The Governance of Financial Intermediaries

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In this paper we investigate financial intermediaries, such as banks, life insurance companies and pension funds that are governed by their depositors. Using a simple OLG Diamond Dybvig model we find that the risk sharing capacity of such institutions is severaly limited due to the temptation to renegotiate the institution's asset buffer. Unless an institution can verify the age of its depositors, mutually owned intermediaries cannot improve on the market economy.

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1. Introduction

Providing insurance against liquidity risk is one of the main services that financial intermediaries are purported to provide. Seminal papers of Edgeworth (1888), Bryant (1980) and Diamond and Dybvig (1983), show how coalitions can share risk in economies were production comes with gestation lags and assets are illiquid. Diamond and Dybvig (1983) show that if agents face the risk of having to consume before a long term asset pays off, a coalition, interpreted as a bank, is able to offer a consumption schedule in which early diers are *ex post* subsidized by late diers. Jacklin (1987) and Bhattacharya and Gale (1987) however show that the presence of a market constrains the risk sharing ability of intermediaries. An extensive literature now exists on bank risk sharing.[1]

An important strand in the risk sharing literature considers the role of financial intermediaries in overlapping generations (OLG) economies. A common feature of the studied models is that financial intermediaries hold buffers of liquid and illiquid productive assets to smooth consumption (Allen and Gale, 1997), or to take better advantage of productive technologies (Qi, 1994).

In this paper we argue that the buffer holding intermediaries are potentially subject to a renegotiation of the stationary payout and investment rules governing the institution. We model the governance of perpetual institutions by letting depositors change the investment and payout schedules. We show that if the intermediary's asset buffer becomes to large, depositors may unanimously decide to liquidate and ostracize future generations. If we take the overlapping generations concept seriously, and do not rely on altruism or an infinitely lived enforcement mechanism, we need to consider the threat of such liquidations.

Payoff renegotiations are not confined to banks. They loom in all infinitely lived financial intermediaries that transfer assets between agents of different generations. Insurance

[1] E.g. Haubrich and King (1990), Hellwig (1994) and von Thadden (1997, 1998) provide additional critique on Diamond Dybvig risk sharing. Wallace (1988), Gorton and Penachi (1990), and Diamond (1997) suggest conditions under which banks can offer superior risk sharing.

mutuals, defined benefit pension plans, social security schemes and endowment funds are particularly prone to renegotiations. Such coalitions regularly see renegotiations, also in periods when their assets are at unusually high levels. Indeed, the recent failure of Equitable Life, the world's oldest life insurance firm, and the pension fund crisis that is currently plaguing many industrialized economies are generally attributed to excessive generosity during good times.[2]

To illustrate the consequences of payoff renegotiations, we study an overlapping generations model in which periodically born generations can invest their endowments in a two-period productive technology that returns $R > 1$, or in a storage technology. Each generation's agents face liquidity risk in that they may either live either one or two periods. This model, sometimes referred to as the OLG Diamond Dybvig model, has been studied by Qi (1994), Bhattacharya and Padilla (1998), and Fulghieri and Rovelli (1998), among others. Like abovementioned papers we analyze under what conditions a coalition can improve on the allocation that obtains in a market economy.

To answer this basic question we first look for the market equilibrium in which secondary claims on one year old projects are traded between different generations. We corroborate the stationary benchmark equilibrium allocation $\{\sqrt{R}, R\}$ established by earlier papers, and then look for an arrangement in which agents transfer their endowments to a coalition that maximizes agents' *ex ante* utility. The problem is the natural lack of constraints for the coalition's maximization program. Without constraints, the optimal mechanism accumulates unlimited wealth and pays out unlimited amounts to an infinite sequence of early and late-diers. To deal with this OLG paradox, we assume decreasing returns to scale on the productive technology.[3]

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[2] See for example, "Equitable Life: The blame game" (The Economist, April 14th, 2005, p. 70) "When the spinning stops - actuaries and the pensions crunch" (The Economist, January 26th 2006, p. 75)

[3] The problem of an infinite equilibrium space, typical of boundless OLG models (Shell, 1971), has received ample attention in the literature. See e.g. Hendricks et al., (1980), Esteban and Milan (1990), Esteban and Sakovics (1993), or Prescott and Rios-Lull (2000).

The novel feature of this paper is that we specifically model the governance structure of the intermediary. We assume that intermediaries are governed by their members, and who periodically vote for alternative payoff- and investment- regimes.

We define a (perpetual) financial intermediary as a combination of an asset buffer, and a *mechanism* that stipulates periodic investment and depositor payoffs. Between periods, a randomly chosen depositor is given the right to propose an alternative feasible mechanism. We assume that a feasible mechanism is accepted if it receives unanimous support from the living depositors.

We show that the temptation to liquidate the assetbuffer to the benefit of the living depositors constrains the risk sharing ability of the intermediary. In particular, we identify a renegotiation constraint and a convex permissible set of allocations where the institution will not be raided.

We find that the market allocation is on the boundary of the permissible set. However, the market allocation may not be the welfare maximizing allocation within the permissible set. We show that a coalition can achieve an allocation that is superior to the market allocation if it can prohibit the rolling over and selling of deposits.

Our renegotiation constraint is similar to the no restart condition that Prescott and Rios-Rull (2000) suggest as a solution to the empty 'core' paradox of OLG economies (Esteban, 1986). Their condition requires that no young generation should be better off by ostracizing old generations and restarting the economy. Prescott and Rios-Rull call their restart-proof equilibrium an *organizational equilibrium*. In our model, parent coalitions are in control, and the danger looms that future generations are abandoned.

Apart from the bank raid constraint our analysis also finds that the market economy allocation is inherently cyclical.[4] The one-periodic stationary allocation studied in the literature is merely a special case, of a wide set of endogenously cyclical equilibriums that are only governed by the no-arbitrage condition.

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[4] Bhattacharya, Fulghieri and Rovelli (1998) mention the existence of one such cyclical equilibrium, $\{(1,R),(R,R)\}$, in a footnote.

Other papers that analyze the merits of a banking system in OLG Diamond Dybvig economies include Qi (1994), Bhattacharya and Padilla (1996), and Fulghieri and Rovelli (1998). These papers all (silently) assume that returns to scale imply that the Golden Rule investment level is unity. In addition, Qi (1994) suggests a roll-over constraint, and finds that the equilibrium bank offers an allocation of the form $\{r,r^2\}$ where $r > \sqrt{R}$. Bhattacharya and Padilla (1996) however point out that if interbank deposits are allowed, Qi's allocation is not sustainable, because coalitions would invest with each other instead of in the technology. They then show that a government that taxes late diers and subsidizes newborns can achieve allocations superior to the market allocation, and even the first best allocation.[5] Fulghieri and Rovelli (1998) show that the first best allocation can also be achieved if intermediaries can discriminate on age.

In this paper we argue that neither taxation nor age verification is sufficient to ensure the first best allocation. The problem with the allocations suggested in the extant literature is that they are not renegotiation proof. In a genuine overlapping generations economy there is no long-lived enforcer who can prevent the living coalition members from raiding the coalition's assets.

Other papers that study the OLG Diamond Dybvig model are Bencivenga and Smith (1991) and Dutta and Kapur (1998) who are concerned with economic growth. In Bencivenga and Smith (1991) newborns are endowed with labor only, and equilibrium wages grow perpetually because a fixed proportion of it is invested in capital goods. In their model, banks accelerate growth by providing intragenerational risk sharing. Dutta and Kapur (1998) assume that due to adverse selection, entrepreneurs cannot sell unfinished investments and want to hold a liquid security (bank-notes or 'money') alongside the illiquid assets. In an inflationary environment asset-backed bank-money is the optimal means of payment, as it avoids the negative return on fiat money.

The next section describes the OLG economy. In section 3 we examine the market economy equilibrium. We need to do this to provide a benchmark for the coalition

[5] Bhattacharya and Padilla (1996) consider different tax-subsidy regimes. A subsidy that is linear in investment substantially improves welfare, but it cannot achieve the first best allocation. To achieve first best, an age dependent tax-subsidy scheme is required.

equilibrium, and to analyze the agents' outside option. In section 4 we describe the coalition, including its governance structure, and analyze the equilibrium. Section 9 concludes. Proofs are in the appendix.

2. The OLG Diamond Dybvig Model

The object of our study is an infinite horizon economy with a boundless sequence of overlapping generations of atomistic agents. A new generation, of size normalized to one, is born on every date. Agents are born with an endowment of one unit of a homogeneous good that can be used for consumption or as an input for production. Agents who enter the economy at date t can be of two types: with probability ε agents are *impatient* and live for one period only; with probability 1-ε they remain *patient* and live for two periods. Impatient agents born at t consume at t+1. Patient agents consume at t+2. All agents have expected utility preferences, with an instantaneous utility functions $U(\cdot)$ that is increasing, strictly concave and twice-continuously differentiable. Agents born at date t learn their type after t but before t+1. Types may be verifiable. We assume that the population is large enough so that there is no uncertainty on the aggregate distribution of agents in the population.[6] Hence, at any date t, the population contains 3-ε agents: 1-ε patient agents born at t-2, 1 agents born at t-1 who know their type, and 1 newborns who do not know their type yet. In between dates there are 2-ε agents: 1 young, and 1-ε old.

