

On the Relationship Between Government Spending Multiplier and Welfare

Ksenia Koloskova *

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Abstract

Aggregate output effect of fiscal stimulus, summarized by the size of the multiplier, has been extensively studied in recent years, however little attention has been given to understanding the welfare content of this statistic. In this paper I address the question whether relationship between government spending multiplier and welfare is monotone, i.e. if a higher multiplier implies a higher welfare gain from a particular policy. I compare a representative agent economy to an economy in which agents face idiosyncratic shocks and markets are incomplete. In the latter case, welfare implications of the multiplier depend on the distribution of welfare gains across heterogeneous households. I find that certain combinations of structural parameter values can produce a higher cumulative multiplier but also a larger dispersion of welfare gains, with the poorest households losing the most. The real interest rate behavior is the main factor defining how gains and losses are divided between wealth rich and wealth poor. This result is in contrast to a representative agent model, in which the cumulative output effect of government spending is a good indicator for welfare change.

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*Bocconi University. *E-mail:* ksenia.koloskova@phd.unibocconi.it. I thank my advisors Nicola Pavoni and Roberto Perotti for their guidance and support, as well as Arpad Abraham, Alberto Alesina, Todd Mattina, Pascal Michallait, Marcos Poplawski-Ribeiro, Daniele Siena, Frank Smets, and seminar participants at Bocconi University for helpful comments and discussions.

1 Introduction

Fiscal issues have been in the spotlight of academic and policy debate in recent years, with particular focus set on the size of the government spending multiplier, i.e. the amount of extra GDP generated per unit increase in government spending. Substantial attention has been given to the problem of identifying exogenous changes in government spending and taxation in the data, as various identification schemes produce alternative results¹. Many authors explored multipliers in structural models with different features². However, little attention has been given to understanding the welfare content of the multiplier statistic. It remains unclear whether the size of the multiplier can provide broader information on the impact of fiscal policy.

The present work aims at establishing a relation between the size of the spending multiplier and the change in social welfare, induced by increase in government spending. In particular, the paper assesses whether there exists monotonicity between the change in welfare and change in aggregate output so that a higher multiplier implies a higher welfare gain from a particular fiscal policy. The type of government spending I study is government purchases of goods and services³. An important assumption is that these purchases enter private utility in a separable manner, and they do not affect productivity of private resources in production.

In a representative agent model, adopted by many studies on the multiplier, the effects of increase in spending on aggregate output are easily mapped into changes in consumption and hours, and therefore into welfare of the representative agent. I start my analysis by demonstrating that in this environment change in welfare is proportional to the long-run cumulative multiplier⁴ with the size of the labor wedge. The intuition behind this positive monotone relationship is that increase in government spending pushes output up, bringing it closer to the efficient level, and thus reducing welfare losses from the causes that made output inefficient in the first place (monopoly pricing or distortionary taxation in my model).

While the representative agent framework establishes a clear relationship between

¹Blanchard and Perotti (2002), Ramey and Shapiro (1998), Ramey (2009), Perotti (2011) are just a few examples.

²See, for example, Galí et al. (2007), Monacelli and Perotti (2008), Bilbiie (2009), Uhlig (2010), Christiano et al. (2009), Drautzburg and Uhlig (2011).

³In general government spending includes government purchases, transfers and interest payments.

⁴A long-run cumulative multiplier captures overall dynamic effect of a fiscal expansion. In a model with departures from Ricardian equivalence, such as the one studied in this paper, the short- and the long-run multipliers generally differ from each other.

the multiplier and welfare, it does not take into account heterogeneity and market incompleteness. Both features are realistic and have been found relevant by previous literature⁵. Redistributive issues are also an important part of the recent policy discussion, as austerity measures in Europe and debate over marginal tax rates for the rich in the US have brought into light the problem of winners and losers of fiscal adjustments. If the economy is characterized by an unequal distribution of capital and labor income across households, then changes in current and future taxation as well as wages and real interest rates induce uneven distribution of gains and losses from change in fiscal policy. Naturally, in such environment the aggregate output response to a policy might be not be sufficient to draw welfare-related conclusions.

I use a framework with heterogeneity across agents (Aiyagari-Huggett-Bewley) and distortionary taxation. The fiscal shock is a persistent increase in government spending, financed by increase in public debt with delayed debt stabilization via labor income tax. I study multipliers and expected welfare gains for different combinations of intertemporal elasticity of substitution and Frisch elasticity of labor supply. These parameters, unlike for example the level of the price mark-up or share of government spending in GDP, cannot be easily computed from the data, and there is little agreement in the literature about their values. Their choice turns out to have important implications for the relation between multiplier and welfare gains of a particular fiscal policy.

Depending on the values of structural parameters, the same policy can result in different aggregate and redistributive effects. What matters for the redistribution of wealth is the real interest rate behavior. A smooth path of output and earnings over the transition after the spending shock results in a large increase in the real interest rate because the desire of agents for self-insurance via accumulation of a buffer stock of savings is moderate so they have to be compensated more for holding government debt. Redistribution from wealth poor to wealth rich is high even though the long-run cumulative multiplier might be large. On the contrary, if output expands strongly in the short run but also declines significantly in the future as taxes increase, then agents' desired buffer stock of saving is high, which eliminates the need for the interest rate to increase dramatically. Therefore a lower long-run cumulative multiplier does not necessarily correspond to a more unequal distribution of gains nor lower welfare gains at the bottom.

The main message of the paper is that the size of government spending multiplier, even if one looks at its cumulative long-run value, can be of limited use in evaluating

⁵See, for example, Attanasio and Davis (1996).

welfare implications of a temporary fiscal expansion. There exist plausible combinations of structural parameter values, across which monotonicity between the multiplier and welfare in a heterogeneous agent environment is not preserved. It is possible that a higher cumulative multiplier can be associated with a larger welfare loss for the poorest households in the economy. Therefore, the policymakers should evaluate gains and losses for different groups of population, on top of evaluating aggregate output expansion.

Although the welfare consequences of government spending have great importance for policy analysis, little effort has been made so far to relate the study of (predominantly short-run) effects of fiscal policy on aggregate activity to private welfare. Previous research in this area has been limited to few studies (Woodford (2011), Mankiw and Weinzierl (2011)), and has been restricted to a representative agent framework with lump-sum taxation. Woodford (2011) establishes two benchmark results. First, he shows that in a static model without frictions optimal government purchases satisfy equality of marginal utility of public and private consumption. Second, Woodford (2011) shows that there is a scope for fiscal stabilization policy if output is suboptimal (in his example output is below efficient in a time of recession due to inability of prices and wages to react), and change in welfare is proportional to the change in output with the size of the wedge in consumption/leisure optimality condition.

The most related to the current analysis is the paper by Mankiw and Weinzierl (2011). In a two-period model with sticky prices and zero lower bound they examine alternative fiscal policies (government spending vs. investment subsidy financed by lump-sum taxes) aimed at restoring full employment. Mankiw and Weinzierl (2011) find that the policy that is best for welfare, which includes optimal mix of increase in spending and large increase in investment subsidy in the first period, is worst according to the multiplier metric. An important message of the paper is that the "bang-for-the-buck" calculations do not take into account the composition of GDP.

The work by Monacelli and Perotti (2011) on the consequences of tax burden distribution for the size of the multiplier is another important study relevant for current analysis. The main result of the paper is that in environment with sticky prices it matters for the size of output response to a government spending shock which type of agent bears the major part of taxation. The multiplier is larger if (lump sum) taxes are levied mainly on the unconstrained savers as opposed to credit constrained borrowers.

This paper differs from the previous literature in several aspects. First, the relationship between the multiplier and welfare is the primary focus of this paper, while previous work focused on the size of the multiplier (Woodford (2011)) or on the optimal

fiscal (Mankiw and Weinzierl (2011)) and mix of monetary and fiscal policies (Woodford (2011)). Second, a distinctive feature of my analysis is taking into account heterogeneity of agents and importance of distributional effects. Third, instead of doing comparisons across alternative fiscal policies (Mankiw and Weinzierl (2011), Monacelli and Perotti (2011)), I explore the consequences of uncertainty about structural parameters. Instead of focusing on the composition of GDP as the culprit of poor welfare performance of the multiplier I bring forward redistributive aspect of fiscal expansion.

The paper is organized as follows. Section 2 sets up the heterogeneous agent environment. Benchmark findings from a representative agent model are established in Section 3. Section 4 presents welfare decomposition in a heterogeneous agent framework. I proceed with describing numerical analysis and its results in Section 5. Finally, Section 6 concludes.

2 Model

I use a framework similar to Huggett (1993). There is continuum of infinitely lived agents of measure 1, who receive idiosyncratic shocks to labor income against which they cannot fully insure. Agents maximize their expected discounted utility by choosing optimal amounts of consumption and labor supply. Savings are invested in one-period government debt which yields a risk-free return. Since the paper is focused on the effects of increase in government spending, the model does not feature any other sources of aggregate risk such as productivity shock. The shock to government spending is a one-time unexpected shock with a deterministic transition back to the steady state.

2.1 Households

Each agent's productivity $s \in S = \{s^1, \dots, s^N\}$ evolves according to an N-state Markov process. I denote the transition probability matrix Π , where $\Pi_{ij} = Pr(s_{t+1} = s^j | s_t = s^i)$ is the probability that next period productivity is s^j given that current productivity is s^i . Period t productivity level is realized before period t decisions are made. Let $s^t = \{s_0, \dots, s_t\}$ denote a history of idiosyncratic shocks from date 0 to date t , originating from s_0 , and $P(s^t)$ denote the probability of this history.

Denote the set of possible values for individual wealth a_t as $E = [-\bar{a}, a_{max}]$. Denote $X = E \times S$ the set all possible individual states. An element of this set x is a pair of individual (endogenous and exogenous) states (a, s) , characterizing each agent's position at each point in time. Unconditional distribution of (a, s) pairs is $\lambda_t(x) = \text{Prob}(a_t =$

$a, s_t = s$). The probability measure $\lambda(x)$ is defined over the Borel σ -algebra of X .

At time 0 each agent is characterized by her initial wealth and initial productivity level, summarized by $x_0 = (a_0, s_0)$. Agents have identical preferences over consumption, hours and government consumption sequences, described by the following expected discounted utility

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t(x_0, s^t), l_t(x_0, s^t), g_t) \quad (1)$$

where $\beta \in (0, 1)$, $c_t(x_0, s^t)$ is level of consumption, $l_t(x_0, s^t)$ is proportion of time devoted to working activities, and g_t is government consumption.

The budget constraint is given by

$$c_t(x_0, s^t) + a_{t+1}(x_0, s^t) = (1 + r_t)a_t(x_0, s^{t-1}) + (1 - \tau_t^l)s_t w_t l_t(x_0, s^t) + \Gamma_t \quad \forall s^t, t, \quad (2)$$

where r_t is the risk-free interest rate, Γ_t is the profits of firms redistributed to households as dividends, and $(1 - \tau_t^l)s_t w_t l_t(x_0, s^t)$ is the after tax labor income of an agent.

Financial markets are incomplete, and the only asset agents can use to smooth consumption is a one-period risk-free government bond, which they trade subject to a borrowing constraint

$$a_{t+1}(x_0, s^t) \geq -\bar{a}, \quad \bar{a} > 0 \quad (3)$$

where $\bar{a} = \min \left\{ b, \sum_{j=0}^{\infty} \frac{\Gamma_{t+j}}{\prod_{i=0}^j (1+r_{t+i})} \right\}$ with b being an arbitrary "ad hoc" borrowing limit, and $\sum_{j=0}^{\infty} \frac{\Gamma_{t+j}}{\prod_{i=0}^j (1+r_{t+i})}$ being the "natural" borrowing limit.

Measure λ_0 describes the distribution of agents across the joint individual state (a, s) at time 0. The social welfare function is defined as

$$V = \sum_{x_0} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t(x_0, s^t), l_t(x_0, s^t), g_t) \lambda_0(x_0) \quad (4)$$

Equation (4) describes the average lifetime discounted utility, where each individual's utility is given the same weight. Therefore it can be considered a utilitarian welfare function. This welfare criterion can be also thought of as ex-ante welfare of a household at the steady state, i.e. welfare of a household before it learns its initial asset position and productivity level⁶.

⁶For more discussion about this welfare criterion see Aiyagari and McGrattan (1998).

2.2 Firms

A. FINAL GOOD PRODUCER. A perfectly competitive firm produces the final good using differentiated varieties y_{it} with the following technology

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \quad (5)$$

The production function is a CES function, where $\theta > 1$ is the elasticity of substitution across intermediate varieties.

