House Prices and Time on the Market in Competitive Search *

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Abstract

Some salient stylized facts of the housing sector are hard to reconcile with the Walrasian market paradigm. These include the high volatility of prices and sales relative to GDP, the co-movement of these variables and their negative correlation with time on the market, the high volatility of housing construction and its slow response to changes in prices and vacancies. We build a search equilibrium model of the housing market which is consistent with these facts. Our model captures the illiquidity of housing assets. Throughout their lives, households experience idiosyncratic shocks that affect how much they value their residence (e.g. their job or their size changes over time). When hit by a shock, households become mismatched and seek to move. Potential buyers take time to locate an appropriate unit. Competitive market forces operate, whereby owners of vacant units may post low prices in order to get more visits and sell their property faster. We characterize a steady-state equilibrium in the short-run (when the housing stock is fixed) and in a long-run where housing construction is endogenous but sluggish. We calibrate our economy to reproduce selected aggregate statistics of the U.S. economy. We find that our model economy is able to reproduce some features of the observed joint behavior of prices, sales and time on the market.

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1 Introduction

A household's residence is usually its largest single asset. Household real net housing wealth is about half of the level of tangible financial assets held by households on average (according to the Board of Governors of the Federal Reserve System's data for 2006). Residential real estate also represents a large share of both total wealth and GDP–33 and 11 percent, respectively, in the US in 2005 (see Merlo, Ortalo-Magne, and Rust 2007). There are some salient stylized facts of the housing sector that have been emphasized in the literature. In particular, housing prices are more volatile than GDP (displaying positive auto-correlation at high frequencies), and sales are even more volatile and they co-move with prices (see Sanchez-Marcos and Ríos-Rull 2007). Housing construction levels are also highly volatile and persistent over time (see Glaeser and Gyourko 2006), but construction reacts relatively slowly to changes in prices and vacancy rates (see Wheaton 1990).

Despite the importance of the housing sector, our understanding of many aspects of this sector is, at best, incomplete. Attempts to build an equilibrium model of the housing market that is consistent with the stylized facts mentioned above have proven difficult. One reason is that the housing market displays some key features that are hard to reconcile with the Walrasian market paradigm.

First and foremost, housing units are *illiquid* long-lived assets. They are costly to sell and buy, they can be partially financed, and they are not movable.[1] Incorporating these frictions into the Walrasian set-up helps to match the observed price volatility (see, for instance, Nakajima 2005 or VanNieuwerburgh and Weill 2006), but it is still hard to explain the positive co-movement of prices and sales (see, for instance, Sanchez-Marcos and Ríos-Rull 2007).

The illiquidity of houses, moreover, has an additional dimension. Anyone who has gone through the process knows it takes time and resources to buy and sell housing units. In particular, there are costs of acquiring relevant information in each potential transaction. For instance, buyers' valuations typically depend not only on attributes of the units for sale which are listed together with the price by sellers or realtors, or verifiable at a negligible

[1] See Diaz and Luengo-Prado (2006) for a model of tenure choice with housing adjustment costs and collateralized borrowing constraints, for instance.

cost, say via a phone call or an internet search (e.g. size, number of bedrooms and bad rooms, neighborhood, proximity to public transportation, age, floor, ...). Buyers' valuations also depend on idiosyncratic features of the unit that can only be verified by visiting and inspecting them. For instance, buyers may want to visit units for sale to check how well-kept the property is, whether there are sources of noise such as heavy traffic or loud neighbors, etc. In general, a typical buyer visits several units prior to purchasing one. Thus, time to sell is a key variable that we have to take into account to understand the joint movement of prices and sales.

This liquidity (e.g., time to sell) also seems to vary widely over time, and it co-moves with prices and sales. In "hot" real estate markets, price and sales are high and so is liquidity (e.g. properties sell fast and time-on-the-market is low), while in "cold" real estate markets prices, sales and liquidity are low, as reported by Krainer (2001). Thus, there is evidence that adjustments to changes in housing market conditions take place not only through prices and quantities, as in a Walrasian world, but also through the degree of liquidity; i.e., how easy/hard it is to sell a property.

In this paper we build a search equilibrium model of the housing market. Our model captures the aforementioned illiquidity of the housing market since, in order to trade, buyers and sellers must search for trading partners. We also assume that buyers' valuations have an idiosyncratic match specific component that is realized when buyers visit and inspect the units for sale. Sometimes buyers will visit units that they do not like, and will then move on with their search.

Following Wheaton (1990), we consider a symmetric dynamic environment where households experience shocks throughout their life-time that affect how much they value their residence. We interpret these shocks as changes in a job or in the number of household members (e.g. through marriage, birth or divorce) that render the households mismatched with their current residence. When a shock occurs a household who was formerly matched with its residence may become mismatched. Such a household may then seek to buy an appropriate unit and sell the current one. For instance, if an agent living and working in NYC in period t gets a job in LA in period $t + 1$, the agent will become mismatched and may seek to buy a new residence and sell the current one. We study both a short-run equilibrium where the housing stock is fixed, and a long-run equilibrium where the housing stock is endogenous.[2]

Buyers and sellers who choose to participate the housing market meet at random in bilateral pairs. A standard matching function describes this random meeting process. When a buyer finds a seller with a suitable unit, they bargain over the match surplus according to the bargaining rule in Hosios (1990). It is well-known that this bargaining rule attains an efficient division of the surplus (internalizing the search externalities). There is a competitive markets story that serves as a rationale for this bargaining rule (see Montgomery (1991), Peters 1991, Moen (1997), Shimer (1996), and Burdett, Shi and Wright (1997), among others). There, prior to the search process, sellers compete by posting price offers in other to attract buyers. Buyers then observe all the posted offers and restrict their search to the set of sellers posting the most attractive offer. In general, sellers who post lower prices on average will attract more buyers. There is evidence that competitive forces are present in the housing market in developed economies. For instance, Merlo and Ortalo-Magne (2004) provide evidence for the UK that, by listing lower prices, sellers increase the number of visits and offers they get, and sell their property faster.[3]

We characterize the steady state equilibrium in the short run and derive analytically some comparative static results. Next, we turn to calibrate our model economy to reproduce some selected statistics of the US economy, namely, the vacancy rate, the expected tenure length and the mean time needed for a successful search in the market. Next, we conduct some quantitative exercise to analyze the steady state effects of changes in the housing stock and the population size. The first remarkable feature of our model economy is that sales and time on the market are always negatively correlated. That is, when the volume of sales is high, the mean time needed to sale a unit is low, and vice versa. Second, when the housing stock is given, the elasticity of the price is very high to changes in exogenously given factors.

[2]For simplicity, we assume that all housing units are owned by the households and there is no rental housing market. We also assume that households want to consume at most one unit (the value they assign to additional units is zero).

[3]Our model is too simple to capture the details of the strategic interaction between buyers and sellers (see, however, Merlo, Ortalo-Magne, and Rust 2007). We implicitly make a strong commitment assumption that all transactions take place at the listed price each period (there is no renegotiation). Merlo and Ortalo-Magne (2004) report that sometimes bargaining reduces the sale price relative to the listed price. Yet, in Merlo and Ortalo-Magne (2004)'s sample, properties sell at about 96 percent of their current listed price (a similar number is provided for the US by the National Association of Realtors).

For instance, if population falls so that the vacancy rate rises from 1.5 percent to 4 percent, the price falls by a factor of 16. Time to sell increases from 8 weeks to almost 5 years.

Next, we allow agents to produce housing units and calibrate the economy to reproduce additional features of the U.S economy, such as the ratio of residential investment to the housing stock. We see that the existence of production reduces significantly the range of fluctuations of sales, prices and time on the market. We also formalize the Walrasian counterpart of this model economy, where search frictions are absent. Next, we make some steady state comparative statics. Some lessons can be learnt from comparing our economy with search frictions with its Walrasian counterpart. First, sales and time on the market do not comove any longer. Moreover, sales and prices fluctuate much less than in the economy with search frictions.

Our benchmark model abstracts away from several important issues, including financial frictions, rental housing markets, and population growth (e.g. immigration).[4] In future work, we plan to embed our model of the housing market in general equilibrium to study these issues. We view the model in this paper as a simple benchmark that, while leaving out several important factors that affect the housing market, is a step forward in that it can account for the co-movement of prices, sales and liquidity we see in the data. In our simple model this co-movement arises solely from the interaction of search and matching frictions with competitive forces.

2 Related literature

Other authors have suggested that search models are a natural benchmark to study the housing market (see Wheaton (1990), Yavaş (1992), Williams (1995), Arnold (1999), Krainer (2001), Albrecht, Anderson, Smith, and Vroman (2007), Yui and Zhang (2007), among others). Our focus here different, however. Our objective is to build a fully calibrated equilibrium model that is consistent with the stylized facts of the housing sector mentioned above. Also, a key difference with respect to the existing search models has to do with the equilibrium notion we adopt. In most of these models, prices are determined by Nash

[4]The interaction of financial constraints and search frictions are studied, for instance, by **?**), and **?**).

bargaining after buyers and sellers meet. By contrast, in our model competitive forces operate, whereby sellers can affect the arrival rate of buyers and sell their property faster by posting low prices prior to the search process. As we have already noted, there is evidence that these competitive forces are present in the housing market. This difference is important for policy analysis since the equilibrium notion we adopt generates a constrained efficient allocation, while the equilibrium with Nash bargaining is typically constrained inefficient.

The papers by Krainer (2001) and Yui and Zhang are the closest to our paper. Both papers generate a positive co-movement in prices, sales and liquidity, though the models are not calibrated to the data. The mechanism operating there are slightly different. In Krainer (2001), households' valuations are subject (not only to idiosyncratic but also) to aggregate shocks, and the housing stock is fixed. He finds that, provided there is enough persistence in the aggregate shock, prices, sales and liquidity are higher in the high state (when all valuations are higher) than in the low state. In our model, more frequent idiosyncratic shocks and increases in the population also have this effect of heating up the market. This mechanism arises from competitive search forces and it is not present in Krainer's model (e.g. there more frequent idiosyncratic shocks increase prices but decrease liquidity and sales).

