic fraction of public debt for a sample of 43 countries. Data refer to the last quarter of 2013.

important component of public debt. Even for small economies like Austria and Ireland the domestic to total debt ratio is above 0.25.

Default Incidence

According to the conventional wisdom, domestic debt is senior to external debt and therefore episodes of outright defaults on domestic debt are rare. Empirical evidence, however, does

not fully support this belief. Table 1 groups the 49 default episodes that happened since 1980 into different categories according to the jurisdiction of the debt titles involved in the default episode.⁴ Surprisingly, more than 45% of the default episodes were not limited to external debt, but also included defaults on the domestic component of debt. The case of Argentina is emblematic. Argentina defaulted three times since 1980. Once on domestic debt in 1989 and twice on both external and domestic debt in 1982 and 2001.

Table 1. Incidence of Sovereign Defaults

	Defaults	External Only	External & Internal	Internal Only	De Facto
Obs.	49	26	12	8	3
Pct.	-	53%	24%	17%	6%

Table 1 displays the incidence of sovereign default episodes. Episodes are grouped according to the class of lenders that were involved in the default episode. De facto defaults are episodes of hyperinflation with inflation of more than 100% per month

Defaults and Output Contraction

Countries typically incur in output losses around default episodes. Table 2 shows the evolution of normalized output around default for the 49 default episodes identified between 1980 and 2005 distinguishing between pure external defaults and defaults that also involved domestic debt. Two patterns emerge. First, domestic default episodes are typically associated with stronger output contractions during the years that precede the default. Output contracts by about 3% in the three years preceding a domestic default, while it increases by 17% in the wake of an external default. Second, the evolution of output appears to differ also in the aftermath of a default. On the year of default output contracts by 7% following a domestic default episode, while it contracts by only 4% following an external default. Patterns remain similar when I de-trend output and study the evolution of the output gap. Once again movements in the output gap appear to be stronger around domestic default episodes.

⁴The complete list of default episodes is reported in Table 9 in the Appendix.

Table 2. Output Dynamics around Default

Time	Domestic Defaults		External Defaults	
	Output	Output Gap	Output	Output Gap
t-3	100	0.03	100	0.02
t-2	98	0.04	111	0.03
t-1	97	0.02	117	0.07
t	90	-0.06	112	-0.03
t+1	88	-0.11	116	-0.03
t+2	108	-0.01	127	0.00
t+3	125	0.02	135	0.02

Table 2 describes the evolution of output around default episodes. The first two columns display the evolution of normalized output levels and the output gap around episodes of domestic default. The third and the fourth column display the evolution of normalized output and the output gap around default episodes that only involve external creditors.

Output dynamics reported in Table 2 are confirmed by similar estimates presented in [Reinhart and Rogoff \(2008\)](#). The two authors also find that output contracts more around domestic default episodes. They also confirm that the impact of external defaults on output is modest.

Defaults and Credit Contraction

Understanding why defaults on domestic debt is associated with greater output losses requires a deep understanding of the channels relating the sovereign bond market and the real economy. The theoretical literature has typically related output contraction to external factors such as the exclusion from trade and from financial markets. However, this view has found little empirical support. Empirical studies (e.g. [Sandleris, 2012](#)) have shown that the reduction of trade volumes observed in the aftermath of sovereign default episodes is a consequence, not a cause, of output contraction. Similarly, the length of the exclusion time from financial markets is too short to explain output fall.⁵

⁵The median exclusion length has dropped from four years in the eighties to two years in the nineties ([Gelos et al., 2011](#))

Table 3. Credit Supply around Default

Time	Domestic Defaults		External Defaults	
	<i>Credit/Y</i>	% Change	<i>Credit/Y</i>	% Change
t-3	28.02		24.91	
t-2	27.46	-2.00%	25.27	1.45%
t-1	27.58	-1.57%	26.48	6.30%
t	27.01	-3.60%	27.56	10.64%
t+1	24.49	-12.60%	27.6	10.80%
t+2	24.48	-12.63%	26.96	8.23%
t+3	26.09	-6.89%	26.81	7.63%

Table 3 describes the evolution of credit around default episodes. The first two columns display the evolution of the credit to GDP ratio and the cumulated percentage change of the credit to GDP ratio around episodes of domestic default. The third and the fourth column repeat the analysis for external default episodes. Data are based on a sub sample of 29 defaults episodes for which credit data is available.

Recent developments in the empirical literature have underlined the importance of internal factors to determine output contraction around defaults (De Paoli et al., 2009). The contraction of internal (Albertazzi et al., 2014) and external (Arteta and Hale, 2008) credit in particular appears to play a key role. Gennaioli et al. (2014) find that financial intermediaries curtail credit supply more when their exposure to government debt titles is stronger. Intuitively, the greater exposure towards domestic debt titles, the bigger the loss of financial intermediaries upon default.⁶

Table 3 displays the evolution of the credit to GDP ratio around default episodes.⁷ Credit is measured as the loans supplied by financial intermediaries to the private sector.⁸ Default episodes are divided in domestic and internal defaults according to the jurisdiction of the titles involved in the default. Credit contraction is sharper during episodes of domestic default than in external default episodes. In the three years preceding sovereign a domestic

⁶This finding is also confirmed by Sandleris (2012).

⁷Data about the credit market is taken from the Financial Structure Dataset (FSD) created by Beck et al. (2009). A complete list of the episodes surveyed is contained in Table 9.

⁸Data about alternative sources of funding for firms –such as the private credit to the private sector– are scarce and incomplete. A few countries (i.e. Argentina) report information about alternative sources of credit. However, alternative credit channels are small and unimportant for the overall evolution of credit dynamics.

default credit contracts by 3.6% while it rises by more than 10% in the wake of an external default. Consequences of a domestic defaults are also stronger. Credit contracts by 9% in the two years following a domestic default episode and by only 2.5% in the aftermath of an external default.

3 Model

In this section I extend a standard quantitative sovereign default model to allow for the contemporaneous presence of domestic and external government debt. The model economy is composed of four sectors: Firms, Domestic Households/Investors, External Investors and the Government.

3.1 Firms

Output y is produced by perfectly competitive firms that use labor N as the only input factor and are subject to a productivity shock z :

$$y = zf(N). \tag{1}$$

Firms are subject to a working capital constraint as in [Mendoza and Yue \(2012\)](#) and need to pay a fraction γ of labor costs before output is realized. Anticipated wage payments are financed with intra-temporal loans that firms receive from domestic investors against the payment of an interest rate r^L . Firms' demand for credit L is determined by the working capital constraint:

$$L = \gamma wN. \tag{2}$$

Where w is the wage rate and N is the labor demand. Labor demand is chosen to maximize profits:

$$\max_N \{zf(N) - wN - r^L\gamma wN\} \tag{3}$$

The first order condition associated with the maximization problem of the firm is:

$$N : w = \frac{zf_N(N)}{1 + \gamma r^L}, \quad (4)$$

Equation (4) relates wages to the marginal product of labor and to the prevailing conditions on the credit market. High loan rates r^L make the financing of anticipated wage payments more expensive. As r^L increases firms lower wages to counterbalance the higher cost of credit.

3.2 Households/Domestic Investors

The economy is populated by a continuum i of households with mass one. Each household is composed of a worker and a investor. Workers and investors pool income and consumption. Households value consumption c and dislike labor N according to the flow utility $u(c, N)$. Future is discounted at the rate β . The problem for the household is that of making contingent plans for consumption, labor supply and domestic bond holdings b^H so as to maximize her lifetime utility. Let π be the profits of the firm, π^I the profits from investment and $B^H \equiv \int_0^1 b^H di$ aggregate domestic bond holdings. The maximization problem of each individual household is

$$\begin{aligned} \max_{c, N, b^H} V^H(z, B, B^H, b^H) &= U^H(c, N) + \beta E(1 - def') V'^H(z', B', B'^H, b'^H | z) \\ &\quad + \beta E def' V'^H(z', 0, 0, 0 | z) \\ \text{s.t. } c + T(z, B, B^H) &= wN + \pi + \pi^I. \end{aligned} \quad (5)$$

Budget constraint (5) states that household consumption c and lump-sum tax payments T are financed pooling resources from workers and investors. Workers contribute labor income wN and firms profits π . Investors contribute their profits π^I . To ease the exposition I divide the household problem in the problem of the representative workers and the problem of the representative investor.

Workers

Workers choose consumption and labor taking firms profits and investor's profits as given. The maximization problem is intra-temporal. The representative worker solves:

$$\max_{c,N} U^H(c, N)$$

$$s.t. (5). \tag{6}$$

The first order condition with respect to consumption and labor are:

$$c : U_c = \lambda; \tag{7}$$

$$N : -\frac{U_N}{U_c} = w; \tag{8}$$

Investors

Investors maximize the contribution of profits from investment to the utility of the household. Investors have access to two different markets: the market for government bonds and the market for loans to firms. The access to the two markets is sequential. The recursive problem of investors is separated in two interim times: morning and afternoon.

In the morning investors learn about the realization of the shock z and about the default decision of the government. In absence of default households receive liquidity from maturing bonds. Liquidity can be either stored for consumption or it can be transformed into an inter-temporal loan l according to the loan production function:

$$l = g(b^H). \tag{9}$$

Loans to the private sector pay an interest rate r^L in the afternoon.⁹ The credit supply

⁹As consumption is only possible in the afternoon and the inter-temporal rate r^L is always positive and there is no risk of default on intra-temporal loans, investors always transform the entire liquidity b^H from maturing bonds in loans that.

function (9) captures, in a reduced form, the role of public debt as a liquidity provider for the private sector as described by [Holmstrom and Tirole \(1998\)](#). Equation (9) links sovereign debt market and the credit market. Upon default loan supply contracts generating a credit crunch that pushes up the interest rates on loans. Higher interest rates reduce the wage rate according to equation (4) as well as the equilibrium supply of labor. Equation (9) can be interpreted as a reduced form for a more complex model in which the credit supply fluctuates with the net worth of financial intermediaries.¹⁰ In Appendix B, I present an extension of the model inspired by the work of [Boz et al. \(2014\)](#) and [Guerrieri et al. \(2013\)](#) that includes financial intermediaries which are subject to a capital requirement. This extended model provides a microfoundation to equation (9).

In the afternoon -the second and last interim period- investors receive interest rate payments $(1 + r^L)l$ and purchase government bonds. The profits function combines morning and afternoon cash flows:

$$\pi^I = r^L l - qb^H \quad (10)$$

The maximization problem of investors becomes:

$$\begin{aligned} \max_{b^H} \Pi^I(z, B, B^H, b^H) &= \pi^I(b^H) + EQ [(1 - P(def')) \Pi^I(z', B', B'^H, b^H|z)] \\ &\quad + EQ [P(def') \Pi^I(z', 0, 0, 0|z)] \\ s.t. Q &= \beta \frac{U_c(c', N')}{U_c(c, N)}; \end{aligned} \quad (11)$$

$$s.t. (9); (10).$$

Where equation (11) defines the stochastic discount factor. In equilibrium aggregate domestic investments and the individual ones coincide:

$$B^H \equiv \int_0^1 b^H di = b^H.$$

¹⁰Several papers studying the impact of sovereign defaults on the banking sector adopt similar reduced form equations to explain the interaction between the sovereign debt market and the credit supply (e.g. [Gennaioli et al., 2014](#), [Sosa Padilla, 2014](#) and [Bolton and Jeanne, 2011](#)).The liquidity role of public debt is also discussed extensively in the recent paper by [Angeletos et al. \(2013\)](#).

The same applies for aggregate and individual loans:

$$L \equiv \int_0^1 l \, di = l$$

In standard sovereign default models the representative agent framework paired with the Ricardian equivalence make internal claims completely irrelevant. Consumption smoothing is achieved with external debt while internal debt becomes irrelevant. In this model, instead, the Ricardian equivalence does not hold. The domestic fraction of government debt influences output levels through the credit supply. Low domestic debt levels are associated with low credit supply and low output.

