Optimal Fiscal Policy when Agents Fear Government Default

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Abstract

We consider a model in which a benevolent government has to choose optimally distortionary taxes on labor income and uncontingent debt, in order to finance an exogenous stream of public expenditure. We compare the optimal fiscal plan in two frameworks. In the first one households are fully confident about government solvency. In the second, households believe that there is a positive default probability which is positively related to the level of debt. While in the first framework a temporary bad shock translates into a permanent increase in the debt level, in the second one the increase in government debt is only temporary. The result provides a theoretical rationale for the policy recommendations made by policy analysts for copying with the high debt inherited from the recent crisis. More generally, we aim to derive the optimal strategy for a policymaker which does not consider default as a viable policy option but has to design its fiscal policy by taking into account both private agents' default expectations and macroeconomic dynamics.

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"Only thing we have to fear is fear itself"

F. D. Roosevelt

1 Introduction

To contrast the severe global recession of 2009, governments in most advanced countries performed expansionary fiscal policies. These interventions have led to a steep increase in debt levels. According to the IMF, in the advanced economies of the G20 the debt to GDP ratio is projected to rise from 78% in 2007 to 118% in 2014. Many policy analysts[1], fearing this massive accumulation of debt, have called for a substantial debt reduction in the years to come.

This policy suggestion, although reasonable, is difficult to justify from an optimal fiscal policy perspective.[2] For example, Aiyagari et al. (2002) show that when lump-sum taxes are unavailable and financial markets are incomplete, a benevolent government should optimally finance an adverse fiscal shock by increasing deficit. This policy minimizes the costs associated to distortionary taxation. In similar fashion, Marcet and Scott (2008) show that a temporary adverse shock translates into a permanent increase in the level of debt.

This conclusion is derived in a framework in which agents understand that the primary deficits during the increase in government expenditure are matched by higher future primary surpluses later on, in a way that the fiscal plan is sustainable. Because they realize that the government is always solvent, they do not require a default premium to hold government bonds. However, this implication of the model seems at odds with the recent post-crisis experience, where the market price of risk has increased significantly in several advanced countries. A natural question then arises: how to set fiscal policy in a context in which agents fear government default?

In this paper we answer this question. As in Aiyagari et al. (2002), we consider a closed production economy with no capital and infinitely lived agents. Public spending follows an exogenous stochastic process. The problem of the representative household is to maximize her lifetime expected utility subject to the flow budget constraint. The government acts under full commitment, i.e. it always fulfills its promises about future taxes. The government is also benevolent: it chooses the level of debt and distortionary taxes on labor income to maximize households' expected utility subject to the feasibility constraint, households' beliefs and optimality conditions. But differently from Aiyagari et al. (2002) households believe that with a positive probability the government could default on its own debt. In particular households

[1] See, e.g., IMF (2010).

[2] The optimal taxation literature is immense and offering a comprehensive survey goes beyond the scope of this paper. See Barro (1979, 1989, 1995, 1997), Bohn (1990), Kydland and Prescott (1980), Lucas and Stokey (1983), Chari et al. (1994), Chari and Kehoe (1999), Aiyagari et al. (2002), Zhu (1992) among many other.

believe that there is a positive relation between the probability of default and the amount of outstanding debt. This assumption is reasonable, and seems to be supported by the empirical evidence on yield spread. For example, figure (1) points to a positive relation between the amount of government debt and yield spread, a proxy for the sovereign risk premium, for 10 euro area countries in the period $2000 - 2009$.

Over time households update their estimates of this relation as new data on government behavior become available.

We study the impact of distorted expectations about government default on the optimal fiscal policy in two different set-ups. In the first one, when in the initial period the fiscal authority sets its plans agents are already skeptical about the government capability/willingness to honor its debt obligations. In the second one, agents are instead fully confident about debt repayment, but they may start fearing default if the government uses debt to absorb an adverse shock. These two cases are meant to capture two different situations. The first one refers to the post crisis situation, characterized by high debt levels and significant sovereign risk premia: here the government's problem is to design an optimal "exit strategy". The second one instead is meant to capture both the pre-crisis and the post-crisis period (crisis is modeled here as a very high decrease in productivity and output). The main problem here is to understand whether a "fiscal stimulus" in times of crisis, implying higher deficits and debts, is consistent with an optimal fiscal plan.

Our main findings are the following. First, when agents fear government default, a post-crisis fiscal consolidation becomes optimal. The intuition is that the interest rate on government debt is relatively high due to distorted expectations about government default. Therefore the marginal cost of higher distortionary taxes today is more than compensated by the expected future marginal benefits of lower distortionary taxes tomorrow. This mechanism is stronger ι) the more pessimistic agents are about government solvency and $\iota\iota$) the higher the post-crisis debt level. As in Caprioli (2010) the agents' initial beliefs have an effect on the long-run mean value of the tax rate and debt. Second, as in Aiyagari et al. (2002), optimal policy still prescribes to increase debt to absorb the negative shock. But differently from Aiyagari et al. (2002), the possibility of a negative shock leads the government to run much higher primary surpluses before it materializes. Indeed, the probability of facing a bad shock in the future makes optimal to create "fiscal room" in advance.

The paper proceeds as follows. Section 2 characterizes the optimal fiscal policy, and in section 3 we solve it numerically. In Section 4 we characterize the fiscal plan in the case of an unexpected adverse shock. Section 6 concludes.

