

# Financial Innovation in Sovereign Borrowing and Public Provision of Liquidity

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## Abstract

This paper studies how the introduction of new assets in sovereign debt markets can increase a country's level of investment and welfare. I propose a model where public debt has a liquidity purpose for the domestic private sector and is demanded as a saving vehicle by more patient international investors. The government commits to repay but is constrained by its fiscal capacity which is low when the private sector needs outside liquidity. The main result of the paper is that the government can increase domestic investment by tranching its fiscal capacity, increasing the number of assets supplied and introducing state-contingency or safe assets. The intuition for this result is that these new assets decrease the cost of liquidity hoarding for the private sector. The optimal asset design allocates residual fiscal capacity to foreigners and exploits their different rationale to hold public assets. The comparative statics of the model are consistent with recent changes in investor base for public debt as a whole and for relatively safer debt instruments. Using a dataset on public debt ownership of advanced economies I test the predictions of the model regarding the effect of domestic collateral constraints and risk premia on the proportion of public debt held abroad.

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# 1 Introduction

The set of instruments that governments all over the world issue is large and has expanded over time. Governments issue debt with different maturities, bonds indexed to inflation or to some reference interest rate and some countries issue debt in different currencies. Financial innovation has transformed sovereign debt markets of advanced and emerging economies.

This process of financial innovation is still ongoing. To give a recent example, the United States approved in July 2013 the issuance of Floating Rate Notes (FRNs) indexed to the 13-week US Treasury bill auction rate and the first auction of this type of securities is expected to be held in January 2014.

The timing, circumstances and country characteristics of governments introducing financial innovations in sovereign debt markets differ widely. For instance, inflation-indexed bonds are issued by emerging economies as well as advanced economies. Some of them started issuing them in the nineties and 2000s and others as early as the forties. Moreover there is no systematic distinction in the timing across advanced and emerging economies ([Borensztein et al. \(2004\)](#)). A big proportion of emerging markets' borrowing is done in foreign currency but several advanced economies also issue part of their debt in a foreign currency. See Appendix ([D.1](#)) for some examples.

Another relevant characteristic of sovereign debt markets is that they are open to a large variety of investors. A common distinction made is between domestic and foreign holders of debt and within each of these whether it is the official sector, mostly Central Banks; the financial sector or the non-financial sector. These investors might differ in their degree of patience or in their rationale to hold public debt: as a vehicle to save, as a way to store liquidity or as a policy tool.

This paper combines the two previous observations and studies how the composition of public debt investor base, in particular local vis-a-vis foreign debt holders, can shape the government's financial innovations in sovereign debt markets.

I propose a model where the local private sector uses public debt to hoard liquidity for a future and uncertain liquidity shock in the spirit of [Holmstrom and Tirole \(1998\)](#) and [Woodford \(1990\)](#). I assume that the government's future fiscal capacity is uncertain and that sovereign debt markets are open to more patient risk-neutral international investors who demand the public assets as a savings vehicle.

The main result of the paper is that the government can increase domestic investment if instead of issuing one bond which pays off its risky fiscal capacity in the next period, it tranches its fiscal capacity and issues two different assets. For instance a safe and a risky asset or two Arrow-Debreu securities. The intuition for this result is that this new asset

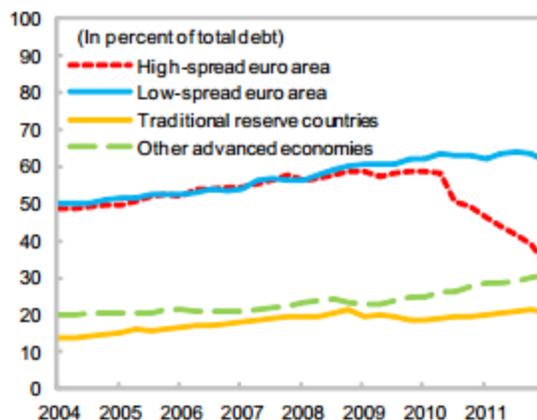


Figure 1: Share of foreign investors in percentage of total debt for advanced economies (Source: [Arslanalp and Tsuda \(2012\)](#))

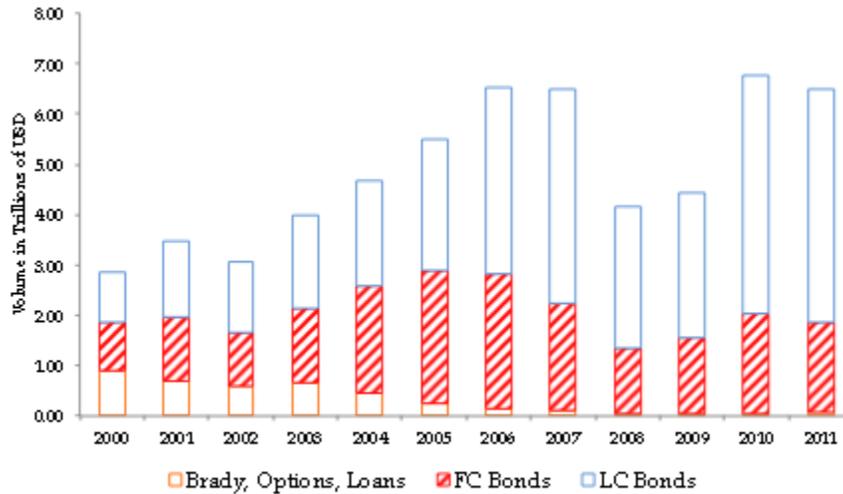
combinations lower the cost of liquidity hoarding for the private sector. The residual fiscal capacity that is not allocated to the domestic private sector is designed to attract risk-neutral international investors who do not have a liquidity motive for holding debt.

This model can shed light on recent changes in ownership of public debt as a whole and differences in the investor composition for different debt instruments. In particular, it is consistent with two sets of stylized facts.

First, the sudden reversal in the share of government debt held by non-residents for the Euro periphery as reported in [Arslanalp and Tsuda \(2012\)](#) and [Merler and Pisani-Ferry \(2012\)](#) and shown in figure (1). Through the lens of the model this can be explained by a drop in government's fiscal capacity or by a tightening of domestic collateral constraints since both would bring about an increase in domestic demand for public debt.

Second, it is consistent with recent sizeable shifts in relative ownership of different debt instruments. [Du and Schreger \(2013\)](#) report that the share of local-currency (LC) debt in total emerging market debt trading volume has increased from 35% in 2000 to 71% in the 2011. This increase has been partly due to an increase in LC debt held by foreigners and partly due to a drop in the amount of foreign-currency (FC) held abroad especially from 2006 onwards as we see figure (2).

Similarly, the proportion of US inflation-indexed securities (TIPS) held abroad has increased sharply in the recent years. Between 2000 and 2008 this fraction fluctuated between 5% and 20% of total outstanding TIPS ([Bondwave \(2010\)](#)). As we see in figure (3) after 2008 this proportion has increased steadily. It rose to almost 30% in 2009 and 2010 and has reached 36% in 2012. The proportion of public debt as a whole held abroad also increased but at a



Source: Annual Debt Trading Volume Survey (2000-2011) by Emerging Market Trading Association (EMTA)

Figure 2: Emerging markets offshore trading volume by instrument type, in Trillions of USD (Source: Du and Schreger (2013))

much slower pace.<sup>1</sup>

The facts on LC debt and TIPS taken together point to a shift of riskier debt instruments towards foreign investors.<sup>2</sup> From the perspective of the model, increases in liquidity needs or drops in fiscal capacity would be consistent with patterns like this. Both increase the domestic demand for assets that will provide liquidity when it is needed such as relatively safer assets, shifting the ownership of riskier debt instruments abroad.

In the empirical front this paper tests the relationships delivered by the benchmark model about proportion of debt held abroad and domestic liquidity conditions. Country-level data for advanced economies between 2004 and 2011 on investor base composition, credit conditions and risk premia confirm the predictions of the model.

The paper is organized as follows. The remainder of this section discusses the related literature. Section (2) introduces the model and presents the benchmark scenario with only one public bond. Section (3) presents the general financial innovation problem that a benevolent government solves. This section also proposes combination of assets that will implement the optimal allocation from the planner’s problem. Section (4) highlights the benefits of finan-

<sup>1</sup>In particular, it rose from 32% in 2007 to 38% in 2012 (Arslanalp and Tsuda (2012)).

<sup>2</sup>LC debt is riskier than FC debt because in bad states of the world, when the government has difficulties in meeting its debt obligations it is more likely that LC debt will be inflated away since the domestic government only has control over its own currency. TIPS are the relatively riskier debt instrument with respect to nominal debt because they pay less in periods of deflation when the prices of other assets are also depressed.

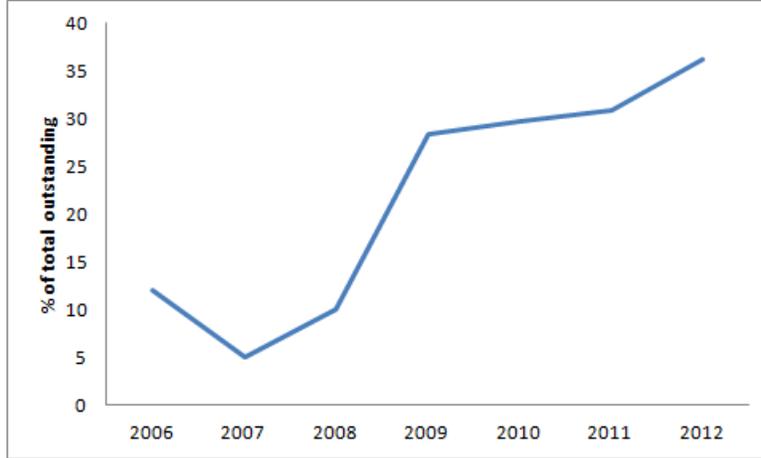


Figure 3: Foreign ownership of US Treasury Inflation-Protected Securities (TIPS) (Data source: US Treasury Direct and Treasury International Capital (TIC) system)

cial innovation in sovereign borrowing and its complementarities with financial integration. Section (5) presents some comparative statics results when the government issues more than one public asset. Section (6) tests empirically some of the comparative statics results from the benchmark model and the differential effect on financial innovators in sovereign debt markets and finally section (7) concludes.

## 1.1 Literature Review

This paper is related most directly to two strands of literature. First, the model I present builds upon the models about public provision of liquidity such as [Holmstrom and Tirole \(1998\)](#), [Holmstrom and Tirole \(2011\)](#) and [Woodford \(1990\)](#). Second, in the spirit of [Allen and Gale \(1994\)](#) I study how financial innovation can decrease the cost of liquidity hoarding and hence increase investment.

To the best of my knowledge there has been little work about financial innovation in sovereign borrowing. Papers such as [Sandleris et al. \(2011\)](#) and [Hatchondo and Martinez \(2012\)](#) have studied GDP-linked bonds regarding sovereign debt sustainability, default incentives and risk-sharing benefits. However these models allow for sovereign default and consider a particular financial innovation, GDP-linked bonds, and their objective is not to solve a general financial innovation problem for the government. This paper instead does not allow for default and imposes commitment on the government's side but studies a general financial innovation framework. Also, contrary to the papers mentioned above, the model presented in this paper features domestic debt and a liquidity purpose of public debt.