Yui and Zhang (2007) characterize a long-run equilibrium with free entry of sellers. Their equilibrium notion is halfway between Nash bargaining and competitive search. While prices are determined by ex post Nash bargaining, there are different "submarkets" where sellers can choose to trade ex ante. [5] Since they must be indifferent among all active submarkets, in equilibrium liquidity is higher in submarkets with lower prices. The authors find a positive co-movement between in prices, sales and liquidity when buyers differ in their waiting costs and their search intensity is endogenous (but without the latter the movement in liquidity is counterfactual). Our model delivers this co-movement with symmetric buyers and sellers and no choice of search intensity. Also, we model housing construction explicitly, in a way that is consistent with its sluggish response to changes in prices and vacancy rates.

[5] Buyers, however, cannot move across submarkets as in a competitive search equilibrium (see Moen (1997)).

3 The model economy

3.1 Environment

Time is infinite and discrete. There is a measure N of symmetric households who derive utility from the services of indivisible housing units. For simplicity, we assume that households want to consume the services of a single unit each period (they assign zero value to additional units). Utility is transferable, so implicit is the existence of a divisible good which yields constant marginal utility to all households.

Throughout their lives, households experience exogenous idiosyncratic shocks that affect their valuation for the units where they reside. Each period t, households living in a housing unit can be in one of two idiosyncratic states, $s = 0, 1$. In state 1 the household's flow (per period) utility from the services provided its residence is $\bar{v} > 0$. By contrast, in state 0 this flow value is only $\underline{v} \in [0, \bar{v})$. We shall say that households in state 1 are "matched" and households in state 0 are "mismatched". For instance, many agents prefer to live and work in nearby locations to avoid long commutes. On the other hand, agents usually change jobs over time. In this example, agents are matched if their residence is close to their job, and are mismatched otherwise. Similarly, households with many (few) members prefer to live in larger (smaller) units. Yet, the number of members in the household changes over time (e.g. through marriage and divorce). In this example, households are matched if the size of their residence is appropriate for the number of household members, and are mismatched otherwise. In general, in state 1 households assign a high value to their residence and in state 0 if the value assigned is low. In our environment, matched households (in state 1) will become mismatched (move to state 0) when they are hit by an idiosyncratic shock (e.g. their job or family size changes).

In our benchmark model, all units are owned by the households and there is no rental market for housing. In the short run, the stock H of housing units is exogenously fixed. For simplicity, we assume that households may own either one or two units. That is, $H \in (N, 2N)$, so there is a measure $H - N$ of households who own two units. As we have already noted, these households derive no value from their second unit.

Matching frictions arise in this environment because households change states over time. Specifically, we assume that matched households who own one unit become mismatched with probability $\alpha \in (0,1)$ each period t (so their state changes from $s = 1$ to $s = 0$). As usual, we assume that these preference shocks are realized in such a way that α is also the fraction of these households who become mismatched each period (the Law of Large Numbers holds). Such households will then seek to buy an appropriate unit and sell the current one.[6]

Housing units are traded bilaterally in the housing market. In this market, households seeking to buy or sell a unit meet potential trading partners at random. In Section 3.2 we describe in detail the random meeting process and the price determination process in the housing market.

We assume that the flow (per period) value a household assigns to a unit visited at random is a random variable v. This variable measures the quality of a match between the household and the unit, and is realized when the household visits the unit. For simplicity, we assume that v can take two values $\{\bar{v}, 0\}$ with respective probabilities $q \in (0,1)$ and $1 - q$. That is, the household either likes the unit or not. This assumption captures the fact, in addition to the usual characteristics of the unit that can be verified over the phone or co-move (such as location, size,...), household valuations depend also on idiosyncratic features of the unit that can only be verified by visiting an inspecting it. For instance, some households may be looking for a unit of an ideal size and location that is also quiet (e.g. if they have trouble sleeping at night). Such households may want to visit units for sale to check for possible sources of noise such as heavy traffic or loud neighbors. The lower the value of q, the larger the number of units households need to visit until they find one they like (because they are more "picky"). The parameter $q \in (0,1)$ then captures the extent of the matching frictions.

Given the above, households can be in one of three states each period: matched with one unit (not seeking to trade), mismatched with one unit (seeking to buy an appropriate

[6] The assumption that only matched households owning one unit change state is for simplicity. Under this assumption, the only way for mismatched households become matched again is by purchasing an appropriate unit. We could alternatively assume that any household can change type. Then a mismatched household can become matched if it fails to purchase an appropriate unit but it changes state again. Our results generalize to a version of the model where all households face preference shocks. But there the equations are more cumbersome.

unit), and matched with two units (seeking to sell the unit they do not value).[7] Denote the measure of households in each of these states by n_t, b_t and s_t, where

$$N = n_t + b_t + s_t. \tag{1}$$

Because households in the first two states own one unit, while households in the third state own two units, each period t,

$$H = n_t + b_t + 2s_t. \tag{2}$$

Our benchmark model assumes that households who become mismatched first buy a new unit, and then try to sell the old one (they are never homeless). With this assumption, the measure of units potentially for sale at time t is simply the measure of vacant units:

$$s_t = H - N. \tag{3}$$

We could consider a variant of the model with the reverse timing;i.e. where households first sell their old unit and then buy a new one. There, households would pay a cost (rent or hotel fee) in the interim while they do not own a unit. In both variants the qualitative results are the same, but the predictions of the model presented here (in terms of how long it takes to sell a unit and how often people move on average) are more aligned with the data.[8]

Buying and selling units in the housing market takes time because of search frictions. Let $\pi_t^b \in [0,1]$ denote the probability that a mismatched household purchases a unit in period t. Similarly, let $\pi_t^s \in [0,1]$ denote the probability that a vacant unit is sold in period t. These probabilities will be endogenously in equilibrium, but for the moment we take them as given. Of course, the number of units purchased in the housing market is equal to the number of

[7] We assume that the utility of being homeless is $-\infty$, so no household will sell a unit unless they have another unit to live in. We also assume that it is not feasible for households to own more than two units. We then do not allow households to buy a third unit, say, when prices are low and try to sell the unit when prices are high. Mismatched households, however, may engage in this kind of arbitrage trading–buying a second home when prices are low and selling the first one when prices are high. For simplicity, we assume that matched households with one unit do not trade. In the steady state all these restrictions are irrelevant since prices are constant. They are only relevant in our analysis of the transitional dynamics.

[8] We could even allow agents to choose between the two timings. This should not alter the main predictions of the model, but it makes the analysis more complicated.

units sold each period. This aggregate feasibility condition may be written as[9]

$$\pi_t^b b_t = \pi_t^s s_t. \tag{4}$$

We may now describe the evolution of the composition of the population over time. Given the composition of the population in period t, $\{n_t, b_t, s_t\}$, the measure of households who do not seek to trade in period $t+1$ is

$$n_{t+1} = (1-\alpha)n_t + \pi_t^s s_t. \tag{5}$$

These are the households who were matched with one unit at the end of period t and do not change state at the start of $t+1$. These households either did not seek to trade in period t (e.g. families living in a large unit), or they were sellers who sold their vacant unit that period (e.g. families living in a large unit who sold their former small residence in period t).

The measure buyers in period $t+1$ is

$$b_{t+1} = (1-\pi_t^b)b_t + \alpha n_t. \tag{6}$$

This includes those buyers who did not trade in period t, and continue to search for a unit in $t+1$. It also includes those households who were matched with one unit at the end of period t and become mismatched at the start of $t+1$ (e.g. a couple living in a small unit in period t who has a baby in period $t+1$).

Finally, remember that the measure of sellers is constant over time when H is fixed:

$$s_{t+1} = (1-\pi_t^s)s_t + \pi_t^b b_t = H - N. \tag{7}$$

Those sellers who did not trade in period t continue to be sellers in $t+1$. Those buyers who bought a unit in period t and put up for sale their old unit in $t+1$ are also sellers in $t+1$ (e.g. families who bought a large unit in period t and seek to sell their former small residence in $t+1$).

[9] Again, we assume that the Law of Large Numbers holds.

3.2 Search Frictions and Price Determination

In this section, we describe the random meeting process that brings buyers and sellers together in the housing market. We also describe how house prices are determined in equilibrium.

Buyers and sellers who choose to participate the housing market meet at random in bilateral pairs. For simplicity, we assume that each trader experiences at most one match each period. Let b_t and s_t be the measures of buyers and sellers in the market in period t. A standard matching function $\mathcal{M}(b_t, s_t)$ determines the total numbers of matches in period t as a function of the measures of buyers and sellers. As usual, $\mathcal{M} : R_+^2 \to R_+$ is continuously differentiable, increasing in both arguments, strictly concave, and homogeneous of degree one. Also, the total number of matches cannot exceed the number of traders in the short side of the market, so $\mathcal{M}(b_t, s_t) \leq \min\{b_t, s_t\}$. In particular, $\mathcal{M}(0, s_t) = \mathcal{M}(b_t, 0) = 0$.

The probabilities with which buyers and sellers meet potential trading partners depend on the ratio of buyers over sellers in the market:

$$\theta_t \equiv b_t/s_t. \tag{8}$$

This ratio is usually referred to as the degree of "congestion" or "market tightness". By the Law of Large Numbers, the probability that a seller meets a buyer in period t is

$$m^s(\theta_t) = \frac{\mathcal{M}(b_t, s_t)}{s_t} = \mathcal{M}(\theta_t, 1), \tag{9}$$

where $m^s : R_+ \to [0, 1)$ is continuously differentiable, strictly increasing and strictly concave, with $m^s(0) = 0$ and $\lim_{\theta_t \to \infty} m^s(\theta_t) = 1$. Intuitively, the higher the ratio θ_t the easier it is for sellers to meet buyers. As this ratio goes to infinity (zero) the probability that a seller meets a buyer goes to one (zero). Similarly, the probability that a buyer meets a seller in period t is

$$m^b(\theta_t) = \frac{\mathcal{M}(b_t, s_t)}{b_t} = \mathcal{M}(1, {\theta_t}^{-1}), \tag{10}$$

where $m^b : R_+ \to [0, 1)$ a continuously differentiable, strictly decreasing, with $\lim_{\theta_t \to 0} m^b(\theta_t) =$

1 and $\lim_{\theta_t \to \infty} m^b(\theta_t) = 0$. In this case, the higher the ratio θ_t the harder it is for buyers to meet sellers. As this ratio goes to zero (infinity) the probability that a buyer meets a seller goes to one (zero).

