Youth unemployment, labour market institutions and educational systems

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Abstract

This paper studies the relation between labor market institutions and educational systems and how this relation affects youth unemployment rates. By constructing a signalling model in which education can be used as a signal of workers' unobserved productivity and firms face firing costs, we find that the structure of the educational system may affect the school-to-work transition of young unexperienced workers. In particular, different costs of education can lead to different unemployment rates, specially for high-skilled individuals. Besides, the framework presented in this paper allows to investigate how the existence of minimum wages affects individual's education decision and helps to explain some of the observed empirical regularities.

1 Introduction

The high unemployment rates in European countries have traditionally been explained by the strict regulation of European labour markets. Generous unemployment benefits, restrictions on hiring and firing, and restrained wage competition are often blamed responsible for the poor performance of European countries. In particular, Bentolila and Bertola (1990) propose a partial equilibrium analysis of firms' optimal employment with linear adjustment costs, and find that firing costs affect the firing policy much more than the hiring policy. They also find that the reduction of the firing costs should not significantly rise the firms' propensity to hire while strongly affecting their willingness to fire. Also, Blanchard and Landier (2002) argue that the introduction of fixed-term contracts with lower firing costs than permanent contracts have the main effect of higher turnover in entry-level jobs and lead to higher unemployment. Thus, these works, and others such as Canziani and Petrongolo (2001), rely on the existence of firing costs to explain youth and long-term unemployment. Nevertheless, if one observes youth unemployment rates across the OECD countries they seem to be independent of the costs of separation. Figure 1 shows that the ratio of youth to adult unemployment rate in the US is higher than in the UE15 and the OECD average. Besides, higher youth unemployment rates are difficult to justify, specially for high ability individuals, on the grounds of minimum wages since in general new generations have higher educational attainments.

This paper studies the relation between labour market institutions and the educational system and its effects on youth unemployment rates. We ask whether it is possible that the design of the educational system may alter youth unemployment rates, and conversely, if labour market institutions can influence individuals' educational decisions. Education decisions can be regarded as investment decisions. An individual decides to study if the expected revenue of increasing her educational level is greater than the cost of education.

The expected revenue depends on the expected wage, the sort of contract she will be hired on, the probability of being unemployed and the severance pay arrangements to be received in case of firing. On the other hand, the cost of education includes the monetary cost and the effort to be exerted to obtain the degree. We include these elements in a signalling model with lay-off costs in which young workers use education as a signal of their unobservable ability level in the school-to-work transition and show that youth unemployment can arise even in the absence of firing costs.

One of the critical elements of the school-to-work transition is the asymmetry of information between firms and workers about their true productivity. Since workers in their early stages of labour life do not have any significant past labour experience for specific jobs, firms have to infer their productivity from indicators such as the educational level. Some of the main issues concerning school-to-work transitions are related with the role of education as a mean to overcome successfully that asymmetry information problem and thus education has an important role as a signalling mechanism. Since the seminal work of Spence (1973) non compulsory education can be viewed both, as a way used by workers to signal their quality to firms and as a way used by firms to discriminate ex-ante the productivity of workers. This mechanism has also been included in Greenwald (1986), Gibbons and Katz (1991) and Riordan and Staiger (1993). Along these lines Canziani and Petrongolo (2001) study how imperfect information about workers' productivity is affected by the presence of firing costs and show that a firing tax reduces the re-employment prospects of dismissed workers. This paper includes a signalling game in which education is used as a signal of worker's unobserved productivity when firing is costly. Workers can increase their educational level by investing at the beginning of their life, this decision is going to depend on the cost of education and the expected revenues of the investment. Two main results are obtained when a fixed-term contract of two periods is considered for young workers. First, the signalling mechanism reduces unemployment by providing a signal to high ability individuals, moreover, in the limit a continuum of signals would lead to a zero unemployment rate. Second, a bad design of the signalling mechanism may increase youth unemployment rate, specially for high ability individuals. This may occur when the cost of education is too low and the signalling mechanism does not allow to discriminate enough among individuals. Following this argument, it is shown that there exists an education cost function that minimizes firing costs and leads to a zero unemployment rate.

A relatively consistent conclusion of a majority of studies in labour markets, as Neumark and Wascher (2007) points out in their survey, is the negative employment effects of minimum wages, specially for low-skilled individuals. Neumark and Wascher (1995b ,1996b ,2003) also found that a higher minimum wage leads to a significant decline in the proportion of active teenagers increasing the number of them who are neither in school nor employed. It has also been observed that a higher minimum wage may influence the relative wage of high-skilled groups. In relation to this "spill-over" effect of minimum wages, Lee (1999), DiNardo, Fortin and Lemieux (1996), Green and Paarsch (1996), and Neumark, Schweitzer and Wascher (2004), among others, find that the minimum wage affects not only the earnings of workers on the minimum wage but also workers with higher wages. This "spill over" effect has been mainly associated with a substitution effect between low-skilled labour and high skilled labour, or based on efficiency arguments, as Grossman (1983) points out. By increasing the relative price of low-skilled labour minimum wages may lead to an increased demand for more skilled workers. This explanation partially collides with the discouraging effect on education enrollment rates mentioned above, since higher wages for skilled workers should act as an incentive to increase the education level. In this paper, we provide an alternative explanation based on the role of education as a signal that overcomes this limitation.

In our setting, the existence of a minimum wage distorts the signalling mechanism in two ways. On the one hand, firms will be reluctant to hire low skilled workers if they have to pay a wage higher than their expected productivity. On the other hand, it discourages workers with intermediate abilities from obtaining education since, in terms of wages differentials, the gains of costly education are lower. In turn this implies that only high ability individuals increase their educational level raising their average productivity and consequently their wage. The establishment of a minimum wage therefore undermines the signalling mechanism. If the signalling mechanism fails, firms do not hire low skilled workers because their expected productivity is lower than the minimum wage, however, they continue to offer jobs to workers with education and, as it is showed in our model, at a new wage. The sign of this "spill-over" effect on the wages of skilled workers depends on the density of high-skilled individuals and is due to the discouraging effect that the establishment of a minimum wage has on the education of intermediate ability workers, which makes that only high ability individuals decide to obtain education.

The rest of the paper is organized as follows. Section 2 describes the model with one educational signal and the information structure of the signalling game. The main implications on the unemployment rate and the educational decisions are derived in section 3. In section 4 we introduce a minimum wage and study its effects in the framework developed in section 2. Section 5 concludes.

