Securitization under Asymmetric Information over the Business Cycle

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Abstract

This paper studies the efficiency of financial intermediation through securitization with asymmetric information about the quality of securitized loans. In this theoretical model, I show that, in general, by providing reputation based implicit recourse the issuer of a loan can credibly signal its quality. However, in boom stages of the business cycle, information on loan quality remains private and lower quality loans accumulate on balance sheets. This deepens a subsequent downturn. The longer the duration of a boom, the deeper will be the fall of output in a subsequent recession. I present empirical evidence from securitization deal-level data consistent with this result. In recessions, the model also produces amplification of adverse selection problems on re-sale markets for securitized loans. These are especially severe after a prolonged boom period and when securitized loans of high quality are no longer traded. Finally, the model suggests that excessive regulation requiring higher explicit risk-retention by the originators of loans could adversely affect both quantity and quality of investment in the economy.

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Contents

1 Introduction

Securitization has recently attracted a great deal of criticism due to its role in the financial crisis of the late 2000's (e.g. Bernanke, 2010). Securitization and generally market-based system of financial intermediation grew significantly in importance in the decades preceding the crisis (Adrian and Shin, 2009). The financial crisis of the late 2000's led to intensified research into the problematic aspects of securitization. New research is often very critical about securitization; consider Shleifer and Vishny (2010), who argue that securitization creates systemic risks and inefficiencies in financial intermediation. Currently, regulation of the financial sector is being redrafted and strengthened on national as well as international levels in many developed countries. The new regulation addresses also securitization practices.[1] The agency problems related to securitization to which most of the criticism points are, however, not new. Securitization designs contained tools, such as tranche retention schemes or implicit recourse, that were supposed to limit these negative aspects of securitization. The question is whether these tools worked efficiently in the period prior to the late 2000's financial crisis.

In this paper I show in a dynamic stochastic general equilibrium model that reputation concerns can allow sponsors of securitized products to credibly signal the quality of loans by providing implicit recourse and thus limit the problem of asymmetric information. Implicit recourse is an implicit support provided by the issuer of securitized products to the holders of these assets. This support is not contractual and is enforced in a reputation equilibrium.[2] Typically, there are both pooling and separating equilibria in this signaling game. By applying Intuitive Criterion refinement, I can select a unique separating equilibrium, in which the information about loan quality is transferred, and the outcome is therefore efficient. However, there are limits to the degree of commitment based on reputation and thus also to the efficiency of implicit recourse in eliminating the problem of asymmetric information. Following the empirical evidence in Bloom (2009) and Bloom et al. (2011), who find that second moments of firms' Total Factor Productivity (TFP) in the economy are countercyclical, in this model the relative difference in the productivity of projects' (loans') is also countercyclical. As a result, it turns out that even though in the steady state provision of implicit recourse helps

[1] Pozsar et al. (2012) describe the role of securitization in the "Shadow Banking" and Adrian and Ashcraft (2012) review the proposals of new regulation.

[2] For a review of empirical evidence on implicit recourse, description of its types and discussion of its role in the securitization process, I would like to refer the reader to the literature review.

in achieving a separating equilibrium, in boom stages of the business cycle separation equilibrium would require too high levels of implicit recourse, which cannot be enforced through reputation. Therefore, in boom stages of business cycles, there are only pooling equilibria, in which the information about the quality of loans remains private and the allocation of investment is inefficient. This has only very moderate effects as long as the economy stays in a boom, where relative difference in the productivity of projects (loans) is low. However, the effect of accumulated stock of low quality loans becomes more pronounced in a subsequent downturn of the economy, which is thus amplified. Further, the longer the boom, the larger the share of lower quality loans on the balance sheets and the deeper will be the subsequent downturn.

Results of this paper could have also implications for the related macro-prudential policy which requires higher explicit risk-retention for the originators (issuers) of the securitized products (such as in section 941 of the Dodd-Frank reform). Although none of the frictions in the model is sufficient to rationalize regulation of this sort, the model points to an adverse general equilibrium effect of higher explicit risk-retention. In this model, higher than equilibrium explicit risk-retention, such as the practice of keeping a larger fraction of issued loans on the balance sheet of the issuer, limits the financial intermediation ability of the issuer. Since higher explicit risk-retention restricts the supply of loans, through the general equilibrium effect it increases the equilibrium prices of securitized assets and makes securitization more profitable. Higher prices mean that even the securitization of lower quality loans is profitable. Therefore, when regulation is excessive, any possible benefits of the regulation, which are not modeled here, can be outweighed by the adverse general equilibrium effect, which lowers both the quantity and the quality of the investment in the economy.

In an extension of the model, I also introduce asymmetric information between sellers and buyers of securitized loans on the re-sale market. The model then produces adverse selection which is amplified in a recession. The negative impact on the adverse selection on the market price depends on the share of low quality investments on the balance sheets. Therefore, adverse selection is especially severe in a recession following a prolonged boom period. When a price on resale markets falls low enough, even firms in need of liquidity find it unprofitable to sell high quality loans for low market prices in order to finance new investment opportunities. Ultimately, securitized loans of high quality are no longer traded on the re-sale markets at all.

In the empirical section of the paper, I test hypotheses from the theoretical model on the level of securitization deals using data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. Further, this effect is smaller and may even be overturned for assets issued in a boom stage of the business cycle. This is in line with the higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, however, are statistically insignificant for deals issued in Spain. The difference could be explained by a significant differences in regulatory framework and securitization practices.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis as it replicates some of the securitization market outcomes observed prior to and during the crisis. In the period preceding the crisis, many inefficient investments of unknown quality, were undertaken. While this was not a problem as long as the economy was performing well, the large amount of low quality loans in the economy contributed to the depth of the financial crisis. Also, during the crisis the markets for securitized products were severely strained. The paper also points to some unexpected effects of the newly proposed regulation.

The paper is organized in the following way. Chapter 2 reviews the related literature. Chapter 3 introduces the set-up of the model and shows its solution, the effect of assumed financial frictions and the effect of implicit recourse. For analytical tractability, this chapter focuses on steady state with only idiosyncratic stochasticity and in which the aggregate variables are deterministic. Chapter 4 shows the results of the full-fledged model with aggregate stochasticity obtained using global numerical methods and focuses on the switching between the separating and pooling equilibria over the business cycle. Chapter 5 develops extensions of the model. In particular I discuss the policy implications of the model and produce the adverse selection on re-sale markets. Chapter 6 describes the empirical testing of hypotheses derived in the theoretical model.

2 Literature review

My research is broadly related to several strands of literature. In this chapter I would like to focus on research related to securitization with implicit recourse and to financial intermediation imperfections, information frictions and business cycles.

2.1 Securitization and implicit recourse

Securitization is the process of selling cash flows related to the loans issued by the originator (often called the sponsor). The sale of loans is effectuated in a legally separated entity called a special purpose vehicle (SPV) or special purpose entity (SPE). The entity purchases the right to the cash flows with resources obtained by issuing securities in the capital market. The sponsor and the SPV are "bankruptcy remote" and the sale of loans is officially considered to be complete, i.e., the sponsor should transfer all the risks to the buyers of newly emitted securities. Loans are pooled in a portfolio, which is then usually divided into several tranches ordered by seniority, which have a different exposure to risk. Before the crisis, securitization was perceived mainly as a means of dispersing credit risk and allocating it to less risk-averse investors who would be compensated by higher returns, while highly risk-averse investors could invest into the most senior tranches with high ratings. Due to the role securitization played in the late 2000's financial crisis (e.g. Bernanke 2010), securitization attracted a lot of criticism and the attention of researchers turned more to the set of agency problems present at different stages of the securitization process (Shin, 2009). A detailed review of those agency conflicts has been compiled, for instance, by Paligorova (2009).

Gorton and Pennacchi (1995) were among the first to point to moral hazard problems related to securitization and to address the issue why securitization takes place despite them. Moral hazard problems stem from the fact that if the risk is transferred with a loan from the originator of the loan to the investor, the bank has a reduced incentive to monitor borrowers to increase loan quality. Gorton and Pennacchi (1995) argue that before the 1980s, securitization was very limited. In the 1980s, several regulatory changes took place that effectively increased the cost of deposit funding. One key factor was the imposition of a binding credit requirement for commercial banks.[3] Banks could avoid increased capital requirements by securitization, which moved some of the risky assets off their balance sheets. This view that an important reason for securitization is regulatory arbitrage is shared by many economists (e.g. Gorton and Pennacchi, 1995, Gertler and Kiyotaki, 2010, Gorton and Metrick, 2010). Calomiris and Mason (2004) present some evidence suggesting regulatory arbitrage is effectuated by securitizing banks to increase efficiency of contracting in the situation where capital requirements are unreasonably high rather than to abuse the safety net. The moral hazard problems

[3]"In 1981 regulators announced explicit capital requirements for the first time in U.S. banking history: all banks and bank holding companies were required to hold primary capital of at least 5.5 percent of assets by June 1985." (Gorton and Metrick, 2010, p. 10)

and agency problems in general were then alleviated by the practice of keeping part of the loan in the portfolio on the balance sheet of the originator. Fender and Mitchell (2009) study different tranche retention design and their effect on incentives. But any loan sale, partial or complete, results in lower incentives to monitor borrowers, which of course affects the price investors are willing to pay for the securitized loan. Loan originators thus have incentive to provide implicit recourse.

Implicit recourse is a particular form of implicit support provided by the issuers of securitized products to the holders of these assets. They represent a certain guarantee of the quality of the loan. The guarantee cannot be explicit since then it would have to abide by regulations and to be kept on the balance sheet of the bank. Nevertheless, much evidence suggests that implicit recourse was frequently used during the securitization process ("As the saying goes, the only securitization without recourse is the last." (Mason and Rosner, 2007, p. 38)). Gorton and Souleles (2006) show in a theoretical model that this mutually implicit collusion between investors and originators of the loans can be an equilibrium result in a repeated game due to the reputation concerns of the originator who wants to pursue securitization in the future at favorable conditions. Several empirical studies documented concrete cases of implicit recourse or showed indirect evidence of its presence. Higgins and Mason (2004) study 17 discrete recourse events that were directed to an increase in the quality of receivables sponsored by 10 different credit-card banks. The forms of the support provided were, for instance, adding higher quality accounts to the pool of receivables, removing lower quality accounts, increasing the discount on new receivables, increasing credit enhancement, waiving servicing fee, etc. Higgins and Mason (2004) argue that implicit recourse increases sponsors' stock prices in the short and long run following the recourse. It also improves their long-run operating performance. Recourse may help to signal investors that shocks that made recourse necessary are only transitory.

Another example showing that the risks were not fully transferred during securitization to the SPV is given by Brunnermeier (2009), who argues that when the SPV was subject to liquidity problems which arise from a maturity mismatch between SPV's assets and liabilities and a sudden reduced interest in the instruments emitted by the SPV, the sponsor would grant credit lines to it.

In my model I will concentrate on the relationship between investors and banks, where the latter have better information about the quality of loans, and I will show that, due to reputation concerns, bank has an incentive to signal this quality. This is in line with the suggestion by Higgins and Mason (2004) that implicit recourse is used to as a signaling tool.

2.2 Financial intermediation imperfections, information frictions and business cycles

This paper is related to large literature on financial frictions in macroeconomic models and the role of asymmetric information and reputation in financial intermediation.

In the recent financial crisis we have witnessed important disruptions of financial intermediation. It became clear that frictions in the financial sector are important and should not be omitted from macroeconomic models. The classical papers that endogenize financial frictions on the side of borrowers include Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). These papers introduce an agency problem between borrowers and lenders. The resulting endogenous amplification of the effects of the shocks in the economy is denoted as the "financial accelerator". Some of the recent macroeconomic models with financial frictions directly incorporate securitization. Brunnermeier and Sannikov (2011) find that securitization enables sharing idiosyncratic risks but may be amplifying the systemic risk.

In this paper I will refer often to Kiyotaki and Moore (2012) model of monetary economy with differences in liquidity among different asset classes. Their model features borrowing and re-saleability constraints and stochastic uninsurable arrival of idiosyncratic investment shocks among the market participants. I simplify this model and in order to study the financial intermediation similar to securitization, I introduce asymmetric information and model signaling by provision of reputation based implicit recourse.

There is a large literature on adverse selection in lender-borrower relationships based on asymmetric information which develop the original contribution of Akerlof (1970). In Parlour and Plantin (2008) the intensity of adverse selection on the markets for securitized assets (sold loans) depends on the proportion of liquidity sellers and informed sellers, who want to sell low quality loans. Kurlat (2011) models similar adverse selection problem in an extension of the model by Kiyotaki and Moore (2012) and shows that the proportion of sellers of high quality assets is lower in a recession, which can lead to market shutdowns. Recent papers study the role of asymmetric information on the interbank market. Heider et al. (2009) show that asymmetric information about counterparty risk can produce market breakdowns. Boissay et al. (2013) explain in a model with moral hazard and asymmetric information why interbank market freezes are more likely after a credit boom. While in this paper I focus on the securitization markets, I find similar results, that the liquidity problems on the securitization markets are more severe in recession especially after a prolonged boom period.

One of the major assumption in the model is the existence of a dispersion shock, which is inspired by the empirical evidence on countercyclical cross-sectional variance in TFP of US firm in Bloom (2009) and Bloom et al. (2011). These authors also build models which assume time-varying variance of idiosyncratic TFP shocks and show that higher variance can cause a recession. Bigio (2013) uses similar assumption and shows that a dispersion shock due to the existence of asymmetric information will worsen the adverse selection problem and create a recession. Compared to Bigio (2013) my model features reputation based signaling which is more effective when the dispersion is larger.

There are also several papers which study the importance of reputation in the lender-borrower relationships. Nikolov (2012) introduces reputation in the model of Kiyotaki and Moore (1997) and shows that reputation represents an intangible capital, which is more valuable in boom stage of the business cycle, therefore it further strengthens the collateral amplification mechanism. Ordogñez (2012) argues that unregulated banking disciplined only by reputation forces may be efficient due to saving on regulatory and bankruptcy costs, but is more fragile.

My model is also related to research about the degree of asymmetric information over the business cycle. While some researchers argue that booms are associated with higher degree of trading and therefore more learning (Veldkamp, 2005), others argue that information may be lost in boom periods of business cycles. Gorton and Ordogñez (2012) present a model where assets with unknown value can serve as a collateral for borrowing. In booms none of the parties has the incentive to verify the the value of the asset, the economy saves on information acquisition costs and enjoys a "bliss-full ignorance" equilibrium, while in periods with low aggregate productivity, lenders have incentives to verify the value of collateral, which leads to underinvestment. In my model higher productivity will be also associated with less public information but this would create inefficiencies.

3 Model

To allow for maximum tractability the set-up of the model is rather simple. The economy contains a continuum of financial firms which have stochastic investment opportunities. The problem in this model is to transfer resources from firms without investment opportunities or with low quality investment opportunities to firms with the best investment opportunities. The transfer of funds is possible through securitization which is modeled as a sale of cash flows from the funded projects.[4]

3.1 Model set-up

3.1.1 Investment projects

There are three types of projects available to financial firms and the allocation of firms to projects is stochastic through an i.i.d. shock:

- $(1-\pi)$ share of firms (subset $\mathcal{Z}_t$) don't have access to new investment projects,
- $\pi\mu$ share of firms (subset $\mathcal{H}_t$) have access to high quality projects with high gross profit per unit of capital $r_t^h = A_t^h K_t^{\alpha-1}$,
- $\pi(1-\mu)$ share of firms (subset $\mathcal{L}_t$) have access to low quality projects with low gross profit per unit of capital $r_t^l = A_t^l K_t^{\alpha-1}$.

This shock cannot be insured.

Assumption 1: I assume that the relative difference in gross profits from high and low quality projects is countercyclical:

$$\frac{\partial}{\partial A_t}\frac{A_t^h - A_t^l}{A_t^l} < 0, \tag{3.1}$$

where A_t is the aggregate component of the total factor productivity (TFP) of the projects.

This assumption is inspired by the empirical evidence on countercyclical cross-sectional variance in TFP of US firm in Bloom (2009) and Bloom et al. (2011).[5] In this model the TFP of the projects has an aggregate component A_t and a type-specific component Δ_t^h and Δ_t^l resp.: $A_t^h = A_t\Delta_t^h$ and $A_t^l = A_t\Delta_t^l$. To satisfy the assumption in eq. 3.1 the ratio of type-specific TFP components has to be countercyclical $\partial\left(\Delta_t^h/\Delta_t^l\right)/\partial A_t < 0$.

[4] To keep the model simple I do not model alternative means of transferring funds like debt. Kuncl (2013) presents an extension of this model, where different types of debt such as deposits or interbank loans are considered, and replicates the main qualitative results of this paper.

[5] Bloom (2009) and Bloom et al. (2011) depart from the empirical evidence and build models which assume time-varying variance of idiosyncratic TFP shocks and show that higher variance can cause a recession.

Some of the basic features of the model are inspired by Kiyotaki and Moore (2012). Similarly to Kiyotaki and Moore (2012), agents are subject to an i.i.d. investment shock, and face constant returns to scale, i.e., they take r_t^h, resp. r_t^l, as given, however, on the aggregate level there are decreasing returns to scale:

$$Y_t = r_t^h H_t + r_t^l L_t = \left(A_t^h \frac{H_t}{K_t} + A_t^l \frac{L_t}{K_t} \right) K_t^\alpha$$

where $K_t = H_t + L_t$ and $H_t\,(L_t)$ are aggregate holdings of high (low) quality capital.[6]

3.1.2 Frictions

Two **core frictions** are assumed in the model:

- Investing firms, which sell securitized loans, have to keep ***"skin in the game"***, i.e., at least $(1-\theta)$ fraction of the investment on their balance sheet. This means they can sell at most θ fraction of the current investment and the rest have to be financed from their own resources. For simplicity, θ is taken throughout most of the paper as a parameter. However, in chapter 5 this friction is endogenized by the existence of a moral hazard problem.

- There is an ***asymmetry of information*** about the above described allocation of investment opportunities among firms. Each firm knows the type of the project it is assigned to in the current period, but it is not aware of the allocation of projects among other firms.

