

# Cointegrated TFP Processes and International Business Cycles\*

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## Abstract

A central puzzle in international macroeconomics is that observed real exchange rates are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations, and can generate only one-fourth of the real exchange rate volatility observed in the data. Typically, IRBC models are solved assuming that total factor productivity (TFP) processes are stationary. In this paper, we first show that TFP processes for the U.S. and the “rest of the world” have a unit root, are cointegrated, and can be jointly characterized with a vector error correction model (VECM). Then, we explore the implications of extending an otherwise standard international real business cycle model that allows for cointegrated technology shocks. We show that the model can account for the high real exchange rate

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volatility observed in the data without having to rely on any particular nominal or real friction. Also, we show that the increase of relative volatility of the real exchange rate with respect to output in the last 20 years can be explained by changes in the parameter estimates of the VECM.

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# 1. Introduction

A central puzzle in international macroeconomics is that observed real exchange rates are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations. For instance, Heathcote and Perri (2002) simulate a two-country, two-good economy with total factor productivity (TFP) shocks and find that the model can only explain less than a fourth of the observed volatility in real exchange rates for U.S. data. An important feature of their model, following the seminal work of Backus, Kehoe, and Kydland (1992), is that it considers stationary TFP shocks that follow a VAR process.

In this paper we provide evidence that TFP processes for the U.S. and a sample of main industrialized trade partners have a unit root and are cointegrated. Motivated by this empirical finding, we introduce technology shocks that follow a vector error correction model (VECM) process into an otherwise standard two-country, two-good model. Engle and Granger (1987), Engle and Yoo (1987), and LeSage (1990) indicate that if the system under study includes integrated variables and cointegrating relationships, then this system will be more appropriately specified as a VECM rather than a VAR in levels. As Engle and Granger (1987) note, estimating a VAR in levels for cointegrated systems leads to ignoring important constraints on the coefficient matrices. Although these constraints are satisfied asymptotically, small sample improvements are likely to result from imposing them in the cointegrating relationships. Failing to impose them affects the small sample estimates and the implied dynamics.

The presence of cointegrated TFP shocks requires restrictions on preferences, production functions, and the law of motion of the shocks in order to have balanced growth. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy. However, in a two-country model, an additional restriction on the cointegrating vector relating the TFP processes is needed. In particular, we need the cointegrating vector to be  $(1, -1)$ , which means the ratio of TFP levels (or, equivalently, the difference of the log-levels of TFP) across countries is stationary. After presenting evidence for this additional restriction, we show that the VECM specification for TFP processes solves a large part of the real exchange rate volatility puzzle without affecting

the good match for domestic variables. In particular, we show that our model can generate a real exchange rate volatility more than four times larger than an equivalent model with stationary shocks calibrated as in Heathcote and Perri (2002).

Why does a model with cointegrated TFP shocks generate higher relative volatility of the real exchange rate than a model with stationary shocks? The reason is that the VECM parameter estimates imply higher persistence and lower spillovers than the traditional stationary calibrations. As we briefly explain below, and later in more detail, higher persistence and lower spillovers imply higher volatility of the real exchange rate and lower volatility of output.

The intuition behind this result is as follows. In the standard IRBC model with no spillovers, when productivity increases at home, home households feel richer and output, consumption, and investment increase, while labor rises because of the upsurge in marginal productivity. As output at home grows, the demand for intermediate goods produced in the foreign country also soars. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, the terms of trade and the real exchange rate rise, reflecting greater world scarcity of the foreign intermediate good relative to the home one. As the persistence of TFP shocks increases, home country households feel richer and supply less labor and capital. This has two main effects. First, it lowers the initial increase of home output and, hence, its volatility. Second, it causes a larger demand increase for the foreign intermediate good and, hence, a larger terms of trade and real exchange rate depreciation. As a result, higher persistence in TFP shocks implies higher relative volatility of the real exchange rate with respect to output.

When spillovers of TFP shocks across countries are introduced in the model, a “news” channel arises. This channel has the opposite effect than the one described above. Since foreign country households know that productivity will eventually increase in their country, they feel richer and supply less labor and capital but demand more consumption goods and, therefore, demand more intermediate good from the home country. Thus, the foreign intermediate good is, relatively, less scarce and the real exchange rate would tend to depreciate less than in a model with no spillovers. Faster spillovers amplify these effects and lead to a lower relative volatility of the real exchange rate with respect to output.

Therefore, the mechanism we just described requires high persistence of each of the TFP

processes, as well as high persistence in their difference (i.e., a slow transmission of shocks across countries), in order to explain high relative volatility of the real exchange rate with respect to output. This is what comes out of our parameter estimates. Estimating a VECM introduces a unit root in the system. What is also crucial for our results is that we estimate a very slow speed of convergence to the cointegrating relationship, implying that the second largest root of the system is also very close to, but inside, the unit circle.

Another very well documented empirical fact is the substantial decline in the volatility of most U.S. macroeconomic variables during the last 20 years. That change in the cyclical volatility is known as the “Great Moderation.”<sup>1</sup> In this paper, we report that, for most industrialized countries, the Great Moderation has not affected the real exchange rate as strongly as it has affected output. As a result, the ratio of real exchange rate volatility to output volatility has increased. We also show that the increase in the relative volatility of the real effective exchange rate of the U.S. dollar coincides in time with a weakening of the cointegrating relationship of TFP shocks between the U.S. and the “rest of the world.”<sup>2</sup> More important, we confirm that if we allow for a fading in the cointegrating relationship of the size estimated in the data, the model can jointly account for the observed increase in the relative volatility of the real exchange rate and the substantial decline in the volatility of output.

An important problem of IRBC models is that the comovement between the real exchange rates and the ratio of consumptions does not match the one observed in the data (Backus and Smith, 1993). Even when considering cointegrated TFP shocks the model still generates a correlation close to one, while in the data the correlation is negative and close to zero. For this reason, we introduce two extensions of the benchmark model that allow us to better fit this correlation without affecting relative volatility of the real exchange rate. In the first extension, we add a taste shock as in Heathcote and Perri (2008). The second extension considers an investment-specific technology shock as in Greenwood, Hercowitz, and Krusell (1997) and Fisher (2006). As expected, the taste shock allows us to solve Backus and Smith’s puzzle without affecting the model’s ability to explain the relative volatility of the real exchange

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<sup>1</sup>Some early discussion of the Great Moderation can be found in Kim and Nelson (1999). A discussion of different interpretations for this phenomenon and some international evidence can be found in Stock and Watson (2002) and Stock and Watson (2005), respectively.

<sup>2</sup>In section 4 we describe the set of countries that compose our definition of “rest of the world.”

rate (see Rabanal and Tuesta, 2006, and Heathcote and Perri, 2008). Also, as shown by Rabanal and Rubio-Ramírez (2008), investment-specific technology shocks accomplish the same objective.

Our paper relates to two important strands of the literature. On the one hand, it connects with the literature stressing the importance of stochastic trends to explain economic fluctuations. King, Plosser, Stock, and Watson (1991) find that a common stochastic trend explains the comovements of main U.S. real macroeconomic variables. Lastrapes (1992) reports that fluctuations in real and nominal exchange rates are due primarily to permanent real shocks. Engel and West (2005) show that real exchange rates manifest near-random-walk behavior if TFP processes are random walks and the discount factor is near one, while Nason and Rogers (2008) generalize this hypothesis to a larger class of models. Aguiar and Gopinath (2007) show that trend shocks are the primary source of fluctuations in emerging economies. Alvarez and Jermann (2005) and Corsetti, Dedola, and Leduc (2008a) highlight the importance of persistent disturbances to explain asset prices and real exchange rate fluctuations, respectively.

On the other hand, our paper also links to the literature analyzing different mechanisms to understand real exchange rate fluctuations. Some recent papers study the effects of monetary shocks and nominal rigidities. Chari, Kehoe, and McGrattan (2002) are able to explain real exchange rate volatility in a monetary model with sticky prices and a high degree of risk aversion. Benigno (2004) focuses on the role of interest rate inertia and asymmetric nominal rigidities across countries. Other papers use either non-tradable goods, pricing to market, or some form of distribution costs (see Corsetti, Dedola, and Leduc 2008a, 2008b; Benigno and Thoenissen 2007, and Dotsey and Duarte 2007). Our model includes only tradable goods with home bias. Our choice is guided by evidence that the relative price of tradable goods has large and persistent fluctuations that explain most of the real exchange rate volatility (see Engel 1993 and 1999). Fluctuations of the relative price of non-tradable goods accounts for, at most, one-third of the real exchange rate volatility (see Betts and Kehoe 2006, Burnstein, Eichenbaum, and Rebelo 2006, and Rabanal and Tuesta 2007).

The rest of the paper is organized as follows. Section 2 documents the increase of the real exchange rate volatility with respect to output for most industrialized countries. In Section 3 we present the model with cointegrated TFP shocks. In Section 4 we report estimates for

the law of motion of the (log) TFP processes of the United States and a “rest of the world” aggregate. In Section 5 we present the main findings from simulating the model, leaving Section 6 for concluding remarks.

## 2. The Great Moderation and Real Exchange Rate Volatility

In this section, we present evidence that in the period known as “the Great Moderation,” the relative volatility of the real exchange rate (measured as the real effective exchange rate) with respect to output (measured as real GDP) has increased in the United States, the United Kingdom, Canada, and Australia. In Figures 1 and 2 we present the standard deviation of the HP-filtered output, the standard deviation of the HP-filtered real exchange rate, and the ratio of the two for these four countries. We compute the standard deviation of rolling windows of 40 quarters.<sup>3</sup>

Let us first focus on the U.S. economy. Figure 1 shows a substantial decline in the volatility of output, from 2.3 percent standard deviation in the window 1973:1-1982:4, to 0.8 percent in the window 1997:3-2007:2. This decline in output volatility is what is typically referred to as “the Great Moderation.” The volatility of the real exchange rate has experienced a different path: the standard deviation was at about 4.5 percent for the window 1973:1-1982:4; thereafter, it increased to values above 7 percent for the window 1980:1-1989:4 and declined to a value of 4.3 percent for the window 1997:3-2007:2.

So what is the behavior of the ratio of volatilities between the two series? The ratio has increased in a non-monotonic way from 1.96 percent to 4.5 percent in the period we study. Hence, the volatility of the real exchange rate has more than doubled relative to that of output.

What has been the experience with the other main currencies? As Figures 1 and 2 show, the pattern that arises with the United Kingdom, Canada, and Australia is also quite similar: dramatic declines in the volatility of output, erratic behavior of the absolute volatility of the real exchange rate, and dramatic increases in the relative volatility of the real exchange rate

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<sup>3</sup>Therefore, the first data point in 1982:4 denotes the standard deviation of HP-filtered output for the period 1973:1-1982:4. The last data point 2007:2 denotes the standard deviation of HP-filtered output between 1997:3-2007:2.

with respect to output.