We shall see below that a key parameter in the determination of the terms of trade is the elasticity of the buyers' matching probability $m^b(\theta_t)$:

$$\eta(\theta_t) = \frac{-m^{b\prime}(\theta_t)\theta_t}{m^b(\theta_t)} \in [0, 1]. \tag{11}$$

It is commonly assumed that this elasticity is non-decreasing in θ_t.

Given the above, the probabilities with which buyers and sellers trade in the housing market are:

$$\pi^b(\theta_t) = m^b(\theta_t)q, \tag{12}$$

$$\pi^s(\theta_t) = m^s(\theta_t)q = m^b(\theta_t)\theta_t q, \tag{13}$$

respectively. That is, buyers locate a vacant unit with probability $m^b(\theta_t)$, and like this unit with probability q. Similarly, sellers are contacted by a buyer with probability $m^b(\theta_t)\theta_t$ who likes their unit with probability q.

Denote the values of buyers, sellers, and non-traders by W_t^b, W_t^s and W_t^n, respectively. Denote the discount factor by $\beta \in (0, 1)$. By symmetry, all bilateral transactions in the housing market are identical. In particular, there will be a single equilibrium price p_t at which all units traded are sold each period.

The Bellman equation of a buyer is

$$W_t^b = \pi^b(\theta_t)[\max\{\bar{v} - p_t + \beta W_{t+1}^s, \underline{v} + \beta W_{t+1}^b\}] + (1 - \pi^b(\theta_t))[\underline{v} + \beta W_{t+1}^b]. \tag{14}$$

If buyers locate a vacant unit that they like and choose to purchase the unit, they get utility $\bar{v}$, pay a price p_t for it, and become sellers in the following period. Otherwise, buyers get utility $\underline{v}$ from their current unit and continue to be buyers in period $t + 1$.

The Bellman equation of a seller is

$$W_t^s = \bar{v} + \pi^s(\theta_t)[\max\{p_t + \beta W_{t+1}^n, \beta W_{t+1}^s\}] + \beta(1 - \pi^s(\theta_t))W_{t+1}^s. \tag{15}$$

Sellers always get utility $\bar{v}$ for the unit they occupy, whether or not they trade. If they meet a buyer who likes their vacant unit and choose to trade with the buyer, they receive the price p_t and exit the housing market the following period. Otherwise, sellers do not trade and continue to be sellers in period $t + 1$.

Finally, the Bellman equation of a non-trader is

$$W_t^n = \bar{v} + \beta[(1 - \alpha)W_{t+1}^n + \alpha W_{t+1}^b]. \tag{16}$$

Non-traders get flow utility $\bar{v}$ for their only unit and do not participate in the housing market in period t. However, with probability α they become mismatched at the start of period $t + 1$ and seek to buy a unit during that period.

We assume that the bilateral match surplus is positive (and avoid the uninteresting autarky case). Since buyers and sellers get a positive share of this surplus, the equilibrium price p_t will be such that both buyers and sellers want to trade when the buyer likes the unit. The Bellman equations (14)-(16) can then be written as

$$W_t^b = \underline{v} + \beta W_{t+1}^b + \max\{S^b(\theta_t, p_t), 0\} = \underline{v} + \beta W_{t+1}^b + S^b(\theta_t, p_t), \tag{17}$$

$$W_t^s = \bar{v} + \beta W_{t+1}^s + \max\{S^s(\theta_t, p_t), 0\} = \bar{v} + \beta W_{t+1}^s + S^s(\theta_t, p_t), \tag{18}$$

$$W_t^n = \bar{v} + \beta W_{t+1}^n - \beta\alpha(W_{t+1}^n - W_{t+1}^b), \tag{19}$$

where $S^b(\theta_t, p_t) > 0$ and $S^s(\theta_t, p_t) > 0$ are the expected trade surpluses for buyers and sellers in period t as a function of the price p_t and the number θ_t of buyers per seller in the housing market:

$$S^b(\theta_t, p_t) = \pi^b(\theta_t)[\bar{v} - \underline{v} - p_t + \beta(W_{t+1}^s - W_{t+1}^b)], \tag{20}$$

$$S^s(\theta_t, p_t) = \pi^s(\theta_t)[p_t + \beta(W_{t+1}^n - W_{t+1}^s)]. \tag{21}$$

When they trade, buyers get an increase $\bar{v} - \underline{v}$ in current (flow) utility from housing services net of the price p_t. The buyers' continuation utility also changes since they become sellers in period $t+1$. Similarly, when sellers trade, they receive the price p_t. Their continuation utility also changes since, at the start of $t+1$, they remain in a matched state and exist the housing market with probability $1-\alpha$, and become mismatched (buyers) with complementary probability.

The expressions of the expected trade surpluses in equations (20) and (21) highlight the fact that, in the presence of search frictions, traders care not only about prices but also about the expected time it takes then to trade in the housing market. Buyers prefer low prices and fast purchases (S^b is decreasing in p_t and θ_t). Sellers prefer high prices and fast sales (S^s is increasing p_t and θ_t). This trade-off between prices and trading delays is key in the division of the surplus between buyers and sellers in our search equilibrium notion.

The search equilibrium notion we adopt assumes that each period t buyers and sellers bargain over the match surplus according to the bargaining rule in Hosios (1990). The match surplus is the sum of the ex-post surplus of the buyer and the seller when the buyer likes the unit:

$$G_t = \bar{v} - \underline{v} - p_t + \beta(W^s_{t+1} - W^b_{t+1}) + p_t + \beta(W^n_{t+1} - W^s_{t+1}) \tag{22}$$

$$= \bar{v} - \underline{v} + \beta(1-\alpha)(W^n_{t+1} - W^b_{t+1}). \tag{23}$$

The total bilateral surplus reflects the instantaneous gain from a match, $\bar{v} - \underline{v}$, as well as the gain in terms of the expected change in the discounted continuation utilities of buyers and sellers, $\beta(1-\alpha)(W^n_{t+1} - W^b_{t+1})$. This gain is realized when the agents trade, so the buyer becomes a seller and the seller becomes a no-trader.

The bargaining rule in Hosios allocates a share $\eta(\theta_t)$ of the match surplus to sellers and a share $1-\eta(\theta_t)$ of the match surplus to buyers; i.e.,

$$\bar{v} - \underline{v} - p_t + \beta(W^s_{t+1} - W^b_{t+1}) = (1-\eta(\theta_t))G_t, \tag{24}$$

$$p_t + \beta(W^n_{t+1} - W^s_{t+1}) = \eta(\theta_t)G_t. \tag{25}$$

Equivalently, the equilibrium prices satisfy the following equation:

$$\frac{\bar{v} - \underline{v} - p_t + \beta(W^s_{t+1} - W^b_{t+1})}{p_t + \beta(W^n_{t+1} - W^s_{t+1})} = \frac{1 - \eta(\theta_t)}{\eta(\theta_t)}. \tag{26}$$

It is well-known that this bargaining rule attains an efficient division of the surplus (internalizing the search externalities). For a given θ, the elasticity $\eta(\theta)$ measures the contribution of sellers to the matching process, while $1 - \eta(\theta)$ measures the contribution of buyers to this process. Condition (26) says that buyers and sellers get a share of the surplus that measures their contribution to the matching process. In particular, unlike in the case of the Nash bargaining solution, here the division of the surplus then depends on the relative numbers of buyers and sellers in the market. In particular, because $\eta(\theta_t)$ is non-decreasing, the higher the number of buyers per seller the higher the fraction of the surplus received by sellers in equilibrium (and vice versa).

There reason why we assume that prices are determined according to the above rule is that there is a competitive markets story that serves as a rationale it. This competitive notion of search equilibrium originates in the work of Montgomery (1991) and Peters 1991, and is further developed by Moen (1997), Shimer (1996), and Burdett, Shi and Wright (1997), among others. The "competitive search process" can be described as follows. Prior to the search process, sellers compete by simultaneously posting price offers (and committing to these offers). Buyers then observe all the posted offers and direct their search to the set of sellers posting the most attractive deal (possibly randomizing over offers if they are indifferent). All sellers who post an offer and all buyers who seek that offer meet at random according to the matching function $\mathcal{M}$. When a buyer and a seller meet, the buyer inspects the unit and, if she likes it, she buys the unit at the posted price. Otherwise, there is no trade.

The key to the "competitive search process" is that, in choosing among the posted offers, buyers have rational expectations about the number of buyers that will be attracted by each offer and hence about the degree of congestion θ that the offer will generate. In choosing among the posted offers, buyers will then trade off higher prices for faster trades. In particular, the above authors show that any price offer p_t posted by sellers in equilibrium and the ratio θ_t of buyers per seller associated with that offer must solve the following program:

$$\bar{S}_t^s = \max_{(\theta_t, p_t) \in R_+ \times R} S^s(\theta_t, p_t) \text{ s.t. } \bar{S}_t^b = S^b(\theta_t, p_t). \tag{27}$$

Here, $\bar{S}_t^b$ and $\bar{S}_t^s$ denote the expected surpluses that buyers and sellers receive in equilibrium. Put differently, any posted price p_t and associated congestion θ_t must maximize the expected surplus of the sellers while ensuring that buyers get a common expected surplus in equilibrium. Hence, the equilibrium outcome is constrained efficient.

The above result is intuitive. First, since all buyers are symmetric, they must receive a common expected surplus $\bar{S}_t^b$ no matter what price offer they choose. Similarly, sellers are also symmetric, so they must receive the same expected surplus $\bar{S}_t^s$ no matter what price offer they post. Second, sellers cannot gain by posting deviating price offers. If a small mass of sellers deviates and posts a different offer p_t' that attracts buyers, that offer cannot yield more than $\bar{S}_t^b$ (otherwise, other sellers would profitably undercut this offer). Given the common utility received by buyers $\bar{S}_t^b$, any deviating offer p_t' would then attract θ_t' buyers per seller, where $\bar{S}_t^b = S^b(\theta_t', p_t')$. That is, buyers will switch to this offer p_t' until they are indifferent between trading with the deviating sellers and trading with the non-deviating sellers (and getting $\bar{S}_t^b$). It follows that sellers cannot gain by posting deviating offers if and only if the equilibrium values of θ_t, p_t solve program (27).

