

# Mortgage Amortization and Welfare

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## Abstract

In the pre-crisis period the U.S. housing market experiences a huge increase in mortgage lending due – at least partially – to the government housing policies aimed at expanding mortgage credit to low-income households. Such an expansion was made possible by the introduction of nonstandard mortgage products characterized by low down payments and reduced (initial) repayments. In this paper, we build a dynamic stochastic general equilibrium model with housing and endogenous default on mortgages to study the welfare effects of alternative mortgage contracts. We find that nonstandard mortgage contracts with low down payments and reduced amortization make credit-constrained borrowers *worse off* and unconstrained households (savers) *better off* at least in the long run. Our results can be explained *only* in the light of general equilibrium effects. When mortgage amortization rates and down payments are lower, housing demand is higher as well as housing prices. To afford housing at the higher prices borrowers must cut their non-durable consumption and work more. Therefore they are worse off. Our results contribute to the recent debate on the reforms of the mortgage market.

Keywords: Housing; Mortgage default; Mortgage Amortization

JEL Codes: E32, E44, G01, R31

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# 1 Introduction

Starting in the mid 1990s, the U.S. Department of Housing and Urban Development (HUD) was required to formulate affordable housing goals for the Government Sponsored Enterprises (GSEs) and monitor their performance in achieving such goals. The GSEs affordable housing goals of 1993-2008 set by HUD focused on low- and moderate-income borrowers as well as very-low income borrowers living in low-income areas, requiring a substantial increase in the share of mortgage purchases by GSEs in these categories.<sup>1</sup> Indeed home ownership, which had been stable for almost thirty years, boomed during the period 1994 to 2008, going from 64% in 1994 to 69% in 2004. Originations increased sharply over the period 2000 to 2006. Outstanding home mortgage liabilities of households and nonprofit organizations increased by 3.9 trillion USDs over that period, going from 46% to 73% of GDP.

Such large increase in mortgage credit was achieved by reaching income groups that had not been able to take mortgages before. Using micro-level data Mian and Sufi (2009) document that subprime areas experienced rapid growth in mortgage credit from 2002 to 2005 despite a decline in relative or even absolute income growth. At the same time, the expansion in mortgage credit to subprime areas is closely correlated with the increase in securitization of subprime mortgages. To expand mortgage credit to these income groups, financial institutions started offering nonstandard mortgage products. Bernanke (2010) points to several changes in the methods of housing finance in the run up to the financial crisis and in particular to the appearance of alternative mortgage products such as interest-only Adjustable Rate Mortgages (ARMs), 40-year balloon ARMs, negative amortization ARMs, and pay-option ARMs. These nonstandard mortgage products share a feature: the reduction in the monthly payments, especially the initial ones, relative to conventional Fixed-Rate Mortgage (FRM) contracts. Bernanke (2010) suggests that initial monthly payments could be as low as 14% of a comparable fixed-rate mortgage payment for a negative amortization ARM and even lower for a pay-option ARM. The percentage of ARMs originated with various nonstandard features increased rapidly during the U.S. housing boom. Figure 1 shows the percentage of nonstandard mortgage products (including interest-only ARMs, pay-option ARMs and 40-year balloon mortgages) as a percentage of total originations from 2004Q4 to 2010Q4. The incidence of nonstandard mortgages among

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<sup>1</sup>See Weicher (2010) for a detailed analysis.

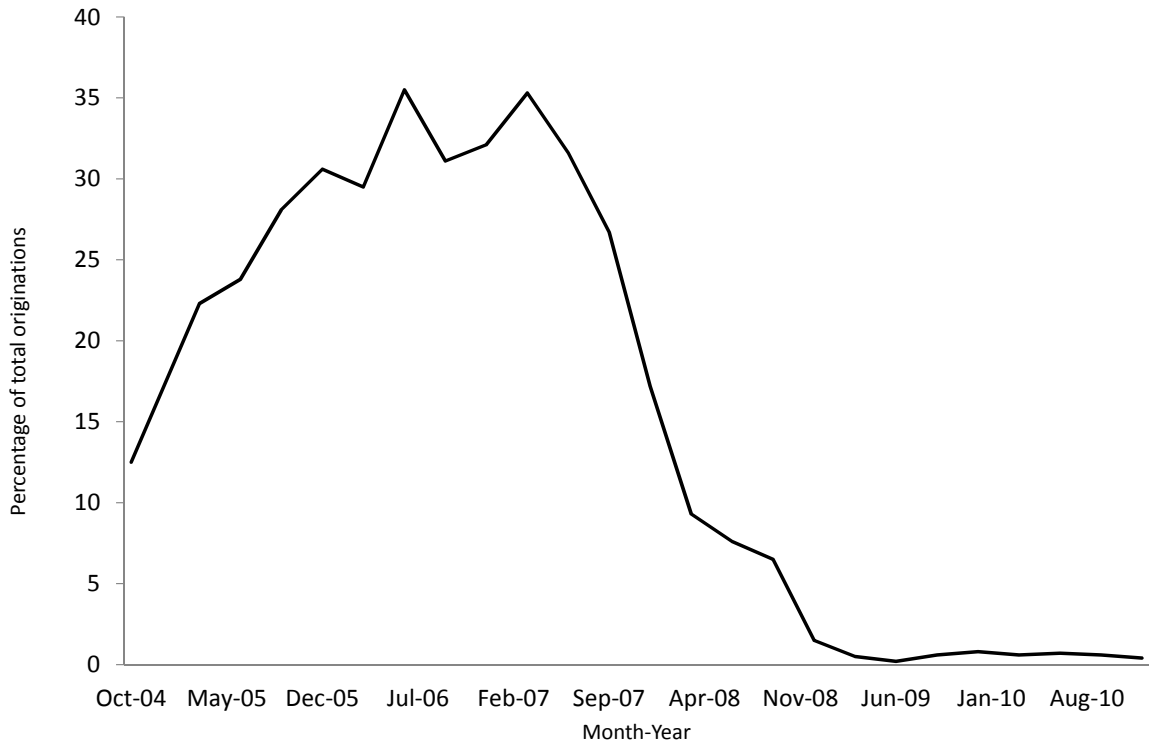


Figure 1: U.S. Alternative Mortgage Share. Total alternative mortgage divided by originations. Source: Inside Mortgage Finance Publications

total originations tripled in just nine quarters, going from 12.5% in 2004Q4 to 35.3% in 2007Q1. Another feature shared by most nonstandard mortgage products that became popular during the housing boom was higher-than-standard Loan-to-Value (LTV) ratios. Over the period 1996 to 2008, the percentage of GSEs' home purchase volume with a combined LTV above 97% went from 5 to almost 40 percentage points.<sup>2</sup>

The mechanism is simple: by offering mortgages that required lower-than-standard monthly payments and down-payments financial institutions could expand the mortgage credit to lower-income households. The maintained assumption behind these government housing policies, we believe, was that these income groups would *benefit* from expanding home ownership. In the finance literature, some recent contributions have highlighted the benefits of nonstandard mortgage products. For example, Cocco (2012) argues that alternative mortgage contracts have been a valuable tool for households who expect higher future income but wish to transfer

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<sup>2</sup>See Pinto (2011).

resources to the present; Piskorski and Tchistyi (2010) and Lacour-Little and Yang (2008) stress that deferred amortization is optimal for households facing income risk or when house prices are expected to go up. Then according to these papers nonstandard mortgage contracts should have beneficial effects for borrowers for a quite intuitive reason: these financial instruments allow credit-constrained households to better smooth consumption in response to shocks and over time.

By contrast, in this paper we show that nonstandard mortgages characterized by reduced and deferred amortization and lower initial payments unequivocally make borrowers *worse off*. In our model borrowers are credit constrained and cannot properly smooth consumption. Therefore we could expect that allowing for nonstandard mortgages improves borrowers' welfare even in our set up. Instead we find that the opposite is true. The reason behind our counter-intuitive results is general equilibrium. When mortgage contracts entail a reduced amortization (initial or constant) the demand for housing and loans by borrowing-constrained households is higher. In fact, this is exactly what the government housing policies aimed to achieve by expanding mortgage credit. But higher housing demand implies higher house prices. To afford housing at higher prices, borrowing-constrained households must reduce their non-durable consumption and work more, which reduces their welfare. Perhaps surprisingly, non-borrowing-constrained households, which we label savers, are better off because they experience a positive, significant income effect.

We build a two-sector DSGE model with housing. There are two households that differ in terms of their discount factor. Savers have a higher discount factor and lend to Borrowers, who have a lower discount factor. Household preferences are defined over non-durable consumption, housing, and hours worked. Borrowers pledge their homes as collateral for mortgages. We model the structure of mortgage repayments so as to encompass different amortization schedules. Our model features one- and two- period mortgage contracts, infinite-period mortgage contracts with a constant amortization rate, mortgages with a very aggressive amortization and mortgages with deferred amortization. We assume that loan contracts are non-recourse in our model, as it is the case in many U.S. states. This means that lender's recovery in case of default is strictly limited to the collateral. Every period Borrowers experience an idiosyncratic housing shock that is private information. Borrowers that experience low realizations of the idiosyncratic shock default on their debts; non-defaulting Borrowers pay an adjustable rate on their mortgages.

Savers pay a monitoring cost and seize the houses of defaulting Borrowers. The spread between the adjustable mortgage rate and the rate on risk-free loans is the external finance premium paid by Borrowers.

To make our point clear, we focus first on the steady state and we analyze the effects of changes in the rates of amortization, which we take as given. Lower monthly repayments and deferred amortization *endogenously* increase the LTV ratio in our model. Borrowing-constrained households demand larger mortgages and become more leveraged. Because home equity is now lower for these agents, they are more likely to experience underwater mortgages, i.e. mortgages where the value of the house is below its liability, and default. This raises the adjustable mortgage rate and the external finance premium. Increased housing demand raises house prices and residential investment. Output increases also in the non-durable sector thanks to higher demand by Savers. The welfare of Borrowers falls as amortization is reduced. Intuitively, borrowing-constrained agents are adversely affected by the increase in housing prices stemming from their additional borrowing. Savers, on the other hand, are better off. Social welfare defined as the appropriately weighted average of Borrowers' and Savers' welfare does not change much with changes in the amortization schedule. Hence, the welfare effects are mainly redistributive.

In the second part of the paper we let Borrowers choose optimally the amortization rate. We show that borrowing-constrained agents always choose the lowest feasible mortgage repayment. Why would Borrowers choose an amortization schedule that makes them worse off? The answer lies once again with general equilibrium. Borrowing-constrained households take house prices as given when they choose the amortization schedule and thereby fail to internalize the adverse effect of higher mortgages on housing prices and their own welfare.

The results on welfare discussed so far abstract from the presence of shocks. In the last part of the paper we allow for shocks and compare the unconditional welfare costs of alternative amortization schedules. Our results confirm the steady-state results obtained in the earlier sections of the paper.

Finally we show that the general equilibrium effect highlighted above is not specific to our model, but it arises in a wide class of models featuring borrowing-constrained agents. To this purpose we consider an exogenous LTV ratio as in Iacoviello (2005) and show that the same mechanism holds in this simpler setting. This means that the mechanism at play is not driven

by the presence of endogenous default and LTV ratios but rather by general equilibrium effects.

The result in our paper has important policy implications. It shows that policies designed to expand mortgage credit of borrowing-constrained households will end up, at least in the long run, making these very same agents worse off.<sup>3</sup> This argument is independent of the fact that nonstandard mortgage products have been identified by many as the main source of the recent financial crisis. In this light, recent proposals aiming at reducing the maximum LTV ratio admissible for mortgages and the maximum length of the amortization period can be possibly interpreted as a move in right direction from the perspective of mortgage holders.

