

# SWITCHING BUBBLES: FROM OUTSIDE TO INSIDE BUBBLES

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## Abstract

The United States experienced two different bubbles: the Dot-Com and the Housing Bubbles. These bubbles had very different effects on investment and debt of manufacturing firms. In this paper I develop a framework to understand the differential effect of two types of rational bubbles. I distinguish between *(i) Outside* Bubbles, which I define as investors buying and selling useless and costless assets among themselves and *(ii) Inside* Bubbles, which I define as investors buying an input of production only as a store of value. The model is an OLG economy with investors and entrepreneurs. Investors save to consume when they are old. Entrepreneurs can borrow to produce output but they face a collateral constraint. I show that the size of an Inside Bubble is larger. I also show that when the economy switches from an Outside to an Inside Bubble, investment and debt are lower, consistent with the U.S. experience. Finally, I show that even though steady-state consumption with an Outside Bubble is higher, a social planner would prefer an Inside Bubble when the productivity of entrepreneurs is low.

Keywords: Rational bubbles, collateral constraint, investment, debt.

JEL Classification: E22, E44.

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# 1 Introduction

There exists a widespread agreement that during the last decade the United States experienced two different bubbles: the Dot-Com and the Housing Bubbles. Figure 1 represents these two bubbles using the S&P-500 and the Case-Shiller House Price Index (both indices are in real terms).<sup>1</sup>

These bubbles had very different effects on the economy. Figure 2 represents how residential and non-residential investment evolved during the last fifteen years. Residential investment was above average when there was the Housing Bubble. In contrast, non-residential investment was higher during the Dot-Com Bubble and it was below average during most of the Housing Bubble episode. Given this evidence, it seems that the relationship between investment and bubbles depends on the type of bubble.

The funding of firms was also differently affected by the two bubbles. Figure 3 represents the evolution of liabilities of the manufacturing industry in the United States. The liabilities of manufacturing firms were higher during the Dot-Com Bubble. Moreover, they were above average when the Dot-Com Bubble was the largest (2000) and below average when house prices peaked (2006). This evidence also points to the need of considering different types of bubbles to understand the effect of bubbles on the funding of firms.

Throughout the paper I will distinguish between two types of bubbles: *Outside* and *Inside* Bubbles. I define an Outside Bubble as a bubble which does not directly affect the real economy. That is, investors buy and sell useless and costless assets among themselves (for example, stocks during the Dot-Com Bubble). In contrast, I define an Inside Bubble as a bubble which does directly affect the real economy. In this case, investors buy an input of production only as a store of value (for example, land during the Housing Bubble).

The goal of this paper is to provide a framework to understand why the effects of rational bubbles on investment and debt depend on the type of bubble. I also analyze what happens when an economy switches between bubbles. Finally, I study which bubble is better and when a social planner would prefer one type of bubble over another.

The model is summarized in Figure 4. There are two agents in the economy: investors and entrepreneurs. When there are no bubbles, investors lend all their money to entrepreneurs. These entrepreneurs purchase capital, which they need to produce the output. Once production is completed, entrepreneurs return the loans to investors. When there is an Out-

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<sup>1</sup>See, for example, Case and Shiller (2003), Shiller (2005), Kraay and Ventura (2007) or Laibson and Mollerstrom (2009), for a discussion on these bubble episodes in the United States.

side Bubble, investors still lend to entrepreneurs, but a fraction of their savings (the bubble) is invested among themselves. The young investors buy a useless asset and sell it to the next generation of investors when they are old. Finally, when there is an Inside Bubble, investors, instead of coordinating among themselves, decide to purchase capital directly.

The paper yields three main results. The first result is that, consistent with the suggestive evidence shown in Figures 2 and 3, when the economy switches from an Outside to an Inside Bubble, investment and debt decline. The intuition is that borrowing (and investment) of entrepreneurs depend on the value of their collateral (capital). When the economy switches from an Outside to an Inside Bubble, it has two effects on investment. On the one hand, the price of capital increases because there is more demand for capital, which increases investment. On the other hand, the capital that entrepreneurs can purchase decline because they are competing with investors, which reduces investment. I show that the second effect always dominates.

The second result is that the size of an Inside Bubble is always larger than the size of an Outside Bubble. The reason is that when there is an Inside Bubble, the amount that entrepreneurs can borrow declines (as explained above). This fall in borrowing, reduces the supply of assets in the economy. Since a bubble is defined as the difference between the supply and demand of assets, the size of the bubble is larger when there is an Inside Bubble.

The third result is that even though steady-state consumption is higher with an Outside Bubble, a social planner would choose an Inside Bubble when productivity is low. Steady-state consumption is higher with an Outside Bubble because this bubble is less distorting. However, a social planner also needs to take into account the welfare of the first generation. There is a trade-off between the first and next generations. The first generation prefers an Inside Bubble because they have already made their investment decisions and the bubble only increases the value of their capital. The next generations prefer an Outside Bubble because it is less distorting. When productivity is low, the loss of next generations for having an Inside Bubble is low, thus, the Inside Bubble is better.

The model is an OLG economy with two types of agents: entrepreneurs and investors. The main difference between both agents is that entrepreneurs have ideas (investment opportunities) and produce, whereas investors do not have ideas, they just invest their money. Investors live two periods. When they are young they receive an endowment, which they invest to consume when they are old. Entrepreneurs live three periods. When they are young, they receive an endowment, which they use to invest in capital. When they are middle-aged, they invest everything to consume when they are old. Finally, when they are old, they consume everything and die.

An important feature of the model and the source of rational bubbles is that entrepreneurs

face an endogenous borrowing constraint. Entrepreneurs cannot promise to repay more than the future value of their capital. Moreover, only a fraction of capital can be collateralized. This type of constraint is obtained if there is, for example, limited enforcement.

Section 2 presents the model and solves the steady-state equilibrium without bubbles. First, I solve the fundamental equilibrium and show that rational bubbles can appear in this model. In particular, rational bubbles can appear if the fraction of capital than can be collateralized decreases. It could be rationalized in terms of investors losing confidence with the entrepreneurs and asking for more collateral.

Section 3 solves the steady-state equilibrium with the two bubbles. First, I consider the Outside Bubble, that is, young investors buy the bubble from the old investors and sell it when they are old to the next generation of investors. Then, I compute the steady-state equilibrium with an Inside Bubble, that is, young investors decide to buy capital to sell it when they are old, instead of lending money to old investors.

Section 4 studies what happens when the economy switches between the Outside and the Inside Bubble. I show that investment and debt of entrepreneurs are lower. In particular, the investment of both middle-aged and young entrepreneurs are lower. Young entrepreneurs do not have any capital, therefore, they invest less because the price is higher. Middle-aged entrepreneurs increase their investment at impact (if I assume that they are leveraged) but they end up with less capital. The reason is that net worth changes in two opposite directions. On the one hand, it increases because the price of capital increases. On the other hand, it falls because when they were young they could invest less. The second effect dominates, therefore, investment falls. Similarly, borrowing of young entrepreneurs does not change because the value of capital remains the same. However, borrowing of middle-aged entrepreneurs has the same dynamics that investment.

Finally, in section 5 I do welfare analysis. First, I show that the size of the Inside Bubble is larger. The size of the bubble is defined as the difference between the demand and supply of assets. When there is an Inside Bubble, entrepreneurs can borrow less, this reduces the supply of assets, which increases the size of the bubble. I also show that the steady-state consumption is higher with an Outside Bubble and that a social planner would choose an Inside Bubble over an Outside Bubble when productivity is low. The intuition for the first result is that an Outside Bubble distorts less than an Inside Bubble. The intuition for the second result is that, even though the steady-state consumption is higher with an Outside Bubble, the first generation prefers an Inside Bubble because the increase in the price of capital is higher. Moreover, when productivity is low, the benefit of having a less distortive bubble is lower.