The second friction is motivated by the reality of the securitization market and by the mentioned criticism of securitization, which takes the asymmetric information as the source of most of the agency problems (for details see the literature review). The first friction can be also observed in reality, but the main reason I include it in this otherwise simple model is that despite competition among financial firms, a binding "skin in the game" constraint increases equilibrium prices above the costs of investment, and therefore makes the securitization process profitable. Only when securitization is profitable, there exists a reputation equilibrium with implicit recourse, where losing reputation of providing implicit recourse is costly. As I explain later a firm without

[6] Kiyotaki and Moore (2012) obtain this result by including labor in the production function and requiring a competitive wage to be paid to workers in order to run a project. Here for simplicity I omit the workers from the model, but I use the results of constant returns to scale on individual level and decreasing returns to scale on aggregate level by assumption.

the reputation of providing implicit recourse will be unable to securitize and sell the projects, in which they have invested, and therefore it would loose the profits from securitization. [7]

3.1.3 Firms' problem

Each financial firm (indexed by i) chooses the control variables $\{c_{i,t+s},\ i_{i,t+s},\ \{a_{i,j,t+s+1}\}_j,\ h^S_{i,t+s+1},\ l^S_{i,t+s+1},\ r^G_{i,t+s+1},\ \chi_{i,t+s}\}_{s=0}^{\infty}$ to maximize the expected discounted utility from future consumption stream:

$$\sum_{s=0}^{\infty} \beta^s u\left(c_{i,t+s}\right),$$

where $u\left(c_{i,t+s}\right) = \log\left(c_{i,t+s}\right)$. The budget constraint for all firms is

$$\begin{aligned} c_{i,t} + i_{i,t}\left(1 - q^G_{i,t}\right) + \sum_{j\in\mathcal{I}_t} a_{i,j,t+1} q^G_{j,t} + h^S_{i,t+1} q^h_t + l^S_{i,t+1} q^l_t \quad &+ \quad \chi_{i,t} cir_{i,t} \\ = \sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) \quad &+ \quad h^S_{i,t}\left(r^h_t + \lambda q^h_t\right) + l^S_{i,t}\left(r^l_t + \lambda q^l_t\right)\ \forall i, \forall t, \end{aligned}$$

and firms with no investment opportunities face additional constraint $i_{i,t} = 0\ \forall i \in \mathcal{Z}_t$. This constrained maximization problem describes the following options of firms. The resources of firms consists of stochastic gross profits from projects financed in the past and the market value of non-depreciated part λ of those projects. They consume $c_{i,t}$ part of those resources. If they have an investment opportunity, they can invest at unit costs into new project $i_{i,t}$.[8] I denote the subset of firms that decide to invest into new projects (issue new loans) by $\mathcal{I}_t$. They can also buy securitized cash flows from newly financed projects on the primary market $\{a_{i,j,t+1}\}_j$ for prices $\{q_{j,t}\}_j$ or securitized cash flows from older projects of known high (low) quality on the secondary (re-sale) market $h^S_{i,t+1}\ \left(l^S_{i,t+1}\right)$ for the price $q^h_t\ \left(q^l_t\right)$, where $j \in \mathcal{I}_t$ and superscripts h, l denote the known quality of the traded asset. Investing firms can securitize and and sell cash flows

[7] I assume that it is possible to commit to not buying securitized assets from a particular firm and show that such commitment can be credible if the related incentive compatibility constraint holds. But I assume that it is not possible to prevent a particular firm from buying securitized assets from others, i.e., a threat of complete autarky is not possible. I believe this assumption corresponds to the reality of securitization markets.

[8] Gertler and Kiyotaki (2010) in their study of the interbank market, based on the same modeling approach by Kiyotaki and Moore (2012), refer to investments into projects as loans to entrepreneurs who run those projects. Entrepreneurs are able to offer a perfectly state contingent debt and since financial firms (banks) have all bargaining power, they can extract entire profits from entrepreneurs. Following this approach, I will sometimes refer to the investment into projects as loans too, and later calibrate this model on the performance of mortgage backed securities.

from the newly issued projects. If they sell part of their investment[9], they can provide implicit recourse to buyers of these newly securitized assets in the form of promise of minimum gross profit per unit of capital next period $r^G_{i,t+1}$. An asset with implicit recourse is traded for a market price $q^G_{i,t}$, which depends on the information structure in the equilibrium, i.e., on the beliefs of buyers about the type of the sold asset. Each firm can decide whether to default on the implicit recourse from previous period or not, which is represented by χ_t.[10] If a firm honors the implicit recourse, it has to spend part of the resources on covering related costs $cir_{i,t}$. The details on the cost of implicit recourse and the choice of default is discussed in detail in subchapter 3.2.4. The timing of shocks and choice of controls by firms withing each period is shown in figure 3.1.

Note that since the profits (cash flows) are observed and Δ^h, Δ^l, A_t are public information, the uncertainty about the quality of financed projects is resolved at latest in the period following the investment in the project. Therefore, depending on the particular equilibrium the quality of assets traded on the primary market may be either public or private information, while when these assets are traded next period on the secondary market, the quality is already public information. Therefore we can collapse all assets issued prior in past periods into two categories of high and low quality assets h^S, l^S.[11] Laws of motion for high and low quality assets traded on re-sale markets are

$$\begin{aligned} H^S_{t+1} &= \sum_i h^S_{i,t+1} = \sum_i \sum_{j \in \mathcal{H}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda h^S_{i,t}, \\ L^S_{t+1} &= \sum_i l^S_{i,t+1} = \sum_i \sum_{j \in \mathcal{L}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda l^S_{i,t}. \end{aligned}$$

Since the uncertainty about project quality lasts only for one period, for simplicity and tractability I will also restrict the guarantee to the performance of the loans to one period after the issuance.

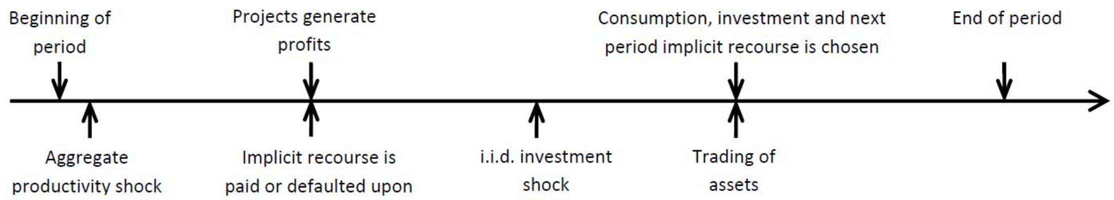
Since utility is logarithmic and budget constraints are linear in individual holdings of assets, the policy functions will be also linear in individual holdings of wealth. Due to logarithmic utility all firms will always consume a constant fraction of their current wealth (for derivation see appendix 8.2):

[9]The amount of new loans kept on the balance sheet is the difference between investment i_t and next period holdings of assets of firm i issued by the firm i: $a_{i,i,t+1}$, while $i_t - a_{i,i,t+1} \geqq 0$.

[10]χ_t takes value 1 in case of no-default and 0 in case of default.

[11]In chapter 5.3 I relax this assumption and introduce asymmetric information on secondary markets also.

Figure 3.1. Timing of shocks and choice of firm's controls withing each period



$$c_{i,t} = (1-\beta)\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}\left(r^h_t + \lambda q^h_t\right) + l^S_{i,t}\left(r^l_t + \lambda q^l_t\right)\right) \ \forall i.$$

Linear policy functions and i.i.d. investment opportunities enable easy aggregation. Application of the law of large numbers implies that the aggregate quantities and prices do not depend on the distribution of wealth across individual firms.

3.1.4 Goods and and asset markets

The model features a market for consumption good and for capital goods (securitized cash flows from projects). Every period all projects generate gross profits in the form of a consumption good. Consumption goods must be either consumed or converted into capital goods by investment into new projects. Consumption goods markets clear when all current output Y_t is consumed or invested: $Y_t = C_t + I_t$.

Capital goods are traded on asset markets. There is a secondary market on which assets of known quality are traded and a primary market for newly issued assets whose quality is either known or not depending on the type of the equilibrium. As derived in appendix 8.2 the conditions for clearing of assets markets come from the first order conditions of firms, which buy on asset markets (subset $\mathcal{S}_t$), which we will call saving firms $i \in \mathcal{S}_t$. These conditions imply that the discounted return of all assets traded on markets have to be equal to 1, and that in equilibrium saving firms will be indifferent between holding different assets.

Asset markets clearing conditions:

$$
\begin{aligned}
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^{\hat{G}}_{t+1}+\lambda q_{j,t+1}}{q^{G}_{j,t}}\right] &= 1\ \forall i\in\mathcal{S}_t, \forall j\in\mathcal{I}_t,\\
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^{h}_{t+1}+\lambda q^{h}_{t+1}}{q^{h}_{t}}\right] &= 1\ \forall i\in\mathcal{S}_t,\\
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^{l}_{t+1}+\lambda q^{l}_{t+1}}{q^{l}_{t}}\right] &= 1\ \forall i\in\mathcal{S}_t.
\end{aligned}
$$

Recall that all assets depreciate over time, so the law of motion for capital (stock of projects) is $K_{t+1} = \lambda K_t + I_t$. [12]

3.2 Model solution in special cases

To demonstrate the effect of the core frictions in the model, I will first briefly show in this subchapter the behavior and solution of the model without frictions, then I will successively introduce a binding "skin in the game" and the asymmetric information. I show that when the "skin in the game" is binding, there exists a reputation equilibrium, where implicit recourse can be provided. In the next subchapter I will show the solution of the the model in the case of interest, where both frictions hold and the provided implicit recourse can signal quality of the securitized cash flows from projects and result in a separating equilibrium, where the inefficiency related to asymmetric information is eliminated.

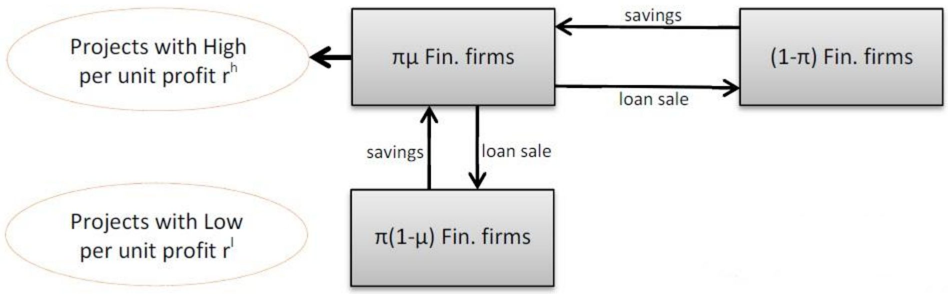
To show the results analytically, I will, in the next subchapters, mostly refer to the case with constant aggregate productivity $A_t = A$. In chapter 4, I report numerical results from the fully stochastic case.

3.2.1 Case with no financial frictions - first best

If none of the two frictions are present, i.e., project allocation is public information and the "skin in the game" constraint is not binding, in equilibrium only firms with high quality investment opportunities will invest, securitize loans and sell them to firms with low or unproductive investment opportunities. Since there is no asymmetric information and only high quality projects are being financed, there is only one type of asset traded

[12] Similar laws hold for both types of capital (low quality and high quality): $H_{t+1} = \lambda H_t + I^h_t$, $L_{t+1} = \lambda L_t + I^l_t$. Similarly to Kiyotaki and Moore (2012), I assume that the subjective discount factor exceeds the share of capital left after depreciation: $\beta > \lambda$.

Figure 3.2. Case without frictions - First best case



Note: In the first best case only firms with access to projects with high profit per unit of capital invest and they sell some of these projects to remaining firms.

in the economy. When I omit the variables which turn out to be zero in equilibrium, the budget constraints of individual firms with different investment opportunities are:

$$
\begin{aligned}
c_{i,t} + i_{i,t} + (h_{i,t+1} - i_{i,t})\, q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{H}_t, \\
c_{i,t} + h_{i,t+1} q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{L}_t, \\
c_{i,t} + h_{i,t+1} q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{Z}_t.
\end{aligned}
$$

Because of competition among firms with high quality investment opportunities, the price of loans is equal to the unit costs of financing the project (issuing the loan), $q^h = 1$.

Combining the aggregate consumption function, the goods market clearing condition and the law of motion for capital we obtain[13]:

$$
r^h + \lambda = \frac{1}{\beta}. \tag{3.2}
$$

The current period gross profit per unit of invested capital plus the value of non-depreciated assets is equal to the time preference rate, therefore the amount of investment is indeed first best.

[13] For details see appendix 8.1.1

3.2.2 Introducing the "skin in the game" constraint

In this chapter I show that a binding "skin in the game" constraint (at most θ fraction of new loans can be sold) increases the equilibrium prices above the replacement rate, which makes securitization profitable. As noted above, only when securitization is profitable, can a reputation equilibrium can exist. The "skin in the game" constraint is also a usual practice observed in securitization contracts in the form of tranche retention schemes[14]. This constraint can be motivated and endogenized by a moral hazard problem, which is derived in chapter 5. Chapter 5 also discuses some potential policy implications of making θ a policy parameter. In this chapter I assume for simplicity a constant θ.

By lowering θ we limit the capacity of firms with access to high quality projects to issue new investments. When this capacity is lower than the demand for new investments at the zero-profit price $q^h = 1$, then the "skin in the game" constraint becomes binding and the price has to increase above the unit costs of investment to clear the market. Securitization becomes profitable.

If the "skin in the game" is binding in equilibrium for firms with access to high quality projects, i.e., their holdings of newly issued assets represent $(1-\theta)$ fraction of their investment $h_{i,t+1} = a_{i,i,t+1} = (1-\theta)\, i_{i,t}\ \forall i \in \mathcal{H}_t$[15], we can rewrite their budget constraint to:

$$c_{i,t} + \frac{\left(1-\theta q_t^h\right)}{(1-\theta)} h_{i,t+1} = h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l)\ \forall i \in \mathcal{H}_t. \tag{3.3}$$

Combining these two equations and the consumption function we can find the level of investment of the constrained firm with access to high quality projects:

$$i_{i,t}^h = \frac{\beta\left(h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l)\right)}{\left(1-\theta q_t^h\right)}\ \forall i \in \mathcal{H}_t. \tag{3.4}$$

All policy functions are again linear, therefore can be easily aggregated and as appendix 8.1.2 shows we can obtain the following proposition.

Proposition 1. *If "skin in the game" is sufficiently large to be binding, i.e., θ is sufficiently low to satisfy*

[14] For simplicity I do not model the existence of different tranches. "Skin in the game" constraint is analogous to keeping a "vertical slice" of all tranches.

[15] I show below that for a subset of parameters firms with access to low quality projects will be investing and securitizing loans in equilibrium too. They may also face the binding "skin in the game" constraint, i.e., $l_{i,t+1}^l = a_{i,i,t+1} = (1-\theta)\, i_{i,t}^l\ \forall i \in \mathcal{L}_t$.

$$\theta < 1 - \frac{\pi\mu}{1-\lambda},$$

then in the deterministic steady state:

(i) the price of high quality assets q^h exceeds 1;

(ii) the steady state level of output and capital is lower than in the first best case.

The above proposition is analogue to Claim 1 in Kiyotaki and Moore (2012), but for a complete characterization of the model's steady state we also need the following proposition.

Proposition 2. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following cases:*

Case H: only firms with access to high quality projects issue credit and securitize ($q^l < 1$);

Case M: firms with access to low quality loans use mixed strategy and issue credit with probability ψ, ($q^l = 1$);

Case B: all firms with access to high and low quality projects issue credit and securitize ($q^l > 1$).

The above cases are ranked from the lest restricted ($q^l < 1$), where output and capital levels are relatively the closest to first best case, to the most restricted ($q^l > 1$), where output and capital is the lowest:

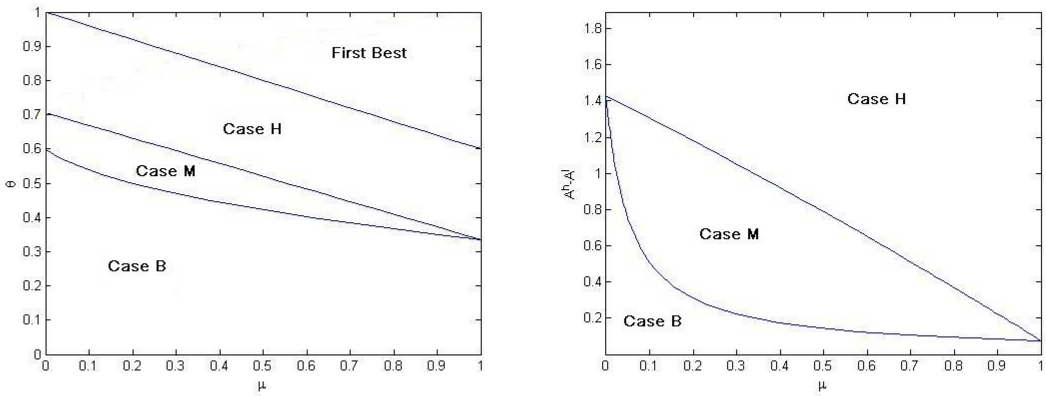
$$\begin{aligned} Y_{FB} &> Y_H > Y_M > Y_B, \\ K_{FB} &> K_H > K_M > K_B, \end{aligned}$$

where subscript FB denotes first-best case, subscript H, M and B denote the above described cases.

Proofs of the above propositions are in appendices (8.1.2 and 8.1.3).

Figure 3.3. shows the effect of selected parameter values on the type of the steady state. On the left panel we can see that lowering θ or μ moves the steady state from unrestricted first-best case to more restricted cases. The right panel shows that lowering the difference in productivity of the two types makes it more likely that low quality projects would be financed in the steady state.

Figure 3.3. Type of deterministic steady state depending on selected parameter values



3.2.3 Introducing asymmetric information

In this subchapter, I describe the consequences of introducing asymmetric information about the allocation of investment opportunities among firms on the model solution. I focus on the effect of asymmetric information between issuers of securitized assets and their first buyers, therefore at this point I do not consider asymmetric information on re-sale markets.[16]

Unless the difference in qualities is large enough, firms with access to low quality projects mimic firms with access to high quality projects. Since it is not possible to distinguish between the projects, saving firms, which want to diversify their portfolio, buy both high and low quality securitized assets at the rate corresponding to the probabilities of their arrival. This means that in equilibrium μ fraction of investment is allocated to high quality and $1-\mu$ fraction to low quality projects.

Proposition 3. *Compared to public information case the allocation of capital is generally less efficient (more in favor of low quality projects), therefore, the capital is less productive and in the steady state the amount of capital and output is lower.*

For proof see appendix 8.1.4.

The public information case will be equal to the private information case only if the difference in the qualities is large enough. The firm with low quality investment

[16] I assume that past projects are not anonymous, therefore, the quality of all existing projects becomes public information in the period following their securitization. In chapter 5.3., I relax this assumption and show that, if asymmetric information exists in general between the buyer and seller on the re-sale markets, there can be partial markets shutdowns similar to those found by Kurlat (2011).

opportunities will avoid mimicking firms with high quality investment opportunities as long as the return from buying high quality assets exceeds the return from mimicking:

$$R \mid buying\,high\,loans \quad > \quad R \mid mimicking$$

As shown in appendix 8.1.5 in the steady state this condition implies

$$\frac{A^h}{A^l} > \frac{(1-\theta)\,q^h}{1-\theta q^h} = \frac{(1-\pi\mu)\,(1-\lambda)\,(1-\theta)}{\pi\mu\lambda + (1-\lambda)\,\theta\pi\mu}. \tag{3.5}$$

If the ratio of the high and low productivity does not satisfy the equation 3.5, the resulting pooling equilibrium will be less efficient than the public information case. The separation condition can be also rewritten as

$$q^l < \frac{1-\theta q^h}{1-\theta}. \tag{3.6}$$

Since, by Proposition 1, $q^h > 1$, eq. 3.6 implies that a necessary condition for the existence of a separating equilibrium is that the equilibrium price of low quality assets is lower than costs of investing $q^l < 1$.

