# A Model of Stigma in the Fed Funds Market*

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**Abstract**

It is often the case that banks in the US are willing to borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing at the Fed's discount window. This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. We provide a model where market participants may regard discount window borrowing as a negative signal about the financial strength of the borrowing bank. Under certain conditions, which we explicitly identify and discuss, some banks in our model, just as in the data, are willing to borrow in the market at higher rates than the ones they could obtain at the discount window.

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# 1 Introduction

It is often the case that banks in the US are willing to borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing from the central bank, at the Fed's discount window (Peristiani, 1998, Furfine, 2001, Darrat el Al. 2004). This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. The general argument is that market participants eventually identify with some accuracy which banks have borrowed at the discount window and take such activity as a sign of weakness in the financial conditions of the borrowing institution.[1]

While most policymakers and empirical researchers consider the stigma hypothesis plausible, no formal treatment of the issue has ever been provided in the literature. In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; (2) informational frictions limit those trade opportunities; and (3) under some specific conditions which we clearly identify, a stigma effect like the one observed in the data may arise.

Banks in our model obtain loans in over-the-counter markets, involving search, bilateral matching, and negotiations over the terms of the loans. To repay these loans, banks sell assets of heterogeneous quality to outside investors. When asset quality is not perfectly unobservable, information about the actions taken by banks in the credit market may influence the price at which they can sell their assets. In particular, under some conditions, discount window borrowing may be regarded as a negative signal of the quality of the borrower's assets. In such cases, some of the banks in our model, just as in the data, are willing to accept loans in the interbank market at higher rates than the ones they could obtain at the discount window.

Understanding the reluctance of banks to use the discount window is important as, for example, it limits the ability of the central bank to effectively implement a "hard ceiling" on the range of interest rates observable in the interbank market. Partly in an effort to address this perceived reluctance to borrow from the window, in 2003, the Federal Reserve completely changed the terms of operation of its credit facilities (Furfine, 2005). More recently, the creation of the Terms Auction Facility (TAF), and some of its particular organizational features, can be regarded as trying to limit the possibility of stigma from accessing this source of central bank liquidity. In the context of our formal model, we discuss specific conditions under which stigma may arise, and use the resulting insights to suggest arrangements and policy options that may be useful to reduce the incidence of stigma in the interbank market.

We make some stark assumptions in our model. It is fairly easy to see that many of them could be readily generalized. However, our main objective here is to formalize in as simple a framework as

[1] Journalist Mathew Cowley expresses this popular view succinctly in his column at Dow Jones Newswires: "*There's traditionally been a stigma associated with borrowing [from the Fed's discount window], which is initiated by the financial institution and is therefore regarded as a sign of weakness.*" (1 August 2008).

possible an argument that is often used to explain certain apparently abnormal trading patterns in the US interbank market for funds. Abstracting from some realistic features allow us to better capture the mechanism at play. After stripping the problem from inessential features, some previously unappreciated components get revealed. Market frictions and bilateral negotiations, for example, play a prominent role in our formal explanation but not necessarily on the more heuristic one used in policy circles. We believe that identifying these and other features is one of our main contributions in this paper.

Our model combines several elements that have become common in payments economics. First, as in Freeman (1996) and the large literature that followed, spatial separation plays a key role in limiting the ability of some agents (banks) to trade with some other agent (investors) at certain points in time. Second, search and bilateral negotiations determine the terms of trade of assets in the economy, as in Duffie et. Al (2005) and Lagos and Rocheteau (2009). Ashcraft and Duffie (2007) argue that these are realistic features of the US interbank market for funds.

The paper is organized as follows. In the rest of this section we discuss some evidence that has often been regarded as indicating the presence of stigma in lending from the Fed's discount window. Then, in the next section, we introduce our formal model of the interbank market, based on intertemporal trade with (physical and informational) frictions. In Section 3 we study equilibrium in the basic framework when the discount window is not available to banks. This section is intended to provide a basic description of how our modeled economy works. In Section 4 we introduce discount window lending and derive the equilibrium conditions for this, more complicated case. In Section 5, then, we study an equilibrium in which discount window lending becomes a negative signal and, hence, results in stigma. In that context, we discuss the particular conditions that can give rise to such phenomenon. We draw some conclusions in Section 6.

## 1.1 Interest rates in the fed funds market and the discount window

On January 2003, the US Federal Reserve started operating a standing facility, offering discount window loans to eligible depository institutions at a penalty interest rate (initially 100 basis points above fed funds target rate). Under this new regime, no other restriction or especial supervisory scrutiny was associated to lending from the discount window. In principle, under this system, the rate at the discount window (plus the implicit cost of collateral) should act as a ceiling for the fed funds market rate. However, banks often borrow from another bank at a higher rate than the one they could get at the Fed's discount window. This and other evidence seems to indicate a reluctance of US banks to borrow from the discount window.

For example, Furfine (2003) compares the amount of borrowing at the discount window after January 2003 with the volume that one would have predicted in advance, given the historical (pre-standing facility) empirical distribution of cross-bank interest rates paid. He finds that borrowing from the primary credit facility was significantly lower than what one might have expected. Also, he finds that each day during the period from 9 January 2003 to 31 March 2003, on average, there is more than 57 times more activity in the fed funds market at rates equal to or higher than the rate offered by the discount window than at the primary credit facility.

