Foreign Currency Debt and Optimal Monetary Policy: Is There a Role for the Exchange Rate in Completing Markets?

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Abstract

This paper characterizes the optimal monetary and exchange rate policy for a small open economy with an incomplete domestic financial market. The existence of a domestic financial market arises from the existence of different generations within the domestic economy. The source of market incompleteness is the inability of domestic generations to trade financial claims other than a nominal, non-contingent security denominated in foreign currency. The decentralized equilibrium of the model features a tight relationship between the consumption of different generations and the aggregate debt-to-GDP ratio. The stabilization of this ratio therefore increases the degree of risk-sharing across generations. The optimal monetary policy rule arises from a tradeoff between the stabilization of the debt-to-GDP ratio and the traditional objectives of price and output gap stabilization. Given that debt is denominated in foreign currency, the former requires active control of the nominal exchange rate. For the calibration considered, price and output stabilization dominate risk-sharing in the optimal policy rule. This indicates that the objective of improving risk sharing is excessively costly from the point of view of the policymaker. The optimal policy rule is found to be closest to Producer Price Index Inflation Targeting than to Consumer Price Index Inflation Targeting or an Exchange Rate Peg. Finally, the model admits an extension of the result by Cole and Obstfeld[8, 1991] for risk-sharing across domestic households under financial autarky.

JEL *Classification*: E52, E58, F41

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Introduction

This paper aims to characterize the optimal monetary and exchange rate policies that a small open economy should follow when there is an incomplete domestic financial market in which financial instruments

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(assets and liabilities) are denominated in foreign currency.

The importance of the issues raised by the pervasiveness of foreign currency financial instruments in an economy with market incompleteness is illustrated by the recent experience of household debt in several Eastern European economies such as Poland, the Czech Republic, Slovakia, Serbia and, most especially, Hungary (see Szpunar and Glogowski[23]). Since the beginning of the past decade, and perhaps due to lower interest rates and to the expectations created by a tight channel for the nominal exchange rate, Hungarian households started a process of rapid accumulation of both assets and liabilities denominated in currencies other than the Hungarian Forint (HUF), principally Swiss Francs (CHF).

The ratio of foreign currency household debt to GDP in Hungary increased from virtually zero in 1999 to slightly above 29% at the beginning of 2009. According to Balás and Nagy[2, 2010], close to 90% of this debt was denominated in CHF, and only 7% was denominated in Euros. Since then, borrowing in foreign currency has ground to a halt and the outstanding balances have unravelled quickly in a context of financial turbulence for households, nominal depreciation and extreme measures taken by Hungarian authorities with the aim of limiting issuance of this type of liabilities (see Balogh et al[1, 2013]). At the beginning of 2013, payments on about 20% of mortgages denominated in foreign currency in Hungary were overdue (see WESP[24, 2013]); the government was publicly discussing with the financial system the possibility to introduce differentiated exchange rates for households making prepayments of debt in foreign currency.

The struggle of Hungarian households and of the Hungarian government is indicative of the importance of monetary and exchange rate strategies in a context where financial contracts in foreign currency are pervasive, in particular taking into account (among others) the effect of inflation and the nominal exchange rate on the financial health of domestic agents.

This paper characterizes optimal monetary and exchange rate policies for an open economy where financial claims are denominated in foreign currency. The first section of the paper builds a New Keynesian model of the small open economy which is standard in the literature except for the following two key features. The first consists of abandoning the representative agent framework in the domestic economy with the aim of introducing a domestic financial market. This market is incomplete in the sense that *domestic households* will only be able to lend and borrow to each other in a nominal, non-contingent debt instrument[1]. This form of incompleteness will necessarily require some form of incompleteness in the international financial market as well (that is, in the market where households *of different countries* will exchange financial claims)[2]. The second key non-standard feature of the model is that the financial instrument traded by households of the domestic economy is denominated in foreign currency. As was

[1] In particular, the model assumes an overlapping generations (OLG) structure with different generations of households that will borrow and lend from each other as a consequence of the fact that individual income is characterized by a standard life-cycle pattern. The incompleteness of the domestic financial markets will have the well-known implication that risk-sharing across generations is potentially suboptimal (from the point of view of a benevolent policymaker). The potential effects of monetary and exchange rate policies on risk-sharing across the generations of the model will be a key part of the story told by the model.

[2] As will be seen, the model presented below will assume the most extreme form of international market incompleteness, financial autarky. This implies that domestic households will not be able to share risk with foreign households. In such a way, the model 'stacks the cards' against risk sharing by domestic households. This assumption, however, goes against the results of the model as eliminating it would only weaken the incentive of optimal monetary policy to be proactive in the promotion of risk-sharing across households if this had already been achieved by other means.

the case for Hungary in recent years, the denomination of financial instruments will imply that the nominal exchange rate plays a crucial role in the degree to which the households of the domestic economy share risk. Importantly, the only driving force of the model is shocks to the productivity of labour.

Naturally, part of the strategy of monetary policy may include the decision to allow domestic agents to take positions among themselves in foreign currency in the first place. In other words, it is clear that the issues raised by the pervasiveness of foreign currency debt would disappear had the policymaker forbidden households to take this positions. This paper will not consider the decision of the policymaker with regard to regulating ex-ante the ability of households to trade financial instruments in foreign currency, but will rather focus on optimal policy *once* financial contracts denominated in foreign currency have become a dominant feature in the financial system of the economy[3].

Beyond the effect of the nominal exchange rate on risk-sharing across domestic agents, monetary policy is affected by the introduction of these new elements in several ways. In a closed economy setting, for example, Sheedy[22, 2013] has demonstrated that the introduction of market incompleteness in an economy with heterogeneous agents may potentially render the strategy of inflation targeting suboptimal when compared to a Nominal GDP targeting rule. The intuition for this result is straightforward and, given its importance to understand the results of this paper, it merits a brief consideration.

The basic model by Sheedy[22, 2013] considers an endowment economy with flexible prices, where the only existing friction is the financial market incompleteness, where households can trade only a nominal, non-contingent bond. This incompleteness implies that ex-post, aggregate risk is potentially unevenly shared across households. For instance, when inflation is constant (say, because of the Central Bank following an Inflation Targeting rule), creditors are relatively isolated from any productivity shock (they hold a non-contingent bond in real terms) whereas debtors are overly exposed to risk (their capacity to fulfil financial promises is affected by changes in income). From the point of view of ex-ante efficiency and given that households are risk-averse, a benevolent policymaker would like to devise a mechanism to transfer wealth from creditors to debtors after a particularly bad shock[4]. That mechanism is inflation, which changes the real burden of debt for debtors. The wealth transfers induced by inflation (which are not arbitrary but specifically engineered as part of an optimal monetary policy strategy) improve on risk-sharing and increase welfare from an ex-ante perspective.

The combination of market incompleteness with debt instruments denominated in foreign currency introduces additional considerations for a policymaker concerned about ex-ante risk-sharing, price stability and output gap stability. First, on the risk-sharing front, nominal depreciation emerges as an additional mechanism to transfer wealth across households. However, the ability of monetary policy to steer the nominal exchange rate and inflation in potentially different directions is related to its ability to control the real exchange rate, the terms of trade and output. In principle, a negative productivity shock should trigger nominal appreciations (and vice versa) in order to reduce the value of outstanding debt liabilities, and thus transfer wealth from creditors to debtors. However, the desire of the policymaker to improve on risk-sharing this way clashes with its objective to stabilize output and prices. In this sense,

[3]For an account of a risk-sharing argument to explain the high pervasiveness of foreign currency financial contracts in economies that recently achieved price stability, see Rappoport[18, 2009].

[4]In general, from an ex-ante perspective the policymaker would like to transfer wealth from low marginal utility-households to high marginal utility-households after any aggregate shock. A negative (positive) productivity shock will imply a relatively low (high) consumption -high (low) marginal utility- for debtors.

there is a trade-off for monetary policy between risk-sharing and the standard macroeconomic objectives, which is essentially a quantitative matter.

Optimal monetary and exchange rate policies are characterized analytically in the second section of this paper by means of the Linear-Quadratic method. This is common to a great portion of the literature on optimal policy in New Keynesian models. From this characterization, the model provides a natural extension to the parametric condition studied by Cole and Obstfeld[8, 1991]. In particular, it is found that the same condition studied by those authors (namely, unitary elasticity of substitution across goods produced in different countries) is a sufficient condition for full risk-sharing across domestic households independently of monetary policy.

The third section of the paper studies the response of key variables of the model to a negative productivity shock under the optimal policy rule compared to a set of alternative policy regimes[5]: Producer Price Index (PPI) Inflation Targeting, Consumer Price Index (CPI) Inflation Targeting and an Exchange Rate Peg. For the calibration considered, the optimal monetary policy rule ascribes most of the weight in the abovementioned trade-off to the standard macroeconomic objectives of price and output stabilization. To this purpose, the policymaker will therefore mostly sacrifice risk-sharing considerations, which indicates that the objective of improving on risk-sharing is excessively costly from the point of view of the policymaker. In addition, some risk sharing takes place automatically if the calibration is close to the parametric condition of Cole and Obstfeld[8, 1991], which reduces the incentive of the policymaker to sacrifice the stability of inflation and output in favour of risk-sharing. As a consequence, the optimal policy rule is found to be closest to PPI Inflation Targeting than to Consumer Price Index Inflation Targeting or an Exchange Rate Peg, due to the ability of the former to replicate the flexible price equilibrium allocation.

Despite optimal policy mostly sacrificing risk-sharing considerations, the nominal exchange rate will still play an important role in creating ex-post redistributions of wealth. The fourth section of the paper calculates the relative welfare losses imposed on households by an Exchange Rate Peg. It is found that a Peg can be significantly more harmful in terms of welfare in the overlapping generations model of Section 1 than in standard, representative agent models of the open economy, the reason being the need for the exchange rate to respond actively to productivity shock in a context where risk is unevenly shared across the small open economy.

Related Literature The model of this paper combines distinct elements from two separate areas of the literature.

The first strand of the literature focuses on the study of optimal policy in New Keynesian models of the small open economy. The review by Corsetti, Dedola and Leduc[9, 2011] summarizes the issues at hand in characterizing optimal monetary policy in this setting[6]. A seminal work that inspires the spirit of this paper is Galí and Monacelli[13, 2005], who derive optimal policy for a small open economy under internationally complete markets. Di Paoli[16, 2009] abandons this last assumption to study optimal policy under different international financial market structures, whereas Benigno and Benigno[3, 2006]

[5]Following Galí and Monacelli[13, 2005].

[6]Key papers among this literature include Obstfeld and Rogoff[15, 1995], McCallum and Nelson[14, 2001], Clarida, Galí and Gertler[7, 2001], Corsetti and Pesenti[10, 2001] and De Fiore and Liu[12, 2005].

abandon the small open economy setting to explore the international coordination aspects of monetary policy. All these papers assume that, at the national (domestic) level, either the economy is populated by a representative agent or by a continuum of identical households with perfect risk-sharing across its members. This paper abandons this assumption to postulate instead the existence of a domestic financial market and imperfect risk-sharing at the national level.