Denoting p_t the price of good Y_t and p_{it} the price of y_{it} , demand for each variety from the final good producer, derived from profit maximization problem, is

$$y_{it}^d = \left(\frac{p_{it}}{p_t} \right)^{-\theta} Y_t \quad (6)$$

B. INTERMEDIATE GOODS PRODUCERS. There is monopolistic competition in the intermediate goods sector. Under the assumption of flexible prices, each producer sets the price according to the profit maximization problem

$$\begin{aligned} \max_{p_{it}} \quad & p_{it}y_{it} - W_t l_{it}, \\ \text{s.t.} \quad & y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\theta} Y_t, \\ & y_{it} = l_{it}, \end{aligned}$$

where Y_t is the final good, p_t is the aggregate price level, and W_t is the nominal wage, and technology is linear in labor.

If the firm is able to choose its price freely, the optimal price is set as a constant mark-up over the nominal wage

$$p_{it} = \frac{\theta}{\theta-1} W_t = \mu W_t.$$

Under assumption that all firms are symmetric, they will charge the same price, therefore $p_{it} = p_{kt} = p_t$. The real wage is then simply the inverse of the mark-up

$$w_t = \frac{W_t}{p_t} = \frac{\theta-1}{\theta} = \frac{1}{\mu}$$

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C. MARK-UP BEHAVIOR. Galí et al. (2007), Monacelli and Perotti (2008) and Woodford (2011), among others, point out that response of the mark-up to the business cycle can

affect the size of the multiplier. In particular, if one assumes that nominal prices of intermediate goods are sticky and nominal wages are flexible, the real wages increase in reaction to a positive government spending shock while firms' mark-ups decline. This mechanism facilitates producing a positive consumption multiplier, found in empirical literature⁷, because increase in real wages makes it possible for the marginal utility of consumption to decrease as hours expand⁸. This mechanism could be potentially important for my question, because the size of output expansion affects aggregate welfare gain from a policy and increase in real wages also has redistributive implications for households with different productivity.

To take into account this effect of fiscal policy on real wages I allow for movements in the mark-up in response to government spending shock. However I use a short cut for modeling this part of the environment. Similar to Hall (2009), I assume that the mark-up is a constant elasticity function of output:

$$\mu_t = \tilde{\mu} Y_t^{-\omega}. \quad (7)$$

A positive ω implies a countercyclical mark-up. If one has in mind a model with sticky goods prices⁹ and flexible nominal wages, parameter ω captures both the degree of price rigidity and the degree of monetary policy accommodation in response to a fiscal shock. Note that this formulation is compatible with any explanation of a negative relationship between output and mark-ups.¹⁰

Hall (2009) compares multipliers from a model using the stylized mark-up equation and a New Keynesian model with monetary policy and price rigidity and concludes that the functional form (7) is adequate for inferring effects of government spending on output and consumption. Given that building a large empirically relevant model is beyond the scope of this paper, the simple functional form above suffices for the question I address.

⁷Blanchard and Perotti (2002), Perotti (2008), Galí et al. (2007) are a few examples.

⁸Recall the first order condition: $-u_l(c, l) = w u_c(c, l)$.

⁹The relationship between sticky prices and countercyclical mark-ups has been studied, for example, by Rotemberg and Woodford (1992).

¹⁰It remains an open question whether in reality mark-ups show a countercyclical behavior. Bils (1987) finds that mark-up of price over marginal cost decreases in booms and goes up in recessions. Nekarda and Ramey (2013) revisit this finding with new data and arrive to a conclusion that mark-ups behave procyclically.

2.3 Government

The government collects taxes to finance an exogenously given level of government spending and interest payments on outstanding government debt. The government budget constraint in real terms is

$$\tau_t^l w_t L_t + B_{t+1} = (1 + r_t) B_t + g_t. \quad (8)$$

Government consumption behaves according to the following process

$$g_t = (1 - \rho) \bar{g} + \rho g_{t-1} \quad (9)$$

$$g_0 = \bar{g} + \epsilon_0 \quad (10)$$

where ϵ_0 is the unexpected shock to spending at the beginning of period 0. After the shock occurs, the transition back to the steady state is deterministic.

The government issues short-term debt to finance higher level spending, adjusting the taxes according to the rule $\tau_t^l = \phi_b B_t$. This tax rule is similar to the one used by Uhlig (2010). The difference is that Uhlig (2010) specifies the rule in terms of overall tax revenue from labor income tax, while here the rule is specified in terms of the tax rate.

2.4 Market clearing

Capital market equilibrium implies that total asset holdings (total net saving) of the private sector is equal to the government debt

$$A_{t+1} = \sum_x \sum_{s^t} P(s^t) a_{t+1}(x_0, s^t) \lambda_t(x) = B_{t+1}.$$

Labor market equilibrium analogously implies equality of total effective hours supplied by households to total labor demand by intermediate goods producers

$$L_t^s \equiv \sum_x \sum_{s^t} P(s^t) s_t l_t(x_0, s^t) \lambda_t(x) = \int_0^1 l_i di \equiv L_t^d.$$

The final goods market equilibrium condition follows from the two conditions above and integration of individual budget constraints

$$Y_t = C_t + g_t,$$

where $C_t = \sum_x \sum_{s^t} P(s^t) c_t(x_0, s^t) \lambda_t(x)$.

3 Representative agent benchmark

It is useful to start with multiplier and welfare analysis in a representative agent model. In this simple model there is clear link between the size of the output response and change in welfare if output is below its efficient level¹¹.

3.1 Welfare

The representative agent maximizes her value function by choosing optimal sequences of consumption, hour worked and asset holdings. All variables are functions of ϵ_0 , the shock to government spending process, meaning that whenever $\epsilon_0 = 0$ they are at their steady state level, and whenever ϵ_0 is different from zero, they take the values of the t -th period of transition. The problem of the representative agent is:

$$\begin{aligned}
V_0^*(\epsilon_0) &= \max_{\{C_t(\epsilon_0), L_t(\epsilon_0), A_{t+1}(\epsilon_0)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t(\epsilon_0), L_t(\epsilon_0), g_t(\epsilon_0)) \\
\text{s.t. } C_t(\epsilon_0) + A_{t+1}(\epsilon_0) &= (1 + r_t(\epsilon_0))A_t(\epsilon_0) + (1 - \tau_t^l(\epsilon_0))w_t(\epsilon_0)L_t(\epsilon_0) + \Gamma_t(\epsilon_0) \\
L_t(\epsilon_0) &\in [0, 1] \\
C_t(\epsilon_0) &\geq 0 \\
A_0 &\text{ is given} \\
\lim_{T \rightarrow \infty} \frac{A_{T+1}}{(1 + r)^{T+1}} &\geq 0.
\end{aligned}$$

The maximum value of the problem above is given by

$$V_0^*(\epsilon_0) = \sum_{t=0}^{\infty} \beta^t u(C_t^*(\epsilon_0), L_t^*(\epsilon_0), g_t(\epsilon_0)), \quad (11)$$

where $C_t^*(\epsilon_0)$ and $L_t^*(\epsilon_0)$ satisfy optimality condition

$$(1 - \tau_t^l(\epsilon_0))w_t(\epsilon_0)u_{C_t^*(\epsilon_0)} + u_{L_t^*(\epsilon_0)} = 0. \quad (12)$$

I use $u_{C_t^*}$ and $u_{L_t^*}$ as a short notation for $u_c(C_t^*, L_t^*, g_t)$ and $u_l(C_t^*, L_t^*, g_t)$ respectively. Furthermore, $C_t^*(\epsilon_0)$ and $L_t^*(\epsilon_0)$ satisfy the market clearing condition¹²

$$L_t^*(\epsilon_0) = C_t^*(\epsilon_0) + g_t(\epsilon_0) \quad (13)$$

¹¹Subsection 3.1 extends some baseline results in Woodford (2011), Sections 5.1 and 5.2., who relates the change in welfare to the output effect of fiscal expansion.

¹²Recall that technology is linear, i.e. $Y_t^*(\epsilon_0) = L_t^*(\epsilon_0)$.

In what follows I drop the superscript *, keeping in mind that all variables' sequences are optimal choices of the representative agent. I will use a variable *without* subscripts to indicate its value at $\epsilon_0 = 0$, i.e. its steady state value, $C = C(0)$, and I will use a variable *with* subscript t to indicate the value of the variable at the t -th period of transition, $C_t = C_t(\epsilon_0)$.

The change in welfare of a representative agent can be derived by differentiating the maximum value of the problem with respect to ϵ_0 , having substituted into it the budget constraint, and applying the envelope theorem. The resulting expression is

$$\frac{dV_0}{d\epsilon_0} = \sum_{t=0}^{\infty} \beta^t \frac{dg_t}{d\epsilon_0} (u_g - u_C) + u_C \sum_{t=0}^{\infty} \beta^t \left(1 + \frac{u_L}{u_C}\right) \frac{dY_t}{d\epsilon_0} \quad (14)$$

The first term $\sum_{t=0}^{\infty} \beta^t \frac{dg_t}{d\epsilon_0} (u_g - u_C)$ is related to the difference between marginal utilities of public and private consumption. In a model without inefficiencies the optimal level of government spending is such that $u_g = u_C$ (Woodford (2011)), which means that marginal reallocation of resources between public and private consumption should not affect welfare. If agents do not value government spending, $u_g = 0$ and this term represents a pure welfare loss due to taking resources away from private consumption and wasting them.

The term $v \equiv \left[1 - \left(-\frac{u_L}{u_C}\right)\right]$ is the difference between the marginal rate of transformation $f'_L = 1$ and the marginal rate of substitution $-\frac{u_L}{u_C}$, which represents a wedge in the first order condition for consumption/leisure choice. Efficiency implies this wedge should be zero, i.e. $MRS=MRT$. In a model with distortions, like the one considered here, this difference is positive. The size of $v = (1 - (1 - \tau_l)/\mu)$ reflects the level of inefficiencies in the economy, which stem from monopoly power of firms, and distortionary taxation if taxes are proportional. There exists space for welfare improvement due to increase in spending in this case. Potential welfare gains come from the reduction of the dead weight loss, arising from distortions, as output moves closer to its efficient level¹³.

While the first term depends only on how agents value welfare and the size of the shock, the second term is proportional to the change in output. Notice that this decomposition of welfare change does not depend on the type of taxation nor on the way of financing increase in spending.

¹³These results can be related to the discussion in Hendren (2013). He shows that welfare impact of a policy includes a causal effect of the behavioral response to the policy change on government's budget, which is a term similar to $v \frac{dY_t}{d\epsilon_0}$. This effect is related to the presence of a fiscal externality, since the agent does not take into account the effect of her behavior on the government's budget constraint.

3.2 Multipliers

Government spending multiplier is defined as the change in GDP per unit change in government spending. I distinguish between a short-run multiplier, by which I mean impact multiplier, defined as

$$M_0 = \frac{dY_0}{d\epsilon_0}, \quad (15)$$

and a cumulative multiplier, computed according to

$$M_t = \frac{\sum_{s=0}^t (1+r)^{-s} \frac{dY_s}{d\epsilon_0}}{\sum_{s=0}^t (1+r)^{-s} \frac{dg_s}{d\epsilon_0}}. \quad (16)$$

I define the long-run multiplier as the cumulative multiplier after a sufficiently high number of periods T , when the transition after the shock is over and the economy is approximately back at the steady state

$$M_T = \frac{\sum_{s=0}^T (1+r)^{-s} \frac{dY_s}{d\epsilon_0}}{\sum_{s=0}^T (1+r)^{-s} \frac{dg_s}{d\epsilon_0}}. \quad (17)$$

With the multiplier definitions above, $\frac{dY_t}{d\epsilon_0}$ can be written as

$$\frac{dY_t}{d\epsilon_0} = (1+r)K_{t-1} [M_t - M_{t-1}] + \rho^t M_t, \quad K_t = \sum_{i=0}^t (1+r)^i \rho^{t-i}.$$

Then the welfare decomposition (14) takes the following form

$$\frac{dV_0}{d\epsilon_0} = \sum_{t=0}^{\infty} \beta^t \frac{dg_t}{d\epsilon_0} (u_g - u_C) + u_C v (1 - \beta(1+r)) \lim_{T \rightarrow \infty} \sum_{t=0}^{T-1} \beta^t K_t M_t + u_C v \lim_{T \rightarrow \infty} \beta^T K_T M_T.$$

In a representative agent model the steady state interest rate satisfies $\beta(1+r) = 1$, therefore the term $u_C v (1 - \beta(1+r)) \lim_{T \rightarrow \infty} \sum_{t=0}^{T-1} \beta^t K_t M_t$ is equal to zero. Given that M_T is the long-run multiplier, summarizing the whole transition of output back to the steady, it remains unchanged as T increases. Then it can be shown that

$$u_C v \lim_{T \rightarrow \infty} \beta^T K_T M_T = u_C v \frac{1+r}{1+r-\rho} M_T.$$

Proposition 1. *Let v be the labor wedge and M_T the long-run multiplier. Then the change in welfare is*

$$\frac{dV_0}{d\epsilon_0} = \sum_{t=0}^{\infty} \beta^t \frac{dg_t}{d\epsilon_0} (u_g - u_C) + u_C v \frac{1+r}{1+r-\rho} M_T. \quad (18)$$

Corollary 1. *In a representative agent model with suboptimal output ($v > 0$) the change in welfare due to a government spending shock is proportional to the long-run multiplier with a positive coefficient $v \frac{1+r}{1+r-\rho}$, where v is the labor wedge.*

Welfare decomposition 18 allows to address the question whether the threshold of 1 for the output multiplier has a welfare content. This value has been widely discussed in the fiscal policy literature. From the resource constraint $Y_t = C_t + g_t$ it follows that if the multiplier is greater than one, an increase in government spending has a positive effect on consumption. This increase in consumption might lead one to a false conclusion that in this situation increase in government spending is welfare-improving, even if it is a pure waste.

