Competition and Strategic Control of a Central Counterparty: When Lower Risk Increases Profit*

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Abstract

We model the strategic behavior of dealers in Over-the-Counter (OTC) derivatives markets where a small number of dealers trade with a continuum of heterogeneous clients (hedgers). Imperfect competition and (endogenous) default induces a familiar trade-off. Increasing the number of dealers servicing the market decreases prices for hedgers but lowers revenue for dealers, increasing the probability of a default. Restricting entry maximizes welfare when dealers's efficiency is high relative to their market power. A Central Counter-Party (CCP) offering novation tilts the trade-off toward more competition. Free-entry is optimal for all level of dealers' efficiency if the can constrain risk-taking by its members. Importantly, dealers can frame CCP rules to restrict entry and increase their benefits. Moreover, dealers strategically impose binding risk constraints to increase revenues at the expense of the hedgers. In other words, risk controls serve as a coordination device and dealers can commit to lower degree of competition. These results have important implications for ongoing international reforms to increase the role of CCPs.

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"Bank of New York Mellon has been trying to become a so-called clearing member since early this year. But three of the four main clearinghouses told the bank that its derivatives operation has too little capital, and thus potentially poses too much risk to the overall market."

A Secretive Banking Elite Rules Trading in Derivatives, New York Times, 2010.

1 Introduction

Over-the-counter (OTC) markets are decentralized markets where dealers offer intermediation services to hedgers or investors. In practice, a small number of dealers provide customized intermediation services to a large numbers of diverse investors. This immediately raises questions about the competitive environment and the extent to which OTC dealers can exercise market power. The answers hinge on the nature of any barriers to entry into the different infrastructure underlying OTC markets (e.g., trading platforms, clearinghouses). Moreover, the interaction between the level of competition and the level of counterparty risk in OTC markets raises other questions. As the introductory quote illustrates, existing clearinghouse members often claim that restrictions to entry are necessary to control counterparty risk. The financial crisis of 2007-2009 revealed that counterparty credit risk in OTC markets was concentrated in the hands of large global dealers, with significant consequences for the stability of OTC markets (Duffie, 2010b). In response, regulators are pursuing the increased use of Central Counter-Party (CCPs) to enhance the resilience of OTC markets.[1]

We consider an OTC market where a small number of dealers meet a continuum of risk-averse agents with an uncertain endowment (i.e., hedgers). Dealers offer hedgers a swap contract that promises a fixed payment in returns of the random endowment. Dealers are differentiated and have market power. For instance, hedger's utility from trading can vary across dealers because of preferences, differentiated ancillary services, or because the costs of establishing a relationship vary across dealers. We use the well known Salop (1979) circular city model to study competition with horizontal differentiation. Although the swap contract transfers the endowment risk to the dealer, it does not eliminate risk for hedgers since the transaction introduces default risk. Dealers have limited liability, they face an idiosyncratic shock, and they may default. The reduction of the risk for hedgers depends on the scale of dealers' idiosyncratic shock. We interpret the reduction of risk as a measure of dealers'

[1] The leaders of the G-20 group of nations stated in September 2009 that all standardized OTC derivatives contracts should be cleared through CCPs. Norman (2010) discusses how CCPs effectively managed counterparty risk in some markets during the crisis. Duffie et al. (2010) discuss the policy response to problems in the OTC derivatives market during the crisis.

ability to transfer risk efficiently. For instance, efficient dealers may have access to hedging strategies that transfer most of the risk arising from the swap contract. On the other, less efficient dealers may have access to imperfect hedging strategies that involves higher basis risks or other residual risks (e.g., operational risks).

First, we find that increasing the number of dealers alters the industrial structure of the OTC market. The trade-off is the following. Increasing the number of dealers improves welfare of hedgers via two channels. A higher number of dealers intensifies competition, lowering the price paid by hedgers. More dealers also implies that the pool of dealers is more diverse, lowering transaction costs for hedgers. On the other hand, a lower price reduces revenue and increases the probability of a dealer's default. This reduces welfare. Then, the socially optimal level of entry involves trading-off the benefits of a lower price and of lower transaction costs against the consequences of a higher default probability. Depending on model parameters, restrictions to entry may be optimal to balance the conflicting effect of competition.[2] Hedgers prefer free-entry when dealers' market power is high relative to their ability to transfer risk efficiently. The gains from competition, in terms of lower prices and lower transaction costs, exceed its costs. Otherwise, hedgers prefer a situation where entry is restricted and competition is inhibited.

Idiosyncratic default risk justifies the introduction of novation via a CCP to reap efficiency gains from diversification (Koeppl and Monnet, 2010). Novation means that the CCP stands between hedgers and dealers, becomes a party to every trade, and absorb counterparty risk in the OTC market.[3] As in Koeppl and Monnet (2010) novation is not a guarantee. Instead, a CCP can only pool the resources available from its members to fullfill its obligations. We analyze the effect of novation on the competitive environment, the trading strategies of each agent and the endogenous (joint) distribution of dealers' default. The CCP offers novation but may also control access to the market, via membership requirements, and affect the probability of losses due to dealers' default.[4] We then ask how these risk controls affect the optimal level of entry. We also ask what controls and requirements dealer-members would

[2]This results is related to a vast literature that assesses the interaction between competition and stability in the banking industry.Competition may reduce or promote stability. See Keeley (1990), Boyd and De Nicolò (2005), Martinez-Miera and Repullo (2010) and the review in Vives (2010). However, our results show that market infrastructure, such as CCPs, introduces an important difference between market-based intermediation and banking.

[3]CCPs have long played an important role in markets for exchange-traded securities but have been absent from most OTC derivatives markets. Counterparty credit risk is more salient in derivative markets since contractual relationships between counterparties are long-lasting and, therefore, the interaction between controlling default risk and competition is especially relevant.

[4]CCPs constrain and mitigate counterparty risk via different means, including margins, haircuts, default funds, mutualisation, multilateral netting and others.

choose if they controlled the CCP.[5]

Our second contribution establishes that a CCP that maximizes its members' profits favors strict membership rules to achieve a lower level of entry relative to the social optimum. Membership requirements thus act as a barrier to entry. Realistically, the number of dealers in the market may be determined from history and the CCP cannot reduce their number. In this case, the CCP chooses to implement stringent risk controls on its members' trading activities. This is perhaps surprising since, individually, dealers with limited liability prefer to avoid restrictions on their ability to take risk. But CCP members as a group prefer binding risk-controls. In effect, controlling the CCP allows dealers to commit to lower competition and obtain a more favorable price in equilibrium. In contrast, a social planner that controls the CCP favors free-entry membership requirements to increase diversity and enhance competition between dealers.[6]

Third, we find that the introduction of novation tilts the social optimum toward a greater level of competition. Indeed, free-entry may become optimal if the benefits from the diversification of default risk are sufficiently high. Each case arises because the diversification benefits affect the trade-off associated with increasing the number of dealers. Novation reduces the welfare costs of an increase in dealers default probability associated with higher level of competition. This argument can be extended and applied to other aspects of CCPs that reduce the marginal welfare costs of default associated with new entrants. Consistent with general results in Koeppl and Monnet (2010), a CCP that offers novation improves welfare. They study the benefits of CCP's novation and mutualization with a continuum of dealers. We emphasize the effect of novation on the trade-off underlying the optimal level of entry.

Finally, free-entry is socially optimal if the CCP has the ability to directly limit dealers' risk. This results is similar to the previous one but where the ability of the CCP to control the effect of competition on default is very high. The CCP's ability to constrain dealers' default risk implies that the effect of competition cannot reduce the contract's price excessively, and that dealers' revenues must be consistent with the default probability imposed by the CCP. On the other hand, free-entry eliminates any further mark-up of price due to market power. Hence, hedgers reap the benefits from greater competition and from lower default risk. The key choice for a CCP is to control the probability and the consequences of default by its

[5] Rules and institutions designed to mitigate the effect of a dealer's default on the market are the focus of this paper but are not the only barriers to entry. For instance, the required technology to offer intermediation services requires substantial fixed investments.

[6] Analog results exist in the literature on the industrial organization of firms. Dixit (1980) and Kreps and Scheinkman (1983) find that firms can strategically choose ex-ante small levels of productive capacity in order to decrease competition ex-post.

members. The CCP chooses a lower level of risk when dealers are more efficient.

To our knowledge, this paper is the first detailed analysis of the interactions between CCP rules, competition and counterparty risk in an OTC market. But the relevance of this issue has been highlighted by Duffie (2010a) and Pirrong (2011).[7] Barriers to entry into a CCP have practical relevance and have started to attract the attention of policy-makers. For instance, LCH.Clearnet's SwapClear facility for interest rate swaps currently requires that members have \$5 billion in equity capital and a \$1 trillion derivatives book. The result is that few dealers are eligible to clear directly at SwapClear.[8] Unsurprisingly, not all market participants agree with these rules. Many argue that incumbents influence CCPs' access and risk management policies to maintain and capture the economic rent associated with clearing services. In response, the US CFTC has proposed rules that would cap minimum capital requirements at \$50 million. More recently, the US Department of Justice has extended its probe of derivative markets to investigate "the possibility of anticompetitive practices in credit derivatives clearing" among others (Reuters, June 8, 2012).

A recent literature studies deep frictions that justify the introduction of a CCP. Koeppl and Monnet (2010) considers the benefits of novation and loss mutualization in a market with a continuum of hedgers and dealers. Acharya and Bisin (2009) emphasize that with asymmetric information, two parties to a trade cannot commit to limit risky trading and maintain their default risk in the future — the counter-party risk externality. Another strand of the nascent literature discusses the effect of different CCP configurations on counterparty risk. Haene and Sturm (2009) model the optimal balance between default fund and margin contributions in capitalizing a CCP, and its effect on the incentives of CCP users. Rausser et al. (2010) suggest that a CCP is not capable of internalizing all the benefits it creates and therefore should be run as a public-private partnership. Jackson and Manning (2007) use simulations to examine alternative CCP configurations and argue that tiered membership may be optimal. Duffie and Zhu (2010) consider the effect of inefficient netting by CCPs on the counterparty risk of members. Finally, Renault (2010) models the optimal number of CCPs for a market. Interestingly, there is some evidence that membership restrictions may not address the most salient sources of risk. In particular, Jones and Perignon (2012)

[7] Duffie (2010a), comment 4: *"Large CMs* [clearing members], *which tend to be dealers, may have a conflict of interest in claiming that smaller CMs are unsafe merely because of being small.* [...] *Allowing smaller CMs introduces more competition between large dealers and smaller market participants."* Pirrong (2011), page 28: *"One way to exercise this market power for the benefit of members is to limit membership to an inefficiently small number through the imposition of unduly restrictive membership requirements. Therefore, it cannot be ruled out that CCPs will utilize membership requirements for strategic, competitive purposes."*

[8] Membership requirements for other major OTC derivatives CCPs such as ICE and CME Clearing are similar. See Lazarow (2011) for details on the governance structure of some large CCPs. All these CCPs are in the process of revising their membership requirement in response to recent and anticipated actions by regulators.

argue that a substantial share of the systemic risk faced by the CME clearinghouse is due to proprietary trading on the part of its members. Still, the empirical literature remains thin, in large part due to limited access to data. Finally, our paper is related to the market microstructure literature with imperfect competition (see the seminal work of Kyle (1985) and Kyle (1989)).

2 Model

We consider a trading environment similar in spirit to that of Duffie et al. (2005) and Lagos and Rocheteau (2009) where hedgers and dealers (described below) meet to trade an OTC contract. Of course, our focus and the market environment differ. First, we study how risk and competition change as the number of dealers changes. Hence, we abstract from search frictions, where results rely on a law of large numbers applied to a continuum of homogenous dealers. One implication is that dealers' market power does not arise from the costs that hedgers face to search for an alternate dealer. Instead, we rely on horizontal differentiation across dealers where dealers provide different levels of ancillary services which are reflected by heterogenous trading costs borne by hedgers. Second, dealers carry an inventory of residual risks and, eventually, may find themselves unable to fulfill their side of the contract. This residual, or basis risk, plays a key role in our analysis. We interpret a low level of residual risk (low variance) as a proxy for the ability for dealers to transfer risk efficiently. Nonetheless, dealers's default is endogenous and the joint distribution of defaults is a key result in the analysis. Finally, we introduce a CCP that novates all trades, restricts entry via membership rules, and controls the risk arising from its members' trading activities. Clearly, real-world CCPs also control risk via, for instance, margin calls, a default fund and the mutualization of losses. All of these additional features could add to the trade-off between competition and default risk but would not change the thrust of our results.

2.1 Over-the-Counter Market

There are two types of agents : a unit measure of non-atomistic risk-averse agents, called hedgers, and a small number, $n \in \mathbb{N}$, of specialized agents, called dealers. There is one asset and one numéraire good. Each hedger owns m unit of the numéraire and one unit of the asset. The asset produces a random endowment of the consumption good, $\tilde{e}$, with

$$\widetilde{e} = \begin{cases} e > 0 & \text{with probability} \quad q \\ 0 & \text{with probability} \quad 1-q. \end{cases}$$

Following, e.g., Lagos and Rocheteau (2009) and Koeppl and Monnet (2010), hedgers have quasi-linear preferences over the consumption good and the numéraire,[9]

$$u_h(\widetilde{e}, m) = \log(1 + \widetilde{e}) + m.$$

The asset is not tradable – hedgers are exposed to a common endowment shock.[10] Dealers are specialized agents that offer a contingent contract s with payoffs $\widetilde{s}$ correlated with the endowment shock, $\widetilde{e}$. Dealers have linear preferences but with limited liability. The exchange between dealers and hedgers is an over-the-counter (OTC) financial market. All dealers offer the same contract but they also offer differentiated ancillary services to their clients. For example, cash accounts, accounting services and other customized services. Alternatively, establishing a relationship between a hedger and a dealer may be costly or dealers may have varying knowledge about different hedgers. This can also be seen as an approximation to an environment with search frictions.