Having presented some evidence for the main industrialized countries, in this paper we focus only on the relationship between the U.S. economy and the “rest of the world.” Hence, we build a two-country, two-good model that we calibrate using standard parameters of the IRBC literature and estimated parameters of a vector error correction model (VECM) using TFP processes for the U.S. and a “rest of the world” aggregate. In Section 5 we show that it is possible to explain the observed increase in relative volatility of the real exchange rate with respect to output with changes in the estimated parameters of the VECM.

### 3. The Model

In this section, we present a standard two-country, two-good IRBC model similar to the one described in Heathcote and Perri (2002). The main difference with respect to the standard IRBC literature is the definition of the stochastic processes for TFP. In that literature, the TFP processes of the two countries are assumed to be stationary or trend stationary in logs, and they are modelled as a VAR.<sup>4</sup> In this paper, we consider instead (log) TFP processes that are cointegrated of order  $C(1,1)$ . This implies that (log) TFP processes are integrated of order one but a linear combination is stationary. According to the Granger representation theorem,<sup>5</sup> our  $C(1,1)$  assumption is equivalent to defining a VECM for the law of motion of the log differences of the TFP processes. The VECM is defined in more detail in section 3.2.3. Our cointegration assumption has strong and testable implications for the data. The empirical evidence supporting our assumption will be presented in section 4.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use local capital and labor in the production process. The final good can be locally consumed or invested by

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<sup>4</sup>See Backus, Kehoe and Kydland (1992), Kehoe and Perri (2002), and Heathcote and Perri (2002). Interestingly, Baxter and Crucini (1995) estimate a VECM using TFP processes for the United States and Canada, but they dismiss this evidence when simulating their model.

<sup>5</sup>See Engle and Granger (1987).

consumers. The stock of the local capital good can be increased only by using the domestic final good. Thus, all trade between the countries is in intermediate goods. In addition, consumers trade across countries an uncontingent international riskless bond denominated in units of domestic intermediate goods. No other financial asset is available. In each period of time  $t$ , the economy experiences one of many finite events  $s_t$ . We denote by  $s^t = (s_0, \dots, s_t)$  the history of events up through period  $t$ . The probability, as of period 0, of any particular history  $s^t$  is  $\pi(s^t)$  and  $s_0$  is given.

In the remainder of this section, we describe the households' problem, the intermediate and final goods producers' problems, and the VECM process. Then, we explain market clearing and equilibrium. Finally, we discuss the conditions for the existence of a balanced growth path and explain how to transform the variables in the model to achieve stationarity.

### 3.1. Households

In this subsection, we describe the decision problem faced by home-country households. The problem faced by foreign-country households is similar, and hence it is not presented. The representative household of the home country solves

$$\max_{\{C(s^t), L(s^t), X(s^t), K(s^t), D(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \frac{\left\{ C(s^t)^\tau [1 - L(s^t)]^{1-\tau} \right\}^{1-\sigma}}{1 - \sigma} \quad (1)$$

subject to the following budget constraint

$$\begin{aligned} P(s^t) [C(s^t) + X(s^t)] + P_H(s^t) \bar{Q}(s^t) D(s^t) \leq \\ P(s^t) [W(s^t) L(s^t) + R(s^t) K(s^{t-1})] + P_H(s^t) D(s^{t-1}) - P_H(s^t) \Phi(D(s^t)), \end{aligned} \quad (2)$$

and the law of motion for capital

$$K(s^t) = (1 - \delta) K(s^{t-1}) + X(s^t), \quad (3)$$

The following notation is used:  $\beta \in (0, 1)$  is the discount factor,  $L(s^t) \in (0, 1)$  are hours worked in the home country,  $C(s^t) \geq 0$  are units of consumption of the final good,  $X(s^t) \geq 0$

are units of investment,  $K(s^t) \geq 0$  is the capital level in the home country at the beginning of period  $t + 1$ .  $P(s^t)$  is the price of the home final good, which will be defined below,  $W(s^t)$  is the hourly wage in the home country, and  $R(s^t)$  is the home-country rental rate of capital, where the prices of both factor inputs are measured in units of the final good.  $P_H(s^t)$  is the price of the home intermediate good,  $D(s^t)$  denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good in period  $t + 1$  regardless of the state of nature, and  $\bar{Q}(s^t)$  is its price, measured in units of the home intermediate good. Finally, the function  $\Phi$  is an arbitrarily small cost of holding bonds measured in units of the home intermediate good.<sup>6</sup>

We assume, following the existing literature, that  $\Phi$  takes the following functional form  $\Phi[D(s^t)] = \frac{\phi}{2}A(s^{t-1}) \left[ \frac{D(s^t)}{A(s^{t-1})} \right]^2$ . Note that we need to include the level of TFP in the home country,  $A(s^{t-1})$ , in the adjustment cost function, both dividing  $D(s^t)$  and multiplying  $\left[ \frac{D(s^t)}{A(s^{t-1})} \right]^2$ . The reason is that since  $A(s^{t-1})$  is an integrated process,  $D(s^t)$  will grow at the rate of growth of  $A(s^{t-1})$  along the balanced growth path, making the ratio  $\frac{D(s^t)}{A(s^{t-1})}$  stationary. Also, since all home real variables will also grow at the rate of growth of  $A(s^{t-1})$  along the balanced growth path, we need to make the adjustment cost (measured in units of home intermediate good) also grow at the same rate in order to induce stationarity. As was the case for the cost of adjusting the capital level, the cost of holding debt grows quadratically as the normalized debt level moves away from its steady state level of zero.<sup>7</sup>

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<sup>6</sup>The  $\Phi(\cdot)$  cost is introduced to ensure stationarity of the level of  $D(s^t)$  in IRBC models with incomplete markets, as discussed by Schmitt-Grohé and Uribe (2003).

<sup>7</sup>Our results will also hold for any other assumption for the steady state level of debt, but assuming balanced trade in the steady state is convenient to solve for the levels of imports and exports in both economies.

## 3.2. Firms

### 3.2.1. Final goods producers

The final good in the home country,  $Y(s^t)$ , is produced using home intermediate goods,  $Y_H(s^t)$ , and foreign intermediate goods,  $Y_F(s^t)$ , with the following technology:

$$Y(s^t) = \left[ \omega^{\frac{1}{\theta}} Y_H(s^t)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F(s^t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (4)$$

where  $\omega$  denotes the fraction of home intermediate goods that are used for the production of the home final good and  $\theta$  controls the elasticity of substitution between home and foreign intermediate goods. Therefore, the representative final goods producer in the home country solves the following problem:

$$\max_{Y(s^t) \geq 0, Y_H(s^t) \geq 0, Y_F(s^t) \geq 0} P(s^t) Y(s^t) - P_H(s^t) Y_H(s^t) - P_F^*(s^t) Y_F(s^t)$$

subject to the production function (4).

### 3.2.2. Intermediate goods producers

The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves:

$$\text{Max}_{L(s^t) \geq 0, K(s^{t-1}) \geq 0} P_H(s^t) [Y_H(s^t) + Y_H^*(s^t)] - P(s^t) [W(s^t) L(s^t) + R(s^t) K(s^{t-1})]$$

subject to the production function

$$Y_H(s^t) + Y_H^*(s^t) = A(s^t)^{1-\alpha} K(s^{t-1})^\alpha L(s^t)^{1-\alpha} \quad (5)$$

where  $Y_H(s^t)$  is the amount of home intermediate goods sold to the home final goods producers,  $Y_H^*(s^t)$  is the amount of home intermediate goods sold to the foreign final goods

producers, and  $A(s^t)$  is a stochastic process affecting TFP of home intermediate goods producers, which we will characterize below.

### 3.2.3. The processes for TFP

As mentioned above, we depart from the standard assumption in the IRBC literature and consider processes for both  $\log A(s^t)$  and  $\log A^*(s^t)$  that are cointegrated of order  $C(1,1)$ . Equivalently, we specify the following VECM for the law of motion driving the log differences of TFP processes for both the home and the foreign country:

$$\begin{aligned} \begin{pmatrix} \Delta \log A(s^t) \\ \Delta \log A^*(s^t) \end{pmatrix} &= \begin{pmatrix} c \\ c^* \end{pmatrix} + \rho^1 \begin{pmatrix} \Delta \log A(s^{t-1}) \\ \Delta \log A^*(s^{t-1}) \end{pmatrix} + \rho^2 \begin{pmatrix} \Delta \log A(s^{t-2}) \\ \Delta \log A^*(s^{t-2}) \end{pmatrix} \\ &+ \begin{pmatrix} \kappa \\ \kappa^* \end{pmatrix} [\log A(s^{t-1}) - \gamma \log A^*(s^{t-1}) - \log \xi] + \begin{pmatrix} \varepsilon^a(s^t) \\ \varepsilon^{a,*}(s^t) \end{pmatrix} \end{aligned} \quad (6)$$

where  $\rho^1$  and  $\rho^2$  are  $2 \times 2$  coefficient matrices,  $(1, -\gamma)$  is called the cointegrating vector,  $\xi$  is the constant in the cointegrating relationship,  $\varepsilon^a(s^t) \sim N(0, \sigma^\varepsilon)$  and  $\varepsilon^*(s^t) \sim N(0, \sigma^{\varepsilon,*})$ ,  $\varepsilon^a(s^t)$  and  $\varepsilon^*(s^t)$  are correlated,  $\Delta$  is the first-difference operator, and either  $\kappa$  or  $\kappa^*$  are different from zero and of the right sign.<sup>8</sup>

This VECM representation implies that deviations of today's log differences of TFP with respect to its mean value depend not only on lags of home and foreign log differences of TFP but also on a function of the ratio of lag home and foreign TFP,  $A(s^{t-1}) / [\xi A^*(s^{t-1})^\gamma]$ . Thus, if the ratio  $A(s^{t-1}) / A^*(s^{t-1})^\gamma$  is larger than its long-run value,  $\xi$ , then  $\kappa < 0$  and  $\kappa^* > 0$  will imply that  $\Delta \log A(s^t)$  would fall and  $\Delta \log A^*(s^t)$  would rise, driving both series toward their long-run equilibrium values. The VECM representation also implies that  $\Delta \log A(s^t)$ ,  $\Delta \log A^*(s^t)$ , and  $\log A(s^{t-1}) - \gamma \log A^*(s^{t-1}) - \log \xi$  are stationary processes.

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<sup>8</sup>Here we restrict ourselves to a VECM with two lags. This assumption is motivated by the empirical results to be presented in section 4, where only two lags are significant. In any case, we recognize that the Granger representation theorem states that the series are  $C(1,1)$  if and only if there exists a VECM representation of some lag length. In our theoretical model we could use any finite number of lags.

