Policy at the Zero Bound*

Isabel Correia
Banco de Portugal, Universidade Catolica Portuguesa and CEPR;

Emmanuel Fahri
Harvard University

Juan Pablo Nicolini
Minneapolis Fed and Universidad Di Tella

Pedro Teles
Banco de Portugal, Universidade Catolica Portuguesa and CEPR.

March, 2010

Abstract

Key words: Zero Bound; Liquidity Trap; Fiscal policy; Monetary policy; Sticky prices.

JEL classification: E31; E40; E52; E58; E62; E63.

1 Introduction

Arbitrage between money and bonds restricts nominal interest rates from becoming negative. One could imagine circumstances in which, in the event of a potential recession, it is optimal for the Central Bank to lower the nominal interest rate. If the interest rate is very close to zero to begin with, the constraint may be binding. This is the "zero bound" problem of monetary policy.

Considerable attention has been placed on this issue in recent times, following the outbreak of the financial crisis in 2007-2008. Nominal interest rates have indeed been very close to zero in the US, the EMU, the UK and other countries. There has been work on public spending multipliers, showing that these can be very large at the zero bound (see Christiano, Eichenbaum, Rebelo, 2009, Woodford (2010), Mertens and Ravn (2010)). There has also been work on tax policy. Eggertsson (2009) considers different

*Preliminary and incomplete. Correia and Teles gratefully acknowledge financial support of FCT and Nicolini, of the Agencia Nacional de Promoción Científica y Tecnológica.

alternative taxes and assesses which one is the most desirable. All this work is done in standard sticky price models, where the zero bound on interest rates can indeed be a serious challenge to policy.

The zero bound is also a key component in the numerical work presented in Romer and Bernestein (2009) as well as in the reply by Coglan, Cwick, Taylor and Wieland (2010). It is always a main concern in Blanchard, Dell'Ariccia and Mauro (2010), in which they argue for a better integration between monetary and fiscal policy, an issue that is directly addressed here.

In this paper, we move further, and show that the zero bound constraint on interest rates is non-binding if consumption taxes are used to stabilize the economy. Since the zero bound is relevant only in exceptional circumstances, it is natural to think that consumption taxes can be used, as exceptional measures. The argument that consumption taxes neutralize the effects of the zero bound on the policy response is very simple. As in the mentioned papers, we consider an environment in which the objective of policy is to lower real rates. If nominal rates cannot be lowered real rates can still be low if expected inflation is high. Getting all prices to move together in response to aggregate conditions - so expected inflation is high - may come at a cost. Note that the relevant inflation to consider is producer price inflation. Indeed, it may be costly to get all producers in the economy to raise all future prices uniformly. But inflation arising from a reduction on current consumption taxes (or increases in future consumption taxes) is easy to achieve. Can be announced and implemented at zero cost, and will certainly bring down real interest rates.

Movements in consumption taxes would in general distort other margins. For this reason we have to use a model where those decisions are explicitly modelled. We first analyze a standard new-Keynesian model similar to the one in Eggertsson (2009). We show that once monetary policy and taxes are jointly considered, the zero bound on nominal interest rates is not a binding constraint on policy even during a severe recession. We show in a particular example how the appropriate choice of consumption taxes and labor income taxes, can implement the same allocation that would be achieved if nominal interest rates could be reduced following a negative shock. This is true for any value of the nominal interest rate at the beginning of the contraction, even if it is zero. We then analyze the same economy but with capital accumulation. We show that the main results extend to this case as long as we allow for flexible capital income taxes.

Correia, Nicolini, and Teles (2002 and 2008) consider Ramsey taxation in monetary models with labor only similar to the one in Lucas and Stokey and Chari, Christiano and Kehoe, but with sticky prices. They show that sticky prices do not matter for policy, provided both monetary and fiscal policy are used for stabilization. Since the zero bound is irrelevant under flexible prices, it follows that it is also irrelevant under sticky prices. Eggertsson and Woodford (2004b) consider both monetary and fiscal policy at the zero bound also in a Ramsey taxation model with labor only. They allow for two consumption taxes, such that prices would be set before one but after the other. With the two taxes they have the result of Correia, Nicolini and Teles. They find them to be highly unrealistic and move on to analyze the case of a single consumption tax. That paper was motivated by the need for a policy response to the prolonged recession that Japan experienced during the 90' s, while nominal interest rates were at their lower bound. In Eggertsson and Woodford (2003) and Eggertsson and Woodford (2004a) they consider deviations from the Taylor rule under normal times that would minimize the effects of the recession. In particular they propose a policy that keeps the nominal interest rate for a long period at zero in order to generate inflation. Inflation is associated with price dispersion and therefore it is costly. Instead, the policy we propose generates price stability and can implement the first best. Poterba, Rotemberg and Summers (1986) also approach the issue of how consumption taxes interact with sticky prices in a paper aimed at testing nominal rigidities.

Our policy recommendation requires flexibility of taxes. It has been argued that fiscal instruments are not as flexible as monetary policy instruments. Whether this argument applies to stabilization policy during a "great moderation" period could be argued about. However, we believe it does not apply to the either the recent crisis or to the Japanese economy in the nineties, precisely because the need to use fiscal instruments is exceptional. There have been recent policy proposals in this direction by Bob Hall and Susan Woodward[1], and earlier on, by Feldstein (2003), intended at Japan. Both of them suggested lowering consumption taxes as a way to fight the crisis. Our model formalizes these proposals and highlights the way other taxes must be jointly used at the optimum. The integration of fiscal and monetary instruments in the toolbox of policy analysis is one of the messages in the discussion of the future of macroeconomic policy in Blanchard et. al. (2010).

[1] An article by Justin Lahart in the Wall Street Journal, January 5, 2009, "State Sales-Tax Cuts: Get Another Look", comments on the proposals of Hall and Woodward in their blog.

2 The Model

The model we analyze is a standard new-Keynesian model with capital, similar to the one analyzed by Eggertsson and Woodford (2003) and (2004b), and Eggertsson (2009)[2]. As it has become standard in the New Keynesian literature, the economy is cashless.