Evidence of similar behavior can be found before the change in the system in 2003. For example, Furfine (2001) studies activity in the fed funds market during the operation of a temporary Lombard-type Fed lending facility (the SLF) put in place from October 1, 1999 to April 7, 2000 to respond to possible spikes in liquidity demand associated with the general Y2K phenomenon. He finds that whenever market interest rates rose noticeably, borrowing in the overnight federal funds market at 150 or more basis points above the Fed's target rate dwarfed lending at the SLF. Furfine, then, concludes that commercial banks were extremely reluctant to borrow from the Fed.

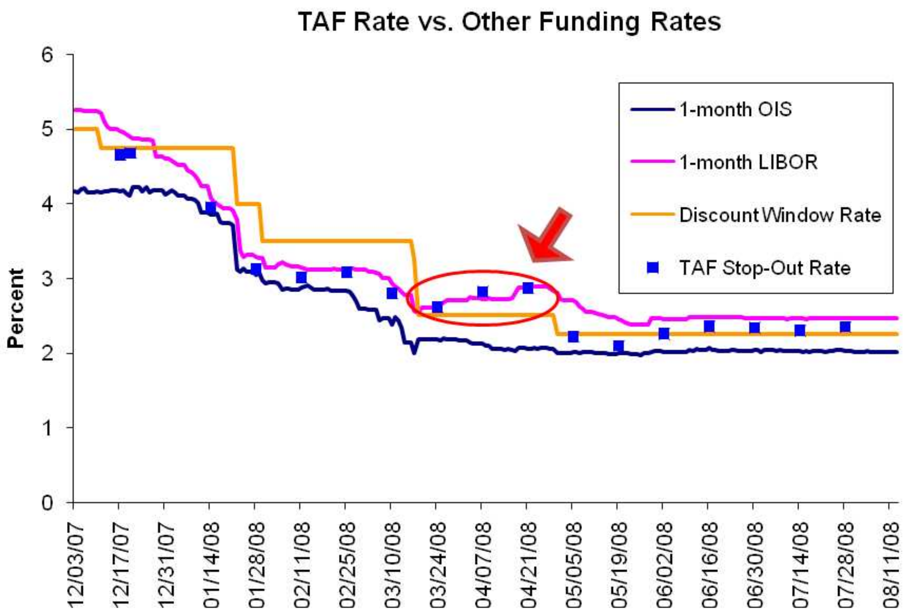


Figure 1

In August 2007, as a response to the liquidity crisis, the Fed allowed eligible institutions to borrow funds at the discount window for up to thirty days (instead of overnight). The change generated little to no additional borrowing. In December 2007, the Fed created the TAF (a biweekly auction of a fixed amount of 28-day funding for depository institutions eligible to obtain primary credit through the discount window). From its inception, borrowing at the TAF was in high demand. Interestingly, during March and April 2008 the stop-out rate at the TAF was higher than the primary credit rate at the discount window (see Figure 1). That is, banks preferred to borrow at the TAF at a higher rate than that at the discount window.[2]

[2] This tendency to borrow at the TAF and not at the discount window is even more evident when the effective cost of discount window funding is computed as the 30-day OIS rate plus the primary credit spread. See Armantier, Krieger, and McAndrews (2008).

| Date | Daily effective fed funds rate | Range | | Standard Deviation |
|---|---|---|---|---|
| | | Low | High | |
| 09/08/08 | 1.92 | 1.00 | 2.50 | 0.22 |
| 09/09/08 | 1.96 | 1.62 | 2.19 | 0.07 |
| 10/09/08 | 2.12 | 1.50 | 2.62 | 0.18 |
| 09/11/08 | 2.00 | 1.00 | 2.50 | 0.19 |
| 09/12/08 | 2.10 | 1.75 | 2.94 | 0.10 |
| 09/15/08 | 2.64 | 0.01 | 7.00 | 1.72 |
| 09/16/08 | 1.98 | 0.01 | 6.00 | 1.10 |
| 09/17/08 | 2.80 | 0.25 | 6.00 | 0.78 |
| 09/18/08 | 2.16 | 0.00 | 6.00 | 1.13 |
| 09/19/08 | 1.48 | 0.00 | 3.50 | 1.01 |
| 09/22/08 | 1.51 | 0.00 | 3.50 | 0.92 |
| 09/23/08 | 1.46 | 0.06 | 3.00 | 0.69 |
| 09/24/08 | 1.19 | 0.01 | 3.25 | 0.66 |
| 09/25/08 | 1.23 | 0.00 | 3.00 | 0.66 |
| 09/26/08 | 1.08 | 0.00 | 3.00 | 0.69 |

Table 1

Many days in September 2008, the (average) interest rate in the fed funds market was above the discount window rate. The rate for discount window primary credit was 2.25 during that period. As is clear from Table 2, the high average and standard deviation during the week of September 15 imply that a relatively high proportion of the trade volume was executed at rates significantly above the discount window rate.

## 2 The model

The economy last for three periods, $t = 1, 2, 3$. There is a large number of banks and investors which can interact with each other over time, subject to some specific limitations described below. Each bank owns an asset which pays a return $v$ in period 3 if held to maturity (and zero in periods 1 and 2). The return $v$ can take one of two values, $R$ or 0. If the return of the asset is $R$ we say that the asset is high quality. If it is 0, then we say that the asset is low quality. The probability that the asset is high quality is $p$. Then, with probability $1 - p$ the asset is low quality. Asset quality is realized in period 1 even though the return only becomes available in period 3.