Note also that increasing the "skin in the game", i.e., lowering θ will only increase the lower bound for the ratio of productivities in the condition 3.5, and therefore make mimicking more likely. This result is driven by the general equilibrium effect. Lower θ increases the prices in the economy, and therefore makes mimicking more profitable.

Proposition 4. *Under private information, increasing the "skin in the game", i.e., lowering θ, makes pooling equilibrium, in which firms with low quality investment opportunities mimic firms with high quality investment opportunities, more likely.*

3.2.4 Introducing implicit recourse and reputation equilibrium case

Proposition 3 implies that the outcome of a private information case is generally inefficient compared to a public information case. Firms with high quality investment opportunities have incentives to distinguish themselves from low quality investment firms. However, under Proposition 4 we can see that retaining higher "skin in the game" does not lead to a separating equilibrium.

It turns out that by providing **implicit recourse**, a firm with high quality investment opportunities can distinguish itself without restricting its investment potential.

Under this strategy, the issuing firm promises minimum gross profit per unit of invested capital r_t^G to the buyers of securitized loans. Should the actual gross profits in the following period fall below this minimum, the issuing firm would reimburse the difference. This promise is not enforced by any explicit contract, rather it is a result of collusion between issuers of loans and their buyers[17]. Implicit recourse can be enforced in a reputation equilibrium, where securitizing firms aim to keep reputation of sticking to the promise and firms buying securitized projects enforce this promise by punishing the issuing firms in case of default on the implicit recourse. I assume a trigger strategy punishment that prevents a firm without reputation of honoring implicit recourse from selling securitized assets on the market. The punishment has to be credible, therefore in this reputation equilibrium buyers of securitized products with implicit support aim to keep a reputation of being "tough investors", i.e., reputation of always punishing firms that did not fulfill the promise.

At this point it is convenient to write the problem recursively:

$$
\begin{aligned}
V^{ND}\left(\bar{s}, w-cir; \bar{S}\right) &= \pi\left(\mu V^{ND,h}\left(\bar{s}, w-cir; \bar{S}\right) + (1-\mu) V^{ND,l}\left(\bar{s}, w-cir; \bar{S}\right)\right) \\
&\quad + (1-\pi) V^{ND,z}\left(\bar{s}, w-cir; \bar{S}\right), &(3.7)\\
V^{D}\left(\bar{s}, w; \bar{S}\right) &= \pi\left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + (1-\mu) V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + (1-\pi) V^{D,z}\left(\bar{s}, w; \bar{S}\right), &(3.8)\\
V^{ND,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c,i,\left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}, r^{G\prime}} \left[\log(c) + \beta E\left[\max\left(V^{ND}\left(\bar{s}', w'-cir'; \bar{S}'\right), V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right)\right]\right], &(3.9)\\
V^{D,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c,i,\left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}} \left[\log(c) + \beta E V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right], &(3.10)
\end{aligned}
$$

where $V^{ND}\left(V^{D}\right)$ are the value functions for the firm, that never defaulted (has already defaulted) on implicit recourse. w is individual wealth before deducting costs of implicit recourse cir, $\bar{s} = \left\{\left\{a_j\right\}_j, h^S, l^S\right\}$ is a vector of individual state variables, $\bar{S} = \{K, \omega, A\}$ is a vector of aggregate state variables and superscript k, which can take values $\{h, l, z\}$, represents the type of investment opportunity that the firm faces in the current period.

The equations (3.7) and (3.8) show the investment shock that takes place after the realization of aggregate productivity shock and decision on (non)default on implicit

[17] In this paper I do not compare the advantages of implicit and explicit guarantees. Based on the observed empirical evidence I model only the implicit guarantee. Reasons for provision of implicit rather than explicit guarantees can be various. Regulatory arbitrage is probably the major reason. Also the individual as well as social costs of default on an implicit guarantee (costs of punishment) could be lower than costs of default on explicit guarantee, which can be represented by liquidation costs (Ordogñez (2012) mentions the second reason).

recourse from previous period. After the investment shock firms choose optimally the level of consumption, the quantity of securitized loans they buy on the primary and secondary market and if they have an investment opportunity, they choose the optimal level of investment into new projects, securitization of their cash flows, fraction of the new investment which is sold and the implicit recourse which they provide.[18] This problem is described by the equations (3.9) and (3.10) for firms with reputation of having never defaulted on implicit recourse and without this reputation, respectively.

The above problem is constrained by the budget constraints which take the following form for investing firms for which the "skin in the game" constraint is binding (e.g. in case of firms with high investment opportunities):

$$c_{i,t} + \frac{(1-\theta q^G_{i,t})}{(1-\theta)} h_{i,t+1} + cir_{i,t} = \sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{H}_t,$$

where the price of securitized loans issued by firm j: $q^G_{j,t}$ depends on the information structure, i.e., on the beliefs of buyers about the type of the sold asset $\varphi_{j,t} \mid r^G_{j,t}$. When the "skin in the game" is binding, the costs of implicit recourse are given by:

$$cir_{i,t+1} = \theta i_{i,t}\left(r^G_{i,t} - r^k_t\right)\ \forall i \notin \mathcal{S}_t, k \in \{h,l\}$$

The incentive compatible constraints (ICCs), which have to be satisfied in equilibrium for the existence of reputation based implicit recourse are the following:

$$V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) > V^{D}\left(\bar{s}, w; \bar{S}\right) \tag{3.11}$$

$$V^{P}\left(\bar{s}; \bar{S}\right) > V^{NP}\left(\bar{s}; \bar{S}\right) \tag{3.12}$$

where V^P, V^{NP} are the value functions for the firm that always punished for default on implicit recourse, and failed to punish for default, respectively. The condition 3.11 determines the level of implicit recourse that can be credibly provided, i.e., it is not defaulted upon, given the trigger strategy punishment rule. The trigger punishment strategy has to be credible, therefore the saving firm which observes default on implicit recourse has to be better of punishing the investing firm that defaulted rather than not

[18] Recall that the timing of shocks and choice of controls by firms withing each period is shown in figure 3.1.

punishing it. This corresponds to the condition 3.12.[19]

Definition 1. A recursive competitive equilibrium consists of prices $\{q^h(\bar{S}),\ q^l(\bar{S}),$ $\{q_j^G(\bar{S})\}_j\}$ and gross profits per unit of capital $\{r^h(\bar{S}), r^l(\bar{S})\}$, individual decision rules $\{c(\bar{s};\bar{S}),\ h^{S\prime}(\bar{s};\bar{S}),\ l^{S\prime}(\bar{s};\bar{S}),\ r^{G\prime}(\bar{s};\bar{S}),\ \{a_j'(\bar{s},r_j^G,\varphi_j \mid r_j^G;\bar{S})\}_j,\ \chi(\bar{s},r^G;\bar{S})\}$, value functions $\{V^{ND}(\bar{s};\bar{S}), V^{ND,k}(\bar{s};\bar{S}), V^D(\bar{s};\bar{S}), V^{D,k}(\bar{s};\bar{S}), V^{NP}(\bar{s};\bar{S}), V^P(\bar{s};\bar{S})\}$ and law of motion for $\bar{S}=\{K,\omega,A,\Sigma\}$ such that: (i) individual decision rules and value functions solve each firm's problem taking prices, gross profits per unit of capital and law of motion for $\bar{S}=\{K,\omega,A\}$ as given; (ii) both asset and good markets clear and (iii) the law of motion for $\bar{S}=\{K,\omega,A\}$ is consistent with the individual firms' decisions.

3.2.5 Public information case with implicit recourse

Although one might think that the public information case is uninteresting, it is an important benchmark. If issuing firms could coordinate, they wouldn't be providing implicit recourse in this case, where it does not serve as a tool that would distinguish the firm type. However, due to competition firms tend to out-bet each other.

Should the promises be always credible, the optimal level of implicit recourse would be determined by the following F.O.C. (note that individual firm ignores the effects of this choice on aggregate variables):

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND\prime}}{\partial (w' - cir')}\frac{\partial (w' - cir')}{\partial r^G} = 0.$$

I show in appendix 8.1.7 that this condition implies that $q^j = 1$, which means that as far as there are positive profits from securitization, the competition will drive the level of implicit recourse so high that profits from securitization are zero. However, when profits from securitization are zero, the punishment has zero costs, and the original non-defaulting incentive compatibility constraint (3.11) is not satisfied. This leads us to the following conclusion.

Proposition 5. *As long as the implicit recourse is credible, firms find it optimal to increase it up to the level, where $q^j = 1$. So the level of implicit recourse is defined by the maximum, which can be sustained by the no-default condition (3.11).*

For details on the derivation see appendix 8.1.7. The steady state in this case is characterized by the following propositions.

[19] I show that this condition holds in appendix 8.1.6.

Proposition 6. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following cases:*

Case 1: only firms with access to high quality projects issue credit, securitize loans and provide implicit recourse $r^G_{h,cred}$ *(*$q^h > 1$*,* $q^l < 1$*,*$G^h_{cred} \geq r^h$*);*

Case 2: firms with access to high quality projects issue credit, securitize loans and provide implicit recourse $r^G_{h,cred}$*, firms with access to low quality projects use a mixed strategy and issue credit with probability* ψ *and provide implicit recourse* $r^G_{l,cred}$ *(*$q^h > 1$*,* $q^l = 1$*,* $r^G_{h,cred} \geq r^h$*,* $r^G_{l,cred} = r^l$*);*

Case 3: all firms with access to high and low quality projects issue credit, securitize and provide implicit recourse $r^G_{h,cred}$ *and* $r^G_{l,cred}$ *resp. (*$q^h > 1$*,* $q^l > 1$*,* $r^G_{h,cred} \geq r^h$*,* $r^G_{l,cred} \geq r^l$*).*

Note that $r^G_{k,cred}$ is the maximum implicit recourse, that can be credibly provided by firms with with $k \in \{h, l\}$ type of investment opportunity.

Proposition 7. *Compared to the public information case without implicit recourse, the amount of capital and output is higher, the allocation of capital is more in favor of high quality projects, and the wealth is less concentrated inside firms with investment opportunities. This holds in all cases except when the provided implicit recourse has no value (*$r^G_{h,cred} = r^h$*), and the two cases are identical.*

3.3 Case of interest: Implicit recourse as a signal of loan quality

In this chapter I analyze the case of interest, where the "skin in the game" constraint is binding, there is asymmetric information about the allocation of firms to investment opportunities and where the implicit recourse can signal the type of the investment opportunity.

As proved in the subchapter 3.2.4, the implicit recourse can be credibly provided in a reputation equilibrium. Under asymmetric information, implicit recourse can be interpreted as a signal of the loan quality. Investing firms (subset $\mathcal{I}_t$) sell securitized cash flows from newly financed projects and provide implicit recourse $r^G_{j,t+1} \in (0, \infty)$. The fact that a particular firm sells securitized cash flows and provides $r^G_{j,t+1}$is the message that this firm is sending to potential buyers of its securitized cash flows. Saving firms (subset $\mathcal{S}_t$) observing any message sent with positive probability use Bayes' rule to compute the posterior assessment that the message comes from each type. Without restriction on out of equilibrium beliefs (beliefs about the types conditioned on observing messages that are not sent in equilibrium) there is a multiplicity of Perfect Bayesian Equilibria, generally both pooling and separating. I use the Intuitive Criterion (Cho and Kreps, 1987) as a refinement to eliminate the dominated equilibria with unreasonable out of equilibrium beliefs.

Pooling Equilibria: In pooling equilibria both firms with access to high and low quality investment opportunities choose to provide the same level of implicit recourse given beliefs of investors. They both provide r^{G*} with probability 1. Saving firms observe this message and use the Bayes' rule to compute the posterior assessment that messages is sent by each type:

$$\varphi\left(j \in \mathcal{H}_t \mid r_j^G = r^{G*}\right) = \frac{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1}{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1 + \varphi\left(j \in \mathcal{L}_t\right) \cdot 1 + \varphi\left(j \in Z_t\right) \cdot 0} = \frac{\mu\pi}{\mu\pi + (1-\mu)\pi} = \mu.$$

Under no aggregate stochasticity there are several candidates for the pooling Perfect Bayesian Equilibria (PBE):

Case 1: Firms with access to both high and low quality projects select with probability 1: $r^{G*} = r^G_{l,cred,p}$, where $r^G_{l,cred,p}$ is the maximum implicit recourse, that can be provided by firms with low quality assets under pooling. Out of equilibrium beliefs of the saving firms that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r^G_{l,cred,p} < r_j^G < r^G_{h,cred,s}\right) = 0$ and unrestricted for intervals $0 < r_j^G < r^G_{l,cred,p}$ and $r_j^G > r^G_{h,cred,s}$. $r^G_{h,cred,s}$ is the maximum level of implicit recourse that can be promised credibly in a separating equilibrium (see below). In this equilibrium none of the firms defaults. None of the firms has incentive to unilaterally decrease implicit recourse or increase it.

Note that choosing $r_j^G < r^G_{l,cred,p}$ is not an equilibrium since both types will have incentives to increase implicit recourse to $r_j^G = r^G_{l,cred,p}$ due to competition, no matter what are the beliefs of investors, since both types would fulfill the implicit recourse in this interval.

Case 2: Firms with access to both high and low quality projects select $r_j^G = r^{G*}$ s.t.:

$$r^G_{lb,p} \leq r^{G*} \leq \min\left(r^G_{minsep}, r^G_{h,cred,p}\right).$$

Out of equilibrium beliefs of the saving firms that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r^{G*} < r_j^G < r^G_{h,cred,s}\right) = 0$ and $\varphi\left(j \in \mathcal{H}_t \mid 0 < r_j^G < r^{G*}\right) \leq \mu$

and unrestricted for the interval $r_j^G > r_{h,cred,s}^G$.

r_{minsep}^G is the minimum level of implicit recourse which the low types would not mimic under any beliefs (see derivation in appendix 8.1.9). $r_{lb,p}^G$ is the lower bound on r^G, where firms with high quality investments do not have incentives to deviate to $r_{l,cred,p}^G$. The fact that for r^G such that $r_{l,cred,p}^G < r^G < r_{lb,p}^G$, both types have incentives to decrease implicit recourse to $r_j^G = r_{l,cred,p}^G$, is due to equilibrium defaults on implicit recourse of firms with low investment, which bring investors lower utility, than when $r^G = r_{l,cred,p}^G$. And this negative effect on price together with potentially higher costs of higher implicit recourse (when $r^G > r^h$) outweighs the positive effect of higher implicit recourse on the price.

Separating Equilibria: There is potentially a continuum of separating equilibria, where firms with access to low quality projects save and buy securitized assets from firms with access to high quality projects. Firms with access to high quality projects invest, securitize and provide implicit recourse $r^{G*} \in \left(r_{minsep}^G, r_{h,cred,s}^G\right)$ with probability 1, where r_{minsep}^G is the minimum implicit recourse which prevents mimicking by firms with low investment opportunities. Saving firms observe this message and use the Bayes' rule to compute the posterior assessment that message is sent by each type:

$$\varphi\left(j \in \mathcal{H}_t \mid G_j = G^*\right) = \frac{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1}{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1 + \varphi\left(j \in \mathcal{L}_t\right) \cdot 0 + \varphi\left(j \in Z_t\right) \cdot 0} = \frac{\mu\pi}{\mu\pi} = 1.$$

Out of equilibrium beliefs of the saving firms that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r^{G*} < r_j^G < r_{h,cred}^G\right) = 0$ and unrestricted for intervals $0 < r_j^G < r^{G*}$ and $r_j^G > r_{h,cred,s}^G$.

Application of Intuitive Criterion: If a separating equilibrium exists, then all pooling equilibria are dominated, and therefore fail the Intuitive Criterion. In particular, due to competition among firms with access to high quality investments, Intuitive Criterion selects only one separating equilibrium, where firms with access to high quality investments invest, securitize and provide the maximum credible implicit recourse $r^{G*} = r_{h,cred,s}^G$.[20] So after application of Intuitive Criterion, there is either one unique separating equilibrium left, or one or multiple pooling equilibria.

The condition for the existence of a separating equilibrium:

Thanks to Proposition 5 we know that firms have incentives to unilaterally increase the provided implicit recourse up to the maximum credible level. But then, if low quality firms are already at the maximum credible level, where cost of defaulting and keeping

[20] This case is shown in figure 3.3.

