

Sentiment Shocks as Drivers of Business Cycles

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Abstract

This paper studies the role of sentiment shocks as a source of business cycle fluctuations. Considering a standard New Keynesian model of the business cycle, it introduces agents that update beliefs about the parameters of their forecasting models using newly observed data and exogenous sentiment shocks. The resulting learning model fits U.S. data better than its non-sentiment version and than its rational expectations counterpart. Sentiment is found to be an important driver of economic fluctuations, accounting for up to half of the forecast error variance of aggregate variables at business cycle frequencies. Furthermore, sentiment displays a common pattern for real GDP, investment and consumption growth, where a significant part of the sluggish recovery following a recession can be attributed to the persistent pessimistic views of agents. Sentiment also explains a substantial fraction of the high inflation experienced during the 70's and early 80's.

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'[...] a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most [...] of our decisions to do something positive [...] can only be taken as the result of animal spirits - a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.

[...] often falling back [...] on whim or sentiment or chance.' Keynes, 1936¹

1 Introduction

The rational expectations hypothesis (RE hypothesis) is the predominant approach for imputing expectations in macroeconomic applications. A number of recent studies show, however, that relatively small deviations from RE, such as the ones implied by adaptive learning (AL), can significantly improve the fit of business cycle models with the data (e.g. Slobodyan and Wouters (2012a, b), Milani (2007)). By endowing agents with subjective beliefs about their forecasting models, these studies arrive at alternative explanations for macroeconomic fluctuations, challenging the standard roles played by demand, technology and mark-up shocks and open a promising new field of research.

The standard approach in the AL literature consists of assuming that beliefs move only in response to economic outcomes or economic fundamentals, neglecting other important determinants of agents' expectations, such as purely subjective components of beliefs (see Akerlof and Shiller (2009)). This paper models these subjective views about the future as sentiment shocks, defined as shocks to the beliefs agents entertain about their forecasting models, and explores their empirical importance for business cycle fluctuations in the context of an estimated New Keynesian model à la Smets and Wouters (see Smets and Wouters (2007)) with adaptive learning.

After accounting for the different degrees of freedom, I show that the model with sentiment shocks fits the data significantly better than the model without sentiment shocks. In particular, the model with AL and sentiment shocks has an improved ability to match the observed data covariances. Both results also hold when compared to the model under RE. A forecast error variance decomposition exercise shows the large importance of sentiment shocks as drivers of economic fluctuations in the United States: they account for up to 50%

¹Keynes (1936), Chapter 12, The State of Long-term Expectation.

of all variability in aggregate variables at business cycle frequencies. The concrete role played by sentiment is explored in a simple counterfactual exercise in which historical shocks are fed back into the estimated model. This exercise suggests that sentiment shocks display a common pattern for real variables, amplifying their fluctuation over the cycle, acting as alternating waves of optimism and pessimism. In particular, this role appears to be stronger during recessions when agents persistent pessimistic views take time to revert, thereby slowing down the subsequent recovery. Furthermore, the counterfactual exercise suggests that sentiment shocks also played an important role in the historic evolution of price inflation. While these shocks accounted, on average, for about a third of inflation deviations from steady state over the pre-Volcker period, they appeared to have remained largely at bay during the 'Great Moderation', reinforcing the idea that inflation is largely expectations-driven.

There is a vast universe of potential forces that may drive expectations away from what past data or *market fundamentals* suggest. Keynes, for example, emphasised the role of speculation and of what he famously termed as 'animal spirits'. More recently, Akerlof and Shiller (2009) addressed the effects that emotional and psychological factors have on economic decision making. They revisit Keynes' idea of animal spirits and identify several different categories, including the state of confidence, money illusion and the role of stories for shaping behaviour, among others. Moreover, there are other factors that, perhaps today more than ever, reinforce or exacerbate these sentiments, such as media and their ability to shape and coordinate public opinion. As mentioned above, in this paper, and borrowing the term proposed by Milani (2013), these emotional, psychological or social mood drivers are broadly defined as sentiment and they are modelled as shocks shifting the beliefs agents entertain about the reduced form models they use to form expectations; thus, affecting agents' forecasts about future economic variables.

The model is estimated using Bayesian techniques and U.S. canonical data. However, given the nature of the model at hand standard Bayesian estimation approaches cannot be employed directly. Therefore, the paper employs to a new estimation strategy proposed in Arias and Rancoita (2013). The standard approach to estimate models with learning consists of abstracting from all uncertainty in the expectation formation mechanism model, i.e. sentiment shocks, thereby rendering the model de facto linear and, thus, allowing the likelihood to be computed with the standard Kalman Filter.² Arias and Rancoita (2013) propose an alternative estimation approach that relies on linearizing the belief updating

²By approximating all uncertain states in the non linear parts of the model by 'certain' estimates, beliefs behave as time-varying parameters.

equations.³ This allows making use of the Kalman Filter and can accommodate sentiment shocks. This paper is the first to introduce such an approach in a medium-size learning model. For completeness, I also compare the standard estimation approach with the new one devised in Arias and Rancoita (2013).

This paper is closely related to Milani (2013) and Slobodyan and Wouters (2012). Milani (2013) is concurrent work, attempting to answer a similar question as the one studied here. It proposes a two step estimation procedure, which consists of first identifying sentiment shocks in the U.S. business cycle, using the difference between survey data and what the model under AL suggests expectations should have been. It then incorporates these shocks as exogenous sentiment shocks into the estimation of the model. It finds evidence suggesting that these type of shocks play a significant role in the U.S. economy, in particular sentiment shocks related to investment decisions.

The present paper addresses the question in a different way. First, it relies on a single step estimation approach that estimates sentiment shocks and other model parameters jointly. In addition, the set of sentiment shocks considered here is significantly larger and includes sentiment related to all forward-looking variables that need to be forecasted in the model instead of restricting it to the ones related to investment, consumption and inflation.⁴ Another important difference is that, the expectation formation mechanism follows Slobodyan and Wouters (2012), where agents are assumed to use small forecasting models and update them using Bayes rule, instead of the reduced form models of the same form as the minimum state variable (MSV) solution and constant-gain learning algorithm adopted in Milani (2013). The reason for this choice is twofold: first the more simple small forecasting models simplify the estimation considerably; and second, they have a better empirical performance. As Slobodyan and Wouters (2012) show, in the context of a standard new Keynesian model, this learning scheme considerably improves the model's fit to the data relative to other more elaborate forms, including the MSV solution.

Another strand of related literature is represented by Cogley and Sargent (2008) and Suda (2013). Cogley and Sargent (2008) studies the role of particularly pessimistic initial priors in a Bayesian learning version of a simple asset pricing model. It suggests that an event like the Great Depression may alter beliefs, generating pessimistic initial conditions, in a way that can explain part of the equity premium puzzle in the postwar U.S. data.

³The validity of such an approximation rest largely on the validity of the original log-linearization around the model's steady state and some stability conditions for the learning dynamics, analogous to the ones found in the learning literature; see Arias and Rancoita (2013).

⁴The reason behind this restriction in Milani's paper is that the estimation makes use of data on expectations from the Survey of Professional Forecasters which is limited to those variables.

Suda (2013) builds on the same idea and studies the effects of one time 'shattered' beliefs, consequence of one time events such as the Great Depression. He does this in the context of a standard equilibrium business cycle model where agents learn about the probabilities characterizing productivity in the economy, via Bayesian methods. He finds that sufficiently large shocks to beliefs of agents can have a quantitatively important and persistent impact in the macroeconomy. Both works provide evidence of the importance of non-rational beliefs as drivers of macroeconomic dynamics and in particular of subjective interpretations of events as 'shifters' of agents beliefs. The paper at hand tries to build a framework where these shocks can be appropriately studied.⁵

The rest of the paper is structured as follows. Section 2 describes the core of the model and the expectation formation mechanism. Section 3 discusses the estimation methodology and the data. Section 4 presents and discusses the results. Section 5 concludes.

2 The Model

The paper explores the role of shocks to beliefs in business cycle fluctuations in the context of a standard New Keynesian model (see Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007)), featuring both nominal and real rigidities. The version adopted here follows Slobodyan and Wouters (2012), who replace the traditional rational expectations hypothesis with adaptive learning. This not only makes the information assumption on agents more realistic but, as they show, it improves the fit of the model to the data, as measured by the marginal data density and the implied models' posterior odds - indicators of the relative likelihood of the models to generate the observed data and used to compare models in a Bayesian framework. In addition, learning helps reducing the degree of dependency on highly autocorrelated exogenous stochastic structures and mechanical sources of endogenous persistence, which are generally accepted as drawbacks of the rational expectations literature (Milani (2012)). This is consequence of the endogenous propagation introduced by learning and that manifests in the gradual update of agents beliefs.

The model consists of three main sectors: Households, Firms and a Government. In the first sector, representative households seek to maximize their lifetime utility subject to a

⁵The literature on 'News shocks' , although more distantly, is also related to this project as it allows expectations to depend on information about future events, which is generally revealed with some uncertainty. News shocks could be interpreted as subjective views on future events which may or may not be realized. Similar considerations could be extended to the literature on sunspots in RE.

budget constraint, where the former may rise either from consumption (relative to a habit component) or from leisure (agents choose how much to work). Each period, they also decide how much to invest in capital, taking into account its adjustment costs, and bonds and choose the utilization rate of capital, depending on its return. Households provide their labor to a union which, then, sets wages subject to a Calvo pricing scheme with indexation. This is consequence of the monopolistic power the union creates by differentiating the labor provided by the households. In the second sector, firms are further divided into two sub-sectors: an intermediate goods sub-sector, where firms, choosing capital and labor, produce differentiated goods, creating a monopolistic-competition market where prices are set subject, also, to a Calvo scheme with indexation; and a final goods sub-sector, in which firms combine the intermediate goods into a final good that, in a perfectly competitive market, sell to consumers, investors and the government. Finally, the third sector is given by the monetary authority which, by means of a Taylor type rule, determines the short-term nominal interest rate as a function of inflation and output deviations from their respective targets.