Specifically, hedger i incurs a costs of $c_{i,j}$, in terms of the numéraire, to trade with dealer j. These costs can be seen as a transaction costs, or costs arising from participating in the market. In which case, they are incurred directly by the hedgers. They can also be seen as paid upfront to cover for the costs incurred by dealers to offer customized services. We use the circular road model of Salop (1979) to model price competition with horizontal differentiation in the OTC market. The Salop circle is an analytical device where heterogeneity is captured by the location of hedgers and dealers on the circular road (see figure in appendix). In this representation of the market, hedgers are uniformly dispersed around a circle with length $H = 1$ and the n dealers are equally spaced along the circle. Horizontal differentiation is captured by varying trading costs with the distance between hedgers and dealers. Hedger i's costs of trading with dealer j located at a distance of $d_{i,j}$ on the road is given by $c_{i,j} = d_{i,j} \cdot t$ with $t > 0$.[11]

2.2 Dealers' Intermediation

The timing of events is the following. First, hedgers trade a swap contract with dealers at a given price, p_j, and pay the transaction $c_{i,j}$. Second, all shocks are realized, the contract's

[9] As in the extant literature, studying an economy with log-linear preferences yields greater insight in the underlying mechanism due to its analytical tractability. However, it implies that wealth effect from hedging the risk are absent.

[10] Hedgers have the same exposure and there is no gain from trading between themselves. We abstract abstract from idiosyncratic shocks to hedgers. The common shock is the hedgers' systemic exposure remaining after diversification benefits have been exhausted.

[11] We exclude the case where hedgers can trade with each others. Dealers are truly specialist of offer services that hedgers cannot replicate easily. We will consider entry of additional, competing, dealers below.

payoff is determined and some dealers may default.[12] The contingent contract is a fixed-for-floating swap. Both legs of the transaction occur in the second period. The terms of the contract specify the exchange of a fixed quantity, known today, against an uncertain quantity. It is natural to set the variable payment to be $\widetilde{e}$ and we the fixed payment to the mean of $\widetilde{e}$, $\overline{e} = qe$. The net payoff to hedgers from this contract is $\widetilde{s} = \overline{e} - \widetilde{e}$. The distribution of $\widetilde{s}$ is given by:

$$\widetilde{s} = \begin{cases} (q-1)e & \text{with probability} \quad q \\ qe & \text{with probability} \quad 1-q. \end{cases}$$

Dealers and hedgers exchange an additional transfer of the numéraire, p, which is the price of the contract. The price is paid upon settlement of the contract in the second period.[13] In equilibrium, hedgers are willing to pay a non-negative price to buy this contract from dealers, to receive a fixed payment and eliminate uncertainty. Risk-neutral investors would be indifferent to take either side of this swap with $p = 0$.

Dealers incur losses from the swap contract whenever $\widetilde{s} > p$. For simplicity, we do not endow dealers with capital to cover for these losses. Instead, dealer j has access to different trading strategies in other markets where he can trade to hedge its risk. In our model, a dealer is not a bank or an insurance but a trading specialist in financial markets.[14] In particular, dealer j can establish trading strategies, $\widetilde{h}$ to hedge the risk. The net payoff from the combination of a swap contract and the hedge position is $\widetilde{\Delta} = \widetilde{s} - \widetilde{h}$ with distribution given by:

$$\widetilde{\Delta} = \begin{cases} (q-1)\sigma e & \text{with probability} \quad q \\ q\sigma e & \text{with probability} \quad 1-q, \end{cases}$$

and $\sigma \geq 0$. The shocks $\widetilde{e}$ and $\widetilde{\Delta}$ are uncorrelated and we fix $P(\widetilde{\Delta} = (q-1)\sigma e) = P(\widetilde{s} = (q-1)e) = q$ for parsimony. This parametrization implies that $E[\widetilde{\Delta}] = 0$ and, more importantly, that the relationship between the variance of the swap and of the hedged positions is given by:

$$Var[\widetilde{\Delta}] = \sigma^2 Var[\widetilde{s}].$$

[12] Hedgers do not default in equilibrium since they only promise to deliver their random endowment. We assume that m is large enough so there are not income effects. Any costs incurred to observe m reliably, and possibly monitor its usage, can be included in the trading costs t without loss of generality.

[13] The fixed payment in a typical swap or futures transaction is set such that there is no initial exchange of money. This is purely a matter of notional convention. We could define the fixed payment as $\overline{e} - p$ with no loss of generality. Our approach highlights the role of risk on the exchanges between hedgers and dealers.

[14] We refrain from including capital for two reasons. First, our focus is on varying the number of competing dealers in the market. This raises important general equilibrium questions about how much capital is available in the economy to cover default risk in the OTC market and to allocate it among dealers (incumbents and new entrants) as we vary the number of dealers n. Second, although capital could be modeled by adding a constant payoff to the hedging strategies, this addition would not alter the main results below.

The hedge contract mirrors the payoff from the swap contract but with the spread $\widetilde{\Delta}_j$ that represents residual risk, or basis risk. The parameter σ controls the variance reduction that results from hedging activities. It is a deep parameters that represents how efficiently can dealers transfer the risk borne by hedgers. In particular, the hedge is perfect if $\sigma = 0$. In this case, we have that dealers never default as long as $p \geq 0$, since $\widetilde{\Delta}_j = 0$ with probability one, and hedgers face no risk in equilibrium. On the other end, $\sigma = 1$ corresponds to the case where dealers do not reduce variance. Even if σ is close to one, and the reduction of risk is low, the hedging strategy may still offer diversification benefits since the shock is uncorrelated with the endowment. The deviations from a perfect hedge, $\widetilde{\Delta}_j$, can represent risk that simply cannot be traded away by dealers. For instance, if the best hedging strategy is not perfectly correlated with the payoff from the swap contract. The deviations may also correspond to gains or losses incurred by dealer j from trading strategies in other markets or business lines.[15] In any case, the residual risk introduces the possibility that a dealer may default on its obligations. This differs from the common assumption in the literature on OTC markets where a dealer has access to a frictionless markets to trade its risk away entirely (see e.g., Duffie et al. 2005).

2.3 Dealers' Default Probability

Dealer j defaults whenever the price obtained from entering the swap contract, p_j, is less than the loss incurred from its partially hedged position.[16] The probability that dealer j will default is given by:

$$D_j(p) = \Pr\left(p_j < \widetilde{\Delta}_j\right) = \begin{cases} 0 & \text{if} \quad \overline{p} \leq p_j \\ 1-q & \text{if} \quad \underline{p} \leq p_j < \overline{p} \\ 1 & \text{if} \quad p_j < \underline{p}. \end{cases} \tag{1}$$

The thresholds $\overline{p} \equiv \sigma q e$ and $\underline{p} \equiv \sigma(q-1)e$ define three distinct regions:

- **Low Risk Region:** $D_j(p) = 0 \quad \forall p_j \geq \overline{p}$
- **High Risk Region:** $D_j(p) = 1-q \quad \forall p_j \in [\underline{p}, \overline{p})$
- **Default region:** $D_j(p) = 1 \quad \forall p_j < \underline{p}$.

[15] The interaction with hedgers does not affect the distribution of these gains or losses (i.e., the distribution of $\widetilde{\Delta}_j$ is exogenous).

[16] One important consideration is whether hedging by dealers is consistent with their preferences. We show in Appendix B that, for any price p_j, the hedgers' demand is higher when dealers hedge their risk, even if only partially.

The economically relevant cases are the low-risk and high-risk regions. The default region, where dealers always default is irrelevant in equilibrium since it is never optimal for dealers. The hedgers's demand is zero in this case (see below). Using a simple distribution for the shock $\tilde{\Delta}$ allows us to derive clear analytical results related to the joint probability distribution of events where some, many or all dealers default. This is crucial when studying losses within the CCP. It may appear simplistic that the probability of default is zero in the low risk region. Adding one point or more to the support of $\widetilde{\Delta_j}$ would lead to one or more risk regions but the extra cases that we then must consider in the analysis only complicate the notation and lengthen the propositions. Similarly, extending the length of the support, e, without bound would eliminate regions where $D_j(p) = 0$ and where $D_j(p) = 1$. However, much of the tractability would be lost without changing the thrust of the paper.

2.4 Novation by a CCP

The OTC market equilibrium is not efficient where there are $n \geq 2$ dealers servicing the market. This follows because a dealer's default is an idiosyncratic risk and, therefore, re-allocating the losses across hedgers can improve welfare (Koeppl and Monnet, 2010). Novation by a CCP is one mechanism to implement this re-allocation. Novation implies that every contract between a hedger and a dealer is superseded by two contracts: one between the hedger and the CCP, and one between the CCP and the dealer.

In case of a default, the CCP fulfills its obligations as follows. Consider the event where $k < n$ of the n dealers face a low realization of their hedge strategy and default on their contracts with the CCP. Then, the CCP can use the proceeds from contracts with the $n-k$ surviving dealers to fulfill a pro-rata share $(n-k)/k$ of its obligations toward hedgers. We assume that, in returns, hedgers pay the same pro-rata share of the agreed price. Then, the net quantity of the consumption good obtained by one hedger when k of the n dealers default, $F_{\tilde{e}}(k)$, is given by:

$$F_{\tilde{e}}(\widetilde{k}) = 1 + \widetilde{e} + \frac{n - \widetilde{k}}{n} \left(qe - \widetilde{e}\right).$$

The hedger entering a swap contract has utility given by:

$$\log\left(F_{\tilde{e}}(\widetilde{k})\right) - \frac{n - \widetilde{k}}{n} p_j - d_j t, \tag{2}$$

if a CCP operates in the market and the expected utility is given by:

$$E[u_h^{CCP}(p_j; n)] = qE\left[\log(F_e(k)\right] + (1-q)E\left[\log(F_0(k)\right] - E\left[\frac{n-k}{n} p_j\right] - d_j t.$$

For instance, $F_{\tilde{e}}(\widetilde{k}) = 1 + \widetilde{e}$ if all dealers default and $F_e(k) = 1 + \bar{e}$, if no dealer defaults. These cases correspond to the hedger's payoffs in the absence of a CCP. We can derive the the joint distribution of dealers' default. Dealer j defaults when $p_j < \widetilde{\Delta}_j$. The payoffs $\widetilde{\Delta}_j$ are independent of the endowment $\widetilde{e}$, and they have independent bernoulli distributions across dealers. Therefore, the random variable $\widetilde{k}$, the number of default among n dealers, follows a binomial distribution with parameters $D_j(p)$ and n.[17]

Since it improves welfare, the CCP's emergence can be seen as endogenous or following a mandate by the regulatory authority to clear all trades.[18] Moreover, as in Koeppl and Monnet (2010), novation is not a guarantee. The CCP can only re-distribute the resources from dealers that did not fail and use them to fulfil its obligations to hedgers.[19] In returns, hedgers only pay of pro-rata share of the price. This guarantees that the CCP can fulfill its obligation to surviving dealers but, yet, that no resources remains idle within the CCP and all the benefits of diversification are re-distributed outside of the CCP. Introducing a fixed default funds, or other exogenous resources does change the nature of the trade-off between risk and competition.

2.5 Risk Controls and Membership Rules

The CCP can implement membership rules to restrict entry and select the number of dealers that become members. Hedgers are not clearing members but, instead, hedgers use indirect clearing and the CCP still novates all of hedgers' trades. A CCP can also imposes ex-ante rules on the level of risk that each member can carry. Specifically, it can set the number of members n freely, as well as the default constraint parameter α such that,

$$\Pr\left(p_j < \widetilde{\Delta}_j\right) \leq \alpha. \tag{3}$$

Note that real-world CCPs do not impose a probability constraint directly. However, several of the practices that CCPs put in place are used to minimize the probability of a large loss following a member's default. For instance, initial margins, variations margins and haircuts on collateral secure the CCP's claim on each dealers and directly reduces the risk of a loss following default. Members' contribution to a default fund, mutualization of losses, and other institutional features of a CCP also reduce the probability of losses.

[17] A random variable $\widetilde{x}$ with a Binomial distribution with parameters p and n has a mean np and variance $np(1-p)$.

[18] The Financial Stability Board has recommended the imposition of central clearing mandates for OTC derivatives. Note that we only for one CCP. Economies of scale and network effects present in clearing – but not modeled in this paper – push towards a single CCP for each market.

[19] Hedgers do not default.

3 Equilibrium in the OTC Market

This section describes the equilibrium when the OTC market is unregulated and there is no CCP. We first consider the case where n is low, and dealers are monopolist. In this case, the equilibrium does not "cover" the market: some hedgers will not trade since the sum of the price and of trading costs is too high. We label this case the uncovered equilibrium. With more dealers in the market, the equilibrium eventually covers the market and there is imperfect competition in the market. We label this case the covered equilibrium. This is perhaps the economically relevant case for many real-world OTC markets. In this case, a low level of competition yields a high price and the equilibrium lies in the low-risk region while more competition leads to lower price and the equilibrium eventually reaches the high-risk region.