Figure 3.4. Case where Intuitive Criterion selects unique Separating Equilibrium

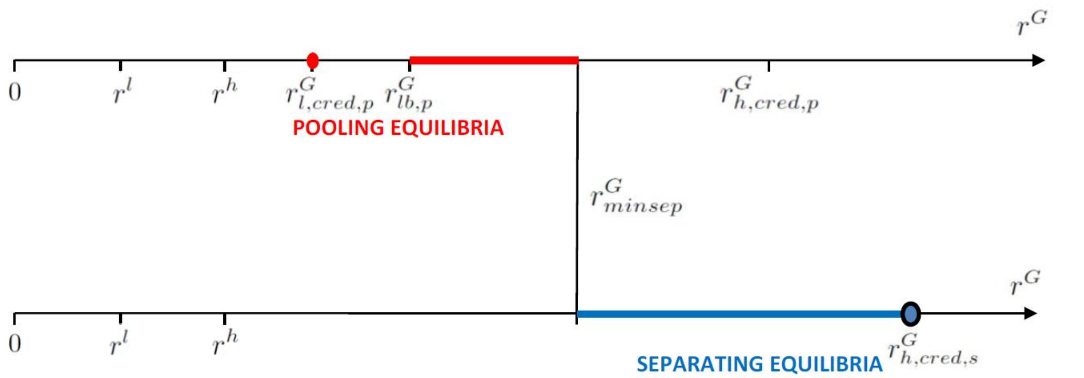


Figure 3.5. Case where there is no Separating Equilibrium

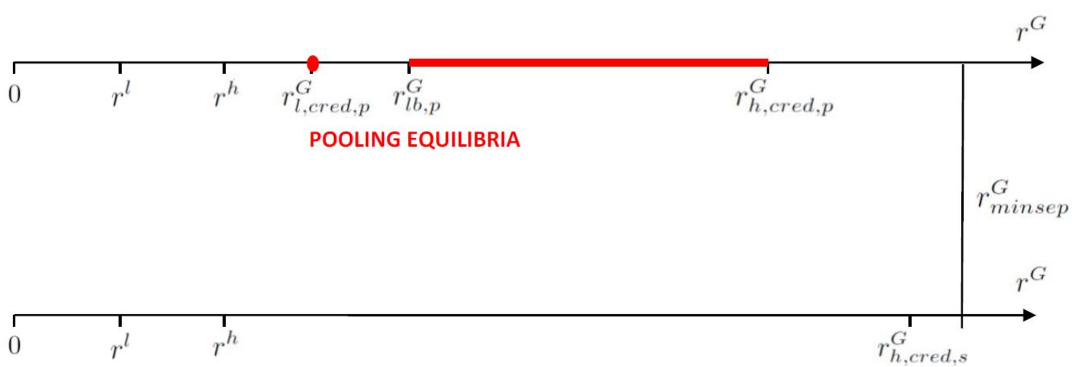
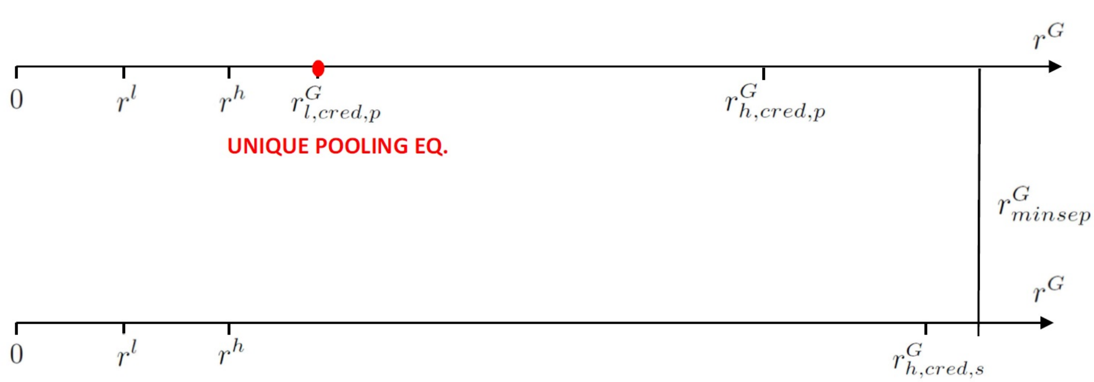


Figure 3.6. Case with unique Pooling equilibrium



the implicit recourse is equalized, they are better of if they increase the implicit recourse without increasing the cost further, but potentially benefiting from being mistaken for a firm with access to high quality projects. Therefore, no separating equilibrium in which firms with low quality investment will provide a different level of implicit recourse can exist. Firms with low quality investments always prefer mimicking firms with high quality investments to providing a lower implicit recourse and disclosing their quality.

Therefore, separation can take place only when the costs of mimicking become so large that investing into high quality assets is preferred. Under deterministic case this condition can be expressed analytically. The implicit recourse r^G have to be high enough to satisfy:

$$V^l \mid mimicking < V^l \mid buying\ high\ loans \tag{3.13}$$

This brings us to one of the main findings in this paper.

Proposition 8. *Under asymmetric information a separating equilibrium is possible in the deterministic steady state if and only if*

$$\frac{A^h}{A^l} > \frac{(1-\theta B)\, q^h}{1-\theta B q^h}, \tag{3.14}$$

where $B \equiv \frac{q^G}{q^h} = \frac{r^G+\lambda q^h}{r^h+\lambda q^h}$ *is the price premium for the equilibrium implicit guarantee. This implies that separating equilibrium:*

(i) exists if and only if the level of aggregate productivity does not exceed the threshold level $\bar{A}$*;*

(ii) exists if and only if $q^l < 1$*;*

(iii) is more likely in the presence of reputation based implicit recourse.

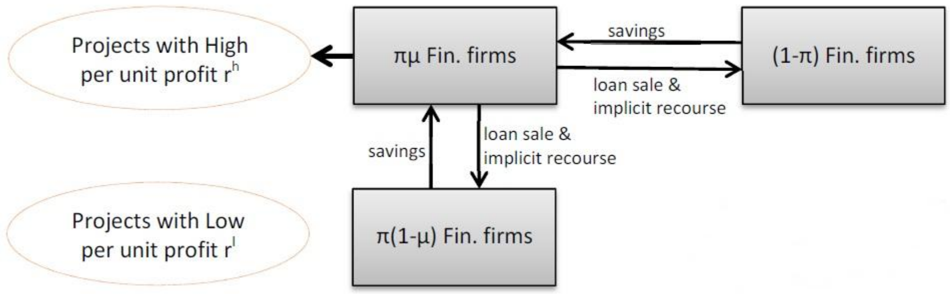
In a separating equilibrium, firms with low quality investment projects save and buy securitized assets from firms with high investment opportunities.

Sketch of proof: The derivation of the equation 3.14 comes directly from no-mimicking condition 3.13.[21] The point (i) comes directly from the Assumption 1 about the countercyclical relative difference of cash flows from projects of different quality. Since the ratio of TFP on the LHS of the eq. 3.14 increases with aggregate TFP A, the mentioned threshold is defined $\Delta^h\left(\bar{A}\right)/\Delta^l\left(\bar{A}\right) = (1-\theta B)\, q^h/\left(1-\theta B q^h\right)$.

Crucially, as I show in appendix 8.1.8, in a separation equilibrium both q^h and B and, therefore, also the whole RHS of 3.14 are independent of the realizations of

[21] See appendix 8.1.8 for derivation.

Figure 3.7. Private information case with implicit recourse: Separating equilibrium



Note: In the separating equilibrium the implicit recourse provided by the firms with access to high quality projects is high enough so that it is not profitable for firms with access to low quality projects to mimic them. They are better off buying the high quality projects.

aggregate productivity A, and are uniquely determined by the intensity of frictions and the punishment for default on implicit recourse.

After substitution of the share of TFP by ratio of prices from the asset market clearing condition, the condition 3.14 can be rewritten to:

$$q^l < \frac{1 - \theta B q^h}{1 - \theta B},$$

which implies that in a separating equilibrium $q^l < 1$ since by Proposition 1 $q^h > 1$.

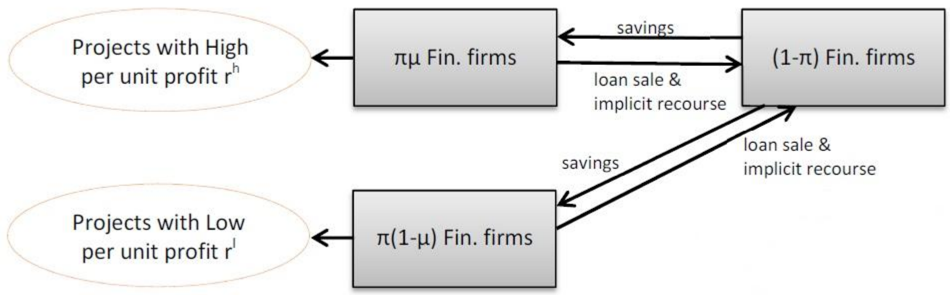
Finally, when comparing the lower bound on the TFP ratio consistent with separating equilibrium in case without implicit recourse (eq. 3.5) and in case with implicit recourse (eq. 3.14) we can show that the latter is lower. This implies that in the case with implicit recourse the separation condition (eq. 3.14) is more likely to be satisfied.[22]

Uniqueness of pooling equilibrium:

When a separating equilibrium does not exit, there is generally a continuum of pooling equilibria. However, it turns out that for a large set of parameter space there is only one pooling equilibrium with $r^{G*} = r^G_{l,cred,p}$, independent on a specific form of out of equilibrium beliefs.[23] I calibrate the model to have only one pooling equilibrium. The advantage of this is besides having a unique equilibrium, that punishment is never triggered in equilibrium. It still provides the disciplining role, but the dynamic results

[22]Complete proof is in appendix 8.1.8.
[23]Figure 3.6 shows this case with a unique pooling equilibrium.

Figure 3.8. Private information case with implicit recourse: Pooling equilibrium



Note: In the pooling equilibrium both firms with access to high and low quality projects provide the same level of implicit recourse. They are indistinguishable and therefore both firms invest into projects and sell them to firms with no investment opportunities.

are not influenced by exercise of a particular punishment rule.

To obtain such an equilibrium, in general I have to find values of parameters such that $r^G_{lb,p} > r^G_{h,cred,p}$, i.e., the minimum level of implicit recourse for which it pays off to provide recourse higher than $r^G_{l,cred,p}$ is not credible in equilibrium, since it exceeds $r^G_{h,cred,p}$.

It turns out that this condition is satisfied for a low enough share of high quality investment opportunities μ and a high enough difference in type specific TFP in a pooling equilibrium:

$$\mu < \frac{1-\theta q^l}{q^h - \theta q^l}.$$

For details see appendix 8.1.9.

4 Dynamics and numerical examples

In this chapter I show the solution of the fully stochastic version of the model with asymmetric information, binding "skin in the game" and implicit recourse. The allocation of projects to firms is still driven by an i.i.d. shock. The aggregate productivity for simplicity follows a 2 state Markov chain $A_t \in \left(A^H, A^L\right)$[24] with a transition matrix

[24]Note that capital superscripts H, L refer to the aggregate state of the economy and not to the type of investment opportunity.

$P = [p, 1-p; 1-p, p]$.[25]

In the analysis of the dynamic properties of the model I focus on the switching between the separating and pooling equilibria over the business cycle. Even though in the steady state there is a separating equilibrium, when the aggregate productivity increases and the economy is in boom stage of the business cycle $A_t = A^H$, the separating equilibrium is no longer sustainable, and the economy is in the pooling equilibrium, where both types of firms provide the same level of implicit support and both invest into new projects. This follows directly from Proposition 8. The intuition behind the result is the following. As the aggregate productivity increases, the relative difference in productivity of the two nonzero profit project types is reduced. Therefore, a higher implicit recourse is needed to satisfy the separation condition (3.13). Intuitively, following Proposition 8, the condition says that $q^l < \left(1 - \theta B q^h\right) / (1 - \theta B) < 1$ is necessary for separation, but in a boom even the quality of low type projects is relatively high, and therefore one has to provide high implicit recourse to drive the prices of low quality projects low enough. At some point, the level of implicit recourse required to achieve separation exceeds the maximum level that can be credibly provided and the economy switches to the pooling equilibrium.

Calibration of parameters: Since I extend the model of Kiyotaki and Moore (2012), I use the same level of parameters for: $\alpha = 0.4$, $\beta = 0.99$ and $\pi = 0.05$. Persistence parameter for the productivity process is $p = 0.86$.[26] Parameters A^H, A^L are chosen to match the annual standard deviation of GDP in the USA, which is 2.8%.[27] The remaining parameters are chosen to replicate the performance (delinquency rates) of securitized assets which has been at the core of the recent debates about the efficiency of securitization - subprime residential mortgage backed securities issued in the USA: $\mu = 0.63$, $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right) = 0.94$ and $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right) = 0.71$.[28] The annual depreciation $\lambda = 0.78$ is chosen to replicate the weighted average life (WAL) for residential MBS of 54.5 months (Centorelli and Peristiani, 2012). And finally the fraction of loans that can be sold is set to $\theta = 0.75$ to allow for the switching between pooling and separating equilibrium over the business cycle, which is supported by the empirical analysis in chapter 6.

[25] The case when A_t follows a Markov chain is easier to calibrate, but is not crucial for the results. Earlier version of this paper works with an AR(1) process for the aggregate TFP.

[26] This corresponds to an autocorrelation of TFP shocks at the quarterly frequency of 0.95.

[27] Similar approach is used in Nikolov (2012)

[28] For details see appendix 8.3.

Solution method: The fully stochastic model is solved using the global numerical approximation. method. In particular I find the price functions and the value functions by iterating on the grid of state variables until convergence.[29]

Impulse responses: Figure 4.1 shows how the economy behaves in a particular episode of three periods in state with high aggregate TFP followed by three periods in state with low aggregate TFP. Then the productivity shocks are switched off and the economy converges to the steady state.[30] The point of this exercise is to show the switch from separating equilibrium to pooling and back and its effects on the output. For comparison, on the graph I report impulse responses[31] of the constrained model under private information, with binding "skin in the game" and with implicit recourse provision as well as the unconstrained and efficient first-best case. Note that the graph depicts deviations from each model's steady state. Only the share of high quality assets on the balance sheets (ω) is shown in absolute value. So even though on the graph both first-best and constrained case start at the same point, the first-best case is characterized by higher absolute levels of steady state output and capital.

The figure demonstrates that, as the constrained economy moves to the boom stage of the business cycle, the separating equilibrium changes to pooling equilibrium, i.e., the share of high quality projects(ω) decreases, while ω remains constant in the first best case at 100%. Lower share of high quality projects in the constrained case slows slightly the growth of output and accumulation of capital already in the boom, but the effect is small, since in boom stage the difference in the two qualities is rather small. However, the inefficiency in allocation of capital continues to accumulate. As the economy exogenously moves to a recession with higher difference in qualities, one can see that the accumulated inefficiency in the allocation of capital is more pronounced. Therefore, booms have almost the same relative size in constrained and first-best case, but busts following a boom stage are much deeper in a constrained case.

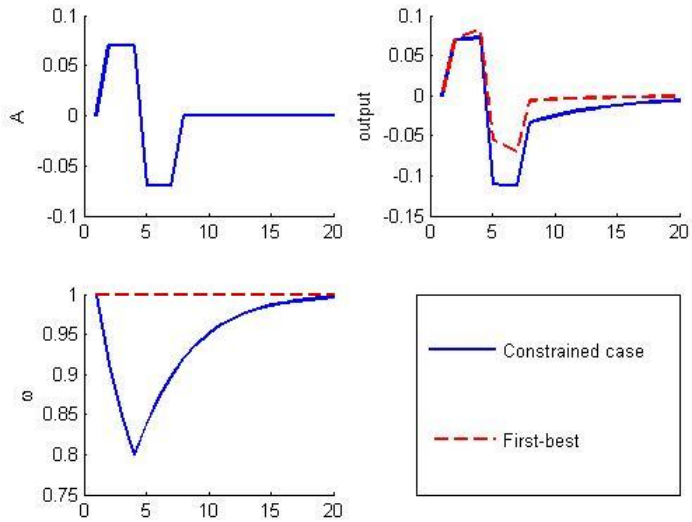
Figure 4.2. shows the result directly following from the switching property of the model - the fact that the longer is the boom period preceding the recession, the larger

[29]Details are in appendix 8.4.

[30]In this case with a Markov chain for aggregate productivity, the steady state productivity $\bar{A}$ is defined as the mean of the ergodic distribution across $\left(A^H, A^L\right)$ and in this zero-probability steady state the expectations about the occurrence of either state is set to 50%.

[31]The impulse responses start from a steady state to which they converge after a long period of zero productivity shocks, i.e. aggregate productivity stays at the steady state productivity $\bar{A}$. Then I introduce the described sequence of productivity shock, after which the shocks are zero again.

Figure 4.1. Impulse responses



the fraction of low quality assets accumulated in the pooling equilibrium and the larger is the difference in the depth of a recession compared to the first best case (recession gap).

5 Extensions

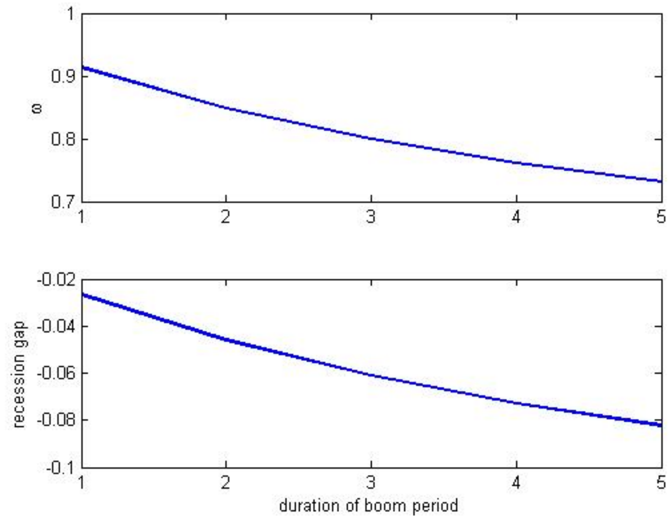
5.1 Endogenizing the "skin in the game"

So far the "skin in the game" (or equivalently the share of loans which could be sold, θ) was taken as an exogenous parameter. In this chapter I will sketch a simple moral hazard problem, which would aims to justify the existence of this constraint.

Consider that firms can divert funds from the sale of current period loans needed to cover the unit investment costs. This cannot be immediately verified. To eliminate this problem investors require the issuing firms to retain a sufficiently large "skin in the game" $(1-\theta)$, i.e., to finance a fraction $1-\theta$ of funds in the project from their own resources. The incentive compatible constraint then points down a sufficiently high θ that prevents this moral hazard problem[32]:

[32]It is intuitive to assume that if a firm would divert funds, other firms will use at least the same punishment tools as for the case of implicit recourse default.

Figure 4.2. The longer the boom stage, the deeper the subsequent recession



$$V^{D}\left(w\beta R' \mid diverting\, funds\right) \leq V^{ND}\left(w\beta R' \mid investing\, properly\right),$$

where return from diverting funds is $R' \mid diverting\, funds = \left(\frac{\theta q^G}{(1-\theta)}\right)^x$, with x being the number of times the individual recycles the returns from this operation to issue and sell new "castles-in-the-air" projects. Since I do not restrict the practice of sequential issuance of loans, which is technically needed even under proper investing, the ICC will always fail unless $\theta q^G < (1-\theta)$, which translates to

$$\theta < \frac{1}{q^G + 1}. \tag{5.1}$$

Thus, the higher the sale price of loans q^G, the higher "skin in the game" $(1-\theta)$ is required to prevent the mentioned moral hazard problem.

Note that in this version of the model I have two sources of asymmetric information. First is the potential diversion of resources needed to make investment properly, which cannot be immediately observed. The "skin in the game" is found to be an efficient tool to prevent this behavior, while the loss of reputation and subsequent punishment is not so efficient. The second source of information asymmetry is the unobserved allocation of investment opportunities among firms. In this case according to Proposition 4 the "skin in the game" is not an efficient tool, while the reputation based implicit support can overcome the related inefficiencies.

Even with endogenous “skin in the game”, the main qualitative result of the paper, which is the endogenous switching between the pooling and separating equilibrium, remains unchanged.[33]

5.2 “Skin in the game” as a policy parameter

The “skin in the game” can be considered as a potential policy parameter. For instance, the section 941 of the Dodd-Frank Reform already requires minimum explicit risk retention of 5%.

If, as in this model, the “skin in the game” is determined endogenously by a moral hazard problem, and securitization is the only means of financial intermediation, policy which tries to increase the “skin in the game” beyond the endogenously determined value would not improve the efficiency of financial intermediation. The reasons are twofold.

First, higher "skin in the game" increases the profits from securitization and lowers the aggregate quantity of investment (this follows from Proposition 1 and 2). Second, higher profits also make issuance and sale of loans profitable even for firms with lower quality projects, which would otherwise be buyers of high quality projects (this holds both in the symmetric information case from Proposition 2 as well under asymmetric information since pooling equilibrium is more likely see Proposition 4 and Proposition 8). Therefore, both quantity as well as quality of investment is lower with higher "skin in the game" than with the level of this constraint determined by the market.