The model is completed by the expectation formation mechanism, which will be introduced next, and the stochastic structure of the model. In particular, the latter can be divided in two to parts: first, the standard structural shocks used to match the data in these type of models and which remains the same as in Smets and Wouters (2007) and Slobodyan and Wouters (2012); and second, the shocks given by agents' sentiment about the future. In the paper the former are referred as standard shocks, while the latter as sentiment shocks. The model, then, comprises 7 different standard shocks use to match the 7 U.S. data series used in the estimation: a total factor productivity shock (TFP), a risk-premium shock, a government expenditure shock, an investment-specific technical change shock (IST), a monetary policy shock, a price mark-up shock and a wage mark-up shock. The first five shocks are modeled as AR(1) processes while the last two as i.i.d. shocks.⁶ In addition, government expenditure is further affected by the innovation of TFP, since in the estimation government expenditure also includes net exports, which can be affected by productivity movements. The model is briefly presented in its log-linearized form in Appendix I.

⁶Under RE both mark-up shocks are usually modeled as a persistent process, e.g. as ARMA(1,1). However, adaptive learning generates sufficient endogenous persistence to abstract from such a structure.

2.1 Expectations Formation

According to the model, agents need to forecast the future value of seven endogenous variables to take decisions: consumption, investment, hours worked, inflation, the price and the return of capital and real wages.⁷ To construct these expectations, they are assumed to use small reduced-form AR(2)-forecasting models. However, the parameters of this processes are unknown to them and need to be estimated. The assumption here is that agents entertain beliefs about those parameters in the form of some distribution and, as new information becomes available, they update this distribution using Bayes rule. Beliefs, then, encompass the parameters' distribution, how this distribution is updated each period, and how it relates to the data agents observe (i.e. through the specific AR(2) models. See equations (1) and (2))

This type of adaptive learning deviates from the traditional rational expectations assumption in three ways. First, the resulting probability measure used to forecast future variables does not need to coincide with the one implied by the model. Second, this probability measure, reflected in the parameters of the forecasting models, evolves over time inducing further dynamics. Third, the information set that agents use to form expectations is smaller than under RE. In particular, it is also smaller than the standard way adaptive learning is modelled. Under adaptive learning, agents are generally endowed with knowledge of the correct form of the rational expectations equilibrium of interest. A common rationale for this is given by the assumption that agents know the model in the same way as the researcher but do not know the values the parameters take, which prevents them from deriving the RE equilibrium.⁸ The reason for using small reduced-form forecasting models is twofold. On the one hand it considerably simplifies the estimation costs and on the other, it can be justified from an empirical perspective. Slobodyan and Wouters (2012), show that these type of small forecasting models significantly improve the fit of the model to the data and help produce impulse response functions in line with DSGE-VAR models.

The following state space model describes how agents perceive the law of motion of forward-looking variables and the way parameters evolve,

$$y_t^f = X_t \beta_t + u_t \tag{1}$$

$$\beta_t = (1 - \rho) \bar{\beta} + \rho \beta_{t-1} + v_t, \tag{2}$$

⁷See Appendix I.

⁸Following this logic, RE would further assume that agents also know the value of the parameters of the model. Therefore, they would know more than the researcher.

where y_t^f , is the vector containing the seven forward variables agents need to forecast each period and $X_t \in \mathbb{R}^{7 \times 21}$ is a matrix that contains for each forward looking variable its first two lags and a constant, i.e. the AR(2) process. Agents believe the parameters characterizing that model, $\beta_t \in \mathbb{R}^{21 \times 1}$, follow an autoregressive process around $\bar{\beta}$, where $\rho \leq 1$. Then, departing from Gaussian distributions for the initial states and assuming Gaussian errors, u_t and v_t , agents use observations of y_t^f to construct the distribution of β_t . Since the model is linear, this can be optimally done by means of the familiar Kalman Filter and belief evolution is fully characterized by the dynamics of their first two moments,

$$\beta_{t|t} = \beta_{t|t-1} + \overbrace{P_{t|t-1} X_{t-1}^T [\Sigma + X_{t-1} P_{t|t-1} X_{t-1}^T]^{-1}}{:=K_t} (y_t^f - X_{t-1} \beta_{t|t-1}) \quad (3)$$

$$P_{t|t} = P_{t|t-1} - K_t X_{t-1} P_{t|t-1} \quad (4)$$

where $\beta_{t|t-1} = (1 - \rho)\bar{\beta} + \rho\beta_{t-1|t-1}$ is the predicted mean and $P_{t|t-1} = \rho^2 P_{t-1|t-1} + V$ the predicted covariance matrix of the states. V is the covariance matrix of v_t and Σ the covariance matrix of u_t . Furthermore, K_t is the Kalman gain, which optimally determines how much past beliefs need to be adjusted in the direction of the forecast error, $y_t^f - X_{t-1}\beta_{t|t-1}$, by considering the uncertainty of the latter relative to the uncertainty of the prior. Equations (3) and (4) are then referred to as the beliefs updating equations.

Intuitively, each period t agents need to construct expectations about next periods forward looking variables, i.e. $E_t(y_{t+1}^f)$. For this, and even though y_t^f is assumed to be known at time t ,⁹ agents use information up to period $t - 1$ to update their beliefs about the distribution of the parameters of their forecasting models. Then, agents use the updated distributions, summarized by $\beta_{t|t-1}$, to generate the necessary expectations to take their economic decisions, $E_t(y_{t+1}^f) = X_t \beta_{t|t-1}$.¹⁰ This closes the model. Plugged into equations (8)-(21) the model becomes backward looking and it can be estimated.

The main innovative point of this paper is that agents may deviate from the way the researcher models how beliefs are updated, i.e. deviate from (3) and (4). At any given period, agents may condition their beliefs on subjective information that pushes them away from what the data suggests. In the spirit of Milani (2013), these drivers are defined as sentiment and encompass a wide range of factors, from psychological and social ones to plain speculation. In the benchmark model these deviations take a simple form, identically and

⁹This is the standard assumption of 'time- t ' dating, which avoids simultaneity between y_t^f and $\beta_{t|t}$. It is done for technical simplicity, see Evans and Honkapohja (2001).

¹⁰Remember, X_t contains information up to $t - 1$.

independently distributed shocks to the constant of the respective reduced form models. Equation (3) is then extended and written as,

$$\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1} X_{t-1}^T \left[\Sigma + X_{t-1} P_{t|t-1} X_{t-1}^T \right]^{-1} \left(y_t^f - X_{t-1} \beta_{t|t-1} \right) + \xi_{t+1}, \quad (5)$$

where ξ_{t+1} is vector of appropriate size containing the sentiment shocks of agents.

3 Estimation Methodology

Adaptive learning introduces non-linearities in an otherwise linear model. Not only the beliefs updating equations are non-linear (see (4) and (5)), but the function used by agents to form expectations becomes non-linear as well, $E_t \left(y_{t+1}^f \right) = X_t \beta_{t|t-1}$. This is because the parameters of the reduced-form models used to forecast forward variables, and that under RE were constants, are now dynamic states, and the researcher needs to keep track of their evolution. In principle, Bayesian estimation of such a model would have to rely on some non-linear filter, such as Particle Filters or Quadrature Filters (Arulampalam et al. (2002)). However, these types of filters suffer from the so-called *curse of dimensionality*. That means that the computational costs increase exponentially with the dimension of the model. With learning the computational problem is further exacerbated since the state space is augmented to include agents' beliefs. In the model at hand the computational cost of such an estimation approach becomes prohibitive.

The literature proposes a simple way of circumventing the problem. By abstracting from all uncertainty in the beliefs updating equations, the evolution of beliefs can be computed deterministically such that beliefs behave as time-varying parameters. This makes the model effectively linear in the states and its marginal likelihood can be computed with the simple Kalman Filter.¹¹ To do this, the method relies on approximating the distribution of the unobserved forward looking variables that enter the beliefs updating equations, y_t^f , X_{t-1} , by a unit mass distribution at their last estimated mean (which in turn is affected by this approximation).

Since the objective of this paper is to study sentiment shocks and these are a source of uncertainty in beliefs, abstracting from uncertainty in the latter is not a feasible strategy here. Hence, this paper adopts the method introduced in Arias and Raincoita (2013) for the

¹¹Examples of this approach can be found in Milani (2005, 2007, 2013) and Slobodyan and Wouters (2007, 2012).

estimation of DSGE models under AL. The method is based on the linearization around the steady state equilibrium under rational expectations of what, in any case, is a largely linear model. Then, the marginal likelihood of the resulting linear model can be easily computed by means of the Kalman Filter. The validity of such a linearization is a direct consequence of the accuracy of the log linearization done on the optimality conditions of the model in the first place (equations (8)-(21)).

The particular linearization point has the advantage that it renders the forecast error, $y_t^f - X_{t-1}\beta_{t|t-1}$, equal to zero. This implies that one can neglect the dynamics of the matrix estimating the second moments of the beliefs, P . Then, after linearization, the beliefs updating equation can be written as,

$$\beta_t = (1 - \rho)\bar{\beta} + \rho \left\{ \beta_{t-1} + M \left(y_{t-1}^f - \bar{X}\beta_{t-1} - X_{t-2}\bar{\beta} \right) \right\} + \xi_t, \quad (6)$$

where $M = P^*X^{T*} [\Sigma + X^*P^*X^{T*}]^{-1}$ ¹². Equation (6) is now the single equation describing the evolution of beliefs.