The second strand of the literature is relatively younger, its main feature being the abandonment of the representative agent assumption to study optimal monetary policy under incomplete markets in a closed economy, New Keynesian setting. Besides the work by Sheedy[22, 2013] mentioned above, Pescatori[17, 2007] studies optimal policy as it relates to ex-post wealth redistribution across rich and poor individuals. Finally, Schmitt-Grohé and Uribe[21, 2004] introduce nominal rigidities into the otherwise classical framework of incomplete markets and optimal fiscal policy of Chari and Kehoe[6, 1999]. They discover that optimal monetary policy will not perform a great deal of ex-post wealth redistribution between the government and households as this would imply extremely volatile inflation. The results of this paper are similar to theirs in the sense of monetary policy being relatively passive to risk-sharing considerations in order to avoid inflation variability, but in this paper tax smoothing plays no role, redistribution is made across households and the key to the desirability of risk-sharing is risk aversion on the side of the latter.

This paper therefore bridges the gap between these two strands of the literature by borrowing the open economy insights of the former and combining them with the domestic incomplete markets of the latter. The resulting framework is expanded to include foreign currency denominated financial claims and in such way to most closely resemble the environment of those group of economies (discussed above) most affected by the penetration of foreign currency debt in incomplete markets.

1 The Model

The model follows closely the open economy structure of Corsetti, Dedola and Leduc[9, 2011] and the incomplete markets framework of Sheedy[22, 2013]. The model postulates a world economy populated by a continuum of households of measure 1. A fraction n of these individuals reside in country H (Home), and the remaining fraction $1-n$ reside in country F (Foreign). There is an international bond market in which the only financial instrument available is a nominal, one period, non-contingent bond denominated in the currency of country F. While the Home economy will be characterized by a generational structure that gives rise to a domestic financial market, the Foreign economy will be modeled as a standard, representative agent economy.

1.1 Households

1.1.1 Home Households

The Home economy is populated by a continuum of households of measure n. Each of these households lives for three periods. In the first period, the household is young, and his choice variables are indexed by y. In the second period, the household is middle-aged (m), and in the third period the household is old (o). At a given time, three generations (or cohorts) exist, each of which has a measure $\frac{n}{3}$. It is thus assumed that, at each period, a new cohort of young households is born in a measure that exactly replaces the measure of old households that die, in such a way that the demographic structure of the

population stays invariant over time. The problem faced by every new generation of young households is given by:

$$\max_{\{C_{i,t},H_{i,t}\}} U_t \equiv \left\{ \ln C_{y,t} - \alpha_y^{-\eta} \frac{H_{y,t}^{1+\eta}}{1+\eta} \right\} + \beta E_t \left\{ \ln C_{m,t+1} - \alpha_m^{-\eta} \frac{H_{m,t+1}^{1+\eta}}{1+\eta} \right\} + \beta^2 E_t \left\{ \ln C_{o,t+2} - \alpha_o^{-\eta} \frac{H_{o,t+2}^{1+\eta}}{1+\eta} \right\} \tag{1}$$

where $C_{i,t}$ represents the consumption at time t of a basket of Home and Foreign goods by a household at period i of his life, $H_{i,t}$ his individual supply of labour, and $\eta > 0$. The set of parameters α_i is related to the disutility of work for generation i. These parameters are specified in such a way that, in equilibrium, the profile of total income over the lifetime of an individual resembles a traditional life cycle pattern (i.e., relatively low income when young and old, and relatively high income when middle-aged). The consumption basket is defined by the following CES-type aggregator:

$$C_{i,t} = \left[a_H^{\frac{1}{\phi}} \left(C_{i,t}^H \right)^{\frac{\phi-1}{\phi}} + (1-a_H)^{\frac{1}{\phi}} \left(C_{i,t}^F \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

with $C_{i,t}^j$ denoting the consumption of goods produced in country j (in what follows, "j goods") at time t by a household in period i of his life. The parameter a_H captures the degree of "home bias" in consumption, and it can also be interpreted as a measure of the "openness" of the economy. The parameter ϕ represents the elasticity of substitution across H and F goods. The consumption of H and F goods is itself a CES-type aggregate of infinite varieties produced in the respective country with a common elasticity of substitution ϵ, as follows:

$$C_{i,t}^H = \left[\int_0^n \left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} C_{i,t}^H(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \qquad C_{i,t}^F = \left[\int_n^1 \left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} C_{i,t}^F(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $C_{i,t}^x(j)$ denotes consumption of variety j produced in country x, and $\epsilon > \phi$ is assumed[7]. The price of the consumption basket, or equivalently, the Consumer Price Index (CPI) of the Home economy, is given, following standard results from CES-type aggregators, by:

$$P_t = \left[a_H \left(P_t^H \right)^{1-\phi} + (1-a_H) \left(P_t^F \right)^{1-\phi} \right]^{\frac{1}{1-\phi}} \tag{4}$$

where P_t^j represents the nominal price of a good produced in country j at time t measured in currency units of the Home economy, and P_t^x is the price of a composite good produced in country x, defined by:

$$P_t^H = \left[\left(\frac{1}{n} \right) \int_0^n P_t^H(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \qquad P_t^F = \left[\left(\frac{1}{1-n} \right) \int_n^1 P_t^F(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \tag{5}$$

[7]This assumption implies that varieties produced within a country are more similar (more substitutable) than goods produced in different countries. See Corsetti, Dedola and Leduc[9, 2011].

P_t^H and P_t^F will be referred to in what follows as the Producer Price Indices (PPI) of the Home and Foreign economies, respectively, as they measure the price level of those goods produced within a given country. Finally, the allocation of the composite good produced in each country among each variety j is:

$$C_{i,t}^H(j) = a_H \left(\frac{1}{n}\right) \left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} \left(\frac{P_t}{P_t^H}\right)^{\phi} C_{i,t} \tag{6}$$

$$C_{i,t}^F(j) = (1-a_H) \left(\frac{1}{1-n}\right) \left[\frac{P_t^F}{P_t^F(j)}\right]^{\epsilon} \left(\frac{P_t}{P_t^F}\right)^{\phi} C_{i,t} \tag{7}$$

The budget constraints faced by a household at each of the three stages of his life, expressed in units of the Home currency, are given by:

$$P_t C_{y,t} + Q_t B_{y,t} e_t + \frac{M_{y,t}}{1+i_t} \leq W_{y,t} H_{y,t} + \alpha_y P_t J_t - P_t T_{y,t} \tag{8}$$

$$P_t C_{m,t} + Q_t B_{m,t} e_t + \frac{M_{m,t}}{1+i_t} \leq W_{m,t} H_{m,t} + \alpha_m P_t J_t - P_t T_{m,t} + B_{y,t-1} e_t + M_{y,t-1} \tag{9}$$

$$P_t C_{o,t} \leq W_{o,t} H_{o,t} + \alpha_o P_t J_t - P_t T_{o,t} + B_{m,t-1} e_t + M_{m,t-1} \tag{10}$$

Q_t is the price of a nominal, one period, non-contingent bond at time t. As this price is measured in currency units of country F currency, the nominal exchange rate e_t is also part of the budget constraint. One unit of this bond purchased at t promises the bearer the payment of one unit of foreign currency at $t+1$. The quantity of bonds purchased by a household is denoted by B. Note that the nominal exchange rate e_t is an important determinant of the burden of debt to be paid (or collected) by the middle-aged and old generations at the beginning of each period. There is a Home Central Bank that produces money, M. Households can deposit their holdings of money at the Central Bank at the riskless nominal interest rate i. Individuals are ex-ante homogeneous in the sense of having the same preferences, the same life cycle evolution of their endowment, and the same (zero) initial wealth. $W_{i,t}$ is the nominal wage and α_i also represents the proportion of total profits received by individuals of generation i[8]. Finally, J_t is the aggregate profits of firms in country H and $T_{i,t}$ represents lump-sum levied charged on generation i by the government (in units of the composite good C). For future reference, let C_t^H and C_t^F denote aggregate home economy consumption of home and foreign goods respectively, defined as:

$$nC_t^H = \frac{n}{3} C_{y,t}^H + \frac{n}{3} C_{m,t}^H + \frac{n}{3} C_{o,t}^H \tag{11}$$

$$nC_t^F = \frac{n}{3} C_{y,t}^F + \frac{n}{3} C_{m,t}^F + \frac{n}{3} C_{o,t}^F \tag{12}$$

and aggregate consumption as:

$$nC_t = \frac{n}{3} C_{y,t} + \frac{n}{3} C_{m,t} + \frac{n}{3} C_{o,t} \tag{13}$$

Optimality Conditions

The necessary first order conditions of the optimization problem (1) subject to (8)-(10) are reduced to

[8]These parameters α_i coincide with the parameters of the disutility of labour in (1). It will be shown this structure implies constancy of the shares of aggregate income received by each generation.

the following set of Euler and intratemporal equations:

$$\frac{\beta}{Q_t} E_t \left[\frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} \left(\frac{C_{y,t}}{C_{m,t+1}} \right) \right] = 1 \tag{14}$$

$$\frac{\beta}{Q_t} E_t \left[\frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} \left(\frac{C_{m,t}}{C_{o,t+1}} \right) \right] = 1 \tag{15}$$

$$\beta E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{y,t}}{C_{m,t+1}} \right) \right] = \frac{1}{1+i_t} \tag{16}$$

$$C_{i,t} \left(\frac{\alpha_i}{H_{i,t}} \right)^{-\eta} = w_{i,t} \tag{17}$$

where $w_{i,t} = {}^{W_{i,t}}/_{P_t}$ denotes the real wage (expressed in units of the composite good).

1.1.2 Foreign Households

The Foreign economy is populated by a continuum of identical, infinitely-lived households of measure $1-n$. The problem of the representative household of this economy is given by:

$$\max_{\{C_t^*, H_t^*\}} U_t^* = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\ln C_{t+\tau}^* - \frac{\left(H_{t+\tau}^*\right)^{1+\eta}}{1+\eta} \right] \tag{18}$$

where '$*$' will refer in what follows to variables of the foreign economy. C_t^* is a basket of goods produced in the Home and the Foreign country analogous to (2) with a bias parameter a_H^*. This parameter indicates the preference of the foreign households for H goods and represents a measure of the "foreign bias" of the Foreign economy. The expressions for the consumption aggregators, CPI, PPI and optimal consumption allocation of the Foreign economy are analogous to (4)-(7) replacing respective terms with C_t^{H*}, C_t^{F*}, $C_t^{H*}(j)$, $C_t^{F*}(j)$, P_t^*, P_t^{H*}, P_t^{F*}, $P_t^{H*}(j)$, $P_t^{F*}(j)$, where price levels are denominated in units of the foreign currency.

The budget constraint of the representative household of the foreign economy (in units of the foreign currency) is given by:

$$P_t^* C_t^* + Q_t B_t^* + \frac{M_t^*}{1+i_t^*} = W_t^* H_t^* + P_t^* J_t^* - P_t^* T_t^* + M_{t-1}^* + B_{t-1}^* \tag{19}$$

where the nominal exchange rate is not included as long as the prices and quantities of bonds/securities are denominated in the currency of the foreign country. There is also a Foreign Central Bank who produces money M^* and riskless deposits for money at the nominal interest rate i^*. The remaining variables have the analogous interpretation as in the Home economy.