The rest of the paper is organized as follows. Section 2 describes the mortgage contract and the problem solved by borrowing-constrained agents; for simplicity, the rest of the model is relegated in Appendix. Section 3 presents the calibration and section 3 derives the steady-state results. The analysis with shocks is carried out in section 5 and section 6 concludes.

## 2 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing services and work. There is idiosyncratic housing risk so that endogenous default on mortgages arises in equilibrium, as in Forlati and Lambertini (2011). Here we introduce infinite-period mortgages and allow for different amortization schedules. Households supply a continuum of labor services that are imperfect substitute across sectors. There are two unions – one for each sector – and Calvo wage setting. There are intermediate and final good producers in both sectors. Intermediate goods markets are monopolistic competitive. In the non-durable sector prices adjust according to a Calvo mechanism. By contrast, prices are flexible but firms face adjustment costs in production in the housing sector. Final goods markets are competitive and characterized by flexible prices and a constant return to scale technology. Monetary policy follows a Taylor rule that targets inflation in the non-durable sector and features interest-rate smoothing.

Our framework incorporates many standard features of the New-Keynesian literature with housing. In particular, the imperfect substitutability of labor services between the non-durable and housing sectors, the Calvo wage setting and the adjustment costs in the housing production

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<sup>3</sup>Our paper does not solve for the transitional dynamics.

are known to be crucial to achieve two purposes. First, they avoid fast and unrealistic rebounds in the housing sector in response to aggregate shocks, due to large labor movements across sectors. Second, they generate sectoral co-movements in output that are consistent with the VAR evidence on monetary shocks.<sup>4</sup>

We describe the new features of the mortgage contract in the next Section, while we relegate the full description of the model to the Appendix.

## 2.1 Households

The economy is populated by a continuum of households distributed over the  $[0, 1]$  interval. A fraction  $\psi$  of identical households has discount factor  $\beta$ , while the remaining fraction  $1 - \psi$  has discount factor  $\gamma > \beta$ . We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively more than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.

### Borrowers

Borrowers have a lifetime utility function given by

$$\sum_{t=0}^{\infty} \beta^t E_0 \{U(C_t, H_{t+1}, N_{C,t}, N_{H,t})\}, \quad 0 < \beta < 1, \quad (1)$$

where  $C_t$  denotes consumption of non-durable goods,  $H_{t+1}$  denotes consumption of housing services,  $N_{C,t}$  is hours worked in the non-durable sector and  $N_{H,t}$  is hours worked in the housing sector. We assume the following period utility function:

$$U(C_t, H_{t+1}, N_{C,t}, N_{H,t}) = (1 - \alpha) \ln C_t + \alpha \ln H_{t+1} - \frac{\nu}{1 + \varphi} \left[ N_{C,t}^{1+\xi} + N_{H,t}^{1+\xi} \right]^{\frac{1+\varphi}{1+\xi}}, \quad \varphi, \xi \geq 0, \quad (2)$$

where  $\alpha$  is the share of housing in the overall consumption. Our specification for the disutility of labor follows Iacoviello and Neri (2010) in assuming that hours in the non-durable and housing sector are imperfect substitutes, as consistent with the evidence by Horvath (2000). For  $\xi = 0$

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<sup>4</sup>On the literature on the co-movement puzzle see Barsky, House and Kimball (2007), Erceg and Levin (2006), Carlstrom and Fuerst (2006), Monacelli (2009) and Sterk (2010).

hours in the non-durable and housing sector are perfect substitutes. On the other hand, positive values of  $\xi$  result in wages not being equalized in the two sectors and the substitution of hours across sectors in response to wage differentials being reduced. The parameter  $\varphi$  is the inverse of the Frisch labor supply elasticity.

## The Mortgage Contract

The Borrower household consists of many members. At the beginning of every period  $t$ , all the members receive equal resources to purchase new houses and negotiate current mortgage contracts. More precisely, at  $t$  the household decides total housing investment  $H_{t+1}$  and the state-contingent mortgage rate  $R_{Z,t}$ . In every period  $t$  the  $i$ -th member takes a new loan  $L_{t+1}$ , purchases the housing stock  $H_{t+1}^i$ , where  $\int_i H_{t+1}^i di = H_{t+1}$ , and he negotiates the mortgage contract using  $H_{t+1}^i$  as a collateral, following the household's instructions. The loan  $L_{t+1}$  is reimbursed with infinitely many installments;  $x$  is the rate of amortization in the first period and  $\phi$  is the rate of amortization from the second period onwards. Hence, the mortgage contract specifies the new loan amount,  $L_{t+1}$ , the housing stock  $H_{t+1}$  used as a collateral, the state-contingent mortgage rate  $R_{Z,t}$ , the outstanding debt  $D_{t+1}$ , the amortization rate in the first period,  $x$  and the amortization rate from the second period onwards,  $\phi$ . We assume  $x \in (0, 1)$  and  $\phi \in (0, 1)$ . Given the mortgage specification, the outstanding debt of the Borrower household evolves following the law of motion:

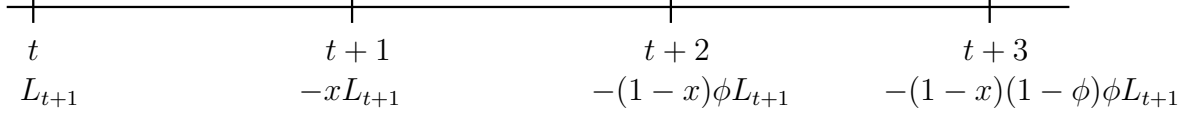
$$D_{t+1} = (1 - \phi)D_t - (x - \phi)L_t + L_{t+1}. \quad (3)$$

According to the law of motion (3), the current debt  $D_{t+1}$  is the outstanding debt accumulated up to the beginning of the period,  $D_t$ , minus the current mortgage installment,<sup>5</sup>  $\phi D_t + (x - \phi)L_t$ , plus the new loan  $L_{t+1}$ . In the rest of the paper we will refer to mortgage installment and repayment interchangeably. The law of motion in (3) is the direct consequence of the mortgage contract, which specifies the repayment flows for the new loan  $L_{t+1}$  as shown in the Figure below.

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<sup>5</sup>Here, as in the rest of the paper, the mortgage installment is net of interest repayments unless specified otherwise.





If the Borrower's members borrow  $L_{t+1}$  at time  $t$ , the repayment of  $L_{t+1}$  due at time  $t + n$  is equal to  $-(1-x)(1-\phi)^{n-2}\phi L_{t+1}$ , where a negative sign denotes a repayment. This structure of repayments implies that the outstanding debt at time  $t$  is determined as follows:

$$D_{t+1} = L_{t+1} + (1-x)L_t + (1-x)(1-\phi)L_{t-1} + (1-x)(1-\phi-(1-\phi)\phi)L_{t-2} + \dots$$

$$D_{t+1} = L_{t+1} - (x-\phi)L_t + (1-\phi)(L_t + (1-x)L_{t-1} + (1-x)(1-\phi)L_{t-2} + \dots)$$

$$D_{t+1} = L_{t+1} - (x-\phi)L_t + (1-\phi)D_t,$$

which corresponds exactly to the law of motion (3). This structure of repayments encompasses different types of amortization schedules: i)  $x = 1$  corresponds to a one-period mortgage because  $L_{t+1}$  is fully repaid in  $t + 1$ ; ii)  $x$  close to 1 is an extreme case of high early amortization where most of the repayment occurs at the end of the first period ; iii)  $x = 0$  means that new loans are not amortized in the first period; iv)  $x$  close to zero represents the case where a small fraction of new loans are repaid in the first period while most of principal is repaid from the second period onwards; v)  $\phi = 1$  corresponds to a two-period mortgage, since the  $L_{t+1}$  is completely reimbursed by  $t + 2$ ; vi)  $\phi$  close to zero implies a very long amortization period; vii)  $x = \phi$  means that the mortgage is repaid with a constant amortization rate equal to  $\phi$ ; viii)  $x = \phi = 0$  implies that the mortgage contract is a console.

### The Incentive Compatibility Constraint

Housing investment is risky. After the mortgage contract is finalized, each member experiences an idiosyncratic shock that affects his housing value. More specifically, in period  $t + 1$ ,  $i$ -th household member experiences an idiosyncratic shock  $\omega_{t+1}^i$  such that the value of the house in period  $t + 1$  is  $\omega_{t+1}^i P_{H,t+1} (1 - \delta) H_{t+1}^i$ , where  $P_{H,t+1}$  is the nominal house price in period  $t + 1$ . Idiosyncratic housing risk captures the fact that housing prices display geographical variation even in the absence of aggregate shocks. Alternatively, one can think of idiosyncratic effects to the housing stock such as damages and (un-modeled) home improvements. The random

variables  $\omega_{t+1}^i$  are i.i.d. across members of the same group and log-normally distributed with a cumulative distribution function  $F_{t+1}(\omega_{t+1}^i)$ , which obeys standard regularity conditions.<sup>6</sup> The mean and variance of  $\ln \omega_{t+1}^i$  are chosen so that  $E_t(\omega_{t+1}^i) = 1$  at all times. This implies that while there is idiosyncratic risk at the household-member level, there is no risk at the household level and  $E_t(\omega_{t+1}^i H_{t+1}^i) = H_{t+1}$ . We are going to assume that housing investment riskiness can change over time, namely that the standard deviation  $\sigma_{\omega,t}$  of  $\ln \omega_{t+1}^i$  is subject to an exogenous shock and displays time variation. The random variable  $\omega_{t+1}^i$  is observed by the  $i$ -th member and by the household but can only be observed by lenders after paying a monitoring cost. As typically done in the matching literature, we assume that the housing stock and mortgages left after default are equally redistributed among all members in the household.

After the idiosyncratic shock is realized, the household member decides whether to pay the mortgage installment or to default. Intuitively, loans connected to housing stocks that experience high realizations of the idiosyncratic shock are repaid while loans connected to housing stocks with low realizations are defaulted on. Let  $\bar{\omega}_{t+1}$  be the threshold value of the idiosyncratic shock for which the Borrower member is willing to pay the mortgage installment. The incentive compatibility constraint in period  $t + 1$  is

$$\bar{\omega}_{t+1}(1 - \delta)P_{H,t+1}H_{t+1} = (1 + R_{Z,t+1})D_{Z,t+1}. \quad (4)$$

The right-hand side of (4) is the value of the outstanding liability at period  $t + 1$ .  $R_{Z,t+1}$  is the state-contingent adjustable rate that non-defaulting Borrowers pay on the debt  $D_{Z,t+1}$  that has survived default until period  $t$ . The left-hand side of (4) is the housing value of the marginal member, namely the member who experiences  $\bar{\omega}_{t+1}$  and is indifferent between keeping the house and fulfilling the mortgage contract and defaulting. The housing value is the housing stock  $H_{t+1}^i$  purchased in period  $t$ , net of depreciation  $\delta$ , evaluated at current house price  $P_{H,t+1}$  and at the realization of the idiosyncratic shock  $\bar{\omega}_{t+1}^i$ . Loans connected to  $\omega_{t+1}^i \in [\bar{\omega}_{t+1}, \infty]$  are repaid. On the other hand, loans connected to  $\omega_{t+1}^i \in [0, \bar{\omega}_{t+1})$  are *underwater* mortgages, namely mortgages for which the current value of the house is lower than the liability associated

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<sup>6</sup>The c.d.f. is continuous, at least once-differentiable, and it satisfies

$$\frac{\partial \omega h(\omega)}{\partial \omega} > 0,$$

where  $h(\omega)$  is the hazard rate.

to it. These members have negative equity in their houses and, as a result, they default on these loans. Lenders pay a monitoring cost to assess and seize the collateral connected to the defaulted mortgage. It is the presence of monitoring that induces Borrowers to truthfully reveal their idiosyncratic shock and justifies the incentive compatibility constraint (4).<sup>7</sup> The household members that default on their mortgages lose their housing stocks.

