MANAGING INTEREST RATE RISK: THE NEXT CHALLENGE?

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Abstract

Are the managers of financial institutions ready for the small but increasingly significant risk of inflation in the near future, due to the unprecedented fiscal and monetary responses of the U.S. government to prevent an economic collapse? This paper addresses this important issue by reviewing important findings in the area of interest rate risk management. We discuss five classes of models in the fixed income literature that deal with hedging the risk of large, non-parallel yield curve shifts. These models are given as M-Absolute/M-Square models, duration vector models, key rate duration models, principal component duration models, and extensions of these models for fixed income derivatives, for valuing and hedging bonds, loans, demand deposits, and other fixed income instruments. These models can be used for designing various hedging strategies such as portfolio immunization, bond index replication, duration gap management, and contingent immunization, to protect against changes in the height, slope, and curvature of the yield curve. We argue that the current regulatory models proposed by the U.S. Federal Reserve, the Office of Thrift Supervision, and the Bank of International Settlements, may understate the true interest rate risk exposure of financial institutions, if sharp increases in interest rates lead to higher default risk.

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This draft: April 2009

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High volatility in oil and commodity prices and the unfolding credit crisis in United States and Europe have made financial institutions all around the world deeply concerned about the effects of movements in interest rates on their profitability and capital solvency ratios. Increases in interest rates can significantly erode the equity values of highly leveraged institutions such as universal banks and hedge funds. Many of these institutions hold a large percentage of their assets in mortgage loans and mortgage-backed securities, which are likely to experience a significant increase in the average maturity or *duration* as the values of the prepayment options plummet due to rising interest rates. Though sharp reductions in the discount rate and significant purchases of Treasury securities by the U.S. Federal Reserve has buffered the losses in the financial sector, significant risks exist for additional losses in this sector due to future increase in the interest rates. Such increases may also lead to additional costs tied to provisions for losses in other sectors, as the creditworthiness of corporate customers may deteriorate due to higher borrowing costs. These additional losses coming on top of the losses already incurred by financial institutions have the potential to further destabilize the global financial system. On the positive side, to the extent that many banks now have a bigger percentage of earnings tied to non-interest income, they will be somewhat immune to future increases in the interest rates.

Though much talk about explicit “socialization” of the financial sector has centered around nationalization of big banks, significant movement of private sector funds from the risky stock and bond markets into the U.S. Treasury securities, together with a significant increase in participation of the U.S. government in the financial sector implies an implicit socialization of the financial sector (through more than a trillion dollars in purchases of toxic securities and hundreds of billions in direct equity purchases, by the U.S. Treasury, and trillions of dollars in swaps by the U.S. Federal Reserve). As investors flee from risk and leverage, government is forced to own more risk and assume more leverage. Such risk shifting is based on a hidden assumption: that risk and leverage can be shifted to the government, if the government happens to be that of the most powerful nation, economically and militarily. Will risk shifting of such enormous magnitude from private sector to public sector lead to unforeseen consequences? Though it would be considered highly speculative to consider a default by the U.S. government, the cost of insuring the U.S. Treasury debt as revealed by the credit default swaps (CDS) written on U.S. Treasuries rose seven-fold in a short span of one year from March 2008 to March 2009 to approximately $100,000 to ensure $10 million of Treasury debt, before falling to about $40,000 in April 2009.

The unusually large fiscal and monetary responses by the U.S. government come with a small but significant risk of inflation. Will there be sufficient political will to cut fiscal spending when the economy begins to recover? Will sharp reversals in the currently high levels of money supply – expected in response to an economic recovery - lead to large increases in the short term interest rates, and similar decreases in the slope and the curvature of the yield curve? Though higher inflationary risk exists with respect to an economic rebound, the risk of stagflation is not irrelevant under the opposite scenario of an economic collapse. Will the Chinese government capitulate and liquidate a significant share of its U.S. Treasury holdings in order to diversify and/or stimulate China's domestic economy, leading to significant increases in the interest rates in the U.S.? Could the U.S. dollar collapse and the U.S. Treasury rates rise due to the small chance of default reflected in the CDS premiums on U.S. Treasury debt, and the small chance of International Monetary Fund's SDRs to become the new global reserve currency?

Though somewhat speculative, such questions underscore the nature of the basic imbalance in the U.S. economy which is that the savings rate in the United States plummeted in the past decade, as huge increases in the domestic consumption were financed by the credit bubble. With the collapse of the credit bubble, it is likely that the savings and consumption will be brought back into balance by increases in the U.S. Treasury rates, regardless of whether this happens with an economic rebound or with an economic collapse. Hence, regardless of which direction the U.S. economy takes, it is likely that U.S. Treasury rates will rise in the near to intermediate term from their record low levels.

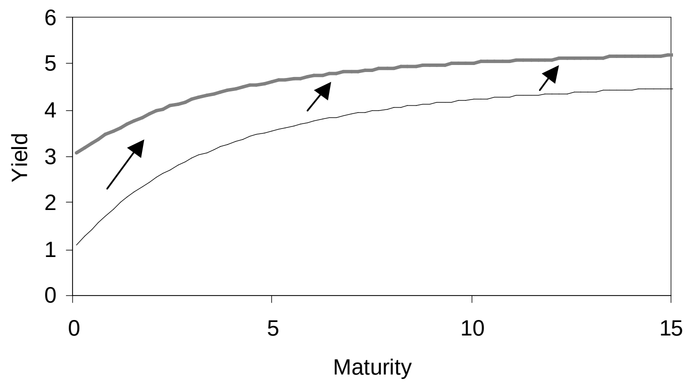

Figure 1. Non-parallel yield curve shift

Near term increases in the interest rates are likely to be non-parallel rising more at the shorter end, as the shapes of the LIBOR and U.S. Treasury yield curves flatten out. It is well known that the traditional duration and convexity risk measures (see Lacey and Nawalkha [1993] and Nawalkha and Latif [2004]) are valid only when the whole yield curve moves in a parallel fashion. If short rates increase more than the long rates, then the slope of the yield curve will experience a negative shift, while the curvature will most likely experience a positive shift (from a high negative curvature to a low negative curvature) as shown in Figure 1. Though such a combination of height, slope, and curvature shifts is more likely based on casual empiricism, other scenarios may lead to other types of shifts in the yield curve.

