Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy

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Abstract

We develop a closed economy model to study the interactions among sovereign risk premia, fiscal limits, and fiscal policy. The stochastic fiscal limit, which measures the ability and willingness of the government to service its debt, arises endogenously from dynamic Laffer curve. The distribution of fiscal limits is country-specific, depending on the size of the government, the degree of countercyclical policy responses, economic diversity, and political uncertainty, among other characteristics. The model rationalizes different sovereign ratings across developed countries. A nonlinear relationship between sovereign risk premia and the level of government debt emerges in equilibrium, consistent with the empirical evidence that once risk premia begin to rise, they do so rapidly. Default risk premia for long-term bonds move ahead of those for short-term bonds, providing early warnings of increasing probabilities of sovereign defaults.

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1 INTRODUCTION

Developed countries are facing unprecedent fiscal challenges. In the wake of deteriorating public finances, rating agencies downgraded the sovereign debt in several developed countries in 2009 and 2010. Ireland and Spain were downgraded by two notches, Portugal by three notches, and Greece by five notches.[1] The spread between 10-year Greek government bonds and equivalent German bonds widened to more than 900 basis points in September 2010. Government debt in developed countries, buoyed by anti-crisis measures, is projected to rise from an average of about 73 percent of GDP in 2007 to about 108 percent of GDP in 2015. More broadly, age-related spending in many developed countries will further raise government indebtedness in the coming decades.

Historical data shows that developed countries have been frequently penalized when concerns arise about the riskiness of government debt, even though these countries have no history of default in the post-war period. Figure 1 depicts the sovereign downgrades of some developed countries since 1975. There is also tremendous diversity in the levels of debt at which downgrades occur. Figure 2 compares six countries in three groups, New Zealand and Canada, Italy and Belgium, and Sweden and Japan. The rating of New Zealand government debt was reduced by three notches from AAA to AA- as its gross debt climbed from 58% to 75% of the GDP.[2] The Canadian government, on the other hand, was able to keep its AAA rating until its debt hit 90% of the GDP.

The theoretical analysis of fiscal policy in developed countries has been largely abstracted from sovereign default risk by assuming that sovereign debts are always honored, which may be an innocuous assumption at normal times. However, when the financial markets have brought fiscal concerns to the front page, understanding the interaction between sovereign default risk and fiscal policy in a dynamic stochastic general equilibrium (DSGE) framework can be very useful for policymaking, which is at the heart of this paper. Using a nonlinear model, we analyze how the maximum level of debt that the government is able and willing to service, which we call *fiscal limit*, may depend on the macroeconomic fundamentals, and what the quantitative impact of sovereign default risk upon the economy may be.

In the literature of sovereign default in emerging market economies, see Eaton and Gersovitz (1981) and Arellano (2008) among many other papers, sovereign default has been modeled as an outcome of the optimal and strategic decision by the government, which may be a reasonable assumption for emerging market economies. However, the predicted level of

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[1] All the sovereign ratings come from Standard & Poor's.

[2] Appendix A provides an explanation of the data.

government debt at which the sovereign default occurs is much lower than the debt level at which the sovereign risk premia are observed in developed countries, making it difficult to use those models for policymaking in developed countries. In this paper, we do not model the sovereign default as a strategic decision, but instead assume that the government defaults when it is bounded by the fiscal limit.

In an attempt to match the sovereign risk premia observed in developed countries, Juessen *et al.* (2009) endogenize the sovereign defaulting by assuming that investors may stop lending whenever the government runs a Ponzi scheme. The distorting tax rate is assumed to be *fixed and exogenous.* After solving the model through a second-order approximation, the authors show that the predicted sovereign risk premia under plausible calibrations fall far short of those observed in the data.

Our paper contributes to the literature by providing a DSGE framework that is suitable for policymaking in developed countries. First, we allow for an *endogenous* fiscal limit. The distribution of fiscal limits arises endogenously from the dynamic Laffer curve, which depend on macroeconomic fundamentals. The predicted distributions of fiscal limits for developed countries are consistent with sovereign downgrades in the data. Second, we allow for *endogenous* tax policy. The government raises the tax rate when government debt rises and, therefore, affects the outcome of sovereign default. Third, we solve the full nonlinear model using the monotone mapping method and find that the model can produce substantial risk premia under plausible calibrations. The nonlinear solution is important as the fiscal limit can be far from the steady state.

We consider a closed economy in which the government finances an exogenous level of purchases and countercyclical lump-sum transfers to households by collecting distorting taxes and issuing non-state-contingent bonds. The bond contracts are not enforceable. The government may partially default if the debt level reaches the fiscal limit, which is constrained by the economy's dynamic Laffer curve and the country's political willingness to make fiscal adjustments. Dynamic Laffer curves, which arise endogenously from distorting taxes, constrain the government's ability to service its debt. If the tax rate is on the "slippery" side of the Laffer curve, then the government is unable to raise more tax revenue through higher tax rates, even if it has the will to raise rates. On the other hand, even if it is economically feasible to increase revenues, the government may be unwilling to raise rates. We treat the willingness as a political decision unrelated to economic fundamentals.

The distribution of the fiscal limit is a set of maximum levels of debt that government is able and willing to service, given the random disturbances hitting the economy. At each period, an effective fiscal limit is drawn from the endogenous distribution. If the level of government debt surpasses the effective fiscal limit, then the government reneges on a fraction

of its debt. Households are assumed to know the distribution of the fiscal limit. Using this information, they can decide the quantity of government bonds that they are willing to hold and the price at which they are willing to purchase the bonds.

Several results emerge. First, the distribution of fiscal limits, which is country-specific, depends on the underlying macroeconomic fundamentals. A government with a heavy burden of transfer payments or government purchases has a higher probability of reaching the peak of the Laffer curve and may face a significantly lower fiscal limit. Strong countercyclical spending policies, which arise from large automatic stabilizers or discretionary countercyclical fiscal policies, can produce a more dispersed distribution of fiscal limits, as they exacerbate the deterioration of government budgets when tax revenue is low. An economy that is less vulnerable to exogenous shocks may face less uncertainty about the government's ability to service its debt and, therefore, a less disbursed distribution of fiscal limits. The distribution of fiscal limits also hinges on political willingness to service debt. Higher political risk may reduce the fiscal limit. We calibrate the model to three groups of countries, Canada and New Zealand, Belgium and Italy, and Japan and Sweden, and find that the predicted distributions of fiscal limits are qualitatively consistent with the observed sovereign downgrades in these countries, as illustrated in Figure 2.

Second, the default risk premium, reflecting the probability of sovereign default, is a non-linear function of the level of the government debt. Risk premium begins to emerge as the level of government debt approaches the lower end of the distribution of fiscal limits. Non-linearity between the sovereign risk premia and government indebtedness is widely identified in empirical literature (see Alesina *et al.* (1992), Bernoth *et al.* (2006), among others). We also find that risk premia for the long-term bonds move ahead of those for short-term bonds, providing early warning signals of rising default probabilities. Under a plausible calibration to the economy of Greece, the model predicts sovereign risk premia comparable to those observed in the data.

The remainder of the paper is organized in four sections. Section 2 presents the model, and Section 3 discusses the distributions of fiscal limits. The nonlinear model is solved in Section 4. Section 5 closes the paper.