Optimality Conditions

The necessary first order conditions of the optimization problem of the foreign household (18) subject to the sequence of constraints (19) are given by:

$$\beta E_t \left[\frac{P_t^*}{P_{t+1}^*} \left(\frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \right] = Q_t \tag{20}$$

$$\beta E_t \left[\frac{P_t^*}{P_{t+1}^*} \left(\frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \right] = \frac{1}{1+i_t^*} \tag{21}$$

$$C_t^* \left(H_t^* \right)^{\eta} = w_t^* \tag{22}$$

where $w_t^* = {}^{W_t^*}/_{P_t^*}$ is the real wage of the foreign economy.

1.1.3 Interest Rate Parity

From the optimality conditions of households across the world it is possible to derive a version of the Uncovered Interest Rate Parity Condition (UIP). From (20) and (21), it is the case that:

$$Q_t = \frac{1}{1+i_t^*} \tag{23}$$

From (14) and (16), using (23), the UIP condition is:

$$\left(1+i_t^*\right) E_t \left[\frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} \left(\frac{C_{y,t}}{C_{m,t+1}} \right) \right] = E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{y,t}}{C_{m,t+1}} \right) \right] \left(1+i_t\right) \tag{24}$$

1.2 Terms of Trade, Real Exchange Rate and Demand Functions

Let S_t denote the terms of trade of the Home economy, defined as the relative price of foreign goods in terms of home goods:

$$S_t \equiv \frac{P_t^F}{P_t^H} \tag{25}$$

Let Ω_t denote the real exchange rate of the home economy, defined as the ratio between the CPI of the foreign and the home economies, expressed in the same currency:

$$\Omega_t \equiv \frac{e_t P_t^*}{P_t} \tag{26}$$

In what follows, a law of one price for individual varieties produced in both countries is assumed[9]. For goods produced in country x:

$$P_t^x \left(j\right) = e_t P_t^{x*} \left(j\right) \tag{27}$$

It is straightforward to demonstrate that (27) implies:

$$e_t P_t^{H*} = P_t^H \qquad e_t P_t^{F*} = P_t^F \tag{28}$$

The real exchange rate is therefore rewritten as:

$$\Omega_t \equiv \left[\frac{a_H^* \left(P_t^H\right)^{1-\phi} + \left(1-a_H^*\right) \left(P_t^F\right)^{1-\phi}}{a_H \left(P_t^H\right)^{1-\phi} + \left(1-a_H\right) \left(P_t^F\right)^{1-\phi}} \right]^{\frac{1}{1-\phi}} \tag{29}$$

The real exchange rate collapses to 1 if $a_H = a_H^*$. As will be seen, the ability of monetary policy to alter real outcomes in the Home economy is tightly linked to its ability to control the real exchange rate. Therefore, for the rest of the paper it is assumed that $a_H \neq a_H^*$[10].

[9] This result can also be endogenously derived assuming producer-currency pricing on the side of firms, given the common elasticity of substitution ϵ across varieties in local aggregators.

[10] Notice this assumption need not imply the existence of home bias, which is observed when $a_H > 1/2$.

1.2.1 Demand Functions

For the specification of the problems of firms, it will be useful to calculate the aggregate demand for a given variety produced in a given country. For varieties produced at Home and in the Foreign economy respectively, these will be given by:

$$\begin{aligned} Y_t^{H,d}(j) &= nC_t^H(j) + (1-n)\,C_t^{H*}(j) \\ Y_t^{F,d}(j) &= nC_t^F(j) + (1-n)\,C_t^{F*}(j) \end{aligned}$$

Using (6), (7) and (11)-(13), these can be rewritten as:

$$Y_t^{H,d}(j) = \left(\frac{P_t}{P_t^H}\right)^{\phi}\left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} Y_t^d \tag{30}$$

$$Y_t^{F,d}(j) = \left(\frac{P_t}{P_t^F}\right)^{\phi}\left[\frac{P_t^F}{P_t^F(j)}\right]^{\epsilon} Y_t^{*d} \tag{31}$$

where $Y_t^d = a_H C_t + \left(\frac{1-n}{n}\right) a_H^* \Omega_t^{\phi} C_t^*$ is the total demand faced by the Home economy and $Y_t^{*d} = \left(\frac{n}{1-n}\right)(1-a_H)\,C_t + (1-a_H^*)\,C_t^* \Omega_t^{\phi}$ is the total demand faced by the Foreign economy.

1.3 Firms

1.3.1 Home Firms

A typical firm in the Home economy operates in a monopolistically competitive environment, producing a differentiated good (j) with the following linear technology in labour:

$$Y_t^H(j) = A_t N_t(j) \tag{32}$$

A_t represents the total productivity of labour and constitutes the only exogenous stochastic process of the economy. Following the literature on wage stickiness in New Keynesian models (in particular Erceg et al[11, 2000]), the demand for labour from different generations is aggregated using a Cobb-Douglas specification[11]:

$$N_t(j) = A N_{y,t}(j)^{\frac{\alpha_y}{3}} N_{m,t}(j)^{\frac{\alpha_m}{3}} N_{o,t}(j)^{\frac{\alpha_o}{3}} \tag{33}$$

with $A = \left[\left(\frac{\alpha_y}{3}\right)^{\frac{\alpha_y}{3}}\left(\frac{\alpha_m}{3}\right)^{\frac{\alpha_m}{3}}\left(\frac{\alpha_o}{3}\right)^{\frac{\alpha_o}{3}}\right]^{-1}$, and where $N_{i,t}(j)$ is the employment of hours of labour by individuals of generation i. Firms receive a proportional wage bill subsidy τ on labour costs. The firm solves a problem of allocating labour from different generations analogous to the one faced by the households when allocating components of a composite consumption basket. The cost-minimizing generational labour demand functions are given by:

$$\frac{\alpha_i}{3}\frac{w_t}{w_{i,t}} N_t(j) = N_{i,t}(j) \qquad i = y, m, o. \tag{34}$$

[11] Erceg et al[11, 2000] employ a general CES aggregator for heterogeneous labour; the specification used in this paper borrows the idea of using consumption-style aggregators.

where $w_t = w_{y,t}^{\frac{\alpha_y}{3}} w_{m,t}^{\frac{\alpha_m}{3}} w_{o,t}^{\frac{\alpha_o}{3}}$ is the real wage index of the Home economy. The problem of the home firm is to maximise the present value of lifetime instantaneous profits in real terms:

$$J_t^H(j) = \frac{P_t^H(j) Y_t^H(j)}{P_t} - (1-\tau)\left[w_{y,t} N_{y,t}(j) + w_{m,t} N_{m,t}(j) + w_{o,t} N_{o,t}(j)\right] \tag{35}$$

The absence of intertermporal elements makes the cost minimization problem essentially static. Using (32), (34) and (30), the problem of firms can be redefined as:

$$\max_{P_t^H(j)} J_t^H(j) = \left\{ \frac{P_t^H(j)}{P_t} \left(\frac{P_t}{P_t^H}\right)^{\phi} \left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} - (1-\tau) x_t \left(\frac{P_t}{P_t^H}\right)^{\phi} \left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} \right\} Y_t^d \tag{36}$$

where $x_t = \frac{w_t}{A_t}$ is the aggregate marginal cost and P_t, x_t, P_t^H and Y_t^d are taken as given. The solution of this problem depends on the price formation mechanisms of the economy.

Price Stickiness: Different Information Sets

It is assumed that all firms set prices in the currency of the producer country (referred to in the literature as Producer Currency Pricing, PCP). This paper considers a form of price rigidity in which different firms have random access to a different set of information. In particular, a fraction $1-\kappa$ of firms in the Home economy sets an optimal price with information up-to-date at the moment of making choices (that is, P_t, x_t, P_t^H and Y_t^d are observed at the moment of solving problem (36)). The remaining fraction κ sets an optimal price at t with the information set of period $t-1$, and must therefore rely on forecasts of the relevant variables P_t, x_t, P_t^H and Y_t^d . Notice that all firms are allowed to change prices between time periods.

The first order condition of problem (36) is common for all firms operating under full information. The optimal price will be therefore common among this group and equal to:

$$\hat{P}_t^H = x_t \mu (1-\tau) P_t \tag{37}$$

where $\mu = \frac{\epsilon}{\epsilon - 1}$, with $\mu(1-\tau)$ being the "gross effective markup" (net of the wage subsidy) charged by firms over marginal cost in nominal terms.

All firms with lagged information, on the other hand, solve the following problem:

$$\max_{P_t^H(j)} E_{t-1} J_t^H(j) \tag{38}$$

where $P_t^H(j) = E_{t-1}\left[P_t^H(j)\right]$. The first order condition of this problem is:

$$E_{t-1}\left\{ \left(\frac{\check{P}_t^H}{P_t^H}\right)^{-\epsilon} \left(\frac{P_t}{P_t^H}\right)^{\phi} Y_t^d \left[\frac{\check{P}_t^H}{P_t} - \mu(1-\tau) x_t\right] \right\} = 0 \tag{39}$$

where $\check{P}_t^H$ is the common price chosen by these firms. From the definition of the PPI (5):

$$\left(P_t^H\right)^{1-\epsilon} = \left(\frac{1}{n}\right) \int_0^n P_t^H(j)^{1-\epsilon} dj = (1-\kappa)\left(\hat{P}_t^H\right)^{1-\epsilon} + \kappa \left(\check{P}_t^H\right)^{1-\epsilon} \tag{40}$$

Letting $\hat{p}_t^H = \frac{\hat{P}_t^H}{P_t^H}$ and $\check{p}_t^H = \frac{\check{P}_t^H}{P_t^H}$, we can relate the prices set by firms belonging to different groups as follows:

$$\begin{aligned} 1 &= (1-\kappa)\left(\hat{p}_t^H\right)^{1-\epsilon} + \kappa\left(\check{p}_t^H\right)^{1-\epsilon} \\ \hat{p}_t^H &= \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \end{aligned} \tag{41}$$

These results allow the pricing equations to be rewritten as follows for the relative prices under full information:

$$\hat{p}_t^H = x_t \mu (1-\tau) \frac{P_t}{P_t^H} \tag{42}$$

and for the relative prices under outdated information:

$$E_{t-1}\left\{\left(\check{p}_t^H\right)^{-\epsilon}\left(\frac{P_t}{P_t^H}\right)^{\phi-1} Y_t^d \left(\check{p}_t^H - \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}\right)\right\} = 0 \tag{43}$$

Importantly, (42) and (43) depend on the ratio between the CPI and the PPI at Home, which is related (as will be shown) to the terms of trade, S_t. This observation will result in the Phillips curve of the Home economy depending on its terms of trade.

