

Financial Integration and Sovereign Default

Elena Rancoita*

November 29, 2013

Abstract

I show that private incentives for international portfolio diversification can lead to socially inefficient sovereign bond portfolios, whenever governments face a commitment problem regarding debt repayment. I illustrate this using a two-country model with rational and atomistic investors who can invest in domestic and foreign sovereign debt. Within this framework, the equilibrium allocation is characterized by excessive foreign debt holdings and too much (costly) default relative to the social optimum. The allocation with fully integrated sovereign debt markets may even be dominated by the one in which sovereign debt cannot be traded across borders. Furthermore, I show that - consistent with the predictions of the model - sovereign default spreads in the Euro area are positively correlated with the share of sovereign debt owned by non-residents.

*Elena Rancoita, University of Mannheim. Email: elena.rancoita@gess.uni-mannheim.de. I would like to thank K. Adam, B. Born, A. Ciccone, A. Erce, T. Lee, C. Manger, C. Santos and M. Tertilt for their useful comments and suggestions.

1 Introduction

Do private incentives for portfolio diversification lead to socially efficient allocations when agents can invest in domestic and foreign sovereign debt? This question appears highly relevant in light of the substantial increase in the share of sovereign debt held by non-residents following the creation of the European Monetary Union (EMU). With the constitution of the EMU, the transaction costs (e.g. exchange rate risk, brokerage commissions and non-harmonized taxation) of trading financial instruments within the Euro area has fallen, allowing investor to increase their foreign bond holdings.¹ On average, the share of non-resident sovereign debt holdings increased between 1999 and 2007 from 20 to 56 percent.² An important question arising in this context is to what extent this phenomenon is socially desirable.

Many financial market observers have argued that the increase in sovereign debt holdings by foreigners was driven by the (ultimately wrong) perception that, following the creation of the EMU, government bonds of different member countries have become very close substitutes:

A significant change in European bond market is under way. Europe's decade-long "convergence" play, in which investors bet that over time bond yields across the euro zone would come together, is unraveling. Investors who had assumed an almost equal risk of default among euro-zone countries are now relying on emerging-markets desks to help them understand the credit risk they are taking.
(From The Economist, 'That sinking feeling', May 2010)

While such optimism about the quality of European sovereign debt may have driven part of the increase in the international holdings of EMU sovereign bonds, I show here that even with fully rational expectations, there is a tendency for portfolio diversification to generate a suboptimally high level of sovereign bond holdings by foreign investors.

Even if international trade in financial instruments is generally desirable because it allows to share risk across borders, there is a fundamental difference when considering the trade of private versus sovereign financial instruments: with sovereign instruments the enforcement of repayment is much more difficult, especially, when dealing with a foreign sovereign state (Panizza, Sturzenegger and Zettelmeyer (2009)). Moreover, the sovereign's incentive to default likely depends on who its creditors are. In particular, sovereigns may care more about repaying if domestic agents are holding the debt, compared to a situation where foreign

¹A deep discussion on the integration of financial markets within the EMU can be found in Pagano and Von Thadden (2004) and Manganelli and Wolswijk (2009).

²While debt holding by non-residents have decreased after Q4:2008 they are today still substantially above the levels observed in 1999 at the start of the European Monetary Union.

agents hold its debt (Broner and Ventura (2011)). Since atomistic private investors fail to internalize this effect in their investment decisions, international portfolio diversification may give rise to socially undesirable incentives for (costly) default choices by sovereigns.

The contribution of this paper is to show that private incentives for international diversification of bond portfolios can lead to excessive investments in foreign sovereign debt and excessive default when viewed from a social perspective. To formalize this idea, I present a two-country model with non-committed governments in which private agents face a portfolio choice between domestic and foreign sovereign debt. To my knowledge, the link between the diversification of sovereign bond investments and the incentives to default on sovereign debt has not been studied yet. The structure of the model is also new to the literature on sovereign default. In the present model, each government strategically decides about default not only taking into account the investment decisions of private agents, but also the default decision of the foreign government.

The existing literature on sovereign default has analyzed the trade-off between *reputation* and debt *repudiation* in a context where one government borrows from solely foreign investors (Eaton and Gersovitz (1981), Bulow and Rogoff (1988), Kletzer and Wright (2000), Arellano (2008)). This literature considers infinitely repeated games, focusing on the repeated trade-off between default and repayment in a setting where creditors temporarily exclude governments from bond markets after default (Kletzer and Wright (2000) and Arellano (2008)). Moreover, while in those papers debt is created in order to smooth taxes and consumption over time, in the present model, debt is exogenously given in order to focus on the portfolio problem and its implications for the strategic default incentives. More closely related to the present model are Cooper, Kempf and Peled (2008) and Cooper, Kempf and Peled (2009) who study regional debt repayment in a multi-region economy. In their framework, however, bonds of different sovereigns are perfect substitutes, which prevents them from studying portfolio decisions.

I present a two-period model with two symmetric countries within integrated financial markets (i.e. with no transaction costs). The model is a two-period game where in the first period private agents determine their portfolio allocations and in the second period governments decide on whether or not to default. In the first period, governments issue debt to finance an exogenous public expenditure; private agents are atomistic and can invest in domestic and foreign bonds and consume. Interest rates are determined by the rational anticipation of second period events. In the second period, governments observe the portfolio allocation of investors and decide upon repayment. Governments are benevolent toward their own citizens which creates an incentive to default if foreigners hold its debt, but default is costly, as in the infinite-horizon models described above. Indeed, in the *baseline scenario*, if a

government decides to default (*strategic default*), then this generates a utility cost for its own citizens. This cost is unknown in the first period when debt is issued, but observed in the second period, before the default decision takes place. The default cost could be interpreted as the change in the continuation value resulting from default e.g. due to (non-modeled) exclusion from sovereign bond markets.

In this setting, absent other motives for international bond diversification, the equilibrium is described by agents investing exclusively in domestic sovereign bond markets, even if bond markets are perfectly internationally integrated. To see why this is the case, suppose, by contradiction, that in equilibrium agents diversified their portfolio. Then for sufficiently low realization of the default cost for one government and a sufficient high cost for the other government, only one government would repay. This would allow the defaulting government to cut taxes by expropriating the non-resident holders. But non-resident investors face tax risk and hence should prefer to invest in domestic debt because it perfectly hedges against this risk.

Therefore, to generate a motive for international portfolio diversification in equilibrium, I introduce a small *disaster risk*, which is defined as a situation where domestic income is disrupted, exogenously, so that the government cannot raise sufficient taxes to repay. Consequently, it is forced to default (*forced default*).

The integration of financial markets leads to the following effects: private agents can hedge, at least partially, against the disaster risk by buying foreign debt. However, this increases the strategic default risk of the foreign country. Yet, since private investors are atomistic, they fail to internalize this. As I show for a sufficiently small probability of disaster, the equilibrium allocation with perfectly integrated bond markets is even Pareto dominated by the allocation with financial autarky. However, the autarky allocation is not first best either, as it fails to capture gains from internationally sharing disaster risk.

In this environment it is socially optimal (first best) that the disaster risk is shared equally by all investors and that states do not default. The first best allocation would be implementable via a political union in which the consolidated government cares about all citizens when it comes to repaying debt. Of greater political interest may be the second best solution, in which the strategic incentives for governments to default act as a constraint on the social planner. In this case, the social planner can only determine the portfolio allocation of private agents, while governments act strategically and determine default decisions. I show that the second best allocation is characterized by a lower share of investments in foreign debt than in the equilibrium with privately optimal portfolio choices and by a lower default probability in equilibrium. This allocation could be easily implemented by a sovereign institution taxing the returns of foreign bonds.

The remainder of the paper is structured as follows. Section 2 reports some stylized facts about the Euro area sovereign debt. Section 3 presents the model. In section 4, I solve the model in a setting with financial autarky, before showing, in section 5, the results in a setting with perfect financial integration. Section 6 discusses the inefficiency of the competitive equilibrium allocation with integrated markets. Section 7 concludes.

2 Sovereign Spreads and Non-Residents Holdings: Facts

The objective of this section is to document a number of stylized facts regarding sovereign spreads and non-resident bond holdings in the Euro area. It shows that there has been a substantial increase in the share of sovereign debt held by non-residents in the run-up to the Monetary Union and that this share is still considerably higher now than in the year 1999 for several countries. Moreover, I find a positive correlation between the interest rate spreads (measured for 10-year government bonds and relative to the German Bund) and the share of public debt held by non-residents. This positive correlation indicates that governments of countries with a high level of debt held by non-residents are more likely to default than those with a low level. The correlation is statistically significant and increases when controlling for country characteristics like the degree of political stability and the debt-to-GDP ratio.

I collected quarterly data for a panel of EMU countries and Denmark from the early 1990's to the end of 2012.³ The analysis is carried out on a group of countries with common currency and independent fiscal authorities, hence, controlling for the exchange rate risk. Indeed, starting from 1979, most nations of the European Economic Community became member of the European Monetary System and agreed on keeping their foreign exchange rates within certain bands with respect to the European Currency Unit. Therefore, even before the official start of the EMU, the spread does not include any significant exchange rate risk. The database includes the following variables: debt-to-GDP ratio ($DGDP$), fraction of debt held by non-residents (DNR), a proxy for the political instability ($Instability$), the current-account-deficit-to-GDP ratio (CA) and the spread of 10-year government bonds against the German benchmark ($Spread$).

The variables that I collected with the exception of the DNR are commonly considered good predictors for the spreads. Several studies find that fiscal variables like the debt to GDP or the announcement of fiscal deficits can explain part of the yield differential between Euro area government bonds (see Schuknecht, von Hagen and Wolswijk (2010), Attinasi, Checherita and Nickel (2009)). Schuknecht, von Hagen and Wolswijk (2010) find that yield spreads can be largely explained by economic variables and that the correlations with fiscal

³For more details on the data, see Appendix A.1

indication has become much larger after the financial crisis. Attinasi, Checherita and Nickel (2009) look at three possible determinants of the spread of 10-year governments bonds against the German Bund and they find significant evidence of a positive correlation with fiscal variables. Other results indicating a positive correlation of fiscal variables with interest rates levels for the EMU countries can be found in Faini (2006), Hallerberger and Wolff (2008) and Bernoth, von Hagen and Schuknecht (2012). In addition there is a wide literature showing that the US risk premia and fiscal variables like the public debt level and the fiscal deficit are positively correlated (Goldstein and Woglom (1992), Bayoumi, Goldstein and Woglom (1995) and Poterba and Rueben (2001)) .

The trade balance (measured here with the current account to GDP) is a proxy for the competitiveness of a country and, thus, of future growth and debt solvency (Maltritz (2012)). Indeed, the recent Euro area debt crisis has been characterized by large current account deficits of the Southern European countries, which reflected heterogeneous developments in the unit labor costs and spending patterns across the currency union (Sinn and Wollmershaeuser (2012)).

I consider in my analysis the degree of political stability over the last 20 years too. This choice is motivated by the fact that Southern Euro area countries are characterized by very high political instability and this might play a role in the implementation of reforms which boost growth. The degree of political stability has been already considered in the existing literature as explanatory variable for the risk premia on sovereign government bonds of developing countries. Political instability lead to short-sighted governments. The lower discounted value of future consumption increases the default incentives of the government, who cannot commit to repay. This leads to higher interest rate spreads for any given borrowing level (Cuadra and Sapriza (2008)). The literature has considered several proxies for the measurement of the degree of political stability: the changes in the head of government and changes in the governing group (Brewer and Rivoli (1990)), the number of changes of government over a five-year period (Citron and Nickelsburg (1987) and Balkan (1992)), the cabinet reshuffles involving key policymakers (Moser (2007)). All these proxies for the degree of political stability are found to be statistically significant variables in explaining the probability of sovereign default. In my analysis the variable *Instability* is the average of two measures of political instability. The first one equals the number of months between two elections over the total number of months of a legislation. The second one consists of the number of months that a prime minister is in charge over the the total number of months of a legislation.

Another relevant variable that explains the spread between 10-year government yields is liquidity, as documented in Attinasi, Checherita and Nickel (2009) , Favero, Pagano and Von Thadden (2010) and Manganelli and Wolswijk (2009). Liquidity is generally measured with

the bid/ask spreads or the share of debt of a country over the total debt of the Euro area. The literature has interpreted this variable also as a measure of financial integration. However, these indicators capture the willingness of investors to buy bonds and not necessarily international financial market integration. In my analysis, I discriminate between domestic and foreign investors and use the *DNR* as proxy for financial market integration.

Figure 1 shows the evolution of the share of public debt held by non-resident investors over time. The average share of debt held by non-residents increased on average from 20% to 56% between 1997 and 2008. In Ireland, in 2008, the non-resident holdings reached 94%. With the beginning of the crisis in the end of 2009, the share of non-resident holdings slightly decreased, but it is still today around 45%.

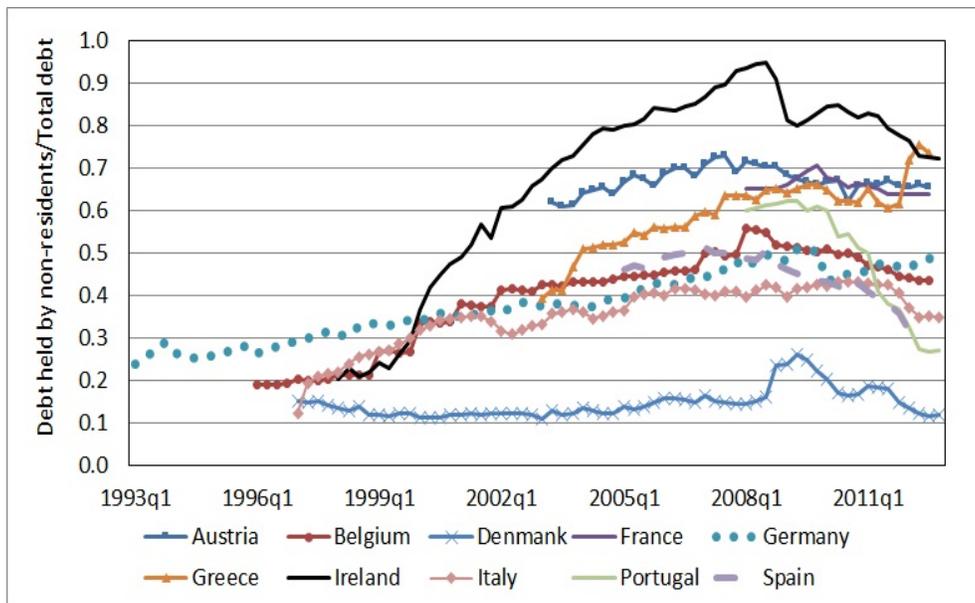


Figure 1: Debt held by non-resident over total debt. Quarterly data 1993q1-2012q4. Data sources: National Central Banks.