*Related literature* This paper relates to different strands of the literature. There exists a large literature on the efficiency of the market equilibrium and the role of assets without fundamental value. It includes, among others, the seminal paper of Samuelson (1958), Cass (1972) and Diamond (1965). As discussed in these papers, under certain conditions, the market equilibrium may be inefficient and assets without fundamental value, bubbles, may improve the market allocation.

The literature on rational bubbles is rich and diverse. I consider an overlapping generation economy as in the seminal paper of Tirole (1985). However, as emphasized in Santos and Woodford (1997), the distinction between an infinitely lived agent and an overlapping generation is not crucial for the existence of rational bubbles. In my model, bubbles are possible because of the financial market imperfection represented by the collateral constraint.

The interaction between financial market imperfections and bubbles was also studied in, for example, Azariadis and Smith (1993), Caballero and Krishnamurthy (2006), Farhi and Tirole (2009), Aoki and Nikolov (2011), Arce and López-Salido (2011) and Martin and Ventura (2011). The closest paper is Arce and López-Salido (2011), which also have two bubbles.<sup>2</sup> They build a housing model and show how bubbles can appear in borrowing constrained economies. They perform welfare analysis with two bubbles: a pure asset price and a housing bubble. The only agents in their economy are households that can buy or rent houses (the stock of houses is fixed). My model features investors and entrepreneurs. By having entrepreneurs, I can study how different bubbles have different effects on investment and debt of firms. Another difference is that whereas in their setup the size of the bubble does not depend on the type of bubble, I show that the size of an Inside Bubble is always larger.

Finally, there is a large literature that studies the relationship between pure asset price bubbles (Outside Bubbles in the model) and investment. This list includes, among others, Tirole (1985), Caballero et al (2006), Farhi and Tirole (2009) and Martin and Ventura (2011). Standard models of bubbles, like in Tirole (1985), find a negative relationship between bubbles and investment. The reason is that bubbles are possible when the economy is investing too much. Therefore, the emergence of bubbles reduces this inefficiency and investment declines. New models of bubbles have tried to find a positive comovement. For example, in Martin and Ventura (2011) bubbles increase investment because inefficient entrepreneurs buy the bubble, which favors efficient entrepreneurs. Therefore, another departure from the related literature is to show that Inside and Outside Bubbles have different effects on investment and debt of manufacturing firms, as observed in the data.

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<sup>2</sup>Basco (2012) also studies the differential effect of two bubbles. The goal in that paper is to study how the effect of globalization on house prices depends on the type of bubble.

## 2 Model

This section develops the model and solves the fundamental steady-state equilibrium. I also derive the conditions under which rational bubbles can appear in equilibrium.

### 2.1 Setup

I consider an OLG economy with two types of agents: entrepreneurs and investors. The entrepreneurs are an OLG version of the farmers in Kiyotaki and Moore (1997). I assume that there are three generations of entrepreneurs: young, middle-aged and old. Each generation consists of a continuum of mass one. When they are young they receive an endowment, which they use to purchase capital. Middle-aged and old entrepreneurs do not receive any endowment. Both the endowment and population of entrepreneurs are constant over time. I also assume, without loss of generality, that entrepreneurs only consume when they are old.

The life-time utility of an entrepreneur born at time  $t$  is

$$U_t = u(c_{t+2}^{E,o})$$

where  $E, o$  stands for old entrepreneur. I assume that  $u'(c) > 0$ .

The timing of events of an entrepreneur born at time  $t$  is as follows.

1. At time  $t$  the entrepreneur is born. She receives an endowment  $E$  and purchases capital  $k_{t+1}^{E,m}$  to produce output when middle-aged.
2. The young entrepreneur can borrow against the future value of capital. However, I assume that there is limited enforcement. At time  $t+1$ , the entrepreneur should pay  $R_t d_t^{E,y}$ . However, the entrepreneur could avoid repayment by paying a fraction  $\theta$  of the value of capital. The lender will anticipate this and she will lend to the borrower only up to the point when  $R_t d_t^{E,y} \leq \theta p_{t+1} k_{t+1}$ .<sup>3</sup>
3. At time  $t+1$ , the entrepreneur is middle-aged, she repays the debt, produces output and purchases more capital to produce when she is old. She can also borrow and she faces the same collateral constraint.  $R_{t+1} d_{t+1}^{E,m} \leq \theta p_{t+2} k_{t+2}$ .
4. At time  $t+2$ , the entrepreneur is old, she repays the debt, produces output and consumes the rest.

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<sup>3</sup>This borrowing constraint is the same as in Monacelli (2009). It is also similar to Kiyotaki and Moore (1997) with the difference that they assume  $\theta = 1$ .

Therefore, the budget constraints for young, middle-aged and old entrepreneurs, respectively, are

$$E + d_t^{E,y} \geq p_t k_{t+1}^{E,m}, \quad (1)$$

$$d_{t+1}^{E,m} + F(k_{t+1}^{E,m}) + p_{t+1} k_{t+1}^{E,m} \geq R_t d_t^{E,y} + p_{t+1} k_{t+2}^{E,o}, \quad (2)$$

$$F(k_{t+2}^{E,o}) + p_{t+2} k_{t+2}^{E,o} \geq R_{t+1} d_{t+1}^{E,m} + c_{t+2}^{E,o}, \quad (3)$$

and the collateral constraints for young and middle-aged entrepreneurs, respectively, are

$$R_t d_t^{E,y} \leq \theta p_{t+1} k_{t+1}, \quad (4)$$

$$R_{t+1} d_{t+1}^{E,m} \leq \theta p_{t+2} k_{t+2}. \quad (5)$$

I assume, for simplicity, that the production function is linear. In particular, I assume that  $F(k) = \alpha k$ , where  $\alpha > 0$  is the productivity. I also assume that capital does not depreciate.

Investors live two periods. Each generation consists of a mass of one. Only young investors receive an endowment  $W$ . Both the endowment and population of investors are constant.

I assume that the life-time utility of an investor born a time  $t$  is

$$U_t = \ln u(c_t^I) + \beta \ln u(c_{t+1}^I)$$

where  $I$  denotes investor.

The timing of events of an investor born at time  $t$  is

1. At time  $t$ , the investor is born, she receives an endowment  $W$  and she chooses how much to save  $-d_t^I$ .
2. At time  $t+1$ , the investor is old, she consumes the returns on her savings,  $-R_t d_t^I$ , and she dies.

Therefore, the budget constraints of young and old entrepreneurs can be written as

$$\begin{aligned} W + d_t^I &\geq c_t^I, \\ 0 &\geq c_{t+1}^I + R_t d_t^I. \end{aligned}$$

Capital market equilibrium requires that the demand of capital of entrepreneurs equals

the supply of capital. I assume, without loss of generality, that the supply of capital is fixed<sup>4</sup>,

$$K_t^S = \bar{K} \quad \forall t.$$

The only asset in this economy are the bonds, which are in zero net supply. Therefore, in equilibrium, savings of investors must equal borrowings of entrepreneurs.

## 2.2 Bubbleless Equilibrium

In this subsection I solve the steady-state bubbleless equilibrium of the model.