3.3 International Investors

International investors are risk neutral agents with deep pockets. They have access to two different inter-temporal investment opportunities. The first investment opportunity is a risk-free asset that pays the risk-free interest rate r^f in every possible contingency. The second investment opportunity is the risky government bond that pays 1 when the government defaults and 0 otherwise. As international investors are risk-neutral, the price of government bonds is derived by arbitrage and is equal to:

$$q(z, B', B^H) = \frac{1 - P(def'|z)}{1 + r^f}. \quad (12)$$

The price q of government debt depends on the future probability of default and on the risk-free rate. The higher the probability of default the higher sovereign yields as they need to compensate investors for the risk. Similarly the higher the risk-free rate, the higher government yields to ensure that the arbitrage condition is respected. The future probability of default depends on the realization of three states. The productivity z , the overall government debt B' and domestic debt levels B^H .

Equation (12) defines the market clearing price of government bonds for international risk neutral investors. Under the assumption that government cannot operate selective defaults and that the marginal lender has deep pockets, equation (12) defines the price for both the

internal and external component of public debt. Under this assumption an externality is introduced in the economy as I will explain in detail in section 5.3.

3.4 Government

Government budget constraint is:

$$G + B = q(z, B', B'^H) B' + T. \quad (13)$$

Where G is the exogenous stream of government spending and T are non-distortionary lump-sum taxes. B is total government debt which is the sum of domestic debt B^H and external debt B^* :

$$B \equiv B^H + B^*. \quad (14)$$

The government also decides whether to default or not. The default decision is taken comparing households value function in the default scenario and in the non-default scenario. Let:

$$V^{nd}(z, B, B^H) = U(c, N) + \beta E(V'(z', B', B'^H)|z); \quad (15)$$

and

$$V^d(z, 0, 0) = U(c, N) + \beta^H E[(1 - \lambda)V^d(z', 0, 0) + \lambda V^{nd}(z', B', B'^H)] \quad (16)$$

be domestic investors' value function in the default scenario and in the non default scenario respectively. The default decision of the government can be expressed as follows:

$$def = \begin{cases} 0 & \text{if } V^{nd}(B, z, B^H) \geq V^d(0, z, 0) \\ 1 & \text{otherwise} \end{cases}.$$

Following a default the government loses access to domestic and international financial

markets. Still the government may regain access to capital markets with probability λ .

4 Analytical Model

In this section I develop a simple two periods model describing analytically the mechanisms operating in the model economy and illustrating the existence of a pecuniary externality that distorts debt composition. While the structure of the economy remains exactly the same as the one presented in Section 3, for simplicity it is assumed that the economy only lasts two periods: $t = 0$ and $t = 1$. Functional forms are also chosen to simplify calculations as much as possible. In particular the production function is assumed to be linear in labor: $y = zN$. The utility function is chosen to be linear in both labor and consumption: $U(c, N) = c - N$ and finally the loan production function is just linear in domestic bond holding: $l = b^H(1 - def) + \Gamma$.

Combining equations (2), (4), (8), (9) and the simplified functional forms specified above I obtain:

$$l = b^H(1 - def) + \Gamma; \quad (17)$$

$$N = \frac{b^H(1 - def) + \Gamma}{\gamma}; \quad (18)$$

$$w = \frac{b^H(1 - def) + \Gamma}{N\gamma}; \quad (19)$$

$$r^L = \frac{zN}{b^H(1 - def) + \Gamma} - \frac{1}{\gamma}. \quad (20)$$

Equations (17), (18), (19), (20) express credit and labor market equilibrium quantities and prices as a function of domestic bond holdings only. Equation (18) outlines the non-Ricardian nature of this economy. Larger domestic holding of government debt are associated with more labor in equilibrium. Upon default the wage rate falls –equation 19– and labor contracts. Equation (20) describes how domestic loan rates are determined. Two forces determine the intra-temporal interest rate r^L . On the one side higher productivity levels increase the labor

demand and the demand for credit pushing up the interest rate on loans. On the other hand, domestic purchases of government debt increase the supply of credit reducing the interest rate. As I will show in the quantitative exercise the supply factors tend to prevail in both normal and crisis times. The interest rate on loans is found to be countercyclical and the countercyclicity is even more pronounced around defaults. In the following sections I will make use of the equilibrium relations (2), (4), (8) and (9) to determine the optimal domestic debt exposure b^H , the optimal overall debt level B and the optimal default decision, Equations (17), (18), (19), (20) express credit and labor market equilibrium quantities and prices as a function of domestic bond holdings only. Equation (18) outlines the non-Ricardian nature of this economy. Larger domestic holding of government debt are associated with more labor in equilibrium. Upon default the wage rate fall (equation 19) and labor contracts. Equation (20) describes how domestic loan rates are determined. Two diverging forces determine the intra-temporal interest rate r^L . On the one side higher productivity levels increase the labor demand and the demand for credit pushing up the interest rate on loans. On the other hand, domestic purchases of government debt increase the supply of credit reducing the interest rate. As I will show in the quantitative exercise the supply factors tend to prevail in both normal and crisis times. The interest rate on loans is found to be countercyclical and the countercyclicity is even more pronounced around defaults. In the following sections I will make use of the equilibrium relations (2), (4), (8) and (9) to determine the optimal domestic debt exposure b^H , the optimal overall debt level B and the optimal default decision,

4.1 Time $t=1$

I solve the two periods model working backward from the last period: $t = 1$. The market for sovereign bonds is closed in $t = 1$ so the government cannot issue new debt in the last period: $b_2^H = B_2 = 0$. Substituting firms' profits (3), investors' profits (10) and the government budget constraint (13) in the Household's budget constraint (5) I obtain:

$$c_1 = b_1^H - B_1 - G + z_1 N_1. \quad (21)$$

Substituting (18) in (21), the budget constraint becomes:

$$c_1 = b_1^H - B_1 - G + z_1 \frac{b_1^H(1 - def) + \Gamma}{\gamma}. \quad (22)$$

Upon default budget constraint (22) becomes:

$$c_1 = -G + z_1 \frac{0 + \Gamma}{\gamma}. \quad (23)$$

As the economy lasts only two periods, household value function in period 1 is equal to the flow utility. I distinguish between the default and the non-default scenarios. In the non-default scenario:

$$V_1^{nd}(z_1, B_1, b_1^H) = c_1 - N_1$$

$$s.t. \text{ (22)}$$

In the default scenario:

$$V_1^d(z, 0, 0) = c_1^d - N_1^d$$

$$s.t. \text{ (23)}$$

By symmetry, the individual and the aggregate choices coincides: $B_1^H = \int_0^1 b_1^H di = b_1^H$. Comparing $V_1^{nd}(z_1, B_1, B_1^H)$ and $V_1^d(z, 0, 0)$ it is possible pin down the optimal default rule of the benevolent government:

$$def(z_1, B_1, B_1^H) = \begin{cases} 0 & \text{if } z_1 \geq \bar{z} \equiv \frac{\gamma B_1 - B_1^H}{B_1^H} \\ 1 & \text{otherwise} \end{cases}. \quad (24)$$

The default rule is defined on the triplet (z_1, B_1, B_1^H) . It depends on the productivity, on the aggregate size of government debt and also on the domestic share of total debt.

4.2 Time $t=0$

Price of Government Bonds

Investors are born in $t = 0$ with initial financial wealth b_0^H while government are born with initial debt B_0 . Without loss of generality it is assumed that defaults cannot happen in $t = 0$. International investors are fully rational and determine debt price by arbitrage:

$$q_1(z_0, B_1, B_1^H) = \frac{1 - Pr(\text{def}(z_1, B_1, B_1^H))}{1 + r^f} = \frac{1 - \Phi(z_1 \leq \bar{z}|z_0)}{1 + r^f}$$

Where $\Phi(\cdot)$ is the cumulative density function of z . Proposition 1 deals with then properties of the price function q .

Proposition 1. *The price of government debt is:*

1. *increasing with productivity levels:* $\frac{\partial q(B_1, z_0, B_1^H)}{\partial z_0} \geq 0$
2. *decreasing with aggregate debt levels:* $\frac{\partial q(B_1, z_0, B_1^H)}{\partial B_1} \leq 0$
3. *increasing with domestic debt exposure:* $\frac{\partial q(B_1, z_0, B_1^H)}{\partial B_1^H} \geq 0$.

Proof. See Appendix C □

Domestic Holdings of Government Debt

I now proceed deriving the optimal domestic debt quantity B_1^{H*} for a given government debt level B_1 both when investors internalize the impact of their purchases on the government borrowing terms (the competitive equilibrium) and when they don't (the constraint Pareto optimum).

The competitive equilibrium allocation of internal debt $b_1^{H,CE*}$ is solution to:

$$V_0(z_0, B_0, B_0^H, B_1^H) = \max_{b_1^{H,CE}} c_0 - N_0 + \beta E [(1 - P(def)) V_1^{nd} + P(def) V_1^d] \quad (25)$$

$$s.t. c_0 + G = z_0 \frac{B_0 + \Gamma}{\gamma} - q_1(z_0, B_1, B_1^H) (b_1^{H,CE} - B_1) + (B_0^H - B_0) - \frac{\nu}{2} (b_1^{H,CE})^2.$$

The extra term $\frac{\nu}{2} (b_1^{H,CE})^2$ is a quadratic cost which is merely introduced in the model to simplify the algebra.

The constrained efficient level of domestic debt $B_1^{H,PO*}$ is solution to:

$$V_0 = \max_{B_1^H} c_0 - N_0 + \beta E [(1 - P(def)) V_1^{nd} + P(def) V_1^d] \quad (26)$$

$$s.t. c_0 + G = z_0 \frac{B_0 + \Gamma}{\gamma} - q_1(z_0, B_1, B_1^H) (b_1^H - B_1) + (B_0^H - B_0) - \frac{\nu}{2} (b_1^H)^2;$$

$$s.t. q_1(z_0, B_1, B_1^H) = \frac{1 - Pr(def)}{1 + r^f}.$$

The constrained Pareto optimal problem (26) differs from the competitive equilibrium one (25) in that the set of constraints also includes the asset pricing equation for bonds. The government internalizes the impact of domestic holdings of government debt on the borrowing terms q . Proposition 2 compares $B_1^{H,CE*}$ and $B_1^{H,PO*}$ and outlines existence of a pecuniary externality.

Proposition 2 (Domestic Under-Lending). *Let $B_1^{H*,PO}$ and $B_1^{H,CE*}$ be solution to the Pareto optimal problem 26 and the competitive equilibrium problem 25 respectively. Domestic holdings of government debt are bigger in the Pareto Optimum than in the Competitive Equilibrium: $B_1^{H,PO*} \geq B_1^{H,CE*}$.*

Proof. See Appendix C □

Proposition 2 states that the competitive equilibrium allocation is not efficient. For a given level of aggregate debt B_1 domestic holdings of government debt are larger in the Pareto optimum than in the competitive equilibrium. Smaller domestic bond holdings affect the price of government debt according Proposition 1. Sovereign yields are higher in the competitive equilibrium than in the Pareto optimum. Proposition 3 deals with the consequences of price distortion on aggregate debt quantities B_1 . For a given internal debt quantity B_1^{H*} , the optimal government debt level is solution to the following maximization problem:

$$V_0(z_0, B_0, B_0^H) = \max_{B_1} c_0 - N_0 + \beta E [(1 - P(def)) V_1^d + P(def) V_1^d] \quad (27)$$

$$s.t. c_0 = z_0 \frac{B_0 + \Gamma}{\gamma} - q_1(z_0, B_1, B_1^{H*}) (B_1^{H*} - B_1) - \frac{\nu}{2} (B_1^{H*})^2;$$

$$s.t. q_1(z_0, B_1, B_1^H) = \frac{1 - Pr(def)}{1 + r^f};$$

Proposition 3 (Government Under-Borrowing). *Let B_1^{PO*} be the aggregate debt levels that solve the maximization problem 27 when B_1^{H*} is set equal to the optimal internal debt levels in the Pareto optimum. Let B_1^{CE*} be the aggregate debt levels that solve the maximization problem 27 when \bar{B}_1^H is equal to the internal debt of the competitive equilibrium problem. Then:*

$$B_1^{PO*} \geq B_1^{CE*}$$

whenever $\gamma \geq \beta E(1 - z_1) \frac{(1+r^f)}{r^f}$.