### 3.3. Market Clearing

The model is closed with the following market clearing conditions in the final goods markets

$$C(s^t) + X(s^t) = Y(s^t), \quad (7)$$

$$C^*(s^t) + X^*(s^t) = Y^*(s^t), \quad (8)$$

and the bond markets

$$D(s^t) + D^*(s^t) = 0. \quad (9)$$

### 3.4. Equilibrium

#### 3.4.1. Equilibrium definition

Now we are ready to define the equilibrium for this economy. Given our law of motion for (log) TFP shocks defined by (3.2.3), an equilibrium for this economy is a set of allocations for home consumers,  $C(s^t)$ ,  $L(s^t)$ ,  $X(s^t)$ ,  $K(s^t)$ , and  $D(s^t)$ , and foreign consumers,  $C^*(s^t)$ ,  $L^*(s^t)$ ,  $X^*(s^t)$ ,  $K^*(s^t)$ , and  $D^*(s^t)$ , allocations for home and foreign intermediate goods producers,  $Y_H(s^t)$ ,  $Y_H^*(s^t)$ ,  $Y_F(s^t)$  and  $Y_F^*(s^t)$ , allocations for home and foreign final goods producers,  $Y(s^t)$  and  $Y^*(s^t)$ , intermediate goods prices  $P_H(s^t)$  and  $P_F^*(s^t)$ , final goods prices  $P(s^t)$  and  $P^*(s^t)$ , rental prices of labor and capital in the home and foreign country,  $W(s^t)$ ,  $R(s^t)$ ,  $W^*(s^t)$ , and  $R^*(s^t)$  and the price of the bond  $\bar{Q}(s^t)$  such that (i) given prices, household allocations solve the households' problem; (ii) given prices, intermediate goods producers allocations solve the intermediate goods producers' problem; (iii) given prices, final goods producers allocations solve the final goods producers' problem; (iv) and markets clear.

#### 3.4.2. Equilibrium conditions

At this point, it is useful to define the following relative prices:  $\tilde{P}_H(s^t) = \frac{P_H(s^t)}{P(s^t)}$ ,  $\tilde{P}_F^*(s^t) = \frac{P_F^*(s^t)}{P^*(s^t)}$  and  $RER(s^t) = \frac{P^*(s^t)}{P(s^t)}$ . Note that  $\tilde{P}_H(s^t)$  is the price of home intermediate goods in terms of home final goods,  $\tilde{P}_F^*(s^t)$  is the price of foreign intermediate goods in terms of foreign final goods, which appears in the foreign country's budget constraint, and  $RER(s^t)$

is the real exchange rate between the home and foreign countries. In our model the law of one price holds; hence, we have that  $P_H(s^t) = P_H^*(s^t)$  and  $P_F(s^t) = P_F^*(s^t)$ .

Let us now determine the equilibrium conditions implied by the first order conditions of households, intermediate and final goods producers in both countries, as well as the relevant laws of motion, production functions, and market clearing conditions. The marginal utility of consumption and the labor supply are given by:

$$U_C(s^t) = \lambda(s^t), \quad (10)$$

$$\frac{U_L(s^t)}{U_C(s^t)} = W(s^t), \quad (11)$$

where  $U_x$  denotes the partial derivative of the utility function  $U$  with respect to variable  $x$ . The first order condition with respect to capital delivers an intertemporal condition that relates the marginal rate of consumption to the rental rate of capital and the depreciation rate:

$$\lambda(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \lambda(s^{t+1}) [R(s^{t+1}) + 1 - \delta], \quad (12)$$

where  $\pi(s^{t+1}|s^t) = \frac{\pi(s^{t+1})}{\pi(s^t)}$  is the conditional probability of  $s^{t+1}$  given  $s^t$ .

Finally, the law of motion of capital is:

$$K(s^t) = (1 - \delta) K(s^{t-1}) X(s^t), \quad (13)$$

The analogous expressions for the foreign country are as follows. All starred variables denote the foreign-country analogous to the home-country variables (i.e., if  $C$  is consumption of the final home good, then  $C^*$  denotes consumption of the final foreign good, and so on).

$$U_{C^*}(s^t) = \lambda^*(s^t), \quad (14)$$

$$\frac{U_{L^*}(s^t)}{U_{C^*}(s^t)} = W^*(s^t), \quad (15)$$

$$\lambda^*(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \lambda^*(s^{t+1}) [R^*(s^{t+1}) + 1 - \delta], \quad (16)$$

and

$$K^* (s^t) = (1 - \delta) K^* (s^{t-1}) + X^* (s^t). \quad (17)$$

The optimal choice by households of the home country delivers the following expression for the price of the riskless bond:

$$\bar{Q} (s^t) = \beta \sum_{s^{t+1}} \pi (s^{t+1}|s^t) \frac{\lambda (s^{t+1}) \tilde{P}_H (s^{t+1})}{\lambda (s^t) \tilde{P}_H (s^t)} - \frac{\Phi' [D (s^t)]}{\beta}. \quad (18)$$

The risk-sharing condition is given by the optimal choice of the households of both countries for the riskless bond:

$$\sum_{s^{t+1}} \pi (s^{t+1}|s^t) \left[ \frac{\lambda^* (s^{t+1}) \tilde{P}_H (s^{t+1})}{\lambda^* (s^t) \tilde{P}_H (s^t)} \frac{RER (s^t)}{RER (s^{t+1})} - \frac{\lambda (s^{t+1}) \tilde{P}_H (s^{t+1})}{\lambda (s^t) \tilde{P}_H (s^t)} \right] = - \frac{\Phi' [D (s^t)]}{\beta} \quad (19)$$

From the intermediate goods producers' maximization problems, we obtain the result that labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

$$W (s^t) = (1 - \alpha) \tilde{P}_H (s^t) A (s^t)^{1-\alpha} K (s^{t-1})^\alpha L (s^t)^{-\alpha}, \quad (20)$$

$$R (s^t) = \alpha \tilde{P}_H (s^t) A (s^t)^{1-\alpha} K (s^{t-1})^{\alpha-1} L (s^t)^{1-\alpha}, \quad (21)$$

$$W^* (s^t) = (1 - \alpha) \tilde{P}_F^* (s^t) A^* (s^t)^{1-\alpha} K^* (s^{t-1})^\alpha L^* (s^t)^{-\alpha}, \quad (22)$$

and

$$R^* (s^t) = \alpha \tilde{P}_F^* (s^t) A^* (s^t)^{1-\alpha} K^* (s^{t-1})^{\alpha-1} L^* (s^t)^{1-\alpha}. \quad (23)$$

From the final goods producers' maximization problem, we obtain the demands of intermediate goods, which depend on their relative price:

$$Y_H (s^t) = \omega \tilde{P}_H (s^t)^{-\theta} Y (s^t), \quad (24)$$

$$Y_F (s^t) = (1 - \omega) \left( \tilde{P}_F^* (s^t) RER (s^t) \right)^{-\theta} Y (s^t), \quad (25)$$

$$Y_H^* (s^t) = (1 - \omega) \left( \frac{\tilde{P}_H (s^t)}{RER (s^t)} \right)^{-\theta} Y^* (s^t), \quad (26)$$

and

$$Y_F^* (s^t) = \omega \tilde{P}_F^* (s^t)^{-\theta} Y^* (s^t). \quad (27)$$

Finally, good, input, and bond markets clear. Thus:

$$C (s^t) + X (s^t) = Y (s^t), \quad (28)$$

$$C^* (s^t) + X^* (s^t) = Y^* (s^t), \quad (29)$$

$$Y (s^t) = \left[ \omega^{\frac{1}{\theta}} Y_H (s^t)^{\frac{\theta-1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} Y_F (s^t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (30)$$

$$Y^* (s^t) = \left[ \omega^{\frac{1}{\theta}} Y_F^* (s^t)^{\frac{\theta-1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} Y_H^* (s^t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (31)$$

$$Y_H (s^t) + Y_H^* (s^t) = A (s^t)^{1-\alpha} K (s^{t-1})^\alpha L (s^t)^{1-\alpha}, \quad (32)$$

$$Y_F (s^t) + Y_F^* (s^t) = A^* (s^t)^{1-\alpha} K^* (s^{t-1})^\alpha L^* (s^t)^{1-\alpha}, \quad (33)$$

and

$$D (s^t) + D^* (s^t) = 0. \quad (34)$$

The law of motion of the level of debt

$$\begin{aligned} \tilde{P}_H (s^t) \bar{Q} (s^t) D (s^t) &= \tilde{P}_H (s^t) Y_H^* (s^t) - \tilde{P}_F^* (s^t) RER (s^t) Y_F (s^t) \\ &\quad + \tilde{P}_H (s^t) D (s^{t-1}) - \tilde{P}_H (s^t) \Phi [D (s^t)] \end{aligned} \quad (35)$$

is obtained using (2) and the fact that intermediate and final goods producers at home make zero profits. Finally, the productivity shocks follow the VECM described in section 3.2.3.

### 3.5. Balanced Growth and the Restriction on the Cointegrating Vector

Equations (10) to (35) and the VECM process for (log) TFP characterize the equilibrium in this model. Since we assume that both  $\log A (s^t)$  and  $\log A^* (s^t)$  are integrated processes,

we need to normalize the equilibrium conditions in order to get a stationary system more amenable to study. Following King, Plosser, and Rebelo (1988) we divide the home-country variables that have a trend by the lagged domestic level of TFP,  $A(s^{t-1})$ , and the foreign-country variables that have a trend by the lagged foreign level of TFP,  $A^*(s^{t-1})$ . In appendix A.1, we detail the full set of normalized equilibrium conditions. (See equations (39) to (64).)

For the model to have balanced growth we require some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy real business cycle (RBC) model. However, in our two-country model, an additional restriction on the cointegrating vector is needed if the model is to exhibit balanced growth. In particular, we need the ratio  $A(s^{t-1})/A^*(s^{t-1})$  to be stationary.

In order to understand why the international dimension of the model requires this additional restriction, let us focus, for example, on equation (54) of the set of normalized equilibrium conditions defined in appendix A.1. This equation is the normalized condition for the demand of imported foreign-produced intermediate goods by the home country:

$$\widehat{Y}_F(s^t) = (1 - \omega) \left[ \widetilde{P}_F^*(s^t) RER(s^t) \right]^{-\theta} \widehat{Y}(s^t) \frac{A(s^{t-1})}{A^*(s^{t-1})}$$

where  $\widehat{Y}_F(s^t) = Y_F(s^t)/A^*(s^{t-1})$  while  $\widehat{Y}(s^t) = Y(s^t)/A(s^{t-1})$ . Since  $\widetilde{P}_F^*(s^t)$  and  $RER(s^t)$  are stationary, if the ratio between  $A(s^{t-1})$  and  $A^*(s^{t-1})$  were to be non-stationary, the ratio between  $\widehat{Y}_F(s^t)$  and  $\widehat{Y}(s^t)$  would also be non-stationary and balanced growth would not exist. A similar argument must hold for the following normalized equilibrium conditions: (55), (59), (60), (63), and (64).