The economy is endowed with two technologies to produce goods over time. The first technology, *storage*, allows agents to costlessly transfer consumption from one period to the next. The second technology, the *long term* technology allows agents to convert one unit of consumption at date t into R>1 units of consumption at t+2. This technology cannot be interrupted at t+1. The maximum amount that can be invested, per period, in the long term technology is X>1. In the following we will look for equilibrium *allocations* $\{C_t^1, C_t^2\}_{t\in Z}$, where C_t^i denotes the consumption of i-year olds at time t.

[6] This is usually justified in terms of the law of large numbers. Duffie and Sun (2007) provide a rigorous formulation of independent random matching for a continuum population such that the law of large numbers holds exactly.

2.1. Pareto Efficiency

In this economy, the social optimal allocation has $C_t^1 = C_t^2 = X(R-1)+1$, for all t. This allocation is attained by periodically investing the maximum, *Golden Rule*, amount X in the long term technology. The periodic consumption can be found by subtracting from the periodic good inflows, XR+1 (XR from production and 1 from endowments), the periodic investment outflows X.

If the economy has a starting date, the Pareto optimal allocation cannot be obtained immediately even if we can somehow force the consumption of early diers to equal the consumption of late diers. The reason for this is that the first generation does not have enough endowment to invest, to generate the Pareto optimal consumption for all generations. To obtain Pareto optimality early generations need to build an asset buffer to obtain the optimal periodic investment. Although startibility is not the central theme of our paper, it will be discussed later in this paper.

3. The market economy

In this section we analyze the equilibrium allocation in an economy where agents of different types and generations trade securities that are backed by one period old investments in the production technology. We shall call these securities *projects*. In the following we let p_t denote the date t price of a project started with one unit of consumption good at t-1, and which hence pays R goods at t+1. Due to their atomistic nature, agents are price takers.

In the market economy agents have access to three investment vehicles: the production technology, projects, and storage. They can invest when born, and when they remain patient on their first birthday. We define the key decision variables as follows: x_t^0 is the number of projects bought by a newborn agent at date t, x_t^1 is the number of projects bought by a patient one-year-old agent at date t. Similarly, y_t^0 and y_t^1 denote the amount invested in the production technology by newborn and patient one-year-olds respectively, while z_t^0 and z_t^1 denote the amounts stored by newborn and patient one-year-olds at t.[7] Naturally, all surviving agents choose their investments so as to maximize utility. We formulate an agent's problem recursively, and start the analysis with the problem of a patient one year old. Let $m_t^1 = x_{t-1}^0 R + y_{t-1}^0 p_t + z_{t-1}^0$ denote the wealth, in number of goods, of a one year old agent at t. We further define the value function $V(m_t^1)$ as the maximum expected utility a patient one year old can obtain as follows:

$$V(m_t^1) = \max_{x_t^1, y_t^1, z_t^1} U(x_t^1 R + y_t^1 p_{t+1} + z_t^1) \tag{1}$$

$$m_t^1 \geq x_t^1 p_t + y_t^1 + z_t^1 \tag{2}$$

The maximand in (1) is the patient agent's utility from consumption $U(C_{t+1}^2)$, expression (2) is her budget constraint. Because impatient agents consume on their first birthday, we have that the newborn's maximization problem is:

$$\max_{x_t^0, y_t^0, z_t^0} \varepsilon U(x_t^0 p_{t+1} + y_t^0 + z_t^0) + (1-\varepsilon) V(x_t^0 p_{t+1} + y_t^0 + z_t^0) \tag{3}$$

$$1 \geq x_t^0 p_t + y_t^0 + z_t^0 \tag{4}$$

The arguments in the U() and V() functions in (3) is a t-born agent's first birthday wealth, m_{t+1}^1, expression (4) is his/her budget constraint. In equilibrium, all agents always maximize expected utility and markets clear. The market clearing condition requires that the aggregate investment on date t equals the aggregate holdings of projects on date t+1:

$$\boldsymbol{y}_t \equiv y_t^0 + (1-\varepsilon) y_t^1 = x_{t+1}^0 + (1-\varepsilon) x_{t+1}^1 \quad \forall t \tag{5}$$

[7] Throughout this article, superscripts denote generations, and subscripts denote decision, transaction, and consumption dates.

We define a market equilibrium as follows:

Definition: An equilibrium for the market economy is a sequence of prices $\{p_t\}_{t\in Z}$ and investment decisions $\{x_t^0, x_t^1, y_t^0, y_t^1, z_t^0, z_t^1\}_{t\in Z}$ such that (i) for all t, all agents of all generations maximize their expected utility and (ii) markets clear.

There exist infinite stationary two-periodic equilibriums, with the following properties:

PROPOSITION 1 (Market Equilibrium):

In any market equilibrium we have, for all $t \in Z$:

(i) $p_t \in [1, R], p_{t+1} = \dfrac{R}{p_t}$

(ii) $C_t^1 = p_t,\ C_t^2 = R$

(iii) *If* $p_t > 1$ *then* $\boldsymbol{y}_t = 1 - \varepsilon \dfrac{R - p_t}{R - 1}$ *and* $z_t^0 = z_t^1 = 0$

Note that there is a continuum of equilibriums, all of which have two-periodic prices and allocations. The requirement that $p_{t+1} = R/p_t$ can be seen as a no arbitrage condition: the one period return on primary market investments must be equal to the one period return in the secondary market. In the interior equilibria ($p_t \in (1,R)$), aggregate investment is determinate, and there is no storage. If we impose that patient one-year olds do not invest in the production technology ($y_t^1 = 0 \ \ \forall \ t$) then the other asset allocation decisions (y_t^0, x_t^0, x_t^1) are determinate and strictly positive. In the appendix we show that the corner price equilibrium with $p_t = \{..,1,R,1,..\}$ can be supported by infinitely many investment processes, and storage may obtain.

Although the OLG Diamond Dybvig model has been studied in the literature, the inherent two-periodicity has not been documented before.[8] Previous papers that investigate the

[8] Bhattacharya, Fulghieri and Rovelli (1998) mention mention two price equilibriums, the one-periodic one ($p_t = \sqrt{R} \ \ \forall \ t$), and the corner equilibriuim $p_t = \{..,1,R,1,..\}$. We show that any price

OLG DD model focus on the one-periodic special equilibrium, where $p_t = \sqrt{R}\ \forall\ t$, which offers all agents an allocation of $\{C_t^1, C_t^2\} = \{\sqrt{R}, R\}$. We assume that it is the social attractiveness of the one-periodic equilibrium that lead Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) to disregard the cyclical equilibriums. Indeed, as long as agents' relative risk aversion coefficient is higher than unity, the one-periodic equilibrium offers the highest overall welfare. Note however that the market equilibrium cannot offer the Pareto optimal allocation, and that the *Golden Rule* investment level, $\boldsymbol{y}_t = X$, is not obtained.

3.1. The startable market equilibrium

If the economy has a starting period ($t = 0$), the first generation determines which of the above mentioned stationary equilibria is played. Since there is no secondary market at date zero, and the only alternative to investing is storing, the first generation solves:

$$\max_{y_0} \varepsilon U\left((1-y_0)+y_0 p_1\right)+(1-\varepsilon)U\left((1-y_0)\frac{R}{p_1}+y_0 R\right) \tag{3}$$

The first derivative of (3)'s maximand is positive for all $p_1 > 1$, which means that p_1 cannot be greater than unity, because it would entice all agents to invest their entire endowment in the technology. For a startable equilibrium we thus find:

PROPOSITION 2 (startable market equilibrium)
In the $Z^+ \cup \{0\}$ *economy we have* $p_{odd} = 1$, $p_{even} = R$, $y_0 = y_{even} = 1$, $y_{odd} = 1-\varepsilon$.

Proposition 2 shows that even though the agents' concave utility function makes the one-periodic stationary equilibrium the most desirable from an overall welfare perspective, it is the least desirable, most cyclical, equilibrium that obtains.

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process with $p_t p_{t+1} = R$ is an equilibrium price process.

