

Optimal Policy in Collateral Constrained Economies (Job Market Paper)

Nina Biljanovska[†]
Frankfurt University

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Abstract

This paper examines optimal policy in a macroeconomic model with collateral constraints. Binding collateral constraints yield inefficient competitive equilibrium allocations because they distort the optimal utilization of real resources and also result in externalities operating through market prices. I identify the set of policy instruments that can be used by a Ramsey planner to achieve the first-best and the second-best (constrained) allocations. A system of distortionary taxes on capital and labor income and lump-sum transfers among borrowers and lenders replicates the first-best outcome. I show analytically and numerically that the capital income tax is strictly positive in the long-run and it can be replicated by a tax on borrowing or by a loan-to-value limit, which underscores the necessity for financial regulation when the economy faces binding collateral requirements. In absence of lump-sum transfers, however, only second-best equilibrium outcomes are attainable. The Ramsey planner still sets positive capital income taxes or tightens loan-to-value ratios, but regulating labor is ambiguous since the policy instruments have the additional role to implement implicit income transfers. I also derive the optimal policy in response to real and financial shocks, and show how the policy recommendations differ depending on the nature of the shock.

JEL: E60, H21, H23, H25

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[†]Department of Money and Macroeconomics, House of Finance, Goethe University Frankfurt. E-mail: biljanovska@wiwi.uni-frankfurt.de. Website: <https://sites.google.com/site/biljanovska/>

1 Introduction

What should optimal policy look like in models with collateral constraints? Chamley (1986) and Judd (1985) derived the theorem of a steady state zero tax on capital and a positive tax on labor income in an economy without market distortions. This result has been invalidated in models featuring uninsurable idiosyncratic risk since agents exhibit precautionary savings motive in response to uncertainty and build up capital (savings) that is in excess of the first-best level.¹ However, not many studies have investigated the optimal policy in environments where agents face collateral constraints.

This type of models, nonetheless, have been extensively used to examine the amplifications of real shocks due to financial frictions (for example, Kiyotaki and Moore, 1997) or to study the effect of financial shocks, like those observed in the offset of the Great Recession, on real economic activity (for example, Jermann and Quadrini, 2012). This paper identifies distinct inefficiencies induced by collateral constraints and derives the set of optimal regulatory tools that can be used to restore the efficient allocations. Furthermore, I distinguish between structural policies in the long-run and cyclical policies in response to real and financial shocks.

I carry out the design of optimal policy using a model similar to the one in Jermann and Quadrini (2012) and Perri and Quadrini (2013). There are two types of agents; households that are the savers (lenders) and entrepreneurs that are the borrowers in the economy. Entrepreneurs invest in capital and purchase labor services from households to produce a consumption good. They fund their investment either by the profits accruing from the previous period's investment or by borrowing from households. However, there is an inability to commit to repay the loan, and therefore entrepreneurs cannot borrow more than a fraction of their capital investment, which is pledged as collateral. In addition, entrepreneurs enjoy a tax benefit on the borrowed funds, given that interest expenses are tax-deductible. This induces higher demand for borrowing and results in binding collateral constraints.²

¹See for example Hubbard and Judd (1986), Aiyagari (1995), Imrohorglu (1998). In the model in Aiyagari (1995), the positive tax rate in the long-run reduces capital accumulation and brings the pre-tax return on capital to equality with the rate of time preference.

²Instead of providing entrepreneurs with a tax advantage on debt, one could assume that they are more impatient

I show that binding collateral constraints result in two (distinct) types of inefficiencies compared to an unconstrained economy. First, they distort the optimal marginal conditions with respect to capital investment and labor utilization, i.e. the factors of production are not used efficiently. Second, they introduce a wedge across agents' shadow costs of wealth, meaning that (pecuniary) externalities, operating through prices (wages and interest rates), will have welfare implications.³

Given the distortions in the private optimal conditions, induced by the binding collateral constraint, the factors of production are not efficiently used. In particular, the capital stock, in addition to its value as a productive asset, embeds a *collateral value* because an incremental unit of capital can relax the collateral constraint [see Geanakoplos and Zame (2014) and Fostel and Geanakoplos (2008) for a discussion of the collateral premium]. Moreover, entrepreneurs' effective cost of borrowing is lower than the actual borrowing costs due to the tax advantage they enjoy. Thus, the preference for capital coupled with the lower effective cost of borrowing pushes the marginal product of capital below the socially efficient level, i.e. the actual cost of funds. On the other hand, the cost of labor, in general, is above the socially efficient level. This is the case when working capital needs to be financed by debt, which discourages entrepreneurs from purchasing labor, and pushes the marginal product of labor, i.e. the wage rate, up.

In addition, the marginal valuations of wealth (i.e. the shadow costs of wealth) of the two agents differ due to the binding collateral requirement. This divergence creates an externality because any price change, induced by private marginal decisions, will not only affect agents' budget constraints, but it will also affect aggregate welfare. The latter outcome emerges because the utility gain of one group of agents, induced by the price change, is not exactly balanced with the utility loss of another group of agents, induced by the same price change. Consequently, any private decisions affecting market prices will also have implications for welfare. As pointed out by Dávila (2014), a planner can modify agents' allocations to induce price changes that bring (closer or exactly) to equality the shadow costs of wealth of the different agents.⁴

than households as in Kiyotaki and Moore (1997) or Iacoviello (2005). The two approaches yield equivalent results in terms of optimal policy, since they introduce the same distortions, as I show in the paper.

³Pecuniary externalities, i.e. externalities operating through relative prices, are always present, but only have welfare implications when markets are not complete or in other words when the shadow values of wealth of agents differ. See Stiglitz (1982) and Dávila (2014) for a more detailed discussion.

⁴It is important to note that the externality present in this setup exists regardless of if the collateral constraint depends on endogenously determined asset prices. As shown in Bianchi (2011), Bianchi and Mendoza (2012) and Jeanne and Korinek (2010), among others, pecuniary externalities may arise because of the presence of an endogenous

The goal of the Ramsey planner is to choose policy to maximize the aggregate welfare of the economy. It is important to emphasize that the planner has a single mandate, i.e. to minimize the distortions and the externalities arising from the binding collateral constraint, rather than to finance any exogenously given government expenditure.⁵ In this setup, the Ramsey planner needs two types of tools to recover the efficiency. On one hand, it needs corrective (to be defined later), distortionary capital income and payroll taxes to affect the private marginal decisions and, on the other hand, it needs lump-sum transfers in order to equalize the marginal valuations of agents, i.e. to eliminate the externality. Then, the Ramsey allocations replicate the first-best outcome. However, when the planner does not have access to lump-sum transfers, the levels of the distortionary taxes are biased. This bias arises since the policy tools attempt to achieve a dual objective, i.e. to correct the private marginal decisions and at the same time to close the gap in the valuations of wealth among the agents. The tax system, in this case, can replicate only the allocations of a constrained social planner.⁶

The tax on capital income that the planner optimally chooses is always positive in the long-run regardless if there are lump-sum transfers available. A positive tax on capital income reduces the demand for capital and increases its return rate, bringing it closer to the lending rate. The two rates are exactly equal when the planner has lump-sum transfers on its disposal; for a given calibration, the tax rate that supports the allocations equals 12.30%. On the other hand, the payroll tax can be either positive, zero or negative depending on the availability of lump-sum transfers. With lump-sum transfers, the payroll tax is negative (a subsidy) since entrepreneurs are compensated for the extra costs incurred by the binding collateral constraint; for the same calibration, the level of the subsidy that supports these allocations is 4.67%. However, without lump-sum transfers, the

relative price in the collateral constraint. In those environments, when collateral constraints become operative, any agents' impact on market prices not only affects other agents by amending their budget constraints, but also by affecting the tightness of the collateral constraints they face. This type of externality, Dávila (2014) refers to as a *collateral externality*. In the current paper, however, the externality arises because of the difference in agents' valuation of wealth. This type of externality, Dávila (2014) refers to as a *terms-of-trade externality*.

⁵As argued by Diamond and Mirrlees (1971), the planner can still use its tax powers to improve real allocations. In their framework, the optimal redistribution, induced by the tax policy, will depend on the trade-off between the improvements gained from the redistribution and the efficiency losses from the taxation.

⁶One may argue that a direct redistribution could be a too powerful policy instrument used to achieve any possible outcome, including the first-best. In this way the planner can affect the market structure of the economy by completing any missing asset markets. Therefore, I consider the allocations that can be achieved by a constrained social planner, following Dávila et al. (2012), who does not have the power to change the structure of the competitive economy, but unlike the private agents, it internalizes the competitive pricing decisions.

level of the payroll tax may be positive in order to bring the level of labor closer to the socially optimal one. The tractability of the model, allows me to also provide analytical solutions of the steady state levels of all policy instruments.

Further, I show that the capital income and the payroll taxes can also be replicated by alternative regulation. Given that the economy's inter- and intra-period margins are distorted there will always be the need for policy tools that can affect both margins. The tax on capital income can alternatively be replaced by a tax on borrowing or a loan-to-value ratio limit. These two instruments replicate the allocations achieved by the capital income tax because both of them affect the no-arbitrage condition, i.e. the joint investment/borrowing decision determining the profit margin. Consequently, these policies should be seen as financial regulation operating either through prices, i.e. making borrowing more expensive, or through quantities, i.e. restricting further the amount of borrowing per unit of collateral. The payroll tax can be replaced by a tax on labor imposed on households. Although, the levels of the tax rates of the different policy regimes differ, they all achieve the same allocations.

In the short run, I examine the responses of the Ramsey plan to a productivity and a financial shock. The positive productivity shock induces the tax rate to decrease from its steady state level, encouraging entrepreneurs to exploit the opportunity and build up capital they use as a collateral. The tax rate also decreases in response to a financial shock that reduces the financial value of the collateral, allowing entrepreneurs to recover faster from the bad shock by investing in capital and increasing the capital stock. Nonetheless, the same conclusions do not endure when lump-sum transfers are available.

Relation to the literature – This paper focuses on collateral constraints, deriving from borrowers' ability to repudiate their debt obligations. From a policy point of view, collateral constraints attract attention since they can affect the efficiency of the market outcome through real resource effects, as well as through effects via market prices (externalities). The main contribution of this paper nests in the normative analysis of the competitive economy facing binding collateral requirements.

A paper that perhaps most closely relates to mine, in terms of the environment and the approach in designing optimal policy, is Itskhoki and Moll (2014). They study optimal Ramsey

policies in a standard growth model with financial frictions. The rationale for policy intervention in their model is the presence of a dynamic pecuniary externality, operating via wages. Households do not internalize that supplying more labor decreases wages and relaxes the collateral constraint, which in turn, increases entrepreneurs' wealth accumulation and future productivity. In the small open economy version of their model, they find it is optimal to subsidize labor. The main difference between this paper and theirs is that I focus on both how collateral constraints distort the efficient use of the factors of production and how they induce externalities. Moreover, I consider a closed economy, where a social planner cares about the welfare of *both* households and entrepreneurs, and I derive the set of tools necessary to replicate the first-best and the second-best equilibrium outcomes.

Further, the paper more generally relates to the literature on Ramsey optimal policy in general equilibrium models. Some of the leading examples in this literature are Lucas and Stokey (1983), Chari et al. (1994) and Aiyagari (1995). However, those papers rule out lump-sum taxation. I consider a Ramsey planner, who cares about redistribution of wealth among agents, and therefore, in addition to distortionary taxes, it has also access to lump-sum transfers. The lump-sum transfers of the Ramsey planner are not restricted in any way and they are set optimally. They are indeed needed to set to equality the marginal valuations of wealth, and thus to implement the first best allocations. In a similar fashion, Bhandari et al. (2013) let the Ramsey planner choose optimal transfers. However, in their model with heterogenous agents, they do not allow transfers to depend on agents' personal identities. Consequently, those lump-sum transfers are not powerful enough to complete the markets and bring the efficiency of the competitive economy to the first-best level.

The paper also relates to the literature studying optimal capital income taxation. Chamley (1986) and Judd (1985) established the result of zero capital taxation in the long run, which rests critically on the possibility of shifting consumption across periods through perfect capital markets. This result has been invalidated, as already discussed, in models with uninsurable idiosyncratic risk and/or borrowing limits, as well as in life-cycle model frameworks.⁷ In the current paper, I find that tax on capital income is positive in the long run. It is required to decrease the capital accumulation and bring the rental rate of capital in equality with the borrowing rate (no-arbitrage

⁷See Conesa et al. (2009) for the result of a positive tax on capital income (in the long run) within a life-cycle model framework.

condition), which is in line with the finding by Aiyagari (1995).

The theoretical framework I use builds on a variant of the models presented in Jermann and Quadrini (2012) and Perri and Quadrini (2013). Both are dynamic stochastic general equilibrium models, populated by two types of agents, households and corporate entities (or entrepreneurs) facing an enforcement constraint. Jermann and Quadrini (2012) introduce an additional financial friction to the model, namely a dividend adjustment cost, and study the effect of financial shocks to the real economy. The framework of Perri and Quadrini (2013) considers investors and firms instead of a single entrepreneur, in a two country model. None of these papers studies policies aimed at alleviating the effects of financial frictions on real allocations.

The rest of the paper is structured as follows. Section 2 outlines the model and defines the equilibrium of the market economy. Further, it characterizes the steady state and the first-best allocations. Section 3 discusses the design of optimal policy and derives the analytical results. Sections 4 summarizes the calibration of the model and provides a numerical characterization of the steady state and analyzes the policy plans in response to shocks. Finally, section 5 concludes.

2 The Model Economy

This section outlines the economy featuring a collateral constraint. I consider a model economy, populated by two types of infinitely-lived atomistic agents: *households/workers* (lenders) and *entrepreneurs/firms* (borrowers). Households consume, supply labor hours, and invest in entrepreneurs' one period corporate bonds. Entrepreneurs are endowed with capital and make optimal investment, borrowing, and labor demand decisions. Further, they produce a homogeneous good by hiring labor from the households and employing the capital stock. In financing production and investment in capital, entrepreneurs borrow from households and their borrowing capacity is limited by a collateral constraint. I proceed by describing each agent's problem and their optimal equilibrium decisions.

2.1 Households / Workers

The economy is populated by a continuum of households, in the $[0, 1]$ interval, who maximize the following sum of discounted utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t), \quad (1)$$

where c_t is consumption, $1 - l_t$ is leisure with l_t denoting labor hours, and β is the subjective discount factor. The utility function, $u(\cdot)$, is concave and increasing in both consumption and leisure and has the constant-relative-risk-aversion (CRRA) form.

Households take the market price of the consumption good, chosen to be a numeraire, the market wage and the borrowing/lending rate as given. They can borrow/save in a one period, risk-free corporate bond, issued by the entrepreneurs, which is the sole instrument available to households for transferring resources inter-temporally. This framework leads to the following per-period budget constraint of households

$$c_t + \frac{b_{t+1}^h}{1 + r_t} + Q_t \leq b_t^h + w_t l_t, \quad (2)$$

where b_t^h is the one period bond, paying a net interest rate r_t , w_t is the wage rate and Q_t is a lump-sum transfer from households to entrepreneurs to finance the proceeds from the tax advantage, which will be defined later. Households choose the following sequence $\{c_t, l_t, b_{t+1}^h\}_{t=0}^{\infty}$ to maximize (1) subject to (2). The equilibrium conditions that characterize their problem are given by the intra-temporal arbitrage condition between labor supply and consumption, given by

$$-\frac{u_{l,t}}{u_{c,t}} = w_t, \quad (3)$$

and the Euler condition

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} (1 + r_t), \quad (4)$$

where $u_{c,t}$ and $u_{l,t}$ denote the partial derivatives at time period t of the utility function with respect to consumption and labor, respectively. I will preserve the same notation in denoting partial derivatives of functions with respect to its arguments throughout the paper, which will become clear from the context.

2.2 Entrepreneurs / Firms

There is a continuum of entrepreneurs, in the $[0, 1]$ interval, who maximize the following sum of discounted utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t u^e(c_t^e), \quad (5)$$

where c_t^e denotes entrepreneurs' consumption and β is their subjective discount factor. The utility function $u^e(\cdot)$ is concave and increasing in c_t^e , and it is of the CRRA form.

Entrepreneurs own the capital stock in the economy, which evolves according to the law of motion $k_{t+1} = i_t + (1 - \delta)k_t$, where i_t is investment and δ is the depreciation rate. In addition, they own a production technology that uses labor, n_t , and capital, k_t , to produce a unit of output

$$F(z_t, k_t, n_t) = z_t k_t^\alpha n_t^{1-\alpha}, \quad (6)$$

which can immediately be turned into consumption. Here, z_t denotes the level of productivity, following a stochastic process, and α denotes the capital share used in the production of output.

Entrepreneurs face the following budget constraint

$$c_t^e + w_t n_t + b_t^e + k_{t+1} \leq F(z_t, k_t, n_t) + \frac{b_{t+1}^e}{R_t} + (1 - \delta)k_t, \quad (7)$$

where w_t is the wage paid for labor and R_t is the gross interest rate paid on borrowing from households, denoted b_{t+1}^e . I assume borrowing is tax deductible. Therefore, the effective gross interest rate entrepreneurs face is given by $R_t \equiv 1 + r_t(1 - \tau)$, where τ denotes the tax benefit of debt and r_t denotes the market interest rate. The tax advantage of debt is financed lump-sum by households, amounting to $Q_t \equiv \frac{b_{t+1}^e}{R_t} - \frac{b_{t+1}^h}{1+r_t}$.

Since there is a probability of default and lenders cannot enforce repayment, entrepreneurs' ability to borrow is bounded by a collateral constraint, given by⁸

$$\varepsilon_t k_{t+1} \geq F(z_t, k_t, n_t) + \varepsilon_t \frac{b_{t+1}^e}{1 + r_t}, \quad (8)$$

where ε_t is the liquidity value of entrepreneurs' capital, net of borrowing. This variable evolves

⁸In fact, if entrepreneurs were free to borrow at the market interest rate, they would exhibit a tendency to accumulate debt either indefinitely (rendering the steady state of the economy indeterminate) or up to the natural borrowing limit.

stochastically and I consider it to be the financial shock.⁹

The collateral constraint, (8), is derived in Jermann and Quadrini (2012) and Perri and Quadrini (2013) from a negotiation process between borrowers and lenders, and it takes the current form because of two crucial assumptions. First, it is assumed entrepreneurs need to acquire a working capital loan (i.e. an intra-period loan); second, the working capital loan is also collateralized.¹⁰

The working capital loan is acquired to finance the flow of funds mismatch between the payments due and the realization of revenues. Since this loan is repaid within the same period, it bears no interest. It can be shown that the intra-period loan is equal to the revenues of production, $F(z_t, k_t, n_t)$, because all payments outstanding, including wages, interest payments on borrowing, consumption and investment expenditures, are due before revenues have been realized.¹¹ The working capital loan is collateralized because of the timing assumption that entrepreneurs' decision to default arises *after* the realization of revenues, but *before* the working capital loan has been repaid. Since entrepreneurs can easily divert their liquidity (i.e. the revenues from production), the only asset that can be posted as collateral is the capital stock, k_{t+1} .