To estimate the model, seven US time series over a period ranging from the first quarter of 1965 to the fourth quarter of 2013 are used: real GDP, short-term nominal interest rate (Federal Funds rate), real consumption, real investment, hours worked, inflation and real wages. This gives rise to the following measurement equation,

$$\begin{pmatrix} dlGDP_t \\ FEDFUNDS_t \\ dlCons_t \\ dlINV_t \\ lHours_t \\ dlP_t \\ dlWage_t \end{pmatrix} \equiv O_t = \underbrace{\begin{pmatrix} \gamma \\ r \\ \gamma \\ \gamma \\ l \\ \pi \\ \gamma \end{pmatrix}}_{:=constant} + \begin{pmatrix} \Delta y_t \\ r_t \\ \Delta c_t \\ \Delta i_t \\ l_t \\ \pi_t \\ \Delta w_t \end{pmatrix}$$

which together with equations (8)-(21) and (6) complete the model.¹³

The whole model can be brought to state space form and succinctly written as,

¹² $\beta_{t|t-1} = (1 - \rho)\bar{\beta} + \rho\beta_{t-1|t-1}$ has been used and the subindices have been simplified, $t-1 | t-1 \equiv t-1$.

¹³ γ denotes the period trend growth rate of real GDP, consumption, investment and wages; π denotes the periods steady state inflation rate as l and r do the same for hours worked and the nominal interest rate respectively. dl stands for log first difference and l for log.

$$\begin{cases} O_t = & Z_t^{obs} \\ Z_t = \mu + & G \cdot Z_{t-1} + V \cdot \epsilon_t, \end{cases} \quad (7)$$

where $Z_t = [Y_t', \omega_t', Y_{t-1}^{f'}, Y_{t-2}^{f'}, \text{dobs}_t', \beta_t']$ is an appropriately stacked vector of $Y_t = [k_t, y_t, r_t, c_t, i_t, l_t, \pi_t, q_t, r_t^k, w_t]'$, $\omega_t = [\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^q, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w]$, observables and beliefs¹⁴.

The likelihood of the model is computed with the Kalman Filter and the posterior distributions of the parameters are generated by means of a Metropolis-Hastings algorithm. The priors for the parameters of the model are taken from Smets and Wouters (2007) and additionally include the priors for the standard deviations of the sentiment shocks (see Tables 3, 4 and 5 in Appendix II). Following Slobodyan and Wouters (2012), σ_0 , the parameter setting the proportion to $(X \Sigma^{-1} X)^{-1}$ of the initial covariance matrix of the belief coefficients around which the linearization is done it is set to 0.03. The remaining parameters are estimated. Initial beliefs for each parameter draw are set to the implied rational expectations value. The estimation starts with the search for the mode of the log-posterior distribution of the parameters, which combines the log-likelihood of the data conditional on the model and the parameters with the log-prior knowledge about the parameters.

Four different expectation formation mechanisms are considered throughout the paper. First, the model derived under rational expectations, RE, the predominant assumption in macroeconomics and hence a natural benchmark. This specification corresponds to the model in Smets and Wouters (2007) and is the only one that requires more persistent processes to model the mark-up shocks. As shown in Slobodyan and Wouters (2012), models under adaptive learning generate enough endogenous persistence so that mark-up shocks are correctly captured by i.i.d. processes. As previously discussed, the model with sentiment shocks is based on the one with small-forecasting reduced form models introduced in Slobodyan and Wouters (2012). To incorporate these type of shocks, the model's expectation formation mechanism is linearized. Therefore, to better understand the contribution and role of sentiment and disentangle them from the effects of the linearization relative to the non-linear adaptive learning scheme, two other specifications are estimated: a model with non-linear adaptive learning, AL, which coincides with the model in Slobodyan and Wouters (2012) and the linearized version of it, without sentiment shocks, LAL. Finally, the core of this paper, the model with sentiment shocks, which, is also linearized, LALwS. The only difference of the model under RE and under AL from their respective original versions is that they are estimated using larger and updated samples, extending until the last quarter of 2013.

¹⁴The state space form of the model is derived in the appendix.

4 Results

Sentiment shocks were introduced, in section 2.1, as i.i.d., however, after a first estimation, the resulting historical innovations presented some significant correlations, both between sentiment shocks and also between sentiment shocks and some *standard* shocks. To cope with this issue, and guided by the results in the first estimation, the stochastic structure is adjusted (see Appendix II). The new specification allows the sentiment shocks on consumption and hours worked to depend on the risk premium innovation in the same way as the exogenous spending process is allowed to depend on TFP. Similarly, sentiment on investment, price and return on capital are allowed to depend on the innovation of the IST shock and sentiment about wage inflation may depend on the innovation of the wage mark-up shock. Finally, the sentiment shock for price inflation is allowed to depend on the innovations of both the price and the wage mark-up shocks. Furthermore, to account for the correlation observed among some sentiment shocks, a feature that is in fact desired, since optimism and pessimism are likely to be contagious, a series of dependencies between the sentiment shocks are estimated. For all of these parameters, their priors are set to be a beta distribution, adjusted to the interval $[-1, 1]$ with mean 0 and standard deviation 0.45; for the parameters accounting for the dependency of price inflation sentiment on price mark-up and of wage inflation sentiment on wage mark-up, these priors are further restricted to the interval $[0, 1]$. This final version of the stochastic structure is set to be the baseline and the one used in the rest of the paper.

Tables 3, 4 and 5 in Appendix II report the posterior distributions statistics for all model specifications. Standard test were used to determine the identification of the different parameters estimates in the model, including Geweke (1992) convergence tests, the plotting of the Metropolis-Hastings draws, the testing for different means in sub-samples of the draws yielded by the MH algorithm, and the plotting of the likelihood as a function of each parameter. Results indicate that all parameters are identified using 400,000 draws and burning the first 10%.

Structural parameters remain mostly unchanged and are robust to the expectations formation mechanism assumed. Although, the mean of the posterior distributions do change for some parameters, there is a strong overlapping between their respective 5th - 95th quantiles intervals.

There are, however, some noteworthy exceptions. First, the inverse of the intertemporal elasticity of substitution (IES) for consumption, σ_c , when sentiment shocks are present, it is estimated to be much lower (0.43). As the willingness to shift consumption across time

increases with the higher IES, so does the effect of the interest rate on consumption and in turn on the whole economy. In addition, increases in the hours worked now lead to positive and larger increases in current consumption, opposite to the small and negative effect that it has in the models without sentiment (see equation (9)).

Second, estimating the model with sentiment shocks delivers an IST shock which is less volatile and persistent (see σ_q and ρ_q in Table 1). The smaller size of the IST shocks seem to be compensated by two effects of the higher IES that help match the data: First, the overall impact of the IST shock on investment becomes larger. Second, the lower σ_c , increases the impact of the sentiment shock about investment on the economy (in particular on real output, consumption and investment).

All together, sentiment shocks are estimated to be small relative to the other standard shocks present. Notwithstanding, they enter through a quite persistent belief process with a mean autoregression coefficient estimate of 0.97 (ρ in last row of Table 3). Their relevance will become apparent once the forecast error variance decomposition is considered later on.

A last point to notice regards the posterior estimates of the non-linear and linear versions of the adaptive learning model without sentiment shocks, they present two main differences. First, under linear adaptive learning, the risk-premium shock is estimated to be less volatile and considerably less persistent. This goes in line with the featured stronger habits in consumption that make the latter more persistent and less responsive to changes in the interest rate. In addition, the value of the capital stock becomes more sensitive to the risk-premium shock, partially compensating for its smaller variability. Second, the elasticity of the capital adjustment cost function, φ , returns to the levels of rational expectations when the adaptive learning scheme is linearized, reducing the effect of the current value of the capital stock on current investment and increasing the sensitivity of the capital accumulation dynamics to how efficient investment is (captured by the IST shock).

4.1 Marginal Data Density

One important dimension to evaluate when comparing models is the degree in which they are able to explain the data. Comparing the marginal data density of different models is the most accepted way of comparing two different models as shown by the ample literature on the matter (e.g. Fernandez-Villaverde and Rubio-Ramirez (2004), Milani (2007), Slobodyan and Wouters (2012)). This indicator is an average over the parameter space of the value of the likelihood of the data conditional on the model, and that accounts for the different degrees

of freedom of the different models. In particular, it can be used to compute two different models' posterior odds, which indicate how more likely a model is to generate the observed data, relative to another. Alternatively, the marginal data density can be interpreted as reflecting the model's out of sample prediction performance. Table 1 shows the marginal data density, expressed in log-points, estimated for the four types of expectation formation mechanisms that are considered in the paper.¹⁵

RE	AL	LAL	LALwS
-1134.9	-1106.7	-1117.4	-1104.5

As the fourth column of the table shows, the model with adaptive learning and sentiments shocks fits the data better than all other specifications. Furthermore, confirming the results in Slobodyan and Wouters (2012) and Milani (2007), adaptive learning, both in its linear and non-linear form, improves the model's fit to the data with respect to its rational expectations counterpart. In terms of the models' posterior odds, the linearized adaptive learning model, LAL, is 17.5 log-points more likely to produce the observed data than the RE model; which in posterior probabilities terms means that the RE model is assigned zero probability relative to any of the learning models (since the other learning specifications present even larger marginal data densities).¹⁶ Moreover, the model with sentiment shocks, LALwS, presents a significant improvement relative to the non-linear and linear adaptive learning versions without sentiment shocks. The evidence is again substantial in favour of sentiment shocks, as even when compared to the non-linear version of adaptive learning without sentiment, AL, the model LALwS is about 9 times more likely to produce the observed data. This is an important results as it provides strong evidence suggesting that sentiment shocks are an important feature of economic expectations. Interestingly, comparing the AL and the LAL

¹⁵The marginal likelihood of the data, for this and most of interesting models, is approximated by the weighted harmonic mean of the posterior likelihoods generated with the Metropolis-Hastings algorithm. Weights are given by a truncated multivariate normal evaluated at the corresponding de-measured parameter draw following Geweke (1998). The marginal data density is defined as

$$\int_{\Theta} \mathcal{L}(y^T | \theta, M) \pi(\theta) d\theta$$

, where M denotes a particular model, $\theta \in \Theta$ the parameters of the model, y^T the data, $\pi(\theta)$ the prior of the parameters and $\mathcal{L}(y^T | \theta, M)$ the likelihood of the data conditional on the parameters and the model.