It is easy to check that the solution (θ_t, p_t) of this convex program is unique and interior for given $\bar{S}_t^b, \bar{S}_t^s > 0$.[10] This means that all units are traded at the same price and with identical probability. The solution is characterized by the following tangency condition:

$$\frac{\frac{\partial S^b(\theta_t, p_t)}{\partial \theta_t}}{\frac{\partial S^b(\theta_t, p_t)}{\partial p_t}} = \frac{\frac{\partial S^s(\theta_t, p_t)}{\partial \theta_t}}{\frac{\partial S^s(\theta_t, p_t)}{\partial p_t}}. \tag{28}$$

Differentiating (20) and (21), substituting into (28), and using equations (12) and (13), we obtain (26).

Remark 1: While we have assumed a general matching function, the urn-ball matching

[10] Substituting z as a function of θ from the constraint into the objective function yields an strictly concave function (because $m^s(\theta)$ is strictly concave). Also, because $\bar{S}^{is}, \bar{S}^{ib} > 0$, the unique solution must satisfy $0 < \theta < \infty$.

process is specially suitable for our environment. Peters (1991) constructs a price-posting game that generates this matching process endogenously when the number of traders is large.[11] In this case,

$$m^s(\theta_t) = 1 - \exp\{-\theta_t\}. \tag{29}$$

$$m^b(\theta_t) = \frac{1 - \exp\{-\theta_t\}}{\theta_t}, \text{ and} \tag{30}$$

$$\eta(\theta_t) = \frac{\exp\{\theta_t\} - (1 + \theta_t)}{\exp\{\theta_t\} - 1}, \tag{31}$$

so $\eta'(\theta_t) > 0, \lim_{\theta_t \to 0} \eta(\theta_t) = 0$,and $\lim_{\theta_t \to \infty} \eta(\theta_t) = 1$. For this reason, we use this matching function in our quantitative analysis.

Also of interest is the limiting competitive situation where search frictions vanish so $q = 1$ and matching is efficient:

$$\mathcal{M}(b_t, s_t) = \min b_t, s_t, \tag{32}$$

$$m^s(\theta_t) = \min\{\theta_t, 1\}, \tag{33}$$

$$m^s(\theta_t) = \min\{0, \theta_t^{-1}\}. \tag{34}$$

In this frictionless competitive case, agents on the short side of the market trade with probability one, while agents on the long side of the market are rationed. There are two cases. If $\theta_t < 1$, there are fewer buyers than sellers, so $m^b(\theta_t) = 1$ and $m^s(\theta_t) = \theta$. Also, competition among sellers drives their surplus to zero, so buyers get all the surplus. That is, p_t is pinned down by setting $S^s(\theta_t, p_t) = 0$ or $S^b(\theta_t, p_t) = G_t$. If $\theta_t > 1$, there are more buyers than sellers, so $m^s(\theta_t) = 1$, $m^s(\theta_t) = \theta^{-1}$, and p_t is given by $S^b(\theta_t, p_t) = 0$ or $S^s(\theta_t, p_t) = G_t$ (so sellers get all the surplus).[12]

[11] In Peters (1991), sellers post and commit to ex ante price offers to attract buyers to match with them. Buyers then observe all the posted offers and simultaneously select a seller as a potential trading partner (their selection strategy being described as a probability measure on the set of sellers posting offers). A symmetric equilibrium is characterized where all buyers and all sellers play identical strategies. The matching function (29) describes how buyers' selection strategies respond to unilateral price deviations by sellers in the symmetric equilibrium.

[12] In the short-run, where the number of sellers is constant, $\theta_t \neq 1$. Setting $b_t = s_t = H - N$ implies that n_t is constant (a contradiction). However, in the long-run–where s_t is endogenously determined as a result of the equilibrium level of housing construction–the pricing rule in 27 implies $\theta_t = 1$ (as we shall see). Given $\theta_t = 1$, any p_t that splits the surplus between the buyer and the seller generates a constrained efficient outcome (i.e., it solves 27 for some value of $S^b \in [0, G_t]$). As we shall see, the long-run equilibrium price p_t is pinned down by a zero profit condition in housing construction.

Remark 2: Our benchmark model assumes that there are neither taxes nor intermediation costs. Suppose buyers and sellers had to pay an extra cost, F^b and F^s (and these are not too high, so $F^b + F^s$ does not exceed the bilateral match surplus). For instance, these costs could be proportional to the price and/or include a fixed part: $F^i = (1 + f^i)p_t + \bar{F}^i, i = b, s$. In this case, the total price paid by buyers p_t^b is different from the net price p_t^s received by sellers : $p_t^b = p_t + F^b, p_t^s = p_t - F^s$. Hence, the equilibrium will be characterized as above except that (26) is now replaced by

$$\frac{\bar{v} - \underline{v} - p_t - F^b + \beta(W_{t+1}^s - W_{t+1}^b)}{p_t - F^s + \beta(W_{t+1}^n - W_{t+1}^s)} = \frac{1 - \eta(\theta_t)}{\eta(\theta_t)}. \tag{35}$$

4 Short-Run Search Equilibrium

We are now ready to define a search equilibrium for the housing market in the short run (where there is no housing construction).

Definition 1: A short-run **search equilibrium** is a set

$$\left\{n_t, b_t, s_t, p_t, \theta_t, \pi_t^b, \pi_t^s, W_t^b, W_t^s, W_t^n, S_t^b, S_t^s\right\}_{t=0}^{\infty}$$

that satisfies the system of equations (1), (2), (5)–(6), (8)-(13), (17)–(21) and (26), given an initial composition of the population $(n_0, b_0, H - N)$.

The steady state is characterized by the following system of equations

$$\begin{aligned}
N &= n+b+s & (36)\\
H &= n+b+2s, & (37)\\
b &= \theta s, & (38)\\
\alpha n &= m^b(\theta)\theta q s, & (39)\\
(1-\beta)W^b &= \underline{v} + m^b(\theta)q[\bar{v} - \underline{v} - z + \beta(W^s - W^b)], & (40)\\
(1-\beta)W^s &= \bar{v} + m^b(\theta)\theta q[z + \beta(W^n - W^s)], & (41)\\
(1-\beta)W^n &= \bar{v} - \beta\alpha(W^n - W^b), & (42)\\
\frac{\bar{v} - \underline{v} - z + \beta(W^s - W^b)}{z + \beta(W^n - W^s)} &= \frac{1-\eta(\theta)}{\eta(\theta)}, & (43)\\
\eta(\theta) &= \frac{-m^{b\prime}(\theta)\theta}{m^b(\theta)}, & (44)
\end{aligned}$$

Equation (36) is the adding-up condition for the population. Equation (37) describes the ownership distribution among households in different states. Equation (38) gives the ratio θ of buyers over sellers in the economy. Equation (39) ensures that the flows in and out of the non-trading state are equal. A fraction α of households flows out of this state each period. Households flowing into this state are sellers who successfully sold their vacant unit. Equation (40) gives the flow value of a buyer as the sum of the flow value from housing services when mismatched and the expected trade surplus in a steady state. Equation (41) is a similar expression for sellers. Equation (42) describes the flow value of a no-trader as the sum of the flow value from housing services when matched and the discounted expected loss of becoming mismatched in the following period. Equation (43) thus says that the buyers' (sellers') ex post surplus is a share $1-\eta(\theta)$ ($\eta(\theta)$ respectively) of the total bilateral trade surplus. Equation (44) says that the share of the surplus received by sellers is equal to the elasticity of m^b evaluated at the equilibrium ratio θ.

Remember that s is constant in the short-run. Equations (36)-(38) yield b and n as a function of θ :

$$\begin{aligned} s &= H - N, \quad (45) \\ b &= \theta(H - N) \quad (46) \\ n &= 2N - H - \theta(H - N). \quad (47) \end{aligned}$$

Using (39), θ is then given by

$$\alpha(\frac{2N - H}{H - N} - \theta) = m^b(\theta)\theta q. \quad (48)$$

Given θ it is then direct to determine p, W^b, W^s, W^n, G using the remaining equations. For instance, housing prices can be expressed as as a function of G and θ

$$p = \frac{1}{1-\beta}(\bar{v} - \underline{v}) + G[\frac{\beta}{1-\beta} m^b(\theta) q[(1+\theta)\eta(\theta) - 1] - 1 + \eta(\theta)]. \quad (49)$$

In turn, the match surplus G is determined by

$$\begin{aligned} G &= \bar{v} - \underline{v} + \beta(1-\alpha)(W_n - W_b) \text{ where} \quad (50) \\ W_n - W_b &= \frac{(\bar{v} - \underline{v})[1 - m^b(\theta)q(1 - \eta(\theta))]}{1 - \beta(1-\alpha) + \beta m^b(\theta)q(1 - \eta(\theta))]} > 0. \quad (51) \end{aligned}$$

When buyers and sellers trade, the gain in terms of the expected change in the discounted continuation utilities of buyers and sellers is given by $\beta(1-\alpha)(W_n - W_b)$. Note that this gain is always positive, and hence so is the match surplus. Finally, given (51), the values W^n, W^b and W^s can be calculated as

$$\begin{aligned} W^n &= \frac{\bar{v} - \beta\alpha(W_n - W_b)}{1-\beta} \quad (52) \\ W^b &= \frac{\underline{v} + m^b(\theta)q(1 - \eta(\theta))[\bar{v} - \underline{v} + \beta(W_n - W_b)]}{1-\beta} \quad (53) \\ W^s &= \frac{\bar{v} + m^b(\theta)\theta q \eta(\theta)[\bar{v} - \underline{v} + \beta(W_n - W_b)]}{1-\beta} \quad (54) \end{aligned}$$

It is easy to check that both $W_n - W_b$ and G increase with θ (other things equal). This is intuitive. When θ is higher, it takes more time to buy a unit. This increases the loss from experiencing a shock that renders the agent mismatched (measured by $W_n - W_b$), and hence the gains from bilateral trade. It is also easy to check that W^s increases with θ. Intuitively, the value of being a seller increases when θ increases because expected time on the market decreases (units sell faster) and the seller's trade surplus is higher (since the total surplus G is higher and sellers get a higher fraction of G). By contrast, W^n and W^b fall with θ. The value of being a buyer decreases because it takes longer on average to purchase a unit and (even though the total bilateral surplus is higher) the buyers' share of the surplus is lower. The value of non-traders also falls because, in the event they become mismatched in the future, their value will be lower. The effect on the price p of changes in θ is ambiguous in general.