The preferences of the households are described by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t, \xi_t\right) \tag{1}$$

where

$$C_t = \left[\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2}$$

where c_{it} is consumption of variety $i \in [0,1]$, N_t is total labor, and ξ_t is a preference shock.

Aggregate government purchases G_t,

$$G_t = \left[\int_0^1 g_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

are exogenous. The government minimizes the expenditure on the individual goods, for a given aggregate, and finances it with time varying taxes on consumption, τ_t^c, and labor income, τ_t^n. As is standard in the new-Keynesian literature, we also allow for lump-sum taxes, T_t, which is a residual variable that adjusts so that the government budget constraint is satisfied.[3]

If we let

$$P_t = \left[\int_0^1 {p_{it}}^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \tag{4}$$

where p_{it} is the price of variety i, then, the minimization of expenditure on the individual goods, implies

$$\frac{c_{it}}{C_t} = \left(\frac{p_{it}}{P_t}\right)^{-\theta}, \tag{5}$$

and

$$\frac{g_{it}}{G_t} = \left(\frac{p_{it}}{P_t}\right)^{-\theta}. \tag{6}$$

[2] These models are with labor only.

[3] This assumption substantially simplifies the discussion. As we show in Appendix XX, all the results go through if we restrict lump sum taxes to be non-positive. We also show that the poliocy we proposeis revenue neutral. THIS APPENDIX TO BE DONE

The budget constraints of the households can be written in terms of the aggregates as

$$\frac{1}{R_t}\overline{B}^h_{t+1}+\sum_{s^{t+1}/s^t} Q_{t,t+1}B_{t,t+1}=\overline{B}^h_t+B^h_{t-1,t}+(1-\tau^n_t)\,W_tN_t+\Pi_t-(1+\tau^c_t)\,P_tC_t-T_t \tag{7}$$

together with a no-Ponzi games condition, $lim_{T\to\infty}Q_{0,T}\left[\overline{B}_T+\sum_{s^{T+1}/s^T}Q_{T,T+1}B_{T,T+1}\right]\geq 0$. $B_{t,t+1}$ represent the quantity of state contingent bonds that pay one unit of money at time $t+1$, in state s^{t+1} and $\overline{B}^h_{t+1}$ are risk free nominal bonds. $Q_{t,t+1}$ is the price of the state contingent bond and $\frac{1}{R_t}$ is the price of the riskless bond - so R_t is the gross nominal interest rate. W_t is the nominal wage and Π_t are profits.

Households problem The marginal conditions of the households problem that maximizes utility (1) subject to the budget constraint (7) with respect to the aggregates are

$$-\frac{u_C(C_t,N_t,\xi_t)}{u_N(C_t,N_t,\xi_t)}=\frac{(1+\tau^c_t)\,P_t}{(1-\tau^n_t)\,W_t}$$

$$\frac{u_C(C_t,N_t,\xi_t)}{P_t\,(1+\tau^c_t)}=\beta R_tE_t\frac{u_C\left(C_{t+1},N_{t+1},\xi_{t+1}\right)}{P_{t+1}\left(1+\tau^c_{t+1}\right)}$$

Firms The production function of each good i, y_{it}, uses labor, n_{it}, according to

$$y_{it}=A_tn_{it}$$

where A_t is an aggregate productivity shock.

We assume that prices are set as in Calvo (1983). Every period, a firm is able to revise the price with probability $1-\alpha$. The lottery that assigns rights to change prices is *i.i.d.* over time and across firms. Since there is a continuum of firms, $1-\alpha$ is also the share of firms that are able to revise prices. Those firms choose the price p_t to maximize profits

$$E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\,Q_{t,t+j}\left[p_ty_{t+j}-W_{t+j}n_{t+j}\right]$$

where output y_{t+j} must satisfy the technology constraint and the demand function

$$y_{t+j}=\left(\frac{p_t}{P_{t+j}}\right)^{-\theta}Y_{t+j}.$$

obtained from (5) and (6), where $y_{t+j} = c_{t+j} + g_{t+j}$, and $Y_{t+j} = C_{t+j} + G_{t+j}$. The optimal price set by these firms is

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \tag{8}$$

where $\eta_{t,j} = \frac{(\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau^C_{t+j}\right)} (P_{t+j})^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau^C_{t+j}\right)} (P_{t+j})^{\theta-1} Y_{t+j}}$.

The price level can be written as

$$P_t = \left[(1 - \alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{9}$$

Equilibria Market clearing for each variety implies that

$$c_{it} + g_{it} = A_t n_{it} \tag{10}$$

while market clearing in the labor market implies

$$N_t = \int n_{it} di. \tag{11}$$

Using the demand functions (5), (6), it follows that

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t N_t. \tag{12}$$

An equilibrium for $\{C_t, N_t\}$, $\{p_t, P_t, W_t\}$, and $\{R_t, \tau_t^c, \tau_t^n\}$ is characterized by

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n) W_t}, \tag{13}$$

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c) P_t} = E_t \left[R_t \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{\left(1 + \tau_{t+1}^c\right) P_{t+1}} \right], \tag{14}$$

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \tag{15}$$

$$P_t = \left[(1 - \alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{16}$$

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t N_t. \tag{17}$$

ϖ_j is the share of firms that have set prices j periods before, $\varpi_j = (\alpha)^j(1-\alpha)$, $j = 0, 2, ..., t$, and $\varpi_{t+1} = (\alpha)^{t+1}$, which is the share of firms that have never set prices so far. We assume that they all charge an exogenous price p_{-1}.

We do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

For now we abstract from the issue of how a particular equilibrium is implemented. In what follows we characterize the efficient allocation and the policy variables and prices that are consistent with that allocation. Later, we present a linearized version of the model in which we explicitly consider an interest rate rule and we discuss how this rule determines a unique local equilibria.

3 Efficient allocations, policy variables and prices

The first best allocation is the one that maximizes utility (1) subject to the technology constraints (2), (3), (10) and (11).

These monopolistic competition models with Dixit-Stiglitz aggregators have the property that optimality conditions are "nested", in the sense that one can solve for the optimal allocation of labor across varieties given values for the aggregates, and then solve for the optimal values of the aggregates. We now describe the optimal conditions using this two-step procedure.