This paper also contributes to the debate about financial innovation in public debt where

there have been several proposals to make governments' securities more state-contingency which would improve risk-sharing between debtors and creditors. The most relevant ones have proposed to make debt contingent on commodity prices or another external variable relevant to the country (Caballero (2002, 2003)) or to create securities indexed to GDP (Shiller (1993, 2003)). State-contingency in this model is advisable in this model due to the existence of two different types of investors who hold public debt for two different rationales: liquidity hoarding and saving.

This paper is also related to the literature on shortage of safe assets (Caballero and Krishnamurthy (2009), Caballero and Farhi (2013), Gourinchas and Jeanne (2012)) since the government increases domestic investment and welfare by issuing safe assets. Finally this paper relates to recent papers on public debt ownership such as Broner et al. (2010), Brutti and Saure (2013), Broner et al. (forthcoming), Erce (2012) and Gennaioli et al. (forthcoming).

## 2 One Defaultable Bond

### 2.1 Set-up

We consider a three period economy with time indexed  $t = 0, 1, 2$  and a single good. There are three types of agents: an entrepreneur, a consumer and a foreign investor. All agents are risk-neutral and get utility from consumption in all three periods. The first two agents have a discount factor of  $\beta = 1$  whereas the foreign investor's discount factor is denoted by  $\beta^* > 1$ .

At date 0 the entrepreneur invests in a project of variable scale and chooses initial investment scale  $I$ . Her initial net worth equals  $A$ . The consumer has a big endowment that can lend to the entrepreneur. At date 1 the project is hit by a liquidity shock  $s$  which makes the project require an injection of  $s$  units of good per unit of initial investment to continue. The liquidity shock can take two values  $\{s_H, s_L\}$ , where  $s_H > s_L$ , with respective probabilities  $\{\lambda, 1 - \lambda\}$ . The entrepreneur after the project is hit by the liquidity shock can decide the continuation scale of the project  $i(s) \in [0, I]$ . At date 2 the project gives a private return to the entrepreneur of  $R > 1$  for each unit of investment that was carried through to date 2. From this return only  $\rho < R$  is pledgeable to consumers.

The entrepreneur's chooses initial investment  $I > A$ . Under  $R > 1$  the project earns a higher return than the market rate. Therefore the entrepreneur wants to be a net borrower and invest more than her own wealth. At date 0 the entrepreneur borrows from consumers  $I - A$  as well as the cost of insuring for the future liquidity shock. The foreign investor cannot lend to the domestic entrepreneur.

**Assumption 1.** *International investors cannot lend directly to domestic entrepreneurs. Only domestic consumers can do so.*

This assumption is a reduced form way to capture that domestic consumers have a comparative advantage in lending to domestic entrepreneurs with respect to foreign investors. Borrowing from abroad would be too costly for entrepreneurs. This might capture domestic consumers having better information about the entrepreneur's project or the domestic conditions in which the entrepreneur's project is carried out, better monitoring capacity or a stronger protection by domestic bankruptcy laws.

At date 0 the government issues public bonds which give a return at date 1. The supply is fixed and normalized to one. The domestic entrepreneur will demand the bond as a way to insure against the future liquidity shock. Foreign investors also demand these bonds in an integrated sovereign debt market but they do not have a liquidity motive for holding debt. Instead, they buy bonds as a saving vehicle.

**Assumption 2.** *The only available security for the entrepreneur to insure against the liquidity shock is the public bond. The supply of the public bond is fixed and it is issued in an integrated debt market, where entrepreneurs compete for the asset with more patient foreign investors.*

The first part of this assumption acts only as a simplification. All the results of the model would still hold even if entrepreneurs had access to other assets as long as the value of the assets is not enough to fulfill the entrepreneur's liquidity needs. This simplification is especially applicable to financial crises when other asset prices collapse and sovereign debt becomes a highly valued asset due to its safety and liquidity (IMF (2012), Krishnamurthy and Vissing-Jorgensen (2012)).

The supply of bonds being fixed can be interpreted as the public borrowing needed to cover an exogenous and fixed level of government expenditures. The focus of this paper is the relative holdings of public debt between internationals and domestics and how the existence of both types of investors shapes the introduction of financial innovation in sovereign debt markets. Thus, we are going to abstract from the bond supply decision and take it as exogenous and fixed.

The last part of assumption (2), the fact that international investors are more patient than domestics, should not be taken literally. It is a reduced form to capture a higher foreign willingness to hold sovereign debt. When performing comparative statics, an increase in the international discount factor can be interpreted as an increase in world risk or as an increase in the available income internationally to invest in sovereign debt.

The government issues the bond at date 0 and receives  $q$  units of good per bond which it transfers to consumers. It commits to repay and redeems the bond at date 1 by taxing consumers and repaying bond holders the face value of their bond. The government's taxation power or fiscal capacity at  $t = 1$  is uncertain and perfectly correlated with the liquidity shock that hits the entrepreneur's project. In particular at date 1, the government can tax  $\bar{\eta}$  when  $s = s_L$  and  $\underline{\eta}$  when  $s = s_H$ , where  $\underline{\eta} < \bar{\eta} \leq 1$ .

**Assumption 3.** *The fiscal capacity shock and the private sector liquidity shock are perfectly correlated.*

Since the government commits to repay the bond issues in date 0, the payoff structure of the public bond is given by  $(\underline{\eta}, \bar{\eta})$  in states  $(s_H, s_L)$  respectively. This is a risky bond that pays less in the state of the world where liquidity needs are high. In other sections we will introduce more than one asset with different payoff structures.

Assumption (3) is a reduced form way to capture the observed temporal connection between banking crises and sovereign debt crises reported in [Reinhart and Rogoff \(2009\)](#), [Arellano and Kocherlakota \(2012\)](#), [Sosa-Padilla \(2011\)](#), [Balteanu and Erce \(2012\)](#) and [Borensztein and Panizza \(2008\)](#).<sup>3</sup>

According to this assumption in one of the states of the world the financial sector requires liquidity, which absence intervention could result in a banking crisis where profitable projects would need to be liquidated. It is in that same state of the world that the government imposes a higher haircut on public debt issued the period before, the bond repays  $\underline{\eta} < \bar{\eta}$ , resembling a sovereign debt crisis.<sup>4</sup> In this sense the set-up is similar to [Bolton and Jeanne \(2011\)](#) where they treat default as exogenous and then microfound it with a productivity shock. Here the shock that microfounds a lower repayment is a fiscal shock.

## 2.2 Demand from Investors

### 2.2.1 Demand from Domestic Investors

The entrepreneur maximizes her expected net return of the project. Since the entrepreneur wants to maximize the initial investment scale of the project it is optimal to assign all

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<sup>3</sup>Several of these papers explore whether banking crises preceded sovereign debt crises or viceversa. For the purpose of this model we assume that both happen simultaneously.

<sup>4</sup>Strictly speaking it is not a sovereign debt crisis because the government does not impose a haircut on investors. In this model there is commitment and  $(\underline{\eta}, \bar{\eta})$  are the assets' payoffs. It is not a decision to default on a part of the government debt. However, at date 1 these are observationally equivalent: the asset only pays  $\underline{\eta} < \bar{\eta}$ .

pledgeable returns  $\rho$  to consumers, keeping the illiquid return for herself:  $R - \rho$  of the amount that is carried through.

Denoting by  $q$  the price of the liquid asset at  $t = 0$ , the entrepreneur's problem is given by:

$$\begin{aligned} \max_{\{I, i(s_H), i(s_L), z\}} \quad & (R - \rho)(1 - \lambda)i(s_L) + (R - \rho)\lambda i(s_H) \\ \text{s.t.} \quad & (\rho - s_L)(1 - \lambda)i(s_L) + (\rho - s_H)\lambda i(s_H) + (\lambda\eta + (1 - \lambda)\bar{\eta})z \geq I - A + zq \\ & i(s_H)(s_H - \rho) \leq \eta z \end{aligned}$$

where  $i(s_L)$  and  $i(s_H)$  are the continuation scales in both states and  $z$  is the amount of bonds bought at  $t = 0$ .

The first constraint is the entrepreneur's budget constraint by which the entrepreneur's initial investment scale plus the purchase of the assets  $qz$  need to be less or equal than the entrepreneur's initial wealth plus the expected net return from the project and the expected return from the asset. It corresponds to the consumer's participation constraint. In order for the consumer to be willing to lend to the entrepreneur at date 0 its expected return from the project must be at least what the entrepreneur borrowed at date 0,  $I - A + zq$ .<sup>5</sup>

The second constraint of the problem is the collateral constraint which imposes that the outside funds required for reinvestment in the high liquidity shock state are less or equal to the return from the liquid asset in that state of the world.

I assume that  $s_L < \rho < s_H < R$  which implies that when the liquidity shock is low the project is self-financed and entrepreneur's inside liquidity is enough to withstand the liquidity shock. Instead when the liquidity shock is high the project needs prearranged financing. Thus in state  $L$  full continuation is always optimal,  $i(s_L) = I$  while in state  $H$  full continuation might not be optimal. Denoting  $\frac{i(s_H)}{I} \equiv \chi$  and  $\lambda\eta + (1 - \lambda)\bar{\eta} \equiv \Pi$  we can rewrite the entrepreneur's problem as:

$$\begin{aligned} \max_{\{I, \chi, z\}} \quad & (R - \rho)(1 - \lambda + \lambda\chi)I \tag{1} \\ \text{s.t.} \quad & (\rho - s_L)(1 - \lambda)I + \lambda\chi(\rho - s_H)I + \Pi z \geq I - A + zq \\ & \chi I(s_H - \rho) \leq \eta z \end{aligned}$$

When  $q > \Pi$ , both constraints bind. Therefore, the collateral constraint expresses the amount of bonds demanded by entrepreneurs at  $t = 0$  in terms of the initial investment scale

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<sup>5</sup>The contract between entrepreneur and consumer will always assign all liquid or pledgeable returns to the consumer in order to maximize the initial investment scale of the project. Thus, in state  $s = s_L$  the consumer will get a repayment of  $\eta z + (\rho - s_L)i(s_L)$  and in state  $s = s_H$  the consumer obtains  $\eta z + (\rho - s_H)i(s_H)$  which corresponds to the left hand side of the budget constraint.

$I$  and the continuation scale  $\chi$  as well as parameters:

$$z = \frac{\chi I(s_H - \rho)}{\eta} \quad (2)$$

Intuitively the amount of bonds demanded is increasing in the continuation scale  $i(s_H)$  and decreasing in the bond's repayment fraction in the high liquidity need state of the world,  $\eta$ . Using this expression for  $z$  in the budget constraint, the initial investment is given by:

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda\chi(\rho - s_H) + \frac{\chi(s_H - \rho)}{\eta} [q - \Pi]} \quad (3)$$

From (3) we see that  $I'(\chi) < 0$ , so the entrepreneur faces a scale-liquidity trade-off as in [Holmstrom and Tirole \(1998\)](#). If the entrepreneur wants to hold more liquidity to withstand the future liquidity shock she has to choose a lower initial investment scale since both liquidity hoarding and initial investment scale are chosen at date 0.