1.3.2 Foreign Firms

A typical firm in the Foreign economy operates in a monopolistically competitive environment, but (for simplicity) it is assumed that it is not subject to any form of price stickiness. The production function of a Foreign firm is given by:

$$Y_t^{F*}(j) = N_t^*(j) \tag{44}$$

where the total productivity of labour has been set to 1. Foreign firms use only one type of labour, supplied by the representative household of the foreign economy. Following an analogous procedure as in the previous section, the relative price set by all firms in the Foreign economy is given by:

$$\hat{p}_t^{F*} = x_t^* \mu (1-\tau) \frac{P_t^*}{P_t^{F*}} = 1 \tag{45}$$

where $x_t^* = w_t^*$, $\hat{p}_t^{F*} = \frac{\hat{P}_t^{F*}}{P_t^{F*}}$ and the remaining variables have an analogous interpretation to those in the Home economy.

1.4 Governments

The only role of the governments in both the Home and Foreign economies is to transfer lump-sum taxes levied on households to firms as wage subsidies with the sole purpose of eliminating market power inefficiencies from the side of firms that are not perfectly competitive. This means that the wage bill

subsidy rate τ will be engineered in such a way that the gross effective markup is one: $\tau = \epsilon^{-1}$. The budget constraint of the Home government is given by:

$$nT_t = \frac{n}{3}T_{y,t} + \frac{n}{3}T_{m,t} + \frac{n}{3}T_{o,t} = \epsilon^{-1}\int_0^n \left[w_{y,t}N_{y,t}(j) + w_{m,t}N_{m,t}(j) + w_{o,t}N_{o,t}(j)\right] dj \tag{46}$$

Assume that the proportion of aggregate government revenue coming from each generation is equal to the disutility parameter α_i: $T_{i,t} = \alpha_i T_t$. For the Foreign government,

$$(1-n)\,T_t = \epsilon^{-1}\int_n^1 w_t^* N_t^*(j)\,dj \tag{47}$$

1.5 Aggregate Equilibrium Conditions

The components described so far allow the construction of a simple condition that relates aggregate demand to aggregate supply in each of the two economies of the model. For the case of the Home economy, condition (30) implies that an individual variety market clearing condition can be written as:

$$Y_t^H(j) \quad = \quad \left(\frac{P_t}{P_t^H}\right)^{\phi}\left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} Y_t^d \tag{48}$$

for all j. In what follows, let $nY_t = \int_0^n \frac{P_t^H(j)Y_t^H(j)}{P_t}dj$ denote the real GDP in the Home country in terms of the composite good. Then, by (48), the aggregate equilibrium condition of the Home economy is:

$$Y_t \quad = \quad Y_t^d\left(\frac{P_t}{P_t^H}\right)^{\phi-1} \tag{49}$$

where we have used (5). The left hand side of expression (49) corresponds to the total supply of the economy. This can be transformed into an aggregate production function of the Home economy using the equilibrium conditions of the labour markets:

$$\int_0^n N_{i,t}(j)\,dj \quad = \quad \frac{n}{3}H_{i,t} \tag{50}$$

Letting $nN_t = \int_0^n N_t(j)\,dj$ denote the aggregate demand for labour of the Home economy, and using (32) and (49) in (48), the aggregate production function of the economy is obtained:

$$Y_t \quad = \quad A_t N_t\left(\frac{P_t}{P_t^H}\right)^{-1}\bigstar_t \tag{51}$$

where $\bigstar_t = \left\{(1-\kappa)\left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}} + \kappa\left(\check{p}_t^H\right)^{-\epsilon}\right\}^{-1}$ captures the distorsions created by price dispersion in the Home economy.

Finally, for the Foreign economy, letting $(1-n)\,Y_t^* = \int_n^1 \frac{P_t^{F*}(j)Y_t^{F*}(j)}{P_t^*}dj$ denote the foreign real GDP, the equilibrium conditions imply:

$$Y_t^* \quad = \quad Y_t^{*d}\Omega_t^{-\phi}\left(\frac{P_t^*}{P_t^{F*}}\right)^{\phi-1} \tag{52}$$

and the aggregate production function is obtained following a similar procedure as above (the foreign labour market equilibrium condition being $\int_n^1 N_t^{F*}(j)\,dj = (1-n)\,H_t^*$):

$$Y_t^* \quad = \quad H_t^*\left(\frac{P_t^*}{P_t^{F*}}\right)^{-1} \tag{53}$$

1.5.1 Non-Financial Household Income

The structure of the model described so far implies that the equilibrium non-financial income of a given generation at Home is a constant share of aggregate nominal GDP. Using (34), (50), and the definition of aggregate profits at Home $nJ_t^H = \int_0^n J_t^H(j)\,dj$:

$$W_{i,t}H_{i,t} + \alpha_i P_t J_t - P_t T_{i,t} \quad = \quad \alpha_i P_t Y_t \tag{54}$$

Following a similar case, the total nominal non-financial income of the foreign household is equal to $P_t^* Y_t^*$. As discussed above, the parameters α_y, α_m, and α_o are set such that the profile of total non-financial income over the lifetime of an individual resembles a traditional life cycle pattern. Following Sheedy[22, 2013], this pattern is reduced to a structural parameter γ that relates the set of α_i as follows:

$$\alpha_y = 1-\beta\gamma \qquad \alpha_m = 1+(1+\beta)\,\gamma \qquad \alpha_o = 1-\gamma \tag{55}$$

with $0 \leq \gamma \leq 1$ representing the slope of the life-cycle income pattern. Total non-financial income will be therefore maximum while middle-aged and minimum while old. The structure of the Home economy can be reduced to a simple representative economy framework by setting $\gamma = 0$. In that case, all generations are exactly the same ex-ante and ex-post, and there will be no trade in the domestic financial market. The domestic financial market will emerge as a result of different generations having different incomes $(\gamma > 0)$, different propensities to save and/or borrow and a desire to smooth consumption across their lifetime.

1.6 Marginal Cost and the Phillips Curve

The Phillips curve of the Home economy can be derived from the first order conditions of households and from the pricing equations. From (17):

$$C_{i,t}^{\frac{1}{1+\eta}}\left(w_t N_t\right)^{\frac{\eta}{1+\eta}} \quad = \quad w_{i,t} \tag{56}$$

Using the definition of the real wage and the aggregate production function (51), the real marginal cost can be rewritten as:

$$x_t \quad = \quad \frac{C_{y,t}^{\frac{\alpha_y}{3}} C_{m,t}^{\frac{\alpha_m}{3}} C_{o,t}^{\frac{\alpha_o}{3}}}{\left(A_t\right)^{1+\eta}\left[\frac{Y_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)\right]^{-\eta}} \tag{57}$$

Using (41), (57) and $\mu(1-\tau)=1$, the pricing equation (42) can be reexpressed as:

$$\left[\frac{1}{1-\kappa}-\frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = \frac{C_{y,t}^{\frac{\alpha_y}{3}}C_{m,t}^{\frac{\alpha_m}{3}}C_{o,t}^{\frac{\alpha_o}{3}}}{(A_t)^{1+\eta}\left[\frac{Y_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)\right]^{-\eta}}\frac{P_t}{P_t^H} \tag{58}$$

Equation (58) represents the non-linear Phillips curve of the Home economy. The left hand side of this expression includes elements related to the degree of price stickiness and to inflation, whereas the right hand side relates to the real side of the economy (recall the ratio between P_t and P_t^H is related to the terms of trade, a real variable). For the foreign economy, the analogous expression is:

$$1 = \frac{C_t^{*\sigma}}{\left[Y_t^{F*}\left(\frac{P_t^*}{P_t^{F*}}\right)\right]^{-\eta}}\frac{P_t^*}{P_t^{F*}} \tag{59}$$

1.7 The Equilibrium of a Small Open Home Economy under Financial Autarky

The system that characterizes the world equilibrium is composed of equations (8)-(10), (14)-(16), (19), (21), (49), (52), (43), (58), (59), and a market clearing condition for the international bond market:

$$\frac{n}{3}\left(B_{y,t}+B_{m,t}\right)+(1-n)\,B_t^*=0 \tag{60}$$

using (23) to replace Q_t everywhere. Following di Paoli[16, 2009], the Home economy can be reduced to a small open economy by taking the limit of the system of equations when $n \to 0$ and $a_H^* \to 0$. For simplicity and to preserve international trade, the additional assumption of $\left(\frac{1-n}{n}\right)a_H^* \to 1-a_H$ is imposed.

It is also assumed that the Foreign economy is in a steady state with zero inflation and zero initial wealth, which in equilibrium implies $B^*=0$ for all t. The latter implies the following financial autarky condition:

$$B_{y,t}=-B_{m,t}=B_t \tag{61}$$

which indicates that any resources borrowed by the young generation must be lent by the middle-aged generation. It is in this sense that the financial market is domestic, under the assumption of financial autarky in international markets. The latter implies that the only possibility for domestic (that is, Home economy) households to share risk is through a domestic financial market. International risk sharing is not allowed in the model[12]. Adding up the set of budget constraints of the Home economy (8)-(10), using (54) and (55), the following simple trade balance condition is derived.

$$C_t=Y_t \tag{62}$$

[12]In a world without financial autarky, domestic households could potentially trade financial instruments and share risk with households in other countries. Thus, as mentioned before, the assumption of financial autarky "stacks the cards" against the ability of Home households to share risk and insure against productivity shocks. As will be seen in the section on optimal monetary policy, the policymaker does not have a strong incentive to improve on risk-sharing relative to the decentralized equilibrium. Therefore, if anything, the assumption of financial autarky goes against the results of the paper, insofar as eliminating it would reduce further the incentive of the policymaker to take action to improve risk-sharing across households of the Home economy, as households would be able to share risk by other means (with the reset of the world). As a consequence, introducing some possibility of international risk sharing would only strengthen the conclusions of the simulations of optimal monetary policy presented below.

A similar condition for the Foreign economy ($C^* = Y^*$) is derived directly from (52) using $a_H^* \to 0$ and $P^* = P^{F*}$.

The condition $a_H^* \to 0$ also implies the following relationships between the terms of trade and the real exchange rate and the ratio between the CPI and the PPI in the Home economy:

$$\Omega_t = \left[a_H S_t^{\phi-1} + (1-a_H)\right]^{\frac{1}{\phi-1}} \tag{63}$$

$$\frac{P_t}{P_t^H} = \left[a_H + (1-a_H)\, S_t^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{64}$$

Equation (62) can be combined with (49), (63), (64) and the trade balance condition in the Foreign economy to obtain:

$$\left[a_H S_t^{\phi-1} + (1-a_H)\right]^{\frac{\phi}{\phi-1}} \Delta_t = S_t^{1-\phi} \tag{65}$$

where $\Delta_t = \frac{Y^*}{Y_t}$ is a measure of relative incomes across countries. Equation (65) determines the terms of trade of the Home economy.