How do the managers of financial institutions hedge their portfolios composed of fixed income securities and their derivatives, against the effects of non-parallel yield curve shifts? This article addresses this important issue by reviewing the important findings in the area of interest rate risk management. We discuss five classes of models given in the fixed income literature that deal with this issue. The next four sections discuss four classes of models given as M-Absolute/M-Square models, duration vector/M-vector models, key rate duration model, and principal component duration model that allow interest rate risk hedging for regular bond portfolios. These models are derived as extensions to the traditional duration-convexity model which assumes parallel yield curve shifts (with the unfortunate implication that riskless arbitrage opportunities are allowed under non-infinitesimal yield curve shifts).

As the first extension, Fong and Vasicek [1983, 1984] obtain a lower bound on the change in the future value of a duration-immunized bond portfolio over a given horizon, as a linear function of the portfolio's M-square. By minimizing portfolio's M-square subject to the duration constraint not only immunizes the portfolio against height shifts, but also minimizes its exposure to slope, curvature, and other higher order shifts in the yield curve. Unlike M-square model, that requires two risk measures for hedging (i.e., both duration and M-square), Nawalkha and Chambers [1996] derive the M-absolute model, which only requires one risk measure for hedging against the non-parallel yield curve shifts. Even with one risk measure, Nawalkha and Chambers demonstrate M-absolute model to reduce more than 50% of the immunization risk inherent in the traditional duration model.

Further improvements in the immunization performance are made possible by the next class of models given as the duration vector model of Chambers, Carleton, and McEnally [1988], and the M-vector model of Nawalkha and Chambers [1997] and Nawalkha, Soto, and Zhang [2003]. These models derive separate risk measures of different types of term structure shifts, such as shifts in the height, slope, curvature, etc. of the yield curve. Unlike M-absolute/M-square models which disallow short positions, the duration vector model and the M-vector model allow short positions in the bond portfolio, and obtain significant improvements in the immunization performance.

Ho's [1992] "key rate durations" represents the next class of model for hedging against non-parallel yield curve shifts. The key rate duration model divides the term structure of interest rates in separate segments, and then immunizes the bond portfolio against changes in each segment of the term structure. The immunization performance of the key rate duration model is similar to that of the duration vector model and the M-vector model. However, unlike the duration vector model and the M-vector model, which require at most three to five risk measures, the number of duration measures to be used and the corresponding division of the term structure into different key rates, remain quite arbitrary under the key rate model. For example, Ho [1992] proposes as many as *eleven* key rate durations to effectively hedge against interest rate risk.

To reduce the dimensionality of hedging problem, a number of researchers have proposed the next class of models known as the principal component duration models. These models capture the changes in the entire yield curve by applying a statistical technique called *principal component analysis* (PCA) to the past interest rate changes. The use of PCA in the Treasury bond markets has revealed that three principal components (related to the height, the slope, and the curvature of the yield curve) are sufficient in explaining almost all of the variation in interest rate changes. However, unlike the earlier models, the major shortcoming of the principal component model is that it assumes a *stationary* covariance structure of interest rate changes.

The four classes of hedging models described above are generalized in the subsequent section which discusses interest rate risk hedging models for default-prone bonds/loans, demand deposits, and fixed income derivatives. As an important finding, this section argues that the current regulatory models proposed by the U.S. Federal Reserve, the Office of Thrift Supervision, and the Bank of International Settlements, may understate the true interest rate risk exposure of financial institutions, if sharp increases in interest rates can lead to higher default risk.

The variety of interest rate risk models reviewed in this paper can be used for designing various interest rate risk hedging strategies such as portfolio immunization, bond index replication, duration gap management, and contingent immunization, to protect against changes in the height, slope, and curvature of the yield curve. For a detailed discussion of these models, we refer the reader to Nawalkha, Soto, and Beliaeva [2005]. The final section summarizes and concludes the paper.

M-ABSOLUTE AND M-SQUARE MODELS

Consider a bond with cash flows C_t, payable at time t. The bond sells for a price P, and is priced using a term structure of continuously compounded zero-coupon yields given by $y(t)$. The traditional duration model can be used to approximate percentage change in the bond price as follows[1]:

$$\frac{\Delta P}{P} \cong -D\Delta y \tag{1}$$

where $D = \text{Duration} = \sum_{t=t_1}^{t=t_N} t\ w_t$, and $w_t = \left[\frac{C_t}{e^{y(t)\cdot t}}\right] / P$.

Duration is given as the weighted average time to maturity of the cash flows, where the weights are defined as the present values of the cash flows divided by the bond price. The duration model given in equation (1) assumes that the yield curve experiences infinitesimal and parallel shifts. Hence, the change in the yield Δy, is assumed to be *equal* for all bonds regardless of their coupons and maturities. However, we know that shorter maturity rates are more volatile than the longer maturity rates, so the assumption of parallel yield curve shifts is obviously false. Due to the violation of this

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[1] Under discrete compounding, equation (1) becomes $\frac{\Delta P}{P} \cong -D_M \Delta y$, where D_M is the modified duration of the bond defined as $D_M = \frac{1}{1+y}\sum_{t=t_1}^{t=t_N} t\ w_t$, with $w_t = \left[\frac{C_t}{(1+y)^t}\right] / P$.

assumption, the duration model given above explains only about 70% of the ex-post return differentials on riskless bonds.

As shown in Nawalkha and Latif [2004], the approximation given in equation (1) can be extended to include non-linearities due to convexity. Though at first high convexity seems like a win-win proposition, since regardless of the direction of the parallel yield curve shifts higher convexity portfolios outperform lower convexity portfolios, Hegde and Nunn [1980] show the gains due to convexity are extremely trivial. This can be seen by extending the approximation in equation (1) as follows:

$$\frac{\Delta P}{P} \cong -D\Delta y + \frac{1}{2} CON(\Delta y)^2 \qquad (2)$$

For example, even for a *large* 50 basis points change in the yield, the gain due to convexity on a bond with a price of $1000 and a convexity of 50 (for example, a 10-year coupon bond) is only $\frac{1}{2} \times 50 \times (0.0050)^2 \times 1000 = 0.625$ dollars or about 0.06% of the $1,000 value. Moreover, Lacey and Nawalkha [1993] show that high convexity in a bond portfolio can introduce significant risks when the slope of the yield curve experiences a *positive* shift.