4.1 Short-Run Steady-State Comparative Statics

It is illustrative to perform some steady-state comparative statics at this point. In our quantitative analysis we study also the transitional dynamics in the short-run. The results below follow directly from the characterization of a steady state. In this characterization, (48) is a key equation which captures how the number θ of buyers per seller in the housing market changes due to changes in the population N, the housing stock H, the probability α that households become mismatched, and the extent of matching frictions as measured by q.

Proposition 1. *Changes in the housing stock H have the following effects on a steady-state equilibrium:*

$$
\begin{aligned}
\frac{\partial \theta}{\partial H} &< 0; \ \frac{\partial \pi^s}{\partial H} < 0; \ \frac{\partial \pi^b}{\partial H} > 0; \ \frac{\partial s}{\partial H} = 1; \ \frac{\partial b}{\partial H} < 0; \\
\frac{\partial n}{\partial H} &> 0, \frac{\partial (\pi^s s)}{\partial H} > 0 \ \textit{if} \ \theta > \theta_0, \ \ \frac{\partial n}{\partial H} < 0, \frac{\partial (\pi^s s)}{\partial H} < 0 \ \textit{if} \ \theta_0 < \theta; \\
\frac{\partial G}{\partial H} &< 0; \ \textit{and} \ \frac{\partial p}{\partial H} < 0 \ \textit{if} \ \theta > \theta_1 > \theta_0 \ \textit{(otherwise the effect on } p \textit{ is ambiguous).}
\end{aligned}
$$

where the thresholds θ_0 *and* θ_1 *are defined by*

$$\begin{aligned} (1+\theta_0)\eta(\theta_0) &= 1, & (55) \\ \frac{\beta}{1-\beta} m^b(\theta_1) q[(1+\theta_1)\eta(\theta_1) - 1] &= 1 - \eta(\theta_1), & (56) \end{aligned}$$

As new vacant units are added, the ratio θ of buyers over sellers decreases (from (48)). This increases the time it takes to sell a vacant unit, and decreases the time it takes to buy one (π^s falls and π^b increases). The number s of households matched with two units increases one-to-one with H. The number of mismatched household b falls. The total bilateral surplus decreases because the loss from being mismatched is now lower. The effect on the remaining variables depends on whether θ is sufficiently large or not. Note that when θ is very low, there is no action in the housing market because it takes a very long time to find a buyer. We interpret the range $\theta < \theta_0$ as being in a "frozen" market. Above this threshold the number n of matched households with one unit increases with the housing stock, and so do sales. (Below this threshold both variables fall with H). Also, when θ is sufficiently high (the market is sufficiently active), the price decreases when H increases. On this range then prices, sales co-move (they both fall with H), and display negative correlation with time on the market (which increases with H). This prediction is consistent with the stylized facts described in the introduction. Below the threshold θ_1 the price effect is ambiguous. In the next section, we use numerical simulations to explore this ambiguous effect.

An increase in the population has essentially the opposite effect than an increase in the housing stock

Proposition 2. *Changes in the population* N *have the following effects on a steady-state equilibrium:*

$$\begin{aligned} \frac{\partial\theta}{\partial N} &> 0; \frac{\partial\pi^s}{\partial N} > 0; \frac{\partial\pi^b}{\partial N} < 0; \frac{\partial b}{\partial N} > 0; \frac{\partial s}{\partial N} = -1; \\ \frac{\partial n}{\partial N} &< 0, \frac{\partial(\pi^s s)}{\partial N} < 0 \text{ if } \theta > \theta_2; \frac{\partial n}{\partial N} > 0, \frac{\partial(\pi^s s)}{\partial N} > 0 \text{ if } \theta < \theta_2 \\ \frac{\partial G}{\partial N} &> 0; \frac{\partial z}{\partial N} > 0 \text{ if } \theta > \theta_1 \text{ (otherwise the effect on } p \text{ is ambiguous).} \end{aligned}$$

where the threshold $\theta_2 > \theta_0$ *is given by* $(2+\theta_2)\eta(\theta_2) = 2$.

The difference is that the threshold θ_2 above which the number of non-traders and sales decrease with the population is different (in fact higher than θ_0). While units sell faster, there are fewer vacant units in this case, that is why sales fall on this range. If $m^b(\theta)q\frac{\beta}{1-\beta} > 1$ (e.g. β sufficiently close to one), then $\theta_0 < \theta_1 < \theta_2$ (otherwise $\theta_0 < \theta_2 < \theta_1$). In this case, there is again a range where prices and volume co-move (both increase when N increases), and they display negative correlation with time on the market (which falls with N).

Proposition 3. *Changes in the probability q that a mismatched household likes a unit visited at random have the following effects on a steady-state equilibrium:*

$$\frac{\partial\theta}{\partial q} < 0; \frac{\partial\pi^s}{\partial q} > 0; \frac{\partial\pi^b}{\partial q} > 0; \frac{\partial b}{\partial q} < 0; \frac{\partial s}{\partial q} = 0; \frac{\partial n}{\partial q} = -\frac{\partial b}{\partial q} > 0; \frac{\partial(\pi^s s)}{\partial q} > 0;$$
$$\frac{\partial G}{\partial q} < 0.$$

The effect on p is ambiguous.

When q increases, it becomes easier for buyers to find and purchase a unit they like. Such a change could reflect for instance an improvement in the intermediation technology (e.g. through new channels like the internet). It could also reflect (in an ad-hoc way) better access to credit, so it is easier for households to get a loan. This increase in q reduces the number of buyer per seller in the market in the steady state. While the probability that a potential buyer likes the house is higher, sellers are now less likely to find a buyer. It is easy to check that the first effect dominates and average time on the market decreases and sales increase. Also, fewer households remain in a mismatched state. The total bilateral surplus decreases because the loss from being mismatched is now lower. Yet, in general, the effect on the price is ambiguous. Our simulations show that this effect is negative when one restricts to ranges of α that are consistent with the data (e.g. how long houses stay on the market and how often households move on average).

Proposition 4. *Changes in the rate α at which matched households's change type have the following effects on a steady-state equilibrium:*

$$\frac{\partial\theta}{\partial\alpha} > 0, \frac{\partial\pi^s}{\partial\alpha} > 0, \frac{\partial\pi^b}{\partial\alpha} < 0, \frac{\partial b}{\partial\alpha} > 0, \frac{\partial s}{\partial\alpha} = 0, \frac{\partial n}{\partial\alpha} < 0, \text{ and } \frac{\partial(\pi^s s)}{\partial\alpha} > 0.$$

The effect on p and G is ambiguous.

An increase in the rate at which household's change type (e.g. more frequent changes in job location or in the family structure) increases the number of buyers per seller in a steady state. This decreases average time on the market and increases total sales. The steady state number of mismatched households increases. In general the effect on the price and the match surplus is ambiguous. Yet, our simulations in the next section show that, for reasonable parameter values, both effects are positive.

4.2 Calibration of the model economy

We want to conduct several quantitative exercises to check the ability of our model to account for the observed co-movements of the price, sales and time on the market. First of all, we need to calibrate our model economy.

We assume that the model period is a week. The discount factor β, in annual terms is set equal to 0.96. We calibrate the model economy to account for the following set of observations: according to the U.S. Housing Vacancy Survey the average quarterly vacancy rate for the period 1985-2001 was about 1.5 percent. The National Association of Realtors conducts an annual survey, the *Profile of Home Buyers and Sellers.* There it is reported that buyers typically search for 8 weeks and that they plan to stay in their home for about 10 years. We use these three observations to calibrate H, q and α. Notice that, in our steady state, the vacancy rate is just $(H-N)/H$. The mean number of weeks needed in order to buy a house is $1/\pi_b(\theta)$. Moreover, after some calculations it can be shown that the mean number of tenure periods is $(1-\alpha)/\alpha + 1/\pi_b(\theta)$. We use the matching function in Peters (1991), so $\pi_b(\theta) = q\frac{1-\exp\{-\theta_t\}}{\theta_t}$. With these observations, we find that $H/N = 1.0152, q = 0.1972$, and $\alpha = 0.0019$. Since we do not have any guidance to set values for the utility of being matched and mismatched, we assume that $\overline{v} = 1$, and $\underline{v} = 0.1\,\overline{v}$.

At the steady state, we find that $\theta = 0.993$. That is, the number of buyers and sellers is very close (because α is pretty low). Thus, the mean number of weeks needed to sell a house is 8.05. The equilibrium price is 814.57 which is 1.04 times the seller's utility. The seller gets 41.56 percent of the bilateral match surplus in a transaction.

Now we turn to study the quantitative effect of changes in various parameters. Our objective is to study the responsiveness of the price, sales, time on the market to such changes.

4.3 Across steady state comparisons

In our first exercise, we study the steady state effects of increases in the housing stock. This is equivalent to study a decrease in the population size. As stated in Proposition 1, Figure 1 shows that the market tightness θ falls, as well the aggregate surplus and the seller's surplus. Total sales increase, though sales as a fraction of the housing stock fall. The price falls sharply. For instance, when the stock as a fraction of the population, H/N, increases from 1.0152, as in the benchmark, to 1.0417, the vacancy rate grows from 1.5 percent to 4 percent. The price falls dramatically from 814.57 to 52.16. Time on the market, the mean time to sell, increases from 8 weeks to 5.5 months.

Next, we turn to examine the effect of mobility at the steady state. We could understand changes in α as changes in the geographical mobility of people. A change in α could also reflect a change in age of the typical buyer. For instance, the *Profile of Home Buyers and Sellers* reports that younger households expect a tenure length of 7 years, whereas older households expect to stay for about 12 years. In Figure 2 we have plotted the steady state for different values of α. The lowest value of α implies that households are hit by the mobility shock every 20 years, on average. The largest value of α implies that households become mismatched every 6 years. The housing stock as a fraction of population is $H/N = 1.0152$, that is, the number of vacancies is 1.5 percent. As shown in Proposition 4, as α increases sales increase and time on the market (the time needed to sell a house) decreases. Moreover, prices also increase. Hence, the larger the degree of mobility, the "hotter" the market and the lower its liquidity. The point is that a larger α implies a larger number of buyers. This increases the price, the volume of sales and lowers the time needed to sell a house. Notice the wild increase in the price when the expected tenure length falls from 10 years to 7 years: the price increases from 814.57 in the benchmark to 5409.8, a factor of almost 7.