2 The Model

We consider an infinite horizon economy with an infinitely lived representative consumer and a benevolent fiscal authority. The government finances an exogenous stream of public consumption levying a proportional tax on labor income and issuing a one-period non state-contingent bond, which is the only financial asset in the economy. The government has a full commitment technology and always repays its debt. There are two sources of aggregate uncertainty, represented by a government expenditure shock and a technology shock. In subsection 2.1 we briefly review optimal fiscal policy under the assumption that households are at any moment fully confident about government solvency, as in Aiyagari et al. (2002). In subsection 2.2 we modify this benchmark model assuming that households assign a positive probability to the event of government default. We show how the way in which households form their expectations change the constraints faced by the fiscal authority and consequently the optimal fiscal policy.

2.1 The rational expectations benchmark

Time is discrete and indexed by $t = 0, 1, 2....$ At the beginning of each period there is a realization of a stochastic state $s_t = (g_t, \theta_t) \in S = G \times \Theta$. Let us define the history of events up to time t as $s^t = (g^t, \theta^t)$, where $g^t = (g_0, g_1, .., g_{t-1}, g_t)$, $\theta^t = (\theta_0, \theta_1, .., \theta_{t-1}, \theta_t)$, and the conditional probability of s^r given s^t as $\pi(s^r|s^t)$; s_0 is non-stochastic.

The Private Sector. - A representative household is endowed with one unit of time which can be used for leisure, l_t, or labor, n_t,

$$n_t(s^t) + l_t(s^t) = 1 \ \forall t \geqslant 0, \forall s^t \in S^t, \tag{1}$$

He chooses consumption $c_t(s^t)$, leisure $l_t(s^t)$ and bond holdings $b_t(s^t)$ to maximize his lifetime discounted expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) = \sum_{t=0}^{\infty} \sum_{s^t}^{\infty} \beta^t u(c_t(s^t), l_t(s^t)) \pi(s^t|s_0)^3 \tag{2}$$

subject to the period-by-period budget constraint

$$b_{t-1}(s^{t-1}) + (1 - \tau_t(s^t)) w_t(s^t)(1 - l_t(s^t)) = c_t(s^t) + p_t(s^t) b_t(s^t) \tag{3}$$

where β is the discount factor, $\tau_t(s^t)$ is the state-contingent labor tax rate, $w_t(s^t)$ is the wage rate and p_t is the price of the one period bond.

The household's optimality conditions are

$$\frac{u_{l,t}(s^t)}{u_{c,t}(s^t)} = w_t(s^t)(1 - \tau_t(s^t)) \tag{4}$$

[3]The utility function satisfies the usual standard assumptions, i.e. $u_{c,t} > 0$, $u_{l,t} > 0$, $u_{cc,t} < 0$, $u_{ll,t} < 0$.

$$p_t(s^t) = \beta \frac{E_t u_{c,t+1}}{u_{c,t}(s_t)} \tag{5}$$

where for notational simplicity we denote from now on $u_{l,t}(s^t)$ and $u_{c,t}(s^t)$ as the marginal utility of labor and consumption in state s^t.

There is only one non-storable good, produced by a representative price-taker firm with a linear production technology given by:

$$y_t(s^t) = \theta_t n_t(s^t).$$

Output, y_t, can be used either for private consumption or public consumption (g_t). Equilibrium in the good market and in the labor market requires:

$$y_t(s^t) = c_t(s^t) + g_t \tag{6}$$

$$\theta_t = w_t(s^t) \tag{7}$$

The Government. - The government finances the exogenous sequence of government expenditures levying taxes and issuing debt. Its policy $(\tau_t(s^t), b_t(s^t))_{t \geqslant 0}$ satisfies the period by period budget constraint:

$$b_{t-1}(s^{t-1}) + g_t = \tau_t(s^t) w_t(s^t)(1 - l_t(s^t)) + p_t(s^t) b_t(s^t).$$

The initial level of debt b_{-1} is given. Aiyagari et al. (2002) show that the dynamic optimal taxation problem of the government is equivalent to the problem of maximizing:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{8}$$

under the following constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c,0}(s_0) b_{-1} \tag{9}$$

$$E_t \sum_{j=0}^{\infty} \beta^j (u_{c,t+j} c_{t+j} - u_{l,t+j}(1 - l_{t+j})) = u_{c,t}(s^t) b_{t-1}(s^{t-1}), \ \forall t \geqslant 0, \, \forall s^t \tag{10}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^j (u_{c,t+j} c_{t+j} - u_{l,t+j}(1 - l_{t+j}))}{u_{c,t}(s^t)} < \overline{M} \ , \ \forall t \geqslant 0, \, \forall s^t \tag{11}$$

$$\theta_t (1 - l_t(s^t)) = c_t(s^t) + g_t \tag{12}$$

Constraints (9) and (10) require that for any period and any state, the inherited level of debt is equal to the stream of expected future primary surpluses. They are equivalent to the intertemporal consumer budget constraint with both prices and taxes replaced using the households' optimality conditions, (4) and (5). If financial markets were complete, constraints (10) would be satisfied by choosing appropriately the vector of state-contingent bond, so they would not constrain the optimal choice of taxes. However, under incomplete markets, the government cannot adjust the inherited stock of debt in response to the current realization of the shock. Therefore, constraints (10) captures the idea that in any period the future path of taxes depends on the current state. Constraints (11) requires that debt limits be respected.

It can be shown that the solution to the government problem satisfies

$$\tau_t = T(s_t, \psi_{t-1}, b_{t-1}) \forall t > 0 \tag{13}$$

$$b_t = D(s_t, \psi_{t-1}, b_{t-1}) \forall t > 0 \tag{14}$$

Equations (13) and (14) are the optimal policy rules for the labor tax rate and for bond holdings respectively. Both of them are time invariant functions of the current state s_t, the inherited bond holding b_{t-1} and the auxiliary state variable ψ_{t-1} which is equal to the sum of past lagrange multipliers, from period 0 till $t-1$, associated to the intertemporal budget constraints (10).[4]

Two observations are worth noting. First, by including the costate variable ψ_{t-1} in the vector of state variables the problem becomes recursive and standard solution techniques can be applied. Second, the presence of ψ_{t-1} and b_{t-1} makes the allocation and the cost of distortionary taxation state and history-dependent.