The equivalent to maximizing the expected return to the investment is to minimize the unit cost of investment

$$\min_{\{\chi\}} c(\chi, q, \eta, \bar{\eta}, \lambda, \Theta) \equiv \frac{1 + s_L(1 - \lambda) + s_H\lambda\chi + \frac{\chi(s_H - \rho)}{\eta} [q - \Pi]}{1 - \lambda + \lambda\chi}$$

where  $\Theta \equiv (s_H, s_L, \rho)$  is a vector of parameters regarding the project. The solution to this problem depends on the price of the bond  $q$ :

$$\chi(q, \eta, \bar{\eta}, \lambda, \Theta) = \begin{cases} 1 & \text{if } q \in [\Pi, q^{max}) \\ \in (0, 1) & \text{if } q = q^{max} \\ 0 & \text{if } q > q^{max} \end{cases}$$

where  $q^{max}$  is given by  $\frac{\partial c(\chi, q^{max}, \eta, \bar{\eta}, \lambda, \Theta)}{\partial \chi} = 0$ .<sup>6</sup>

Thus, the demand for liquidity from local investors is given below and it is denoted by  $z^L$ . It is decreasing in its price  $q$ :

$$z^L(q, \eta, \bar{\eta}, \lambda, \Theta) = \begin{cases} 0 & \text{if } q > q^{max} \\ \frac{\chi I(s_H - \rho)}{\eta} & \text{if } q = q^{max}, \text{ where } \chi \in (0, 1) \\ \frac{I(s_H - \rho)}{\eta} & \text{if } q \in (\Pi, q^{max}) \end{cases}$$

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<sup>6</sup>Solving this derivative we obtain that the expression for  $q^{max}$  is given by  $q^{max} = \lambda\eta + (1 - \lambda)\bar{\eta} + \frac{\lambda(1 + s_L(1 - \lambda) - s_H)\eta}{(s_H - \rho)(1 - \lambda)}$ .

where  $I$  is given by substituting the price  $q$  in 3.

### 2.2.2 Demand from International Investors

The demand from foreign investors,  $z^F$ , is given by their valuation of the bond, which is determined by their discount factor and the bond's expected payoff  $\lambda\underline{\eta} + (1 - \lambda)\bar{\eta} \equiv \Pi$ . Their demand for bonds is perfectly elastic at  $q = \beta^*\Pi$ , they will demand any positive amount of bonds as long as  $q = \beta^*\Pi$ .

## 2.3 Market Clearing

Market clearing in the bond market at  $t = 0$  implies that  $z^L(q, \underline{\eta}, \bar{\eta}, \lambda, \Theta) + z^F(q, \underline{\eta}, \bar{\eta}, \lambda) = 1$ . Necessarily  $q \geq \beta^*\Pi$ , otherwise the demand from international investors would be infinite. There are two possible equilibria, one where all debt is held domestically and one where foreign investors hold part of it. Whether we are in one or the other will depend on parametric conditions. We concentrate on the case where foreign investors hold part of the debt and leave the case where all debt is held at home for Appendix (A).

In the equilibrium where international investors hold part of the supplied public bonds, their valuation pins down the price of debt:  $q = \beta^*\Pi$ . The bond is sold at a premium, that is,  $q - \Pi > 0$ , because  $\beta^* > 1$ . This means that the bond's price is always larger than its expected value  $q > \Pi$ . We make parametric assumptions such that also  $q = \beta^*\Pi < q^{max}$ .<sup>7</sup>

Thus, at date 0 the demand from international investors is given by the section of the domestic demand for bonds where  $q \in (\Pi, q^{max})$ . In that case  $\chi = 1$ , the project is fully continued in both states of the world. Substituting this and the price for the public asset in (3) we obtain the initial scale of investment which is given by

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\bar{\eta}}{\underline{\eta}})} \quad (4)$$

and is proportional to the entrepreneur's initial wealth  $A$ . It is multiplied by the equity multiplier  $\frac{1}{\left[1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\bar{\eta}}{\underline{\eta}})\right]}$   $> 1$  which defines the maximum the entrepreneur can leverage its own initial capital.

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<sup>7</sup>The parametric assumption  $\beta^*\Pi < q^{max} = \Pi + \frac{\lambda(1 + s_L(1 - \lambda) - s_H)\underline{\eta}}{(s_H - \rho)(1 - \lambda)}$  can be rewritten as an upper bound on the foreign discount rate  $\beta^*$  as  $\beta^* < 1 + \frac{\lambda(1 + s_L(1 - \lambda) - s_H)\underline{\eta}}{(s_H - \rho)(1 - \lambda)\Pi}$ . Intuitively, foreigners should value public debt more than local entrepreneurs but should not value public debt "too much" so that their demand does not drive bond prices too much.

We see that the maximum leverage per unit of own capital is increasing in the pledgeable return  $\rho$ . It is decreasing in the total expected cost of the project  $1 + s_L(1 - \lambda) + s_H\lambda$  and the cost of liquidity hoarding which is the given by last sumand in the denominator in (4).

Two points are worth highlighting regarding the cost of liquidity hoarding. First, for the liquidity hoarding to have an effect on the equity multiplier and decrease investment it is key that foreign investors are more patient than domestics. If  $\beta^*$  were to equal 1 the level of investment would equal  $\frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$  and would not be affected by the demand and cost of liquidity. The intuition for this is that if foreign investors were as patient as domestics, since they do not demand public debt as a way to hoard liquidity but in order to save, they would drive the premium at which public debt is sold,  $q - \Pi$ , to 0. The domestic entrepreneurs would be able to buy liquidity at no cost. This would increase the level of investment.

Second, the novel relationship that this model delivers is that the investment level is decreasing in the ratio of fiscal capacities in the low and high liquidity need states. With only one public bond the  $\frac{\bar{\eta}}{\eta}$  ratio parametrizes the amount of wasted liquidity. Wasted liquidity is the amount of useless liquidity that the entrepreneur is forced to purchase when she does not need it ( $\bar{\eta}$ ) for each unit of liquidity she buys for the state when she does need the return ( $\eta$ ). Equilibrium investment is decreasing in the wasted liquidity, the lower this quantity is, the higher investment. The intuition for this is that wasted liquidity increases the cost of liquidity hoarding.

Finally the amount of bonds demanded by local entrepreneurs is  $\frac{I(s_H - \rho)}{\eta}$  where substituting  $I$  for its expression from (4) and rearranging we obtain:

$$z^L = \frac{A(s_H - \rho)}{\eta [1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)] + (s_H - \rho)(\beta^* - 1)\Pi} \quad (5)$$

which is decreasing in the asset returns  $\eta$  and  $\bar{\eta}$ .

At price  $q = \beta^*\Pi$  international investors will demand any residual bonds not demanded by locals. By market clearing the quantity of bonds demanded by internationals is the following:

$$z^F = 1 - z^L \quad (6)$$

For this to be the equilibrium it must be that  $z^L < 1$  so that both domestic entrepreneurs as well as foreign investors hold the public bond. The condition that  $z^L < 1$  boils down to a lower bound on the bond return in the state when  $s = s_H$ :

$$\eta > \frac{(s_H - \rho)(A - (1 - \lambda)\bar{\eta})}{1 - (\rho - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)} \quad (7)$$

Condition (7) is intuitive. For foreign investors to hold part of the publicly supplied bond which is fixed to 1 it must be that the liquidity provided by the bond when  $s = s_H$  is high enough to meet the reinvestment needs. In Appendix (A) I discuss the equilibrium where (7) does not hold,  $z^L = 1$  and all debt is held at home by domestic entrepreneurs.

## 2.4 Comparative Statics

A number of comparative statics are interesting to understand the workings of the model and will be relevant for the empirical analysis. An increase in  $s_H$  and a decrease in  $s_L$  such that the total cost per unit of investment,  $1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)$ , remains constant brings about an increase in the amount of debt held by locals and by market clearing, a decrease in the amount of debt in the hands of international investors. An increase in  $s_H$  is akin to a tightening of domestic collateral constraints which increases the need for public liquidity, increasing the demand for bonds at home. Initial investment scale  $I$  decreases because of the scale-liquidity trade-off: the higher the reinvestment shock in bad times, the higher the liquidity provision that the entrepreneur must make at date 0 and thus the lower the initial investment scale the entrepreneur can choose.

For the purpose of the comparative statics we can set  $\bar{\eta} = 1$ . This implies that in the good state when liquidity needs are low the government can fully redeem the public bond by taxing consumers. In the bad state when private liquidity needs are high the government bond pays less, the bond pays  $\eta \equiv \eta < 1$ . An increase in the repayment fraction  $\eta$  decreases the amount of bonds held by domestics, since local demand for public bonds is decreasing in the amount the bonds repay. By market clearing the amount of debt held by international investors increases. The total amount of liquidity held by domestic entrepreneurs,  $\eta z_L$  also increases since

$$\eta z_L = \frac{A(s_H - \rho)}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + \frac{1-\lambda}{\eta})}$$

when  $q = \beta^* \Pi$ . This amount is increasing in  $\eta$ . Investment increases because of the wasted liquidity force described before. An increase in  $\eta$  implies a return at  $s = s_H$  closer to 1 which lowers wasted liquidity purchased by the entrepreneur for state  $s = s_L$ .

Finally, an increase in the patience of international investors parametrized by an increase in  $\beta^*$  decreases the local demand for bonds and lowers domestic investment. A higher international discount rate increases the price of the public bond for domestic entrepreneurs too because debt markets are integrated (Assumption 2). Domestic investment is also lower when international investors become more patient. The increased demand from international

investors crowds-out domestic demand for bonds and domestic investment. The comparative statics with  $\beta^*$  is a reduced form way to capture an increase in the foreign demand for domestic public bonds.

### 3 Financial Innovation

The comparative statics for  $\beta^*$  implied that the higher the patience from international investors is, the bigger the crowding-out effect on investment and domestic demand for the public asset. This suggests that the government might be able to improve the allocation if it lowers the competition for public liquidity coming from abroad.<sup>8</sup>

In this section we suppose that the government issues two assets and chooses payoffs  $(x_1^H, x_1^L)$  and  $(x_2^H, x_2^L)$  respectively in states  $s = s_L$  and  $s = s_H$  to maximize total welfare.

We will see that indeed the government can improve welfare with respect to the case with only one bond if the securities designed still attract international investors and if the government tranches its fiscal capacity to reduce the amount of wasted liquidity.

#### 3.1 Entrepreneur's Problem

We now write the entrepreneur's problem with two assets available as liquidity vehicles, which is a generalization of Problem (1):

$$\begin{aligned}
 \max_{\{\chi, z_1, z_2, I\}} \quad & (R - \rho)(1 - \lambda + \lambda\chi)I & (8) \\
 \text{s.t.} \quad & (\rho - s_L)(1 - \lambda)I + \lambda\chi(\rho - s_H)I + \Pi_1 z_1 + \Pi_2 z_2 \geq \\
 & I - A + z_1 q_1 + z_2 q_2 \\
 & \chi I (s_H - \rho) \leq x_1^H z_1 + x_2^H z_2
 \end{aligned}$$

where  $\Pi_1$  and  $\Pi_2$  denote the expected payoffs of both assets:  $\Pi_1 = \lambda x_1^H + (1 - \lambda)x_1^L$  and  $\Pi_2 = \lambda x_2^H + (1 - \lambda)x_2^L$ .