**Definition** *A competitive equilibrium is a sequence of prices of capital  $p_t$ , interest rate  $R_t$ , choices of capital  $k_t$ , debt  $d_t^i$ , consumption  $c_t^i$  for  $i \in \{I, E\}$  for all  $t > 0$ , given initial endowments of investors and entrepreneurs and exogenous supply of capital, such that entrepreneurs maximize their utility given their income, investors maximize their utility given their income and all markets clear.*

The problem of entrepreneurs is

$$\begin{aligned} \max_{\{d_t^{E,y}, d_{t+1}^{E,m}, k_{t+1}^{E,m}, k_{t+2}^{E,o}, c_{t+2}^{E,o}\}} U_t &= u(c_{t+2}^{E,o}) \\ &s.t. (1) \text{ to } (5). \end{aligned}$$

I solve the equilibrium by assuming that the collateral constraint is binding and then I check that this is the optimal choice.<sup>5</sup> Therefore, the optimal choices of entrepreneurs are

$$\begin{aligned} k_{t+1}^{E,m} &= \frac{E}{p_t - \frac{\theta}{R}p_{t+1}}, & d_t^{E,y} &= \frac{\theta p_{t+1}}{R p_t - \theta p_{t+1}} E, \\ k_{t+2}^{E,o} &= \frac{(\alpha + p_{t+1})k_{t+1}^{E,m} - R d_t^{E,y}}{p_{t+1} - \frac{\theta}{R}p_{t+2}}, & d_{t+1}^{E,m} &= \frac{\theta}{R} p_{t+1} k_{t+1}^{E,o}, \\ c_{t+2}^{E,o} &= \alpha k_{t+2}^{E,o} + (1 - \theta) p_{t+2} k_{t+2}^{E,o}. \end{aligned}$$

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<sup>4</sup>In a similar environment Basco (2012) allows for an endogenous supply of capital. If I allow for a positive supply elasticity, the fall in investment with an Inside Bubble with respect to an Outside Bubble would be lower but it would still be positive.

<sup>5</sup>Investment is better than saving when  $\frac{p_{t+1} + \alpha}{p_t} \geq R_t$ . Given assumption A1 (below), in the steady-state we will have  $R \leq 1$ . Thus, since we have assumed that  $\alpha > 0$ , it follows that investment is always better and the collateral constraint will bind in equilibrium.

The first line is the investment and borrowing choices of young entrepreneurs. Their only income is the endowment  $E$  and they use it to invest. The key equation is the investment of middle-aged entrepreneurs  $k_{t+2}^{E,o}$ . Note that this equation is almost identical to the investment of farmers in Kiyotaki and Moore (1997). The only difference is that  $\theta < 1$ . However, it is still the case that if they are leveraged (i.e.,  $\alpha k_{t+1}^{E,m} < R d_t^{E,y}$ ), a proportional increase in the present and future price of capital increases investment. Finally, consumption of old entrepreneurs depends on the productivity of their technology ( $\alpha$ ) and the value of their capital.

Next, I turn to the problem of investors. An investor born at time  $t$  solves the following program

$$\begin{aligned} \max_{\{c_t^I, c_{t+1}^I\}} U_t &= \ln u(c_t^I) + \beta \ln u(c_{t+1}^I) \\ \text{s.t. } c_t^I + \frac{c_{t+1}^I}{R_t} &= W. \end{aligned}$$

Therefore, the optimal choices of the investor are given by

$$\begin{aligned} c_t^I &= \frac{1}{1+\beta} W, & d_t^I &= -\frac{\beta}{1+\beta} W, \\ c_{t+1}^I &= \frac{R\beta}{1+\beta} W. \end{aligned}$$

The young investors save a constant fraction of their endowment and they will consume the return of their savings the next period.

Having solved the problems of the entrepreneurs and investor, I can now find the steady-state equilibrium. There are three markets in this economy. By Walras' Law I can focus on only two of them. I choose the capital and bond markets.

The capital market clears when the supply of capital equals the demand of capital of entrepreneurs. Therefore, the capital market clearing condition is

$$K^m(R, \theta) + K^o(R, \theta) = \bar{K}.$$

Since there is a net zero supply of bonds, the aggregate borrowing demand must be zero. Thus, the bond market clearing condition can be written as

$$d^{E,y} + d^{E,m} + d^I = 0.$$

Using the two market clearing conditions, it follows that the unique equilibrium interest

rate  $R^{NB} = r(\theta, \beta, W, E, \bar{K})$  is implicitly defined by<sup>6</sup>

$$\frac{R^{NB}}{\theta} \frac{\beta}{1 + \beta} \frac{W}{\bar{K}} = \Phi(R^{NB}, \theta, \frac{\bar{K}}{E}).$$

It is easy to check that the equilibrium interest rate is increasing with  $\theta$ ,  $E$  and  $\bar{K}$  and decreasing with  $W, \beta$ . The reason is that an increase in either the endowment of investors ( $W$ ) or their patience ( $\beta$ ), directly increases the demand of bonds, which reduces the interest rate. Analogously, an increase in either how much can be collateralized ( $\theta$ ) or the endowment of entrepreneurs ( $E$ ), directly increases how much entrepreneurs can borrow. The supply of bonds is equal to the borrowings of entrepreneurs, therefore, the interest rate raises. The more subtle comparative statics is on  $\bar{K}$ . As the supply of capital increases, the price of capital falls, which increases how much entrepreneurs can borrow.

Once I have found the interest rate, all the other equilibrium outcomes can be derived.

$$\begin{aligned} p &= \frac{R^{NB}}{\theta} \frac{\beta}{1 + \beta} \frac{W}{\bar{K}}, \\ k^{E,m} &= \frac{E}{(1 - \frac{\theta}{R^{NB}})p^{NB}}, \quad k^{E,o} = \frac{\alpha + (1 - \theta)p^{NB}}{(1 - \frac{\theta}{R^{NB}})p^{NB}} k^{E,m}, \\ c^E &= [\alpha + (1 - \theta)p^{NB}] k^{E,o}, \quad c^I = \frac{R^{NB}\beta}{1 + \beta} W. \end{aligned}$$

To guarantee that this is a solution and to allow the possibility of rational bubbles to appear in equilibrium, I make the next assumption on the pledgeable fraction of capital  $\theta$ .

### Assumption A1

$$\theta < \theta^* \quad \text{where} \quad \frac{1}{\theta^*} \frac{\beta}{1 + \beta} \frac{W}{\bar{K}} = \Phi(1, \theta^*, \frac{\bar{K}}{E}).$$

This assumption implies that there is a shortage of assets in equilibrium. The intuition behind this assumption is that the problem of limited enforcement is severe enough to make lenders ask a fraction higher than  $1 - \theta^*$  of capital as collateral.

Given assumption A1,  $R^{NB} < 1$  and the collateral constraint is binding. I have a negative real interest rate because there is no growth in the economy. In this type of economies it is not unusual to have a negative real interest rate (see, for example, Kocherlakota 2009).

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<sup>6</sup>The capital market clearing condition can be written as  $\left[ \frac{R}{R-\theta} \frac{E}{p} \right] \left[ 1 + \frac{R}{R-\theta} \frac{\alpha + (1-\theta)p}{p} \right] = \bar{K}$ , which defines a relationship between price of capital and interest rate,  $p = \Phi(R, \theta, \frac{\bar{K}}{E})$ . It is easy to check that  $\Phi(\cdot)$  is decreasing with  $R$ . Similarly, the bond market clearing condition can be written as  $p = \frac{R}{\theta} \frac{\beta}{1 + \beta} \frac{W}{\bar{K}}$ , which is increasing with  $R$ . Therefore, there is a unique equilibrium interest rate  $R^{NB}$ .