2 INFINITE-HORIZON MODEL

In this section, we lay out an infinite-horizon model in which the fiscal limit, a measurement of the government's ability and willingness to service its debt, arise endogenously from dynamic Laffer curve. Consider a closed economy with linear production technology, thereby the output depends on the level of productivity (A_t) and the labor supply ($1 - L_t$). The household

consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint,

$$c_t + g_t = A_t(1 - L_t) \tag{1}$$

where the level of productivity follows an $AR(1)$ process with A representing the steady-state technology level,

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2). \tag{2}$$

Government: The government finances lump-sum transfers to households (z_t) and exogenous and unproductive purchases by collecting tax revenue and issuing one-period bonds (b_t). The tax revenue is raised through a time-varying tax rate (τ_t) on labor income. q_t is the price of the bond in units of consumption at time t. For each unit of bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At time t, the government may partially default on its liability (b_{t-1}) by a fraction of Δ_t. The post-default government liability is denoted as b_t^d. The default scheme depends on the distribution of the fiscal limit, which arises endogenously from the dynamic Laffer curve. Section 2.1 provides a further discussion of the fiscal limit and default scheme.

$$\tau_t A_t(1 - L_t) + b_t q_t = \underbrace{(1 - \Delta_t) b_{t-1}}_{b_t^d} + g_t + z_t \tag{3}$$

We assume that the government follows a simple tax rule, which is an abstraction designed to capture the tax policy in the real world as fiscal authorities tend to increase tax rate when government debt rises.[3] We refer to the parameter of γ as the "tax adjustment parameter". A larger γ means that the government is more willing to retire debt by raising the tax rate. τ and b represent the tax rate and government debt level at the steady state.

$$\tau_t - \tau = \gamma\left(b_t^d - b\right) \qquad (\gamma > 0) \tag{4}$$

In addition, we assume that the lump-sum transfers are countercyclical and the government purchases follows an exogenous $AR(1)$ process. The assumptions follow from the estimation that the elasticity of real transfers with respect to real GDP per worker (ζ^z) is negative for most developed countries from 1971 to 2007, varying from -0.093 to -2.22. On the other hand, the estimated elasticity of real government purchases with respect to real

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[3]The tax rule specification is similar to Schmitt-Grohe and Uribe (2007).

GDP per worker is inconclusive, varying from +2.5 to -2.5 across developed countries.[4]

$$\ln \frac{z_t}{z} = \zeta^z \ln \frac{A_t}{A} \qquad (\zeta^z < 0) \tag{5}$$

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2). \tag{6}$$

where z and g represent the lump-sum transfers and government spending at the steady state.

Household: With access to the sovereign bond market, a representative household chooses consumption (c_t), leisure (L_t), and bond purchases (b_t) according to,

$$\max \qquad E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{7}$$

$$s.t. \qquad A_t(1 - \tau_t)(1 - L_t) + z_t - c_t = b_t q_t - (1 - \Delta_t) b_{t-1} \tag{8}$$

with policies and prices (τ_t, z_t, Δ_t) taken as given. E_t is the mathematical expectation conditional on the information available at time t, including the sovereign default information. The utility function $(u(c, L))$ is strictly increasing in terms of consumption and leisure. β is the discount factor.

The household's first-order condition requires that the marginal rate of substitution between consumption and leisure equates to the after-tax wage. The government bond price in Equation 10 reflects the household's expectation about the probability and magnitude of sovereign default in the next period.

$$\frac{u_L(t)}{u_c(t)} = A_t(1 - \tau_t) \tag{9}$$

$$q_t = \beta E_t\left((1 - \Delta_{t+1})\frac{u_c(t+1)}{u_c(t)}\right) \tag{10}$$

The optimal solution to the household's maximization problem must also satisfy the following transversality condition,

$$\lim_{j \to \infty} E_t \underbrace{\beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)}}_{Q_{t+j+1}} \underbrace{(1 - \Delta_{t+j+1}) b_{t+j}}_{b_{t+j+1}^d} = 0. \tag{11}$$

Q_{t+j+1} is the stochastic discounted factor from time t to time $t + j + 1$.

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[4]The key results of this paper don't change even if we assume both lump-sum transfers and government purchases are countercyclical, see Bi and Leeper (2010) for a case study of Sweden.

2.1 Fiscal Limit and Laffer Curve   The proportional tax on labor income distorts a household's behavior as it lowers the after-tax wage and may induce household to work less. Depending on the existing rate, an increase in the tax rate on labor income may or may not raise the tax revenue. In general, higher rates increase the revenue when the existing tax rate is low and, on the other hand, reduce the revenue when the existing rate is high, which is the basis for the Laffer curve. Laffer curves are usually dynamic in the sense that the shape of the Laffer curve depends on the state of the economy.[5]

For a given exogenous state (A_t, g_t), a tax rate exists such that higher rates do not raise more revenue. This point is the peak of the dynamic Laffer curve, denoted as $\tau^{max}(A, g)$, at which the government can raise the maximum level of fiscal surplus, because the government purchases and the lump-sum transfers only depend on the exogenous state. The ceiling of the fiscal surplus constrains the maximum level of debt that the government is able to pay back, which is the expected sum of the discounted maximum fiscal surplus in all future periods.[6]

However, the government may not be willing to raise the fiscal surplus at the maximum level, depending on political considerations. Standard & Poor's (2008) states that "... the stability, predictability, and transparency of a country's political institutions are important considerations in analyzing the parameters for economic policymaking." Without resorting to a structural political economy model, we introduce a political risk parameter (θ_t) into the model. Higher political risk (a lower θ_t), as a result of a lack of political consensus or a turnover of power across political parties with different ideologies, implies a lower proba-bility for the government to raise fiscal surplus at the maximum level. We assume that the maximum fiscal surplus that the government is willing to raise is proportional to the polit-ical risk parameter of θ_t. For a given state of the economy (A_t, g_t), the maximum primary fiscal surplus that government is willing to raise (s_t^{max}) depends on the political risk in the following way,

$$s_t^{max} = \theta_t(T_t^{max} - g_t - z_t) \qquad (0 \leq \theta_t \leq 1). \qquad (12)$$

where T^{max} represents the total tax revenue when the tax rate is at the peak of Laffer curve (τ^{max}). Equation 12 implies that if the political risk is extremely high and θ_t is zero, then the government simply rolls over its debt and the primary fiscal surplus is zero in that period. On the other hand, if the political risk is extremely low and θ_t is one, then the government is willing to raise the maximum fiscal surplus.

To this end, we can define the fiscal limit as the maximum level of debt that the govern-

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[5]Trabandt and Uhlig (2009) compute Laffer curves for the United States and 15 European countries using a neoclassical model.

[6]Cochrane (2010) emphasizes the impact of the fiscal limit upon monetary policy in the discussion of monetary and fiscal policy in the financial crisis of 2008-2009.

ment is able and willing to service.

$$\mathcal{B}^* = E_0 \sum_{t=0}^{\infty} Q_t^{max} \theta_t (T_t^{max} - g_t - z_t) \tag{13}$$

Q^{max} represents the discounted rate when the tax rate is at τ^{max}. Appendix B describes how to Markov Chain Monte Carlo simulation to produce the distribution of the fiscal limit, which is approximated to a normal distribution, denoted as $\mathcal{N}(b^*, \sigma_b^2)$.

It is important to emphasize that the fiscal limit, defined in Equation 13, is independent of the equilibrium conditions of the model. Given the structural parameters of the model and the specification of shock processes, the unique mapping between the peak of dynamic Laffer curve (τ^{max}) and the exogenous state of the economy (A, g) determines the distribution of the fiscal limit, regardless of the equilibrium conditions of the model.

2.2 DEFAULT SCHEME   The default scheme depends on the realization of the effective fiscal limit (b_t^*), which is a random draw from the distribution of the fiscal limit. If the debt surpasses the effective fiscal limit, then the government partially defaults. The assumption of a partial default is an abstraction from debt renegotiation. Looking at the historical data, Reinhart and Rogoff (2009) show that creditors can often get a significant share of what they are owed after contentious debt negotiations. The random nature of the effective fiscal limit reflects the fact that debt renegotiation involves political considerations from which we abstract.

We assume that the default rate (δ) depends on the distribution of the fiscal limit in the following way,

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta \equiv \frac{2\sigma_b}{b^*} & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

The more dispersed the distribution, the higher the default rate becomes. Because a more dispersed distribution implies a higher uncertainty of the government's ability and willingness to service its debt. If the level of government debt equals the mean of the distribution (b^*), then the government would default enough to bring the debt level below the lower boundary of the distribution, defined as two standard deviations below the mean ($b^* - 2\sigma_b$).