First, the efficient allocation will have the marginal rate of technical substitution between any two varieties equal to one, meaning that there is production efficiency. This means that

$$c_{it} = C_t\,;\, g_{it} = G_t$$

The efficiency conditions for the aggregates (C_t, N_t) are fully determined by:

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{1}{A_t}, \tag{18}$$

and

$$C_t + G_t = A_t N_t. \tag{19}$$

By comparing the efficiency conditions with the equilibrium conditions we can describe the prices and policy variables that are consistent with the efficient allocation.

Monetary Policy under normal times In this section, we will restrict to policies and prices such that the consumption tax is zero, so $\tau_t^c = 0$. On the other hand, we impose no restrictions to the nominal interest rate; in particular, we will ignore the zero bound restriction. In the next section we will show that the nominal interest rate is a redundant instrument if one allows for time and state varying consumption taxes.

First, in order to achieve production efficiency, conditions (5) and (6) imply that prices must be the same across firms $\frac{p_{t-j}}{P_t} = 1$. That can only be the case if firms start at time zero with a common price, p_{-1},[4] as we assume, and if firms that can subsequently change prices choose that common price, so that $p_t = P_t = p_{-1}$. This means that the price level must be constant across time and states. It therefore follows that the aggregate resource constraint (17) becomes (19).

When $P_t = P$, then from (15), we have that

$$P = \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t}. \tag{20}$$

so the nominal wage must move with productivity so as to maintain the nominal marginal cost constant.

From (13), it must be that

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c) \frac{\theta}{(\theta-1)}}{(1 - \tau_t^n) A_t}, \tag{21}$$

implying that $\frac{1+\tau_t^c}{1-\tau_t^n} = \frac{\theta-1}{\theta}$. Let the consumption tax be zero, $\tau_t^c = 0$. Therefore the labor income tax will have to be $1 - \tau_t^n = \frac{\theta}{\theta-1}$. There is a subsidy to labor that removes the mark up distortion. Note that the subsidy is constant over time and states.

From (14), we have that

$$u_C(C_t, N_t, \xi_t) = R_t E_t \left[\beta u_C \left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right]. \tag{22}$$

so the nominal interest rate must move with the real rate to satisfy the intertemporal condition.

We have implicitly assumed that the zero bound constraint on the nominal interest rate was not binding. The purpose of this exercise is to understand how the constraint affects optimal policy. We proceed to do this in

[4] This is the standard assumption. Yun (2005) analyzes the case with initial price dispersion.

two steps. First, we show that the optimal allocation can be implemented with constant arbitrary nominal interest rates if one allows for consumption taxes to respond to the state of the economy. Second, we consider a special case of the model - the same considered by Egertsson (2009) and Christiano, Eichenbaum and Rebelo (2009) - and show how tax policy can replicate optimal monetary policy after a shock that can move the economy away from the first best and that makes the zero bound constraint binding.

Equivalence betwen tax policy and interest rate policy. In this section we describe policies and prices that are consistent with the firs best allocation, when one restricts the nominal interest rate to be any arbitrary constant, say $R_t = \widehat{R}$.

The particular values of the consumption tax and the nominal interest do not affect the intra-variaties decision, so in order to achieve production efficiency it is still the case that prices must be the same across firms $\frac{p_{t-j}}{P_t} = 1$ so the price level must be constant across time and states.

Now, the intertemporal condition becomes

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = \widehat{R} E_t \left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\right]$$

so, given future taxes, this equation solves for the current consumption tax rate τ_t^c.

The labor income tax will have to move accordingly to compensate for the movements in the consumption tax according to

$$\frac{\left(1+\tau_t^c\right)\frac{\theta}{(\theta-1)}}{\left(1-\tau_t^n\right)A_t} = \frac{1}{A_t},$$

satisfying condition (40) above.

4 Policy at the zero bound

We now consider a special case of the model and study the optimal policy after a particular shock. The exercise we perform is the same as in Egertsson (2009) and Christiano, Eichenbaum and Rebelo (2009). The main difference is that while they mainly consider the effect of government expenditures, we focus on tax policy exclusively.[5]

[5] Eggertsson also considers tax changes, but only one at a time. As we show, it is key to be able to change the two taxes - consumption and labor income - jointly.

First, we specialize preferences as

$$u(C_t, N_t, \xi_t) = u(C_t, N_t)\,\xi_t$$

In this way, the shock does not affect the marginal rate of substitution between consumption and leisure. It will, however, affect the marginal rate of substitution between consumption at time t and consumption at time $t+1$.

Second, we assume that $G_t = G$, $A_t = 1$. Thus, the only shock we consider is the preference shock.

Note that in this case, the conditions for an efficient allocation (18) and (19) imply that the first best satisfies

$$-\frac{u_C(C_t, N_t)}{u_N(C_t, N_t)} = \frac{1}{A},$$

and

$$C_t + G = AN_t,$$

and it is therefore constant. In particular, note that the preference shock does not affect the efficient allocation.

4.1 Optimal policy with flexible prices

With flexible prices, all monopolist will set the same price, since they are all identical.[6] Thus,

$$p_{it} = \frac{\theta}{(\theta-1)}\frac{W_t}{A} = P_t, \text{ for all } i$$

so

$$c_{it} = C_t, \text{ for all } i$$

The equilibrium conditions are therefore summarized by

$$C_t + G = AN_t.$$

$$-\frac{u_C(C_t, N_t)}{u_N(C_t, N_t)} = \frac{(1+\tau_t^c)}{(1-\tau_t^n)}\frac{\frac{\theta}{(\theta-1)}}{A},$$

[6] The Calvo lottery is the only source of - ex post - hetoregenity on these family of models.

$$\frac{u_C\left(C_t,N_t\right)\xi_t}{\left(1+\tau_t^c\right)P_t}=R_t\beta E_t\left[\frac{u_C(C_{t+1},N_{t+1})\xi_{t+1}}{\left(1+\tau_{t+1}^c\right)P_{t+1}}\right].$$

Note then that, as long as

$$\frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^n\right)}\frac{\theta}{\left(\theta-1\right)}=1,\ \text{for all } t$$

the first two equations imply that the equilibrium is the first best. Thus, the preference shock is inessential with flexible prices. As long as the monopolistic distortion is eliminated by a labor (or consumption) subsidy, the first best is obtained.