Proof. See Appendix C □

According to proposition 3 domestic under-lending introduces a pecuniary externality in the economy through the price of government debt. Distorted government debt prices lead to

government under-borrowing.

5 Equilibria and Externality

In this section I formally characterize the recursive competitive equilibrium and the recursive constrained Pareto optimum. While in standard sovereign default models the competitive equilibrium is Pareto optimal, the introduction of domestic debt brakes the equivalence between the two. As discussed by [Krusell and Jr. \(2003\)](#), typically, there is a problem of multiplicity of equilibria in infinite-horizon economies. To avoid this problem I analyze the equilibrium that arises as the limit of the finite-horizon economy as suggested by [Hatchondo et al. \(2010\)](#).

5.1 Recursive Competitive Equilibrium

In this section I define the recursive competitive equilibrium proceeding in three steps. First, I define the optimal individual and aggregate domestic investment policy. I then turn to the definition of the optimal government policy and finally I characterize the recursive competitive equilibrium.

Individual Domestic Investment Policy: The optimal individual domestic investment policy is a set of prices $\{w^{CE}, r^{L,CE}\}$ and quantities $\{b^{H,CE}\}$ with associated consumption and labor plans $\{c^{CE}, N^{CE}\}$ such that, for the given states of the economy $\{z, B, B^{H,CE}, b^{H,CE}\}$, for a given government policy $\{T^H, def\}$ and for a given sovereign debt price $q(z, B', B^{H,CE})$, prices and quantities solve the maximization problems of firms and individual households as described in [Section 3.1](#) and in [Section 3.2](#). The first order condition that determines the optimal domestic debt quantity in the competitive equilibrium is:

$$b^H : q(z, B', B^H)_c = \beta E \left[\frac{U_{c'}(c', N')}{U_c(c, N)} r'^L g'_L \right]. \quad (28)$$

In equilibrium individual and aggregate domestic bond holdings coincide: $B^{H,CE} = \int_0^1 b^{H,CE} di =$

$b^{H,CE}$.

Optimal Government Policy: The optimal government policy is defined by the borrowing rule $B'^{CE}(z, B, B^{H,CE})$ and the default decision $def^{CE}(z, B, B^{H,CE})$ that maximize the welfare of the economy given the optimal aggregate investment policy $B'^{H,CE}$.

The optimal borrowing rule in the non default scenario is obtained maximizing household's utility function for a given investment policy $B'^{H,CE}(z, B, B^H)$:

$$W^{nd} = \max_{B'} V^{nd}(z, B, B^{H,CE})$$

subject to:

$$V^{nd}(z, B, B^{H,CE}) = U(c, N) + \beta EV^{nd}(z', B', B'^{H,CE}|z); \quad (29)$$

$$c + G + q(B'^{H,CE} - B') - (B^{H,CE} - B) = zf(N); \quad (30)$$

$$N \frac{U_N}{U_c} = \frac{g(B^{H,CE})}{\gamma}; \quad (31)$$

$$q(z, B', B'^{H,CE}) = \frac{1 - P(def)|z}{1 + r^f}. \quad (32)$$

Equation (30) is the resource constraints of the economy and is derived combining the household budget constraint (5), the government budget constraint (13) and the profit function (10) Equation (31) is derived from equations (2), (4), (8) and (9). The first order condition with respect to B'^{CE} is:

$$B'^{CE} : q(z, B', B'^{H,CE}) + \frac{\partial q(z, B', B'^{H,CE})}{\partial B'} (B'^{H,CE} - B) = E \left[\beta \frac{U_{c'}(c', N')}{U_c(c, N)} \right] \quad (33)$$

When the government decides to default the value function of the benevolent government becomes:

$$W^d = V^d(z, 0, 0)$$

subject to:

$$V^d(z, 0, 0) = U(c, N) + \beta^H E \{(1 - \lambda)V^d(z', 0, 0|z) + \lambda V^{nd}(z', 0, 0|z)\} \quad (34)$$

$$c + G = zf(N); \quad (35)$$

$$N \frac{U_N}{U_c} = \frac{g(0)}{\gamma}; \quad (36)$$

If the government decides to default, it gets excluded from financial markets. However, it can be re-admitted with an exogenous probability λ as described by the continuation value in equation (34). Equations (35) and (36) are respectively the resource constraints of the economy and the condition that expresses the labor market equilibrium.

The default decision is taken comparing households welfare in the default scenario and in the non-default scenario:

$$W(z, B, B^H) = \max_{def \in \{0,1\}} (1 - def)W^{nd} + defW^d \quad (37)$$

Where W^{nd} and W^d are respectively the households' value function in the non-default and in the default scenario evaluated at the optimal borrowing level $B'(z, B, B^H)$ and given aggregate domestic holdings $B^{H,CE}(z, B, B^H)$.

Recursive Competitive Equilibrium: A recursive Competitive Equilibrium is a borrowing rule $B'(z, B, B^H)$, and an individual investment rule $b^{H,CE}(z, B, B^H)$ and a default rule $def(z, B, B^H)$ with associated household consumption and labor plans $\{c, N\}$, equilibrium prices $\{w, r^L\}$ and equilibrium asset pricing equation $q^{CE}(z, B', B^{H,CE})$ for sovereign bonds such that:

- The competitive equilibrium investment policy $b^{H,CE}(z, B, B^H, b^H)$ solves the maximization problem of the atomistic households and the representative firms for a given government policy $\{B', def\}$, and for a given price $q(z, B', B^{H,CE})$.
- The atomistic investment decision and the aggregate one coincide: $B^{H,CE*} \equiv \int_0^1 b^{H,CE*} di = b^{H,CE*}$.
- Atomistic loan supply equals the aggregate one: $L \equiv \int_0^1 g(b^{H,CE*}) di = l$.
- The borrowing rule for aggregate government debt $B'(z, B, B^H)$ and the default rule $def(z, B, B^H)$ solve the government problem 37 for a given aggregate investment rule $B^{H,CE}(z, B, B^H)$.
- The pricing equation for government debt satisfies equation (12): $q = \frac{1-Pr(def')}{1+r^f}$.
- The loans market and the labor market clear at prices $\{w, r^L\}$.
- The taxation rule $T(z, B, B^H)$ satisfies the government budget constraint for a given borrowing, investment and default rule and for a given price of sovereign bond.

5.2 Recursive Constraint Pareto Optimum

I now turn to the definition of the recursive constraint Pareto Optimum which is derived in two steps. First I define the optimal government policy. I then characterize the recursive constrained Pareto optimum. While in competitive equilibrium domestic bond holdings are determined by domestic investors, in the Pareto optimum the government chooses simultaneously domestic holdings of government debt $B^{H,PO*}$ and government debt size B^{PO*} .

Optimal Government Policy: The optimal government policy is defined by the borrowing rule $B'(z, B, B^H)$, the optimal rule $B^H(z, B, B^H)$ and the default decision $def(z, B, B^H)$ that jointly maximize the welfare of the economy.

The optimal borrowing and investment rules in the non-default scenario are obtained maximizing household's utility function:

$$W^{nd} = \max_{B', B^{H,PO}} V^{nd}(z, B, B^H)$$

subject to:

$$V^{nd}(z, B, B^H) = U(c, N) + \beta E (V'(z', B', B'^{H,PO})|z); \quad (38)$$

$$c + G + q(B'^{H,PO} - B') - (B^H - B) = zf(N); \quad (39)$$

$$N \frac{U_N}{U_c} = \frac{g(B^H)}{\gamma}; \quad (40)$$

$$q(z, B, B^H) = \frac{1 - P(def'|z)}{1 + r^f}; \quad (41)$$

Equation (39) describes the resource constraints of the economy while equation (40) expresses labor as a function of domestic debt. As the government chooses simultaneously both aggregate domestic bond holdings $B'^{H,PO}$ and aggregate debt levels B' there are now two first order conditions associated with the government problem:

$$B'^{H,PO} : q(z, B', B'^{H,PO}) + \frac{\partial q}{\partial B'^H} (B'^H - B') = \beta E \left[\frac{U_{c'}(c', N')}{U_c(c, N)} r'^L g'_L \right]. \quad (42)$$

$$B' : q(z, B', B'^{H,PO}) + \frac{\partial q(z, B', B'^{H,PO})}{\partial B'} (B'^{H,PO} - B) = E \left[\beta \frac{U_{c'}(c', N')}{U_c(c, N)} \right]. \quad (43)$$

The comparison between equations (28) and (42) shows that domestic government bond holdings in the competitive equilibrium are not efficient whenever the price of government debt is sensitive to changes in domestic debt quantities: $\frac{\partial q(z, B', B'^{H,CE})}{\partial B'} \neq 0$. This introduces a pecuniary externality in the economy through the price of government bonds as I discuss extensively in the section 5.3.

When the government decides to default the value function of the benevolent government becomes:

$$W^d = V^d(z, 0, 0)$$

subject to:

$$V^d(z, 0, 0) = U(c, N) + \beta^H E \{(1 - \lambda)V^d(z', 0, 0) + \lambda V^{nd}(z', B', B^H)\} \quad (44)$$

$$c = zf(N(0)); \quad (45)$$

$$N \frac{U_N}{U_c} = \frac{g(0)}{\gamma}. \quad (46)$$

If the government decides to default it may be re-admitted to financial markets with an exogenous probability λ as described in equation (44). Equation (45) describes the resource constraints of the economy in the default scenario while equation (46) expresses labor market equilibrium condition.

The default decision is taken comparing households welfare in in the default scenario and in the non-default scenario:

$$W(B, z, B^H) = \max_{def \in \{0,1\}} (1 - def)W^{nd} + defW^d. \quad (47)$$

Where W^{nd} and W^d are respectively the households' value function in the non-default and in the default scenario evaluated at the optimal borrowing level $B'(z, B, B^H)$ and at the optimal investment level $B^H(z, B, B^H)$.

Constraint Pareto Optimal Equilibrium: A recursive constraint Pareto optimal equilibrium is a borrowing rule $B'(z, B, B^H)$, and an investment rule $B^{H,PO}(z, B, B^H)$ and a default rule $def(z, B, B^H)$ with associated household consumption and labor plans $\{c(z, B, B^H), N(z, B, B^H)\}$ and equilibrium pricing equation $q(z, B, B^H)$ for sovereign bonds such that:

- The borrowing, investment and default rule solve Problem (47) for a given price $q(z, B', B^H)$.
- The price function for government debt satisfies: $q = \frac{1 - Pr(def')}{1 + r^f}$
- The loans market and the labor market clear at prices $\{w, r^L\}$.

- The taxation rule $T(z, B, B^H)$ satisfies the government budget constraint for a given borrowing, investment and default rule and for a given price of sovereign bond.