Note also that  $\theta^*$  depends on  $W$ . It is easy to check that  $\theta^*$  increases with the endowment of investors  $W$ . It means that for a given  $\theta$ , the larger is the endowment of investors, the more likely is that A1 holds. Therefore, with a constant  $\theta > \theta^*$ , if the economy experiences a capital inflow (an increase in  $W$ ), A1 may hold. In this paper I abstract from the reasons behind A1 and I consider a closed economy.<sup>7</sup>

### 3 Equilibrium with Bubbles.

In this section I show that if assumption A1 holds, rational bubbles can appear in equilibrium. In particular, subsection 3.1 computes the steady-state equilibrium with an Outside Bubble and subsection 3.2 with an Inside Bubble.

#### 3.1 Outside Bubble

In this subsection I find the steady-state equilibrium with an Outside Bubble. I define an Outside Bubble as investors engaging in a Ponzi game. Young investors buy a useless and costless asset and they sell it when they are old to the next generation of investors.

Rational bubbles are possible if two conditions are satisfied. The first condition is the arbitrage condition. Investors must be indifferent between investing in the bubble and buying a bond. Since both the endowment of investors and population are constant, the return on the bond, the interest rate, must be one.

The second condition is that there must be a shortage of assets. It means that the supply of assets must be smaller than the demand of assets.

Therefore, rational Outside Bubbles are possible if these two equations are satisfied.

$$d^{E,y}(R = 1) + d^{E,m}(R = 1) + d^I(R = 1) + B = 0$$

and  $B > 0$ .

I solve the equilibrium assuming that these two conditions are satisfied and then I check that  $B > 0$ .

The capital market clearing condition in steady-state can be written as

$$K^m(1, \theta) + K^o(1, \theta) = \overline{K}.$$

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<sup>7</sup>See, for example, Caballero et al (2006) or Basco (2012), for a model in which capital inflows are conducive to bubbles.

The bond market clearing condition in steady-state can be written as

$$d^{E,y}(R = 1) + d^{E,m}(R = 1) + d^I(R = 1) + B = 0.$$

Plugging the optimal choices of entrepreneurs and investors in the market clearing conditions we find that

$$B = \frac{\beta(1 - \theta)W - (1 + \beta)\theta\psi E}{(1 + \beta)(1 - \theta)} > 0,$$

$$p = \frac{1}{1 - \theta} \frac{E}{\bar{K}} \psi,$$

where  $\psi = 1 + \left[1 + \alpha \frac{\bar{K}}{E}\right]^{1/2}$ .

The intuition for the existence of the Outside Bubble can be seen in Figure 5. This figure represents the steady-state bond market. The vertical line is the demand of assets. The downward slopping line is the supply of assets. Given assumption A1, the equilibrium interest rate without bubbles is below one. Therefore, when the interest rate is one the supply of assets is lower than the demand of assets and a bubble may appear. In other words, the size of the bubble is the horizontal distance between the demand of funds (savings of investors)  $-d^I$  and the supply of funds (borrowings of entrepreneurs)  $d^{E,y} + d^{E,m}$  when  $R = 1$ .

Finally, the equilibrium capital and consumption allocations are

$$K^m = \frac{1}{\psi} \bar{K}, \quad K^o = \frac{\psi - 1}{\psi} \bar{K},$$

$$C^o = \frac{\psi - 1}{\psi} [\alpha \bar{K} + \psi E].$$

### 3.2 Inside Bubble

In this subsection I compute the steady-state equilibrium when there is an Inside Bubble. When there is an Inside Bubble, I assume that investors, instead of coordinating among themselves, decide to invest in capital. Since investors do not have any production technology, they just store the capital and sell it when they are old.

The return on investing on capital is the price appreciation  $\left(\frac{p_{t+1}}{p_t}\right)$ . Therefore, the two conditions under which an Inside Bubble can appear are the same as for an Outside Bubble.

$$d^{E,y}(R = 1) + d^{E,m}(R = 1) + d^l(R = 1) + B = 0$$

and  $B > 0$ .

I proceed in the same manner as in subsection 2.3. I claim that the bubble exists and then I check that it is indeed the case. Note that the bond market clearing condition is the same.

$$d^{E,y}(R = 1) + d^{E,m}(R = 1) + d^l(R = 1) + B = 0.$$

However, the capital market clearing condition is different because investors now use a part of their savings to invest in capital.

$$K^m(1, \theta) + K^o(1, \theta) + \frac{B}{p} = \bar{K}.$$

By plugging the optimal choices of entrepreneurs and investors into the market clearing conditions, we find that<sup>8</sup>

$$\begin{aligned} p &= \phi\left(\theta, \frac{\bar{K}}{E}, \frac{W}{E}\right), \\ B &= \frac{\beta}{1+\beta}W - \theta p [K^m(1, \theta) + K^o(1, \theta)], \\ K^m &= \frac{E}{(1-\theta)p}, \quad K^o = \frac{\alpha + (1-\theta)p}{(1-\theta)p} K^m. \end{aligned}$$

It is easy to check that the size of the bubble is also positive. However, I defer this discussion until section 5 where I compare the size of the two bubbles.

## 4 Switching Bubbles: From Outside to Inside Bubble

This section studies the effects on investment and debt of firms when an economy switches between bubbles.

I assume that the economy starts with an Outside Bubble. However, investors fail to coordinate and the Outside Bubble is immediately replaced by an Inside Bubble. One inter-

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<sup>8</sup>It can be shown that  $p = \phi\left(\theta, \frac{\bar{K}}{E}, \frac{W}{E}\right) = \frac{(1-\theta)b + \left[(1-\theta)^2 b^2 + 4(1-\theta)\alpha \frac{\bar{K}}{E}\right]^{1/2}}{2(1-\theta)\frac{\bar{K}}{E}}$  where  $b = 2 + \frac{\beta}{1+\beta} \frac{W}{E}$ .

pretation is that young investors, instead of buying the piece of paper that old investors are selling (Outside Bubble), they buy capital (Inside Bubble).

In this exercise I make two assumptions. The first assumption is that prices are fully flexible. It implies that the price of capital immediately adjusts to the new steady-state. The second assumption is that entrepreneurs are leveraged. It means that the value of production is lower than the debt (i.e.,  $\alpha K^m < D^{E,y}$ ).<sup>9</sup>

## 4.1 Investment

In this subsection I study the effect on investment. Panel *a* of Figure 6 shows that prices immediately adjust to the new steady-state. Prices with an Inside Bubble are higher than with an Outside Bubble because now there is an extra demand for capital (investors).

Panel *b* of Figure 6 shows the evolution of the investment of young entrepreneurs. The investment of young entrepreneurs decreases and capital adjusts directly to the new steady-state. The reason is that young entrepreneurs have no wealth, therefore, all the effect is through the user cost of capital. Therefore, with higher prices they purchase less capital. Remember that the capital of middle-aged entrepreneurs is

$$K^m = \frac{E}{(1 - \theta)p}. \quad (6)$$

Panel *c* of Figure 6 shows the evolution of the investment of middle-aged entrepreneurs. The investment of middle-aged entrepreneurs increases at impact and then they converge to the new steady-state. The investment of middle-aged entrepreneurs is

$$K^o = \frac{(\alpha + p)K^m - D^y}{(1 - \theta)p}. \quad (7)$$

At impact, the amount of capital that they purchased when they were young,  $K^m$ , and the debt,  $D^y$ , is fixed. Therefore, an increase in prices will increase investment because the increase in net worth (the numerator) is higher than the increase in the user cost of capital (denominator). This is true because of the assumption that entrepreneurs are leveraged (i.e.,  $\alpha K^m < D^{E,y}$ ).