3.1 Hedgers' Demand

3.1.1 Demand in a Covered Equilibrium

The covered equilibrium corresponds to the area of the parameter space in Salop (1979) where dealers are competing with each other. As in Duffie et al. (2005), dealers are ex-ante homogenous and it is natural to consider an equilibrium where a unique price prevails across the OTC market. Hence, we follow Salop (1979) and focus on the symmetric equilibrium. To derive hedgers' demand, compare the hedger's utility obtained from trading with different dealers. The utility of a hedger from trading with the nearest dealer j located at a distance d_j is $\log(1+\bar{e}) + m - p_j - d_j t$ if the dealer does not default, and $\log(1+\widetilde{e}) + m - d_j t$ if the dealers default. Then, the expected utility is given by:

$$E[u_h(p_j)] = \begin{cases} \Pi + q\log(1+e) - p_j - d_j t & \text{if} \quad \overline{p} \leq p_j \\ q(\Pi + \log(1+e) - p_j) - d_j t & \text{if} \quad \underline{p} \leq p_j < \overline{p} \\ q\log(1+e) - d_j t & \text{if} \quad p_j < \underline{p} \end{cases}, \tag{4}$$

where Π is the surplus derived from trading,

$$\Pi = \log(1+\bar{e}) - q\log(1+e) > 0, \tag{5}$$

since hedgers are risk-averse with respect to the consumption good. In a symmetric equilibrium, all dealers but dealer j set their price to $\widehat{p}$. Consider the second nearest dealer, $j-1$ say, located at distance $\frac{1}{n} - d$ on the other side of the hedger. The utility from trading with dealer $j-1$ is given by:

$$E[u_h(\hat{p})] = \begin{cases} \Pi + q\log(1+e) - \hat{p} - (\frac{1}{n} - d_{j-1})t & \text{if} \quad \overline{p} \le \hat{p} \\ q(\Pi + \log(1+e) - \hat{p}) - (\frac{1}{n} - d_{j-1})t & \text{if} \quad \underline{p} \le \hat{p} < \overline{p} \\ q\log(1+e) - (\frac{1}{n} - d_{j-1})t & \text{if} \quad \hat{p} < \underline{p} \end{cases}, \tag{6}$$

and, therefore, the hedger will prefer to trade with to dealer j if

$$E[u_h(p_j)] \ge E[u_h(\hat{p})] \Leftrightarrow \begin{cases} \frac{-p_j+\hat{p}+\frac{1}{n}t}{2t} \ge d_j & \text{if} \quad \overline{p} \le p_j, \hat{p} \\ \frac{-qp_j+q\hat{p}+\frac{1}{n}t}{2t} \ge d_j & \text{if} \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ \frac{1}{2n} \ge d_j & \text{if} \quad p_j, \hat{p} < \underline{p} \end{cases}. \tag{7}$$

We obtain a similar condition for those hedgers between dealer j and $j+1$. Then the demand schedule for dealer j is given by,

$$y_c(p_j, \widehat{p}) = \begin{cases} H\frac{-p_j+\widehat{p}+\frac{1}{n}t}{t} & \text{if} \quad \overline{p} \le p_j, \hat{p} \\ H\frac{-qp_j+q\widehat{p}+\frac{1}{n}t}{t} & \text{if} \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ 0 & \text{if} \quad p_j, \hat{p} < \underline{p}, \end{cases} \tag{8}$$

where $y_c(p_j, \widehat{p})$ is the quantity of contracts sold by dealer j as function of its own price, p_j, and the price of other dealers, $\widehat{p}$. This results confirms the irrelevance of the case $\hat{p} < \underline{p}$, which corresponds to the Default region above, since dealers face zero demand in this case.[20]

3.1.2 *Demand in the Uncovered Equilibrium*

A hedger do not trade with any dealer if its expected utility is less than its reservation value,

$$R = \log(1+e)q + \log(1)(1-q) + m = q\log(1+e) + m. \tag{9}$$

Comparing Equation 4 and Equations 9, shows that some hedgers choose not to trade if the prices offered by dealers or the trading costs, t, are large enough. In this case, each dealer is a local monopoly, corresponding to the monopolistic area of the parameter space in Salop (1979). The demand schedule in the uncovered case is

$$y_u(p_j) = \begin{cases} \frac{2H}{t}\max(\Pi - p_j, 0) & \text{if} \quad \overline{p} \le p_j \\ q\frac{2H}{t}\max(\Pi - p_j, 0) & \text{if} \quad p_j \in [\underline{p}, \overline{p}) \\ 0 & \text{if} \quad p_j < \underline{p}, \end{cases} \tag{10}$$

[20] Strictly speaking, the demand schedule also exists and is well-defined for cases where the prices of two neighboring dealers are located on each side of $\overline{p}$. However, we do not present the results for this case since it is irrelevant for the determination of a symmetric equilibrium.

3.1.3 Social Welfare

The total expected utility of hedgers trading a swap contract is given by:

$$E[U_h] = 2n \int_0^{d^*} E[u_{h_i}(p^*)]di \tag{11}$$

where the integral is over all hedgers on one side of a given dealer, up to distance, d^*, where the marginal hedger is located. This distance in given in equilibrium. Multiplying by two includes hedgers on both sides of this dealer, and multiplying by n homogenous dealers gives the total expected utility of hedgers in equilibrium.

3.2 Equilibrium without a CCP

All dealers choose a price simultaneously taking into account the hedgers's demand. The expected utility of individual dealers, is defined by:

$$E[u_d] = E[\max(y(p_j - \widetilde{\Delta}_j), 0)], \tag{12}$$

where y is the quantity of contract sold to hedgers.[21] The following Proposition characterizes the equilibrium price, as well as the equilibrium expected utility of hedgers and dealers. We normalize the mass of hedgers to one, $H = 1$, and consider cases where dealers are sufficiently efficient to reduce risk for hedgers, $0 \leq \sigma \leq \bar{\sigma}$. Then, the sequence of equilibrium type as n increases is the following: *(i)* monopoly in the low-risk region, *(ii)* imperfect competition in the low-risk region and, *(iii)* imperfect competition in the high-risk region.[22] All proofs in can be found in the Appendix for the more general case where $H > 0$ and $\sigma \geq 0$. If the scale of the dealers' risk is too high, $\sigma > \bar{\sigma}$, then the equilibrium lies in the high-risk region for all n because the hedgers' surplus from trading is not sufficiently high.

Proposition 1 ***Equilibrium in the OTC Market without CCP***
Fix $H = 1$ and consider cases where dealers' efficiency ranges in $0 \leq \sigma \leq \bar{\sigma} = \frac{\Pi}{2qe}$. The subscripts u and c designate the uncovered equilibrium and the covered equilibrium, respectively. The superscripts lr and hr designate the low risk and high risk regions, respectively.

Low-Risk Uncovered Equilibrium :

For any $1 \leq n \leq n_u^{lr}$, where $n_u^{lr} = \frac{t}{\Pi}$, a symmetric equilibrium exists, is unique, and

[21] Dealers' utility is linear in the numéraire. Implicitly, dealers have access to a spot market where they can exchange any surplus $p_j - \widetilde{\Delta}_j > 0$ in the form of the numéraire. Alternatively, dealers' utility is linear in the commodity and the numéraire.

[22] When $\sigma < \bar{\sigma}$ the efficiency of dealers to reduce risk is too low reach the low-risk region.

lies in the low-risk uncovered region (dealers are monopolies). This equilibrium is such that,

$$2p_u^{lr} = \frac{\Pi}{2}$$
$$E[U_{h,u}^{lr}] = n\frac{\Pi}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right).$$

Low-Risk Covered Equilibrium :

For any $n \in [n_c^{lr}, n_c^*]$, where $n_c^{lr} = \frac{2t}{\Pi}$, $n_c^* = \frac{t}{q\sigma e}$, a symmetric equilibrium exists, is unique, and lies in the low-risk covered region (dealers compete). This equilibrium is such that,

$$p_c^{lr} = \frac{t}{n}$$
$$E[U_{h,c}^{lr}] = \left(\log(1+qe) - \frac{5t}{4n}\right),$$

and dealers's expected profits decreases with n (see Appendix).

High-Risk Covered Equilibrium :

For any $n > \max(n_c^{hr} n_c^*)$, where $n_c^{hr} = \frac{2t}{q(\Pi-(q-1)\sigma e)}$, a symmetric equilibrium exists, is unique, and lies in the high-risk covered region (dealers compete). This equilibrium is such that,

$$p_c^{hr} = \frac{t}{qn} + (q-1)\sigma e$$
$$E[U_{h,c}^{hr}] = \left(q(\Pi - (q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right),$$

and dealers's expected profits decreases with n (see Appendix).

The monopolistic equilibrium exists only if $n_u^{lr} = t/\Pi \geq 1$. That is, if the costs of differentiation, measured by t, is larger than the surplus from a trade, Π. Then, some hedgers do not to trade and the number of dealers n does not affect the equilibrium. Eventually, as the number of dealers increase, or if differentiation is low enough, the equilibrium is covered and there is imperfect competition between adjacent dealers.[23] Then, each dealer trades with

[23] The demand schedule for all values of p_j has a kink at the point between the uncovered and the covered cases. This kink reflects the effect of competition on the price elasticity of hedgers (compare the slope of the hedgers' demand between the uncovered case and the covered case). The transition between an uncovered equilibrium toward a covered equilibrium, as we vary n from n_u^{lr} to n_c^{lr} can be made smooth by proper choice of the parameter values. For instance $n_c^{lr} - n_u^{lr} \leq 1 \Leftrightarrow H \leq \Pi$. Moreover, moving away from monopolies reduces the price, and $p_u^{lr} \geq p_c^{lr}$, if $H = 1$.

all hedgers located on either of the dealer's side but within a maximum distance of $1/2n$. The quantity traded by every dealer is $y^* = H/n$.

The equilibrium price balances market power and default risk. When the number of dealers is relatively low, and the equilibrium still lies in the low-risk region, the contract's price exceeds the expected payoff ($p_u^{lr} \geq p_c^{lr} > E[\tilde{s}] = 0$). However, as n rises beyond n_c^{hr}, and we reach the high-risk region, the equilibrium price reflects the probability that a dealer may default. Eventually, as competition increases, default risk dominates market power and the contract price declines below the expected payoff to compensate hedgers for losses due to default. The threshold, n_c^{hr}, between the low-risk and the high-risk regions compares the level of differentiation t and the dealers' risk σqe. If dealers are efficient (σ is small) or if differentiation is high, then a larger increase in the number of dealers competing in the market is required before the equilibrium reaches the high-risk region ($n_c^{hr} \gg n_c^{lr}$).

3.3 Equilibrium with Novation

The OTC market equilibrium above is not efficient whenever $n > 1$ since a dealer's default is idiosyncratic and, therefore, re-allocating the losses across hedgers can improve welfare (Koeppl and Monnet, 2010). Consistent with previous results, we find the introduction of novation via a CCP increases welfare. One way to see the benefits of novation is to assess the diversification benefits for the (risk-averse) hedgers directly. Using the binomial distribution of dealers' default, a hedger's expected utility in the high-risk region is given by:

$$E[u_h^{CCP}(p_j; n)] = v_h(n) - qp_j - d_j t, \tag{13}$$

where $qp_j = E[\frac{n-k}{n} p_j]$. The first term, $v_h(n)$, corresponds to the expected utility from the consumption goods, given by:

$$\begin{aligned} v_h(n) &\equiv E[\log(F_{\tilde{e}}(k))] \\ &= qE\left[\log(F_e(k))\right] + (1-q)E\left[\log(F_0(k))\right]. \end{aligned} \tag{14}$$

It follows that for a given price, the expected utility, increases with n due to the benefits of diversification. The effect of n works through its effect on $F_{\tilde{e}}(\widetilde{k})$. The means of $F_e(k)$ and $F_0(k)$ do not change with n,

$$\begin{aligned} E(F_e(k)) &= 1 + qe + e(1-q)^2 \\ E(F_0(k)) &= 1 + qe - eq(1-q), \end{aligned}$$

but their variances decrease strictly with n,

$$Var(F_e(k)) = e^2 \frac{(1-q)q}{n}(1-q)^2$$
$$Var(F_0(k)) = e^2 \frac{(1-q)q}{n} q^2,$$

and reach zero in the limit. Therefore, the distribution of consumption when the number of dealers is $n+1$ can be re-written as as the sum of the distribution when the number of dealers is n and a mean-preserving spread. Then, from standard results, $v_h(n)$ is increasing in n because the log function is concave (see e.g., the review in Levy (1992)). Next, define Π^{CCP} as the surplus from trading when the CCP provides diversification benefits,

$$\Pi^{CCP}(n) \equiv \frac{v_h(n) - q\log(1+e)}{q}.$$

This definition ia an analog to the notation previously introduced for the hedger's surplus. The difference between $\Pi^{CCP}(n)$ and Π measures the benefits from diversification. Indeed, there is no diversification benefits when $n = 1$ and we have that, $\Pi^{CCP}(1) = \Pi$. Moreover, the benefits from diversification increases with the number of members since $\Pi^{CCP}(n) > \Pi$ for all $n > 1$ ($v_h(n)$ increases with n). In turns, hedgers, expected utility increases with n,

$$E[u_h^{CCP}(p_j; n)] = \begin{cases} \Pi + \log(1+e) - p_j - d_j t & if \quad p_j \geq \overline{p} \\ q(\Pi^{CCP}(n) + \log(1+e) - p_j) - d_j t & if \quad p_j \in [\underline{p}, \overline{p}) \end{cases}, \tag{15}$$

where, again, the case $p_j < \underline{p}$ is irrelevant. For a given price, the hedger's expected utility increases with n in the high-risk region but there is no effect in the low-risk region since default is absent. Of course, the net effect also depends on the equilibrium price. The following Proposition builds on Proposition 1 and verifies that expected utility also increases in Equilibrium. We focus on the effect of novation. The effect of entry restrictions and that of risk controls will be analyzed in the following Section.