Which is the effect of a preference shock? Imagine that the monopolistic distortion os taken care by constant tax rates, so $\tau_t^c=\tau^c$, $\tau_t^n=\tau^n$. Then, the third equation, once we replace the values of the first best allocation implies that

$$\frac{\xi_t}{P_t}=R_t\beta E_t\left[\frac{\xi_{t+1}}{P_{t+1}}\right].$$

Thus, a negative shock ξ_t, can be accommodated either with a reduction in the price level or by a reduction in the nominal interest rate.

With flexible prices, it is irrelevant which variable adjusts. However, this is not the case with sticky prices, where changes in the price level affects the relative prices across varieties and creates production inefficiencies.

4.2 Optimal Policy with Sticky prices

In order to achieve production efficiency, the price level must be constant

$$P_t=P.$$

So the equilibrium conditions are

$$C_t+G=AN_t.$$

$$-\frac{u_C\left(C_t,N_t\right)}{u_N\left(C_t,N_t\right)}=\frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^n\right)}\frac{\frac{\theta}{(\theta-1)}}{A},$$

$$\frac{u_C\left(C_t,N_t\right)\xi_t}{\left(1+\tau_t^c\right)}=R_t\beta E_t\left[\frac{u_C(C_{t+1},N_{t+1})\xi_{t+1}}{\left(1+\tau_{t+1}^c\right)}\right].$$

As with flexible prices, the first best can be implemented only if

$$\frac{(1+\tau_t^c)}{(1-\tau_t^n)}\frac{\theta}{(\theta-1)}=1, \text{ for all } t$$

If, as before, we also assume constant taxes, the last equation - again, evaluated at the first best allocation - reads

$$\xi_t = R_t \beta E_t \left[\xi_{t+1}\right].$$

So the interest rate must move to compensate any movements in ξ_t. The zero bound problem arises when the shock ξ_t takes a value low enough such that

$$\frac{\xi_t}{\beta E_t \left[\xi_{t+1}\right]} < 1$$

since this call for negative nominal interest rates.

Note that as long as the preference shock is such that ratio is always larger than one, monetary policy - movements in R_t - can fully isolate the economy from this shock. However, for values of the shock where that ratio becomes lower than one, monetary policy runs out of bullets.

It is clear from the previous equations that changes in G will affect the equilibrium allocation. But is also the case that those changes cannot achieve the first best. On the other hand, if we allow for flexible taxes, the first best can always be achieved.

In this case, as long as

$$\frac{(1+\tau_t^c)}{(1-\tau_t^n)}\frac{\theta}{(\theta-1)}=1, \text{ for all } t$$

the first two equilibrium conditions imply that the allocation is the first best. The third condition, evaluated at the first best is given by

$$\frac{\xi_t}{(1+\tau_t^c)} = R_t \beta E_t \left[\frac{\xi_{t+1}}{(1+\tau_{t+1}^c)}\right].$$

Thus, as long as

$$\frac{\xi_t}{\beta E_t \left[\xi_{t+1}\right]} \geq 1$$

the first best only requires a flexible monetary policy, while taxes can be constant. However, the optimal allocation can also be implemented with flexible taxes.

In particular, consider the first best in which prices are constant and $R\beta = 1$, so in a steady state without shocks[7] the nominal interest rate is equal to the real and prices are constant. Then, consumption taxes must evolve according to

$$\frac{\xi_t}{(1+\tau_t^c)} = E_t\left[\frac{\xi_{t+1}}{\left(1+\tau_{t+1}^c\right)}\right].$$

Let us consider a particular deterministic example. This example is the deterministic version of the ones performed by Egertsson (2009) and Christiano, Eichenbaum and Rebelo (2009). In their models, it is this shock - interacting with the zero bound - that generates a potentially big recession.

Assume that ξ_t evolves exogenously according to

$$\begin{aligned}\frac{\xi_t}{\xi_{t+1}} &= \frac{1}{\eta} < 1 \text{ for } t = 0, 1, 2, .., T-1, \\ \frac{\xi_t}{\xi_{t+1}} &= 1 \quad \text{for } t = T, T+1, T+2, ..\end{aligned}$$

and assume that the interest rate is at the zero bound.

Then, the solution for consumption taxes is given by

$$\frac{\left(1+\tau_{t+1}^c\right)}{(1+\tau_t^c)} = \eta > 1, \text{ for } t = 0, 1, 2, .., T-1$$

Thus, a policy that is consistent with the first best is to reduce current consumption taxes and increase future taxes. Note that the growth rate of taxes must be positive as long as the growth rate of the shock is also positive. The interpretation in Egertsson and CER is that T is the duration of the crisis.

This policy resembles the proposal by Hall and Woodward at the end of 2008. However, to implement the first best, it is important to note that the condition

$$\frac{(1+\tau_t^c)}{(1-\tau_t^n)}\frac{\theta}{(\theta-1)} = 1, \text{ for all } t$$

[7] Most of the literature solves the log-linearized version of this model and considers deviations form this same steay sate. We consider an equilibrium where the nominal interest rate satisfies the same condition for comparison with the literature. It should be clear from the previous discussion that flexible taxes are consistent with the first best for *any* value of the nominal inrterest rate - although the particular values of the taxes will, of course, depend on which is the nominal interest rate in equilibrium.

must also hold, which means that

$$\frac{\left(1+\tau_{t+1}^{c}\right)}{\left(1+\tau_{t}^{c}\right)}=\frac{\left(1-\tau_{t+1}^{n}\right)}{\left(1-\tau_{t}^{n}\right)}=\eta$$

Thus, as consumption taxes go up, labor income taxes must go down, so as not to distort the consumption labor decisions.