We also assess the effect of the idiosyncratic friction q. Remember that q measures the probability that a buyer likes the house offered by the seller. We could think of q as a parameter capturing informational frictions about the suitability of the house found by the buyer. Figure 2 shows the steady state for different values of q, assuming a number of vacancies equal to 1.5 percent of the stock of houses. As we can see, the price has a non-monotonic behavior. This is so because q affects both the effective demand and supply of houses. For very low values of q, the market is frozen because it is very hard to find an appropriate unit. The effective demand of houses increases with q (as buyers locate appropriate units faster). This in turn increases the price. After some threshold level a larger q implies that the supply of houses suitable for any given buyer increases. Hence, the price must decrease. For instance, if $q = 3$ the price is 26.96 of the price in our benchmark case. As noted in Proposition 3, time on the market falls with q, and sales increase.

Finally, we have included an exercise where the value $\overline{v}$ of being matched increases. We could understand changes in $\overline{v}$ as capturing changes in the business cycle in the economy. In periods of expansions, utility of having a suitable house increase and, hence, buyers are willing to pay a higher price for it. Our analysis shows that θ is independent of households' valuations, so that the only variable affected is the price. This is reflected in Figure 4. Actually, changes in $\overline{v}$ translate almost in a one-to-one basis to the price. This is so because the stock of houses is given in the short run, but things will be different in the long run.

4.4 Transitional dynamics

To be written.

5 Endogenous Housing Stock

In this section, we characterize a long-run equilibrium in the housing market by making endogenous the housing stock H. The description of equilibrium is almost identical to that in Section 3.2 except that now we allow for depreciation of the housing stock and we assume that each period t households who do not trade can build new units at a cost.

The way we model this is the following. There are two stages within one period, which we could call day and night. At the first stage, sellers might be hit by a shock that makes the additional house they hold to disappear. This shock has probability λ to be realized. If a seller is hit by this shock he turns into a non trader at night. Each non-trader of type i can build a unit by paying a cost c. This cost has two parts. One of them is fixed, f. The second part is meant to capture congestion effects due to decreasing returns in construction (e.g. fixed land). If we call π^n the fraction of non traders that do build a unit, the variable cost is $\kappa\ (\pi_n\, n)^{\xi}$. Hence, the total cost of building one unit is $c = f + \kappa\ (\pi_n\, n)^{\xi}$.

Notice that in this new setup sellers are subject to an idiosyncratic shock that may turn then into non traders. Thus, the value functions would differ depending on the stage of he period we consider. It will be useful to state the low of motion of population. At the beginning of the period, in the morning, the triplet $\left(\widetilde{n}_t, \widetilde{b}_t, \widetilde{s}_t\right)$ describes the distribution of population. Next, sellers are subject to their idiosyncratic shock and a fraction λ of them loss their additional houses. Non traders decide whether to build a house or not and a fraction π_t^n do so. Hence, at the beginning of the second stage, (at night) the distribution of population is

$$n_t = (1 - \pi_t^n)\, \widetilde{n}_t + \lambda\, \widetilde{s}_t, \tag{57}$$

$$b_t = \widetilde{b}_t, \tag{58}$$

$$s_t = (1 - \lambda)\widetilde{s}_t + \pi_t^n\, \widetilde{n}_t. \tag{59}$$

Population at the next period's first stage is

$$\widetilde{n}_{t+1} = (1 - \alpha) n_t + \pi^s\,(\theta_t)\, s_t, \tag{60}$$

$$\widetilde{b}_{t+1} = \alpha\, n_t + \left(1 - \pi^b\,(\theta_t)\right) b_t, \tag{61}$$

$$\widetilde{s}_{t+1} = \pi^b\,(\theta_t)\, b_t + (1 - \pi^s\,(\theta_t))\, s_t, \tag{62}$$

where the market tightness $\theta_t = b_t/s_t$. Now need to differentiate between day and night. Let us denote as $\widetilde{W}_t^i$ the value function of agent i at the first stage of period t and W_t^i the utility function at the second stage, or night. Thus, in the morning, the value function of sellers is

$$\widetilde{W}_t^s = \lambda\, W_t^n + (1 - \lambda) W_t^s. \tag{63}$$

Buyers are not active at the first stage, hence

$$\widetilde{W}_t^b = W_t^b. \tag{64}$$

Non traders have to decide whether to build a housing unit or not. We assume that they open a market for lotteries, as in Rogerson (1988), and that they decide the probability with which they actually build the unit. This probability is denoted as π_t^n. Thus, the problem solved is

$$\widetilde{W}_t^n = \max_{\pi_t^n \in [0,1]} \left(1 - \pi_t^n\right) W_t^n + \pi_t^n \left(W_t^s - c\right). \tag{65}$$

The value functions at night are

$$W_t^s = \bar{v} + \pi^s(\theta_t)[\max\{p_t + \beta\widetilde{W}_{t+1}^n, \beta\widetilde{W}_{t+1}^s\}] + \beta(1 - \pi^s(\theta_t))\widetilde{W}_{t+1}^s, \tag{66}$$

$$W_t^b = \pi^b(\theta_t)[\max\{\bar{v} - p_t + \beta\widetilde{W}_{t+1}^s, \underline{v} + \beta\widetilde{W}_{t+1}^b\}] + (1 - \pi^b(\theta_t))[\underline{v} + \beta\widetilde{W}_{t+1}^b], \tag{67}$$

$$W_t^n = \bar{v} + \beta[(1 - \alpha)\widetilde{W}_{t+1}^n + \alpha\widetilde{W}_{t+1}^b]. \tag{68}$$

Notice that in equilibrium, in an interior solution

$$W_t^s - c = W_t^n. \tag{69}$$

This is a free entry condition.

5.1 Calibration of the model economy

Here we follow our initial strategy and calibrate our economy so that its steady state matches some selected statistics for the U.S. economy. As before, we want to calibrate our model economy so that the vacancy rate is 1.5 percent, the mean tenure is 10 years and the mean time to buy is 8 weeks. These observations imply values form q and λ identical to those calibrated in the economy with a exogenously given housing stock. We keep assuming $\overline{v} = 1$. We also assume that f, the fixed component of building a house is equal to $\overline{v}$. We have three parameters left, λ, κ and ξ, the elasticity of the cost with respect to the number new houses built. At the steady state, investment in new houses as a fraction of the stock is equal to total depreciation. We assume that this fraction is 4 percent in annual terms. This number is taken from Díaz and Luengo-Prado (2006). This implies a value for $\lambda = 0.0523$. With respect to ξ, we assumed that, implicitly, each non trader has a Cobb-Douglas technology that uses capital and land to produce one housing unit. If the amount of land is given, the cost of producing one housing unit must be increasing in the number of units, which at the steady state is $\pi^n n$. The factor ξ would be just the ratio of the land elasticity to capital elasticity. We take those numbers from Davis and Heathcote (2005), which are $\xi = 0.1060/0.1831 = 0.5789$. Given these choices for f and ξ, the free entry condition shown in (69) implies $\kappa = 3.3039$.

5.2 The steady state

Notice that, at the steady state, the market tightness $\theta = 0.9930$, as in the economy with a given housing stock. As in that economy, the vacancy rate is 1.5 percent. Moreover, the expected time needed to buy a house as the expected tenure are also 8 weeks and 10 years, respectively. Moreover, the expected time to sell is also 8 weeks. The difference lies in the price. Now, the price is 16.2974. The associated cost is 73.53 percent of the price. Moreover, the seller surplus is also 41.56 percent of aggregate surplus. Moreover, the aggregate surplus is the same, 11.90. Hence, making endogenous the housing stock does not alter the distribution of the surplus, nor the surplus itself.

In order to quantify the effect of search frictions we want to calculate the steady state in the Walrasian counterpart of the economy described. to do so, we assume that $\pi^s(\theta) = \min\{\theta, 1\}$ and, as before, $\pi^b = \pi^s/\theta$. Thus, we are also assuming that buyers are perfectly informed about the characteristics of houses and $q = 1$. Thus, we find that the price is $p_w = 4.33$. That is, search frictions make the price to multiply by a factor of 3.76. In this case the markup over the cost is zero. That is, the cost of production also falls. This is so because the housing stock is smaller. As a matter of fact, in the benchmark economy with housing production the vacancy rate is 1.5 percent, whereas in the economy without search frictions the vacancy rate is 0.1938 percent, almost nihil. Notice also that sales, as a fraction of the stock, is very similar, 0.1862 and 0.1938 percent, respectively, in the economies with and without search frictions.

5.3 Across steady state comparisons

Here, as before, we study steady state effects of changes in some parameters of interest. We start by studying changes in population. Figure 5 shows steady state values for market tightness, the price and other variables, in the search environment as well as in the economy without search frictions, for different values of the population size. The first thing we notice is that, in absence of search frictions, the price increases, since we are assuming that the cost of producing a house unit increases with population but it does not affect sales, neither time on the market nor the vacancy rate. It is interesting to note that the price multiplies by a factor of 3 when N goes from 50 to 150, in either environment, with and without search frictions. Sales, the vacancy rate and the surplus do not change significantly with big changes in population.

This is why we turn to analyze effects of changes in α. Notice in Figure 6 that the range of values of α considered implies the expected tenure to go from 20 to 6 years. Thus, small changes in α produce large changes in mobility. We could interpret an economy where α is low as an economy with relatively old population, whereas in an economy where all agents are very young should correspond in our framework with an economy with large α. Hence, the price is larger in economies with younger population. The overall change goes from 10.39, when expected tenure is 20 years to 17.08, when the expected tenure is 6 years, a 65 percent change. In absence of search frictions, the price would increase from 2.73 to 4.64, implying a 69 percent increase. Moreover, the search premium, the ratio of the price in the economy with search frictions to the price in the Walrasian economy ranges from 3.67 to 3.8. These magnitudes are very similar to those found above, when we analyzed the effect of changes in the population size. We also can see that time to sell is a bit more responsive to changes in α than to changes in N. Thus, the most significant difference is that the vacancy rate increases with α, as population becomes younger, and falls with N. For instance, in this case, when the expected tenure is 20 years the vacancy rate is 0.83 percent, whereas is 2.31 percent when the expected tenure is 6 years. Notice, also, that in absence of search frictions, changes in the vacancy rate are almost nihil.