Figure 2 reports the evolution of the spreads over time.⁴ In the first part of the sample, all series follow a downward trend. From 2006 to 2011, the spread increased and then decreased.

⁴I do not report the spread for Greek sovereign bonds, because they are very high in comparison with the other spreads. A figure including Greek data can be found in Appendix A.1.

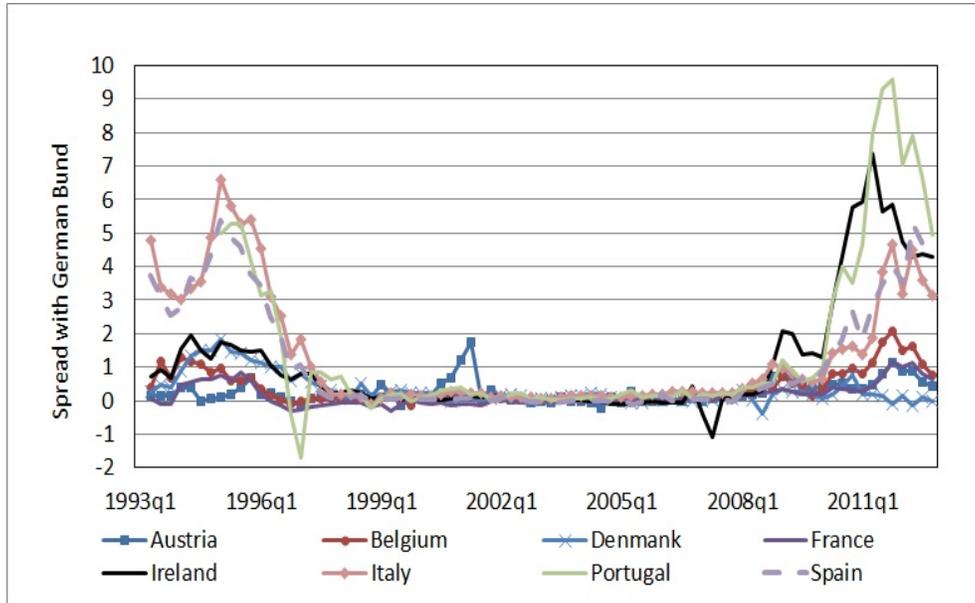


Figure 2: Spread of 10-year government bonds against German Bund. Quarterly data 1993q1-2012q4. Data sources: Datastream.

Figure 3 illustrates the evolution of the debt-to-GDP ratio over time. As shown in Figure 3, the debt-to-GDP ratio declined, on average, until the end of 2007 when governments have enacted large fiscal stimulus packages to offset the reduction in private sector demand caused by the crisis.

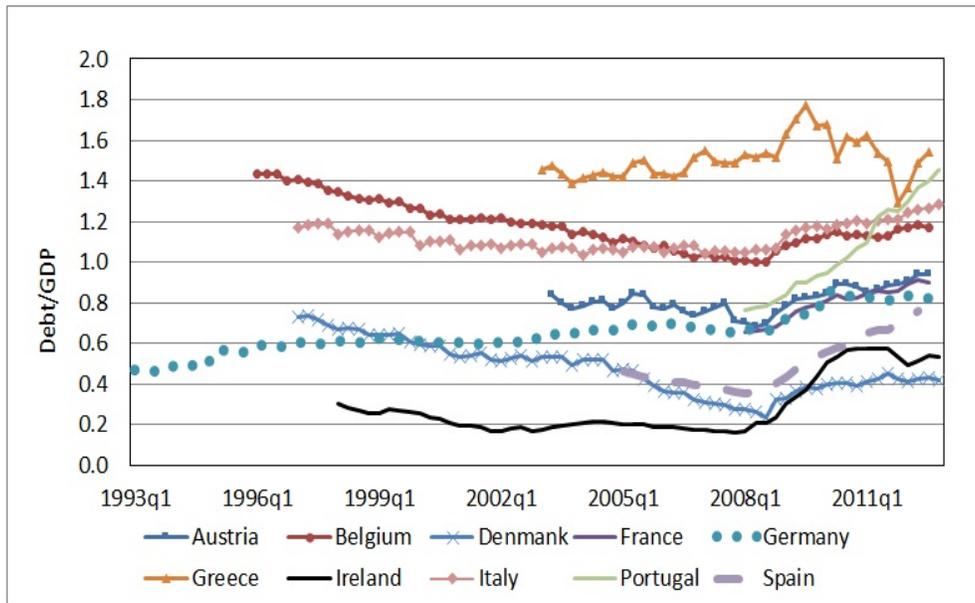


Figure 3: debt-to-GDP ratio. Quarterly data 1993q1-2012q4. Data sources: National Central Banks

From figures 1 and 2, we observe that for each country the correlation over time between the *DNR* and the spread is higher before the debt crisis and after 2011. For each country I computed the correlation between the *DNR* and the spread over a rolling window of 5 years starting from 2003q4 in order to isolate the effect of the crisis on the correlation between the two time series. The same exercise repeated for some lags and leads of the *DNR* (-2, -1, 0, 1, 2) shows that, on average, the highest correlation is obtained when the fraction of debt held by non-resident investors leads the growth rate of the spread by two quarters. For this latter case, the correlations over different windows are reported in Figure 4. With the exception of Denmark, the correlation between the *DNR* and the spread is the highest for all countries during the pre-crisis period. With the beginning of the debt crisis in 2010, the correlation becomes negative and it stops declining towards the end of the sample. This pattern indicates that the correlation of the latest observations of the sample is higher than when the debt crisis exploded.

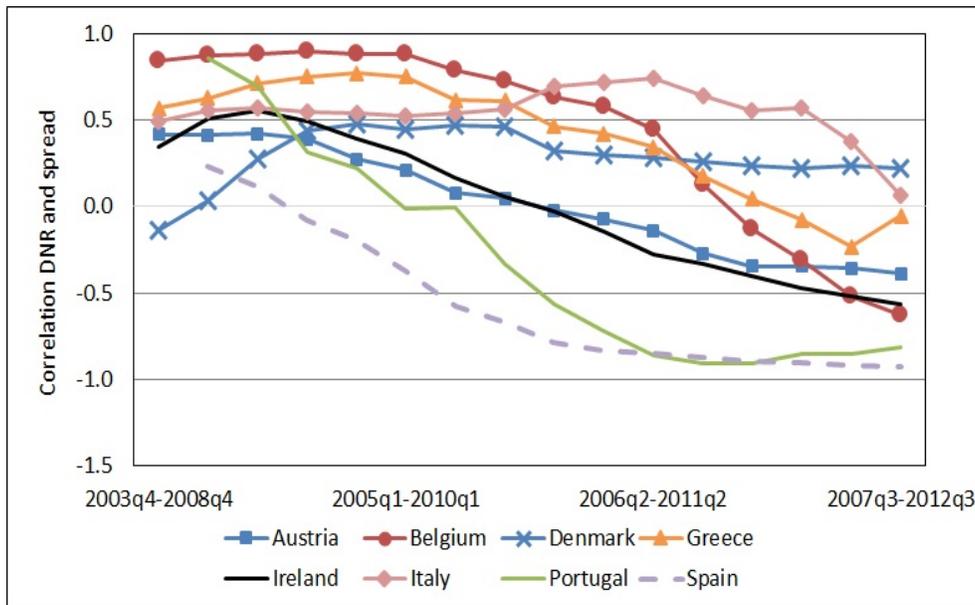


Figure 4: Correlation over time between *DNR* and spread. Quarterly data 1993q1-2012q4. Data sources: National Central Banks and Datastream

In figure 4 it was shown that there is a positive correlation over time between the *DNR* and the spread especially in the beginning of the sample. Evidence for a positive correlation between the *DNR* and the spread arises also at cross sectional level. Starting from the year 2003, I pooled the cross-sectional data of each quarter with the consecutive one so to have at least 14 observation for each cross section. Then, I performed a regression where the spread is explained by a constant and the *DNR*. I report the slope (blue line) and the R squared (red line) of the regression in Figure 5. Figure 5 confirms that there exists a positive

correlation at cross-section level between the DNR and the spread and it shows that the positive correlation has increased over time. Starting from 2011 the slope of the intercept becomes larger than 8 and still can explain about 15% of the variation in the spread.

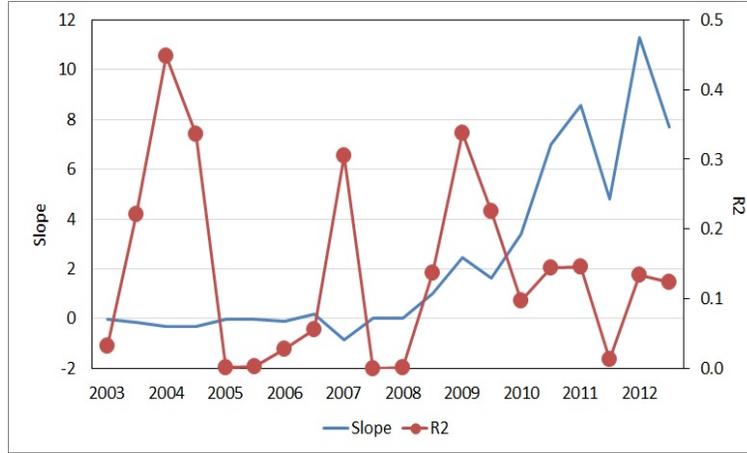


Figure 5: Left axis: Slope of the trend of the cross section. Right axis: R squared of the trend of the cross section. Data source: National Central Banks and Datastream.

The simple scatter plots analysis does not control for other variables which might also be correlated with the growth rate of the spread. Controlling for those factors might reduce the importance of the debt held by non-residents in explaining the spread. As shown in Appendix A.2, I selected the following model after having tested several different specifications. I started with a specification of the model in first-differences including DNR , $DGDP$, $Instability$ and CA and their interactions as regressors. Then I eliminated one by one the most non-significant regressors. The final model obtained is given by:

$$\begin{aligned} \Delta Spread_{it} = & \alpha_0 + \alpha_1 \Delta DNR_{it} + \alpha_2 \Delta DGDP_{it} + \alpha_3 \Delta DNR_{it}^* \Delta DGDP_{it} \\ & + \alpha_4 \Delta (DGDP_{it}^* Instability_{it}) + \alpha_5 \Delta CA_{it}^* \Delta DGDP_{it} + u_{it} \end{aligned} \quad (1)$$

Table 1 reports the results for the first-difference regression in (1). The regression was performed with centered data, for a clearer interpretation of the results.

	Estimate	Std. Error	Pr(> t)
Intercept	-0.03	0.02	0.045 **
ΔDNR_{it}	3.96	1.03	0.000 ***
$\Delta DGDP_{it}$	5.15	0.67	0.000 ***
$\Delta DNR_{it}^* \Delta DGDP_{it}$	10.37	1.68	0.000 ***
$\Delta DGDP_{it}^* \Delta Instability_i$	10.56	3.23	0.000 ***
$\Delta DGDP_{it}^* \Delta CA_{it}$	2.06	1.21	0.088 *
RSS = 48.31 ; R2adj = 0.38			

Table 1: Results from the first-difference regression in (1).

From the results reported in Table 1 we can infer that the growth rate of the fraction of debt held by non-residents is positively correlated with the growth rate of the spread even when controlling for other possible explanatory variables. The share of debt held by non-resident investors has alone a coefficient of 3.96 and of 10.37 when interacted with the debt-to-GDP ratio.

According to these results, an additional 10% of public debt held by foreign investors is associated with an increase in the spread of 40 basis points for countries with 80% of debt-to-GDP. For countries with 120% of debt-to-GDP, an increase of 140 basis points is associated with an additional 10% of public debt held by non-residents.

In the first different regression of table 1, I consider contemporaneous values of the DNR and the spread. This positive correlation can be read in both directions which are both encompassed by the model: higher shares of debt held by foreign investors lead to higher spreads and higher spreads lead investors to increase their holdings in foreign bonds seeking high returns. Using lagged values of the DNR allows to show that higher shares of debt held by non-resident investors lead to higher spreads. In the following table, I show the same regression in (1) for values of DNR lagged of two quarters

	Estimate	Std. Error	Pr(> t)
Intercept	-0.02	0.018	0.185
ΔDNR_{it-2}	2.03	1.09	0.062**
$\Delta DGDP_{it}$	6.12	0.68	0.000 ***
$\Delta DNR_{it-2}^* \Delta DGDP_{it}$	5.21	1.87	0.000 ***
$\Delta DGDP_{it}^* \Delta Instability_i$	16.48	3.26	0.000 ***
$\Delta DGDP_{it}^* \Delta CA_{it}$	3.44	1.29	0.008 ***
RSS = 51.03 ; R2adj = 0.34			

Table 2: Results from the first-difference regression in (1) using lagged values of the *DNR*.

As we can see in Table 2, when the *DNR*'s growth rate leads the spread's growth rate by two quarters, the positive correlation is still significant, although smaller, than when considering contemporaneous values of both variables.

In line with the empirical literature on government-bond yield differentials, the debt-to-GDP ratio is also positively correlated with the spread. The positive correlation is even larger for countries with higher than average *DNR* (like in Ireland, Greece, Belgium, France) and/or political *Instability* (like in Italy, Ireland and Austria).

To summarize, I have shown that there is a positive correlation between the *DNR* and the spread both at cross sectional and time series levels. The first difference regression indicates that this correlation is positive and significant even when controlling for the debt-to-GDP ratio and the degree of political stability. Moreover, the correlation is stronger for countries with a debt-to-GDP ratio larger than 80%.