¹⁶This is true if one departs, as it is done here, from an agnostic point of view in which both models are given the same prior probabilities of being the true one, i.e. 0.5.

models shows how the linearizing strategy seems to reduce the ability of the model to fit the data - relative to the non linear adaptive learning strategy.

In spite of the apparent cost of linearizing the expectation formation mechanism relative to the costs of abstracting from uncertainty in beliefs when sentiment is not present, the linearization strategy significantly outperforms the non-linear adaptive learning model in matching the variance in the observed variables - it also outperforms the RE model in this respect.¹⁷ The non-linear version tends to generate too much variance, in particular, for hours worked, real investment growth and the interest rate. Furthermore, the fact that it is a linear model allows for a correct (conditional on the linearization) variance decomposition, an exercise that is presented next.

4.2 Forecast Error Variance Decomposition

To better understand the main drivers of economic fluctuations, Figures 1 and 2 present the forecast error variance decomposition for real GDP and investment growth; and for the price inflation and the Federal Funds rate respectively (the decomposition of the other observed variables is not shown, but briefly described). Several horizons are considered, ranging from 1 quarter to 25 years, which is taken as the unconditional forecast error and denoted with ∞ . It is worth mentioning, that this variance decomposition can not be defined for the non-linear adaptive learning model, and is only possible because of the linearization strategy used to estimate the learning models with sentiment.

Again results show the importance of sentiment shocks. In the shorter horizons, the three model specifications present a roughly similar role for the different shocks. Sentiment shocks, taken together, account for about 10 percent of all variations, and even though this is a significant amount, the usual main sources observed without sentiment are in place. Adaptive learning does not seem to introduce important changes relative to rational expectations in the short run either. The two main differences are given by the change in the persistence's perception of the mark-up shocks, that results in the price mark-up shock becoming the only driver of inflation variations in the short run and the expanded role of the risk-premium shock relative to the IST shock that becomes even more important for real GDP, consumption and investment.

¹⁷Not reported.

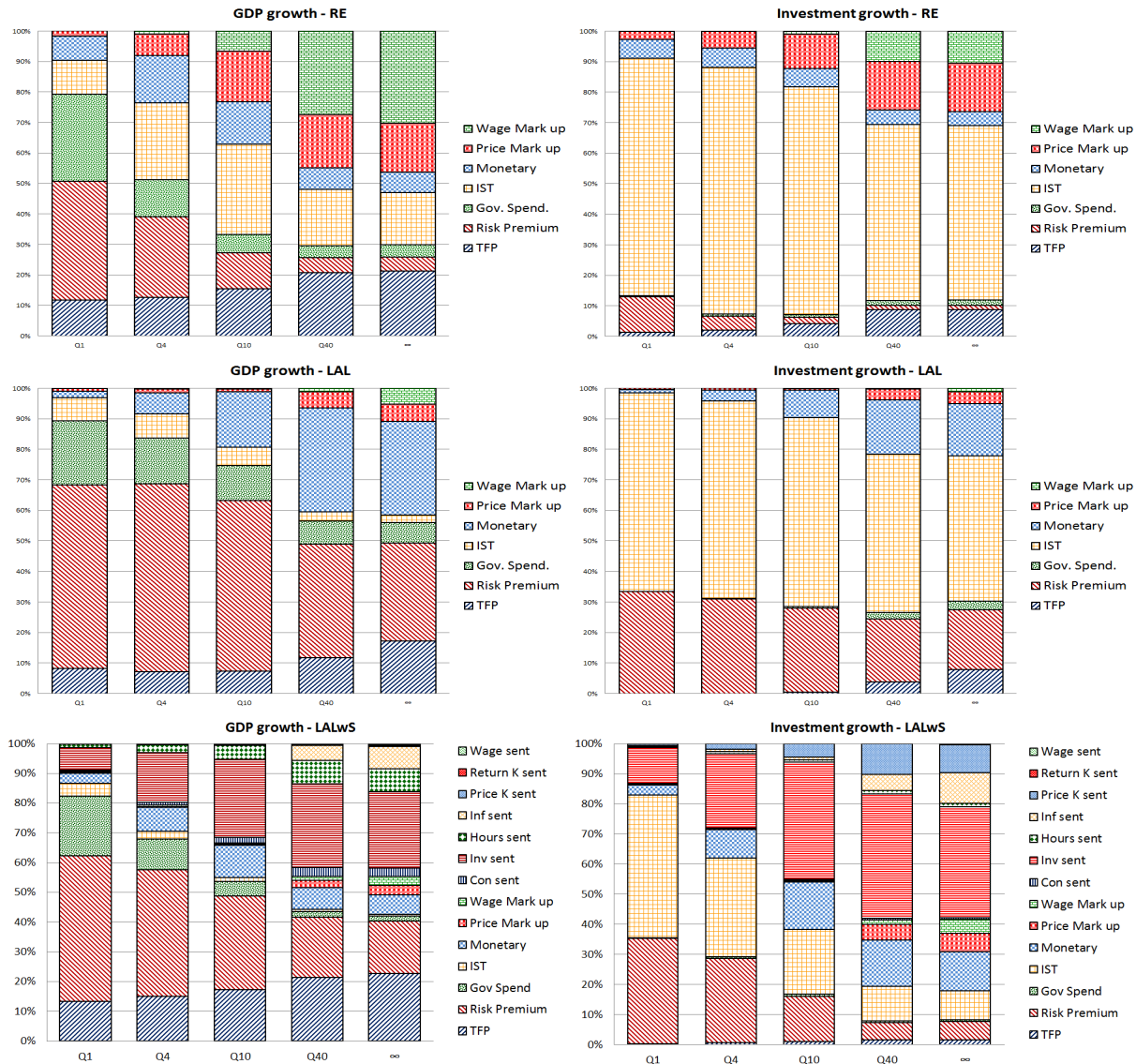


Figure 1: Forecast Error Variance Decomposition

In the medium and long run the picture changes. The contribution of sentiment shocks considerably expands, showing their importance as an explanation for economic dynamics. In the longer run, except for wage inflation, where they account for about a fifth of its total variation (still a significant fraction), the role of sentiment shocks becomes largest and accounts for about half of all variations. This corresponds, in particular, to the long-lived effect that they have through the beliefs updating mechanism which is a highly persistent process. In line with the findings of Milani (2013), sentiment shocks related to investment appear to be particularly important for real variables. This shock is the main source of

variation of real investment in the medium and long run, explaining more than 30 percent of it. In addition, it accounts for approximately a quarter of the variation in GDP where the risk-premium and TFP shocks still play a relevant role.

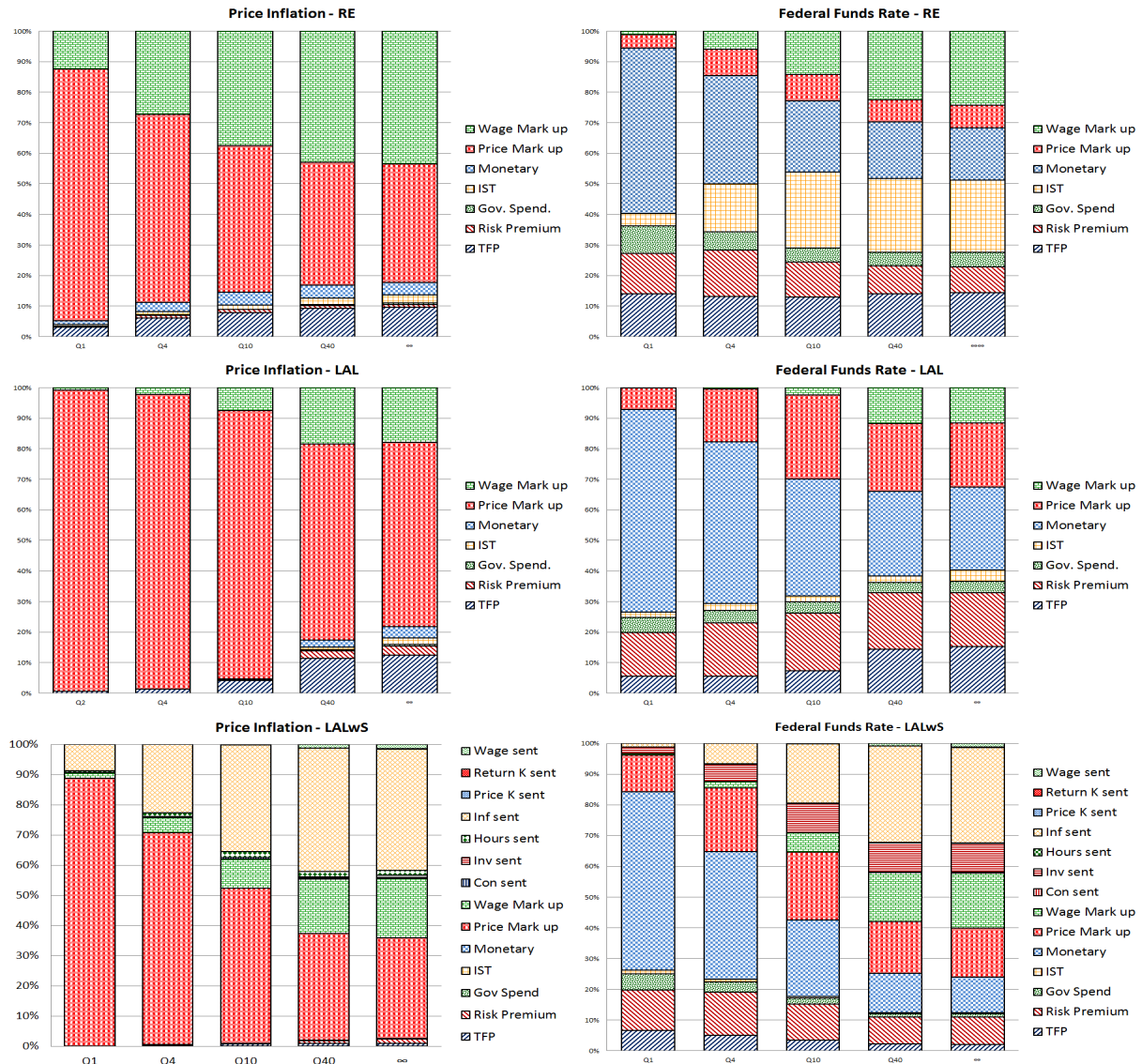


Figure 2: Forecast Error Variance Decomposition

Consumption, which in the long run under rational expectations was largely explained by wage mark-up shocks, under learning is mostly explained by the risk-premium and monetary shocks. Once sentiment is introduced, this decomposition changes further, giving room to the above mentioned investment sentiment shock and to hours sentiment shock, two shocks

that directly affect the intertemporal consumption Euler equation. Finally, the sentiment shock related to price inflation turns into the major source of variation in inflation in the long run, followed by the standard price and wage mark-up shocks, which continue to play an important part.