Corollary 2. *GDP multiplier of pure waste government spending above 1 does not imply that increase in spending improves welfare.*

If the government spending is not valued by agents, marginal utility of government spending is zero ($u_g = 0$). Using $\frac{dg_t}{d\epsilon_0} = \rho^t$, we can rewrite welfare decomposition (18) as follows

$$\frac{dV_0}{d\epsilon_0} = u_C \frac{1+r}{1+r-\rho} [vM_T - 1].$$

It is straightforward to see that the condition for welfare improvement in this case is

$$M_T > \frac{1}{v}. \quad (19)$$

The welfare-improving size of the multiplier therefore depends on the size of the inefficiencies. The size of the labor wedge has been estimated by Shimer (2005) at about 0.40 in normal times and 0.45 in recessions. This size can be reproduced in a model with inefficiencies stemming from mark-ups and distortionary taxes like the one presented here. If the share of profit is 20% and the labour income tax is 28%, the wedge $1 - \frac{1-\tau_l}{\mu} = \frac{1.2-0.72}{1.2}$ is 0.40. The multiplier should be above 2.5 to obtain an increase in welfare.

Indeed, (19) makes it clear that the only possibility for the multiplier of 1 to be threshold for a positive effect of policy on welfare is if $v = 1 - \frac{1-\tau_l}{\mu}$ is equal to 1, which is only possible if either $\tau_l = 1$ or $\mu = \infty$. Neither is a realistic case. Therefore, the question of whether the short-run multiplier is greater or smaller than 1 can only be related to the effect of government spending on aggregate consumption, while it is not a relevant number for welfare considerations even in the simplest model.

I proceed with discussing the determinants of the short-run and long-run multipliers. The size of the multiplier depends on the type of taxation the government uses. I wish to start with a short discussion of the multipliers when government spending is financed by lump-sum taxes to establish some benchmark results. I then proceed by studying multipliers and welfare in a model with labor income tax.

A. LUMP-SUM TAXATION. In a representative agent model with lump sum taxes Ricardian equivalence holds, which implies that the timing of taxes does not matter. All debt is held by the representative agent, and what affects agent's optimization problem is only the total amount of government expenditure, which needs to be financed. The multiplier and the welfare decomposition do not depend on the presence of debt.

In a model with lump-sum taxes therefore the impact multiplier is equal to multipliers at all other horizons, i.e.

$$M_0 = M_t = M_T, \quad \forall t.$$

Corollary 3. *In a representative agent model with lump-sum taxes and suboptimal output the change in welfare due to a government spending shock is proportional to the **short-run** multiplier with a positive coefficient $v \frac{1+r}{1+r-\rho}$.*

The proof follows from the welfare decomposition (18) and the equality of the short-run multiplier to the long-run multiplier.

The expression for the multiplier under lump-sum taxes is¹⁴

$$M_0^{ls} = \frac{\sigma}{\sigma + (1 - s_g) [\psi - \omega]}, \quad (20)$$

where σ is the inverse of intertemporal elasticity of substitution (IES) of consumption, ψ is the inverse of Frisch elasticity of labor supply, and s_g is the share of government spending in output.

One important conclusion from a model with lump sum taxes is that the set of factors, affecting the multiplier such as the monetary policy or parameters of the model, and the set of factors, affecting the change in welfare, given the multiplier, such as the size of the wedge $v = 1 - 1/\mu$, the steady state interest rate r , and persistence of the shock ρ , do not intersect. Therefore higher multiplier implies a higher positive effect on welfare. Thus, in a representative agent model with lump-sum taxes and suboptimal output the relationship between multiplier and change in welfare is positive monotone.

¹⁴For the derivation of the multiplier in a representative agent model with lump-sum taxes one can refer to Hall (2009), who also provides a comprehensive discussion of the determinants of the size of the multiplier in that model.

B. DISTORTIONARY TAXATION. *i. Static model.* I start by presenting multipliers in a static model. In this model the only choice an agent makes is between consumption and leisure, and the only way to finance increase in spending is to collect taxes in the same period. This simple case provides us with a useful starting point to build on.

In a static model the impact multiplier is equal to multipliers at all other horizons

$$M_0 = M_t = M_T, \quad \forall t,$$

therefore the change in welfare due to a government spending shock is proportional to the short-run multiplier.

The expression for the multiplier under distortionary taxes is

$$M_0^{dist} = \frac{(1 - \tau^l)\sigma - \mu(1 - s_g)}{(1 - \tau^l)\sigma + (1 - s_g)[\psi(1 - \tau_l) - \omega - \tau_l]}. \quad (21)$$

The sign and the size of the multiplier under distortionary taxes depend on the combination of parameters. However, three structural parameters σ , ψ and ω do not affect the monotonicity of multiplier and welfare relationship, because it is only the multiplier that depends on them. Any variation in one of these parameters, which drives up the multiplier, also increases welfare.

On the other hand, both the multiplier and the labor wedge depend on the size of the product mark-up μ and the labor tax τ^l . Variation in these parameters can potentially move the multiplier and the change in welfare in opposite directions.

Numerical results suggest this can be a relevant matter. I use the following benchmark values for parameters: $\sigma = 2$, $\psi = 1$, $\omega = 0.5$. The remaining three parameters, $\{s_g, \mu, \tau^l\}$ are calibrated jointly, because the government budget constraint imposes a relationship between them: $\tau^l/\mu = s_g$. I set $s_g = 0.25$ and $\mu = 10/9$ and then find value for τ_l , which makes the budget constraint hold.

The impact of varying μ , and adjusting τ_l to balance the budget¹⁵ on the multiplier and welfare depends on the combination of structural parameter values, such as σ , ψ , and ω . One plausible combination of parameters, under which monotonicity breaks, is $\sigma = 4$, $\epsilon = 1$ and $\omega = 0.5$. Higher steady state mark-up decreases the multiplier, but increases the labor wedge to the extent that the overall effect on the welfare is positive¹⁶. Alternatively, I keep the mark-up constant and vary the labor tax τ_l , allowing the share

¹⁵Another possibility is to allow s_g , the share of government spending, to adjust in response to μ . The results are qualitatively the same.

¹⁶The intuition behind this non-monotonicity is the following. Increase in μ , accompanied by an increase in τ_l , increases the wedge, and therefore the change in welfare for a given multiplier. However, higher steady state mark-up implies lower steady state real wage. Increase in the tax rate $\frac{d\tau_0^l}{d\epsilon_0}$, needed

of government spending adjust to balance the budget. The computations suggest that in this case monotonicity is preserved for combinations of realistic structural parameter values.

The break of monotonicity across the size of the mark-up is a potentially important case, showing that the same factors that lead to a smaller expansion in output can also increase the scope for welfare improvement as aggregate output expands, resulting in overall positive effect on welfare despite decline in the multiplier. However, this might be not the most empirically relevant problem. The wedge depends on parameters, which one can calculate from the data, such as the mark-up level (defines the share of profits in final output) and the labor income tax. This way one knows the scope for welfare improvement, given the multiplier. A more interesting case, related to variety in estimates of structural parameter values, such as σ and ψ , can be explored in a dynamic framework.

ii. Dynamic model. In a model with distortionary taxes the presence of government debt makes a difference in two ways. First, if some amount of debt exists in the steady state, even if the government keeps it constant after increase in spending, variation in the real interest rate might call for a higher increase in income tax rate, because the government needs to finance not only higher spending but also higher interest payments. Unlike in the lump-sum taxes case, under distortionary taxation higher taxes affect optimal consumption and hours choice, and therefore affect the multiplier.

Second, timing of taxes matters due to intertemporal substitution effect on labor supply, allowing for time-dependent dynamic multipliers. Positive short run effect of increase in spending financed by a temporary budget deficit, reflected in a relatively high impact multiplier, can be turned over by negative effect of higher tax burden in the future, which should be captured by the long run multiplier.

As I have shown earlier, what matters for the change in welfare in a representative agent environment is the long-run multiplier, which summarizes all output dynamics before the economy returns back to the steady state. In a dynamic environment with non-lump-sum taxes the size of the multiplier varies across different horizons, therefore the short-run multiplier might be a misleading statistic for welfare evaluation, unless it is a good predictor for the welfare-relevant long-run multiplier. I compute short-run and long-run multipliers numerically¹⁷.

to finance higher spending, is proportional to the steady state wage and hours (tax base), meaning that the lower is the wage, the higher is the needed increase in taxes. A higher increase in tax then translates into a lower multiplier due to the dampening effect on labor supply.

¹⁷The system of equations which is solved numerically is presented in Appendix B.

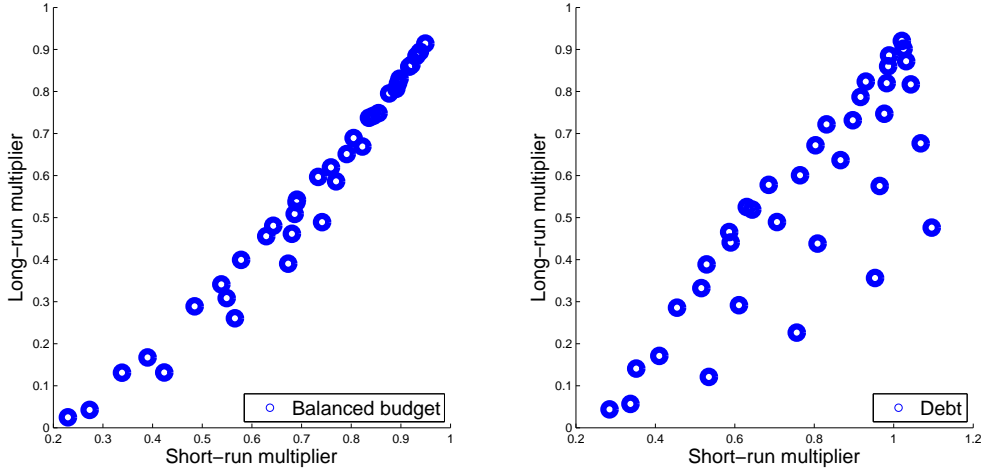


FIGURE 1. SR and LR multiplier across σ and ψ

Figure 1 shows short-run and long-run multipliers for different pairs of two structural parameters, σ and ψ . Other parameters take values: $\mu = 10/9$, $\tau_l = 0.28$, $s_g = 0.25$, $\phi_b = 0.5$, $\beta = 0.9$, $\rho = 0.9$, $\omega = 0.5$. The analysis in this Section considers two types of financing the increase in spending, balanced budget or issuing debt.

The left panel on Figure 1 represents the balanced budget case, while the right one shows the debt case. Under balanced budget long-run multiplier is typically higher than the short-run multiplier because taxes are front-loaded. The opposite is true for the deficit situation, because the negative effect of taxes kicks in later and is not reflected in the impact multiplier. Therefore depending on how government reallocates tax collection across time the short-run multiplier might under- or overestimate the welfare cost of the policy.

The choice of the two structural parameters, σ and ψ , is motivated by the fact that they cannot be readily computed from the data, and their estimates vary substantially across empirical studies. If the multipliers depended only on parameters easy to recover from the data, such as the level of the mark-up or share of government spending in GDP, it would have been straightforward to evaluate the welfare content of the short-run multiplier, once the size of the wedge and the fiscal and monetary policy is known. This vast uncertainty in structural parameters however poses a problem.

Empirical work has found Frisch labor supply elasticities as low as 0.1 (MaCurdy (1981)) and as high as 4 (Imai and Keane (2004)). Estimates using household level data typically find lower values, in range of 0.2 to 1. Domeij and Floden (2006) argue that the true value of elasticity might be twice the estimated value if the econometrician does

not take into account the presence of borrowing constraints. Studies taking into account movements in and out of employment and labor force (Rogerson and Wallenius (2009)) usually find high macro elasticity. I set a range for ψ between 0.25 to 5, implying a range for Frisch elasticity between 0.2 and 4. Studies of intertemporal elasticity of substitution (the inverse of σ) have little agreement on its value as well. Attanasio et al. (1995) use US household survey data and estimate IES around 0.7 with a relatively large standard error. Barsky (1997) find low elasticity around 0.2, while Guvenen (2006) suggests that this elasticity can be 1 for certain agents. I pick a range for σ from 1.5 to 6, which corresponds to intertemporal elasticity of substitution between slightly below 0.2 to 0.67.