In contrast to some other models of securitization, such as Gorton and Pennacchi (1995), my model does not feature continuous monitoring or effort level. I only have an option of funds diversion which is observed only with a time lag. At a high level of abstraction this can be understood as the analogy to costly monitoring in Gorton and Pennacchi (1995), where the level of monitoring would take only two values (no monitoring or full monitoring). This moral hazard problem indeed points down the optimum level of "skin in the game". Given that everyone is rational, not only there is no reason to increase the "skin in the game" above the level determined by the equilibrium, but increasing the "skin in the game" would have negative effects on the

[33] For the proof see appendix 8.1.10. Also note that the assumption of moral hazard problem is absolutely essential since without it the solution would be first best even under asymmetric information. Under first best, securitization is not profitable, therefore firms with access to low quality investment do not have any incentives to mimic firms with high quality investments. Therefore, neither reputation equilibria nor implicit recourse would take place.

economy as described above.[34]

One could possibly introduce additional frictions which would create benefits of the mentioned regulation. However, those possible benefits can be outweighed by the mentioned adverse general equilibrium effect especially when the regulation is too excessive.

5.3 Adverse selection on re-sale markets

So far we have considered the asymmetry of information between the originators of securitized assets and buyers of these assets. In this section I extend the asymmetry of information to the re-sale market. In particular I assume that the holder of the asset can learn the quality of the underlying asset, while the buyer cannot. This leads to the typical adverse selection on the re-sale market.

The new result in this paper comes from the interaction of the adverse selection on re-sale markets with the switching between pooling and separating equilibria. The severity of the adverse selection on the secondary markets depends on the difference in qualities but as well on the share of low quality assets on the balance sheets. Therefore, intuitively the adverse selection is more important in a recession than in a boom. But also the longer is the boom period which precedes the recession, the larger is the share of low quality loans on the market and the more acute the adverse selection issue becomes. If adverse selection is strong enough, securitized loans of high quality stop being traded on the re-sale markets altogether, which further deepens the recession.

The motivation for including this section are the problems witnessed on the securitization markets during the late 2000's financial crisis.

The assumption of asymmetric information on re-sale markets has the following impact on the model behavior. First, when an asset is re-sold, there is a unique price which is independent on the quality of this asset q_t^s. If an asset is not re-sold, the owner who knows its quality will value high quality asset q_t^h and low quality asset q_t^l, but this is not the market price. Second, prices depend on the share of high quality assets on the re-sale market.[35] Every period firms find out the quality of assets on their balance sheets and sell all low quality assets. Unlike original issuers in the period when investment is made, they no longer have the technology to provide implicit recourse. High assets on the market are sold only by firms with investment opportunities which

[34] It can be argued that this model is too simplistic to inform policy recommendations. That is why I reproduce the above results in a richer framework with debt as well as deposit financing and study the optimal mix of macro-prudential policy in Kuncl (2013).

[35] See appendix 8.1.11 for details.

are in the need for liquidity.

Therefore, the share of high quality assets on the re-sale market is

$$f_t^h = \frac{\pi\mu\omega_t}{\pi\mu + (1-\pi\mu)(1-\omega_t)}$$

in case of a separating equilibrium and

$$f_t^h = \frac{\pi\omega_t}{\pi + (1-\pi)(1-\omega_t)}$$

in case of a pooling equilibrium.

If, due to the adverse selection, the price of assets on the re-sale market drops low enough, even firms which sell assets due to liquidity reasons will stop selling high quality assets. The price is so low that the return from taking advantage of the investment opportunity would not compensate for the cost of selling a valuable asset at a low market price. In a deterministic steady state this situation takes place if:

$$R^h > q^s \frac{R^h - \theta R^G}{1 - \theta q^{IR}},$$

where $R^h = r_{t+1}^h + \lambda\pi\mu q_{t+1}^s + \lambda(1-\pi\mu) q_{t+1}^h$ and $R^G = r_{t+1}^G + \lambda\pi\mu q_{t+1}^s + \lambda(1-\pi\mu) q_{t+1}^h$. As shown in appendix 8.1.11, this condition implies that the share of high quality assets traded on the re-sale market has to be low enough to satisfy:

$$f^h < 1 - \frac{q^h - 1}{(q^h - q^l)(1 - \theta B)}.$$

If this conditions is satisfied, there will not be complete market shutdowns since low quality assets would be still sold at a fair price, but the volume of sales would greatly diminish by the absence of high quality assets and the level of overall investment in the economy would be also significantly lower.

6 Empirical analysis

The main results of the theoretical model are the prediction that providing implicit support can signal the quality of the underlying loans and the prediction that this signaling is less efficient for loans issued in boom stages of the business cycle. This section presents empirical tests of these hypotheses. The results are in line with the model predictions.

Due to the implicit nature of the reputation based support there is no data which would measure directly the level of implicit support. However, when the implicit support is activated for instance in periods of lower than expected cash flows (higher than expected delinquency rates) from the securitized products, it can be observed and often appears in the data.[36] Even using the data on support provided by the originator (credit enhancement) when it is actually explicitly provided, we can test the hypotheses contained in the theoretical model.

The empirical literature on the relationship between credit enhancements and the quality of the loans (typically approximated by the delinquencies on the collateral) is limited. The most relevant paper is the work by Mandel et al. (2012), where the authors test the signaling and the buffer hypotheses of credit enhancement (credit protection provided to holders of the securitized assets). The signaling hypothesis, which is already described in this paper, predicts a negative correlation of credit enhancements and delinquencies on the collateral. According to the buffer hypothesis credit enhancement does not serve as a signal of high quality of collateral but is rather provided as a buffer against observable risk. In this case securitized assets with poor quality of collateral will need higher credit enhancement, and therefore, it will imply a positive relationship between credit enhancements and delinquency rates.

6.1 Hypotheses

I perform two tests: first tests the signaling hypothesis (with the alternative being the buffer hypothesis) and the second tests the hypothesis of lower efficiency of signaling (switching to pooling equilibria) when loans are issued in boom periods of the business cycle.

H1: Credit enhancement signals the quality of collateral If the signaling hypothesis is correct, then more support would be positively correlated with the quality of the securitized products. Therefore, this hypothesis would suggest a negative ef-

[36]As an anecdotal evidence let me cite the example reported originally by Mandel et al. (2012) on the increase in credit enhancement by Chase Issuance Trust. The originator of the securitized assets increases credit enhancement on both future issuance as well as all outstanding securitized products. Note that they had no contractual obligation to provide higher credit enhancement on loans products issued in the past, so this is a typical case of implicit support that appears in the data only at the time when the implicit support is activated. Fitch: Chase Increases Credit Enhancement in Credit Card Issuance Trust (CHAIT)," http://www.reuters.com/article/2009/05/12/ idUS260368+12-May-2009+BW20090512.

fect of lagged credit enhancements on the delinquency rates of the collateral. If the relationship is opposite then the buffer effect dominates.

H2: For loans issued in the boom stage of the business cycle a pooling equilibrium is more likely, therefore signaling is less efficient If the signaling is less strong for assets originated in the boom period of the business cycle as predicted by the model due to higher likelihood of the pooling equilibrium, the positive correlation between credit enhancements and quality of collateral should be smaller or even become negative for this particular subset of products. I construct a dummy for securitized products issued in boom stage of the business cycle. This hypothesis would suggest that and interaction term of lagged credit enhancements with the dummy for deals issued in the boom should have a positive effect (an increase) on delinquency rates of the collateral.

6.2 Data description

I use the database Performance Data Services (PDS) provided by Moody's, which contains the data on delinquency rate of collateral in the pool as well as on the credit enhancement provided to back securitized products. I have access to the part of the database which covers Residential Mortgage Backed Securities (RMBS) issued in Europe.[37]

As a proxy for quality of collateral (mortgage loans) which backs the securitized products I use 90plus delinquency rate which is defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance. The support provided to securitized products is captured by credit enhancement which is the amount of credit protection available to the holders of securitized assets in the form of subordination, overcollateralization, reserve funds, letters of credit, spread accounts, cash collateral accounts and other non-guaranteed funds. The data is available for individual tranches.

Since the quality of collateral is available only on the level of the pool, I need to aggregate credit enhancement data. I aggregate on the level of deals. A deal is typically backed by a pool of collateral and consists of several tranches. I drop the observations where more pools back the same deal or more deals are backed by the same pool of loans

[37] I would like to thank the European Central Bank for providing me with the access to this part of the PDS database.

since I do not have information needed to do proper aggregation. The data on credit enhancement is available on tranche level, therefore I compute a weighted average. Credit enhancement is expressed as total amount of credit protection as a fraction of current pool balance. I winsorize both delinquency rates and the credit enhancement rate at the 2.5%-level to account for data errors and limit the effect of potential outliers.

The real output data for the respective countries are obtained from Eurostat. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

6.3 Panel regression results

I run the following fixed effect regression:

$$\begin{aligned} DelinquencyRate_{i,t} &= \alpha_i + \alpha_t + \beta\, CERatio_{i,t-1} + \gamma\, CERatio_{i,t-1} \times D\left\{boom\right\}_{i,t} \\ &\quad + \delta\, CERatio_{i,t-1} \times D\left\{originated\, in\, boom\right\}_{i,t} \\ &\quad + \iota\, Deal\, age_{i,t} + \kappa\, Output\, gap_{i,t} + \varepsilon_{i,t} \end{aligned}$$

on data with quarterly frequency, where $CERatio_{i,t-1}$ is the ratio of total credit enhancement to current pool balance lagged one period in time[38]; $D\left\{boom\right\}$ is the dummy variable for boom period in the country of issuance; $D\left\{originated\, in\, boom\right\}$ is the dummy variable for deals issued in a boom period of the respective country; $Deal\, age$ is the number of quarters since the closing date of the deal; and $Output\, gap = \ln\left(GDP\right) - \ln\left(GDP_{HP}\right)$, where GDP_{HP} is the smoothed level of respective real Gross Domestic Product obtained by the HP filter.

Table 1 shows the results for the four largest European countries by securitization activity for residential mortgage loans: the United Kingdom (UK), Netherlands (NL), Spain and Italy. I show results for the whole subset and for the UK and Spain separately. I use fixed effects for deals and time and report Huber-White robust standard errors. Standard errors are clustered by deals. I report the results on the maximum sample period, but also on the period excluding the recent crisis. The results are con-

[38]Note that I use the variable credit enhancement lagged by one quarter. This is because contemporaneous correlation between credit enhancements and and loan quality could be positive due to a trigger of some implicit support in times of temporary distress. However, this does not contradict the signaling hypothesis. In fact it is a part of the signaling story developed in this model. On the other hand if the signaling hypothesis is correct then the lagged credit enhancement should be negatively correlated with current quality of the collateral.

sistent for both periods. I also checked the results when initial periods with relatively few observations are excluded and the results are still consistent. Although I do not claim that the relationships found are necessarily causal, I still find that analyzing the magnitude of the relationship is interesting and informative.

For the whole sample of four countries (UK, NL, Spain and Italy) the results are in line with the signaling hypothesis (coefficient of $CERatio$ is significantly negative), and also in line with the hypothesis, that signaling in case of loans issued in periods of boom is much weaker (coefficient of $CERatio \times D\{originated\,in\,boom\}$ is significantly positive). Finally, the coefficient of $CERatio \times D\{boom\}$ is significantly negative. This would suggest that the signaling effect is stronger in the boom period for all loans irrespective of the time of issuance. However, I would offer a slightly different interpretation. Following the model presented in the previous chapters, since the guaranteed minimum cash flows is not conditional on the state of the economy, implicit support is most likely to be activated and therefore appear in the data in a recession. The lower the quality of the asset the higher the support (additional credit enhancement) needed to keep to the expected implicit obligation. This is an analogue to the buffer effect mentioned in Mandel et al. (2012). Both signaling and buffer effect are likely to operate all the time. However, in recession the buffer effect might be stronger; that is why the effect of credit enhancements on delinquencies is less negative.

I also analyzed selected countries individually. The UK and Spain had the highest number of observations, so I report these results. In the UK the results are qualitatively the same as for the whole sample. However, in Spain the credit enhancement has no significant effect on delinquencies. I believe that this result is due to a very different regulation of securitization in both countries. Unlike in other countries, in Spain the regulator treated off-balance sheet assets (i.e. all securitized products) in the same way as if they remained on the balance sheet.[39] Therefore, the securitization practice in Spain was very different from other countries. Securitization was not used to transfer risk, but rather to obtain more liquidity. Consistent with this, Almazan et al. (2013) reports that securitization in Spain was used mainly by small banks which had difficulties obtaining debt financing. Following the evidence from Almazan et al. (2013), in Spain securitization was not related to adverse selection problems, which were so typical for practice in other countries. As a result credit enhancement did not serve as a signaling tool. Consistently with this I cannot find any significant relationship between credit

[39] See Acharya and Schnabl (2009) for detailed description of the regulatory practice in different countries.

enhancements on the delinquencies on the collateral in Spain.

To conclude, the results of the panel regressions are consistent with the signaling hypothesis as well as the lower efficiency of the signaling for loans issued in a boom period for countries, where securitization was related to a transfer of risk, such as the United Kingdom. However, in countries, such as Spain, where the risk primarily remained on the balance sheet of the originators, no significant relationship between credit enhancement and the quality of loans is found.

Table 1. Panel Regression Results (Dependent variable: Delinquency rate) [a]

Countries	UK, NL, Spain, Italy	UK, NL, Spain, Italy	UK	UK	Spain	Spain
Time period	1998q3-2013q2	1998q3-2007q2	2000q2-2013q2	2000q2-2007q2	1998q3-2013q2	1998q3-2007q2
$CERatio(-1)$	-0.0191	-0.0107	-0.0212	-0.0118	0.0031	0.0058
	[4.40]***	[2.17]**	[3.86]***	[2.41]**	[0.70]	[1.22]
$CERatio(-1)\times$	-0.0039	-0.0061	-0.0033	-0.0052	-0.0014	-0.0015
D_{boom}	[4.65]***	[2.79]***	[1.91]*	[1.68]*	[0.67]	[1.72]*
$CERatio(-1)\times$	0.0115	0.0200	0.0144	0.0301	-0.0034	-0.0047
$D_{origin\,in\,boom}$	[2.31]**	[3.63]***	[2.31]**	[4.83]***	[0.52]	[0.99]
Deal age	0.0032	-0.0027	-0.042	-0.068	0.003	-0.004
	[1.57]	[0.33]	[4.12]***	[5.23]***	[1.47]	[1.88]*
Output gap	0.15	-5.13	omitted[b]	omitted[b]	omitted[b]	omitted[b]
	[0.03]	[0.45]				
Observations	15826	4664	4210	1184	5717	1707
Number of deals	747	399	197	129	227	122
R^2(w/b/o)[c]	0.13/0.19/0.13	0.14/0.08/0.10	0.28/0.04/0.00	0.32/0.00/0.00	0.12/0.01/0.01	0.12/0.04/0.06

[a] Robust t-statistics appear in brackets. Time dummies are not reported. Variables are defined in text. ***/**/* - Statistically significant at 1/5/10 percent level.
[b] Ouput gap for individual country varies only over time, so cannot be included due to time fixed effects.
[c] Reports R^2 within/between/overall.

7 Conclusions

In this paper I show that, in general, reputation concerns allow sponsors of securitized products to signal the quality of securitized loans by providing implicit recourse and thus they limit the problem of private information typical for securitization. However, there are limits to the efficiency of these particular reputation based tools, which become more pronounced in boom stages of the business cycles. The level of sufficiently high implicit recourse that would not be mimicked by firms with investment projects of lower quality exceed the level which can be credibly promised. In the resulting pooling equilibrium, the information about the quality of loans is lost and investment allocation becomes more inefficient. Due to this mechanism, large inefficiencies in the allocation of capital can be accumulated in the boom stage of the business cycle. The accumulated inefficiencies can then amplify a subsequent downturn of the economy. Additionally, the longer the duration of the boom stage of the business cycle the deeper will be the fall of output in a subsequent recession.

The results of this paper also have implications for related macro-prudential policy, which requires higher explicit risk-retention ("skin in the game"). In this model, such requirements restrict the supply of loans and, through the general equilibrium effect, make securitization more profitable. As a result, this regulation lowers both the quantity and the quality (higher likelihood of pooling equilibria) of investment in the economy.

In an extension of the model, I also introduce asymmetric information on the re-sale market for securitized loans. The model predicts amplified adverse selection in a recession, particularly if the recession is preceded by long boom period. If the adverse selection is severe enough, high quality securitized loans are no longer traded at all.

In the empirical section, I test hypotheses from the theoretical model on the level of securitization deals using data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. The effect is smaller and may even be overturned for assets that have been issued in a boom stage of the business cycle. This is in line with higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, however, are not statistically significant for deals issued in Spain. The difference could be explained by significant differences in regulatory framework and practice of securitization.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis, as it describes the experience of securitization markets prior to and during the recent financial crisis. In the period preceding the crisis, many inefficient investments of unknown were undertaken. While this was not problematic as long as the economy was performing well, the large amount of low quality loans in the economy ultimately contributed to the depth of the financial crisis and caused severe strain on the markets for securitized products. The paper also points to some unexpected negative effects of the newly proposed regulation.

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8 Appendix

8.1 Proofs

8.1.1 First best case

Due to logarithmic utility firms always consume $1-\beta$ fraction of their wealth: $c = (1-\beta)\, h\left(r^h + \lambda\right)$. This policy function is linear, so it is trivial to aggregate it across the continuum of firms to obtain the equation describing the evolution of aggregate variables: $C = (1-\beta)\, H\left(r^h + \lambda\right)$.

From the market clearing condition we know that $I = Y - C = Hr^h - C$. And from the law of motion for capital we know that in the steady state $I = (1-\lambda)\, H$. Combining these two conditions we obtain:

$$Hr^h - C = (1-\lambda)\, H.$$

Substituting there for aggregate consumption we get:

$$\begin{aligned} Hr^h - (1-\beta)\, H\left(r^h + \lambda\right) &= (1-\lambda)\, H, \\ r^h + \lambda &= \frac{1}{\beta}. \end{aligned}$$

8.1.2 Proof of Proposition 1

In the first best allocation $q^h = 1$. Should the "skin in the game" be binding the $q^h > 1$. Let's consider the least restrictive case where still only the firm with access to high quality loans is issuing credit and securitizes these loans and the "skin in the game" is not high enough to allow firm with access to low quality investment opportunities to profitably issue loans $q^l < 1$.