3 Model

The model is a two-period game where in the first period private agents choose their investment allocations and in the second period governments decide about repayment. The model is different from the existing literature in that I also allow domestic agents to purchase sovereign debt of their own government. In the literature starting with Eaton and Gersovitz (1981), risk neutral foreign investors are the sole buyer of government debt. A second innovation compared to the existing literature is that I study the interaction between two governments' default decisions in a setting where there are cross border bond holdings.

Consider two symmetric countries (Home and Foreign) in a two period world populated by atomistic private agents which, for simplicity, have mass one. Agents live for two periods, are risk averse and maximize their utility over consumption. In the first period of their life, agents earn an exogenous endowment which they consume or invest in home and foreign sovereign bonds. In the second period, agents again earn an exogenous endowment, receive the returns of their investments and pay taxes. Governments are benevolent only toward their own citizens, i.e., they maximize the utility of their own citizens. In the first period, each government finances an exogenous public expenditure with sovereign debt. Public debt is supposed to be repaid in the second period by raising income taxes. In the second period, two scenarios can realize: the *baseline scenario* and the *disaster scenario*. In the baseline scenario, governments decide whether to default (*strategic default*) after observing the realization of a stochastic default cost, which is detrimental for citizens. Strategic default is costly because it generates a utility loss for the citizens of the defaulting government. Default might also be *forced* when a disaster occurs. A disaster is defined as an unexpected and exogenous low income realization such that governments cannot raise taxes and, consequently, cannot repay their debt (e.g., an earthquake). The model timing is summarized in Figure 6.

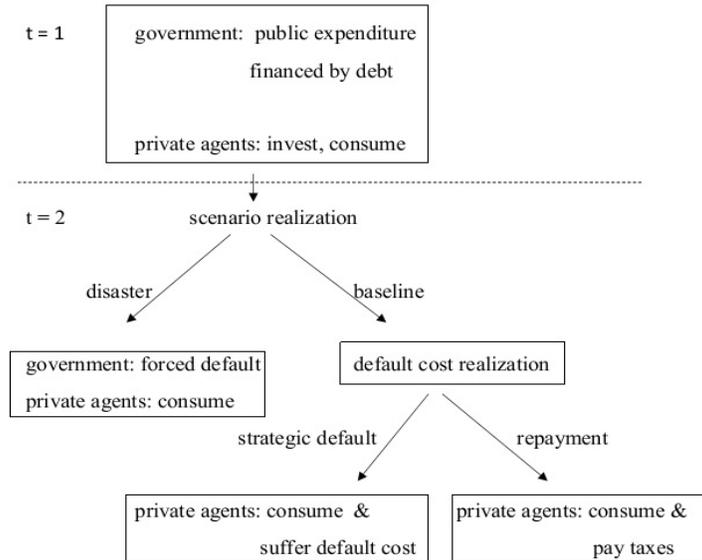


Figure 6: Model timing

The model consists of two subgames. In the first period, private agents of both countries buy a portfolio of sovereign bonds on the financial market. In the second period, if the baseline scenario occurs, governments decide strategically whether to default having observed the portfolio allocations and the realization of their own utility cost.

3.1 Private agents

Private agents are symmetric across countries and maximize the following expected utility function:

$$U(c_1^i, c_2^i) = c_1^i + E[u(c_2^i) - \psi^i I_{[sd]}] \quad \text{for } i = \{H, F\}, \quad (2)$$

s.t. :

$$c_1^i = e - b^i(H) - b^i(F), \quad (3)$$

$$c_2^i = \begin{cases} e + R^H b^i(H) I_{[rep\ H]} + (1 - \rho) R^F b^i(F) I_{[rep\ F]} - T^i I_{[rep\ i]} & \text{if baseline,} \\ [e - Q] + R^F b^i(F) I_{[rep\ F]} & \text{if disaster,} \end{cases} \quad (4)$$

where H and F denote home and foreign, respectively, c_1^i stands for the consumption in the first period, c_2^i stands for the consumption in the second period. The random variable $\psi^i \sim U[0, \Psi]$ represents the utility default cost and $I_{[sd]}$ is an indicator function which equals one when a strategic default occurs. The parameter $e > Q > 0$ represents the exogenous endowment, $b^i(j)$ is the amount of debt issued by country j held by investors resident in i , R^i is the gross return on bonds issued by country i , T^i are the income for taxes charged by government i , the indicator function $I_{[rep\ i]}$ equals one when government i repays the debt and Q is the income loss generated by the disaster.

For simplicity, I assume that the first period utility function is linear with respect to consumption, while the second period utility function is increasing and strictly concave: $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The expected utility is computed with respect to the distribution of the scenario (probability of baseline and disaster) and to the distribution of the utility cost of default ψ^i .

In both periods, agents earn an exogenous endowment $e > Q > 0$, sufficiently large such that consumption is always positive. In period 1, agents invest their endowments in bonds of country Home, $b^i(H)$, and of country Foreign, $b^i(F)$. In period 2, two possible scenarios can occur: the baseline scenario, with probability $\varepsilon \in [0, 1]$, and the disaster scenario, whose probability is $1 - \varepsilon$.

In the baseline scenario, the citizens of the defaulting government suffer a utility loss $\psi^i \sim U[0, \Psi]$ with $\Psi > 0$ if a strategic default (*sd*) occurs. This default cost is a practical shortcut to the infinite horizon models (e.g. Eaton and Gersovitz (1981), Bulow and Rogoff (1988), Kletzer and Wright (2000), Arellano (2008)), but allows to work with a simpler two period models. The classical literature on sovereign debt deals with repeated games where a defaulting government is punished with, for example, the temporary exclusion from financial markets. This punishment implies a utility loss, because risk averse agents are no more able

to smooth consumption over time through public debt. In a similar way, the utility cost can be interpreted as the present value of the future utility after default (see Cooper, Kempf and Peled (2008)). The additive specification of the utility cost of default is only dictated by the tractability of the model. For simplicity, I also assume that the utility costs of the two countries are independent: $\psi^H \perp \psi^F$. A further assumption is that domestic investors do not suffer any utility cost due to a strategic default of the foreign government. In this case, domestic investors only lose the returns on foreign bond investments, if the foreign government strategically defaults. The relaxation of this assumption would be an elegant way to introduce contagion in the model.

In the baseline scenario, the government decides whether to repay the debt and imposes taxes T^i accordingly. Taxes are neutral and represent an incentive for default because they reduce agents' income. On the basis of the previous discussion on the default cost, in the baseline scenario, governments face a trade-off between repayment and default. While default allows to reduce taxation, it also generates a loss of utility.

If a disaster occurs, an amount $Q \in [0, e]$ of agents' income is destroyed, their government cannot repay the debt and cannot impose taxes⁵.

The goal of this paper is to show that financial integration leads to over-investment in foreign sovereign bonds and thereby distorts the probability of sovereigns to default. The variable $\rho \in [0, 1]$ indicates the degree of integration of financial markets. This definition reflects the so called *law of one price*: financial markets are fully integrated when assets with identical risk and return characteristics are priced identically regardless of where they are traded (Adam, Jappelli, Menichini, Padula and Pagano (2002)). Markets might be non-integrated when, for example, transaction costs are very high (non-harmonized taxation, asymmetry of information etc.), i.e. $\rho = 1$. I will call this case *financial autarky*. Markets are instead *fully integrated* whenever transaction costs are zero, i.e. $\rho = 0$.

3.2 Governments

Governments are benevolent toward their citizens and can strategically default in the baseline scenario. Governments decide whether to default after having observed the debt allocation between domestic and foreign investors, the scenario realization, the corresponding interest rate and the utility cost realization. In order to decide upon default, governments maximize the utility function of their own citizens in (2) with respect to the default/repayment decision,

⁵One could have also modeled endogenously that the government cannot raise taxes and, thus, defaults. However, this complication would make the model intractable without adding any interesting insight.

subject to the budget constraints in eq. (3)–(4) and the additional budget constraints:

$$b(i) = g^i \quad \forall i \in \{H, F\}, \quad (5)$$

$$T^i = \begin{cases} R^i b(i) & \text{if no default,} \\ 0 & \text{if default,} \end{cases} \quad (6)$$

and the following market clearing condition

$$b(i) = b^H(i) + b^F(i) \quad \forall i \in \{H, F\}. \quad (7)$$

The first budget constraint means that in the first period the public expenditure is fully financed by government debt. In the second period, each government sets taxes equal to the total bond returns if it repays, 0 if it defaults. I call A^i with $i = \{H, F\}$ the set of actions available to each government and a^i an element of that set. Governments can either repay or strategically default. Formally, this means that the set of actions available to each government is given by: $A^i = \{Rep^i, Def^i\}$. The model is a two period game where in the first period only private agents play and in the second period only governments play. Therefore, the government's optimal choice is described by a strategy which is conditional on the possible "histories" of the game. In game theory, a "history" is defined as a sequence of past decisions and realizations of stochastic events occurring before a choice. If one represents the game as a tree (see Figure 6), a "history" is the sequence of branches and nodes until the node where decisions are taken. In this model, governments decide to default after observing the realization of the exogenous expenditure ($g^i = b(i)$), the investors' portfolio allocations ($b^i(H), b^i(F)$) and the realization of the default cost (ψ^i). Conditional on this history, governments play a simultaneous game. The decision of the government is a strategy such that, conditional on each possible history, each government plays the best reply function to the action played by the other government. Calling the strategy of each government s^i , we can write:

$$s^i : \{b^i(H) \times b^i(F) \times b(i) \times a^j \times \psi^i\} \rightarrow A^i. \quad (8)$$

One could rewrite the strategy of the government in a simpler way which underlines which events are known (the history) at the decision moment

$$s_{y(i)}^i : A^j \rightarrow A^i, \quad (9)$$

where $y(i)$ indicates that the decision is conditional on the history ($b^i(H), b^i(F), b(i), \psi^i$) defined above.

3.3 Equilibrium definition

The model consists of two subgames. In the first period agents choose their portfolio allocation on a competitive financial market. This means that the allocation of bonds and the equilibrium interest rates are determined by a competitive equilibrium. Atomistic agents internalize correctly that the repayment probability is endogenously determined, but they know that their individual decision does not affect this probability. In the second period, governments play strategically taking into account the portfolio allocations. In the baseline scenario, governments take decisions after observing the public expenditure, the investment decisions of private agents and the realization of the cost of default. The equilibrium in the second period of the game is a Nash equilibrium conditional on each possible realization of the history. In summary,

Definition 1. *The equilibrium of the model is a set of bond allocations*

$\{\hat{b}^H(H), \hat{b}^F(H), \hat{b}^H(F), \hat{b}^F(F)\}$, *interest rates* $\{\hat{R}^H, \hat{R}^F\}$ *and default strategy profiles* $\{\hat{s}_{y(H)}^H, \hat{s}_{y(F)}^F\}$ *such that:*

- *the bond allocations* $\{\hat{b}^H(H), \hat{b}^F(H), \hat{b}^H(F), \hat{b}^F(F)\}$ *maximize the agents' utility function in (2),*
- *the interest rates* $\{\hat{R}^H, \hat{R}^F\}$ *clear the market (eq. (8)),*
- *the strategies* $\{\hat{s}_{y(H)}^H, \hat{s}_{y(F)}^F\}$ *are such that the governments play a Nash equilibrium in period 2 for all possible equilibrium allocations of bonds of* $\hat{b}^H(H), \hat{b}^H(F), \hat{b}^F(H)$ *and* $\hat{b}^F(F)$ *and all the possible realization of the default costs* (ψ^H, ψ^F) .

In the next section, I analyze the simple case of financial autarky ($\rho = 1$). In this case, the transaction costs are so high that agents hold only domestic debt. I will use the autarky solution as a benchmark for the analysis, in section 5, of the solution with fully integrated financial markets.

4 Benchmark: Financial Autarky

Let us for the moment assume that financial markets are not integrated because there exist very high transaction costs: $\rho = 1$. In this case, agents will never hold foreign bonds and

their maximization problem can be written as follows

$$U(c_1^i, c_2^i) = c_1^i + E[u(c_2^i) - \psi^i I_{[sd]}] \quad \text{for } i = \{H, F\}, \quad (10)$$

s.t. :

$$c_1^i = e - b(i), \quad (11)$$

$$c_2^i = \begin{cases} e + R^i b(i) I_{[rep \ i]} - T^i I_{[rep \ i]} & \text{if baseline,} \\ e - Q & \text{if disaster.} \end{cases} \quad (12)$$

From the assumption of non-integrated financial markets, it follows that domestic agents buy the whole domestic debt in equilibrium: $\hat{b}^i(i) = b(i) = g^i$, where $\hat{b}^i(i)$ indicates the equilibrium amount of domestic debt held by domestic investors. It is possible to show that, when markets are not integrated, the probability of repayment/default of each country depends only on its own scenario realization and on the domestic bond demand. For simplicity of notation, I indicate with $pr(Rep^i|base^i)$ ($pr(Def^i|base^i)$) the probability of repayment (default) given that the baseline scenario occurs. In principle, the probabilities of repayment and default depend also on the total amount of debt issued, but I leave this implicit. The same holds for the probability of repayment (default) given that a disaster occurs, $pr(Rep^i|dis^i)$ ($pr(Def^i|dis^i)$). The FOC of agents with respect to the fraction of domestic bonds is given by:

$$b(i) : \quad -1 + (1 - \varepsilon) pr(Rep^i|base^i) u'(e + R^i b(i) - T^i) R^i = 0. \quad (13)$$

The probability of repayment, $pr(Rep^i|base^i)$, is determined by the best reply of the government. Conditional on the realization of the baseline scenario, a government is indifferent between repayment and default if the second period utility from repayment and from default are equal. This hinges on the realization of the utility cost ψ^i :

$$u(e + R^i b(i) - T^i) = u(e) - \hat{\psi}^i, \quad (14)$$

where $\hat{\psi}^i$ is the threshold level of the default cost such that the government is indifferent between repayment and default. Realizations of the stochastic default cost higher than $\hat{\psi}^i$ make default too costly. Therefore only for values of the stochastic default shock smaller than this threshold level, a government defaults. Eq.(14) implies that in autarky a government will never default because after having substituted for the taxes $T^i = R^i b(i)$, we obtain $\hat{\psi}^i = 0$. This result implies that the probability of repayment in the baseline scenario is given by $pr(\widehat{Rep^i|base^i}) = 1 - \int_0^{\hat{\psi}^i} d\psi^i = 1 \ \forall b^i(i)$. By construction, we have that $pr(\widehat{Rep^i|dis^i}) = 0 \ \forall b^i(i)$. Thus, the equilibrium strategy of each government is $\widehat{s}_{y(i)}^i(s_{y(j)}^j) = Rep^i \ \forall s^j, \psi^i, \psi^j, b^i(i), b^j(j)$ with $j \neq i$. Then, from (14), the equilibrium

interest rate in autarky is given by

$$\hat{R}^i = \frac{1}{(1 - \varepsilon) u'(e)}, \quad (15)$$

as shown in more detail in Appendix B.1. Under the restriction that agents must hold only domestic debt, the equilibrium is given by:

$$\left\{ \hat{b}^H (H), \hat{b}^F (F), \hat{R}^H, \hat{R}^F \hat{s}_{y(H)}^H, \hat{s}_{y(F)}^F \right\}. \quad (16)$$

5 Integrated markets

The autarky case analyzed in the previous section constitutes a simple benchmark for understanding the effects of market integration on welfare. On the one hand, the autarky equilibrium is characterized by the absence of the strategic default risk. On the other hand, however, the disaster risk cannot be diversified away. Financial market integration allows private agents to hedge against the disaster risk, which constitutes the motive for diversification in this model. Absent other motives for international bond diversification, the equilibrium would be described by agents investing exclusively in domestic sovereign bond markets, even if these markets were perfectly internationally integrated. Suppose, for example, that agents diversified their portfolio in equilibrium even in the absence of a disaster risk. Then, under a sufficiently low realization of the default cost for one government and a sufficiently high for the other government, only one government would repay. In this case, the defaulting government would expropriate the foreign bond holders and thereby reduce taxes. But foreign investors would anticipate this and would prefer to buy only domestic debt to hedge tax risk. Under the assumption that a disaster can occur with very low probability, fully integrated markets lead to positive cross bond holdings.