Though convexity leads to higher returns for large and parallel shifts in the term structure of interest rates, as pointed out in the introduction, this “convexity view” is somewhat naïve and has been challenged both theoretically and empirically in the fixed income literature. An alternative view of convexity, which is based upon a more realistic economic framework, relates convexity to *slope shifts* in the term structure of interest rates. This view of convexity was proposed by Fong and Vasicek [1983, 1984] and Fong and Fabozzi [1985] through the introduction of the new risk measure, M-square, which is a linear transformation of convexity. The M-square of a bond portfolio is given as the weighted average of the squares of the distance between cash flow maturities and the planning horizon of the portfolio:

$$M^2 = \sum_{t=t_1}^{t=t_N} (t-H)^2 \cdot w_t \tag{3}$$

where the weights are defined in equation (1) with C_t redefined as portfolio's cash flow at time t, P redefined as the value of the portfolio, and H is the planning horizon.

M-square allows obtaining a limit on the change in the terminal value of a bond portfolio whose duration equals the planning horizon and hence, it is immunized against parallel movements of the term structure of interest rates. This lower bound is given as:

$$\frac{\Delta P_H}{P_H} \geq -\frac{1}{2} \kappa M^2 \tag{4}$$

where κ is a constant.

A bond portfolio selected with minimum M-square has cash flows clustered around the planning horizon date and hence, protects the portfolio from immunization risk resulting from non-parallel yield curve shifts. Though both convexity and M-square measures give similar information about the riskiness of a bond or a bond portfolio (since one is a linear function of the other), the developments of these two risk measures follow different paths. Convexity emphasizes the *gain* in the return on a portfolio, against large and parallel shifts in the term structure of interest rates. On the other hand, M-square emphasizes the *risk exposure* of a portfolio due to slope-shifts in the term structure of interest rates. Hence, the "convexity view" and the "M-square view" have exactly opposite implications for bond risk analysis and portfolio management. Lacey and Nawalkha [1993] empirically investigate the convexity view versus the M-square view and find strong support for the M-square view. They find that high convexity (which is the same as high M-square) adds risk but not return to a bond portfolio using U.S. Treasury bond price data over the period 1976-1987.

Unlike M-square model, that requires two risk measures for hedging (i.e., both duration and M-square), Nawalkha and Chambers [1996] derive the M-absolute model, which only requires one risk measure for hedging against the non-parallel yield curve shifts. The M-absolute of a bond portfolio is given as the weighted average of the absolute distances between cash flow maturities and the planning horizon of the portfolio.

$$M^A = \sum_{t=t_1}^{t=t_N} \left|t - H\right| \cdot w_t \tag{5}$$

where the weights are defined in equation (1), and H is the planning horizon.

The lower bound on the change on the terminal value of the bond portfolio depends linearly on M-absolute as follows:

$$\frac{\Delta P_H}{P_H} \geq -\kappa M^A \tag{6}$$

The essential difference between the duration model and the M-absolute model can be summarized as follows. The duration model completely immunizes against the height shifts but ignores the impact of slope, curvature, and other higher order term structure shifts on the future target value of a bond portfolio. In contrast, the M-absolute model immunizes only partially against the height shifts, but it also reduces the immunization risk caused by the shifts in the slope, curvature, and all other term structure shape parameters by selecting a minimum M-absolute bond portfolio with cash flows clustered around its planning horizon date. The relative desirability of the duration model or the M-absolute model depends on the nature of term structure shifts expected. If height shifts completely dominate the slope, curvature, and other higher order term structure shifts, then the duration model will outperform the M-absolute model. If, however, slope, curvature, and other higher order shifts are relatively significant – in comparison with the height shifts – then the M-absolute model may outperform the traditional duration model. Using McCulloch's term structure data over the observation period 1951 through 1986, Nawalkha and Chambers [1996] find the M-absolute model reduces the immunization risk inherent in the duration model by more than half.

DURATION VECTOR MODELS

Though both M-absolute and M-square risk measures provide significant enhancement in the immunization performance over the traditional duration model, perfect immunization is not possible using either of the two measures except for the trivial case in which the portfolio consists on a zero-coupon bond maturing at the horizon date. Further gains in immunization performance have been made possible by the duration vector model, which using a vector of higher-order duration measures immunizes against changes in the shape parameters (i.e., height, slope, curvature, etc.) of the term structure of interest rates. Various derivations to the duration vector model have been given by Chambers [1981], Granito [1984], Chambers, Carleton, and McEnally [1988], Prisman and Tian [1994], Nawalkha [1995], Nawalkha and Chambers [1997], and Grandville [2001]. According to them, the percentage change in the bond price is approximated as a product of the duration vector $D(m)$ and a shift vector $\Delta Y(m)$, as follows:

$$\frac{\Delta P}{P} \cong -D(m)\Delta Y(m) \tag{7}$$

where the duration vector is defined as,

$$D(m) = \sum_{t=t_1}^{t=t_N} t^m \cdot w_t = H^m, \text{ for } m = 1,2,3,...,Q \tag{8}$$

with the weights defined in equation (1), and H is the planning horizon. The shift vector elements $\Delta Y(1)$, $\Delta Y(2)$,…, $\Delta Y(Q)$ approximate both the linear and non-linear changes in the height, slope, curvature, and other shape parameters of the term structure of interest rates measured as a polynomial function. The duration vector of a portfolio of bonds can be obtained by taking a weighted average of the duration vectors of individual bonds. About three to five duration vector constraints (i.e., Q = 3 to 5) have shown to almost perfectly immunize a bond portfolio against the risk of non-parallel yield curve shifts.