**Proposition 4.** *When  $q_1 > \Pi_1$  and  $q_2 > \Pi_2$  both constraints bind.*

*Proof.* To see this, note that  $q_1 \geq \Pi_1$  and  $q_2 \geq \Pi_2$ . The price of the assets can never go below their expected values, otherwise consumers who are assumed to have a big endowment

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<sup>8</sup>In appendix (B) we show that imposing capital controls and banning all competition from abroad will not increase welfare. As we discuss there, the reason for this is that there are no pecuniary externalities in the entrepreneur's collateral constraint.

would want to postpone all their consumption to date 1. This would drive the price of the liquid assets to their date 1 values, which are the expected values. If  $q_1 > \Pi_1$  or  $q_2 > \Pi_2$ , only entrepreneurs will demand the assets since they have a higher valuation for the assets. To see that if  $q_1 > \Pi_1$  and  $q_2 > \Pi_2$  the budget constraint must bind we rewrite it as:

$$\rho(1 - \lambda + \lambda\chi)I - s_L(1 - \lambda)I - s_H\lambda\chi I \geq I - A + (q_1 - \Pi_1)z_1 + (q_2 - \Pi_2)z_2 \quad (9)$$

Since  $\rho(1 - \lambda + \lambda\chi)I$  enters negatively the entrepreneur's objective function, she will make this term as small as possible choosing to just satisfy the constraint. The collateral constraint binds for  $q_1 > \Pi_1$  and  $q_2 > \Pi_2$  because the entrepreneur will choose  $z_1$  and  $z_2$  just enough to cover the liquidity shock in the high liquidity shock state. Demanding more than this amount would imply that the right-hand side of the budget constraint in (9) increases. Since the budget constraint binds this would increase  $\rho(1 - \lambda + \lambda\chi)$ , which is not in the entrepreneur's interest given her objective function. Hence, from now on we will consider  $q > \Pi$  for both assets and both constraints will bind.  $\square$

Denote by  $\ell$  the unit cost of liquidity. The entrepreneur will choose the asset that will minimize her unit cost of liquidity, that is:  $\ell = \min \left\{ \frac{q_1 - \Pi_1}{x_1^H}, \frac{q_2 - \Pi_2}{x_2^H} \right\}$ . Suppose for concreteness that asset  $j$  minimizes  $\ell$  and that asset  $j$  provides enough liquidity in state  $s = s_H$  to cover all reinvestment needs. In that case  $z_{-j} = 0$  since the entrepreneur does not want to purchase the liquidity using the asset that provides it at a higher cost.

We also know that  $q_j > \Pi_j$  because  $q_j \geq \beta^*\Pi_j$ , with strict equality if international investors also hold part of asset  $j$  issued. Therefore as we proved above both constraints hold with equality. From collateral constraint we obtain the local demand for the relatively cheap asset:

$$\frac{\chi I (s_H - \rho)}{x_j^H} = z_j \quad (10)$$

Substituting this and  $z_{-j} = 0$  in the budget constraint and solving for investment we obtain:

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda\chi(\rho - s_H) + \chi(s_H - \rho)\ell} \quad (11)$$

The entrepreneur's unit cost minimization problem is given below:

$$\min_{\{\chi\}} \frac{1 + s_L(1 - \lambda) + s_H\lambda\chi + \chi(s_H - \rho)\ell}{1 - \lambda + \lambda\chi} \equiv c(\ell, \chi)$$

The solution for the continuation scale  $\chi$  is the following:

$$\chi = \begin{cases} 1 & \text{if } \ell \in (0, \ell^{max}) \\ \in (0, 1) & \text{if } \ell = \ell^{max} \\ 0 & \text{if } \ell > \ell^{max} \end{cases}$$

where  $\ell^{max}$  is a threshold value. To see that this is the schedule for the continuation scale note that because the problem (8) is linear in  $I$  we only need to evaluate the utility levels corresponding to  $\chi = 0$  (continuing only when the shock is low) and  $\chi = 1$  (always continuing). The unit cost for  $\chi = 0$ ,  $c(\chi = 0, \ell) = \frac{1+s_L(1-\lambda)}{1-\lambda}$  and  $c(\chi = 1, \ell) = 1 + s_L(1 - \lambda) + s_H\lambda + (s_H - \rho)\ell$ . Comparing these we obtain that  $c(\chi = 1, \ell) < c(\chi = 0, \ell)$  if and only if

$$\ell < \frac{\lambda}{(1-\lambda)(s_H - \rho)} + \frac{(s_L - s_H)\lambda}{s_H - \rho} \equiv \ell^{max} \quad (12)$$

which equals  $\frac{\partial c(\chi, \ell^{max})}{\partial \chi} = 0$ . Therefore when  $\ell < \ell^{max}$ , investment as a function of  $\ell$  is given by the following expression:

$$I(\ell) = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\ell} \quad (13)$$

From expression (13) we see that  $\frac{\partial I(\cdot)}{\partial \ell} < 0$ , that is investment level is decreasing in the unit cost of liquidity. We will impose throughout that  $\ell < \ell^{max}$  and the project is continued at full scale in both states of the world.

### 3.2 Planner's Objective and Constraints

Domestic welfare  $W$  is given by the utility of consumption in the three periods for both types of agents in the economy, entrepreneurs and consumers,  $U^E$  and  $U^C$ . Both agents have linear utility of consumption and do not discount future payoffs. We assume for this section that the government always wants to fully continue in both states of the world  $\chi = 1$ .<sup>9</sup>

Entrepreneurs consume the expected rent from their investment at date 2. Consumers lend a part  $A - I + q_1 z_1 + q_2 z_2$  of their endowment  $E$  to finance the project initially and for the entrepreneur to prearrange for the future liquidity need. They obtain a return of  $x_1^L z_1 + x_2^L z_2 + (\rho - s_L)I$  when the liquidity shock is low and obtain  $x_1^H z_1 + x_2^H z_2 + (\rho - s_H)I$  when

<sup>9</sup>As in section (2) for this to be optimal we impose an upper bound on the foreign discount factor  $\beta^*$  which will ensure that the prices of the public assets are not too high. In particular, if we assume  $\beta^* - 1 < \frac{1+(s_L-s_H)(1-\lambda)}{(s_H-\rho)(1-\lambda)}$ ,  $\chi = 1$  will be optimal for the two financial innovations problems that we study in this section.

the liquidity shock is high. Also, at date 0 they obtain the proceeds from the total asset issuance  $q_1$  and  $q_2$  and are taxed the face value of both assets at date 1.

Thus, the utility expressions are given by:

$$\begin{aligned} U^E &= (R - \rho)I \\ U^C &= E + A - I - q_1 z_1 - q_2 z_2 + (\rho - s_L)(1 - \lambda)I + (\rho - s_H)\lambda I \\ &\quad + (q_1 - \Pi_1) + (q_2 - \Pi_2) + \Pi_1 z_1 + \Pi_2 z_2 \end{aligned}$$

and total welfare is given by

$$\begin{aligned} W &= E + A + [R(1 - \lambda + \lambda\chi) - s_L(1 - \lambda) - s_H\lambda\chi - 1]I \\ &\quad + (q_1 - \Pi_1)(1 - z_1) + (q_2 - \Pi_2)(1 - z_2) \end{aligned} \quad (14)$$

By market clearing  $1 - z_1$  equals the amount of asset held by international investors,  $z_1^F$  and similarly for asset 2.

$$\begin{aligned} W &= E + A + [R(1 - \lambda + \lambda\chi) - s_L(1 - \lambda) - s_H\lambda\chi - 1]I \\ &\quad + (q_1 - \Pi_1)z_1^F + (q_2 - \Pi_2)z_2^F \end{aligned} \quad (15)$$

The welfare expression in (15) is intuitive. The government wants to maximize the total net surplus from the investment and the liquidity premia,  $q_1 - \Pi_1$  and  $q_2 - \Pi_2$  obtained from international investors. The liquidity premium paid by entrepreneurs to consumers is a transfer across agents which cancels out in the welfare calculation and only the premia coming from abroad matter for welfare in the economy.

The government is constrained by its fiscal capacity, thus asset payoffs must satisfy:

$$x_1^L + x_2^L = 1 \quad (16)$$

$$x_1^H + x_2^H = \eta \quad (17)$$

### 3.2.1 Monotonicity Requirement

If payoffs satisfy monotonicity it must be the case that for both assets:

$$x^L \geq x^H \quad (18)$$

We start our analysis without considering this restriction in section (3.3). Then, we add

the more realistic assumption that public assets pay less in the state of the world that fiscal capacity is low.

In our discussion of assumption (3) in the set-up of the benchmark model in section (2) we argued that the state where fiscal capacity is low and the private liquidity shock is high is a state which would correspond to a state of twin crisis, banking and sovereign debt crisis.

Sovereign debt crises are resolved with a sovereign debt restructuring process. We know that after a debt restructuring process haircuts on debt instruments are positive. Allowing for violations of (18) would imply that haircuts on some debt instruments are negative. Although there is no systematic data about haircuts at the debt instrument level, this seems unrealistic.

We can imagine assets affected differently after a sovereign debt restructuring. For example, we can expect long-term debt more affected than short-term. Long-term debt due date can be adjourned and the payments will be rescheduled. It is more likely that short-term will be paid-off quicker and hence experience no haircut or a small one. In any case, it seems unlikely that it will have a negative haircut.

Also bonds issued under different laws might differ in the final recouped investment. Those under local law are normally hit stronger by a sovereign debt restructuring than those issued under the UK or US law where creditor litigation has increased dramatically the amount of recouped investment (Schumacher et al. (2013)). Again, however, bonds under UK or US law do not recoup more than they invested after a sovereign debt restructuring process which would be the implication of violating (18).

### 3.3 Planner's Problem without Monotonicity Requirements

The planner maximizes expected welfare (15) choosing the asset payoffs and the minimum cost of liquidity hoarding for the entrepreneur,  $\ell$ . In doing so the government is subject to the fiscal capacity constraints and internalizes the effect that its choice has on investment,

demand for the public assets and prices. The government solves the following problem:

$$\begin{aligned} \max_{\{x_1^H, x_1^L, x_2^H, x_2^L, \ell\}} \quad & W \equiv E + A + [R - s_L(1 - \lambda) - s_H\lambda - 1] I(\ell) \\ & + (q_1 - \Pi_1)(1 - z_1(\ell)) + (q_2 - \Pi_2)(1 - z_2(\ell)) \end{aligned} \quad (19)$$

*s.t.*

$$x_1^L + x_2^L \leq 1 \quad (20)$$

$$x_1^H + x_2^H \leq \eta \quad (21)$$

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\ell} \quad (22)$$

$$\ell = \min \left\{ \frac{q_1 - \Pi_1}{x_1^H}, \frac{q_2 - \Pi_2}{x_2^H} \right\} \quad (23)$$

$$0 \leq z_j(\ell) \leq 1 \quad (24)$$

$$q_j \geq \beta^* \Pi_j \quad \text{with inequality only if } z_j(\ell) = 1 \quad (25)$$

where  $j = \{1, 2\}$  and  $\Pi_j$  are assets' expected payoffs defined above.

Constraints (20) and (21) are the fiscal capacity constraints. Equation (22) gives the expression for investment from the entrepreneur's problem which depends on the unit cost of liquidity  $\ell$  which is defined in (23) and is also a constraint on the planner's problem.

Constraints (24) and (25) impose market clearing considerations in the planner's problem and short-selling constraints. Equation (24) imposes that the local demand for asset  $j$ ,  $z_j(\ell)$  which depends on the cost of liquidity, cannot be bigger than the total supply of asset  $j$  which is normalized to 1. Also,  $z_j(\ell) \geq 0$  because agents cannot short-sell the public assets. Equation (25) is just saying that asset prices will be pinned down by international investors' valuation if they hold the asset, that is if  $z_j(\ell) < 1$  and will be strictly above international investors' valuation only when all of the supplied asset  $j$  is held domestically ( $z_j(\ell) = 1$ ).