3 DISTRIBUTION OF FISCAL LIMIT

As shown in Equation 13, the distribution of fiscal limits is country-specific and depends on the underlying parameters including but not limited to political uncertainty, the degree of

countercyclical transfers, the size of the government, and shock processes.[7] In this section, we first compute the distribution of the fiscal limit by calibrating the model to a 'typical' developed country in the sense that all of the parameters are set to the average of all developed countries in the sample. Then we change one parameter at a time in order to understand how each parameter affects the distribution of the fiscal limit. Finally, we move a step further by calibrating the model to six different countries in three groups, Canada and New Zealand, Belgium and Italy, and Japan and Sweden. We find that the predicted distributions of fiscal limits are qualitatively consistent with the observed sovereign downgrades in those countries.

3.1 BENCHMARK CALIBRATION   The model is calibrated at annual frequency. The household discount rate is 0.95 and the net interest rate is 5.26%.[8] The utility function is assumed to be $u(c, L) = \log c + \phi \log L$. The leisure preference parameter (ϕ) is calibrated in such a way that the household spends 25% of time working and the Frisch elasticity of labor supply is 3. The total amount of time and the productivity level at the steady state are normalized to 1.

Following Arteta and Galina (2008), we use the International Country Risk Guide's (ICRG) index of political risk to calibrate the parameter of θ_t. The ICRG index stays fairly stable across time for most developed countries. Therefore, we let θ_t fixed across time and calibrate it to the average ICRG index, which has been available since 1984. The higher the ICRG index, the lower the political risk. As the index is measured at the scale of 0 to 100, we calibrate θ_t to ICRG index scaled by 100. The calibration of the other variables are explained in Appendix A.

We simulate the distribution of fiscal limits using Markov Chain Monte Carlo method, following the steps in Appendix B.

3.2 GENERAL COMPARISON   In the benchmark case, we calibrate the model to a 'typical' developed country. The government purchase-GDP ratio (g/y) is calibrated to the average share of the real government purchases over the GDP across all countries in the sample between 1971 and 2007, which is 21.3%. Similarly, the average lump-sum transfers are 15.7% of the GDP and the average tax rate is 0.362. The average of the estimated ζ^z is -0.947, implying that a decrease of productivity by 10% raises the lump-sum transfers by 9.47%. Using a HP filter, both the productivity and the government purchase shocks have an average persistence of 0.553 and an average standard deviation of 0.02. The ICRG index of political risk is 83 on average.

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[7]In an open economy model, fiscal limits can potentially be affected by other parameters, for instance, the currency of the debt and the portion of external debt out of total debt.

[8]In the related literature, Uribe (2006) uses an annual real interest rate of 6%.

Figure 3 shows the distribution of the fiscal limit. Following Equation 13, the fiscal limit ($\mathcal{B}^*$) is computed in terms of the level of government debt. However, in order to provide a meaningful illustration, the simulated fiscal limit is scaled by the steady-state output in Figure 3. In each plot, the horizontal axis represents the fiscal limit scaled by the steady-state output, while the vertical axis represents the distribution density. The solid blue line in each plot shows the distribution under the benchmark calibration, which has a mean (b^*) of 0.328, equivalent to 131.2% of the steady-state output, and a standard deviation (σ_b) of 0.0113.

We then change one parameter at a time to understand how the different parameters affect the distribution of fiscal limits.

Government Size: The average share of government purchases over the GDP is 21.3% across all countries in the sample. The highest average share is 29% in Sweden and the lowest average share is 13.7% in Switzerland. In the top left panel of Figure 3, we keep all of the other variables the same as in the benchmark case, but vary the share of government purchases over the GDP from 29% to 21.3% and 13.7%. The variance of the distribution does not change much, but the mean rises dramatically from 30% of the steady-state output to 236%. A lower government purchases increases the maximum feasible fiscal surplus at each period and, therefore, significantly raise the fiscal limit. Similar dramatic changes are observed in the top right panel of Figure 3, as the lump-sum transfers change from 22.4% of the GDP (Austria) to 15.7% (the benchmark case) and 8.4% (Australia).

Countercyclical Lump-sum Transfers: In the left panel of the second row, we let the elasticity of lump-sum transfers with respect to productivity (ζ^z) change from -2.22 (Sweden) to -0.947 (the benchmark case) and -0.093 (Austria), while calibrating the other parameters to the benchmark case. The mean of the distribution doesn't change, but the standard deviation is doubled from 0.0076 to 0.017. Strong countercyclical fiscal transfers, arising from a large automatic stabilizer or discretionary countercyclical policy, can aggravate the volatility of the fiscal limit, as the government transfers more resources to households when tax revenue is low.

Political Risk: In the right panel of the second row, when θ decreases from 0.96 (Netherland) to 0.83 (the benchmark case) and 0.59 (Greece), the mean of the distribution decreases from 152% of the steady-state output to 130% and 93%.[9] A lower political risk (a higher θ) indicates that the government is more willing to service its debt.

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[9]The ICRG index of political risk is quite stable over time for developed countries. The highest index in the sample is from Netherland in 2001, and the lowest is from Greece in 1986.

Shock Process: In the third row of Figure 3, we compare the distributions when the persistence and standard deviation of the productivity shock change. The shock persistence varies from 0.747 (Spain) to 0.553 (the benchmark case) and 0.342 (Italy) in the left panel, while the standard deviation changes from 0.034 (New Zealand) to 0.02 (the benchmark case) and 0.014 (France) in the right panel. The standard deviation of the distribution is reduced from 0.019 to 0.008 in both panels. The economy may face a more dispersed distribution of fiscal limits if the productivity shock is more persistent or volatile.

The shock persistence of government purchases varies from 0.726 (Canada) to 0.553 (the benchmark case) and 0.2 (Denmark) in the left panel of the bottom row, while the standard deviation changes from 0.0288 (Ireland) to 0.02 (the benchmark case) and 0.0147 (France) in the right panel. However, the distributions of fiscal limits barely change. This outcome is model-specific. Due to the logarithmic utility function of leisure, the wealth and substitution effects from a change of government purchases are canceled out.

3.3 COUNTRY ANALYSIS   In this section, we calibrate our model to three groups of countries, Canada and New Zealand, Belgium and Italy, and Japan and Sweden, so that we can compare the predicted distributions of fiscal limits to the observed sovereign downgrades. Table 1 compares the key parameters across the three groups.

	Canada	New Zealand	Belgium	Italy	Japan	Sweden
τ	32.4	31	43	36	26	49
g/y	23	20.5	23.8	20	16.2	29
z/y	12	13.4	18.8	17.8	10	19.5
ζ^z	-1.25	-0.82	-0.63	-0.73	-1.15	-2.22
θ	0.84	0.85	0.8	0.7	0.84	0.86
ρ^A	0.6	0.6	0.5	0.5	0.56	0.66
σ_A	0.02	0.04	0.018	0.018	0.018	0.015

Table 1: Country Analysis

3.3.1 CANADA VS. NEW ZEALAND   Canada and New Zealand are very similar in terms of average tax rates, average government spending shares, average transfer shares, and average ICRG indexes of political risk. The estimated elasticities of lump-sum transfers with respect to productivity are also comparable for the two countries .

However, the two countries have been treated differently by the rating agencies, as illustrated in the top row of Figure 2. From 1983 to 1992, the rating of New Zealand government debt was reduced by three notches from AAA to AA- as its gross debt climbed from 58% to

75% of the GDP. The Canadian government, on the other hand, was able to keep its AAA rating until the gross debt hit 90% of the GDP in 1992. The rating was reduced by one notch to AA+ and then stayed at this level until 2001.

An important reason for this discrepancy is that the economy of New Zealand is less diversified than Canada. For instance, Standard & Poor's (2007) sees the economic structure of New Zealand as relatively narrowly-based and heavily reliant on agriculture as it reports that agriculture accounted for close to 7% of New Zealand's GDP and 58% of its export receipts. This type of economic structure makes New Zealand particularly vulnerable to international commodity price fluctuations and global economic slowdowns. During the period between 1970 and 2007, the standard deviation of the detrended real GDP per worker in New Zealand is about twice as large as that of Canada.