2 The model

In this section we present the framework of analysis used in the paper. We consider a signalling model in which education is used as a signal of the unobserved productivity of unexperienced workers. In this model workers with no experience in the labour market may decide to acquire education in order to send a signal of their true productivity, which is unknown to firms. The two main problems that this group of workers have to obtain a first job are the lack of experience and the lack of information that potential employers have on their ability to perform job tasks. Thus, since we want to address the difficulties that young unexperienced workers face when they enter the labour market, we consider a fixed-term contract for the school-to-work transition that applies to this group of workers. These sort of contracts have been used in several recent works as in Blanchard and Landier (2002) and Canziani and Petrongolo (2001). Besides, Booth, Francesconi and Frank find empirical evidence that fixed-term contracts are a stepping stone to permanent work.

We consider a three period model in which the ability of a worker is denoted by θ and is distributed on the interval $[\underline{\theta}, \overline{\theta}]$. The sequence of events is depicted in figure 1. At the beginning of the first period workers decide whether to increase their educational level or not, once this decision has been taken they enter the labour market where they can be hired by a firm. This implies that there are two types of workers in the labour market, those who have studied and those who have not. As we will show later, in equilibrium there are two different wages one for educated workers w_{HS} and other for uneducated workers w_{LS}. Right after the educational decision has been taken workers are hired. At that moment the ability of a worker is unknown to firms and they are offered a two-period contract with constant wage. It is not until the end of the first period that the ability level is revealed to the firm that hires the worker, but it remains unknown for the rest of the firms until the end of the second period. As in Gibbons and Katz (1991) we assume that a worker's current employer is better informed about the worker's productivity than prospective employers. Consequently, workers hired at the beginning of period 1 receive a wage w_{HS} or w_{LS} depending on their educational level, but when their ability is revealed at the end of the first period they may be fired at a cost F for the firm. If we consider a linear technology in which the productivity of each worker is given by $k_1\theta$, uneducated workers with ability $\theta < \frac{w_{LS}-F}{k_1}$ and educated workers with ability $\theta < \frac{w_{HS}-F}{k_1}$ are fired at the end of the first period and by assumption remain unemployed for one period.

In the second period the true ability of each worker is only known to the firm that employs her and it is assumed that the wage is the same than in the first period. Only workers whose ability is over the firing threshold remain in the firm. In the third period, the ability of workers is common knowledge and they receive a wage equal to their productivity, but productivity in the third period depends on past history. Workers that have not been fired have a productivity equal to $k_2\theta$ while workers that were fired after the first period remain unemployed for one period, after this period their productivity is $k_3\theta$ which is also common knowledge and receive a wage equal to it. Experience gained in the first two periods may increase productivity and therefore we assume that $k_2 \geq k_3 > k_1$.[1] It is important to stress that in this model the only purpose of education is to send a signal of the true ability and that being educated does not imply a higher productivity. One may also consider this two-period contract as an apprenticeship contract in which workers need two periods to acquire the knowledge needed to implement all their potential productivity. This process takes place in the form of on the job training formation. Once this has been done and given that the ability level is known, compensation is equal to productivity. On the other hand, firms have no information on workers ability before hiring them and therefore the wage paid at the beginning of the contract is equal to the ex ante expected productivity of a worker.

2.1 Workers

In our model economy all job positions, which can be created at cost zero, can be filled by both type of workers, besides, workers are assigned to positions by an auction mechanism where competitive risk neutral firms offer higher wages for a given worker until the expected profit is equal to zero. Under this assumption, the wages of workers whose ability is common knowledge is equal to their productivity and they keep their position permanently. Workers with no experience can only opt for apprenticeship contracts since their ability is unknown and lack job-specific skills.

We assume that agents are risk neutral and discount future at a rate β_1 for the second period and β_2 for the third period. The different discount rates can also be interpreted as the relative length of period 2 and 3. Before entering the labour market workers decide whether to increase their educational level or not. In order to take this decision agents take into account the wages paid by firms w_{HS}, w_{LS}, the firing cost F and the cost of education $C(\theta)$ which is a continuous and strictly decreasing function of the ability level θ and always positive. Moreover, since workers know their own ability and equilibrium wages w_{HS} and w_{LS}, they can predict if they will be fired in the second period. Following this argument, a worker will choose to increase her educational level if the amount of additional earnings due to higher education is greater than the cost of education. The next proposition states the cases in which a worker decides to obtain education.

Proposition 2.1 : *A worker with ability θ decides to increase her educational level only in the following cases:*

i) *When the worker is never fired and*

$$(1+\beta_1)(w_{HS}-w_{LS}) > C(\theta).$$

ii) *When the worker is always fired and*

$$(w_{HS}-w_{LS}) > C(\theta).$$

iii) *When the worker is fired only if she decides to be educated and*

$$(w_{HS}-w_{LS}) - \beta_1(w_{LS}-F) > C(\theta).^2$$

Proof : The worker decides to obtain education if the earnings when educated less the cost of education are greater than the earnings when she has no education, in case i) this occurs when

$$w_{HS}+\beta_1 w_{HS}+\beta_2 k_2\theta - C(\theta) > w_{LS}+\beta_1 w_{LS}+\beta_2 k_2\theta.$$

[1] In what follows we will assume that $k_2 = k_3$ since the qualitative predictions of the model are not affected by this assumption.

[2] Notice that the third case of the proposition satisfies $w_{HS}+\beta_1 F > w_{LS}(1+\beta_1)$ which means that despite being fired, the income is higher when she decides to increase her education.

Rearranging terms we obtain the expression in proposition 2.1. Similar arguments lead to cases ii) and iii).

If we denote P_{HS}^F as the indicator function of the event "be fired" if the agent chooses to increase her educational level and P_{LS}^F as the indicator function of the event "be fired" if not, we can summarize the three conditions in proposition 2.1 as,

$$\begin{aligned} E(w_{LS}, w_{HS}, \theta) &= (w_{HS} - w_{LS}) + \beta_1(w_{HS} - w_{LS})(1 - P_{HS}^F) \\ &\quad -\beta_1(w_{HS} - F)P_{HS}^F(1 - P_{LS}^F) - C(\theta) > 0. \end{aligned} \tag{1}$$

Only if $E(w_{LS}, w_{HS}) > 0$ does a worker decide to study. Condition (1) can be depicted graphically for given values of w_{HS} and w_{LS} as illustrated in figure 2. There it can be observed that (1) has two discontinuity points, the left one is the firing threshold for non-educated workers while the right one is the firing threshold for educated workers.