5.3 Externality

In standard sovereign default models (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008) the competitive equilibrium and the Pareto optimum coincide. This is not the case anymore when domestic debt is added to the framework. Domestic and external debt are intrinsically different. External debt determines the size of government transfers while domestic debt interacts with credit supply and output as in Holmstrom and Tirole (1998). Still, the price is the same for the two components of debt and defined by equation (12) under the assumption that international investors have deep pockets. It follows that the market clearing price does not equate the marginal cost and the marginal utility of holding government debt for domestic investor and domestic investors fail to internalize the effects of their purchases B^H on the equilibrium price $q(z, B', B^H)$. In other words, there is a missing market for domestic debt and the resulting internal-external debt composition is sub-optimal. The sub-optimality of debt composition in the competitive equilibrium can be illustrated comparing equations (28) and (42), which are respectively the first order conditions for domestic bonds in the competitive equilibrium and in the Pareto optimum. Whenever domestic debt quantities matter to determine the price of government bonds –that is: whenever $\frac{\partial q}{\partial B^H} \neq 0$ –, the two first order conditions differ one from the other. Sovereign defaults weaken investor’s balance sheet causing a contraction of credit and a fall in output. As a result the price of government debt is increasing in domestic debt levels. In the competitive equilibrium agents fail to internalize that they can improve terms of borrowing for the government by purchasing more domestic debt. As a result they consume too much and lend too little to the government. The sub-optimal debt internal-external debt composition also affects the aggregate government debt management through the prices $q(z, B', B^H)$. Lower domestic holdings of government debt worsen terms of borrowing for the government. Lower debt quantities become sustainable in equilibrium and the probability of default increases.

6 Calibration

6.1 Functional Forms

Three functional forms need to be specified in this model. The first one is the household's utility function $U(c, N)$, the second one is firm's production function $f(z, N)$ and the third one is the loan supply function $g(B^H)$. I choose preferences that satisfy [Zhu \(1992\)](#) conditions: utility is separable in consumption and labor and is CRRA with respect to consumption:

$$U^C(c, N) = \frac{(c - \frac{N^\omega}{\omega})^{(1-\sigma)}}{1 - \sigma}. \quad (48)$$

Firms' production function is a standard Cobb-Douglas with decreasing returns to scale with respect to labor:

$$y = zN^{1-\alpha}. \quad (49)$$

The productivity shock z follows a standard AR(1) process: $\log z_t = \rho \log z_{t-1} + \epsilon_t$. Where ϵ_t is a normally distributed idiosyncratic shock. Finally the loans supply function $L = g(B^H)$ is assumed to be linear in domestic bonds as it is suggested in the microfundation described in [section B](#) of the Appendix.

$$L = l = B^H + \Gamma. \quad (50)$$

6.2 Parameters

The calibration aims to replicate the behavior of the Argentinean economy around the 2001 sovereign debt crisis. [Table 4](#) reports parameter values that were used to simulate the model and the corresponding target statistics. Data sources for the target statistics are listed in [Table 8](#) in the Appendix. Parameters are either set equal to standard values of the real business cycle literature, or they are calibrated directly from Argentinean data.¹¹ Re-entry probability λ is set equal to 0.185 which is consistent with the mean exclusion length of 5.4 years found by [Gelos et al. \(2011\)](#). The risk-free rate is calibrated to the average quarterly yield of a 5 years US government bond and set equal to 0.017 as in [Arellano \(2008\)](#).

¹¹Sources for Argentinean data are listed in [Table 8](#)

Table 4. Calibration

Calibrated Parameter		Value	Target Statistics/Source
Loan share final good production	α	0.3	Standard RBC
Discount factor	β^B	0.8	Standard RBC
Re-entry probability	λ	0.185	Exclusion length = 5.4 years
Autocorrelation of TFP shocks	ρ	0.96	Standard RBC
Coefficient of relative risk aversion	σ	2	Standard RBC
Frisch elasticity	ω	1.455	Standard RBC
Risk Free rate	r^f	0.017	US 5 years bond return
Working capital parameter	γ	0.42	Credit supply/GDP = 24.96%
Government spending	G	0.13	Gov't spending/GDP = 0.2
Investors' endowment	Γ	0.06	Domestic Investors exposure = 0.41

Table 4 contains values that are used for the calibration of the model and the associated target statistic/source

The working capital parameter γ is set to match the average size of the Argentinian credit market between 1991-2000: 24.96%. Government spending parameter G is set equal to 0.1 to match the average government spending to GDP ratio observed in 1985-2000: 0.2. Investor's endowment Γ is set equal to 0.06 to match the 21% average banks exposure observed between 1985 and 2001. Domestic investor's exposure is defined as the ratio between domestic debt and total assets held by financial intermediaries.

7 Positive Analysis

Quantitative results are arranged in two different sections. Here I present the positive implications of introducing domestic debt in a sovereign default model. In section 8, instead, I comment on the normative aspects.

7.1 Competitive Equilibrium: Default Risk, Government Yields and Domestic Debt

I begin the positive analysis looking at the default sets for the model economy. Panel A of Figure 2 displays the default set as a function of total public debt and productivity levels. Consistently with standard results, I find that defaults happen when productivity is low and simultaneously the public debt is big. The introduction of domestic debt adds an additional dimension to the government default decision. The probability of default also depends on domestic debt. Panel B shows how the default set changes as domestic debt levels change. The default set becomes the bigger the smaller the domestic component of public debt. The intuition is as follows. When the domestic component of debt is big, a non-discriminatory sovereign default on both domestic and external debt weakens investor's balance sheet causing a contraction of credit and a fall in output. The contraction of the economy is the stronger the bigger the initial domestic exposure. Internal debt levels, therefore, discipline government default decision determining default costs. Properties of the default set map into the price of government debt as established by equation (12). Figure 3 describes the price function for government debt. Panel A draws the sovereign debt price q as a function of government debt for three different productivity levels. The price (yield) of government debt is increasing (decreasing) with productivity and decreasing (increasing) in the size of total debt. Panel B instead describes the relation between the price of government debt and internal debt levels. The price (yield) of government debt is increasing (decreasing) in the size of internal debt reflecting the disciplinary action that domestic debt exert on the default risk. Results reported here are consistent with those derived analytically in Proposition 1.

Figure 4 describes the policy functions for internal debt quantities. Internal debt is increasing in the productivity levels and in the size of aggregate debt while it is decreasing in the probability of default confirming the analytical results of in Corollary 1.¹² When the economy is more productive households have more resources available for the purchase of domestic bonds and hence domestic bond holdings expands. Similarly, when government debt is big and the default risk is low more resources are dragged in to the economy and both consumption and domestic bond holdings expand. Finally the probability of default reduces the internal component of debt. Domestic investors do not purchase government debt when it becomes too risky. The inter-play between government debt size and the probability of default is captured by the humped shape of the policy function that I draw for low TFP realization drawn in Figure 4. Internal debt is increasing in total debt size for low

¹²Debt is a negative quantity

Figure 2. Defaults Set

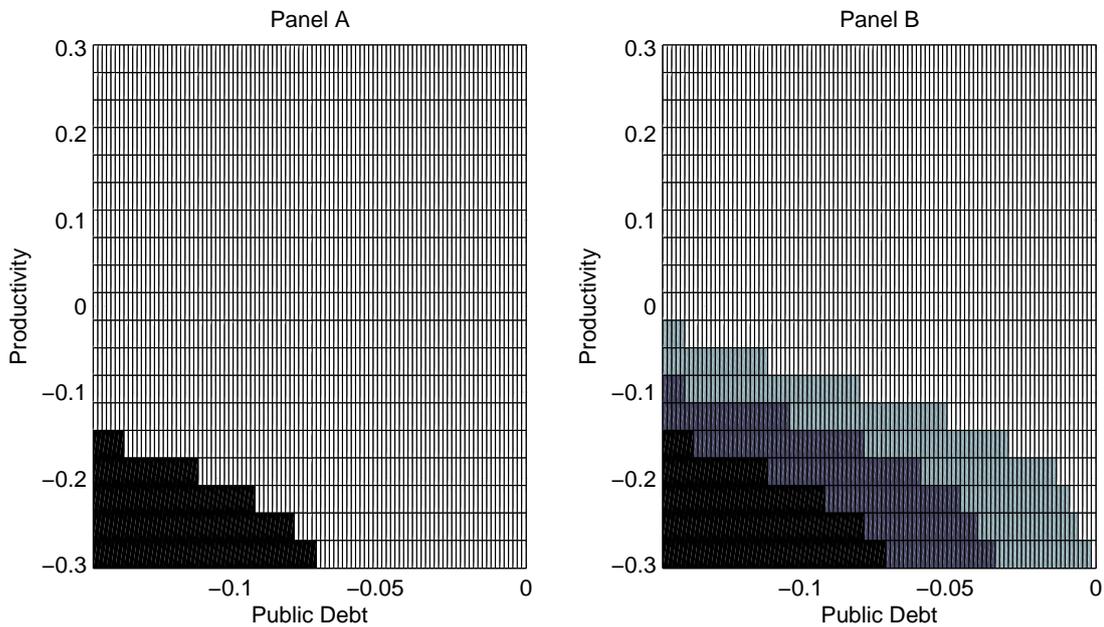


Figure 2 describes the default set for the model economy. Panel A draws the default set for a given level of internal debt. Total public debt is increasing from right to left on the horizontal axis. Productivity levels are instead reported on the vertical axis. The black shaded area corresponds to combinations of productivity shocks and debt levels that trigger a default. Panel B draws the default set for three different levels of internal debt. The default area in black is associated with high levels of domestic debt exposure. The gray and the cyan areas are associated with intermediate and low levels of debt respectively. The default set expands as domestic debt becomes smaller

and intermediate levels of government debt. However, when total debt is large and the probability of default is high, domestic investors reduce their exposure to government debt to avoid potential losses.

7.2 Competitive Equilibrium: Simulations

I now turn to discuss the quantitative predictions of the model in terms of matching the data for the Argentinean economy. The model economy is simulated 100 times for 10,000 periods. Results of this exercise are contained in Table 5. The first column of Table 5 contains data

Figure 3. Public Debt Pricing Function

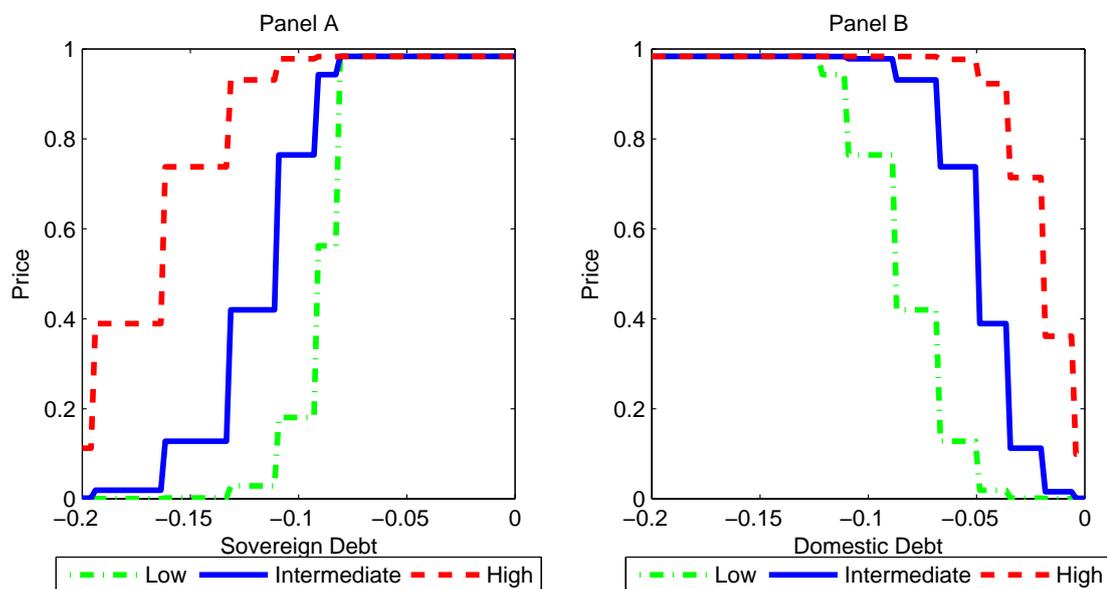


Figure 3 describes the price function for government debt in the model economy. Panel A draws prices as a function of total debt for three different productivity levels. Total public debt is increasing from right to left on the horizontal axis. Panel B draws prices as a function of internal debt for three different productivity levels and for a given level of total public debt. Internally held public debt is increasing from right to left on the horizontal axis.

moments.¹³ The second column reports simulated moments for the competitive equilibrium, while the third column contains simulation results for the Pareto optimum equilibrium that are commented in the Section 8. Panel A compares data and simulated moments for statistics that were not directly targeted by the calibration exercise. Panel B, instead, reports results for moments that were directly addressed by the simulation exercise.