The asset has some degree of specificity that determines that, if sold to another bank, its return $v$ becomes much lower. Investors, instead, have the ability to manage the asset appropriately after buying it from a bank, and hence to maintain the potential return virtually unaffected. In principle, some banks could become good managers of purchase assets. We maintain this stark distinction between banks and investors just for simplicity.[3]

For simplicity, Investors have deep pockets and can always store their funds from one period to the next in a technology that gives a return $i$ per unit stored.

At the beginning of period 1, banks get a liquidity shock: Some banks need to make a payment of size 1 and, hence, become illiquid; and some banks receive a payment of size 1 which makes them liquid. For simplicity, we assume that half the banks become liquid and the other half, illiquid. Liquid banks have access to the same storage technology than investors. An illiquid bank that is unable to make the required payment by the end of period 1 suffers a (non-pecuniary) penalty $\rho$, with $i < \rho < R$.

In period 1, after the quality of the assets and the liquidity shocks are realized, banks can interact in an over-the-counter market for interbank funds. Illiquid banks search for liquid banks to obtain immediate funding. Investors cannot participate in this market. An illiquid bank finds a liquid bank with probability $\sigma$. When two banks match, the liquid bank can costlessly verify the quality of the asset held by its illiquid counterparty. The two banks in the match then decide whether or not to enter in a lending agreement with each other and, finally, bargain over the terms of the loan. For simplicity, we assume that the outcome of the negotiations is determined according to Nash bargaining with $\theta$ being the bargaining power of the lender. A bank that does not pay back a loan has to surrender its asset to the lender at the time that the payment was due.

In period 2, banks and investors participate in a centralized market in which participants can trade funds and assets with each other, and make payments to each other, in a frictionless environment. Each bank has a probability $\alpha$ that the quality of its asset become publicly observable at the beginning of period 2. With probability $1-\alpha$ the quality of the asset remains unknown to investors and other banks. At the end of period 2 all banks and investors part ways and, consequently, there are no possible (business) interactions in the economy in period 3.

We assume that investors in period 2 cannot observe whether a bank has borrowed from another bank in period 1. If borrowing were observable, then it could also act as a (potentially imperfect) signal of the quality of the asset in period 2. For example, if being able to take a loan in period 1 were regarded as a positive signal of the quality of the asset, then even a liquid bank with no urgent need for funds may want to take a loan in period 1 when holding a bad asset. Of course, if the liquidity position of the bank were observable, this would undermine the previous strategy. If not only the loan, but also the terms of the loan were observable, this information could become fully revealing in some cases. All this different specifications seem interesting and would be worth studying formally. However, to keep the focus and simplicity of the analysis, we will assume here that the loans in period 1 are not observable to investors in period 2.

Basically, the set of frictions that characterize our environment are designed to capture a situation

[3] In principle, the basic economics of our simple three-period model appears suitable to be incorporated into a general equilibrium framework like the one studied by Lagos and Rocheteau (2007) following the technical innovations in Lagos and Wright (2005).

where some banks own illiquid assets but have an immediate need for funding. While, in principle, there is enough funds in the economy (in investors hands) to cover all immediate needs, banks cannot access such liquidity directly. Instead, in the short run, illiquid banks can only trade with other banks in a market with frictions. Trade in this market is based on the premise that banks will have access to investors' funds in the medium term. We are interested in studying the implications of private information in this process of intertemporal reallocation of funds via borrowing and trading.

# 3 Equilibrium

We will solve for a Perfect Bayesian Equilibrium of this economy. To do so, we proceed by backward induction. We start by computing asset prices in period 2 given investors beliefs about the trading strategies of banks. Then, in period 1, illiquid banks look for liquid banks and when matched, negotiate over the terms of a loan taking into account their equilibrium prediction about asset prices in period 2.

Two important properties of the equilibrium loan contracts result directly from the assumed agent isolation in period 3. First, all contract among banks involve one-period loans; and second, upon default in period 2, the lending bank will take possession of the borrower assets and sells them immediately to investors in the market.

## 3.1 Observable asset quality

Suppose that $\alpha = 1$; that is, investors in period 2 can perfectly observe the quality of the assets being sold in the market. Then, since low quality assets give zero return in period 3, investors are not willing to pay any positive amount for low quality assets in period 2. On the other hand, due to competition among investors, a high quality asset can be sold at price $R$ in period 2.

In period 1, if an illiquid bank holding a low quality asset finds a liquid bank, the former will not be able to borrow from the latter. The lender in this case will anticipate that the borrower will have no funds to pay back the loan in period 2. Furthermore, by taking possession of the asset in period 2, the lender cannot sell the asset for any positive amount. In other words, an illiquid bank holding a low quality asset has no borrowing capacity in period 1 and, hence, will get no loan.

The situation is different if the illiquid bank is holding a high quality asset. In this case, if the illiquid bank finds a liquid bank then it will be able to take a loan from the liquid bank. After agreeing on a loan, the two banks will bargain over the interest rate, denoted by $r_H$. In particular, the interest rate will solve the following problem:

$$\max_{r_H \leq R} (r_H - i)^{\theta} \left((R - r_H) - (R - \rho)\right)^{1-\theta}$$

Since $\rho < R$ the solution to this problem is given by $r_H = i + \theta(\rho - i)$. Note that the interest rate is increasing on the bargaining power of the liquid bank and is always below the penalty rate $\rho$.

In equilibrium, only illiquid banks that find a liquid bank and hold a high quality asset are able to take loans in the interbank market in period 1. Hence, the interbank market interest rate is given by $r_H$. Note that whenever $\theta$ is positive $r_H$ is greater than $i$, the risk-free opportunity cost of funds in period 1. This interest rate premium is the result of bargaining power by lenders and not default risk. In this model, banks that could be expected to default do not get loans in period 1 and, hence, do not influence the observed interest rates in the interbank market.