The graph shows that although there is a positive correlation between the two multipliers conditional on the policy mix, the short-run multiplier is not a perfect predictor of the long-run multiplier. Consider an example. If $\sigma = 1.5$ and $\psi = 1$ the short-run multiplier is 0.76, while its long-run counterpart is 0.23. If instead σ is set to 4 and ψ to 2, the short-run multiplier is the same as in the previous case and equals to 0.76, while the long-run multiplier at 0.60 is almost three times as high.

The intuition behind these numbers relies on the opposite effects of ψ and σ on the multiplier. For a given σ , increase in ψ means that labor becomes less elastic, compressing the size of the hours response. For a given ψ , increase in σ lowers intertemporal elasticity of substitution so consumption is depressed less. Another implication of high σ , which follows from a limited decline in consumption, is that the real interest rate path is more favorable, calling for less increase in taxes due to the need to finance payments on government debt. Increase in both, σ and ψ , leaves the short-run multiplier almost unchanged, suggesting that the opposing driving forces, i.e. modest reaction of hours vs. modest reaction of consumption, compensate each other, while the third effect did not arrive yet due to delayed taxation. As time goes by and taxes start to increase the third effect gains more importance, allowing for a limited decline in output in the long run if σ is high. This last effect is further supported by low labor supply elasticity (high ψ), because hours don't react strongly to increase in taxes.

4 Heterogeneous agents

In this Section I study marginal welfare impact of change in fiscal policy in a model with heterogeneity and market incompleteness and show that this framework allows for additional welfare effects of increase in government spending compared to the

representative agent framework.

The welfare criterion (4) assigns an equal weight to each agent's welfare. Assuming that the weights are not affected by changes in government spending, the aggregate marginal welfare change is the weighted average of individual marginal welfare changes. Thus, I start with evaluating welfare impact of increase in government spending for an agent with state $x_0 = (a_0, s_0)$ at the time of the shock $t = 0$. The maximization problem of this agent in sequential form is

$$\begin{aligned}
V_0^*(x_0, \epsilon_0) &= \max_{\{c_t, l_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t(x_0, s^t, \epsilon_0), l_t(x_0, s^t, \epsilon_0), g_t(\epsilon_0)) \\
\text{s.t. } c_t(x_0, s^t, \epsilon_0) + a_{t+1}(x_0, s^t, \epsilon_0) &= (1 + r_t(\epsilon_0))a_t(x_0, s^{t-1}, \epsilon_0) \\
&\quad + (1 - \tau_t^l(\epsilon_0))s_t l_t(x_0, s^t, \epsilon_0) w_t(\epsilon_0) + \Gamma_t(\epsilon_0) \\
a_{t+1}(x_0, s^t, \epsilon_0) &\geq -\bar{a}, \quad \bar{a} > 0 \\
l_t(x_0, s^t, \epsilon_0) &\in [0, 1] \\
c_t(x_0, s^t, \epsilon_0) &\geq 0 \\
a_0, s_0 &\text{ are given.}
\end{aligned}$$

For convenience I denote the right hand side of the budget constraint as

$$\begin{aligned}
Y_t(x_0, s^t, \epsilon_0) \\
&\equiv (1 + r_t(\epsilon_0))a_t(x_0, s^{t-1}, \epsilon_0) + (1 - \tau_t^l(\epsilon_0))s_t l_t(x_0, s^t, \epsilon_0) w_t(\epsilon_0) + \Gamma_t(\epsilon_0). \quad (22)
\end{aligned}$$

All variables are functions of ϵ_0 , the initial shock to government spending. If $\epsilon_0 = 0$, the economy is at the steady state, and all variables are at their steady state levels¹⁸. When ϵ_0 is different from zero, the variables take their values of the t -th period of transition. After the shock occurs, the transition back to the steady state is deterministic. The sequences of prices, profits and taxes at all periods are deterministic, and known to agents.

The consumer problem is therefore a maximization problem with a parameter, and I study how the maximum value of the problem changes with the parameter. To do this, I differentiate V_0^* , the value function of an agent with initial state $x_0 = (a_0, s_0)$ at time $t = 0$, with respect to the parameter ϵ_0 which is the initial shock to spending.

¹⁸Notice, however, that in this model the steady state does not imply that individual variables, such as consumption or labor supply, are constant. Due to idiosyncratic shocks agents adjust their consumption and labor supply decisions every period even in the absence of aggregate shocks. The key difference from the transition is that in the steady state individual decisions follow time invariant decision rules.

The maximum value of the agent's problem is

$$V_0^*(x_0, \epsilon_0) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0))$$

where $\{c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0)\}_{t=0}^{\infty}$ are optimal sequences for consumption and labor supply. To simplify notation, in what follows I denote

$$u_t^*(x_0, s^t, \epsilon_0) \equiv u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0)),$$

$$u_{c_t}^*(x_0, s^t, \epsilon_0) \equiv \frac{\partial u_t^*(x_0, s^t, \epsilon_0)}{\partial c_t^*(x_0, s^t, \epsilon_0)}.$$

The welfare impact of increase in government spending is¹⁹

$$\frac{\partial V_0^*(x_0, 0)}{\partial \epsilon_0} = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) \left\{ u_{g_t}^*(x_0, s^t, 0) \frac{\partial g_t(\epsilon_0)}{\partial \epsilon_0} + u_{c_t}^*(x_0, s^t, 0) \frac{\partial \tilde{Y}_t(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} \right\}$$

where $\frac{\partial \tilde{Y}_t}{\partial \epsilon_0}$ stands for the change in Y_t due to change in variables which the agent takes as given (prices, taxes, dividends).

The welfare evaluation above shows that the increase in welfare stems from direct increase in utility due to increase in valued government consumption g , and from increase in total resources available for consumption due to changes variables taken as given by the consumer.

Let $\Lambda_t(x_0, 0) = \tilde{\phi}_t \sum_{i=t+1}^{\infty} \beta^{i-t} R^{i-t-1} \sum_{s^i} P(s^i) u_{c_i}^*(x_0, s^i, \epsilon_0) \frac{s_i l_i^*(x_0, s^i, 0)}{L_i(0)}$, where $\tilde{\phi}_t = \phi w_t(0) L_t(0)$, and $R = 1 + r_t(0) - \tilde{\phi}_b$. It represents the marginal private utility cost in period t of a unit increase in government spending at time t for an agent with initial state x_0 after history s^t , given that the taxes are collected via a proportional labor income tax and government finances increase in spending by running a short-run budget deficit.

Private net benefit of G . The first term in the welfare decomposition relates the marginal utility of public and private consumption:

$$\sum_{t=0}^{\infty} \beta^t \frac{\partial g_t(\epsilon_0)}{\partial \epsilon_0} [u_{g_t}^*(x_0, s^t, 0) - \Lambda_t(x_0, 0)]$$

It can be interpreted as the net willingness to pay for an additional unit of government spending. The marginal benefit is proportional to the expected marginal utility of government consumption. The marginal cost $\Lambda_t(x_0, 0)$ is the individual expected utility cost of a unit increase in tax revenues, which the government has to collect to finance

¹⁹See Appendix C for derivation.

increase in spending. If taxes are lump-sum, and the level of government debt does not change in response to policy, this cost is simply the expected marginal utility of 1 unit of foregone consumption. Under proportional labor income taxation the cost of unit increase in tax revenue is unequally distributed across individuals and is higher for those with high working hours. If the government responds to increase in spending by running budget deficits in the short-run, the individual cost of g is lower. Ricardian equivalence does not hold in this model, and delayed taxation is favorable for agents' welfare as the cost is discounted²⁰.

Redistribution due to change in the real interest rate and the real wage. The next two terms describe redistributive effects due to changes in prices:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \frac{\partial r_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) a_t^*(x_0, s^{t-1}, 0) - \Lambda_t(x_0, 0) B_t(0) \right] \\ & + \sum_{t=0}^{\infty} \beta^t \frac{\partial w_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) (1 - \tau_t^l(0)) s_t l_t^*(x_0, s^t, 0) \right. \\ & \quad \left. - L_t(0) \left(\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) - \tau_t^l(0) \Lambda_t(x_0, 0) \right) \right] \end{aligned}$$

If the interest rate increases, agents with low asset holdings suffer a welfare loss, while wealthier agents gain. The expected marginal welfare change is related to the difference between $a_t^*(x_0, s^{t-1}, 0)$, individual asset holdings, and $B_t(0)$, the amount of government debt equal to the average asset holdings. An increase in the interest rate provides more resources for consumption (given that $a_t^*(x_0, s^{t-1}, 0) > 0$), while it also implies that the government has to pay higher interest on her debt and needs to increase taxation. The first effect is proportional to the individual asset holdings, while the second is related to the level of government debt (average asset holdings). The difference across agents stems not only from being a borrower or a saver, but from having assets above or below than average, which is typically greater than 0. Borrowers lose the most. Similarly, if the real wage goes up, agents who work relatively more hours (wealth poor) gain, while those working relatively less lose. This difference comes from the fact that increase in wage increases resources for consumption proportionately to individual hours worked, while it decreases aggregate profits proportionately to aggregate hours.

Impact on government revenue and firms' profits. Finally, the last term is similar to

²⁰The model has a steady state property $\beta(1 + r) < 1$

the wedge term, discussed in a representative agent framework:

$$\sum_{t=0}^{\infty} \beta^t \frac{\partial L_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) (1 - w_t(0)) + \tau_t^l(0) w_t(0) \Lambda_t(x_0, 0) \right]$$

It captures the behavioral impact of increase in spending on government tax revenue and firms profits. Increase in aggregate output, caused by a unit increase in spending, raises aggregate profits by $(1 - w_t(0)) \frac{\partial L_t(0)}{\partial \epsilon_0}$ and raises aggregate tax revenues by $\tau_t^l(0) w_t(0) \frac{\partial L_t(0)}{\partial \epsilon_0}$. If profits were zero and taxes were lump sum, this term would not be present, because the response of hours to the shock would not affect any of the two. The presence of mark-ups and proportional taxes make the right hand side of the individual budget constraint depend on the response of aggregate hours to policy.

The overall expected marginal welfare increase is given by

$$\frac{\partial V_0^*(x_0, 0)}{\partial \epsilon_0} = \underbrace{\sum_{t=0}^{\infty} \beta^t \frac{\partial g_t(\epsilon_0)}{\partial \epsilon_0} [u_{g_t}^*(x_0, s^t, 0) - \Lambda_t(x_0, 0)]}_{\text{Private net benefit of G}} \quad (23)$$

$$+ \underbrace{\sum_{t=0}^{\infty} \beta^t \frac{\partial r_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) a_t^*(x_0, s^{t-1}, 0) - \Lambda_t(x_0, 0) B_t(0) \right]}_{\text{Redistribution due to change in real interest rate}} \quad (24)$$

$$+ \underbrace{\sum_{t=0}^{\infty} \beta^t \frac{\partial w_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) (1 - \tau_t^l(0)) s_t l_t^*(x_0, s^t, 0) - L_t(0) \left(\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) - \tau_t^l(0) \Lambda_t(x_0, 0) \right) \right]}_{\text{Redistribution due to change in real wage (cont'd)}} \quad (25)$$

$$+ \underbrace{\sum_{t=0}^{\infty} \beta^t \frac{\partial L_t(\epsilon_0)}{\partial \epsilon_0} \left[\sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, 0) (1 - w_t(0)) + \tau_t^l(0) w_t(0) \Lambda_t(x_0, 0) \right]}_{\text{Impact on government revenue and firms' profits}}. \quad (26)$$

I integrate individual welfare changes with respect to the stationary distribution $\lambda_0(x_0, 0)$ to get change in aggregate ex ante welfare.

5 Quantitative analysis

5.1 Parameterization

The model period is one quarter. Table 1 presents parameter values in quarterly terms. The model parameters and calibration targets are chosen to match US data.

A. PREFERENCES. Utility function is separable and isoelastic in consumption, hours and government purchases

$$u(c, l, g) = \frac{c^{1-\sigma}}{1-\sigma} - \gamma \frac{l^{1+\psi}}{1+\psi} + \chi \log(g),$$

where $\sigma > 0$ is the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution, $\psi > 0$ is the inverse of the Frisch elasticity of labor supply, $\gamma > 0$ defines the disutility of work, and χ captures how agent values public purchases of goods and services.

The discount factor β is calibrated to deliver a yearly interest rate of 2% in the steady state. Parameter γ is set to match average hours of work to be equal to 0.40. Relative preference for government consumption χ is set to 0, i.e. government spending is a pure waste. Since the focus of the study is on monotonicity properties between the multiplier and change in welfare, the size of χ does not matter for the main results of the paper because utility is separable in government consumption.