Investment is lower in the new steady-state. The reason is that there are two effects. On the one hand, a rise in the price of capital raises the net worth of entrepreneurs. On the other hand, a rise in the price of capital reduces the investment that the entrepreneur made when she was young. The second effect dominates, therefore, the investment of middle-aged

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<sup>9</sup>Assumption A1 does not guarantee that the leverage of entrepreneurs is positive in the steady-state.

entrepreneurs is lower with an Inside Bubble.<sup>10</sup>

Note that if I did not assume that the net worth of entrepreneurs is negative the only difference would be that, at impact, investment would increase.

In the next subsection I study the effect on the debt of entrepreneurs and the size of the bubble.

## 4.2 Debt and Size of the Bubble

Panel *d* of Figure 6 represents the debt of young  $D^y$  and middle-aged entrepreneurs  $D^m$ . Borrowing of entrepreneurs is limited by the collateral constraint, therefore, it is proportional to the value of capital. Note that the borrowing of young entrepreneurs is the same with the two types of bubbles. From equation (6) it is clear that  $pK^m$  is constant. The intuition is that the wealth of young entrepreneurs is the endowment that they receive, which does not depend on the price of capital.

The amount that middle-aged entrepreneurs borrow increases at impact. However, in the steady-state, the borrowings with the Inside Bubble are lower than with the Outside Bubble. From equation (7), we can see that the value of capital has two components. The value of capital purchased when young  $pK^m$  and the leverage  $\alpha K^m - D^y$ . At impact,  $K^m$  and  $D^y$  are constant, therefore an increase in price raises the value of capital purchased when young, which increases borrowing. Note that, as opposed to the real investment  $K^o$ , this result does not depend on the sign of the second component (leverage).

In the new steady-state the debt of middle-aged entrepreneurs is lower. The intuition is that the value of capital purchased when young is constant but the leverage decreases. The value of production  $\alpha K^m$  is lower with the Inside Bubble, whereas the borrowing of young entrepreneurs  $D^y$  is the same.

Panel *e* of Figure 6 shows how the size of the bubble changes. Remember that the size of the bubble is defined as the difference between the demand and the supply of bonds. The demand of bonds, savings of investors, is a fraction of the endowment, which is constant. The supply of bonds is the total debt of entrepreneurs.

Given that the demand of bonds does not change when we switch between bubbles, the change in the size of the bubble is the mirror image of the change in the borrowings of middle-aged entrepreneurs. Therefore, the size of the bubble drops at impact (it seems a bust instead of a switch between bubbles). However, in the steady-state the size of the bubble is larger when there is an Inside Bubble.

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<sup>10</sup>Capital of old-aged entrepreneurs can be written as  $k^o(p) = \frac{\alpha + (1-\theta)p}{(1-\theta)p} k^m(p)$ . Taking derivative of  $k^o(p)$  with respect to  $p$  and noting that  $\frac{\partial}{\partial p} k^m(p) < 0$ , it follows that  $k^o(p)$  decreases with the price of capital.

### 4.3 Does the model fit the data?

In the introduction we have seen that non-residential and residential investment followed different trends during the two bubble episodes (Figure 2). Non-residential investment was below average during the Housing Bubble. In contrast, residential investment was above average during the same period. If I assume that investors purchasing capital is residential investment, the model is able to qualitatively explain the data. When the economy switched from an Outside (Dot-Com) to an Inside Bubble (Housing), investors start buying capital for non-productive purposes, which increased residential investment but had a negative effect on non-residential investment.

We have also seen in Figure 3 that the liabilities of manufacturing firms were lower during the Housing Bubble. The model can also qualitatively explain the fall in the debt of manufacturing firms. When the economy switches to an Inside Bubble, investors start competing with entrepreneurs for capital. This competition has two effects on the borrowings of entrepreneurs. On the one hand, the price of capital increases, which raises the value of capital, thereby increasing their borrowing. On the other hand, entrepreneurs can purchase less capital, which reduces how much they can pledge, and, thus, borrow. In the model, the second effect dominates and debt of entrepreneurs decreases.

## 5 Welfare Analysis

This section performs welfare analysis. First, I show that the size of the bubble depends on the type of bubble. Second, I show that the steady-state consumption is always higher with an Outside Bubble. Finally, I analyze which bubble a social planner would choose and when one type of bubble is better.

### 5.1 Size of the bubble

In this section I show that the size of the bubble with an Inside Bubble is always larger than with an Outside Bubble.

**Proposition 1** *The Inside Bubble is larger than the Outside Bubble.*

**Proof.** To prove this result note that the size of bubble can be written as  $B^j = \frac{\beta}{1+\beta}W - \theta E\xi(p^j)$  for  $j = \{In, Out\}$ , where  $\xi(p^j) = p^j [K^{m,j}(1, \theta) + K^{o,j}(1, \theta)] = \frac{2(1-\theta)p^j + \alpha}{(1-\theta)^2 p^j}$  and  $In$

and  $Out$  stand for Inside and Outside Bubble, respectively. It is immediate to show that  $\xi'(p^j) < 0$ . Therefore, since  $p^{In} > p^{Out}$ , we have that  $B^{In} > B^{Out}$ . ■

The intuition for this result can be seen in Figure 8. It represent the steady-state bond market for the two bubbles. The vertical line is the demand of bonds which is the fraction of the endowment that investors save. This demand of bonds does not depend on the type of bubble. The downward slopping lines, the supply of bonds, are the borrowing choices of entrepreneurs. As we have seen above, the debt of middle-aged entrepreneurs depends negatively on the price of capital. When there is an Inside Bubble, investors purchase capital, which increases the price of capital and reduces the borrowings of middle-aged entrepreneurs. Therefore, the supply of bonds falls (shifts to the left). Since when there is a bubble the interest rate is equal to one, the difference between demand and supply of assets (the bubble) is higher with an Inside Bubble.

This result does not depend on the zero supply elasticity of capital. If I allowed for a positive supply elasticity, the rise in prices would be lower but it would still be the case that the Inside Bubble is larger than the Outside Bubble. Note from the proof that only when  $\alpha = 0$ , the two bubbles have the same size. The reason is that, in this case, debt is independent of the price of capital.

## 5.2 Which bubble a social planner would choose?

This subsection proves that steady-state consumption is higher with an Outside Bubble and then analyzes when a social planner is more likely to choose one type of bubble over another.

**Proposition 2** *Steady-state consumption is higher with an Outside Bubble.*

**Proof.** To prove this result, remember that the interest rate is one with both types of bubbles. Therefore, investors are indifferent between both bubbles. The consumption of an old entrepreneur is  $C^j = [\alpha + (1 - \theta)p^j] K^{old,j} = \Gamma(p^j, \theta)$  for  $j = \{In, Out\}$ , where  $\Gamma(p^j, \theta) = \left[1 + \frac{\alpha}{(1-\theta)p^j}\right]^2 E$ . It is immediate to show that  $\Gamma(p^j, \theta)$  is decreasing with  $p^j$ . Therefore, since  $p^{In} > p^{Out}$ , it follows that  $C^{Out} > C^{In}$ . ■

The reason why an Outside Bubble is better is as follows. There are two effects: a price and a quantity effect. On the one hand, when there is an Inside Bubble, the extra demand for capital raises the price of capital, which increases consumption. On the other hand, with the Inside Bubble, the competition for capital, reduces the amount of capital that entrepreneurs can purchase, which reduces consumption. The second effect always dominates, therefore, steady-state consumption is higher with an Outside Bubble.