Under binding "skin in the game" constraint the aggregate investment into higher quality project will be (obtained as an aggregation of eq. 3.4):

$$I_t^H = \pi\mu \frac{\beta\left(H_t\left(\left(A_t + \Delta^h\right) K_t^{\alpha-1} + \lambda q_t^h\right) + L_t\left(\left(A_t + \Delta^l\right) K_t^{\alpha-1} + \lambda q_t^l\right)\right)}{\left(1 - \theta q_t^h\right)}. \tag{8.1}$$

Prices of particular assets are determined from Euler equations of saving firms. In equilibrium these firms are indifferent between investing in high or low quality projects:

$$E_t\left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\left(\omega_{t+1}\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1-\omega_{t+1})\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}\right)}\right] = 1 \tag{8.2}$$

$$E_t\left[\frac{\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}}{\left(\omega_{t+1}\frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t}+(1-\omega_{t+1})\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}\right)}\right]=1, \tag{8.3}$$

where ω_t is the share of high quality projects in the overall assets in the economy$\omega_t = {}^{H_t}/_{K_t}$. The derivation of these conditions can be found in appendix 8.2.

Finally goods market clearing condition has to hold too:

$$Y_t = C_t + I_t. \tag{8.4}$$

Steady state conditions (8.1, combination of 8.2 and 8.3, 8.4) in the steady state become the following:

$$(1-\lambda)\left(1-\theta q^h\right) = \pi\mu\beta\left(r^h+\lambda q^h\right)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}$$

$$r^h = (1-\lambda) + (1-\beta)\left(r^h+\lambda q^h\right).$$

Combining these equations we can obtain

$$q^h_H = \frac{(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta+\pi\mu\lambda} \tag{8.5}$$

$$K_H = \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta+\pi\mu\lambda}}{\beta A^h}\right]^{\frac{1}{\alpha-1}}.$$

As long as $q^h = 1$, we would obtain $K_H = \left[\frac{1}{A^h}\left(\frac{1}{\beta}-\lambda\right)\right]^{\frac{1}{\alpha-1}}$ which is the first best optimal level of capital (compare with (3.2)). If $(1-\lambda)(1-\pi\mu) > (1-\lambda)\theta+\pi\mu\lambda$ then $q^h > 1$. Deterministic steady state level of capital is then lower than in the first best case:

$$K_H = \left[\frac{(1-\lambda)+(1-\beta)\lambda q^h_H}{\beta A^h}\right]^{\frac{1}{\alpha-1}} < \left[\frac{(1-\lambda)+(1-\beta)\lambda}{\beta A^h}\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

8.1.3 Proof of Proposition 2

Proposition 2 claims that there are three possible types of steady state depending on the parameter values. In the proof of Proposition 1 above I described already the least restricted case where only firm with access to high quality projects will be issuing and securitizing loans. By continuing to tighten the "skin in the game" constraint we will increase the price of low quality asset to 1 ($q^l = 1$). At this point the firms with access to low quality loans will be indifferent between buying high quality securitized assets or issue and securitize their own loans. Credit to low quality projects counterweights the effect of tightening "skin in the game" constraint and therefore the price stay at the same levels ($q^l = 1$, $q^h = A^h/A^l$). For an interval of θ there will be an steady state in which firms with access to low quality investment will play a mixed strategy when giving credit with probability ψ. As θ decreases ("skin in the game" rises), ψ increases all the way up to 1, where a third type of steady state takes place. In this firms with access to both high and low quality projects will be all issuing credit and securitizing always.

Steady state conditions are the following:

$$(1-\lambda)\left(1-\theta q^h\right)\omega = \pi\mu\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right) \tag{8.6}$$

$$(1-\lambda)\left(1-\theta q^l\right)(1-\omega) = \pi(1-\mu)\psi\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right) \tag{8.7}$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l} \tag{8.8}$$

$$q^l = 1 \tag{8.9}$$

$$\omega r^h + (1-\omega) r^l = (1-\lambda) + (1-\beta)\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right). \tag{8.10}$$

Let's define

$$q \equiv \frac{q^h}{A^h} = \frac{q^l}{A^l} \tag{8.11}$$

and

$$D \equiv \omega A^h + (1-\omega) A^l. \tag{8.12}$$

Using (8.11), (8.12) and combining equations (8.6), (8.7) and (8.8):

$$(1-\lambda)(1-\theta q D) = \pi\left(\mu+\varphi(1-\mu)\right)\beta D\left(K^{\alpha-1}+\lambda q\right)$$

$$(1-\lambda) - \pi\left(\mu+\psi(1-\mu)\right)\beta D K^{\alpha-1} = qD\left[(1-\lambda)\theta + \pi\left(\mu+\psi(1-\mu)\right)\beta\lambda\right] \tag{8.13}$$

We can also rewrite (8.10):

$$\beta D K^{\alpha-1} = 1-\lambda+(1-\beta) D\lambda q \tag{8.14}$$

Combining (8.13), (8.14) we get

$$q_M = \frac{(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}\frac{1}{D} \tag{8.15}$$

Substituting (8.15) back into (8.14) we get:

$$K_M = \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D}\right]^{\frac{1}{\alpha-1}} \tag{8.16}$$

Deterministic steady state is defined by:

$$(1-\lambda)\left(1-\theta q^h\right)\omega = \pi\mu\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right) \tag{8.17}$$

$$(1-\lambda)\left(1-\theta q^l\right)(1-\omega) = \pi(1-\mu)\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right) \tag{8.18}$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l} \tag{8.19}$$

$$\omega r^h+(1-\omega) r^l = (1-\lambda)+(1-\beta)\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right). \tag{8.20}$$

Using (8.11), (8.12) and combining equations (8.17), (8.18) and (8.19):

$$(1-\lambda)(1-\theta q D) = \pi\beta D\left(K^{\alpha-1}+\lambda q\right)$$

$$(1-\lambda)-\pi\beta D K^{\alpha-1} = qD\left[(1-\lambda)\theta+\pi\beta\lambda\right] \tag{8.21}$$

We can also rewrite (8.20):

$$\beta D K^{\alpha-1} = 1-\lambda+(1-\beta) D\lambda q \tag{8.22}$$

Combining (8.21), (8.22) we get

$$q_B = \frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}\frac{1}{D} \tag{8.23}$$

Substituting (8.23) back into (8.22) we get:

$$K_B = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}} \tag{8.24}$$

Second part of proposition claims that $K_H > K_M > K_B$.

To show this lets first focus on the in the brackets part of the formulas for capital:

Since in Case 1 $q_H^l < 1$ then $q_H^h < \frac{A^h}{A^l}$. And since $q_M^l = 1$ then $\frac{(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda} = \frac{D_M}{A^l}$. The following inequality then holds

$$\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h} < \frac{(1-\lambda)}{\beta A^h} + (1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\,\lambda\frac{1}{\beta A^l} = \frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M}.$$

This implies that

$$K_H = \left[\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h}\right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M}\right]^{\frac{1}{\alpha-1}} = K_M.$$

Similarly we can show that $K_P > K_B$. Since $w_B < w_P$ then $D_B < D_P$. Also $q_B^l > 1$ then $\frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda} > \frac{D_B}{A^l}$. This implies that

$$\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} = \frac{(1-\lambda)}{\beta D_M}+(1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_B}+(1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B},$$

$$K_M = \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M}\right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B}\right]^{\frac{1}{\alpha-1}} = K_B$$

8.1.4 Proof of Proposition 3

Even when "skin in the game" constraint is not binding enough to influence aggregate quantities and prices, the capital and output levels are lower than in the first best case due to the inefficient allocation of capital. When the "skin in the game" constraint is not binding average gross profit from one unit of invested capital in the economy equals

$$\bar{r} = \mu r^h + (1-\mu) r^l = \frac{1}{\beta} - \lambda.$$

The level of capital K_P is determined by:

$$K_P = \left[\frac{1}{\mu A^h + (1-\mu)A^l}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} < \left[\frac{1}{A^h}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

Suppose $(1-\pi)(1-\lambda) > \pi\lambda + (1-\lambda)\theta$, in which case the "skin in the game" constraint starts to bind in this case of private information. The deterministic steady state conditions then collapse to the two following equations in (K, q):

$$(1-\lambda)(1-\theta q) = \pi\beta\left(\mu r^h + (1-\mu)r^l + \lambda q\right),$$

$$\mu r^h + (1-\mu)r^l = (1-\lambda) + (1-\beta)\left(\mu r^h + (1-\mu)r^l + \lambda q\right),$$

where $q = \mu q^h + (1-\mu)q^l$. From this we can easily derive:

$$q = \frac{(1-\pi)(1-\lambda)}{\pi\lambda + (1-\lambda)\theta} \tag{8.25}$$

$$K = \left[\frac{(1-\lambda) + (1-\beta)\lambda q}{\beta\left(\mu A^h + (1-\mu)A^l\right)}\right]^{\frac{1}{\alpha-1}}.$$

In the proof of Proposition 1 and 2 we already proved that $K_{FB} > K_H > K_M > K_B$. To prove Proposition 3 it suffices to prove that $K_B > K_{private}$, where $K_{private}$ is the level of capital under private information about the allocation of investment opportunities. To obtain $K_B > K_{private}$, we need:

$$K_B^{\alpha-1} < K_{private}^{\alpha-1}$$

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\omega A^h + (1-\omega)A^l\right)} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\mu\Delta A^h + (1-\mu)A^l\right)}$$

$$\omega > \mu.$$

Writing equations (8.17) and (8.18) in a ratio we obtain:

$$\frac{(1-\lambda)\left(1-\theta q^h\right)\omega}{(1-\lambda)\left(1-\theta q^l\right)(1-\omega)} = \frac{\pi\mu\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right)}{\pi(1-\mu)\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right)}.$$

Since $q^h > q^l$ we can obtain:

$$\frac{\omega}{(1-\omega)} = \frac{\left(1-\theta q^l\right)}{\left(1-\theta q^h\right)}\frac{\mu}{(1-\mu)} > \frac{\mu}{(1-\mu)},$$

and this implies that $\omega > \mu$.

8.1.5 Proof of proposition 4

Under private information case, firms with low quality investment opportunities prefer to buy high quality loans rather than to mimic firms with high quality investment opportunities if:

$$\begin{aligned}
R \mid mimicking &< R \mid buying\, high\, loans, \\
\frac{r^l + \lambda q^l}{\frac{1-\theta q^h}{1-\theta}} &< \frac{r^h + \lambda q^h}{q^h}, \\
\frac{(1-\theta)\, q^h}{1-\theta q^h} &< \frac{r^h + \lambda q^h}{r^l + \lambda q^l} = \frac{q^h}{q^l}, \\
q^l &< \frac{1-\theta q^h}{1-\theta}.
\end{aligned}$$

Substituting for q from (8.5) and using $\frac{A^h}{q^h} = \frac{A^l}{q^l}$, we get

$$\frac{A^h}{A^l} > \frac{(1-\pi\mu)(1-\lambda)(1-\theta)}{\pi\mu\lambda + (1-\lambda)\,\theta\pi\mu}.$$

8.1.6 Credibility of the trigger punishment strategy

A necessary condition for the existence of the reputation equilibrium in which implicit recourse is being provided is the credibility of the punishment rule. The saving firm which observes default on the implicit recourse has to be prefer punishing the defaulting firm to non-punishing even ex-post. This condition is expressed in condition (3.12). I will express analytically both elements of that inequality in the case of the separating deterministic steady state, where level of aggregate TFP is constant. In fully stochastic version this can be solved numerically. Following the same steps as in appendix 8.1.9 we can find that the value function of the firm that always punished and therefore has a reputation of being a "tough investor" is:

$$V^P(w) = \frac{\log\left[(1-\beta)\, w\right]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi\mu \log\left(R^{h,IR}\right) + (1-\pi\mu)\log\left(R^s\right)\right),$$

and the value function of the firm that failed to punish and therefore lost reputation of being a "tough investor" is:

$$V^{NP}(w) = \frac{\log\left[(1-\beta)\, w\right]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi\mu \log\left(R^{h,IR}\right) + (1-\pi\mu)\log\left(R^{s,NP}\right)\right).$$

If a firm looses the reputation of being "tough investor", other firms will expect that this firm will never punish in the future and as a consequence they will never provide implicit support to this firm anymore. So when a firm without reputation of "tough investor" buys assets with implicit support issued in the primary market, its return is $R^{s,NP} = \frac{r^h+\lambda q^h}{q^G}$. While firms with reputation of "tough investors" have return $R^{s,NP} = \frac{\hat{r^G}+\lambda q^h}{q^G}$. If firms without reputation of "tough investors" buy assets without implicit recourse on the secondary (re-sale) markets, they are also in a disadvantageous position. When firms with reputation of "tough investors" sell a high quality assets to firms with reputation, they charge a market price q^h. However, if firms without reputation have the outside option only buying on the primary market, they will be willing to buy high quality asset even for the price q^G. The price for which a high quality asset is sold on the secondary market to the firms without reputation is somewhere on the interval $q^{h,NP} \in \left(q^h, q^G\right)$ depending on the bargaining power of sellers and buyers. Unless all the bargaining power is on the side of firms without reputation, then $q^{h,NP} > q^h$. This implies that $R^{s,NP} < R^s$ and therefore saving firms are better of punishing and the equation (3.12) is satisfied.

It is well known that trigger strategies are often not renegotiation-proof. While in this paper I do not address this problem in detail and rule out renegotiation by assumption, it can be shown that for large set of parameter space and relative bargaining power of different agents in the economy renegotiation is not optimal. Therefore, trigger strategy will be robust even in the case when renegotiation is allowed.

Suppose one firm decides to default on the implicit support (which is the case that is relevant for the ICC for non-defaulting, eq. 3.11). Other firms decide whether to punish this firm and face lower returns in the future $R^{s,NP}$ as shown above or whether not punish and negotiate with the defaulted firms better terms, i.e., buy from them the assets for a lower price $q^{h,RN} < q^h$, giving it a return $R^{s,RN} > R^s$. However, those benefits from renegotiation are limited by the fact that the defaulted firm would be selling the assets only with probability $\pi\mu$ and the quantity of assets the firm can sell is limited and proportional to its equity. Even if quantity of the assets sold by the defaulted firm is large enough, renegotiation would not be optimal as long as

$$R^s > \pi\mu R^{s,RN} + (1-\pi\mu)\, R^{s,NP}.$$

This depends on the prices $q^h, q^{h,NP}, q^{h,RN}$, which themselves depend on the relative bargaining power of different agents in the economy.

8.1.7 Proofs of proposition 5

I claimed that if the implicit recourse would be credible, the optimal level of promise would mean $q^j = 1$ and therefore zero profit for securitizing firms. The relevant F.O.C. can be transformed in the following way (Let's consider F.O.C. for firms with high quality investment opportunities. The remaining would not invest at all.):

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial\left(w'-cir'\right)}{\partial r^G} = 0.$$

$$\begin{aligned}
\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^G}\frac{\left(1-\theta\right)\beta w\left(r^{j'}+\lambda q^j\right)-\theta\beta w\left(r^G-r^j\right)}{1-\theta q^G} &= 0\\
\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^G}\frac{\beta w\left(r^{j'}+\lambda q^j-\theta\left(r^{G'}+\lambda q^j\right)\right)}{1-\theta q^G} &= 0.
\end{aligned}$$

After substituting in this case with constant aggregate productivity $q^{G,j} = \frac{r^{G'}+\lambda q^j}{r^{j'}+\lambda q^j}q^j$ this condition implies that

$$\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^G}\frac{\beta w\left(r^{j'}+\lambda q^j\right)\left(1-\theta\frac{q^{G,j}}{q^j}\right)}{1-\theta q^{G,j}} = 0,$$

and since $\frac{\partial V^{ND'}}{\partial(w'-cir')} > 0$, $\frac{\partial q^{G,j}}{\partial r^G} > 0$ the above condition simplifies to

$$\frac{\partial}{\partial q^{G,j}}\frac{\left(1-\theta\frac{q^{G,j}}{q^j}\right)}{1-\theta q^{G,j}} = \frac{\theta\left(q^j-1\right)}{q^j\left(1-\theta q^{G,j}\right)^2} = 0.$$

This implies $q^j = 1$.

Note that for when the level of r^G satisfies this condition, return from investing and securitizing is equal to the return from investing but not securitizing, i.e., securitization does not increase the return:

$$\begin{aligned}
R \mid investing\,\&\,securitizing &= R \mid investing\\
\frac{\left(r^j+\lambda q^j-\theta\left(r^G+\lambda q^j\right)\right)}{1-\theta\frac{r^G+\lambda q^j}{r^j+\lambda q^j}q^j} &= \frac{r^j+\lambda q^j}{1}
\end{aligned}$$

When you substitute in the above condition $q^j = 1$, the condition is exactly satisfied for all parameter values.

8.1.8 Proof of Proposition 8

To complete the proof of Proposition 8 sketched in the main text I first need to derive from eq. 3.13 the eq. 3.14 and show that the RHS of the equation (3.14), i.e., the variables B and q^h are independent of the level of aggregate productivity A.