Assumption 1 : *For any given value of the pair* (w_{HS}, w_{LS})*:*

a) *For all* θ *such that* $\underline{\theta} \leq \theta \leq \frac{w_{LS}-F}{k_1}$ *it must be that* $C(\theta) > w_{HS} - w_{LS}$

b) $C(\overline{\theta}) < (1 + \beta_1)(w_{HS} - w_{LS})$.

Part a) of assumption 1 implies that workers that are always fired have no incentive to acquire education. Part b) guarantees that there are individuals for which increasing the educational level is profitable. These two assumptions guarantee that there exist an interior solution as defined in the next proposition.

Proposition 2.2 : *For any given value of* (w_{HS}, w_{LS})*, assumption 1 implies that there exists a unique ability level* $\theta^* \in [\underline{\theta}, \overline{\theta}]$ *which determines that workers with ability* $\theta \leq \theta^*$ *decide not to increase their educational level while workers with* $\theta > \theta^*$ *decide to increase their educational level. Moreover, one can define a continuous function* $f : \Re_+^2 \rightarrow [\underline{\theta}, \overline{\theta}]$ *that assigns a unique value* θ^* *in* $[\underline{\theta}, \overline{\theta}]$ *to any pair* $(w^{HS}, w^{LS}) \in \Re_+^2$*.*

Proof : This result comes from assumption 1 and the continuity and strict monotonicity of $C(\theta)$.

Proposition 2.2 implies that for any given pair (w_{HS}, w_{LS}), the equilibrium value θ^* is given by the following equation,

$$E(w^{LS}, w^{HS}, \theta) = 0. \tag{2}$$

2.2 Firms

Before hiring a young unexperienced worker her true ability is unknown to the firm. We represent this uncertainty by assuming a differentiable distribution function for workers' ability $H(\theta)$ with density function $h(\theta)$ defined on the interval $[\underline{\theta}, \overline{\theta}]$. For expositional purposes we present the problem of the firm first for the case in which there is no education signal.

2.2.1 No education signal

When there is no signal firms cannot discriminate between workers since all of them have the same educational level and therefore there is a common wage for all the workers with no experience. We define the expected profit for a position given the wage w as

$$V(w) = E[k_1\theta - w + \beta_1 max\{k_1\theta - w, -F\}] \tag{3}$$

which can also be written as

$$V(w) = k_1E[\theta] - w + \beta_1 \int_{\frac{w-F}{k_1}}^{\overline{\theta}} (k_1\theta - w)dH(\theta) - \beta_1 \int_{\underline{\theta}}^{\frac{w-F}{k_1}} FdH(\theta).$$

In a competitive setting firms offer a common wage w in period 1 and 2 that makes their expected profit equal to zero. In the third period firms have zero profits because the ability of workers is common knowledge and they are paid their productivity. The equilibrium wage w^* will be such that (3) is equal to zero. The equilibrium value w^* can also be obtained as the solution to the following equation

$$w^* = k_1 E[\theta] + \beta_1 \int_{\frac{w^*-F}{k_1}}^{\overline{\theta}} (k_1\theta - w^*) dH(\theta) - \beta_1 \int_{\underline{\theta}}^{\frac{w^*-F}{k_1}} F dH(\theta). \tag{4}$$

LEMMA 2.3 : *There exists a unique w^* that satisfies equation (4).*

PROOF : The expected profit $V(w)$ in (3) is a continuous and strictly decreasing function of w. Moreover, we have that $V(0) > 0$ and that for w high enough $V(w) < 0$. By Bolzano's theorem there exists a w^* for which $V(w^*) = 0$ and since $V(w)$ is a strictly decreasing function w^* is unique.

If no worker is fired w^* is the usual competitive solution when there is imperfect information $w^* = k_1 E[\theta]$. This can only happen when the firing cost is high enough. More precisely, no worker is fired when $F \geq k_1(E[\theta] - \underline{\theta})$. For lower values of F, w^* may be greater than the unconditional expected productivity $k_1 E[\theta]$ and therefore workers with ability $\theta < \frac{w^*-F}{k_1}$ are fired in the second period.[3]

2.2.2 Education as a signal

Now we consider the case in which individuals can send a signal to potential employers of their ability level acquiring education at a cost $C(\theta)$ at the beginning of the first period. As already pointed out, in this model education only has value as information since it is costly and observable while the ability level is unobservable. We define the expected profit for a firm when it hires an educated worker as

$$V(w_{HS}|\theta > \theta^*) = E[k_1\theta - w_{HS} + \beta_1 max\{k_1\theta - w_{HS}, -F\}|\theta > \theta^*], \tag{5}$$

given an equilibrium value for the educational threshold θ^*. In a similar way the expected profit when the worker has no education is,

$$V(w_{LS}|\theta \leq \theta^*) = E[k_1\theta - w_{LS} + \beta_1 max\{k_1\theta - w_{LS}, -F\}|\theta \leq \theta^*]. \tag{6}$$

Let us define the probability density function of workers that decide to increase their educational level as $m(\theta) = \frac{h(\theta)}{1-H(\theta^*)}$ and its corresponding distribution function as $M(\theta)$. In the same manner one can define the probability density function of workers that decide not to buy education as $b(\theta) = \frac{h(\theta)}{H(\theta^*)}$ and its corresponding distribution function as $B(\theta)$. From the previous two equations and following similar arguments as in the case with no signal, the equilibrium wage for educated workers given θ^* is the solution to,

$$w^*_{HS} = k_1 E[\theta|\theta > \theta^*] + \beta_1 \int_{\frac{w^*_{HS}-F}{k_1}}^{\overline{\theta}} (k_1\theta - w^*_{HS}) dM(\theta) - \beta_1 \int_{\theta^*}^{\frac{w^*_{HS}-F}{k_1}} F dM(\theta). \tag{7}$$

In the same line, the equilibrium wage for uneducated workers is given by,

$$w^*_{LS} = k_1 E[\theta|\theta \leq \theta^*] + \beta_1 \int_{\frac{w^*_{LS}-F}{k_1}}^{\theta^*} (k_1\theta - w^*_{LS}) dB(\theta) - \beta_1 \int_{\underline{\theta}}^{\frac{w^*_{LS}-F}{k_1}} F dB(\theta). \tag{8}$$

PROPOSITION 2.4 : *For a given value of the educational threshold θ^* there exists a unique value for (w^*_{HS}, w^*_{LS}) that satisfies equations (7) and (8). Moreover, it can be defined a continuous function $g : [\underline{\theta}, \overline{\theta}] \rightarrow \Re^2_+$ that assigns a unique value to the pair (w_{HS}, w_{LS}) for any possible value of θ^* in $[\underline{\theta}, \overline{\theta}]$.*

PROOF : Similar arguments as for lemma 2.3 lead to this result.