A few comments on our assumptions about the repayment process are in order at this point. First, mortgages are nonrecourse in our model. This means that mortgages are secured by the pledge of collateral (the house) and the lender's recovery is strictly limited to the collateral. Defaulting Borrowers are not personally liable for the difference between the loan and the collateral value. This is a natural assumption in our model because housing is the only asset held by Borrowers. In addition to this, nonrecourse debt is broadly applicable to most U.S. states, especially those that experienced soaring mortgage delinquencies, and the focus of our paper is on the United States. Second, in our model the monitoring cost is rebated back to Savers in a lump-sum fashion<sup>8</sup> and, as in Bernanke, Gertler and Gilchrist (1999), is equal to a fraction  $\mu$  of the housing value. Under this assumption, the foreclosure cost is proportional to the value of the house under seizure. Third, in our model, as often assumed in the literature on matching frictions, there is perfect insurance among members in the same household. In fact, consumption of non-durable goods and housing services are ex-post equal across all Borrower household members. Hence, Borrower household members are ex-post identical.

Given the incentive compatibility constraint in (4), we can write the law of motion of the outstanding debt after default as:

$$D_{Z,t+1} = [1 - F_t(\bar{\omega}_t)] [(1 - \phi)D_{Z,t} - (x - \phi)L_t] + L_{t+1}, \quad (5)$$

where  $1 - F_t(\bar{\omega}_t)$  is fraction of mortgage installments that are been repaid at time  $t$ . Equation (5) states that the outstanding debt after default at time  $t$ ,  $D_{Z,t+1}$ , is equal to the fraction of the debt that survived default until time  $t-1$ ,  $D_{Z,t}$ , net of the period  $t$  repayment,  $\phi D_{Z,t} + (x - \phi)L_t$ , plus the new loans,  $L_{t+1}$ .

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<sup>7</sup>See the seminal work of Townsend (1979).

<sup>8</sup>We could alternatively assume that monitoring implies housing destruction. However, under this assumption an increase in the mortgage default generates (rather unrealistic) rebounds in housing demand.

At this point we can formulate the budget constraint at period  $t$  for the Borrower household:

$$\begin{aligned} & P_{C,t}C_t + P_{H,t}H_{t+1} + [1 - F_t(\bar{\omega}_t)][(R_{Z,t} + \phi)D_{Z,t} - (\phi - x)L_t] \\ & = L_{t+1} + W_{C,t}N_{C,t} + W_{H,t}N_{H,t} + (1 - \delta)[1 - G_t(\bar{\omega}_t)]P_{H,t}H_t. \end{aligned} \quad (6)$$

On the left-hand side of the budget constraint we find the use of resources, which includes the purchase of consumption goods with price  $P_{C,t}$ , housing, mortgage amortization and the interest payment on the debt.  $R_{Z,t}$  is the state-contingent interest rate paid on the mortgages by non-defaulting Borrowers. On the right-hand side of the budget constraint we find resources.  $W_{C,t}$  is the nominal wage in the consumption good sector and  $W_{H,t}$  is the nominal wage in the housing sector. Borrower's resources include the new loans  $L_{t+1}$  as well as the housing stock accumulated in the previous periods. This is the initial stock  $H_t$  net of depreciation and net of the fraction  $G_t(\bar{\omega}_t)$  lost to default in period  $t$ . We explicitly derive this term below.

### The Participation Constraints

Our mortgage contract guarantees lenders a pre-determined rate of return. As in Bernanke et al. (1999), the idea is that Savers have access to alternative assets that pay a risk-free rate return that pins down the return on mortgages. Savers make new loans  $L_{t+1}$  to Borrowers and demand the net rate of return  $R_{D,t}$ . This rate of return is non-state contingent and pre-determined at  $t$ . Hence, the time  $t$  participation constraint of lenders for the outstanding debt is given by:

$$\begin{aligned} (R_{D,t} + \phi)D_{t+1} - (\phi - x)L_{t+1} &= \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1 - \mu)(1 - \delta)P_{H,t+1}H_{t+1}f_{t+1}(\omega)d\omega \\ &+ \int_{\bar{\omega}_{t+1}}^{\infty} [(R_{Z,t+1} + \phi)D_{Z,t+1} - (\phi - x)L_{t+1}]f_{t+1}(\omega)d\omega \end{aligned} \quad (7)$$

where  $f_{t+1}(\omega)$  is the probability density function of  $\omega$ . The interest payment  $R_{D,t}D_{t+1}$  plus the mortgage installment is equal to the housing stock net of monitoring costs and depreciation seized from defaulting Borrower members (the first term on the right-hand side of (7)) and the mortgage repayments by non-defaulting members (the second term on the right-hand side of (7)). After idiosyncratic and aggregate shocks have realized, the threshold value  $\bar{\omega}_{t+1}$  and the state-contingent mortgage rate  $R_{Z,t+1}$  are determined so as to satisfy the participation

constraint above. Hence, our mortgage contract is characterized by adjustable mortgage rates. The participation constraint holds state by state and not in expected terms. An aggregate state that raises  $\bar{\omega}_{t+1}$  and thereby default generates, *ceteris paribus*, an increase in the adjustable rate  $R_{Z,t+1}$  paid by non-defaulting members in order to satisfy the participation constraint (7) in that state.

Let

$$G_{t+1}(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f_{t+1}(\omega) d\omega \quad (8)$$

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value  $\bar{\omega}_{t+1}$ , multiplied by the probability of default, and let

$$\Gamma_{t+1}(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{j,t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f_{t+1}(\omega) d\omega + G_{t+1}(\bar{\omega}_{t+1}). \quad (9)$$

Using these definitions, the law of motion (3) and the participation constraints (7) the Borrower budget constraint in real terms can be rewritten as

$$C_t + p_{H,t} H_{t+1} + \frac{(1 + R_{D,t-1})}{\pi_{C,t}} d_t = d_{t+1} + (1 - \delta)[1 - \mu G_t(\bar{\omega}_t)] p_{H,t} H_t + w_{C,t} N_{C,t} + w_{H,t} N_{H,t} \quad (10)$$

where  $p_{H,t}$  is the relative price of houses in terms of non-durable consumption at  $t$ ,  $\pi_{C,t}$  is non-durable inflation and  $w_{C,t}, w_{H,t}$  are real wages in the  $C$  and  $H$  sector, respectively, in terms of  $P_{C,t}$ .  $d_{t+1} \equiv D_{t+1}/P_{C,t}$  is the real debt outstanding at  $t$ .

Making use of definitions (8), (9), the incentive compatibility constraint (4) and the law of motion of debt (3), the participation constraint at  $t$  can be written in real terms as follows

$$\frac{(1 + R_{D,t})}{\pi_{C,t+1}} d_{t+1} = [\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})](1 - \delta) p_{H,t+1} H_{t+1} + d_{t+2} - d_{Z,t+2}. \quad (11)$$

According to the constraint (11), the total return on outstanding debt – which corresponds to the one of one-period bond – is equal the share of housing net of monitoring costs seized by the lender plus the term  $d_{t+2} - d_{Z,t+2}$ , which represents the compensation to the lender for the losses on the residual debt generated by Borrowers' default.

We define the loan-to-value (LTV henceforth) ratio as

$$LTV \equiv \frac{(1 + R_{D,t})d_{t+1}}{E_t \{ \pi_{C,t+1}(1 - \delta)p_{H,t+1}H_{t+1} \}}. \quad (12)$$

The LTV ratio measures the total mortgage (including interests) as a fraction of the expected net housing value. Models with housing which do not allow for endogenous default on mortgages usually feature an exogenous borrowing constraint instead of a participation constraint.<sup>9</sup> In terms of our notation, such borrowing constraint could be written as:

$$(1 + R_{D,t})d_{t+1} = \chi(1 - \delta)E_t \{ p_{H,t+1}H_{t+1}\pi_{C,t+1} \}, \quad (13)$$

which implies a constant LTV equal to  $\chi$ . Conversely, in our model the LTV ratio varies endogenously.

## 2.2 Exogenous Shocks

There are four exogenous shocks in our model. Aggregate productivity in the two sectors and the monetary policy shock evolve according to the following first-order autoregressive processes

$$\ln A_{C,t} = \rho \ln A_{C,t-1} + \epsilon_{C,t}, \quad \rho_C \in (-1, 1), \quad (14)$$

$$\ln A_{H,t} = \rho \ln A_{H,t-1} + \epsilon_{H,t}, \quad \rho_H \in (-1, 1), \quad (15)$$

$$\ln A_{M,t} = \rho \ln A_{M,t-1} + \epsilon_{M,t}, \quad \rho_M \in (-1, 1), \quad (16)$$

where  $\epsilon_C, \epsilon_H, \epsilon_M$  are i.i.d. innovations with mean zero and standard deviation  $\sigma_C, \sigma_H, \sigma_M$ , respectively, and  $\rho_C, \rho_H, \rho_M$  are the persistence parameters. Regarding the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that  $\omega_{j,t}$  is distributed log-normally:

$$\ln \omega_t \sim N\left(-\frac{\sigma_{\omega,t}^2}{2}, \sigma_{\omega,t}^2\right). \quad (17)$$

As stated earlier, the mean of the distribution is chosen so that  $E_t(\omega_{t+1}) = 1$ . We believe that housing investment risk increased with the entrance in the mortgage market of subprime debtors. Our model captures this increase in riskiness as an exogenous time variation in the

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<sup>9</sup>See for instance Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009).

standard deviation of idiosyncratic housing investment risk: we assume that the standard deviation of  $\ln \omega_t$  follows the first-order autoregressive process:

$$\ln \frac{\sigma_{\omega,t}}{\sigma_{\omega}} = \rho_{\sigma} \ln \frac{\sigma_{\omega,t-1}}{\sigma_{\omega}} + \varepsilon_{\sigma_{\omega,t}}, \quad (18)$$

where  $\varepsilon_{\sigma_{\omega,t}}$  is an i.i.d. shock with mean zero and finite standard deviation  $\sigma_{\sigma_{\omega}}$  and  $\rho_{\sigma}$  is the serial correlation coefficient.

Private agents know these exogenous processes and use them to form rational expectations.

### 3 Benchmark Calibration

Table 1 summarizes the parameter values for our benchmark calibration. We follow Monacelli (2009) in choosing the rate of depreciation for housing and the elasticity of substitution between non-durable goods and housing services. We choose an annual depreciation rate for housing of 4 percentage points, implying  $\delta = 0.01$ . The Saver's discount factor  $\gamma$  is set equal to 0.99, which pins down the steady-state risk-free interest rate at  $R_D = 0.0101$  on a quarterly basis. This implies an annual risk-free interest rate equal of 4.01 percentage points. The Borrowers' discount factor  $\beta$  is set equal to 0.98 as in Iacoviello (2005).

U.S. private fixed investment in structures, residential and nonresidential, has been on average 5 percent of GDP from 1960 to 2009, while during the period 2000 to 2007 it averaged 8 percent of GDP. We set the parameter  $\alpha$  that measures the share of housing in the consumption bundle equal to 0.16, so that the housing sector represents 8 percent of total output at the steady state. The inverse of the Frisch elasticity of labor supply  $\varphi$  is set equal to one, as in Barsky et al. (2007) and as typical in the macro literature. As for the parameter  $\xi$  that measures the degree of substitutability between hours worked in the two sectors, we set it equal to 0.7251. This is the appropriate weighted average of the  $\xi$  for Borrowers and Savers estimated by Iacoviello and Neri (2010).