Proposition 2 ***Equilibrium in the OTC Market with Novation***
Fix $H = 1$ and consider cases where $n > 1$ and $0 \leq \sigma \leq \bar{\sigma} = \frac{\Pi}{2qe}$. The subscripts u and c designate the uncovered equilibrium and the covered equilibrium, respectively. The superscripts lr and hr designate the low risk and high risk regions, respectively.

The introduction of novation,

- *does not affect the equilibrium price given in Proposition 1 (Equilibrium without a CCP),*

- *does not affect dealers' expected profit,*
- *but novation improves welfare. Hedgers' total utility is given by*

$$E[U^{hr}_{h,c}] = \left(q(\Pi^{CCP}(n) - (q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n} \right)$$
$$for \quad n^{hr}_{c,CCP} < n < +\infty.$$

where $n^{hr}_{c,CCP}$ is given in the Appendix.

Novation does not affect the price in a covered equilibrium. Due to competition, dealers cannot raise their price beyond a level consistent with the extent of differentiation. Novation does not affect the price in the uncovered equilibrium either since diversification has no effect in the low-risk region.[24] Therefore, dealers' profit is unchanged. On the other hand, hedgers' expected utility increases due to novation ($\Pi^{CCP}(n) \geq \Pi$) and hedgers capture all of the welfare gains due to novation. Therefore, the introduction of novation improves social welfare.[25] For our purpose, this framework captures the key effect of novation. As the number of dealers increase, and competition intensifies, novation reduces the welfare declines associated with the increased in individual dealer's default risk. This trade-off determines the optimal level of entry on the OCT market.

4 Novation, Restrictions to Entry and Risk Controls

4.1 Socially Optimal Level of Entry

We first study the optimal level of entry in a CCP that only offers novation but does not control risk. Consider the free-entry equilibrium where new dealers enter the market as long as expected profits are positive. Free-entry leads to an equilibrium where dealers have no market power but which lies in the high-risk region. Therefore, free-entry may not be socially optimal since the lower price associated with a higher degree of competition may not warrant the higher default probability in the high-risk region.[26] The following Proposition characterizes the free-entry equilibrium with and without novation. In each case, there is a role for a CCP to restrict entry. The analysis yields two results. First, hedgers prefer free-entry

[24] This is an artefact of our specification where the dealers' hedging shocks only take two, bounded, values. Otherwise, monopolistic dealers would capture part of the diversification benefits from novation.
[25] The CCP only affect the price in the high-risk region, i.e., $\Pi^{CCP}(n) = \Pi$ in the low-risk region.
[26] The same trade-off arises if we consider the sum of dealers and hedgers' expected utility as a criteria for social welfare. For instance, if hedgers hold all the share of the dealer's firm.

when dealers' market power is high relative to their ability to transfer risk efficiently. Otherwise, hedgers would prefer a situation where entry is restricted and competition is inhibited. Second, novation improves tilts the optimal level of entry toward more competition.

Proposition 3 ***Free-Entry Equilibrium***

Define the social welfare as the sum of all hedger' utility. Define σ^*_{OTC} and σ^*_{CCP} as,

$$\sigma^*_{OTC} = \frac{4(1-q)\Pi}{9qe - 4q^2e})$$
$$\sigma^*_{CCP} = \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e},$$

with $0 < \sigma^*_{CCP} < \sigma^*_{OTC} < \bar{\sigma}$.

- *In the free entry equilibrium,* $n \to +\infty$, *and in the limit we have that*

$$p^{hr}_c \to (q-1)\sigma e, \quad E[u^{hr}_{d,c}] \to 0, \quad \textit{and} \quad D \to 1-q.$$

- *Hedgers' utility without and with novation is given by*

$$E[U^{hr}_{h,c}] \to q(\Pi - (q-1)\sigma e) + q\log(1+e)$$
$$E[U^{hr}_{h,c}] \to \left(q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1+e)\right),$$

  respectively.

- *In the OTC market without novation , the social welfare is maximized by restricting entry to* $n^*_{OTC} \le n^*_c$ *if* $\sigma < \sigma^*_{OTC} \le \bar{\sigma}$. *The equilibrium lies in the low-risk region with imperfect competition. Otherwise, if* $\sigma > \sigma^*_{OTC}$, *then the social welfare in the OTC market without CCP is maximized in the free-entry equilibrium.*

- *In the OTC market with novation, the social welfare is maximized by restricting entry to* $n^*_{CCP} \le n^*_c$ *if* $\sigma < \sigma^*_{CCP}$. *Otherwise, if* $\sigma > \sigma^*_{CCP}$ *the social welfare in the OTC market with a CCP offering novation is maximized in the free-entry equilibrium.*

The free-entry equilibrium reaches the high-risk region and dealers default with probability $1-q$.[27] Whether free entry is optimal depends on the following trade-off. Increasing the

[27] The implication that $n \to +\infty$ should not interpreted literally. The number of new entrants is bounded if they must incur a fixed cost. Then, the Proposition still holds unless the level of fixed cost is so high that competition cannot materially reduce risk.

number of dealers improves welfare via two channels. A higher number of dealers intensifies competition, lowering the price paid by hedgers. More dealers also implies that the pool of dealers is more diverse, lowering transaction costs for hedgers. On the other hand, a lower price reduces revenue and increases the probability of a dealer's default. This reduces welfare. Whether the trade-off favors more ore less depends in a significant way on the ability of dealers to reduce risk for hedgers. When dealers are sufficiently efficient, $\sigma < \sigma^*_{OTC}$, a relatively large number of new entrants is required to reach the high-risk region. At this point, just before crossing into the high-risk region, competition is sufficiently intense and the potential benefit of further entry, via a lower price, does not compensate for the increased default risk. Hedgers would prefer that entry be restricted. Higher efficiency implies that the optimal level of entry is higher.

Novation increases the hedgers' expected utility in the free-entry equilibrium. But still, free-entry many not still optimal. New entrants are beneficial as long as their effect on the equilibrium default risk is less than the benefits due lower price. Novation pushes this threshold lower, $\sigma^*_{CCP} < \sigma^*_{OTC}$ and tilts the optimum toward greater competition in two ways. If restrictions to entry are still optimal, novation increases the number of dealers needed before reaching a point where the benefits of higher competition do not compensate for the higher default risk, raising the optimal level of entry. Moreover, if $\Pi - q\Pi^{CCP}(+\infty)$ is sufficiently large, then the introduction of novation can switch the social optimum to the free-entry equilibrium.

4.2 Social Optimal Level of Entry with Risk Controls

Next, we consider how the introduction of risk controls within the the CCP, via the constraint in Equation 3, affects the optimal level of entry.

Proposition 4 ***Optimal CCP Rules***
Define the social welfare as the sum of all hedger' utility and σ^+_{CCP},

$$\sigma^*_{CCP} < \sigma^+_{CCP} = \frac{\Pi - q\Pi^{CCP}(+\infty)}{q(2-qe)}.$$

- *Free-entry to a CCP that controls risk is socially optimal.*
- *If $\sigma < \sigma^+_{CCP}$, then the optimal risk control is $\alpha^+ < 1-q$ and the equilibrium is in the low-risk region. Otherwise, the optimal risk control is $\alpha^+ = 1-q$ and the economy is in the high-risk region.*

Free-entry is optimal in the presence of risk controls and the level of dealers' efficiency dictates the level of the CCP's risk controls. As above, the free-entry entry equilibrium in the high-risk region is socially optimal for low level of dealers' efficiency. The cost of reaching the low-risk regions are too high via entry restrictions and greater dealers' revenues is too high. But free-entry is also optimal for high-level of efficiency. The ability to control risk directly allows the CCP to maintain the equilibrium in the low-risk regions for lower level of dealers' efficiency. The binding constraint implies that dealers must seek a higher price to limit default risk. But free-entry eliminates any mark-up of prices due to market power. Hence, hedgers reap the benefits of lower-risk and greater competition.

4.3 Dealers' Optimal CCP Rules

This Section contrasts the choice of CCP's rule by dealers that can coordinate their actions to maximize their total average expected profits,

$$\frac{1}{n}\sum_{i=1}^{n} E[u_d^*]. \tag{16}$$

By symmetry, this is equivalent to maximizing each dealer's utility.[28] We consider two cases. In the first case, dealers can set both n and α. Alternatively, dealers must take the initial number incumbents as given from history and they cannot exclude any incumbent member (but they could allow for new entrants). The following Proposition summarizes the results.

Proposition 5 ***Dealers' Optimal CCP Rules***

- *If members can determine n and α, then they choose either n_u^{lr} or the minimum n where a covered equilibrium exists. The choice of α is irrelevant.*
- *If n is exogenous and cannot be reduced then members exclude new entrants and choose $\alpha < 1 - q$.*

Dealers' choices of n and α differ from the social optimum. If the dealers control the CCP structure, they can maximize expected profit by choosing a low number of members n, in which case the risk constraint becomes irrelevant. Dealers choose to nearly cover the market but nonetheless remain local monopolist. Otherwise, if n is given exogenously, dealers can maximize profit by setting a binding risk constraint while restricting further entry. Incumbent members set binding risk constraints ex-ante to dampen the effect of competition, leading to higher profits.

[28] There is a large literature that studies stability of cartels and other coalition structures. See, e.g., Donsimoni et al. (1986), Bloch (1996) or Thoron (1998).

5 Conclusion and Extensions

Clearing houses and Central Counter-Parties (CCPs) have for long played a key role in securities markets and their importance is going to be increasing the next years. In order to manage risk, a CCP typically imposes stringent membership requirements and other risk controls. Our analysis focused on the implications that these restrictions have for the competitive structure of the financial markets that the CCP serves. We have shown that maximizing welfare implies allowing free-entry to the CCP. This minimizes members' rent and maximize variety. We also find that the optimal risk controls trades-off the costs to hedgers arising from a default of the default against the costs from increasing stability. A very different outcome arises when members determines the CCP rules and regulations. Members use risk controls to commit to a lower degree of competition and increase their profit. This has positive implications for the stability of the market but at a greater costs to hedgers relative to the social optimum. Hedgers would give up some degree of stability in favor of lower costs and greater diversity. This will give some satisfaction to proponents of privately-held CCPs since member them choose stringent risk control. However, an even lower level of risk may be available at lower welfare costs when a regulator can promote entry of new members to the CCP.

Other aspects of a CCP's structure could be brought within the model. Margins, default funds, mutualization could affect the trade-off between competition and default risk. However, the thrust of the message would likely remain. A CCP can use these risk-controls to achieve the benefits from competition without the negative effect on default risk but dealers use these risk-controls to achieve high profits and block new entrants. Similarly, considering fixed costs to entry would affect the results so long as these costs are not so prohibitive that lowering risk is not economically feasible.

The social planner's problem ignored potential externalities in other sectors that follows from different choice of competition and default risk. For instance, a negative externaliy from default of too-big-to-fail dealers or positive externality a having a robust pool of diverse dealers may tilt the social optimum away from or close to competition, respectively. Also, the level of dealers' hedging efficiency is homogenous and exogenous. A broader perspective would ask what would be the dealers' optimal choice of hedging strategy. The interaction between dealers behavior and the CCP's structure would lead to ask how can the planners' implement free-entry equilibrium in low-risk regions.

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A Proofs of Propositions

A.1 Proof of proposition 1

We probe a general case of the proposition for any value of H and any value of $\sigma \geq 0$.

A.1.1 Uncovered equilibrium case

For the dealers, the expected utility is

$$\begin{aligned} E[u_{d,u}] &= E[y_u(p_j)(p_j - \widetilde{\Delta}_j)/y_u(p_j)(p_j - \widetilde{\Delta}_j) \geq 0] \cdot \Pr(y_u(p_j)(p_j - \widetilde{\Delta}_j) \geq 0) = \\ &= y_u(p_j)E[p_j - \widetilde{\Delta}_j/p_j - \widetilde{\Delta}_j \geq 0] \cdot \Pr(p - \widetilde{\Delta}_j \geq 0) \end{aligned}$$

which can be rewritten as

$$E[u_{d,u}] = \begin{cases} y_u(p_j)p_j & if \quad p_j \geq \overline{p} \\ y_u(p_j)(p_j - (q-1)\sigma e)q & if \quad p_j \in [\underline{p}, \overline{p}) \\ 0 & if \quad p_j < \underline{p} \end{cases}$$

If we assume that the equilibrium price is in the low risk region ($\overline{p} \leq p_j$), and also that $p_j \leq \Pi$ we have that

$$E[u_{d,u}(p_j = p_u^{lr})] = \frac{2H}{t}\left(\Pi - p_j\right)p_j$$

This is a concave quadratic function, therefore, first order conditions are necessary and sufficient to find an optimum which is equal to

$$p_u^{lr} = \frac{\Pi}{2} < \Pi$$

In order to satisfy $\overline{p} \leq p_u^{lr}$ we need

$$q\sigma e \leq \frac{\Pi}{2} \Leftrightarrow \sigma \leq \frac{\Pi}{2qe}$$

The utility of the dealer in the low risk region is

$$E[u_{d,u}^{lr}] = y_u(p_u^{lr})p_u^{lr} = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$$

If we assume that the uncovered equilibrium price is in the high risk region, then equilibrium price p_u^{hr} must satisfy the following first order conditions:

$$\frac{\partial E[u_{d,u}^{hr}(p_j = p_u^{hr})]}{\partial p_j} = \frac{\partial y_u}{\partial p_j}(p_u^{hr} - (q-1)\sigma e)q + y_u q$$
$$= -\frac{2qH}{t}(p_u^{hr} - (q-1)\sigma e)q + \frac{2qH}{t}(\Pi - p_u^{hr})q = 0$$

Therefore, p_u^{hr} is equal to

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2}$$

where $p_u^{hr} < p_u^{lr}$. Again, $y_u(p_j)(p_j - (q-1)\sigma e)q$ is a concave quadratic function, therefore, first order conditions are necessary and sufficient to find an optimum.