[3] The unemployment rate also depends on the shape of the distribution function $H(\theta)$.

2.3 Equilibrium

Once we have laid out the behavior of individuals and firms we study the existence and uniqueness of the equilibrium of the model economy. First, we define the equilibrium for the economy.

Definition 1 : *For the set of parameter values $\{k_1, k_2, \beta_1, \beta_2, F\}$ and distribution function $H(\theta)$, a separating competitive Bayesian equilibrium is defined by a set of values for wages $(w^{eq}_{HS}, w^{eq}_{LS})$ and education threshold θ^{eq} that satisfy equations (2), (7) and (8).*

In this model the equilibrium is given by three equations (2), (7) and (8). The main difficulty in solving it is that equation (2) is not continuous and hence non-differentiable and therefore it is not possible to obtain a closed-form solution for all the parameter space. In the next theorem it is shown that a unique separating competitive Bayesian equilibrium does exist.

Theorem 2.5 : *Under assumption 1 there exists a separating competitive Bayesian equilibrium defined by $(w^{eq}_{HS}, w^{eq}_{LS}, \theta^{eq})$.*

Proof : By proposition 2.2 and proposition 2.4 it can be defined a continuous function $z(\theta) = f(g(\theta))$ such that $z : [\underline{\theta}, \overline{\theta}] \rightarrow [\underline{\theta}, \overline{\theta}]$. The existence of a fixed point is given by Brower's fixed point theorem.

Theorem 2.6 : *If the ability level is uniformly distributed in the interval $[\underline{\theta}, \overline{\theta}]$ the equilibrium defined in theorem 1 is unique.*

Proof : See the appendix

3 The role of education signalling and the firing costs

In this section we study how firing costs and the existence of the educational signal affect the equilibrium of the economy and the unemployment rate. First, we are going to consider the model economy with no frictions, that is, with no firing costs and perfect information. In this benchmark economy the wage is equal to worker's productivity, which is always known, and no one decides to study since education does not increase productivity. Also, the unemployment rate is zero since there are no frictions or uncertainty in the labour market. The introduction of firing costs does not alter these results since perfect information on workers ability eliminates any risk for firms when hiring a new worker. All this changes when the assumption of perfect information is dropped. To study the effects of this assumption we consider now an economy in which there are no firing costs, no education signal is available but workers' ability is unknown to firms. Given the sort of contract available for young workers, the equilibrium wage in the first two periods is given by equation (4) with $F = 0$. Hence, if we denote w^{eq}_0 as the equilibrium wage of this economy when there is no education signal and no firing costs, we have that w^{eq}_0 is the solution to

$$w^{eq}_0 = k_1 E[\theta] + \beta_1 \int_{\frac{w^{eq}_0}{k_1}}^{\overline{\theta}} (k_1\theta - w^{eq}_0) dH(\theta).$$

Now, because there is a common wage for all workers, the productivity of some of them is lower than the equilibrium wage w^{eq}_0 and once their productivity is known they are fired. In the second period workers with ability level $\theta < \frac{w^{eq}_0}{k_1}$ are laid off and the unemployment rate in the economy is $U^{eq}_0 = \int_{\underline{\theta}}^{\frac{w^{eq}_0}{k_1}} dH(\theta)$. In this model unemployment is due to the lack of information on workers' ability. In the rest of this section we study how firing costs and the signalling mechanism may help to reduce the unemployment rate among unexperienced workers.

3.1 A model with firing costs

When firing costs are introduced in the model with asymmetric information the equilibrium wage for the first two periods of the contract is given by equation (4) and we denote it as w_F^{eq}. The firing threshold and the unemployment rate differ from the case with no firing costs since the firing thresholds change and in the second period workers with ability level $\theta < \frac{w_F^{eq} - F}{k_1}$ are fired and the unemployment rate is $U_F^{eq} = \int_{\underline{\theta}}^{\frac{w_F^{eq}-F}{k_1}} dH(\theta)$. It can be shown that the existence of firing costs reduces both the equilibrium wage and the unemployment rate. To see this we have from equation (3) that $V_F(w) \leq V_0(w)$ for any value of w, where $V_0(w)$ is the expected profit of a position when there is no education signal and no firing costs and $V_F(w)$ is the expected profit when there are firing costs. Since the equilibrium wage must satisfy that $V(w^{eq}) = 0$ and both $V_F(w)$ and $V_0(w)$ are decreasing functions of w, this implies that the equilibrium wage with firing costs is not higher than with no firing costs. It is readily obtained that $\frac{w_0^{eq}}{k_1} \geq \frac{w_F^{eq}-F}{k_1}$ which leads to $U_0^{eq} \geq U_F^{eq}$.

When the contract is signed at the beginning of the first period firms do not know the ability of workers and all workers are paid the same wage. Hence, high ability workers receive a compensation lower than their productivity while low ability workers receive more than they produce. Once the true ability of workers is revealed in the second period those with low productivity are fired. The introduction of firing costs implies that it is not profitable for firms to fire so many workers in the second period and the equilibrium wage must decrease to compensate for the lower average productivity, the unemployment rate also decreases. If firing costs are high enough the unemployment rate may be zero and the wage drops to the average level of the labour force $k_1 E[\theta]$. In the next section we consider the existence of a minimum wage, in that case the effects of firing costs on the unemployment rate may be softened because wages have a lower bound and if this bound is high enough unemployment may arise, even with high firing costs.

3.2 A model with education signal and firing costs

Now we consider the case in which workers can increase the educational level at the beginning of their working life and firing is costly. Under this setting different outcomes are possible, since firms are risk neutral and in equilibrium profits are zero we define a benchmark economy as that in which there is no unemployment.[4] A second issue is the choice of a welfare function to evaluate which income distribution is more desirable, in particular, if a common wage is more desirable than a situation in which each worker receives a wage equal to her productivity. We are not going to deal with this second problem and we will only focuss on the unemployment rate.

As already pointed out, in a setting with imperfect information unemployment may arise, however, zero unemployment can be achieved by implementing a system that provides a continuum of signals such that each worker can send a different signal to potential employers. Then each individual would choose to acquire the education signal corresponding to her ability level and everyone would be paid its own productivity, also the unemployment rate would be zero.[5] Nonetheless, the implementation this system may be costly and other solutions as unemployment benefits may be preferable.