We assume that housing prices are fully flexible. For non-durable goods,  $\theta_C$  is set equal to 0.67 to imply that firms in the non-durable sector change their prices on average every nine months. We set  $\varrho = \tilde{\varrho}$ , the Calvo probability for wages in the  $C$  and  $H$  sector, equal to 0.75. This implies that, on average, wages are changed less often than prices in the non-durable

Parameter	Value	Description
$\gamma$	0.99	Discount factor of Savers
$\beta$	0.98	Discount factor of Borrowers
$\psi$	0.5	Relative size of Borrower group
$\delta$	0.01	Rate of depreciation for housing
$\varepsilon$	7.5	Elasticity of substitution for intermediate goods
$\varepsilon_w$	21	Elasticity of substitution for labor services
$\varsigma$	3	Elasticity of substitution across labor inputs
$\zeta$	0.5	Share of Borrower labor in the production function
$\xi$	0.7251	Elasticity of substitution across labor types
$\alpha$	0.16	Share of housing in consumption bundle
$\nu$	2.5	Disutility from work
$\eta$	1	Elasticity of substitution between $C$ and $H$ goods
$\varphi$	1	Inverse of elasticity of labor supply
$\theta_C$	0.67	Calvo probability in $C$
$\varrho$	0.75	Calvo probability wages in $C$ and $H$
$\phi_\pi$	1.5	Taylor-rule coefficient on inflation
$\phi_r$	0.9	Taylor-rule coefficient on past nominal interest rate
$\sigma_\omega$	0.22	Standard deviation of idiosyncratic shocks
$\rho$	0.9	Serial correlation of shocks
$\mu$	0.43	Monitoring cost
$\phi$	0.0149	Amortization rate from the second installment onward
$x$	$\phi$	Amortization rate of the first installment
$\kappa$	0.2	Adjustment cost in housing production

Table 1: Benchmark Calibration

and housing sectors. For monetary policy we set  $\phi_\pi = 1.5$ , as standard in the literature. For the benchmark calibration we set  $\phi_r = 0.9$  because interest-rate inertia mimics the zero lower bound, which was reached in 2009Q1. We assume that the Borrower and Saver groups have equal size so that  $\psi = 0.5$  as in Monacelli (2009).

For technology, we follow Calza, Monacelli and Stracca (2011) and set the elasticity of substitution among intermediate goods  $\varepsilon$  equal to 7.5 in each sector. Labor inputs are imperfect substitutes in production and the elasticity of substitution across Borrower's and Saver's labor is  $\varsigma = 3$ . We assume that the share of Borrower's labor in the production function  $\zeta$  is equal to 0.5. We follow Altig, Christiano, Eichenbaum and Linde (2011) and assume a steady-state markup of wages over the marginal rate of substitution between leisure and consumption of 5 percent and we set  $\varepsilon_w = 21$ . We believe that housing risk shocks are persistent but there is no previous work we can rely on. Christiano, Motto and Rostagno (2009) estimate the persistence



Variable	
Amortization rate from the second installment onward	0.0149
Amortization rate of the first installment	0.0149
Loan-to-Value Ratio	79
Leverage Ratio	85.42
Default Rate †	2.5
Mortgage Interest Rate †	6.9
External Finance Premium †	2.85

Percentage points

†Annual

Note: The Leverage Ratio is calculated as  $d/(d + w_C N_C + w_H N_H)$

Table 2: Steady State under the Benchmark Calibration

of the idiosyncratic productivity shock for the United States to be 0.85. We set  $\rho_\sigma = 0.9$ . The serial correlation of the productivity shocks in the non-durable and housing sectors are chosen to be  $\rho_C = 0.9$  and  $\rho_H = 0.9$ , respectively.

Regarding the mortgage market, we need to specify values for the parameters  $\phi, x, \sigma_\omega$  and  $\mu$ ; at the same time we want to match the pre-crisis delinquency rate and the LTV ratio. We set  $\phi = x = 0.0149$  which implies a first-period amortization rate of 1.49% and a residual debt after 30 years equal to 16.51% of total debt for our benchmark scenario. The U.S. LTV ratio was equal to 77 percentage points on 2006Q4 and its average value between 1973Q1 and 2010Q4 was 76 percentage points.<sup>10</sup> According to the National Delinquency Survey of the Mortgage Banker Association, seriously delinquent mortgages are all mortgages more than 90 days past due or in foreclosure. U.S. seriously delinquent mortgages averaged 2.3 percent of total mortgages between 1979Q1 and 2010Q4 and they represented 2.2 percent of total mortgages in 2006Q4. In our model, the LTV ratio and the delinquency rate are non-linear functions of  $\sigma_\omega$  and  $\mu$ . Higher monitoring costs reduce loans and thereby the LTV ratio and the default rate; higher idiosyncratic volatility lowers mortgage loans and the LTV ratio and raises the default rate. We choose the standard deviation of idiosyncratic housing price shocks  $\sigma_\omega$  to be equal to 0.22 at the steady state. Given the chosen value for  $\sigma_\omega$ , we set monitoring costs  $\mu$  equal to 0.43. Table 2 reports the steady-state value of mortgage market variables under the benchmark calibration. For  $x = \phi = 0.0149$  the LTV ratio is 79 percentage points. The mortgage market parameters

<sup>10</sup>Source: Terms on Conventional Single-Family Mortgages, Monthly National Averages, All Homes, Federal Housing Finance Agency.

generate a steady-state default rate of 2.5 percent under the benchmark calibration.

Given the parameters  $\mu$  and  $\sigma_\omega$  of the mortgage market, the difference in discount factors between Savers and Borrowers determines the adjustable mortgage rates. In our model the adjustable mortgage rate on one-period loans is 6.85 percentage points (annualized). Between 1980 and 2009 the Adjustable Rate Mortgage Index<sup>11</sup> has averaged 8.7 percentage points, which implies an average real rate of 5.3 percentage points. In the data, the spread measured as the difference between the bank prime loan rate and the 6-Month certificate of deposit was equal to 3 percentage points in 2006Q4 and it averaged 2 percentage points between 1964Q3 to 2011Q3. The external finance premium is 2.85 percentage points. The leverage ratio for Borrowers at the steady state is calculated as

$$\text{Leverage Ratio} = \frac{d}{d + w_C N_C + w_H N_H},$$

which measures the fraction of total expenses financed by total loans, namely consumption of  $C$  and  $H$  plus loan repayment over loans. The leverage ratio captures the dependence of Borrowers from external funding.

## 4 Steady-State Analysis

### 4.1 Comparative Static

What are the steady-state effects of a reduction of the amortization rates  $x$  and  $\phi$ ? Figures 2 and 3 show the steady state of our model as  $x$  and  $\phi$  vary in the intervals  $(0, 1)$  and in  $(0.008, 0.1)$ <sup>12</sup> respectively. In Figure 2 we consider three calibrations for  $\phi$ :  $\phi = 0.00149$ , which is the benchmark calibration (the dark blue dash-dot line);  $\phi = 0.999$  (the light blue solid line), which is close to a two-period mortgage; and  $\phi = 0.0008$  (the red dotted line) which implies a lower amortization rate relative to the benchmark calibration. Similarly, Figure 3 considers three calibrations of  $x$ :  $x = \phi$ , the benchmark calibration (the light blue solid line, not visible

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<sup>11</sup>Data from the Finance Board's Monthly Survey of Rates and Terms on Conventional Single-Family Non-farm Mortgage Loans and reported by the Federal Housing Finance Agency.

<sup>12</sup>We choose this interval for two reasons. First, when  $\phi = 0.008$  the LTV is above 150%. Values of  $\phi$  below 0.008 would lead to even higher LTVs. Second, values of  $\phi$  greater than 0.1 imply a very low LTV compared to the empirical evidence.

because it is very close to the dark blue dash-dot line);  $x = 0.001$  (the dark blue dash-dot line), which corresponds to a low amortization rate that almost approximates a console; and  $x = 0.99$  (the red dotted line), which is close to the one-period mortgage case.

Figures 2 and 3 clearly indicate that, once we allow for contracts that last more than one period, the steady state of our economy differs significantly from that of a one-period mortgage. Moreover, these figures exhibit some common patterns. A reduction in  $x$  or  $\phi$  reduces the mortgage installment and lengthen the duration of the mortgage, thereby increasing the LTV ratio. A higher LTV ratio has three consequences. First, it raises the demand for new loans because it allows more leveraging. Borrowers, in fact, are credit constrained and always inclined to borrow up to their borrowing limit since the interest rate they are willing to pay is higher than the market rate (i.e.,  $\frac{1}{\beta} > \frac{1}{\gamma}$ ). By borrowing more, they can increase current consumption, which is too low from their perspective. Second, the rise in the LTV reduces the Borrowers' home equity. Given the standard deviation of idiosyncratic risk, more Borrowers are likely to experience negative equity in their housing and default. As a result, the adjustable mortgage rate and the external finance premium soar. Third, as the LTV increases, Borrowers raise their demand for new houses in order to increase the collateral available for taking new loans. As housing demand increases, housing prices increase too. As a result, Borrowers cut their non-durable consumption and increase their labour supply to afford housing at the new, higher prices. Notwithstanding the rise in housing prices, the housing demand of Savers increases as well. The expansion in mortgage lending induces a strong positive income effect on Savers' budget constraint.<sup>13</sup> In fact, Savers raise their demand for the non-durable good as well as leisure. As amortization rates decline, output increases both in the housing and in the non-durable sector, thereby leading to an expansion of aggregate output.

Figures 2 and 3 allow us to compare the steady-state effects of alternative amortization schedules. Such a comparison would not be possible in models in which mortgages last one period. In our model, there little differences between one- or two-period mortgages.<sup>14</sup> This can be seen in Figure 2, noting that the steady states that emerge with  $\phi = 0.999$ , no matter the value taken

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<sup>13</sup>This positive income effect is due to the increase in the interest payments and in the lump-sum transfers associated to rebating monitoring costs. If we alternatively assume that monitoring costs are lost, our main results remain unchanged.

<sup>14</sup>However, in an extension of our model in which mortgages can be renegotiated only every two periods we show that there are significant steady-state effects induced by a change in the amortization schedule from one- to two-period mortgages. This extension will be included in future drafts of the paper.

by  $x$ , is identical to the steady state that emerges when  $x = 0.999$ . However, the first-period amortization rate matters for  $\phi$  sufficiently low. As  $x$  decreases, the LTV increases and it induces an expansion in both the housing and non-durable sectors according to the mechanism just described above. Interestingly, Figure 2 shows that the largest effects arise when  $x$  is close to 1. Indeed, when the mortgage amortization rate is close to that of a one-period mortgage – as assumed by the vast majority of the literature – and the amortization rate from the second period onwards is low, decreasing the fraction of loans repaid with the first installment reduces significantly the mortgage installment, much more so relative to the case where  $\phi$  is high. Intuitively, when  $\phi$  is low,  $x$  controls the amortization rate.

Figure 3 compares the effects of a change in  $\phi$  under the baseline amortization schedule (the light blue solid line) and a very aggressive amortization schedule where 99.9 percent of the loans is repaid in the first period (the red dotted line). Not surprisingly, a change in  $\phi$  has stronger effects under the baseline calibration relative to the case where  $x = 0.999$ . If loans are almost completely repaid with the first-period installment the effects of a change in the amortization rate from second period onwards are necessarily small.