Under separation steady state conditions are the following:

$$
\begin{aligned}
(1-\lambda)\left(1-\theta q^{h,IR}\right) &= \pi\mu\beta\left(r^h+\lambda q^h\right) && (8.26)\\
r^h &= (1-\lambda)+(1-\beta)\left(r^h+\lambda q^h\right) && (8.27)\\
\frac{r^G+\lambda q^h}{q^G} &= \frac{\left(A+\triangle^h\right)K^{\alpha-1}+\lambda q^h}{q^h} && (8.28)\\
V^{ND}\left(w'-cir'\right) &= V^D\left(w'\right) && (8.29)
\end{aligned}
$$

Using the following property given by the logarithmic utility function:

$$
\begin{aligned}
V(w) &= \log\left((1-\beta)w\right)+\beta\log\left((1-\beta)\beta Rw\right)+\beta^2\log\left((1-\beta)\beta^2R^2w\right)+\beta^3\log\left((1-\beta)\beta^3R^3w\right)\ldots\\
&= \frac{1}{1-\beta}\log(w)+\log\left((1-\beta)\right)+\beta\log\left((1-\beta)\beta R\right)+\beta^2\log\left((1-\beta)\beta^2R^2\right)+\beta^3\log\left((1-\beta)\beta^3R^3\right)\ldots\\
&= \frac{1}{1-\beta}\log(w)+V(1),
\end{aligned}
$$

we can transform the no-default condition expressed in eq. (8.29) in the following way:

$$
\begin{aligned}
V^D\left(w'\right) &= V^D\left(w\beta\frac{(1-\theta)\left(r^h+\lambda q^h\right)}{\left(1-\theta q^G\right)}\right)=V^D(w)+\frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h+\lambda q^h\right)}{\left(1-\theta q^G\right)}\right)\\
V^{ND}\left(w'-cir'\right) &= V^{ND}\left(w\beta\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{\left(1-\theta q^G\right)}\right)\\
&= V^{ND}(w)+\frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{\left(1-\theta q^G\right)}\right)
\end{aligned}
$$

For simplicity lets express value functions separately from the individual wealth in the following way, which is easy to do given the log utility:$V(w)=V(1)+\frac{1}{1-\beta}\log(w)$. And we can find solutions for value functions with wealth normalized to unity which we can denote simply $V=V(1)$.

$$
\begin{aligned}
V^{ND} &= \log(1-\beta)+\beta\left(\pi\mu V^{ND}\left(\beta R^{h,IR}\right)+\pi(1-\mu)V^{ND}\left(\beta R^l\right)+(1-\pi)V^{ND}\left(\beta R^z\right)\right)\\
&= \log(1-\beta)+\beta\left(\frac{\pi\mu\log\left(\beta R^{h,IR}\right)}{1-\beta}+\pi(1-\mu)\frac{\log\left(\beta R^l\right)}{1-\beta}+(1-\pi)\frac{\log\left(\beta R^z\right)}{1-\beta}+V^{ND}\right)\\
&= \frac{\log(1-\beta)}{1-\beta}+\frac{\beta\log(\beta)}{(1-\beta)^2}+\frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,IR}\right)+\pi(1-\mu)\log\left(R^l\right)+(1-\pi)\log\left(R^z\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
V^{D} &= \log(1-\beta)+\beta\left(\pi\mu V^{D}\left(\beta R^{h,D}\right)+\pi(1-\mu)V^{D}\left(\beta R^l\right)+(1-\pi)V^{D}\left(\beta R^z\right)\right)\\
&= \log(1-\beta)+\beta\left(\frac{\pi\mu\log\left(\beta R^{h,D}\right)}{1-\beta}+\pi(1-\mu)\frac{\log\left(\beta R^l\right)}{1-\beta}+(1-\pi)\frac{\log\left(\beta R^z\right)}{1-\beta}+V^{D}\right)\\
&= \frac{\log(1-\beta)}{1-\beta}+\frac{\beta\log(\beta)}{(1-\beta)^2}+\frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,D}\right)+\pi(1-\mu)\log\left(R^l\right)+(1-\pi)\log\left(R^z\right)\right)
\end{aligned}
$$

Substituting the above derived conditions into the no-default condition (eq. 8.29) and canceling the terms equal for both value functions we obtain:

$$\log\left(\beta(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,IR}\right)=\log\left(\beta(1-\theta)\left(r^h+\lambda q^h\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right),$$

where LHS shows the utility from consumption when the wealth is reduced by repayment of implicit recourse and the future discounted benefit of having good reputation. The RHS then shows higher immediate utility from saving on implicit recourse, but the future utility is lower since the firm cannot longer issue and sell new loans. This equation can further be simplified using (8.28) and substituting for the returns:

$$\begin{aligned}
\log\left(\frac{r^h+\lambda q^h-\theta\left(r^G+\lambda q^h\right)}{(1-\theta)\left(r^h+\lambda q^h\right)}\right) &= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,IR}}{R^{h,D}}\right)\\
&= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{(1-\theta q^G)}\frac{1}{\left(r^h+\lambda q^h\right)}\right)\\
&= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{r^h+\lambda q^h-\theta\left(r^G+\lambda q^h\right)}{r^h+\lambda q^h-\theta q^h\left(r^G+\lambda q^h\right)}\right)
\end{aligned}$$

Now for let's denote the price premium for the equilibrium implicit guarantee $B\equiv\frac{q^G}{q^h}=\frac{r^G+\lambda q^h}{r^h+\lambda q^h}$, then we can express the above equation as follows:

$$\log\left(\frac{1-\theta B}{1-\theta}\right) = \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta B q^h}{1-\theta B}\right), \tag{8.30}$$

which is an equation in two unknown endogenous variables $\left(B,q^h\right)$ depending on time preference parameters β and parameters defining the strength of the financing frictions (π,μ,θ).

We can express a second steady state condition in two endogenous variables $\left(B,q^h\right)$ combining two remaining conditions for the steady state (8.26, 8.27):

$$(1-\lambda)\left(1-\theta B q^h\right)=\pi\mu\left(1-\lambda+\lambda q^h\right). \tag{8.31}$$

Combining the two equations (8.30, 8.31) we can obtain the solution to both the price of the high quality asset q^h and the price premium for the equilibrium implicit guarantee B. Crucially the solution does not depend on the level of aggregate productivity A. Which is one step we needed to show to complete the proof of Proposition 8.

When defaulting on the implicit recourse is optimal, which is the case since in a separation equilibrium $r^{G*}>r^G_{l,cred,s}$

Second step is the derivation of equation 3.14 from 3.13. Note that in the separating equilibrium selected by the Intuitive Criterion mimicking forms with access to low quality projects would find optimal to default on implicit recourse since in a separation equilibrium $r^{G*}>r^G_{l,cred,s}$.

Similarly as with the condition 8.29 we can transform the following condition for separation (eq. 3.13):

$$
\begin{aligned}
V^{l}\left(mimicking\,\&\,default\right) &< V^{l}\left(buying\;high\;loans\right)\\
\log\left(\frac{\beta\left(1-\theta\right)\left(r^{l}+\lambda q^{l}\right)}{\left(1-\theta q^{G}\right)}\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right) &< \log\left(\beta\frac{\left(r^{h}+\lambda q^{h}\right)}{q^{h}}\right)+\beta\pi\mu\log\left(R^{h,IR}\right)\\
-\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,IR}}{R^{h,D}}\right) &< \log\left(\frac{\left(1-\theta q^{h,IR}\right)}{\left(r^{l}+\lambda q^{l}\right)\left(1-\theta\right)}\frac{\left(r^{h}+\lambda q^{h}\right)}{q^{h}}\right)\\
\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta Bq^{h}}{1-\theta B}\right) &< \log\left(\frac{\left(1-\theta Bq^{h}\right)}{\left(1-\theta\right)}q^{l}\right)
\end{aligned}
$$

Using the equation 8.30 and the the preceding transformations we can replace LHS to get:

$$
\begin{aligned}
\log\left(\frac{1-\theta B}{1-\theta}\right) &< \log\left(\frac{\left(1-\theta Bq^{h}\right)}{\left(1-\theta\right)}q^{l}\right)\\
q^{l} &< \frac{1-\theta Bq^{h}}{1-\theta B} \qquad (8.32)
\end{aligned}
$$

If we divide eq. 8.32 by q^{h} and substitute ratio of prices by the steady state asset market clearing condition $A^{h}/q^{h}=A^{l}/q^{l}$ then we obtain:

$$
\frac{A^{h}}{A^{l}}>\frac{\left(1-\theta B\right)q^{h}}{1-\theta Bq^{h}}.
$$

Proposition 8 (iii) also claims that the inequality in eq. 3.5 is less likely to be satisfied than in eq. 3.14. To prove that let's first rewrite the denominator of the eq. 3.5 using eq. 8.5, which says:

$$
\left(1-\theta q^{h}\right)\left(1-\lambda\right)=\pi\mu\left(1-\lambda+\lambda q^{h}\right),
$$

to obtain

$$
\frac{A^{h}}{A^{l}}>\frac{\left(1-\theta\right)\left(1-\lambda\right)}{\pi\mu\left(\frac{1-\lambda}{q^{h}}+\lambda\right)}.
$$

Similarly let's rewrite the denominator of eq. 3.14 using eq. 8.31 to obtain:

$$\frac{A^h}{A^l} > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)}.$$

We can show that

$$\frac{1-\lambda}{\pi\mu} = \frac{(1-\theta)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)} \mid no\, implicit\, recourse > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)} \mid implicit\, recourse,$$

because price premium for implicit recourse B is by definition higher than one and $q^h \mid no\, implicit\, recourse > q^h \mid implicit\, recourse$. The latter comes directly form comparison of the eq. 8.5 and eq. 8.31, which when combined give:

$$\frac{1-\lambda+\lambda q^h}{1-\theta q^h} \mid no\, implicit\, recourse = \frac{1-\lambda+\lambda q^h}{1-\theta B q^h} \mid implicit\, recourse.$$

And this can be satisfied only if $q^h \mid no\, implicit\, recourse > q^h \mid implicit\, recourse.$

8.1.9 Other derivations from subchapter 3.4.3

Conditions for the minimum level of implicit recourse needed for separation G_{minsep}:

At G_{minsep}, firms with low quality investments are indifferent between mimicking and separating:

$$\begin{aligned}
V^l \mid mimicking\,\&\, default &= V^l \mid buying\; high\; loans \\
\log\left(\frac{\beta(1-\theta)\left(r^l+\lambda q^l\right)}{(1-\theta q^G)}\right) + \beta\pi\mu\log\left(R^{h,D}\right) &= \log\left(\beta\frac{\left(r^h+\lambda q^h\right)}{q^h}\right) + \beta\pi\mu\log\left(R^{h,IR}\right) \\
-\beta\pi\mu\log\left(\frac{1-\theta B_{min}}{1-\theta}\right) &= \log\left(\frac{\left(1-\theta B_{min} q^h\right)}{(1-\theta)} q^l\right) \qquad (8.33)
\end{aligned}$$

Combining equation (8.33) with the following equilibrium investment condition

$$(1-\lambda)\left(1-\theta B_{min} q^h\right) = \pi\mu\left(1-\lambda+\lambda q^h\right), \qquad (8.34)$$

where $B_{min} \equiv \frac{q^G}{q^h} = \frac{(A+G_{minsep})K^{\alpha-1}+\lambda q^h}{r^h+\lambda q^h}$, we can obtain the $\left\{G_{minsep}, q^h, B_{min}\right\}$.

Conditions for a unique pooling equilibrium:

A necessary condition for firms to have incentives to increase G above $G^l_{cred,p}$ is that it must be considered as profitable to at leas individually deviate above $G^l_{cred,p}$. The following condition should, therefore, be satisfied:

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND}}{\partial R^{h,IR}} \frac{\partial R^{h,IR}}{\partial r^G} > 0$$

Since $\frac{\partial V^{ND}}{\partial R^{h,IR}} > 0$, this becomes:

$$\frac{\partial R^{h,IR}}{\partial r^G} = \frac{\partial}{\partial r^G} \frac{\left(\left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right)\right) + \lambda q^h\right)(1-\theta)}{1 - \theta \frac{\left(\mu r^G + (1-\mu) r^l\right) + \lambda\left(\mu q^h + (1-\mu) q^l\right)}{r^h + \lambda q^h} q^h} > 0$$

Taking the derivative we obtain:

$$\begin{aligned} -\theta K^{\alpha-1} \left(1 - \theta \frac{\left(\mu r^G + (1-\mu) r^l\right) + \lambda\left(\mu q^h + (1-\mu) q^l\right)}{r^h + \lambda q^h} q^h\right) & \\ + \frac{\theta \mu q^h K^{\alpha-1}}{r^h + \lambda q^h} \left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right) + \lambda q^h\right)(1-\theta) & > 0 \end{aligned}$$

$$\begin{aligned} \left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right) + \lambda q^h\right)(1-\theta)\mu q^h &> r^h + \lambda q^h - \theta\left(\mu r^G + (1-\mu) r^l\right) \\ &\quad + \lambda\left(\mu q^h + (1-\mu) q^l\right) q^h \\ \left(\mu q^h - 1\right)\left(r^h + \lambda q^h\right) &> \theta q^h (\mu - 1)\left(r^l + \lambda q^l\right). \end{aligned} \tag{8.35}$$

As long as $\left(\mu q^h - 1\right) > 0$ the condition (8.35) always holds since $\mu < 1$. When $\left(\mu q^h - 1\right) < 0$, then we get

$$\left(r^h + \lambda q^h\right) < \theta \frac{q^h (1-\mu)}{(1 - \mu q^h)} \left(r^l + \lambda q^l\right),$$

which is not satisfied if:

$$\frac{A^h}{A^l} > \theta \frac{q^h (1-\mu)}{(1 - \mu q^h)}$$

or when rewritten:

$$\mu < \frac{1 - \theta q^l}{q^h - \theta q^l}.$$

This implies that share of high quality assets have to be low enough or in a pooling equilibrium the relative difference in TFP has to be large enough.

8.1.10 Endogenizing the "skin in the game"

If we endogenize the "skin in the game" with the moral hazard problem described in chapter 5 we obtain the incentive compatible constraint (5.1). In this subchapter I would like to show briefly that the main results concerning the provision of implicit recourse and the endogenous switching between the pooling and separating equilibrium hold.

First, we have to check whether firms have incentive to provide implicit support. The check is equivalent to the proof of Proposition 5 as discussed in chapter 8.1.7 and which boils down to showing that

$$\frac{\partial}{\partial q^{G,j}} \frac{\left(1-\theta\frac{q^{G,j}}{q^j}\right)}{1-\theta q^{G,j}} = \frac{\left(q^j-1\right)}{q^j\left(1-\theta q^{G,j}\right)^2}\frac{\partial \theta q^{G,j}}{\partial q^{G,j}} \geq 0.$$

Since $\frac{\partial \theta q^{G,j}}{\partial q^{G,j}} = \frac{\partial}{\partial q^{G,j}}\frac{q^{G,j}}{q^{G,j}+1} = \frac{1}{\left(q^{G,j}+1\right)^2} > 0$, the above condition corresponds again to $q^j \geq 1$. This means that in equilibrium implicit recourse will be provided.

Given 5.1 the separating equilibrium in the deterministic steady state is defined by:

$$\log\left(\frac{1-\theta B}{1-\theta}\right) = \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta q^h B}{1-\theta B}\right), \tag{8.36}$$

$$\begin{aligned}
\left(1-\lambda\right)\left(1-\theta B q^h\right) &= \pi\mu\left(1-\lambda+\lambda q^h\right) \\
\log\left(\frac{1-\theta B}{1-\theta}\right) &= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta q^h B}{1-\theta B}\right) \\
\theta &= \frac{1}{Bq^h+1}.
\end{aligned}$$

Which simplifies to two equations in which are independent on the level of TFP A:

$$\begin{aligned}
\left(1-\lambda\right)\left(\frac{1}{Bq^h+1}\right) &= \pi\mu\left(1-\lambda+\lambda q^h\right) \\
\log\left(\frac{B\left(q^h-1\right)+1}{Bq^h}\right) &= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1}{B\left(q^h-1\right)+1}\right).
\end{aligned}$$

The conditions for the existence of a separating equilibrium (3.14) becomes:

$$\frac{A^h}{A^l} > q^h\left(B\left(q^h-1\right)+1\right).$$

8.1.11 Adverse selection on re-sale markets

We derive the pricing conditions from the F.O.C. of saving firms. In the case of a separating equilibrium they are the following. The value of a high quality asset q_t^h

reflects the expected gross profit next period and the value of the asset next period which is q^h_{t+1} is the firm has no investment opportunities and keeps the asset on the balance sheet or q^s_{t+1} if the firms has and investment opportunity and sells the asset:

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}}{q^h_t}\right]=1.$$

The value of the low quality asset reflects the expected next period gross profits and the expected next period resale price since low assets are always sold on the re-sale market.

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^l_{t+1}+\lambda q^s_{t+1}}{q^l_t}\right]=1.$$

The price of the newly issued asset with implicit support in a separating equilibrium and the price of an asset sold on re-sale market satisfy the following:

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^G_{t+1}+\lambda f^h_t\left(\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}\right)}{q^G_t}\right]=1,$$

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{f^h_t r^h_{t+1}+\left(1-f^h_t\right)r^l_{t+1}+\lambda f^h_t\left(\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}\right)+\lambda\left(1-f^h_t\right)q^s_{t+1}}{q^s_t}\right]=1,$$

where

$$\begin{aligned}\Xi_{t+1} &= I_t\frac{r^G_{t+1}+\lambda q^s_{t+1}}{q^G_t}+\lambda K_t[(\pi\mu+\left(1-\pi\mu\right)\left(1-\omega_t\right))\,\frac{f^h_t r^h_{t+1}+\left(1-f^h_t\right)r^l_{t+1}+\lambda q^s_{t+1}}{q^s_t}\\ &\quad+\left(1-\pi\mu\right)\omega_t\frac{r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}}{q^h_t}].\end{aligned}$$

Also note that $q^s_t=f^h_t q^h_t+\left(1-f^h_t\right)q^l_t$.

For investing firms to prefer keeping their high quality loans to selling them and investing such obtained liquidity the following condition has to be satisfied in the deterministic steady state:

$$R^h>q^s\frac{R^h-\theta R^G}{1-\theta q^G},$$

where $R^h=r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}$ and $R^h=r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}$. This can be transformed as follows:

$$
\begin{aligned}
R^h - \theta q^h R^G &> q^s R^h - \theta q^s R^G \\
R^h \left(1 - q^s\right) &> \theta R^G \left(q^h - q^s\right).
\end{aligned}
$$

Substituting $q^s = f^h q^h + \left(1 - f^h\right) q^l$ and $B = {}^{R^G}/_{R^h}$ we get

$$
\begin{aligned}
1 - f^h q^h - \left(1 - f^h\right) q^l &> \theta B \left(1 - f^h\right) \left(q^h - q^l\right) \\
\frac{1 - f^h q^h}{1 - f^h} &> \theta B q^h + \left(1 - \theta B\right) q^l \\
f^h \left(q^l - q^h\right) \left(1 - \theta B\right) &> \theta B q^h - 1 + \left(1 - \theta B\right) q^l \\
f^h &< 1 - \frac{q^h - 1}{\left(q^h - q^l\right)\left(1 - \theta B\right)}.
\end{aligned}
$$

8.2 Derivation of firms' policy functions

In this chapter I will derive in detail the policy functions of firms in the most general case. It is convenient to rewrite the firm's problem characterized in subchapter 3.1.3 in a recursive formulation:

$$
\begin{aligned}
V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) &= \pi\left(\mu V^{ND,h}\left(\bar{s}, w - cir; \bar{S}\right) + \left(1 - \mu\right) V^{ND,l}\left(\bar{s}, w - cir; \bar{S}\right)\right) \\
&\quad + \left(1 - \pi\right) V^{ND,z}\left(\bar{s}, w - cir; \bar{S}\right), \\
V^{D}\left(\bar{s}, w; \bar{S}\right) &= \pi\left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + \left(1 - \mu\right) V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + \left(1 - \pi\right) V^{D,z}\left(\bar{s}, w; \bar{S}\right), \\
V^{ND,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c, i, \left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}, r^{G\prime}} \left[\log\left(c\right) + \beta E\left[\max\left(V^{ND}\left(\bar{s}', w' - cir'; \bar{S}'\right), V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right)\right]\right], \\
V^{D,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c, i, h', l'} \left[\log\left(c\right) + \beta E V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right],
\end{aligned}
$$

subject to the budget constraints which take the following form for investing firms for which "skin in the game" constraint is binding:

$$
c_{i,t} + \frac{\left(1 - \theta q^G_{i,t}\right)}{\left(1 - \theta\right)} h_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,
$$

$$
c_{i,t} + \frac{\left(1 - \theta q^G_{i,t}\right)}{\left(1 - \theta\right)} l_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.
$$

The incentive compatible constraints, which has to be satisfied in equilibrium for the existence of reputation based implicit recourse are the following:

$$
\begin{aligned}
V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) &> V^{D}\left(\bar{s}, w; \bar{S}\right),\\
V^{P}\left(\bar{s}; \bar{S}\right) &> V^{NP}\left(\bar{s}; \bar{S}\right),
\end{aligned}
$$

where $V^{ND}, V^{D}, V^{P}, V^{NP}$ are the value functions if firm, never defaulted, when firm defaulted, when firm always punished a default on a promise on gross profits and when firm failed to punished respectively.