Diebold, Ji, and Li [2006] obtain a three-factor duration model (exponential-based duration vector) which measures the level, slope, and curvature risks as defined from a reformulation of the Nelson and Siegel [1987] model. They find some parallelism between their model with the traditional duration vector model, and with the principal component model of Soto [2004] introduced later in this paper. They confirm the immunization performance of their model to be similar to that of the traditional duration vector model, but superior to that of the Macaulay duration model using CRSP data from 1971 to 2001.

A more general derivation to the duration vector model is given by the M-vector model of Nawalkha and Chambers (1997).[2] Unlike the traditional approach, the M-vector approach does not restrict the term structure shifts to be of a polynomial functional form. This approach is based upon a Taylor series expansion of the bond return function with respect to the cash flow maturities around a

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[2] This model allows the risk measures to be set to zero. In contrast, the M-square model of Fong and Vasicek [1984] and the M-absolute model of Nawalkha and Chambers [1996] are derived to minimize the risk measures subject to portfolio constraints.

given planning horizon. Similar to the duration vector model, the M-vector model leads to duration measures that are linear in t, t^2, t^3, etc. where t is the maturity of the cash flow. Extending the earlier empirical work, Nawalkha and Chambers find near perfect immunization performance using a five element M-vector model, using McCulloch's term structure date over the period 1951 to 1986.

Ventura and Pereira [2006] test the M-vector model using Portugal bond data from August 1993 to September 1999. They obtain results similar to those of Nawalkha and Chambers [1997] using the first three elements of the M-vector. Additional M-vector constraints beyond three did not lead to further improvement in immunization performance, a result also found by Soto [2004]. Also, consistent with Soto [2001, 2004], bond portfolios including a bond maturing near the end of the planning horizon provide the best immunization performance.

Since the shifts in the height, slope, curvature, and other parameters of the term structure of interest rate shifts are generally larger at the shorter end of the maturity spectrum, it is possible that an alternative set of duration measures that are linear in $g(t)$, $g(t)^2$, $g(t)^3$, etc., and which put relatively more weight at the shorter end of the maturity spectrum due to the specific choice of the function $g(t)$, may provide enhanced immunization performance. Consistent with this intuition, Nawalkha, Soto, and Zhang [2003] derive a class of *generalized M-vector models* using a Taylor series expansion of the bond return function with respect to specific functions of the cash flow maturities. This paper finds that $g(t) = t^{0.25}$ or $g(t) = t^{0.5}$ perform significantly better than the traditional M-vector for short planning horizons when at least three risk measures are used. Overall, though the duration vector, the M-vector, and the generalized M-vector models, significantly outperform the M-absolute and M-square models, the improvement in performance comes at the cost of higher portfolio rebalancing costs required by these models.

KEY RATE DURATION MODELS

The key rate duration model of Ho [1992] describes the shifts in the term structure as a discrete vector representing the changes in the *key* spot rates of various maturities. That is:

$$TSIR\ shift = \left(\Delta y(t_1), \Delta y(t_2) \ldots, \Delta y(t_m)\right) \tag{9}$$

where $y(t_i)$ is the zero-coupon rate for term t_i and $y(t_1)$, $y(t_2)$, ..., $y(t_m)$ define the set of m key rates. Interest rate changes at other maturities are derived from these values via linear interpolation. The linear interpolations, together with an initial term structure give the new term structure as shown in Figure 2.

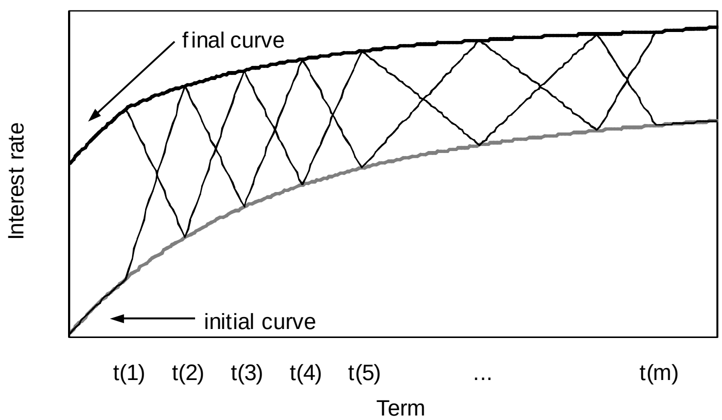

Figure 2. The term structure shift under the key rate model

Key rate durations are then defined as the sensitivities of the percentage change in bond price to key rates at m different points across the term structure. The key rate duration model can be considered an extension of the traditional duration model given in equation (1), as follows:

$$\frac{\Delta P}{P} \cong -\sum_{i=1}^{m} KRD(i) \cdot \Delta y(t_i) \tag{10}$$

where the yield curve is divided into *m* different key rates. The key rate durations of a portfolio of bonds can be obtained by taking weighted averages of the key rate durations of individual bonds.

Similar to the duration vector models, an appealing feature of the key rate model is that it does not require a *stationary* covariance structure of interest rate changes (unless performing a VAR analysis). Hence, it doesn't matter whether the correlations between the changes in interest rates of different maturities increase or decrease or even whether these changes are positively or negatively correlated. Also, the model allows for any number of key rates, and therefore, interest rate risk can be modeled and hedged to a high degree of accuracy.

However, unlike the duration vector models, which require at most three to five duration measures, the number of duration measures to be used and the corresponding division of the term structure into different key rates, remain quite arbitrary under the key rate model. For example, Ho [1992] proposes as many as *eleven* key rate durations to effectively hedge against interest rate risk. Further, unlike the duration vector model, where the higher order duration measures serve as linear as well as non-linear risk measures (for example, *D(2)* simultaneously gives the linear exposure to slope shifts, as well as non-linear exposure to height shifts), the key rate durations give only the linear exposures to the key rates. To measure non-linear exposures to the key rates, key rate convexity measures are required (see Ho, Chen, and Eng [1996]). Hedging against a large number of key rate durations and convexities, implies larger long and short positions in the portfolio, which can make this approach somewhat expensive in terms of the transaction costs associated with portfolio construction and rebalancing.