We consider now an intermediate solution which is the model already explained with only one educational signal. The first result is that in the separating equilibrium defined by $(w_{HS}^{eq}, w_{LS}^{eq}, \theta^{eq})$, for a given level of firing costs it must be that $w_{HS}^{eq} > w_F^{eq} > w_{LS}^{eq}$. To see this, let us consider the first inequality. We have that the equilibrium value w_F^{eq} is obtained

[4]Notice that since education does not increase productivity output is maximized when all workers are employed.

[5]The proof has the following argument. When there is available a continuum of signals, individuals with ability above the average level would increase their educational level in order to obtain a wage above the average. This would reduce the average ability level of workers with no signal and their wage. This would make some of these workers willing to acquire education. Repeating this argument recursively shows that only the individual with the lowest ability level is indifferent to increase the educational level.

when $V_F(w) = 0$ and from (5) the equilibrium value of w_{HS}^{eq} is obtained when $V(w_{HS}|\theta > \theta^{eq}) = 0$. Besides, for any value of w and θ^* it is satisfied that $V(w|\theta > \theta^*) > V_F(w)$. Since both $V(w|\theta > \theta^*)$ and $V_F(w)$ are strictly decreasing functions of w and both take a strictly positive value at $w = 0$ it follows that $w_{HS}^{eq} > w_F^{eq}$. Similar arguments lead to $w_F^{eq} > w_{LS}^{eq}$.

As we have already explained, for this economy high enough firing costs can lead to a zero unemployment rate. Hence we define the minimum level of the firing costs that lead to zero unemployment. First we consider the case when there is no educational signal.

Definition 2 : *When there is no educational signal the minimum level of firing costs that leads to zero unemployment is* $F_{min} = k_1(E[\theta] - \underline{\theta})$.

When individuals can increase the educational level there are two bounds for the firing costs, one for educated workers and other for non-educated workers. We define these two bounds for a given educational threshold θ^* as follows.

Definition 3 : *When there is an educational signal the minimum level of firing costs that leads to zero unemployment for the educated workers is* $F_{min}^{HS}(\theta^*) = k_1(E[\theta|\theta \geq \theta^*] - \theta^*)$.

Definition 4 : *When there is an educational signal the minimum level of firing costs that leads to zero unemployment for the non-educated workers is* $F_{min}^{LS}(\theta^*) = k_1(E[\theta|\theta \leq \theta^*] - \underline{\theta})$.

It is easy to check that these bounds satisfy the following properties: $F_{min}^{HS}(\overline{\theta}) = F_{min}^{LS}(\underline{\theta}) = 0$ and $F_{min}^{HS}(\underline{\theta}) = F_{min}^{LS}(\overline{\theta}) = F_{min}$. Besides, we have that $F_{min}^{LS}(\theta^*)$ is a strictly increasing function of θ^* and for regular distribution functions $F_{min}^{HS}(\theta^*)$ is a strictly decreasing function of θ^*.[6] All these properties are depicted in figure 3 for a uniform distribution function.

Now we turn to how the existence of a signalling mechanism affects the unemployment rate. Due to this mechanism the labour force is divided in educated and non-educated workers and consequently the aggregate unemployment rate is the sum of the unemployment rates in both groups,

$$U_S^{eq} = \int_{\underline{\theta}}^{\frac{w_{LS}^{eq} - F}{k_1}} dH(\theta) + \int_{\theta^{eq}}^{\frac{w_{HS}^{eq} - F}{k_1}} dH(\theta).$$

As pointed out above, when there is a continuum of signals the unemployment rate in the economy is zero. This effect of the signalling mechanism is also present even if there is only a finite number of signals. In the next proposition we study an economy with only one signal and show that the unemployment rate when the signalling mechanism is available is lower than when there is no signal. We assume for this proof that the ability of workers is uniformly distributed on the interval $[\underline{\theta}, \overline{\theta}]$.[7]

Proposition 3.1 : *If the ability of workers is uniformly distributed on the interval* $[\underline{\theta}, \overline{\theta}]$ *the unemployment rate when the education signal is available is not greater than when there is no educational signal.*

Proof : see the appendix.

In this model the government has two instruments to influence the equilibrium, the firing cost and the cost of education, we denote this policy vector as $\{F, C(\theta)\}$. If there is no signalling mechanism setting a firing cost equal to F_{min} leads to a zero unemployment rate at the cost of reducing the wage of more skilled workers. When education is introduced two instruments are available. In figure 3 $\widehat{F}$ is the minimum level of firing costs that

[6] The uniform distribution function satisfies this property, also any distribution function with negative slope in the density function for values $\theta > \theta^*$ satisfies this property.

[7] When there is a continuum of signals the unemployment rate is zero independently of the class of distribution function considered. However, if the number of signals is increased in a finite number the reduction of the unemployment rate may depend on the form of the distribution function and the cost function and is not necessarily strictly monotone in the number of signals.

yields a zero unemployment rate with an the educational threshold $\widehat{\theta}$, but this will only be achieved if an appropriate cost function consistent with $\widehat{\theta}$ is implemented. In the next proposition we show that the government can implement a policy $\{\widehat{F}, \widehat{C}(\theta)\}$ that leads to a zero unemployment rate and an equilibrium educational threshold $\widehat{\theta}$.

PROPOSITION 3.2 : *Under assumption 1 there exists at least one optimal policy $\{\widehat{F}, \widehat{C}(\theta)\}$ which leads to a zero unemployment rate with an equilibrium educational threshold $\widehat{\theta}$. Such optimal policy is characterized by the following cost function,*

$$\widehat{C}(\theta) = \begin{cases} \widehat{w}_{HS} - \widehat{w}_{LS} - \beta_1(\widehat{w}_{LS} - \widehat{F}) + \varepsilon & \text{for} \quad \theta < \widehat{\theta} \\ 0 & \text{for} \quad \theta \geq \widehat{\theta} \end{cases}$$

where $\varepsilon > 0$, $\widehat{w}_{HS}$ and $\widehat{w}_{LS}$ are the equilibrium wages when $\{\widehat{F}, \widehat{C}(\theta)\}$ is implemented and the equilibrium educational threshold is $\widehat{\theta}$.