We simulate the distributions of fiscal limits for both countries. In order to illustrate the impact of the variance of the productivity shock upon the distribution, we keep all the other parameters the same for two countries. The average tax rate is set to 0.32, average government purchase-GDP ratio to 0.21, average transfer-GDP ratio to 0.13, average political risk to 0.85, and the elasticity of transfers with respect to productivity to -1.25. The standard deviation of the productivity shock is estimated to be 0.04 in New Zealand and 0.02 in Canada, while the persistence is estimated to 0.6 for both countries. Since the shock process of the government purchases does not have much of an impact on the distribution, we let it be constant.

The top panel of Figure 4 shows that the distribution of fiscal limits is much more dispersed in New Zealand than in Canada. Given the same level of government debt, a less diversified economy, which is more vulnerable to exogenous shocks, may face tighter fiscal restrictions.

3.3.2 BELGIUM VS. ITALY   Both Italy and Belgium accumulated massive public debt in the 1990s, well above 100% of the GDP, as shown in the middle row of Figure 2. However, only the Italian government received persistent downgrades, while Belgium's rating has been stable. One possible explanation is that Belgium demonstrated strong political will to be fiscal responsible by making great strides in reforming the welfare program and reducing lump-sum transfers by about 10% of the GDP since 1980. On the contrary, a high level of public debt has been sustained in Italy since 1980 in spite of fiscal consolidation attempts that have occurred periodically in the country.

The two countries are very similar in regard to average transfer shares and the elasticity of transfers with respect to productivity . The processes of productivity shocks feature a similar standard deviation and fairly comparable persistence . They are also fairly comparable in

terms of average tax rates and average government spending shares. However, the average ICRG index of political risk is lower in Italy than in Belgium during the period between 1988 and 1995 when the Italian government bond was downgraded.

In the simulation, we set the average tax rate to 0.4, average government spending-GDP ratio to 0.225, average transfer-GDP ratio to 0.18, and the elasticity of transfers with respect to productivity to -0.7 in both countries. The standard deviation of the productivity shock is set to 0.018 and the persistence to 0.5. The parameter of political risk, nevertheless, is calibrated at 0.7 in Italy and 0.8 in Belgium.

The middle panel of Figure 4 shows that the mean of the distribution of the fiscal limit is lower in Italy than in Belgium. A lack of political will to act on the deterioration of public finances raises political uncertainty, which is often an important factor in sovereign downgrades.

3.3.3 Japan vs. Sweden

The comparison of Japan and Sweden sheds some light on how the size of a government and the degree of countercyclical transfers affect the distribution of fiscal limits. The two countries are similar in terms of political risk and the estimated processes of productivity shocks. However, the two countries are very different in terms of average tax rates, average government spending shares, average transfer shares, and the elasticities of transfers with respect to productivity.

A repeated warning from the rating agencies is that Sweden's large general government sector constrains its fiscal flexibility and put Sweden in an unfavorable position relative to its peers (see Standard & Poor's (1997, 2009)). On the other hand, Standard & Poor's (2001) claims that Japan still could maintain fiscal flexibility in the medium term when the government gross debt climbed up to 140% of the GDP in 2001.

We simulate the distribution of fiscal limits in both countries by setting the political parameter to 0.86, the persistence of productivity shock to 0.6, and the standard deviation to 0.018. All of the fiscal variables are calibrated to match the data. The bottom panel of Figure 4 illustrates that a large fiscal transfer program generates small fiscal margins for maneuver and places a short leash on the level of government debt.

4 Sovereign Risk Premia: Nonlinear Solution

Based on the country comparisons in section 3, we have analyzed how the fiscal limit depends on macroeconomic fundamentals. In this section, we take a step further to study the quantitative impact of sovereign risk premia upon the economy by calibrating the model to the economy of Greece, which experienced a severe fiscal crisis in 2010. Due to the existence of a fiscal limit, the model can not be solved through linearization. Instead, we resort to the

monotone mapping method and solve the nonlinear model.

4.1 METHOD   The solution method, based on Coleman (1991) and Davig (2004), con-jectures candidate decision rules that reduce the system to a set of expectation first-order difference equations. In this model, the decision rule maps the state at period t, denoted as $\psi_t = \{b_t^d, A_t, g_t\}$, into the stock of government debt in the same period (b_t). The mapping is denoted as $b_t = f^b(\psi_t)$. The state variable of the post-default government liability (b_t^d) incorporates the information of the effective fiscal limit at time t (b_t^*) and the pre-default government liability (b_{t-1}).

   The complete model consists of a system of nonlinear equations, including the first-order conditions from the household's maximization problem, Equations 9 and 10; the government budget constraint, Equation 3; the specifications of fiscal policy, Equations 4 and 5; the aggregate resource constraint, Equation 1; the specifications of shock processes, Equation 2 and 6; the transversality condition, Equation 11; and the specification of the default scheme, Equation 12.

   After substituting into the conjectured rule, the core equation of the model is,

$$
\frac{b_t^d + g_t + z(\psi_t) - \tau(\psi_t)A_t\Big(1 - L(\psi_t)\Big)}{f^b(\psi_t)}
$$
$$
= \beta E_t\left\{\Big(1 - \Delta(f^b(\psi_t), A_{t+1}, g_{t+1}, b_{t+1}^*)\Big)\frac{u_c(f^b(\psi_t), A_{t+1}, g_{t+1}, b_{t+1}^*)}{u_c(\psi_t)}\right\} \tag{14}
$$

The expectation in the right-hand side is evaluated using a numerical quadrature. Equation 14 is solved for each set of state variables defined over a discrete partition of the state space. The decision rule $(f^b(\psi))$ is updated at every node of the state space. The procedure is repeated until the iterations update the current decision rule by less than some $\epsilon > 0$ (set to $1e - 8$).

   After finding the decision rule for government bonds $(f^b(\psi))$, I can solve the pricing rule $(q = f^q(\psi))$ using the government budget constraint, Equation 3. The interest rate on government bonds can also be solved using $R_t = 1/q_t$, denoted as $f^R(\psi)$.

4.2 CALIBRATION TO THE ECONOMY OF GREECE   The fiscal parameters are calibrated in order to match the Greek data from 1971 to 2007. At the steady state, the government purchases are set to 16.7% of the GDP, the lump-sum transfers to 13.34% of the GDP, and the government debt to 40% of the GDP.[10] The resulting tax rate is 0.32 at the steady state, which is slightly higher than the average tax rate of 0.28 in the data. The estimated

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[10]The data of government debt is from European Commission (2009).

elasticity of real lump-sum transfers with respect to real GDP per worker (ζ^z) is -0.45. The tax adjustment parameter (γ) is estimated at 0.42.[11]

The ICRG index of political risk for Greece represents a regime-switching process. It stayed low and stable during the period between 1984 and 1993, then rose from the level of 60 to 80 between 1994 and 1996, and stayed at the high level until the financial crisis erupted in 2008. The regime switching in the middle 1990s is due to the establishment of the European Union, while the recent switching is related to the political uncertainty in the wake of financial crisis. In this model, we calibrate θ_t as a two-state symmetric Markov regime-switching process. The low state (θ_L) is calibrated to 0.61, which is the average ICRG index of political risk between 1984 and 1993 scaled by 100, and the high state (θ_H) is 0.78, which is the average ICRG index from 1994 to 2007 scaled by 100. We assume that the probability of switching between the two states is 1/13, as Greece enjoyed a stable and high ICRG index within 13 years from 1994 to 2007.

Using a HP filter, the productivity shock has a persistence of 0.45 and a standard deviation of 0.0328, while the government purchase shock has a persistence of 0.426 and a standard deviation of 0.0294. Both shock processes are discretized following the method in Tauchen (1986). All of the other parameters are calibrated in the same way as the benchmark calibration in Section 3.1.