PRINCIPAL COMPONENT MODELS

The principal component model assumes that the yield curve movements can be summarized by a few composite variables. These new variables are constructed by applying a statistical technique called *principal component analysis* (PCA) to the past interest rate changes. The use of PCA in the Treasury bond markets has revealed that three principal components (related to the height, the slope, and the curvature of the yield curve) are sufficient in explaining almost all of the variation in interest rate changes. An illustration of the impact of these components on the yield curve is shown in Figure 3.

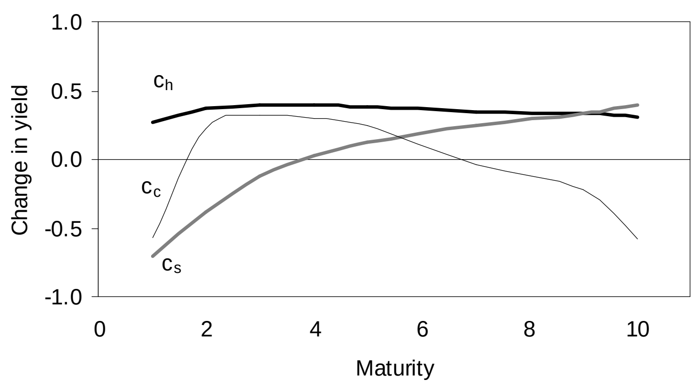

Figure 3. Shape of the Principal Components

The first principal component c_h, basically represents a parallel change in the yield curve, which is why it is usually named the level or the height factor. The second principal component c_s, represents a change in the steepness or the slope, and is named the slope factor. This factor is also called the "twist factor" as it makes the short-term rates and long-term rates move in the opposite directions. The third principal component c_c, is called the curvature factor, as it basically affects the curvature of the yield curve by inducing a butterfly shift. This shift consists of short rates and long rates moving in the same direction and medium-term rates moving in the opposite direction.

The yield changes can be given as weighted linear sums of the principal components as follows:

$$\Delta y(t_i) \approx l_{ih}\Delta c_h + l_{is}\Delta c_s + l_{ic}\Delta c_c \quad i = 1, \ldots, m \tag{11}$$

where Δc_h is the change in the first component, Δc_s is the change in the second component, and Δc_c is the change in the third component.

The variables l_{ih}, l_{is}, and l_{ic}, are the factor sensitivities (or loadings) of the yield change $\Delta y(t_i)$ on the three principal components respectively. They correspond to the lines in Figure 3. The sensitivities of the percentage change in the bond price to these three risk factors is measured by principal component durations (*PCDs*) given as follows:

$$\frac{\Delta P}{P} \cong -\sum_{i=h,s,c} PCD(i) \cdot \Delta c_i \tag{12}$$

The first three principal component durations given in equation (12) explain anywhere from 80 to 95% of the ex-post return differentials on bonds, depending on the time period chosen. To measure non-linear exposures, Nawalkha, Soto, and Beliaeva [2005] derive principal component convexity measures. Besides the benefit of reduction in the dimensionality (when compared with other models such as the key rate model), the principal component model is able to produce orthogonal risk factors. This feature makes interest rate risk measurement and management a simpler task because each risk factor can be treated independently.

Also, since the principal component model explicitly selects the factors based upon their contributions to the total variance of interest rate changes, it should lead to some gain in hedging efficiency. Further, in situations where explicit or implicit short positions are not allowed, the duration vector or the key rate duration model cannot give a zero immunization risk solution, except for some trivial cases. With short positions disallowed, significant immunization risk is bound to remain in the portfolio, and this risk can be minimized with the knowledge of the factor structure of interest rate changes using a principal component model.

The major shortcoming of the principal component model is that it assumes a stationary covariance structure of interest rate changes. The use of the model will lead to hedging errors in situations where this assumption is violated. Recall that the M-absolute/M-square models, the duration vector model, and the key rate duration model do not require this assumption,[3] and therefore the hedging performance of these models is invariant to the non-stationarities in the covariance structure of rate changes. All that is required is that the shifts in the term structure remain smooth in order to be captured by a small number of risk measures under these models.

Furthermore, since the three most important principal components that drive the interest rate movements resemble the changes in height, slope, and curvature of the yield curve measured by the first three elements of the duration vector, the duration vector model is not inconsistent with a principal component model for hedging. As Soto [2004] demonstrates, the specific yield curve shifts corresponding to the first three duration vector elements capture as much as the variance captured by the first three principal components of the interest rate changes.

INTEREST RATE RISK HEDGING MODELS FOR FIXED INCOME DERIVATIVES

The revolution in finance unleashed by the option pricing models of Black and Scholes [1973] and Merton [1973] allows any derivative security to be expressed as a portfolio of traded securities in a complete market, simply by absence of arbitrage. Though expected or average maturity defines the sensitivity of the percentage price changes of a *fixed-coupon*, *default-free* bond to interest rate changes, such a concept does not define the sensitivity of the percentage price changes of a security with embedded options to interest rate changes. If we define "duration" as the sensitivity of percentage price

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[3] In its generality, the key rate model does not require a stationary factor structure of interest rate changes. However, for performing a VaR analysis, the factor structure of the key rate changes must remain stationary.

changes of a security (with or without embedded options) to interest rate changes, then the duration of a security with embedded options may or *may not* have a relation with the expected or average maturity of that security. For example, a call option written on any fixed income instrument may mature in 3 months, but the duration of the call option (defined as the sensitivity of the percentage price changes of the option to interest rate changes) can be 10 or even 100. Because the definition of duration coincides with expected (or average) maturity for the case of a default-free fixed-coupon bond, researchers sometimes assume incorrectly, that this relationship holds even for securities with default risk, or with floating payments, or with embedded options.