Matching simultaneously default rates and debt to GDP ratios has proven to be extremely difficult for quantitative sovereign default models. Models á la [Eaton and Gersovitz \(1981\)](#) typically deliver counter intuitively low default rates while more sophisticated models, as [Arellano \(2008\)](#), are able to match the default incidence only at the cost of underestimating the the default probability. [Arellano \(2008\)](#) for instance predicts an average debt to GDP ratio for Argentina of 5.95% which is far from the 48% observed in the data. The introduction

¹³Table 8 lists data sources for each moment analyzed.

Figure 4. Domestic Debt Policy Function

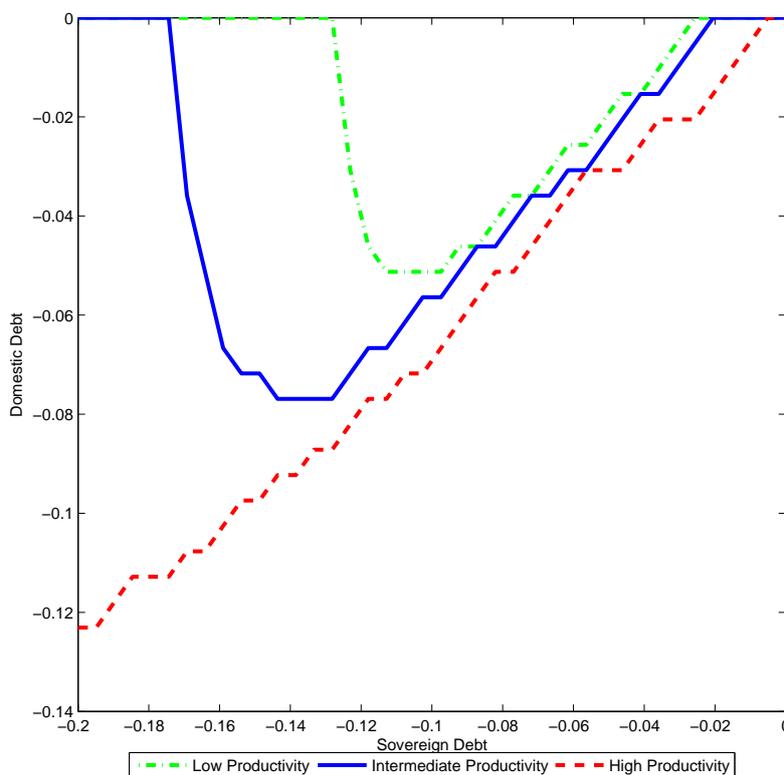


Figure 4 shows the domestic debt policy function as a function of aggregate debt levels for three different productivity levels. Total public debt size is reported on the horizontal axis and it is increasing from right to left Domestic debt levels are reported on the vertical axis.

of the domestic debt improves the quantitative performance of the model. Output losses are the greater the larger the fraction of government debt which is held internally. As domestic debt levels tend to expand simultaneously with the expansion of aggregated government debt, the cost of default rises together with the size of debt making the default option less appealing for a benevolent government. The expansion of domestic debt holdings that accompanies an increase of government debt disciplines government against defaults. When movements in domestic debt quantities are taken into account, a larger fraction of debt to GDP ratio can be sustained in equilibrium for a given default incidence.

The model predicts an average default rate of 2.9% which is not far from the 2.5% observed for the Argentinean economy. At the same time the average debt to GDP ratio is predicted to be 28% which is still far from the 48% observed in the data, but it is still a sizable improvement

compared to the 5.95% in [Arellano \(2008\)](#) and the 18% in [Aguiar and Gopinath \(2006\)](#). The model under-estimates the size of domestic debt. It predicts an average internal debt to GDP ratio of 16% while it is about 30% in the data. The average debt composition instead is predicted rather accurately. The model predicts that 56% of the total debt is held internally. Not far from the 58% average internal debt ratio of Argentina for the 1985-2001 period. The model also accounts for about half of the observed average spread between Argentinean five years bonds and the 5-years treasury bills between 1985 and 2001. The simulated average spread is 636 base points while the observed one is 1,016. I next investigate how well the model predicts correlations and second moments. The model matches broadly well the relative volatility of consumption and output and the negative relation between the spread and both output and consumption. The model also captures the crowding-out effect discussed by [Ardagna et al. \(2004\)](#) and [Evans \(1985\)](#). As sovereign spreads increase, credit to the private sectors contracts.¹⁴ The simulation exercise also matches the negative correlation between spreads and the internal share of domestic debt confirming that spreads tend to be higher when internal debt levels are low.

Finally, I evaluate the performance of the model at predicting the evolution of output and credit around default. In the year following the sovereign default the Argentinean economy contracted by 14% and in the same period of time credit to the private sector contracted by 27%. The model matches well these two regularities. Output is predicted to drop by 14% in the four quarters after the default, while the credit is expected to contract by 25%. [Figure 9](#) in [Appendix A](#) depicts the evolution of credit and output around default.

7.3 Sensitivity Analysis

In this section I perform a sensitivity analysis to gain a better understanding of how a set of key moments in the baseline calibration reacts to changes in the underlying parameters. [Table 6](#) summarizes the findings of this exercise.

I first investigate how the model reacts to a changes in working capital parameter γ . Parameter γ regulates the demand for credit in the economy and it is pivotal to determine the magnitude of output contraction upon default. The smaller γ , the smaller the demand for

¹⁴The model also predicts that the correlation between the interest rates on domestic loans r^L and spreads is positive. This is consistent with empirical finding that borrowing costs raise for the private sector when spreads are higher. The correlation between output and r^L is instead predicted to be negative which is also consistent with findings about the procyclicality of credit in the short-run [Becker and Ivashina \(2011\)](#).

Table 5. Simulations

Panel A: Non Targeted Moments			
	Data	Model CE	Model PO
Moments:			
Default Rate	2.5%	2.9%	1.7%
Debt/GDP ratio	0.48	0.28	0.29
Domestic Debt/GDP ratio	0.28	0.16	0.17
Internal/External Debt ratio	0.58	0.56	0.57
Spread	1,016	636	297
$\sigma(c)/\sigma(y)$	1.17	1.08	1.02
$\rho(\text{spread}, y)$	-0.89	-0.66	-0.61
$\rho(\text{spread}, c)$	-0.88	-0.63	-0.60
$\rho(L, \text{spread})$	-0.28	-0.30	-0.42
$\rho(B^H/B, y)$	0.06	0.38	0.30
$\rho(B^H/B, \text{spread})$	-0.26	-0.39	-0.51
Behavior around default:			
Mean GDP loss	-14%	-15%	-21%
Mean Credit contraction	-27%	-25%	-35%
Panel B: Targeted Moments			
	Data	Model CE	Model PO
Moments:			
Credit supply/GDP	0.25	0.25	0.26
Gov't spending/GDP	0.20	0.21	0.20
Domestic Investors Exposure	0.28	0.28	0.29

Table 5 reports moments obtained from model simulation. The first column reports empirical moments while the second and the third columns reports moments obtained from the model. Moments are obtained simulating the model 100 times for 10,000 periods and averaging across the simulations

credit. When I reduce the value of γ from 0.42 to 0.3, the probability of default more than doubles. Intuitively as γ decreases, firms dependence on credit from domestic investors is reduced. Consequences of a default are milder in terms of output loss and credit contraction and a benevolent government can afford to default more frequently. As the probability of default increases refinancing costs surges and the average debt to GDP ratio decreases from 0.28 to 0.16. When γ increases from 0.42 to 0.5, firms become more dependent on credit markets. Default is more expensive in terms of both output loss and credit contraction and

the probability of default drops to zero. The equilibrium debt to GDP ratio increases.

Table 6. Sensitivity Analysis

	Default Rate	B/Y	B^H/Y	B^H/B	L/Y	% GDP loss	% Credit fall
Data	2.5%	0.48	0.28	0.58	0.25	-14%	-27%
Benchmark	2.9%	0.28	0.16	0.56	0.25	-14%	-25%
W-K constraint. Benchmark value: $\gamma = 0.42$							
$\gamma = 0.3$	6.74%	0.20	0.09	0.47	0.19	-8%	-14%
$\gamma = 0.5$	0.00%	0.25	0.19	0.78	0.29	-	-
Re-entry Prob. Benchmark value: $\lambda = 0.185$							
$\lambda = 0.1$	1.9%	0.28	0.15	0.54	0.25	-18%	-30%
$\lambda = 0.25$	3.0%	0.28	0.16	0.58	0.26	-11%	-20%
Investors Endowment. Benchmark value: $\Gamma = 0.06$							
$\Gamma = 0.03$	0.0%	0.25	0.19	0.79	0.24	-	-
$\Gamma = 0.1$	12%	0.023	0.020	0.89	0.20	-1%	-2%

Table 6 reports moments obtained from model simulation for a number of different parameters. Moments are obtained simulating the model for 10,000 periods and averaging across 100 simulations

Second, I evaluate how the model responds to changes in the re-entry probability λ . I find results are not very sensitive to changes in λ . As λ increases from 0.185 to 0.25 the average duration of the financial autarky decreases making the default option more appealing. Higher λ values generate slightly higher default rates and lower output and credit contraction upon. Conversely low λ levels associate with more severe exclusion costs and governments become less prone to default.

Finally, I check how the model reacts to changes of the exogenous households endowment Γ . This happens for instance when domestic investors are less exposed to government debt. When this is the case the impact of a sovereign default on the loan supply is milder. Credit contracts less upon default and so does output. The disciplinary role of domestic debt is reduced and the government will default more in equilibrium. The average debt to GDP ratio contracts.

8 Normative Analysis

8.1 Constrained Pareto Optimum: Default Risk, Government Yields and Internal Debt

I now turn to the analysis of the constraint Pareto Optimum defined in Section 5 and compare it with the competitive equilibrium. Panel A in Figure 5 compares the policy functions for domestic holdings of government debt in the Pareto Optimum and in the Competitive Equilibrium. The difference between the two policy functions is explained by the impact of domestic holdings of government debt on the probability of a sovereign default. *Ceteris paribus* as total debt grows bigger the probability of default rises lowering the price of debt. In the Pareto optimum domestic investors are aware that the rise of government yields can be partially offset purchasing more government bonds and tilting debt composition towards the domestic component. In competitive equilibrium, instead, investors do not internalize the consequences of their purchases on government bonds price q . It follows that their domestic bond purchases are inefficiently low as suggested by Proposition 2 in Section 4. The competitive equilibrium is Pareto optimal only for low government debt quantities. When government debt is very small the probability of default is low and close to zero regardless of the composition of debt and the externality that drives the two equilibria apart disappears.

The sub-optimal management of internal debt has implications also for external debt management through the price q . Panel B in Figure 5 compares policy functions for aggregate debt in the Pareto optimum and in the competitive equilibrium. Optimal internal debt management reduces the risk of default and improves government's borrowing capacity. Total government debt is bigger in the Pareto optimum than in the competitive equilibrium as suggested by Proposition 3. The constrained Pareto optimum and the competitive equilibrium coincide when productivity levels are so low that the probability of default is not influenced by the composition of debt.