From first order conditions we can obtain the following Euler equations in case when the “skin in the game” is binding for all investing firms:

$$
\begin{aligned}
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^{\hat{G}}+\lambda q_{j,t+1}}{q_{j,t}^{G}}\right] &= 1\ \forall i\in\mathcal{S}_t, \forall j\in\mathcal{I}_t, && (8.37)\\
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^{h}+\lambda q_{t+1}^{h}}{q_{t}^{h}}\right] &= 1\ \forall i\in\mathcal{S}_t, && (8.38)\\
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^{l}+\lambda q_{t+1}^{l}}{q_{t}^{l}}\right] &= 1\ \forall i\in\mathcal{S}_t, && (8.39)\\
E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^{h}+\lambda q_{t+1}^{h}}{\frac{\left(1-\theta q_t^{G}\right)}{(1-\theta)}}\right] &= 1\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t, && (8.40)\\
E_t\left[\beta\frac{c_{t}^{l}}{c_{t+1}^{l}}\frac{r_{t+1}^{l}+\lambda q_{t+1}^{l}}{\frac{\left(1-\theta q_t^{G,l}\right)}{(1-\theta)}}\right] &= 1\ \forall i\in\mathcal{L}_t\cap\mathcal{I}_t. && (8.41)
\end{aligned}
$$

I guess and verify that all investing firms provide the same level of implicit support $r_{j,t+1}^{G} = r_{t+1}^{G}\ \forall j\in\mathcal{I}_t$ (see discussion in chapter 3.3. for details). Then I guess and verify that policy functions have the following form.

Due to the logarithmic utility function all firms consume $(1-\beta)$ fraction of their wealth:

$$
c_{i,t} = (1-\beta)\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}}+\lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^{h}+\lambda q_t^{h}\right) + l_{i,t}^{S}\left(r_t^{l}+\lambda q_t^{l}\right)\right)\ \forall i.
$$

Under binding "skin in the game", firms with access to high quality investment opportunities $\mathcal{H}_t$ invest all non-consumed part of wealth into new project and sell the maximum fraction of investment θ to saving firms:

$$h_{i,t+1} = a_{i,i,t+1} = \frac{\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{\frac{\left(1-\theta q_{i,t}^{G}\right)}{(1-\theta)}} \; \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,$$

$$l_{i,t+1} = 0 \; \forall i \in \mathcal{H}_t \cap \mathcal{I}_t.$$

In the pooling equilibrium firms with access to low quality investment opportunities $\mathcal{L}_t$ also invest all non-consumed part of wealth into new project and if the "skin in the game" constraint is binding they sell the maximum fraction of investment θ to saving firms:

$$l_{i,t+1} = a_{i,i,t+1} = \frac{\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{\frac{\left(1-\theta q_{i,t}^{G}\right)}{(1-\theta)}} 1 \; \forall i \in \mathcal{L}_t \cap \mathcal{I}_t,$$

$$h_{i,t+1} = 0 \; \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.$$

If the economy is in a separating equilibrium the intersection $\mathcal{L}_t \cap \mathcal{I}_t = \emptyset$ is an empty set and firms with access to low quality investment opportunities $\mathcal{L}_t$ are not investing into new projects, but rather buy securitized assets from other firms $\mathcal{L}_t \subset \mathcal{S}_t$.

Saving firms $\mathcal{S}_t$ are in equilibrium indifferent between investing into different types of assets. All of them try to diversify their investment, so I guess and verify that in equilibrium all will allocate the same fraction of wealth into different types of assets:

$$h_{i,t+1}^{S} = \frac{\zeta^{hS}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^h} \; \forall i \in \mathcal{S}_t,$$

$$l_{i,t+1}^{S} = \frac{\zeta^{lS}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^t} \; \forall i \in \mathcal{S}_t,$$

$$
\begin{aligned}
h_{i,t+1}^{P} &= \sum_{j\in\mathcal{H}_t\cap\mathcal{I}_t} a_{i,j,t+1} \\
&= \frac{\zeta^{hP}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}}+\lambda q_{j,t}\right)+h_{i,t}^{S}\left(r_t^h+\lambda q_t^h\right)+l_{i,t}^{S}\left(r_t^l+\lambda q_t^l\right)\right)}{q_t^G}\ \forall i\in\mathcal{S}_t,
\end{aligned}
$$

$$
\begin{aligned}
l_{t+1}^{P} &= \sum_{j\in\mathcal{L}_t\cap\mathcal{I}_t} a_{i,j,t+1} \\
&= \frac{\zeta^{lP}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}}+\lambda q_{j,t}\right)+h_{i,t}^{S}\left(r_t^h+\lambda q_t^h\right)+l_{i,t}^{S}\left(r_t^l+\lambda q_t^l\right)\right)}{q_t^G}\ \forall i\in\mathcal{S}_t,
\end{aligned}
$$

where $\zeta^{hS}+\zeta^{lS}+\zeta^{hP}+\zeta^{lP}=1$.

The consumption of the firms in the following period depends on the return from their investment:

$$
\begin{aligned}
c_{i,t+1} &= (1-\beta)\,[h_{i,t+1}^{S}\left(r_{t+1}^h+\lambda q_{t+1}^h\right)+l_{i,t+1}^{S}\left(r_{t+1}^l+\lambda q_{t+1}^l\right) \\
&\quad +h_{i,t+1}^{P}\left(r_{t+1}^{\hat{G},h}+\lambda q_{t+1}^h\right)+l_{t+1}^{P}\left(r_{t+1}^{\hat{G},l}+\lambda q_{t+1}^l\right)]\ \forall i\in\mathcal{S}_t, \\
c_{i,t+1} &= (1-\beta)\left(h_{i,t+1}\left(r_{t+1}^h+\lambda q_{t+1}^h\right)\right)\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t, \\
c_{i,t+1} &= (1-\beta)\left(l_{i,t+1}\left(r_{t+1}^l+\lambda q_{t+1}^l\right)\right)\ \forall i\in\mathcal{L}_t\cap\mathcal{I}_t.
\end{aligned}
$$

Using these guesses and substituting in equations (8.40) and (8.41) we can see that these conditions always hold.

The remaining Euler equations (8.38), (8.39) and (8.37) after substitutions can be rewritten into:

$$
E_t\left[\frac{\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}}{\Xi_{t+1}}\right]=1
$$

$$
E_t\left[\frac{\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}}{\Xi_{t+1}}\right]=1.
$$

$$
E_t\left[\frac{\frac{r_{t+1}^{\hat{G}}+\lambda q_{t+1}}{q^G}}{\Xi_{t+1}}\right]=1,
$$

where $\Xi_{t+1} = \zeta^{hS} \frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t} + \zeta^{lS} \frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t} + \zeta^{hP} \frac{r^{\hat{G},h}_{t+1}+\lambda q^h_{t+1}}{q^G_t} + \zeta^{lP} \frac{r^{\hat{G},l}_{t+1}+\lambda q^l_{t+1}}{q^G_t}$.

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both primary and secondary markets for high and low projects.

$$\begin{aligned}
\lambda H_t &= \zeta^{hS}\beta \sum_{i\in\mathcal{S}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right) \\
\lambda L_t &= \zeta^{lS}\beta \sum_{i\in\mathcal{S}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right)
\end{aligned}$$

$$\begin{aligned}
\theta \quad & \frac{\beta \sum_{i\in\mathcal{H}_t\cap\mathcal{I}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right)}{\left(1-\theta q^G_t\right)} \\
&= \frac{\zeta^{hP} \sum_{i\in\mathcal{S}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right)}{q^G_t}
\end{aligned}$$

$$\begin{aligned}
\theta \quad & \frac{\beta \sum_{i\in\mathcal{L}_t\cap\mathcal{I}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right)}{\left(1-\theta q^G_t\right)} \\
&= \frac{\zeta^{lP} \sum_{i\in\mathcal{S}_t} \left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t} \left(r^{\hat{G}}_{j,t} + \lambda q_{j,t} \right) + h^S_{i,t} \left(r^h_t + \lambda q^h_t \right) + l^S_{i,t} \left(r^l_t + \lambda q^l_t \right) \right)}{q^G_t}
\end{aligned}$$

And the goods market clears too $Y_t = C_t + I_t$.

8.3 Calibration of the parameters used in chapter 4

In chapter 4 I explain the choice of most of the model parameters. Here I would like to specifically comments on the choice of the share of high quality investment opportunities μ and the dispersion of the type specific component of the high and low quality projects in the two states $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right)$, $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right)$.

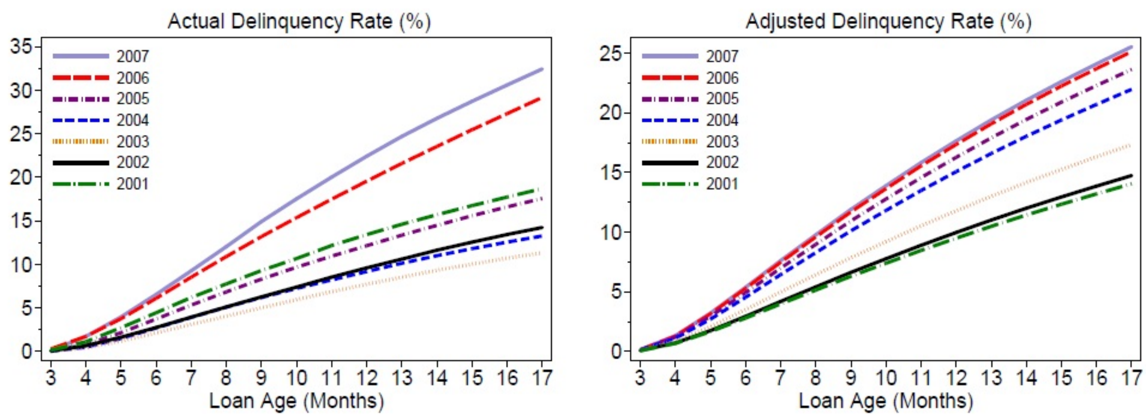
I choose these parameters to replicate the performance (delinquency rates) of securitized assets which has been at the core of the recent debates about the efficiency of securitization - subprime residential mortgage backed securities. Demyanyk and Hemert (2011) study the delinquency rates of the subprime mortgage loans. In figure 8.1, which is taken from Demyanyk and Hemert (2011), they report the actual delinquency rates of these loans in the left panel and in the right panel the delinquency rates adjusted by the effect of various observable characteristics of the loans and the economy. They conclude that the quality of the loans measured by the adjusted delinquency rates has deteriorated significantly since 2004. This finding is consistent with the switching mechanism presented in this paper. As you can see on the left panel of the figure 8.2 the US emerged from a recession in 2003 and in 2004 the output reached again the potential. The model predicts that as the economy moves to the boom stage of the business cycle the equilibrium in the signaling game becomes pooling and as a consequence low quality loans start to be financed. As shown on the right panel of the figure 8.1 boom period of 2004-2007 are associated with lower quality loans and economic downturn of 2001-2003 is associated with higher quality loans.

I used the reported delinquency rates by Demyanyk and Hemert (2011) to calibrate the model parameters.[40] I particular I want to match the delinquency rate of high quality loans after 12 months in the low state to the delinquency of the 2001 vintage which is 12.5%, the delinquency rate of high quality loans after 12 months in the high state to the average of the delinquency of the 2002 and 2003 vintage which is approx. 7%, the delinquency rate of the mix of high and low quality loans after 12 months in the high state to the delinquency of the 2005 vintage which is 9.5% and the delinquency rate of the mix of high and low quality loans after 12 months in the low state to the delinquency of the 2007 vintage which is 22.5%. This gives me: $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right) = 0.94$ and $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right) = 0.71$.

Calibration of the share of high quality investment opportunities μ is more complicated since I do not have disaggregated data for the USA. However, assuming that the growth in the volume of subprime mortgage loans between 2003 and 2004 is driven mainly by the entry of the firms with access to low quality loans to the market we would obtain $\mu = 0.6$. Since this estimate is rather rough, I use the loan level data from the Moody's PDS database for the UK, which according to the empirical analysis in chapter 6 seems to be in line with the model predictions. When you compare the delinquency rates of the collateral of the RMBS in the period with lowest output gap, i.e., in period 2009q3 for loans issued in previous boom stage of the business cycle, i.e., in 2005q3-2008q1 (left panel) with those of loans issued in previous recession, i.e., in periods 2001Q3-2003Q2 and 2004Q3-2005Q2, you can find a significant difference. In particular, it seems that you can distinguish in the subset of RMBS issued in the boom period relatively clear cut two groups. One has very low delinquency rates (below 4%) and other has sometimes much higher delinquency rates. When I use the thresh-

[40] The model presented in this paper does not model loans repayments explicitly. If I assume that delinquent fraction of loans/projects do not generate cash-flows in the current period, then I can compute the ratio of gross profits of the two types of projects.

Figure 8.1. Actual and adjusted delinquency rated for subprime mortgages by Demyanyk and Hemert (2011)



Note: Demyanyk and Hemert (2011) p.1 describe their figure: "The figure shows the age pattern in the actual (left panel) and adjusted (right panel) delinquency rate for the different vintage years. The delinquency rate is defined as the cumulative fraction of loans that were past due 60 or more days, in foreclosure, real-estate owned, or defaulted, at or before a given age. The adjusted delinquency rate is obtained by adjusting the actual rate for year-by-year variation in FICO scores, loan-to-value ratios, debt-to-income ratios, missing debt-to-income ratio dummies, cash-out refinancing dummies, owner- occupation dummies, documentation levels, percentage of loans with prepayment penalties, mortgage rates, margins, composition of mortgage contract types, origination amounts, MSA house price appreciation since origination, change in state unemployment rate since origination, and neighborhood median income."

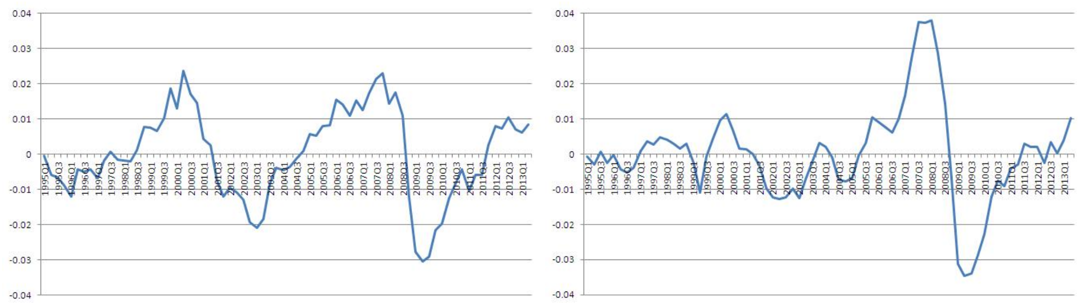
old delinquency rate of 4% to identify high and low quality assets and combine the reported frequency with volumes, I find the share of high quality investment opportunities $\mu = 0.63$. This is approximately consistent with my initial guess for the subprime mortgage loans in the USA, so I use this level of the parameter.

8.4 Numerical solutions of the fully stochastic dynamic model

To solve the fully stochastic dynamic model I use global numerical approximation methods. Since depending on the state variables the economy is switching between separating and pooling equilibrium I am using global approximation methods. In particular I look for the values of the following functions:

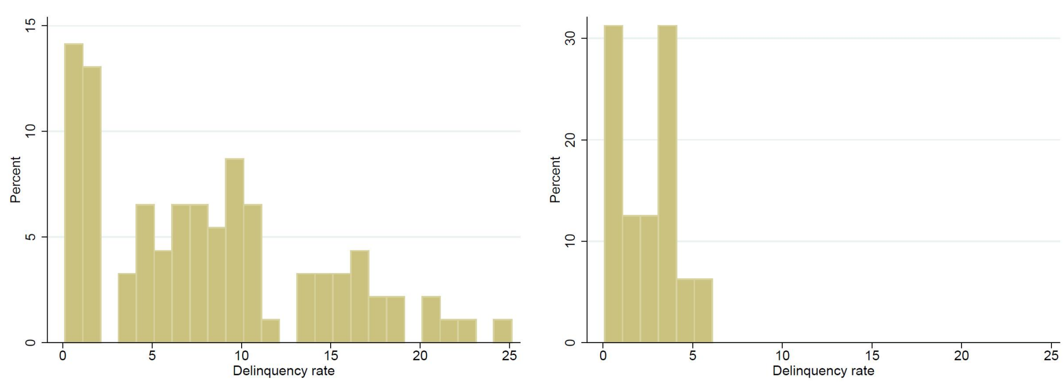
$$\begin{aligned} q_t^h &= \Gamma_1 (A_t, K_t, \omega_t) \\ q_t^l &= \Gamma_2 (A_t, K_t, \omega_t) \\ V^{ND} - V^D &= \Gamma_3 (A_t, K_t, \omega_t) \end{aligned}$$

Figure 8.2. Log of output gap in the USA (left panel) and the UK (right panel)



Note: Data is from Eurostat for the UK and from FRED (St.Louis FED) for the USA. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

Figure 8.3. Histograms of delinquency rate for collateral of the RMBS issued in the UK in 2009q3 for loans issued in the boom (left panel) and loans issued in bust (right panel)



Note: Figure shows histograms of the delinquency rate of the collateral for the RMBS, which is defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance (in %). The source of the data is the PDS database by Moody's. The left panel shows the delinquency rate for the subset of RMBS issued in the boom periods 2005q3-2008q1 and the right panel RMBS issued in recessions in periods 2001Q3-2003Q2 and 2004Q3-2005Q2.

I construct a grid for the three aggregate states A, K, ω and start with the guess equal to the steady-state values. Then I iterate using the set of equilibrium conditions to find the updated values of $(\Gamma_1, \Gamma_2, \Gamma_3)$ until the updated values are close to the previous guesses:

$$\begin{aligned} & \mid q_t^h\,(iter) - q_t^h\,(iter-1) \mid + \mid q_t^l\,(iter) - q_t^l\,(iter-1) \mid \\ & + \mid V^{ND}\,(iter) - V^{ND,h}\,(iter-1) \mid + \mid V^D\,(iter) - V^D\,(iter-1) \mid \quad < \quad \varepsilon. \end{aligned}$$

During iteration at each point of the grid it is evaluated whether the economy is in separating or pooling equilibrium. The values of $(\Gamma_1, \Gamma_2, \Gamma_3)$ out of grid are obtained by trilinear interpolation.