A common fallacy of this type occurs in the case of a default-prone bond, which can be modeled as a default-free bond minus a put option written on the firm's underlying assets. For example, Chance [1990] derives the duration of a zero-coupon default-prone bond using such a framework (see Merton's [1973, 1974]) and finds that the duration represents "expected maturity" of the bond under the risk neutral measure. Since the possibility of default reduces the expected maturity of the bond, Chance claims that the duration of the default-prone zero-coupon bond is *always* less than or equal to the duration of the default-free zero-coupon bond with identical maturity. A simple example can demonstrate the fallacy of this argument. Consider a firm with its assets priced at $100, financed by $8 of equity and $92 of zero-coupon debt maturing in one year with a face value of $100. Assume that the duration of the firm's assets equals 12. Thus, a one percent parallel increase in the yield curve reduces the firm's asset value to approximately $88. Since the default-prone zero-coupon bond must lose at least $4 (i.e., $92 - $88 = $4), the duration of this bond must be at least 4 (which is *four times* its maturity), resulting in about 4% loss due to a 1% rise in the yield curve.

The reason Chance's model does not account for this possibility is because his model implicitly assumes that the duration of the firm's assets is zero. However, the duration of assets of most financial institutions is rarely close to zero. Hence, default-prone bonds issued by these institutions can have durations that are *longer* than the durations of equivalent default-free bonds, if the duration of the assets is very high, due to the interaction of default risk and interest rate risk. Consistent with this argument Nawalkha [1996], Jacoby and Roberts [2003], and Nawalkha, Soto, and Beliaeva (NSB) [2005], show that duration of a default-prone bond can be either higher or lower than the duration of the equivalent default-free bond. Other researchers including Fooladi, Roberts, and Skinner [1997] and Jocoby [2003] also model the risk aversion of the investors and a delay period in the recovery (due to the default process) and find that the duration of a default-prone bond can be higher than the duration of the equivalent default-free bond. Jacoby and Roberts [2003] and Duffee [1998] also show that failing to control for call risk (which reduces the duration of the default-prone bond) can lead to a spurious empirical conclusion that default risk reduces the duration of the bond, when, in fact, it may not reduce, and even increase the duration of the bond.

In the following, we provide a framework which allows computing the interest rate sensitivity measures of a derivative security with embedded options by modeling it as a portfolio of securities or assets without embedded options. Such a framework avoids confusing the expected or average maturity of the derivative security with the duration or price sensitivity of that security. The framework is general enough and can be applied to a variety of securities with embedded options like default-prone bonds, callable bonds, naked options, mortgage-backed securities with prepayment options, etc. The framework also allows extending the four classes multifactor interest rate risk models given in the earlier sections to the large class of fixed income derivative securities, with a simple inspection of terms.

Let P^* represent the price of the derivative security with embedded options, and let P_1, P_2,…, P_M , represent the prices of M number of replicating securities or assets *without* embedded options that dynamically replicate P^*, in a complete market, such that:

$$P^* = N_1 P_1 + N_2 P_2 + \ldots + N_M P_M \tag{13}$$

where,

$$N_i = \partial P^* / \partial P_i \tag{14}$$

is the delta sensitivity of the derivative security with respect to the i^{th} replicating security or asset, for $i = 1,2,\ldots M$.

An instantaneous percentage change in price P^* can be approximated as follows:

$$\frac{\Delta P^*}{P^*} \cong w_1 \frac{\Delta P_1}{P_1} + w_2 \frac{\Delta P_2}{P_2} + \ldots + w_M \frac{\Delta P_M}{P_M} \tag{15}$$

where,

$$w_i = \frac{\partial P^* / \partial P_i}{P^* / P_i} \tag{16}$$

is the elasticity of the derivative security with respect to the i^{th} replicating security or asset.

Note that if the replicating securities are all given as default-free bonds, then substitution of equations (1), (2), (4), (6), (7), (10), and (12), immediately generalizes the four classes of multifactor interest rate risk models (i.e., M-absolute/M-square models, duration vector model, key rate duration model, and principal component duration model) given in the earlier sections to the derivative security. If the replicating security or asset is not a default-free bond, then the given class of multifactor model can be used to first derive the respective duration measures of the replicating security or asset, and then appropriate substation in equation (15) can immediately provide the duration measures of the derivative security. Nawalkha [1995] uses the above framework to derive duration vectors of call and put options written on default-free bonds, and NSB [2005] use this framework to derive duration vectors of forward rate agreements, interest rate swaps, caps, floors, collars, receiver swaptions, payer swaptions, and bond options among other securities. Nawalkha [1996] and NSB [2005] also use the above framework to derive the duration measure of a default-prone zero-coupon bond, under the models of Merton [1974], Longstaff and Schwartz [1995], and Collin-Dufresne and Goldstein [2001], and demonstrate that the duration of a default-prone bond is higher (lower) than the duration of the equivalent default-free bond, if the duration of the firm's assets is higher (lower) than the duration of the equivalent default-free bond. In general, the above framework can be used to derive duration measures of any derivative security with embedded options that can be replicated as a portfolio of traded securities or assets without embedded options.

The above framework is also useful for computing interest rate risk measures for demand deposits and credit card loans. These financial instruments are valued by Hutchison and Pennacchi [1996] and Jarrow and Deventer [1998] using models that allow banks to determine the deposit rates and credit card loan rates using imperfect competition. Jarrow and Denventer show that these instruments can be represented as a replicating portfolio of two assets: a shortest term default-free zero coupon bond and an exotic interest rate swap representing the net present value of the financial instrument. In another class of models, Frauendorfer and Schürle [2003, 2006] and Kalkbrener and Willing [2004] replicate the demand deposits as a portfolio of assets by optimizing a specific objective criterion, subject to the constraint that the volume of optimal portfolio mimics the volume of the demand deposits. The portfolio weights, which are similar to the weights defined in equation (16), determine the duration of the replicating portfolio, which represents the duration of the demand deposits. Though the duration estimates from these models are very sensitive to the objective criterion used, Dewachter, Lyrio, and Maes [2006] find that these estimates are in line with those obtained using the discounted-cash flow approach if the standard deviation of the margin between the optimal portfolio return and the deposit rate is minimized.