Our VECM implies that the ratio between  $A(s^{t-1})$  and  $A^*(s^{t-1})^\gamma$  is stationary. Therefore, a sufficient condition for balanced growth is that the parameter  $\gamma$  equals one or, equivalently, that the cointegrating vector equals  $(1, -1)$ .

## 4. Estimation of the VECM

In this section, after describing our constructed TFP series for the U.S. and the “rest of the world,” we perform three exercises. First, we show that our assumption that the TFP processes are cointegrated of order  $C(1,1)$  cannot be rejected in the data. By the Granger representation theorem this implies that our VECM specification is valid. Second, we also show that the restriction imposed by balanced growth, i.e., that the parameter  $\gamma$  is equal to one, cannot be rejected in the data either. Finally, we estimate the parameters driving our VECM in order to simulate our model in the next section.

### 4.1. Data

In order to estimate our VECM we use data for the U.S. and an aggregate for the “rest of the world.” For the U.S., we obtain quarterly output data from the Bureau of Economic Analysis and employment data from the Payroll Survey from 1973:1 to 2007:3. The “rest of the world” aggregate contains nominal output and employment data for the 12 countries of the Euro Area (using Eurostat and the Area Wide Model data set maintained at the European Central Bank), the United Kingdom, Canada, Japan, and Australia (using national sources data). This group accounts for about 50 percent of the basket of currencies that the Federal Reserve uses to construct the real exchange rate for the U.S. dollar. Given some restrictions on employment data necessary to build TFP shocks, our sample period for the “rest of the world” goes from 1980:1 to 2007:3. Ideally, one would want to include additional countries that represent an important and increasing share of trade with the United States, such as China, but long quarterly output and employment figures are not available.<sup>9</sup>

We aggregate the nominal outputs of the “rest of the world” using PPP exchange rates to convert each national nominal output to current U.S. dollars, and then use the output deflator of the United States to convert the “rest of the world” nominal output to constant U.S. dollars. We obtain aggregate “rest of the world” employment data by simply aggregating

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<sup>9</sup>We also included in our definition of the rest of the world Mexico and South Korea, which resulted in a shortening of the starting point to 1982:3, which is when the Korean employment series starts. The results were similar including these two countries, but to take advantage of the longer time series in our subsample analysis, we decided to exclude them.

the number of employees in each country.

Since capital stock series are not available at a quarterly frequency for most countries, we estimate the TFP shock as follows:

$$\log A(s^t) = [\log Y(s^t) - (1 - \alpha) \log L(s^t)] / (1 - \alpha)$$

$$\log A^*(s^t) = [\log Y^*(s^t) - (1 - \alpha) \log L^*(s^t)] / (1 - \alpha)$$

where  $\alpha$  is the capital share of output. Backus, Kehoe, and Kydland (1992) and Heathcote and Perri (2002, 2008) use a similar approach when constructing TFP series for the United States and a “rest of the world” aggregate. Figure 3 plots the constructed (log) TFP processes for the U.S. and “rest of the world.”

## 4.2. Integration and Cointegration Properties

In this section, we present evidence supporting our assumption that the (log) TFP processes for the U.S. and the “rest of the world” are cointegrated of order  $C(1,1)$ . First, we will empirically support the unit root assumption for the univariate processes. Second, we will test for the presence of cointegrating relationships using the Johansen (1991) procedure. Both the trace and the maximum eigenvalue methods support the existence of a cointegrating vector.

Univariate analysis of the log TFP processes for the U.S. and the “rest of the world” strongly indicates that both series can be characterized by unit root processes with drift. Table 1 presents results for the U.S. log TFP process using the following commonly applied unit root tests: augmented Dickey-Fuller (Dickey and Fuller, 1979, and Said and Dickey 1984); the DF-GLS and the optimal point statistic ( $P_TGLS$ ), both of Elliott et al. (1996); and the modified  $MZ_\alpha$ ,  $MZ_t$ , and MSB of Ng and Perron (2001). The lag length is chosen using the modified Akaike Information criterion (MAIC) as Ng and Perron (1995) recommend.<sup>10</sup> In each case a constant and a trend are included in the specification and data from 1973:1 to 2007:4 are used. Table 1 also presents the same unit root test results for the “rest of the world” log TFP process using data from 1980:1 to 2007:3. None of the test statistics are even

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<sup>10</sup>We have tried different maximum lag lengths with similar outcomes. At the end we chose to report results with a maximum of two lag lengths because of the short sample we have available.

close to rejecting the null hypothesis of unit root at the 5 percent critical value and only the augmented Dickey-Fuller t-test rejects it at the 10 percent critical value for the U.S. Using the same statistics, unit root tests on the first difference of the log TFP processes for the U.S. and the “rest of the world” are stationary. For the U.S. all the tests reject the null hypothesis of unit root at the 1 percent critical value. For the “rest of the world” augmented Dickey-Fuller,  $P_TGLS$ , and MSB reject the null hypothesis of unit root at the 5 percent critical value, while the DF-GLS and  $MZ_\alpha$  tests reject it at the 10 percent value.

Once we have presented evidence that strongly indicates that the (log) TFP for the U.S. and the “rest of the world” is well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. Table 2 presents some statistics calculated from an unrestricted VAR with five lags and a trend for the two-variables system  $[\log A(s^t), \log A^*(s^t)]$  for the sample period 1980:3 to 2007:3.<sup>11</sup> Table 2 shows the two eigenvalues with the largest absolute value for the VAR implied by the point estimates. If  $\log A(s^t)$  and  $\log A^*(s^t)$  share one common stochastic trend (balanced growth), the estimated VAR has to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. As shown in Table 2, point estimates are in accord with this prediction: the highest eigenvalue equals one, while the second highest is less than one. But this is not a formal test of cointegration. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). The cointegration tests are run for the sample period 1981:2 to 2007:3 and assume a constant in the cointegrating vector. Clearly, the data strongly support a single eigenvalue.

### 4.3. The VECM Model

In the last subsection, we presented evidence that  $\log A(s^t)$  and  $\log A^*(s^t)$  are cointegrated of order  $C(1,1)$ . In this subsection we provide four additional results. First, we show that the null hypothesis of  $\gamma = 1$  cannot be rejected by the data using a likelihood ratio test. This is very important because a cointegrating vector  $(1, -1)$  implies that the balanced growth path hypothesis cannot be rejected.

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<sup>11</sup>We choose the number of lags using the AIC criterion.

In the IRBC literature, it is typically assumed that the coefficients driving TFP processes are symmetric across countries. Thus, we also use the likelihood ratio test to present evidence supporting the following three null hypothesis: (1) whether the coefficients related to the speed of adjustment in the cointegrating vector are equal and of opposite sign, i.e.,  $\kappa = -\kappa^*$ , (2) whether the coefficients of the constant terms are the same, i.e.,  $c = c^*$ , and (3) we also check for symmetry in the coefficients of the VAR. Since the lag coefficient matrices are

$$\rho^1 = \begin{pmatrix} \rho_{11}^1 & \rho_{12}^1 \\ \rho_{21}^1 & \rho_{22}^1 \end{pmatrix} \text{ and } \rho^2 = \begin{pmatrix} \rho_{11}^2 & \rho_{12}^2 \\ \rho_{21}^2 & \rho_{22}^2 \end{pmatrix},$$

the restrictions we check are:  $\rho_{11}^1 = \rho_{22}^1$ ,  $\rho_{11}^2 = \rho_{22}^2$ ,  $\rho_{12}^1 = \rho_{21}^1$ , and  $\rho_{12}^2 = \rho_{21}^2$ .

Finally, after imposing the above-described restrictions, i.e., balanced growth path, symmetric constant terms, symmetric speed of adjustment parameters, and symmetric coefficients of the VAR, we estimate our VECM.

In Table 4, we present the outcome of the four likelihood ratio tests. Note that the tests are incremental. The first important result is that the restriction that the cointegrating vector is  $(1, -1)$ , i.e.,  $\gamma = 1$ , is not rejected by the data. Second, we cannot reject that the coefficients on the speed of adjustment are the same in absolute value across countries. Third, we cannot reject that the constant term is equal across countries. Finally, the symmetry in the coefficients restriction is marginally rejected by the data at the 5 percent level. The above evidence allows us to follow the usual practice in the literature and simulate our model with all the restrictions in place.

In the final step, we estimate a restricted VECM. The estimated restricted model delivers the parameter estimates reported in Table 5. The results are as follows. First, it is worth noting that the coefficient of the speed of adjustment, while significant, is quantitatively small, denoting that TFP processes converge slowly over time. This finding is going to be important in explaining our results. Second, the coefficient on the own first lag implies significant but low autocorrelation. The crossed second lag is also significant. Third, the rest of the coefficients are not significant. Finally, we estimate the standard deviation of the innovations  $\sigma^\varepsilon$  and  $\sigma^{\varepsilon,*}$  to be around 0.0082. When simulating our model, we calibrate the stochastic process using the point estimates reported in Table 5 for the significant parameters,

including those for  $\sigma^\varepsilon$  and  $\sigma^{\varepsilon,*}$ . We also assume that  $\sigma^\varepsilon$  and  $\sigma^{\varepsilon,*}$  are uncorrelated, since the null hypothesis could not be rejected in the data.

## 5. Results

### 5.1. Parameterization

Our baseline parameterization follows that in Heathcote and Perri (2002) closely. Table 6 summarizes the parameter values. The discount factor  $\beta$  is set equal to 0.99, which implies an annual rate of return on capital of 4 percent. We set the consumption share,  $\tau$ , equal to 0.34 and the coefficient of risk aversion,  $\sigma$ , equal to 2. Backus, Kehoe, and Kydland (1992) assume the same value for the latter parameter. We assume a cost of bond holdings,  $\phi$ , of 100 basis points (0.01). Parameters on technology are fairly standard in the literature. Thus, the depreciation rate,  $\delta$ , is set to a quarterly value of 0.025, the capital share of output is set to  $\alpha = 0.36$ , and home bias for domestic intermediate goods is set to  $\omega = 0.9$ , which implies the observed import/output ratio in steady state. We set the adjustment cost of investment such that the volatility of investment over GDP is roughly as in the data. We assume two possible values for the elasticity of substitution between intermediate goods,  $\theta = 0.85$  and  $\theta = 0.62$ . The first value is based on Heathcote and Perri (2002); the second one is used by Corsetti, Dedola, and Leduc (2008b). The baseline technology process is calibrated as described in Table 5. For the stationary case, we set the parameters of the TFP shocks as in Heathcote and Perri (2002).