### 3.2 Unobservable asset quality

Suppose now that the quality of the asset held by a bank becomes observable in period 2 only with a probability less than one. That is, assume that $0 \leq \alpha < 1$. As in the previous section, when the quality of the asset becomes observable the price is equal to $R$ if the asset is high quality and zero if the asset is low quality.

The more interesting case is when the quality of the asset is not observed. In this case, pricing in period 2 will depend on the beliefs of investors about the relative prevalence of high and low quality assets in the market. Let $q$ be the (equilibrium) belief of investor that a given asset being sold in the market in period 2 is high quality. Then, the price of an asset of unobserved quality, $P_U$, will be equal to $qR$.

We need to determine now the possible equilibrium values of $q$. The first thing to note is that whenever $q < 1$ all liquid banks holding assets of high unobserved quality will not want to sell them to investors in period 2. Similarly, whenever $q \geq 0$ all banks (liquid and illiquid) holding assets of unobserved low quality will want to sell their assets in the market. What makes equilibrium determination nontrivial is the action of illiquid banks holding assets with high unobserved quality. These banks may or may not take a loan in period 1 depending on the value of $q$. In turn, whether these banks take a loan or not determines the relative prevalence of high quality assets in the market and, hence, the values of $q$ consistent with equilibrium.

**Proposition 1.** When $(1-\alpha)R > \rho - i$ there is an equilibrium with $q = 0$.

**Proof:** Recall that we are assuming that $R > \rho > i$. Then, we can rewrite the condition in the proposition as $\alpha R - (R - \rho) < i$. We will now show that when this condition holds and $q = 0$ no loans are made in period 1. The reason for this is as follows. If an illiquid bank with a high quality asset does not take a loan in period 1, his payoff is $R - \rho$. Hence, this bank should get at least as much in expected terms from entering a loan contract. Since $q = 0$ the maximum obtainable from the asset in expected terms is $\alpha R$. Then, a borrower can get a maximum expected repayment equal to $\alpha R - (R - \rho)$, but he can get $i$ from not making the loan. So, under the condition of the proposition, if banks expects that investors will not be willing to pay for an asset of unobserved quality (i.e., if $q = 0$), it is not possible to have both the liquid and illiquid bank agreeing on a feasible loan contract. But then, since illiquid banks with assets of high unobserved quality do not have a loan to repay, they have no reason to sell their assets (they get zero from doing so, instead of $R$). Therefore, only low quality assets can be expected to be put for sale in period 2, which is consistent with the belief expectation $q = 0$. #

The proposition gives us a condition under which the asset market in period 2 could shut down and, in anticipation of that fact, illiquid banks get screened out of the loan market in period 1 even when they are holding a high quality asset. It is interesting to note that the condition is more likely to hold when the probability $\alpha$ that the quality of an asset will become observable in period 2 is low; that is, when the information frictions in the asset market are expected to be large.[4]

This "no credit" equilibrium does not exist if $\rho - i - (1-\alpha)R > 0$. Furthermore, even if the condition of the proposition is satisfied, another equilibrium with credit in the interbank market may be possible under certain conditions. We study the equilibrium with credit next.

Suppose now that $q > 0$ in equilibrium. In that case, we know that in period 2 there will be $(1-\alpha)(1-p)$ low quality assets in the market. Furthermore, the high quality assets of illiquid banks that manage to obtain a loan in the interbank market will be put up for sale in period 2. This is the case even if the borrowing bank does not manage to pay the loan, since in such eventuality the asset will be sold by the liquid bank which acquires it upon default.[5] Hence, the total amount of high quality assets in the market will be $(1-\alpha)\frac{1}{2}p\sigma$ and the equilibrium belief will be given by:

$$q^* \equiv \frac{p\sigma}{2(1-p) + p\sigma}.$$

Let $TS(q) = \rho - i - (1-\alpha)(1-q)R$ be the expected total surplus from a loan relationship between a liquid bank and an illiquid bank holding a high quality asset. As we will see, if $TS(q^*) > 0$ an illiquid bank holding high quality assets will be able to obtain a loan from a liquid bank whenever the two of them match in period 1. The negotiated value of $r_H$ will determine how this surplus gets divided between the two parties.[6] Given a value of $r_H$ the surplus for the borrower is given by:

$$S_b(r_H, q^*) = \alpha(R - r_H) + (1-\alpha)\max\{q^*R - r_H, 0\} - (R - \rho)$$

and the surplus for the lender is given by:

$$S_l(r_H, q^*) = \alpha r_H + (1-\alpha)\min\{q^*R, r_H\} - i.$$

Note, of course, that $S_b(r_H, q^*) + S_l(r_H, q^*) = TS(q^*)$. The equilibrium interest rate, then, solves

[4] Changes in the quality of the asset, as reflected by changes in the return $R$, have two opposing effects. On the one hand, an increase in $R$ increases the availability of funds for repayment, but, on the other hand, it increases the outside option for the potential borrower, reducing his incentives to take the loan. In our setup, the second effect dominates and, as a consequence, increases in $R$ make the possibility of a shut-down of the interbank credit market more likely.

[5] Alternatively, we could assume that whenever indifferent the borrower will sell the asset in the market and try to pay as much as possible of it debt.