Finally Figure 6 draws the default sets for Pareto optimum and the Competitive equilibrium for a given level of internal debt. The default set is bigger in the competitive equilibrium. Sub-optimal management of aggregate and internal debt does not only reduce the borrowing ability of the government. It also undermines the sustainability of government debt. The set of productivity and debt levels that trigger a sovereign default episode expands.

Figure 5. Policy Functions for Internal and Total Public Debt

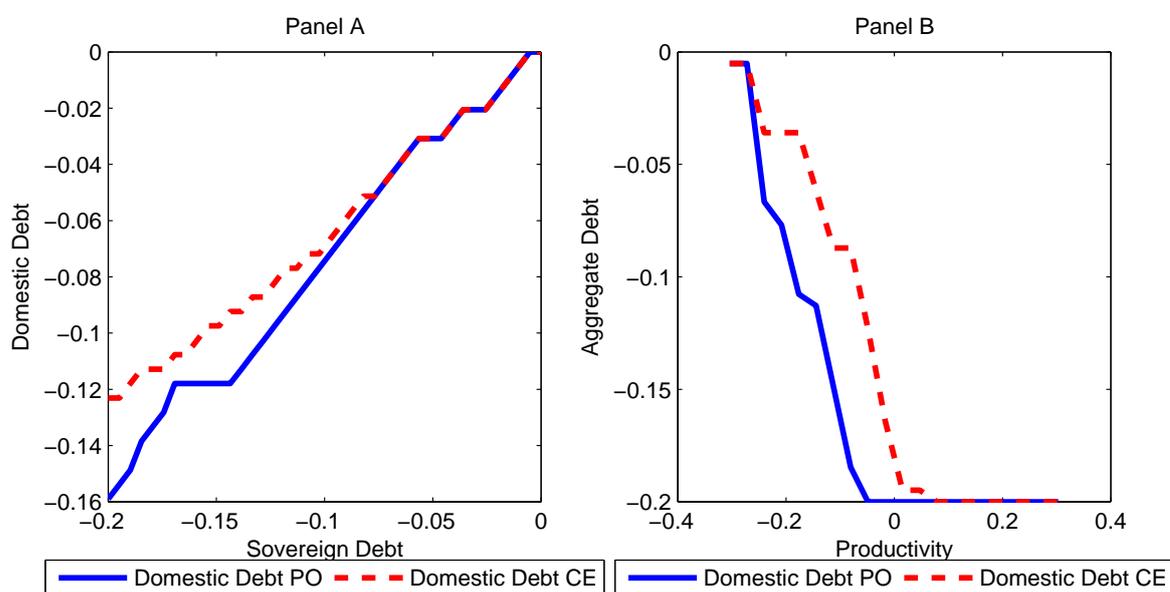


Figure 5 describes the policy functions for internal and total public debt in the model economy. Panel A compares the domestic debt policy function in the competitive equilibrium and in the Pareto optimum. Total public debt size is reported on the horizontal axis and it is increasing from right to left. Domestic debt levels are reported on the vertical axis. Panel B compares the total debt policy function in the competitive equilibrium and in the Pareto optimum. Productivity levels are reported on the horizontal axis. Aggregate debt levels are reported on the vertical axis

8.2 Simulations

I repeat here the simulation exercise discussed in Section 7.2. The model is calibrated again to the Argentinean economy using the same parameters presented in Table 4 and simulated 100 time for 10,000 periods. Figure 7 compares the distribution of domestic holdings of government debt in the competitive equilibrium and in the Pareto optimum. The distribution of domestic bond holding in the competitive equilibrium is similar to the one in the Pareto optimum but is shifted to the right meaning that domestic bond holdings are smaller in the competitive equilibrium. Moreover, the distribution for competitive equilibrium has a greater mass of observations at zero. This confirms that defaults are more frequent in the competitive equilibrium than in the Pareto optimum.

Figure 6. Default Set Po and CE

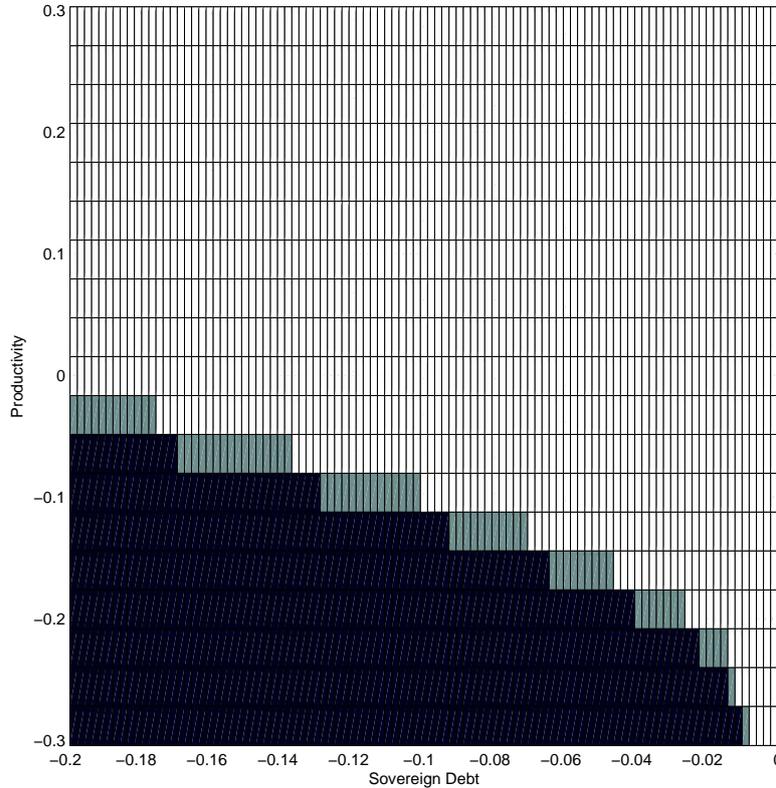


Figure 6 compares the default set in the Pareto optimum (black) and in the competitive equilibrium (grey). Total public debt size is reported on the horizontal axis and it is increasing from right to left. Productivity levels are increasing on the horizontal axis.

The third column of Table 5 summarizes the main results of the model simulation. Comparing the second and the third column of Table 5 it is possible to appreciate quantitatively the differences between the competitive equilibrium and the Pareto optimum. The simulation confirms the results anticipated in the previous section. The default rate is lower in the competitive equilibrium. Optimal management of debt composition reduces default risk from 2.9% down to 2.5%. The internal component of debt is higher. Internal debt accounts for 57% of total debt in the Pareto optimum while it is 56% in the competitive equilibrium. The optimal management of internal debt improves government borrowing ability. Debt to GDP ratio increases from 0.28 to 0.29 in the Pareto optimum. One interesting result emerges from the inspection of the covariance terms. The covariance between the spread and the output is reduced. A more efficient management of the internal debt reduces spreads volatility and dampens the negative correlation between output and spreads.

Figure 7. Domestic Holdings of Government Debt

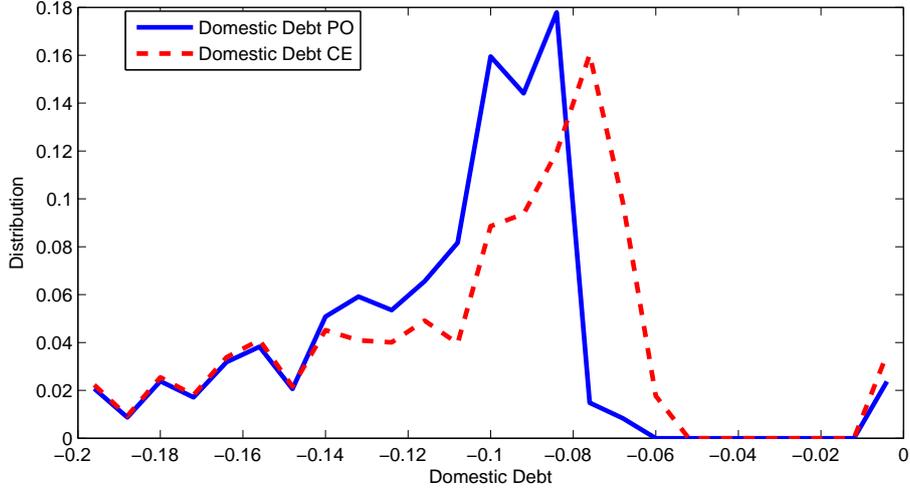


Figure 7 compares the distribution of domestic holdings of government debt in the Pareto optimum and in the competitive equilibrium

Finally I turn to the analysis of model predictions about the evolution of the economy around default. The model predicts sharper contractions of GDP and credit in the Pareto optimum. Default episodes are rare when the social planner induces the optimal allocation of debt, but, when they actually happen, they are more harmful for the economy. Figure 9 in Appendix A depicts the evolution of credit and output around default.

8.3 Optimal Pigouvian Taxation

Domestic investors fail to achieve the social optimum as they fail to internalize the consequences of their own decision on the price q of government debt. Investors under-lending leads to aggregate under-borrowing, high default rates and high government yields. Government intervention, however, can restore optimality setting the correct incentives for domestic bond purchases. In particular the government can introduce an prudential subsidy scheme τ that restores efficiency.

Let τ be a Pigouvian tax on domestic debt holdings. After the introduction of τ investors profits (10) becomes:

$$\pi^I(z, B, B^H) = r^L l - (q + \tau). \tag{51}$$

Substituting equation (9) in (51) and deriving with respect to domestic bond holding B'^H , I obtain:

$$B'^H : U_c(c, N)(q + \tau) = \beta EU_c(c', N') r^L g'_l$$

Comparing (42) and (52), I retrieve the optimal prudential tax rate τ that restores efficiency:

$$\tau = -\frac{\partial q}{\partial B'^H} (B' - B'^H). \quad (52)$$

The optimal Pigouvan tax is a subsidy to domestic bond purchases. Whenever government debt prices are not affected by domestic bond holdings ($\frac{\partial q}{\partial B'^H} = 0$), the pecuniary externality disappears and the optimal tax rate is zero.

Figure 8 displays the optimal prudential tax τ as a function of aggregate debt level and for a given productivity level z . For very low levels of debt, the optimal tax rate is zero. This is because the probability of default is null when debt is very low regardless of size of the internal debt composition. As domestic debt increases, the optimal tax rate becomes negative. Governments subsidize domestic purchases of government debt titles. Finally for very high debt levels the probability of default is equal to one regardless debt composition. In this case debt price is not influenced by internal debt levels and the optimal prudential tax falls back to zero.

The subsidy scheme τ induces the efficient internal-external debt composition. The default set shrinks, as depicted by Figure 6 and the probability of default reduces. At the same time the welfare of the economy also improves as reported in Table 7.

Table 7 reports statistics for the optimal prudential tax τ . The average optimal tax rate is -0.06 and it appears to be pro-cyclical. Subsidies are higher during bad times and lower in good times. The correlation between spreads and taxation is instead negative. The average welfare gain, measured in permanent units of consumption, of achieving the Pareto optimum is 1.6%. Welfare gains of achieving the efficient allocation are typically very contained, while in this case they appear to be quite large¹⁵ The reason is that moving from the competitive equilibrium to the Pareto optimum benefits the economy along three dimensions. First

¹⁵Bianchi (2011) evaluates the welfare gains of eliminating over-borrowing in small open economies and finds that the average welfare gain is only 0.1%.