4. The coalition

We now investigate whether a coalition can improve on the market allocation. Following the literature, we assume that a coalition offers its members, in exchange for their endowment, a demandable debt security that can be exchanged for r_1 by impatient depositors on period after making a deposit, or for r_2 by patient agents after two periods. In this paper we specifically model the governance of the institution. In particular we assume that it is governed by its living members, who periodically decide on the investment and the pay out schedule $\{r_1,r_2\}$. In the following we shall limit our attention to stationary coalitions, and denote their periodic investment *y*. A coalition is thus described by a vector $\{y,r_1,r_2\}$. Given that it is governed by its ex-ante identical depositors, any equilibrium solves:

$$\max_{y,r_1,r_2} \; \varepsilon U(r_1)+(1-\varepsilon)U(r_2) \tag{4}$$

We shall limit our analysis to institutions who make promises $\{r_1,r_2\}$ that are feasible in the short and long term and conjecture that institutions (if there are more than one), are of constant size.[9] Hence, maximization problem (4) is subject to the *internal budget constraint*:

$$\varepsilon r_1 + (1-\varepsilon)r_2 + y \leq 1 + yR \tag{5}$$

and the *external budget constraint*:

$$y \leq X \tag{6}$$

The left hand side of (5), which will naturally be binding in a stationary equilibrium, gives the period outflows, to impatient and patient depositors, and for investment. The right hand side gives the periodic inflow, from new depositors and from maturing projects. Between periods, the coalition holds 2*y* projects: *y new* projects that are recently

[9] That is, we rule out Ponzi schemes. The reason that coalition sizes are constant is that they have asset buffers with stationary pay-off vectors which cannot be offered to a increasing number of depositors, without diluting the current depositors.

initiated, and *y mature* projects that are about to pay off. The external budget constraint (6) follows from our simple modeling of decreasing returns to scale.

If we solve (4)-(6) we will see that the external constraint (6) will bind. This implies that in the intermediated economy the first best allocation may be obtained. However, in the next section we will see that the temptation of the institutions' living members to liquidate and distribute the institutions' assets leads to additional constraints on the intermediary's risk sharing potential.

4.1. The coalition's governance structure

Following the incumbent literature, we treat the coalition as a *mechanism* and an asset buffer. In the stationary Diamond Dybvig model, the coalition only determines the withdrawal rules, which are unanimously agreed upon, at the foundation. Because the withdrawal rights cannot be renegotiated once established, the DD-bank can be interpreted as an automatic cash dispenser.[10]

For an OLG economy, the cash-dispenser interpretation is problematic because OLG institutions must not only dispense cash, but also offer payment schedules to future generations, accept new deposits, and make additional investment decisions. Up until this point, it was assumed that these decisions were taken so as to provide welfare to all the institution's (present and future) depositors. In practice, such institutions (banks, insurance companies, governmental organizations, pension funds, *etc.*) have some kind of internal organization that governs how those decisions are made, and in particular, many such institutions give their depositors (explicitly or implicitly) some degree of control over the functioning of the institution. Our focus is on those institutions that are exclusively governed by their living members.

The main decisions of these institutions concern the uses of the investment payoffs (how much to allocate to depositors and how much to reinvest), how to raise additional depositor financing (what to offer to potential new depositors). The primary source of conflict within perpetual organizations derives from the conflict of interest between the

[10] The cash dispenser interpretation was first suggested by Wallace (1988).

current and future members. Depositors of an institution with a large asset buffer will be tempted to use their control rights to liquidate the buffer and divide the proceeds amongst themselves.

To model the coalition's control rights we assume that in between payoff dates, general depositors' meetings take place, in which every depositor has a vote proportional to their initial investment. We study whether a randomly appointed chairman can suggest an alternative liquidating payoff vector $\left(r_2^t, r_1^t, r_2^{t+1}\right)$, designating the payoffs in period t to the two year olds and to the impatient one year olds, and in period t+1 to the two year olds, which will obtain unanimous support.

The alternative payoff schedule needs to be *feasible*, which means that it can be financed by the dividends of terminated projects and a sale of the intermediate assets owned by the institution. In the following we let ξ stand for the number of intermediate projects sold by a renegotiating institution, and $p(\xi)$ the price clearing price per project obtained.

Clearly, if there exists a feasible liquidating vector which Pareto improves the welfare of the living depositors, the institution that gives rise to such a renegotiation is not sustainable in equilibrium. Hence we define a stationary coalition equilibrium as follows:

DEFINITION: A stationary renegotiation proof coalition equilibrium is characterized by a vector $\left(y^*, r_1^*, r_2^*\right)$ that meets the following conditions:

$$\varepsilon r_1^* + (1-\varepsilon) r_2^* = 1 + y^*(R-1)\,, \tag{7}$$

$$\varepsilon U(r_1^*) + (1-\varepsilon) U(r_2^*) \quad \geq \quad \varepsilon U(r_1^t) + (1-\varepsilon) U(r_2^{t+1}) \tag{8}$$

and $$U(r_2^*) \quad \geq \quad U(r_2^t) \tag{9}$$

$\forall\ r_2^t,\ r_1^t,\ r_2^{t+1}$ that meet the following feasibility constraints:

$$(1-\varepsilon) r_2^t + \varepsilon r_1^t \quad \leq \quad y^* R + \xi p(\xi) \tag{10}$$

$$(1-\varepsilon) r_2^{t+1} \quad \leq \quad \left(y^* - \xi\right) R \tag{11}$$

$\forall\ \xi \in [0, y^*]$

Equations (7) and (8) require that is impossible to make all coalition members better off by staging a feasible bank raid. The set of feasible deviations is constraint by the coalition's existing asset base, characterized by y^*, and the market outside the coalition. The left hand side of inequality (10) denotes the maximum payout in the period immediately following a renegotiation, for a given number of intermediate projects sold ξ. Inequality (11) gives the maximum payout, to patient two year olds, two periods after the renegotiation.

Notice that our equilibrium definition specifies equilibrium allocations instead of equilibrium strategies. Naturally, the equilibrium strategies that support the equilibrium allocation are that the chairman only suggests proposals that will be accepted and that agents only vote in favor of a proposal if it is feasible and makes them weakly better off.

Clearly, the temptation to renegotiate constrains the coalition's allocation. We shall call the resulting constraint the *renegotiation constraint*. Essentially, this constraint requires that the chairman should not be able to profit from making the following proposal:

> *"Next period, all two-year olds receive r_2 goods. One-year olds can choose between $\frac{r_2}{R}$ projects or r_1 goods. Anything left over is for me. "*

Clearly, all agents would (weakly) accept the above proposal. Under the proposal, the young will self-select into patient and impatient types at the next date, and will consume r_1 if impatient, and r_2 if patient, the same as their scheduled consumption in the coalition. Hence, in case of a renegotiation, the two-year olds would be left with a minimum of $yR - \varepsilon r_1$ goods and $y - (1-\varepsilon)\frac{r_2}{R}$ projects. Since the projects need to be exchanged for consumption goods we denote:

$$\xi \equiv y - (1-\varepsilon)\frac{r_2}{R} = \frac{\varepsilon r_1 + (1-\varepsilon)r_2 - 1}{R-1} - (1-\varepsilon)\frac{r_2}{R} = \frac{\varepsilon R r_1 + (1-\varepsilon)r_2 - R}{R(R-1)} \tag{12}$$

To arrive at the final term, we eliminate y from by using the binding internal budget constraint (5). To avoid a renegotiation, the proceeds of this sale together with the left over goods must be less than the scheduled payment under the coalition. Hence, the renegotiation constraint can be written as:

$$\xi p(\xi) + yR - \varepsilon r_1 \leq (1-\varepsilon) r_2 \tag{13}$$

The price $p(\xi)$ that a coalition can achieve for its intermediate assets depends on the industry structure. The simplest case is that of a competitive economy. In such an economy, the price that a renegotiating institution can fetch for its intermediate projects is the reservation value for depositors of competing institutions. If however, the coalition is a monopolist, the only agents a renegotiating coalition can sell intermediate goods to are the newborns. We will consider this case in the next section. First we consider an economy with many coalitions, each of small size and types are not verifiable.

4.2.. The renegotiation constraint in a competitive economy.

In a competitive economy with many coalitions where types are not verifiable, the hypothetical sale price of a coalition's intermediate projects is determined by the shadow price for one-year investments for patient depositors of other institutions. Or, the price a coalition can fetch must be such that patient members of other institutions are indifferent between withdrawing early and acquiring intermediate projects or staying at their institution. If they take the former action, their payoff is $\frac{r_1}{p} R$, if they stay with the institution, they consume r_2. Hence we find that in an economy with many coalitions we have

$$p(\xi) = \frac{r_2}{r_1} R . \tag{14}$$

Substituting this, (5) and (12) in (13) gives, after some algebra:

PROPOSITION 3: (renegotiation-proof banking with competition)

If, conditional on unanimous support, coalition members can renegotiate a financial intermediary and sell its assets, the institution's payoff schedule is limited to a convex permissible set of (r_1, r_2) *combinations described by:*

$$r_2 \leq \frac{1}{2(1-\varepsilon)}\left(R - r_1 + \sqrt{R^2 + r_1^2 + 2Rr_1(1-2\varepsilon) - 4Rr_1^2\varepsilon(1-\varepsilon)}\right) \tag{15}$$

The one periodic market allocation $\{\sqrt{R}, R\}$ *is on the frontier of this set.*

The key insight of proposition 3 is that the threat or renegotiation poses a serious threat on the buffer holding capacity of intermediaries. Figure 1 illustrates the renegotiation constraint and constraint optimal allocation alongside the market allocation and several allocations suggested in the literature, for parameter values $R = 4$, $\varepsilon = \frac{1}{3}$, X = 1.2, and CRRA coefficient $\gamma = 4$.