Entrepreneurs choose the following sequence $\{c_t^e, n_t, b_{t+1}^e, k_{t+1}\}_{t=0}^{\infty}$ to maximize (5) subject to (7) and (8). Denoting λ_t^e to be the Lagrange multiplier on the budget constraint (7) and $\mu_t u_{c,t}^e$ to be the Lagrange multiplier (scaled by the marginal utility of entrepreneurial consumption) on the collateral constraint (8), the conditions characterizing the optimal choices of the entrepreneurs are given by the optimal labor demand decision

$$(1 - \mu_t) F_{n,t} = w_t \tag{9}$$

and the Euler conditions with respect to borrowing and capital, respectively

$$\frac{1}{R_t} - \frac{\varepsilon_t \mu_t}{1 + r_t} = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \tag{10}$$

⁹Since ε_t affects the tightness of the collateral constraint and entrepreneurs' ability to borrow, in the literature it has been referred to as a "financial shock." For more detailed description of its implications, see Jermann and Quadrini (2012).

¹⁰In the rest of the paper, I will refer to the working capital loan and the intra-period loan interchangeably.

¹¹Using the budget constraint (7), one can show that $wkl_t = F(\cdot) = c_t^e + w_t n_t + b_{t+1}^e + k_{t+1} - (1 - \delta) k_t - \frac{b_{t+1}^e}{R_t}$, where wkl_t denotes the working capital loan. Jermann and Quadrini (2012) show that there are alternative ways of formalizing the working capital loan. A key feature of this formulation is that the working capital loan is related to the production scale.

$$1 - \varepsilon_t \mu_t = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} [(1 - \mu_{t+1}) F_{k,t+1} + 1 - \delta]. \quad (11)$$

Here $F_{k,t}$ is the marginal product with respect to capital and $F_{l,t}$ is the one with respect to labor at t . The corresponding complementarity slackness condition is given by

$$\mu_t \left(\varepsilon_t k_{t+1} - \varepsilon_t \frac{b_{t+1}^e}{R_t} - F(k_t, l_t) \right) = 0, \quad \mu_t \geq 0. \quad (12)$$

The optimality conditions of the entrepreneurs' problem, in presence of a binding collateral constraint, can be interpreted as follows. First, consider equations (4) and (10), i.e. the consumption-borrowing Euler equations of households and entrepreneurs, respectively. A binding collateral constraint ($\mu_t > 0$), coupled with the tax advantage of debt, results in a divergence between the inter-temporal marginal utilities of consumption between the two types of agents. In other words, the borrowing rate (faced by entrepreneurs) differs from the lending rate (faced by households).

Second, consider the consumption-capital Euler condition, (11). Given the formulation of the collateral constraint, (8), this inter-temporal decision yields two effects. On one hand, foregoing one unit of consumption and accumulating one unit of capital today, relaxes the collateral constraint. In other words, using the terminology in Geanakoplos and Fostel (2008) and Geanakoplos and Zame (2014), the capital stock embodies a *collateral value*.¹² Consequently, entrepreneurs increase their investment in capital compared to the case when $\mu_t = 0, \forall t$. On the other hand, the new unit of capital reduces the value to the entrepreneurs because the capital that was posted as collateral at t , is used in the production of the final good in the following period. The production activity requires an intra-period loan, which tightens the collateral constraint. If the working capital loan were not collateralized, this effect would have been absent.

Finally, consider equation (9). Hiring one more unit of labor yields additional (to the wage) cost for the entrepreneurs since, via the working capital loan, it tightens the collateral constraint. If the working capital loan were not collateralized, the optimal labor decision would not have been affected by the presence of the collateral requirement.

Overall, a binding collateral constraint of the form (8) has an effect on all marginal decisions

¹²More specifically, Geanakoplos and Zame (2014) emphasize the importance of the collateral asset by arguing that the type of the asset posted as a collateral determines the price of the security since "the same promise backed by a different collateral constitutes a different security and might trade at a different price because it might give rise to different realized deliveries."

of the entrepreneurs. On one hand, the incentive for entrepreneurs to over-accumulate capital, with the purpose to relax the collateral constraint, may result in equilibrium levels of capital return that are lower than those in the unconstrained economy ($\mu_t = 0, \forall t$). On the other hand, the additional costs associated with the labor decision, when the collateral constraint binds, may result in lower levels of equilibrium labor and higher wages compared to the same benchmark (i.e. $\mu_t = 0, \forall t$). These considerations are discussed in more detail in section 2.6.

2.3 Remarks About Modeling Assumptions

This section discusses the modeling assumptions.

Collateralized working capital loan. Following Jermann and Quadrini (2012), the present formulation of the collateral constraint assumes that the inter-period loan and the working capital loan are both collateralized.¹³ This assumption captures the idea that some businesses lack the credibility to borrow *short-term*, and therefore posting collateral provides security for their borrowing.

An alternative specification of the collateral constraint would be as in Kiyotaki and Moore (1997). This specification assumes there is no need of a working capital loan; hence only inter-period borrowing is collateralized. As discussed in section 2.2, the requirement of a collateralized working capital loan induces effects on both the capital investment and the labor demand decisions of the entrepreneurs, which would be absent otherwise. Further, to some extent, this formulation has implications for the policy design, as it will become clear in section 3.3.

Risk averse entrepreneurs. This assumption captures the idea that entrepreneurs are not more prone to risk-taking than other agents, but they do possess the ability to spot opportunities. Having entrepreneurs as risk averse agents induces the planner to consider aggregate welfare rather than households' welfare alone. This assumption has certain policy implications. In particular, as it will become clear later, if entrepreneurs were risk neutral, the planner would focus on fixing marginal decisions only, but not on implementing any income redistribution between the agents necessary to maximize social welfare. Moreover, the optimal policies will be functions of the planner weights (i.e. the Pareto-Negishi weights) in the aggregate welfare function. Only in the case when entrepreneurial utility weight equals zero, the optimal policy recommendations will coincide with

¹³For an alternative specification of a collateralized working capital loan, see Mendoza (2010).

the scenario of risk-neutral entrepreneurs.

Debt advantage. The assumption of tax advantage of debt suggests that it is cheaper for entrepreneurs (investors) to finance their activities by debt than by cutting down on their consumption. This assumption results in a rate on borrowing (for entrepreneurs) that is lower than their subjective discount factor. Consequently, entrepreneurs become borrowers and households become lenders in the economy. The same assumption can be replaced by introducing a different discount factors between the two types of agents, as in Iacoviello (2005) and Kiyotaki and Moore (1997).¹⁴ Both assumptions result in a binding collateral constraint, but they do so because of different reasons. The policy recommendations derived in this paper are invariant to this modeling choice.

Closed economy. Opposite to a small open economy, I consider a closed economy setup.¹⁵ This assumption generates implications for policy similar to those discussed above for risk averse entrepreneurs. In a closed economy, the planner considers the welfare of both agents rather than the one of borrowers, as particular for small open economies. Similarly as before, the planner then does not only focus on fixing agents' marginal decisions, but also cares about income redistribution between the agents. Again, optimal policies depend on the Pareto-Negishi weights. Only in the case when households' (opposite to entrepreneurs' before) utility weight equals zero, the optimal policy recommendations will coincide with those for a small open economy.

2.4 Market Equilibrium

Since there are no idiosyncratic shocks, I focus on the symmetric equilibrium where all households and entrepreneurs are alike. Then, the competitive equilibrium of the economy can be defined as follows. First, labor market clearing requires that the supply of labor by households equals the demand for labor by entrepreneurs

$$l_t = n_t, \quad \forall t \tag{13}$$

Second, given the closed economy setup, in equilibrium, borrowing and lending is equalized

$$b_{t+1}^h = b_{t+1}^e = b_{t+1}, \quad \forall t \tag{14}$$

¹⁴Kumhof et al. (2014) show that such assumption can also be micro-founded.

¹⁵For optimal policy in a small open economy setup, see Itskhoki and Moll (2014) and also Bianchi (2011).

Finally, combining the individual budget constraints of the two agents, the resource constraint of the economy is given by

$$c_t + c_t^e + k_{t+1} = F(z_t, k_t, l_t) + (1 - \delta)k_t, \quad \forall t. \quad (15)$$

Definition 1. A price system is a sequence $\{r_t, w_t\}_{t=0}^{\infty}$. An allocation is a sequence $\{c_t, c_t^e, l_t, n_t\}_{t=0}^{\infty}$. An asset profile is a sequence $\{k_{t+1}, b_{t+1}^h, b_{t+1}^e\}_{t=0}^{\infty}$.

Definition 2. For given initial values $k_{-1}, b_{-1}^h, b_{-1}^e$ and exogenous processes $\{z_t, \varepsilon_t\}_{t=0}^{\infty}$, a competitive equilibrium for the economy with a collateral constraint is a sequence of allocations $\{c_t, c_t^e, l_t, n_t\}_{t=0}^{\infty}$, an asset profile $\{k_{t+1}, b_{t+1}^h, b_{t+1}^e\}_{t=0}^{\infty}$, and a price system $\{r_t, w_t\}_{t=0}^{\infty}$, such that

1. given the price system $\{r_t, w_t\}_{t=0}^{\infty}$, the allocations and the asset profile solve households' and entrepreneurs' problems, and
2. markets clear, satisfying conditions (13), (14) and (15).

2.5 Steady State

This section examines the deterministic steady state of the competitive equilibrium, as defined in section 2.4, where allocations, prices and asset profiles are time invariant. It can easily be shown that the steady state is unique and it can be portrayed by proposition 1 below.

Proposition 1. The steady state of the model can be summarized by the following equations, where the capital stock, k , and labor, l , are the solutions to

$$1 - \mu = \frac{-u_l(k, l) / u_c(k, l)}{F_l(k, l)} \quad (16)$$

$$1 - \bar{\varepsilon}\mu = \beta [(1 - \mu) F_k(k, l) + 1 - \delta] \quad (17)$$

where $\mu = \bar{\varepsilon}^{-1} [(1 - \beta) - (1 - \beta)(1 - \tau)]$.

Proof. See Appendix.

Equations (16) and (17) show that the steady state of the model can be characterized as a function of only two variables, the capital stock k , and the equilibrium level of labor l .

Corollary 1. If $\tau > 0$, then $\mu > 0$ and the collateral constraint binds in the steady state.

Proof. The deterministic steady state version of equation (10), obtained by removing the time

subscripts of the variables, implies $\frac{1}{R} - \frac{\bar{\varepsilon}\mu}{1+r} = \beta$, where $\bar{\varepsilon}$ denotes the mean value of the financial shock. Using the definition for r , obtained from the invariant steady state version of eq. (4), and the definition of $R \equiv 1 + r(1 - \tau)$ (see pg. 9), yields $\mu = \bar{\varepsilon}^{-1} [(1 - \beta) - (1 - \beta)(1 - \tau)]$. It follows that if $\tau > 0$ then $\mu > 0$. Q.E.D.

Insofar as $\tau > 0$, the borrowing constraint binds in the steady state. From the demonstrated proof, its tightness in the steady state is determined by the level of the tax advantage. If entrepreneurs enjoy a higher tax advantage, the collateral constraint will be tighter (i.e. the Lagrange multiplier will be larger) since they would like to borrow as much as possible due to the actual borrowing rate being smaller than their own discount rate. This result is important because from here it follows that, in the long run, the borrowing constraint (8) holds with equality, which pins down borrowing in the steady state.¹⁶

Therefore, the steady state of the competitive economy will be characterized by a lower marginal product of capital and with a higher marginal product of labor (compared to the case when $\mu = 0$) because entrepreneurs' decisions that characterize the short-run also carry through the long-run.

2.6 Economy's Efficient Allocations: First-Best Allocations

This section defines the first-best allocations, employed to identify the distortions, induced by the collateral constraint.

The first-best outcome for this economy can be obtained as a solution to the following planning problem

$$\max_{\{c_t, c_t^e, l_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \omega u(c_t, l_t) + (1 - \omega) u^e(c_t^e) \} \quad (18)$$

$$s.t. \ c_t + c_t^e + k_{t+1} \leq F(\cdot) + (1 - \delta) k_t, \quad (19)$$

where the objective function is the infinite sum of agents' average utilities, weighted by (exogenous) Negishi-Pareto weights, where ω denotes the planner's weight placed on households' utility. The planner does not choose the level of borrowing, but it chooses agents' consumption levels. In this case, households fully insure entrepreneurs and the shadow price of wealth of the agents are

¹⁶Note that outside the steady state, however, the constraint may or may not always bind since entrepreneurs, in anticipation of bad realizations, may accumulate enough capital such that the constraint does not bind. In the appendix, I demonstrate that the constraint binds with a probability of 95% outside the steady state.

equalized. The capital Euler equation is as in (11) with $\mu_t = 0 \forall t$, and the optimal labor decision is $-\frac{u_{l,t}}{u_{c,t}} = F_{l,t}$. Therefore as $\mu_t \rightarrow 0 \forall t$ and the collateral constraint becomes loose, the competitive economy converges to its first-best outcome (with exogenous Pareto-Negishi weights). On the other hand, as long as the collateral constraint binds, it distorts all marginal decisions of the entrepreneurs (compared to the decisions characterizing the first-best solution), and exerts effects on both short run dynamics and the deterministic steady state of the economy.

Moreover, the binding collateral constraint results in a divergence between the agents' shadow prices of wealth. To see this, we can rewrite the collateral constraint for the competitive economy as $(1 - \delta)k_t - c_t^e - w_t n_t - b_t^e \geq \frac{b_{t+1}^e}{1+r_t}$, obtained by substituting *in* the entrepreneurs' budget constraint. Taking first order conditions with respect to entrepreneurial consumption yields $u_{c,t}^e (1 - \mu_t) = \lambda_t^e$, where $u_{c,t}^e \mu_t$ and λ_t^e are the Lagrange multipliers on the corresponding constraints, defined above. It follows, as long as $\mu_t > 0$, the shadow cost of wealth of entrepreneurs will be higher than the one of households. This divergence, in addition to the distortions discussed above, induces an externality, which cannot be pinned down from the comparison of the competitive economy's allocations to those of the first-best. As pointed out in the introduction, a planner can engineer welfare improvements by manipulating allocations such that agents' marginal valuations of wealth come to equality. I will revisit this point in section 3.3, when considering the allocations of a constrained social planner (to be defined later). Hence the competitive equilibrium is trivially not efficient and policy may be used to restore efficiency.

3 Optimal Policy

In this section, I examine the design of optimal policy that can correct for the distortions and the externalities induced by the binding collateral constraint. To this end, I design a Ramsey plan where the planner's objective is to maximize a utilitarian welfare function given a set of policy instruments. In particular, I consider two types of policy instruments: distortionary capital income and payroll taxes, aiming at correcting the distortions in the agents' marginal decisions; and lump-sum transfers that can be used for income redistribution such that the shadow prices of wealth among the agents are equalized.¹⁷ Further, the distortionary taxes are of *corrective* nature, i.e.

¹⁷As initially argued by Lipsey and Lancaster (1956) and recently shown by Park (2014), in a model of uninsurable idiosyncratic risk and limited commitment, distortionary taxes may have efficiency gains given that there are pre-

they are imposed on the side of the entrepreneurs and their revenues are returned in a lump-sum fashion back to the same agent. The payroll tax alternatively can be replaced by a labor income tax imposed on households; and the capital income tax can be replaced by a borrowing tax or a loan-to-value ratio limit.

To examine the performance of the policy instruments in terms of efficiency, I consider two benchmarks. I compare the allocations of the competitive economy with corrective instruments in place to the *first-best* and to the *second-best* allocations. The first-best allocations are those defined in section 2.6. In other words, these allocations can be obtained by a planner, who possesses the power to completely relax the binding collateral constraint and to implement redistribution among agents. I show that all three instruments, i.e. the distortionary capital income and payroll taxes, as well as, the lump-sum transfer are necessary to replicate the first-best outcome. Following the definition in Dávila et al. (2012), I refer to the second-best allocations as those achieved by a constrained social planner, who faces the same market structure as the competitive economy, but, unlike the private agents, it incorporates that individual decisions affect equilibrium prices.¹⁸ The comparison between these allocations and those of the competitive economy points out to the externalities induced by the binding collateral constraint, which cannot be identified by simply contrasting the competitive economy's optimality conditions to those of the first-best. I also show that the distortionary capital income and payroll taxes alone are sufficient to replicate the constraint planner's allocations.

The rest of the section is organized as follows. Section 3.1 presents and incorporates the policy instruments in the competitive economy with a collateral constraint. Section 3.2 outlines the Ramsey plan. Section 3.3 discusses the properties of the optimal taxation and the externalities, and finally section 3.4 examines the alternative instruments that can be used in the replications of the planners' solutions.

3.1 Policy Instruments

I propose the following tax system: tax on capital income (τ_t^k), payroll tax (τ_t^n), and lump-sum transfer (\mathcal{T}_t). The capital income tax is in place to correct the distortions present in the non-existing distortions in the economy, as it is the case in the present model.

¹⁸In the following subsection, I formally define the problem of the constrained social planner and outline its allocations.

arbitrage (i.e. the inter-temporal optimality) condition. The payroll tax is in place to correct for the distortions present in the intra-period optimal condition. And finally, the lump-sum transfer is in place to implement any necessary income redistributions between the private agents to equalize their marginal valuations of wealth. As we will see later, alternative policy systems can implement the same allocations. Since both taxes are imposed on the side of the entrepreneurs and they are of corrective nature, the tax revenues are returned lump-sum as a gift to the entrepreneurs in the same period.

The policy system introduced modifies entrepreneurs' and households' budget constraints, (7) and (2), respectively to

$$c_t^e + k_{t+1} + b_t^e + \tau_t^p w_t n_t \leq \left(1 - \tau_t^k\right) [F(k_t, n_t) - w_t n_t] + \frac{b_{t+1}^e}{R_t} + (1 - \delta) k_t + T_t^e + \mathcal{T}_t, \quad (20)$$

where $T_t^e = \tau_t^k [F(k_t, n_t) - w_t n_t] + \tau_t^p w_t n_t$ is entrepreneurs' transfer stemming from the linear, corrective taxes, and

$$c_t + \frac{b_{t+1}^h}{1 + r_t} + Q_t + \mathcal{T}_t \leq b_t^h + w_t l_t. \quad (21)$$

With the policy system in place, the optimality conditions of households remain the same as in section 2.1 and those of entrepreneurs change to

$$1 - \mu_t = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} \left[\left(1 - \mu_{t+1} - \tau_{t+1}^k\right) F_{k,t+1} + (1 - \delta) \right], \quad (22)$$

$$\left(1 - \mu_t - \tau_t^k\right) F_{n,t} = w_t \left(1 + \tau_t^p - \tau_t^k\right), \quad (23)$$

where (22) is the capital Euler equation and (23) is the optimal labor demand decision. In what follows, I define the policy distorted competitive equilibrium (PDCE) with a collateral constraint.