Figure 8. Taxation Rate

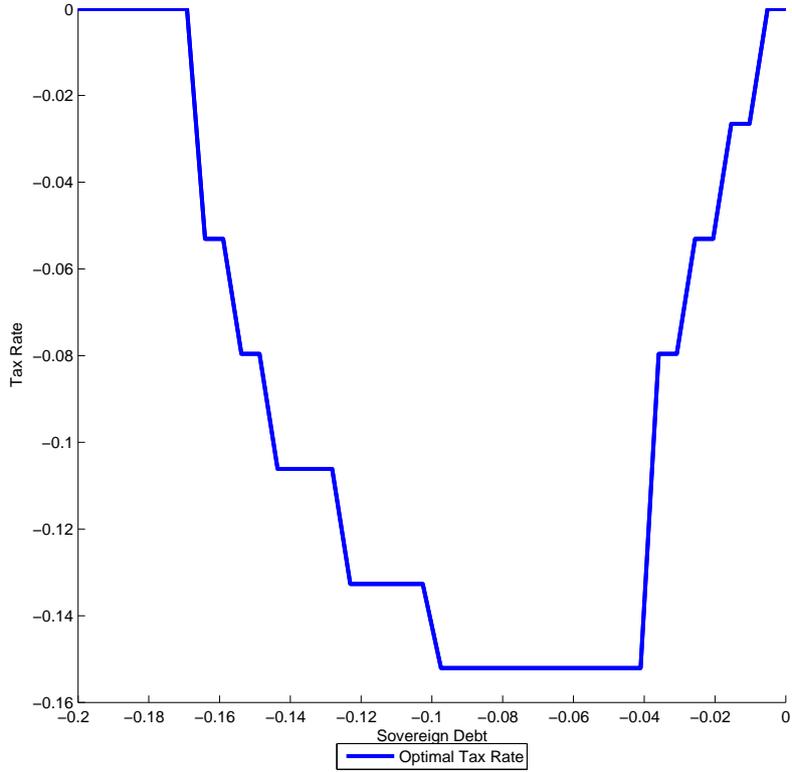


Figure 8 shows optimal prudential tax as a function of total public debt. Total public debt size is increasing from right to left on the horizontal axis

Table 7. Optimal Tax Rate: Simulations

	$mean(\tau)$	$\sigma(\tau)$	$\rho(\tau, y)$	$\rho(\tau, spread)$	W. Gain	max W. Gain
Model Simulations:	-0.07	0.39	0.28	-0.21	1.6%	46%

Table 7 reports moments for the optimal tax rate τ . Moments are obtained simulating the model 100 times for 10,000 periods and averaging across the simulations.

consumption smoothing is improved. As the government gains better access to international financial market it can smooth consumption more efficiently. Second as the probability of default drops, the economy incurs less often in deep crisis episodes. Finally as the economy moves towards the Pareto optimal allocation, domestic holdings grow larger, credit expands and output also increases.

9 Conclusions

The academic and policy debate concentrate on debt size and cycle fluctuation to explain sovereign default risk. While these two dimensions are certainly crucial to determine the risk of default, the domestic versus external composition of debt is equally important. In particular empirical evidence suggests that (i) Domestic debt is big. (ii) Defaults on domestic debt do happen rather often. (iii) Output and (iv) credit contract more around episodes of domestic defaults compared to external defaults. Based on these regularities I construct a dynamic stochastic general equilibrium model with endogenous default risk á la [Eaton and Gersovitz \(1981\)](#) which also includes domestic debt.

I show that the composition of debt matters to assess the sustainability of public balances. I find that domestic debt interacts with default costs and reduces incentive to defaults. Consequently, the optimal management of sovereign debt should not be restricted to the management of debt size and to the optimal default decision. Optimal debt management should also include tools that influence the composition of debt. This is especially true when markets are not able to deliver the efficient debt composition.

I believe that the study of the domestic debt and its implication for the sustainability of public balances is a promising area for future research. In particular, the long-run relation between domestic debt and government default risk has not yet received the deserved attention with some few exception as [Reinhart et al. \(2012\)](#) and [Pescatori et al. \(2014\)](#). Similarly very little research has been produced on the relation between the different components of public debt and the default risk. In this paper I focused on the nationality of bonds holders, but there are worth exploring. One is obviously the maturity structure of debt ([Arellano and Ramanarayanan, 2012](#)). A second one would be the demographic and economic characteristics of sovereign bonds holders ([D'Erasmus and Mendoza, 2013](#)).

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A Tables and Graph

Table 8. Data Sources

Variable	Data Source
$\sigma(c)/\sigma(y)$	Sosa Padilla (2014)
$\rho(tb, y)$	Arellano (2008)
$\rho(spread, y)$	Arellano (2008)
$\rho(spread, c)$	Arellano (2008)
Default Rate	Arellano (2008)
Domestic Credit Provided by the Financial Sector	World Bank (World Development Indicators)
Government to GDP ratio	World Bank (World Development Indicators)
Exposure rate	Financial Structure Database
Equity/Assets ratio	Financial Structure Database: ROA/ROE
Gross Domestic Product	World Bank (World Development Indicators)
Total Debt	Reinhardt and Rogoff "This Time is Different" Database
External Debt	World Bank (International Debt Statistics)
Internal Debt (Argentina)	Difference between internal and external debt
Spread	Arellano (2008)

Table 8 lists source database for the moments reported in the quantitative analysis

Table 9. Default Episodes

Country	Domestic Default	External Default	Credit Data	Country	Domestic Default	External Default	Credit Data
Argentina	1982,1989,2001	1982,2001	yes	Jordan		1989	no
Belize	2005	2005'	yes	Morocco		1983	yes
Bolivia	1983	1980,1986	yes	Mexico		1982	yes
Brazil	1986,1990	1983	yes	Nigeria		1982	yes
Chile		1983	yes	Pakistan		1998	yes
Costa Rica		1981	yes	Panama		1983	yes
Croatia	1993		yes	Peru		1983	yes
Domenican Rep.	1982,2003	1982,2003	yes	Paraguay	2003	1986,2003	yes
Ecuador	1999,2005	1982,1999,2005	yes	Romania		1986	yes
El Salvador	1981		yes	Russia	1998		yes
Egypt		1984	yes	Trinidad & Tob.		1988	yes
Grenada	2004	2004	yes	Turkey		1982	yes
Guatemala		1986	yes	Ukraine	1998	1998	yes
Indonesia		1998	yes	Uruguay	2003	1984,1990,2003	yes
Jamaica		1981,1987	yes	Venezuela	1995	1983,1990,1995,2003	yes

Table 9 reports the list of default episodes observed between 1980 and 2005. Following [Reinhart and Rogoff \(2008\)](#) and [Kohlscheen \(2009\)](#), I adopt Standard and Poor's identification of defaults. Whenever two defaults are separated by less than three years, I consider them a single default

Figure 9. Output and Credit Around default

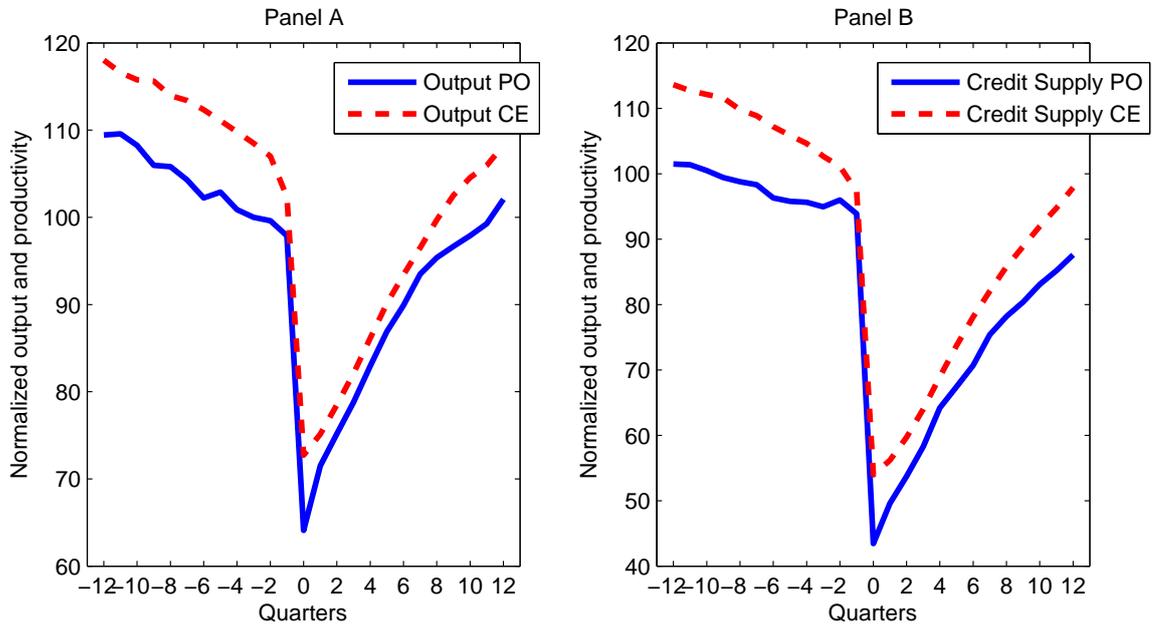


Figure 9 describes the evolution of Credit and Output around default in the competitive equilibrium and in the Pareto optimum. Panel A compares the evolution of output in the competitive equilibrium and in the Pareto optimum in the 12 quarters before and after a default episode. Time is on the horizontal axis, while scaled output levels are on the vertical axis. Output is scaled using competitive equilibrium's output levels in the quarter before the default. Panel B compares the evolution of credit in the competitive equilibrium and in the Pareto optimum. Time is on the horizontal axis, while scaled credit levels are on the vertical axis. Credit levels are scaled using competitive equilibrium's credit levels in the quarter before the default.

B Microfundation

In this section I extend the model presented in Section 3 to provide the microfundation of the loan supply function described in equation (9). While the firm, the government and the international investors sectors remain the same, the household problem is extended to include banks as well as workers in the economy. A household is composed of a representative worker and a representative banker. The representative banker and the representative worker pool income and consumption. Households value consumption c and dislike labor N according to the flow utility $u(c, N)$. Future is discounted at the rate β . The problem for the household is that of making contingent plans for consumption, labor supply and domestic bond holdings b^H so as to maximize lifetime utility of workers and bankers. Bankers take prices as given as well as the aggregate domestic debt levels B^H . Workers receive net dividend payments \mathcal{D} from banks and have access to an intra-temporal deposit D that pay an interest rate r^D . The representative household solves:

$$\begin{aligned} \max_{c, N, b^H} V^H(z, B, B^H, b^H) &= U^H(c, N) + \beta E(1 - def') V'^H(z', B', B'^H, b'^H | z) \\ &\quad + \beta E def' V'^H(z', 0, 0, 0 | z) \\ \text{s.t. } c + T(z, B, B^H) &= wN + r^d D + \mathcal{D} + \pi^B(z, B, B^H, b^H). \end{aligned} \quad (53)$$

Budget constraint (53) states that household consumption and tax payments are financed by the combined income of workers – wage payments wN , interest rates payments on deposits $r^d D$, and firm' profits π – and bankers net dividends \mathcal{D} .

For simplicity the household maximization problem is divided in two sub-problems. The problem of the worker and the problem of the banker. Workers solve an intra-temporal problem and choose the optimal labor supply N taking banker's dividend payments \mathcal{D} as given. The first order condition with respect to labor is standard:

$$N : -\frac{U_N}{U_c} = w. \quad (54)$$

The problem of the banker, instead, is an inter-temporal one and it determines each house-

hold's exposure towards domestic debt b^H .