The approach to solve this problem is to solve a slightly modified version with fewer constraints and a modified objective and then check that the solution obtained satisfies initial constraints and that the objective takes the same value in the original objective. Thus, we start by modifying the objective  $W$ .

From constraint (25) we see that the government is not free to choose any quantity for the liquidity premia coming from foreigners. The maximum the government can obtain from foreigners for each asset  $j$  is  $(\beta^* - 1)\Pi_j$ . A liquidity premium higher than that would imply that all the supplied assets are held domestically ( $z_j(\ell) = 1$  for both assets) and the liquidity premia coming from foreigners goes to zero. Note also that  $\Pi_j \leq 1$ . This implies that we can bound the liquidity premia coming from international investors:

$$\begin{aligned}
0 &\leq (q_1 - \Pi_1)(1 - z_1(\ell)) + (q_2 - \Pi_2)(1 - z_2(\ell)) \\
&\leq (\beta^* - 1)(1 - z_1(\ell)) + (\beta^* - 1)(1 - z_2(\ell))
\end{aligned} \tag{26}$$

where the first inequality in (26) holds with equality only when  $z_1 = z_2 = 1$ .

Therefore we can write a slightly modified welfare objective  $\tilde{W}$  which is an upper bound on  $W$  which is given by:

$$\begin{aligned}
\tilde{W} &\equiv E + A + [R - s_L(1 - \lambda) - s_H\lambda - 1]I + \\
&\quad (\beta^* - 1)(1 - z_1(\ell)) + (\beta^* - 1)(1 - z_2(\ell))
\end{aligned} \tag{27}$$

The constraint on asset prices contained in (24) also imposes a lower bound on the unit cost of liquidity  $\ell$ , namely  $\ell \geq \frac{(\beta^* - 1)\Pi_j}{x_j^H} = \frac{(\beta^* - 1)\Pi_j}{x_j^H} = (\beta^* - 1) \left( \lambda + (1 - \lambda) \frac{x_j^L}{x_j^H} \right)$  where I have used the definition of  $\Pi_j$ . The lower bound on  $\ell$  is given by:

$$\ell \geq \lambda(\beta^* - 1) \tag{28}$$

with equality only when  $x_j^L = 0$  for asset  $j$ .

Using the modified objective (27) and the lower bound on unit cost of liquidity (28) we can rewrite the previous planner problem (23) in terms of the implemented fiscal capacity allocation between domestics and foreigners in both states of the world.

Denote fiscal capacity allocations by  $F$ . In  $(F_D^L, F_F^L, F_D^H, F_F^H)$  superscripts denote state of the world and subscripts denote the holder. Then  $F_D^L$  denotes how much of the fiscal capacity in  $s = s_L$  is held domestically and  $F_F^L$  denotes how much of that capacity is held abroad and similarly for  $F_D^H$  and  $F_F^H$ . Finally denote  $(\ell^L, \ell^H)$  as the unit cost of liquidity in both states of the world.

$$\begin{aligned} \max_{\{\ell^H, F_F^L, F_D^H, F_F^H\}} \quad & E + A + [R - s_L(1 - \lambda) - s_H\lambda - 1]I + \\ & \lambda(\beta^* - 1)F_F^H + (1 - \lambda)(\beta^* - 1)F_F^L \end{aligned} \quad (29)$$

*s.t.*

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\ell^H} \quad (30)$$

$$F_D^H \geq I(s_H - \rho) \quad (31)$$

$$\ell^H \geq \lambda(\beta^* - 1) \quad (32)$$

$$F_D^H + F_F^H \leq \eta \quad (33)$$

$$F_D^L + F_F^L \leq 1 \quad (34)$$

The planner's problem given by (35) above maximizes the returns from investment and the liquidity premia coming from abroad subject to the behavior coming from the entrepreneur's problem. In particular (30) gives the expression for investment in terms of parameters and the liquidity premium in state  $s = s_H$ . Constraint (31) rewrites the collateral constraint in terms of the new variables: it is just saying that the amount of fiscal capacity held by domestics in state  $s = s_H$  has to be greater or equal than the reinvestment need in that state of the world. Constraint (32) gives the lower bound for the liquidity premium that we obtained above. Finally the government needs to satisfy the fiscal capacity constraints (33) and (34).

To solve this problem, first note that the objective function is increasing in investment  $I$ . Since  $I'(\ell^H) < 0$  the planner chooses the minimum possible  $\ell^H$ , that is:  $\ell^H = \lambda(\beta^* - 1)$  from (32) with equality. This pins down investment  $I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\lambda(\beta^* - 1)} \equiv I^{**}$ . The planner's aim is to make  $F_D^H$  as small as possible. From (33) we see that increasing  $F_D^H$  necessarily decreases  $F_F^H$  because fiscal capacity is limited and this lowers the objective function. Then (37) will hold with equality,  $F_D^H = I^{**}(s_H - \rho)$ . Finally, it is optimal to make  $F_D^L = 0$  in order to maximize the liquidity premium coming from abroad as we see from (40). Finally fiscal constraints (39) and (40) also hold with equality or else fiscal capacity would be wasted and the planner's objective would be lower.

The optimal fiscal capacity allocations are given by:

$$\begin{aligned}
F_D^H &= I^{**}(s_H - \rho) \\
F_D^L &= 0 \\
F_F^H &= \eta - I^{**}(s_H - \rho) \\
F_F^L &= 1
\end{aligned}$$

where  $I^{**}$  is defined above.

The allocation wants to provide just enough liquidity domestically to attain the desired level of investment. When the government is not constrained by monotonicity this implies that the government does not supply any liquidity to domestics when the private liquidity is enough to achieve the desired level of investment. Hence the null allocation of liquidity in the low-liquidity need state of the world when the project is self-financed. The rest is allocated abroad to maximize the capital flows coming from abroad. This result is summarized in the following proposition.

**Proposition 5.** *If the government is not constrained by monotonicity considerations, it is optimal to allocate no fiscal capacity to domestics in the state of the world when private liquidity suffices and allocate all the fiscal capacity abroad.*

### 3.4 Planner's Problem with Monotonic Asset Payoffs

The problem when the planner is not subject to monotonicity requirements is simply (19) including a monotonicity constraint. To solve the problem in terms of fiscal capacity allocations we still solve a slightly modified version of the problem where we impose an upper bound welfare  $\tilde{W}$  like in (27).

However the lower bound on unit cost of liquidity is now smaller than (28). To see this note that  $\ell \geq \frac{(\beta^*-1)\Pi_j}{x_j^H} = \frac{(\beta^*-1)\Pi_j}{x_j^H} = (\beta^* - 1) \left( \lambda + (1 - \lambda) \frac{x_j^L}{x_j^H} \right)$  which under monotonicity requirements  $x_j^L \geq x_j^H$  cannot be smaller than  $\ell \geq \beta^* - 1$ . This condition holds with equality when  $x_j^L = x_j^H$ . We see that when the government is constrained by monotonicity the minimum payoff in state  $s = s_L$  it can choose is the same as in  $s = s_H$ , which increases the minimum unit cost of liquidity it can attain.

The problem in terms of fiscal capacity allocations is the following:

$$\begin{aligned} \max_{\{\ell^H, F_F^L, F_D^H, F_F^H\}} \quad & E + A + [R - s_L(1 - \lambda) - s_H\lambda - 1]I + \\ & \lambda(\beta^* - 1)F_F^H + (1 - \lambda)(\beta^* - 1)F_F^L \end{aligned} \quad (35)$$

s.t.

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\ell^H} \quad (36)$$

$$F_D^H \geq I(s_H - \rho) \quad (37)$$

$$\ell^H \geq \beta^* - 1 \quad (38)$$

$$F_D^H + F_F^H \leq \eta \quad (39)$$

$$F_D^L + F_F^L \leq 1 \quad (40)$$

$$F_D^L \geq F_D^H; F_F^L \geq F_F^H \quad (41)$$

The planner's problem given by (35) is very similar to the one with no monotonicity requirements. The only differences are a higher upper bound on the liquidity premium (38) and the monotonicity constraints (41). They require the amount of fiscal capacity held by both types of agents, domestics and foreigners, in the low liquidity shock state be greater or equal to what they hold in the high liquidity shock state.

The reasoning to obtain the solution is also very similar to above. The government wants to minimize  $\ell^H$  since  $I'(\ell^H) < 0$ . Thus the planner chooses the minimum possible that is:  $\ell^H = \beta^* - 1$  from (38) with equality. This pins down investment  $I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)} \equiv I^*$ . The planner's aim is to make  $F_D^H$  as small as possible. From (39) we see that increasing  $F_D^H$  necessarily decreases  $F_F^H$  because fiscal capacity is limited and this lowers the objective function. Then (37) will hold with equality.

Also (41) for domestics must hold with equality  $F_D^L = F_D^H$ . The argument is similar to above, making  $F_D^L$  higher than just necessary would lower  $F_F^L$  from (40) which again lowers the objective function. As before fiscal constraints (39) and (40) hold with equality. Under this allocation the monotonicity constraint for foreigners is slack.

Thus, the fiscal capacity allocation that maximizes social welfare is the following:

$$F_D^H = F_D^L = I^*(s_H - \rho)$$

$$F_F^H = \eta - I^*(s_H - \rho)$$

$$F_F^L = 1 - I^*(s_H - \rho)$$

where  $I^*$  is the level of investment given above and pinned down by  $I(\ell^H = \beta^* - 1)$ .

This allocation is intuitive, the government wants to provide just enough liquidity domestically to attain the desired level of investment. The rest is allocated abroad since this is a positive income flow for consumers who obtain the value of both assets from abroad at date 0 and then are taxed the payoff that the government needs to repay which is lower than what they obtained because of the liquidity premia. This result is summarized in the following proposition.

**Proposition 6.** *Under monotonicity, the optimal liquidity allocation provides equal liquidity in both states of the world to domestics. The government provides just enough liquidity to attain the desired level of investment. The residual fiscal capacity is allocated abroad in order to maximize the liquidity premia coming from foreigners.*

### 3.5 Assets that Implement Optimal Fiscal Capacity Allocations

A payoff structure that implements the fiscal capacity allocation under no monotonicity corresponds to the Arrow-Debreu securities given by:

$$\begin{aligned} x_1^H &= \eta, x_1^L = 0 \\ x_2^H &= 0, x_2^L = 1 \end{aligned}$$

in which fiscal capacity in each state of the world is supplied in the form of one asset. Under monotonicity the payoff structure that implements the fiscal capacity allocation is a safe and a risky asset:

$$\begin{aligned} x_1^H &= \eta, x_1^L = \eta \\ x_2^H &= 0, x_2^L = 1 - \eta \end{aligned}$$

#### 3.5.1 Arrow-Debreu Securities

This combination implements the allocation obtained in subsection (3.3). In this case the entrepreneur will only demand asset 1,  $\Pi_1 = \eta\lambda$  which makes  $\ell = \lambda(\beta^* - 1)$ . The investment level chosen by the entrepreneur is given by  $I^{**} = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)\lambda(\beta^* - 1)}$ . The domestic demand for asset 1 equals  $z_1^D = \frac{I^{**}(s_H - \rho)}{\eta}$  and  $z_2^D = 0$ . The foreign demand for both assets equal  $z_1^F = 1 - z_1^D = 1 - \frac{I^{**}(s_H - \rho)}{\eta}$  and  $z_2^F = 1$ .

Plugging these and the payoffs in (42) and (43) for both types of investors we obtain the optimal fiscal capacity allocations.