Definition 3. A policy distorted competitive equilibrium (PDCE), given a system of instruments $\{\tau_t^k, \tau_t^p, T_t^e, \mathcal{T}_t\}_{t=0}^\infty$ consists of a sequence of:

- (i) allocations and asset profiles for households, $\{c_t, l_t, b_{t+1}^h\}_{t=0}^\infty$,
- (ii) allocations and asset profiles for entrepreneurs, $\{c_t^e, n_t, k_{t+1}, b_{t+1}^e\}_{t=0}^\infty$, and
- (iii) prices $\{r_t, w_t\}_{t=0}^\infty$, such that,

the allocations and the asset profiles satisfy the following conditions

$$\varepsilon_t k_{t+1} \geq F(z_t, k_t, n_t) + \varepsilon_t \frac{b_{t+1}^e}{1 + r_t}, \quad (24)$$

$$\left(\varepsilon_t k_{t+1} - F(z_t, k_t, n_t) - \varepsilon_t \frac{b_{t+1}^e}{1 + r_t} \right) \mu_t = 0, \quad (25)$$

$$\mu_t \geq 0, \quad (26)$$

$$-\frac{u_{l,t}}{u_{c,t}} = w_t, \quad (27)$$

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} (1 + r_t), \quad (28)$$

$$n_t = l_t, \quad (29)$$

$$b_{t+1}^h = b_{t+1}^e = b_{t+1}, \quad \forall t \quad (30)$$

$$\frac{1}{R_t} - \frac{\varepsilon_t \mu_t}{1 + r_t} = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \quad (31)$$

$$T_t^e = \tau_t^k [F(k_t, n_t) - w_t n_t] + \tau_t^p w_t n_t, \quad (32)$$

(20), (21), (22) and (23).

3.2 The Ramsey Plan

Given policy instruments, the objective of the Ramsey planner is to maximize the social welfare function subject to the constraints constituting a PDCE (with a collateral constraint). The objective of the Ramsey planner is the utilitarian welfare function, which cannot be defined unambiguously, given that it depends on exogenously given Pareto-Negishi weights, ω for households and $(1 - \omega)$ for entrepreneurs.¹⁹ I assume that the planner has a technology to commit to the policy

¹⁹Models featuring representative agents, as in Bianchi (2011) for example, do not encounter the problem of unambiguously defining a welfare criterion. In contrast, when the economy is populated by heterogeneous agents, aggregate welfare can be defined by setting Pareto-Negishi weights (see for e.g. Monacelli, 2006; and Bhandari et al., 2013 more recently). The drawback with this approach is that all allocations, prices and policies will be contingent on the exogenously imposed weights. A way around this obstacle has been proposed by Werning (2007) and Park (2014), for an economy with idiosyncratic heterogeneity, who choose optimal Pareto-Negishi weights that are associated with the efficient consumption and labor allocations and that can be supported by a competitive equilibrium. As I will explain later, I follow a slightly different approach. Namely, I incorporate lump-sum transfers in the tax system and reverse engineer the weights that can replicate the first-best efficient allocations in the steady state. Alternatively, following Lorenzoni (2008) for example, the objective may be defined as the utility function of one agent subject to the constraint that the utility of the other agent does not go below some specified level (e.g. the level of the

chosen at the beginning of time, and I formally define the Lagrangian of the Ramsey problem as follows.

Definition 4. Let $\Lambda \equiv \{\lambda_t^k\}_{t=0}^\infty, \forall k = 1, 2, 3, \dots$ represent Lagrange multipliers on constraints (20) - (32), respectively. For given stochastic processes $\{z_t, \varepsilon_t\}_{t=0}^\infty$, plans for the control variables $\Xi \equiv \{c_t, l_t, b_{t+1}^h, c_t^e, n_t, k_{t+1}, b_{t+1}^e, r_t, w_t, \tau_t^k, \tau_t^p, \mathcal{T}_t\}_{t=0}^\infty$ and for the co-state variables $\{\lambda_t^k\}_{t=0}^\infty$ represent a second-best (i.e. first-best constrained) allocation if they solve the following maximization problem

$$\min_{\Lambda} \max_{\Xi} E_0 \sum_{t=0}^{\infty} \beta^t \{ \omega u(c_t, l_t) + (1 - \omega) u^e(c_t^e) \} \quad (33)$$

subject to conditions (20) - (32).²⁰

In characterizing the Ramsey allocations, I follow a similar approach to Schmitt-Grohe and Uribe (2012). Essentially, the strategy consists in first dropping entrepreneurs' distorted optimality conditions, (22), (23) and (31), and then showing that the allocations solving the Less Restricted Ramsey (LRR) plan also satisfy the full set of constraints constituting a PDCE. Intuitively, this result arises since the planner can choose policy instruments such that the constraints involving those instruments are not binding. The following proposition summarizes the equivalence result between the *more* and the *less* restricted Ramsey plans, followed by the proof. I refer to the more restricted Ramsey plan as the one satisfying the full set of PDCE conditions, (20) - (32); and the less restricted Ramsey plan as the one satisfying conditions (20) - (21), (24) - (30) and (32).

Proposition 2. *Ramsey plans that satisfy the constraints of the less restricted Ramsey plan also satisfy the conditions describing the PDCE and vice versa.*

Proof. First, to show that a sequence of allocations, $\{c_t, c_t^e, l_t, n_t, k_{t+1}\}_{t=0}^\infty$, satisfying the more restricted Ramsey plan also satisfies the LRR plan is trivial since it requires that a set of allocations that respect a larger number of constraints also honors a subset of those constraints. Next, I show that allocations, $\{\check{c}_t, \check{c}_t^e, \check{l}_t, \check{n}_t, \check{k}_{t+1}\}_{t=0}^\infty$, satisfying the LRR plan also satisfy the full set of equations constituting a PDCE. Using the LRR plan allocation of \check{c}_t and the pricing kernel, (28), derive r_t such that the equation holds with equality. Next, using the allocations from the LRR plan for \check{c}_t

competitive economy).

²⁰Since eq. (22), (28), and (31) incorporate expectations of future variables, the problem of the planner is not intrinsically recursive. This problem can be overcome by introducing pseudo-state variables, following the approach by Marcet and Marimon (2011), or by setting initial conditions of the values for the marginal utilities, following the approach by Kydland and Prescott (1980).

and \check{l}_t , derive w_t such that eq. (27) is satisfied. Given \check{c}_t^e and \check{k}_{t+1} , choose τ_{t+1}^k such that condition (22) is not binding in the Ramsey plan. Similarly, given \check{l}_t and \check{k}_t , choose τ_t^p such that (23) is not binding. Through the no-arbitrage condition, eq. (31) will not bind for the planner. Consequently, the set of relevant conditions for the Ramsey planner reduces to (20) - (21), (24) - (30) and (32). Q.E.D.

The LRR problem, along with the first order conditions summarizing the planner's allocations, is summarized in section 7.2 in the appendix. By contrasting these conditions to those characterizing the competitive economy's equilibrium, one can identify the externalities induced by the collateral constraint. Before I move to the next section to discuss these considerations, I briefly revisit two additional equivalence results mentioned in the beginning of this section.

3.2.1 Equivalence Results

In this section, I establish two equivalence results. The first one posits that the allocations of the Ramsey planner, endowed with distortionary, corrective capital income and payroll taxes, replicates the allocations of a constrained social planner. Second, the allocations of a Ramsey planner, endowed with the same policy tools as before, but in addition lump-sum transfers, replicates the first-best allocations.

To formally define the constrained social planner, I follow Dávila et al. (2012).

Definition 5. *The problem of the constrained social planner consists in maximizing the social welfare function of households and entrepreneurs subject to their individual budget constraints and the collateral constraint of entrepreneurs, while recognizing that $w(c_t, l_t) = -\frac{u_{l,t}}{u_{c,t}}$ and $1 + r(c_t, c_{t+1}) = \frac{u_{c,t}}{\beta u_{c,t+1}}$.*

The definition suggest that the second-best allocations can be found as solutions to the less restricted Ramsey (LRR) problem with the only difference that $\mathcal{T}_t = 0$ (outlined in section 7.2 of the appendix). Given that the problem of the constrained social planner and the one of the LRR planner are exactly the same, then the allocations that will characterize their equilibria will be equivalent. The following proposition summarizes this result.

Proposition 3. *The Ramsey planner's problem with a sequence of policies $\{\tau_t^k, \tau_t^p, T_t^e\}_{t=0}^\infty$ is equivalent to the one of the constrained social planner and they yield the same optimal allocations.*

It is important to note that the equivalence result between the Ramsey plan and the constrained

social planner’s problem is contingent on the set of instruments, available to the planner. In the case when the Ramsey planner does not have on its disposal tools that can exactly offset the distortions in the optimality conditions arising from the collateral constraint, the equivalence result does not apply. More specifically, the equivalence result will hold as long as the Ramsey planner is endowed with an instrument that can affect the no-arbitrage condition (i.e. the inter-temporal margin) and one that can affect the intra-temporal margin. Itskhoki and Moll (2014) design a Ramsey plan, which they juxtapose to the constrained planner’s problem in Dávila et al. (2012) and Caballero and Lorenzoni (2007). However, they do not prove the equivalence between the two planning problems. In their environment, a small open economy stochastic growth model, only one instrument is sufficient to replicate the constrained planner’s solution due to the small open economy setup.²¹

Next, I turn to the equivalence between the Ramsey planner’s allocations and those of the first-best.

Proposition 4. *The Ramsey planner’s problem with a sequence of policies $\{\tau_t^k, \tau_t^p, T_t^e, \mathcal{T}_t\}_{t=0}^\infty$ is equivalent to a social planner, who maximizes the aggregate utility function of the two agents subject to the resource constraint, and they yield the same optimal allocations.*

Proof. Consider the first order conditions of the LRR plan in section 7.2 in the appendix. The presence of lump-sum transfers, \mathcal{T}_t , imposes an equality between the marginal values of wealth of the agents ($\lambda_t^{RP,e} = \lambda_t^{RP,h}$ in the appendix). Honoring this equality in the remaining first order conditions, we derive the exact same conditions as those of the planner, outlined in section 2.6. Q.E.D.

3.3 Properties of Optimal Taxation

This section discusses how the optimal marginal decisions, characterizing the equilibrium of the competitive economy with the collateral constraint, differ from those of the Ramsey planner (with and without lump-sum transfers) and how taxes can be chosen optimally to correct for the inefficiencies. As noted before, the presence of the binding collateral constraint also results in externalities, which private agents do not take into account. This section elaborates more on this point. I will

²¹In an extension of their model they show that in the case of a closed economy both labor and capital income are taxed.

focus on the steady state outcomes and provide analytical characterizations of the optimal policy. Section 4.3 shows how the results discussed here survive when the economy undergoes a series of shocks.

The Ramsey planner chooses corrective taxes to eliminate the distortions in the privately optimal decisions to invest in capital and to employ labor. The optimal tax rates allow the planner to affect the distortions in the optimality conditions induced by the binding collateral constraint, but they will not be sufficient to equalize the marginal valuations of wealth between the agents. Therefore, the levels of the optimal corrective taxes will be biased in a sense that they will simultaneously implement transfers in an attempt to equalize agents' marginal valuations of wealth, which in turn depend on the planner's weights. One can circumvent this bias by introducing lump-sum transfers. In this way, the lump-sum transfers would allow the planner to transfer resources between the agents such that it equalizes the shadow values of wealth in the economy, whereas the corrective taxes would take care of the distortions present in the equilibrium conditions of the competitive economy.²² I show below that when lump-sum transfers are not available, the tax on capital income is always positive, but its level depends on the marginal valuations of wealth and the weights in the social welfare function. On the other hand, the tax on labor can be positive or negative, but it is always negative if lump-sum transfers are available.

3.3.1 Tax on Capital Income

First, to understand the externality arising from the capital investment/borrowing decision, affecting the level of the tax rate when lump-sum transfers are not available, consider the no-arbitrage conditions characterizing the competitive and the social planner's equilibrium, respectively

$$1 - \left(\frac{1}{R} - \beta\right)(1+r) = \beta \left[\left(1 - \frac{\left(\frac{1}{R} - \beta\right)(1+r)}{\bar{\varepsilon}}\right) F_k + 1 - \delta \right], \quad (34)$$

$$1 - \Phi \left(\frac{1}{R} - \beta\right)(1+r) = \beta \left[\left(1 - \Phi \frac{\left(\frac{1}{R} - \beta\right)(1+r)}{\bar{\varepsilon}}\right) F_k + 1 - \delta \right]. \quad (35)$$

²²Note that lump sum transfers are not needed to achieved the first best outcomes if one considers a model where entrepreneurs are firms owned by households as in Jermann and Quadrini (2012). Nevertheless, binding collateral constraints will still induce distortions in the optimality conditions of the firm, and thus, the corrective (distortional) taxes reported in this section will be the same.

Here eq. (34) represents the no-arbitrage condition of the competitive economy obtained by combining the steady state capital-consumption and borrowing-consumption Euler conditions, i.e. conditions (10) and (11) in the steady state. Eq. (35) represents the no-arbitrage condition of the planner's problem derived combining the same corresponding conditions (69) and (71) in the steady state, where $\Phi \equiv \frac{\lambda^{RP,e} - \lambda^{RP,h}}{\lambda^{RP,e}}$. The only difference between the two conditions is the presence of Φ , indicating that the planner internalizes that the marginal valuations of wealth (i.e. the shadow costs of wealth) among the agents are not equalized. As argued before, the planner can manipulate allocations such that the gap between agents' shadow costs of wealth shrinks. Before proceeding further with the analysis, let's examine the sign of Φ .

Lemma 1. Φ is positive and strictly less than one, i.e. $0 \leq \Phi < 1$.

Proof. Given that $\lambda^{RP,e}$ and $\lambda^{RP,h}$ represent the Lagrange multipliers on the budget constraints of the agents, they are always positive. Further, given that entrepreneurs are the ones facing the collateral constraint, their marginal value of wealth, as shown before, will be always higher than the marginal value of wealth of households, i.e. $\lambda^{RP,e} > \lambda^{RP,h}$, and $\Phi = 1 - \frac{\lambda^{RP,h}}{\lambda^{RP,e}} < 1$ and positive. When transfers are allowed, $\lambda^{RP,e} = \lambda^{RP,h}$ and $\Phi = 0$. Q.E.D.

Consider the case when lump-sum transfers are available such that $\Phi = 0$ and the marginal values of wealth between the two agents are equalized. Then, eq. (35) reduces to

$$1/\beta = F_k + (1 - \delta), \tag{36}$$

where $R_t^k \equiv F_k + (1 - \delta)$ denotes the gross return on capital. In this case, the planner invests in capital up to the point where the rate of return equals the lending rate, determined by households' valuation of inter-temporal consumption and given by $1/\beta = 1 + r$. In absence of financial frictions, entrepreneurs' would invest up to this point. However, as explained in section 2.2, (by the no-arbitrage condition) the tax advantage on debt coupled with the binding collateral constraint, result in a return rate on capital that is lower than the lending rate.²³ In this case, the first-best allocations can be replicated solely by imposing a corrective policy aimed at bringing the return rate on capital in par with the lending rate.

²³Recall that the financial frictions present in the competitive economy not only introduce a wedge between the lending rate and the rate of return on capital, but, more generally, a different valuation of consumption between the two agents across periods, which affects all inter-temporal margins.

On the contrary, when the shadow values of wealth between the two agents are not equalized, i.e. when no lump-sum transfers are available ($0 < \Phi < 1$), the planner has only one tool, i.e. the corrective policy, to take care of both the distortions in the optimality decision and the discrepancy in the marginal valuations of wealth. Consequently, the level of the tax rate will in general differ from the one when $\Phi = 0$ since it will include a component that will implicitly redistribute income between the agents. At last, the corrective policy will not be able to bring the rate of return on capital exactly equal to the rate on lending. These considerations can be seen more clearly from the optimal tax on capital income set by the Ramsey planner, which is characterized in the following proposition.

Proposition 5. *The optimal tax on capital income in the steady state set by the Ramsey planner, whose only instruments are corrective taxes, is strictly positive and given by²⁴*

$$\tau_k^{SB} = (1 - \Phi) \left(\frac{1}{R} - \beta \right) (1 + r) \left(\frac{1}{\beta F_k} - \frac{1}{\bar{\varepsilon}} \right). \quad (37)$$

Proof. The tax on capital income can be derived from the no-arbitrage condition of the competitive economy with taxes and the planner's economy. In order to eliminate the Lagrange multiplier in the equilibrium conditions of the competitive economy with corrective taxes, combine the steady state consumption-borrowing, (31), and the consumption-capital Euler, (22), conditions of entrepreneurs to get the no-arbitrage condition

$$1 - (1 + r) \left(\frac{1}{R} - \beta \right) = \beta \left[1 - \delta + F_k \left(1 - (1/\bar{\varepsilon}) (1 + r) (1/R - \beta) - \tau^k \right) \right].$$

Similarly, derive the same condition from the planner's equilibrium conditions, using (69) and (71), to get

$$1 - \Phi (1 + r) \left(\frac{1}{R} - \beta \right) = \beta [1 - \delta + F_k (1 - \Phi (1/\bar{\varepsilon}) (1 + r) (1/R - \beta))].$$

Subtracting one from the other and solving for τ_k^{SB} , yields eq. (37). Q.E.D.

The proposition above confirms that as long as $0 < \Phi < 1$, the corrective tax will depend on the marginal values of wealth of the two agents. To see this more clearly, notice that (37) can be written as $\tau_k^{SB} = \lambda^h / \lambda^e (1/R - \beta) (1 + r) ((\beta F_k)^{-1} - 1/\bar{\varepsilon}) = \lambda^h / \lambda^e \cdot \tau_k^{FB}$, where τ_k^{FB} , defined in the

²⁴I use τ_k^{SB} and τ_k^{FB} to denote the tax rates of capital income (τ^k) that replicate the second- and the first-best allocations. I use equivalent notation for the payroll tax.

corollary below, denotes the level of the corrective tax when $\Phi = 0$. Therefore, the capital income tax is scaled by the ratio of the marginal values of wealth, which can be equalized in the presence of lump-sum transfers. Further, given that $\lambda^{RP,e} > \lambda^{RP,h}$, the level of the tax that the planner chooses would be lower than the first-best level of τ_k^{FB} . Moreover, the higher the shadow value of income of entrepreneurs compared to the one of households, the lower the tax would be. Intuitively, this means that the planner aims at correcting the externality described above by imposing a positive tax, but its value would be smaller when the planner cares more about the entrepreneurs, given that lump-transfers are not available to transfers resources directly.

Corollary 2. *The optimal tax on capital income in the steady state set by the Ramsey planner, endowed with corrective taxes and lump-sum transfers, is strictly positive, fixed and independent of agents' shadow prices of wealth, $\tau_k^{FB} = (1/R - \beta)(1 + r) [(\beta F_k)^{-1} - 1/\bar{\varepsilon}]$.*

Proof. From (37), set $\Phi = 0$. Then, it follows that the tax on capital income is positive since the effective borrowing cost to the entrepreneurs, R , is lower than the lending rate, $1 + r = 1/\beta$, hence $1/R - \beta > 0$. In addition, for any plausible calibration of $\bar{\varepsilon}$ such that a portion of capital can be used as a collateral, for which $\beta F_k < \bar{\varepsilon}$, we have $1/(\beta F_k) - 1/\bar{\varepsilon} > 0$. Q.E.D.