Wage inflation remains largely determined by the wage mark-up shock, which under adaptive learning becomes even more relevant than under rational expectations.

4.3 Historical Variance Decompositions

Table 2 presents the historical variance decomposition obtained from the model with sentiment shocks (Tables 6 and 7 present the corresponding results for the RE and LAL versions respectively - AL is a non-linear model and thus cannot be used to linearly decompose the variance in a comparable fashion).

Table 2: LAL with Sent. Shocks - variance decomposition in %

	Δy	r	Δc	Δi	l	π_p	π_w
Structural Shocks:	86.5	61.5	83.4	75.7	45.4	58.7	96.1
TFP	12.0	2.1	6.2	0.5	4.1	1.0	0.3
Risk Premium	40.8	8.1	66.6	27.4	24.2	1.2	1.7
Gov. Exp.	24.7	1.1	5.3	0.4	3.4	0.1	0.1
IST	4.3	0.3	1.4	38.1	1.1	0.1	0.0
Monetary	4.7	12.7	5.0	7.6	8.8	0.1	0.2
Price markup	0.9	15.4	1.0	1.7	2.4	33.8	6.6
Wage markup	0.6	18.2	0.3	0.7	4.5	20.4	89.2
Sentiment Shocks:	13.5	38.5	16.6	24.3	54.6	41.3	3.9
Consumption	0.7	0.2	3.0	0.2	5.4	0.4	0.2
Investment	9.7	9.4	7.4	19.3	28.5	0.6	0.6
Hours Worked	1.7	0.1	3.7	0.4	12.4	1.8	0.2
Inflation	0.9	28.1	0.9	2.1	7.3	37.2	0.1
Price of Capital	0.1	0.1	0.4	2.1	0.5	0.1	0.0
Return of Capital	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wage	0.0	1.2	0.1	0.0	0.8	1.5	1.9

It shows that sentiment accounted for a substantial fraction of the overall observed variance in GDP, Consumption and Investment growth, explaining from 13.5 to 24.3 percent, and in the Fed Funds rate, hours worked and price inflation, where the contribution is even

higher, ranging from 38.5 up to 54.6 percent. Agents' sentiment about investment and inflation seems to play the larger roles. Specifically, agents' sentiment about price inflation accounts for about a third of the variance in inflation and about a quarter of the variance in the interest rate, while sentiment about investment is responsible for approximately a fifth of the variance in real investment growth and more than a quarter of the one in hours worked. On the other hand, sentiment related to consumption and wage inflation play a smaller role, while sentiment about the price and the return of capital are almost negligible. When sentiment is included in the model, the contribution of the risk-premium shock becomes lower in line with its overall estimated lower impact on the economy. A similar case is presented for the IST and monetary shocks, which are estimated to have smaller innovations and persistence. Most of this lower variability is then taken over by sentiment shocks.

The remaining differences of the learning model with sentiment from its RE counterpart are largely inherited from the learning dynamics. Therefore, they are studied in the context of the learning model without sentiment.

First, a similar result as in Slobodyan and Wouters (2012) is found when comparing the contributions of the price and wage markup shocks to inflation, between the RE and the LAL versions. Under learning, both shocks are estimated as iid, consequently, agents do not distinguish between their persistence, yielding a relatively larger role for the price mark-up shock, which under RE was the least persistent one. The adaptive learning dynamics further affect two other shocks and their contributions in terms of the variance generated. The risk-premium shock and the IST shock are estimated to be less volatile and less persistent respectively. However, a higher estimate for the habits component increases the effect of the risk-premium shock on real investment and on the price of the capital stock compensating for the smaller shocks, which explains why the risk-premium shock becomes more important under learning. Finally, the smaller contribution of the monetary shock also reflects the larger role played by habits under learning, as they make consumption less sensitive to changes in the interest rate in the corresponding Euler equation.

4.4 Counterfactual Exercise

After having established the large importance that sentiment shocks play at business cycle frequencies, this section presents a simple counterfactual exercise aiming at studying the concrete role of sentiment shocks over the cycle.

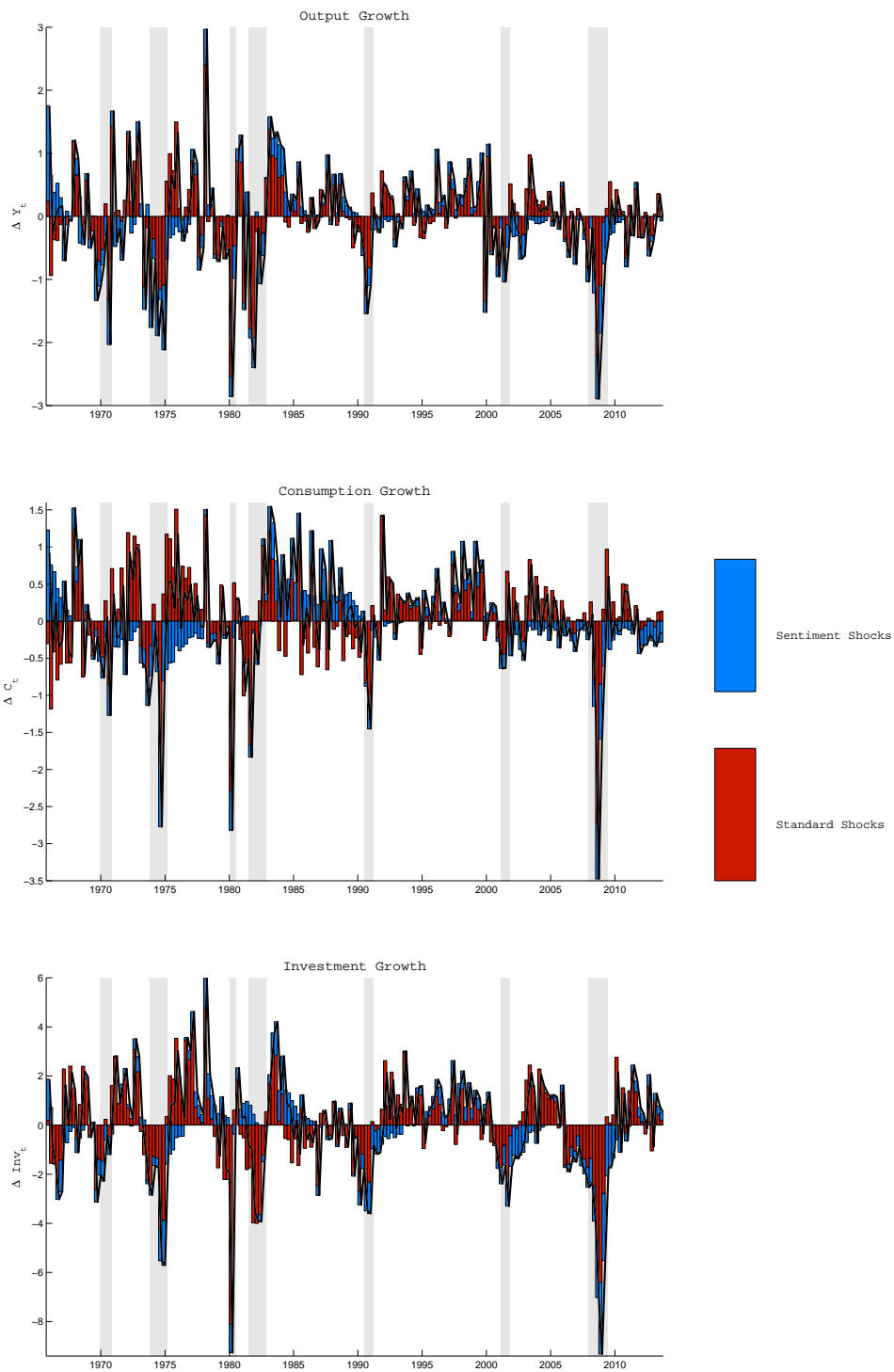


Figure 3: Counterfactual, shock decomposition. Real variables.

It describes the US economy in the hypothetical scenario in which there were no sentiment

shocks. How would GDP growth, investment growth, consumption growth and price inflation have been if agents would have only used economic fundamentals to form beliefs? Figures 3 and 4 plot the evolution of those variables under the virtual scenario. The red bars depict the evolution of the variables when sentiment shocks are turned off; the blue bars show their quarterly contribution, which is linearly added to yield the black line, which depicts the actually observed path for the variables.