5 The Model with Capital

The model can easily be extended to allow for capital accumulation. However, to achieve the first best, the tax policy must be enriched to include a tax on income from capital. To do so, assume that investment, I_t, is also an aggregate of the individual varieties

$$I_t=\left[\int_0^1 i_{it}^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}. \tag{23}$$

Aggregate investment increases the capital stock according to

$$K_{t+1}=(1-\delta)K_t+I_t. \tag{24}$$

Minimization of expenditure on the individual investment goods implies

$$\frac{i_{it}}{I_t}=\left(\frac{p_{it}}{P_t}\right)^{-\theta}, \tag{25}$$

The budget constraints of the households now reads

$$\begin{aligned} &\frac{1}{R_t}\overline{B}_{t+1}^h+\sum_{s^{t+1}/s^t}Q_{t,t+1}B_{t,t+1}+P_tK_{t+1} \qquad (26)\\ =\;&\overline{B}_t^h+B_{t-1,t}^h+U_tK_t+(1-\delta)P_tK_t-\\ &\tau_t^k\left(U_tK_t-\delta P_tK_t\right)+\left(1-\tau_t^n\right)W_tN_t+\Pi_t-\left(1+\tau_t^c\right)P_tC_t-T_t\end{aligned}$$

U_t is the rental cost of capital. Note that the tax τ_t^k has an allowance for depreciation. We believe this is the most natural assumption. As we will show, it will have implications on the behavior of this tax rate when implementing the optimal allocation.

The marginal condition with respect to capital is

$$P_t=\sum_{s^{t+1}/s^t}Q_{t,t+1}\left[P_{t+1}+\left(1-\tau_{t+1}^k\right)\left(U_{t+1}-\delta P_{t+1}\right)\right],\ t\geq 0 \tag{27}$$

Firms The production function of each good i, y_{it}, uses labor, n_{it}, and capital and is given by

$$y_{it} = A_t F\left(k_{it}, n_{it}\right),$$

where A_t is an aggregate productivity shock and the production function is constant returns to scale.

The firms choices must satisfy

$$\frac{U_t}{W_t} = \frac{F_k\left(\frac{k_{it}}{n_{it}}\right)}{F_n\left(\frac{k_{it}}{n_{it}}\right)}$$

Let the corresponding cost function be $C_t = C\left(y_{it}; U_t, W_t\right)$. This is linear in y_{it}, so that marginal cost is a function of the aggregates only.

The optimal price set by these firms is

$$p_t = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} C_y\left(U_{t+j}, W_{t+j}\right), \tag{28}$$

where $C_y(.)$ is marginal cost.

Equilibria Market clearing for each variety implies that

$$c_{it} + g_{it} + i_{it} = A_t F\left(n_{it}, k_{it}\right) \tag{29}$$

while market clearing for capital implies

$$K_t = \int k_{it} di. \tag{30}$$

Using the demand functions (5), (6), it follows that[8]

$$C_t + G_t + I_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t}\right)^{-\theta} di\right]^{-1} A_t F\left(K_t, N_t\right). \tag{31}$$

An equilibrium for $\{C_t, N_t, K_t\}$, $\{p_t, P_t, W_t, U_t\}$, and $\left\{R_t, \tau_t^c, \tau_t^n, \tau_t^k\right\}$ is characterized by $(13), (14), (16)$ and

$$\frac{U_t}{W_t} = \frac{F_k\left(\frac{K_t}{N_t}\right)}{F_n\left(\frac{K_t}{N_t}\right)} \tag{32}$$

[8] Since the production function is constant returns to scale, $F\left(k_{it}, n_{it}\right) = F_k\left(\frac{k_{it}}{n_{it}}\right) k_{it} + F_k\left(\frac{k_{it}}{n_{it}}\right) n_{it}$ and $\frac{k_{it}}{n_{it}}$ is the same across firms, $\frac{k_{it}}{n_{it}} = \frac{K_t}{N_t}$.

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} C_y \left(U_{t+j}, W_{t+j}\right), \tag{33}$$

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = \sum_{s^{t+1}/s^t} \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} \left[1 + \left(1 - \tau_{t+1}^k\right)\left(\frac{U_{t+1}}{P_{t+1}} - \delta\right)\right], \tag{34}$$

$$C_t + G_t + K_{t+1} - (1-\delta) K_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t}\right)^{-\theta}\right]^{-1} A_t F\left(K_t, N_t\right). \tag{35}$$

As before, we do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

Efficient allocations As before, the efficient allocation will have the marginal rate of technical substitution between any two varieties equal to one, so

$$c_{it} = C_t\,;\; g_{it} = G_t\,;\; i_{it} = I_t.$$

The efficiency conditions for the aggregates are:

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{1}{A_t F_n\left(K_t, N_t\right)}, \tag{36}$$

$$u_C\left(C_t, N_t, \xi_t\right) = \sum_{s^{t+1}/s^t} \beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\left[A_{t+1} F_k\left(K_{t+1}, N_{t+1}\right) + 1 - \delta\right] \tag{37}$$

and

$$C_t + G_t + K_{t+1} - (1-\delta) K_t = A_t F\left(K_t, N_t\right). \tag{38}$$

Policy variables and prices with variable interest rates We fist set $\tau_t^c = 0$. so to achieve production efficiency, the price level must be constant across time and states. The aggregate resource constraint (35) becomes (38). When $P_t = P$, (33) becomes

$$P = \frac{\theta}{(\theta-1)} C_y\left(U_t, W_t\right). \tag{39}$$

so that nominal marginal cost must be constant. Since $C_y\left(U_t, W_t\right) = \frac{U_t}{A_t F_k} = \frac{W_t}{A_t F_n}$, from (13), it must be that

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{\left(1+\tau_t^c\right)\frac{\theta}{(\theta-1)}}{\left(1-\tau_t^n\right) A_t F_n\left(K_t, N_t\right)}, \tag{40}$$

implying that $\frac{1+\tau_t^c}{1-\tau_t^n} = \frac{\theta-1}{\theta}$ so the labor income tax will have to be $1-\tau_t^n = \frac{\theta}{\theta-1}$. The nominal wage will be such that (13) is satisfied.