I evaluate multipliers and changes in welfare for three values for the coefficient of relative risk aversion, $\sigma = \{2, 4, 6\}$, and four values for the Frisch elasticity of labor supply, $1/\psi = \{0.2, 0.5, 1, 4\}$. Parameters are recalibrated for each combination of σ and ψ . In Table 1 only two sets of parameter values, for $(\sigma, 1/\psi) = (2, 1)$ and $(\sigma, 1/\psi) = (4, 0.5)$, are presented.

B. IDIOSYNCRATIC PRODUCTIVITY PROCESS AND CREDIT MARKET. Following Floden and Linde (2001), I assume that the idiosyncratic productivity process follows an $AR(1)$ in logs

$$\begin{aligned} \log(s_t) &= \varpi + \vartheta_t \\ \vartheta_t &= \rho \vartheta_{t-1} + \eta_t \end{aligned}$$

where ω is a permanent component, and ϑ_t is a temporary component which evolves stochastically over time with persistence ρ , η_t is *i.i.d.* $N(0, \sigma_\eta^2)$ and ϖ and ϑ_t are orthogonal. I assume the permanent component is absent, i.e. $\varpi = 0$, thus individual productivity shocks are purely transitory shocks²¹. Realizations s_t are independent across agents, therefore the cross-sectional distribution of idiosyncratic productivity at any point in time and in any aggregate state is log normal with mean 1.

Following Floden and Linde (2001) estimate individual wage process using yearly PSID data, and find a coefficient of autocorrelation for the transitory component to

²¹In general this is not the case, because the permanent component might be related to age, skill level, etc. I do not include these features in my model.

TABLE 1. Parameter values

Parameter ($\sigma, 1/\psi$)	Value (2, 1) (4, 0.5)		Calibration target
β	0.9848	0.9799	annual interest rate $r = 2\%$
γ	14.44	292.03	average hours are 0.40
χ	0		pure waste spending
ρ	0.9777		persistence of prod-ty process
σ_η	0.0117		variance of prod-ty process
$pr(s' \in S s \in S)$	0.9		prob of keeping positive prod-ty
$pr(s' = 0 s = 0)$	0.5		prob of keeping zero prod-ty
ϕ	-1.0678	-1.6002	share with neg wealth is 25%
θ	10		share of profits is 10%
ω	0.5		Hall (2009)
\bar{g}	0.08		share of G in Y is 20%
τ_l	0.2333		annual debt-to-GDP ratio is 50%
ϕ_b	0.5		response of tax rate to debt

be 0.9136 and variance to be 0.0426. The individual productivity process at quarterly frequency is calibrated to match these moments in yearly data. The continuous productivity process is approximated by a 7 state Markov chain using Tauchen (1986) method.

Individual productivity process also allows for state in which the productivity is equal to zero. This state can be interpreted as unemployment. Transition to and from this state is calibrated following Krusell et al. (1998). The transition probabilities imply an average duration of unemployment spell of 1.5 quarters²², and an average duration of a job of 2.5 years.

The borrowing limit ϕ is set such that the share of agents with zero or negative wealth is about 25%, which roughly corresponds to US data in 2007 (not taking into account illiquid wealth). The share of agents close to the borrowing limit is important for both the size of the multiplier and the evaluation of welfare. For the former it matters because agents with low wealth have stronger wealth effects. For the latter it matters because changes in the interest rate affect these agents differently than savers.

²²Duration is computed according to the formula $D = \frac{1}{1-Prob}$

C. TECHNOLOGY AND MARK-UPS. Intermediate goods technology is assumed to be linear in labor

$$y = l \quad (27)$$

The elasticity of substitution across intermediate varieties θ is set to 10, which corresponds to a 10% share of profits in final output in the steady state. I set $\omega = 0.5$ in line with Hall (2009).

D. FISCAL POLICY. Pre-shock level government spending is set to 20% of final output, corresponding to the historical share of total US government spending in GDP. The labour income tax τ_l was chosen to be 0.2333, which delivers government debt-to-GDP ratio of 50%. This value for the tax is roughly in line with historical US data. The shock to government spending that I consider is 1 % of GDP.

The choice of ϕ_b , which governs the delay in tax collection, is important for the results. To the best of my knowledge the literature does not provide a definitive suggestion on how to chose this parameter, so I set it to have a persistent debt dynamics and insure stability at the same time.

E. SOLUTION METHOD. The model is solved using policy function iteration on an endogenous grid. The details are discussed in Appendix D.

F. WELFARE MEASURES. Together with the expected marginal welfare change, I compute a welfare gain of a household with initial state $x_0 = (a_0, s_0)$ which is defined as the constant percentage increase in consumption in the case the economy stays in the steady state that gives the household the same expected utility as when the government temporarily increases spending²³. Let $c_t^*(x_0, s^t, \epsilon_0)$ be the optimal equilibrium consumption of a household with initial state x_0 if there is an increase in government spending at $t = 0$, and $c_t^*(x_0, s^t, 0)$ the same thing in case the economy stays in the steady state. Then the welfare gain δ_{x_0} solves the following equation:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0)) \\ &= \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u((1 + \delta_{x_0})c_t^*(x_0, s^t, 0), l_t^*(x_0, s^t, 0), g_t(0)) \end{aligned}$$

The welfare gains for households with different initial wealth and productivity are computed by creating a large artificial population, where each household starts with a

²³For example, see Domeij and Heathcote (2004).

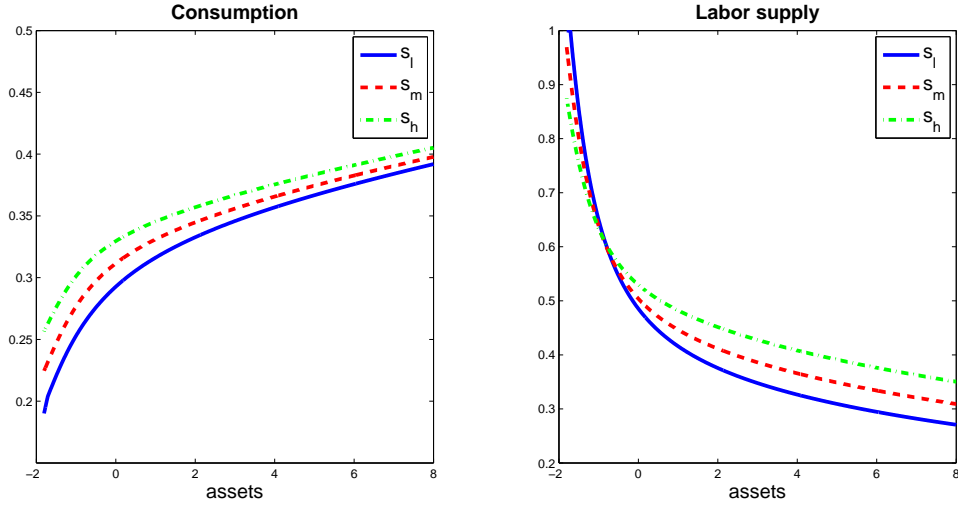


FIGURE 2. Optimal consumption and labor in the steady state

different combination of assets and productivity, and simulating the economy forward under two scenarios: 1) the economy stays in the steady state forever; 2) there is a temporary increase in government spending.

The average welfare gain in the economy is defined as the constant percentage increase in consumption (the same for everybody) if the economy stays in the steady state that delivers the same aggregate utility under welfare criterion (4) as when there occurs a temporary change in fiscal policy. The average welfare gain δ solves the following equation:

$$\begin{aligned} & \sum_{x_0} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0)) \lambda_0(x_0) \\ &= \sum_{x_0} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u((1 + \delta)c_t^*(x_0, s^t, 0), l_t^*(x_0, s^t, 0), g_t(0)) \lambda_0(x_0) \end{aligned}$$

5.2 Steady state

I briefly discuss the properties of agents' policy functions in the steady state. Figure 2 presents optimal consumption and labor supply decision of agents in the steady state as a function of their asset holdings. Policy functions are plotted for the lowest positive (s_l), medium (s_m) and highest (s_h) levels of productivity.

There are substantial differences in consumption and labor supply across agents with different levels of assets. agents at the top of the wealth distribution consume almost twice as much and work three times as little as those at the bottom. Policy functions'

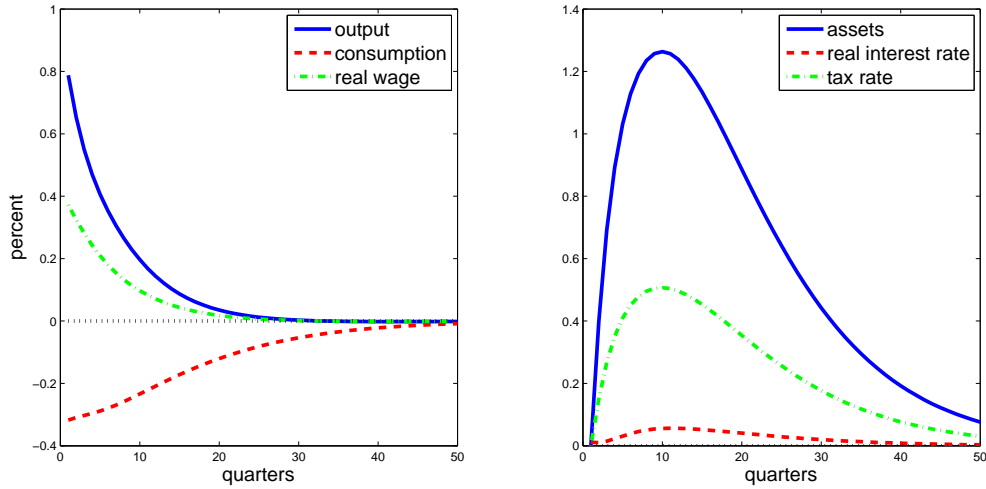


FIGURE 3. Transition dynamics of main variables

curvature also changes along the distribution. While the consumption optimal decision is almost linear in wealth for agent with a high buffer stock of saving, it becomes concave as we move down the asset distribution. This is a typical feature of buffer stock models, as was shown by Carroll and Kimball (1996). Consumption and labor supply are related through the intratemporal optimality condition, therefore the concavity of the consumption policy function translates into convexity of the policy function for hours. For the wealth poor a higher productivity is associated with a lower labor supply because the income effect of higher wage dominates the substitution effect. For richer agent the substitution effect turns out to be larger, and higher wage (stemming from high productivity) makes the agent be willing to work more.

5.3 Results

A. TRANSITION PATH. Figure 3 describes the behavior of output, consumption, assets, real interest rate and real wage during a transition after a persistent increase in government spending following a 1 percent of GDP shock ϵ_0 in period 0, financed by increase in labor income tax starting from period 1, with tax following a rule $\partial \tau_t^l / \partial \epsilon_0 = \phi_b \partial B_t / \partial \epsilon_0$. This an example of a typical behavior of the main variables during the transition path, with parameter values chosen for this exposition being $(\sigma, \psi) = (4, 2)$.

Output peaks at $t = 0$ and goes back to the steady state as the impact of the the shock dies out²⁴. Consumption mirrors output dynamics, falling at the time of the

²⁴Under some parameterizations, which imply a strong reaction of hours to after-tax real wage, output

shock and then recovering gradually. Asset holdings in equilibrium are equal to the amount of government debt, which builds up at the the beginning of transition and starts decreasing when the tax rate adjusts to bring debt back to its steady state level. The interest rate shows a similar pattern: as the level of debt increases, the return on it goes up to make agents willing to hold this quantity of debt.

Both the real wage and the real interest rate reach their respective peaks in the short-run. The wage response is highest on impact, while the real interest rate reaches its maximum point in about 10 quarters. This suggest that most of the redistribution due to prices happens in the short-run. Given the persistence of productivity process, the initial position of the agent in the distribution over productivity and wealth is important in determining individual expected welfare gains.

B. MULTIPLIER AND WELFARE. Central results from the model with heterogeneous agents are summarized in Figures 4 and 5. Figure 4 describes the relationship between short- and long-run multipliers and average welfare gains (top panel) and the standard deviation of individual welfare gains (bottom panel). The short-run multiplier has a non-monotone relationship with both average welfare gain and the dispersion of individual gains. On the other hand, the average welfare gain is monotone increasing in the long-run multiplier for most of parameter values (the exception is $1/\psi = 0.2$, which implies very inelastic labor supply). Some parameterizations are associated with high multipliers at all horizons and low variation of welfare gains across agents, while others result in low multipliers and highly uneven distribution of gains. However, there are several cases in between, for which the relationship between short-run multiplier, long-run multiplier and distribution of welfare gains changes with parameter values. Figure 5 suggest why this can happen. While the short-run multiplier characterizes output response on impact and the long-run multiplier takes into account its cumulative change, the distribution of welfare gains depends on the distribution of taxation burden, and changes in real wage and real interest rate. While transition dynamics of the wage repeats the shape of output response and therefore is related to the multiplier, the behavior of the interest rate is determined by the pattern of government debt and saving decisions of agents.