These observations permit the reduction of the system that characterizes the equilibrium of the Home economy to the following equations (lower-case variables have been scaled by Y_t):

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}}\right)\right] = 1 \tag{66}$$

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{m,t}}{c_{o,t+1}}\right)\right] = 1 \tag{67}$$

$$\beta E_t \left[\frac{I_t}{\Pi_{t+1} G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}}\right)\right] = 1 \tag{68}$$

$$c_{y,t} + l_t = 1 - \beta\gamma \tag{69}$$

$$c_{m,t} - l_t = 1 + (1+\beta)\,\gamma + d_t \tag{70}$$

$$c_{o,t} = 1 - \gamma - d_t \tag{71}$$

$$\left[a_H S_t^{\phi-1} + (1-a_H)\right]^{\frac{\phi}{\phi-1}} \Delta_t = S_t^{1-\phi} \tag{72}$$

$$E_{t-1}\left\{\left(\check{p}_t^H\right)^{-\epsilon} Y_t^H \left(\check{p}_t^H - \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}\right)\right\} = 0 \tag{73}$$

$$\bigstar_t^{-\eta} (A_t)^{-\eta-1} Y_t^{(1+\eta)} c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \left[a_H + (1-a_H)\, S_t^{1-\phi}\right]^{\frac{1+\eta}{1-\phi}} = \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{74}$$

$$\left\{(1-\kappa)\left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}} + \kappa\left(\check{p}_t^H\right)^{-\epsilon}\right\}^{-1} = \bigstar_t \tag{75}$$

$$R_t = \frac{\bar{I}^* \xi_t}{\Pi_t} \tag{76}$$

where $c_{y,t} \equiv \frac{C_{y,t}}{Y_t}$, $c_{m,t} \equiv \frac{C_{m,t}}{Y_t}$, $c_{o,t} \equiv \frac{C_{o,t}}{Y_t}$, $G_t = \frac{Y_t}{Y_{t-1}}$, $\xi_t = \frac{e_t}{e_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $l_t = \frac{Q_t B_t e_t}{P_t Y_t}$, $d_t = \frac{B_{t-1} e_t}{P_t Y_t}$, $I_t = 1 + i_t$ and $\bar{I}^*$ is the gross steady state nominal interest rate of the Foreign economy. In this setting, l_t denotes the end-of-period debt-to-GDP ratio, whereas d_t denotes the ratio between debt maturing at the beginning

of time t and GDP. This beginning-of-period debt-to-GDP ratio will be the focus of the analysis of optimal monetary policy undertaken below, for it is intuitively the variable through which redistributions of wealth ex-post across borrowers and lenders operate. Equation (76) describes the ex-post real interest rate of the nominal, non-contingent bond traded within the domestic financial market of the Home economy[13]. As mentioned above, changes in this ex-post real interest rate (caused by changes in inflation or nominal depreciation) will create ex-post redistributions of wealth across lenders and borrowers, and therefore will be potentially important in the degree of risk-sharing across generations within the Home economy.

The system has the following endogenous variables: $c_{y,t}$, $c_{m,t}$, $c_{o,t}$, d_t , l_t, ξ_t , Π_t, I_t, R_t, $\bigstar_t$, $\check{p}_t^H$, S_t and Y_t. To this system of equations it is necessary to append the definition $d_t = l_{t-1}\frac{R_t}{G_t}$ and the policy rule. Following the optimal monetary policy literature, the policy rule will be provided by the solution to the optimal monetary policy problem in the form of a targeting rule for a set of key variables.

1.7.1 The Equilibrium of the Loglinearized System

Following standard practice, the non-linear system of equations (66)-(76) will be solved after being re-expressed in the form of logarithmic deviations from a deterministic steady state. In what follows, let $\tilde{x}_t = \ln\left(x_t/\bar{x}\right)$ denote the logarithmic deviation of variable x from its deterministic steady state value $\bar{x}$. The following proposition characterizes the deterministic steady state of the Home economy.

Proposition 1. *Assuming $\bar{A} = 1$ and $\bar{\Delta} = 1$, there exists a symmetric steady state where $\bar{c}_y = 1$, $\bar{c}_m = 1$, $\bar{c}_o = 1$, $\bar{l} = -\beta\gamma$, $\bar{d} = -\gamma$, $\beta\left(1+\bar{r}\right) = 1$, $\bar{S} = 1$, $\bar{Y} = 1$, $\bar{\check{p}}^H = \bar{\bigstar} = 1$ and $\bar{\xi} = \bar{\Pi}$.*
Proof: See Online Appendix 1.

The loglinearized system can be written as follows:

$$
\begin{aligned}
E_t\tilde{R}_{t+1} + \tilde{c}_{y,t} - E_t\tilde{G}_{t+1} - E_t\tilde{c}_{m,t+1} &= 0 && (77)\\
E_t\tilde{R}_{t+1} + \tilde{c}_{m,t} - E_t\tilde{G}_{t+1} - E_t\tilde{c}_{o,t+1} &= 0 && (78)\\
\tilde{I}_t &= E_t\tilde{\xi}_{t+1} && (79)\\
\tilde{c}_{y,t} &= \beta\gamma\tilde{l}_t && (80)\\
\tilde{c}_{m,t} + \beta\gamma\tilde{l}_t &= -\gamma\tilde{l}_{t-1} - \gamma\left(\tilde{R}_t - \tilde{G}_t\right) && (81)\\
\tilde{c}_{o,t} &= \gamma\tilde{l}_{t-1} + \gamma\left(\tilde{R}_t - \tilde{G}_t\right) && (82)\\
\tilde{d}_t &= \tilde{l}_{t-1} + \tilde{R}_t - \tilde{G}_t && (83)\\
\tilde{S}_t &= -\psi\tilde{Y}_t && (84)\\
-\frac{\kappa}{1-\kappa}\left(E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H\right) &= (1-\eta)\left(\tilde{Y}_t - \tilde{A}_t\right) + \left(\frac{\alpha_y}{3}\tilde{c}_{y,t} + \frac{\alpha_m}{3}\tilde{c}_{m,t} + \frac{\alpha_o}{3}\tilde{c}_{o,t}\right) && (85)\\
&\quad + (1-\eta)\left(1-a_H\right)\tilde{S}_t \\
\tilde{\bigstar}_t &= 0 && (86)\\
\tilde{R}_t &= -a_H\psi\left(\tilde{Y}_t - \tilde{Y}_{t-1}\right) && (87)
\end{aligned}
$$

[13]It is important to distinguish between the ex-post real interest rate of the economy, R, and the ex-ante real interest rate, which is related to the expectation of R and is crucial in the intertemporal allocation of consumption as indicated in equations (66)-(67).

with $\psi = [1 - \phi(1 + a_H)]^{-1}$. The following set of observations are in order.

First, equation (84) pins down the log-deviation of the terms of trade from its steady state only from the log-deviation of GDP. This relationship comes directly from equation (72).

Second, equation (87) reveals that the evolution of GDP also determines the log-deviation of the real interest rate. This equation is derived from the definition of the real interest rate (which in log-deviation corresponds to $\tilde{R}_t = \tilde{\Xi}_t - \tilde{\Pi}_t$), using equation (84) and the definitions of the log-deviations of S_t and Π_t:

$$\tilde{S}_t - \tilde{S}_{t-1} \equiv \tilde{\xi}_t - \tilde{\Pi}_t^H \qquad \tilde{\Pi}_t = a_H \tilde{\Pi}_t^H + (1 - a_H)\,\tilde{\xi}_t$$

with $\Pi_t^H = {}^{P_t^H}/_{P_{t-1}^H}$ being the PPI inflation rate of the Home economy. In other words, when expressed in loglinear deviations from the deterministic steady state, changes in real GDP are related to changes in the terms of trade that in turn create changes in the real interest rate through changes in the nominal depreciation and PPI inflation of the Home economy.

Third, equation (85) corresponds to the Phillips curve of the Home economy, and is derived as a log-linear approximation of the pricing equation (58) around the deterministic steady state, using the fact that $\tilde{p}_t^H = E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H$.

Finally, equation (79) is the Uncovered Interest Rate Parity Condition in log-deviation form.

1.7.2 The Natural Allocation and Risk-Sharing

The system of equations (77)-(87) is easily solved taking into account that the subsystem composed by equations (77)-(82) coincides exactly with the system of equations of a closed economy described in Sheedy[22, 2013]. The solution of this subsystem is therefore established immediately from the following proposition.

Proposition 2 *The solution of the system of equations (77)-(87) is given by:*

$$E_t \tilde{d}_{t+1} = \lambda \tilde{d}_t \tag{88}$$

$$\tilde{R}_t = \tilde{d}_t + \frac{\tilde{d}_{t-1}}{\theta} + \tilde{G}_t \tag{89}$$

$$\tilde{c}_{y,t} = -\frac{\gamma\beta}{\theta}\tilde{d}_t \qquad \tilde{c}_{m,t} = -\gamma\left(1 - \frac{\beta}{\theta}\right)\tilde{d}_t \qquad \tilde{c}_{o,t} = \gamma\tilde{d}_t \tag{90}$$

with θ and λ denoting combinations of structural parameters of the economy described in Online Appendix 2.

Proof: Sheedy[22, 2013].

A particular type of equilibrium allocation that will be useful is given by studying a hypothetical small open Home economy where markets are complete (that is, where there is full risk sharing) and prices are flexible ($\kappa = 0$). This allocation will be referred to as the "natural" allocation of the small open economy, and is characterized by the following proposition.

Proposition 3 *The natural allocation of the small open economy subsystem of equations (77)-(87) is given by:*

$$\begin{aligned} \tilde{d}_t^n &= 0 \\ \tilde{R}_t^n &= \tilde{G}_t^n \end{aligned}$$

$$\tilde{c}_{y,t}^n = \tilde{c}_{m,t}^n = \tilde{c}_{o,t}^n = 0$$

where "n" denotes the "natural" allocation.

Proof: Sheedy[22, 2013].

Equation (90) indicates that the fluctuations of consumption across the different generations of the Home economy are tightly linked in equilibrium to the fluctuations of the debt-to-GDP ratio, $\tilde{d}_t$. Proposition 3 indicates that, under complete markets at Home, $\tilde{d}_t$ is fully stabilized at zero. Therefore, the degree of risk-sharing across generations (that is, the degree in which the decentralized equilibrium replicates the complete markets allocation) depends crucially on the degree in which the debt-to-GDP ratio is stabilized. From equation (89), the stabilization of $\tilde{d}_t$ is related in equilibrium to the degree to which the real interest rate responds (in the same direction) to changes in the growth rate of the economy. That is, a recessionary shock should trigger a fall in the real interest rate, and vice versa. From equation (84), this latter response is related to the endogenous reaction of output fluctuations to shocks to labour productivity, $\tilde{A}_t$.

Considering only the objective of replicating the complete markets allocation and improving on risk-sharing across generations, the required response of the real interest rate to productivity shocks also highlights the role of the nominal exchange rate in completing markets. Given PPI inflation, equation (76) indicates that the fall in the real interest rate required after a recessionary shock in order to help replicate the complete markets allocation can be brought about only through a nominal appreciation, which in turn implies an increase in S_t. This observation lends a key role to the nominal exchange rate in risk-sharing in a world where financial transactions are denominated in foreign currency: given PPI inflation, nominal appreciation reduces the real burden of debt and redistributes wealth from the creditor generation to the debtor generation. The quantitative relevance of this role will be explored in detail below.