Figure 5 shows that the distribution of the fiscal limit is more dispersed if the political risk is a Markov regime-switching process than if fixed at either low or high level. We focus on the Markov regime-switching case in the following sections. The mean of the distribution (b^*) is 0.4255, around 170% of the steady-state output, and the standard deviation (σ_b) is 0.0266. Following the default scheme described in Section 2.2, the default rate (δ) is 0.125.

4.3 DECISION RULE   The pricing rule of the interest rate maps a 3-dimension state space to a 1-dimension state space, $R_t = f^R(b_t^d, A_t, g_t)$. The rule is a 4-dimension surface in a single graph. For simplicity, we plot slices of the surface by fixing two variables at a time.

By keeping the government purchases at the steady state, the top panel of Figure 6 compares the response of net interest rate to the current government liability (b_t^d) under different levels of productivity. In order to provide a meaningful illustration, the horizontal axis shows the debt level scaled by the steady-state output (b_t/y), instead of the debt level itself. The two green vertical lines, labeled b^* and b_{2std}^*, are the mean and lower boundary of the distribution, with the lower boundary being defined as the debt level that is lower than the mean by two standard deviations ($b_{2std}^* = b^* - 2\sigma_b$). The dashed red, solid blue, and dashed-dotted black line represent the responses in interest rate when the level of productivity

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[11]γ is estimated through linear regression of the tax rate over government debt-GDP ratio during the period of 1971 to 1995, because the debt-GDP ratio is almost flat from 1995 to 2007.

is respectively two standard deviations lower than the steady state (Low A), at steady state, and two standard deviations higher than the steady state (High A).

Two findings follow from the comparison. First, the net interest rate rises with government liability in a nonlinear way. The interest rate rises sharply as government liability enters the lower end of the distribution of the fiscal limit. The financial market starts to demand a premium for the government bond as the probability of sovereign default begins to emerge from nil. Second, the level of productivity has a substantial impact on default risk premia. For a given level of current government liability, the lower the level of productivity, the higher the interest rate, i.e. the dashed red line lies above the dashed-dotted black line. In a recession, tax revenue is slashed and the government has to issue more debt in order to finance its expenditures. A higher debt level can substantially raise the sovereign default probability and, therefore, risk premium for the government bond.

The bottom panel of Figure 6 compares the responses of the interest rate to the post-default government debt under different levels of government purchases, while keeping the level of productivity at the steady state. Again, the interest rate rises sharply as the government liability enters the lower end of the distribution of the fiscal limit. However, different levels of government purchases do not have much of an impact on the interest rates, as the dashed, solid and dashed-dotted lines are very close. Government purchases affect the level of government debt in two opposite directions. Higher purchases drive up the government liability on one hand. On the other hand, they crowd out private consumption and force the household to work more due to an income effect. A higher output raises the tax revenue and helps to finance government purchases. Overall, different government purchases do not have a significant impact on the interest rate due to the logarithmic utility function.

4.4 NONLINEAR SIMULATION   Figure (7) illustrates the nonlinear simulation. At the period $t = 1$, the government debt is set to 115% of the GDP, similar to the debt level that the Greek government has accumulated by 2009. In the following five periods, the economy receives a sequence of positive government spending shocks and negative productivity shocks. The simulated paths of productivity and government purchases over GDP ratio, summarized in the following table, are meant to capture the countercyclical government spendings in a severe economic downturn.

	t=1	t=2	t=3	t=4	t=5	t= 6
Productivity	-4.88%	-8.61%	-9.97%	-6.67%	-4.21%	-1.92%
Government Spending-GDP	20.35%	21.68%	21.81%	21.08%	20.29%	19.52%

As shown in Figure 7, the representative household works and consumes less when the

productivity is low. The lower tax revenue, as a result of a lower labor supply and productivity, and higher government purchases raise the government debt. The interest rate rises partly because the household substitutes away from future consumption and partly because the government liability approaches the effective fiscal limit (the green line (b_t^*)) which is randomly drawn from the distribution of the fiscal limit, $\mathcal{N}(b^*, \sigma_b^2)$. The government budget is further deteriorated through rising interest payments.

In order to disentangle the effect of the intertemporal substitution from the risk premium in the rising interest rate, Figure 7 compares the default model with a default-free economy in which the default rate (δ) is zero. The solid black lines represent the default model, while the dashed blue lines represent the default-free economy under the same sequences of shocks. The risk premium of a one-period bond, defined as the interest rate difference between the two economies, is around 60 basis points over a prolonged period.

4.5 LONG-TERM BOND   In practice, the interest rate spreads of long-term bonds, rather than short-term bonds, are used to measure sovereign default risk premia. An n-period bond can be priced as,

$$Q_t^n = \beta^n E_t \left((1 - \Delta_{t+n}) \frac{u_c(t+n)}{u_c(t)} \right).$$
(15)

We use a finite-element method to approximate the decision rules of a one-period bond. The multiple-period expectation is calculated using Markov Chain Monte Carlo simulation.

Conditional on the same sequences of shocks as in Figure 7, Figure 8 illustrates that the default risk premia for the long-term bonds move ahead of those for the short-term bonds, which provides early warnings of increasing probability of sovereign default. The expectation of higher government indebtedness in the future results in a rise in current risk premia for long-term bonds, even those for short-term bonds are still flat. The longer the bond maturity, the earlier the default risk premia begin to emerge.

5   CONCLUSION

We present a general equilibrium framework in which an endogenous and stochastic fiscal limit measures the government's ability and willingness to service its debt. The distribution of the fiscal limit depends on underlying economic fundamentals, such as the size of the government, the degree of the countercyclical fiscal policy, economic diversity, and political uncertainty. Due to the existence of the fiscal limit, the model produces a nonlinear relationship between the default risk premia and the level of government debt. The risk premia start to emerge as the level of debt approaches the fiscal limit.

This DSGE framework, which predicts distributions of fiscal limits consistent with the

observed sovereign ratings in developed countries, can shed light to policy debates around the world in different dimensions. First, what are the tradeoffs between short-run stabilization and long-run substainability when the perceived riskiness of government debt depends, in part, on the current and expected fiscal environment in place? Will a fiscal consolidation be possibly expansionary when the government debt is close to the fiscal limit? This question is partially addressed in the paper by Bi and Leeper (2010) in the case study of Sweden in the financial crisis of the early 1990s.

Another important extension is to study the interaction of monetary policy and fiscal policy in the presence of a fiscal limit. Will monetary policy's ability to control inflation be jeopardized? More broadly, what will be the optimal monetary policy responses in a monetary union when some member country faces sovereign default risk? This line of analysis will relate to recent work by Sims (2004, 2009), Cochrane (2010), Davig *et al.* (2010), and Daniel and Shiamptanis (2010).

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A DATA APPENDIX

Unless otherwise noted, all of the fiscal variables are calibrated to the data from the OECD Economic Outlook No. 84 (2009) for the period between 1971 and 2007. The gross debt is defined as all financial liabilities of general government, typically mainly in the form of government bills and bonds. The average tax rate is defined as the ratio of the total tax revenue over the GDP, including social security, indirect and direct taxes. The total government purchases include government consumption of fixed capital and government final consumption of expenditures. Lump-sum transfers are defined as the sum of social security payments, net capital transfers and subsidies. Using a Hodrick and Prescott (1997) filter, we detrend the data of the real GDP per worker from Penn World Table Version 6.2 (see Heston *et al.* (2006)) and estimate the shock process of productivity. The elasticity of lump-sum transfers with respect to productivity (ζ^z) is estimated using the detrended data of real lump-sum transfers and real GDP per worker.

The sample includes Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, United Kingdom, and United States.