[6] Since the maximum that an illiquid bank could repay is given by $R$, we can restrict the analysis, without loss of generality, to consider only values of $r_H \leq R$.

the following Nash bargaining problem:

$$\max_{r_H} S_l(r_H, q^*)^\theta S_b(r_H, q^*)^{1-\theta}$$

subject to $S_l(r_H, q^*) \geq 0$ and $S_b(r_H, q^*) \geq 0$. Define the functions $\widetilde{r}_H(\theta, q^*) \equiv i + \theta TS(q^*)$ and $\widehat{r}_H(\theta, q^*) \equiv \frac{1}{\alpha}[i - (1-\alpha)q^* R] + \frac{\theta}{\alpha} TS(q^*)$. Then, we have that the solution to the Nash bargaining problem is given by:

$$r_H(\theta, q^*) = \begin{cases} \widetilde{r}_H(\theta, q^*) & \text{if} \quad \theta < \theta^T(q^*) \\ \\ \widehat{r}_H(\theta, q^*) & \text{if} \quad \theta \geq \theta^T(q^*) \end{cases}$$

where $\theta^T(q^*) = \max\{0, \min\{(q^* R - i)/TS(q^*), 1\}\}$.[7]

(insert here figure of $S_l$ and $S_b$ as functions of $r_H$)

Note that when $\theta^T(q^*) \in (0,1)$ we have that $\widetilde{r}_H(\theta^T(q^*), q^*) = \widehat{r}_H(\theta^T(q^*), q^*) = q^* R$. After some algebra, it can be shown that $S_b(r_H(\theta, q^*), q^*) = (1-\theta)TS(q^*)$ and $S_l(r_H(\theta, q^*), q^*) = \theta TS(q^*)$. Hence, as long as $TS(q^*)$ is positive both the liquid and illiquid bank in a match will agree to participate in a loan agreement. Define the threshold value $q^T$ as:

$$q^T \equiv \max\left\{0, \frac{(1-\alpha)R - (\rho - i)}{(1-\alpha)R}\right\},$$

such that $TS(q) > 0$ for all $q \geq q^T$. For concreteness, let us assume that whenever indifferent, banks enter a loan relationship. Then, we have the following proposition that provides conditions on parameter values such that an equilibrium with interbank credit exist.

**Proposition 2.** When $q^* \geq q^T$ there is an equilibrium with interbank credit.

**Proof:** Follows from the argument above. #

Note that in this equilibrium, illiquid banks holding low quality assets may or may not receive credit depending on whether $(1-\alpha)q^* R$ is greater than or less than $i$, respectively. If we denote by $r_L$ the interest rate arranged by these banks in a loan agreement, then it only make sense to consider values of $r_L$ less than or equal to $q^* R$, since this is the maximum that a lender could obtain in period 2 from a borrower holding a low quality asset. But, then, it is clear that the lender in expected terms can only get no more than $(1-\alpha)q^* R$ from the borrower, and if this quantity is less than $i$, the lender would not agree to participate in the loan. Note, also, that a loan may not take place even if the surplus from the loan agreement $\rho - i$ is positive, as we assume it is.

**Corollary 1.** When $q^* \geq q^T > 0$ there are two equilibria, one with interbank credit and one where the interbank credit market shuts down.

[7] Note that since $R > \rho$ we have that $r_H(\theta, q^*) < R$ for all $(\theta, q^*)$ and, in consequence, $r_H(\theta, q^*)$ is always reasonable in the sense that there are some borrowers that are able to pay as much.

**Proof:** Note that the parameters determining $q^*$ are different from those determining $q^T$. Then, we can easily find parameters such that the conditions in the corollary hold. In such case, since $q^T > 0$ implies that the condition in Proposition 1 holds, we have that a "no credit" equilibrium exists. Furthermore, since $q^* \geq q^T$, by Proposition 2, an equilibrium with interbank credit also exist. That is, there are multiple equilibria. #

This corollary tells us that, for a set of the parameter values, the model is consistent with multiple equilibria. Furthermore, these equilibria have significantly different implications for the outcomes on the interbank credit market. If banks become pessimistic about their ability to sell assets to outside investors, they may not be willing to enter into loan relationships, and this shutdown of the credit market, in turn, will result in a (selective) reduction in the participation in the asset market that would justify the initial pessimism.

**Corollary 2.** When $(1-\alpha)R \leq \rho - i$ we have that $q^T = 0$ and there is a unique equilibrium with interbank credit. The equilibrium price of the unobserved quality asset is $P_U = q^* R$.

**Proof:** Since $(1-\alpha)R \leq \rho - i$ we have that $TS(q) \geq 0$ for all $q \in [0,1]$. Then, high quality illiquid banks that find a match in the interbank credit market always establish a loan relationship. This implies that those banks will put their asset for sale in the market in period 2 and, hence, that the only consistent equilibrium value of $q$ is $q^*$. #

**Lemma 1.** When the threshold value $q^T$ is greater than zero, it is increasing in $R$ and $i$ and decreasing in $\rho$ and $\alpha$. The equilibrium value $q^*$ is increasing in $p$ and $\sigma$.

**Proof:** The results can be obtained by simple differentiation of the expression for $q^T$ and $q^*$. #

The results on this lemma can be given interesting interpretations. For example, according to the lemma, higher values of $\alpha$ make the credit equilibrium consistent with a larger set of parameter values. More broadly, then, we can say that less informational frictions in the asset market make interbank credit more likely in our model. Similarly, since $q^*$ is increasing in $\sigma$, we can say that the potential for a more liquid credit market makes the credit equilibrium more likely to arise. Perhaps somewhat less intuitive is the case of alternative values of the return $R$ of the high quality asset. Higher values of $R$ make the credit equilibrium less likely. The reason for this fact is that when a bank holding a high quality asset enters a credit relationship, it anticipates that with some probability it will have to sell the asset when its quality is not observed. This sale entails a loss proportional to $R$ which discourages credit relationships ex ante.