This exercise shows the important role played by sentiment in the determination of economic fluctuations, which, if not present, would have yielded quite different economic dynamics. This role presents a distinctive and common pattern for the three observed real variables in the model. During recessions sentiment shocks reinforce the downturn by further driving the growth rates downwards, acting as a pessimistic wave. This situation, however, is not necessarily immediately reverted and can endure for several years slowing the recovery down - even though economic fundamentals may have already recovered. Except for the ones experienced during the 80's, this seems to have been the case for almost all recessions since the seventies, where the pessimistic views of agents played an important part in avoiding a quick exit. This observation is in line with the persistent effect of sentiment on beliefs, suggesting that it takes time before agents perceptions about the future are reverted into a neutral or optimistic state. During the last two recessions these pessimistic views have been particularly strong. Sentiment remained pessimistic long after the recession was over, particularly for consumption. This contrasts with the situation experienced during the 90's and the 80's. After the recession at beginning of the decade sentiment bounced back relatively quickly and became positive until the next recession.

Figure 4 shows this counterfactual exercise for the price inflation rate. The results show an interesting explanation for the evolution of price inflation and a dependency on the economic cycles. According to the counterfactual scenario, a significantly large fraction of the high inflation experienced during the seventies and beginning of the eighties, the 'Great Inflation', was due to subjective beliefs of agents. Pessimistic views in the form of sentiment, which this time translate into positive contributions, account on average for 37% of the quarterly price inflation deviations from steady state over that period. A point that underscores the idea that inflation is mainly driven by expectations.

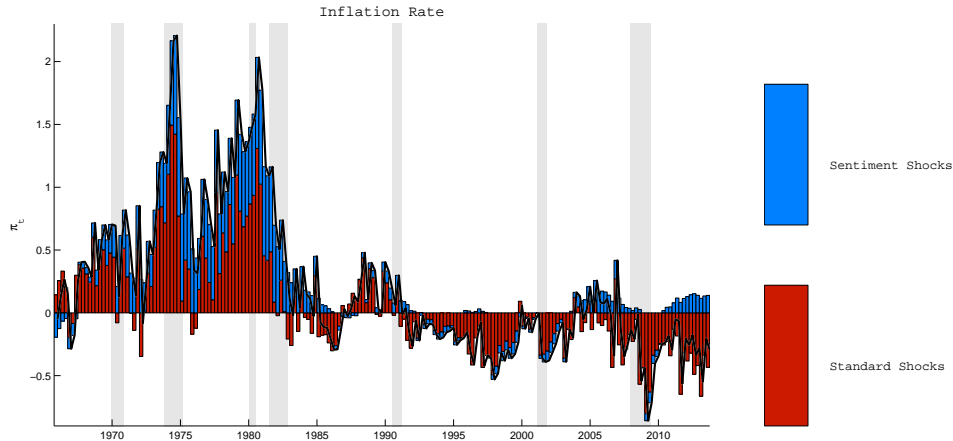


Figure 4: Counterfactual, shock decomposition. Inflation

This period, which commenced its decline with the appointment of Paul Volcker as chairman of the Federal Reserve Board, was then followed by a period of relatively low and stable inflation. The counterfactual experiment points out that one important difference between both periods was that during the 'Great Moderation' sentiment affecting price inflation was also relatively small. Hence, the monetary authority was able to transmit confidence to the agents regarding the evolution of prices in the economy.

5 Concluding remarks

This paper studies the role of shocks to beliefs as source of business cycle fluctuations. In the context of a new Keynesian Model with adaptive learning and applying a novel estimation methodology it presents strong evidence suggesting that sentiment shocks are important drivers of economic fluctuations and therefore, that they need to be considered when studying DSGE models. The relevance of this result is strengthened by the ability of the learning model to fit the data considerably better when sentiment is present than otherwise - and, in turn, better than when rational expectations are assumed. Agents' subjective views used to form beliefs challenge the standard sources of macroeconomic dynamics. In particular, sentiment is responsible for a substantial portion of the medium and long run variability in real GDP, investment and consumption growth as well as in inflation, hours worked and the nominal interest rate, accounting for up to almost half of it. Coinciding with the results in Milani (2013), investment is the most important sentiment shock driving the real variables in the model, which brings Keynes' idea of animal spirits to mind. In a straightforward

counterfactual exercise, a particular role of beliefs shocks over the cycle is identified for real GDP, investment and consumption growth. Evidence suggests that agents tend to become pessimistic during and well after a recession, slowing the subsequent recovery down. Also, sentiment shocks are found to play an important role in the historic evolution of price inflation: a role that seems to have changed after the high inflation period of the 70's and beginning of the 80's. The large impact of sentiment during that period, which accounted for more than a third of the quarterly inflation deviations from steady state on average over the period, has reduced dramatically in line with the lower and more stable inflation observed during the 'Great Moderation'.

Appendix I

Model

The model is briefly presented in its log-linearized form around the stationary steady state and consists of 14 endogenous variables in the same number of equations and 7 exogenous shocks.¹⁸

The economy's aggregate resource constraint captures how output (y_t) is allocated either to consumption (c_t), investment (i_t), the cost of adjusting the utilization level of capital (u_t) or to the exogenous government spending (ε_t^g):^{19,20}

$$y_t = \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r_*^k k_*}{y_*} u_t + \varepsilon_t^g, \quad (8)$$

where $\frac{c_*}{y_*}$, $\frac{i_*}{y_*}$, $\frac{r_*^k k_*}{y_*}$ denote the steady state shares of consumption, investment, and cost of changing capital utilization relative to output, respectively.²¹

A typical consumption Euler equation,

$$c_t = (1 - c_1)c_{t-1} + c_1 E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1}) + \varepsilon_t^b, \quad (9)$$

describes current consumption's (c_t) dependence on past and expected future consumption, on expected hours worked growth (l_t), and on the real interest rate. ε_t^b stands for the exogenous process followed by the risk premium. The parameters are given by $c_1 = \frac{1}{1+\eta/\gamma}$, $c_2 = \frac{c_1(\sigma_c-1)w_*L_*/C_*}{\sigma_c}$, and $c_3 = \frac{1-\eta/\gamma}{\sigma_c}$, where η is the habit formation parameter and σ_c denotes the inverse of the intertemporal elasticity of substitution.

The investment Euler equation,

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^q, \quad (10)$$

with parameters $i_1 = (1 + \beta\gamma^{1-\sigma_c})^{-1}$ and $i_2 = \frac{i_1}{\gamma^2\varphi}$ characterizes the dependence of current investment (i_t) on past and next periods expected investment and on the real value of the capital stock, (q_t). It also depends on an investment-specific technological (IST) change

¹⁸Later on, the exogenous stochastic structure will be augmented to include sentiment shocks.

¹⁹For a detailed step by step derivation see the appendix contained in Smets and Wouters (2007). The description here largely follows Smets and Wouters (2007) and Milani (2013).

²⁰Includes net exports.

²¹All lower case variables represent log-deviations from their respective steady state value unless stated otherwise.

shock, ε^a . As usual, β denotes the discount factor and φ represents the adjustment costs in investment.

The evolution of the value of capital is given by,

$$q_t = (1 - q_1)E_t q_{t+1} + q_1 E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) + q_2 \varepsilon_t^b, \quad (11)$$

where $q_1 = \frac{r_*^k}{r_*^k + (1 - \delta)}$, $q_2 = \frac{\sigma_c(1 + \eta/\gamma)}{1 - \eta/\gamma}$, δ is the depreciation rate, and r_*^k the steady-state rental rate of capital. It is a function of its expected future value, of the expected rental rate of capital, and of the return on assets held by households.

Aggregate supply is given by a Cobb-Douglas production function,

$$y_t = \phi_p(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a). \quad (12)$$

Output is produced using capital services k_t^s and labour l_t as inputs, with shares determined by α . ε^a is total factor productivity (TFP), while ϕ_p reflects the existence of fixed costs in production and corresponds to the price mark up in steady state.

Capital services, in turn, are a fraction of the capital stock in the previous period (capital is assumed to need one quarter to become operational), determined by the degree of capital utilization u_t .

$$k_t^s = k_{t-1} + u_t. \quad (13)$$

Moreover, the degree of capital utilization is a positive function of the rental rate of capital,

$$u_t = \frac{1 - \psi}{\psi} r_t^k, \quad (14)$$

where ψ is the elasticity of the capital utilization cost function.

Capital accumulation dynamics are given by,

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^q, \quad (15)$$

where $k_1 = \frac{1 - \delta}{\gamma}$ and $k_2 = (1 - k_1)(1 + \beta\gamma^{1 - \sigma_c})\gamma^2\varphi$. The capital stock net of depreciation increases with investment but depends on how efficient those investments are, ε^q .

Price mark up is defined as the difference between the marginal product of labour and wages (w),

$$\mu_t^p = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t \quad (16)$$

While inflation is determined by the following New Keynesian Phillips curve,

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \quad (17)$$

where $\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c\iota_p}}$, $\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c\iota_p}}$ and $\pi_3 = \frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} \frac{1}{1+\beta\gamma^{1-\sigma_c\iota_p}}$, with ι_p denoting the indexation to past inflation of those prices that were not re-optimized, ξ_p the Calvo parameter regulating the price stickiness and ε_p the curvature of the Kimball goods market aggregator. Current inflation depends on both lagged and expected future inflation and also on the price mark-up and a price mark-up disturbance, ε^p .

The rental rate of capital depends negatively on the capital to labor ratio and positively on the real wage,

$$r_t^k = -(k_t^s - l_t) + w_t. \quad (18)$$

In the labor market the wage markup is characterized by,

$$\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1-\eta/\gamma} \left(c_t - \frac{\eta}{\gamma} c_{t-1} \right) \right), \quad (19)$$

i.e. the difference between the real wage and the marginal rate of substitution between consuming and working. σ_l denotes the inverse of the Frisch elasticity of labor supply.