Finally, instead of imposing a tax on capital income, the planner could choose alternative corrective instruments that can affect the inter-temporal margin, including a tax on borrowing or a loan-to-value (LTV) ratio such that they brings the return rate on capital and the lending rate closer to each other. All three instruments would make the marginal unit invested more expensive by distorting the no-arbitrage condition from different angles. I show in section 3.4 and 4.2 below that both optimal tax on borrowing or the LTV ratio are equivalent to a tax on capital income.

3.3.2 Payroll Tax

First, to understand the externality arising from the labor decision, affecting the level of the tax rate when lump-sum transfers are not available, consider the optimal labor conditions characterizing the competitive economy and the one of the social planner, respectively

$$\left[1 - \frac{(\frac{1}{R} - \beta)(1 + r)}{\bar{\varepsilon}} \right] F_l = -\frac{u_l}{u_c}, \quad (38)$$

$$\left[1 - \Phi \frac{(\frac{1}{R} - \beta)(1+r)}{\bar{\varepsilon}}\right] F_l = -\frac{u_l}{u_c} \left[\Phi + \frac{\omega}{1-\omega} \frac{u_c}{u_c^e} \right] - \Phi \left(\frac{\partial \frac{u_l}{u_c}}{\partial l} \right) l. \quad (39)$$

Here, (38) is obtained by substituting (10) in (9). Similarly, condition (39) is obtained by combining (69) and (70). As before, since Φ is present in the planner's optimality condition, the planner internalizes the difference in agents' valuation of wealth. Begin by first comparing eq. (38) and (39) for $\Phi = 0$. Recall that this allocation is implied by the presence of lump-sum transfers. Then, the planner's condition (39) becomes

$$F_l = -\frac{u_l}{u_c}, \quad (40)$$

since $(1-\omega)u_c^e = \lambda^{RP,e} = \lambda^{RP,h} = \omega u_c$ for $\Phi = 0$. In this case, the planner sets the marginal product of labor equal to the wage, utilizing labor efficiently. On the contrary, as pointed out in section 2.2, entrepreneurs in the competitive economy stop demanding labor at a stage prior to the point where the marginal product is equalized to the wage that households demand since more labor tightens the collateral constraint. Then, the planner's allocations can be replicated by a corrective policy that compensates entrepreneurs for the extra costs incurred by the binding collateral constraint.

As before, when the shadow costs of wealth between the agents are not equalized, i.e. in absence of lump-sum transfers ($0 < \Phi < 1$), the policy rate placed by the planner, in addition to correcting for the distortions in the competitive economy, will also implicitly transfer resources in order to bring closer the shadow costs of agents' wealth. Hence, the corrective policy that can implement the planner's allocations will not exactly be able to equate the marginal product of labor to the marginal rate of substitution, as it will be shown below.

The following proposition derives the corrective, optimal payroll tax. I show in sections 3.4 and 4.2 that this is equivalent to imposing a tax on labor income on the side of households, which is redistributed back to entrepreneurs.

Proposition 6. *The payroll tax in the steady state is given by*

$$\tau_p^{SB} = \frac{\left[\frac{(\frac{1}{R} - \beta)(1+r)}{\bar{\varepsilon}} + \tau^k \right] F_l}{\frac{u_l}{u_c}} + \tau^k + \left(\Phi + \frac{\omega}{1-\omega} \frac{u_c}{u_c^e} - 1 \right) + \Phi \frac{u_{ll}}{u_l}. \quad (41)$$

Proof. The payroll tax can be derived from the equilibrium steady state labor conditions in the PDCE, (23), and the planner's economy, (70). In order to eliminate the Lagrange multipli-

ers in these conditions, substitute *in* the consumption-borrowing Euler equations, (31) and (69), respectively. Subtracting one condition from the other and expressing τ^p , yields eq. (41). Q.E.D.

The sign on the tax on labor cannot be unambiguously determined in the case when lump-sum transfers are not available. Indeed, as I show in section 4.2, for a calibrated version of the economy, the sign of the tax rate changes from negative to positive as the planner considers the points along the grid of agent weights. On the other hand, when lump-sum transfers are available, the planner chooses unambiguously a negative payroll tax, as shown in the following corollary.

Corollary 3. *In presence of lump-sum transfers labor is subsidized. The subsidy is constant, positive and independent of agents' shadow prices of wealth*

$$\tau_p^{FB} = -\frac{\left(\frac{1}{R} - \beta\right)(1+r)}{\bar{\varepsilon}}. \quad (42)$$

Proof. The proof is straightforward from (41), by setting $\Phi = 0$ and recognizing that $F_l = -\frac{u_l}{u_c}$ from (40) and that $\frac{\omega}{1-\omega} \frac{u_c}{u_c^e} = 1$ when transfers are allowed. Q.E.D.

This result suggests that given the lump-sum transfers, the planner chooses the payroll tax solely to address the distortions arising from the collateral constraint. A negative payroll tax is a subsidy on labor demand by entrepreneurs, suggesting that the level of labor in the first-best economy is higher than in the competitive economy with financial friction. The subsidy on labor demand encourages entrepreneurs to hire more labor, and thus, it exactly equates the marginal product of labor to the marginal rate of substitution between consumption and labor, i.e. the wage rate.

To sum up, there is a difference in the way that the planner uses distortionary taxes to correct for the inefficiencies in the competitive economy when transfers are available and when they are not. In the latter case, the planner faces a trade-off between using the taxes to address the distortions in the optimal conditions and using them to indirectly induce desirable income transfers. This trade-off is eliminated once lump-sum transfers are available, rendering corrective taxes independent from agents' shadow costs of wealth. It may sound surprising that the Ramsey planner needs distortionary taxes to obtain the first-best equilibrium outcome and cannot do this with lump-transfers alone. The reason is that the collateral constraint introduces distortions in the optimality conditions of agents, which lump-sum transfers cannot fix since they do not affect

marginal decisions. The following proposition shows the optimal level of the lump-sum transfers chosen by the planner. As expected, they depend on the weights in the social welfare function.

Proposition. *The optimal lump-sum transfer from the households to the entrepreneurs, in the steady state, is*

$$\mathcal{T} = \frac{\left[\omega \left(\Upsilon^{\frac{\alpha}{\alpha-1}} - \delta \Upsilon^{\frac{1}{\alpha-1}} \right) \right]}{\left[1 + \frac{\omega \psi}{1-\alpha} \left(1 - \frac{\delta}{\Upsilon} \right) \right]} \left(1 + \frac{\psi}{(1-\alpha) \Upsilon^{\frac{\alpha}{\alpha-1}}} \Xi \right) - \Xi, \quad (43)$$

where $\Upsilon \equiv \frac{1/\beta - 1 + \delta}{\alpha}$, $\Xi \equiv (1-\alpha) \Upsilon^{\frac{\alpha}{\alpha-1}} + \beta^{-1} \varphi (1+\varphi)^{-1} \left(\Upsilon^{\frac{\alpha}{\alpha-1}} - \frac{1}{\Xi} \Upsilon^{\frac{1}{\alpha-1}} \right)$ and $\varphi \equiv (1/\beta - 1)(1-\tau)$.

Finally, even though one may consider that the planner with lump-sum transfers is too powerful and would rather focus attention to second-best outcomes, omitting the lump-sum transfers poses the risk of using distortionary taxes as a tool to indirectly transfer resources among agents. By comparing the taxes implementing the second-best to those implementing the first-best allocations, one can disentangle the part of the tax that is being used to fix the distortions and the part that is being used to implicitly transfer resources in the second-best economy (i.e. to fix the externality). At last, it should be noted that there exist weights in the social welfare function for which the planner chooses zero lump-sum transfers. At this point, the first-best and the second-best outcomes coincide and only distortionary taxes are needed to restore first-best efficiency.²⁵

3.4 Alternative Implementation

This section discusses the alternative instruments that can be used to implement the planner's allocations. In the previous sections, I focus specifically on corrective taxes, which are imposed on and distributed back to the same agent. However, one can consider alternative instruments that are not necessarily fiscal in nature or corrective per se. I show analytically the equivalence between the system of capital income and payroll taxes to alternative policy systems, but I postpone the computation and the discussion of their optimal levels until section 4.2, where I study a calibrated version of the economy.

The choice of instruments, however, is constrained by two considerations. First, since both the intra-temporal and the inter-temporal margins are distorted, there should be at least two

²⁵For a calibrated version of the economy, I show this level of weights in the quantitative analysis of the model economy.

instruments that can affect each margin individually. Second, if the instruments considered are redistributive taxes in particular, i.e. taxes that are imposed on one agent and their proceeds distributed back to the other, it also matters who gets the revenues from those taxes when direct lump-sum transfers are not available. In the present model, since entrepreneurs are the ones facing the financial frictions, it is important that either their decisions are appropriately *corrected* or that funds are *distributed* to them, such that the distortions are offset.

In what follows, I will show both analytically and numerically that the corrective tax on capital income can be substituted with either a corrective tax on borrowing or with a loan-to-value ratio, optimally chosen by the planner. In addition, the payroll tax can be replaced by a tax on labor income, imposed on the side of the households, but its revenues are distributed back to the entrepreneurs. The redistribution is not needed when the planner can make lump-sum transfers directly, but it is necessary to replicate the second-best equilibrium allocations in absence of it.

First, I consider a corrective tax on borrowing, τ_t^b , which is in place instead of the tax on capital income, τ_t^k . The borrowing tax is imposed on the side of entrepreneurs and its proceeds are distributed back to the same agents at the end of the period. Since the borrowing tax is corrective in nature, the budget constraint of the entrepreneurs (and all other equilibrium conditions) remain the same as in section 3.1 with $\mathcal{T}_t = 0$, but the consumption-borrowing Euler becomes

$$\frac{1 - \tau_t^b}{R_t} - \frac{\mu_t}{1 + r_t} = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \quad (44)$$

or in the steady state

$$\frac{1 - \tau^b}{R} - \frac{\mu}{1 + r} = \beta. \quad (45)$$

Proposition 7. (*Equivalent Tax Systems*) *Given a system, consisting of capital income and payroll taxes, $\{\tau_t^k, \tau_t^p\}_{t=0}^\infty$, there exists a system consisting of borrowing and payroll taxes, $\{\tau_t^b, \tilde{\tau}_t^p\}_{t=0}^\infty$, such that the two competitive equilibria with the corrective taxes are equivalent in both economies.*

Proof. See appendix.

The proof shows that given a system of prices and taxes $\{r_t, w_t, \tau_t^k, \tau_t^p\}_{t=0}^\infty$, one can construct a system of prices and taxes $\{\tilde{r}_t, \tilde{w}_t, \tau_t^b, \tilde{\tau}_t^p\}_{t=0}^\infty$ such that both economies yield the same allocations. Therefore, the planner can impose a tax on borrowing to substitute for the one on capital income.

The role of the tax on borrowing is to bring the borrowing costs for entrepreneurs and households closer to together. Both tax on capital income and borrowing would make the marginal unit invested more expensive, since they would distort the no-arbitrage condition from different angles. The following corollary reports the borrowing tax in the steady state.

Corollary 4. *The steady state value of the tax on borrowing is positive and it is given by*

$$\tau^b = \frac{\beta R_t F_k \tau^k}{(1+r) \left(1 - \frac{\beta F_k}{\varepsilon}\right)} \quad (46)$$

Alternatively, the planner could impose quantity limits on assets instead of affecting the marginal return (cost) of investment (borrowing) thorough taxes. To examine this possibility, I consider an optimal loan-to-value (LTV) ratio, instead of the corrective tax on capital income or the borrowing tax, in order to replicate the planner's allocations. The LTV ratio differs from the remaining tools because it restricts the quantity of the asset that serves as a collateral directly instead of affecting it indirectly via its price. Also, unlike the tax on capital income, which is a fiscal instrument, the LTV ratio can be considered as a financial policy tool. To define the LTV ratio in the model economy, I add an additional constraint to the entrepreneurs' problem, given by

$$\eta_t k_{t+1} \geq \frac{b_{t+1}}{1+r_t}, \quad (47)$$

where η_t denotes a time-varying LTV ratio. Setting $\lambda_t^e \chi_t$ to be the Lagrange multiplier on this constraint, the inter-temporal optimality conditions of the entrepreneurs become

$$1 - \varepsilon_t \mu_t - \eta_t \chi_t = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} [(1 - \mu_{t+1}) F_{k,t+1} + 1 - \delta], \quad (48)$$

$$\frac{1}{R_t} - \frac{\varepsilon_t \mu_t + \eta_t \chi_t}{1+r_t} = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}. \quad (49)$$

All other equations remain the same as in section 3.1, with $\mathcal{T}_t = 0$.

Corollary 5. *(Equivalence between Fiscal and Financial Instruments) Given a system, consisting of capital income and payroll taxes, $\{\tau_t^k, \tau_t^p\}_{t=0}^\infty$, there exist a system consisting of a combination of fiscal (tax) and financial (LTV) policy instruments, $\{\eta_t, \tilde{\tau}_t^p\}_{t=0}^\infty$, such that the two competitive equilibria with the different policy systems yield equivalent allocations.*

The proof of this corollary can be derived following the exact same steps showing the equiva-

lence between the taxes on capital income and borrowing, outlined above. Therefore, both equivalence results show that the tax on capital income is essentially a financial tool. Its equivalence with the corrective tax on borrowing suggests that it can be used to affect the marginal investment/borrowing (i.e. the no-arbitrage) condition and therefore it can be used as a price-based policy tool. On the other hand, its equivalence with the LTV regulation suggests that the tax on capital income can be used to directly affect the quantities of the assets (i.e. the capital stock) and therefore acts as a quantity-based policy tool. At the end, regardless of if the planner uses price- or quantity-based policy tools, it aims at correcting the discrepancy between the borrowing cost and the lending rate (stemming from the combination of the tax advantage and the binding collateral constraint), which is the underlying reason that entrepreneurs continue to borrow and invest up to the point that their collateral constraint binds.²⁶

Finally, I consider a redistributive tax on labor income, which is imposed on households and its proceeds are redistributed back to the entrepreneurs. The redistributive tax on labor can be used to replicate the planner's allocations in combination with any of the policy instruments, affecting the no-arbitrage condition, discussed above. It is important to note that the tax on labor income has to be redistributive and not corrective. The reason is that the policy tool, required to correct the intra-temporal decision, would need to address the distortions present in the entrepreneurs' side of the economy. In this way entrepreneurs' decisions would not be corrected, but they would instead be compensated by a lump-sum transfer from households, i.e. the proceeds from the labor tax.

Introducing the redistributive tax on labor income on the side of the households results in the following changes of the equations in the PDCE

$$c_t + \frac{b_{t+1}^h}{R_t} + T_t^h = (1 - \tau_t^l) w_t l_t + b_t^h \quad (50)$$

²⁶This intuition and the type of optimal policy is general and carries through to cases where the collateral constraint binds for different reasons than a tax advantage on debt to the entrepreneur. For example, the entrepreneur could have been more impatient than the household as in Kiyotaki and Moore (1997) or Iacoviello (2005). Then, the planner would still impose a tax on capital to correct for the discrepancy between the subjective discount factors of the two agents (the results are not presented here and are available upon request). In general, the planner internalizes the externalities from the different way that the two agents discount a unit of borrowed funds, which results in binding collateral constraints and lower return on capital. A tax on capital income is levied to correct this distortion in the no-arbitrage condition.

$$u_{c,t} (1 - \tau_t^l) w_t = -u_{l,t} \quad (51)$$

$$c_t^e + w_t l_t + b_t^e + k_{t+1} = F(k_t, l_t) + (1 - \delta) k_t + \frac{b_{t+1}^e}{R_t} + T_t^h \quad (52)$$

where $T_t^h = \tau_t^l w_t l_t$. All other equations remain intact, as in section 3.1 with $\mathcal{T}_t = 0$.

Proposition 8. (*Equivalence between Labor and Payroll Taxes*) *Given a system consisting of capital income and payroll taxes (imposed on entrepreneurs), $\{\tau_t^k, \tau_t^p\}_{t=0}^\infty$, there exist a system consisting of capital and labor income taxes (the latter imposed on households), $\{\tilde{\tau}_t^k, \tau_t^l\}_{t=0}^\infty$, such that the two competitive equilibria with the different policy systems yield equivalent allocations.*²⁷

The proof for this proposition can be shown following a similar approach as the one showing the equivalence between the tax on capital income and the LTV limit or the tax on borrowing. However, at the last step, in order to derive τ_t^l , one would need, in addition, to combine the marginal product of labor and the marginal rate of substitution for the two economies.

To sum up, the tools that the planner can use to implement the optimal allocations fall into two broad categories. First, tools that operate on the inter-temporal marginal decision of the entrepreneurs, i.e. the no-arbitrage investment/borrowing decision. Second, tools that operate on the intra-period marginal decision of either the entrepreneurs or the households to demand or supply labor, respectively. The reason is that both inter- and intra-period margins are distorted in the competitive economy due to binding collateral constraints. The next section computes the optimal level of the policy tools discussed for a calibrated version of the economy both in the steady state and in response to shocks.

4 Quantitative Analysis

The goal of this section is to evaluate the optimal policy *in* and outside the steady state for a calibrated version of the model economy. To perform the outside the steady state analysis, I consider the responses of a subset of variables and the policy instruments to a productivity and a financial shock. In the computations, I conjecture that the collateral constraint is always binding, which is indeed the case in the steady state. In the appendix, I perform an exercise that shows that, in presence of productivity and financial shocks, the collateral constraint binds with probability of

²⁷In this corollary, I consider the tax on capital income as a policy tool to affect the no-arbitrage condition. However, the corrective tax on borrowing or the LTV ratio regulation would produce the same results.

95%. In the rest of this section, I proceed by first discussing the calibration of the economy, and then outlining the results in the steady state and in response to shocks.

4.1 Calibration

In this section I lay out the functional forms assumed for the preferences and technology and I assign values to the parameters in those functional forms.

Agents' preferences. The time period is a quarter. The subjective discount factor is set to $\beta = 0.9805$ so that the annual steady-state rate on borrowing for non-financial corporate businesses equals the one observed in the data.²⁸ The preferences of both agents are characterized by a CRRA utility function with a relative risk aversion parameter of 1. The utility function of the household takes the logarithmic form $u(c_t, l_t) = \log(c_t) + \psi \log(1 - l_t)$ with $\psi = 2.2$, generating a steady-state value of $l \approx 0.3$, and similarly the utility function of the entrepreneur takes the form of $u^e(c_t^e) = \log(c_t^e)$.

Production. The production function of entrepreneurs is Cobb-Douglas $F(\cdot) = z_t k_t^\alpha n_t^{1-\alpha}$ with $\alpha = 0.36$. The quarterly depreciation rate is set to $\delta = 0.025$.