In order to satisfy $(q-1)\sigma e \leq p_u^{hr} < q\sigma e$ we need

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2} < q\sigma e \Leftrightarrow \sigma > \frac{\Pi}{(1+q)e}$$

and

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2} \geq (q-1)\sigma e \Leftrightarrow \sigma \geq -\frac{\Pi}{(1-q)e}$$

which is always satisfied because by assumption $\sigma \geq 0$.

The utility of the dealer in the high risk region is

$$E[u_{d,u}^{hr}] = y_u(p_u^{hr})(p_u^{hr} - (q-1)\sigma e)q = \frac{2q^2H}{t}\left(\frac{\Pi - (q-1)\sigma e}{2}\right)^2$$

The expected utility for dealers in the high risk region is lower than in the low risk region if the following condition is satisfied:

$$E[u_{d,u}^{hr}] = \frac{2q^2H}{t}\left(\frac{\Pi - (q-1)\sigma e}{2}\right)^2 \leq E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2 \Leftrightarrow q\frac{\Pi - (q-1)\sigma e}{2} \leq \frac{\Pi}{2} \Leftrightarrow$$
$$q\Pi - q(q-1)\sigma e \leq \Pi \Leftrightarrow (q-1)\Pi \leq q(q-1)\sigma e \Leftrightarrow$$
$$\Pi \geq q\sigma e \Leftrightarrow \sigma \leq \frac{\Pi}{qe}$$

Therefore, if the two solutions exists, this condition selects the one that is optimal for the dealers.

Since it is true that $\frac{\Pi}{(1+q)e} \leq \frac{\Pi}{2qe} \leq \frac{\Pi}{qe}$, the previous inequalities can be summarized as follows:

- if $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe}$ then $p_u^{lr} = \frac{\Pi}{2} > \overline{p}$ is the uncovered equilibrium price, and

- if $\sigma > \overline{\sigma} = \frac{\Pi}{2qe}$ then $p_u^{hr} = \frac{\Pi+(q-1)\sigma e}{2} < \overline{p}$ is the uncovered equilibrium price

Now we need to find extra conditions for this equilibrium to exist. In equilibrium in the low risk region, every dealer sells $y_u(p_u^{lr}) = \frac{2H}{t}\frac{\Pi}{2}$ which must be lower than $\frac{H}{n}$ by definition of the uncovered equilibrium. As n increases, eventually $y_u(p_u^{lr}) > H/n$ and there will not be uncovered equilibrium. When $n > n_u^{lr}$ this equilibrium does not exist. The expression of n_u^{lr} is given by

$$\frac{\Pi H}{t} = \frac{H}{n_u^{lr}} \Leftrightarrow n_u^{lr} = \frac{t}{\Pi}$$

For the high risk region, we can find n_u^{hr} equal to

$$\frac{2qH}{t}\frac{\Pi-(q-1)\sigma e}{2} = \frac{H}{n_u^{hr}} \Leftrightarrow n_u^{hr} = \frac{t}{q(\Pi-(q-1)\sigma e)}$$

Finally, the utility of hedgers covered by one dealer is equal to the integral of utilities of hedgers covered by that dealer, where we integrate over the distance d_j. For the low risk region, hedgers trade with a dealer if

$$\Pi + q\log(1+e) - p_j - d_j t \geq q\log(1+e) \Leftrightarrow$$
$$\frac{\Pi - p_j}{t} \geq d_j$$

For the low risk region, this is equal to

$$2H\int_0^{\frac{\Pi-p_u^{lr}}{t}} \left[\Pi + q\log(1+e) - p_u^{lr} - zt\right] dz = 2H\left((\Pi + q\log(1+e) - p_u^{lr})\frac{\Pi - p_u^{lr}}{t} - \left[\frac{z^2 t}{2}\right]_{z=0}^{z=\frac{\Pi-p_u^{lr}}{t}}\right)$$
$$= 2H\left((\Pi + q\log(1+e) - p_u^{lr})\frac{\Pi - p_u^{lr}}{t} - \frac{(\Pi - p_u^{lr})^2}{2t}\right) =$$
$$= 2H\frac{\Pi - p_u^{lr}}{t}\left(\Pi + q\log(1+e) - p_u^{lr} - \frac{\Pi - p_u^{lr}}{2}\right)$$
$$= 2H\frac{\Pi - p_u^{lr}}{t}\left(\frac{\Pi - p_u^{lr}}{2} + q\log(1+e)\right)$$

Simplifying furthermore the expression, we have

$$\frac{2H}{t}\frac{\Pi}{2}\left(\frac{\Pi}{4} + q\log(1+e)\right)$$

And the total utility of hedgers would be n times this quantity:

$$E[U_{h,u}^{lr}] = n\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right)$$

Since prices do not depend on n in the uncovered case, this expression is strictly increasing in n.

In the high risk region we have similar derivations:

$$
\begin{aligned}
&2H \int_{0}^{q\frac{\Pi - p_u^{hr}}{t}} \left[q(\Pi + \log(1+e) - p_u^{hr}) - zt\right] dz \\
&= 2H \left(\left(q(\Pi - p_u^{hr}) + q\log(1+e)\right) q\frac{\Pi - p_u^{hr}}{t} - \left[\frac{z^2 t}{2}\right]_{z=0}^{z=q\frac{\Pi - p_u^{hr}}{t}} \right) \\
&= 2H \left(\left(q(\Pi - p_u^{hr}) + q\log(1+e)\right) q\frac{\Pi - p_u^{hr}}{t} - \frac{q^2(\Pi - p_u^{hr})^2}{2t} \right) \\
&= 2Hq\frac{\Pi - p_u^{hr}}{t} \left(q(\Pi - p_u^{hr}) + q\log(1+e) - q\frac{\Pi - p_u^{hr}}{2} \right) \\
&= 2H\frac{\Pi - p_u^{hr}}{t} \left(q\frac{\Pi - p_u^{hr}}{2} + q\log(1+e) \right)
\end{aligned}
$$

Simplifying furthermore the expression, we have

$$
\frac{2H}{t}\frac{\Pi - (q-1)\sigma e}{2}\left(q\frac{\Pi - (q-1)\sigma e}{4} + q\log(1+e)\right)
$$

And the total utility of hedgers would be n times this quantity:

$$
E[U_{h,u}^{hr}] = n\frac{H}{t}(\Pi - (q-1)\sigma e)\left(q\frac{\Pi - (q-1)\sigma e}{4} + q\log(1+e)\right)
$$

Since prices do not depend on n in the uncovered case, this expression is strictly increasing in n.

A.1.2 Covered equilibrium case

Similar to the uncovered case, the expected utility can be written as

$$
E[u_{d,c}] = \begin{cases} y_c(p_j, \widehat{p})p_j & if \quad \overline{p} \le p_j, \hat{p} \\ y_c(p_j, \widehat{p})(p_j - (q-1)\sigma e)q & if \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ 0 & if \quad p_j, \hat{p} < \underline{p} \end{cases}
$$

If we assume that the equilibrium price is in the low risk region, we have that the symmetric equilibrium price p_c^{lr} must be given by the first order conditions

$$
\frac{\partial E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j} = \frac{\partial y_c}{\partial p_j}p_c^{lr} + y_c = 0
$$

Since in the symmetric covered equilibrium all firms charge the same price we have $y_c = H/n$. Therefore, we have

$$\frac{\partial E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j} = -\frac{H}{t}p_c^{lr} + \frac{H}{n} = 0 \Leftrightarrow p_c^{lr} = \frac{t}{n}$$

In order for p_c^{lr} to be an equilibrium, the second order conditions must be verified. These are given by

$$\frac{\partial^2 E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j^2} = \frac{\partial^2 y_c}{\partial p_j^2}p_c^{lr} + \frac{\partial y_c}{\partial p_j} + \frac{\partial y_c}{\partial p_j}$$

Second derivative of $y_c(p_j, \widehat{p})$ is zero. Therefore, second order conditions are satisfied at the optimum:

$$\frac{\partial^2 E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j^2} = 2\frac{\partial y_c}{\partial p_j} = -2\frac{H}{t} \leq 0$$

Also, in order to have $\overline{p} \leq p_c^{lr}$ we need to satisfy the following inequality:

$$q\sigma e \leq \frac{t}{n} \Leftrightarrow n \leq n_c^* = \frac{t}{q\sigma e}$$

When n increases, p_c^{lr} decreases, and eventually the equilibrium price will be in the high risk region. The equilibrium price p_c^{hr} in the high risk region is given by

$$\begin{aligned}\frac{\partial E[u_{d,c}(p_j = p_c^{hr}, \widehat{p} = p_c^{hr})]}{\partial p_j} &= \frac{\partial y_c}{\partial p_j}(p_c^{hr} - (q-1)\sigma e)q + y_c q = 0 \\ &= -\frac{qH}{t}(p_c^{hr} - (q-1)\sigma e)q + \frac{H}{n}q = 0 \\ &\Leftrightarrow \\ p_c^{hr} &= (q-1)\sigma e + \frac{t}{qn}\end{aligned}$$

Second order conditions are again satisfied:

$$\begin{aligned}\frac{\partial^2 E[u_{d,c}(p_j = p_c^{hr}, p_k = p_c^{hr})]}{\partial p_j^2} &= \frac{\partial^2 y_c}{\partial p_j^2}(p_c^{hr} - (q-1)\sigma e)q + \frac{\partial y_c}{\partial p_j}q + \frac{\partial y_c}{\partial p_j}q = \\ &= 2\frac{\partial y_c}{\partial p_j}q = 2\frac{-Hq}{t}q \leq 0\end{aligned}$$

Note that the condition for $p_c^{hr} < q\sigma e$ is

$$p_c^{hr} = (q-1)\sigma e + \frac{t}{qn} < q\sigma e \Leftrightarrow \frac{t}{q\sigma e} = n_c^* < n$$

The utility of every dealer in every region is

$$E[u_{d,c}^{lr}] = y_c(p_c^{lr})p_c^{lr} = \frac{Ht}{n^2}$$
$$E[u_{d,c}^{hr}] = y_c(p_c^{hr})(p_c^{hr} - (q-1)\sigma e)q = \frac{Ht}{n^2}$$

Also, for this equilibrium to exist it must be that the marginal hedger in $d = H/n$ is having an utility that is greater than the outside value. For the low risk region we have

$$\log(1+qe) - p_c^{lr} - dt \geq q\log(1+e) \Leftrightarrow \Pi - \frac{t}{n} - \frac{H}{n}t \geq 0 \Leftrightarrow$$
$$n \geq n_c^{lr} = \frac{t(1+H)}{\Pi}$$

Therefore, it must be that $n \geq n_c^{lr}$ for the covered equilibrium to exist in the low risk region.

Also, for the high risk region we have a similar condition

$$q(\Pi - p_c) + q\log(1+e) - dt \geq q\log(1+e) \Leftrightarrow q(\Pi - (q-1)\sigma e - \frac{t}{qn}) - \frac{H}{n}t \geq 0 \Leftrightarrow$$
$$n \geq n_c^{hr} = \frac{t(1+H)}{q(\Pi - (q-1)\sigma e)}$$

We can compare n_c^{hr} and n_c^{lr}:

$$n_c^{hr} \leq n_c^{lr} \Leftrightarrow \frac{t(1+H)}{q(\Pi-(q-1)\sigma e)} \leq \frac{t(1+H)}{\Pi} \Leftrightarrow$$
$$\Pi \leq q(\Pi - (q-1)\sigma e) \Leftrightarrow q(q-1)\sigma e \leq (q-1)\Pi \Leftrightarrow$$
$$q\sigma e \geq \Pi \Leftrightarrow \sigma \geq \frac{\Pi}{qe}$$

Given these inequalities, the covered equilibrium in the low risk region will exist as long as $n_c^{lr} < n_c^*$ therefore, it exists when $n \in [\min(n_c^{lr}, n_c^*), n_c^*]$. Similarly, the covered equilibrium in the high risk region will exist when $n > \max(n_c^{hr}, n_c^*)$.

If $H = 1$ and $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe} < \frac{\Pi}{qe}$ then $n_c^{lr} = \frac{2t}{\Pi}$ and since $n_c^* = \frac{t}{q\sigma e}$ then we have that $n_c^{lr} < n_c^*$ because we have assumed that $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe}$. Therefore, in this case there exists a low risk covered equilibrium in the interval $[n_c^{lr}, n_c^*]$.