The top panel of Figure 5 plots multipliers against the impact on revenues and profits (component 26 in welfare decomposition), the panel in the middle shows the net benefit of increase in government spending (component 23), and finally the bottom panel describes redistributational components (components 24 and 25). All components are scaled by the average marginal utility of consumption. Impact on revenues and

is going below the steady state when taxes increase.

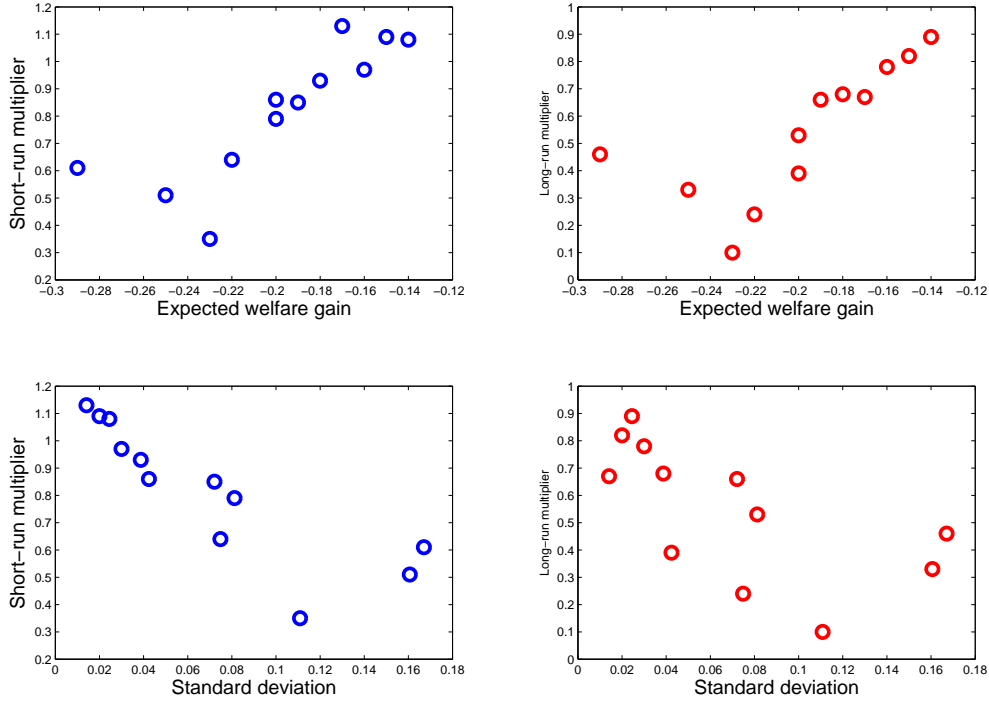


FIGURE 4. Multipliers and expected average welfare gain, and standard deviation of individuals gains

profits has a positive monotone relationship with the long-run multiplier. This is the only component related to the size of the multiplier in the representative agent model. Net benefit of government spending and redistribution due to changes in prices have a non-monotone relationship with the long-run multiplier. Why is this so?

i. Frisch elasticity of labor supply. More elastic labor supply leads to a stronger output reaction on impact, as hours increase by more in response to higher real wage and relatively low income tax rate. The size of the long-run multiplier depends on the relative strength of two opposing effects. First, as taxes start to increase in response to higher debt level, hours fall by more if labor supply is elastic. Second, if initial increase in output was large, the level of debt accumulated in the short-run is lower, which leads to a lower tax rate increase. This mitigates the negative effect of taxes on hours. Under parameterizations I use in this paper, the second effect appears to be larger and more elastic labor results in both short- and long-run multipliers being higher.

Since multipliers are higher at all horizons, the transition is characterized by a lower level of public debt and a smaller increase of the real interest rate. This path for the real

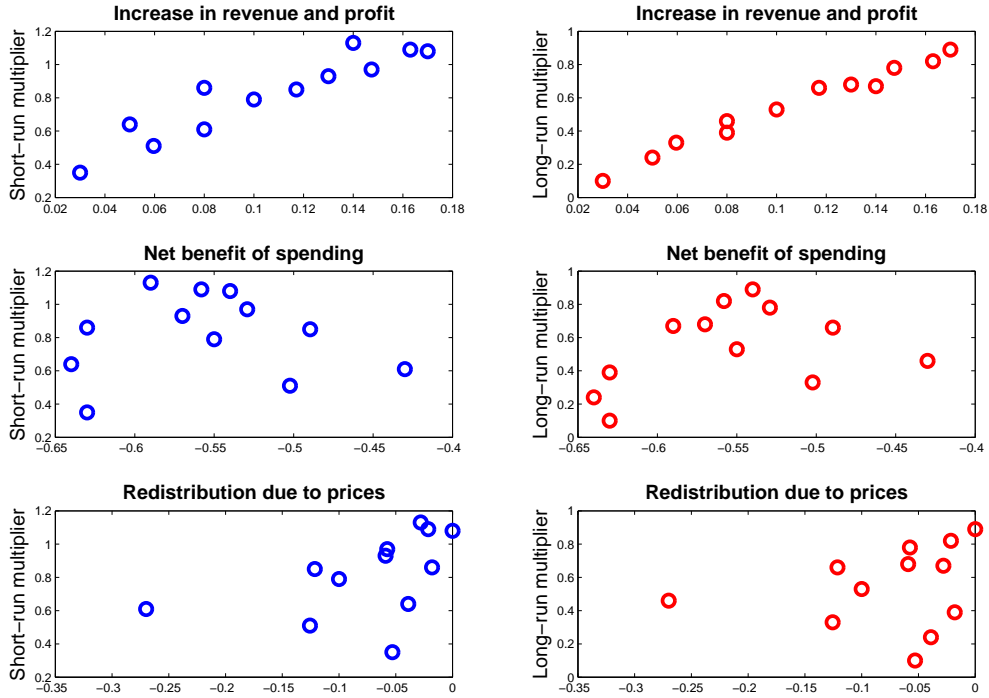


FIGURE 5. Multipliers and components of expected marginal welfare gain

rate is more favorable for the wealth poor than for wealth rich, resulting in a narrower distribution of welfare gains. Therefore a higher Frisch elasticity is associated to a less negative redistributational component.

Higher Frisch elasticity also corresponds to a larger net benefit of spending at lower level of σ , but to a lower net benefit when σ is high. More elastic labor supply implies that in the steady state hours of work are more unequally distributed across agents with different asset holdings, with relatively poorer agents working more hours. The cost of financing increase in spending is higher for poorer agents. First, the tax is proportional to the labor supply, therefore in absolute terms wealth poor agents pay more income taxes than wealth rich (given the same productivity and wage). Second, agents with little assets have lower steady state consumption than those with more assets, meaning that an equal decrease in consumption translates into a higher utility cost for them. Thus, the poor bear a disproportionately larger cost of financing fiscal policy than the rich. The size of decline in marginal utility is proportional to the initial consumption, which is more equal across agents with different levels of wealth if the Frisch elasticity is high, since they adjust hours rather than consumption in response to idiosyncratic

shocks. This mitigates the effect of elastic labor supply on the utility cost of the poor, since consumption is more equally distributed. The size of the two effects depends on the strength of consumption smoothing motive. If this motive is weak, i.e. σ is low, inequality in hours is low and the second effect (utility cost) dominates. On the contrary, if agents prefer to keep their consumption profile smooth across time and states, then hours vary substantially across asset levels. In this case, the first effect (taxation cost) is larger, and more elastic labor supply implies more negative net benefit of government spending.

ii. Intertemporal elasticity of substitution. A smaller value of the intertemporal elasticity of substitution (IES) results in a higher short-run multiplier. This parameter affects the income effect on consumption, which is large when IES is low. The long-run multiplier is high as well because the initial large expansionary effect on output leads to a lower level of debt and taxation over the transition, which delivers a higher path for output despite a larger negative income effect of higher taxes. Although a smaller IES generates a lower debt transition path, it is accompanied by a stronger reaction of the real interest rate. This can be explained by the tendency of agents to smooth consumption, which is strong when IES is low. As taxes increase to stabilize debt, agents prefer to dissave rather than decrease consumption. They therefore require a higher interest rate to be willing to hold the same quantity of debt compared to the situation when the desire for consumption smoothing is weak.

The stronger real interest rate response explains why the redistributive component of change in welfare is more negative when IES is low. Lower IES also corresponds to a higher net benefit of spending because consumption inequality in the steady state is smaller (consumption smoothing over states) which reduces the utility cost of financing increase in spending at the bottom of the distribution by more than it increases it at the top.

C. EXAMPLE. To explain how choices of parameters interact with each other and why different combinations can produce a non-monotone relationship between the long-run multiplier and welfare gain, I focus on comparing multipliers and expected welfare gains for two sets of structural parameters. Table 2 presents multipliers and expected welfare gains in case when both IES and Frisch elasticity are high ("high elasticity", $(\sigma, 1/\psi) = \{2, 1\}$) and when both elasticities are low ("low elasticity", $(\sigma, 1/\psi) = \{4, 0.5\}$).

The short-run multipliers are similar. The long-run multiplier is 0.37 in the 'high elasticity' case and 0.52 when both elasticities are low. The average welfare gain in the

TABLE 2. Comparison of multipliers and changes in SW

	'High elasticity'	'Low elasticity'
$(\sigma, 1/\psi)$	(2, 1)	(4, 0.5)
Short-run Multiplier	0.86	0.79
Long-run Multiplier	0.37	0.52
Welfare gain, average	-0.20	-0.19
Welfare gain, bottom 10 %	-0.24	-0.30
Welfare gain, top 10 %	-0.12	-0.05

Expected welfare gain is measured as percentage of steady state consumption.

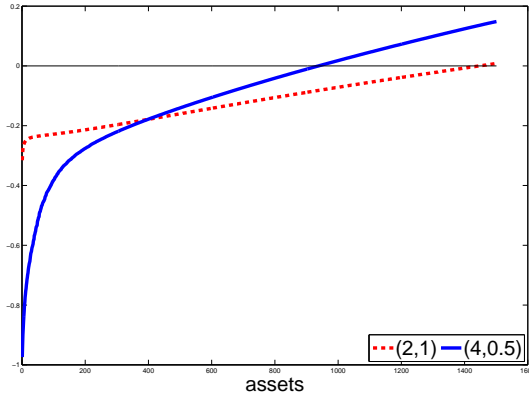


FIGURE 6. Expected welfare gain across asset levels

elastic case is equivalent to a permanent 0.20 % decline in consumption, with decline for the bottom ten percent of agents being 0.24 % and decline for top ten percent being 0.12 %. Under the 'low elasticity' parameterization the average gain in welfare is similar and corresponds to a permanent 0.19 % decline in consumption, but in contrast to the previous case the welfare decline for the bottom ten percent is 0.30 % and decline for the top ten percent is 0.05 %.

Figure 6 shows the distribution of welfare gains across agents with different asset levels. In the 'high elasticity' case the distribution of welfare gains is more equal, agents' expected gains are between approximately 0.30% and 0% permanent loss in consumption depending on their initial wealth level. In the 'low elasticity' case the distribution of gains is more uneven, despite the average being the same. While those at the top of the income distribution gain due to increase in government spending, agents at the bottom lose an equivalent of up to 0.90% of their consumption permanently. Thus, despite the long-run multiplier being higher and the average welfare gain similar when both

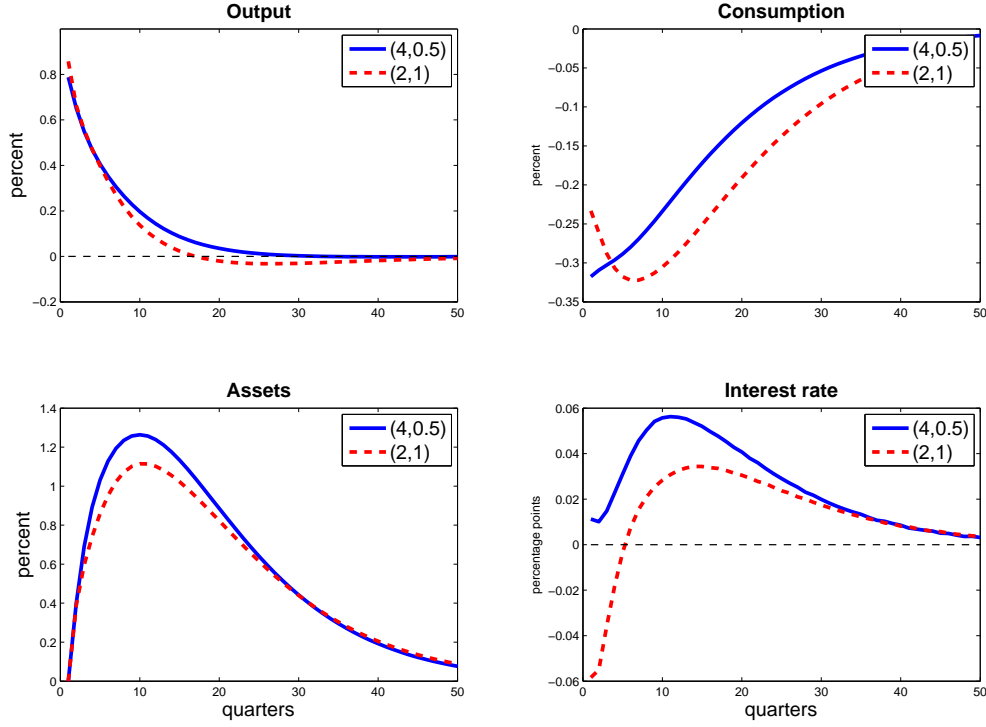


FIGURE 7. Response to a 1 percent of GDP government spending shock

intertemporal substitution and Frisch elasticities are high, individual welfare gains are more dispersed and are larger for the poorest agents in the economy.