Consider the optimization problem of investors when financial markets are integrated, i.e., $\rho = 0$. From (2)–(4) it follows that the demand of home and foreign bonds is defined by the FOCs of investors:

$$\begin{aligned}
b^i(H) : & -1 + (1 - \varepsilon)^2 pr(Rep^H, Rep^F | base^H, base^F) u'(Rep^H, Rep^F | base^H, base^F) R^H \\
& + (1 - \varepsilon)^2 pr(Rep^H, Def^F | base^H, base^F) u'(Rep^H, Def^F | base^H, base^F) R^H \\
& + \varepsilon (1 - \varepsilon) pr(Rep^H, Def^F | base^H, dis^F) u'(Rep^H, Def^F | base^H, dis^F) R^H = 0, \quad (17)
\end{aligned}$$

$$\begin{aligned}
b^i(F) : & -1 + (1 - \varepsilon)^2 pr(Rep^H, Rep^F | base^H, base^F) u'(Rep^H, Rep^F | base^H, base^F) R^F \\
& + (1 - \varepsilon)^2 pr(Def^H, Rep^F | base^H, base^F) u'(Def^H, Rep^F | base^H, base^F) R^F \\
& + \varepsilon (1 - \varepsilon) pr(Def^H, Rep^F | dis^H, base^F) u'(Def^H, Rep^F | dis^H, base^F) R^F = 0, \quad (18)
\end{aligned}$$

for $i \in \{H, F\}$. In eq.(17) and (18), the symbol $(Def^H, Rep^F | dis^H, base^F)$ stands for repayment in country H and default in country F conditional on no disaster in country H and disaster in country F .⁶ The second period best reply functions of the governments are conditional on the investment allocation, the action played by the other government and the realization of the utility default cost. For a given investment allocation and action of the other government, if the utility default cost is low, a government chooses default. Hence, to compute the best reply function of the government, it is sufficient to look at these threshold values of the utility cost ψ^i . Let us call $\tilde{\psi}^i$ ($\bar{\psi}^i$) the utility cost such that a government is indifferent between repayment and default, conditional on the other government repayment (default) and any given investment allocation:

$$\tilde{\psi}^i \equiv u(e + R^j b^i(j)) - u(e + R^i b^i(i) + R^j b^i(j) - T^i), \quad i \neq j \text{ and } i, j \in \{F, H\}, \quad (19)$$

$$\bar{\psi}^i \equiv u(e) - u(e + R^i b^i(i) - T^i). \quad (20)$$

Both $\tilde{\psi}^i$ and $\bar{\psi}^i$ are non-negative by the concavity of the utility function. This means that if government j repays and a default cost $\psi^i \in [0, \tilde{\psi}^i]$ occurs, government i defaults. Instead, if government j defaults and a default cost $\psi^i \in [0, \bar{\psi}^i]$ realizes, also government i defaults. As we can see from (19) and (20), if the amount of domestic sovereign debt held by domestic investors ($b^i(i)$) decreases, government i is more likely to default ($\tilde{\psi}^i$ and $\bar{\psi}^i$ increase).

On the basis of (17)–(20), it is possible to show that

Theorem 1. *With fully integrated financial markets ($\rho = 0$), there exists a unique equilibrium consisting of a bond portfolio allocation $\{\tilde{b}^H(H), \tilde{b}^F(H), \tilde{b}^F(F), \tilde{b}^H(F)\}$, bond interest rates $\{\tilde{R}^H, \tilde{R}^F\}$ and strategy profiles for the government default decision $\{\tilde{s}_{y(H)}^H, \tilde{s}_{y(F)}^F\}$*

⁶Note that the probability of repayment and of default depend on the bond allocations. I don't write this conditioning explicitly in order to keep the notation simple.

s.t.:

1. investors of each country hold a positive amount of foreign sovereign debt $\tilde{b}^i (i) \in (0, b(i))$ for $i \in \{H, F\}$.
2. The equilibrium strategies of the government for each possible default costs realization and bond portfolio allocation is described by the following table:

Default cost of F	Ψ	(Def^H, Rep^F)	(Rep^H, Rep^F)	(Rep^H, Rep^F)
	$\bar{\psi}^F$	(Def^H, Def^F)	(Rep^H, Rep^F)	(Rep^H, Rep^F)
	$\tilde{\psi}^F$	(Def^H, Def^F)	(Def^H, Def^F)	(Rep^H, Def^F)
0		$\tilde{\psi}^H$	$\bar{\psi}^H$	Ψ
		Default cost of H		

where $\tilde{\psi}^i$ and $\bar{\psi}^i$ are defined in eq.(19) and (20), respectively.

Proof. see Appendix B.2. □

Theorem 1 states that with fully integrated financial markets, agents buy in equilibrium a positive amount of foreign sovereign debt. The purchase of foreign debt induces the foreign government to strategically default in equilibrium for some realizations of the default costs. The equilibrium strategy profiles are such that for any portfolio allocation, if in both countries a high cost of default occurs ($\psi^H \geq \tilde{\psi}^H$ and $\psi^F \geq \tilde{\psi}^F$), both governments decide to repay. If in both countries a low cost of default occurs ($\psi^H < \bar{\psi}^H$ and $\psi^F < \bar{\psi}^F$), both governments decide to default. If in one country a high cost of default occurs while in the other a low cost of default occurs, only the government facing a high cost of default repays.

5.1 Comparative statics and comparison with the stylized facts

This section discusses some properties of the current model and compares them to the empirical findings of section 2. Overall, I find that the predictions of the model are qualitatively in line with the results illustrated in section 2.

5.1.1 Debt held by foreign investors

The first-difference regression whose estimates are reported in section 2 predicts that an increase in the share of debt held by non-residents (DNR) is positively correlated with an increase in the spread. In the model, I assumed for simplicity that both countries have the same default risk, which implies no spread between the two bonds. A comparison between the prediction of the data and the ones of the model is not straightforward because the spread between the two bonds considered is zero.

In order to compare the empirical correlation with the predictions of the model, one could look at the spread between the equilibrium interest rate and a risk free rate. To this end, I could either assume that agents can buy another asset in zero net supply which repays in each state of the world or that one country never defaults ($\Psi \rightarrow \infty$). In both these cases, the spread in the model cannot be computed because the equilibrium interest rate with integrated markets has no close form.

Another option is to interpret the spreads as a proxy for the default probability of a government. One could read the positive correlation between the growth rate of the spread and the one of non-resident holdings as that the integration of financial markets should be accompanied by higher default risk. In the current model, financial market integration is represented by lower levels of the parameter $\rho \in [0, 1]$. In sections 4 and 5 it was shown that the equilibrium probability of default is higher with integrated markets than in autarky because governments default strategically. These are however two extreme cases. I show in the following lemma that when the degree of market integration increases smoothly, also the equilibrium default probabilities increase.

Lemma 1. *The equilibrium probability that both governments repay is an increasing function of ρ .*

Proof. See Appendix B.3 □

5.1.2 Total debt

In the empirical analysis of section 2, I showed that the debt-to-GDP ratio is one of the main factors explaining the evolution of the spread in the Euro area. This finding also emerges in large part of the empirical literature on government bond yield differentials. Differently from my model, in reality governments can decide how to finance public expenditure. They can choose either to raise taxes or to postpone tax payment to the future by issuing public debt. In the current model, the total debt issued is constrained to be equal to the total public

expenditure. In the current model, an increase in the public expenditure and, thus, of the total debt issued, has an effect on the default risk through two channels. First, a change in the public expenditure might have an effect on the portfolio allocation of private agents. Second, default is less costly when the public expenditure is high, as shown in the traditional literature on sovereign default. As I show in Appendix B.4, the amount and the share of debt held by non-residents decreases with the first period public expenditure.

Lemma 2. *The amount and the share of sovereign debt held by foreign investors decreases with the first period public expenditure.*

Proof. See Appendix B.4. □

This result is in line with Figures 1 and 2 showing that, starting from the beginning of 2009, the share of debt held by non-residents decreased while the debt to GDP increased. The overall effects on the default risk are in the model quite difficult to disentangle given that there is no closed form solution. In the data, we observe that from 2006q1 to 2008q3 the spreads and the *DNR* increased on average, while the debt-to-GDP ratio decreased. From 2008q3 till 2011q1 the *DNR* was stable, while the spreads and the debt-to-GDP ratio increased. During the last two years, both the spreads and the debt-to-GDP ratio decreased while the debt-to-GDP ratio increased. The most reasonable interpretation of these facts is that before the crisis and since 2011 the *DNR* had a more relevant effect in explaining the spreads, while during the crisis the debt-to-GDP ratio played a bigger role. The long run trends of the debt-to-GDP ratio and the *DNR* are also nested by the model which predicts a negative correlation between the two variables.

6 Inefficiency of the portfolio allocation

The comparison between the equilibrium with non-integrated financial markets and the equilibrium with integrated financial markets allows investigating the effects of integration on the default incentives. From section 5, the integration of financial markets is beneficial for private agents because it allows them to hedge against the disaster risk. At the same time, integration is detrimental for private agents because it gives rise to strategic default in equilibrium, which is absent in autarky (see section 4). In this section, I show that the negative effects of strategic default risk outweigh the positive effect of diversification. Lower shares of non-resident debt holdings are Pareto improving and socially optimal under reasonable assumptions. Therefore, the equilibrium with integrated markets is inefficient. This inefficiency originates from the game between non-strategic investors and strategic sovereigns. A

marginal increase in the percentage of debt held by a representative non-resident investor increases the incentives to default, because governments are benevolent toward their own citizens. On the contrary, both foreign and domestic private agents play non-strategically and correctly estimate equal to zero the effect of their individual investment decision on the probability of default. Figure 6 illustrates the effect of an increase in the debt held by non-resident investors on the threshold levels of the stochastic default cost.

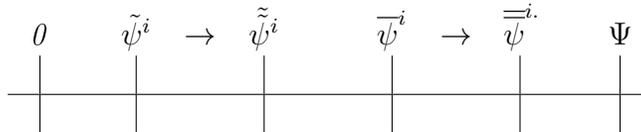


Figure 7: Shift in the threshold values of ψ due to an increase in the fraction of debt held by foreign investors

If the fraction of debt of country i held by non-residents increases, both threshold levels $\tilde{\psi}^i$ and $\bar{\psi}^i$ move to the right. For any realization of the default cost, default is less costly because it allows to expropriate more non-resident investors than in a situation with lower shares of non-resident holdings. This result implies that the probability of default increases in equilibrium. I find that the equilibrium portfolio allocation is characterized by excessive investments in foreign debt with respect to a situation where the social planner imposes the portfolio allocations. Indeed, the autarky allocation of section 4 is a Pareto improving allocation:

Theorem 2. *The autarky allocation is Pareto superior to equilibrium allocation with integrated markets when $\varepsilon \rightarrow 0$ and countries have the same first period expenditure shock.*

Proof. See Appendix C.1 □

Although the autarky equilibrium allocation is Pareto improving upon the equilibrium allocation with integrated markets, autarky is not the first best. The first best is the solution of a social planner problem where the social planner decides about the portfolio allocation and the default strategy. This equilibrium could be implemented with a political union where governments would jointly decide upon default. Intuitively, governments would not strategically default when cooperating. Given this strategy, the portfolio equilibrium allocation would be the same as in the classical portfolio theory: private agents would equally share risk. This means that domestic and foreign investors overall would hold half of each debt.