A potentially important application of equations (13) through (16) is for modeling the durations of default-prone bonds/loans held on the asset side of the balance sheet of banks and other financial institutions. The current regulatory models, such as the U.S. Federal Reserve's Economic Value model (EVM) [1995], the Office of Thrift Supervision's (OTS) Net Portfolio Value model [1994], and the model of Basel Committee on Banking Supervision [2004, 2008], all implicitly use "maturities" of the cash flows from the assets as measures of sensitivities of these cash flows to interest rate changes. Hence, despite their superficial differences, all of the regulatory models implicitly assume that the durations of default-prone loans and bonds can be approximated by their "average" maturities. In the current financial environment, one must be cautious regarding this assumption, since trillions of dollars of spending by the U.S. Treasury, and even larger amounts in swaps by the U.S. Federal Reserve, could potentially lead to high inflation in the next 3 to 5 years, if these policies cannot be reversed quickly. In the event of high inflation or stagflation, the durations of default-prone loans can *exceed* their average maturities, due to the interaction of default risk and interest rate risk, as shown by the example given earlier. This risk is especially relevant in the current financial environment since fixed-rate mortgage loans and other fixed-rate loans have been financed at record low interest rates exposing these loans to significant risk of duration lengthening if interest rates were to rise sharply.

Finally, the imperfect competition-based duration models of demand deposits may overstate the durations of demand deposits in the extreme environment of high inflation or stagflation, since deposit volumes may decline non-linearly if interest rates were to rise sharply. Deposit holders may continue to keep the deposits at banks if interest rates rise gradually (i.e., using the standard argument based on imperfect competition), but in the event of sharp increases in the interest rates, deposit holders would withdraw their deposits at a much faster pace, as opportunity costs of not investing in short term money market instruments would be too high, similar to what occurred in the late 1970s. If banks would rapidly increase the rate of interest offered on demand deposits to counter the fast pace of deposit withdrawal, then the effective duration of the deposits would be lower than that assumed by models based on imperfect competition.

Hence, the durations of assets may be understated and durations of liabilities may be overstated if interest rates were to rise sharply in the near future. This implies that the true duration gap between assets and liabilities may turn out to be higher in a high inflationary environment, both because of the non-linear relation between interest rates and deposit volumes, and because of the interaction between interest rate risk and default risk. Though the risk of high inflation is low at present, this risk must be evaluated carefully by financial institutions to avoid more economic turmoil in the near future.

SUMMARY AND CONCLUSIONS

The unprecedented fiscal and monetary responses of the U.S. government to prevent an economic collapse has led to an increase in the odds of a future inflationary regime, even though the chances of such an occurrence seem low at present. In this paper we ask if the managers of the financial institutions are ready to deal with the challenge of inflation if it were to arrive soon. We discuss five classes of models in the fixed income literature that deal with hedging the risk of large, non-parallel yield curve shifts. These models are given as M-Absolute/M-Square models, duration vector/M-vector models, key rate duration model, principal component duration model, and extensions of these models for fixed income derivatives, for valuing and hedging bonds, loans, demand deposits, and other fixed income instruments. These models can be used for designing various hedging strategies such as portfolio immunization, bond index replication, duration gap management, and contingent immunization, to protect against changes in the height, slope, and curvature of the yield curve. The main conclusions of this paper can be summarized as follows:

- Powerful interest rate risk hedging models exist as alternatives to the traditional duration/convexity model, which assumes parallel yield curve shifts and implies riskless arbitrage opportunities.
- The M-absolute/M-square models can minimize the bond portfolio's exposure to slope, curvature, and other higher order yield curve shifts. The M-absolute model, using a single risk measure, can alone reduce the risk inherent in the traditional duration model by more than 50%.
- Further gains in immunization performance can be made by using a vector of risk measures using the duration vector model and the M-vector model.
- The hedging performance of the key rate duration model is similar to that of the duration vector and M-vector model, but it requires a large number of key rate durations to achieve this objective.
- The principal component duration model can reduce the dimensionality of the interest rate risk hedging problem, but this reduction comes at the cost of the assumption of a *stationary* variance-covariance matrix of rate changes.
- The four classes of interest rate hedging models given above can be extended to hedging default-prone bonds and loans, demand deposits, and other fixed income derivatives by replicating these securities with embedded options, as portfolios of securities or assets without embedded options, using equation (15).
- The current regulatory models proposed by the U.S. Federal Reserve, the Office of Thrift Supervision, and the Bank of International Settlements, may understate the true interest rate risk exposure of financial institutions, if sharp increases in interest rates lead to higher default risk and a quickening of the pace of deposit withdrawals.

REFERENCES

Basel Committee on Banking Supervision, 2004, “Principles for the Management and Supervision of Interest Rate Risk,” *Bank for International Settlements*.

Basel Committee on Banking Supervision, 2008, “Range of Practices and Issues in Economic Capital Modelling,” Consultative Document, *Bank for International Settlements*.

Black, F. and M. Scholes, 1973, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, 81(3), 637-654.

Chambers, D.R., 1981, *The Management of Default-Free Bond Portfolios*, Ph.D. dissertation, University of North Carolina at Chapel Hill.

Chambers, D.R.,W.T. Carleton, and R.M. McEnally, 1988, “Immunizing Default-Free Bond Portfolios with a Duration Vector,” *Journal of Financial and Quantitative Analysis 23(1)*, 89-104.

Chance, D.M., 1990, “Default Risk and the Duration of Zero Coupon Bonds,” *Journal of Finance*, March, 265–274.

Collin-Dufresne, P., R.S. Goldstein, 2001, “Do Credit Spreads Reflect Stationary Leverage Ratios?," *Journal of Finance*, 56, 1929-1958.

Dewachter, H., M. Lyrio, and S. Maes, 2006, “A Multi-Factor Model for the Valuation and Risk Management of Demand Deposits,” *National Bank of Belgium*, Working Paper No. 83.

Diebold, F.X., L. Ji, and C. Li, 2006 "A Three-Factor Yield Curve Model: Non-Affine Structure, Systematic Risk Sources, and Generalized Duration," in L.R. Klein (ed.), Long-Run Growth and Short-Run Stabilization: Essays in Memory of Albert Ando. Cheltenham, U.K. Edward Elgar, 240-274.