To understand why the parameterizations differ in their welfare implications of the size of the multiplier, we first look at the dynamics of output, consumption, assets and the real interest rate after the shock, summarized in Figure 7. In the 'high elasticity' case, the short-run multiplier is (a little) higher and the long-run multiplier is lower because the intertemporal labor substitution is strong. Consumption initially falls less because labor response is stronger, but then it continues to decrease as output falls, and starts rebounding only when output starts to go back to the steady state. This consumption path implies that the interest rate falls on impact, and then shoots above the steady state (see Figure 7). In the 'low elasticity' case the interest rate is above the steady state during the whole transition path, and its peak is above that of the elastic case. The interest rate in this case remains high throughout the transition because the quantity of government debt is higher while the desire for precautionary saving is lower since output path is more smooth. On the contrary, in the elastic case the debt is lower while the desire for accumulating a buffer stock saving is strong, implying that

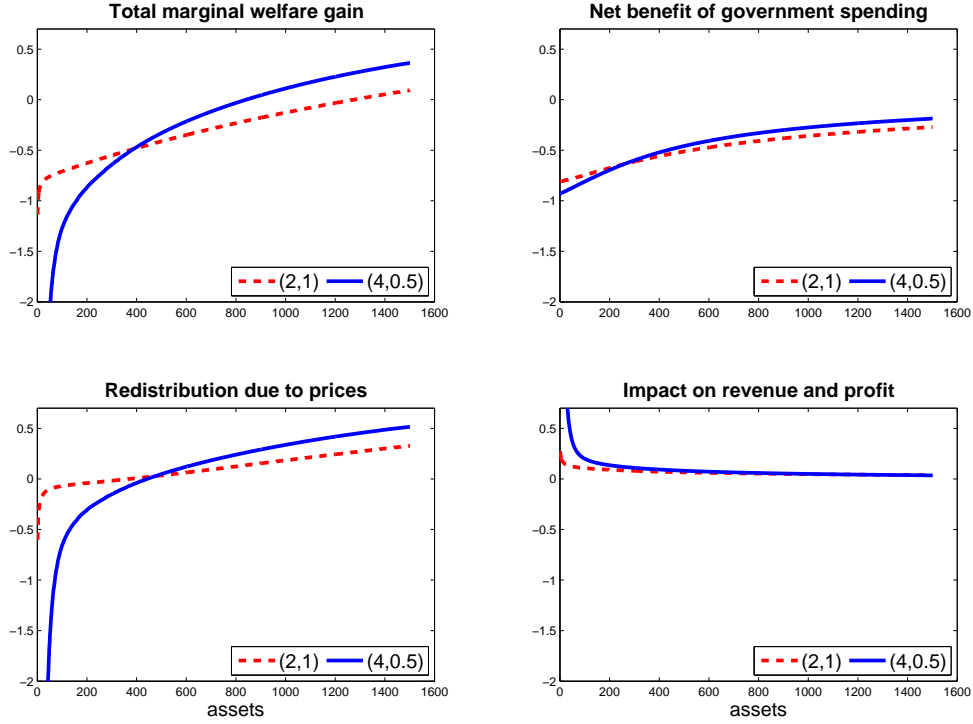


FIGURE 8. Marginal welfare gains

the agents are willing to hold government debt even at a low return.

Figure 8 shows marginal changes in welfare for agents with different levels of asset holdings and productivity at time 0. The net benefit of government spending is lower for wealth poor in both cases, and more equally distributed across agents with different assets in the 'high elasticity' case. However, the impact on revenue and profits is lower, especially for the poor, since the long-run multiplier is low. The largest difference in welfare gains between agents at different ends of the distribution is due to the changes in prices. The reason why welfare gains are lower at the bottom and larger at the top in the 'high elasticity' case is that the interest rate path is more favorable for the wealth poor. The poor benefit from lower real interest rate transition path while the rich lose, which results in an overall more equal distribution of welfare gains.

D. WAGE VS. REAL INTEREST RATE. An agent with high productivity gains when the real wage increases because increase in earnings offsets decline in dividend income, while the agent with low productivity loses for the same reason. A high productivity agent is also trying to accumulate wealth rapidly in order to self-insure against a bad productivity shock, thus expecting to gain from future increase in the interest rate. A

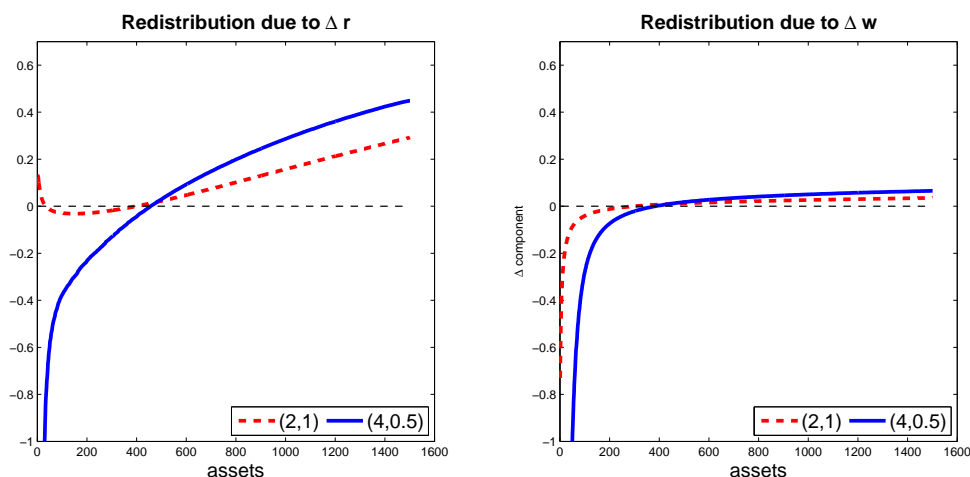


FIGURE 9. Marginal welfare gain, redistributional components

low productivity agent is typically using their wealth to smooth current consumption, and therefore is expected to suffer losses from future interest rate increase. These losses are larger at the bottom of the distribution as the current assets are already low. It is clear from Figure 9 that the movement in the real wage cause little redistribution apart from the very bottom²⁵, and the change in welfare due to it is similar between parameterizations. On the contrary, the main difference in welfare gains between different sets of parameter values is explained by the redistribution stemming from the behavior of the real interest rate.

5.4 Robustness

So far I used a utilitarian welfare criterion, which assigned a weight of unity to each agent. In the light of the previous discussion about distribution of gains and losses from temporary increase in spending I check how results change when the welfare criterion gives different weights to the rich and the poor. I use two alternative weighting schemes. According to the first one each agent's welfare is included in the aggregate with a weight proportional to the square of their initial consumption. This assignment of weights implies that richer agents get a higher weight than poorer ones ('pro-rich'). The second scheme weights each agent proportionally to the inverse of the square of their initial consumption, this way giving more weight to the poor ('pro-poor'). Results are presented in Figure 10.

²⁵The negative marginal welfare gain at the bottom comes from the losses of agents with low/zero productivity, who suffer from a decline in dividends as the wage increases.

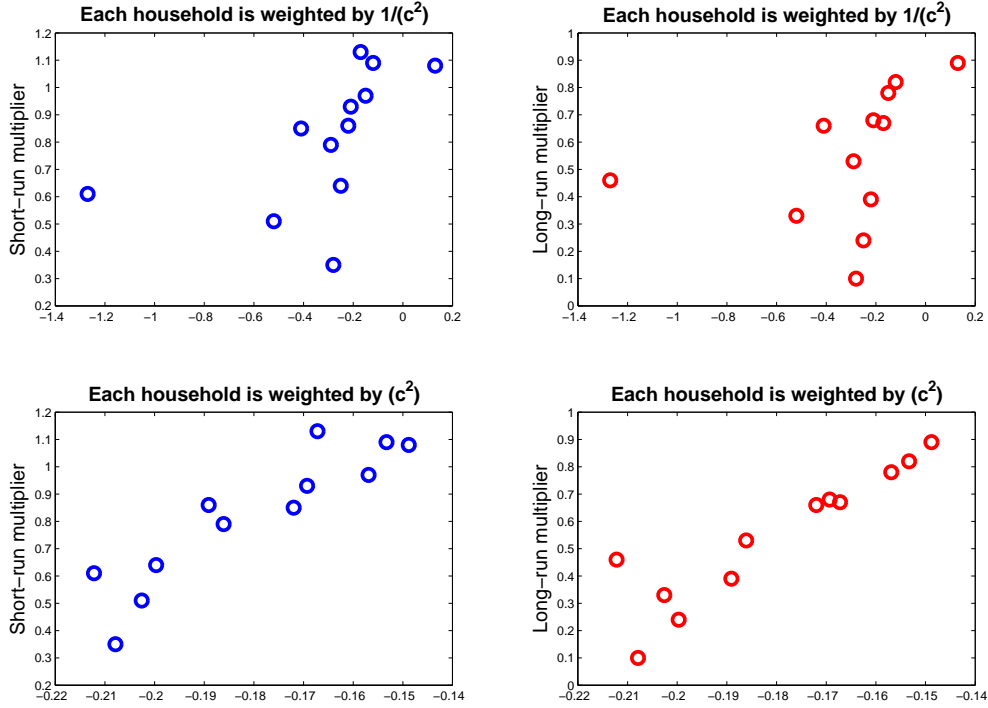


FIGURE 10. Multipliers and expected average welfare gain

The bottom panel shows relationship between short- and long-run multipliers and average expected welfare gain under the 'pro-rich' welfare criterion. Variation in expected welfare gain is moderate, and for most combinations of structural parameter values the welfare gain is monotone increasing in the long-run multiplier. On the contrary, under the 'pro-poor' welfare criterion the average expected welfare gain is more dispersed and non-monotone in the long-run multiplier. This leads to a conclusion that while the long-run multiplier can be informative of the average welfare gain (under most parameterizations), this relationship relies on the welfare gains of the richer part of the economy. The welfare gains at the bottom are less predictable from the high value of the long-run multiplier.

6 Conclusion

This paper looks at the relationship between the size of government spending multiplier in the short and in the long run and welfare gain resulting from increase in spending. I show that while in the representative agent framework the welfare gain is proportional

to the long-run (cumulative present discounted value) multiplier, allowing for differences across agents in terms of their asset holdings, consumption and labor supply can change the welfare implications of the size of the government spending multiplier. Redistributive effects of fiscal policy and its general equilibrium effects might go in opposite directions. There exist plausible combinations of values for Frisch elasticity of labor supply and intertemporal elasticity of substitution, for which a high multiplier might be associated with a lower welfare at the bottom of the wealth distribution (for a given policy). This can result in the average welfare gain being low when the multiplier is high if the welfare of the poor is given enough weight.

The mechanism which accounts for these findings relies on intertemporal response of output, government debt dynamics and precautionary saving behavior of the agents, which together affect the transition dynamics of the real interest rate. A more expansive path for the real rate redistributes wealth from asset poor to asset rich. This can lead to a wider dispersion of expected welfare gains across agents despite higher cumulative multiplier, with the agents at the bottom losing the most.

The main implication of this analysis is that when evaluating fiscal policy, one cannot fully rely on the multiplier statistic. First, if one wants to pick the most welfare-relevant multiplier they should look at the long-horizon present discounted value multiplier, which takes into account the whole transition path of output. Second, even the size of the cumulative multiplier can fail to reflect how welfare gains are distributed across different agents, because the relationship between the two depends on (among other factors) the size of Frisch elasticity of labor supply and intertemporal elasticity of substitution. Given that there is still a large disagreement in the literature on the values of these structural parameters, there is a concern that even within a structural model one cannot precisely evaluate the relative size of redistributive and aggregate effects of government spending.