### 5.2. Matching Real Exchange Rate Volatility

In this subsection we analyze the performance of our model in generating enough real exchange rate volatility. Results are shown in Table 7a. To perform the simulation, we solve the model taking a log-linear approximation around the steady state and calculate the moments of the HP-filtered artificial data.<sup>12</sup>

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<sup>12</sup>One might question the use of the Hodrick-Prescott filter in a model without a stochastic trend. The reason is that we want to replicate patterns studied in the international business cycle literature. Hence, we want to emphasize the fact that the stochastic trend process generates much of the RER variance at business

The first and second rows of Table 7a report the results of the economy with cointegrated TFP and high and low values for the trade elasticity,  $\theta$ , respectively. For comparison with a model such as the one in Heathcote and Perri (2002), we also report the results for the economy with stationary TFP shocks in the next two rows (we use Heathcote and Perri's estimates for stationary TFP shocks).<sup>13</sup> Overall, models with cointegrated shocks generate higher relative volatility of the real exchange rate with respect to output than models with stationary TFP shocks. Note that with high trade elasticity and cointegrated TFP shocks, the relative volatility of the real exchange rate more than doubles with respect to the model with stationary shocks (1.75 versus 0.75). We go from explaining less than 20 percent of the observed relative volatility of the real exchange rate to explaining more than 40 percent. As expected for lower values of the trade elasticity, the relative volatility of the real exchange rate increases under both the stationary and cointegrated models. The striking finding is that the model with cointegrated TFP shocks and elasticity equal to 0.62 is able to closely match the relative volatility of the real exchange rate (4.26 in the model versus 4.28 in the data), while the model with stationary shocks and the same elasticity can get only to 1.41 (which represents only about 30 percent of the fluctuation in the data). Interestingly, even though the model with cointegrated TFP shocks improves significantly in matching the real exchange rate volatility, it does not affect other unconditional moments. Both the stationary and the cointegrated TFP models display very similar volatilities of consumption, hours, and investment relative to output. Also, both models display similar cross-correlations between consumption, hours, and investment relative to output and autocorrelations of real exchange rates (Tables 7a and 7b).

We have also simulated the model under two alternative asset market structures: complete markets and financial autarky. In the first case we assume that agents have access to a full set of state-contingent bonds that pay one unit of the domestic intermediate good in every

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cycle frequencies.

<sup>13</sup>In particular, Heathcote and Perri use the following VAR(1) process:

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \rho_a^* a_{t-1}^* + \varepsilon_t^a \\ a_t^* &= \rho_a a_{t-1}^* + \rho_a^* a_{t-1} + \varepsilon_t^{a,*} \end{aligned}$$

where  $\rho_a = 0.97$ ,  $\rho_a^* = 0.025$ ,  $Var(\varepsilon_t^a) = Var(\varepsilon_t^{a,*}) = 0.0073^2$ , and  $corr(\varepsilon_t^a, \varepsilon_t^{a,*}) = 0.29$ .

state of the world. In the second case, we calibrate the cost of holding bonds  $\Phi' [D(s^t)]$ , to a very large number such that intertemporal trade never occurs. As expected (see, Heathcote and Perri, 2002), the version of the model with complete markets generates lower relative volatility of the real exchange rate (in particular, it goes from 1.75 to 1.11 when  $\theta = 0.85$  and shocks are stationary, and from 4.26 to 1.35 when  $\theta = 0.62$  and shocks are cointegrated), while the version of the model with financial autarky delivers a larger relative volatility of the real exchange rate (in particular, it goes from 1.75 to 2.05 when  $\theta = 0.85$  and shocks are stationary, and from 4.26 to 5.41 when  $\theta = 0.62$  and shocks are cointegrated).

### 5.3. Intuition

In this subsection we explain why our results differ from those obtained with more traditional calibrations of TFP processes. In the typical IRBC model, there are two forces driving relative volatility of the real exchange rate with respect to output. In particular, the model needs high persistence and low spillovers of the TFP processes across countries in order to get high relative volatility of the real exchange rate. Our VECM estimates imply higher persistence and slower spillovers than what is typically assumed in the literature and, therefore, we succeed in matching the high relative volatility of the real exchange rate. Let us now explain how persistence affects the relative volatility of the real exchange rate. Later, we will discuss the effects of spillovers.

To understand the effects of persistence, we simulate our model assuming the following simple processes for TFP

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a, \\ a_t^* &= \rho_a a_{t-1}^* + \varepsilon_t^{a,*} \end{aligned}$$

where the innovations are uncorrelated and there are no spillovers across countries. In Figures 4 and 5 we present the impulse responses to a home TFP shock for different values of  $\rho_a = [0.9, 0.95, 0.975]$  using the same calibration as in Table 6 for the rest of the parameters (for these figures we use  $\theta = 0.62$ ). As Table 8 shows, as we increase the persistence parameter, the relative volatility of the HP-filtered real exchange rate with respect to HP-filtered output

also rises.

For any given persistence parameters, when productivity increases at home, home households consume and invest more, while labor rises because of the increase in the marginal productivity of labor. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, the demand for the foreign intermediate good rises. Therefore, the foreign intermediate good becomes relatively scarce with respect to the home intermediate good, and the terms of trade and the real exchange rate rise (depreciate).

As the persistence of TFP shocks increases, the initial impact and the persistence of the response of the real exchange rates are larger. In Table 8, we confirm that the standard deviation of the HP-filtered real exchange rate increases with  $\rho_a$ . However, output's response is different. Its impact is smaller, but more persistent. Thus, by just looking at the impulse response in Figure 4, the effect on volatility is uncertain. Table 8 shows that in our calibration, these two conflicting forces lead to a lower standard deviation of HP-filtered output as persistence increases. Hence, the relative volatility of the real exchange rate with respect to output rises.

What is the mechanism behind this result? With a higher persistence of TFP shocks, home-country households suffer a larger positive income effect and therefore supply less labor and capital. This income effect has two implications. First, it lowers the initial increase of home output. Second, it leads to home households demanding more consumption goods. Thus, the demand for foreign intermediate goods increases, making foreign intermediate goods even more scarce and the effects on the terms of trade and the real exchange rate are larger.

Let us now analyze the effects of spillover changes on the relative volatility of the real exchange rate. We assume the following simple VECM model:

$$\begin{aligned}\Delta a_t &= -\kappa(a_{t-1} - a_{t-1}^*) + \varepsilon_t^a \\ \Delta a_t^* &= \kappa(a_{t-1} - a_{t-1}^*) + \varepsilon_t^{a,*}\end{aligned}$$

where the innovations are uncorrelated and  $\kappa$  represents the speed of adjustment to the cointegrating relationship. Note that we have switched to a model with one unit root. The reasons behind this choice are twofold. First, it allows a clear mapping with our estimated VECM while changing  $\kappa$ . Second, the behavior of a model with TFP shocks driven by a

VAR, such as the one used above with a persistence coefficient arbitrarily close to one, is numerically very similar to a model with TFP shocks driven by a VECM with a speed of convergence coefficient arbitrarily close to zero.

In Figures 6 and 7 we present the impulse responses to a home TFP shock for different values of  $\kappa = [0.005, 0.05, 0.25]$ . As before, we use the same calibration as in Table 6 for the rest of the parameters and we fix  $\theta = 0.62$ . Now the foreign TFP process also responds over time to a home TFP increase due to the cointegrating relationship. The larger is  $\kappa$ , the faster is the response of foreign TFP to home TFP shocks. The most important consequence of considering cointegration (and, therefore, spillovers) is the fact that there is a “news” channel effect as foreign-country households anticipate the future increase of foreign TFP. When  $\kappa = 0.005$  (slow speed of convergence), the mechanism at work is very similar to that of Figures 4 and 5 with  $\rho_a = 0.975$  because the “news” channel is quantitatively very small. As  $\kappa$  increases, the “news” channel becomes more important as the foreign households feel the income effect associated with it.

When productivity increases at home and spillovers are faster, foreign-country households know that productivity will increase sooner in their country. Hence, because of an income effect, they demand more consumption goods than they would if spillovers were slower. Thus, the demand for home intermediate goods increases because foreign final goods producers substitute away from domestic intermediate goods. As a consequence, foreign intermediate goods become less scarce and the terms of trade and real exchange rate depreciate less than in a model without spillovers. Therefore, faster spillovers in TFP shocks lead to a lower relative volatility of the real exchange rate with respect to output. Table 8 confirms this intuition. As the speed of convergence increases, the volatility of HP-filtered output increases, and the volatility of the HP-filtered real exchange rate decreases. Hence, the relative volatility of the real exchange rate with respect to output decreases.

Note that in the case of  $\kappa = 0.25$ , the relative standard deviation is 0.48, which is much lower than the values obtained under stationary TFP shocks, despite the fact that the VECM has one unit root. Hence, having cointegrated TFP shocks is not enough to solve the real exchange rate puzzle: a very slow speed of convergence is also necessary. Note that we can

write the VECM as a VAR process in levels as follows:

$$\begin{pmatrix} a_t \\ a_t^* \end{pmatrix} = \begin{pmatrix} 1 - \kappa & \kappa \\ \kappa & 1 - \kappa \end{pmatrix} \begin{pmatrix} a_{t-1} \\ a_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^{a,*} \end{pmatrix}. \quad (36)$$

where the eigenvalues of the VAR are  $\lambda_1 = 1$ ,  $\lambda_2 = 1 - 2\kappa$ .<sup>14</sup> Therefore, a small  $\kappa$  means that we need both one unit root and that the second eigenvalue is very close to one. In fact, in the simple VECM with  $\kappa = 0.005$ , the two eigenvalues are  $\lambda_1 = 1$  and  $\lambda_2 = 0.99$  (while our point estimate for  $\kappa$  is 0.0045; see Table 5).

Why is the implied relative volatility of the real exchange rate using our VECM estimates so different from the one obtained using Heathcote and Perri's (2002) calibration? Their estimated VAR has eigenvalues equal to  $\lambda_1 = 0.995$  and  $\lambda_2 = 0.945$  (see footnote 13). While one of the eigenvalues is very close to one (which would imply high relative volatility of the real exchange rate given the results reported in Table 8), the second eigenvalue is farther away from one (implying fast spillovers), which matters for real exchange rate volatility. Also, Heathcote and Perri (2002) find that the correlation of innovations to TFP shocks is 0.29, which acts as a contemporaneous spillover, further reducing the relative volatility of the real exchange rate for the reasons we have just explained.<sup>15</sup>

We have also explored two other popular calibrations in the literature. Backus, Kehoe, and Kydland's (1992) calibration implies eigenvalues of  $\lambda_1 = 0.994$  and  $\lambda_2 = 0.812$  and a correlation between innovations of 0.26, resulting in a relative volatility of the real exchange rate of 0.65. Heathcote and Perri (2008) have two eigenvalues at 0.91 and uncorrelated innovations, which delivers a relative volatility of the real exchange rate of 1.05.