### 3.5.2 Safe and Risky Asset

Indeed this combination implements the fiscal capacity allocations given in subsection (3.4). Given these two assets the entrepreneur will choose to hold asset 1 since it is the only asset that provides liquidity in state  $s = s_H$ , thus minimizing the unit cost of liquidity. The expected payoff  $\Pi_1 = \eta$ , thus  $\ell = \beta^* - 1$ . Investment level chosen by the entrepreneur equals  $I^* = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)}$ .

The fiscal capacity allocated to domestics in both states of the world equals:

$$F_D^H = z_1^D x_1^H + z_2^D x_2^H \quad (42)$$

$$F_D^L = z_1^D x_1^L + z_2^D x_2^L \quad (43)$$

where  $z_1^D$  and  $z_2^D$  is the demand coming from domestic for each asset. Since the entrepreneur does not buy asset 2,  $z_2^D = 0$ . From the collateral constraint (10)  $z_1^D = \frac{I^*(s_H - \rho)}{\eta}$  and the fact that  $x_1^H = x_1^L = \eta$  we immediately see that the fiscal capacity allocation for domestics is the optimal one obtained in subsection (3.4).

The expressions for fiscal capacity allocated to foreigners  $F_F^H$  and  $F_F^L$  are symmetric to (42) and (43) where domestic demand is substituted by foreign demand. The foreign demand for asset 2 is all the asset supplied  $z_2^F = 1$  because domestics hold none of this asset and  $z_1^F = 1 - z_1^D = 1 - \frac{I^*(s_H - \rho)}{\eta}$ . The fiscal capacity allocations that are attained with this combination of assets is exactly the optimal one from above.

## 4 The Benefits of Financial Innovation and Complementarities with Financial Integration

To start, it is worth noting that the planner problems given in subsections (3.3) and (3.4) were subject to the same fiscal capacity constraints as in the scenario with only one defaultable bond. Thus government revenues are identical in all scenarios. In the case with one bond the government revenues are given by  $\beta^*(1 - \lambda + \lambda\eta)$ , which is identical to the level of government revenues in the other two scenarios when we add the revenues accruing from both assets. In the scenario with the safe and the risky asset, the revenues from the risky asset are given by  $\beta^*(1 - \lambda)(1 - \eta)$  and the revenues coming from the safe one are given by  $\beta^*\lambda\eta$  and the case

with Arrow-Debreu securities the revenues coming from the security that pays in  $s = s_L$  equal  $\beta^*(1 - \lambda)$  and the revenues from the one that pays when  $s = s_H$  equal  $\beta^*\lambda\eta$ .

Despite keeping fiscal capacity constant the government is increasing domestic investment when it issues two different securities. In particular investment is highest when the government issues Arrow-Debreu securities and lowest one it issues only one defaultable bond:  $I^{**} > I^* > I$  where these denote the investment levels in the case with Arrow-Debreu securities, one safe and one risky and one defaultable bond respectively. We summarize this important result in the following propositions.

**Proposition 7.** *Financial innovation increases domestic investment. The level of domestic investment is weakly higher when the government issues Arrow-Debreu securities and the inequality becomes strict whenever the fiscal capacity is high enough.* <sup>10</sup>

*If the government issues two assets, one risky and one safe, investment is weakly higher than when it only issues only defaultable bonds. The inequality becomes strict when fiscal capacity is high enough.* <sup>11</sup>

The intuition for these results is that by issuing two different assets the government tranches its fiscal capacity and reduces the wasted liquidity, that is, the amount of unneeded liquidity the entrepreneur purchases per unit of liquidity in the state of the world when liquidity is useful goes down. The lower the wasted liquidity, the higher the domestic investment. We see this by comparing the case with Arrow-Debreu securities and with one safe and one risky asset. In the former case, when the entrepreneur buys asset 1 she does not buy liquidity for the state when she does not need the public liquidity because that asset's payoff is 0 in that state of the world. In the latter case, when the government is issuing a safe asset the entrepreneur has to buy some liquidity for the state she will not use it.

The cost of the wasted liquidity in this model comes from the existence of international investors. The model features a crowding-out effect coming from international investors' demand for public bonds. This high demand from foreign investors is captured by the higher discount factor abroad  $\beta^* > 1$  and drives up the price of the public bond. The government by tranching its fiscal capacity and decreasing wasted liquidity decreases the cost of liquidity hoarding for entrepreneurs without imposing capital controls. See Appendix (B) for a discussion on capital controls in this model. Finally, financial innovation introduces assets especially designed to attract international investors who are risk-neutral and demand the assets as a savings vehicle.

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<sup>10</sup>In particular, when the following condition is satisfied:  $\eta > \frac{(s_H - \rho)(A - (\beta^* - 1)\lambda)}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$ .

<sup>11</sup>In particular, when condition  $\eta > \frac{(s_H - \rho)(A - (\beta^* - 1))}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$  which ensures that the safe asset is held at home and by foreigners.

It is worth noting that in this model the introduction of financial innovation in sovereign debt markets is beneficial when these markets are integrated and open to foreign investors.

**Proposition 8.** *The benefits of financial innovation arise when sovereign debt markets are integrated.*

In Appendix (C) we derive the equilibria when foreign investors cannot buy public debt in sovereign debt markets. In both equilibria, with high and low fiscal capacity, investment would not be affected by financial innovation.<sup>12</sup> In the case where fiscal capacity is large enough, (7) holds, there is no cost of liquidity hoarding because the marginal holder of public debt is the consumer. Thus, the government cannot improve the allocation by changing the payoff structure.<sup>13</sup>

## 5 Comparative Statics

In this section I perform comparative statics for the scenario with one safe and one risky asset. The comparative statics regard the relative holdings of safe to risky asset for different types of investors as well as the domestic investment level.

An increase in  $s_H$  and a decrease in  $s_L$  such that the total cost per unit of investment,  $1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)$ , remains constant brings about an increase in the international relative holdings of risky to safe asset. The intuition for this is that the tightening of collateral constraints increases the domestic demand for the safe asset. By market clearing, the amount of safe asset held by international investors decreases. The international holdings of risky asset are always equal to the amount supplied. Thus, the relative holdings of risky to safe increase because the denominator decreases.

An increase in fiscal capacity  $\eta$  decreases the relative holdings of risky to safe asset. The reason for this is that an increase in  $\eta$  lowers the domestic demand for the safe asset because now every bond pays more. Thus international investors need to hold more of the safe asset which decreases the ratio of risky to safe assets held by foreigners.

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<sup>12</sup>More broadly we can think of sovereign debt markets open to different types of investors with different motives to hold debt and with a higher demand for the public bond. In the model presented in this paper these are foreign investors. This is consistent with the empirical evidence that shows a steady increase in the share of debt held by foreigners for most of the advanced economies in recent years (Arslanalp and Tsuda (2012)).

<sup>13</sup>The same result holds when (7) does not hold. In that case, as we discuss in Appendix (A) investment is increasing in  $\eta$ , the fiscal capacity in the high liquidity shock state. However, the government cannot change its fiscal capacity by introducing financial innovation in sovereign debt markets. It can only change the relative payoffs across states.

An increase in the patience of international investors parametrized in an increase in  $\beta^*$  increases the foreign demand for the safe asset because being more patient they demand more of all assets. This decreases the ratio of risky to safe international holdings since as in the previous scenarios risky asset holdings are fixed and normalized to 1.

## 6 Empirical Analysis

### 6.1 Objective and Data

The objective of this section is to study the effects of tightening of collateral constraints on the proportion of public debt held abroad for a group of advanced economies as well as the effect of changes in international demand. To study the effect of tightening of collateral constraints I explore the effect of the financial crisis on public debt ownership. Furthermore I study whether the financial crisis hit differentially countries whose governments were financial innovators in the year of the crisis or not.

The sample of countries I consider for this empirical analysis is comprised of 22 advanced economies between 2004 and 2011. The countries included in the sample are Australia, Canada, Japan, Korea, Switzerland, UK, US and several countries of the Euro area.<sup>14</sup>

For the proportion of debt held by non-residents and the total level of debt issued I use the dataset from [Arslanalp and Tsuda \(2012\)](#).<sup>15</sup> For local credit I use the annual ratio of private credit provided by deposit money banks and other financial institutions to GDP from [Beck et al. \(2009\)](#) which has data on local credit until 2011. For risk premia I use estimates based on the country ratings assigned by Moodys.<sup>16</sup> For financial openness I use the Chinn-Ito index taken from [Chinn and Ito \(2008\)](#) which is updated now to 2011. Finally, for investment I use the data on gross capital formation from the World Bank. All data is yearly.

To determine whether in a particular sovereign debt market the government has introduced financial innovation or not I use data on the set of available instruments for each country in the sample. This information is summarized in Appendix (D.1) and is obtained from the public debt management offices of each country. As we see there all advanced economies issue debt with different maturities and many of them issue at least another type of bond, indexed to inflation, with a variable coupon or interest rate or in a foreign currency. I consider the countries that only issue bonds with different maturities as not having introduced financial

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<sup>14</sup>The included countries from the Euro area are Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Slovenia, Spain and Sweden.

<sup>15</sup>Combining the data from [Arslanalp and Tsuda \(2012\)](#) with the data on debt ownership from [Merler and Pisani-Ferry \(2012\)](#) does not change the qualitative results from this section.

<sup>16</sup>Estimates of country risk premia are publicly available at <http://people.stern.nyu.edu/adamodar/>.

innovation in sovereign debt markets. These countries are Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland. I label them as non-innovators in sovereign debt markets.<sup>17</sup>

## 6.2 Effect of Collateral Constraints on Debt Held Abroad

To start, I test the comparative statics of the benchmark model regarding tightening of collateral constraints. That section concluded that a tightening of collateral constraints, which in the model is due to an increase in  $s_H$ , decreases the proportion of public debt held abroad because more public debt is needed at home to meet collateral requirements. The empirical counterpart of a tightening of collateral constraints is a lower availability of credit in domestic markets.

Another testable implication from the benchmark model is that an increase in  $\beta^*$  increases the proportion of public debt in foreign hands. As we have already mentioned an increase in  $\beta^*$  should not be interpreted directly as an increase in international patience. Instead it is better understood as an increase in the demand for domestic public debt coming from abroad. In practice this can be captured by decreases in the country risk premium.

The main specification used to test these hypotheses is the following:

$$\% non - resident_{it} = \alpha_i + \beta_1 Local\ credit_{it} + \beta_2 Risk\ Premium_{it} + \gamma X_{it} + \epsilon_t + u_{it} \quad (44)$$

where the dependent variable is the proportion of debt held by international investors and  $X_{it}$  is a vector of controls.

If the previous hypotheses hold in the data it must be that  $\beta_1 > 0$  and  $\beta_2 < 0$ . Specification (44) contains country fixed effects which control for time-invariant country specific factors and time fixed effects which control for common shocks to all countries. It also includes two relevant control variables: financial openness and the level of debt over GDP. Everything else equal, a more financially open country will have a higher proportion of debt held abroad, since public debt is a financial asset too and regarding total amount of debt, a country which has issued more debt is likely to have more debt held by foreigners.

However this specification does not control for all potentially relevant time-varying country specific shocks. Therefore it is just testing whether there is an association in the data that is consistent with the predictions of the model given above. Appendix (D.2) shows the results of estimating equation (44).

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<sup>17</sup>Admittedly this is a crude measure of financial innovation and many improvements can be made to include heterogeneity in the degree of financial innovators. Constructing a better measure of financial innovation goes beyond the scope of this paper.