--- Figure 1 around here ---

The intermediary's external budget constraint, is denoted by *a*. This constraint, given by the line segment $\varepsilon r_1 + (1-\varepsilon) r_2 \leq 1 + X(R-1)$, can be interpreted as a budget constraint for a hypothetical infinitely lived welfare maximizer. Point A gives the Pareto optimal allocation.

Curve *b* depicts renegotiation constraint given by proposition 3. On this line we find the market allocation, denoted *M*, and the renegotiation constrained optimal allocation *B*. Notice that in order to achieve *B*, intermediaries need to be able to rule out *roll over arbitrage*, the practice of withdrawing r_1 after one period, and reinvesting for another period. If this practice cannot be ruled out, coalitions can only offer allocations above line *c*. Qi (1994) suggested that for banks, the roll over cannot be ruled out, so that only *c* and *a* would bind, giving rise to allocation *C*. However our analysis shows that allocation *C* is not renegotiation proof.

Bhattacharya and Padilla (1996) also argue that allocation *C*, which has $r_2 > R$, cannot be obtained in a contestable market because it would invite competing coalitions to invest with each other instead of in the production technology. Bhattacharya and Padilla then show that *C* can still be attained if there is a government that offers proportional investment subsidies and taxes income. If the government can offer age-independent subsidy for the optimal level investment, allocation D can be obtained, and if it can condition the subsidy on the optimal investment level, it can enforce the Pareto optimal allocation A.

As can be seen from the figure, none of the three government transfer schemes suggested by Bhattacharya and Padilla (1996) are renegotiation proof. As long as the renegotiation constraint d lies to the left of the external budget constraint, a motion that calls for an immediate cancellation of a (Bhattacharya Padilla) tax-subsidy scheme would gain the vote of the entire living population, and unravel allocations A, B and C. This is because between periods all living agents have already received the subsidy, while only future, unborn generations benefit from subsequent subsidies.[11]

Fulghieri and Rovelli (1998) show that allocation E can be obtained if coalitions can condition payoffs on the age of the depositor. E is the best allocation attainable within the interbank arbitrage constraint d, which requires $r_1, r_2 \leq R$.[12]

A key result of our analysis is that none of the allocations suggested in the extant literature are renegotiation proof. Notice that also the allocations of Bhattacharya and Padilla are problematic. Because they are enforced by an exogenous government. However, it is reasonable to believe that also the government is ruled, at the end of the day, by mortal agents. Clearly a motion abandon taxes and stop subsidies will receive support from the entire living population.

One reason behind the fragility of the coalitions is that they operate in a competitive market. In the next section we will consider a monopoly intermediary. We will see that also in such a setting, a renegotiation constraint will apply.

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[11] Even if newborns can vote too, a motion could be crafted that gains their support. Such a motion would call for a reduced subsidy (financed by a reduced tax) on the following date, and a complete cancellation of the tax-subsidy scheme thereafter.

[12] In their paper allocation E coincides with allocation A, because they assumed X = 1. In our view, line d does not necessarily pass through A. Whether allocation E provides more or less intergenerational welfare than allocation C or D depends on the model's parameters, in particular on the maximum profitable investment X.

4.3. A monopoly intermediary.

As established in beginning of this section, the key to the renegotiation proof criterion, is, as before, the $p(\xi)$ function, which denotes the price that a coalition can obtain when they liquidate ξ intermediate projects to the benefit of its current members. In the monopoly case this price is more complicated to derive than in a competitive market because now the only potential buyers of second-hand intermediate products are the newborns. If a large monopolist institution offers second hand one year projects for sale to newborns, the price per project will depend on the amount of projects offered. The following lemma gives the per project price that a coalition can fetch by selling ξ projects to newborns.

LEMMA

If ξ_t projects are sold to a population of newborns, of size one, the clearing price per project will be:

$$p = \frac{\varepsilon R}{\xi(R-1)+\varepsilon} \quad \text{if } \xi \leq \varepsilon \tag{16a}$$

$$p = 1 \quad \text{if } \varepsilon < \xi \leq 1 \tag{16b}$$

$$p = \frac{1}{\xi} \quad \text{if } \xi > 1 \tag{16c}$$

Clearly, the price decreases in the supply. The derivation of (16) (see appendix) follows from proposition 1 and 2. First it is established that the newborns will not store but spend their entire endowment on investing in the technology and buying projects. Then we derive the equilibrium price and quantities that maximize individual utility and clear markets. We find that if fewer than ε projects are offered at $t = 0$, the subsequent market equilibrium is two-periodic with $p_0 = p_{2i} = \frac{\varepsilon R}{\xi(R-1)+\varepsilon}$ and $p_1 = p_{1+2i} = \frac{R}{p_0}$ for all $i \in Z^+$. If $\varepsilon < \xi < 1$, the subsequent price process follows $p_{2i} = 1$, $p_{1+2i} = R$. If $\xi > 1$, the post-raid price process is given by $\{\frac{1}{\xi}, R, 1, R, ...\}$.

To find the set of renegotiation constraint for stationary insurance monopolists, we substitute the price function (16) into (13). After some algebra, the resulting inequality gives a constraint on the schedules (r_1, r_2) that intermediary can offer without exposing itself to a renegotiation. We find:

PROPOSITION 4: (monopolist's renegotiation constraint)

If, conditional on unanimous support, coalition members can renegotiate a financial intermediary and sell its assets, the institution's payoff schedule is limited to a convex permissible set of (r_1, r_2) *combinations described by:*

$$r_2 \leq \min\left(R + \varepsilon \frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)}, \frac{R}{1-\varepsilon}\left(1 - \frac{2\varepsilon r_1}{1+R}\right)\right) \tag{17}$$

The one periodic market allocation $\{\sqrt{R}, R\}$ *is on the frontier of this set. For* $r_1 > \sqrt{R}$, *the line given by* (17) *lies above the line given by* (15).

Equation (17) shows that the renegotiation constraint is the most restrictive of two decreasing $r_2(r_1)$ lines in R_+^2 that crosses the *y*-axis in $r_2(0) = \frac{R}{1-\varepsilon}$. It can be established that *iff* $\varepsilon < \frac{1}{2}$, the frontier has a kink.

Renegotiation constraint (17) implies that also a monopolist, such as a government institution, cannot improve on the market allocation $\{\sqrt{R}, R\}$ if it cannot discriminate on age (to avoid roll over arbitrage). However, if age is verifiable, a centralized intermediary can offer higher welfare than the market economy, and, if agents are sufficiently risk averse, higher welfare than an economy with competing coalitions. The latter conclusion derives from the fact that the renegotiation constraint (17) lies above renegotiation constraint (15). This is also illustrated in Figure 2, which depicts both renegotiation constraints.

--- Figure 2 around here ---

Figure 2 illustrates the renegotiation constraint for a monopolist intermediary for the same parameter values as Figure 1. Renegotiation constraint (17) is given by line *e*, and the renegotiation constrained optimal allocation – assuming that the coalition can discriminate on age – by point *E*. As can be seen, the monopolist intermediary can obtain better risk sharing than the competing coalitions, but the resulting equilibrium allocation stills falls significantly short of the Pareto Optimal outcome.

4.4. Equilibrium

So far we only characterized the renegotiation constraints, and the constrained optimal allocations. However, more renegotiation proof coalition equilibriums exist, even though not all allocations in the permissible sets fulfill the equilibrium requirements. First we observe that allocations to the left of line *b* (and *e*, for the monopolist) cannot be equilibria because the depositors can rearrange the payoffs for the young generation without liquidating. This is the case because internal budget constraints have slopes larger than the renegotiation constraints (15) and (17). See figure 3. Similarly, allocations on *b* that lie above *B*, (on *e*, above *F* for the monopoly case), cannot be equilibria because they are associated with higher asset buffers *y* than the optimal allocation *E*. To refine the set of equilibrium allocations we observe that a stationary coalition equilibrium must solve:

$$\max_{r_1, r_2} \ \varepsilon U(r_1) + (1-\varepsilon) U(r_2) \tag{18}$$

subject to internal budget budget constraint (5) and bank raid constraint (15) or (17).

That is, an equilibrium coalition allocation must maximize the expected utility of the young generation, subject to the budget constraint that comes with the stationary investment level y^*. It can be shown that allocations (r_1, r_2) that are unconstraint by the renegotiation threat lie on the 45 degree line, so that the stationary coalition allocation lies either on the 45 degree line, or on the frontier of the permissible set, above the 45 degree line.