B SIMULATION OF FISCAL LIMIT

In this model, the choices of household consumption and labor supply only depend on the income tax rate and the exogenous state variables (A, g). Assume the utility function is $u(c, L) = \log c + \phi \log L$. The household first-order conditions can be written as,

$$1 - L_t = \frac{A_t(1 - \tau_t) + \phi g_t}{A_t(1 + \phi - \tau_t)} \tag{B.1}$$

$$c_t = \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}. \tag{B.2}$$

The tax revenue (T_t) is,

$$
\begin{aligned}
T_t &= \tau_t \frac{A_t(1 - \tau_t) + \phi g_t}{1 + \phi - \tau_t} \\
&= (1 + 2\phi)A_t - \phi g_t - \left(A_t(1 + \phi - \tau_t) + \frac{(1 + \phi)\phi(A_t - g_t)}{1 + \phi - \tau_t} \right).
\end{aligned}
\tag{B.3}
$$

The tax revenue reaches to the maximum level (T_t^{max}) when the tax rate reaches the peak point of the Laffer curve (τ_t^{max}).

$$\tau_t^{max} = 1 + \phi - \sqrt{\frac{(1+\phi)\phi(A_t - g_t)}{A_t}} \tag{B.4}$$

$$T_t^{max} = (1+2\phi)A_t - \phi g_t - 2\sqrt{(1+\phi)\phi A_t(A_t - g_t)} \tag{B.5}$$

Since there exists a unique mapping between the exogenous state space (A, g) to τ^{max} and T^{max}, the fiscal limit $(\mathcal{B}^*)$ can be obtained using Markov Chain Monte Carlo simulation.

First, for each simulation, we randomly draw the shocks of productivity and government purchases for 400 periods. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables following the household first-order conditions and the budget constraints. According to Equation 13 in Section 2.1, we compute the discounted sum of maximum fiscal surplus by discarding the first 200 draws as a burn-in period.

Second, we repeat the simulation for $100,000$ times and obtain the distribution of the fiscal limit. Since the simulated distribution is well approximated by a normal distribution, we report the approximated normal distributions in Figure 3, 4 and 5.
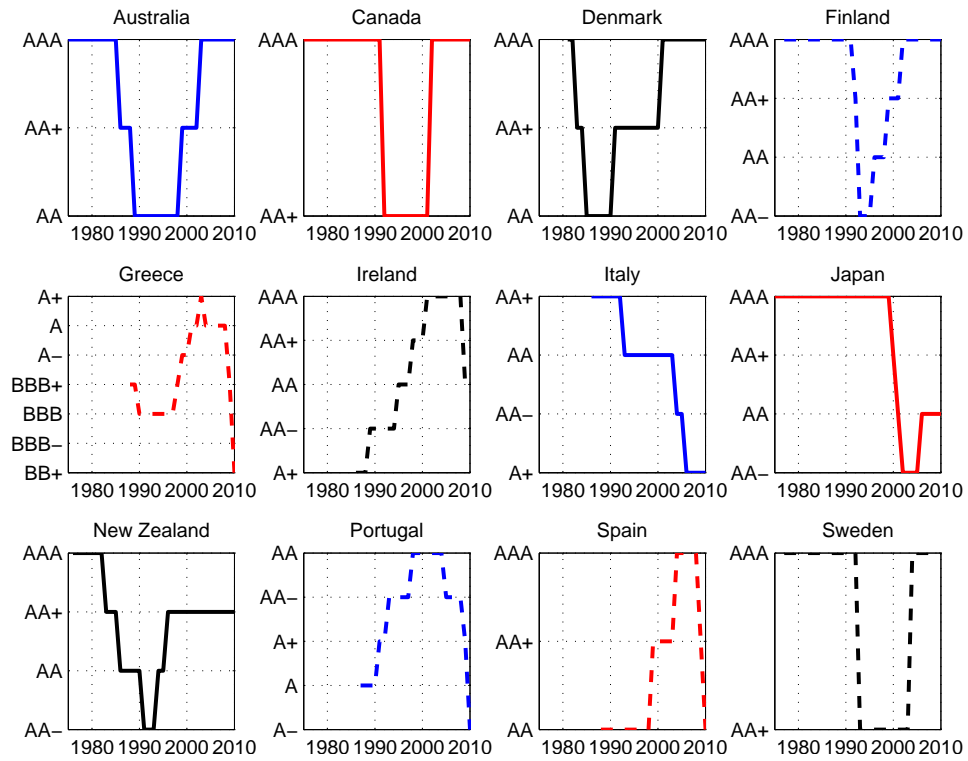
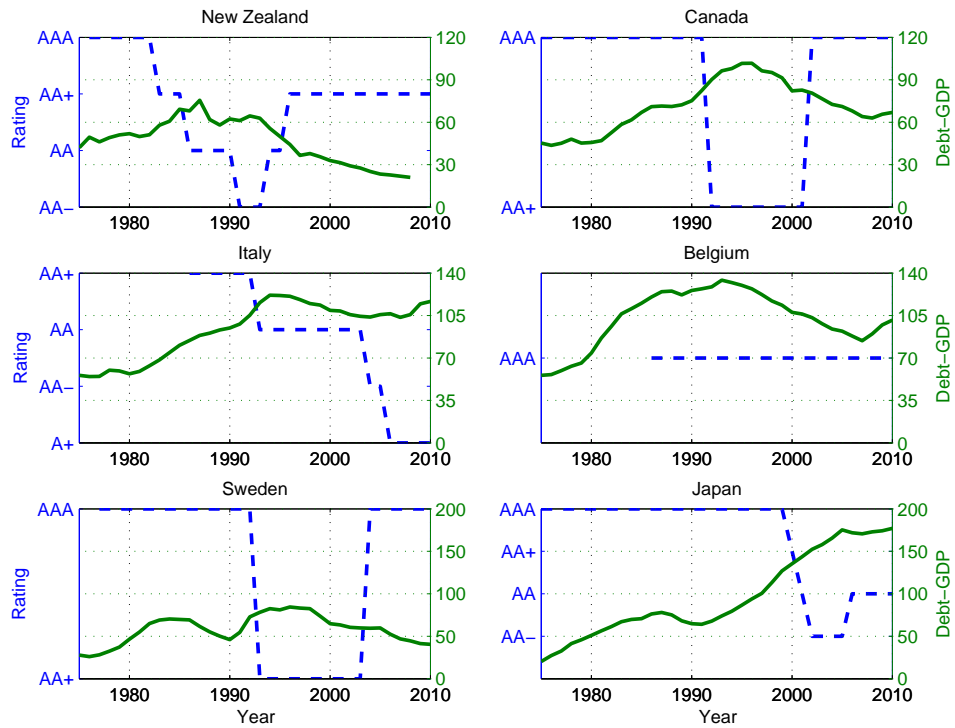Figure 1: Sovereign downgrades in some developed countries from 1975 to 2010 (Standard & Poor's)