In view of (36), the question is: would it be possible to solve the real exchange rate volatility puzzle using TFP shocks driven by a VAR in levels with one lag instead of a VECM?. In principle, it would be possible as long as we calibrate the law of motion of the VAR so that the two eigenvalues are very close to one. The problem is that when we estimate a VAR such as one described above using our data set, we find that the two eigenvalues are 0.999 and 0.952, and there is a correlation between innovations of 0.16. Using this point estimate

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<sup>14</sup> Also note that if  $\kappa = 0$ , then the VAR has two unit roots, consistent with two independent random walks.

<sup>15</sup> Actually, our estimated correlation for the residuals of the VECM is 0.07, and it is not significant.

and the calibration described in Table 6 for the rest of the parameters (with  $\theta = 0.62$ ), the relative volatility of the real exchange rate with respect to output is only 1.67.

#### 5.4. Matching the Increase in Real Exchange Rate Volatility

As described in section 2, the volatility of the real exchange rate with respect to the volatility of output has increased in the last decade for most industrialized economies. If we focus on the U.S., the increase seems to be dated around the early to mid 90's. As Table 7a shows, the volatility of the real exchange rate has gone from less than four times the volatility of output during the period 1980:1 to 1993:4 to more than five times during the period 1994:1 to 2007:3. Using U.S. and "rest of the world" data, in this section we present evidence that relates a decrease in the speed of convergence to the cointegrating relationship, i.e., lower  $\kappa$ , with the increase in the relative volatility of the real exchange rate.

To present our evidence, we estimate our VECM for two non-overlapping sub-samples.<sup>16</sup> The first sample goes from 1980:1 to 1993:4, while the second sub-sample goes from 1994:1 to 2007:3. We split the sample such that we have the same number of observations in both sub-samples. Using data from 1980:1 to 1993:4, we observe three significant changes with respect to the full sample estimates. The value of the speed of adjustment term is larger in absolute value, the first own lag is also somewhat larger, and the second crossed lag is close to zero. In particular,  $\kappa$  moves from  $-0.0045$  to  $-0.0077$ , making the speed of convergence faster and  $\rho_{11}^1$  moves from 0.2041 to 0.2203, increasing the autocorrelation of the process. Also, the standard deviation of the stochastic process for the U.S.,  $\sigma$ , is estimated to be 0.010, while the standard deviation for the "rest of the world,"  $\sigma^*$ , is estimated to be 0.0081.

In the second sub-sample, 1994:1 to 2007:3, the estimated speed of adjustment coefficient dramatically decreases with respect to both the full sample and the first sub-sample: the point estimate is  $-0.0029$ . This means that the catching up process is much slower in the second part of the sample. In addition, the second crossed-lag coefficient gets larger and negative:  $\rho_{12}^2 = -0.4124$ . In this case, the first own lag moves close to zero. These results indicate that the comovement between total factor productivities in the post-1994 period is characterized by short-run negative comovement and slow return to the long-run level.

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<sup>16</sup>We assume that the cointegrating relationship is the same across samples.

Finally, the standard deviations  $\sigma$  and  $\sigma^*$  are estimated to be 0.0062 and 0.0086, respectively. Our sub-sample estimates of  $\sigma$  and  $\sigma^*$  reflect both our sample period and the countries that we include in the “rest of the world.” While the big drop in  $\sigma$  across sub-samples reveals the reduction in output volatility that the U.S. experienced during the 80’s (see Kim and Nelson, 1999, and McConnell and Perez-Quirós, 2000), the stable  $\sigma^*$  exposes that this was not the case for most of the countries in our definition of the “rest of the world” during the considered period. This second finding is in line with those in Stock and Watson (2005).

We now simulate the model under the estimates of the VECM for each of the sub-samples. Tables 7a and 7b report the results. Our results indicate that the change in the estimates of the VECM across samples is an important force behind the increase in the relative volatility of the real exchange rate. While in the data the relative volatility of the real exchange rate increases by 30 percent across samples, our simulations show that the model generates increases in relative volatility of more than 50 percent for both low and high values of  $\theta$ .

### 5.5. The “Backus-Smith Puzzle”

How does the model perform in terms of the correlation between the real exchange rate and the ratio of consumption across countries? As the last column of Table 7b shows, our model implies that the correlation between the real exchange rate and relative consumption is very close to one, whereas in the data this correlation is negative but close to zero. This discrepancy between the models and the data is known as the “Backus-Smith puzzle.”<sup>17</sup> The failure in accounting for the “Backus-Smith puzzle” is typical in standard IRBC models. In recent papers, Corsetti, Dedola, and Leduc (2008a) and Benigno and Thoenissen (2007) show that adding non-tradable goods to a traditional IRBC model helps to solve the “Backus-Smith puzzle.”

As we have shown in Figures 6 and 7, in our model a domestic TFP shock induces an increase in home consumption relative to foreign consumption and at the same time causes a real exchange rate depreciation. Hence, it is hard for our model to generate a negative correlation between the real exchange rate and relative consumption unless another source of fluctuations is considered. One option is to introduce taste shocks that affect the marginal

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<sup>17</sup>Chari, Kehoe, and McGrattan (2002) tag this problem as the consumption-real exchange rate anomaly.

utility of consumption and allow them to break the risk-sharing condition implied by the model. Following this line of research, Heathcote and Perri (2008) introduce taste shocks and show how this simple device accomplishes the objective. We introduced this type of shock in our framework and obtained a negative correlation between relative consumption and the real exchange rate.

Since it is difficult to measure taste shocks in the data, we also consider another avenue. In particular, we introduce investment-specific technology shocks, as in Greenwood, Hercowitz, and Krusell (1997). Thus, we change equation (3), and its foreign-country counterpart, to

$$K(s^t) = (1 - \delta) K(s^{t-1}) + V(s^t) X(s^t) \quad (37)$$

and

$$K^*(s^t) = (1 - \delta) K^*(s^{t-1}) + V^*(s^t) X^*(s^t) \quad (38)$$

where both  $\log V(s^t)$  and  $\log V^*(s^t)$  are cointegrated of order  $C(1, 1)$  and follow the same VECM process as the TFP shocks introduced in section 3.2.3.

Ideally, we would like to estimate the VECM process for the investment-specific technology shocks. In the literature, these shocks have been proxied by the quality-adjusted relative price of investment goods with respect to the price of consumption goods.<sup>18</sup> While the quality-adjusted relative price of investment goods is available for the U.S., it is not for most other countries in our “rest of the world” definition. Hence, we cannot estimate a VECM. To illustrate the potential of this shock to solve the “Backus-Smith puzzle,” we calibrate the VECM process for the investment-specific technology shocks using the same parameterization obtained for the TFP shock (see Table 5). Since we do not have a way to determine the relative importance of these two shocks, in Table 9 we report simulations of the model letting the standard deviation of the investment-specific technology shocks change from one to three times that of the TFP shock. Given the estimates reported in the literature, this appears to be a plausible range.<sup>19</sup>

As can be observed in Table 9, as investment-specific technology shocks become more

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<sup>18</sup>See Fisher (2006).

<sup>19</sup>See Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007).

important, the correlation between relative consumption and the real exchange rate drops dramatically. When we consider only TFP shocks, the correlation is 0.97, but when the standard deviation of the investment-specific technology shocks is three times the standard deviation of TFP shocks, the correlation becomes negative and very similar to that in the data. The good news is that the relative volatility of the real exchange rate with respect to output declines, but only mildly. When the standard deviation of the investment-specific technology shocks is three times the standard deviation of TFP shocks, it drops from 4.26 to 3.82 when only TFP is considered.

Why do investment-specific technology shocks do the job? As an investment-specific technology shock hits the home country, investment increases and consumption decreases at home. Since home investment goods are produced using foreign intermediate goods, the price of intermediate goods produced in the foreign country increases, making the real exchange rate depreciate. Hence, foreign households feel richer because of the terms of trade depreciation and they consume more. As a result, the model generates a negative correlation between the consumption ratio across countries and the real exchange rate.

## 6. Concluding Remarks

In this paper, we document two empirical facts. First, that TFP processes of the U.S. and the “rest of the world” are cointegrated with cointegrating vector  $(1, -1)$  and, second, that the relative volatility of the real exchange rate with respect to output has increased in the United States, the United Kingdom, Canada, and Australia during the last 20 years.

Then, we have shown that introducing cointegrated TFP processes in an otherwise standard IRBC model increases the model’s ability to explain real exchange rate volatility, without affecting the fit to other second moments of the data. We have also documented that if we allow the speed of convergence to the cointegrating vector to change, as it does in the data, the model can also explain the observed increase in the relative volatility of the real exchange rate.

For future research, it would be interesting to introduce cointegrated TFP processes in medium-scale open economy macroeconomic models, which typically include more frictions,

and try to match a larger set of domestic and international variables.<sup>20</sup> Also, Rabanal and Rubio-Ramírez (2008) investigate whether investment-specific technology shocks are cointegrated across countries, and their role in international business cycles models with a focus on the quantity and Backus and Smith puzzles.

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<sup>20</sup>See Adolfson et al. (2007).

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Table 1: Unit Root tests for TFP

log U.S. TFP				
Method	Level		First Difference	
	t-statistic	p-value	t-statistic	p-value
ADF	-3.29	0.07	-3.03	0.13
DF-GLS	-1.30	0.2	-1.16	0.25
$P_T$ -GLS	31.98	6.81*	49.64	6.82*
$MZ_\alpha$	-3.49	-14.2**	-2.00	-14.2**
$MZ_t$	-1.23	-2.6**	-0.97	-2.6**
MSB	0.35	0.19**	0.48	0.18**

log “Rest of the World” TFP				
	Level		First Difference	
	t-statistic	p-value	t-statistic	p-value
ADF	-7.11	0.00	-3.89	0.02
DF-GLS	-2.76	0.00	-6.95	0.07
$P_T$ -GLS	1.75	0.00	4.66	6.8*
$MZ_\alpha$	-51.5	0.00	-14.77	-14.2**
$MZ_t$	-5.06	0.00	-2.48	-2.62**
MSB	0.09	0.00	0.16	0.19**

Notes: ADF stands for augmented Dickey-Fuller test. DF-GLS stands for Elliott-Rothenberg-Stock detrended residuals test statistic.  $P_T$ -GLS stands for Elliott-Rothenberg-Stock Point-Optimal test statistic.  $MZ_\alpha$ ,  $MZ_t$ , and MSB stand for the class of modified tests analyzed in Ng-Perron (2001). p-values for the ADF test are one-sided p-values as in MacKinnon (1996). p-values for the DF-GLS test are as in Elliott-Rothenberg-Stock (1996, Table 1). \* These values do not represent the p-values but the critical values of the test at the 10 percent level as reported in Elliott-Rothenberg-Stock (1996) Table 1. \*\* These values do not represent the p-values but the asymptotic critical values of the test at the 10 percent level as reported in Ng-Perron (2001) Table 1.