Financial frictions. The tax advantage on borrowing is set to $\tau = 0.38$. This parameter is important for the performance of the model since it determines whether the collateral constraint is binding. As shown below, with this calibration of τ , the collateral constraint is binding with a probability of 95% outside the steady state. Following Jermann and Quadrini (2012), the mean value of the financial variable $\bar{\varepsilon} = 0.16$ is chosen such that the ratio of debt to GDP replicates the one observed in the data (≈ 3).²⁹

Exogenous shocks. The model economy is analyzed in response to productivity and financial shocks. The parametrization of the persistence parameters and the shock variances of these variables follows Jermann and Quadrini (2012), estimated from the following autoregressive system

$$\begin{pmatrix} z_{t+1} \\ \varepsilon_{t+1} \end{pmatrix} = A \begin{pmatrix} z_t \\ \varepsilon_t \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^z \\ \epsilon_{t+1}^\varepsilon \end{pmatrix}, \quad (53)$$

²⁸The borrowing rate for the non-financial corporate business was calculated using the data from the Federal Reserve Bank of St. Louis, series "Nonfinancial corporate business; corporate bonds" (code: NCBCBIA027N), for the period 1946-2013.

²⁹To compute this statistics, Jermann and Quadrini (2012) use the average ratio over the period 1984Q1 - 2010Q2 for the non-financial business sector based on the data from the FoF (for debt) and the National Income and Product Accounts (for business GDP).

where z_t and ε_t are log-deviations from the deterministic trend, and ϵ_{t+1}^z and $\epsilon_{t+1}^\varepsilon$ are iid, with standard deviations σ_z and σ_ε , respectively.

Welfare weights. I assume a grid of weights for the planner’s welfare function in the range $\omega = [0.4, 0.85]$. This range of weights provides reasonable results for which the shadow values of wealth for both agents are positive.

The full set of parameters is summarized in table 1 below.

Description	Parameter
Discount factor	$\beta = 0.9805$
Utility parameter	$\psi = 2.2$
Production technology	$\alpha = 0.36$
Depreciation rate	$\delta = 0.025$
Tax advantage	$\tau = 0.38$
Collateral constraint parameter	$\bar{\varepsilon} = 0.16$
Std. of productivity shock	$\sigma_z = 0.0045$
Std. of financial shock	$\sigma_\varepsilon = 0.0098$
Matrix for the shock processes	$A = \begin{bmatrix} 0.9457 & -0.0091 \\ 0.0321 & 0.9703 \end{bmatrix}$
Pareto weight	$\omega = [0.4, 0.85]$

Table 1: Summary of model parameters

4.2 Steady State Results

As discussed, the planner aims at correcting the distortions and the externalities in the labor and capital/borrowing decisions induced by the binding collateral constraint. In particular, it wants to implement a lower marginal product of labor and a higher return on capital compared to those characterizing the competitive equilibrium. The extent to which it can bring the marginal product of labor and the return rate on capital closer to the frictionless levels depends on its ability to levy lump-sum transfers.

Table 2 in the appendix reports the planning outcomes and the competitive equilibrium steady state allocations in presence and absence of lump-sum transfers. I consider two benchmarks for the planner’s weights. When $\omega = 0.5$, both agents are equally important to the planner. When

$\omega = 0.828$, as we will see, the lump-sum transfers are equal to zero and this is the case when the second-best allocations equal those of the first-best in the steady state.

From table 2, a general observation for all planning outcomes is that the level of borrowing is lower compared to the competitive economy with a binding collateral requirement; whereas the rate of return on capital is higher and the marginal product of labor is lower. This suggests that the private agents have an incentive to over-borrow and they favor capital investment, which relaxes their collateral constraint to labor purchase, which tightens it.

In the case when weights are equally set, without lump-sum transfers, the planner can only achieve the second-best allocation, and implements it by choosing positive capital income and payroll taxes. The positive payroll tax, which is substantial, is imposed in order to decrease the labor demand of entrepreneurs since more labor tightens their constraint and reduces their consumption.³⁰ The level of the tax rate, however, is contingent on the valuation of agents' wealth, as well as the Pareto-Negishi weights. In this way, the payroll tax effectively represents a transfer from households to entrepreneurs since the lower equilibrium level of labor results in lower wages for households. Further, the tax on capital income is set to be positive such that it deters entrepreneurs to invest, increasing the return rate on capital, which corrects for the distortion in the inter-temporal margins.

On the other hand, when lump-sum transfers are available, the planner can achieve the first-best outcome, and implements it by setting a positive tax on capital income, a payroll subsidy and a lump-sum transfer from households to entrepreneurs. In this case, any transfer that needs to take place between the agents such that the marginal valuations of wealth are equalized is taken care of by the lump-sum transfer. The linear taxes are imposed to offset the distortions that the binding collateral constraint induces in the marginal decisions. Indeed, there is a direct lump-transfer from the household to the entrepreneur, which amounts to 0.3562 when $\omega = 0.5$. The turn in the sign of the payroll tax arises because, in this case, the planner uses the tax to directly compensate entrepreneurs for the incurred cost from the binding collateral constraint rather than to implement the indirect transfer. Further, the tax on capital income is still positive and set to 12.3%, and it is even higher than when lump-sum transfers are absent (6.67%). The difference between these

³⁰Recall that when the collateral constraint binds, entrepreneurs would need to reduce their consumption in order to finance employment.

two tax levels suggests that the tax on capital income, in absence of lump-sum transfers, is also implicitly used to transfer resources between the agent groups.

Welfare gains, measured by the compensating variation in consumption, are substantial for this set of weights even if one accounts for the transitory dynamics from one steady state to another. Figure 13 in the appendix displays the change in welfare along the transition from the steady state without to the steady state with the tax regime.³¹ The reason for the high levels of the welfare gain is due to the more than double increase in entrepreneurs' consumption, who have a high marginal utility in the competitive equilibrium.

Table 2 also reports the planning outcomes when the weight on households is such that the planner does not need lump-sum transfers to achieve the first-best outcome ($\omega = 0.828$ for this calibration). In this case, the second-best and the first-best coincide and the sole role of the tax instruments is to correct for the distortions in the inter-temporal and the intra-temporal optimality conditions. It is important to emphasize that the capital and the payroll taxes are invariant across the first-best equilibria for the different weights, implying that their sole role is to fix the distortions of the agents' decisions. Moreover, the general result that borrowing is lower, the return rate on capital higher, and the marginal product of labor lower in the planning solution than in the competitive economy still holds. However, when $\omega = 0.828$, the utilization of labor is lower than in the first-best under equal weights. The reason is that the planner cares more about the households, who suffer the disutility from supplying labor. As a result, the investment in capital is lower, since less capital is needed to achieve the same return when labor is lower. Welfare improves, but less, because the gain accruing to the entrepreneurs, who have higher marginal utility, is given a lower weighting.

The channels described above operate smoothly across the different weights in the social welfare function. Figure 2 presents the tax on capital income, the payroll tax and the lump-sum transfers in the first- and in the second-best economies as functions of households' Pareto-Negishi weight. In particular, the tax on capital income, implementing the second-best solution, starts from levels close to zero for a low weight on households and steadily increases up to its first-best level as the weight on households increases. Similarly, the payroll tax starts from high positive levels and steadily

³¹I calculate the compensating variation in average consumption following the approach in Lucas (1987). Since I consider the welfare of both agents the consumption gain is the *average* of the consumption gains of the two agents.

decreases, while turning negative before it reaches its first-best (negative) level. As mentioned, the positive payroll tax is trying to induce an implicit income transfer from households to entrepreneurs in the second-best. Instead, this is achieved with a lump-sum transfer in the first-best, which is decreasing until it reaches zero for higher weights on households. Thus, the decreasing payroll taxes in the second-best and the decreasing lump-sum transfers in the first-best are in par with what the planner is trying to achieve.

Figure 3 displays the marginal products of capital and labor, respectively, as functions of households' weight. These graphs clearly show that the planner would like to increase the return rate on capital and to decrease the marginal product of labor in the competitive economy. In addition, it shows that the second-best allocations crucially depend on the Pareto-Negishi weights. Recall that the planner, without lump-sum transfers, can only choose policy that will tackle the distortions in the marginal conditions and at the same time obey the relative importance of each agent. Thus, when choosing policy optimally, given that the planner takes into account the marginal valuations of wealth that depend on the Pareto-Negishi weights, the adjustment in the marginal rates happens slowly as the weight on households increases.

In conclusion, the level of borrowing is always lower in the planning solution than in the competitive economy and decreasing in the weight placed on households' utility. The level of capital and labor employed can be higher or lower than the competitive economy and their levels will be determined by marginal valuations of wealth, (thereby also the Pareto-Negishi weights) and the tax system in place.

Finally, table 4 shows the rates of the borrowing tax and the LTV limit. When the borrowing tax is combined with the payroll tax, both are positive, however their levels depend on the planner's weights; when $\omega = 0.828$, the payroll tax goes close to zero, suggesting that the tax on borrowing alone is sufficient to replicate the first-best allocations, i.e. to completely relax the collateral constraint. Similarly, the LTV regulation in combination with a payroll tax can be used to replicate the second-best equilibrium outcomes. However, it is not sufficient to replicate the first-best allocations for this calibration. The reason is that as the planner tightens the LTV ratio, the (market) collateral constraint becomes less binding and after a point it becomes slack. This is the case for $\omega > 0.5675$ for this calibration.

4.3 Optimal Policy in Response to Shocks

I solve the model under the assumption that the collateral constraint is always binding. I follow the dual approach in solving the Ramsey problem, so that the planner directly chooses allocations, prices and policies. The optimal policy in response to shocks is computed using first order approximation of the equilibrium conditions for the recursive Lagrangian problem, characterizing the Ramsey plan. This requires, first, computing the stationary allocations that characterize the deterministic steady state of the optimality conditions of the Ramsey plan, then computing first order Taylor approximation of the respective policy functions around the same steady state. Essentially, this implies that the economy has been evolving around the same steady state for a long period of time and policy has been conducted around that steady state.

I compare the responses of the variables in the competitive economy and under the Ramsey plan depending on whether lump-sum transfers are available. In what follows, I will refer to the Ramsey plan with distortionary and lump-sum transfers as the unconstrained planner (first-best economy) and to the Ramsey with distortionary taxes only as the constrained planner (second-best economy). I choose $\omega = 0.828$ because, as argued before, at this point the lump-sum transfers in the steady state equal zero, and thus the second- and the first-best outcomes coincide.

Figure 5 displays the responses of selected variables to a one standard deviation increase in productivity. Looking at the impulse response functions, three general observations can be made. First, the constrained planner's allocations can be replicated by the Ramsey plan with distortionary tax instruments, not only at the steady state, but also outside of it. Second, an increase in productivity leads to an increase in output and investment, but the qualitative and quantitative effects differ across the three equilibria. Third, the planner's solutions with and without lump-sum transfers differ (substantially), indicating that transfers are necessary to achieve the first-best in response to shocks even though they not be needed in the steady state; or, in other words, *outside* the steady state, the planner could not simply choose weights such that it does not need to care about the income distribution.

Further, the competitive and the second-best economies respond qualitatively the same way. In particular, the planner that replicates the second-best outcomes reduces the capital income tax from its steady state level so as to facilitate capital accumulation and relax the collateral

requirement. On the other hand, the positive productivity shock directly tightens the collateral constraint because of the collateralized working capital loan. In addition, labor decrease since the tighter collateral constraint discourages entrepreneurs from hiring on the labor market. Therefore, the planner increases the payroll subsidy (recall that for $\omega = 0.828$, there is a payroll subsidy) to stimulate employment, as can be seen in figure 7. Finally, the level of borrowing declines since the tighter collateral constraint allows for less inter-temporal borrowing in the initial periods after the shock has materialized until enough capital accumulates to serve as collateral.

Indeed, borrowing starts to increase after the fourth quarter. Intuitively, the constrained planner would like to directly transfer funds to entrepreneurs so that they increase their investment in capital immediately after the shock hits, and not gradually waiting for collateral to accumulate. However, it lacks this possibility. This motive of the planner is evident from the impulse response function in the first-best solution. Investment overshoots with the shock and gradually decreases as the shock fades out (see figure 5). Indeed, the planner transfers resources from households to entrepreneurs and implements a higher capital tax to correct for the distortions in the optimality condition for capital (see figure 7).

Thus, it is worth emphasizing that the optimal capital tax policy is fundamentally different depending on whether the planner can levy lump-sum transfers. In the case that transfers are available, they are used to boost investment immediately after the shock and a procyclical capital tax is used to bring investment gradually down. On the contrary when transfers are not available, capital builds slowly, while the planner supports capital accumulation by a countercyclical capital tax.

Similarly, figure 6 displays the responses of selected variables to a one standard deviation negative financial shock for the three economies mentioned above. The general observations made for the productivity shock carry through in this case, as well. However, the negative financial shock affects the economy through a different channel. Once it hits, the collateral requirement becomes tighter in the competitive economy by directly affecting the value of the collateral pledged. Consequently, output is depressed since entrepreneurs find it more difficult to acquire a loan to fund working capital. Further, entrepreneurs also reduce their consumption and their investment in capital. The constrained planner internalizes these effects and adjusts policy in order to stimulate

the economy. In fact, it reduces the tax on capital income from its steady state level to stimulate investment. This relaxes the current and the future collateral constraints by allowing accumulation of capital. Moreover, the planner increases the subsidy on labor to stimulate employment and stabilize output in the initial periods following the shock (figure 8).

As it was the case with the productivity shock, the planner will use lump-sum transfers to mitigate the negative effects of a more binding constraint on investment and output. Indeed, it does transfer resources from households to entrepreneurs so that they can maintain a higher level of investment without the need to increase their borrowing (figure 8). As a result, the unconstrained planner does not need to stimulate investment by substantially reducing the capital tax, which is the case for the constrained planner. The effect on the payroll tax is also mild, since lump-sum transfers can be used to mitigate the effects of the shock.

Figures 9 and 10 illustrate the responses of the policy instruments of the alternative regimes in the constraint planner's economy, i.e. a tax on borrowing combined with a payroll tax, and tax on labor combined with capital income tax, to a positive productivity and a negative financial shock. The figures suggest, despite that the different policy instruments replicate the same allocations, the sign as well as the amplitude of the responses of the policy instruments may differ across the different regimes.³²

Finally, figure 11 outlines the responses of the regime with LTV-limits and payroll taxes, in the constrained planner's economy, in response to a one standard deviation positive productivity shock. Following the shock, the planner initially reduces the required LTV ratio to stimulate investment (as was the case with the capital tax), but then it gradually increases it. Note that the LTV ratio may have unambiguously responded in a procyclical way if it was implemented as part of a regulatory mix implementing the first-best unconstrained optimum (recall that this is the case for the capital tax implementing the first-best, which is described above). However, such an experiment cannot be performed for the current calibration as explained in section 4.2. In response to a negative financial shock, the LTV ratio initially increases at the point when the shock materializes, and then it starts declining to stimulate investment, as it can be seen in figure 12.

³²In interest of space, I am not showing this result here. The figures are available upon request.

5 Conclusion

This paper studies the design of optimal Ramsey policy in environments with lenders and borrowers, in which the latter face collateral constraints. I consider the choice of optimal policy in the long-run (deterministic steady state) and in response to productivity and financial shocks.

The presence of binding collateral constraint distorts the competitive economy's equilibrium decisions, as well as it results in externalities arising from the capital and labor decisions. In particular, atomistic agents tend to over-accumulate capital due to its ability to relax the collateral constraint, and to under-employ labor since it tightens it. The latter outcome is specific to the assumption that working capital is also collateralized. The (pecuniary) externality arises since binding collateral constraints render non-balanced shadow values of wealth between the two types of agents. Therefore, any changes in the levels of the factors of production that affect the equilibrium prices, induced by private choices, have welfare effects.

I show that a Ramsey planner, endowed with corrective, distortive capital income and payroll taxes and a lump-sum transfer can replicate the first-best allocations. Specifically, I find a positive tax on capital income and a payroll subsidy, and a lump-sum transfers from households to entrepreneurs in the long run. The tax on capital income and the payroll subsidy are intended to correct the distortions present in the private agents' optimal decisions, which cannot be affected by the lump-sum transfer. On the other hand, the lump-sum transfer is used to correct for the externality by equalizing the marginal valuations of wealth (i.e. redistributing income from one agent to another such that the utility gains/losses associated with the price change are offset).

Further, I find that, in absence of lump-sum transfers, the levels of the tax rates would be biased because they will not only target the distortions present in the marginal decisions of agents, but they will also attempt to equalize the marginal valuations of wealth. Consequently, the tax levels with and without lump-sum transfers available will differ. I also show that the Ramsey planner with corrective, distortionary instruments, but without lump-sum transfers, replicates the allocations of the constrained social planner (as defined in Dávila et al., 2012) for given weights.

Moreover, the tax system consisting of capital income and payroll taxes can be implemented by alternative tax systems. Particularly, the tax on capital income can be substituted by a tax on borrowing or alternatively by an LTV ratio, demonstrating that taxes on capital income can

serve as price-based (tax on borrowing) or quantity-based (LTV ratio) financial regulation. On the other hand, the payroll tax can be substituted by a tax on labor income imposed on the side of households.

The above results characterize the competitive economy with the collateral constraint in the long-run. Outside the steady state, I find that an optimal combination of capital income and payroll taxes can still replicate the second-best equilibrium outcomes, while the first-best is attainable only if lump-sum transfers are available. Further, the optimal tax policy will depend on the nature of the shock and will be time-varying.

In response to a positive productivity shock, an unconstrained planner, who has access to lump-sum transfers, will implement a procyclical capital tax policy, i.e. it will increase capital taxes and transfer resources directly to entrepreneurs so that they take advantage of the positive shock. On the contrary, a constrained planner will implement a counter-cyclical capital tax policy, i.e. it will reduce the capital tax to motivate entrepreneurs to build up capital, since it cannot transfer resources to them otherwise.

In response to a (negative) financial shock, both constrained and unconstrained planners will reduce the capital tax to allow entrepreneurs to (re)build capital used as collateral. However, the unconstrained planner will implement a milder reduction in the capital tax, since it can also transfer resources to entrepreneurs in a lump-sum fashion.

Overall, a planner would like to correct for the distortions in the optimal utilization of the factors of production by imposing capital and labor taxes. Since the distortions are present both in the long- and in the short-run, these policies are always necessary to achieve the efficient outcomes. However, tax policy will be additionally biased by the implicit redistribution of income when direct income transfers are not available. This highlights the political economy underlying the optimal policy choice especially in constrained environments.

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6 Appendix

6.1 Steady State of the Competitive Economy (Proposition 1)

In the long run, the model is characterized by the following set of equations, which represent the steady state conditions satisfying the competitive equilibrium

$$1 = \beta (1 + r) \quad (54)$$

$$F_l (1 - \mu) = -u_l / u_c \quad (55)$$

$$\bar{\varepsilon} k = \frac{\bar{\varepsilon} b}{1 + r} + F(\cdot) \quad (56)$$

$$c^e + k + wl + b = (1 - \delta) k + F(\cdot) + \frac{b}{R} \quad (57)$$

$$1 - \bar{\varepsilon} \mu = \beta [(1 - \mu) F_k + 1 - \delta] \quad (58)$$

$$\frac{1}{R} - \frac{\bar{\varepsilon} \mu}{1 + r} = \beta \quad (59)$$

$$c + \frac{b}{R} + Q = wl + b \quad (60)$$

Using (54), yields $1 + r = \frac{1}{\beta}$. The Lagrange multiplier can be obtained using (59),

$\mu = \bar{\varepsilon}^{-1} [(1 - \beta) - (1 - \beta)(1 - \tau)]$. The steady state level of borrowing, as a function of capital and labor, can be obtained using the borrowing constraint, which holds with equality, $b(k, l) = \frac{1}{\bar{\varepsilon} \beta} (\bar{\varepsilon} k - F(\cdot))$. Households' consumption can be obtained from (60), and similarly entrepreneurial consumption can be obtained from (57). Then, all variables depend on the steady state levels of capital and labor. The steady state level of the model can then be summarized by only two equations with two unknowns, k and l

$$1 - \mu = \frac{-u_l(k, l) / u_c(k, l)}{F_l(k, l)} \quad (61)$$

$$1 - \bar{\varepsilon} \mu = \beta [(1 - \mu) F_k(k, l) + 1 - \delta] \quad (62)$$

where (61) is obtained by substituting (60) in (55). This completes the proof of proposition 1.