2.2 Modeling fear of government default

In the benchmark model of subsection (2.1) households fully understand the government problem and therefore attach zero probability to the event of a government default, whatever the observed evolution of government debt. In particular, as households understand the risk-free nature of government bonds, they do not require to be compensated for any default risk. In this section

[4]This approach has been pioneered by Marcet and Marimon (1998)

we study what happens if agents abruptly - and wrongly - start to fear that the government might not fulfill the promise of always paying back its own obligations.

In particular, at time t the household believes that at time $t+1$ debt will be honored with probability $\hat{\pi}_t$ and will be instead repudiated with probability $(1-\hat{\pi}_t)$.

In this case, the optimality condition of the household is given by:

$$p_t(s^t,\delta^t)u_{c,t}(s^t,\delta^t) = \beta\sum_{s_{t+1}} u_{c,t+1}(s^{t+1},\delta_{t+1}=1,\delta^t)\tilde{\pi}(s^{t+1},\delta_{t+1}=1,\delta^t|s^t,\delta^t) = \tag{15}$$

$$\beta\sum_{s_{t+1}} u_{c,t+1}(s^{t+1},\delta_{t+1} = 1,\delta^t)\tilde{\pi}(s^{t+1}|s^t,\delta^t)\hat{\pi}_t \tag{16}$$

where $\delta_t \in \{0,1\}$ is equal to 1 if the government does not default on debt in period t and equal to 0 otherwise, and $\hat{\pi}_t$ is the probability that $\delta_{t+1} = 1$ conditional on s^t and δ^t. The relevant expectations ($\tilde{\pi}$) are now with respect to s^t *and* the event of government default.

We make two assumptions about how default expectations evolve. First, the higher the level of outstanding debt, the stronger the fear of government default, and in particular fear of default start to arise when the debt goes above some "psychological" threshold $\bar{b}$[5]:

$$\hat{\pi}_t = \frac{1}{1+\alpha_t \max(0; b_t - \bar{b})} \tag{17}$$

Second, we assume that agents revise their beliefs about the probability of a public default as new evidence about government behavior becomes available. In the literature various ways have been proposed to model agents' learning.[6] We adopt the approach pioneered by Marcet and Sargent (1989). They study agents which are similar to an econometrician, i.e. in each period they estimate recursively those parameters which are relevant for their decision, and whose values they ignore. In our model the only parameter that has to be estimated is α. Let α_t be the agents' estimate of α at time t. If agents use a constant gain algorithm with gain parameter equal to k, a special case of the algorithm studied by Marcet and Sargent (1989)[7], it can be shown that α_t is given by the following expression:

$$\alpha_t = \alpha_{t-1}(1 - kb_{t-1}^2). \tag{18}$$

[5]In the remaining of the paper, we set $\bar{b} = 0$, without loss of generality.

[6]For a comprehensive survey of learning models, see Evans and Honkapohja (2001). Several papers have already used these models to explain real world phenomena. For example, Adam and Nicolini (2006), Carceles and Giannitsarou (2007) and Cogley and Sargent (2008) introduce boundedly rational agents in a standard consumption based asset pricing model to fit some features of asset prices. Marcet and Nicolini (2003) and Adam and Honkapohja (2005) show how learning can be an explanation of hyperinflationary episodes. Kurz et al. (2005), Beaudry and Portier (2004, 2007) and Eusepi and Preston (2008) stress the importance of shifting expectations for business cycle fluctuations.

[7]In any case the economic intuition behind the result is robust to alternative learning schemes.

Several observations are worth-noting. First, equation (17) nests the rational expectation case in which households understand that default cannot happen. In fact, when $\alpha_t = 0$, $\hat{\pi}_t = 1$. Second, under the condition that $|(1 - kb_{t-1}^2)| < 1$ equation (18) is such that α_t converges to its true value, 0.

It is important to stress the fact that the perceived default probability has no impact on the actual default probability, which is always equal to 0. We believe that these features of the model capture the challenges that advanced countries are facing in the aftermath of the huge fiscal stimulus packages put in place to contrast the recent crisis. More generally we aim to derive optimal strategies for policymakers which do not see default as a viable policy option but have to take into account the link between the design of fiscal policy, default expectations and macroeconomic variables.

Definition 1. *Given b_{-1} and a stochastic process for the government expenditure g_t and the technology shock θ_t, a competitive equilibrium is an allocation $\{c_t, l_t, g_t\}_{t=0}^{\infty}$, state-contingent beliefs about government default probabilities $\{\hat{\pi}\}_{t=0}^{\infty}$, a price system $\{p_t, w_t\}_{t=0}^{\infty}$ and a government policy $\{\tau_t, b_t\}_{t=0}^{\infty}$ such that (a) given the price system, the beliefs and the government policy the households' optimality conditions are satisfied; (b) given the allocation and the price system the government policy satisfies the sequence of government budget constraint* (3)*; and (c) the goods and the bond markets clear.*

Define

$$A_t \equiv \prod_{k=0}^{t} \hat{\pi}_{k-1} \tag{19}$$

In the full credibility case A_t is constant and always equal to 1, while under learning it is not, unless the initial beliefs coincide with the rational expectations ones, i.e. unless $\alpha_{-1} = 0$. Using households' optimality conditions to substitute out prices and taxes from the government budget constraint, Aiyagari et al. (2002) show the constraints that a competitive equilibrium imposes on allocations. Using a similar argument, we show that under incomplete markets and bounded rationality the following result holds.