Since in the covered equilibrium, the distance from the hedger indifferent between two dealers and the dealer he is trading with is $1/2n$, then utility for hedgers covered by one dealer for the low risk region is equal to

$$2H\int_0^{1/(2n)} [\log(1+qe)-zt-p_c]\,dz = 2H\left(\log(1+qe)\frac{1}{2n}-p_c\frac{1}{2n}-\frac{t}{8n^2}\right)$$
$$= 2H\frac{1}{2n}\left(\log(1+qe)-\frac{t}{n}-\frac{t}{4n}\right)=\frac{H}{n}\left(\log(1+qe)-\frac{5t}{4n}\right)$$

And for the high risk region is equal to

$$2H\int_0^{1/(2n)} [q(\Pi+\log(1+e)-p_c)-zt]\,dz = 2H\left((q(\Pi-p_c)+q\log(1+e))\frac{1}{2n}-\frac{t}{8n^2}\right)$$
$$= 2H\frac{1}{2n}\left(q(\Pi-p_c)+q\log(1+e)-\frac{t}{4n}\right)=\frac{H}{n}\left(q(\Pi-(q-1)\sigma e-\frac{t}{qn})+q\log(1+e)-\frac{t}{4n}\right)$$
$$=\frac{H}{n}\left(q(\Pi-(q-1)\sigma e)+q\log(1+e)-\frac{5t}{4n}\right)$$

And the total utility of hedgers would be n times this quantity:

$$E[U_{h,c}^{lr}] = H\left(\log(1+qe)-\tfrac{5t}{4n}\right)$$
$$E[U_{h,c}^{hr}] = H\left(q(\Pi-(q-1)\sigma e)+q\log(1+e)-\tfrac{5t}{4n}\right)$$

This expression is strictly increasing in n.

A.2 Proof of proposition 2

We probe a general case of the proposition for any value of H and any value of $\sigma \leq \overline{\sigma}$.

A.2.1 Uncovered equilibrium case

Using identical analysis to the one used in proposition 1, we find that the uncovered equilibrium price in the high risk region is

$$p_{u,CCP}^{hr} = \frac{\Pi^{CCP}(n)+(q-1)\sigma e}{2} > p_u^{hr}$$

This price is in the high risk region as long as

$$\frac{\Pi^{CCP}(n)+(q-1)\sigma e}{2} < q\sigma e \Leftrightarrow \Pi^{CCP}(n)+(q-1)\sigma e < 2q\sigma e$$
$$\Leftrightarrow \sigma > \frac{\Pi^{CCP}(n)}{(1+q)e} > \frac{\Pi}{(1+q)e}$$

The utility of the dealer in the high risk region is

$$E[u_{d,u}^{hr}] = \frac{2q^2H}{t}\left(\frac{\Pi^{CCP}(n)-(q-1)\sigma e}{2}\right)^2$$

This utility is greater than the case without CCP.

Also, the uncovered equilibrium price and utility when we are in the low risk region is identical to the case without novation,

$$p_{u,CCP}^{lr} = \frac{\Pi}{2}, \quad E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$$

where as before we need $\sigma \leq \frac{\Pi}{2qe}$ in order for this price to be in the low risk region.

Note that the expected utility for dealers in the high risk region is lower than in the low risk region if the following condition is satisfied:

$$E[u_{d,u}^{hr}] = \frac{2q^2H}{t}\left(\frac{\Pi^{CCP}(n) - (q-1)\sigma e}{2}\right)^2 \leq E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$$
$$\Leftrightarrow q\frac{\Pi^{CCP}(n) - (q-1)\sigma e}{2} \leq \frac{\Pi}{2} \Leftrightarrow$$
$$q\Pi^{CCP}(n) - q(q-1)\sigma e \leq \Pi \Leftrightarrow q(1-q)\sigma e \leq \Pi - q\Pi^{CCP}(n)$$
$$\Leftrightarrow \sigma \leq \frac{\Pi - q\Pi^{CCP}(n)}{q(1-q)e} < \frac{\Pi}{qe} < \frac{\Pi}{2qe}$$

The expression of n_u^{lr} is given by

$$\frac{\Pi H}{t} = \frac{H}{n_u^{lr}} \Leftrightarrow n_u^{lr} = \frac{t}{\Pi}$$

If the price is in the high risk region, the value n_u^{hr} is obtained as the solution to the following fixed point equation

$$\frac{2qH}{t}\frac{\Pi^{CCP}(n_{u,CCP}^{hr}) - (q-1)\sigma e}{2} = \frac{H}{n_{u,CCP}^{hr}}$$

Note that we have that $n_{u,CCP}^{hr} < n_u^{hr}$.

A.2.2 Covered equilibrium case

Using identical operations from proposition 1, we can find

$$p_c^{lr} = \frac{t}{n}$$
$$p_c^{hr} = (q-1)\sigma e + \frac{t}{qn}$$

Note that, as before, if $n \leq n_c^* = \frac{t}{q\sigma e}$ then $p_c^{lr} \geq \overline{p}$. And if $\frac{t}{q\sigma e} = n_c^* < n$ then $p_c^{hr} < \overline{p}$

Also, like in proposition 1, we can find the value of the coefficient $n^{hr}_{c,CCP}$ that solves

$$q\Pi^{CCP}(n^{hr}_{c,CCP}) - q(q-1)\sigma e = \frac{t(1+H)}{n^{hr}_{c,CCP}}$$

n^{hr}_c does not have a closed form solution. Because $\Pi^{CCP}(n)$ is increasing with n, we have that $n^{hr}_{c,CCP} < n^{hr}_c$.

Similarly, the term $n^{lr}_c = \frac{t(1+H)}{\Pi}$ is identical to the case of proposition 1. The utility of every dealer is

$$E[u^{lr}_{d,c}] = y_c(p^{lr}_c)p^{lr}_c = \frac{Ht}{n^2}$$
$$E[u^{hr}_{d,c}] = y_c(p^{hr}_c)(p^{hr}_c - (q-1)\sigma e)q = \frac{Ht}{n^2}$$

Finally, identical operations from proposition 1 gives the utility of the hedgers for the low and high risk regions.

A.3 Proof of proposition 3

A.3.1 Free entry in the OTC market without novation

When $n \to +\infty$, we are in the high risk region and $y_c = H/n \to 0$. From proposition 1, we also have that

$$p^{hr}_c = (q-1)\sigma e + \frac{t}{qn} \underset{n\to+\infty}{\longrightarrow} (q-1)\sigma e$$

Therefore, in the free entry equilibrium, the price is the lowest possible. Also, with free entry, dealers will enter in the market as long as $E[u^{hr}_{d,c}] \geq 0$, therefore, in the limit we have $E[u^{hr}_{d,c}] \to 0$.

A.3.2 Social optimum in the OTC market without novation

By summarizing the results from proposition 1, we obtain the utility of hedgers for every case equal to

$$E[U^{lr}_{h,u}] = n\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right) \text{ for } n \leq \frac{t}{\Pi}$$
$$E[U^{hr}_{h,u}] = n\frac{H}{t}(\Pi - (q-1)\sigma e)\left(q\frac{\Pi - (q-1)\sigma e}{4} + q\log(1+e)\right) \text{ for } n \leq \frac{t}{q(\Pi - (q-1)\sigma e)}$$
$$E[U^{lr}_{h,c}] = H\left(\log(1+qe) - \frac{5t}{4n}\right) \text{ for } n \in [\min(\frac{t(1+H)}{\Pi}, \frac{t}{q\sigma e}), \frac{t}{q\sigma e}]$$
$$E[U^{hr}_{h,c}] = H\left(q(\Pi - (q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right) \text{ for } n \in (\max(\frac{t(1+H)}{q(\Pi - (q-1)\sigma e)}, \frac{t}{q\sigma e}), +\infty)$$

Since every case gives an utility that is increaseing with n, we need to compare every case at the highest possible n. To simplify the analysis, we treat n here as a real number. This gives the following expected utility values

$$
\begin{aligned}
E[U^{lr}_{h,u}] &= \frac{t}{\Pi}\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right)\\
E[U^{hr}_{h,u}] &= H\left(\frac{\Pi-(q-1)\sigma e}{4} + \log(1+e)\right)\\
E[U^{lr}_{h,c}] &= H\left(\log(1+qe) - \frac{5t}{4\frac{t}{q\sigma e}}\right)\\
E[U^{hr}_{h,c}] &= H\left(q(\Pi-(q-1)\sigma e) + q\log(1+e)\right)
\end{aligned}
$$

First we need to obtain conditions for the inequality $E[U^{lr}_{h,c}] \leq E[U^{hr}_{h,c}]$ to be satisfied:

$$
\begin{aligned}
E[U^{lr}_{h,c}] \leq E[U^{hr}_{h,c}] &\Leftrightarrow \log(1+qe) - \frac{5q\sigma e}{4} \leq q(\Pi-(q-1)\sigma e) + q\log(1+e)\\
&\Leftrightarrow \Pi - \frac{5q\sigma e}{4} \leq q\Pi - q(q-1)\sigma e \Leftrightarrow (1-q)\Pi - \frac{5q\sigma e}{4} \leq q(1-q)\sigma e \Leftrightarrow\\
(1-q)\Pi \leq \left(\frac{5qe}{4} + q(1-q)e\right)\sigma &\Leftrightarrow (1-q)\Pi \leq \left(\frac{5qe+4qe-4q^2e}{4}\right)\sigma \Leftrightarrow\\
(1-q)\Pi \leq \left(\frac{9qe-4q^2e}{4}\right)\sigma &\Leftrightarrow 0 < \frac{4(1-q)\Pi}{9qe-4q^2e} \leq \sigma
\end{aligned}
$$

The covered case in the low risk region exists if $\frac{t(1+H)}{\Pi} \leq \frac{t}{q\sigma e} \Leftrightarrow \sigma \leq \frac{\Pi}{(1+H)qe}$. And we have the condition

$$
\begin{aligned}
\frac{4(1-q)\Pi}{9qe-4q^2e} \leq \frac{\Pi}{(1+H)qe} &\Leftrightarrow \frac{4(1-q)}{9-4q} \leq \frac{1}{(1+H)} \Leftrightarrow 1+H \leq \frac{9-4q}{4(1-q)}\\
&\Leftrightarrow H \leq \frac{9-4q}{4(1-q)} - 1 = \frac{5}{4(1-q)} \equiv H^*
\end{aligned}
$$

Therefore, the comparison between $E[U^{lr}_{h,c}]$ and $E[U^{hr}_{h,c}]$ is only valid when $H \leq H^* = \frac{5}{4(1-q)}$ where H^* is a number strictly greater than one.

Note also that

$$
\begin{aligned}
\frac{4(1-q)\Pi}{9qe-4q^2e} < \frac{\Pi}{2qe} &\Leftrightarrow \frac{4(1-q)}{9-4q} < \frac{1}{2} \Leftrightarrow\\
8-8q < 9-4q &\Leftrightarrow -1 < 4q \Leftrightarrow -\frac{1}{4} < q
\end{aligned}
$$

which is always satisfied.

Also, if we assume that $\sigma \leq \frac{\Pi}{2qe}$, then we need

$$E[U^{lr}_{h,u}] \leq E[U^{hr}_{h,c}] \Leftrightarrow \frac{1}{4}\log(1+qe) + \frac{3}{4}q\log(1+e) \leq q(\Pi - (q-1)\sigma e) + q\log(1+e) \Leftrightarrow$$
$$\frac{1}{4}\log(1+qe) + q\log(1+e) - \frac{1}{4}q\log(1+e) \leq q(\Pi - (q-1)\sigma e) + q\log(1+e) \Leftrightarrow$$
$$\frac{1}{4}\Pi \leq q\Pi - q(q-1)\sigma e \Leftrightarrow (\frac{1}{4} - q)\Pi \leq q(1-q)\sigma e$$
$$\Leftrightarrow \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} \leq \sigma$$

Note that

$$\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} < \frac{\Pi}{2qe} \Leftrightarrow \frac{(1-4q)}{4(1-q)} < \frac{1}{2} \Leftrightarrow$$
$$2 - 8q < 4 - 4q \Leftrightarrow -2 < 4q$$

which is always satisfied. Also it is easy to check that

$$\frac{4(1-q)\Pi}{9qe - 4q^2e} \geq \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} \;\; \Leftrightarrow \frac{1-q}{\frac{9}{4}-q} \geq \frac{\frac{1}{4}-q}{1-q} \text{ for any } q \in [0,1]$$

And if $\sigma > \frac{\Pi}{2qe}$, then it must be true that $E[U^{hr}_{h,u}] \leq E[U^{hr}_{h,c}]$. This is because when $n = +\infty$, we are in the covered high risk region, the price is the minimum possible and equal to $(q-1)\sigma e$ and the distance from the hedger to the dealer is 0. However, in the uncovered region the price is higher and the distance is not zero.

Therefore, we can summarize these conditions the following way:

- If $H \leq H^*$, then if $\sigma \in [\frac{4(1-q)\Pi}{9qe-4q^2e}, +\infty)$ the free entry equilibrium is socially optimum. This is the relevant case to consider when we assume $H = 1$ because $H^* > 1$.
- If $H > H^*$, then if $\sigma \in [\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e}, +\infty)$ the free entry equilibrium is socially optimum.

We define $\sigma^*_{OTC} = \frac{4(1-q)\Pi}{9qe-4q^2e}$ or $\sigma^*_{OTC} = \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e}$ depending on the value of H.

To show where is the optimum when the free entry equilibrium is not socially optimum, we need to check more inequalities. Note that if $\sigma \leq \frac{\Pi}{2qe}$

$$E[U^{lr}_{h,c}] \geq E[U^{lr}_{h,u}] \Leftrightarrow \log(1+qe) - \frac{5q\sigma e}{4} \geq \frac{\Pi}{4} + q\log(1+e) \Leftrightarrow$$
$$\Pi - \frac{5q\sigma e}{4} \geq \frac{\Pi}{4} \Leftrightarrow \frac{3\Pi}{4} \geq \frac{5q\sigma e}{4} \Leftrightarrow \frac{3\Pi}{5qe} \geq \sigma$$

The following inequality is true

$$\frac{3\Pi}{5qe} \geq \frac{4(1-q)\Pi}{9qe-4q^2e} \Leftrightarrow \frac{3}{5} \geq \frac{4(1-q)}{9-4q} \Leftrightarrow 27-12q \geq 20-20q \Leftrightarrow$$
$$8q \geq -7 \Leftrightarrow q \geq -7/8$$

which is always satisfied.