6.2 The Less Restricted Ramsey (LRR) Plan

The problem of the LRR plan is given by

$$\max_{\{c_t, c_t^e, l_t, k_{t+1}, b_{t+1}, T_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \omega u(c_t, l_t) + (1 - \omega) u^e(c_t^e) \} \quad (63)$$

$$s.t. \quad c_t + \frac{b_{t+1}}{R(c_t, c_{t+1})} + T_t \leq w(c_t, l_t) l_t + b_t \quad (64)$$

$$c_t^e + w(c_t, l_t) l_t + b_t + k_{t+1} \leq F(\cdot) + \frac{b_{t+1}}{R(c_t, c_{t+1})} + (1 - \delta) k_t + T_t \quad (65)$$

$$\varepsilon_t k_{t+1} \geq F(\cdot) + \varepsilon_t \frac{b_{t+1}}{1 + r(c_t, c_{t+1})}, \quad (66)$$

where I have substituted (28) and (27) to express r_t , R_t and w_t , respectively and imposed the market clearing conditions, i.e. $b_t^e = b_t^h = b_t$ and $l_t = n_t$, $\forall t$. Then, letting $\lambda_t^{RP,h}$ denote the Lagrange multiplier on (64), $\lambda_t^{RP,e}$ on (65), and $\lambda_t^{RP,e} \mu_t^{RP}$ on (66), the first order conditions describing the equilibrium allocations are given by

c_t :

$$\begin{aligned} & \omega u_{c,t} - \lambda_t^{RP,h} + \left(\lambda_t^{RP,e} - \lambda_t^{RP,h} \right) \frac{\partial}{\partial c_t} \left(\frac{b_{t+1}}{R(c_t, c_{t+1})} \right) \\ & - \left(\lambda_t^{RP,e} - \lambda_t^{RP,h} \right) \frac{\partial w(c_t, l_t)}{\partial c_t} - \lambda_t^{RP,e} \mu_t^{RP} \frac{\partial}{\partial c_t} \left(\frac{b_{t+1}}{1 + r(c_t, c_{t+1})} \right) = 0 \end{aligned} \quad (67)$$

c_t^e :

$$(1 - \omega) u_{c,t} - \lambda_t^{RP,e} = 0 \quad (68)$$

b_{t+1} :

$$1 - \frac{\lambda_t^{RP,e} - \lambda_t^h}{R(c_t, c_{t+1})} - \frac{\varepsilon_t \lambda_t^{RP,e} \mu_t^{RP}}{1 + r(c_t, c_{t+1})} - \beta E_t \left[\lambda_{t+1}^{RP,e} - \lambda_{t+1}^{RP,h} \right] = 0 \quad (69)$$

l_t :

$$\omega u_{l,t} - \left(\lambda_t^{RP,e} - \lambda_t^{RP,h} \right) \left[w(c_t, l_t) + \frac{\partial w(c_t, l_t)}{\partial l_t} l_t \right] + \lambda_t^{RP,e} (1 - \mu_t^{RP}) F_{l,t} = 0 \quad (70)$$

k_{t+1} :

$$\lambda_t^{RP,e} (1 - \varepsilon_t \mu_t^{RP}) - \beta E_t \varepsilon_t \lambda_t^{RP,e} \left[(1 - \mu_{t+1}^{RP}) F_{k,t+1} + 1 - \delta \right] = 0 \quad (71)$$

T_t :

$$\lambda_t^{RP,e} = \lambda_t^{RP,h} \quad (72)$$

6.3 Alternative Implementation (Proposition 7)

In this subsection, I provide the proof for the proposition stating that the allocations obtained by a system of capital income and payroll taxes can be replicated by a tax system consisting of a borrowing and payroll taxes. The proof consists in showing that given an allocation $\{\Theta\}_{t=0}^{\infty}$ and a system of prices and taxes $\{r_t, w_t, \tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, one can construct prices and taxes $\{\tilde{r}_t, \tilde{w}_t, \tilde{\tau}_t^k, \tilde{\tau}_t^p\}_{t=0}^{\infty}$ such that both tax systems support the given allocation. The taxes and prices, supporting the allocation, can be constructed by equating the equilibrium conditions characterizing the two model economies.³³

1. The system of equations in economy I, given $\{\tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, are as follows

$$c_t + \frac{b_{t+1}}{R_t} + T_t = w_t l_t + b_t \quad (73)$$

$$c_t^e + b_t + k_{t+1} + w_t l_t = F(k_t, l_t) + (1 - \delta) k_t + \frac{b_{t+1}}{R_t} + T_t \quad (74)$$

$$\varepsilon_t \left(k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right) \geq F(k_t, l_t) \quad (75)$$

$$u_{c,t} = \beta E_t u_{c,t+1} (1 + r_t) \quad (76)$$

$$w_t u_{c,t} = -u_{l,t} \quad (77)$$

$$(1 - \mu_t - \tau_t^k) F_{l,t} = (1 + \tau_t^p - \tau_t^k) w_t \quad (78)$$

$$\frac{1}{R_t} - \frac{\varepsilon_t \mu_t}{1 + r_t} = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} \quad (79)$$

$$1 - \varepsilon_t \mu_t = \beta E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} \left[(1 - \mu_{t+1} - \tau_{t+1}^k) F_{k,t+1} + 1 - \delta \right] \quad (80)$$

2. The system of equations in economy II, given $\{\tau_t^b, \tilde{\tau}_t^p\}_{t=0}^{\infty}$, are as follows

$$\tilde{c}_t + \frac{\tilde{b}_{t+1}}{\tilde{R}_t} + \tilde{T}_t = \tilde{w}_t \tilde{l}_t + \tilde{b}_t \quad (81)$$

$$\tilde{c}_t + \tilde{b}_t + \tilde{k}_{t+1} + \tilde{w}_t \tilde{l}_t = F(\tilde{k}_t, \tilde{l}_t) + (1 - \delta) \tilde{k}_t + \frac{\tilde{b}_{t+1}}{\tilde{R}_t} + \tilde{T}_t \quad (82)$$

³³Note that the market clearing conditions are taken into account and the collateral constraint is assumed to be binding.

$$\varepsilon_t \left(\tilde{k}_{t+1} - \frac{\tilde{b}_{t+1}}{1 + \tilde{r}_t} \right) \geq F(\tilde{k}_t, \tilde{l}_t) \quad (83)$$

$$u_{\tilde{c},t} = \beta E_t u_{\tilde{c},t+1} (1 + \tilde{r}_t) \quad (84)$$

$$\tilde{w}_t u_{\tilde{c},t} = -u_{\tilde{l},t} \quad (85)$$

$$(1 - \tilde{\mu}_t) F_{\tilde{l},t} = (1 + \tilde{\tau}_t^p) \tilde{w}_t \quad (86)$$

$$\frac{1 - \tau_t^b}{\tilde{R}_t} - \frac{\varepsilon_t \tilde{\mu}_t}{1 + \tilde{r}_t} = \beta E_t \frac{u_{\tilde{c},t+1}^e}{u_{\tilde{c},t}^e} \quad (87)$$

$$1 - \varepsilon_t \tilde{\mu}_t = \beta E_t \frac{u_{\tilde{c},t+1}^e}{u_{\tilde{c},t}^e} \left[(1 - \tilde{\mu}_{t+1}) F_{\tilde{k},t+1} + 1 - \delta \right] \quad (88)$$

Equating the allocations in the two economies, requires setting

$$\tilde{r}_t = r_t, \quad (89)$$

$$\tilde{w}_t = w_t. \quad (90)$$

In addition, note that the Lagrange multipliers on the collateral constraints in the two economies are not equal. Namely, from (87), we can express $\tilde{\mu}_t = \mu_t - \frac{\tau_t^b(1+r_t)}{\varepsilon_t R_t}$. Then, to derive the borrowing rate as a function of the given allocations, prices and taxes, substitute the derived expression for $\tilde{\mu}_t$ and equate (80) and (88). This generates a difference equation in τ_t^b , which can be solved by using an iterative solution method. Here, for simplicity, I solve for the steady state result instead, which is given by

$$\tau^b = \frac{\beta R_t F_k \tau^k}{(1+r) \left(1 - \frac{\beta F_k}{\varepsilon} \right)}. \quad (91)$$

To solve for the payroll tax in the second economy, follow a similar procedure, i.e. substitute the expression for $\tilde{\mu}_t$ and equate (78) and (86). Then, express the payroll tax in the second economy

$$\tilde{\tau}_t^p = \frac{(1 + \tau_t^p - \tau_t^k) \left[1 - \mu_t - \frac{\tau_t^b(1+r_t)}{\varepsilon_t R_t} \right]}{1 - \mu_t - \tau_t^k}, \quad (92)$$

which depends on the prices and taxes in the first economy. Q.E.D.

6.4 Collateral Constraint Outside the Steady State

The equilibrium allocations (at the steady state) and in responses to shocks (outside the steady state) are solved under the assumption that the collateral constraint always binds. Thus, I have omitted the possibility that outside the steady state the collateral constraint, in fact, may not bind. To check for this possibility, as in Faia and Iliopoulos (2011), I perform the following exercise. I let the competitive economy be hit by a suite of stochastic shocks (productivity and financial) during 10,000 periods and store the values of the Lagrange multiplier associated with the collateral constraint. Figure 1 below displays the distribution of the multiplier on the constraint. It is distributed normally and it is centered around its steady state value. The probability that the collateral constraint does not bind coincides with the cumulative distribution for which the multiplier is smaller or equal to zero, i.e. $\mu_t \leq 0$. I find that the constraint does not bind, i.e. that the collateral constrained has surpassed zero, only in 5% of the cases.³⁴ This result allows me to rule out the possibility of non-binding collateral constraint also outside the steady state.

³⁴This is a value that is relatively small and that can be found in the literature of collateral constraints. See for example, Faia and Iliopoulos (2011) and Iacoviello (2005).

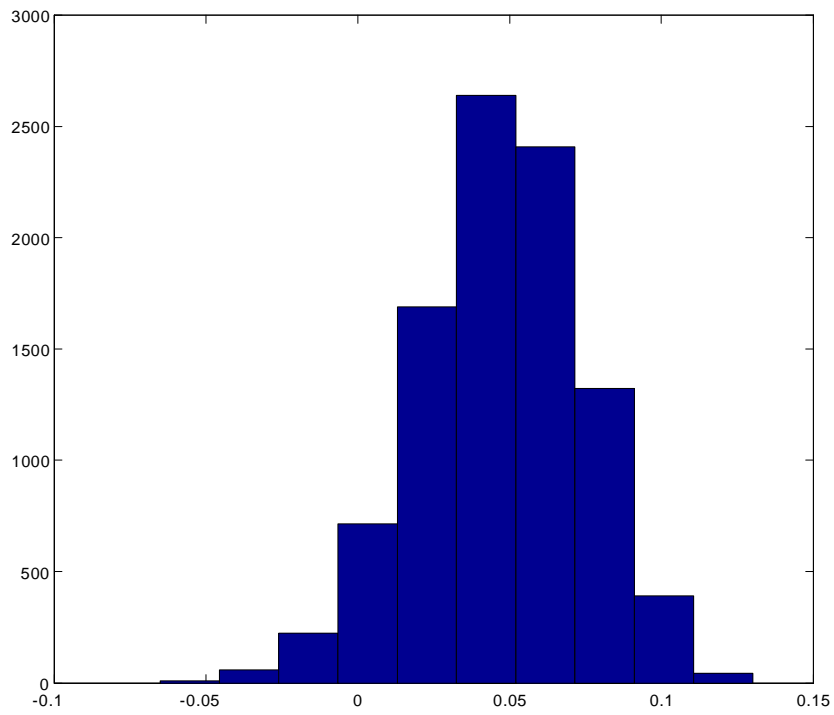


Figure 1: Distribution of the Lagrange multiplier associated with the collateral constraint, μ_t .

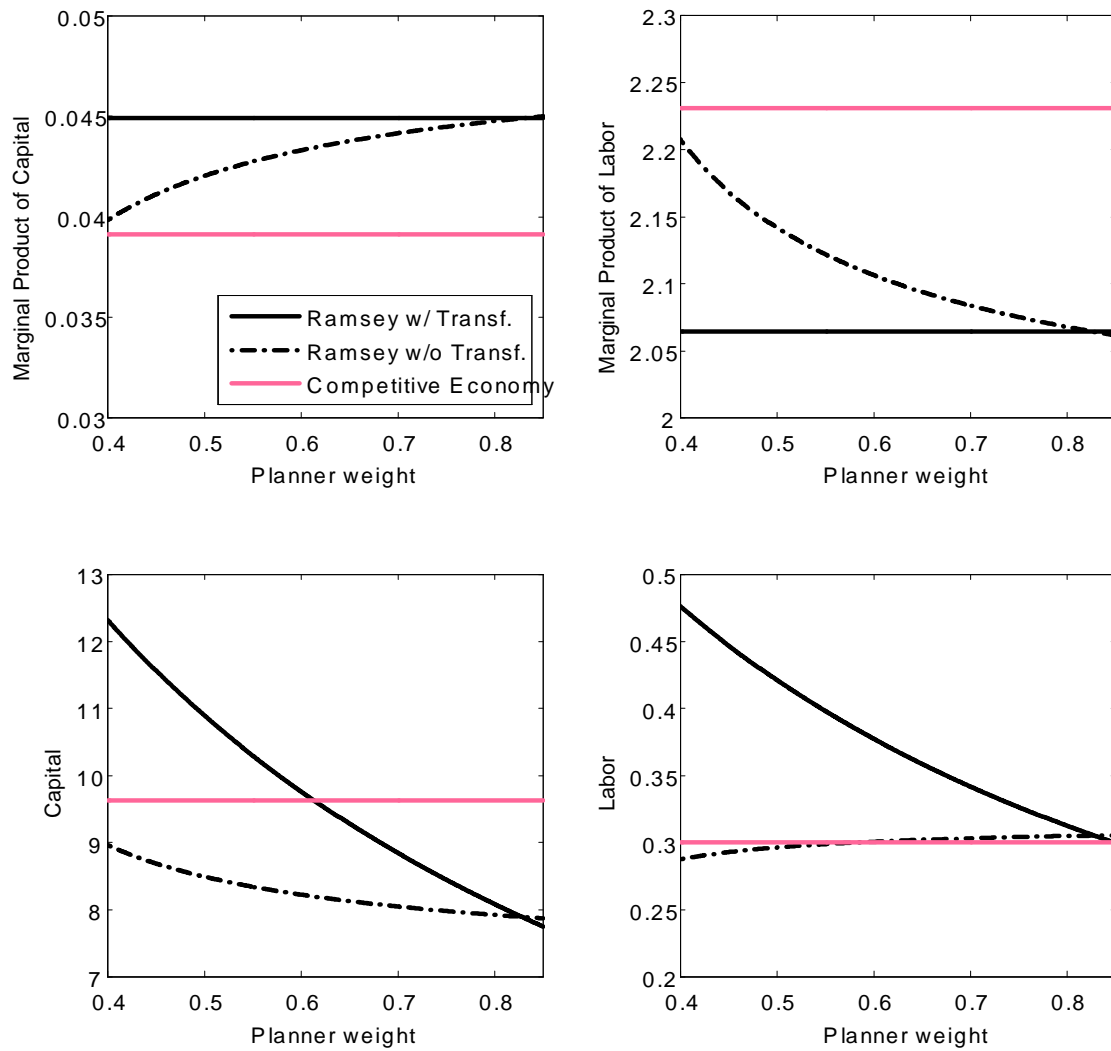


Figure 2: Steady state allocations of the factors of production, as well as their marginal product for a grid of weights $\omega \in [0.4, 0.85]$, where ω denotes the weight placed on households' welfare. The figure compares the values in the competitive economy, the Ramsey plan with and without access to lump-sum transfers.

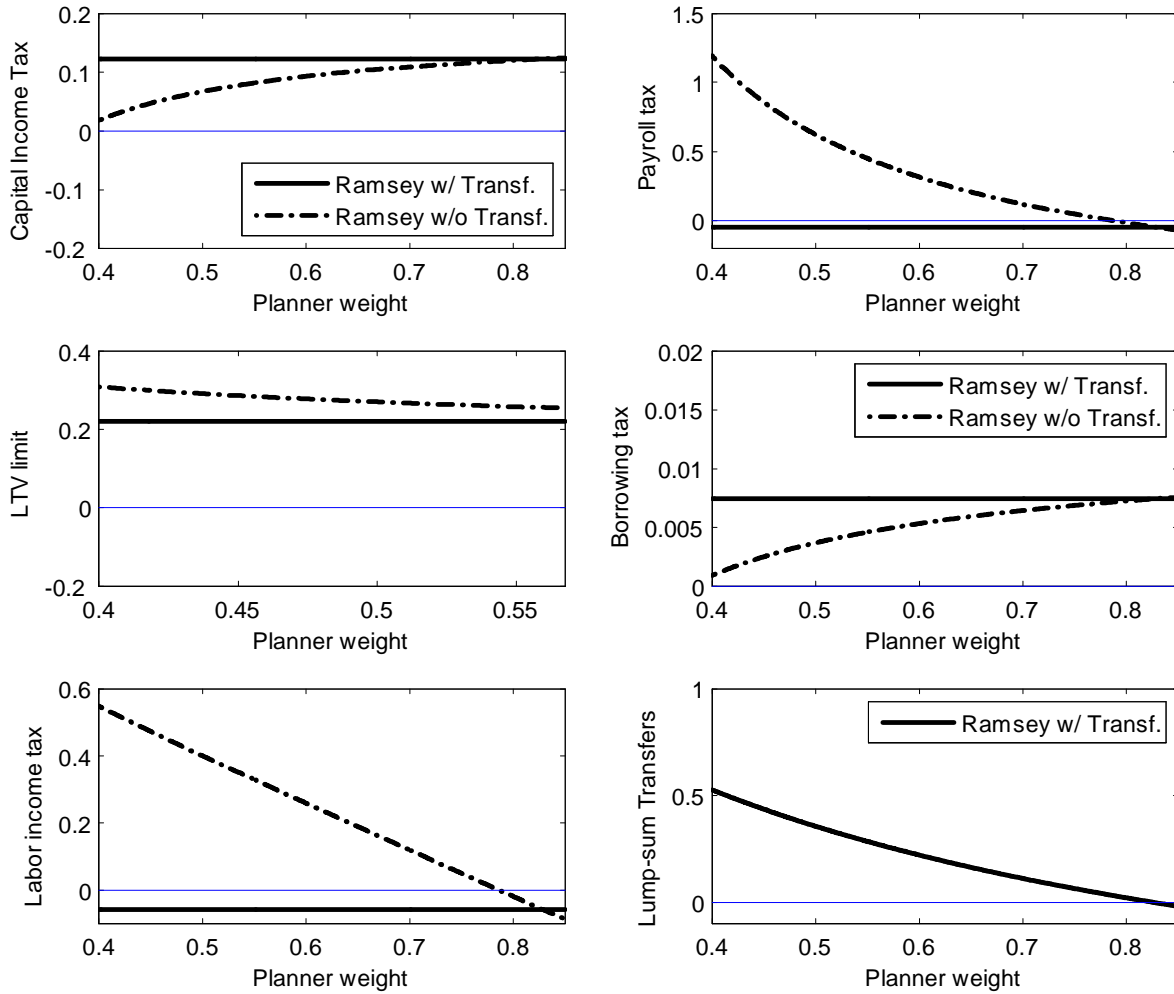


Figure 3: Steady state levels of the various tax rates for a grid of weights $\omega \in [0.4, 0.85]$, where ω denotes the weight placed on households' utility. The figure compares the levels of the rates in the case when the Ramsey planner has access to lump-sum transfers and when it does not.