Proposition 1. *Assume that for any competitive equilibrium $\beta^t A_t u_{c,t} \to 0$ a.s. Given b_{-1} and α_{-1}, a feasible allocation $\{c_t, l_t, g_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if the following constraints are satisfied*

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = A_0 u_{c,0} b_{-1} \tag{20}$$

$$E_t \sum_{j=0}^{\infty} \beta^j A_{t+j} (u_{c,t+j} c_{t+j} - u_{l,t+j}(1 - l_{t+j})) = A_t u_{c,t} b_{t-1} \tag{21}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j}))}{A_t u_{c,t}} < \overline{M} \tag{22}$$

with initial condition $A_{-1} = 1$

Proof. We relegate the proof to the appendix. □

Equation (21) is the bounded rationality version of the intertemporal constraint on the allocation derived by Aiyagari et al. (2002) in a rational expectations framework, given in equation (21). The difference between equations (21) and (10) arises through the effect that government default expectations exert on bond prices. As expectations are not model-consistent, the primary surplus at time t, expressed in terms of marginal utility of consumption, is weighted by the product of one minus the expected default probabilities from period 0 till period t.

2.3 The government problem

Using the so called primal approach to taxation, we can recast the problem of choosing taxes and bond holdings as a problem of directly choosing allocations of consumption and labor, under the constraint that they satisfy the conditions for a competitive equilibrium.

At this point a clarification is needed. When the households and the benevolent government share the same information, they maximize the same objective function. But when the way in which they form their expectations differ, as in this setup, their objective functions differ as well. In what follows we assume that the fiscal authority maximizes the representative consumer's welfare *as if* the latter were rational. Said differently, the government understands how agents behave and form their beliefs, and it understands that these beliefs are distorted.[8]

Definition 2. *The government problem under learning is*

$$\max_{\{c_t, l_t, \alpha_t, A_{t+1}, b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j})) = A_t(s^t, \delta^t) u_{c,t}(s^t, \delta^t) b_{t-1}(s^{t-1}, \delta^{t-1}) \tag{23}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j}))}{A_t(s^t, \delta^t) u_{c,t}(s^t, \delta^t)} < \overline{M} \tag{24}$$

$$A_{t+1} = A_t \hat{\pi}(s^t, \delta^t) \tag{25}$$

[8]The same assumption is made in Karantounias et al. (2007) and Caprioli (2010).

$$\alpha_t(s^t,\delta^t) = \alpha_{t-1}(s^{t-1},\delta^{t-1})(1 - kb_{t-1}(s^{t-1},\delta^{t-1})^2) \tag{26}$$

$$c_t(s^t,\delta^t) + g_t = \theta_t(1 - l_t(s^t,\delta^t)) \tag{27}$$

for given b_{-1} and α_{-1}. Equations (23) and (24) constrain the allocation to be chosen among competitive equilibria. Equation (25) is the recursive formulation for A_t, obtained directly from equation (19). Equation (26) gives the law of motion of beliefs. Equation (27) is the resource constraint. As in equations (23) and (24) appear expectations of future control variables, the problem is not recursive and standard solution techniques cannot be used.

The Lagrangian for the Ramsey problem can be represented as

$$\begin{aligned}\mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t) + \psi_t A_t(u_{c,t}c_t - u_{l,t}(1-l_t)) \\ & - \lambda_t b_{t-1} A_{t-1} u_{c,t} + \gamma_t (A_{t+1} - A_t \hat{\pi}_t) \\ & + \rho_t(\alpha_t - \alpha_{t-1}(1 - kb_{t-1}^2)) + \nu_t(\theta_t(1-l_t) - c_t - g_t)\}\end{aligned}$$

where $\psi_t = \psi_{t-1} + \lambda_t - \epsilon_{1,t} + \epsilon_{2,t}$, where $\beta^t \epsilon_{1,t}$ and $\beta^t \epsilon_{2,t}$ are the Lagrange multipliers attached to the upper and lower debt constraints respectively. Since A_t and α_t have a recursive structure, the problem becomes recursive adding A_t and α_{t-1} as endogenous state variables to the ones in the Aiyagari et al. (2002) model, which are ψ_{t-1} and b_{t-1}.

First order necessary conditions $\forall t \geq 0$ are:[9]

- c_t:

$$u_{c,t} + \psi_t A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t} b_{t-1} u_{cc,t} A_t = \nu_t \tag{28}$$

- l_t:

$$u_{l,t} + \psi_t A_t(u_{l,t} - u_{ll,t}(1-l_t)) = \theta_t \nu_t \tag{29}$$

[9] As standard in the optimal fiscal policy literature, it is not easy to establish that the feasible set of the Ramsey problem is convex. To overcome this problem in our numerical calculations we check that the solution to the first-order necessary conditions of the Lagrangian is unique.

- α_{t+1}:

$$\rho_t - \beta E_t \rho_{t+1}(1 - kb_t^2) + \gamma_t A_t \frac{b_t}{(1+\alpha_t b_t)^2} \tag{30}$$

- b_t:

$$-\beta E_t \lambda_{1,t+1} u_{c,t+1} A_{t+1} + \gamma_t A_t \frac{\alpha_t}{(1+\alpha_t b_t)^2} + 2\beta E_t \rho_{t+1} \alpha_t k b_t = 0 \tag{31}$$

- A_{t+1}:

$$\gamma_t - \beta E_t \gamma_{t+1} \frac{1}{(1+\alpha_{t+1} b_{t+1})} - \beta E_t \lambda_{1,t+1} b_t u_{c,t+1} + E_t \psi_{t+1}(u_{c,t+1} c_{t+1} - u_{l,t+1}(1 - l_{t+1})) = 0 \tag{32}$$