If we assume that the outside opportunity of newborns is the *intra*generational coalition, the set of *inter*generational coalition equilibrium allocations is bounded below by the allocation on the 45 degree allocation which offers agents the reservation utility they can obtain by playing intragenerational Diamond-Dybvig. The following propositions formalizes this observation:

PROPOSITION 5 (stationary equilibrium in a competitive coalition economy)

If a coalition can prevent patient members from rolling over their claims, but cannot prohibit bank raids or newborns from starting up an intragenerational coalition, a stationary coalition equilibrium allocation either satisfies:

$$r_2 \geq r_1,\ r_2 = \frac{1}{2(1-\varepsilon)}\left(R - r_1 + \sqrt{R^2 + r_1^2 + 2Rr_1(1-2\varepsilon) - 4Rr_1^2\varepsilon(1-\varepsilon)}\right),\ \textit{and}\ r_1 \geq r_1^B, r_2 \leq r_2^B \tag{19}$$

or

$$r_2 = r_1,\ r_2 \leq \frac{1}{2(1-\varepsilon)}\left(R - r_1 + \sqrt{R^2 + r_1^2 + 2Rr_1(1-2\varepsilon) - 4Rr_1^2\varepsilon(1-\varepsilon)}\right),\ \textit{and}\ r_1, r_2 \geq r^G \tag{20}$$

Where $\{r_1^B, r_2^B\}$ *is the unique renegotiation constrained allocation that maximizes agents' expected utility in the competitive economy and* $\{r^G, r^G\}$ *is the unique symmetric allocation that offers an agent the same expected utility as the intragenerational Diamond-Dybvig coalition.*

PROPOSITION 4 (stationary equilibrium in a monopoly coalition economy)

If a coalition can prevent patient members from rolling over their claims, but cannot prohibit bank raids or newborns from starting up an intragenerational coalition, a stationary coalition equilibrium allocation either satisfies:

$$r_2 \geq r_1,\ r_2 = R + \varepsilon\frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)},\ \textit{and}\ r_1 \geq r_1^E, r_2 \leq r_2^E \tag{21}$$

or

$$r_2 = r_1,\ r_2 \leq R + \varepsilon\frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)},\ \textit{and}\ r_1, r_2 \geq r^G \tag{22}$$

Where $\{r_1^E, r_2^E\}$ is the unique renegotiation constrained allocation that maximizes agents' expected utility in the monopoly economy, and $\{r^G, r^G\}$ is the unique symmetric allocation that offers an agent the same expected utility as the intragenerational Diamond-Dybvig coalition.

The second component of (21) and (22) is the bank raid constraint. It contains only the first component of (17), because only this component is binding above the 45 degree line.

--- Figure 3 around here ---

Figure 3 illustrates the set of potential equilibrium allocations if $R = 4$, $\varepsilon = \frac{1}{3}$ and $\gamma = 4$, for a coalitions that can avoid roll-over arbitrage. The thick bold line identifies the equilibrium allocations. The thin downward sloping straight lines are the internal budget constraints associated with coalition buffers y. Line f gives the budget constraint of the intragenerational DD coalition, and H marks the optimal allocation for $\gamma = 4$.[13]

5. Summary and discussion.

In this article we re-examine risk sharing in overlapping generations economies. Such risk sharing obtains through intergenerational trade or through financial intermediaries. Incumbent models in the literature show how financial intermediaries can improve on the allocation that obtains in an exchange economy by accumulating a buffer to smooth consumption or exploit productive technologies. We argue that such buffers may tempt contemporary agents to renegotiate the payoffs, to the detriment of successive generations.

We show that if unanimous member approval is sufficient to change the coalition's payoff schedule, a restrictive renegotiation constraint applies to the allocations that coalitions can offer their depositors. We characterize a *renegotiation constraint*, and find that most intergenerational coalition allocations suggested in the literature do not satisfy

[13] In the intragenerational DD equilibrium the coalition stores goods. The G allocation maximizes ex ante utility subject to the budget constraint $\varepsilon r_1 + \frac{1}{R}(1-\varepsilon) r_2 = 1$. See Diamond and Dybvig (1983).

it. Moreover, we find that the one-periodic market equilibrium is barely permissible in the face a bank raid threat.

If intermediaries' claims can be freely traded, or if agents are allowed to make early withdrawals, either to purchase seasoned projects or to open new deposits, coalitions cannot improve overall welfare vis-à-vis the market economy, even if interbank arbitrage is ruled out. Only coalitions that can prevent its members from making early withdrawals, or trade secondary claims can improve on the market equilibrium by carrying a buffer of long term projects between generations. If agents are sufficiently risk averse, the constrained optimal coalition redistributes wealth from early diers to late diers.

Although developed in a classic banking model with liquidity risk, bank raids apply to all forms of financial intermediaries where agents of different generations share risk. We believe that our results are particularly important for pension plans and social security schemes where asset return risk is shared between generations. Studies that analyze risk sharing in OLG models generally favor intergenerational risk sharing because financial systems can build up asset buffers, or borrow to intertemporally smooth consumption. Gordon and Varian (1988) hint at a limit of intergenerational smoothing due to lack of altruism. Allen and Gale (1997) point out that a buffering financial system is fragile because agents will abandon it as soon as it becomes underfunded. Our results suggest that even if agents can be forced to join underfunded coalitions, such institutions are still unable to improve welfare because overfunding will entice contemporaneous generations to carry out bank raids.

Appendix A: Proofs of propositions and lemma

PROOF OF PROPOSITION 1

Clearly, in the suggested set of equilibria all agents maximize expected utility, by (1) and (2). They are the only equilibria because iff $p_t p_{t+1} > (<) \; R$, the solution to (1) is a corner solution with $y_t = y_{t-1} = 1$ ($y_t = y_{t-1} = 0$), in which case (2) leads to $p_t = 0 \; (\infty)$, a contradiction.

To find the periodic investment process, replace $\frac{R}{p_{t-1}}$ by p_t in the market clearing condition (2), and solve for y_t. We find:

$$p_t = \frac{(1-y_t)+(1-\varepsilon)(1-y_{t-1})p_t}{\varepsilon y_{t-1}} \Leftrightarrow y_t = 1+(1-\varepsilon-y_{t-1})p_t \tag{A1}$$

Because aggregate investment has to be two-periodic too, we have

$$y_{t-1} = 1+(1-\varepsilon-y_{t-2})p_{t-1} = 1+(1-\varepsilon-y_t)p_{t-1} \tag{A2}$$

Substitute the rhs of (A2) into the rhs of (A1), then replace $p_t p_{t-1}$ with R, and rewrite:

$$y_t = 1-\varepsilon p_t - (1-\varepsilon)R + y_t R \tag{A3}$$

From which the investment process given in proposition 1 follows immediately. *Q.E.D.*

PROOF OF PROPOSITION 2

The first order condition for the first generation's maximization problem (3) is:

$$\varepsilon\left(p_1-1\right)U'\left(y_0 p_1+(1-y_0)\right)+(1-\varepsilon)\left(R-\frac{R}{p_1}\right)U'\left(y_0 R+\frac{(1-y_0)}{p_1}R\right) \tag{A4}$$

It is positive for all $p_1 > 1$. Hence the first generation invests $y_0 = 1$. The equilibrium price and investment process follows from proposition 1. *Q.E.D.*

PROOF OF LEMMA

First we prove that the first generation agents will not store but spend their entire endowment on buying the x projects and on investing in the technology. Denote z_0 the amount stored, and as before, denote y_0 the amount invested in the technology. The first generation solves:

$$\max_{y_0} \varepsilon U\left(z_0 + y_0 p_1 + (1 - z_0 - y_0)\frac{R}{p_0}\right) + (1-\varepsilon)U\left(y_0 R + \left((1 - z_0 - y_0)\frac{R}{p_0} + z_0 \right)\frac{R}{p_1}\right) \tag{A5}$$

Of which the first derivative with respect to z_0 is:

$$\frac{\partial E[U]}{\partial z_0} = \varepsilon\left(1 - \frac{R}{p_0}\right)U'(\cdot) + (1-\varepsilon)\frac{R}{p_1}\left(1 - \frac{R}{p_0}\right)U'(\cdot) \tag{A6}$$

Which is negative for all $p_0 < R$. This proves that the first generation does not store.

To find the clearing price in a liquidating sale, we first consider an interior solution in which all agents spend their entire endowment on projects and on investing. In such an equilibrium, y_t solves (1) for all t, so that $p_{odd} = \frac{R}{p_0}$ and $p_{even} = p_0$.