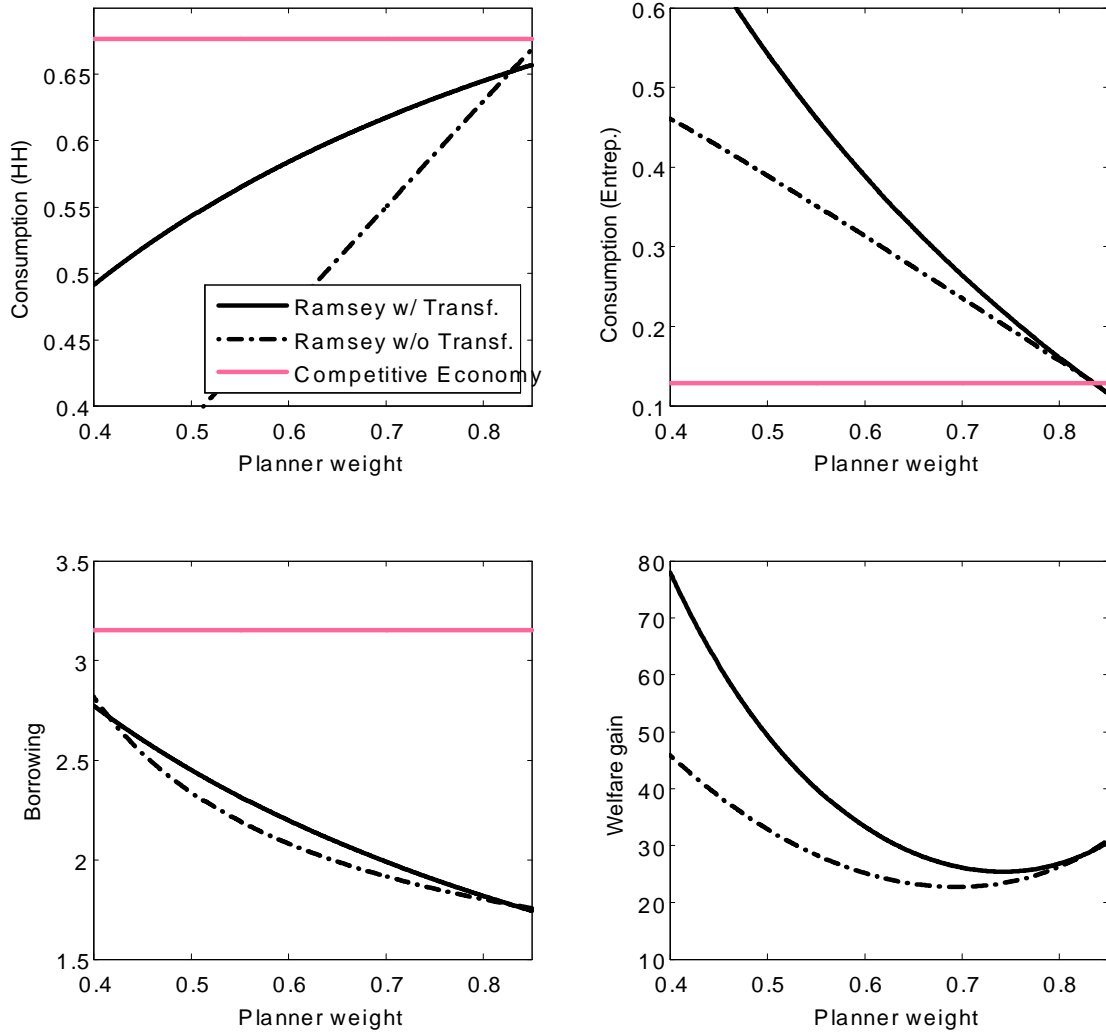


Figure 4: Steady state allocations of consumption and borrowing, as well as, the welfare gain (in terms of agents' average consumption compensation) for a grid of weights $\omega \in [0.4, 0.85]$, where ω denotes the weight placed on households' welfare. The figure compares the variables in the competitive economy and the Ramsey plan with and without access to lump-sum transfers.

Table 2: Steady State Allocations

<i>CE</i>	<i>SB</i>			<i>FB</i>			<i>RP(SB implement.)</i>			<i>RP(FB implement.)</i>		
	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.5$
τ^k	—	—	—	—	—	—	12.30%	—	12.30%	—	12.30%	12.30%
τ^p	—	—	—	—	—	—	-4.67%	—	-4.67%	—	-4.67%	-4.67%
<i>T</i>	—	—	—	—	—	—	—	—	—	—	0	0.3562
<i>b</i>	3.155	1.777	2.340	1.777	2.452	1.777	1.777	2.340	1.777	2.340	1.777	2.452
<i>k</i>	9.629	7.895	8.493	7.895	10.894	7.895	7.895	8.493	7.895	8.493	7.895	10.894
<i>l</i>	0.300	0.305	0.296	0.305	0.421	0.305	0.305	0.296	0.305	0.296	0.305	0.421
F_k	0.039	0.045	0.042	0.045	0.045	0.045	0.045	0.042	0.045	0.042	0.045	0.045
F_l	2.231	2.064	2.142	2.064	2.064	2.064	2.064	2.142	2.064	2.142	2.064	2.064
<i>Welf.Gain</i>	—	28.60%	38.43%	28.60%	49.38%	28.60%	28.60%	32.91%	28.60%	32.91%	28.60%	49.38%

Table 2 presents the first best (FB), second best (SB) and the competitive equilibrium allocations along with the taxes and the lump-sum transfers that implement them. The welfare gain is presented in terms of average consumption compensation of the agents.

Table 3: Steady state allocations and tax rates

	$RP(\tau^k, \tau^p)$		$RP(\tau^b, \tau^p)$		$RP(\eta, \tau^p)$		$RP(\tau^k, \tau^l)$	
	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.5$	$\omega = 0.5675$	$\omega = 0.5$	$\omega = 0.828$	$\omega = 0.5$
τ^k	12.30%	6.67%	—	—	—	—	12.30%	6.67%
τ^p	-4.67%	62.45%	0	71.57%	51.90%	73.90%	—	—
τ^b	—	—	0.74%	0.37%	—	—	—	—
η	—	—	—	—	25.44%	27.01%	—	—
τ^l	—	—	—	—	—	—	—	—
b	1.777	2.340	1.777	2.452	2.154	2.340	1.777	2.452
k	7.895	8.493	7.895	8.493	8.302	8.493	7.895	8.493
l	0.305	0.296	0.305	0.421	0.2995	0.296	0.305	0.296
F_k	0.045	0.045	0.045	0.045	0.043	0.042	0.045	0.042
F_l	2.064	2.142	2.064	2.142	2.12	2.142	2.064	2.142
<i>Welf.Gain</i>	28.60%	32.91%	28.60%	32.91%	28.60%	32.91%	28.60%	32.91%

Table 3 presents the equivalence among the different tax systems. The tax levels are presented for the case when $\omega = 0.5$ and 0.828, respectively. The former pareto weight gives rise to the second best allocations, whereas the latter gives rise to the first best allocations when the economy is at the steady state.

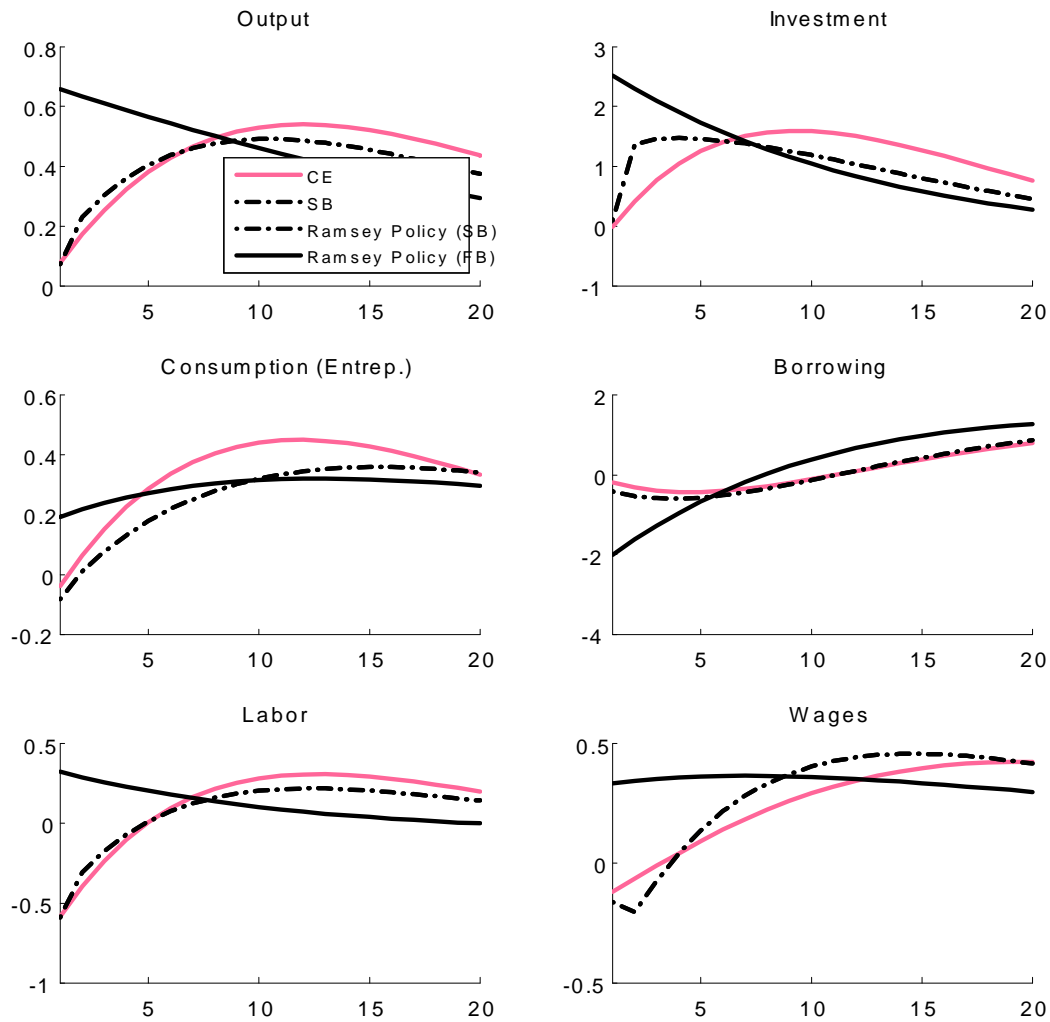


Figure 5: IRFs of selected variables in the competitive (CE) economy and the Ramsey plans replicating the first-best (FB) and the second-best (SB) outcomes to one standard deviation positive productivity shock. I compare the responses (% deviations from steady state) for Pareto-Negishi weight set to $\omega = 0.828$ (for households).

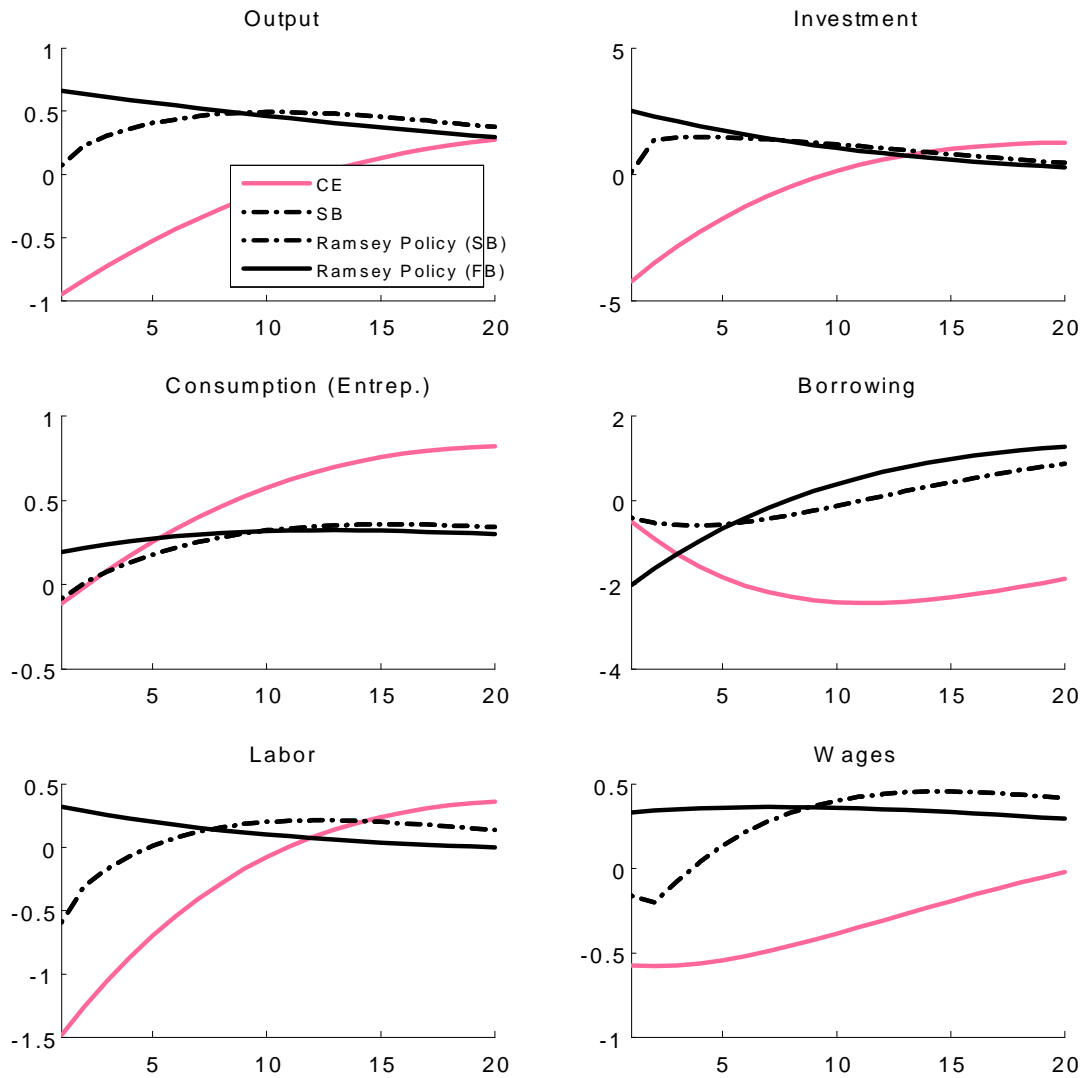


Figure 6: IRFs of selected variables in the competitive (CE) economy and the Ramsey plans replicating the first-best (FB) and the second-best (SB) outcomes to one standard deviation negative financial shock. I compare the responses (% deviations from steady state) for Pareto-Negishi weight set to $\omega = 0.828$ (for households).

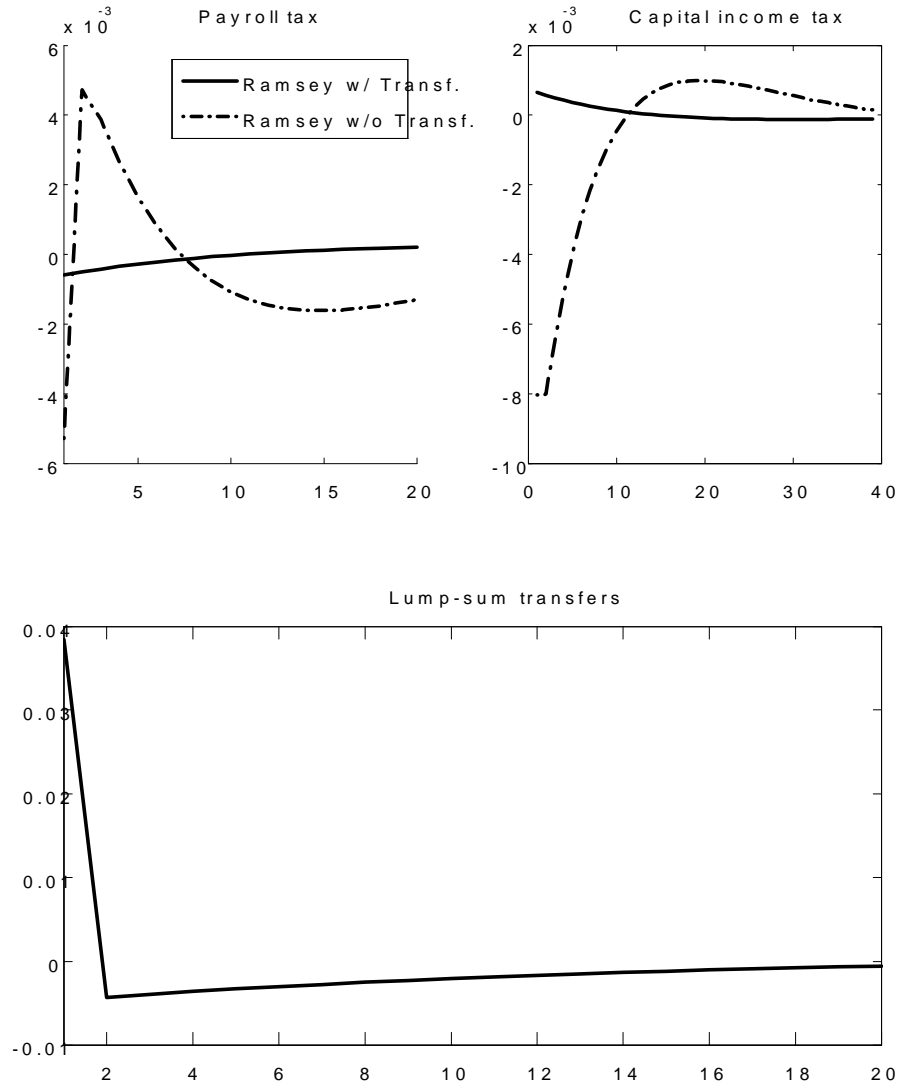


Figure 7: IRFs of the tax policy in response to one standard deviation positive productivity shock. I compare the responses of the policy instruments with and without lump-sum transfers for Pareto-Negishi weight set to $\omega = 0.828$ (for households).