Now I assume that the economy is in a steady-state without a bubble and it can switch to a steady-state with an Inside Bubble or an Outside Bubble. I show that a social planner would choose to switch to an Inside Bubble when productivity is low.

**Proposition 3** *As productivity decreases, an Inside Bubble becomes relatively better than an Outside Bubble.*

I assume that a social planner cares about the consumption of all future generations. In particular, she maximizes

$$Welfare_t = \sum_{t=0}^{\infty} \delta^t \left[ u(c_t^{E,o}) + u(c_t^I) \right],$$

where  $c_t^{E,o}$  is consumption of old entrepreneurs and  $c_t^I$  is consumption of investors. Since the interest rate is the same with both bubbles, the consumption of investors is the same. Therefore, the social planner only needs to maximize the consumption of old entrepreneurs.

If at time  $t$  the economy coordinates to bubble  $j = \{In, Out\}$ , the first generation of old entrepreneurs will see that the price of their capital is  $p^j$ . The capital and price of capital for the next generations will be the steady-state values  $(p^j, K^j)$ . Therefore, a social planner chooses an Inside Bubble if

$$C(p^{In}, K^{NB}) + \frac{\delta}{1-\delta} C(p^{In}, K^{In}) \geq C(p^{Out}, K^{NB}) + \frac{\delta}{1-\delta} C(p^{Out}, K^{Out}).$$

By using the consumption functions derived in section 3, it follows that an Inside Bubble is better if

$$K^{NB} (p^{In} - p^{Out}) \geq \frac{\delta}{1-\delta} \left[ \frac{\alpha}{1-\theta} (K^{Out} - K^{In}) + (p^{Out} K^{Out} - p^{In} K^{In}) \right]. \quad (8)$$

The left-hand side of equation 8 is the benefit of having an Inside Bubble and the right-hand side is the relative loss.

It cannot be proved analytically that under any configuration of the parameter space one type of bubble is better. To give an example, consider the parameter  $\delta$ . When  $\delta = 0$ , the social planner only values the consumption of the first generation, an Inside Bubble is always better. The reason is that the price of capital is higher with the Inside Bubble. On the other extreme, when  $\delta = 1$ , the social planner only values the consumption of the future generations, an Outside Bubble is always better. The reason is that steady-state consumption is higher with an Outside Bubble (Proposition 2).

The comparative statics I want to emphasize is how changes in the productivity of entrepreneurs affect the choice of the social planner.

The answer is that as productivity decreases, an Inside Bubble becomes relatively better than an Outside Bubble. This result can be seen in figure 8. The darker line is the left-hand side of equation 8 and the lighter line is the right-hand side of equation 8. Note that when

productivity is very high (in the figure,  $\alpha = 3$ ), the two lines are at zero. The reason is that at this productivity level the size of the bubble is zero. In this figure the two lines only cross once at  $\alpha = \alpha^*$ . When productivity is below (above)  $\alpha^*$ , the benefit of having an Inside Bubble is larger (lower) than the loss, therefore, the Inside (Outside) Bubble is better. The intuition is as follows.

The benefit of having an Inside Bubble is the price difference  $p^{In} - p^{Out}$ , which is decreasing with the productivity. The reason is that as productivity increases, the borrowing of entrepreneurs increases, which reduces the size of the bubble. Since the size of the bubble is small, the increase in the price of capital when there is an Inside Bubble is smaller. This can also be seen in the proof of Proposition 1.

The relative loss of having an Inside Bubble is the consumption difference in the steady-state, which has an inverse U-shape. When productivity ( $\alpha$ ) is very low, the effect of the price of capital on the borrowing of middle-aged entrepreneurs is small. Thus, the consumption of old entrepreneurs is very similar with both bubbles. Even though the bubble is very large  $p^{Out} \ll p^{In}$ , the value of investment is very similar  $p^{Out} K^{Out} \simeq p^{In} K^{In}$ . When productivity is very large, the two bubbles are very small and the negative effects of the Inside Bubble are also small. That is,  $p^{Out} \simeq p^{In}$  and  $K^{Out} \simeq K^{In}$ . For intermediate values of productivity, the effect of higher prices on the borrowing of middle-aged entrepreneurs is high. In this case, investment falls ( $K^{In} < K^{Out}$ ) and it is not compensated by higher prices. That is,  $p^{Out} K^{Out} > p^{In} K^{In}$ .

Therefore, an Inside Bubble is better when productivity is low. When productivity is low, the bubble is large, which means that the gain of the first generation of old-entrepreneurs is large. Moreover, the loss of next generations is small because the fall in capital is offset by the increase in its price.

In other words, an Inside Bubble is inefficient because some capital instead of being used to produce output, it is purchased for investors as a store of value. This inefficiency is higher, the more productive the entrepreneurs are. Therefore, the more productive an economy is, the worse is to have an Inside Bubble.

### 5.2.1 Complementarity of Productivity and Financial Institutions

In this subsection I ask in which countries a social planner is more likely to choose one type of bubble over another.

In order to answer this question I interpret the tightness of the collateral constraint,  $\theta$ , as an index of the quality of financial institutions. The idea is that in more financially developed countries, the problem of limited enforcement is less severe, therefore, lenders ask borrowers a lower fraction of capital as collateral.

**Proposition 4** *An Inside Bubble is better in less financially developed countries.*

Figure 9 shows how the relative benefit and loss of having an Inside Bubble is affected by the level of financial institutions ( $\theta$ ). The dotted line represents an economy with a lower level of financial development. Note that the Inside Bubble would be chosen for a wider range of productivity values in the economy with a lower level of financial development.

The intuition is that in countries with less financially developed institutions (low  $\theta$ ), the size of the bubble is larger (see Proof of Proposition 1). The gain of having an Inside Bubble increases with the size of the bubble (the price difference is higher). The loss of having an Inside Bubble also increases with the size of the bubble. However, we can see that the threshold of productivity in which the two bubbles give the same utility is higher in the country with lower  $\theta$ .

It is easy to prove that, in the neighborhood of  $\alpha = 0$ , the net gain of having an Inside instead of an Outside Bubble is larger in the country with the less financially developed institutions. The reason is that whereas the consumption of next generations is not affected by the size of the bubble, the price increase in the first generation does depend on the size of the bubble.

Then, by a continuity argument, we can say that the lower is the level of financial institutions  $\theta$ , the wider will be the range of productivity levels ( $\alpha$ ) in which an Inside Bubble is better than an Outside Bubble (see figure 10). In this sense, an Inside Bubble is better in less financially developed countries.

Therefore, this model could rationalize why real estate bubbles tend to appear more often in financially underdeveloped countries and why they are a good solution to their shortage of assets.<sup>11</sup>

## 6 Concluding Remarks

In this paper I developed a framework to understand the differential effect that two types of rational bubbles have on investment and debt. I distinguished between (i) Outside Bubble, which I defined as investors buying and selling useless and costless assets among themselves and (ii) Inside Bubble, which I defined as investors buying an input of production, only as a store of value.

The model is an OLG economy with investors and entrepreneurs. Investors only have an endowment when they are young and they do not have any production technology. Therefore,

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<sup>11</sup>See, for example, Caballero and Krishnamurthy (2006) for a discussion on real estate bubbles in emerging economies.

they need to invest to consume when they are old. Entrepreneurs invest and produce the consumption good. They can borrow to invest but they face a collateral constraint.