Theorem 3. *The first best allocation is characterized by no strategic default and equal risk sharing ($b^i(j) = b^j(i)$).*

The result of Theorem 3, is not a very realistic policy recommendation, because Euro area governments seem far from giving up their sovereign power and agree upon a political union. A more interesting result for policy recommendation comes from the second best solution. I define the second best as a situation where the social planner imposes the portfolio allocations but cannot decide about default. Let us consider the maximization problem of the social planner:

$$\max_{c_1^H, c_1^F, c_2^H, c_2^F} \{c_1^H + E[u(c_2^H) - \psi^H I_{[sd]}] + c_1^F + E[u(c_2^F) - \psi^F I_{[sd]})]\}, \quad (21)$$

where c_1^H , c_2^H , c_1^F , c_2^F are defined as before and the social planner knows also the budget constraints of both governments. Calling $h \in [0, 1]$ the share of public debt held by residents, $h \equiv b^i(i) / b(i)$, the maximization of the social planner's utility function leads to the following result:

Theorem 4. *Let us assume that the utility function in (21) is strictly concave with respect to the share of debt held by non-resident investors and that the public expenditure in the first period is equal across countries. Then, the second best solution is characterized by a higher share of resident debt holdings in both countries.*

- *If it holds:*

$$\frac{u'(e - Q)}{u'(e)} > 1 + u(e) + u(e - Q), \quad (22)$$

then the second best allocation is characterized by a share of resident debt holdings which is higher than in the equilibrium of Theorem 1 and lower than in autarky: $\tilde{h} \in \left(\frac{\tilde{b}^F(H)}{b(H)}, 1\right)$, where \tilde{h} is the optimal share for the social planner and $\tilde{b}^F(H)$ is the optimal amount of non-resident holdings in the equilibrium of Theorem 1.

- *If condition (22) does not hold, then the second best allocation corresponds to the autarky allocation.*

Proof. See Appendix C.2 □

Theorem 4 states that the second best is characterized by a lower share of debt held by non-resident investors. If condition (22) holds, then the social planner would purchase a positive share of foreign debt but lower than in the competitive equilibrium. If condition

(22) does not hold, then the autarky allocation is the second best allocation. Condition (22) is satisfied under the assumption of logarithmic or quadratic utility and reasonable values for the parameters. Intuitively, condition (22) means that the marginal utility in a disaster when a country is in autarky is high. In other words, this means that the gains from diversification are sufficiently high. Graphically, if condition (22) holds, then the utility function looks like in Figure 8: the autarky allocation ($h = 1$) achieves a utility which is higher than the one at the competitive equilibrium. However, the maximum of the utility of the social planner is obtained at a point with positive level of foreign bond holdings $h < 1$, but lower than in the competitive equilibrium.

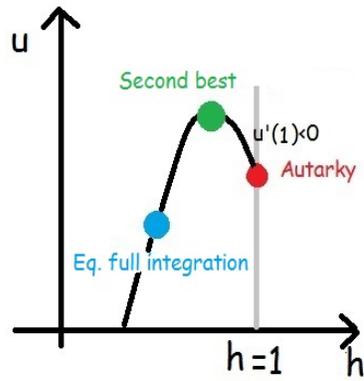


Figure 8: Social planner allocation if condition (22) holds

7 Conclusions

The integration of financial markets in the Euro area was accompanied by a significant increase in the share of public debt held by non-residents. Although with the debt crisis this share decreased, it is still above 50% in many Euro area countries. I ask whether this fact can be explained by a rational model and whether it is socially desirable. Empirical analysis shows that, indeed, the spread of 10-year government bonds are positively correlated with the share of debt held by non-residents. Countries with higher shares of debt held by non-residents are likely to have also higher spreads. This correlation is even higher for countries with a debt-to-GDP ratio higher than 80%.

To formalize these facts, I built a two-period-two-country model and investigated the efficiency of the bond-portfolio allocation in equilibrium. I find that there is a tendency for atomistic investors to over-invest in foreign sovereign debt and this raises the default probability above the social optimum. The reason is that while investors are non-strategic

(atomistic), non-committed governments are benevolent toward their own citizens and decide strategically about default on the basis of the aggregate choice of investors.

I show that with a sufficiently small probability of disaster, the financial autarky equilibrium Pareto dominates the equilibrium with integrated financial markets.

However, the autarky equilibrium is not the first best. It is socially optimal (first best) that the disaster risk is shared equally by all investors and that states do not default voluntarily. The first best could be implemented via a political union, which is desirable, but still unlikely to occur in the near future. A more interesting result for policy recommendation is represented by the second best. In this case, I assume that governments maintain their sovereign power while the portfolio allocation of private agents is dictated by the social planner. The second best solution is characterized by a lower share of non-resident holdings of sovereign debt, and consequently, lower default probability in equilibrium. This could be easily achieved with a taxation on returns of foreign bonds.

My model focuses on the portfolio problem abstracting from the discussion of many other interesting aspects of the debt crisis. Possible extensions could include contagion, the choice on how to finance public debt (debt vs taxes) and the ex-post possibility of a bailout. Further research in this direction is also needed in order to get an idea of how large is the inefficiency discussed here actually is.

References

- [1] ADAM, K., JAPPELLI, T., MENICHINI, A., PADULA, M., AND PAGANO, M. *Analyse, compare, and apply alternative indicators and monitoring methodologies to measure the evolution of capital market integration in the European Union*. University of Salerno, 2002.
- [2] ARELLANO, C. Default risk and income fluctuations in emerging economies. *The American Economic Review* (2008), 690–712.
- [3] ATTINASI, M. G., CHECHERITA, C., AND NICKEL, C. What explains the surge in euro area sovereign spreads during the financial crisis of 2007-09? *Working Paper Series 1131, European Central Bank* (2009).
- [4] BALKAN, E. M. Political instability, country risk and probability of default. *Applied Economics* 24, 9 (1992), 999–1008.
- [5] BAYOUMI, T., GOLDSTEIN, M., AND WOGLOM, G. Do credit markets discipline sovereign borrowers? evidence from us states. *Journal of Money, Credit and Banking* 27, 4 (1995), 1046–1059.
- [6] BERNOTH, K., VON HAGEN, J., AND SCHUKNECHT, L. Sovereign risk premiums in the european government bond market. *Journal of International Money and Finance* 31, 5 (2012), 975–995.
- [7] BREWER, T. L., AND RIVOLI, P. Politics and perceived country creditworthiness in international banking. *Journal of Money, Credit and Banking* 22, 3 (1990), 357–369.
- [8] BRONER, F., AND VENTURA, J. Globalization and risk sharing. *The Review of Economic Studies* 78 (2011), 49–82.
- [9] BULOW, J. I., AND ROGOFF, K. S. Sovereign debt: Is to forgive to forget? *The American Economic Review* 79 (1989), 43–50.
- [10] CARLO FAVERO, M. P., AND VON THADDEN, E.-L. How does liquidity affect government bond yields? *Journal of Finance and Quantitative Analysis* 45 (2010), 107–134.
- [11] CITRON, J.-T., AND NICKELSBURG, G. Country risk and political instability. *Journal of Development Economics* 25, 2 (1987), 385–392.
- [12] COOPER, R., KEMPF, H., AND PELED, D. Is it is or is it ain’t my obligation? regional debt in a fiscal federation*. *International Economic Review* 49, 4 (2008), 1469–1504.

- [13] COOPER, R., KEMPF, H., AND PELED, D. Insulation impossible: Fiscal spillovers in a monetary union. Tech. rep., National Bureau of Economic Research, 2009.
- [14] CUADRA, G., AND SAPRIZA, H. Sovereign default, interest rates and political uncertainty in emerging markets. *Journal of International Economics* 76, 1 (2008), 78–88.
- [15] EATON, J., AND GERSOVITZ, M. Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies* 48, 2 (1981), 289–309.
- [16] FAINI, R. Fiscal policy and interest rates in europe. *Economic Policy* 21, 47 (2006), 443–489.
- [17] GOLDSTEIN, M., AND WOGLOM, G. Market-based fiscal discipline in monetary unions: evidence from the us municipal bond market. *Establishing a Central Bank: Issues in Europe and Lessons from the US* 228 (1992).
- [18] HALLERBERG, M., AND WOLFF, G. B. Fiscal institutions, fiscal policy and sovereign risk premia in emu. *Public Choice* 136, 3-4 (2008), 379–396.
- [19] KLETZER, K. M., AND WRIGHT, B. D. Sovereign debt as intertemporal barter. *The American Economic Review* 90, 3 (2000), 621–639.
- [20] MALTRITZ, D. Determinants of sovereign yield spreads in the eurozone: A bayesian approach. *Journal of International Money and Finance* 31, 3 (2012), 657–672.
- [21] MANGANELLI, S., AND WOLSWIJK, G. What drives spreads in the euro area government bond market? *Economic Policy* 24, 58 (2009), 191–240.
- [22] MOSER, C. The impact of political risk on sovereign bond spreads-evidence from latin america.
- [23] PAGANO, M., AND VON THADDEN, E.-L. The european bond markets under emu. *Oxford Review of Economic Policy* 20, 4 (2004), 531–554.
- [24] PANIZZA, U., STURZENEGGER, F., AND ZETTELMEYER, J. The economics and law of sovereign debt and default. *Journal of Economic Literature* (2009), 651–698.
- [25] POTERBA, J. M., AND RUEBEN, K. S. Fiscal news, state budget rules, and tax-exempt bond yields. *Journal of Urban Economics* 50, 3 (2001), 537–562.
- [26] SINN, H.-W., AND WOLLMERSHAEUSER, T. Target loans, current account balances and capital flows: the ecb' s rescue facility. *International Tax and Public Finance* 19, 4 (2012), 468–508.

- [27] VON HAGEN, J., SCHUKNECHT, L., AND WOLSWIJK, G. Government bond risk premiums in the eu revisited: The impact of the financial crisis. *European Journal of Political Economy* 27, 1 (2011), 36–43.

Appendix A.1: Database

The databases comprises the following countries: Austria (2003q1-2012q4), Belgium (1996q1-2012q4), Denmark (1997q1-2012q4), France (2008q1-2012q4), Germany (1991q1-2012q4), Greece (2003q1-2012q4), Ireland (1998q1-2012q4), Italy (1997q1-2012q4), Portugal (2003q1-2012q4), Spain (2005q1-2012q4).

- Dependent variables: *Spread* between the yield on 10Y government bonds of each country and the interest rate on German Bund. I took the average of the last week of the quarter the estimations. Taking the average over the quarter does change the results. Data source: Datastream
- The independent variables are such that the *Spread* is increasing in their values. In this way, I should avoid problems in the interpretation of the signs of the interaction terms. The independent variables are the following (in parenthesis the name used in the estimates is indicated):
 - Debt to GDP (*DGDP*) (magnitude [0,1]). Data source: National Central Banks for the debt, National Statistical Offices for the GDP.
 - Percentage of public debt held by foreign investors (*DNR*) (magnitude [0,1]). Data source: National Central Banks. Note that when the ECB buys public debt, this is then accounted in the debt held by the national Central Bank.
 - Corruption (*Corruption*) (magnitude [0,1]). This is the inverse of the “Corruption Perceptions Index” (which is higher when corruption is lower). Intuitively, the higher value of “corruption” the more difficult is to grow. Data source: Transparency International.
 - Political instability (*Instability*) (can take values between 0 and infinity, in the sample ranges between (0,2)). This is the inverse of the average of two measures of political stability:
 - * Number of months that a prime minister is in charge over the maximum possible number of months that a minister could be in charge during a legislation. The average was computed for the last 20 years. This might undervalue the political stability because a prime minister might be reelected or the same party might stay in power over different legislations.
 - * Number of months between two different elections over the maximum number of months between two different elections. The average was computed for

the last 20 years. This could overvalue the political stability because often governments are changed without reelections (e.g. change of some ministers).

Intuitively, the higher the political instability, the lower the possibility to promote reforms for growth. This index is similar to the one used in Citron and Nickelsburg (1987) . Data source: national governments websites.

- Net current account over GDP (CA) (magnitude $[0,1]$). A low CA should mirror the lower competitiveness of the production sector, which in the long run is a growth indicator. Data source: National Central Banks.

Appendix A.2: Data Analysis

Additional charts

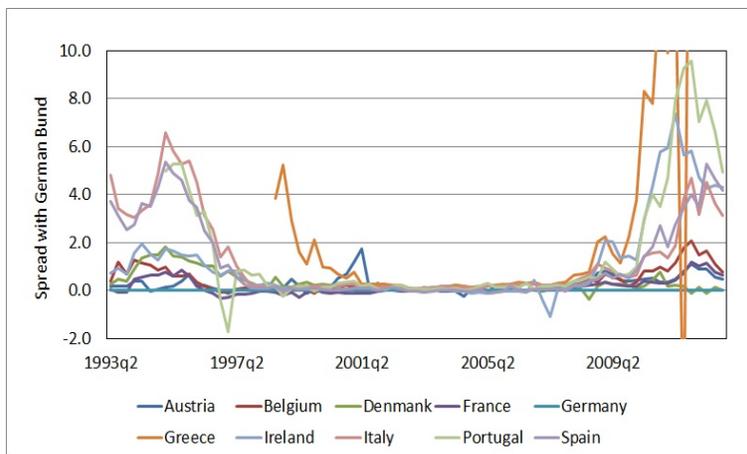


Figure 9: Evolution of the spread over time including Greece. Data source: Datastream.

For completeness I report the correlations over time between the DNR and the spread for the whole samples. Given that in the majority of the countries there are trends, I consider detrended data in order to have consistent correlation estimates.

	AT	BE	DK	GR	EI	IT	PT	ES
$Corr \left(\frac{\Delta DNR_{i,t}}{DNR_{i,t-4}}, \frac{\Delta Spread_{i,t}}{Spread_{i,t-4}} \right)$	-0.28	0.14	0.19	0.21	-0.12	0.04	0.18	-0.09
$Corr \left(\frac{\Delta DNR_{i,t-2}}{DNR_{i,t-6}}, \frac{\Delta Spread_{i,t}}{Spread_{i,t-4}} \right)$	-0.34	0.15	0.08	0.07	0.03	0.31	0.34	-0.12

Table 3: Cross correlation between Y-o-Y growth rates in DNR and Spread. Data sources: National Central Banks and Datastream.

Preliminary analysis

1. First I check for multicollinearity in the regressors: I computed the Variance Inflation Factor predictor of each independent variable against the others.

	R^2 of the variable regressed on the others
<i>Corruption</i>	0.94762
<i>Instability</i>	0.80517
<i>DGDP</i>	0.62555
<i>DNR</i>	0.66602
<i>CA</i>	0.71161

Table 4: Multicollinearity test on the regressors

Values of VIF ($VIF = \frac{1}{1-R^2}$) exceeding 5 are considered evidence of collinearity, that is value of the R^2 larger than 0.8 are evidence of collinearity. From the table above, then *Corruption* and *Instability* could be explained by a linear combination of other factors. Therefore, I eliminate the variable *Corruption* and the remaining variables are non multicollinear

	R^2 of the variable regressed on the others
<i>Instability</i>	0.32678
<i>DGDP</i>	0.51702
<i>DNR</i>	0.41113
<i>CA</i>	0.35993

Table 5: Multicollinearity test a restricted set of regressors

The remaining variables don't present evidence of high correlation.