3 Numerical Solution

Together, the first order conditions and the constraints of the government program imply a stochastic non linear system of difference equations in the variables c_t l_t τ_t b_t ψ_t A_{t+1} and α_t. We solve the system using standard collocation methods both in the case in which there are no doubts about debt repayment and in the case in which agents start to fear a government default. In both cases we consider a truncated AR(1) process for government expenditure and labor productivity:

$$g_t = \begin{cases} \underline{g}, & \text{if } (1-\rho_g)g^{ss} + \rho_g g_{t-1} + \epsilon_t^g < \underline{g} \\ (1-\rho_g)g^{ss} + \rho_g g_{t-1} + \epsilon_t^g, & \text{if } \underline{g} < (1-\rho_g)g^{ss} + \rho_g g_{t-1} + \epsilon_t^g < \overline{g} \\ \overline{g}, & \text{if } (1-\rho_g)g^{ss} + \rho_g g_{t-1} + \epsilon_t^g > \overline{g} \end{cases} \tag{33}$$

where ϵ_t^g is assumed to be normally distributed with zero mean and σ^g standard deviation. Labor productivity has an analogous structure.

Figure (2) shows the path of consumption, primary surplus and government debt over GDP in two economies which are identical except for the fact that in the second one α starts at a value different from 0 (0.01). In both cases g_t and θ_t are constant and equal to their unconditional mean. Both economies start with the same positive level of debt (set equal to 100 % of GDP).[10] Given this parametrization, the initial default probability is equal to 5 %.

[10] Of course, changing the initial value does not affect the qualitative features of the result, as long as $b_{-1}$is above the threshold $\bar{b}$.

In the baseline case, government debt stays roughly constant at its initial value. This result is consistent with the main policy message coming out from the optimal fiscal policy literature. The intuition is that, as lump-sum taxes are not available, the only way to reduce debt is by increasing the distortionary tax rate today, which in turn would allow to reduce tax rates tomorrow. Under this path of taxes, households would initially enjoy less consumption and more leisure, whereas the contrary would be true later on (when the tax rate would be allowed to be lower, thanks to the reduction attained in the burden of debt). However, under standard assumptions on the utility function, households prefer to smooth consumption and leisure over time and states. Therefore a benevolent government keeps distortionary taxes as smooth as possible, and allows debt to fluctuate around the initial value. In other words, a policy of debt reduction is sub-optimal. This policy implication does not hold anymore in a context in which households fear government default. Instead, taxes are increased at the beginning and debt is correspondingly reduced. To get an intuition of this result, it is important to understand the trade-off now faced by the government. On one side, as in the baseline framework, taxes are distortionary and therefore the government would like to keep them as constant as possible. On the other side, the government is aware that the perceived probability of default is higher the higher the debt level. These expectations translate into higher interest rates on government bonds and higher interest payments. Since agents are learning, the only way to manipulate distorted believes is by reducing debt. Fiscal consolidation becomes optimal because it is a way to correct distorted expectations.

Moving from a single realization to a fully-fledged simulation, Table 1 shows the average values for consumption and leisure and for fiscal variables (tax rate, government debt and primary surplus) in our two economies (averages are computed over 1000 simulated realizations of the shocks, for 20 time periods each). The qualitative results are confirmed. While in the rational expectation benchmark the mean value of bond holdings is equal to the initial one, in the economy with fear of default it is equal to 0.14, which means that fiscal consolidation is indeed optimal. Correspondingly, in the second economy taxes and primary surpluses are on on average higher (0.51 instead of 0.49 for taxes, 0.01 instead of 0.004 for the primary surplus). After 20 periods debt over GDP is equal to about 100% in the case of a fully credible government, while it is equal to 35 % in the other scenario.

4 A step backward: are stimulus packages justified?

In section (3) we studied a post-crisis situation, in which the debt has already reached the threshold above which skepticism about government commitment to debt repayment kicks in. In such a context, we showed that doubts about the capability/willingness of the government to pay back debt require a substantial, and possibly quite painful, fiscal consolidation. It is therefore natural to ask whether implementing a fiscal expansion in the event of a crisis can be justified, given that the stimulus might triggers fears of a government default.

To answer this question, in this section we do not focus on the post-crisis period only, but we aim at characterizing the optimal fiscal policy both before and after the crisis.

In particular, we assume that productivity θ_t is uncertain only at time $t = T$, when it can take two values, either θ_L or θ_H, with $Prob(\theta_T = \theta_H) = \pi$ and $Prob(\theta_T = \theta_L) = 1 - \pi$, but it is constant in all other periods: $\theta_0 = \theta_1 = ...\theta_{T-1} = \theta_{T+j} = \theta_L(1-\pi) + \theta_H \pi = 1 \ \forall j \geq 1$.

Figure (3) shows the optimal way to react to a large decrease in the productivity under the rational expectation benchmark. Before period T the government sets a constant tax rate in all periods and runs a balanced budget in all periods. At T, conditional on the bad shock realization, the government runs a primary deficit and issues debt, which from that period onwards is rolled over for ever. After the bad shock the tax rate is higher than before to pay for the higher debt services than before the crisis. But it is not optimal to bring debt to a lower levels.

Things are different when agents fear government default. In particular consider an economy in which debt has been below the "psychological" threshold above which concerns for debt repayment start to appear. The government faces a trade-off concerning the way to cope with the crisis. If the government decides to react to the bad shock by issuing bonds, effects on consumption will be smoothed, but agents will start to fear default, which has costs because it suboptimally increases interest rates and interest payments.

What is the optimal way to respond to the shock in this case? Figure (4) offers a graphical answer to the question, for the case of $\pi = 0.5$, $\theta_H = 1.1$ and $\theta_L = 0.9$. As in the rational expectations benchmark, the optimal fiscal policy implies running a budget deficit in the event of a realization of a bad shock in T. So one could conclude that in adverse circumstances a fiscal stimulus is justified even if it induces fears concerning government debt.