Wage dynamics are determined by,

$$w_t = w_1 w_{t-1} + (1-w_1) E_t (w_{t+1} + \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \quad (20)$$

where $w_1 = (1 + \beta\gamma^{1-\sigma_c})^{-1}$, $w_2 = \frac{1+\beta\gamma^{1-\sigma_c\iota_w}}{1+\beta\gamma^{1-\sigma_c}}$, $w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c\iota_w}}$ and $w_4 = \frac{1}{1+\beta\gamma^{1-\sigma_c\iota_w}} \frac{(1-\beta\gamma^{1-\sigma_c}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}$. As for prices, ι_w is the degree of wage indexation to past inflation, ξ_w is the Calvo parameter regulating wage stickiness and ε_w is the curvature of the Kimball labor market aggregator. The current real wage depends on its past and expected future values; on the past, current and expected future value of inflation; and on the wage mark-up and the wage mark-up disturbance ε^w .

Finally, monetary policy is assumed to follow a Taylor type rule,

$$r_t = \rho_r r_{t-1} + (1-\rho_r) [\chi_\pi \pi_t + \chi_y (y_t - y_t^*)] + \chi_{\Delta y} (\Delta y_t - \Delta y_t^*) + \varepsilon_t^r, \quad (21)$$

where the short-term nominal interest rate is gradually adjusted to changes in inflation, the

output gap and a monetary policy shock, ε^r .²²

The stochastic sub-structure given by the seven shocks introduced above, TFP, risk-premium, government expenditure, investment-specific technical change, monetary, price mark-up and wage mark-up are set in the following way: the first five shocks are modeled as AR(1) processes while the last two as i.i.d. shocks.²³ In addition, government expenditure is further affected by the innovation of TFP, since in the estimation government expenditure also includes net exports, which can be affected by productivity movements.

²²The output gap is defined as the difference between actual output and potential output, $y_t - y_t^*$. Where the latter, y_t^* , in turn, is defined as the output that would prevail in the economy under flexible prices.

²³Under RE both mark-up shocks are usually modeled as a persistent process, e.g. as ARMA(1,1). However, adaptive learning generates sufficient endogenous persistence to abstract from such a structure.

Appendix II

Estimations

This section presents the posterior estimation of parameters.

Table 3: Posterior Estimates: stochastic structure.
- mean and 5% ÷ 95% quantiles reported -

	Prior(<i>mean, std</i>)	RE	AL	LAL	LALwS
σ_a	$\Gamma^{-1}(0.1, 2)$	0.44 0.40÷0.48	0.44 0.41÷0.49	0.44 0.41÷0.49	0.47 0.42÷0.52
σ_b	$\Gamma^{-1}(0.1, 2)$	0.22 0.17÷0.26	0.15 0.12÷0.17	0.10 0.07÷0.15	0.16 0.13÷0.21
σ_g	$\Gamma^{-1}(0.1, 2)$	0.50 0.46÷0.55	0.48 0.44÷0.53	0.48 0.44÷0.52	0.50 0.46÷0.55
σ_q	$\Gamma^{-1}(0.1, 2)$	0.36 0.31÷0.42	0.42 0.37÷0.47	0.35 0.28÷0.42	0.17 0.12÷0.24
σ_r	$\Gamma^{-1}(0.1, 2)$	0.22 0.20÷0.25	0.21 0.20÷0.23	0.21 0.20÷0.23	0.20 0.19÷0.22
σ_p	$\Gamma^{-1}(0.1, 2)$	0.13 0.10÷0.15	0.15 0.13÷0.16	0.12 0.10÷0.15	0.14 0.12÷0.16
σ_w	$\Gamma^{-1}(0.1, 2)$	0.35 0.32÷0.39	0.34 0.31÷0.37	0.36 0.32÷0.40	0.34 0.30÷0.38
σ_c^s	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.05 0.03÷0.10
σ_i^s	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.14 0.05÷0.25
σ_l^s	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.06 0.03÷0.10
σ_π^s	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.04 0.03÷0.06
$\sigma_{p_k}^s$	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.08 0.03÷0.16
$\sigma_{r_k}^s$	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.07 0.03÷0.13
$\sigma_{w_\pi}^s$	$\Gamma^{-1}(0.1, 2)$	—	—	—	0.05 0.03÷0.08
ρ_a	<i>Beta</i> (0.5, 0.2)	0.96 0.94÷0.98	0.99 0.99÷0.99	0.97 0.96÷0.99	0.97 0.95÷0.99
ρ_b	<i>Beta</i> (0.5, 0.2)	0.36 0.21÷0.55	0.64 0.48÷0.76	0.29 0.14÷0.49	0.13 0.04÷0.27
ρ_g	<i>Beta</i> (0.5, 0.2)	0.98 0.96÷0.99	0.97 0.96÷0.99	0.97 0.96÷0.99	0.96 0.95÷0.98
ρ_q	<i>Beta</i> (0.5, 0.2)	0.79 0.72÷0.86	0.38 0.28÷0.49	0.43 0.28÷0.58	0.15 0.05÷0.28
ρ_r	<i>Beta</i> (0.5, 0.2)	0.13 0.05÷0.22	0.15 0.06÷0.25	0.14 0.06÷0.24	0.08 0.03÷0.15
ρ_p	<i>Beta</i> (0.5, 0.2)	0.90 0.82÷0.96	—	—	—
ρ_w	<i>Beta</i> (0.5, 0.2)	0.96 0.94÷0.98	—	—	—
θ_p	<i>Beta</i> (0.5, 0.2)	0.77 0.63÷0.87	—	—	—
θ_w	<i>Beta</i> (0.5, 0.2)	0.93 0.88÷0.97	—	—	—
α_{g^b}	<i>N</i> (0.5, 0.25)	0.51 0.37÷0.64	0.55 0.42÷0.67	0.55 0.42÷0.68	0.46 0.33÷0.59

The models under adaptive learning, following Slobodyan and Wouters (2012), feature a simpler process for both mark-up shocks with respect to the model under RE. While

Table 4: Posterior Estimates: stochastic structure cross-correlations.
- mean and 5% ÷ 95% quantiles reported -

	Prior(<i>mean, std</i>)	RE	AL	LAL	LALwS
θ_{b-c}	<i>Beta</i> (0.5, 0.2)	–	–	–	0.15 –0.53 ÷ 0.74
θ_{b-l}	<i>Beta</i> (0.5, 0.2)	–	–	–	0.19 –0.51 ÷ 0.73
θ_{q-i}	<i>Beta</i> (0.5, 0.2)	–	–	–	0.10 –0.47 ÷ 0.65
θ_{q-p_k}	<i>Beta</i> (0.5, 0.2)	–	–	–	0.03 –0.68 ÷ 0.73
θ_{q-r_k}	<i>Beta</i> (0.5, 0.2)	–	–	–	0.00 –0.72 ÷ 0.73
$\theta_{p-\pi_p}$	<i>Beta</i> (0.5, 0.2)	–	–	–	0.11 0.02 ÷ –0.25
$\theta_{w-\pi_w}$	<i>Beta</i> (0.5, 0.2)	–	–	–	0.19 0.03 ÷ 0.43

under RE these shocks followed an ARMA (1,1) process, under learning they are can be modeled as white noise ($\rho_p, \rho_w, \theta_p, \theta_w$ are the persistence and moving average parameters respectively).²⁴ Furthermore, when sentiment shocks are included a series of shocks cross effects are estimated, namely:

$$\left\{ \begin{array}{l} s_t^c = e_t^c + \theta_{b-c}e_t^b \\ s_t^i = e_t^i + \theta_{q-i}e_t^q \\ s_t^l = e_t^l + \theta_{b-l}e_t^b \\ s_t^{\pi_p} = e_t^{\pi_p} + \theta_{p-\pi_p}e_t^p \\ s_t^{p_k} = e_t^{p_k} + \theta_{q-p_k}e_t^q \\ s_t^{r_k} = e_t^{r_k} + \theta_{q-r_k}e_t^q \\ s_t^{\pi_w} = e_t^{\pi_w} + \theta_{w-l}e_t^w \end{array} \right. \quad (22)$$

The first column of the right hand side is given by the sentiment shocks, the second column is given by the effect of the innovations of the standard shocks on the sentiment shocks.

²⁴ σ : standard deviation of shocks. ρ : AR(1) coefficient. θ : MA(1) coefficient. $a_{\alpha\beta}$: the effect of TFP innovations on exogenous demand.

Table 5: Posterior Estimates: structural parameters.
- mean and 5% ÷ 95% quantiles reported -