The market clearing conditions are:

$$p_0 = \frac{1 - y_0}{x} \tag{A7}$$

$$p_t = \frac{(1 - y_t) + (1-\varepsilon)(1 - y_{t-1})\frac{R}{p_{t-1}}}{\varepsilon y_t} \qquad \forall t > 0 \tag{A8}$$

Equation (A8) implies that y_t is two-periodic. From proposition 1 we know that:

$$y_t = 1 - \varepsilon\frac{R - p_t}{R - 1} \qquad \forall t > 0. \tag{A9}$$

From equation (A8) it also follows that $y_0 = y_{even}$ so that:

$$y_0 = 1 - \varepsilon \frac{R - p_0}{R - 1}. \tag{A10}$$

Substitute (A10) into (A7) to find expression (16a). If $x > \varepsilon$, (16a) results in $p_0 < 1$, and by proposition 1, to a $p_1 > R$. This in turn implies (from (A9)) that $y_1 > 1$, contradicting our assumption of an interior solution.

Trivially, if more than ε projects are offered, the price they fetch will not be less than unity ((16b) of lemma), unless more than one projects are offered. In the latter case all the goods of the first generation goes to buying projects, so that the price per project is $\frac{1}{x}$ ((16c) of lemma). *Q.E.D.*

PROOF OF PROPOSITION 3

We look for a function $r_2(r_1 \mid \varepsilon, R)$ so that (15) holds with equality. This expression is the frontier of the permissible set. If (16a) describes the price equation, we can find the frontier by substituting (16a) in (15):

$$\frac{\left(\frac{\varepsilon r_1 + (1-\varepsilon) r_2 - 1}{R-1} - (1-\varepsilon)\frac{r_2}{R}\right)\varepsilon R}{\left(\frac{\varepsilon r_1 + (1-\varepsilon) r_2 - 1}{R-1} - (1-\varepsilon)\frac{r_2}{R}\right)(R-1) + \varepsilon} + \frac{\varepsilon r_1 + (1-\varepsilon) r_2 - 1}{R-1} R - \varepsilon r_1 - (1-\varepsilon) r_2 = 0 \tag{A11}$$

Which can be written as a quadratic equation in r_2, of which the positive root is:

$$r_2 = R + \varepsilon \frac{\sqrt{4R^2 + r_1^2 (R-1)^2} - r_1(1+R)}{2(1-\varepsilon)} \tag{A12}$$

This gives us the first part of (17). If (16b) is the relevant price equation, we need to substitute this into (15). We obtain:

$$\frac{\left(\varepsilon r_1 + (1-\varepsilon) r_2 - 1\right)}{(R-1)} - (1-\varepsilon)\frac{r_2}{R} + \frac{\left(\varepsilon r_1 + (1-\varepsilon) r_2 - 1\right)}{(R-1)} R - \varepsilon r_1 = (1-\varepsilon) r_2 \tag{A13}$$

Which, after some algebra, becomes:

$$r_2 = \frac{R}{(1-\varepsilon)} - \frac{2\varepsilon R}{(1-\varepsilon)\left(1+R\right)} r_1 \tag{A14}$$

The second part of (17). Finally, if (16c) is the relevant price equation, we get for (15):

$$1 + \frac{\left(\varepsilon r_1 + (1-\varepsilon) r_2 - 1\right)}{(R-1)} R - \varepsilon r_1 \leq (1-\varepsilon) r_2 \tag{A15}$$

Which reduces to:

$$r_2 \leq \frac{1-\varepsilon r_1}{1-\varepsilon} \tag{A16}$$

Observe that the price-equation ((16a)-(16c)) can be written as:

$$p(x) = p_0(x) = \min\left(\max\left(\frac{\varepsilon R}{x(R-1)+\varepsilon}, 1 \right), \frac{1}{x} \right) \tag{A17}$$

Because the number of projects sold in a raid is $y - (1-\varepsilon)\frac{r_2}{R} = \ldots = \frac{R\varepsilon r_1 + (1-\varepsilon) r_2 - R}{R(R-1)}$, which increases in both r_1 and r_2, we may combine (A12), (A14) and (A16) to describe the permissible as follows:

$$r_2 \leq \max\left(\min\left(R + \varepsilon \frac{\sqrt{4R^2 + r_1^2 (R-1)^2} - r_1(1+R)}{2(1-\varepsilon)}, \frac{R}{(1-\varepsilon)} - \frac{2\varepsilon R}{(1-\varepsilon)\left(1+R\right)} r_1 \right), \frac{1-\varepsilon r_1}{1-\varepsilon} \right) \tag{A18}$$

Because the minimum of the first and second term is always greater than the third term (in the relevant region $r_1, r_2 > 0$) we can omit the outer max-operator. The proposition that $\left\{\sqrt{R}, R\right\}$ lies on the frontier can be proven by substitution. *Q.E.D.*

PROOF OF PROPOSITON 4

We prove proposition 4 in four steps. We first show that the problem (21)-(22), if unconstrained by bank raid constraint (17), has a solution with $r_1 = r_2$. Second we prove that the crossing point of the first and second component of (17) lies above the 45 degree line. Third, we proof that the slope of the first component of (17) is greater than the slope of budget constraint (22), and finally we prove uniqueness of point *E*.

Due to non-satiability (22) is binding, so that we rewrite (21) as:

$$\max_{r_1} \ \varepsilon U(r_1) + (1-\varepsilon)U\left(\frac{1+y^*(R-1)}{(1-\varepsilon)} - \frac{\varepsilon}{(1-\varepsilon)}r_1\right) \tag{A19}$$

The first order condition of which is:

$$\varepsilon U'(r_1) - \varepsilon U'\left(y^*(R-1) - \frac{\varepsilon}{(1-\varepsilon)}r_1\right) = 0 \tag{A20}$$

proving that $r_1 = r_2$.

For the second part of the proof, we observe that the second component of (17), $r_2 = \frac{R}{1-\varepsilon} - \frac{2\varepsilon R}{(1-\varepsilon)(1+R)}r_1$, crosses the 45 degree line at $r_1 = \frac{R(1+R)}{1-\varepsilon+R+\varepsilon R}$. We need to show that this is greater than first component of (17), $R + \varepsilon\frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)}$ evaluated at $r_1 = \frac{R(1+R)}{1-\varepsilon+R+\varepsilon R}$. Or, we need to show that:

$$\frac{R(1+R)}{1-\varepsilon+R+\varepsilon R} \geq R + \varepsilon\frac{\sqrt{4R^2 + \frac{R^2(1+R)^2(R-1)^2}{(1-\varepsilon+R+\varepsilon R)^2}} - \frac{R(1+R)^2}{1-\varepsilon+R+\varepsilon R}}{2(1-\varepsilon)} \tag{A21}$$

Multiplying both sides by $1-\varepsilon+R+\varepsilon R$, dividing by *R*, canceling terms, then dividing both sides by ε, and multiplying sides by 2(1-ε) gives:

$$(1+R)^2 - 2(R-1)(1-\varepsilon) \geq \sqrt{4(1-\varepsilon+R+\varepsilon R)^2 + (1+R)^2(R-1)^2} \tag{A22}$$

And hence

$$4(1-\varepsilon+R+\varepsilon R)^2 + (1+R)^2(R-1)^2 - \left((1+R)^2 - 2(R-1)(1-\varepsilon)\right)^2 \leq 0 \tag{A23}$$

Collecting terms eventually gives:

$$-4\varepsilon(R-1)^3 - 4(R-1)^2 \leq 0 \tag{A24}$$

Which clearly holds for all $\varepsilon \in [0,1]$ and $R \geq 1$.

The slope of the first component of (17), is

$$\frac{\partial}{\partial r_1}\left(R + \varepsilon \frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)} \right) = \frac{\varepsilon}{2(1-\varepsilon)}\left(\frac{r_1(R-1)^2}{\sqrt{4R^2 + r_1^2(R-1)^2}} - (1+R) \right) \tag{A25}$$

For a given y', the budget constraint (22) can be written as

$$r_2 = \frac{1 + y'(R-1) - \varepsilon r_1}{(1-\varepsilon)} \tag{A26}$$

The slope of which is $\frac{-\varepsilon}{1-\varepsilon} r_1$.

To prove the third step we thus need to show that:

$$r_1 \geq \frac{1}{2}\left(\frac{r_1(R-1)^2}{\sqrt{4R^2 + r_1^2(R-1)^2}} - (1+R) \right) \tag{A27}$$

or:

$$\left(2r_1 + 1 + R\right)\sqrt{4R^2 + r_1^2(R-1)^2} \geq r_1(R-1)^2 \tag{A28}$$

After squaring both sides and simplifying this becomes:

$$4\left((R-1)^2\left(r_1^4 + (R+1)r_1^3\right) + R(R+1)\left((R+1)\left(r_1^2 + R\right) + 4Rr_1\right)\right) \geq 0 \tag{A29}$$

Which holds for all $r_1 \geq 0$ and $R \geq 1$, because all terms of the left hand side are positive.