The model thus implies a trade-off for monetary policy in the face of technological shocks: the desire by the policymaker to steer the economy towards a more stable debt-to-GDP ratio for more risk-sharing across generations potentially requires a more volatile output and (due to the Phillips curve) more volatile inflation, which generally will also be part of the objective of the policymaker. This trade-off will depend on the specific set of parameter values chosen and is therefore a quantitative matter.

Using the results from Propositions 2 and 3, defining the output gap of the Home economy as $\hat{Y}_t \equiv \tilde{Y}_t - \tilde{Y}_t^n = \tilde{Y}_t - \frac{\tilde{A}_t}{1-(1-a_H)\psi}$ and the terms of trade gap as $\hat{S}_t = \tilde{S}_t - \tilde{S}_t^n = \tilde{S}_t + \frac{\psi}{1-(1-a_H)\psi}\tilde{A}_t$, the system of equations that characterizes the equilibrium of the Home economy can be reduced to:

$$E_t\tilde{d}_{t+1} = \lambda\tilde{d}_t \tag{91}$$

$$(1+a_H\psi)\left(\hat{Y}_t - \hat{Y}_{t-1}\right) = -\tilde{d}_t - \frac{\tilde{d}_{t-1}}{\theta} - \frac{1+a_H\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - \tilde{A}_{t-1}\right) \tag{92}$$

$$-\frac{\kappa}{1-\kappa}\left(E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H\right) = (1+\eta)\hat{Y}_t - \frac{\zeta}{\theta}\tilde{d}_t + (1+\eta)(1-a_H)\hat{S}_t \tag{93}$$

$$\hat{S}_t = -\psi \hat{Y}_t \tag{94}$$

where $\zeta = \left[\frac{1-\beta\gamma}{3}\beta\gamma + \gamma(\theta-\beta)\frac{1+\gamma(1+\beta)}{3} - \gamma\theta\frac{1-\gamma}{3}\right]$. Given the policy rule and a stochastic process for $\tilde{A}_t$, this system of equations provides the solution for the endogenous variables $\tilde{d}_t$, $\hat{Y}_t$, $\hat{S}_t$ and $\tilde{\Pi}_t^H$. This system of equations will constitute the set of constraints on a policymaker seeking to establish and implement an optimal monetary policy strategy. The following section is devoted to the analysis of the problem of the policymaker.

2 Optimal Monetary Policy

The Optimal Monetary Policy strategy will result from a benevolent policymaker/Central Bank who attempts to maximize the following welfare function, which comprises the weighted sum of utilities of every generation living in the Home economy at all times:

$$W_o = E_0\left[\frac{1}{3}\sum_{t=-2}^{\infty}\beta^t U_t\right] \tag{95}$$

subject to the system of equations (91)-(94)[14]. This optimization problem will be solved using the common approach in the New Keynesian literature developed by Rotemberg and Woodford[20, 1998] and Benigno and Woodford[4, 2004], which consists in constructing a second order approximation of the welfare function using the original system of non-linear equilibrium conditions. Online Appendix 3 demonstrates that problem of the policymaker is approximately equivalent to the minimization of the following loss function:

$$\begin{aligned}\mathcal{L}_0 = {} & E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\frac{\chi}{\theta^2}\tilde{d}_t^2 + \frac{(1+\eta)}{2}\hat{Y}_t^2 + \frac{\epsilon}{2}\left(\frac{\kappa}{1-\kappa}\right)\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)^2\right. \\ & + (1-a_H)\hat{S}_t + (1+\eta)(1-a_H)\hat{Y}_t\hat{S}_t + \frac{1}{2}(1-a_H)\left[(\phi+\eta)(1-a_H)+(1-\phi)\right]\hat{S}_t^2 \\ & \left. - \frac{\psi(1-a_H)a_H(1-\phi)}{1-(1-a_H)\psi}\hat{S}_t\tilde{A}_t\right\}\end{aligned} \tag{96}$$

where χ is a combination of structural parameters of the economy (see Online Appendix 2). The policymaker picks optimal sequences for $\tilde{d}_t$, $\hat{Y}_t$, $\hat{S}_t$ and $\tilde{\Pi}_t^H$ subject to the system of linear constraints (91)-(94) given by the first order approximation to the system of non-linear equilibrium conditions. Online Appendix 4 shows that the solution to this optimization problem is reduced to the following system of linear equations:

$$\Theta_\pi\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) = -\frac{\Theta_d}{\theta}\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right) - \frac{\Theta_A\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{97}$$

$$\frac{\kappa}{1-\kappa}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) = (1+\eta)(1-\psi+\psi a_H)\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) - \frac{\zeta}{\theta}\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right) \tag{98}$$

$$(1+a_H\psi)\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) = -\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right) - \frac{1+a_H\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{99}$$

[14][NOTE ON WEIGHTS]

where Θ_π, Θ_d and Θ_A are combinations of structural parameters of the economy (see Online Appendix 2). Equation (97) corresponds to the first order condition of the optimization problem. This condition encapsulates the intuition on optimal policy and risk-sharing that has been discussed throughout the paper. For the calibration employed below, Θ_π, Θ_d and Θ_A are all positive, and $\psi < 0$. Thus, given PPI inflation, the optimal response to a negative productivity shock is to reduce the debt-to-GDP ratio. This is precisely the risk-sharing objective pursued by the policymaker. The linear system of equations provides an analytical solution for the unanticipated responses of the output gap ($\hat{Y}_t - E_{t-1}\hat{Y}_t$), the debt-to-GDP ratio ($\tilde{d}_t - E_{t-1}\tilde{d}_t$) and PPI inflation ($\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H$) as a function of the only driving process of the model, the unanticipated innovation in labour productivity ($\tilde{A}_t - E_{t-1}\tilde{A}_t$). It is also possible to calculate the response of the terms of trade, the real return and the nominal exchange rate. The unanticipated response of the terms of trade follows directly from (94):

$$\hat{S}_t - E_{t-1}\hat{S}_t = -\psi\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) \tag{100}$$

For the real return, recall the definitions of the real return in (87) and the output gap $\tilde{Y}_t = \hat{Y}_t + \frac{\tilde{A}_t}{1-(1-a_H)\psi}$:

$$\tilde{R}_t = -a_H\psi\left(\hat{Y}_t - \hat{Y}_{t-1}\right) - \frac{a_H\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - \tilde{A}_{t-1}\right) \tag{101}$$

The unanticipated response of the nominal exchange rate, key in determining the degree of risk-sharing ex-post across generations, is calculated as the residual expression resulting from the combination of the terms of trade response and the Home inflation response:

$$\tilde{\xi}_t - E_{t-1}\tilde{\xi}_t = \left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + \left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) - \frac{\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{102}$$

2.1 Cole and Obstfeld[8, 1991] extended

Before proceeding to the calculation of the set of unanticipated responses, the model admits an extension of a parametric condition first discussed in Cole and Obstfeld[8, 1991], by which the Home and Foreign economies fully share risk despite being unable to trade financial claims in a world under financial autarky. This result was derived in the context of a model with two economies that feature a representative agent and operate under financial autarky.

The following proposition generalizes the result by showing that the same parametric condition guarantees full risk sharing both across countries and across generations within the Home economy.

Proposition 4 *If the parametric condition $\phi = 1$ is imposed (and countries are assumed to have initial zero net foreign assets), the following are true:*

1. *The optimal monetary policy problem rule achieves:*

$$\hat{Y}_t - E_{t-1}\hat{Y}_t = \tilde{d}_t - E_{t-1}\tilde{d}_t = \tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0 \tag{103}$$

2. *There is full risk-sharing across countries:*

$$\frac{\Omega_t}{\Omega_{t+1}}\frac{C_t^*}{C_{t+1}^*} = \frac{C_t}{C_{t+1}} \tag{104}$$

3. *There is full risk-sharing across generations:*

$$\tilde{c}_{y,t} - \tilde{c}_{m,t+1} = \tilde{c}_{m,t} - \tilde{c}_{o,t+1} \tag{105}$$

Proof:

1. *If $\phi = 1$, $\psi = -a_H^{-1}$ and $\Theta_A = 0$ (see Appendix 2). Thus, equation (99) implies $\tilde{d}_t - E_{t-1}\tilde{d}_t = 0$ (irrespective of monetary policy, the unanticipated response of the debt-to-GDP ratio is zero) and the first order condition (97) implies that the policymaker chooses $\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0$. The Phillips curve (98) implies $\hat{Y}_t - E_{t-1}\hat{Y}_t = 0$ □.*

2. *Under the assumption of zero initial net assets and given that the Foreign economy is assumed to be in steady state, Equation (104) implies: $\tilde{C}_t = \tilde{\Omega}_t = a_H\tilde{S}_t = -a_H\psi\tilde{Y}_t$, where the second equality comes from the loglinear first-order approximation to (63) and the third from (84). If $\phi = 1$, $\psi = -a_H^{-1}$ and the international risk sharing condition collapses to $\tilde{C}_t = \tilde{Y}_t$, which is equation (62) and holds in the equilibrium of the model □.*

3. *From the first part of this proposition, $\tilde{d}_t = E_{t-1}\tilde{d}_t = \lambda d_{t-1}$, where the last equality comes from the equilibrium condition (91). Given $|\lambda| < 1$, $\tilde{d}_t = 0$. The proposition follows from the equilibrium allocation of consumption given in (90) □.*

Intuitively, the condition $\phi = 1$ guarantees full risk-sharing across generations because it guarantees that the output gap (and therefore the real interest rate) will react to productivity innovations in a way that is consistent with the full stabilization of the debt-to-GDP ratio in equation (89). Proposition 4 shows that the condition $\phi = 1$ has generally stronger implications than considered in standard, representative agent models of the open economy, by providing full risk-sharing and allowing the policymaker to achieve full macroeconomic stabilization, defined in this context as a situation in which no inflation, output gap or debt fluctuations occur.

3 Alternative Policy Regimes

This section calculates the set of unanticipated responses that solve the system of equations (97)-(99) for a particular baseline calibration in order to explore the effect of some key parameters and to compare the optimal policy rule with alternative policy regimes. The set of alternative policy regimes that will be studied in this paper follows the seminal work by Galí and Monacelli[13, 2005]:

1. Producer Price Index Inflation Targeting (PPI-IT): A regime of PPI-IT would seek to have:

$$\tilde{\Pi}_t^H = 0$$

for all t. A commitment to follow this policy would imply that the surprise component of domestic inflation is set to zero:

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0 \tag{106}$$

The responses of variables can be obtained by solving the equilibrium system given by (98), (99) and (106).

2. Consumer Price Index Inflation Targeting (CPI-IT): A regime of CPI inflation targeting would seek to have:

$$\tilde{\Pi}_t = 0$$

From the definition of CPI inflation, $\tilde{\Pi}_t = a_H \tilde{\Pi}_t^H + (1 - a_H)\, \tilde{\xi}_t$. Thus, the policy prescribes:

$$\tilde{\xi}_t \quad = \quad -\left(\frac{a_H}{1 - a_H}\right) \tilde{\Pi}_t^H$$

From this equation, it is apparent that a policy regime of CPI-IT postulates a particular relationship between the nominal exchange rate and PPI inflation. This observation will be relevant when discussing the intuition for the suboptimality of CPI-IT below. From (102):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = -\left(1 - a_H\right)\left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + \left(1 - a_H\right) \frac{\psi}{1 - \left(1 - a_H\right)\psi} \left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right)$$

and substituting this into (100):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = \left(1 - a_H\right)\psi\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) + \left(1 - a_H\right) \frac{\psi}{1 - \left(1 - a_H\right)\psi} \left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{107}$$

The responses of variables are now obtained by solving the equilibrium system given by (98), (99) and (107).