Table 2: Cointegration Statistics I

Real	Imaginary	Modulus
1.01	0	1.01
0.97	0	0.97
0.77	0.28	0.82
0.77	0.28	0.82
Log likelihood		752.51

Table 3: Cointegration Statistics II: Johansen's test

Number of Vectors	Eigenvalue	Trace	p-value	Max-Eigenvalue	p-value
0	0.15	19.06	0.01	18.21	0.01
1	0.00	0.855	0.35	0.00	0.35

Note: p-values as reported in MacKinnon-Haug-Michelis (1999)

Table 4: Likelihood ratio tests

Restriction	Likelihood value	Degrees of freedom	p-value
None	744.18	-	-
$\gamma = 1$	743.33	1	0.19
$\kappa = -\kappa^*$	741.71	2	0.09
$c = c^*$	740.43	3	0.06
Symmetry across VAR coefficients	736.51	7	0.032

Table 5: VECM model

1980 – 2007	
$c$	0.0071* (5.83)
$\kappa$	-0.0045* (-2.65)
$\rho_{11}^1$	0.2041* (2.97)
$\rho_{11}^2$	0.1026 (1.54)
$\rho_{12}^1$	0.1035 (1.55)
$\rho_{12}^2$	-0.1497* (-2.40)

t-statistics in parenthesis.

\* denotes significance at the 5 percent level.

Table 6: Calibration

Preferences	Discount factor	$\beta = 0.99$
	Consumption share	$\tau = 0.34$
	Risk aversion	$\sigma = 2$
	Cost of bond holdings	$\phi = 0.01$
Technology	Capital share	$\alpha = 0.36$
	Depreciation rate	$\delta = 0.025$
	Home bias	$\omega = 0.9$
	Elasticity of substitution between intermediate goods	$\theta = [0.85, 0.62]$
	Adjustment cost of investment	varies

Table 7a: Results

Full Sample	$SD(Y)$	$SD(C)^+$	$SD(X)^+$	$SD(N)^+$	$SD(RER)^+$	$\rho(RER)$
Data	1.25	0.80	3.40	0.91	4.28	0.84
Cointegrated TFP, $\theta = 0.85$	0.81	0.63	2.32	0.28	1.75	0.72
Cointegrated TFP, $\theta = 0.62$	0.70	0.62	2.31	0.28	4.26	0.70
Stationary TFP, $\theta = 0.85$	1.19	0.52	2.53	0.32	0.75	0.77
Stationary TFP, $\theta = 0.62$	1.12	0.54	2.51	0.31	1.41	0.75
1980-1993						
Data	1.57	0.80	3.08	0.89	3.97	0.85
Cointegrated TFP, $\theta = 0.85$	1.12	0.63	2.17	0.25	1.33	0.72
Cointegrated TFP, $\theta = 0.62$	0.95	0.65	2.15	0.25	3.17	0.71
1994-2007						
Data	0.83	0.76	4.20	0.96	5.17	0.81
Cointegrated TFP, $\theta = 0.85$	0.64	0.55	2.74	0.38	2.04	0.71
Cointegrated TFP, $\theta = 0.62$	0.62	0.43	3.01	0.42	5.06	0.69

<sup>+</sup> denotes relative to output.

Table 7b: Results

Full Sample	$CORR(Y, N)$	$CORR(Y, C)$	$CORR(Y, X)$	$CORR(RER, C/C^*)$
Data	0.79	0.81	0.91	-0.04
Cointegrated TFP, $\theta = 0.85$	0.94	0.95	0.97	0.95
Cointegrated TFP, $\theta = 0.62$	0.92	0.93	0.95	0.97
Stationary TFP, $\theta = 0.85$	0.97	0.93	0.97	0.99
Stationary TFP, $\theta = 0.62$	0.97	0.93	0.97	0.99
1980-1993				
Data	0.82	0.82	0.93	-0.10
Cointegrated TFP, $\theta = 0.85$	0.93	0.96	0.97	0.95
Cointegrated TFP, $\theta = 0.62$	0.91	0.96	0.96	0.94
1994-2007				
Data	0.71	0.76	0.90	0.12
Cointegrated TFP, $\theta = 0.85$	0.89	0.82	0.94	0.98
Cointegrated TFP, $\theta = 0.62$	0.94	0.78	0.97	0.95

Table 8: Changing  $\rho_a$  and  $\kappa$ 

	$SD(RER)$	$SD(Y)$	$SD(RER)^+$
$\rho_a$			
0.9	1.43	1.33	1.07
0.95	1.96	1.2	1.64
0.975	2.47	1.06	2.33
$\kappa$			
0.005	1.98	0.64	3.1
0.05	1.02	0.82	1.25
0.25	0.71	0.86	0.82

<sup>+</sup> denotes relative to output.

Table 9: Investment-Specific Technology shocks

Scaling	0	1	2	3
$\frac{SD(RER)}{SD(Y)}$	4.26	4.17	4.13	3.82
$CORR(RER, \frac{C}{C^*})$	0.97	0.55	0.19	-0.08

Scaling is the ratio of the standard deviation of the innovation to the investment-specific technology shock with respect to that of TFP shocks. Hence 0 is the model with only TFP shocks, and 3 denotes the model where investment-specific technology shocks are three times as volatile as TFP shocks.

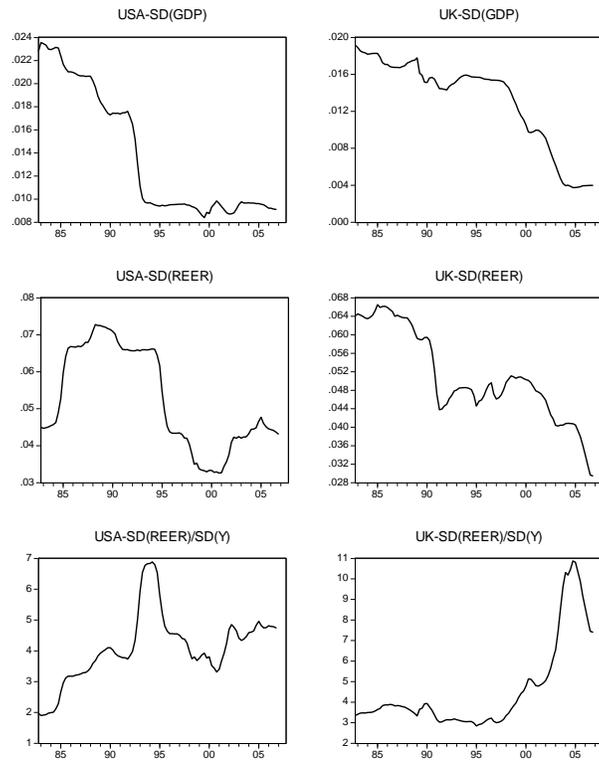


Figure 1: Standard Deviation of HP-Filtered Data. USA and UK.

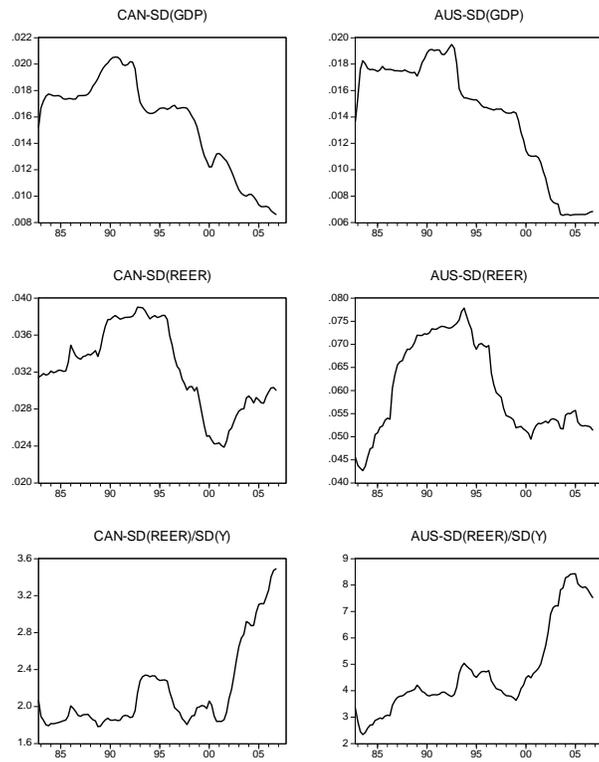


Figure 2: Standard Deviation of HP-Filtered Data. Canada and Australia.

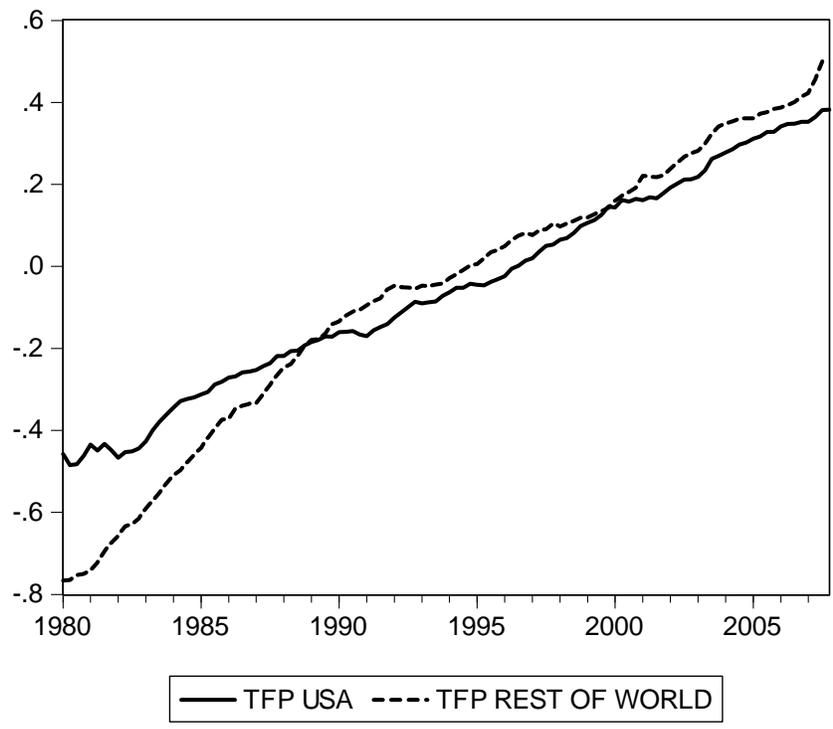


Figure 3: TFP processes for the U.S. and the “rest of the world.”