The data confirms the predictions of the model, the effect of local credit on debt held abroad is positive and high risk premia are associated with lower proportion of debt held by international investors. These coefficients are highly significant even controlling for total debt, financial openness and country and time fixed effects as we see in column (4) in Appendix (D.2).

Specification (44) might suffer from omitted variable bias. For this reason we need an instrument for tightening of collateral constraints. I use the financial crisis as a plausibly exogenous shock to collateral constraints in each of these countries. In order to capture different intensities of the “financial crisis” treatment I rely on a Bartik-style instrumental variable (Bartik (1991)). In particular, I interact the financial crisis shock ( $Post^{09}$ ) with the leverage level of the economy before the crisis hit. What is crucial for identification is that the leverage level of each economy before the shock is exogenous to the financial crisis shock.<sup>18</sup>

The specification I consider in appendix (D.3) is the following:

$$\% non - resident_{it} = \alpha_i + \beta_1 Local\ credit_i^{08} * Post_t^{09} + \gamma X_{it} + \epsilon_t + u_{it} \quad (45)$$

where  $Local\ Credit_i^{08}$  is a variable constructed interacting the level of local credit in 2008 for each country, that is, how exposed the country was to the treatment and  $Post_t^{09}$  is a dummy variable which takes a value of one for years after and including 2009. The vector of controls  $X_{it}$  include financial openness and level of debt to GDP. As before (45) includes country and time fixed-effects.

In this specification we expect  $\beta_1 < 0$ : the shock to collateral constraints causes a repatriation of public debt since more public assets are needed by the domestic private sector to use as collateral. The table in appendix (D.3) confirms this prediction and the coefficient remains significant even when we control for variables such as level of debt to GDP and financial openness.

### 6.3 Differential Effect of Collateral Constraints on Financial Innovators

Finally my objective is to explore whether those countries that have introduced financial innovation in sovereign debt markets are differentially hit by the financial crisis or not. The variables of interest that we consider are percentage of debt held abroad as well as the

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<sup>18</sup>We could be worried about the exogeneity of the leverage of the American financial system to the financial crisis shock. All the results in this section are robust to dropping the United States from our sample.

fraction of investment to GDP. In section (4) we saw that the level of investment under financial innovation in sovereign debt markets is larger than with only one risky bond.

For this I divide our sample of 22 advanced economies in two, those that have introduced financial innovation in sovereign debt markets and those who have not. The latter group includes only Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland and explore the differential effect of the financial crisis on financial innovators and non-innovators.<sup>19</sup>

I run the following econometric specification:

$$y_{it} = \alpha_i + \beta_1 Local\ credit_i^{08} * Post_t^{09} * Financial\ Innovator_i + \gamma X_{it} + \epsilon_t + u_{it} \quad (46)$$

where  $y_{it}$  is either %*nonresident* or gross capital formation to GDP and *FinancialInnovator<sub>i</sub>* is a dummy that takes a value of one if the country is a financial innovator and zero otherwise.

From the results in section (3) and (4) we expect the effects of the crisis to be less severe for financial innovators, that is a smaller public debt repatriation after the financial shock and a smaller hit to country investment. Since the dummy *FinancialInnovator<sub>i</sub>* takes a value of one when the country is a financial innovator and the coefficient of the financial crisis treatment in specification (45) is negative we expect  $\beta_1 > 0$  for both outcome variables.

The results in Appendices (D.4) and (D.5) confirm these predictions and remain highly significant controlling for the level of debt and financial openness for both outcome variables. Furthermore, all regressions are run with country and time fixed effects.

These results cannot be directly attributed to financial innovation in sovereign debt markets because the sample division can be capturing many other time-varying country-specific relevant variables not included in (46). However we can conclude that the main results about the benefits of financial innovation on investment and debt repatriation are consistent with the effect of the financial crisis on investment levels in our sample of advanced economies.

## 7 Conclusion

This paper has presented a model where public debt has a liquidity role, the government's fiscal capacity is risky which renders public debt risky too and different types of investors demand public debt in integrated sovereign debt markets. The paper has shown that financial

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<sup>19</sup>The inclusion of Czech Republic, Austria and Switzerland in the financial non-innovators group and the non-inclusion of other countries in the Euro periphery such as Italy minimizes the concerns about the financial non-innovators capturing other fundamentals. Also, as we will see all the regressions include country and time fixed-effects.

innovation in sovereign debt markets can increase domestic investment and domestic welfare. The key assumption behind the investment increase is the liquidity role of public debt. The driver of this result is that financial innovation can make liquidity for the private sector cheaper by changing the payoff structure of the public assets and lowering liquidity waste. For this to be possible there must be other types of investors willing to hold the residual risk not allocated to domestics.

The financial innovations proposed and the way of approaching the government's financial innovation problem have highlighted that the government can exploit the existence of different types of investors when designing assets. Especially the government can design assets taking advantage of the different degrees of patience in its investor base and the different rationales for holding public debt. The paper has shown that to provide liquidity optimally at home the government does not need to segment markets or tax foreigners. The model has shown that an appropriate asset design can result in market segmentation. The financial innovation problem has also highlighted that when there are investors willing to hold riskier tranches of the public fiscal capacity the government first allocates fiscal capacity to meet liquidity demands at home and after that allocates the residual riskier fiscal capacity to those investors willing to hold it. Finally it has also highlighted the complementarities between financial innovation and financial integration showing that the benefits of financial innovation are higher when sovereign debt markets are more integrated.

The model presented in this paper has delivered comparative statics consistent with sizeable shifts in composition of investor base for public debt as a whole that has been reported extensively in the recent years as well as shifts of relatively riskier debt instruments to foreigners. Moreover, recent data on debt ownership and local credit for a group of advanced economies is consistent with the predictions of the theoretical model and with the differential effects for financial innovators and non-innovators.

The framework and results presented in this paper point to a number of promising avenues for future research. Looking at investor base composition and debt ownership at a smaller level of granularity regarding debt instruments and different types of investors. For instance, thinking about shifts of ownership between financial and non-financial types of investors or with different degrees of risk appetite. Introducing lack of commitment and allowing the government to default on its public debt can introduce relevant trade-offs in the planner's financial innovation problem which have been ignored in this paper.

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## A Equilibria when public bond is held only at home

When the return of the bond in  $s = s_H$  is not big enough, that is when condition (7) does not hold international investors do not purchase the bond. All debt is held by domestic entrepreneurs and by market clearing it must be the case that  $z^L = 1$ , where  $z^L$  is the demand for bonds coming from entrepreneurs.

We concentrate on the case when the project is continued in both states of the world and  $\chi = 1$ . For this to be the case it must be that the price of the bond  $q$  satisfies  $\Pi < q < q^{max}$  where  $q^{max}$  is given by  $q^{max} = \Pi + \frac{\lambda(1+s_L(1-\lambda)-s_H)\underline{\eta}}{(s_H-\rho)(1-\lambda)}$  and  $\Pi$  is the bond's expected payoff.

In this case the demand for bonds from entrepreneurs when  $\Pi < q < q^{max}$  is given by  $z^L = \frac{I(s_H-\rho)}{\underline{\eta}}$ . Combining this expression with (3), equating it to total supply of bonds and solving for the bond price we obtain that:

$$q = \Pi + A - \frac{\underline{\eta}(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho} \quad (47)$$

We now indicate what are the parametric assumptions for  $\Pi < q < q^{max}$  and thus  $\chi = 1$ . First it must be the case that  $q > \Pi$ . Otherwise consumers would be buying the bond domestically because they would pay  $q = \Pi$ . For this to be satisfied in equilibrium  $A$  should be higher than a threshold. In particular  $A > \frac{\underline{\eta}(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho}$ . For  $q < q^{max}$ , we impose an upper bound on  $A$  too:  $A < \frac{\underline{\eta}(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho} + \frac{\underline{\eta}\lambda(1 + s_L(1 - \lambda) - s_H)}{(1 - \lambda)(s_H - \rho)}$ .

Investment is given by:

$$I = \frac{\underline{\eta}}{s_H - \rho} \quad (48)$$

which as we see is increasing in  $\underline{\eta}$  but not in the ratio of fiscal capacities as in the section (2). It is increasing in  $\underline{\eta}$  because when  $\underline{\eta}$  is low the entrepreneur is constrained and cannot hoard as much liquidity as she would want to. Therefore, the slacker this constraint is the higher attainable investment will be.

## B The Case of Capital Controls

As we have seen the equilibrium investment level with international investors given in (4) is decreasing in the international investor's patience. A natural question is whether imposing capital controls which ban the arrival of international flows from abroad can increase welfare at home.

A first approximation to this question would be to calculate the consumption of both types of agents and compute domestic welfare as we discuss in section (3.2). Investment under open financial markets is given by (4). Under autarky since (7) holds the marginal buyer of the public bond is the domestic consumer which implies that the price of the bond has no liquidity premium,  $q = \Pi$  and investment under autarky is simply given by  $I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$ , since liquidity hoarding in autarky is costless. Thus, investment is higher under autarky. However consumers lose from remaining in autarky since they now do not get the premia coming from abroad,  $q - \Pi$  which due to the higher patience abroad will always be greater than zero.

Now we allow for ex ante transfers between the entrepreneur and the consumer. The entrepreneur will transfer part of her initial wealth  $A$  to the consumer in order to make him indifferent between autarky and open financial markets. Then we will compute whether the final investment under autarky is higher than in open financial markets.

For convenience let's denote by  $\bar{R} \equiv R - s_L(1 - \lambda) - s_H\lambda - 1$ , the net return from the domestic investment and by  $k \equiv 1 - (\rho - s_L)(1 - \lambda) - (\rho - s_H)\lambda$ . As in the comparative statics part we simplify notation by assuming  $\bar{\eta} = 1$ . Under open financial markets investment and the transfer to consumers from abroad are given by:

$$I^{Open} = \frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1)(\lambda\eta + 1 - \lambda)} \quad (49)$$

$$T^{Open} = (\beta^* - 1)(\lambda\eta + 1 - \lambda) \left( 1 - \frac{I^{Open}(s_H - \rho)}{\eta} \right) \quad (50)$$

For this to be an equilibrium (7) needs to hold, which we can rewrite in terms of the initial wealth  $A$  being below a threshold:

$$A < \frac{\eta k}{s_H - \rho} + (\beta^* - 1)(\lambda\eta + 1 - \lambda)$$

Therefore let assume that the entrepreneur transfers  $T^{Open}$  to consumers. After this transfer the consumer is indifferent between autarky and open financial markets. In autarky the

entrepreneur's investment choice after transfers is given by:

$$I^{Autarky,T} = \frac{\tilde{A}}{k} = \frac{A - T^{Open}}{k}$$

where the entrepreneur can leverage its initial wealth after transfers  $\tilde{A} < A$  as much as  $1/k$  since in the closed economy when (7) holds the consumer is the marginal holder of the asset who pins down the price. Thus,  $q = \Pi$  and there is no cost of hoarding liquidity for the entrepreneur.

**Proposition.** *Under autarky the consumer is made worse-off, the entrepreneur is made better-off and total welfare is always smaller than under open financial markets. Open financial markets is also better than autarky when the entrepreneur makes a transfer to the consumer to make him indifferent between autarky and open financial markets.*

*Proof.* For the first part, we need to calculate total welfare  $W = E + A + (R - s_L(1 - \lambda) - s_H\lambda - 1)I + (q - \Pi)z^F$  for both scenarios, open financial markets and autarky and show that  $W^{Open} > W^{Autarky}$ . Denote  $\Pi = (\lambda\eta + 1 - \lambda)$ .