	Prior(<i>mean, std</i>)	RE	AL	LAL	LALwS
φ	$N(4, 1.5)$	5.23 3.73÷6.89	2.92 2.09÷4.52	5.59 4.05÷7.30	6.77 5.11÷8.60
σ_c	$N(1.5, 0.37)$	1.29 1.11÷1.51	1.65 1.40÷1.91	1.38 1.16÷1.61	0.43 0.28÷0.65
η	$Beta(0.7, 0.1)$	0.74 0.65÷0.81	0.66 0.58÷0.75	0.84 0.78÷0.88	0.87 0.81÷0.91
σ_l	$N(2.0, 0.5)$	1.40 0.56÷2.28	1.56 0.83÷2.36	1.77 0.94÷2.56	1.28 0.54÷2.03
ξ_p	$Beta(0.5, 0.1)$	0.76 0.68÷0.83	0.74 0.69÷0.79	0.75 0.68÷0.81	0.75 0.61÷0.87
ξ_w	$Beta(0.5, 0.1)$	0.78 0.68÷0.86	0.82 0.77÷0.87	0.79 0.74÷0.85	0.71 0.64÷0.78
ι_p	$Beta(0.5, 0.15)$	0.24 0.11÷0.38	0.32 0.18÷0.49	0.40 0.20÷0.64	0.26 0.12÷0.42
ι_w	$Beta(0.5, 0.15)$	0.64 0.42÷0.83	0.33 0.17÷0.53	0.41 0.22÷0.63	0.39 0.19÷0.61
ψ	$Beta(0.5, 0.15)$	0.72 0.57÷0.86	0.70 0.51÷0.86	0.65 0.45÷0.82	0.66 0.47÷0.83
ϕ_p	$N(1.25, 0.12)$	1.72 1.57÷1.90	1.63 1.47÷1.80	1.63 1.45÷1.83	1.59 1.39÷1.82
ρ_R	$Beta(0.75, 0.1)$	0.82 0.78÷0.86	0.90 0.87÷0.93	0.91 0.88÷0.93	0.86 0.83÷0.89
r_π	$N(1.5, 0.25)$	1.55 1.34÷1.80	1.63 1.33÷1.94	1.61 1.34÷1.92	1.67 1.44÷1.91
r_y	$N(0.12, 0.05)$	0.04 0.02÷0.07	0.11 0.06÷0.16	0.08 0.03÷0.13	0.07 0.03÷0.11
$r_{\Delta y}$	$N(0.12, 0.05)$	0.16 0.12÷0.19	0.12 0.09÷0.15	0.12 0.09÷0.15	0.13 0.10÷0.16
$\bar{\pi}$	$\Gamma(0.62, 0.1)$	0.81 0.64÷0.97	0.62 0.51÷0.73	0.68 0.53÷0.84	0.69 0.53÷0.87
$100(\beta^{-1} - 1)$	$\Gamma(0.25, 0.1)$	0.15 0.07÷0.24	0.14 0.06÷0.24	0.16 0.08÷0.29	0.25 0.13÷0.39
\bar{l}	$N(5.0, 2.0)$	6.62 4.56÷8.63	7.58 6.19÷8.85	6.78 4.53÷8.96	6.91 5.20÷8.84
γ	$N(0.4, 0.1)$	0.41 0.38÷0.43	0.43 0.39÷0.46	0.41 0.37÷0.44	0.42 0.38÷0.45
α	$N(0.3, 0.05)$	0.20 0.18÷0.23	0.18 0.15÷0.21	0.18 0.15÷0.21	0.16 0.13÷0.19
ρ	$Beta(0.5, 0.29)$	—	0.97 0.96÷0.98	0.96 0.90÷0.98	0.97 0.94÷0.99

Historic Variance Decomposition

Table 6: LAL no shocks - variance decomposition in %

Structural Shocks:	Δy	r	Δc	Δi	l	π_p	π_w
TFP	8.4	13.4	0.8	0.3	9.0	11.1	0.0
Risk Premium	52.8	17.7	85.6	32.3	33.6	2.9	0.8
Gov. Exp.	24.1	4.0	0.8	0.2	11.3	0.7	0.0
IST	6.9	4.2	0.4	62.3	6.4	2.5	0.0
Monetary	5.3	26.9	8.7	3.4	27.9	3.4	0.4
Price markup	1.9	21.7	3.3	1.4	5.7	62.0	5.5
Wage markup	0.6	11.6	0.6	0.2	6.0	17.5	93.3

Table 7: RE - var decomposition in %

Structural Shocks:	Δy	r	Δc	Δi	l	π_p	π_w
TFP	9.7	16.1	1.7	2.2	3.8	11.2	0.8
Risk Premium	32.0	8.5	66.9	6.6	7.4	1.0	0.8
Gov. Exp.	21.1	4.9	2.5	0.6	10.1	0.6	0.0
IST	16.8	24.9	3.3	76.4	16.7	2.7	2.1
Monetary	10.2	17.4	13.4	6.0	9.4	4.4	1.5
Price markup	6.4	6.7	4.8	6.7	14.7	38.5	17.0
Wage markup	3.8	21.5	7.4	1.6	37.2	41.7	77.7

Appendix III

State Space Form

This section briefly introduces the model in its state space form which is the basis for the likelihood computation and, in turn, for the Bayesian estimation. It consists of an observation and a process equation. The process equation describes the law of motion of the states, that is the economic model, while the observation equation maps them into the data.

The *structural* model given by equations (8)-(21) constitutes the main building block for our process equation. The model can first be written in its matrix form as

$$Y_t = BY_{t-1} + CY_t + D\hat{E}_t Y_{t+1} + E\omega_t + F\omega_{t-1} \quad (23)$$

$$\omega_t = \rho_\omega \omega_{t-1} + S\epsilon_t \quad (24)$$

where $Y_t = [k_t, y_t, r_t, c_t, i_t, l_t, \pi_t, q_t, r_t^k, w_t]'$, is a vector of endogenous states,

$\omega_t = [\epsilon_t^a, \epsilon_t^b, \epsilon_t^g, \epsilon_t^q, \epsilon_t^r, \epsilon_t^p, \epsilon_t^w]$ the vector containing all seven standard shocks and ϵ_t is a 14×1 random vector of innovations to ω , that also includes the 7 innovations to sentiment shocks. The matrices B , C , D , E , F , ρ_ω and S are then functions of the parameters of the model, θ , of the appropriate size. Notice that the model still includes the subjective expectations operator \hat{E}_t .

Expectations are formed by means of reduced-form models whose coefficients are updated using Bayes rules. In particular, following Slobodyan and Wouters (2012) these models are simple $AR(2)$, i.e. linear on the first two lags of the relevant variable that needs to be forecasted and include a constant. From the perspective of the researcher the coefficients

estimated by the agents each period become states, rendering the model non-linear. After linearizing, expectations are constructed as

$$\hat{E}_t Y_{t+1} = \bar{\beta}_1 Y_t + \bar{\beta}_2 Y_{t-1} + \beta_t$$

and the model can be solved for the current states,

$$\begin{pmatrix} Y_t \\ \omega_t \end{pmatrix} = N\beta_t + T \begin{pmatrix} Y_{t-1} \\ \omega_{t-1} \end{pmatrix} + R\epsilon_t \quad (25)$$

where ²⁵

$$N = \begin{pmatrix} (Id - C - D\bar{\beta}_1)^{-1} D \\ 0_{(7 \times 21)} \end{pmatrix}$$

$$T = \begin{pmatrix} \left[(Id - C - D\bar{\beta}_1)^{-1} (B + D\bar{\beta}_2) \right] & \left[(Id - C - D\bar{\beta}_1)^{-1} (E\rho_\omega + F) \right] \\ 0_{(7 \times 10)} & \rho \end{pmatrix}$$

$$R = \begin{pmatrix} (Id - C - D\bar{\beta}_1)^{-1} ES \\ S \end{pmatrix}$$

The linearized version of the optimal Bayesian updating rules for beliefs can be written as

$$\beta_t = \rho\beta_{t-1} + \rho M \left\{ Y_{t-1}^f - \bar{\beta}_1 Y_{t-2}^f - \bar{\beta}_2 Y_{t-3}^f - \beta_{t-1} \right\} + \Omega\epsilon_t \quad (26)$$

where the exponent f denotes that only the rows corresponding to the forward variables are selected and the matrix M is a composite that includes the Kalman gain evaluated at the REE. Variables appearing with a bar on top denote the corresponding point around which the equation was linearized.

Considering that the mapping of the data to the model is given by,

²⁵In our exercise $\mu = 0$.

$$\begin{pmatrix} dlGDP_t \\ FEDFUNDS_t \\ dlCons_t \\ dlINV_t \\ lHours_t \\ dlP_t \\ dlWage_t \end{pmatrix} \equiv O_t = \underbrace{\begin{pmatrix} \gamma \\ r \\ \gamma \\ \gamma \\ l \\ \pi \\ \gamma \end{pmatrix}}_{:=constant} + \begin{pmatrix} \Delta y_t \\ r_t \\ \Delta c_t \\ \Delta i_t \\ l_t \\ \pi_t \\ \Delta w_t \end{pmatrix}$$

Finally, the state vector is defined as $Z_t \equiv [Y'_t, \omega'_t, Y_{t-1}^{f'}, Y_{t-2}^{f'}, dobs'_t, \beta'_t]'$ and write the model in its state space form

$$\begin{cases} O_t = & Z_t^{obs} \\ Z_t = \mu + & G \cdot Z_{t-1} + V \cdot \epsilon_t \end{cases} \quad (27)$$

where obs denotes that only $dobs_t$ is selected from Z_t , $\mu = (0_{1 \times 31}, trend', 0_{1 \times 7})'$ and

$$G = \begin{pmatrix} T + N\rho M Sel^f & & -N\rho M \bar{\beta}_1 \\ Sel^f & & 0 \\ 0 & & Id_7 \\ (T + N\rho M Sel^f)^{obs} - Sel^{obs} & & (-N\rho M \bar{\beta}_1)^{obs} \\ \rho M Sel^f & & -\rho M \bar{\beta}_1 \\ -N\rho M \bar{\beta}_2 & 0 & N\rho (Id_7 - M) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ (-N\rho M \bar{\beta}_2)^{obs} & 0 & (N\rho (Id_7 - M))^{obs} \\ -\rho M \bar{\beta}_2 & 0 & \rho (Id - M) \end{pmatrix} \quad (28)$$

$$V = \begin{pmatrix} R + N\Omega \\ 0 \\ 0 \\ (R + N\Omega)^{obs} \\ \Omega \end{pmatrix} \quad (29)$$

where Sel^f is a matrix selecting the forward variables in $(Y'_{t-1}, \omega'_{t-1})$ and obs , again,

denotes that only the rows corresponding to the observable variables are selected. The model given by (27) is used to compute the likelihood of the data for each parameter draw generated by the MH algorithm.

Data

The model is estimated using data on seven US macroeconomic variables: real GDP, short term interest rate (Federal Funds rate), real consumption, real investment, hours worked, Inflation and real wages. The data is constructed as in Slobodyan and Wouters (2012) and updated to include the latest releases. For example, real GDP is expressed in billions of chained 2009 dollars. It covers the period ranging from the first quarter of 1966 till the fourth of 2013 and includes a pre sample of 4 quarters starting in 1965. Aggregate real variables are expressed in per capita terms and all series have been adjusted for seasonality. Furthermore, all variables are expressed in percentage points.

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