Also, as we have shown before, if $\sigma > \frac{\Pi}{2qe}$, then it must be true that $E[U^{hr}_{h,u}] \leq E[U^{hr}_{h,c}]$. Therefore, $E[U^{hr}_{h,u}]$ can never be an optimum. Therefore, these results can be summarized as follows:

- If $H \leq H^*$, then if $\sigma \in [0, \frac{4(1-q)\Pi}{9qe-4q^2e})$ the social optimum is in the low risk covered region. This is the relevant case to consider when we assume $H = 1$ because $H^* > 1$.
- If $H > H^*$, then if $\sigma \in [0, \min(\frac{\Pi}{(1+H)qe}, \frac{4(1-q)\Pi}{9qe-4q^2e})$ the social optimum is in the low risk covered region. and if $\sigma \in [\min(\frac{\Pi}{(1+H)qe}, \frac{4(1-q)\Pi}{9qe-4q^2e}), \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e})$ the social optimum is in the low risk uncovered region. Note that depending of the value of some parameters, the low risk uncovered region could not exist.

A.3.3 Free entry in the OTC market with novation

As n increases, we are in the covered case and therefore

$$p^r_{c,CCP} = (q-1)\sigma e + \frac{t}{qn} \underset{n\to+\infty}{\to} (q-1)\sigma e$$

With free entry by dealers and no risk controls, dealers will enter in the market as long as $E[u^{hr}_{d,c}] \geq 0$, therefore, in the limit we have $E[u^{hr}_{d,c}] \to 0$ and dealers trade $y_c = \frac{H}{n} \to 0$.

Therefore, in the free entry equilibrium, the price is the lowest for all n. Since all hedgers pay the lowest possible price and they are infinitely close to a dealer, then their utility is the higest possible.

A.3.4 Social optimum in the OTC market with novation

We proceed using identical calculations as in the case without novation, but due to the diversification from the CCP, the surplus for hedgers is $\Pi^{CCP}(n)$. First we need to have the following condition for the inequality $E[U^{lr}_{h,c}] \leq E[U^{hr}_{h,c}]$ to be satisfied:

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e} \leq \sigma$$

The covered case in the low risk region exists if $\frac{t(1+H)}{\Pi} \leq \frac{t}{q\sigma e} \Leftrightarrow \sigma \leq \frac{\Pi}{(1+H)qe}$. And we have the condition

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} \leq \frac{\Pi}{(1+H)qe} \Leftrightarrow \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9-4q} \leq \frac{\Pi}{(1+H)} \Leftrightarrow$$
$$1 + H \leq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} \Leftrightarrow H \leq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} - 1 \equiv H^*_{CCP}$$

Therefore, the comparison between $E[U^{lr}_{h,c}]$ and $E[U^{hr}_{h,c}]$ is only valid when $H \leq H^*_{CCP}$. Note that because $\Pi^{CCP}(+\infty) \geq \Pi$ then $\Pi - q\Pi^{CCP}(+\infty) \leq (1-q)\Pi$ and

$$H^*_{CCP} = \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} - 1 \geq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi\right)} - 1 = \frac{5}{4\left(1-q\right)} > 1$$

Note also that because $\Pi^{CCP}(+\infty) > \Pi$ we have

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} < \frac{4(1-q)\Pi}{9qe - 4q^2e}$$

And since we probed previously that $\frac{4(1-q)\Pi}{9qe-4q^2e} < \frac{\Pi}{2qe}$, then it is also true that $\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e} < \frac{\Pi}{2qe}$.

Also, if we assume that $\sigma \leq \frac{\Pi}{2qe}$, then for $E[U^{lr}_{h,u}] \leq E[U^{hr}_{h,c}]$ to be satisfied, we need

$$\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} \leq \sigma$$

Again, because $\Pi^{CCP}(+\infty) > \Pi$ we have that

$$\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} < \frac{(\frac{1}{4} - q)\Pi}{q(1-q)e}$$

and since we probed previously that $\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} < \frac{\Pi}{2qe}$, then it is also true that $\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} < \frac{\Pi}{2qe}$.

Also it is easy to check numerically that

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} \geq \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} \quad \text{for any } q \in [0,1]$$

Therefore, we can summarize these conditions like in the case without novation:

- If $H \leq H^*_{CCP}$, then if $\sigma \in [\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}, +\infty)$ the free entry equilibrium is socially optimum. This is the relevant case to consider when we assume $H = 1$ because $H^*_{CCP} > 1$.

- If $H > H^*_{CCP}$, then if $\sigma \in [\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e}, +\infty)$ the free entry equilibrium is socially optimum.

We define $\sigma^*_{CCP} = \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}$ or $\sigma^*_{CCP} = \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e}$ depending on the value of H.

Note that because $\Pi^{CCP}(+\infty) > \Pi$ then we have that $\sigma^*_{CCP} < \sigma^*_{OTC}$.

To show where is the optimum when the free entry equilibrium is not socially optimum, we proceed like in the no novation case and we arrive to the following result:

- If $H \leq H^*_{CCP}$, then if $\sigma \in [0, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e})$ the social optimum is in the low risk covered region. This is the relevant case to consider when we assume $H = 1$ because $H^*_{CCP} > 1$.

- If $H > H^*_{CCP}$, then if $\sigma \in [0, \min(\frac{\Pi}{(1+H)qe}, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e})$ the social optimum is in the low risk covered region. and if $\sigma \in [\min(\frac{\Pi}{(1+H)qe}, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}), \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e})$ the social optimum is in the low risk uncovered region. Note that depending of the value of some parameters, the low risk uncovered region could not exist.

A.4 Proof of proposition 4

To probe proposition 4, we first show the existence and unicity of a default constrained equilibrium where the default probability constraint $\Pr(p_j < \widetilde{\Delta}_j) \leq \alpha$ must be satisfied.

A.4.1 ***Existence and unicity of default-constrained equilibrium***

For any $\alpha < 1 - q$ and assuming that $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe}$, we know from proposition 2 that the equilibrium for any $n < \overline{n} = \max(n^{hr}_{c,CCP}, n^*_c)$ will satisfy the default constraint with strict inequality because we are in the low risk region (and therefore default probability is zero). Therefore, for any $n < \overline{n}$ the default constrained equilibrium is just the uncovered/covered case shown before.

In $n \geq \overline{n}$ the unconstrained equilibrium does not satisfy the default probability constraint because we are in the high risk region and $\alpha < 1 - q$.

Now we probe existence and unicity of this default-constrained equilibrium. For any $\alpha < 1 - q$, price must be in the low risk region, therefore the constraint $\Pr\left(p_j < \widetilde{\Delta}_j\right) \leq \alpha$ is satisfied if and only if $p_j \geq \overline{p}$.

First we check if this equilibrium is uncovered. The uncovered equilibrium problem subject to the constraint is

$$\max_{p_j} y_u(p_j)p_j = 2H\frac{\Pi - p_j}{t}p_j$$

$$st$$

$$p_j \geq \overline{p}$$

The unique optimum of this problem when the constraint does not bind is $\frac{\Pi}{2}$. Since $\overline{p} \leq \frac{\Pi}{2}$ by assumption and the objective function is concave and quadratic, then $\frac{\Pi}{2}$ is the optimum. However, since $n \geq \overline{n}$ this equilibrium cannot exist (because the amount traded by every agent is $> \frac{H}{n}$). Therefore, the default-constrained equilibrium cannot be uncovered.

Now we show existence of constrained covered equilibrium equal to $\overline{p}$. If all other dealers are charging $p = \overline{p}$, then any dealer j has to maximize the low-risk region profits subject to the constraint. If we are in the covered equilibrium, the problem is

$$\max_{p_j} y_c(p_j, \widehat{p} = \overline{p})p_j = H\frac{-p_j + \overline{p} + \frac{1}{n}t}{t}p_j$$
$$st$$
$$p_j \geq \overline{p}$$

The objective function is a concave quadratic function with a maximum at $p_j^* = \frac{1}{2}(\overline{p} + \frac{t}{n})$. If $\frac{1}{2}(\overline{p} + \frac{t}{n}) < \overline{p}$ then the unconstrained optimum for dealer j is lower than $\overline{p}$. Therefore, the optimum for dealer j must be $\overline{p}$. This inequality is satisfied when $\overline{p} > \frac{t}{n}$ or $n > n_c^* = \frac{t}{\overline{p}} = \frac{t}{q\sigma e}$, which is satisfied by assumption (because $n \geq \overline{n} = \max(n_c^{hr}, n_c^*)$). At this equilibrium every firm makes $\frac{H}{n}\overline{p}$.

Finally, we show this equilibrium is unique. Let's assume than there is an equilibrium where all firms set $\widehat{p} > \overline{p}$. This would be a covered equilibrium where the constraint is satisfied with strict inequality (low risk region). Therefore, proposition 2 applies. But from proposition 2, for $n \geq \overline{n}$ the (unconstrained) equilibriumis unique and it is in the high risk region. Therefore, $\widehat{p}$ cannot be an equilibrium.

Therefore, if $n \geq \overline{n}$ there exists a unique default-constrained equilibrium $\overline{p}$.

A.4.2 ***Social Optimum with a CCP***

Let assume that $\sigma \leq \frac{\Pi}{2qe}$. For any $\alpha < 1-q$, we know from proposition 2 that the equilibrium for any $n < \overline{n} = \max(n_c^{hr}, n_c^*)$ will satisfy the default constraint with strict inequality and will be in the low risk region. Also, for $n \geq \overline{n} = \max(n_{c,CCP}^{hr}, n_c^*)$ the equilibrium is default constrained with price equal to $\overline{p} = q\sigma e$. The choice of $\alpha < 1-q$ and $n = +\infty$ gives a higher utility for dealers than any $n < \overline{n}$ because hedgers are in the low risk region in any case, they are infinitely close to a dealer, and they pay the lowest minimum price $\overline{p}$ to be in the low risk region. In this case, hedgers utility is

$$H\left(\log(1 + qe) - \overline{p}\right)$$

We need to compare this amount with the utility of hedgers when $\alpha \geq 1-q$. If $n < \overline{n}$, we are in the low risk region and as discussed above, the utility is lower than the case $\alpha < 1-q$ and $n = +\infty$. If $n \geq \overline{n}$ then the utility is maximized at $n = +\infty$ and we are in the high risk region, where the hedgers obtain

$$H\left(q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1 + e)\right)$$

This is also the utility in the free entry equilibrium. By concavity of the log function, the following inequality must be satisfied:

$$\Pi^{CCP}(+\infty) \equiv \frac{q\log\left(1+qe+e(1-q)^2\right)+(1-q)\log\left(1+qe-eq(1-q)\right)-q\log(1+e)}{q}$$
$$< \frac{\log(1+qe)-q\log(1+e)}{q} = \frac{\Pi}{q}$$

We need to compare these two values:

$$\begin{aligned} H\left(\log(1+qe)-\overline{p}\right) \geq H\left(q(\Pi^{CCP}(+\infty)-(q-1)\sigma e)+q\log(1+e)\right) \Leftrightarrow \\ \log(1+qe)-\overline{p} \geq q(\Pi^{CCP}(+\infty)-(q-1)\sigma e)+q\log(1+e) \Leftrightarrow \\ \Pi-\overline{p} \geq q\Pi^{CCP}(+\infty)+q(1-q)\sigma e \Leftrightarrow \\ \Pi-q\Pi^{CCP}(+\infty) \geq +q(2-q)\sigma e \Leftrightarrow \\ \frac{\Pi-q\Pi^{CCP}(+\infty)}{q(2-q)e} \geq \sigma \end{aligned}$$

And because $q\Pi^{CCP}(+\infty) < \Pi$, we have $\frac{\Pi-q\Pi^{CCP}(+\infty)}{q(2-q)e} > 0$.

Therefore, when $\sigma \leq \sigma^{+}_{CCP} \equiv \frac{\Pi-q\Pi^{CCP}(+\infty)}{q(2-q)e}$ we have that a CCP that limits trade through setting $\alpha < 1-q$ and $n=+\infty$ maximizes total utility of hedgers. And when $\sigma > \sigma^{+}_{CCP}$ then $n=+\infty$ without ris controls (so any $\alpha \geq 1-q$) maximizes total utility of hedgers.

Note that when $\sigma = 0$ there is never default in any case and the free entry (unconstrained) equilibrium maximizes the utility of hedgers.

A.5 Proof of proposition 5

A.5.1 Case n exogenous, choice of α

By proposition 4, if $n \geq \bar{n}$ and $\alpha^* \in [0, 1-q)$, then the equilibrium price is $\overline{p}$. We want to show that if $n \geq \bar{n}$, then by choosing any value $\alpha^* \in [0, 1-q)$ the dealers are better off. For that, we need to compare the dealers expected utility when they are unconstrained, $\frac{Ht}{n^2}$, with the default-constrained utility obtained in proposition 5 (equal to $\frac{H}{n}\overline{p}$). The condition to be satisfied is

$$\frac{Ht}{n^2} < \frac{H}{n}\overline{p} \Leftrightarrow \frac{t}{n} < \overline{p} \Leftrightarrow n^*_c = \frac{t}{q\sigma e} < n$$

This conditions is satisfied by assumption $n \geq \bar{n} = \max(n^{hr}_c, n^*_c)$.