Under open markets,

$$\begin{aligned} I^{Open} &= \frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1)\Pi} \\ W^{Open} &= E + A + \bar{R}I^{Open} + (\beta^* - 1)\Pi \left( 1 - \frac{I^{Open}(s_H - \rho)}{\eta} \right) \end{aligned}$$

where as we see  $W^{Open}$  welfare is comprised of the net return from investment and the financial flows from abroad. For this to be an equilibrium it must be that

$$\eta > \frac{(s_H - \rho)(A - (1 - \lambda))}{1 - (\rho - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)} \quad (51)$$

Investment and welfare in the autarkic economy is given by:

$$\begin{aligned} I^{Autarky} &= \frac{A}{k} \\ W^{Autarky} &= E + A + \bar{R}I^{Autarky} \end{aligned}$$

It follows directly from condition (51) that  $I^{Autarky} > I^{Open}$ . For  $W^{Open} > W^{Autarky}$  it must be the case that

$$\bar{R}(I^{Autarky} - I^{Open}) < (\beta^* - 1)\Pi \left( 1 - \frac{I^{Open}(s_H - \rho)}{\eta} \right) \quad (52)$$

Rearranging (52) we see that it always holds when  $I^{Autarky} > I^{Open}$ , that is when (51) holds.

For the second part of the proposition we need to prove that  $I^{Open} - I^{Autarky,T} > 0$ .

Substituting investment for their expressions we obtain that  $\frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1)\Pi} - \frac{\tilde{A}}{k} > 0$ . Rearranging this expression and substituting  $\tilde{A} = A - T^{Open}$  we obtain  $(\beta^* - 1)\Pi \left( \frac{\eta - I^{Open}}{\eta} \right) \eta k > -k(s_H - \rho)(\beta^* - 1)\Pi$ . Cancelling out terms this expression becomes  $\eta + s_H - \rho > I^{Open}$ . Using the expression for  $I^{Open}$  this is equivalent to  $\frac{A}{\left(\eta k + \frac{s_H - \rho}{\eta}(\beta^* - 1)\Pi\right)(\eta + s_H - \rho)} < 1$  which can be rewritten as

$$A < \eta k + k(s_H - \rho) + (s_H - \rho)(\beta^* - 1)\Pi + \frac{(s_H - \rho)^2(\beta^* - 1)\Pi}{\eta}$$

This condition on  $A$  always holds when  $A < \frac{\eta k}{s_H - \rho} + (\beta^* - 1)\Pi$ , which is the initial condition for an equilibrium given in section (B).  $\square$

Capital controls which ban foreign investors from buying the public asset do not increase welfare in this model because there are no pecuniary externalities. A crucial ingredient for capital controls to be welfare-improving is that a relative price affects constrained individuals, either directly by tightening collateral constraints or by decreasing wealth of constrained individuals (Caballero and Krishnamurthy (2001), Caballero and Krishnamurthy (2004), Aghion et al. (2001), Korinek (2011)).

From the entrepreneur's problem (1) we see that this is not the case here. The price of the public debt affects the investment choice level  $I$  and in turn how much of the public bond available the entrepreneur demands but it does not affect how constrained the entrepreneur is directly nor by changing the entrepreneur's net worth.

## C Autarkic equilibria: Sovereign Debt Markets Closed to Foreign Investors

In this section we derive the solution of the model in autarky. We start by analyzing the case when (7) holds:  $\eta > \frac{(s_H - \rho)(A - (1 - \lambda)\bar{\eta})}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)}$ . In that case we know  $z^L$  coming from domestic entrepreneurs is lower than total supply of public bond. With integrated sovereign debt markets part of the bond would be held by foreign investors.

With sovereign debt markets only open to domestic agents, it will be consumers who will demand the public bond. Since they do not have a liquidity motive to hold debt and their discount factor  $\beta = 1$  in equilibrium:

$$q^{Aut,I} = \Pi$$

When  $q = \Pi$  the entrepreneur is indifferent between holding as much bonds as to cover liquidity needs and infinite:  $z^L \in [\frac{I\chi(s_H - \rho)}{\eta}, \infty)$ . We assume that the entrepreneur just holds enough to cover liquidity needs. Since  $q < q^{max}$  defined in appendix (A), full continuation is optimal in both states  $\chi = 1$  and liquidity needs are given by  $\frac{I(s_H - \rho)}{\eta}$ . Plugging this and the price in the expression for investment (3) we obtain that the equilibrium level of investment is:

$$I^{Aut,I} = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$$

Therefore the level of investment is increasing in the level of entrepreneur's initial wealth and in the equity multiplier  $\frac{1}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$  which defines the maximum leverage per unit of own capital that the entrepreneur can obtain. As in section (2) it is decreasing in the expected cost of the project and increasing in the pledgeable return  $\rho$ .

However, under autarky when  $\eta$  is large enough, investment does not depend on the cost of liquidity hoarding. This is due to the fact that when  $\eta$  is large enough the marginal holder of the public bond is the domestic consumer who drives the liquidity premium to zero.

We now turn to the case when  $\eta \leq \frac{(s_H - \rho)(A - (1 - \lambda)\bar{\eta})}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)}$ . In this case only the domestic entrepreneur will hold the bond. The analysis here will be identical to the case with integrated debt markets but  $\eta$  low derived in Appendix (A). There we found that the price of the bond is given by:

$$q^{Aut,II} = \Pi + A - \frac{\eta(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho}$$

and that the level of equilibrium investment is increasing in  $\eta$ :

$$I^{Aut,II} = \frac{\eta}{s_H - \rho}$$

## D Appendix to the Empirical Analysis

### D.1 Financial Innovation in Advanced Economies' Sovereign Borrowing

Country	(1)	(2)	(3)	(4)	Notes
Australia	✓	✓			
Austria	✓				
Belgium	✓		✓		
Canada	✓		✓	✓	(4) is US dollars
Czech Republic	✓				
Denmark	✓	✓		✓	(4) in euro
Finland	✓			✓	(4) in US dollars
France	✓	✓	✓		(2) to Euro and French inflation
Germany	✓	✓			
Greece	✓				
Ireland	✓				
Italy	✓	✓	✓		(2) to Euro and Italian inflation
Japan	✓	✓	✓		
Korea	✓	✓		✓	(4) in US dollars
Netherlands	✓			✓	(4) in US dollars
New Zealand	✓	✓			Other: Kiwi bonds (retail stock)
Portugal	✓				
Slovenia	✓			✓	(4) in US dollars
Spain	✓				
Switzerland	✓				
United Kingdom	✓	✓			
United States	✓	✓	✓		

(1) Bonds with different maturities, (2) Bonds indexed to inflation,  
(3) Bonds with variable rate/coupon, and (4) Bonds issued in foreign currency  
Source: Debt Management Offices of each country

## D.2 The Determinants of Proportion of Debt Held Abroad

The table presents panel regressions for 22 advanced economies between 2004 and 2011. The dependent variable is the percentage of debt held by foreign investors. Local credit is the ratio of private credit provided by deposit money banks and other financial institutions to GDP, risk premia are estimated from Moody's country ratings, openness refers to financial openness and it is captured by the Chinn-Ito index and debt is the total stock of public debt over GDP. All regressions include country and year fixed effects.

Table 1: Proportion of Debt held Abroad

	(1)	(2)	(3)	(4)
	nonres	nonres	nonres	nonres
Local Credit	0.0770*** (0.0274)	0.102*** (0.0280)	0.0976*** (0.0280)	0.0940*** (0.0283)
Risk Premium		-1.267*** (0.443)	-1.272*** (0.441)	-1.521*** (0.519)
Financial Openness			-4.057 (2.746)	-3.815 (2.761)
Debt				0.0434 (0.0477)
Constant	30.15*** (2.887)	28.20*** (2.895)	37.59*** (6.980)	34.96*** (7.561)
Time FE	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes
No. of Observations	166	166	166	166
No. of Countries	22	22	22	22
R-squared	0.350	0.387	0.397	0.401
F	9.148	9.472	8.818	8.081

Standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

### D.3 Effect of Collateral Constraints on Proportion of Debt Held Abroad

The table presents panel regressions for 22 advanced economies between 2004 and 2011 to gauge the effect of tightening of collateral constraints on proportion of debt held abroad.

The dependent variable is the percentage of debt held by foreign investors. LCPost09 is a variable which measures the intensity of the financial crisis shock for the different countries in the sample by considering their pre-shock level of local credit. All regressions include country and year fixed effects.

Table 2: Effect of Financial Crisis on Proportion of Debt held Abroad

	(1)	(2)	(3)
	nonres	nonres	nonres
Local Credit08 * Post09	-0.0303*	-0.0298*	-0.0377*
	(0.0179)	(0.0178)	(0.0199)
Financial Openness		-4.696	-4.361
		(2.849)	(2.876)
Debt			0.0401
			(0.0453)
Constant	37.80***	48.31***	45.20***
	(0.871)	(6.371)	(7.281)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
No. of Observations	168	166	166
No. of Countries	22	22	22
R-squared	0.333	0.339	0.343
F	6.800	7.707	7.004

Standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## D.4 Differential Effect of Collateral Constraints on Debt Held Abroad for Financial Innovators

The table presents panel regressions for 22 advanced economies between 2004 and 2011 to gauge the differential effect of tightening of collateral constraints on proportion of debt held abroad for financial innovators.

The dependent variable is the percentage of debt held by foreign investors. LCPost09\*Financial Innovation is a variable which measures the intensity of the financial crisis shock for the different countries as in table from section (D.3) for financial innovators (that is, all countries except Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland).

All regressions include country and year fixed effects.

Table 3: Effect of Financial Crisis on Debt held Abroad for Financial Innovators

	(1)	(2)	(3)
	nonres	nonres	nonres
LCPost09 * Financial Innovation	0.0249** (0.0115)	0.0262** (0.0114)	0.0268** (0.0115)
Financial Openness		-5.162* (2.828)	-5.053* (2.853)
Debt			0.0149 (0.0406)
Constant	37.81*** (0.865)	49.35*** (6.325)	48.23*** (7.045)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
No. of observations	168	166	166
No. of countries	22	22	22
R-squared	0.342	0.351	0.352
F	7.075	8.116	7.271

Standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## D.5 Differential Effect of Collateral Constraints on Investment for Financial Innovators

The table presents panel regressions for 22 advanced economies between 2004 and 2011 to gauge the differential effect of tightening of collateral constraints on investment for financial innovators.

The dependent variable is the gross capital formation over GDP. LCPost09\*Financial Innovation is a variable which measures the intensity of the financial crisis shock for the different countries as in table from section (D.3) for financial innovators (that is, all countries except Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland).

All regressions include country and year fixed effects.

Table 4: Effect of Financial Crisis on Investment for Financial Innovators

	(1)	(2)	(3)
	pgcf	pgcf	pgcf
LCPost09 * Financial Innovation	0.0240*** (0.00435)	0.0240*** (0.00434)	0.0173*** (0.00344)
Financial Openness		0.731 (0.543)	3.036*** (0.849)
Debt			-0.138*** (0.0121)
Constant	23.63*** (0.358)	22.16*** (1.149)	24.05*** (2.097)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
No. of observations	254	254	254
No. of countries	22	22	22
R-squared	0.504	0.509	0.799
F	18.66	17.43	53.36

Standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$