The last proposition of this section provides a characterization of the equilibrium interest rate in the interbank market for credit when such a market is open. One aspect that seems interesting to investigate is under what conditions the borrower is able to pay the arranged interest rate when the quality of his asset is not observable by investors. Recall that if the borrower cannot pay the interest rate, the asset changes hands and gets sold by the lender. Whether this happens or not depend on how $r_H(\theta, q^*)$ compares with $q^*R$. If $r_H(\theta, q^*) > q^*R$ then, when the quality of the asset is not observed by investors, the borrower cannot pay the arranged interest rate.

**Proposition 3.** Assume $q^* \geq q^T$. If $\alpha i \geq \rho - (1-\alpha)R$ there is an equilibrium with interbank credit in which the interest rate is given by $r_H(\theta, q^*) = \widehat{r}_H(\theta, q^*) > q^*R$ for all $\theta$. If $\alpha i < \rho - (1-\alpha)R$ there is an equilibrium with interbank credit in which the interest rate is given by the following expressions:

1) If $q^*R \leq i$ then $r_H(\theta, q^*) = \widehat{r}_H(\theta, q^*) > q^*R$ for all $\theta$.

2) If $i < q^*R < \frac{1}{\alpha}[\rho - (1-\alpha)R]$ then $\theta^T(q^*) \in (0,1)$ and

$$r_H(\theta, q^*) \begin{cases} \widetilde{r}_H(\theta, q^*) < q^*R & \text{if} \quad \theta < \theta^T(q^*) \\ \\ \widehat{r}_H(\theta, q^*) > q^*R & \text{if} \quad \theta \geq \theta^T(q^*) \end{cases}$$

(3) If $\frac{1}{\alpha}[\rho - (1-\alpha)R] \geq q^*R$ then $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*) < q^*R$ for all $\theta$.

**Proof:** See appendix. #

(insert here figure in space $(\theta, q)$)

In this section, we have studied the functioning of an interbank market for funds in the presence of frictions that limit the ability of banks to trade with each other. As a result of these frictions, some banks are not able to borrow during period 1 even when they hold high quality assets that have a present value larger than the face value of loan that the bank seeks to obtain. A natural question to ask is how would equilibrium outcomes change if a central bank lending facility is available to all banks in this environment. In the next section, we introduce central bank discount window lending and describe the equilibrium conditions in such case.

# 4 Discount window lending

Assume now that banks have access in period 1 to the central bank's discount window, where they can obtain loans at the interest rate $r_W > 0$. We will assume that discount window lending is perfectly observable by all agents in the economy. This is an extreme, simplifying assumption. All we need for our results is that agents obtain informative signals about the other banks' activity at the discount window.[8]

As before, to solve for a Perfect Bayesian equilibrium it is convenient to start by identifying possible outcomes in the market for assets during period 2. When the quality of the asset becomes

[8] In the U.S., the Federal Reserve announces every other week the amount of discount window lending granted the previous two weeks. While this is potentially a very noisy signal of the participation of particular banks' in the discount window lending activity, sometimes other market participants can put together several pieces of information (like rejected funding request, etc.) which may reveal with some degree of confidence the identity of a particular bank lending at the window.

observable, again the price of the asset is either $R$ or zero depending on whether the asset is of high or low quality, respectively.

When the quality of the asset is not observable, things become more complicated. Whether a bank borrowed at the discount window or not could be an informative signal about the quality of the asset that the bank is holding (and trying to sell). This possibility is the result of two important assumptions in our model. On the one side, banks in the interbank market are able to obtain some accurate information about the quality of the asset held by counterparties, which in turn influences their lending behavior. On the other side, sometimes investors are not able to observe directly the quality of the asset being traded, nor the asset-holder's private dealings in the interbank market, but do get to observe the asset-holder's past transactions with the central bank (or, more generally, an informative signal of those transactions). While our informational assumptions in this respect could be regarded as extreme, they are especially useful in clarifying the basic mechanism that creates the possibility of stigma.

In period 2, then, investors form beliefs about the quality of a given asset of unobserved quality which depend on whether the asset holder has borrowed or not at the discount window. Let $q_W$ be the belief probability that the asset is high quality if the holder has borrowed at the window; and let $q_N$ be the corresponding probability (of high quality) if the holder has not borrowed at the window. These are equilibrium beliefs that will depend also on the decisions taken by all banks, given those beliefs. We study bank decision next.