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{P}{\left(1-\tau_t^n\right) W_t},$$

and the nominal interest rate must move with the real rate to satisfy

$$u_C\left(C_t, N_t, \xi_t\right) = R_t E_t\left[\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right].$$

The rental cost of capital satisfies (32). Finally, the tax rate on capital income must be chosen to satisfy the marginal condition for capital (34).

$$u_C\left(C_t, N_t, \xi_t\right) = \sum_{s^{t+1}/s^t} \beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\left[1+\left(1-\tau_{t+1}^k\right)\left(\frac{\theta-1}{\theta} A_t F_k\left(K_t, N_t\right)-\delta\right)\right].$$

The introduction of capital changes the nature of the tax policy that is consistent with optimal monetary policy, since the optimal capital income tax depends on the allocation, so it must be state dependent. This contrast with most optimal stabilization policy analysis in New Keynesian models, since they assume that monetary policy is "flexible" - time and state dependent - but only consider "inflexible" fiscal policy - one that is time and state invariant.

It is interesting to note, though, that this is the case only because we allowed firms to deduct depreciation from the capital income tax, i.e., the tax is paid on $(U_t - \delta P_t)K_t$ If, on the contrary, we assume that the tax is paid on the gross return $U_t K_t$, the marginal condition for capital would become

$$u_C\left(C_t, N_t, \xi_t\right) = \sum_{s^{t+1}/s^t} \beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\left[1-\delta+\left(1-\tau_{t+1}^k\right)\frac{\theta-1}{\theta} A_t F_k\left(K_t, N_t\right)\right]$$

In this case, setting $\left(1-\tau_{t+1}^k\right)\frac{\theta-1}{\theta} = 1$ will be consistent with the optimal allocation.

Policy variables and prices with constant interest rates We now set $R_t = \widehat{R}$. The price level must be constant, as in the model without capital, so as to achieve production efficiency. The intertemporal condition is as before

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = \widehat{R} E_t\left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\right]$$

so, given future taxes, this equation solves for the current consumption tax rate τ_t^c.

The labor income tax will have to move accordingly to compensate for the movements in the consumption tax, satisfying condition (40) above.

Now the capital income will also have to move to compensate for the changes in the consumption tax.

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = \sum_{s^{t+1}/s^t} \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} \left[1+\left(1-\tau_{t+1}^k\right)\left(\frac{\theta-1}{\theta} A_t F_k\left(K_t, N_t\right)-\delta\right)\right]$$

In this case, the capital income tax must move to compensate for changes in the optimal allocation and changes in the consumption tax rates. Note that, contrary to the case with a flexible interest rate and no consumption taxes, in this case a flexible capital income tax rate is necessary even if the tax base is the gross capital income.

6 The linearized model

If we assume away productivity shocks ($A_t = 1$), assume constant government consumption, $G_t = G$, and assume that $u_C\left(C_t, N_t, \xi_t\right) = u_C\left(C_t, N_t\right)\xi_t$, so that preference shocks do not affect the consumption-leisure margin, the following equations provide a log linear approximation[9] to the model above

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t) + \sigma(E_t \widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c), \tag{41}$$

$$\pi_t = \kappa \widehat{y}_t + \kappa\psi\left(\widehat{\tau}_t^n + \widehat{\tau}_t^c\right) + \beta E_t \pi_{t+1}, \tag{42}$$

where $\pi_t = \ln \frac{P_t}{P_{t-1}}$, $i_t = \ln R_t$, $\widehat{y}_t = \ln \frac{Y_t}{Y}$, $\widehat{\tau}_t^c = \ln \frac{(1+\tau_t^c)}{(1+\tau^c)}$, $\widehat{\tau}_t^n = \ln \frac{(1-\tau_t^n)}{(1-\tau^n)}$, and $r_t = \ln \beta^{-1} + \ln \xi_t - E_t \ln \xi_{t+1}$. Note that i_t and r_t are in levels, while the other variables are in deviations to the steady state. That is only for the convenience of defining the lower bound.

We also assume that monetary policy follows an interest rate rule that explicitly takes into account the lower bound on nominal interest rates

$$i_t = \max\{0, r_t + \phi_\pi \pi_t + \phi_y \widehat{y}_t\}. \tag{43}$$

In this linear version of the model, if the parameters of the interest rate rule satisfy the Taylor principle, then given the tax policy, the interest rule implements the unique local linear solution to the system.

[9] See the Appendix for the derivation of the linear approximation. The linear equations are similar to Eggertsson (2009).

7 Avoiding a recession

7.1 Using monetary policy to avoid a recession

Consider the case where fiscal policy is not used, $\widehat{\tau}_t^c = 0$ and $\widehat{\tau}_t^n = 0$. As long as the lower bound does not bind, movements in the nominal interest rate can fully offset the preference shock ξ_t. Indeed, the interest rate rule is defined so as to fully insulate output and inflation from this shock, so that in equilibrium it may be that $\widehat{y}_t = 0$, and $\pi_t = 0$. The intuition is simple: shocks to the real interest rate should be absorbed one to one by changes in the nominal interest rate. In this way, the shock does not affect prices and therefore there is no change in output. This would be monetary policy in normal times.

Note, on the other hand, that if the interest rate is zero and there is a negative shock to the real interest rate ($r_t < 0$), this could result in deflation and, given the price frictions, output would drop. This is why the zero bound on interest rates can be a cost to policy.

7.2 Using fiscal policy

Fiscal policy can also be used to respond to the shock, and fully stabilize the economy. Suppose the outcome of the interest rate rule is that the nominal interest rate is zero, $i_t = 0$. From (41), it is clear that there will be a conditional growth rate of the consumption tax,

$$E_t \widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c = r,$$

that will satisfy the first equation for $\widehat{y}_t = E_t \widehat{y}_{t+1} = 0$ and $E_t \pi_{t+1} = 0$. From (42), there is an adjustment on the labor income tax,

$$\widehat{\tau}_t^n = -\widehat{\tau}_t^c,$$

that will satisfy the second equation for $\widehat{y}_t = 0$ and $\pi_t = E_t \pi_{t+1} = 0$. The interest rate rule, (43) , is satisfied.

How can fiscal policy be implemented? Suppose the objective is to have the real rate be -4%. Then the future consumption should be four percentage points higher than the tax rate today. This can be accomplished with a cut in the present consumption tax rate of two percentage points, and with an increase in the future tax of two percentage points, or many other possible combinations. If the shock today is persistent, then there would be a need to keep on raising the consumption tax, for as many periods as the real rate were to stay below zero.