Rational bubbles can appear when there is a shortage of assets. Therefore, bubbles can appear when the supply of assets (borrowings of entrepreneurs) is smaller than the demand of assets (savings of investors). I showed that in this environment rational bubbles can appear in equilibrium when the collateral constraint is tight enough.

I computed the steady-state equilibrium for the two types of bubbles. I showed that the size of the bubble is larger when there is an Inside Bubble. The reason is that when investors compete with entrepreneurs, the borrowing of entrepreneurs decreases, which reduces the supply of assets, thereby increasing the size of the bubble.

Then, I analyzed what happens when the economy switches from an Outside to an Inside Bubble. I showed the investment and debt of entrepreneurs decline, which is consistent with the data. Non-residential investment and liabilities of manufacturing firms in the United States were lower during the Housing Bubble (Inside Bubble).

I also performed welfare analysis. I proved that the steady-state consumption is higher with an Outside Bubble. The reason is that the Inside Bubble is a less efficient solution to the shortage of assets of the economy. With an Inside Bubble, investors channel their excess demand of assets to hoard capital, which reduces the capital that entrepreneurs can purchase, instead of engaging in a Ponzi scheme among themselves (Outside Bubble).

Finally, I assumed that the economy was in a bubbleless equilibrium and a social planner could choose a type of bubble. I showed that as productivity decreases, an Inside Bubble becomes relatively better than an Outside Bubble. The intuition is that the loss of having an Inside Bubble is lower, the lower is the productivity of entrepreneurs.

To conclude, in this paper I showed that Inside and Outside Bubbles have different effects on the economy. A natural future step is to ask whether different Inside Bubbles also have different effects. In particular, if some inputs are better as collateral than others, in which input is better to have a bubble? Do different Inside Bubbles have different effects on the efficiency of firms? Which is the best Inside Bubble? I am currently working on answering these questions.

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## 8 Figures

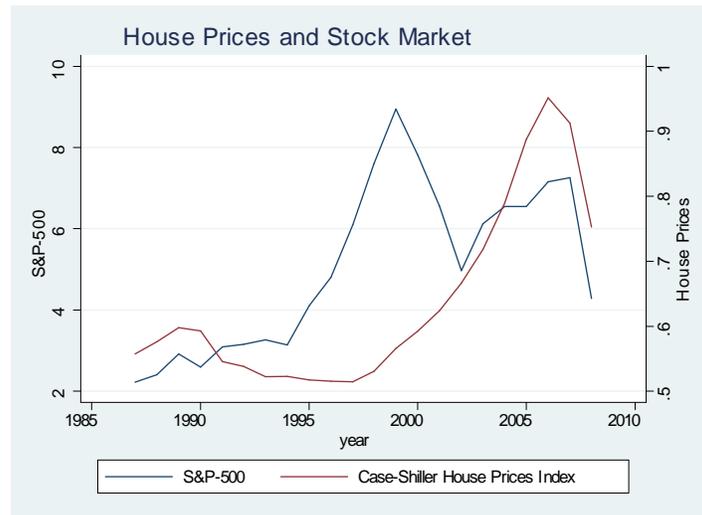


Figure 1: The Dot-Com and the Housing Bubbles.

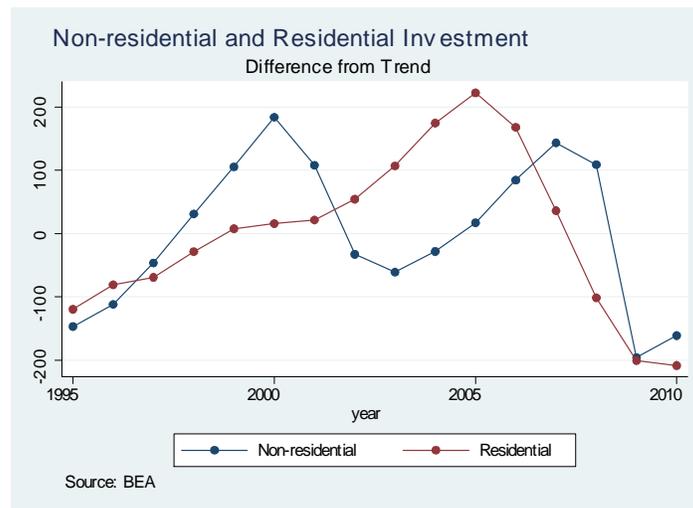


Figure 2: Evolution of residential and non-residential investment in the United States.

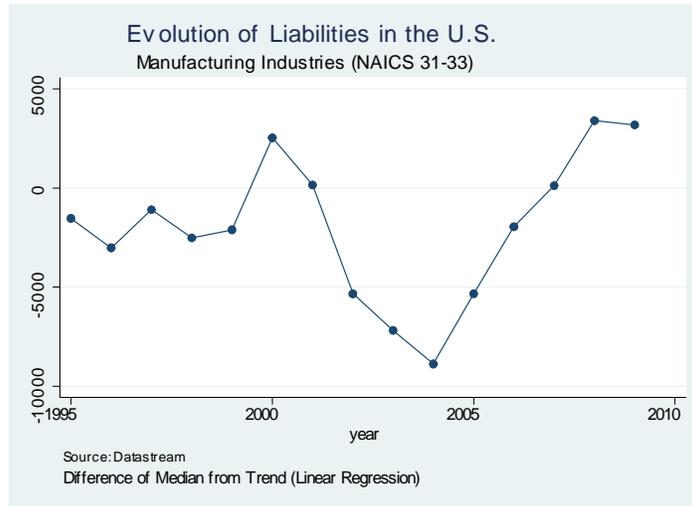


Figure 3: Evolution of liabilities of the manufacturing industry in the United States.

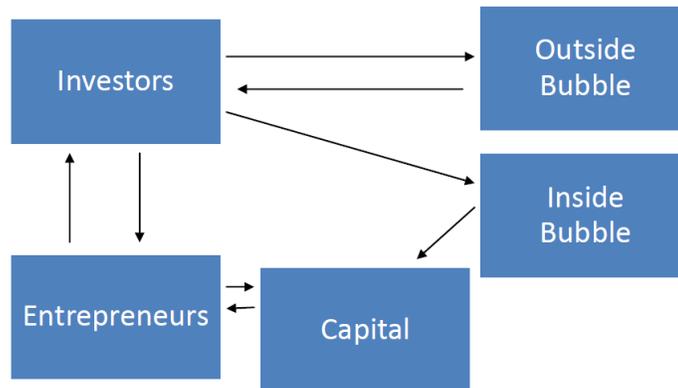


Figure 4: Description of the model.

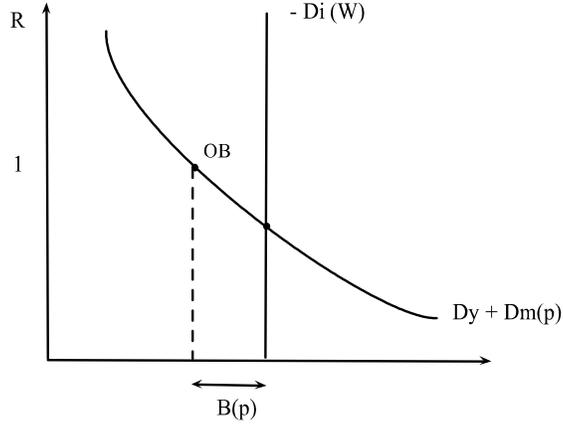


Figure 5: Bond market in steady-state with an Outside Bubble.