Now we check if the optimum for the dealers is also the best for the hedgers. When $\alpha^* \in [0, 1-q)$, then the equilibrium price is $\overline{p}$ and the utility of the hedgers is

$$H\left(\log(1+qe)-\overline{p}-\frac{t}{4n}\right)$$

If $\alpha^* \geq 1-q$ then the utility of the hedgers is from proposition 2

$$H\left(q(\Pi^{CCP}(n)-(q-1)\sigma e)+q\log(1+e)-\frac{5t}{4n}\right)$$

By comparing both values

$$\log(1+qe)-\overline{p}-\frac{t}{4n}\geq q(\Pi^{CCP}(n)-(q-1)\sigma e)+q\log(1+e)-\frac{5t}{4n}\Leftrightarrow$$
$$\Pi-q\Pi^{CCP}(n)+\frac{t}{n}-\overline{p}+(q-1)\overline{p}\geq 0\Leftrightarrow \Pi-q\Pi^{CCP}(n)+\frac{t}{n}\geq(2-q)\overline{p}$$
$$\Leftrightarrow\frac{\Pi-q\Pi^{CCP}(n)+\frac{t}{n}}{(2-q)qe}\equiv\sigma_{CCP}(n)\geq\sigma$$

Note that $\sigma_{CCP}(n)>0$, $\sigma_{CCP}(n)$ is decreasing with n, and $\sigma_{CCP}(n=+\infty)=\sigma^+_{CCP}$ from the previous proposition.

Therefore, when $n\geq\bar{n}$ is exogenous and $\sigma\leq\sigma_{CCP}(n)$ then hedgers are better off with any $\alpha^*\in[0,1-q)$.

A.5.2 Choice of n and α

From the previous section, we can find the optimum α for a given n. If $n\geq\overline{n}$, it is optimum to choose any $\alpha^*\in[0,1-q)$ and every dealer obtains $\frac{H}{n}\overline{p}$. For $n<\overline{n}$, α^* is irrelevant because we are in the low risk region (we are assuming that $\sigma\leq\frac{\Pi}{2qe}$). In order to obtain the optimum n^* that maximizes every dealer utility, we need to compare every case. In the uncovered equilibrium, any dealer makes $E[u^{lr}_{d,u}]=\frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$, independent of n. In the low risk covered case, every dealer makes $E[u^{lr}_{d,c}]=\frac{Ht}{n^2}$, decreasing with n. Therefore the minimum n for the covered equilibrium to exist will maximize the expected utility. This is $n=\min(n^{lr}_{c,CCP},n^*_c)$. Finally, in the low risk covered case $\frac{H}{\overline{n}}\overline{p}$ gives the highest utility.

Now we compare these values. For example, if $n^*_c\geq n^{lr}_{c,CCP}$ then there is low risk region in the covered equilibrium and this implies

$$n^*_c\geq n^{lr}_{c,CCP}\Leftrightarrow\frac{t}{q\sigma e}\geq\frac{t(1+H)}{\Pi}\Leftrightarrow\sigma\leq\frac{\Pi}{(1+H)qe}$$

Since by assumption we have $\sigma\leq\frac{\Pi}{2qe}$, then if $H\leq 1$ we have $\frac{\Pi}{2qe}\leq\frac{\Pi}{(1+H)qe}$ and there exists covered equilibrium in the low risk region. Therefore, the minimum n for the covered equilibrium to exist is $n^{lr}_{c,CCP}=\frac{t(1+H)}{\Pi}$. If $E[u^{lr}_{d,c}]$ exists, then it must be that $E[u^{lr}_{d,c}]\geq\frac{H}{\overline{n}}\overline{p}$. This is because $E[u^{lr}_{d,c}]$ exists at smaller n and $p^{lr}_c\geq\overline{p}$. Using similar arguments, we have that $E[u^{lr}_{d,u}]\geq\frac{H}{\overline{n}}\overline{p}$.

Therefore, we just need to compare $E[u^{lr}_{d,c}]$ at $n^{lr}_{c,CCP}$ with $E[u^{lr}_{d,u}]$:

$$E[u_{d,c}^{lr}] \geq E[u_{d,u}^{lr}] \Leftrightarrow \frac{\Pi^2 H}{t(1+H)^2} \geq \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2 \Leftrightarrow \frac{1}{(1+H)^2} \geq \frac{1}{2}$$
$$\Leftrightarrow \sqrt{2} \geq 1 + H \Leftrightarrow H \leq \sqrt{2} - 1 \approx 0.41$$

Therefore, if $H \in (0, \sqrt{2}-1]$ then, $n_{c,CCP}^{lr}$ maximizes the utility of every dealer, and if $H \in (\sqrt{2}-1, 1]$, n_u^{lr} maximizes the utility of every dealer. Also, if $H > 1$ and $\sigma \leq \frac{\Pi}{(1+H)qe}$, then from the above inequality, we have $E[u_{d,c}^{lr}] < E[u_{d,u}^{lr}]$ so the uncovered low risk equilibrium is optimal.

Finally, if $H > 1$ and $\frac{\Pi}{(1+H)qe} \leq \sigma \leq \frac{\Pi}{2qe}$ we have $n_c^* < n_{c,CCP}^{lr}$ and therefore there is not low risk equilibrium in the covered case. However, since we have previously shown that, $E[u_{d,u}^{lr}] \geq \frac{H}{\overline{n}}\overline{p}$, then n_u^{lr} gives the highest utility for dealers.

Since we have shown that the optimum is in either n_u^{lr} or $n_{c,CCP}^{lr}$, the choice of α is irrelevant because α is only relevant in the high risk covered region.

B Demand of hedgers when dealers cannot hedge the hedgers' risk

Let assume in this section that dealers cannot trade the swap contract $\widetilde{s}$ in the outside market. The utility of hedger trading with a dealer j that does not default is

$$\log(1+\widetilde{e}+\widetilde{s})-p_j-d_jt=\log(1+qe)-p_j-d_jt$$

where $\widetilde{s}=qe-\widetilde{e}$. Dealer defaults when $p_j<\widetilde{s}$. If $p_j\geq qe$ the dealer never defaults. If $p_j\in[(q-1)e,qe)$ the dealer defaults when $\widetilde{e}=0$ (and therefore $\widetilde{s}=qe$) which occurs with probability $1-q$. In that case, the utility of the hedger is

$$\log(1)-d_jt=-d_jt$$

This is the key difference with the case where dealers can hedge the hedgers' risk. Because the variable $\widetilde{\Delta}_j$ is incorrelated with $\widetilde{e}$, then dealer j defaults with exogenous probability and when there is a default,, the hedger obtains $q\log(1+e)+(1-q)\log(1)=q\log(1+e)>0$. However, in the case of no hedging, dealer defaults when $\widetilde{e}=0$ (when the outcome is negative for the dealer) and the hedger obtains $\log(1)=0$. Therefore, the expected utility of the hedger when $p_j\in[(q-1)e,qe)$ is

$$\begin{aligned}E[u_h(p_j)]&=(\log(1+qe)-p_j)\cdot N_j-d_jt\\&=q\left(\Pi+q\log(1+e)-p_j\right)-d_jt\end{aligned}$$

Therefore, the utility of the hedger in every region can be written as

$$E[u_h(p_j)]=\left\{\begin{array}{ccc}\Pi+q\log(1+e)-p_j-d_jt & if & p_j\geq qe\\ q\left(\Pi+q\log(1+e)-p_j\right)-d_jt & if & p_j\in[(q-1)e,qe)\\ q\log(1+e)-d_jt & if & p_j<(q-1)e\end{array}\right.$$

We can compare the utility in the high risk region with the case where dealers can hedge the risk from hedgers:

$$\begin{aligned}q\left(\Pi+q\log(1+e)-p_j\right)-d_jt\leq q\left(\Pi+\log(1+e)-p_j\right)-d_jt\Leftrightarrow\\ q\log(1+e)\leq\log(1+e)\Leftrightarrow q\leq 1\end{aligned}$$

Since it is always true that $q\leq 1$, then in the high risk region, hedgers are worse off when dealers do not trade the risk from hedgers.

The number of trades is given by the number of hedgers that get a higher value than the outside value of no trading, $q\log(1+e)$. Note that in the region $p_j\in[(q-1)e,qe)$, we have

$$\begin{aligned}q\left(\log(1+qe)-p_j\right)-d_jt\geq q\log(1+e)\Leftrightarrow\\ q\frac{\log(1+qe)-\log(1+e)-p_j}{t}\geq d_j\end{aligned}$$

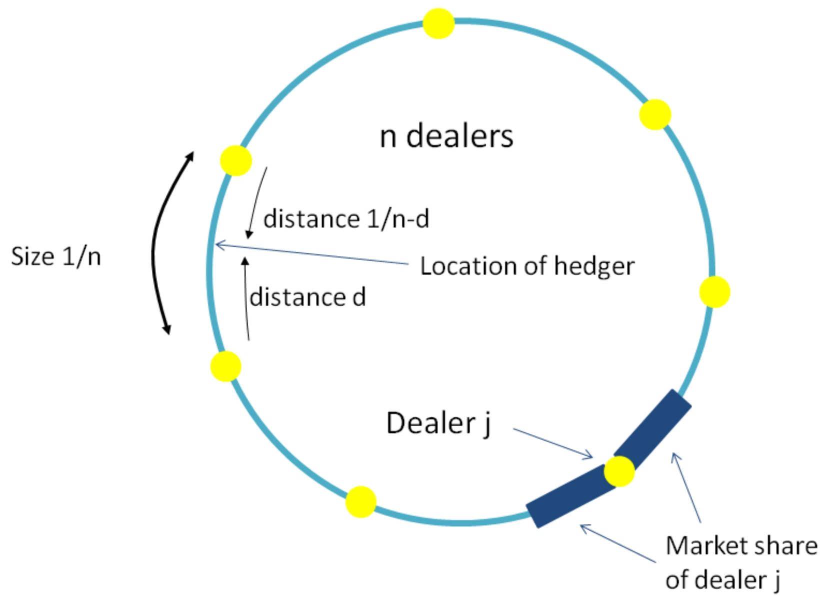
Since $\log(1+qe) - \log(1+e) \leq 0$ for any $q \leq 1$, it is necessary that $p_j < 0$ and negative enough in order to have a strictly positive number of hedgers trading with dealer j ($d_j > 0$). Hedgers need to be paid to buy the contract in order to have a positive number of hedgers willing to trade with a dealer.

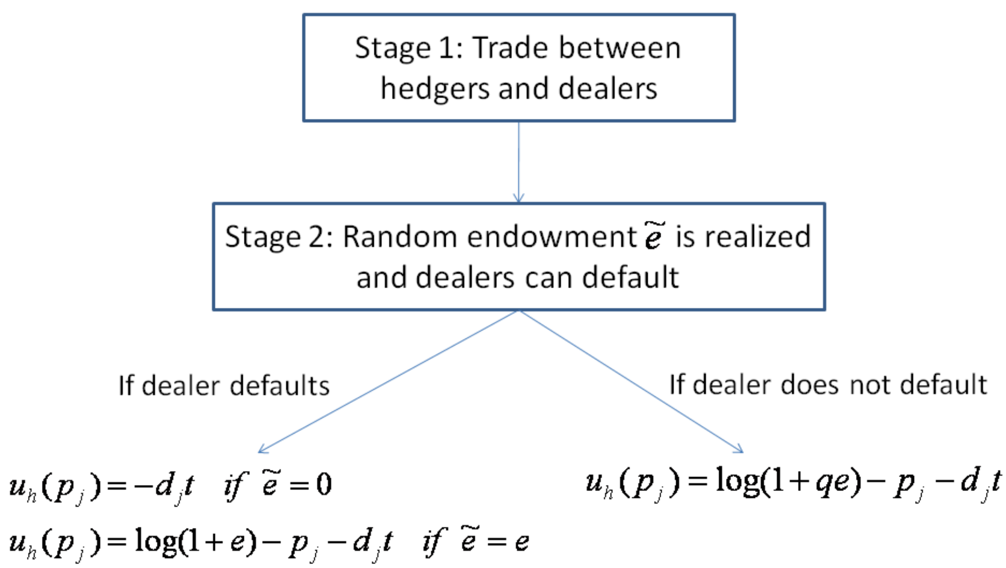
Therefore, the number of swap contracts bought by the hedgers to a dealer for a given price p_j in the uncovered equilibrium when dealers do not hedge the risk is

$$y_{u,nh}(p_j) = \begin{cases} \frac{2H}{t} \max\left(\Pi - p_j, 0\right) & if \quad p_j \geq qe \\ 0 & if \quad p_j \in [\log(1+qe) - \log(1+e), qe) \\ \frac{2qH}{t} \max\left(\log(1+qe) - \log(1+e) - p_j, 0\right) & if \quad p_j \in [(q-1)e, \log(1+qe) - \log(1+e)) \\ 0 & if \quad p_j < (q-1)e \end{cases}$$

Since $\log(1+qe) - \log(1+e) < 0 < \Pi$, we have $y_{u,nh}(p_j) < y_u(p_j)$ for any price p_j. This explain why dealers need to hedge the risk of hedgers. Hedging helps increasing the demand of contrats bought by hedgers.

C Figures


n dealers
distance 1/n-d
Size 1/n
Location of hedger
distance d
Dealer j
Market share
of dealer j


Stage 1: Trade between hedgers and dealers
Stage 2: Random endowment $\tilde{e}$ is realized and dealers can default
If dealer defaults
If dealer does not default
$u_h(p_j) = -d_j t \quad if \; \tilde{e} = 0$
$u_h(p_j) = \log(1+e) - p_j - d_j t \quad if \; \tilde{e} = e$
$u_h(p_j) = \log(1+qe) - p_j - d_j t$