Bankers

Each period is composed of two interim times: morning and afternoon. In the morning bankers receive liquidity from maturing bonds b^H , deposits D and external finance S from workers. Liquidity is employed to produce loans L and purchase government bonds b^H . The morning cash-flow statement is:

$$S + b^H (1 - def) + D = L + qb^H (1 - def) + \phi(S). \quad (55)$$

Following the tradition in the corporate finance literature it is assumed that equity issuance entail some additional costs. These are summarized by the quadratic cost function $\phi(S) = \frac{\zeta}{2}S^2$ as in [Hennessy and Whited \(2005\)](#). def is a binary variable that is equal to one when the government decides to default while it is zero otherwise. Using the balance sheet identity, equity after loans have been extended is defined as:

$$e \equiv L + qb^H (1 - def) - D. \quad (56)$$

Combining equations (55) and (56), I derive the equity equation:

$$e = b^H (1 - def) + S - \phi(S). \quad (57)$$

Equity levels are determined by domestic bond holdings and by fresh external finance injections by domestic investors. Ceteris paribus, a contraction of domestic bond holdings reduces the value of the equity. Following a sovereign default episode domestic bond holdings are canceled and equity is written off accordingly. This is consistent with Argentinean data. Following the 2001 default episode, equity in the financial sector contracted on average by more than 10% in less than a year time.

Following a large body of the literature that introduces financial regulation in DSGE models (i.e. [Angelini et al., 2012](#) and [Boz et al., 2014](#)), it is assumed that banks are subject to a capital adequacy requirement (CAR). Equity needs to be greater than a fraction ϕ of assets'

risk weighted value:

$$e \geq \frac{1}{\psi} (\nu L + \mu b^H (1 - def)) \quad (58)$$

$\psi \in [1, \infty]$ is the capital requirement parameter, while ν is the risk weight on loans and assets. The smaller ψ , the tighter the capital requirement. The risk-weight μ for government bonds is set equal to zero to mimic requirements stated in Basel II and Basel III accords.

Assuming that capital requirement (58) is always binding, equations (55) and (57) can be combined to derive the loan supply function:

$$L^B = (b^H (1 - def) + S - \phi(S)) \frac{\psi}{\nu_1}. \quad (59)$$

Equations (57) and (59) together clarify how sovereign default episodes transmit to the credit market through the financial sector. Following a sovereign default episode the value of the equity contracts and this reduces bankers lending ability.

I now turn to the second interim time. In the afternoon bankers receive interest rate payments for intra-temporal loans and pay interests on deposits. The end-of-the-period cash-flow statement is:

$$(1 + r^L)L = (1 + r^f) D \quad (60)$$

Combining equations (55) and (60) I obtain the profit equation:

$$\pi^B(z, B, B^H, b^H) = (1 - def) (b^H - q(z, B, B^H)b^H) + r^L L - r^D D \quad (61)$$

Banks purchase government bonds b^H taking the price $q(z, B, B^H)$ as given and maximize the expected value of current and future profits. Future profits are discounted according the endogenous stochastic discount factor $Q = \beta \frac{U_c(c')}{U_c(c)}$ and are rebated to the household. The solution of the maximization problem of the banks determines domestic bond holdings in

equilibrium:

$$\begin{aligned} \max_{b^H, S} \Pi^B(z, B, B^H, b^H) &= \pi^B(b^H) + EQ(1 - def') \Pi^B(z', B', B'^H, b^H|z) \\ &\quad + EQ(def') \Pi^B(z', 0, 0|z) \end{aligned}$$

$$s.t. \text{ (61); (59)}$$

Taking the first order condition with respect to S and b^H , I obtain the following conditions:

$$S : S = \frac{1}{\zeta}; \tag{62}$$

Substituting equation (62) in (59) and defining $\Gamma = \frac{1}{2\zeta} \frac{\psi}{\nu_1}$ and $\Phi = \frac{\psi}{\nu_1}$ I obtain:

$$l^B = \Phi b^H + \Gamma. \tag{63}$$

Equation (63) is a function of b^H only and coincides with the functional form in equation (50).

C Proofs

C.1 Proof of Proposition 1

Proof. Let:

$$q_1 = \frac{1 - \Pr(z_1 \leq \bar{z} | z_0)}{1 + r^f}. \quad (64)$$

1. I assume that productivity shocks distribute normally and they are autocorrelated: $z_1 = \rho z_0 + \epsilon_1$. The properties of the normal distribution ensure that whenever $z_0 \leq \hat{z}_0$, then:

$$\Pr(z_1 \leq \bar{z} | z_0) = \Phi(\bar{z} | z_0) \geq \Phi(\bar{z} | \hat{z}_0) = \Pr(z_1 \leq \bar{z} | \hat{z}_0).$$

Hence:

$$q_1(z_0) \leq q_1(\hat{z}_0)$$

whenever $z_0 \leq \hat{z}_0$.

2. Deriving the threshold productivity level $\bar{z} = \frac{\gamma B_1 - B_1^H}{B_1^H}$ with respect to aggregate bonds:

$$\frac{\partial \bar{z}}{\partial B_1} = \frac{\gamma}{B_1^H} \geq 0.$$

Since: $\frac{\partial \Phi_z(\bar{z} | z_0)}{\partial \bar{z}} \geq 0$. Then, using definition (64) for the asset pricing equation, I obtain:

$$\frac{\partial q_1}{\partial B_1} \leq 0.$$

3. Since: $\frac{\partial \Phi_z(\bar{z} | z_0)}{\partial \bar{z}} \geq 0$, the partial derivative of the threshold level \bar{z} with respect to domestic debt levels is:

$$\frac{\partial \bar{z}}{\partial B_1^H} = -\frac{1}{B_1^H} - \frac{\gamma B_1 - B_1^H}{(B_1^H)^2} \leq 0.$$

Given the definition of the asset pricing equation:

$$\frac{\partial q_1}{\partial B_1^H} \geq 0.$$

□

C.2 Proof of Proposition 2

Proof. The First Order Condition of the Competitive equilibrium problem 25 gives:

$$B_1^{H*,CE} = \frac{(1 - Pr(def)) \left(\beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right)}{\nu}. \quad (65)$$

The First Order Condition of the Pareto optimum problem 26, I obtain:

$$B_1^{H,PO*} = \frac{(1 - Pr(def)) \left(\beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right) - \frac{\partial P(def)}{\partial B_1^H} \left(\frac{1}{1+r^f} - \beta \right) B_1}{\nu + \frac{\partial P(def)}{\partial B_1^H} \left(\beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right)}. \quad (66)$$

Comparing equations (65) and (66) I get that the Pareto optimal allocation B_1^{H*} is greater than the competitive equilibrium one \hat{B}_1^{H*} whenever:

$$(1 - Pr(def)) \left(\frac{1}{1+r^f} - \beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) \right)^2 \frac{\partial P(def)}{\partial B_1^H} \leq \nu B_1 \frac{\partial P(def)}{\partial B_1^H} \left(\beta - \frac{1}{1+r^f} \right) \quad (67)$$

As:

1. $\beta \leq \frac{1}{1+r^f}$ to ensure borrowing in equilibrium
2. Proposition 1 ensures that:

$$\frac{\partial q_1}{\partial B_1^H} \geq 0 \Leftrightarrow \frac{\partial P(def)}{\partial B_1^H} \leq 0.$$

Hence, the LHS of (67) is certainly negative while the RHS is certainly positive. Inequality (67) is always respected. □

C.3 Corollary

Corollary 1 (Domestic Lending Properties). *Let $B_1^{H,CE*}$ be solution to competitive equilibrium problem 25 respectively. The following properties apply:*

1. $B_1^{H,CE*}$ is decreasing in the probability of default $P(def)$.
2. $B_1^{H,CE*}$ is increasing in productivity z

Proof. The proof Corollary 1 follows directly from equation (65). Taking the first order derivative of equation (65) with respect to the probability of default $P(def)$ and the productivity z it is possible to verify the corollary. \square

C.4 Proof of Proposition 3

Before proving Proposition 3 I state and demonstrate the following Lemma.

Lemma 1. *If $\gamma \geq \beta E(1 - z_1)^{\frac{(1+r^f)}{r^f}}$, then*

$$\frac{\partial B_1^{H,PO*}}{\partial B_1} \leq \frac{\partial B_1^{H,CE*}}{\partial B_1}.$$

That is: domestic lending is more sensitive to changes in total debt size in the competitive equilibrium than in the Pareto optimum.

Proof. Assuming that cross-partial derivatives are negligible, the first order derivative of the competitive equilibrium equation (65) with respect to competitive B_1 is:

$$\frac{\partial B_1^{H,PO*}}{\partial B_1} = \frac{\frac{\partial Pr(def)}{\partial B_1} \left(\beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right)}{\nu}. \quad (68)$$

The first order derivative of the Pareto optimal equation (66) yields:

$$\frac{\partial B_1^{H,PO*}}{\partial B_1} = \frac{\frac{\partial P(def)}{\partial B_1} \left(\beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right) + \frac{\partial P(def)}{\partial B_1} \left(\frac{1}{1+r^f} - \beta \right)}{\frac{\partial P(def)}{\partial B_1} \left(\frac{1}{1+r^f} - \beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) \right) - \nu}. \quad (69)$$

□

Comparing equations (68) AND (69) inequality $\frac{\partial B_1^{H,PO*}}{\partial B_1^H} \leq \frac{\partial B_1^{H,CE*}}{\partial B_1^H}$ holds whenever:

$$\frac{\partial P(def)}{\partial B_1} \left(\frac{1}{1+r^f} - \beta E \left(\frac{z_1 - 1}{\gamma} + 1 \right) \right) \geq \nu \left(\beta - \frac{1}{1+r} \right). \quad (70)$$

As the LHS is negative $\beta \leq \frac{1}{1+r}$, condition $\gamma \geq \beta E(1 - z_1) \frac{(1+r^f)}{r^f}$ is enough to ensure that the relation (70) is verified.¹⁶

I now turn to the proof of Proposition 3 proper.

Proof. Given the optimal choice B_1^{H*} of domestic investors the objective function of the government becomes:

$$\begin{aligned} \max_{B_1} c_0 - \frac{B_0^H + \Gamma}{\gamma} - q_1 (B_1^{H*} - B_1) - \frac{\nu}{2} (B_1^{H*})^2 \\ + \beta E \left[(1 - P(def)) \left(B_1^{H*} - B_1 + (z_1 - 1) \frac{B_1^H + \Gamma}{\gamma} \right) + P(def) \frac{(z_1 - 1)}{\gamma} \right]. \end{aligned}$$

The FOC with respect to B_1 is:

$$\begin{aligned} B_1 \left(\frac{\partial P(def)}{\partial B_1} \left(\frac{1}{1+r^f} - \beta \right) \right) = q(z, B_1, B_1^{H*}) + \frac{\partial P(def)}{\partial B_1} B_1^{H*} \left(\frac{1}{1+r^f} - \beta E \left(\frac{z-1}{\gamma} + 1 \right) \right) \\ + \frac{\partial B_1^{H*}}{\partial B_1} (1 - P(def)) \left(\beta E \left(\frac{z-1}{\gamma} + 1 \right) - \frac{1}{1+r^f} \right) \quad (71) \end{aligned}$$

¹⁶Inequality $\gamma \geq \beta E(1 - z_1) \frac{(1+r^f)}{r^f}$ is respected also by low values of γ , z_1 is oscillates around 1 in our calibration. Hence the RHS of the inequality is often negative.

Under the assumption that $\gamma \geq \beta E(1 - z_1) \frac{(1+r^f)}{r^f}$ and given $B_1^{H,PO*} \geq B_1^{H,CE*}$ as stated in Proposition 2, it follows that:

1. Default probability is higher when $B_1^{H,PO*} = B_1^{H,CE*}$ and $q(z, B_1, B_1^H) \geq (z, B_1, B_1^H)$ from Proposition 1. This implies that the first term on the LHS of equation (71) is bigger in the Pareto optimum than in the competitive equilibrium.
2. The second term on the LHS of equation (71) is positive and it is greater in the Pareto optimum
3. The last term on the LHS of equation (71) is negative and, according to Lemma 1, it is the smaller the smaller is the Pareto optimum.

Hence, as long as $\frac{\partial P(def)}{\partial B_1}$ is similar in the PO and in the CE, the aggregate level of public debt is higher in the Pareto optimum: $B_1 \geq B_1$ □