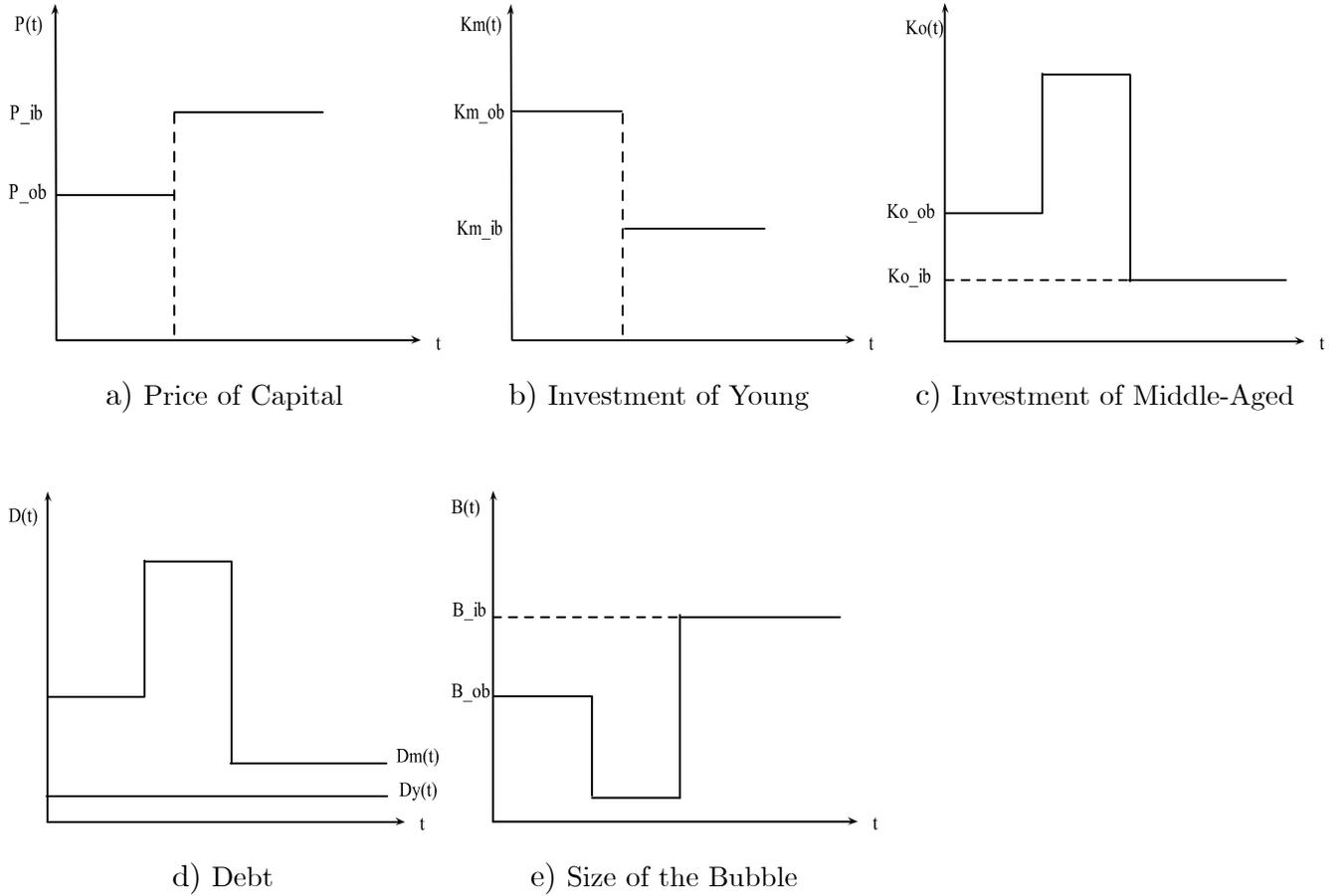


Figure 6: Switching Bubbles: From Outside to Inside Bubbles. I assume that the price of capital directly adjusts to new steady-state.

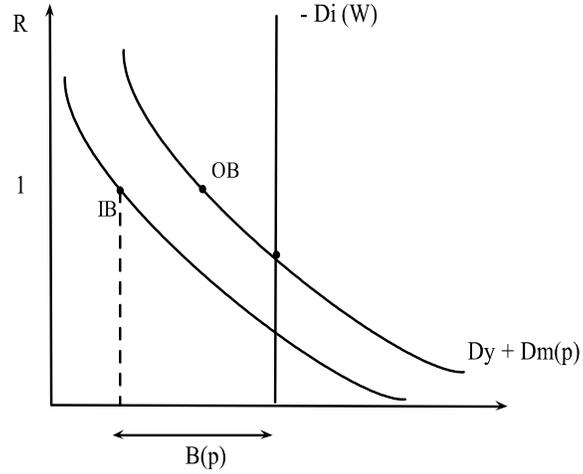


Figure 7: Size of the Bubble: Outside and Inside Bubbles

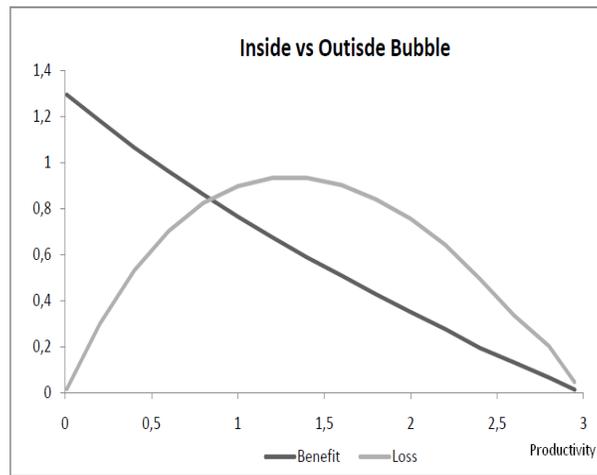


Figure 8: Welfare: Benefit and Loss of an Inside Bubble.

I assume  $\theta = 0.2$ ,  $\beta = 0.6$ ,  $E = 10$ ,  $W = 20$ ,  $\bar{K} = 10$  and  $\delta = 0.6$ ,

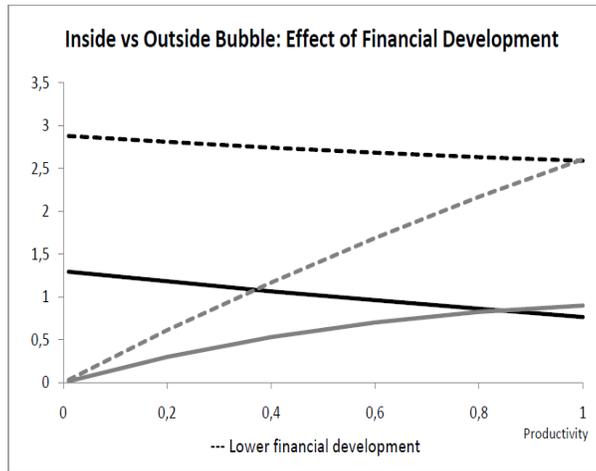


Figure 9: Welfare: Effect of Financial Development.

Same parameters as Figure 8. The dashed line with  $\theta = 0.1$  instead of  $\theta = 0.2$

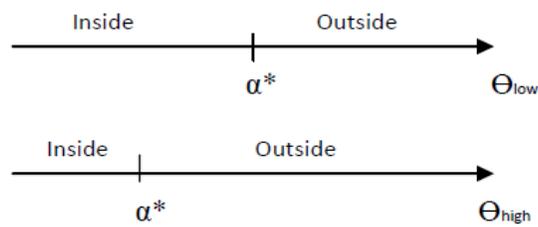


Figure 10: Inside Bubbles are better in less Financially Developed Countries.