PROOF : From figure 2 it follows that when $\{\widehat{F}, \widehat{C}(\theta)\}$ is implemented individuals with ability $\theta < \widehat{\theta}$ do not increase their educational level while those with $\theta > \widehat{\theta}$ decide to study. The equilibrium wages are given by equations (7) and (8).

The unemployment rates of both educated and non-educated workers heavily depend on the structure of the educational system. In figure 2 the number of educated workers unemployed in the second period is given by the distance between the firing and the educational thresholds. If there is a change in the cost function, for instance the cost of obtaining a degree decreases, the function moves down and θ^* drops. This in turn raises w_{LS} and w_{HS} which increases θ^*. However, under a uniform distribution function this second order effect is smaller and the equilibrium educational threshold decreases.[8] Also the effect on the firing threshold of educated workers is lower than in θ^* and the unemployment rate for educated workers increases. Following a similar argument it can be shown that a less expensive educational signal reduces the number of non-educated workers employed but no general prediction can be made about the overall unemployment rate. On the other hand when the signal is too expensive the educational threshold may be above the firing threshold for educated workers and none of them is unemployed.

4 Minimum wages

Imposing a minimum wage reduces the set of wage menus that a firm can choose and also disturbs the signalling mechanism. To understand the effect of such legislation in our economy suppose that a minimum wage w_{min} is introduced when the equilibrium is $(w_{HS}^{eq}, w_{LS}^{eq}, \theta^{eq})$ with $w_{min} > w_{LS}^{eq}$.[9] If the equilibrium is unique as in proposition 2.6, the introduction of a minimum wage lowers the value of $E(w_{LS}, w_{HS})$ which implies that θ^* rises (see figure 2). The rise in θ^* increases both w_{LS} and w_{HS}, but the effect on w_{LS} is not enough to compensate the effect of the minimum wage. This implies that a new complete equilibrium $(w_{HS}^{eq'}, w_{LS}^{eq'}, \theta^{eq'})$ with $w_{LS}^{eq'} > w_{min}$ cannot be reached and therefore firms do not offer any position to non-educated workers. The reason for this is that the expected revenue of a position for low ability workers is lower than the wage paid w_{min} and hence there is only an incomplete equilibrium $(w_{HS}^{eq'}, \theta^{eq'})$ in which firms only offer positions to educated workers with $w_{HS}^{eq'} > w_{HS}^{eq}$. The number of individuals that decide to study is also affected by the minimum wage. Yet, whether enrollment rates increase or not is going to depend on the skewness of the distribution function $H(\theta)$.

The main consequence of the introduction of a minimum wage is that individuals with low ability are not able to enter the labour market. In the absence of a minimum wage there is a complete equilibrium with a pair of wages $(w_{HS}^{eq}, w_{LS}^{eq})$. All workers are hired in the first period although some of them are fired at the beginning of the second period when their ability level is revealed. However, in the presence of a minimum wage firms offer

[8] See the appendix.
[9] If $w_{min} < w_{LS}^{eq}$ the equilibrium is not affected.

no positions to non-educated workers, whose ability is lower than $\theta^{eq'}$, and unemployment arises in the first period of the working life. Thus, the unemployment rate for workers in the first period is

$$U^{eq1}_{w_{min}} = \int_{\underline{\theta}}^{\theta^{eq'}} dH(\theta),$$

while in the second period rises to

$$U^{eq2}_{w_{min}} = \int_{\underline{\theta}}^{\frac{w^{eq'}_{HS} - F}{k_1}} dH(\theta).$$

The existence of a minimum wage hinders the entry of young workers in the labour market and rises their unemployment rate, probably leading them to permanent unemployment. But this is not its only effect, due to minimum wages the signalling mechanism is altered and school enrollment rates are affected. In our model education is only a signal and does not increase the ability of individuals, however, if one regards education as a valuable asset for individuals and society, the effects of minimum wages on enrollment rates have also to be taken into account. To further investigate the implications of minimum wages on enrollment rates let us consider an example. Assume an economy as the one considered so far but with two educational signals instead of only one. Suppose that the cost functions of each signal are $C_1(\theta)$ and $C_2(\theta)$ such that $C_2(\theta) > C_1(\theta)$ for all θ. In the appendix it is shown that an equilibrium for this economy always exists and under similar assumptions as for proposition 2.6 the equilibrium is unique. Figure 4 represents the education decision faced by individuals when there are two signals, in this case there are three equilibrium wages $(w_1^{eq}, w_2^{eq}, w_3^{eq})$ and two educational thresholds $(\theta_1^{eq}, \theta_2^{eq})$. Now consider the introduction of a minimum wage such that $w_3^{eq} > w_{min} > w_2^{eq}$, the existence of this minimum wage renders irrelevant the first educational signal. The reason is that individuals with ability $\theta < \theta_2^{eq'}$ are not willing to pay the cost of the first signal because their expected productivity is lower than the minimum wage and firms do not employ them. For school enrollment rates the most dramatic effect of minimum wages is that the educational structure of the population is polarized. There are individuals with high levels of education, those with ability $\theta > \theta_2^{eq'}$, but those whose ability level does not reach this threshold have no education at all. The main consequence is that in this equilibrium there are no workers with intermediate levels of education since no worker buys the first signal. This example could be extended to an economy with n signals, in that case a minimum wage would eliminate all the signals below the minimum wage and individuals would acquire only the most costly educational signals. These results help to understand the "spill over" effect of minimum wages documented in the empirical literature [e.g. Lee (1999), DiNardo, Fortin and Lemieux (1996), Green and Paarsch (1996), and Neumark, Schweitzer and Wascher (2004)]

5 Conclusion

When the education level can be used as a signal of the unobservable ability level and firms face separation costs several results are obtained. First, depending on the design of the educational system the unemployment rates of young unexperienced workers can be affected. Since workers' productivity is unknown to firms, a fixed-term contract implies a common wage for all the entrants in the labour market. Once firms observe the true productivity of workers they fire those whose productivity is too low. Higher firing costs reduce the number of lay-offs at the cost of decreasing the average wage. At this point the choice of a proper signalling mechanism is crucial since if the cost of studying is very low there are too many educated workers and their unemployment rate increases. This rate can be reduced by imposing a firing tax, however, as it has been extensively studied in the literature the high firing costs may reduce firms' flexibility and have undesirable productive inefficiencies. These problems can be partially overcome by designing a well-structured educational system that minimizes the firing cost necessary to maintain low unemployment rates. In fact, it can be found an education cost function that minimizes the firing cost necessary to have a zero unemployment rate.