2. I perform the following First Difference regression eliminating *Corruption* from the independent variables set:

$$\begin{aligned}
\Delta Spread_{it} = & \alpha_0 + \alpha_1 \Delta DNR_{it} + \alpha_2 \Delta DGDP_{it} + \alpha_3 \Delta CA + \alpha_4 \Delta (DNR_{it}^* DGDP_{it}) + \\
& \alpha_5 \Delta (DNR_{it}^* Instability_i) + \alpha_6 \Delta (DNR_{it}^* CA_{it}) + \\
& \alpha_7 \Delta (DGDP_{it}^* Instability_i) + \alpha_8 \Delta (DGDP_{it}^* CA_{it}) + \\
& \alpha_9 \Delta (Instability_i^* CA_{it}) + u_{it}
\end{aligned} \tag{23}$$

I eliminate one by one the most non significant variables. The coefficients of the regression change during this process namely because there are too many regressors with respect to the number of observations (more or less 360). The final equation is

$$\begin{aligned}
\Delta Spread_{it} = & \alpha_0 + \alpha_1 \Delta DNR_{it} + \alpha_2 \Delta DGDP_{it} + \alpha_3 \Delta (DNR_{it}^* DGDP_{it}) + \\
& \alpha_4 \Delta (DGDP_{it}^* Instability_i) + \alpha_5 \Delta (CA_{it}^* DGDP_{it}) + u_{it}
\end{aligned} \tag{24}$$

Appendix B.1: Equilibrium of the autarky case

$$\max_{c_1^i, c_2^i} \{c_1^i + E [u(c_2^i) - \psi^i I_{[sd]}]\}, \quad \text{for } i \in \{H, F\}, \quad (25)$$

s.t. :

$$c_1^i = e - b(i), \quad (26)$$

$$c_2^i = \begin{cases} e + R^i b(i) I_{[rep \ i]} - T^i I_{[rep \ i]} & \text{if baseline,} \\ e - Q & \text{if disaster.} \end{cases} \quad (27)$$

which implies the FOC

$$b(i) : \quad -1 + (1 - \varepsilon) pr(Rep^i | base^i) u'(e + R^i b(i) - T^i) R^i = 0. \quad (28)$$

The government is indifferent between repayment and default when

$$u(e + R^i b(i) - T^i) = u(e) - \hat{\psi}^i \quad (29)$$

$$\Leftrightarrow \hat{\psi}^i = 0. \quad (30)$$

This implies that the probability of default is given by

$$\frac{1}{\Psi} \int_0^{\hat{\psi}^i} d\psi^i = 0. \quad (31)$$

Using the fact that the probability of repayment equals one, we obtain

$$-1 + (1 - \varepsilon) u'(e + R^i b(i) - T^i) R^i = 0. \quad (32)$$

Substituting the budget constraint of the government $T^i = R^i b(i)$, then

$$h : \quad -1 + (1 - \varepsilon) u'(e) R^i = 0 \quad (33)$$

$$\Leftrightarrow \hat{R}^i = [(1 - \varepsilon) u'(e)]^{-1}. \quad (34)$$

Appendix B.2: Theorem1

With fully integrated financial markets ($\rho = 0$), there exists a unique equilibrium consisting of a bond portfolio allocation $\{\tilde{b}^H(H), \tilde{b}^F(H), \tilde{b}^F(F), \tilde{b}^H(F)\}$, bond interest rates $\{\tilde{R}^H, \tilde{R}^F\}$ and strategy profiles for the government default decision $\{\tilde{s}_{y(H)}^H, \tilde{s}_{y(F)}^F\}$ s.t.:

1. investors of each country hold a positive amount of foreign sovereign debt $\tilde{b}^i(i) \in (0, b(i))$ for $i \in \{H, F\}$.
2. The equilibrium strategies of the government for each possible default costs realization and bond portfolio allocation is described by the following table:

Default cost of F	Ψ	(Def^H, Rep^F)	(Rep^H, Rep^F)	(Rep^H, Rep^F)
	$\bar{\psi}^F$	(Def^H, Def^F)	(Rep^H, Rep^F)	(Rep^H, Rep^F)
	$\tilde{\psi}^F$	(Def^H, Def^F)	(Def^H, Def^F)	(Rep^H, Def^F)
	0	(Def^H, Def^F)	(Def^H, Def^F)	(Rep^H, Def^F)
		$\tilde{\psi}^H$	$\bar{\psi}^H$	Ψ
		Default cost of H		

where $\tilde{\psi}^i$ and $\bar{\psi}^i$ are defined in eq.(19) and (20), respectively.

Proof:

Optimal government strategies

Thresholds for ψ given that the other repays

$$u(e + b^i(j)R^j) - \tilde{\psi}^i = u(e + b^i(i)R^i + b^i(j)R^j - b(i)R^i) \quad (35)$$

$$\Leftrightarrow \tilde{\psi}^i = u(e + b^i(j)R^j) - u(e + b^i(i)R^i + b^i(j)R^j - b(i)R^i) \quad (36)$$

$$= u(e + b^i(j)R^j) - u(e + b^i(j)R^j - \Delta) \quad (37)$$

where $\Delta \equiv b^i(i)R^i - b(i)R^i < 0$. Thus $\tilde{\psi}^i \geq 0$ because $u' > 0$.

Thresholds for ψ given that the other defaults

$$u(e) - \bar{\psi}^i = u(e + b^i(i)R^i - b(i)R^i) \quad (38)$$

$$\Leftrightarrow \bar{\psi}^i = u(e) - u(e + b^i(i)R^i - b(i)R^i) \quad (39)$$

$$= u(e) - u(e - \Delta) \quad (40)$$

Thus $\bar{\psi}^i \geq 0$ because $u' > 0$.

Best replies and strategies By the strict concavity of the utility function and given that $e < eb^i(j)R^j$, we have that $\bar{\psi}^i > \tilde{\psi}^i$. Intuitively, this means that a government is more likely to default given that the other government repays. The marginal utility from a repayment is lower when agents receive already returns from other investments than when agents are left with the exogenous endowment only.

We can study the best responses conditional on the space of the default cost using the following tables

θ	$\tilde{\psi}^H$	$\bar{\psi}^H$	Ψ
	A	B	C
$\tilde{\psi}^F$			
	D	E	F
$\bar{\psi}^F$			
	G	H	I
Ψ			

$(\psi^H, \psi^F) \in \dots$	Best reply
A	$Def^F (Def^H)^*, Def^H (Def^F)^*, Def^F (Rep^H), Def^H (Rep^F)$
B	$Def^F (Def^H)^*, Def^H (Def^F)^*, Def^F (Rep^H), Rep^H (Rep^F)$
C	$Def^F (Def^H), Rep^H (Def^F)^*, Def^F (Rep^H)^*, Rep^H (Rep^F)$
E	$Def^F (Def^H)^*, Def^H (Def^F)^*, Rep^F (Rep^H)^*, Rep^H (Rep^F)^*$
F	$Def^F (Def^H), Rep^H (Def^F), Rep^F (Rep^H)^*, Rep^H (Rep^F)^*$
I	$Rep^F (Def^H), Rep^H (Def^F), Rep^F (Rep^H)^*, Rep^H (Rep^F)^*$

where the stars indicate the Nash equilibria and Rep^i indicates repay while Def^i default. Even if for $(\psi^H, \psi^F) \in E$ there are two Nash equilibria, I select only the equilibrium with the highest utility which increases the parameter space where both countries repay. Using this equilibrium in E, makes the result even stronger because it increases the advantages from integration. That is for each combination of utility costs there exists one and only one strategy of the government which maps from the parameter space to the binary default-repay space.

Bond demand

To keep the notation light, I indicate with r repayment, d default, 0 the baseline scenario, q the disaster scenario.

Let us assume that $pr(r, d|0, 0) > 0$ and $pr(r, d|0, q) > 0$, let consider the FOCs of the investors:

$$\begin{aligned} b^H(H) : \quad & -1 + (1 - \varepsilon)^2 pr(r, r|0, 0) u'(r, r|0, 0) R^H + \\ & (1 - \varepsilon)^2 pr(r, d|0, 0) u'(r, d|0, 0) R^H + \\ & \varepsilon(1 - \varepsilon) pr(r, d|0, q) u'(r, d|0, q) R^H = 0 \end{aligned} \quad (41)$$

$$\begin{aligned} b^F(H) : \quad & -1 + (1 - \varepsilon)^2 pr(r, r|0, 0) u'(r, r|0, 0) R^H + \\ & (1 - \varepsilon)^2 pr(d, r|0, 0) u'(d, r|0, 0) R^H + \\ & \varepsilon(1 - \varepsilon) pr(d, r|q, 0) u'(d, r|q, 0) R^H = 0 \end{aligned} \quad (42)$$

Let us assume that $b(F) \geq b(H)$.

1. Suppose that in equilibrium $b^H(H) = b(H)$ and $b^F(F) \in [0, b(F)]$, so that in equilibrium the autarky allocation would be implemented, then we would get for the second addend of each for

$$\begin{aligned} & (1 - \varepsilon)^2 pr(r, r|0, 0) u'(e + b^H(F) R^F) R^H < \\ & (1 - \varepsilon)^2 pr(r, r|0, 0) u'(e + b^F(F) R^F - b(F) R^F) R^H \end{aligned} \quad (43)$$

and for the rest:

$$\begin{aligned}
& (1 - \varepsilon)^2 pr(r, d|0, 0) u'(e) + \varepsilon(1 - \varepsilon) pr(r, d|0, q) u'(e) - \\
& (1 - \varepsilon)^2 pr(r, d|0, 0) u'(e) - \varepsilon(1 - \varepsilon) pr(r, d|0, q) u'(e - Q) = \quad (44)
\end{aligned}$$

$$\Leftrightarrow \varepsilon(1 - \varepsilon) pr(r, d|0, q) [u'(e) - u'(e - Q)] < 0 \quad (45)$$

by the concavity of the utility function

2. Suppose that $b^H(H) = 0$ and $b^F(F) \in [0, b(F)]$, so that in equilibrium agents would hold only foreign bonds, then we would get

$$\begin{aligned}
& (1 - \varepsilon)^2 pr(r, r|0, 0) u'(e + b^H(F) R^F - b(H) R^H) R^H > \\
& (1 - \varepsilon)^2 pr(r, r|0, 0) u'(e + b^F(F) R^F - b(F) R^F) R^H \quad (46)
\end{aligned}$$

because by the symmetry of the problem at maximum $b^F(F) = 0.5b(F)$ (share equally risk). And

$$\begin{aligned}
& (1 - \varepsilon)^2 pr(r, d|0, 0) u'(e - b(H) R^H) + \\
& \varepsilon(1 - \varepsilon) pr(r, d|0, q) u'(e - b(H) R^H) - \\
& (1 - \varepsilon)^2 pr(r, d|0, 0) u'(e + b(H) R^H) - \\
& \varepsilon(1 - \varepsilon) pr(r, d|0, q) u'(e - Q + b(H) R^H) > 0 \quad (47)
\end{aligned}$$

because

$$(1 - \varepsilon)^2 pr(r, d|0, 0) [u'(e - b(H) R^H) - u'(e + b(H) R^H)] > 0 \quad (48)$$

$$\varepsilon(1 - \varepsilon) pr(r, d|0, q) [u'(e - b(H) R^H) - u'(e - Q + b(H) R^H)] > 0 \quad (49)$$

due to the concavity of the utility function.

Given that the utility of private agents is strictly concave, continuously differentiable there exists only one value of $b^H(H)$ so that such that the FOC takes value zero and it is such that $b^H(H) \in (0, b(H))$. The same can be shown symmetrically for $b^F(F)$.

Appendix B.3: Lemma 1

The equilibrium probability that both governments repay is an increasing function of ρ .

To keep the notation light, I indicate with r repayment, d default, 0 the baseline scenario, q the disaster scenario.

Proof: Suppose that the integration of financial markets is represented by a constraint on the total amount of non-domestic bonds each private agent can hold and that this constraint is binding. Then to study the effects of integration on the equilibrium interest rate and default probability it is sufficient to look at the comparative statics regarding this constraint. Let us consider the FOC of a citizen of Foreign with respect to $b^F(H)$:

$$\begin{aligned} -1 + (1 - \varepsilon)^2 pr(r, r|0, 0) u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) R^H + \\ (1 - \varepsilon)^2 pr(r, d|0, 0) u' (e + b^F(H) R^H) R^H + \\ (1 - \varepsilon) \varepsilon pr(r, d|0, q) u' (e - Q + b^F(H) R^H) R^H = 0 \end{aligned} \quad (50)$$

Let us apply the implicit function theorem:

$$\begin{aligned} \frac{\partial FOC}{\partial b^F(H)} = & (1 - \varepsilon)^2 \frac{\partial pr(r, r|0, 0)}{\partial b^F(H)} u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) R^H + \\ & (1 - \varepsilon)^2 pr(r, r|0, 0) u'' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) (R^H)^2 + \\ & (1 - \varepsilon)^2 \frac{\partial pr(r, d|0, 0)}{\partial b^F(H)} u' (e + b^F(H) R^H) R^H + \\ & (1 - \varepsilon)^2 pr(r, d|0, 0) u'' (e + b^F(H) R^H) (R^H)^2 + \\ & (1 - \varepsilon) \varepsilon \frac{\partial pr(r, d|0, q)}{\partial b^F(H)} u' (e - Q + b^F(H) R^H) R^H + \\ & (1 - \varepsilon) \varepsilon pr(r, d|0, q) u'' (e - Q + b^F(H) R^H) (R^H)^2 < 0 \end{aligned} \quad (51)$$

because the utility is concave and the derivatives of the equilibrium probabilities $pr(r, r|0, 0)$, $pr(r, d|0, 0)$ and $pr(r, d|0, q)$ are negative (see the definition of these probability and the scheme of the governments' Nash equilibria).

$$\begin{aligned}
\frac{\partial FOC}{\partial R^H} = & (1 - \varepsilon)^2 pr(r, r|0, 0) u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) + \\
& (1 - \varepsilon)^2 pr(r, r|0, 0) u'' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) (R^H) b^F(H) + \\
& (1 - \varepsilon)^2 \frac{\partial pr(r, r|0, 0)}{\partial R^H} u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) R^H + \\
& (1 - \varepsilon) pr(r, d|0, 0) u' (e + b^F(H) R^H) + \\
& (1 - \varepsilon)^2 pr(r, d|0, 0) u'' (e + b^F(H) R^H) R^H b^F(H) + \\
& (1 - \varepsilon) \frac{\partial pr(r, d|0, 0)}{\partial R^H} u' (e + b^F(H) R^H) R^H + \\
& (1 - \varepsilon) \varepsilon pr(r, d|0, q) u' (e - Q + b^F(H) R^H) + \\
& (1 - \varepsilon) \varepsilon pr(r, d|0, q) u'' (e - Q + b^F(H) R^H) R^H b^F(H) + \\
& (1 - \varepsilon) \varepsilon \frac{\partial pr(r, d|0, q)}{\partial R^H} u' (e - Q + b^F(H) R^H) R^H > 0
\end{aligned} \tag{52}$$

Therefore we obtain

$$\frac{\partial R^H}{\partial b^F(H)} > 0. \tag{53}$$

If the equilibrium with higher shares of foreign bond holdings is characterized by higher interest rates, then also the equilibrium default probabilities increase.