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A Definition of Equilibrium

An equilibrium constitutes sequences of government purchases $\{g_t\}_{t=0}^{\infty}$ and labor income taxes $\{\tau_t^l\}_{t=0}^{\infty}$, sequences of optimal consumption and labor supply $\{c_t(x_0, s^t), l_t(x_0, s^t)\}_{t=0}^{\infty}$, sequences of prices $\{w_t, r_t\}_{t=0}^{\infty}$ and dividends (profits) $\{\Gamma_t\}_{t=0}^{\infty}$, and a sequence of distributions for asset holdings and productivity levels $\{\lambda_t(x)\}_{t=0}^{\infty}$, such that given the initial distribution λ_0 for each time $t = 0, 1, 2, \dots$ and each history s^t :

1. $\forall x_0$ $c_t(x_0, s^t)$ and $l_t(x_0, s^t)$ solve household maximization problem, given the sequences $\{\tau_t^l\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, $\{r_t\}_{t=0}^{\infty}$ and $\{\Gamma_t\}_{t=0}^{\infty}$;

2. $\{\lambda_t(x)\}_{t=0}^{\infty}$ is induced by $c_t(x_0, s^t)$ and $l_t(x_0, s^t)$ ²⁶, and Π ;
3. the tax rate is determined according to $\tau_{lt} = \phi_b B_t$ and the government budget constraint is satisfied

$$\tau_t^l w_t L_t + B_{t+1} = (1 + r_t) B_t + g_t;$$

4. the real wage is the inverse of product mark-up: $w_t = \frac{1}{\mu_t}$;
5. the markets for assets and labor clear

$$A_{t+1} = \sum_x \sum_{s^t} P(s^t) a_{t+1}(x_0, s^t) \lambda_t(x) = B_{t+1},$$

$$L_t^s = \sum_x \sum_{s^t} P(s^t) s_t l_t(x_0, s^t) \lambda_t(x) = \int_0^1 l_i di = L_t^d.$$

B Multiplier computation an a dynamic model with distortionary taxation

The model is solved in deviations from the steady state under the assumption of perfect foresight. Together with the initial conditions $\frac{dY_{-1}}{d\epsilon_0} = 0$, and $\frac{dB_0}{d\epsilon_0} = 0$, equations (28), (29) (or (30) in case of debt), (31) and (32) below characterize solution for the change in output in each period after the shock

$$\frac{dY_t}{d\epsilon_0} = \frac{\sigma}{\sigma + \psi(1 - s_g)} \frac{dg_t}{d\epsilon_0} + \frac{s_g y}{w(\sigma + \psi(1 - s_g))} \frac{dw_t}{d\epsilon_0} - \frac{s_g y}{(1 - \tau^l)(\sigma + \psi(1 - s_g))} \frac{d\tau_t^l}{d\epsilon_0} \quad (28)$$

$$\frac{d\tau_t^l}{d\epsilon_0} = \frac{1}{wy} \frac{dg_t}{d\epsilon_0} + \frac{B}{wy} \frac{dr_t}{d\epsilon_0} - \frac{\tau^l(1 + \omega)}{y} \frac{dY_t}{d\epsilon_0} \quad \text{if balanced budget} \quad (29)$$

$$\frac{d\tau_t^l}{d\epsilon_0} = \phi_b \sum_{s=0}^{t-1} \left[B \frac{dr_s}{d\epsilon_0} + \frac{dg_s}{d\epsilon_0} - \frac{\tau^l(1 + \omega)}{\mu} \frac{dY_t}{d\epsilon_0} \right] \quad \text{if debt} \quad (30)$$

$$\frac{dr_t}{d\epsilon_0} = \frac{\sigma}{\beta c} \left[\left(\frac{dY_t}{d\epsilon_0} - \frac{dY_{t-1}}{d\epsilon_0} \right) - \left(\frac{dg_t}{d\epsilon_0} - \frac{dg_{t-1}}{d\epsilon_0} \right) \right] \quad (31)$$

$$\frac{dw_t}{d\epsilon_0} = \frac{\omega w}{y} \frac{dY_t}{d\epsilon_0}. \quad (32)$$

The multipliers can be computed according to (16).

²⁶Consumption and labor supply optimal decisions together with the budget constraint fully determine asset holdings optimal decision.

C Welfare decomposition for model with heterogeneous agents

To simplify notation, in what follows I denote

$$u_t(x_0, s^t, \epsilon_0) \equiv u(c_t(x_0, s^t, \epsilon_0), l_t(x_0, s^t, \epsilon_0), g_t(\epsilon_0)),$$

$$u_{c_t}(x_0, s^t, \epsilon_0) \equiv \frac{\partial u_t(x_0, s^t, \epsilon_0)}{\partial c_t(x_0, s^t, \epsilon_0)}.$$

The optimality conditions are

$$u_{c_t}(x_0, s^t, \epsilon_0) \frac{\partial Y_t}{\partial l_t(x_0, s^t, \epsilon_0)} + u_{l_t}(x_0, s^t, \epsilon_0) = 0$$

$$u_{c_t}(x_0, s^t, \epsilon_0) \geq \beta \sum_{s_{t+1}} P(s_{t+1}|s_t) \frac{\partial Y_{t+1}}{\partial a_{t+1}(x_0, s^t, \epsilon_0)} u_{c_{t+1}}(x_0, s^{t+1}, \epsilon_0)$$

Slackness

$$\left[u_{c_t}(x_0, s^t, \epsilon_0) - \beta \sum_{s_{t+1}} P(s_{t+1}|s_t) \frac{\partial Y_{t+1}}{\partial a_{t+1}(x_0, s^t, \epsilon_0)} u_{c_{t+1}}(x_0, s^{t+1}, \epsilon_0) \right] [\bar{a} + a_{t+1}(x_0, s^t, \epsilon_0)] = 0$$

Transversality

$$\lim_{T \rightarrow \infty} \beta^{T+1} \sum_{s_{T+1}} P(s_{T+1}) u_{c_{T+1}}(x_0, s^{T+1}, \epsilon_0) a_{T+1} \leq 0$$

The solution to the problem of an agent is the optimal sequences for consumption, hours and assets $\{c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), a_{t+1}^*(x_0, s^t, \epsilon_0)\}_{t=0}^{\infty}$, such that the conditions above hold. The maximum value of the agent's problem is

$$V_0^*(x_0, s_0, \epsilon_0) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0)).$$

For simplification, again I denote

$$u_t^*(x_0, s^t, \epsilon_0) \equiv u(c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), g_t(\epsilon_0)),$$

$$u_{c_t}^*(x_0, s^t, \epsilon_0) \equiv \frac{\partial u_t^*(x_0, s^t, \epsilon_0)}{\partial c_t^*(x_0, s^t, \epsilon_0)}.$$

Because the budget constraint in each period holds as equality²⁷, I can plug it (evaluated at optimum) into the problem. Then the change of the maximum value of the

²⁷Otherwise it is possible to increase consumption, and hence utility, staying within the constraint. Therefore sequences $\{c_t^*(x_0, s^t, \epsilon_0), l_t^*(x_0, s^t, \epsilon_0), a_{t+1}^*(x_0, s^t, \epsilon_0)\}_{t=0}^{\infty}$ satisfying optimality conditions and the budget constraint as inequality cannot be maximizing the value of the consumer problem.

problem when ϵ_0 changes is (assuming differentiability wrt ϵ_0)

$$\begin{aligned} \frac{\partial V_0^*(x_0, s_0, \epsilon_0)}{\partial \epsilon_0} &= \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, \epsilon_0) \left[\frac{\partial \tilde{Y}_t}{\partial \epsilon_0} + \frac{\partial Y_t}{\partial l_t^*(x_0, s^t, \epsilon_0)} \frac{\partial l_t^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} \right. \\ &\quad \left. + \frac{\partial Y_t}{\partial a_t^*(x_0, s^{t-1}, \epsilon_0)} \frac{\partial a_t^*(x_0, s^{t-1}, \epsilon_0)}{\partial \epsilon_0} - \frac{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} \right] \\ &\quad + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u_{l_t}^*(x_0, s^t, \epsilon_0) \frac{\partial l_t^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u_{g_t}^*(x_0, s^t, \epsilon_0) \frac{\partial g_t(\epsilon_0)}{\partial \epsilon_0} \end{aligned}$$

where $\frac{\partial \tilde{Y}_t}{\partial \epsilon_0}$ is the change in Y_t due to change in variables which the agent takes as given (prices, taxes, dividends).

Rearranging the terms, I get

$$\begin{aligned} \frac{\partial V_0^*(x_0, s_0, \epsilon_0)}{\partial \epsilon_0} &= \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) \frac{\partial l_t^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} \left[u_{l_t}^*(x_0, s^t, \epsilon_0) + u_{c_t}^*(x_0, s^t, \epsilon_0) \frac{\partial Y_t}{\partial l_t^*(x_0, s^t, \epsilon_0)} \right] \\ &\quad + \frac{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} \left[\beta \sum_{s_{t+1}} P(s_{t+1}|s_t) \frac{\partial Y_{t+1}}{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)} u_{c_{t+1}}^*(x_0, s^{t+1}, \epsilon_0) - u_{c_t}^*(x_0, s^t, \epsilon_0) \right] \\ &\quad + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u_{g_t}^*(x_0, s^t, \epsilon_0) \frac{\partial g_t(\epsilon_0)}{\partial \epsilon_0} + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} P(s^t) u_{c_t}^*(x_0, s^t, \epsilon_0) \frac{\partial \tilde{Y}_t}{\partial \epsilon_0} \end{aligned}$$

We can use first order conditions and the budget constraint to eliminate term (1).

Let's show that term (2) is equal to zero as well.

If the borrowing constraint is not binding, i.e. the Euler equation holds as equality, the term is zero. If the constraint is binding, i.e. $\beta \sum_{s_{t+1}} P(s_{t+1}|s_t) \frac{\partial Y_{t+1}}{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)} u_{c_{t+1}}^*(x_0, s^{t+1}, \epsilon_0) - u_{c_t}^*(x_0, s^t, \epsilon_0) > 0$, we want to show that $\frac{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} = 0$.

The slackness condition at the optimum is

$$\underbrace{\left[u_{c_t}^*(x_0, s^t, \epsilon_0) - \beta \sum_{s_{t+1}} P(s_{t+1}|s_t) \frac{\partial Y_{t+1}}{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)} u_{c_{t+1}}^*(x_0, s^{t+1}, \epsilon_0) \right]}_A \underbrace{\left[\bar{a} + a_{t+1}^*(x_0, s^t, \epsilon_0) \right]}_B = 0$$

Differentiating this condition with respect to ϵ_0 yields

$$\left[\frac{\partial(A)}{\partial \epsilon_0} \right] [B] + [A] \frac{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} = 0.$$

Since the borrowing constraint is binding, $I > 0$ and $B = 0$. From the equation above it follows that in this case it must be that $\frac{\partial a_{t+1}^*(x_0, s^t, \epsilon_0)}{\partial \epsilon_0} = 0$.

Dropping the zero terms, and evaluating derivatives at $\epsilon_0 = 0$, we obtain the decomposition of expected marginal welfare change.

D Computation of the steady state and transition

A. STEADY STATE

1. Set calibration targets, guess values for parameters which are calibrated
 - (a) Guess the real interest rate r_0 which clears the bond market
 - (b) Iterate on the Euler equation and first order condition for labor supply to get policy functions for consumption and hours, using Endogenous Grid Point Method (EGM) (see Carroll (2006))
 - (c) Derive the inverse of the bond accumulation policy from the consumption and labor optimal decisions and compute time invariant distribution
 - (d) Compute aggregate bond holdings, consumption and hours
 - (e) Update the guess for the interest rate
2. Update calibrated parameters

B. TRANSITION

1. Choose T large enough so that at $t = T$ the economy is approximately in the steady state (I set $T = 200$)
2. Set consumption and labor supply optimal decisions are at their steady state in T
3. The initial bond distribution at $t = 0$ is the time invariant distribution
4. Guess a path of real interest rates $\{r_t\}_{t=0}^T$ with $r_T = r_{ss}$; guess a path for real wage, profit, tax rate
 - (a) Solve for consumption and hours policies from $t = T - 1$ to $t = 0$ by iterating backward on the Euler equation and the first order condition for labor supply (using EGM)
 - (b) Derive the asset accumulation policy from consumption and hours policies
 - (c) Compute the sequence of distributions from $t = 0$ to $t = T$ starting from distribution at time $t = 0$ using optimal asset accumulation decision
 - (d) Compute aggregate asset holdings, output, government debt paths

5. Update wage and profit path from output path, update tax rate path from government debt path
6. Update the guess for the interest rate path based on the difference between aggregate asset holdings and government debt