## 4.2 Welfare Effects

What are the welfare effects at the steady state of a fall in  $x$  and  $\phi$ ? Figures 5, 6 and 7 show the welfare costs in the absence of shocks for Borrowers, Savers and social welfare. Social welfare is defined as

$$W_t^S = (1 - \beta)\psi W_t + (1 - \gamma)(1 - \psi)\widetilde{W}_t, \quad (19)$$

where  $W_t$  is the lifetime utility of Borrowers and  $\widetilde{W}_t$  is the lifetime utility of Savers. These costs are expressed in consumption equivalent terms<sup>15</sup> and they correspond to the fraction of steady-state consumption that a household needs to be compensated to be as well off in an economy with  $\phi \in (0.008, 0.1)$  and  $x \in (0, 1)$  as in an economy with  $x = 1$  and  $\phi = 0.1$ . Intuitively, one would expect that, by relaxing Borrowers' credit constraint, a reduction of the amortization rates of mortgages improved Borrowers' welfare. Figure 5 shows that exactly the opposite is true. Any parametrization such that  $x < 1$  and  $\phi < 0.1$  entails welfare losses at the steady

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<sup>15</sup>These losses are calculated using the per-period utility. Results does not change if we use the lifetime utility instead.

state relative to the economy where  $x = 1$  and  $\phi = 0.1$ . Moreover for some parametrizations welfare costs are large, reaching 12 percent of steady-state consumption. This result can be explained in the light of the mechanism illustrated in the previous section. As amortization rates fall, mortgage installments are reduced and the LTV ratio increases. Borrowers, who are always willing to borrow more, increase their demands for new loans and new houses. Savers raise their demand for new houses too. As a consequence, housing prices rise, Borrowers reduce non-durable consumption and increase hours worked to afford housing at the new prices. For this reason, as the amortization rates fall, Borrowers are worse off. By contrast, according to Figure 6, a reduction in  $x$  and  $\phi$  makes Savers better off and improves their welfare up to the 16 percent of their steady-state consumption. Indeed, the increase in mortgage lending due to lower amortization rates allows Savers to increase their non-durable consumption, their leisure and their housing. Welfare benefits for Savers are generally larger than those of Borrowers and, according to Figure 7<sup>16</sup> they always more than compensate those of Borrowers in terms of welfare.

A number of conclusions can be drawn from Figures 5, 7 and 7. At the steady state a reduction in the amortization rates can generate strong *redistributive* effects from Borrowers to Savers. In contrast, aggregate welfare effects are small. These results suggest that Borrowers living in economies characterized by high LTV ratios and loose credit conditions are not necessarily better off than those living in economies in which credit conditions are stricter. At the same time policies that influence mortgage characteristics and raise the LTV can have strong redistributive effects in the long run that are detrimental for the welfare of Borrowers and improving for the welfare of Savers.

Finally the general equilibrium mechanism illustrated above is not specific to our model, but it arises in a wide class of models with housing featuring borrowing-constraints with an exogenous LTV as in Iacoviello (2005). To make clear this point we consider a model akin to our in which however Borrowers face a borrowing constraint as the one featured in (13). In Figure 4 we show the steady state of this new model as the LTV varies in the interval  $(0, 1)$ . Results are very similar to the ones obtained with endogenous default on mortgages. The only relevant difference stays in the behavior of Saver housing demand which decreases (and not increase) as the LTV rises. Thus, as it will further be made clear by the results of the next section, the

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<sup>16</sup>We compute the welfare losses in Figure 7.

mechanism at work in our model is not driven by the presence of endogenous default and LTV ratios but rather by general equilibrium effects.

### 4.3 Partial versus General Equilibrium Effects

At a first glance, our findings are counter-intuitive and in contrast with a view recently emerged in the finance literature.<sup>17</sup> This view, shared by a number of contributions, argues that non-standard mortgage contracts can have beneficial effects for Borrowers because they are efficient tools that allow agents to smooth consumption in response to negative income shocks or over the life cycle. In our model we could expect that Borrowers with access to nonstandard mortgages would see their welfare improve for two reasons. First, because these types of contracts should allow Borrowers to better smooth consumption. Indeed Borrowers cannot properly smooth consumption since they are credit constrained and they would like to relax their participation constraint as much as possible so as to anticipate consumption to the current period. Second, because if Borrowers could set  $\phi$  and  $x$  optimally, they would always choose the lowest feasible rates of amortization. We show this result in the Appendix. Borrowers solve their standard problem and they also optimally decide over  $\phi$  and  $x$  under the constraints  $\phi \geq \bar{\phi} > 0$  and  $x \geq \bar{x} > 0$ . The solution is that Borrowers always choose  $\phi = \bar{\phi}$  and  $x = \bar{x}$ .

The apparent contradiction between the intuitive explanation consistent with the finance literature and the findings of the previous section is explained by the general equilibrium effects. When making their optimal decisions, Borrowers take market prices as given. In this case and consistently with a partial equilibrium analysis a fall in  $x$  and  $\phi$  relaxes mortgage credit conditions and improve Borrowers' welfare. Nevertheless, once we allow for general equilibrium effects, a reduction in the amortization rates achieves exactly the opposite and it tightens mortgage credit conditions. In fact, as  $x$  and  $\phi$  fall, the participation constraint of Borrowers becomes more binding. This tightening is due to the increase in the housing price generated by the rise in the housing demand. An increase in housing prices has two direct effects on Borrowers' optimal choices: it relaxes the participation constraint (7) (and then mortgage credit conditions) by increasing the value of housing as a collateral; however it also tightens the budget constraint (6) since more resources are needed for housing purchases. This induces Borrowers

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<sup>17</sup>See, for instance, Cocco (2012), Piskorski and Tchisty (2010) and Keys, Piskorski, Seru and Vig (2012).

to cut non-durable consumption and to increase the demand for new loans, tightening the participation constraint. In equilibrium, this last effect prevails and it more than compensates the direct effect due to the rise in the collateral value.

Our analysis provides a clear insight: general equilibrium effects are potentially important to evaluate the welfare consequences of alternative mortgage amortization schedules and in general of policies that influence directly or indirectly mortgage characteristics.

## 5 Welfare Comparison

The previous analysis emphasizes the importance of steady-state welfare effects abstracting from the presence of shocks. In this section, we allow for shocks and compare the *unconditional* welfare costs of different amortization schedules taking as a benchmark the amortization rates of the baseline calibration, i.e.,  $\phi = x = 0.0149$ .<sup>18</sup> We set  $\sigma_{AC} = 0.01$ ,  $\sigma_{AH} = 0.0193$  and  $\sigma_R = 0.0034$  consistently with the estimates of Iacoviello and Neri (2010). We try different calibration for  $\sigma_{\sigma\omega}$  for which we don't have data and results are not sensitive to a change in this parameter. In the current draft, we calibrate  $\sigma_{\sigma\omega} = 0.0034$ .<sup>19</sup>

<b>x, <math>\phi</math></b>	<b>0.0149</b>	<b>0.0112</b>	<b>0.008</b>	<b>0.999</b>
<b><math>\phi</math></b>	0	2.4	9.25	-2.9013
<b>0.99</b>	-2.51	-2.296	-1.8978	-2.9013
<b>0.01</b>	0	2.4007	9.2353	-2.8910

Table 3: Borrower Welfare Costs.

In Tables 3, 4 and 5 we show the welfare costs for Borrowers, Savers and the economy as a whole. Welfare costs under different calibrations of  $\phi$  ( $x$ ) are listed along the columns (rows). The values of  $\phi$  and  $x$  in the tables are chosen in order to encompass the amortization schedules mentioned in Section 2.1. In fact, i)  $x = 0.99$  entails a very aggressive amortization, close to the that of one-period mortgages; ii)  $x = 0.01$  implies a deferred amortization under the benchmark calibration; iii)  $\phi = 0.999$  is associated with an amortization schedule akin to the one of a two-period mortgage; iv)  $\phi = 0.0112$  and  $\phi = 0.008$  lead to longer amortization schedules than under

<sup>18</sup>The exercise of this section is then slightly different with respect to the one above where we use as a benchmark  $\phi = 0.01$  and  $x = 0.99$ .

<sup>19</sup>We could alternatively calibrate this parameter to match the volatility of the default rate on mortgages.

the baseline calibration. In particular,  $\phi = 0.0112$  aims at capturing the difference between a thirty- and a forty-period mortgage. Indeed under this calibration the fraction of the current debt still due after 40 years is 16.51 percent, the same percentage still due after 30 years under our benchmark scenario; v)  $x = \phi$  implies a constant amortization rate; iv)  $\phi = x = 0.992$  is the closest calibration to that of a console.

Table 3 corroborates the intuitions and the steady-state results discussed above. Even in the presence of shocks, lower amortization rates decrease Borrowers' welfare. For instance, living in an economy in which  $\phi = x = 0.0112$  instead of  $\phi = x = 0.149$  implies a welfare cost of 2.4 percent of the steady-state consumption. A even lower value of  $\phi$  (i.e.,  $\phi = x = 0.008$ ) would generate even larger welfare costs. Similarly, when  $x$  is smaller than  $\phi$  and there is deferred amortization, Borrowers' welfare falls with respect to the benchmark economy. However and not surprisingly welfare losses due to deferred amortization are very small. Conversely a very aggressive amortization schedule akin to that of a one- or two-period mortgage (i.e.,  $x = 0.99$  or  $\phi = 0.999$ ) improve Borrowers' welfare up to approximately the 2.9 percent of the steady-state consumption.

<b>x, <math>\phi</math></b>	<b>0.0149</b>	<b>0.0112</b>	<b>0.008</b>	<b>0.999</b>
<b><math>\phi</math></b>	0	-1.9589	-7.713	2.3812
<b>0.99</b>	2.0683	1.8919	1.5774	2.3812
<b>0.01</b>	-0.01	-1.9589	-7.7022	2.3714

Table 4: Saver Welfare Costs.

Table 4 is the mirror image of Table 3. Savers are better off when amortization rates are lower as in suggested by our results at the steady-state. Interestingly, though, Saver welfare benefits due to lower amortization rates are generally smaller than the Borrower welfare costs. Thus, once shocks are allowed – and as opposed to the steady-state analysis – social welfare costs are most of the times positive when the amortization rates are lower than under the benchmark calibration. Still these costs are quantitatively small.



<b>x, <math>\phi</math></b>	<b>0.0149</b>	<b>0.0112</b>	<b>0.008</b>	<b>0.999</b>
$\phi$	0	0.488	2.244	-0.451
<b>0.99</b>	-0.3908	-0.3606	-0.2904	-0.451
<b>0.01</b>	-0.01	0.4888	2.2444	-0.451

Table 5: Social Welfare Costs.

## 6 Conclusions

Our main results can be interpreted as follows. Non-standard mortgage contracts characterized by amortization rates and low down payments make Borrowers are worse off at least in the long run. This is so because in *general equilibrium* lower down payments raise housing demand and thereby housing prices. Higher housing prices render housing less affordable and induce Borrowers to cut non-durable consumption and increase hours worked. By contrast, under lower amortization rates Savers are better off since they can consume more and enjoy more leisure. These findings have two clear implications. First, general equilibrium effects are important to assess the potential benefits of nonstandard mortgage contracts. Indeed, as the U.S. experience suggests, the introduction of this type of contracts can generate aggregate macroeconomic effects which must be taken into account to weigh up the efficiency of these financial tools. Second, the design of policies and reforms that could affect directly or indirectly mortgage characteristics should evaluate carefully the long-run *redistributive* effects on households' welfare due to an *endogenous* change in housing prices. In particular when policymakers aim at making Borrowers better off, they should consider that lower amortization rates and lower down payments are welfare reducing for Borrowers in the long run.