3. Exchange Rate Peg: A fixed exchange rate regime would seek to have:

$$\tilde{\xi}_t = 0$$

Therefore, from (102):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = -\left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + \frac{\psi}{1 - \left(1 - a_H\right)\psi} \left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right)$$

and from (100):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = \psi\left(\hat{Y}_t^H - E_{t-1}\hat{Y}_t^H\right) + \frac{\psi}{1 - \left(1 - a_H\right)\psi} \left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{108}$$

The responses of variables are now obtained by solving the equilibrium system given by (98), (99) and (108).

The set of baseline parameters employed in this exercise is described below.

3.1 Calibration

Table 1 presents the set of baseline structural parameters employed in the calculations of section 3.2. The baseline values of a subset of the parameters have been borrowed from different papers on open economy models under complete markets and representative agents. This subset includes the parameters ϵ, ϕ and a_H. The baseline value for parameter η has been chosen so that the Frisch elasticity of labour supply is 0.4, in line with recent estimations for the US economy (Reichling and Wahleng[19, 2005]).

The parameters β and γ have been calibrated, to follow the factual motivation described in the introduction, to target some key moments of the Hungarian economy. To perform this calibration, one period of the model is taken to represent 10 years in the data, to make it consistent with the generational interpretation of the physical environment of the model. In particular, β has been chosen to mach a steady state real interest rate of 7%, which is the average real interest rate of mortgage loans denominated in Swiss Francs (CHF) in Hungary during the period 2005-2010. The slope of the life-cycle income γ has been calibrated to match the steady state debt-to-GDP ratio. Given the model's focus on consumption and private debt, γ is chosen to target a ratio of total household debt denominated in foreign currency to private comsumption of 58%, observed at the peak of the penetration of foreign currency debt in Hungary in the first quarter of 2009.

The parameter κ (the fraction of firms that update prices with outdated information) is also subject to the time convention of 10 years in the data corresponding to one period in the model. The inverse of $1-\kappa$ relates to the average duration of a spell of time without a given firm updating the information it uses to set prices (see Sheedy[22, 2013]). This spell is taken to be two years and a half for the baseline calibration. Finally, it is assumed in what follows that the stochastic process $\tilde{A}_t$ is white noise, which implies that $E_{t-1}\tilde{A}_t = 0$.

Parameter	Interpretation	Value	Target/Source
η	Inverse Frisch elasticity	2.5	Frisch elasticity = 0.4 (Reichling and Wahleng[19, 2005])
ϵ	Elasticity of substitution across varieties	10	Benigno and Woodford[5, 2005]
β	Discount factor	0.59	Real Rate on CHF Loans =7% (Hungary)
γ	Slope of life-cycle income	0.29	FC Debt/Consumption = 58% (Hungary)
ϕ	Elasticity of substitution across H and F	1.5	di Paoli[16, 2009]
a_H	Home Bias	0.7	di Paoli[16, 2009]
κ	Fraction of firms that update prices with outdated information	0.2	Information update every 2.5 years

Table 1: Baseline parameters

3.2 Calculation of Responses

Figures 1 and 2 show the responses of the endogenous unanticipated components of macroeconomic variables ($\hat{Y}_t - E_{t-1}\hat{Y}_t$, $\tilde{d}_t - E_{t-1}\tilde{d}_t$, $\hat{S}_t - E_{t-1}\hat{S}_t$ $\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H$, $\tilde{\xi}_t - E_{t-1}\tilde{\xi}_t$ and $\tilde{R}_t - E_{t-1}\tilde{R}_t$) to a negative

one percent shock to labour productivity for different values of ϕ and κ. These results are discussed in turn.

3.2.1 The effect of ϕ

Figure 1 shows the unanticipated response of the set of variables to a negative one percent shock to labour productivity as a function of the elasticity of substitution ϕ, where ϕ is set in the range from 1 to ϵ. The qualitative responses are robust to changes in ϕ (for both the optimal monetary policy rule and the alternative policy regimes).

A negative shock to productivity triggers a fall in the real interest rate in an attempt by the policymaker to stabilize the debt-to-GDP ratio and redistribute wealth from creditors to debtors with the goal of improving ex-ante risk-sharing. This fall in the real interest rate is brought about by a nominal appreciation. The reaction of the nominal exchange rate lies at the heart of the effort of improving on risk-sharing. In doing so, however, the policymaker does not achieve a significant improvement in risk-sharing across generations, as is evident from the fact that the debt-to-GDP ratio is not significantly more stable than in alternative regimes.

The fall in the real interest rate brought about by an increase in the output gap (see equation (87)), which necessarily implies a (relatively small) positive reaction of inflation through the Phillips curve. The volatility of both the output gap and inflation is significantly smaller under the optimal policy rule than under alternative regimes. It is in this sense that the results indicate the policymaker prefers to concentrate on the stabilization of standard macroeconomic variables (the output gap and inflation) at the expense of not being active enough in boosting risk-sharing. The policymaker faces a trade-off between standard macroeconomic objectives and risk-sharing, and the latter proves to be very costly to undertake under the baseline calibration. In other words, the objective of stabilizing the debt-to-GDP ratio would require a significant surprise in terms output and inflation, which the policymaker find suboptimal.

For this reason, the optimal policy rule is closest to PPI-IT than to any other alternative regime: the stabilization of PPI inflation is approximately equivalent to the implementation of the flexible-price equilibrium in a context where only technology shocks drive economic fluctuations. The stabilization of PPI inflation gives priority to standard macroeconomic objectives (output gap and inflation volatility) over risk-sharing considerations.

The suboptimality of CPI-IT and of an exchange rate peg is precisely related to the fact that these regimes create excessive volatility in output and inflation. Under the exchange rate peg, the fall in the real interest rate requires a very strong positive response of inflation, which is related to a strong positive response of output. CPI-IT allows some nominal appreciation and therefore reduces the response of inflation, but the dynamic behavior imposed by CPI-IT on the nominal exchange rate interferes with the role of the latter in contributing to risk-sharing and creates higher volatility of inflation and output.

The effect of the elasticity of substitution ϕ on these responses can be understood from the results described in Proposition 4 and equation (94). When $\phi \to 1$, Proposition 4 indicates that the optimal policy rule achieves full stabilization of output, inflation and the debt-to-GDP ratio. The stabilization of the latter implies that the real return reacts negatively by the exact amount needed to neutralize the effect of shocks to productivity. This negative response is related to a strong nominal appreciation. On

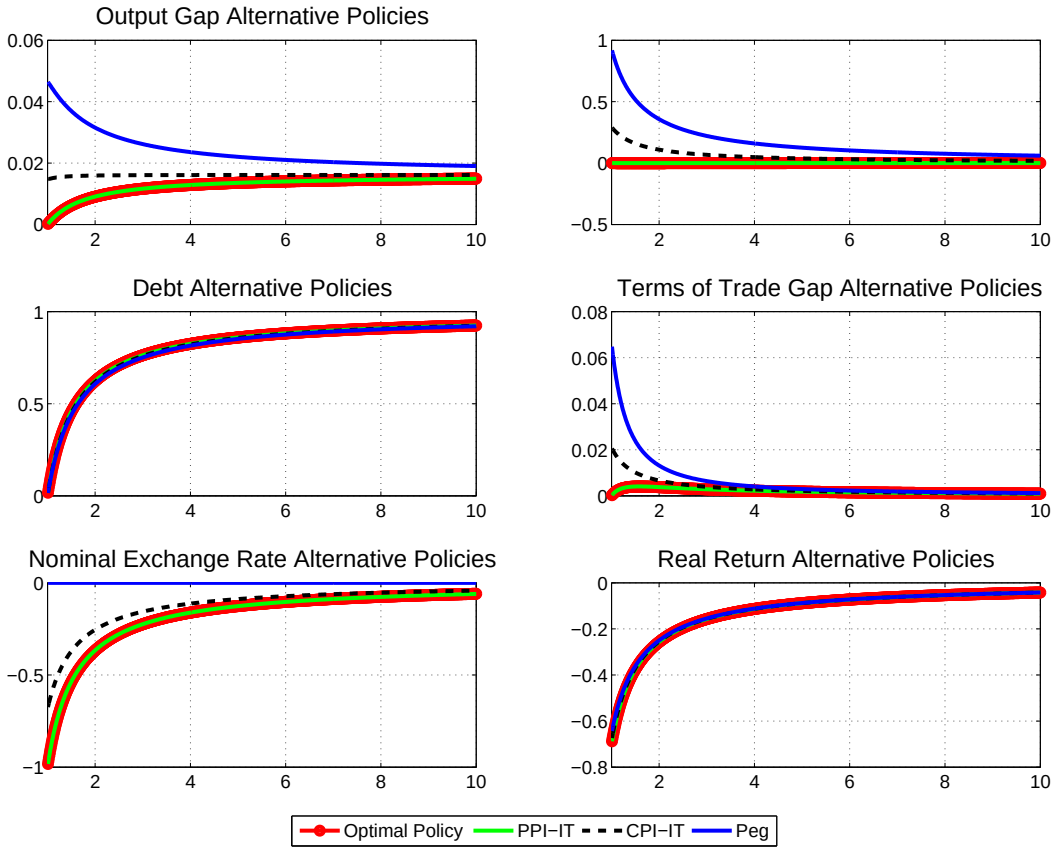

Figure 1: The effect of ϕ

the other hand, taking the limit of the economy when $\phi \to \infty$, equations (87) and (94) imply $\hat{S}_t \to 0$ and $\tilde{R}_t \to 0$. When Home and Foreign goods are perfect substitutes ($\phi \to \infty$), the policymaker does not have any "traction" over the terms of trade or the real interest rate of the economy. Therefore, as ϕ increases, the reaction of the output gap and inflation under the optimal policy rule has to be stronger to generate a smaller fall in the real interest rate, as long as monetary policy is less "powerful". Interestingly, for $\phi > 1$, a fall in the elasticity of substitution ϕ has two effects: it increases the power of monetary policy on key variables (the terms of trade and the real return in particular) but reduces the incentive of the policymaker to take action thanks to the results of Proposition 4. On the other hand, an increase in ϕ reduces the power of monetary policy in a context where a more decisive action from it is required.

3.2.2 The effect of κ

Figure 2 calculates the unanticipated response of the set of variables to a negative one percent shock to labour productivity as a function of the "information updating" parameter κ, where $\kappa \in (0, 1)$. To highlight the differences between the optimal policy rule and PPI-IT (not perceived at the scale of Figure 1), Figure 2 focuses only on these two policy regimes. Similar to the previous case, the qualitative responses are robust to changes in κ for both the optimal monetary policy rule and all the alternative policy regimes.