Illiquid banks that do not find a liquid counterparty in the interbank market have to decide whether or not to borrow from the discount window. For this decision, the bank compares the payoff of taking each possible action. To calculate this payoff, we assume that all banks which have borrowed at the window sell their asset in period 2 to pay back the loan (in full or partially).[9] Define $P(q_W, r_W) \equiv \max\{q_W R - r_W, 0\}$. Then, an illiquid bank that has not found a counterparty and is holding a high quality asset will borrow at the window if:

$$\alpha(R - r_W) + (1-\alpha)P(q_W, r_W) \geq -\rho + R;$$

and a bank holding a low quality asset will borrow at the window if:

$$(1-\alpha)P(q_W, r_W) \geq -\rho + (1-\alpha)q_N R.$$

Illiquid banks that do find a counterparty in period 1 must decide among three possible alternatives: they could either borrow from the liquid bank, from the window, or not borrow at all. If an illiquid bank borrows in period 1, it will have to sell its asset in period 2 to repay (all or some of) the loan. Define $P(q_N, r_j) \equiv \max\{q_N R - r_j, 0\}$ with $j = H, L$, where $r_j$ is the interest rate on

[9] It could be interesting to consider alternative treatments of those borrowers that cannot repay discount window loans in full. In the simple case we study here, discount window lending differs from private lending only in the interest rate, which is exogenously set and is not contingent on asset quality.

a loan from a liquid bank to an illiquid bank holding a $j$ asset. Then, an illiquid bank that finds a liquid counterparty and is holding a high quality asset will agree to take the loan if:

$$S_b(r_H, q_N, q_W) = \alpha(R - r_H) + (1-\alpha)P(q_N, r_H) - \max\{\alpha(R - r_W) + (1-\alpha)P(q_W, r_W), -\rho + R\} \geq 0.$$

An illiquid bank that finds a liquid bank in period 1 and is holding a low quality asset will enter a lending relationship with the liquid bank if:

$$S_b(r_L, q_N, q_W) = (1-\alpha)P(q_N, r_L) - \max\{(1-\alpha)P(q_W, r_W), -\rho + (1-\alpha)q_N R\} \geq 0.$$

Finally, we need to consider the decision of liquid banks upon entering a match with an illiquid bank. If the illiquid bank is holding a high quality asset, then the liquid bank will agree to make a loan if and only if:

$$S_l(r_H, q_N) = \alpha r_H + (1-\alpha)\min\{q_N R, r_H\} - i \geq 0.$$

Similarly, when the illiquid bank is holding a low quality asset, the liquid bank will agree to make a loan if and only if:

$$S_l(r_L, q_N) = (1-\alpha)\min\{q_N R, r_L\} - i \geq 0.$$

Define the total surplus in a match as $TS_j(q_N, q_W) = S_b(r_j, q_N, q_W) + S_l(r_j, q_N)$ for $j = H, L$. Whenever the total surplus in a match is positive, banks will agree to enter a lending relationship and will negotiate over the interest rate. The outcome of such negotiation is the solution to the following Nash problem for $j = L, H$:

$$\max_{r_j} S_l(r_j, q_N)^{\theta} S_b(r_j, q_N, q_W)^{1-\theta}$$

subject to $S_l(r_j, q_N) \geq 0$ and $S_b(r_j, q_N, q_W) \geq 0$. Call the solution to this problem $r_j(\theta, q_N, q_W)$ for $j = L, H$.

In period 2, those banks that have taken a (private or discount window) loan in period 1 will sell their asset in the market. If $q_N > 0$ then all banks holding a low quality asset will sell their asset even if they do not have a loan to repay. If $q_N < 1$ then banks holding a high quality asset that do not have a loan to repay will not sell their asset. These cases exhaust all the possibilities.

A Perfect Bayesian equilibrium, then, can be characterized by a set of beliefs $(q_N, q_W)$, loan agreements with the corresponding interest rates, and asset sales and prices such that: (1) all agents make optimal lending and asset sale decisions given those beliefs (as described above); (2) asset prices reflect those beliefs; and (3) the agents' decisions validate those equilibrium beliefs in the sense that they are the result of applying Bayes Rule on equilibrium outcomes (i.e., a fixed point in beliefs).

We close this section by providing some general lemmas that can be use to simplify certain equilibrium expressions and to facilitate the construction of an equilibrium in our model.

**Lemma 2.** In any equilibrium with private lending to illiquid banks holding low quality asset the following condition holds:

$$\frac{i}{1-\alpha} \leq r_L(\theta, q_N, q_W) \leq q_N R.$$

**Proof:** The maximum amount that an illiquid bank can repay is $q_N R$. Hence, only values of $r_L$ lower that $q_N R$ are relevant. Furthermore, if $(1-\alpha)r_L < i$ the liquid bank will not accept to lend. #

Note that this lemma can be use to simplify the equilibrium expressions for $S_b(r_L, q_N, q_W)$ and $S_l(r_L, q_N)$. The next lemma shows that, in equilibrium, if an illiquid bank holding a high quality asset borrows at the discount window when cannot find a counterparty, then so does an illiquid bank that is holding a low quality asset and is in the same situation (i.e., cannot find a counterparty).

**Lemma 3.** If the condition $\alpha(R - r_W) + (1-\alpha)P(q_W, r_W) \geq -\rho + R$ holds, then condition $(1-\alpha)P(q_W, r_W) \geq -\rho + (1-\alpha)q_N R$ also holds.

**Proof:** The proof follows from the fact that $q_N \leq 1$ and $r_W > 0$. #

In principle, there are many possible configurations of equilibrium outcomes in this model, depending on parameter values. Furthermore, as in the previous section, for some parameter values, multiple equilibria may exist. Studying the different cases can provide interesting insights about the influence of discount window policy on interbank lending activity. In the next section, we take a partial step in this direction. To keep the analysis focus on the issue of stigma, we study equilibrium for a particular range of parameter values for which the possibility of stigma is present.

# 5 Stigma

The objective in this section is to construct an equilibrium where some banks take loans in the interbank market at rates higher than the discount window rate $r_W$; that is, we want to construct an equilibrium in which stigma is attached to lending from the discount window. Doing this, will allow us to identify the condition under which such a situation is theoretically possible and draw some conclusions with respect alternative policy changes that could help to reduce its incidence.