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# A The Model

## Wage Determination

We model imperfectly competitive labor markets that generate a wage-inflation Philips curve as in Schmitt-Grohe and Uribe (2007). Labor decisions are taken by two unions in each household, one for each sector, which monopolistically supply labor to a continuum of labor markets indexed by  $k \in [0, 1]$  in each sector  $j = H, C$ . Here we focus on the unions that supply Borrower's labor to the sectors; the unions that supply Saver's labor to the sectors take decisions in a similar manner. The union that supplies Borrower's labor to sector  $j$  decides the nominal wage to charge in each labor market  $k$  in  $j$  and it is assumed to satisfy demand, namely

$$N_{j,t}(k) = \left( \frac{w_{j,t}(k)}{w_{j,t}} \right)^{-\varepsilon_w} N_{j,t}^d, \quad (20)$$

where, at time  $t$ ,  $w_{j,t}(k)$  denotes the real wage charged by the union in labor market  $k$  in sector  $j$ ,  $w_{j,t}$  is the index of real wages prevailing in sector  $j$ ,  $N_{j,t}^d$  is the aggregate demand for Borrowers' labor by firms in sector  $j$ ,  $\varepsilon_w$  is the elasticity of substitution across labor types, and  $N_{j,t}(k)$  is the supply of labor in market  $k$  of sector  $j$ . This demand is formally derived later in section A.1. The union takes the aggregate demand  $N_{j,t}^d$  and the wage index  $w_{j,t}$  as given when it decides the wage to charge in labor market  $k$ ,  $W_{j,t}(k)$ . Notice that the union decides the nominal wage, even though our model is written in real terms. In addition, the total number of hours supplied in sector  $j$  by Borrowers must be equal to the sum of the hours supplied in each market  $k$ :

$$\psi N_{j,t} = \psi \int_0^1 N_{j,t}(k) dk = N_{j,t}^d \int_0^1 \left( \frac{w_{j,t}(k)}{w_{j,t}} \right)^{-\varepsilon_w} dk. \quad (21)$$

This constraint is taken into account by the household in its maximization problem. A similar constraint holds for the hours supplied by Savers.

We introduce wage stickiness in the model by assuming that each union can optimally set wages only in a fraction  $\varrho_j \in (0, 1)$ ,  $j = C, H$ , of randomly chosen labor markets. In these labor markets, the union can freely set the nominal wage  $W_{j,t}(k)$ ; we assume no wage indexation so that, in the other labor markets, wages remain equal to those of the last period. For simplicity, we assume the same degree of wage stickiness in the two sectors so that  $\varrho_C = \varrho_H = \varrho$ .

The first part of our analysis takes the fraction of one-period loans out of total loans,  $x$ , as a

parameter exogenously given, which can be interpreted as a fixed feature of the mortgage contract. This implies that the Borrower can choose total real loans  $l_{t+1}$  but not its composition. In other words, the Borrower cannot choose the amortization schedule for his mortgage. Borrowers maximize (1) subject to the budget constraint (10), the participation constraint (7), the incentive compatibility constraint (4), and the labor market constraint (21) for sector C and its counterpart for sector H with respect to the variables  $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{t+1}, N_{C,t}, N_{H,t}$ .

## Savers

We denote Savers' variables with a  $\sim$ . Savers maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\tilde{C}_t, \tilde{H}_{t+1}, \tilde{N}_{C,t}, \tilde{N}_{H,t}) \right\}, \quad 0 < \beta < \gamma < 1, \quad (22)$$

We assume that  $\alpha$ , the weight of housing in the consumption index, and the utility function of Savers are identical to those of Borrowers. Savers maximize lifetime utility subject to the sequence of real budget constraints:

$$\tilde{C}_t + p_{H,t} \tilde{H}_{t+1} + \tilde{d}_{t+1} = (1 - \delta) p_{H,t} \tilde{H}_t + (1 + R_{Dt-1}) \frac{\tilde{D}_t}{\pi_{C,t}} + \tilde{w}_{C,t} \tilde{N}_{C,t} + \tilde{w}_{H,t} \tilde{N}_{H,t} + \tilde{\Delta}_t, \quad (23)$$

As for Borrowers, Savers' labor decisions are taken by two unions, one for each sector, which monopolistically supply labor to a continuum of labor markets. Each union chooses optimally nominal wages in a fraction  $\tilde{\varrho}$  of randomly chosen labor markets; in the other markets wages remain unchanged. For simplicity, we assume that the degree of wage stickiness in the two sectors are equal, hence  $\tilde{\varrho}_C = \tilde{\varrho}_H = \tilde{\varrho}$ . The maximization problem faced by Savers' unions is identical to the problem faced by Borrowers' unions and we do not repeat it here.

Savers maximize (22) subject to the budget constraint (23) with respect to  $\tilde{C}_t, \tilde{H}_{t+1}, \tilde{N}_{C,t}, \tilde{N}_{H,t}, \tilde{l}_{t+1}, \tilde{N}_{C,t}, \tilde{N}_{H,t}$ .

### A.1 Firms and Technology

Both the non-durable  $C$  and the housing  $H$  sector have intermediate and final good producers.

## Final Good Producers

Final good producers are perfectly competitive and produce  $Y_{j,t}$ ,  $j = C, H$ . The technology in the  $j$ -th final good sector is given by

$$Y_{j,t} = \left( \int_0^1 Y_{j,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (24)$$

where  $\varepsilon > 1$  is the elasticity of substitution among intermediate goods. Standard profit maximization implies that the demand for intermediate good  $i$  is given by

$$Y_{j,t}(i) = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon} Y_{j,t}, \quad \forall i \quad (25)$$

where the price index is

$$P_{j,t} = \left( \int_0^1 P_{j,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

## Intermediate Good Sectors

There are two intermediate good sectors, one in each sector  $j = C, H$ , and in each intermediate sector there is a continuum of firms, each producing a differentiated good  $i \in [0, 1]$ . These firms are monopolistically competitive. Each firm uses all labor inputs supplied in the economy to produce its own good. The production function (and therefore the labor demand by intermediate firm  $i$  in sector  $j$ ) is:

$$\left[ \zeta^{\frac{1}{\varsigma}} N_{j,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \tilde{N}_{j,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma-1}}$$

where

$$N_{j,t}(i) = \left( \int_0^1 N_{j,t}(k, i)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dk \right)^{\frac{\varepsilon_w}{\varepsilon_w-1}}, \quad \tilde{N}_{j,t}(i) = \left( \int_0^1 \tilde{N}_{j,t}(k, i)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dk \right)^{\frac{\varepsilon_w}{\varepsilon_w-1}}.$$

$N_{j,t}(i)$  and  $\tilde{N}_{j,t}(i)$  are bundles of labor inputs supplied respectively by Borrowers and Savers.  $\zeta \in (0, 1)$  is the labor share of Borrowers in the firm's labor demand and  $\varsigma > 0$  is the elasticity of substitution across labor bundles. When  $\varsigma$  goes to infinity, labor inputs become perfect substitutes. For simplicity these two parameters are assumed to be equal across sectors. In-

intermediate good firms in the non-durable sector adjust their prices according to a Calvo-type mechanism. Hence, in any given period, an intermediate good firm in sector  $C$  may reset its price with probability  $1 - \theta_C$ . Conversely, the prices in the housing sector are fully flexible. We also assume firm-level adjustment costs in the housing sector.

### Non-Durable Sector

Intermediate good firm  $i$  in the  $C$  sector produces according to the following production function:

$$Y_{C,t}(i) = A_{C,t} \left[ \zeta^{\frac{1}{\varsigma}} N_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1 - \zeta)^{\frac{1}{\varsigma}} \tilde{N}_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma-1}}, \quad i \in \{C\}, \quad (26)$$

where  $A_{C,t}$  is the stochastic level of technology in sector  $C$ . In period  $t$  firm  $i$  chooses labor and, if given the possibility, it re-optimizes its nominal price  $P_{C,t}^*(i)$  so as to maximize the expected discount sum of nominal profits over the period during which its price remains unchanged. In our model marginal costs are a CES index of wages net of productivity; since wages are equal across firms in the sector, marginal costs are also equal across firms.

### Housing Sector

Intermediate good  $i$  in sector  $H$  produces according to the following production function:

$$Y_{H,t}(i) = A_{H,t} \left[ \zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1 - \zeta)^{\frac{1}{\varsigma}} \tilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma\kappa}{\varsigma-1}}, \quad i \in \{H\} \quad (27)$$

We follow Topel and Rosen (1988) and assume there are firm-level adjustment costs in the housing sector for intermediate good producers. As for prices in the housing sector, we assume they are fully flexible. In fact Iacoviello and Neri (2010) estimate a DSGE housing model of the United States and find a degree of price stickiness in the housing sector equal to zero. Hence, each period firm  $i$  chooses labor  $N_{H,t}(i)$ ,  $\tilde{N}_{H,t}(i)$  and the price  $p_{H,t}(i)$  so as to maximize the expected discounted value of current and future profits net of adjustment costs.



## A.2 Monetary Policy

We assume that monetary policy follows a Taylor-type rule for the one-period nominal interest rate:

$$\frac{1 + R_{D,t}}{1 + R_D} = A_{M,t} \left[ \pi_{C,t}^{\phi_\pi} \right]^{1-\phi_r} \left[ \frac{1 + R_{D,t-1}}{1 + R_D} \right]^{\phi_r}, \quad \phi_\pi > 1, \phi_r < 1, \quad (28)$$

where  $R_D$  is the steady-state nominal interest rate,  $\phi_\pi$  is the coefficient on the inflation target,  $\phi_r$  is the coefficient on the lagged interest rate, and  $A_{M,t}$  is a monetary policy shock. In our benchmark calibration monetary policy targets inflation in the non-durable goods sector and implements interest-rate smoothing, which mimics the zero lower bound and the impossibility for the central bank to further reduce the policy rate.

## A.3 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good equals aggregate demand:

$$Y_{C,t} = \psi C_t + (1 - \psi) \tilde{C}_t \quad (29)$$

where the last term on the right-hand side are the resources used for financial intermediation. Similarly, equilibrium in the housing market requires

$$Y_{H,t} = \psi [H_{t+1} - (1 - \delta)H_t] + (1 - \psi) [\tilde{H}_{t+1} - (1 - \delta)\tilde{H}_t] + g(Y_{H,t} - Y_{H,t-1}). \quad (30)$$

Equilibrium in the labor market requires

$$\psi N_{j,t} = \psi \int_0^1 N_{j,t}(k) dk = N_{j,t}^d \int_0^1 \left( \frac{w_{j,t}(k)}{w_{j,t}} \right)^{-\varepsilon_w} dk \quad j = C, H \quad (31)$$

$$(1 - \psi) \tilde{N}_{j,t} = \psi \int_0^1 \tilde{N}_{j,t}(k) dk = \tilde{N}_{j,t}^d \int_0^1 \left( \frac{\tilde{w}_{j,t}(k)}{\tilde{w}_{j,t}} \right)^{-\varepsilon_w} dk \quad j = C, H \quad (32)$$

where  $N_{j,t}^d = \int_0^1 N_{j,t}(i) di$  and  $\tilde{N}_{j,t}^d = \int_0^1 \tilde{N}_{j,t}(i) di$ . Equilibrium in the credit market is achieved if

$$\psi l_t = (1 - \psi) \tilde{l}_t. \quad (33)$$

We define total output as

$$Y_t = Y_{C,t} + p_{H,t}Y_{H,t}. \quad (34)$$

Notice that our measurement of total output reflects variations in the relative price of housing.

National account statistics, on the other hand, measure GDP at constant relative prices.