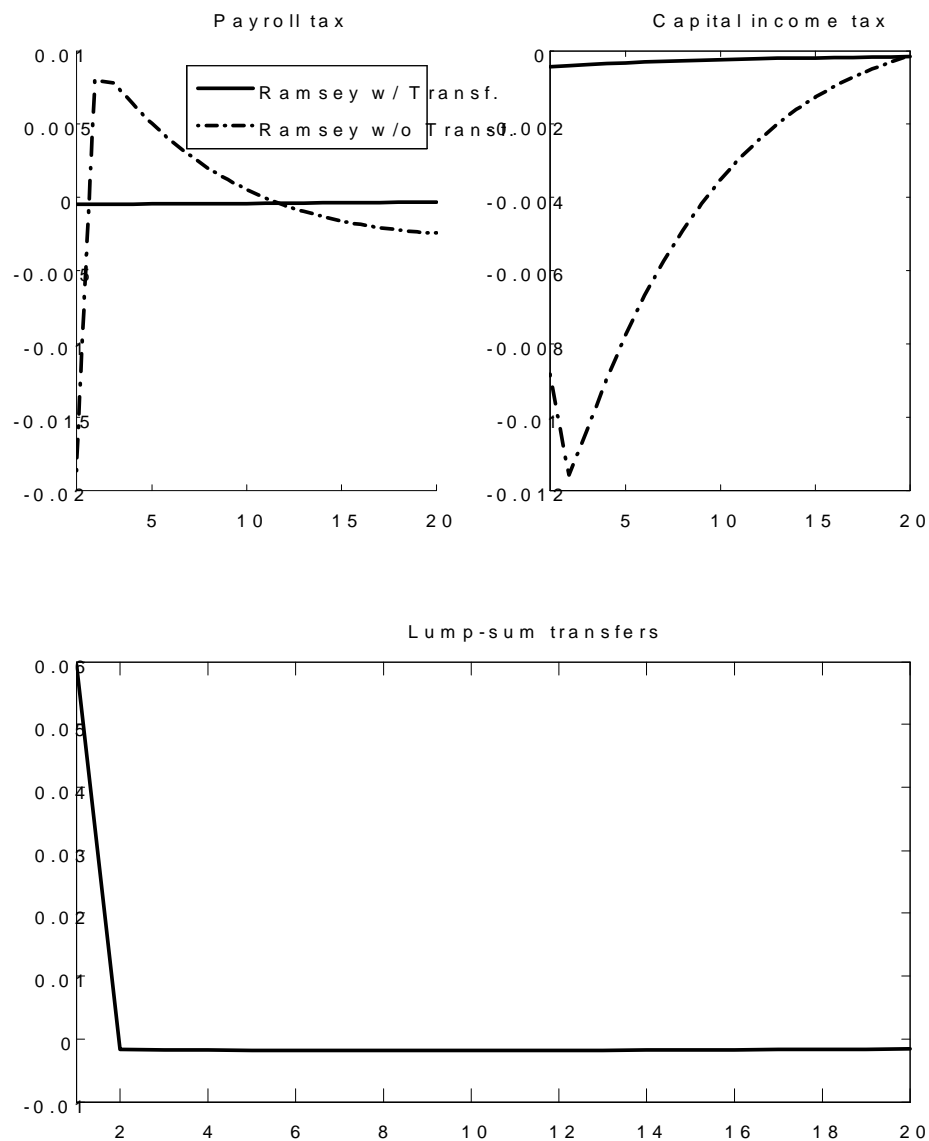


Figure 8: IRFs of the tax policy in response to one standard deviation negative financial shock. I compare the responses of the policy instruments with and without lump-sum transfers for Pareto-Negishi weight set to $\omega = 0.828$ (for households).

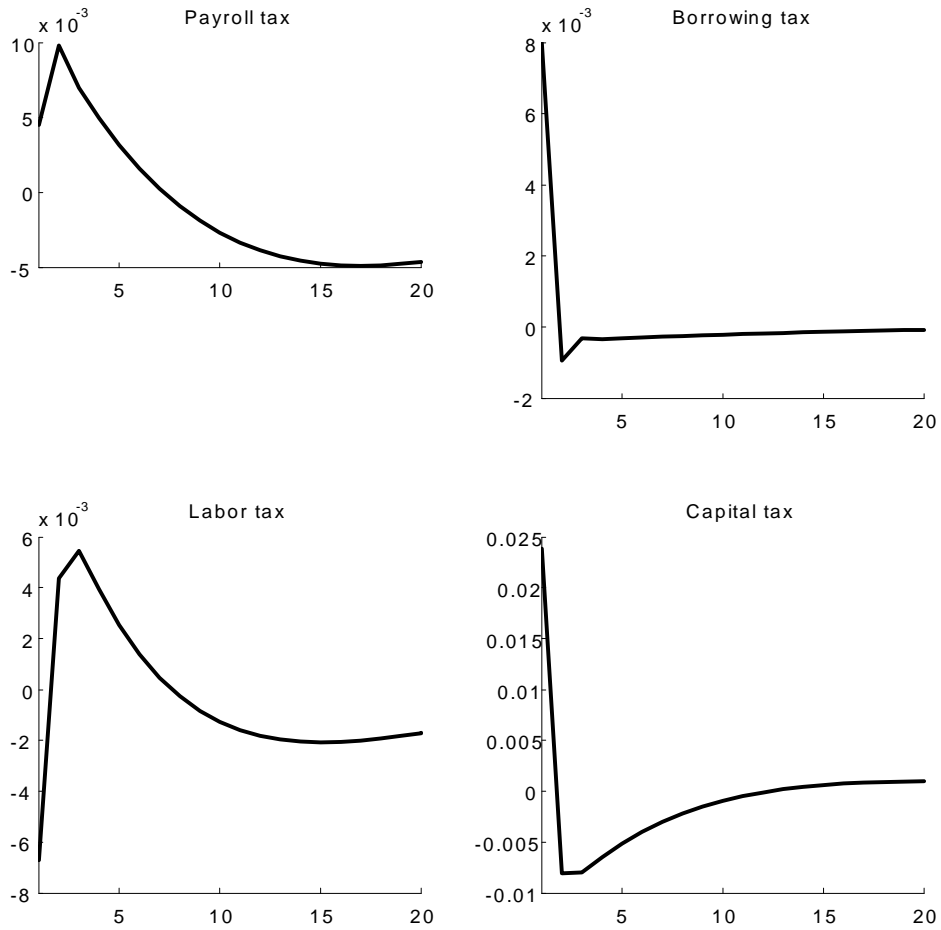


Figure 9: IRFs of the the different policy systems to one standard deviation positive productivity shock. I assume that the planner has no access to lump-sum transfers and that the Pareto-Negishi weight is $\omega = 0.828$ (for households).

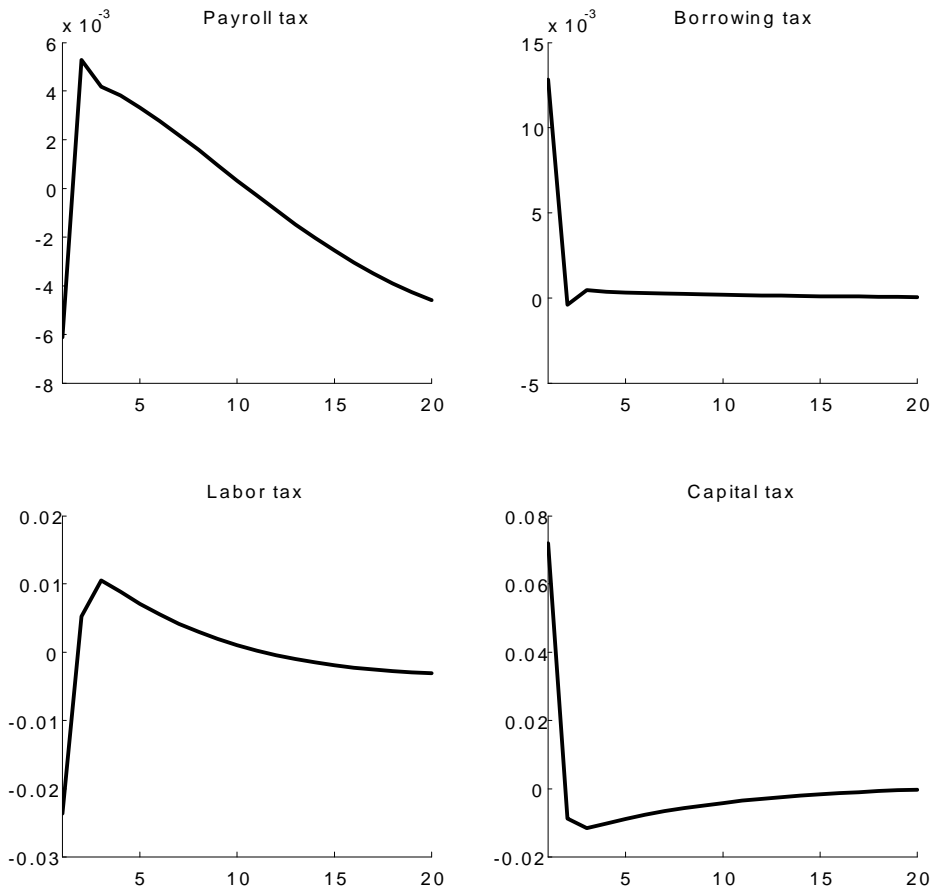


Figure 10: IRFs of the the different policy systems to one standard deviation negative financial shock. I assume that the planner has no access to lump-sum transfers and that the Pareto-Negishi weight is $\omega = 0.828$ (for households).

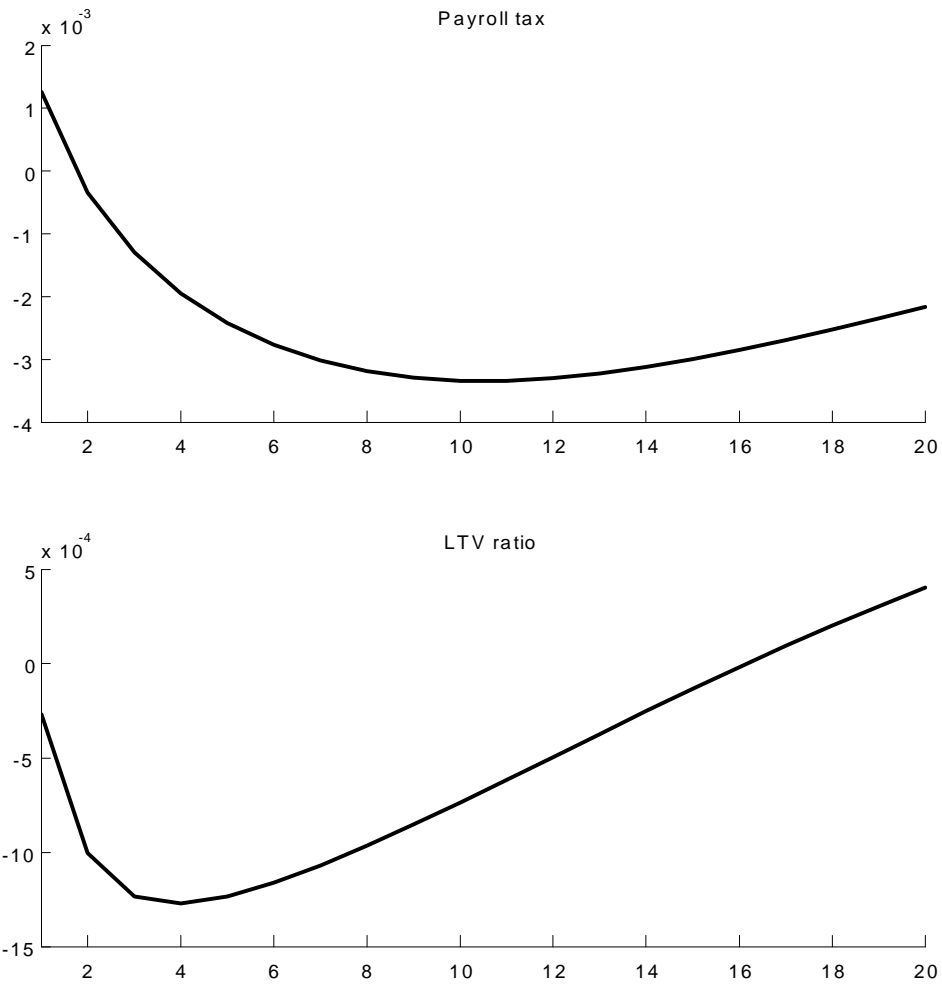


Figure 11: IRFs of the the policy system with LTV ratio limit and a payroll tax to one standard deviation positive productivity shock. I assume that the planner has no access to lump-sum transfers and that the Pareto-Negishi weight is set to $\omega = 0.828$ (for households).

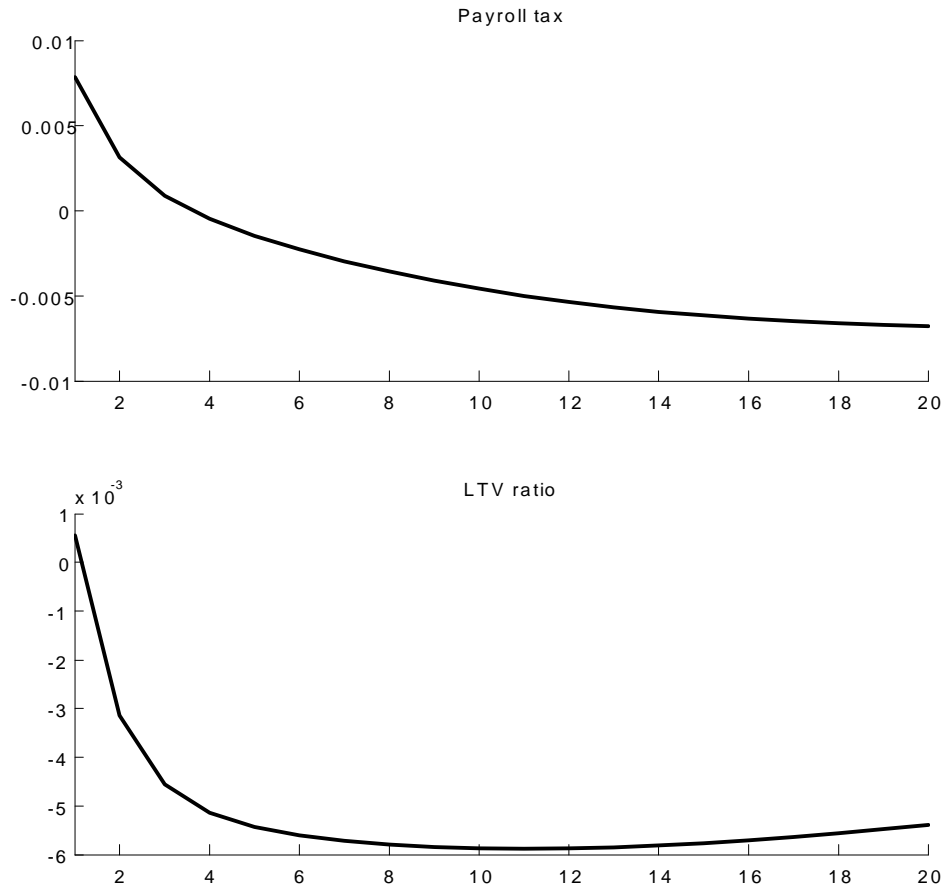


Figure 12: IRFs of the the policy system with LTV ratio limit and a payroll tax to one standard deviation negative financial shock. I assume that the planner has no access to lump-sum transfers and that the Pareto-Negishi weight is set to $\omega = 0.828$ (for households).

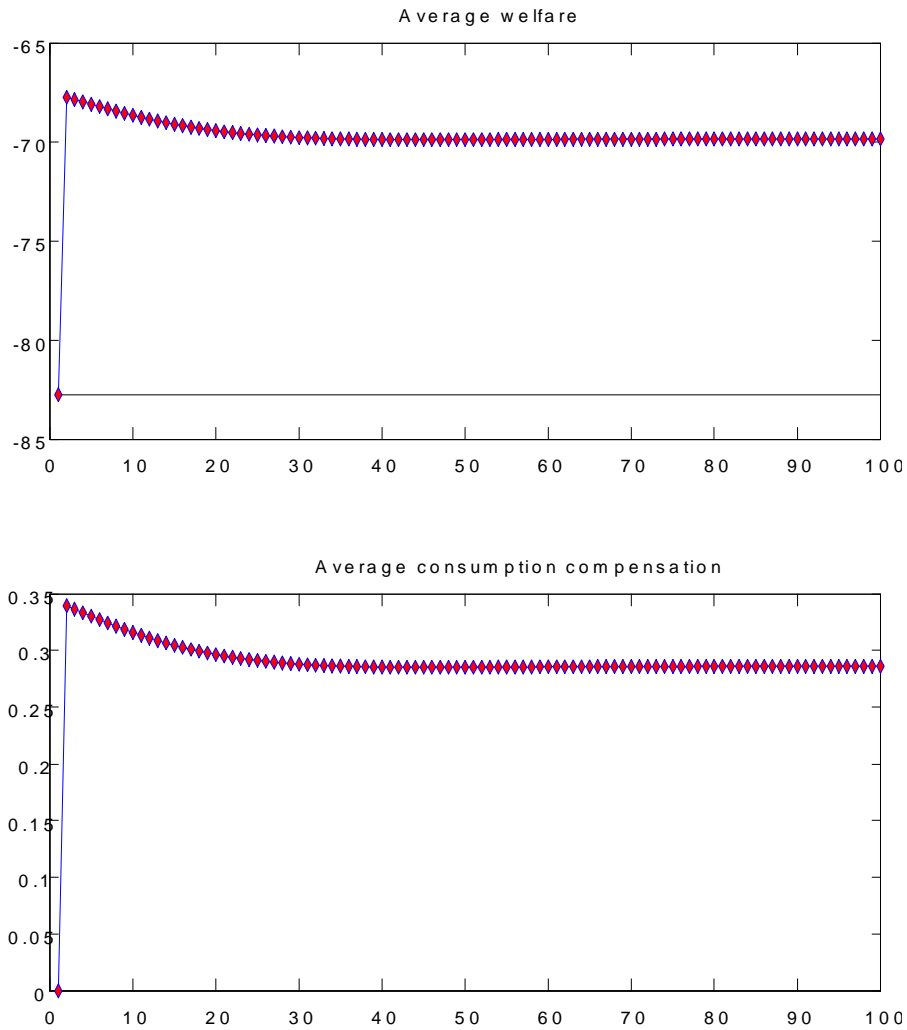


Figure 13: The figure presents the transition from the steady state without to the steady state with capital income and payroll taxes. The first graph (above) shows the transition for the (average) economy-wide welfare; the second graph shows the transition of the welfare in terms of (agents' average) compensation variation.