8 Conclusions

To be written.

References

[1] Blanchard, O., G. Dell'Ariccia and P. Mauro. "Rethinking Macroecocomic Policy", IMF Staff Position Note, 2010.

[2] Calvo, G.. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12: 383-398.

[3] Isabel Correia, Juan Pablo Nicolini, Pedro Teles, 2002. "Optimal fiscal and monetary policy: equivalence results," Working Paper Series WP-02-16, Federal Reserve Bank of Chicago

[4] Correia, I., J. P. Nicolini and P. Teles. 2008. "Optimal Fiscal and Monetary Policy: Equivalence Results." *Journal of Political Economy* 168 (1): 141-170.

[5] Christiano, L., M. Eichenbaum, and S. Rebelo, 2009,

[6] Eggertsson, G. B., 2009, What Fiscal Policy is Effective at Zero Interest Rates?, Federal Reserve Bank of New York.

[7] Eggertsson, G. B., and M. Woodford, 2003, The Zero Bound on Interest Rates and Optimal Monetary Policy, Brookings Papers on Economic Activity 1, 212-219.

[8] Eggertsson, G. B., and M. Woodford, 2004a, Policy Options in a Liquidity Trap, American Economic Review 94, 2: 76-79.

[9] Eggertsson, G. B., and M. Woodford, 2004b, Optimal Monetary and Fiscal Policy in a Liquidity Trap, ISOM conference volume.

[10] Feldstein, M, 2003, Rethinking Stabilization, Kansas City Fed.

[11] Mertens, Karel & Ravn, Morten O., 2010. "Fiscal Policy in an Expectations Driven Liquidity Trap," CEPR Discussion Papers 7931.

[12] Poterba, J. M., J. J. Rotemberg, and L. H. Summers, 1986, A Tax-Based Test for Nominal Rigidities, American Economic Review, 76, 4, 659-675.

[13] Simple analytics of the government spending multiplier. WP, Columbia University, 2010.

[14] ADD KRUGMAN

9 Appendix: The log-linearized version of the model

As productivity shocks play no particular role, we assume that $A_t = 1$ for all t, so (**??**) becomes

$$\frac{p_t}{P_t} = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{P_t}$$

The steady state has

$$\begin{aligned} C_t &= C,\, N_t = N,\, \xi_t = 1, \tau_t^c = \tau^c, \tau_t^n = \tau^n \\ P_t &= p_t = P, R = \beta^{-1} \end{aligned}$$

so that

$$\eta_{t,j} = (1-\alpha\beta)(\alpha\beta)^j \text{, and } \frac{\theta}{(\theta-1)} W = P.$$

We, first, log-linearize equation (**??**), using (**??**) to replace labor. We first write the equation as

$$\frac{u_C(C_t, N_t, \xi_t)}{(1+\tau_t^c)} = E_t \left[R_t \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} \frac{P_t}{P_{t+1}} \right]$$

The log-linearization is given by

$$\lambda \widehat{C}_t + \Gamma \widehat{\xi}_t - \widehat{\tau}_t^c \simeq i_t - \ln \beta^{-1} - E_t \pi_{t+1} + \lambda E_t \widehat{C}_{t+1} + \Gamma E_t \widehat{\xi}_{t+1} - E_t \widehat{\tau}_{t+1}^c \quad (44)$$

where

$$\begin{aligned}
\lambda &= \frac{C}{uc}(u_{CC}+u_{CN}) \\
\Gamma &= \frac{\xi}{u_C}u_{C\xi} = \frac{u_{C\xi}}{u_C} = 1 \text{ if } \ \xi_t \text{ is multiplicative} \\
\widehat{C}_t &= \ln\frac{C_t}{C} \\
\widehat{\xi}_t &= \ln\xi_t \\
\widehat{\tau}_t^c &= \ln\frac{(1+\tau_t^c)}{(1+\tau^c)} \\
\pi_{t+1} &= \ln\frac{P_{t+1}}{P_t} \\
i_t &= \ln R_t
\end{aligned}$$

Linearization of the aggregate resource constraint yields

$$C_t + G_t = \left[\sum_{j=0}^{t+1}\varpi_j\left(\frac{p_{t-j}}{P_t}\right)^{-\theta}\right]^{-1} A_t N_t$$

assuming that government consumption is constant, delivers

$$\frac{C}{C+G}\widehat{C}_t = \widehat{y}_t$$

So, if we let $g^{-1} = \frac{C}{C+G}$, then

$$\widehat{C}_t = g\widehat{y}_t,$$

If we also assume that the shock ξ_t is multiplicative, so $\Gamma = 1$, we can write equation (44) as

$$\lambda g\widehat{y}_t + \widehat{\xi}_t - \widehat{\tau}_t^c \simeq i_t - \ln\beta^{-1} - E_t\pi_{t+1} + \lambda g E_t\widehat{y}_{t+1} + E_t\widehat{\xi}_{t+1} - E_t\widehat{\tau}_{t+1}^c$$

or, letting $\sigma = 1/\lambda g$,

$$\widehat{y}_t \simeq E_t\widehat{y}_{t+1} + \sigma\left[i_t - E_t\pi_{t+1} - \left(\ln\beta^{-1} + \widehat{\xi}_t - E_t\widehat{\xi}_{t+1}\right)\right] - \sigma\left(E_t\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c\right)$$

On the other hand, linearization of (**??**), delivers

$$\ln p_t \simeq \ln\frac{\theta}{(\theta-1)} + \ln E_t\sum_{j=0}^{\infty}\eta_{t,j}W_{t+j}$$

But

$$W_t = \frac{(1+\tau_t^c)\,P_t}{(1-\tau_t^n)}\left[-\frac{u_C\left(C_t,N_t,\xi_t\right)}{u_N\left(C_t,N_t,\xi_t\right)}\right]^{-1}$$

so

$$\ln p_t \simeq \ln\frac{\theta}{(\theta-1)} + \ln E_t \sum_{j=0}^{\infty}\eta_{t,j}\frac{\left(1+\tau_{t+j}^c\right)P_{t+j}}{\left(1-\tau_{t+j}^n\right)}\left[-\frac{u_C\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}{u_N\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}\right]^{-1}$$

or

$$\ln p_t - \ln P_t \simeq \ln\frac{\theta}{(\theta-1)} + \ln E_t \sum_{j=0}^{\infty}\eta_{t,j}\frac{\left(1+\tau_{t+j}^c\right)\frac{P_{t+j}}{P_t}}{\left(1-\tau_{t+j}^n\right)}\left[-\frac{u_C\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}{u_N\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}\right]^{-1}$$