We will consider that case when $r_W = i + \tau$ for $\tau > 0$; that is, lending at the discount window lending involves paying a penalty rate (i.e., greater than the opportunity cost of funds, $i$).

**Proposition 4.** Define $\xi(p) = \frac{p - \sigma p}{1 - \sigma p}$ and assume that $\tau \in (0, A)$ where $A = \min\left\{\rho - i, \frac{\alpha}{1-\alpha} i\right\}$. Then, there exist a threshold value $\overline{p}_\tau < 1$ such that if $p \in (\overline{p}_\tau, 1)$ we have that there exists an equilibrium with both interbank credit and discount window lending in which $q_W = \xi$, $q_N = 1$, and $r_H = i + \theta T S_H(q_N, q_W)$.

**Proof:** See appendix. #

In this equilibrium, only those illiquid banks that find a counterparty in the interbank market and are holding a high quality asset, borrow from another bank. Illiquid banks that do not find a counterparty plus those banks that do but are holding a low quality asset, borrow from the discount window.

**Corollary 3.** When $\theta$ approaches unity, $r_H$ approaches $r_W + (1-\alpha)(q_N - q_W)R > r_W$.

**Proof:** Note that $\theta$ appears only in the condition that determines the surplus splitting rule between the liquid and the illiquid bank holding a high quality asset. Then, the existence of the equilibrium described in Proposition 4 is independent of the value of $\theta$ and we can move $\theta$ as close to one as necessary to obtain our result. #

This corollary demonstrate that when the bargaining power of lenders is high, the equilibrium in our model may involve some banks that are willing to take a loan in the interbank market at a rate higher than the rate that they could obtain at the discount window. That is, under certain conditions, our model predicts that some degree of stigma will be attached to lending from the discount window.

The proportions of banks lending from the window in the equilibrium of Proposition 4 is $1 - \sigma p$. In reality (at least during normal times), most banks do not borrow from the discount window. This regularity would argue for considering cases where the proportion of banks borrowing in the market, $\sigma p$, is high. This, in turn, implies that $q_W$ would be close to one in equilibrium. By our corollary, note that high values of $q_W$ reduce the scope for stigma, which is given by $r_H - r_W = (1-\alpha)(1-q_W)R$, and as $q_W$ approaches unity equilibrium stigma tends to disappear. However, also note that in the data, the stigma component usually amounts to only a few basis points. So, our model predictions may still be consistent with the data.

In the model, the observed interest rate in the market is $r_H$ which does not involve any repayment risk. Banks with low quality assets (which could be considered the risky banks in our setup) do not receive loans from private banks. They are just cut out of the interbank market. In the data, however, not all banks borrow at the same interest rate. In fact, every day, there is a distribution of rates observed in the market. In our model, we could capture these different rates by, for example, introducing some heterogeneity in the bargaining power of different banks. This modification may, in fact, be realistic. For example, banks in need for funds in a given day may find that their usual counterparty has no funds available that day. In that case, they need to search in the market for alternative counterparties, and depending on their network connections, they may find their bargaining power much reduced. In our model, illiquid banks with low bargaining power will pay higher interest rates. In fact, this kind of heterogeneity will be consistent with the fact that, most of the times, only some banks pay interest rates in the market that are higher than the one they could obtain at the discount window.

An interesting feature of our equilibrium is that it requires $p$ to be high. If we think that lower values of $p$ are associated with a general deterioration of asset quality, our model tells us that the kind of equilibrium we are considering will not be possible when asset quality deteriorates beyond some point. The reason why our equilibrium may break down as the value of $p$ becomes lower is that some banks may withdraw from borrowing at the window if the equilibrium value of $q_W$ is too low (as it needs to be if the value of $p$ is low). To be concrete, if the value of $q_W$ is too low, it may be the case, for example, that it is no longer beneficial for banks holding low quality assets to borrow at the window. But then, illiquid banks that find a match will borrow in the market and illiquid banks that do not find a match will borrow at the window. In consequence, the composition of banks borrowing at the window would be the same as that of banks borrowing in the market (a proportion $p$ of banks holding high quality assets and a proportion $1-p$ holding low quality asset). Hence, borrowing at the discount window will no longer be regarded as a negative signal and the equilibrium of the model will no longer be consistent with the possibility of stigma.

Finally, note that higher values of the probability of finding a match in the interbank market, $\sigma$, are associated (*ceteris paribus*) with lower equilibrium values of $q_W$. In fact, very high values of $\sigma$ will undermine the possibility of an equilibrium with stigma of the type we consider in Proposition 4. The reason for this fact is that, in our equilibrium, a bigger proportion of banks finding a match in the interbank market results in a larger (relative) participation of the low-asset-quality-holder portion of those banks in total amount of lending at the discount window. As a consequence, the composition of banks borrowing at the window shifts towards a relative abundance of banks holding low quality assets and, hence, $q_W$ becomes smaller. For small enough values of $q_W$ a bank that finds a match and is holding a low quality asset will actually prefer to take a loan from its private counterparty, an action that is inconsistent with the equilibrium proposed in Proposition 4.[10]

If we think that the likelihood of finding a counterparty in the interbank market is an approximate measure of the liquidity in that market, then we can conclude that the possibility of equilibrium stigma is associated with a moderate degree of liquidity in the interbank market. When the market for interbank loans is very liquid, the equilibrium with stigma studied in this section breaks down.

# 6 Conclusion

To be written

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