Finally, the existence of a minimum wage may distort the equilibrium outcome by introducing asymmetries in the labor market. As already known, a minimum wage destroys the positions with low productivity, yet, in the context of a signalling game this effect can be amplified since workers' productivity is unobservable and individuals are grouped into educated and non-educated workers. Thus, if a minimum wage is imposed it is possible that workers with productivity higher than the minimum wage are unemployed. Moreover, the introduction of a minimum wage may render irrelevant signals for workers with a medium-high ability level dividing the labour force into non-educated workers and highly-educated workers. Also, minimum wage legislation may reduce the enrollment rates of medium and high-ability workers by raising the educational threshold.

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Appendix (preliminary and incomplete)

Proof of theorem 2.6

Proof of proposition 3.1

A model with two educational signals

Consider an economy as the one already studied but with two educational signals instead of one. Suppose that the cost functions of each signal are $C_1(\theta)$ and $C_2(\theta)$ such that $C_2(\theta) > C_1(\theta)$ for all θ and that these two functions satisfy all the properties required to $C(\theta)$. Following similar arguments as in proposition 2.1 we have that in this economy a worker with ability θ decides to buy the first educational signal only in the following cases:

i) When the worker is never fired and

$$(1+\beta_1)(w_2 - w_1) > C_1(\theta).$$

ii) When the worker is always fired and

$$(w_2 - w_1) > C_1(\theta).$$

iii) When the worker is fired only if she decides to be educated and

$$(w_2 - w_1) - \beta_1(w_1 - F) > C_1(\theta).$$

As it is shown below, when there are two signals the second signal is acquired only by workers for whom it is also profitable acquiring the first signal. Then, a worker with ability θ decides to buy the second educational signal only in the following cases:

i) When the worker is never fired and

$$(1+\beta_1)(w_3 - w_2) > C_2(\theta) - C_1(\theta).$$

ii) When the worker is always fired and

$$(w_3 - w_2) > C2(\theta - C_1(\theta)).$$

iii) When the worker is fired only if she buys the signal 2 but not when buying signal 1 and

$$(w_3 - w_2) - \beta_1(w_2 - F) > C_2(\theta) - C_1(\theta).$$

Now we make an extension of assumption 1,

Assumption 2 : *For any given value of the vector (w_1, w_2, w_3):*

a) *For all θ such that $\theta \leq \frac{w_1 - F}{k_1}$ it must be that $C_1(\theta) > w_2 - w_1$*

b) $C_1(\overline{\theta}) < (1+\beta_1)(w_2 - w_1)$

c) *For all θ such that $\theta \leq \frac{w_2 - F}{k_1}$ it must be that $C_2(\theta) > w_3 - w_2$*

d) $C_2(\overline{\theta}) < (1+\beta_1)(w_3 - w_2)$

e) $C_2(\theta) - C_1(\theta) > w_3 - w_2$

Part a) and c) of assumption 2 imply that workers that are always fired have no incentive to study while part b) and d) entail that there are workers for whom increasing the educational level is profitable. The last condition e) involves that not all the individuals that buy signal 1 are willing to buy signal 2. If we denote the equilibrium educational thresholds for signal 1 and 2 as θ_1^{eq} and θ_2^{eq}, it must be satisfied that $\theta_2^{eq} > \theta_1^{eq}$ and that $\theta_2^{eq} > \frac{w_2 - F}{k_1}$.[10]

[10] If $\theta_2^{eq} < \theta_1^{eq}$ some of the workers that do not by signal 2 have incentives to by signal 1 since $C_2(\theta) > C_1(\theta)$, also, if $\theta_2^{eq} < \frac{w_2 - F}{k_1}$ all the workers that buy signal 1 would be fired in the second period.

Proposition 5.1 : *For any given value of the vector* (w_1, w_2, w_3), *assumption 2 implies that there exists a unique pair of ability levels* $(\theta_1^*, \theta_2^*) \in [\underline{\theta}, \overline{\theta}]^2$ *which determines that workers with ability* $\theta \leq \theta_1^*$ *decide not to increase their educational level, workers with ability* $\theta_1^* < \theta \leq \theta_2^*$ *decide to buy the first signal and workers with* $\theta > \theta_2^*$ *decide to buy the second signal. Moreover, one can define a continuous function* $f : \Re_+^3 \rightarrow [\underline{\theta}, \overline{\theta}]^2$ *that assigns a unique pair* (θ_1^*, θ_2^*) *in* $[\underline{\theta}, \overline{\theta}]^2$ *to any vector* $(w_1, w_2, w_3) \in \Re_+^3$.

Proof : First, we have that in equilibrium $\theta_2^* > \theta_1^*$), if not some workers would be willing to buy signal 2 but not signal 1, this contradicts part e) of assumption 2. The existence result comes from assumption 2 and the continuity and strict monotonicity of $C_1(\theta)$ and $C_2(\theta)$.

Now we turn to the problem of the firms. We define the expected profit for a firm when it hires a worker with signal 2 as

$$V(w_3|\theta > \theta^*) = E[k_1\theta - w_3 + \beta_1 max\{k_1\theta - w_3, -F\}|\theta > \theta_2^*], \tag{9}$$

given an equilibrium value for the educational threshold θ_2^*. In a similar way the expected profit when the worker has signal 1 is,

$$V(w_2|\theta \leq \theta^*) = E[k_1\theta - w_2 + \beta_1 max\{k_1\theta - w_2, -F\}|\theta_1^* < \theta \leq \theta_2^*]. \tag{10}$$

The expected profit when the worker has not studied is,

$$V(w_1|\theta \leq \theta^*) = E[k_1\theta - w_1 + \beta_1 max\{k_1\theta - w_1, -F\}|\theta_1^* < \theta \leq \theta_1^*]. \tag{11}$$

Proposition 5.2 : *There exists a unique value for* (w_1^*, w_2^*, w_3^*) *that satisfies equations (9)-(11). Moreover, it can be defined a continuous function* $g : [\underline{\theta}, \overline{\theta}]^2 \rightarrow \Re_+^3$ *that assigns a unique value to the vector* (w_1, w_2, w_3) *for any possible value of* (θ_1^*, θ_2^*) *in* $[\underline{\theta}, \overline{\theta}]^2$.

Proof : Similar arguments as for lemma 2.3 lead to this result.

In the next theorem we prove that for this economy a separating competitive Bayesian equilibrium does exist.