Appendix B.4: Lemma 2

The amount and the share of sovereign debt held by foreign investors decreases with the first period public expenditure.

To keep the notation light, I indicate with r repayment, d default, 0 the baseline scenario, q the disaster scenario.

Proof: In equilibrium it holds that $b(H) = g^H$.

$$\begin{aligned}
 -1 + (1 - \varepsilon)^2 pr(r, r|0, 0) u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) R^H + \\
 (1 - \varepsilon)^2 pr(r, d|0, 0) u' (e + b^F(H) R^H) R^H + \\
 (1 - \varepsilon) \varepsilon pr(r, d|0, q) u' (e - Q + b^F(H) R^H) R^H = 0 \quad (54)
 \end{aligned}$$

Let us apply the implicit function theorem

$$\begin{aligned}
 \frac{\partial FOC}{\partial b(H)} &= (1 - \varepsilon)^2 \frac{\partial pr(r, r|0, 0)}{\partial b(H)} u' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) R^H \\
 &+ (1 - \varepsilon)^2 \frac{\partial pr(r, d|0, 0)}{\partial b(H)} u' (e + b^F(H) R^H) R^H + \\
 &(1 - \varepsilon) \varepsilon \frac{\partial pr(r, d|0, q)}{\partial b(H)} u' (e - Q + b^F(H) R^H) R^H < 0 \quad (55)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial FOC}{\partial b^F(H)} &= (1 - \varepsilon)^2 pr(r, r|0, 0) u'' (e + b^F(H) R^H + b^F(F) R^F - b(F) R^F) (R^H)^2 + \\
 &(1 - \varepsilon)^2 pr(r, d|0, 0) u'' (e + b^F(H) R^H) (R^H)^2 + \\
 &(1 - \varepsilon) \varepsilon pr(r, d|0, q) u'' (e - Q + b^F(H) R^H) (R^H)^2 < 0 \quad (56)
 \end{aligned}$$

This implies that

$$\frac{\partial b^F(H)}{\partial b(H)} < 0. \quad (57)$$

That is when the total amount of debt issued increases, less debt is held by foreigners. It can be shown in the same way that also the SHARE of debt held by non-residents decreases with $b(H)$.

Appendix C.1: Theorem 2

The autarky allocation is Pareto superior to equilibrium allocation with integrated markets when $\varepsilon \rightarrow 0$ and countries have the same first period expenditure shock.

Sketch of the proof:

1. I show that it is sufficient to compare the second period expected utility functions at the equilibrium in autarky and with integrated markets
2. I compare the second period expected utilities without subtracting the default costs for the integrated-markets-case. I show that the expected utility from autarky is larger than the one with integrated markets.
3. The result holds also when subtracting the default costs to the expected utility with integration.

To keep the notation light, I indicate with r repayment, d default, 0 the baseline scenario, q the disaster scenario.

Expected utility in autarky

Let us consider the autarky expected utility function of country $i = \{H, F\}$. In autarky, the equilibrium expected utility is given by

$$e - g^i + (1 - \varepsilon) u(e) + \varepsilon u(e - Q) \quad (58)$$

given that $b(i) = g$.

Expected utility with integrated markets

The first period utility function is given by

$$e - b^i(i) - b^i(j) = \quad (59)$$

$$e - b(i) \quad (60)$$

we know that in equilibrium $b^i(i) = b^j(j)$, by the symmetry of the problem. Thus, the first period utility function becomes equal to the one in autarky.

For the comparison of the expected utility functions, it is necessary to look at the expected second period utility function. Note that if $b^i(i) = b^j(j)$ and $b(i) = b(j)$ then also $R^i = R^j$.

When markets are integrate the expected second period utility function without default costs is given by

$$\begin{aligned}
& (1 - \varepsilon)^2 pr(r, r, |0, 0) u(r, r|0, 0) + (1 - \varepsilon)^2 pr(r, d, |0, 0) u(r, d|0, 0) + \\
& \varepsilon(1 - \varepsilon) pr(r, d|0, q) u(r, d|0, q) + (1 - \varepsilon)^2 pr(d, r, |0, 0) u(d, r|0, 0) + \\
& \varepsilon(1 - \varepsilon) pr(d, r|q, 0) u(d, r|q, 0) + (1 - \varepsilon)^2 pr(d, d, |0, 0) u(d, d|0, 0) + \\
& \varepsilon(1 - \varepsilon) pr(d, d|q, 0) u(d, d|q, 0) + \varepsilon(1 - \varepsilon) pr(d, d|0, q) u(d, d|0, q) + \\
& \varepsilon^2 pr(d, d|q, q) u(d, d|q, q) \tag{61}
\end{aligned}$$

When the probability of involuntary default goes to zero, $\varepsilon \rightarrow 0$, then the second period expected utility without the voluntary default costs becomes

$$\begin{aligned}
& pr(r, r, |0, 0) u(e + R^j b^i(j) + R^i b^i(i) - R^i b(i)) + \\
& pr(r, d, |0, 0) u(e + R^i b^i(i) - R^i b(i)) + \\
& pr(d, r, |0, 0) u(e + R^j b^i(j)) + \\
& pr(d, d, |0, 0) u(e) = \tag{62}
\end{aligned}$$

$$\begin{aligned}
& pr(r, r, |0, 0) u(e + R^j b^i(j) + R^i b^i(i) - R^i b(i)) + \\
& pr(r, d, |0, 0) u(e - R^i b^j(i)) + \\
& pr(d, r, |0, 0) u(e + R^j b^i(j)) + \\
& pr(d, d, |0, 0) u(e) < \tag{63}
\end{aligned}$$

$$\begin{aligned}
& pr(r, r, |0, 0) u(e) + \\
& 2pr(r, d, |0, 0) u(e) + \\
& pr(d, d, |0, 0) u(e) = u(e) \tag{64}
\end{aligned}$$

I have shown that the second period expected utility function in integrated markets is lower than the one in autarky, not taking into account the default costs. Including the default costs would not change the result because they are negative.

Appendix C.2: Theorem 4

Let us assume that the utility function in (21) is strictly concave with respect to the share of debt held by non-resident investors and that the public expenditure in the first period is equal across countries. Then, the second best solution is characterized by a higher share of resident debt holdings in both countries.

- If it holds:

$$\frac{u'(e - Q)}{u'(e)} > 1 + u(e) + u(e - Q), \quad (65)$$

then the second best allocation is characterized by a share of resident debt holdings which is higher than in the equilibrium of Theorem 1 and lower than in autarky: $\tilde{h} \in \left(\frac{\tilde{b}^F(H)}{b(H)}, 1\right)$, where \tilde{h} is the optimal share for the social planner and $\tilde{b}^F(H)$ is the optimal amount of non-resident holdings in the equilibrium of Theorem 1.

- If condition (65) does not hold, then the second best allocation corresponds to the autarky allocation.

Sketch of the proof:

1. I show that the expected utility with integration is strictly decreasing at the autarky allocation.
2. From 2 we know that the autarky allocation achieves a higher utility than integration. Given that the utility function is continuously differentiable, this means that there must be a maximum between the competitive equilibrium allocation and the autarky allocation.
3. Under the assumption that the utility of the social planner (who internalizes also that the probability of repayment depends on the bond allocation) is strictly concave the allocation of point 2) is the first best.

To keep the notation light, I indicate with r repayment, d default, 0 the baseline scenario, q the disaster scenario. *Proof:* In what follows I call $h \equiv \frac{b^H(H)}{b(H)}$ and $f \equiv \frac{b^F(F)}{b(F)}$. Being the countries perfectly symmetric I define also $g \equiv g^H = g^F$ and $R = R^H = R^F$. Let us first compute some variables. Under the assumption of symmetric countries and exogenous expenditure in the first period, the utility function of the social planner can be re-written as:

$$\max_{c_1, c_2} \{c_1 + E[u(c_2) - \psi I_{[sd]}]\} \quad (66)$$

s.t.

$$c_1 = e - hg - (1 - f)g \quad (67)$$

$$c_2 = \begin{cases} e + RhgI_{[rep\ H]} + R(1 - h)gI_{[rep\ F]} - gI_{[rep\ H]} & \text{if baseline} \\ e - Q + R(1 - h)gI_{[rep\ F]} & \text{if disaster} \end{cases} \quad (68)$$

$$\tilde{\psi} = u(e + Rg - hRg) - u(e) \quad (69)$$

$$\bar{\psi} = u(e) - u(e + hRg - Rg) \quad (70)$$

STEP 1

Probabilities

$$\tilde{\psi} = u(e + Rg - hRg) - u(e) \quad (71)$$

$$\tilde{\psi}\Big|_{h=1} = 0 \quad (72)$$

$$\bar{\psi} = u(e) - u(e + hRg - Rg) \quad (73)$$

$$\bar{\psi}\Big|_{h=1} = 0 \quad (74)$$

$$\frac{\partial \tilde{\psi}}{\partial h} = u'(e + Rg - Rgh) \left[\frac{\partial R}{\partial h} - h \frac{\partial R}{\partial h} - R \right] g \quad (75)$$

$$= -Au'(e + Rg - Rgh) < 0 \quad (76)$$

$$\frac{\partial \tilde{\psi}}{\partial h}\Big|_{h=1} = -Au'(e)$$

$$\frac{\partial \bar{\psi}}{\partial h} = -u'(e + hRg - Rg) \left[-\frac{\partial R}{\partial h} + h \frac{\partial R}{\partial h} + R \right] g \quad (77)$$

$$= -u'(e + hRg - Rg) A \quad (78)$$

$$\frac{\partial \bar{\psi}}{\partial h}\Big|_{h=1} = -u'(e) A < 0 \quad (79)$$

where $A \equiv g \left(R + h \frac{\partial R}{\partial h} - \frac{\partial R}{\partial h} \right)$. Then

$$pr(r, r|0, 0) = (1 - \tilde{\psi})^2 \quad (80)$$

$$pr(r, r|0, 0)|_{h=1} = 1 \quad (81)$$

$$\frac{\partial pr(r, r|0, 0)}{\partial h} = -2(1 - \tilde{\psi}) \frac{\partial \tilde{\psi}}{\partial h} \quad (82)$$

$$\frac{\partial pr(r, r|0, 0)}{\partial h} \Big|_{h=1} = -2(-Au'(e)) \quad (83)$$

$$= 2Au'(e) > 0 \quad (84)$$

$$pr(r, d|0, 0) = (1 - \bar{\psi}) \tilde{\psi} \quad (85)$$

$$pr(r, d|0, 0)|_{h=1} = 0 \quad (86)$$

$$pr(d, r|0, 0) = pr(r, d|0, 0) \quad (87)$$

$$\frac{\partial pr(r, d|0, 0)}{\partial h} = -\frac{\partial \bar{\psi}}{\partial h} \tilde{\psi} + (1 - \bar{\psi}) \frac{\partial \tilde{\psi}}{\partial h} \quad (88)$$

$$\frac{\partial pr(r, d|0, 0)}{\partial h} \Big|_{h=1} = -Au'(e) \quad (89)$$

$$\frac{\partial pr(d, r|0, 0)}{\partial h} = \frac{\partial pr(r, d|0, 0)}{\partial h} \quad (90)$$

$$pr(r, d|0, q) = (1 - \bar{\psi}) \quad (91)$$

$$pr(r, d|0, q)|_{h=1} = 1 \quad (92)$$

$$pr(d, r|q, 0) = pr(r, d|0, q) \quad (93)$$

$$\frac{\partial pr(r, d|0, q)}{\partial h} = -\frac{\partial \bar{\psi}}{\partial h} \quad (94)$$

$$= u'(hRg - Rg) A \quad (95)$$

$$\frac{\partial pr(r, d|0, q)}{\partial h} \Big|_{h=1} = u'(e) A \quad (96)$$

$$\frac{\partial pr(d, r|q, 0)}{\partial h} = \frac{\partial pr(r, d|0, q)}{\partial h} \quad (97)$$

$$pr(d, d, 0, 0) = 2(\bar{\psi} - \tilde{\psi})\tilde{\psi} + \tilde{\psi}^2 \quad (98)$$

$$pr(d, d, 0, 0)|_{h=1} = 0 \quad (99)$$

$$\frac{\partial pr(d, d, 0, 0)}{\partial h} = 0 \quad (100)$$

$$pr(d, d, q, 0) = pr(d, d, 0, q) \quad (101)$$

$$= pr(d, d, q, q) \quad (102)$$

$$= pr(d, d, 0, 0) \quad (103)$$

$$\frac{\partial pr(d, d, q, 0)}{\partial h} = \frac{\partial pr(d, d, 0, q)}{\partial h} \quad (104)$$