Figure 2: Comparison of sovereign downgrades (dashed blue line, measured to the left axis) and government debt-GDP ratio (solid green line, measured to the right axis) in selected countries from 1975 to 2010.
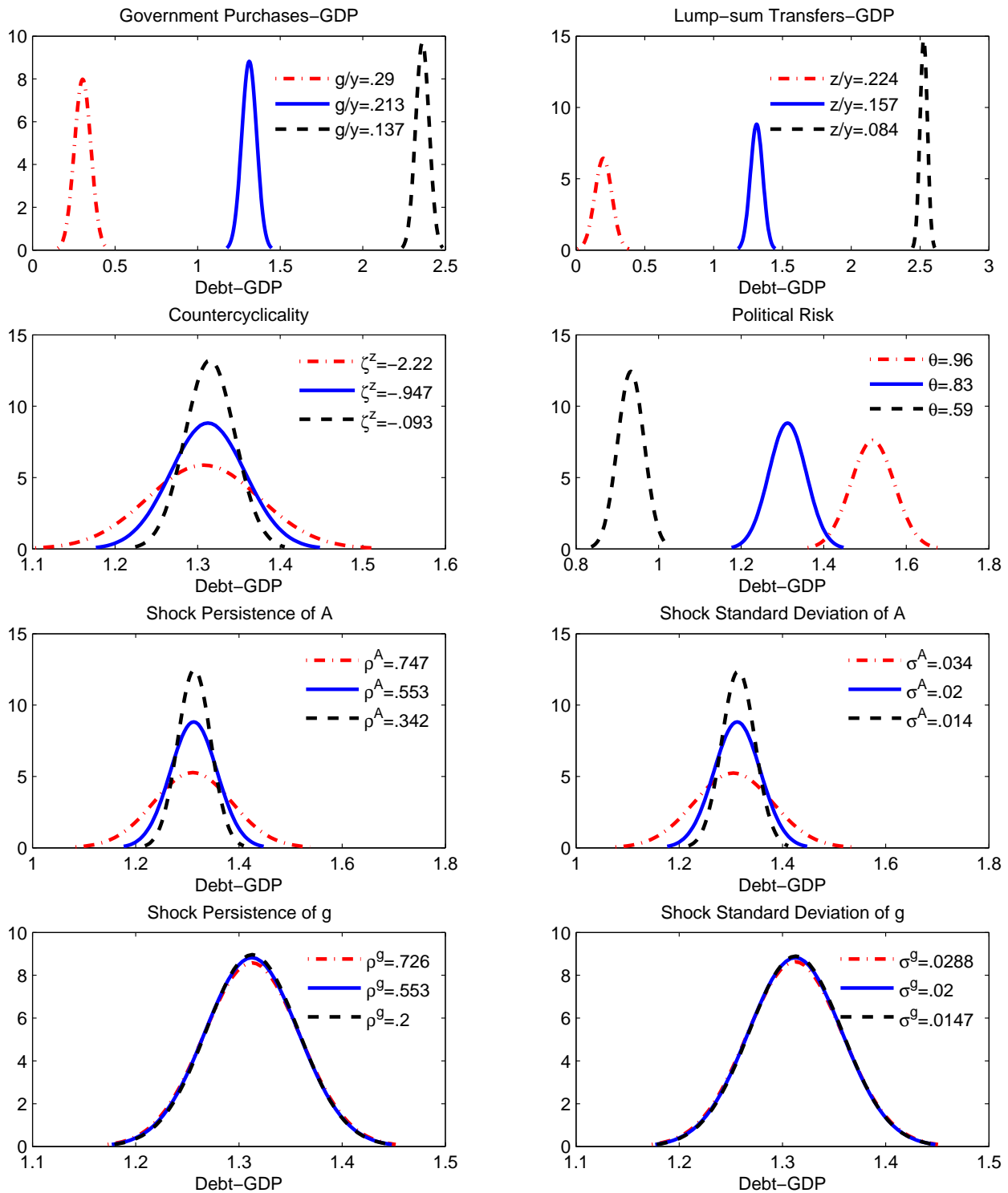
Figure 3: Comparison of the distribution of fiscal limits as one parameter changes in each panel. The solid blue line in each panel is under the benchmark calibration.
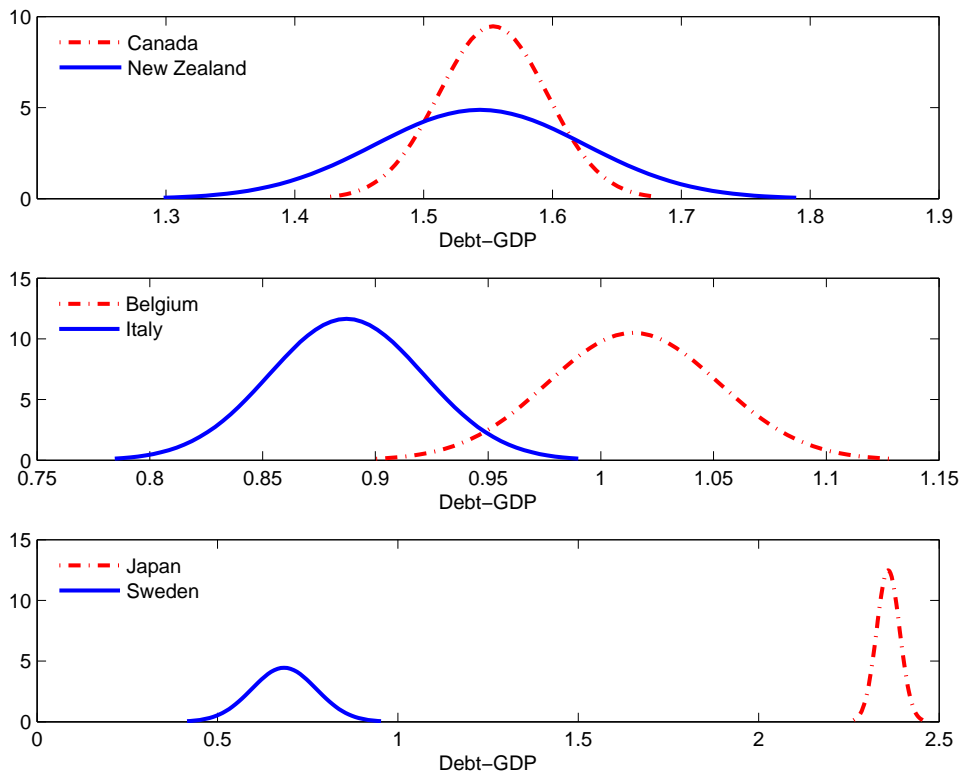
Figure 4: Comparison of the distribution of fiscal limits in different groups of countries: top panel (New Zealand vs. Canada); middle panel (Italy vs. Belgium); bottom panel (Sweden vs. Japan).
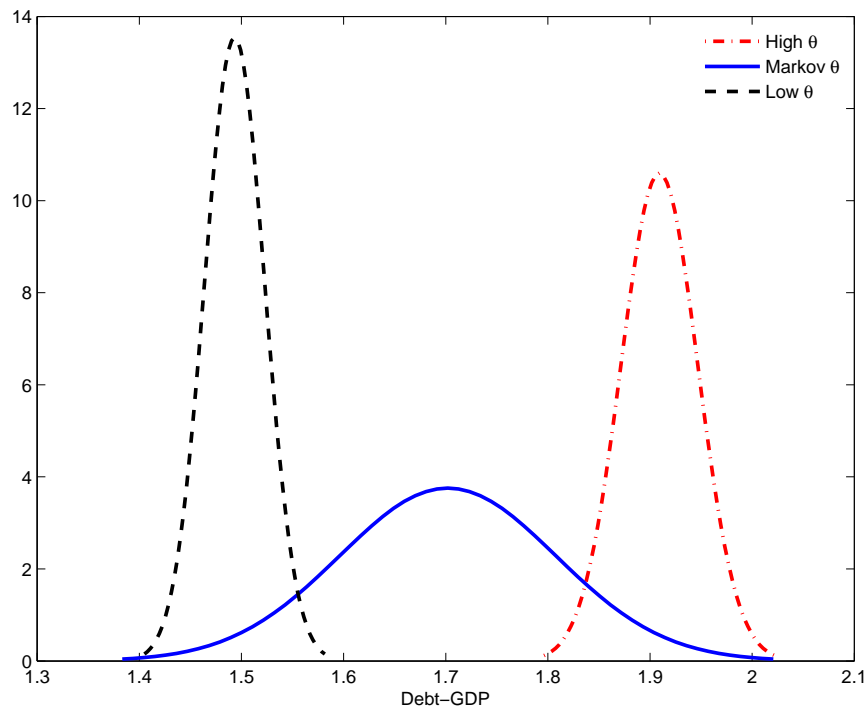
Figure 5: The distributions of fiscal limits in Greece under different specifications of political risk: high political risk (low θ), Markov-switching political risk, and low political risk (high θ).