Theorem 5.3 : *Under assumption 2 there exists a separating competitive Bayesian equilibrium defined by* $(w_1^{eq}, w_2^{eq}, w_1^{eq}, \theta_1^{eq}, \theta_2^{eq})$ *and if the ability levels are uniformly distributed on the interval* $[\underline{\theta}, \overline{\theta}]$ *this equilibrium is unique.*

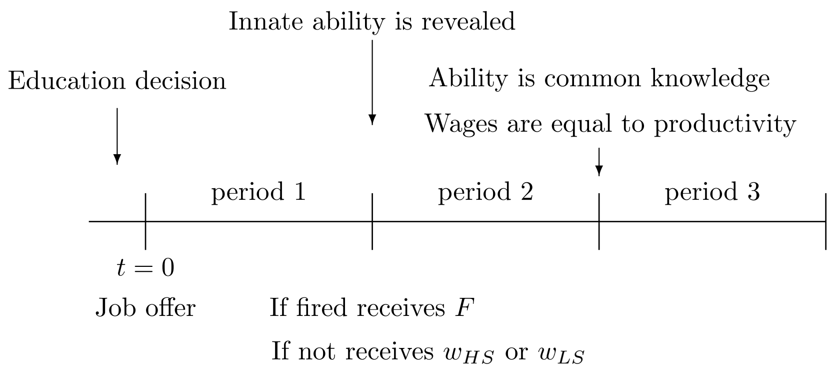

Figure 1: Sequence of events.

Youth vs. adult unemployment rates

Countries	United States		Eur. Union 15		OECD (30 countr.)	
Year	1998	2008	1998	2008	1998	2008
U.R.(%labour force)	10.4	12.8	16.7	14.6	14.8	13.2
Relative U.R. youth/adult	3	2.7	2.3	2.8	2.4	2.8
Relative U.R. low skills/high skills	5.8	4.5	2	2	2.4	2.2

Table 1: Unemployment rates.

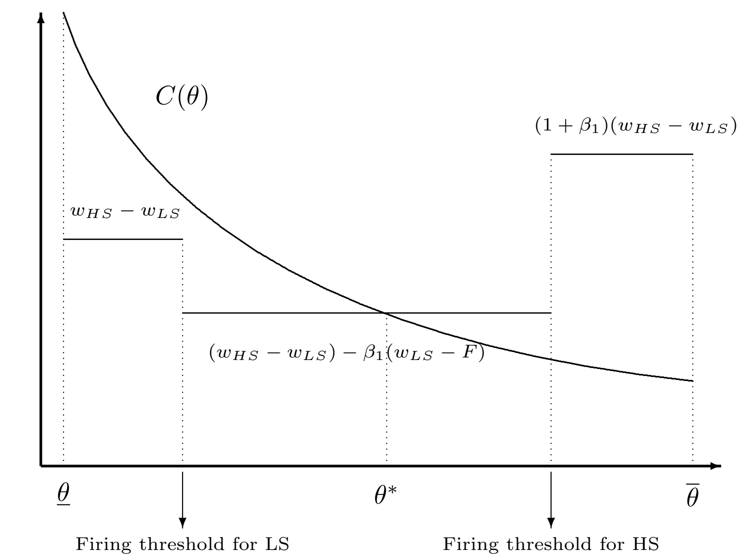

Figure 2: Education decision.

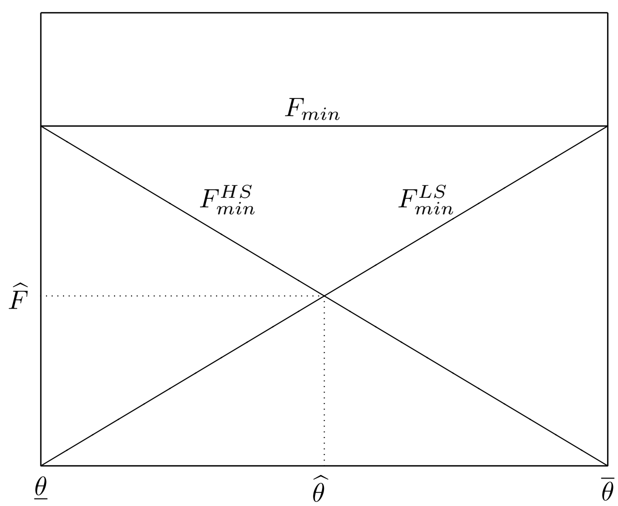

Figure 3: Education decision.

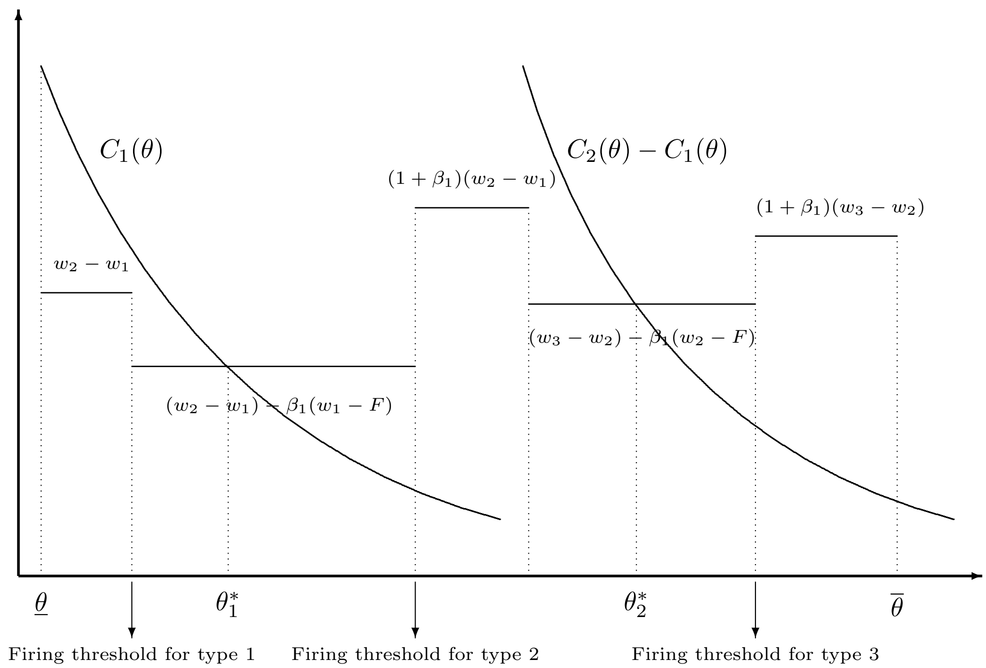

Figure 4: Education decision with two signals.