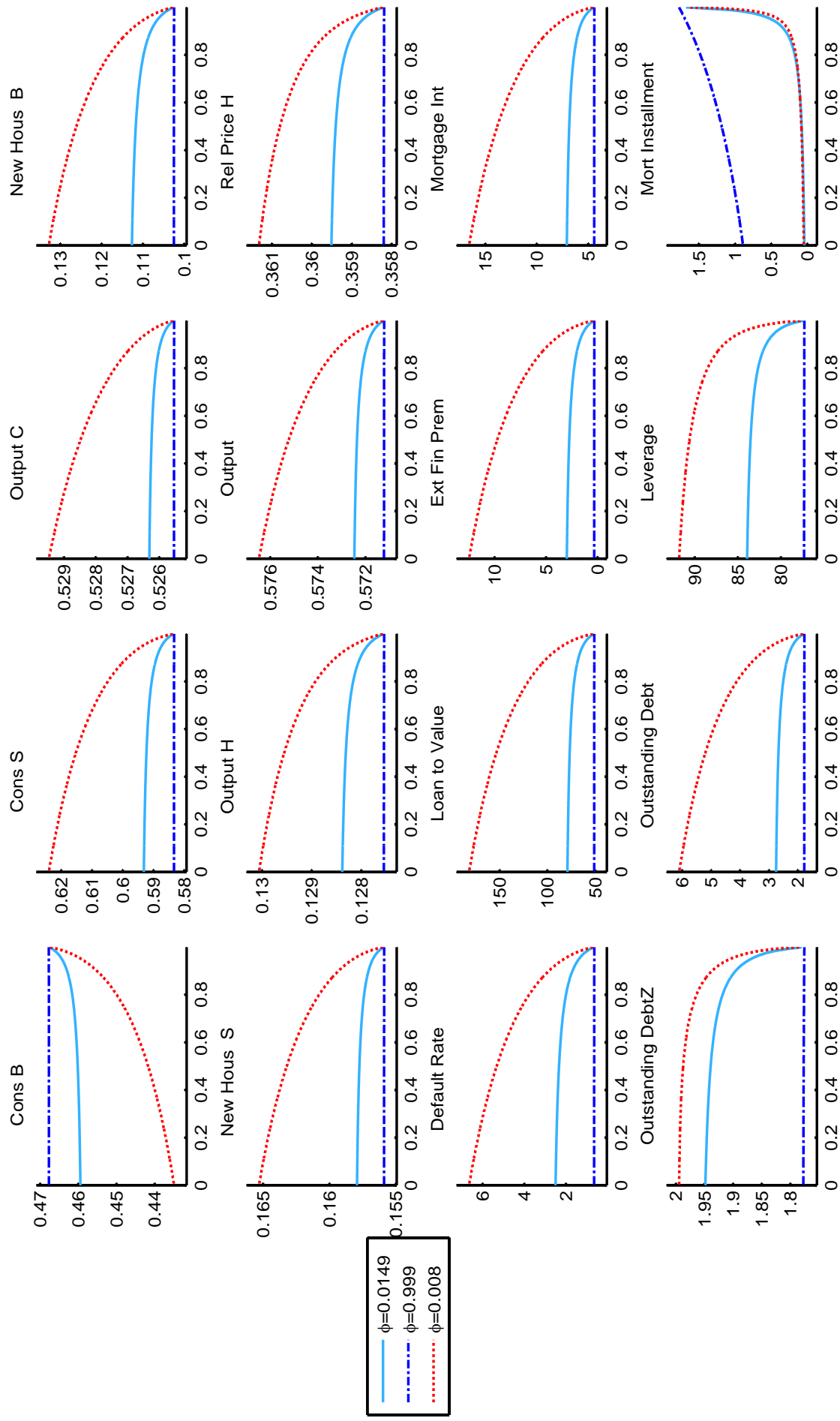


Figure 2: Steady State as  $x$  varies in the interval  $(0, 1)$

Default rates, external finance premium and mortgage interest rate are annual and in percentage points. The loan-to-value and leverage ratios are in percentage points.

[here]

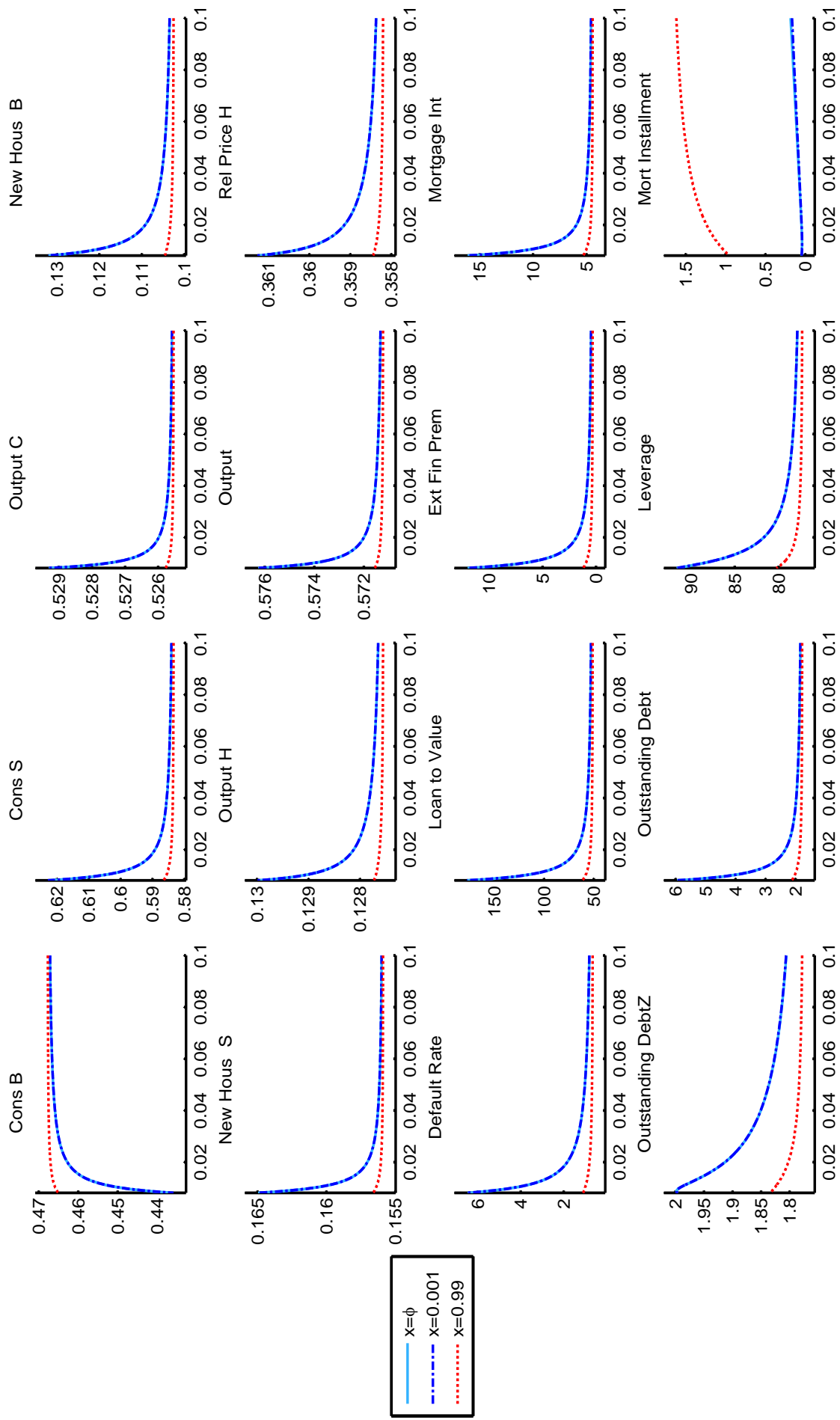


Figure 3: Steady State as  $\phi$  varies in the interval  $(0, 0.1)$

Default rates, external finance premium and mortgage interest rate are annual and in percentage points. The loan-to-value and leverage ratios are in percentage points.