To prove that point E is unique we only need to establish that the second order derivative of the first component of (17) is positive in the relevant range. We find:

$$\frac{\partial^2}{\partial r_1^{\,2}}\left(R+\varepsilon\frac{\sqrt{4R^2+r_1^2(R-1)^2}-r_1(1+R)}{2(1-\varepsilon)} \right)=\frac{\varepsilon(R-1)^2}{2(1-\varepsilon)}\left(\frac{4R^2+r_1^2(R-1)^2-\frac{1}{2}r_1}{\left(4R^2+r_1^2(R-1)^2\right)\sqrt{4R^2+r_1^2(R-1)^2}} \right) \qquad \text{(A30)}$$

Which is clearly positive over the relevant range. *Q.E.D.*

PROOF OF PROPOSITON 5

Uniqueness of the equilibrium follows from the fact that the maximized expected utility of the first generation decreases in y^* while the maximized expected utility of the stationary generation increases in y^*. This proves that there is a y^* where the maximized expected utilities are equal. As long as the point *H* stays on the diagonal, risk aversion does not affect the utility of the stationary generation, but decreases the utility of the first generation. Hence we find that with decreasing risk aversion, the point *H* moves upward along the black line in Figure 2. *Q.E.D.*

Appendix B. Derivation of the Bhattacharya and Padilla transfer schemes for $X \neq 1$.

Bhattacharya and Padilla (1996) show that tax-subsidy schemes can attain Golden Rule allocations that are not attained in a market economy. An age discriminating tax scheme can achieve first best allocation. A tax combined with a subsidy that is proportional to investment can achieve the roll-over proof allocation of Qi (1994), and a tax combined with a subsidy that is conditional on the optimal investment can attain a Golden Rule allocation between first best and Qi's allocation. In the derivation we shall use the terminology of Bhattacharya and Padilla (1996): θ_i^j stands for the number of *j*-aged projects held by *i*-aged agents. The equilibrium project price is given by *p*.

A. The age discriminating tax-subsidy scheme.

If the government can levy a tax *t* on two-year olds to pay a subsidy *s* to newborns, and *z* to one-year olds, the first best allocation $(1+X(R-1),\ 1+X(R-1))$ can be achieved if the transfer schedule is:

$$s = X\left(1+\sqrt{R}-(1-\varepsilon)(R-1)\right)-2+\varepsilon \tag{B1}$$

$$z = 1-X+\sqrt{R}\left((1-\varepsilon)(1+XR)-X(2-\varepsilon)\right) \tag{B2}$$

$$t = \left(1+(R-1)X\right)\left(\sqrt{R}-1\right) \tag{B3}$$

Derivation: Assume, without loss of generality, that only newborns invest in the production technology ($\theta_1^0 = 0$), so that we need $\theta_0^0 = X$. The no arbitrage condition requires $p = \sqrt{R}$, and the newborns' budget constraint is $1+s = \theta_0^0 + \theta_0^1 p = X + \theta_0^1\sqrt{R}$. Impatient one-year olds then consume: $C_1 = (1+s)\sqrt{R}+z$, which in the desired equilibrium should equal 1+(*R*-1)*X*. Patient one-year olds spend $1+(R-1)X$ on projects. Since projects cost $\sqrt{R}$, we have $\theta_1^1 = \frac{1+(R-1)X}{\sqrt{R}}$. Two-year olds thus consume $C_2 = \frac{1+(R-1)X}{\sqrt{R}}R - t$, which should also equal 1+(*R*-1)*X*. Hence tax *t* should be set at $t = \left(1+(R-1)X\right)\left(\sqrt{R}-1\right)$. Market clearing requires $\theta_0^1 + (1-\varepsilon)\theta_1^1 = X$, which implies that

$\theta_0^1 = X - (1-\varepsilon)\frac{1+(R-1)X}{\sqrt{R}}$. Substituting this into the budget constraints of newborns gives $s = X\left(1+\sqrt{R}-(1-\varepsilon)(R-1)\right)-2+\varepsilon$, while for the budget constraint of one-year olds we find $z = 1 - X + \sqrt{R}\left((1-\varepsilon)(1+XR)-X(2-\varepsilon)\right)$. For our example, the transfer schedule is $(s, z, t) = (-0.467, 3.533, 4.6)$ and the obtained allocation is (4.6, 4.6).

The negative subsidy *s*, implies that, if indeed only newborns invest in the technology, they borrow 0.667, pay the negative subsidy and invest *X* = 1.2. An alternative scenario is that the newborns invest 0.533 (and do not borrow or lend) while the patient one-year olds use their subsidy *z* to invest in the technology and spend 2.533 on projects. Taxes, subsidies, prices and *aggregate* investment are not affected.

B. The proportional tax-subsidy

If the government offers a proportional subsidy *s* to the investment made, and taxes dividends likewise, the optimal allocation is $\left(\sqrt{\frac{R-s}{1-s}}, \frac{R-s}{1-s}\right)$, where *s* solves:

$$\frac{1-X+sX}{\sqrt{(R-s)(1-s)}}+\frac{1-\varepsilon}{1-s}=X \tag{B4}$$

Derivation: If the equilibrium investment is *X*, the no arbitrage condition implies $p=\sqrt{(R-s)(1-s)}$). As before assume that $\theta_0^0 = X ; \theta_1^0 = 0$. The newborn's budget constraint is: $1+sX = \theta_0^0 + p\theta_0^1 = X + \theta_0^1\sqrt{(R-s)(1-s)}$ so that $\theta_0^1 = \frac{1-X+sX}{\sqrt{(R-s)(1-s)}}$. The one-year old's budget constraint is $p\theta_0^0 + (R-s)\theta_0^1 = C_1 = p\theta_1^1$. Substituting for θ_0^0 and θ_0^1 gives, after some algebra, $\theta_1^1 = \frac{1}{1-s}$. The market clearing condition gives the proportional subsidy that achieves the Golden Rule investment *X*. For our case we find *s* = 0.3625, C_1= 2.389, and C_2 = 5.706.

C. The fixed conditional tax-subsidy

If the government taxes payoffs by *s*, and offers newborns a subsidy *s*, *conditional* on investing *X*, the optimal allocation is $\left((1+s)\sqrt{R-s}, (1+s)\,(R-s)\right)$, where s solves:

$$X - (1-\varepsilon)(1+s) = \frac{1+s-X}{\sqrt{R-s}} \tag{B5}$$

Derivation: The no arbitrage condition now requires $p = \sqrt{R-s}$. Optimality requires investment to be *X*. Newborns receive a subsidy *s*, *iff* they invest *X* in the technology: $\theta_0^0 = X\,;\theta_1^0 = 0$. The newborns' budget constraint is $1+s = \theta_0^0 + p\theta_0^1 = X + \theta_0^1\sqrt{R-s}$. The budget constraint of the one year olds is $p\theta_0^0 + (R-s)\theta_0^1 = C_1 = p\theta_1^1$. Combining these constraints gives $X + \theta_0^1\sqrt{R-s} = \theta_1^1 = 1+s$. Market clearing requires $\theta_0^1 + (1-\varepsilon)\theta_1^1 = X$, or $\theta_0^1 = X - (1-\varepsilon)(1+s)$. From the newborn's budget constraint we have $\theta_0^1 = \frac{1+s-X}{\sqrt{R-s}}$, so that, if we want to induce $\theta_0^0 = X$, we need *s* to be the solution of (B5). The consumption is then given by $C_1 = (1+s)\sqrt{R-s}$ and $C_2 = (1+s)(R-s)$. For our example, the optimal tax-subsidy is *s* = 0.532. The achieved allocation is (2.853, 5.314)

Appendix C. Calculation of the allocations in the numerical example

For $R = 4$, $\varepsilon = \frac{1}{3}$, and $U(C) = -\frac{1}{3}C^{-3}$, allocation E, $\left(r_1^E, r_2^E\right)$ = (3.010, 3.254) solves:

$$\max_{r_1} \varepsilon U\left(r_1\right) + (1-\varepsilon)U\left(R + \varepsilon \frac{\sqrt{4R^2 + r_1^2(R-1)^2} - r_1(1+R)}{2(1-\varepsilon)} \right) \tag{C1}$$

Diamond-Dybvig allocation G, $\left(r_1^G, r_2^G\right)$ = (1.758, 2.484) solves:

$$\max_{r_1} \varepsilon U\left(r_1\right) + (1-\varepsilon)U\left(R\frac{1-\varepsilon r_1}{1-\varepsilon} \right) \tag{C2}$$

Allocation F, $\left(r^F, r^F\right)$ = (2.121, 2.121) solves:

$$\varepsilon U\left(r^F\right) + (1-\varepsilon)U\left(r^F\right) = \varepsilon U\left(r_1^G\right) + (1-\varepsilon)U\left(r_2^G\right) \tag{C3}$$

First generation allocation (1.923, 2.719), from table I panel A can be found by solving:

$$\max_{s_0} \varepsilon U\left(\frac{1+s_0-y^E}{\varepsilon} \right) + (1-\varepsilon)U\left(\frac{1+(1-s_0)R - \varepsilon r_1^E - y^E}{1-\varepsilon} \right) \tag{C4}$$

where $y^E = \dfrac{\varepsilon r_1^E + (1-\varepsilon) r_2^E - 1}{R-1}$.