However, this conclusion comes with several caveats. First, as we saw in the previous section, after the shock the government starts a fiscal consolidation aimed at reducing debt and increasing its credibility. Second, the jump in debt in T is lower with respect to the benchmark case. Third, the fact that agents may start fearing default at T influences the optimal fiscal policy even before period T. In figure (5) it is shown the dynamics of government debt before the realization of the shock both in the case of a fully credible government and in the case of a non fully credible government. It is apparent that, while starting from the same initial debt levels, the latter reduces debt much more than the former.[11] This provides a theoretical rationale to the policy

[11] The numerical example shown in figure (5) has $\pi = 0$. In this scenario, debt is reduced between 0 and $T-1$ by about 3 per cent by a fully credible government and by about 11 % by a non fully credible government (in both economies the initial debt level has been set equal to 75 % of GDP).

prescription of building "fiscal space" in good times in order to be able to use fiscal policy as a counter-cyclical tool in bad times.

5 Policy Implications for exit strategies: a tale of two countries

In the light of the model described above, how policy suggestions differ across different countries? First, the more investors are skeptical about the government willingness and/or ability to honor its debt, the more the fiscal authorities should pursue fiscal consolidation. Second, countries which are more indebted should act with more strength to reduce the debt burden. In both cases the consequences of distorted expectations are stronger, so more restrictive fiscal policies are required to restore trust in sovereign solvency. We illustrate these insights using the German and the Italian cases. Both countries have been hardly hit by the economic crisis (in both GDP fell by about 5% in 2009), but they have very different public finances (the debt-to-GDP ratio is at about 115% in Italy and about 80% in Germany). Moreover, perceived default risk as reflected in ratings, bond spreads and differences in the cost of credit default swap contracts, is significantly higher in the Italian case.

We calibrate the initial value for α to match the sovereign default expectations implicit in the prices of CDS contracts. We set the initial debt at the 2009 (post-crisis) level in the two countries. Figure (6) shows how debt and primary deficit should evolve in the two countries. The solid line refers to Germany, whereas the dashed line refers to Italy. The country facing an higher debt level and higher default premia runs higher primary surplus and reduces debt quicker than the other one.

6 Conclusions and future research

This paper offers a theoretical rationale for implementing a fiscal consolidation after an economic crisis. Governments have intervened through expansionary fiscal policy in order to moderate the adverse consequences of the economic downturn. These interventions were justifiable but have led to a steep increase in public debts. If agents fully trusted the commitment of governments to always honor their debt obligations, no fiscal consolidation would be required. But if high debt levels induce agents to assign a positive probability to government default, not reducing the debt would imply high risk premia on sovereign bonds, and suboptimally high interest rates. In order to minimize overall distortions, a fiscal consolidation is needed.

The model can be extended in several possible dimensions. First, the assumption that default is not an equilibrium outcome should be relaxed. As our analysis refers to advanced countries, this assumption may be reasonable. Much less so for developing countries. Therefore one important extension would be to include a positive possibility of default in equilibrium. In this kind of model we conjecture that two possible equilibria can arise. When agents assign a low probability to the event of default, the low increase in the interest rate (with respect to the full credibility case) may be not enough to justify actual default. But when agents assign a very high probability of default, then the increase in the interest rate may support their believes because it may be optimal for the government to default. Because of the very high interest rate the cost of a transitory exclusion from the financial markets is lower than the distortionary cost of taxation to repay debt.

Another interesting extension would be to analyze fiscal and monetary coordination. In particular, it would be interesting to understand whether optimality requires that fiscal consolidation precedes or follows monetary tightening in the aftermath of a crisis, and whether a certain amount of inflation tax is an optimal way to pay the fiscal costs of the crisis.

Finally, in the paper we assumed that the government expenditure follows an exogenous stochastic process, as it is customary in the public finance literature. Because of this assumption however, we cannot address the issue of the optimal composition of the post-crisis fiscal adjustment. In particular, should the fiscal authority reduce debt by higher taxes or by lower expenditure? Under standard assumptions on the utility and the production functions the optimal thing to do would probably be a mix of the two.

We leave all these extensions as future research.

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A Appendix

A.1 Proof of Proposition 1

First we show that constraints (3), (4) and (16) imply (21). Consider the period-by-period budget constraint after substituting for the household optimality conditions:

$$b_{t-1} = \frac{u_{c,t}(g_t)s_t(g_t)}{u_{c,t}(g_t)} + \beta E_t \frac{u_{c,t+1}\hat{\pi}_t b_t}{u_{c,t}} \tag{34}$$

where $s_t \equiv c_t - \frac{u_{l,t}}{u_{c,t}}(1-l_t)$, b_t is the amount of bond holdings and $\hat{\pi}_t$ is the perceived probability at time t about government default in $t+1$. Multiplying both sides of (34) by $u_{c,t}A_t$, where $A_t \equiv \prod_{k=0}^{t} \hat{\pi}_{k-1}$ we get

$$b_{t-1}u_{c,t}A_t = A_t(u_{c,t}c_t - u_{l,t}(1-l_t)) + \beta E_t u_{c,t+1}A_t\hat{\pi}_t b_t \tag{35}$$

Notice that A_t has a recursive formulation given by

$$A_t = A_{t-1}\hat{\pi}_{t-1} \tag{36}$$

Forwarding equation (36) one period we get

$$A_{t+1} = A_t\hat{\pi}_t \tag{37}$$

Inserting equation (37) into equation (35) we get

$$b_{t-1}u_{c,t}A_t = A_t(u_{c,t}c_t - u_{l,t}(1-l_t)) + \beta E_t u_{c,t+1}A_{t+1}b_t \tag{38}$$

Keeping iterating forward equation (38) and imposing the transversality condition

$$lim_{t\to\infty}\beta^t A_t b_t u_{c,t} \to 0$$

we get

$$b_{t-1}u_{c,t}A_t = E_t\sum_{j=0}^{\infty}\beta^{t+j}A_{t+1}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1-l_{t+j})) \tag{39}$$

To prove the reverse implication, take any feasible allocation $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$ that satisfies equation (21).