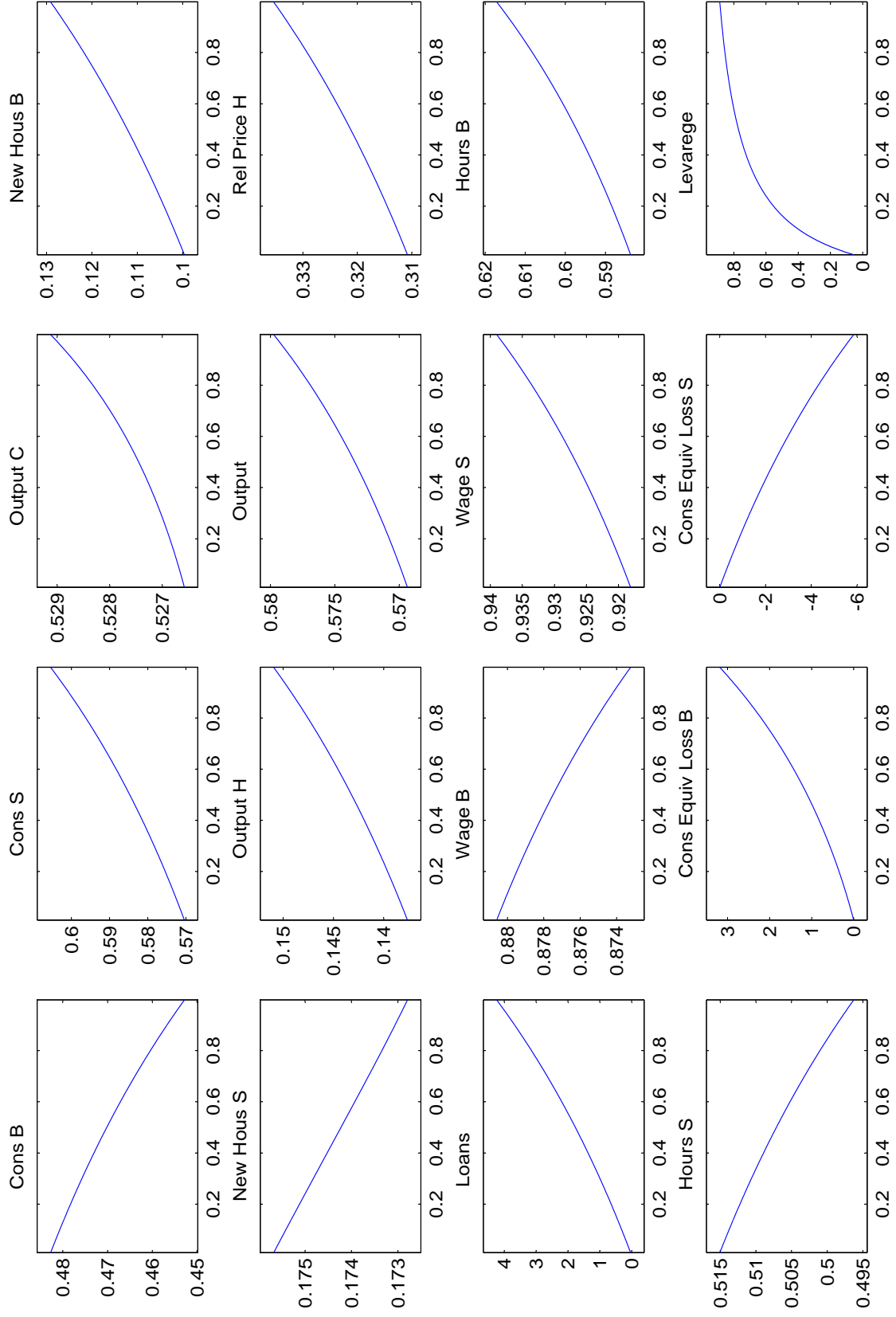


Figure 4: Steady State as  $LTV$  varies in the interval  $(0, 1)$

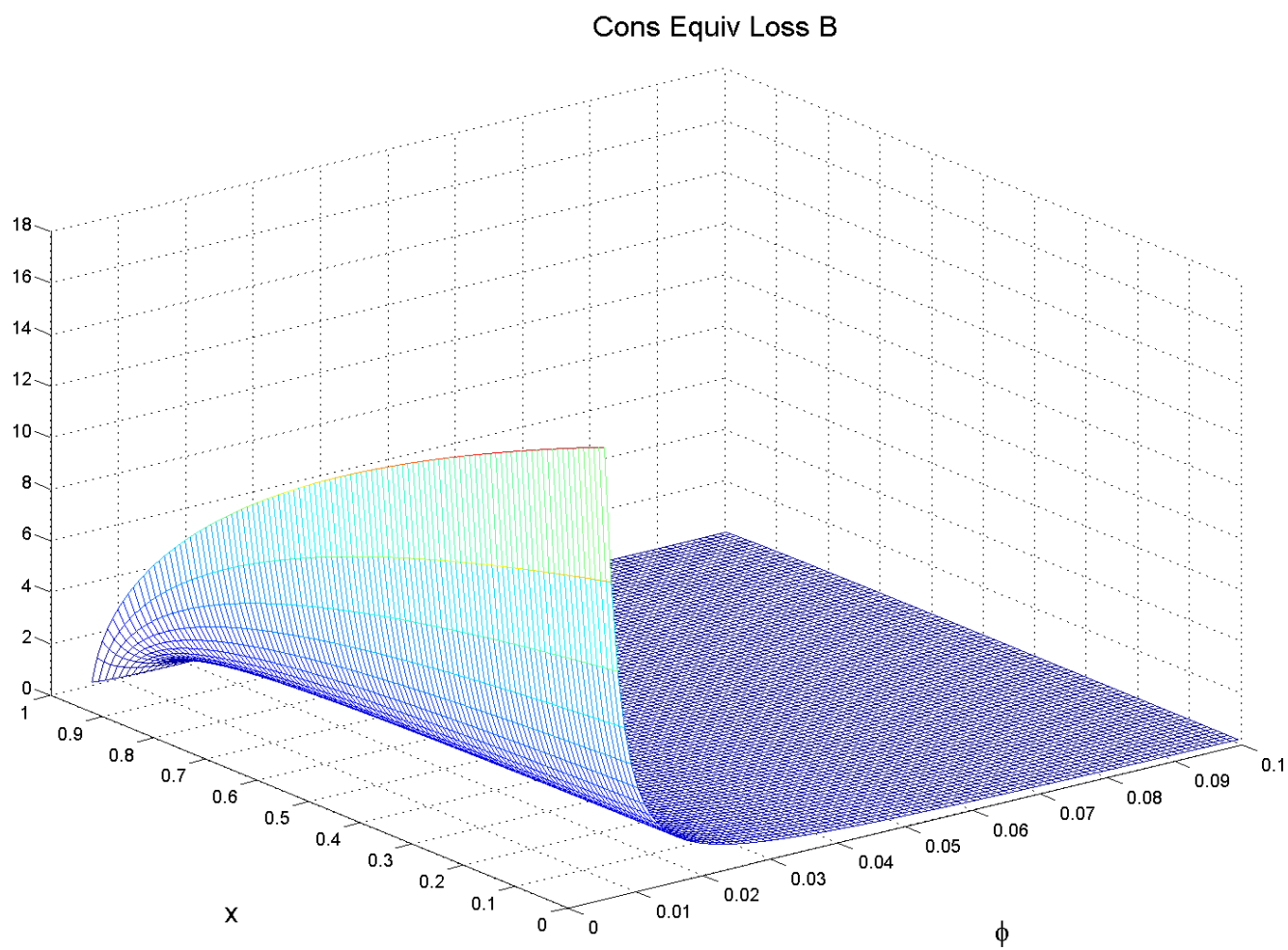


Figure 5: Borrowers' steady-state consumption equivalent welfare losses

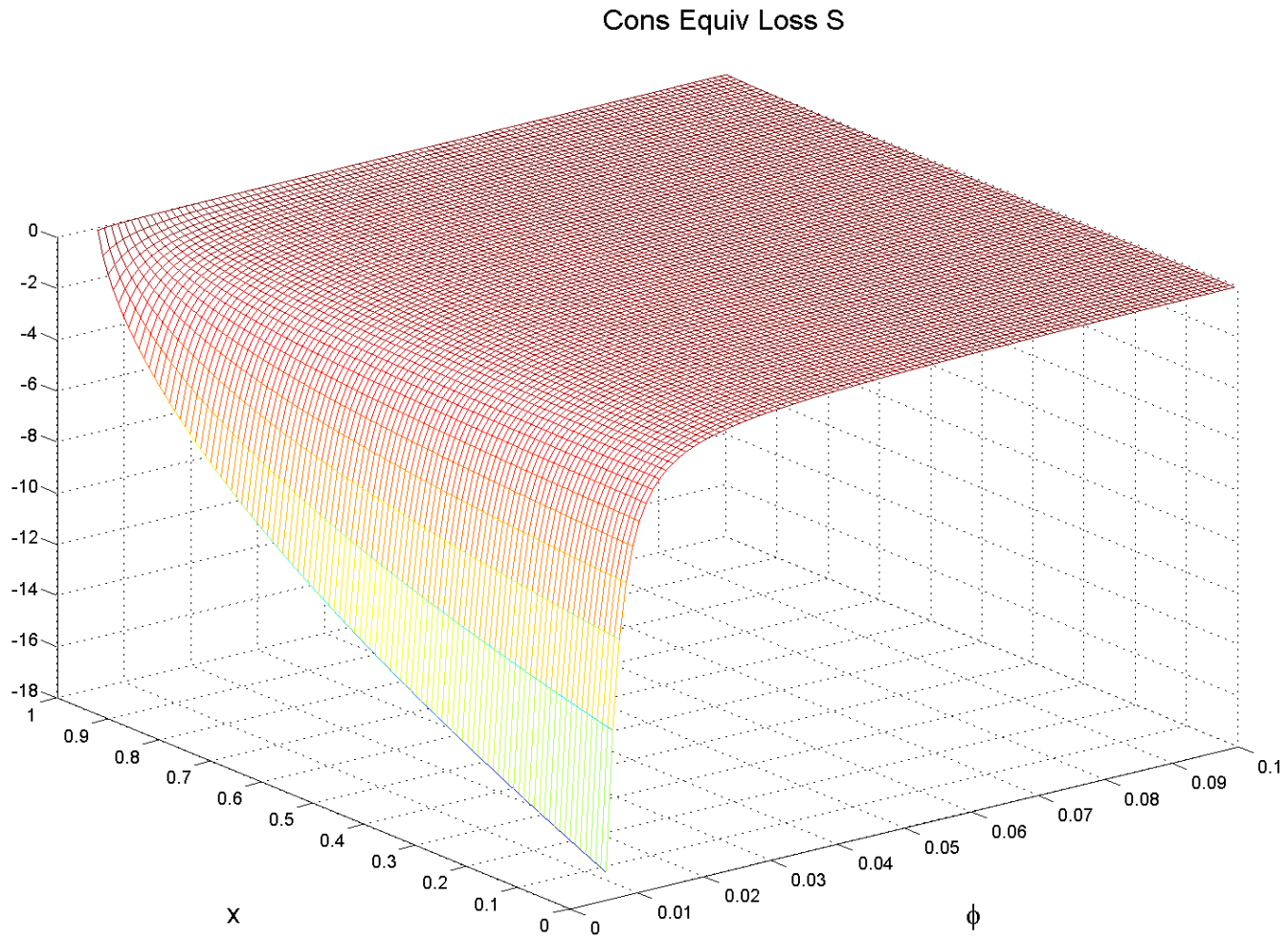


Figure 6: Savers' steady-state consumption equivalent welfare losses

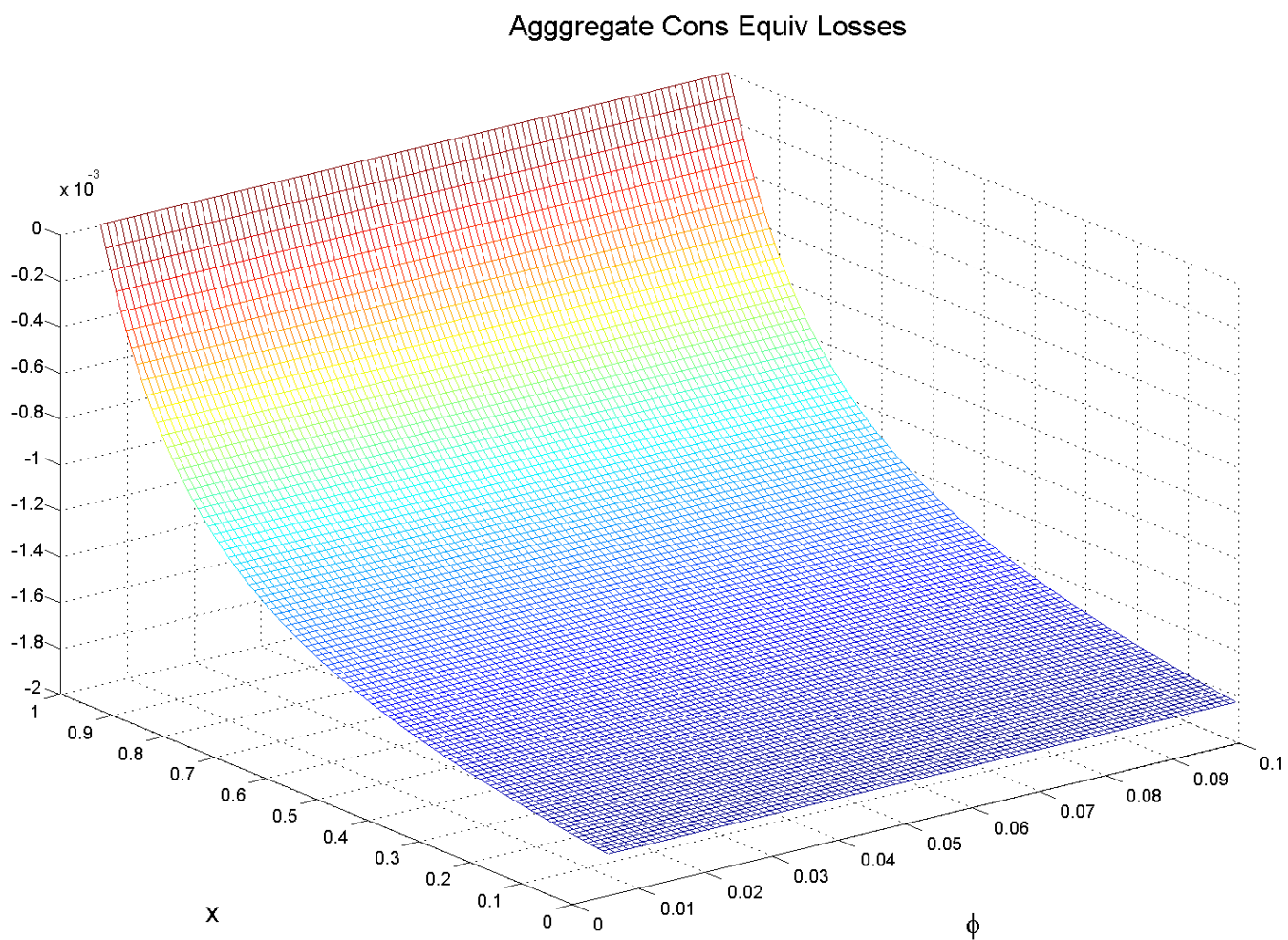


Figure 7: Aggregate steady-state consumption equivalent welfare losses