$$= \frac{\partial pr(d, d, q, q)}{\partial h} \quad (105)$$

$$= \frac{\partial pr(d, d, 0, 0)}{\partial h} \quad (106)$$

Utilities

$$u(r, r|0, 0)|_{h=1} = u(e) \quad (107)$$

$$\frac{\partial u(r, r|0, 0)}{\partial h} \Big|_{h=1} = 0 \quad (108)$$

$$u(r, d|0, 0) = u(e + hRg - Rg) \quad (109)$$

$$u(r, d|0, 0)|_{h=1} = u(e) > 0 \quad (110)$$

$$\frac{\partial u(r, d|0, 0)}{\partial h} = u'(e + hRg - Rg) \left(R + h \frac{\partial R}{\partial h} - \frac{\partial R}{\partial h} \right) \quad (111)$$

$$= Au'(hRg - Rg) \quad (112)$$

$$\frac{\partial u(r, d|0, 0)}{\partial h} \Big|_{h=1} = Au'(e) > 0 \quad (113)$$

$$(114)$$

$$u(r, d|0, q) = u(r, d, 0, 0) \quad (115)$$

$$(116)$$

$$\frac{\partial u(r, d|0, q)}{\partial h} = \frac{\partial u(r, d|0, 0)}{\partial h} \quad (117)$$

$$u(d, r|0, 0) = u(e + Rg - hRg) \quad (118)$$

$$u(d, r|0, 0)|_{h=1} = u(e) \quad (119)$$

$$\frac{\partial u(d, r|0, 0)}{\partial h} = u'(Rg - hRg) \left(\frac{\partial R}{\partial h} - h \frac{\partial R}{\partial h} - R \right) g \quad (120)$$

$$= -Au'(e + Rg - hRg) \quad (121)$$

$$\left. \frac{\partial u(d, r|0, 0)}{\partial h} \right|_{h=1} = -Au'(e) \quad (122)$$

$$u(d, r|q, 0) = u(e + Rg - hRg - Q) \quad (123)$$

$$u(d, r|q, 0)|_{h=1} = u(e - Q) \quad (124)$$

$$\frac{\partial u(d, r|q, 0)}{\partial h} = u'(e + Rg - hRg - Q) \left(\frac{\partial R}{\partial h} - h \frac{\partial R}{\partial h} - R \right) g \quad (125)$$

$$= -Au'(e + Rg - hRg - Q) \quad (126)$$

$$\left. \frac{\partial u(d, r|q, 0)}{\partial h} \right|_{h=1} = -Au'(e - Q) \quad (127)$$

$$u(d, d|0, 0)|_{h=1} = u(e) \quad (128)$$

$$\left. \frac{\partial u(d, d|0, 0)}{\partial h} \right|_{h=1} = 0 \quad (129)$$

$$u(d, d|0, q) = u(e - Q) \quad (130)$$

$$\left. \frac{\partial u(d, d|0, q)}{\partial h} \right|_{h=1} = 0 \quad (131)$$

$$u(d, d|q, 0) = u(d, d|0, 0) \quad (132)$$

$$u(d, d|q, q) = u(d, d|q, 0) \quad (133)$$

Derivative of the expected utility

Voluntary default costs

Let us first consider the expected voluntary default costs:

$$\begin{aligned}
& Cost(d, r|0, 0) + Cost(d, d|0, 0) + Cost(d, d|0, q) = \\
& \int_{\bar{\psi}^F}^{\Psi} \int_0^{\tilde{\psi}^H} \psi^H d\psi^H d\psi^F + \int_0^{\tilde{\psi}^F} \int_0^{\tilde{\psi}^H} \psi^H d\psi^H d\psi^F + \\
& \int_{\tilde{\psi}^F}^{\bar{\psi}^F} \int_0^{\tilde{\psi}^H} \psi^H d\psi^H d\psi^F + \int_0^{\tilde{\psi}^F} \int_{\tilde{\psi}^H}^{\bar{\psi}^H} \psi^H d\psi^H d\psi^F + \\
& \int_0^{\Psi} \int_0^{\bar{\psi}^H} \psi^H d\psi^H = \tag{134}
\end{aligned}$$

$$\begin{aligned}
& 0.5 [\Psi - \bar{\psi}] \tilde{\psi}^2 + 0.5 \tilde{\psi} \tilde{\psi}^2 + \\
& 0.5 [\bar{\psi} - \tilde{\psi}] \tilde{\psi}^2 + 0.5 \tilde{\psi} [\bar{\psi}^2 - \tilde{\psi}^2] + \\
& 0.5 \Psi \bar{\psi}^2 = \tag{135}
\end{aligned}$$

where in the last passages I dropped the country index because $\bar{\psi}^H = \bar{\psi}^F$ and so on. We can simplify (144)

$$\begin{aligned}
& 0.5 \Psi \tilde{\psi}^2 - 0.5 \bar{\psi} \tilde{\psi}^2 + 0.5 \tilde{\psi} \tilde{\psi}^2 + \\
& 0.5 \bar{\psi} \tilde{\psi}^2 - 0.5 \tilde{\psi} \tilde{\psi}^2 + 0.5 \tilde{\psi} \bar{\psi}^2 - 0.5 \tilde{\psi} \tilde{\psi}^2 + \\
& 0.5 \Psi \bar{\psi}^2 = \tag{136}
\end{aligned}$$

$$0.5 \Psi \tilde{\psi}^2 - 0.5 \tilde{\psi}^3 + 0.5 \tilde{\psi} \bar{\psi}^2 + 0.5 \Psi \bar{\psi}^2. \tag{137}$$

The first order derivative with respect to h of the default costs is then

$$\begin{aligned}
& 0.5 \Psi 2 \tilde{\psi} \frac{\partial \tilde{\psi}}{\partial h} - 0.5 (3) \tilde{\psi}^2 \frac{\partial \tilde{\psi}}{\partial h} + 0.5 \tilde{\psi} \left(2 \bar{\psi} \frac{\partial \bar{\psi}}{\partial h} \right) + \\
& 0.5 \bar{\psi}^2 \frac{\partial \tilde{\psi}}{\partial h} + 0.5 \Psi 2 \bar{\psi} \frac{\partial \bar{\psi}}{\partial h} = 0 \tag{138}
\end{aligned}$$

Given that the variation in the default costs is zero, it is sufficient to look at the sign of the positive part of the expected utility.

Expected second period utility

The second period expected utility without default costs with integration is given by

$$\begin{aligned}
& \varepsilon^2 pr(r, r|0, 0) u(r, r|0, 0) + \\
& \varepsilon^2 pr(r, d|0, 0) u(r, d|0, 0) + \varepsilon(1 - \varepsilon) pr(r, d|0, q) u(r, d|0, q) + \\
& \varepsilon^2 pr(d, r|0, 0) u(d, r|0, 0) + \varepsilon(1 - \varepsilon) pr(d, r|q, 0) u(d, r|q, 0) + \\
& \varepsilon^2 pr(d, d|0, 0) u(d, d|0, 0) + \varepsilon(1 - \varepsilon) pr(d, d|0, q) u(d, d|0, q) + \\
& \varepsilon(1 - \varepsilon) pr(d, d|q, 0) u(d, d|q, 0) + (1 - \varepsilon)^2 pr(d, d|q, q) u(d, d|q, q)
\end{aligned} \tag{139}$$

the first derivative with respect to h is then

$$\begin{aligned}
& \varepsilon^2 \frac{\partial pr(r, r|0, 0)}{\partial h} u(r, r|0, 0) + \\
& \varepsilon^2 \frac{\partial pr(r, d|0, 0)}{\partial h} u(r, d|0, 0) + \varepsilon(1 - \varepsilon) \frac{\partial pr(r, d|0, q)}{\partial h} u(r, d|0, q) + \\
& \varepsilon^2 \frac{\partial pr(d, r|0, 0)}{\partial h} u(d, r|0, 0) + \varepsilon(1 - \varepsilon) \frac{\partial pr(d, r|q, 0)}{\partial h} u(d, r|q, 0) + \\
& \varepsilon^2 \frac{\partial pr(d, d|0, 0)}{\partial h} \underbrace{u(d, d|0, 0)}_{=0} + \varepsilon(1 - \varepsilon) \frac{\partial pr(d, d|0, q)}{\partial h} \underbrace{u(d, d|0, q)}_{=0} + \\
& \varepsilon(1 - \varepsilon) \frac{\partial pr(d, d|q, 0)}{\partial h} \underbrace{u(d, d|q, 0)}_{=0} + (1 - \varepsilon)^2 \frac{\partial pr(d, d|q, q)}{\partial h} \underbrace{u(d, d|q, q)}_{=0} + \\
& \varepsilon^2 pr(r, r|0, 0) \underbrace{\frac{\partial u(r, r|0, 0)}{\partial h}}_{=0} + \\
& \varepsilon^2 \underbrace{pr(r, d|0, 0)}_{=0} \frac{\partial u(r, d|0, 0)}{\partial h} + \varepsilon(1 - \varepsilon) \underbrace{pr(r, d|0, q)}_{=1} \frac{\partial u(r, d|0, q)}{\partial h} + \\
& \varepsilon^2 \underbrace{pr(d, r|0, 0)}_{=0} \frac{\partial u(d, r|0, 0)}{\partial h} + \varepsilon(1 - \varepsilon) \underbrace{pr(d, r|q, 0)}_{=1} \frac{\partial u(d, r|q, 0)}{\partial h} + \\
& \varepsilon^2 \underbrace{pr(d, d|0, 0)}_{=0} \frac{\partial u(d, d|0, 0)}{\partial h} + \varepsilon(1 - \varepsilon) \underbrace{pr(d, d|0, q)}_{=0} \frac{\partial u(d, d|0, q)}{\partial h} + \\
& \varepsilon(1 - \varepsilon) \underbrace{pr(d, d|q, 0)}_{=0} \frac{\partial u(d, d|q, 0)}{\partial h} + (1 - \varepsilon)^2 \underbrace{pr(d, d|q, q)}_{=0} \frac{\partial u(d, d|q, q)}{\partial h}.
\end{aligned} \tag{140}$$

Substituting the results found before, we obtain

$$\begin{aligned}
& \varepsilon^2 u(r, r|0, 0) + \\
& \varepsilon^2 \frac{\partial pr(r, d|0, 0)}{\partial h} u(r, d|0, 0) + \varepsilon(1 - \varepsilon) \frac{\partial pr(r, d|0, q)}{\partial h} u(r, d|0, q) + \\
& \varepsilon^2 \frac{\partial pr(d, r|0, 0)}{\partial h} u(d, r|0, 0) + \varepsilon(1 - \varepsilon) \frac{\partial pr(d, r|q, 0)}{\partial h} u(d, r|q, 0) + \\
& \varepsilon(1 - \varepsilon) \frac{\partial u(r, d|0, q)}{\partial h} + \varepsilon(1 - \varepsilon) \frac{\partial u(d, r|q, 0)}{\partial h} = \quad (141)
\end{aligned}$$

$$\begin{aligned}
& \varepsilon^2 2Au'(e)u(e) + \\
& \varepsilon^2 (-A)u'(e)u(e) + \varepsilon(1 - \varepsilon)Au'(e)u(e) + \\
& \varepsilon^2 (-A)u'(e)u(e) + \varepsilon(1 - \varepsilon)Au'(e)u(e - Q) + \\
& \varepsilon(1 - \varepsilon)Au'(e) + \varepsilon(1 - \varepsilon)(-A)u'(e - Q) = \quad (142)
\end{aligned}$$

$$\begin{aligned}
& \varepsilon(1 - \varepsilon)Au'(e)u(e) + \\
& + \varepsilon(1 - \varepsilon)Au'(e)u(e - Q) + \\
& \varepsilon(1 - \varepsilon)Au'(e) + \varepsilon(1 - \varepsilon)(-A)u'(e - Q) = \quad (143)
\end{aligned}$$

$$\varepsilon(1 - \varepsilon)A\{u'(e)[1 + u(e) + u(e - Q)] - u'(e - Q)\} \quad (144)$$

Eq (168) is negative if and only if

$$u'(e)[1 + u(e) + u(e - Q)] - u'(e - Q) < 0, \quad (145)$$

that is

$$\frac{u'(e - Q)}{u'(e)} > 1 + u(e) + u(e - Q) \quad (146)$$

Intuitively, eq. (146) holds if either the utility function is steep between $e - Q$ and e , or the income shock due to a disaster is quite large relative to the exogenous income ($e - Q \sim 0$). If condition (146) does not hold, then the expected utility with integration is strictly increasing

at the autarky allocation.

STEP 2

Given that by assumption the expected utility of the social planner is continuously differentiable (it is sufficient for this that u is continuously differentiable), according to Step 1 two scenarios might occur:

1. If

$$\frac{u'(e - Q)}{u'(e)} > 1 + u(e) + u(e - Q) \quad (147)$$

then there is a maximum between the integration equilibrium and the autarky equilibrium, because we know that the autarky allocation is Pareto superior to the integration equilibrium allocation from 2. This scenario can be represented as in Figure 1:

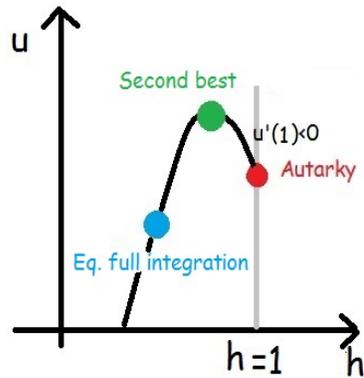


Figure 10: Social planner allocation if condition (147) holds

2. If the condition (147) does not hold, then the autarky allocation is a local maximum

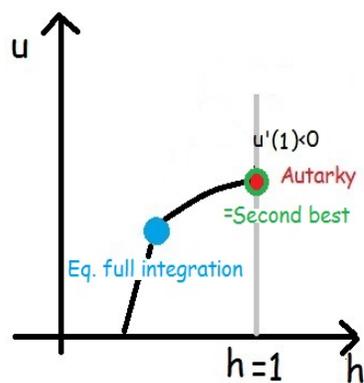


Figure 11: Social planner allocation if condition (147) holds

STEP 3

In Step 1 and 2 it was found that there exists a local maximum characterized by a higher level of home bond holdings than in the competitive equilibrium. Under the assumption of strict concavity of the expected utility function of the social planner with respect to h , this local maximum is also global maximum, that is, first best.