The log-linearization of the second term in the right hand side is given by

$$\ln E_t \sum_{j=0}^{\infty}\eta_{t,j}\frac{\left(1+\tau_{t+j}^c\right)\frac{P_{t+j}}{P_t}}{\left(1-\tau_{t+j}^n\right)}\left[-\frac{u_C\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}{u_N\left(C_{t+j},N_{t+j},\xi_{t+j}\right)}\right]^{-1} \simeq (1-\alpha\beta)E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\Omega_{t+j}\right]$$

where

$$\Omega_{t+j} = \widehat{\tau}_{t+j}^c + \widehat{\tau}_{t+j}^n + \pi_t(j) + \phi\widehat{C}_{t+j} - \gamma\widehat{\xi}_{t+j}$$

where

$$\begin{aligned}
\pi_t(j) &= \ln\frac{P_{t+j}}{P_t} \\
\widehat{\tau}_t^n &= \ln\frac{(1-\tau_t^n)}{(1-\tau^n)}
\end{aligned}$$

and

$$\begin{aligned}
\phi &= (-1)\frac{C}{U_C\left(-U_N\right)}\left[\left(U_{CC}+U_{NC}\right)\left(-U_N\right) - U_C\left(U_{NC}+U_{NN}\right)\right] \\
\gamma &= \frac{-1}{U_N^2}\left[U_{C\xi}U_N - U_C U_{N\xi}\right]
\end{aligned}$$

Note that if, as we will assume, $u\left(C_{t+j},N_{t+j},\xi_{t+j}\right) = u\left(C_{t+j},N_{t+j}\right)\xi_{t+j}$, then $\gamma = 0$. Note also that $\phi > 0$.

Thus, we can write

$$
\begin{aligned}
\widehat{p}_t &\simeq (1-\alpha\beta)E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j} + \widehat{\tau}^n_{t+j} + \pi_t(j) + \phi\widehat{C}_{t+j} - \gamma\widehat{\xi}_{t+j}\right] \\
&\simeq (1-\alpha\beta) \left[\begin{array}{c} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] \\ + (\alpha\beta)\, E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j+1} + \widehat{\tau}^n_{t+j+1} + \pi_t(j) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array} \right]
\end{aligned}
$$

where $\widehat{p}_t = \ln \frac{p_t}{P}$. But note that

$$
\pi_t(j) = \ln \frac{P_{t+j}}{P_t} = \ln \frac{P_{t+1}}{P_t}\frac{P_{t+j}}{P_{t+1}} = \ln \frac{P_{t+1}}{P_t} + \ln \frac{P_{t+j}}{P_{t+1}} = \pi_{t+1} + \pi_{t+1}(j-1)
$$

so we can write the equation as

$$
\begin{aligned}
\widehat{p}_t &\simeq (1-\alpha\beta) \left[\begin{array}{c} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t - \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \\ (\alpha\beta)\, E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j+1} + \widehat{\tau}^n_{t+j+1} + \pi_{t+1} + \pi_{t+1}(j-1) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array} \right] \\
&= (1-\alpha\beta) \left[\begin{array}{c} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\pi_{t+1}\right] + \\ (\alpha\beta)\, E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j+1} + \widehat{\tau}^n_{t+j+1} + \pi_{t+1}(j-1) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array} \right] \\
&= (1-\alpha\beta) \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t \left[\pi_{t+1}\right] + (\alpha\beta)\, E_t\widehat{p}_{t+1}
\end{aligned}
$$

But the log linearization of (9) delivers

$$
\ln P_t \simeq \alpha \ln P_{t-1} + (1-\alpha) \ln p^*_t
$$

so

$$
\ln P_t - \alpha \ln P_t \simeq \alpha \ln P_{t-1} + (1-\alpha) \ln p^*_t - \alpha \ln P_t
$$

or

$$
\widehat{p}_t \simeq \frac{\alpha}{1-\alpha}\pi_t
$$

Replacing above

$$
\frac{\alpha}{1-\alpha}\pi_t \simeq (1-\alpha\beta) \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t \left[\pi_{t+1}\right] + (\alpha\beta)\, E_t \frac{\alpha}{1-\alpha}\pi_{t+1}
$$

or

$$
\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \frac{1-\alpha}{\alpha} (\alpha\beta)\, E_t \left[\pi_{t+1}\right] + (\alpha\beta)\, E_t\pi_{t+1}
$$

so

$$
\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \beta E_t\pi_{t+1}
$$

Finally, recall that

$$\widehat{C}_t = g\widehat{y}_t,$$

so

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi g\widehat{y}_t - \gamma\widehat{\xi}_t\right] + \beta E_t\pi_{t+1}$$

Letting

$$\begin{aligned}\kappa &= (1-\alpha\beta)\frac{1-\alpha}{\alpha}\phi g \\ \psi &= (\phi g)^{-1}\end{aligned}$$

we obtain

$$\pi_t \simeq \kappa\psi\left(\widehat{\tau}_t^c + \widehat{\tau}_t^n\right) + \kappa\widehat{y}_t - \kappa\psi\gamma\widehat{\xi}_t + \beta E_t\pi_{t+1}$$

We assume that the shock ξ_t is multiplicative, so $\gamma = 0$. If we let $r_t = \left(\ln\beta^{-1} + \widehat{\xi}_t - E_t\widehat{\xi}_{t+1}\right)$, the system can be written as

$$\pi_t \simeq \kappa\widehat{y}_t + \kappa\psi\left(\widehat{\tau}_t^n + \widehat{\tau}_t^c\right) + \beta E_t\pi_{t+1}$$

$$\widehat{y}_t \simeq E_t\widehat{y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t) + \sigma(E_t\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c)$$

$$i_t \geq 0$$