Duffee, R.G., 1998, “The Relation between Treasury Yields and Corporate Bond Yield Spreads, *Journal of Finance*, 53, 2225-2241.

Federal Reserve, 1995, Risk-Based Capital Standards: Interest Rate Risk, Docket # R-0802, Washington D.C.

Fong, G. and F.J. Fabozzi. *Fixed Income Portfolio Management*. Dow Jones-Irwin, Homewood, IL (1985).

Fong, G. and O. Vasicek, 1983, Return Maximization for Immunized Portfolios, in G.G. Kaufman, G.O. Bierwag, and A. Toevs, eds., *Bond Portfolio Management: Duration Analysis and Immunization*, JAI Press, Greenwich.

Fong, G. and O. Vasicek, 1984, “A Risk Minimization Strategy for Portfolio Immunization,” *Journal of Finance*, December, 1541-1546.

Fooladi, I.J., G.S. Roberts, and F. Skinner, 1997, “Duration for Bonds with Default Risk,” *Journal of Banking and Finance*, 21, 1-16.

Frauendorfer, K. and M. Schürle, 2003, “Management of Non-Maturing Deposits by Multistage Stochastic Programming,” *European Journal of Operational Research*, 151, 602-616.

Frauendorfer, K. and M. Schürle, 2006, “Dynamic Modeling and Optimization of Non-Maturing Accounts, in *Liquidity Risk Global Best Practices*, John Wiley and Sons.

Grandville, O., 2001, *Bond Pricing and Portfolio Analysis*. MIT Press.

Granito, M., 1984, *Bond Portfolio Immunization*. JAI Press, Greenwich.

Hedge, S.P. and K.P. Nunn, 1988, “Non-Infinitesimal Rate Changes and Macaulay Duration,” *Journal of Portfolio Management*, Winter, 69-73.

Ho, T.S.Y., 1992, “Key Rate Durations: Measures of Interest Rate Risks,” *Journal of Fixed Income*, September, 29-44.

Ho, T.S.Y., M.Z.H. Chen, and F.H.T. Eng, 1996, “VAR Analytics: Portfolio Structure, Key Rate Convexities, and VAR Betas,” *The Journal of Portfolio Management* 23(1), 89-98.

Hutchison, D.E. and G.G. Pennacchi, 1996, "Measuring Rents and Interest Rate Risk in Imperfect Financial Markets: The Case of Retail Bank Deposits," *Journal of Financial and Quantitative Analysis*, 31, 399-417.

Jacoby, G., 2003, “A Duration Model for Defaultable Bonds,” *Journal of Financial Research*, 26.

Jacoby, G. and G.S. Roberts, 2003, "Default and Call-Adjusted Duration for Corporate Bonds," *Journal of Banking and Finance*, 27, 2297–2321.

Jarrow, R.A. and D.R. Van Deventer, 1998, “The Arbitrage-Free Valuation and Hedging of Demand Deposits and Credit Card Loans,” *Journal of Banking and Finance*, 22, 249–272.

Kalkbrener, M. and J. Willing, 2004, “Risk Management of Non-Maturing Liabilities,” *Journal of Banking and Finance*, 28(7), 1547-1568.

Lacey, N.J. and S.K. Nawalkha, 1993, Convexity, Risk, and Returns,” *The Journal of Fixed Income* 3(3), 72-79.

Longstaff, F. and E. Schwartz, 1995, “A Simple Approach to Valuing Risky Fixed and Floating Rate Debt,” *Journal of Finance*, 50, 789-819.

Merton, R.C., 1973, “Theory of Rational Option Pricing,” *Bell Journal of Economics and Management Science*, 4(1), 141-183.

Merton, R.C., 1974, “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance* 29, 449–470.

Nawalkha, S.K., 1995, "The Duration Vector: A Continuous-Time Extension of Default-Free Interest rate Contingent Claims," *The Journal of Banking and Finance* 19(8), 1359-1378.

Nawalkha, S., 1996, A Contingent Claims Analysis of Interest Rate Risk Characteristics of Corporate Liabilities, *Journal of Banking and Finance,* 20(2), 227-245.

Nawalkha, S.K. and D.R. Chambers, 1996, "An Improved Immunization Strategy: M-Absolute," *Financial Analysts Journal*, September–October, 69-76.

Nawalkha, S.K. and D.R. Chambers, 1997, "The M-Vector Model: Derivation and Testing of Extensions to the M-Square Model," *Journal of Portfolio Management* 23(2), 92-98.

Nawalkha, S.K. and S. Latif, 2004, "Measuring True Risk Exposure," *Banking Today*, July/August, 23-27.

Nawalkha, S. K., G.M. Soto, and N.A. Beliaeva, 2005, Interest Rate Risk Modeling: The Fixed Income Valuation Course. Wiley Finance, John Wiley and Sons, New Jersey.

Nawalkha, S.K., G.M. Soto, and J. Zhang, 2003, "Generalized M-Vector Models for Hedging Interest Rate Risk," *Journal of Banking and Finance* 27, 1581-1604.

Nelson, C.R. and A.F. Siegel, 1987, "Parsimonious Modeling of Yield Curves," *Journal of Business*, 60, 473–89

Office of Thrift Supervision, Department of the Treasury, 1994, "The OTS Net Portfolio Value Model."

Prisman E.Z. and Y. Tian, 1994, "Immunization in Markets with Tax Clientele Effect: Evidence from the Canadian Market," *Journal of Financial and Quantitative Analysis*, June, 301-321.

Soto, G. M., 2001, "Immunization Derived from a Polynomial Duration Vector in the Spanish Bond Market," *Journal of Banking & Finance*, 25(6), 1037-1057.

Soto, G.M., 2004, "Duration Models and IRR Management: A question of dimensions?," *Journal of Banking and Finance,* 28, 1089-1110.

Ventura, J.M. and C.M. Pereira, 2006, "Immunization using a stochastic-process independent multifactor model: The Portuguese experience", *Journal of Banking and Finance*, 30, 133–156.