We next turn to analyze changes in q. This parameter only affects the economy with search frictions, thus the Walrasian economy is plotted in Figure 7 for comparison purposes. It is interesting to see that, as q becomes closer to 1, the equilibrium of the economy with search frictions converges to the Walrasian equilibrium. This implies that the type of search friction that we are using here, competitive search, or price posting, imposes a friction that only operates when there is an additional informational friction—which here is captured by q, the probability of a buyer finding a suitable house.

In the following exercise we proceed as in the economy were the stock of houses was given and study the effect of a change in $\overline{v}$, the value of being matched. Figure 8 shows that changes in the value of being matched does not affect the Walrasian price. This is so because the Walrasian price is determined, at the margin, by the cost of production. On the other hand, the effect of the interaction between the increase in the valuation of being matched and search frictions is lessened when there is production. In the range considered of $\overline{v}$ it increases by a factor of 4, whereas the price increase by 35 percent. The counterpart of this price increase is the augment of the housing stock and the vacancy rate.

Thus, summarizing, although the movement in price, sales and time on the market are not as big as in the case with the housing stock exogenously given, their responsiveness is larger than in a Walrasian economy.

6 Final comments

To be added.

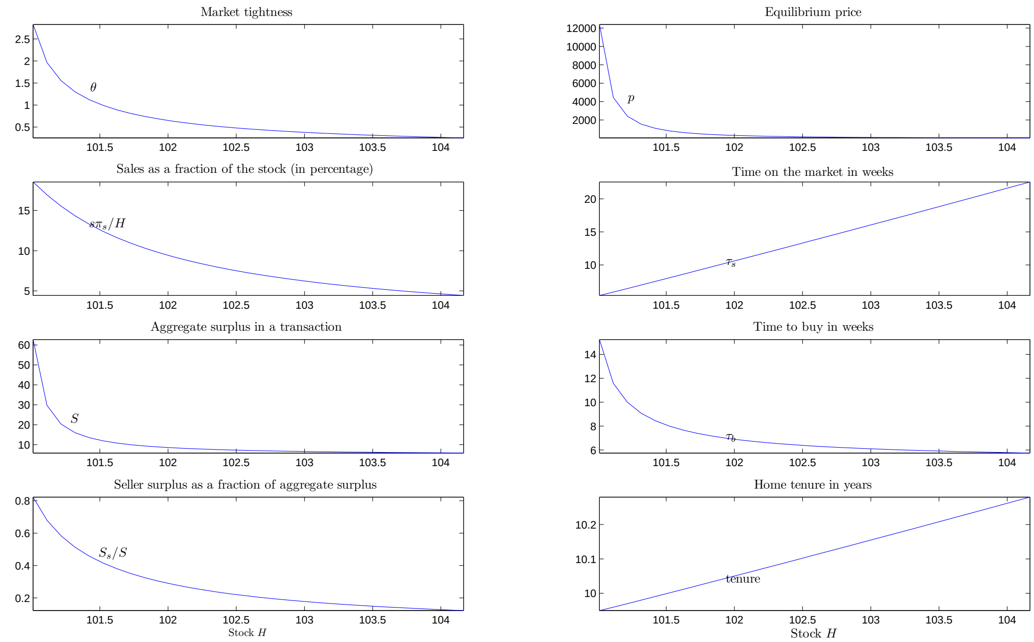

Figure 1: Comparative statics for different values of the housing stock. $\beta = 0.96$ in annual terms, $q = 0.1972$, $\alpha = 0.0019$, and $\overline{v} = 1$.

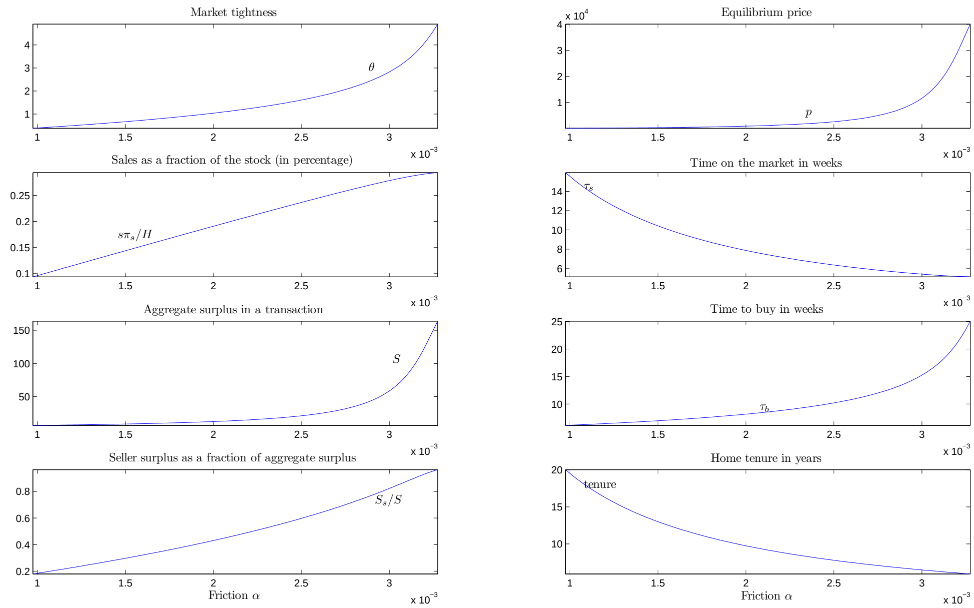

Figure 2: Comparative statics for different values of the mobility parameter α. $\beta = 0.96$ in annual terms, $q = 0.1972$, the vacancy rate is 1.5 percent, and $\overline{v} = 1$.

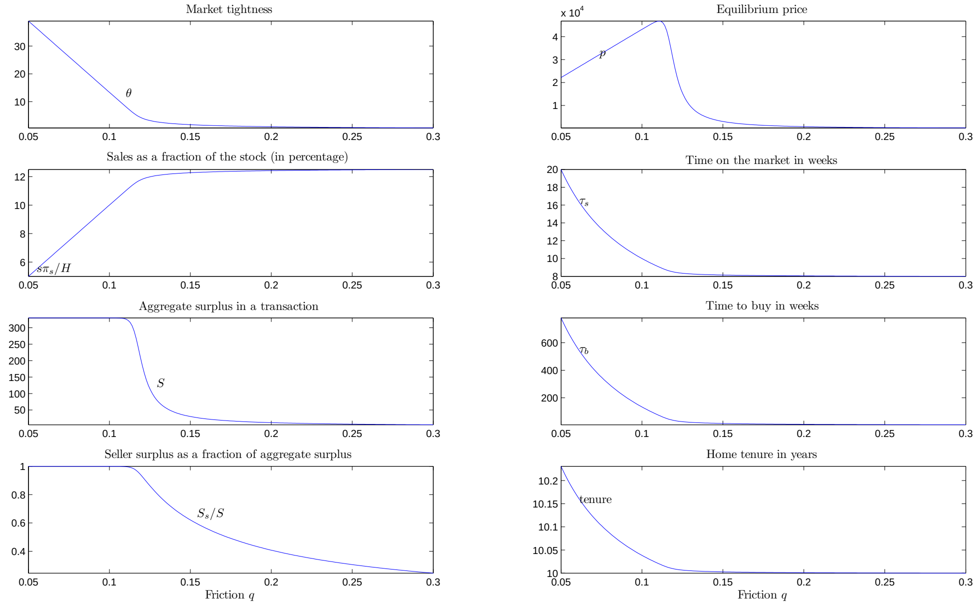

Figure 3: Comparative statics for different values of the friction parameter q. $\beta = 0.96$ in annual terms, $\alpha = 0.0019$, the vacancy rate is 1.5 percent, and $\overline{v} = 1$.

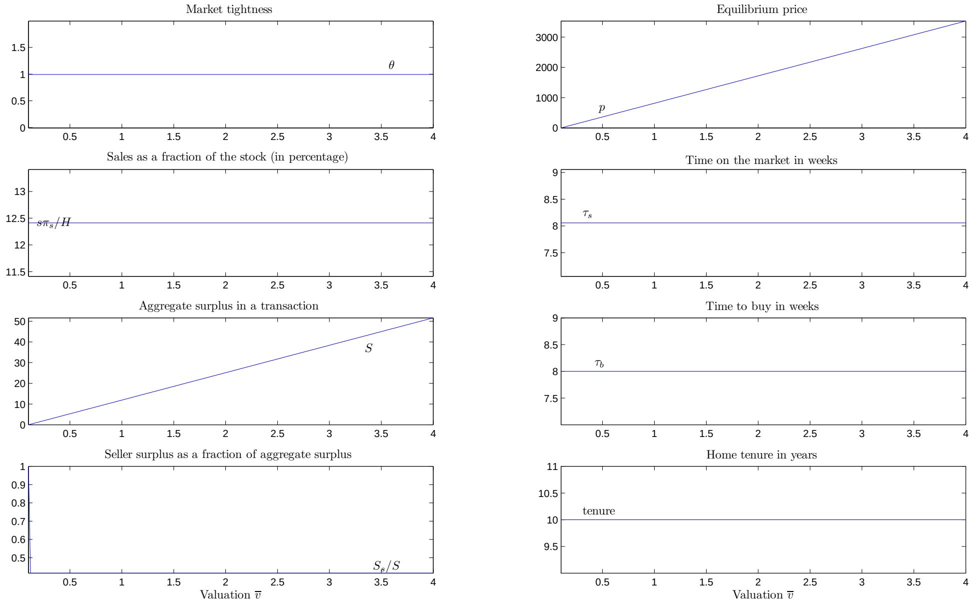

Figure 4: Comparative statics for different values of the valuation parameter $\overline{v}$. $\beta = 0.96$ in annual terms, $\alpha = 0.0019$, $q = 0.1972$, and the vacancy rate is 1.5 percent.