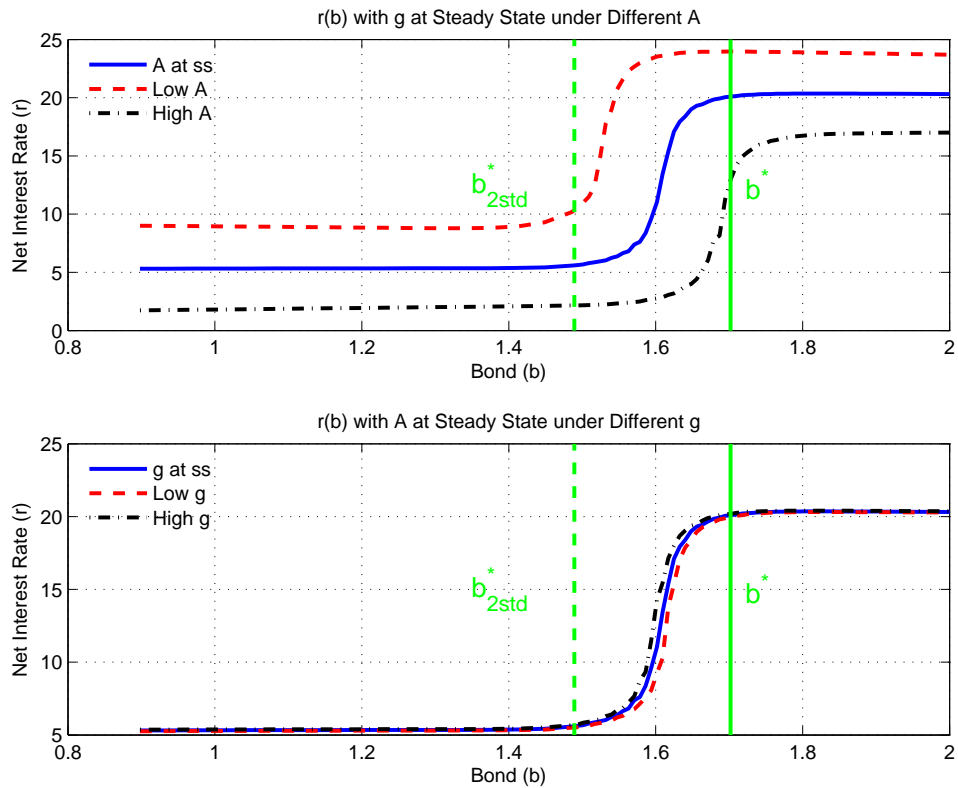
Figure 6: The pricing rule of net interest rate $r(b) = 100(R(b) - 1)$: top panel compares the pricing rules at different levels of productivity while the government purchases are at steady state; bottom panel compares the rules at different levels of government purchases while the productivity is at steady state.
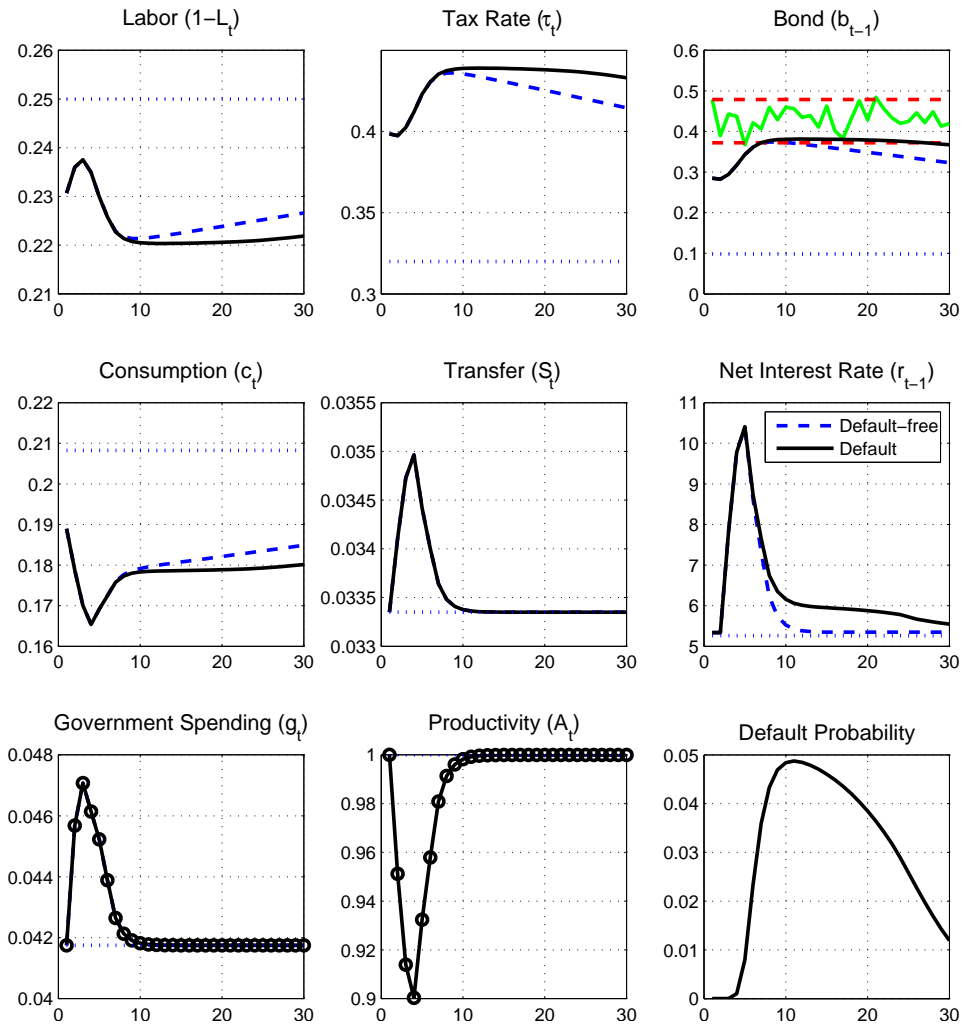
Figure 7: Nonlinear simulations under negative productivity shocks combined with positive government purchase shocks: stochastic default case (solid black line) vs. default-free case (dashed blue line).
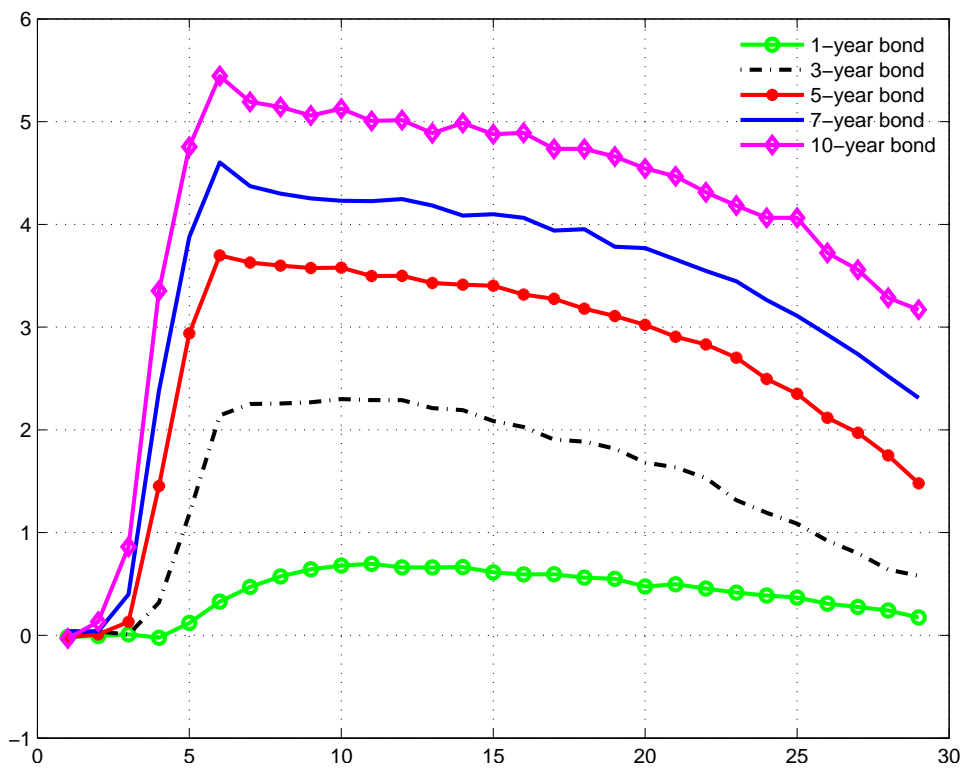
Figure 8: Risk premia for long-term bonds with different maturities conditional on the same path of shocks as in Figure 7.