Similarly, allocation (2.823, 3.991), from table I panel B can be found by solving:

$$\max_{s_0} \varepsilon U\left(\frac{1+s_0-y^F}{\varepsilon} \right) + (1-\varepsilon)U\left(\frac{1+(1-s_0)R - \varepsilon r^F - y^F}{1-\varepsilon} \right) \tag{C5}$$

Finally, allocation (2.320, 3.283) from panel C is be found simultaneously with allocation H, $\left(r_1^H, r_2^H\right) = (2.693,\ 2.693)$ by solving the systems of equations (21)-(25) with all constraints binding and $r_1^* = r_2^*$ (notice that H lies on the diagonal).

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Figure 1

Constraints and coalition allocations under different assumptions in an OLG economy with $\varepsilon = ⅓$, $R = 4$, and CRRA risk aversion parameter $\gamma = 4$. Line a depicts the Golden Rule constraint, assuming that the maximum investment for a return of R is $X = 1.2$, and point *A*, the Pareto Optimal allocation. Curve b represents the renegotiation constraint in a competitive coalition economy, and point *B* the renegotiation constrained optimal allocation. Line c represents the rollover constraint and point *C* the allocation suggested by Qi (1994). Points *D* (Bhattacharya and Padilla, 1996), is the allocation that can be obtained if an infinitely lived government imposes a tax and subsidy scheme where subsidies are only for the young and proportional to investment. Lines *d* denote the interbank arbitrage constraint, and point *E* (Fulghieri and Rovelli, 1998), the allocation that can be obtained by an infinitely lived coalition that can verify agents' ages or types. Point *M* is the one-periodic market allocation. The thin convex curves through points *A* and *B* are utility indifference curves.

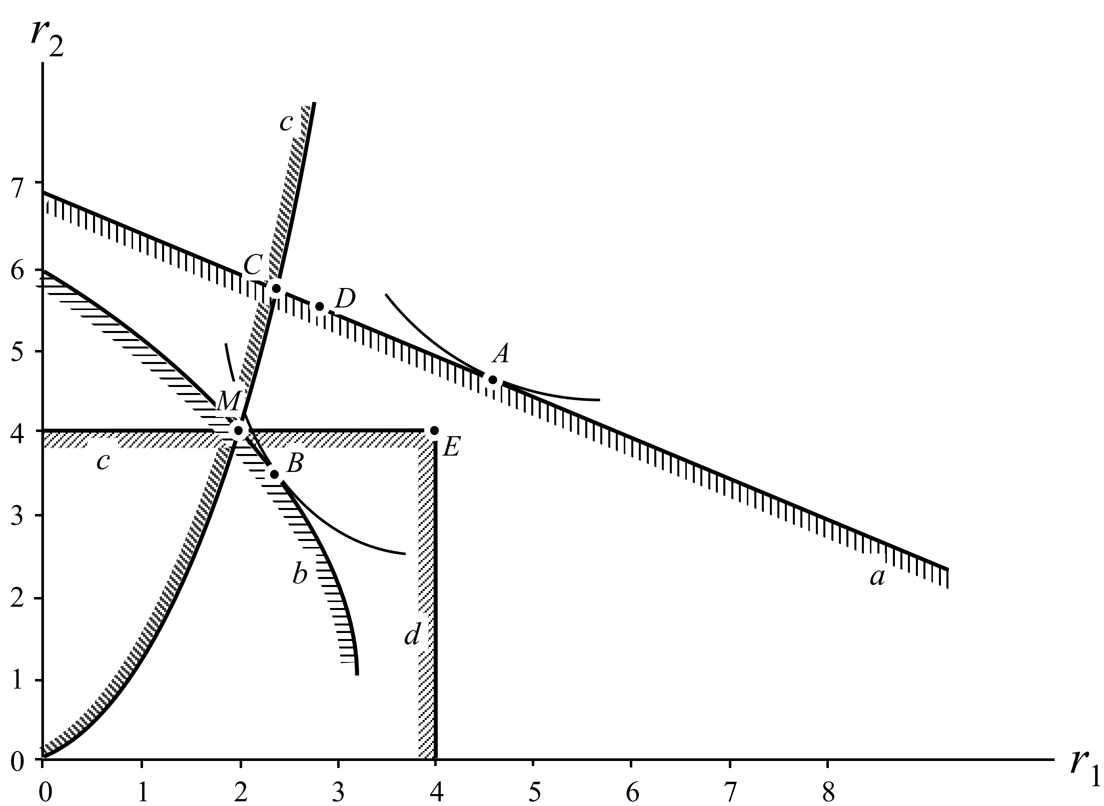

Figure 2

Constraints and coalition allocations under different assumptions in an OLG economy with $\varepsilon = ⅓$, $R = 4$, $X = 1.2$ and CRRA risk aversion parameter $\gamma = 4$. Curve b represents the renegotiation constraint in a competitive coalition economy. Curve e represents the renegotiation constraint for a monopolist and point F the constraint optimal allocation in such an economy.

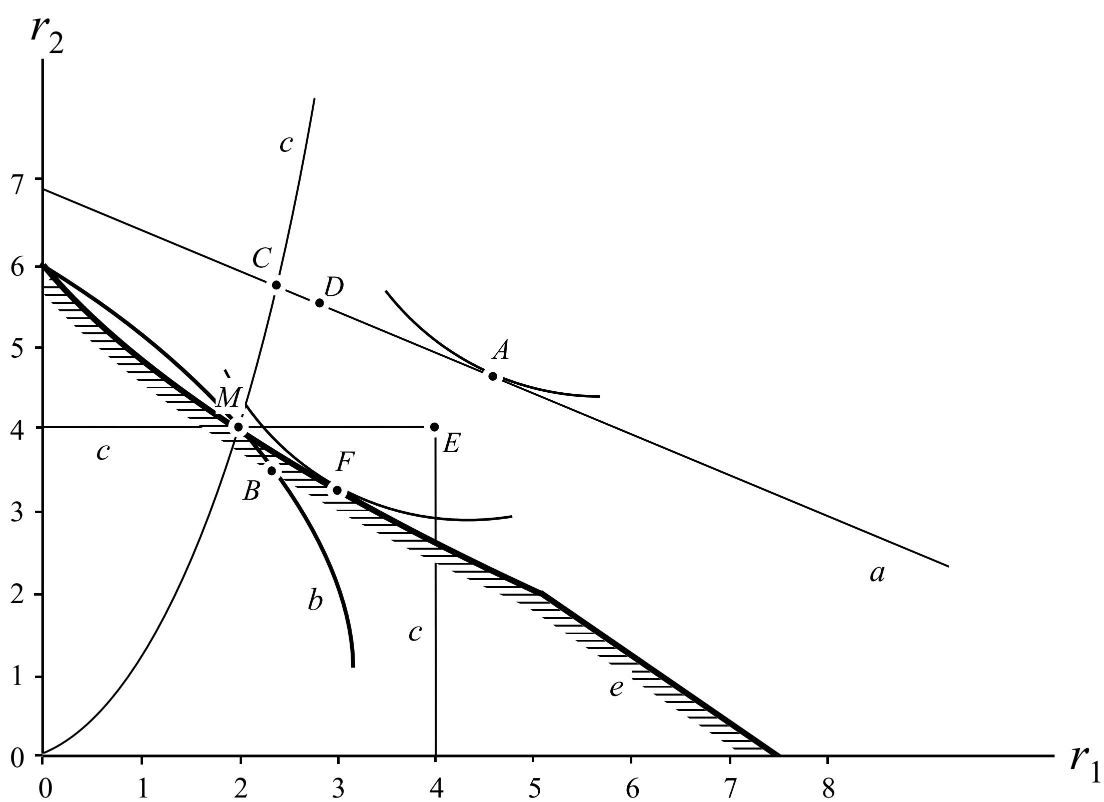

Figure 3

Equilibrium allocations of the stationary coalition game if ε = ⅓, $R = 4$, and CRRA risk aversion parameter $\gamma = 4$, for the competitive (panel A) and monopolist (panel B) coalition economies. The thick black line gives coalition equilibriums. The r_1, r_2 combinations maximize utility for a given inherited asset buffer. The thin downward sloping lines are the internal budget constraints associated with different asset buffers. Line *f* is the budget constraint for the intragenerational coalition. *G* denotes the stationary allocation that offers depositors equal utility as the intragenerational allocation *H*.