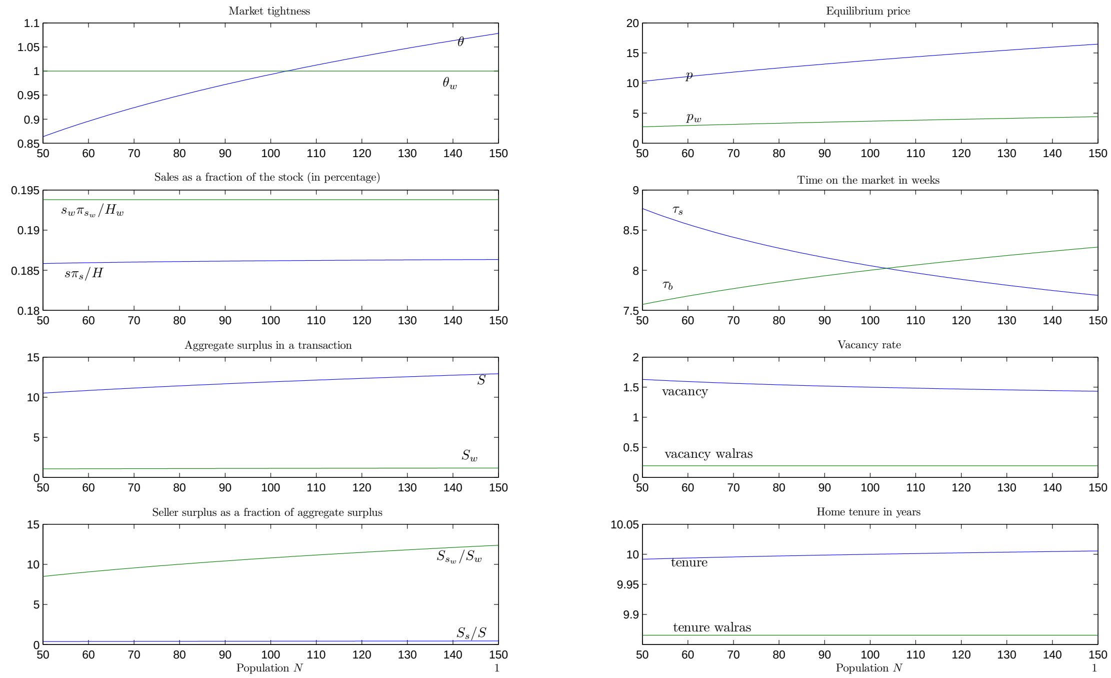

Figure 5: Comparative statics for different population sizes. $\beta = 0.96$ in annual terms, $q = 0.1972$, $\alpha = 0.0019$, $\overline{v} = 1$, $f = \overline{v}$, $\kappa = 3.31$, and $\lambda = 0.0523$.

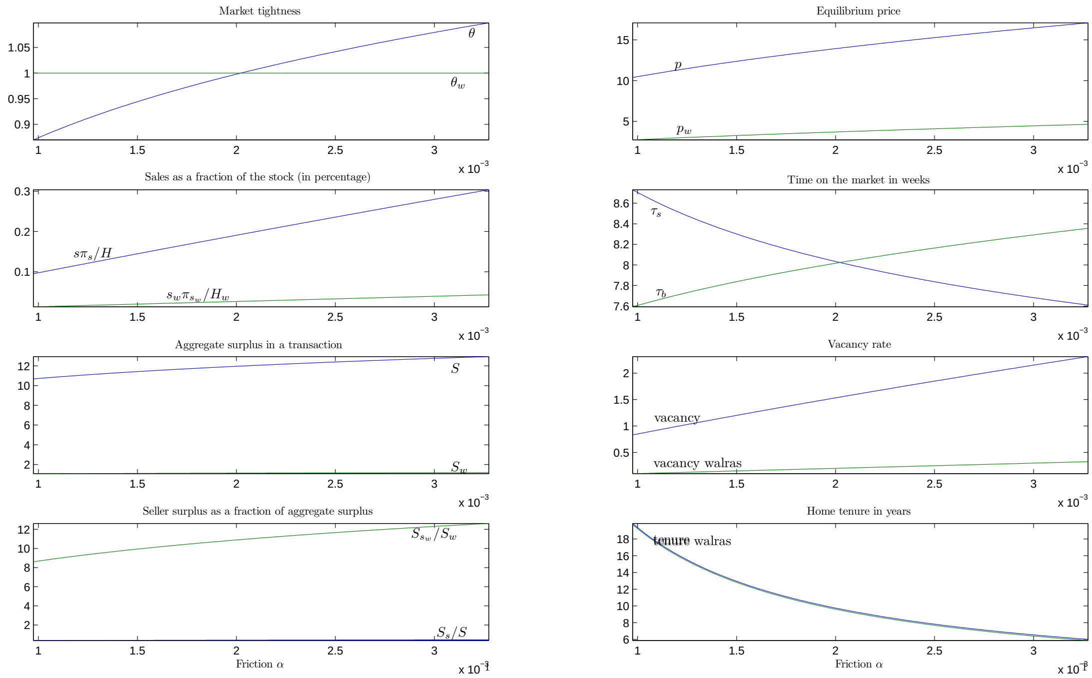

Figure 6: Comparative statics for different values of the mobility parameter α. $N = 100$, $\beta = 0.96$ in annual terms, $q = 0.1972$, $\overline{v} = 1$, $f = \overline{v}$, $\kappa = 3.31$, and $\lambda = 0.0523$.

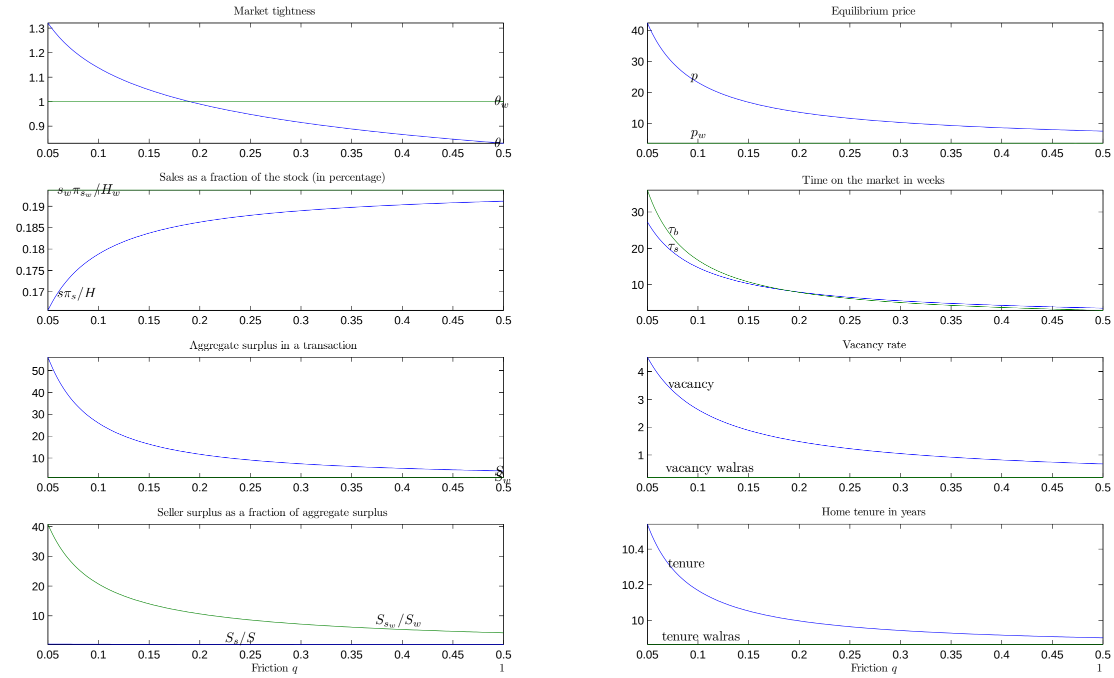

Figure 7: Comparative statics for different values of the friction parameter q. $N = 100$, $\beta = 0.96$ in annual terms, $\alpha = 0.0019$, $\overline{v} = 1$, $f = \overline{v}$, $\kappa = 3.31$, and $\lambda = 0.0523$.

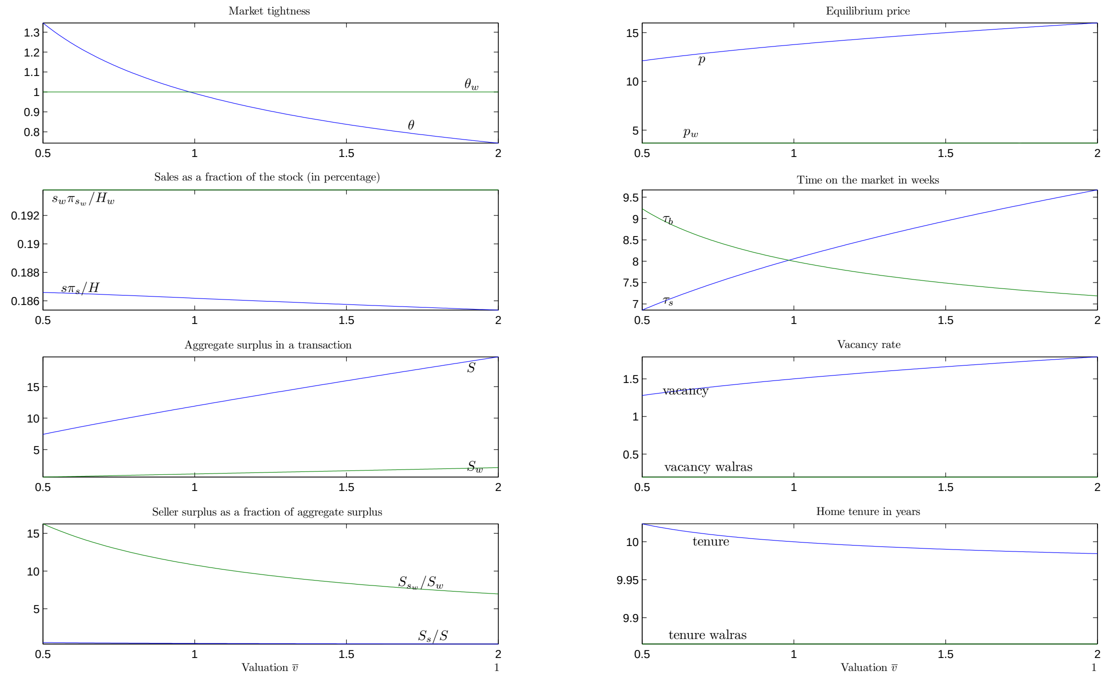

Figure 8: Comparative statics for different values of the valuation parameter $\overline{v}$. $N = 100$, $\beta = 0.96$ in annual terms, $\alpha = 0.0019$, $q = 0.1972$, $f = 1$, $\kappa = 3.31$, and $\lambda = 0.0523$.

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