The intuitive interpretation of the responses in Figure 2 is the same as that described for the case of ϕ in Figure 1, that is, the optimal policy engineers a fall in the real interest rate and a nominal appreciation in order to limit the response of the debt-to-GDP ratio, and to achieve this, a positive reation of the output gap and PPI inflation are required. The latter two are almost negligible from a quantitative point of view, and indicate that regardless of the value of κ, the optimal policy rule achieves a high degree of stability of both variables.

Given ϕ, an increase in κ (which makes the economy more rigid and the Phillips curve flatter) reduces the inflationary cost of a given output gap response. However, the cost of improving on risk-sharing becomes larger as the rigidity of the economy increases. As κ increases, the policymaker finds it less desirable to stabilize the real interest rate and the debt-to-GDP ratio compared to the objective of output gap stabilization, which gains prominence naturally as the greater rigidity of the economy impairs its natural ability to stabilize itself after a negative productivity shock.

The following section focuses again on the remaining alternative regimes and offers a quantitative assessment of the degree of suboptimality created by CPI-IT and an Exchange Rate Peg (which are furthest away from the optimal policy rule).

4 Welfare Losses

The welfare losses attached to each alternative policy regime can be calculated as the unconditional expectation of the loss function, $E(L)$. To calculate the components of the loss function, it is useful to write:

$$\tilde{d}_t = \lambda \tilde{d}_{t-1} + \left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)$$

Using the the standard result for the mean and variance of a stationary AR(1) process:

$$E\left(\tilde{d}_t\right) = 0 \qquad E\left(\tilde{d}_t^2\right) = \frac{V\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)}{1-\lambda^2}$$

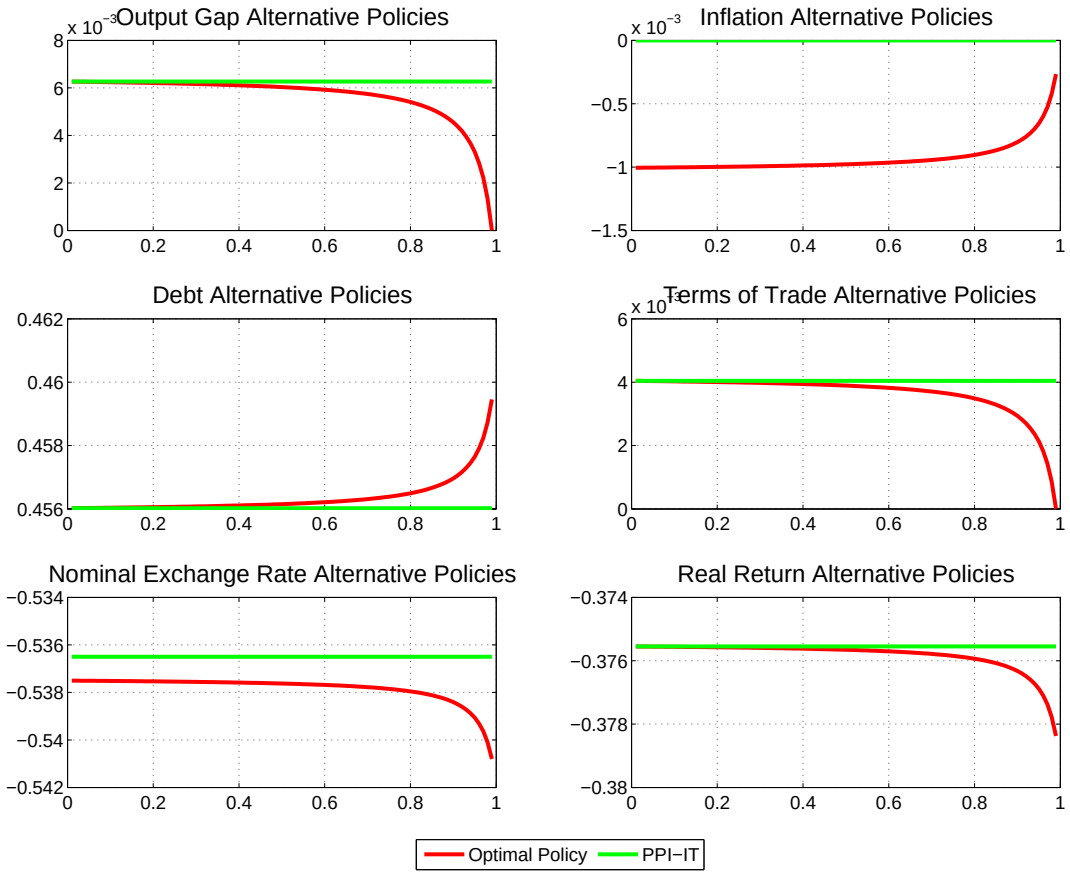

Figure 2: The effect of κ

For the variance of output, we can follow a similar strategy to equation (93).
As $E_{t-1}\hat{Y}_t = -\frac{\zeta}{(1+\eta)(1-\psi+\psi a_H)}\lambda\tilde{d}_{t-1}$, by definition:

$$\begin{aligned}
\hat{Y}_t &= -\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\tilde{d}_{t-1} + \left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) \\
E\left(\hat{Y}_t\right)^2 &= \left[\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\right]^2 \frac{V\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)}{1-\lambda^2} + V\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right)
\end{aligned}$$

As $E\left(\tilde{d}_t\right) = 0$, $E\left(\hat{Y}_t\right) = 0$ and thus:

$$E\left(\tilde{A}_t\hat{Y}_t\right) = cov\left(\tilde{A}_t, \hat{Y}_t\right) = cov\left(\tilde{A}_t - E_{t-1}\tilde{A}_t, \hat{Y}_t - E_{t-1}\hat{Y}_t\right)$$

The welfare criterion is thus:

$$\begin{aligned}
E(L) &= \sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\frac{\chi}{\theta^2}\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2} + \frac{\epsilon}{2}\frac{\kappa}{1-\kappa}V\left(\tilde{\Pi}^{H,s}\right)\right. \\
&\quad \left. + \frac{\delta}{2}\left[\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\right]^2\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2} + \frac{\delta}{2}V\left(\hat{Y}_t^s\right) + \frac{\varrho\psi}{1-(1-a_H)\psi}cov\left(\tilde{A}_t, \hat{Y}_t^S\right)\right\} \quad (109) \\
&= \frac{1}{1-\beta}\left\{\frac{1}{2}\frac{\chi}{\theta^2}\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2} + \frac{\epsilon}{2}\frac{\kappa}{1-\kappa}V\left(\tilde{\Pi}^{H,s}\right) + \frac{\delta}{2}\left[\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\right]^2\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2}\right. \\
&\quad \left. + \frac{\delta}{2}V\left(\hat{Y}_t^s\right) + \frac{\varrho\psi}{1-(1-a_H)\psi}cov\left(\tilde{A}_t, \hat{Y}_t^S\right)\right\} \quad (110)
\end{aligned}$$

where $x_t^s = x_t - E_{t-1}x_t$ for a given variable x. Evaluating the expected loss requires a particular value of $V\left(\tilde{A}_t\right)$ to be specified. But given the interest of this paper in relative welfare losses (compared to alternative policy regimes and other models), no value for this parameter is provided and, instead, expression (**??**) is scaled over $V\left(\tilde{A}_t\right)$, in which form it can be calculated directly from the structural parameters of the economy.

Table 2 compares the welfare losses (relative to the variance of the shock $V\left(\tilde{A}_t\right)$) of the regimes of CPI-IT and the Exchange Rate Peg for different values of κ, ϕ and two different sets of models derived from different parametric conditions: firstly, the "OLG" columns correspond to the baseline calibration shown above for the overlapping generations structure presented. Secondly, the "Representative Agent" columns correspond to a calculation of the welfare criterion under the condition $\gamma = 0$. As discussed above, this condition reduces the model to a standard, representative agent open economy by eliminating the life-cycle pattern of income and therefore the need for domestic financial markets.

The entries in the table are interpreted as permanent reductions in consumption under a given regime relative to steady state scaled by $V\left(\tilde{A}_t\right)$. The tables suggest that the exchange rate peg is generally more costly than the regime of CPI-IT. Besides, as ϕ increases from 1.5 (table 2(a)) to 6 (table 2(b)), the losses from both regimes become much larger than in a representative agent model. This allows to conclude that, given the role of the nominal exchange rate in completing markets in the overlapping generations model, the losses of a Peg and of a regime of CPI-IT are much larger compared to the ones calculated previously for models of the small open economy based on the representative agent assumption (see Galí and Monacelli[13]). Therefore, despite the relatively limited involvement of optimal monetary policy in

	OLG		Representative Agent	
	CPI-IT	Peg	CPI-IT	Peg
$\kappa = 0.2$	3.41	5.15	3.41	5.16
$\kappa = 0.66$	9.79	13.74	9.83	13.82

(a) $\phi = 1.5$

$\phi = 6$	OLG		Representative Agent	
	CPI-IT	Peg	CPI-IT	Peg
$\kappa = 0.2$	7.50	10.20	3.80	5.89
$\kappa = 0.66$	17.60	25.33	14.25	21.58

(b) $\phi = 6$

Table 2: Welfare Losses

risk-sharing (compared to standard macroeconomic objectives), the nominal exchange rate plays a role in ex-post redistributions of wealth that are quantitatively important from the point of view of the welfare losses of households in the Home economy.

5 Concluding Comments

This paper has characterized optimal monetary and exchange rate policies for a small open economy under incomplete markets at the local level and financial instruments denominated in foreign currency. Several conclusions arise from this effort.

The main finding of the paper is that the risk-sharing considerations which arise from market incompleteness introduce a new trade-off for monetary policy under financial autarky. After any productivity shock, the variations in the real exchange rate and the real return required to replicate the complete markets allocation imply excessive volatility in the traditional macroeconomic objectives of output and inflation. Under the calibration considered, optimal policy resolves this trade-off in favour of the traditional objectives. Consequently, the optimal policy is closest to Producer Price Index (PPI) Inflation Targeting than to the more standard Consumer Price Index (CPI) Inflation Targeting or the more extreme Exchange Rate Peg, as the former is closest to the flexible price allocation (the traditional aim of a broad range of New Keynesian optimal policy models). The cost of this strategy is an excessively volatile debt-to-GDP ratio and therefore imperfect risk-sharing across the different generations of the economy.

This result does not imply, however, total passivity of the optimal rule to risk-sharing/financial considerations. In particular, the optimal policy rule prescribes that the nominal exchange rate should appreciate after a negative productivity shock (and vice versa) so that the real burden of outstanding liabilities in foreign currency fall after a bad shock and wealth is transferred from the creditor generation to the debtor generation. Indeed, although the optimal policy is relatively passive when it comes to risk-sharing, being excessively passive (as implied by an Exchange Rate Peg) creates significant welfare losses on households, that significantly exceed those calculated by the literature under representative agent frameworks. The ability of the nominal exchange rate to react to shocks in a specific fashion is

therefore crucial, as far as household welfare is concerned.

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