Define

$$b_{t-1} = E_t\sum_{j=0}^{\infty}\beta^j A_{t+j}u_{c,t+j}s_{t+j}\frac{1}{u_{c,t}A_t} \tag{40}$$

It follows that

$$b_t = E_{t+1} \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{u_{c,t+1} A_{t+1}} \tag{41}$$

$$\begin{aligned}
b_{t-1} &= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + E_t \sum_{j=1}^{\infty} \beta^j A_{t+j} u_{c,t+j} s_{t+j} \frac{1}{u_{c,t} A_t} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \beta E_t \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{u_{c,t} A_t} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \frac{\beta}{u_{c,t} A_t} E_t \{ u_{c,t+1} A_{t+1} [E_{t+1} \frac{\sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j}}{u_{c,t+1} A_{t+1}}] \} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \frac{\beta}{u_{c,t} A_t} E_t \{ u_{c,t+1} A_{t+1} [E_{t+1} \frac{\sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j}}{u_{c,t} A_t}] \} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \frac{\beta}{u_{c,t} A_t} E_t \{ u_{c,t+1} A_{t+1} b_t \} =
\end{aligned} \tag{42}$$

Using equation (37) we get

$$b_{t-1} = s_t + \frac{\beta}{u_{c,t}} E_t (u_{c,t+1} \hat{\pi}_t b_t) \tag{43}$$

Using the households optimality conditions given by (4) and (16), equation (43) coincides with equation (3).

Table 1: Average Allocation

	Full Credibility Model	Partial Credibility Model
consumption	.31	.3
leisure	.38	.39
labor tax rate	.49	.51
bond holding	.2	.14
primary surplus	.004	.01

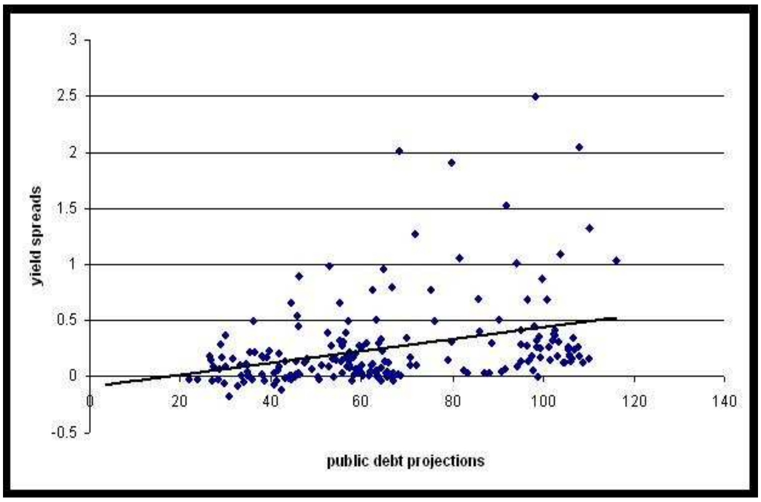

Figure 1: Debt level and yield spread

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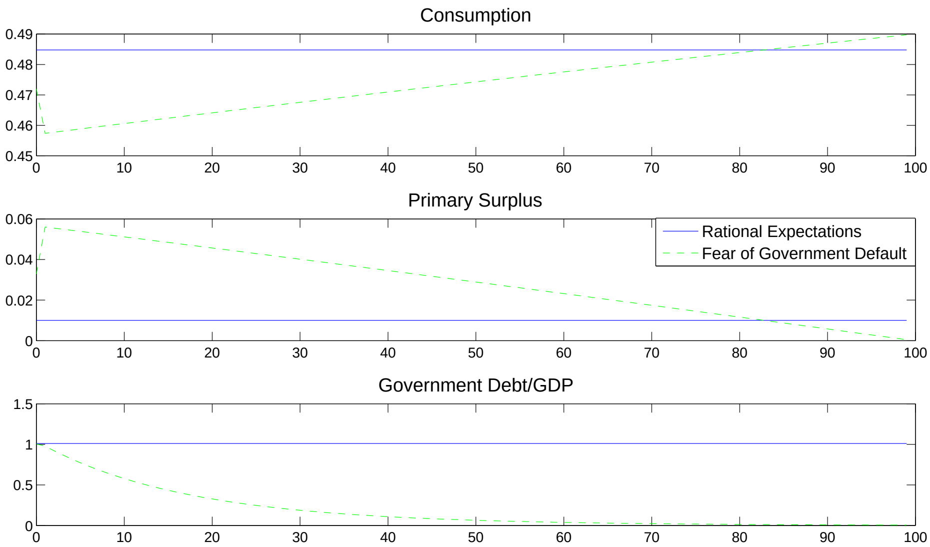

Figure 2: Rational Expectations vs. Fear of Default

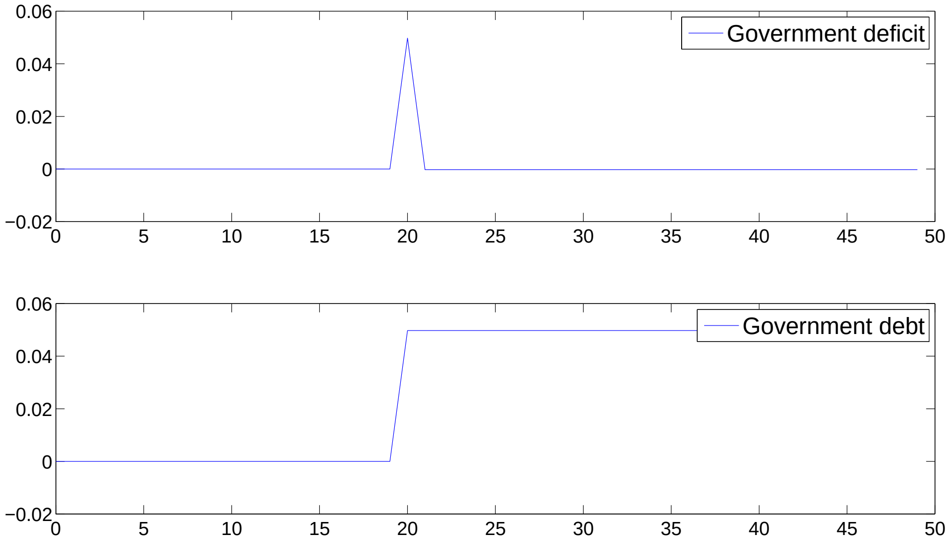

Figure 3: Optimal Response to a bad shock under Rational Expectations

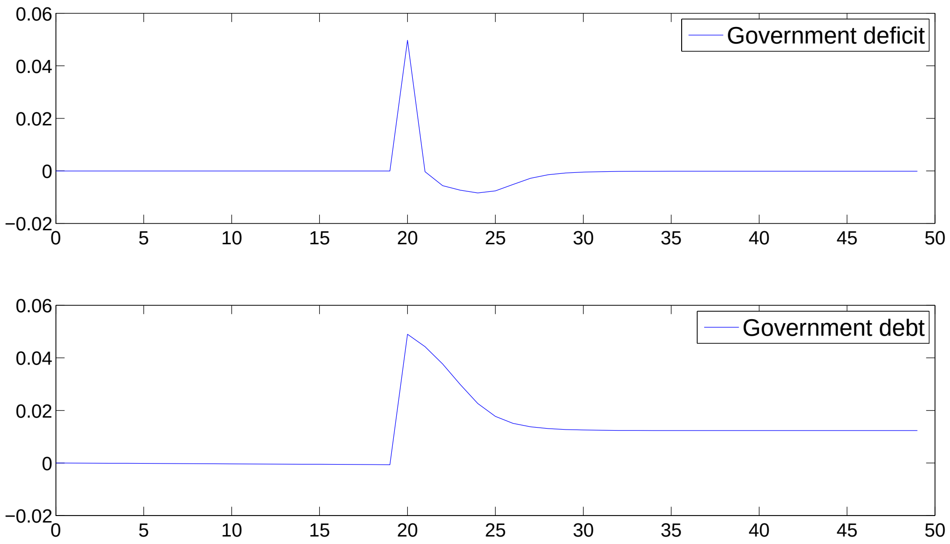

Figure 4: Optimal Response to a bad shock under Fear of Default

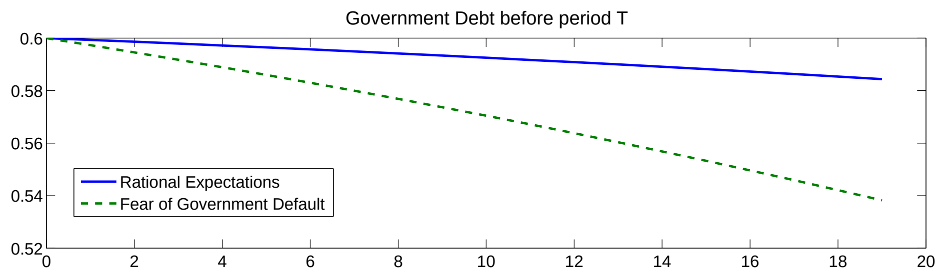

Figure 5: Optimal Froant-loading: Rational Expectations vs Fear of Default

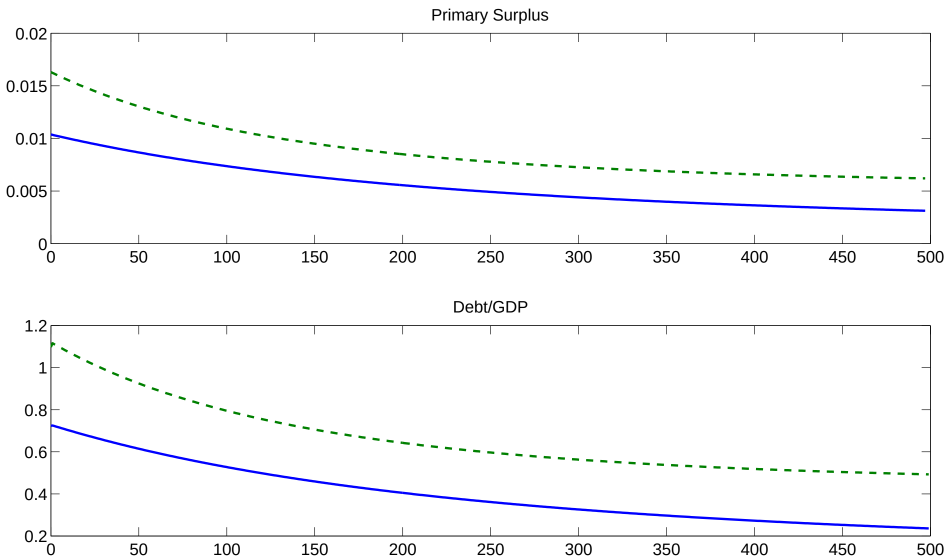

Figure 6: Germany vs. Italy