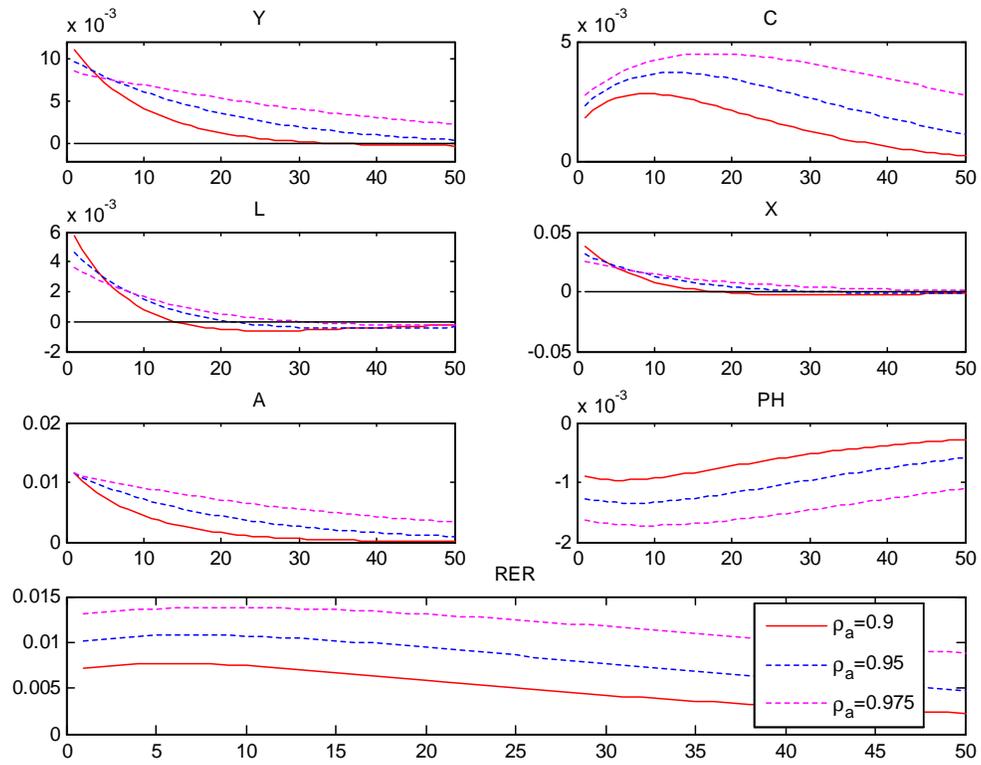


Figure 4: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.

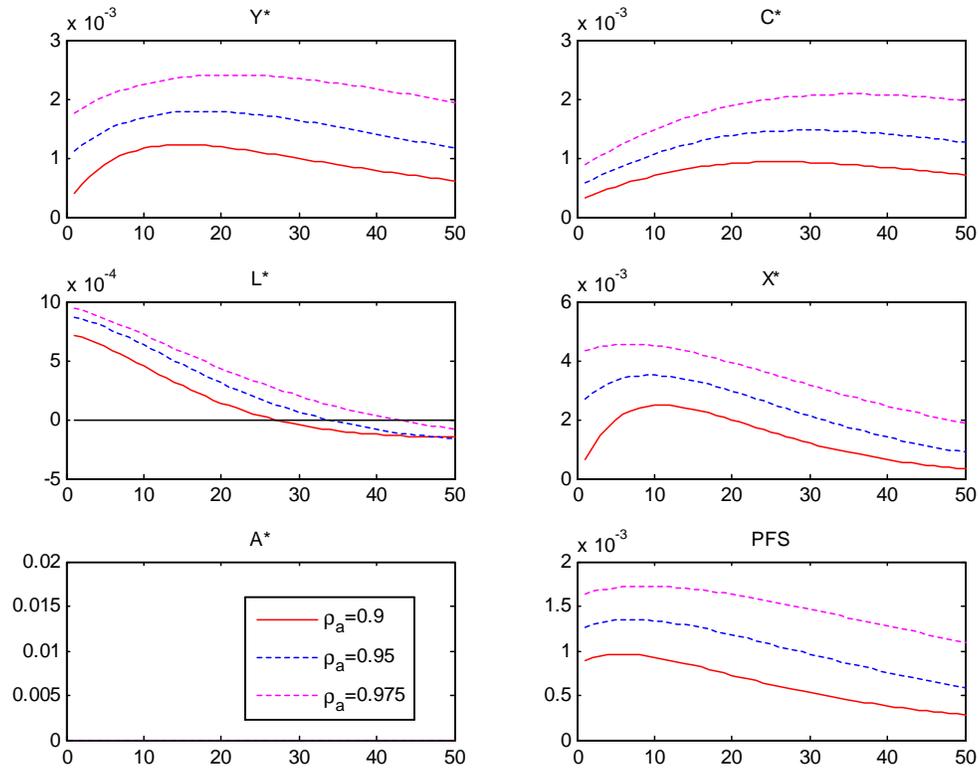


Figure 5: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.

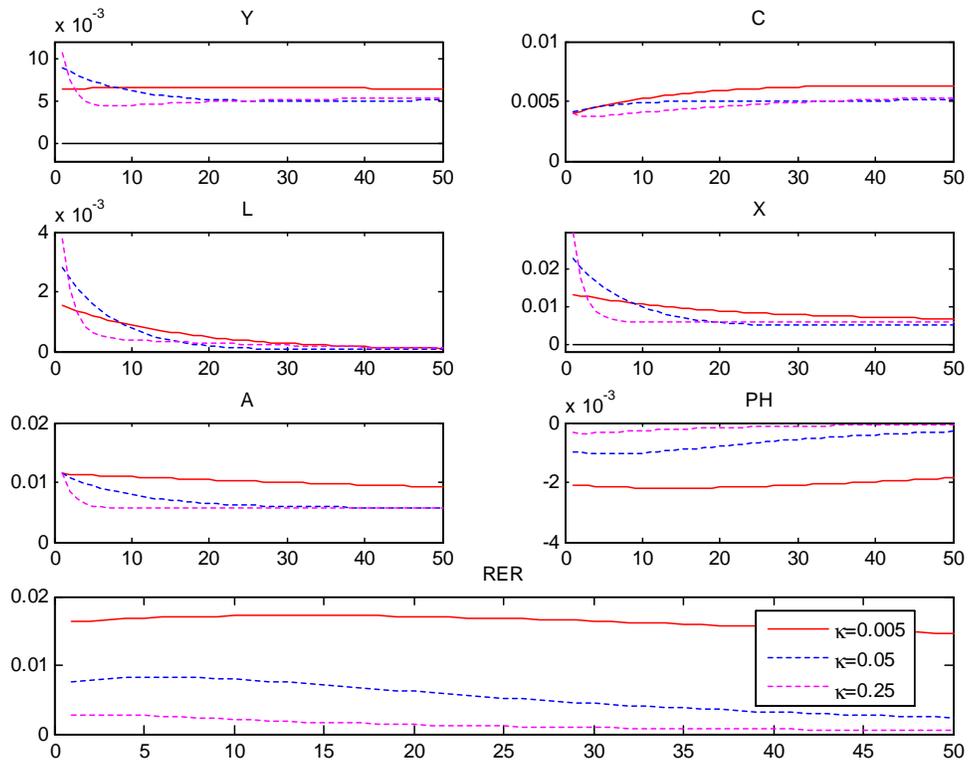


Figure 6: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.

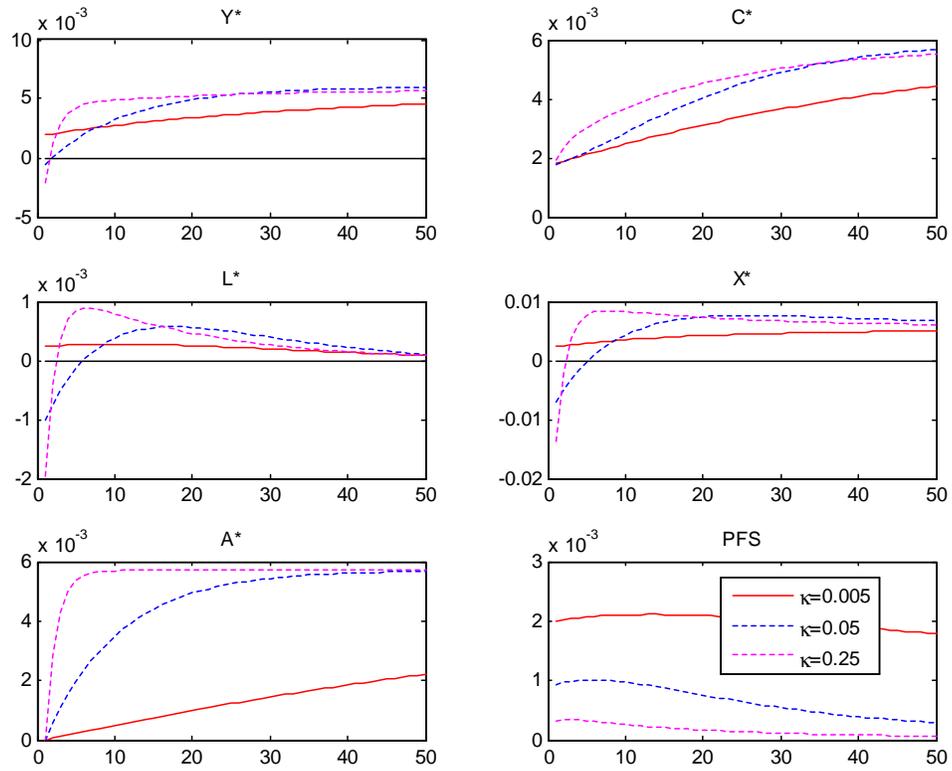


Figure 7: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.

## A. Appendix

### A.1. Normalized Equilibrium Conditions

Equations (10) to (35) characterize equilibrium in this model. Since both  $\log A(s^t)$  and  $\log A^*(s^t)$  are integrated, we now normalize the above-described system in order to get a stationary system more amenable to study. Additional restrictions on the VECM defining the law of motion of the technological processes are required if the model is to exhibit balanced growth. Those restrictions are described in the next subsection.

Let us first define the following normalized variables  $\widehat{Y}_H(s^t) = \frac{Y_H(s^t)}{A(s^{t-1})}$ ,  $\widehat{Y}_H^*(s^t) = \frac{Y_H^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{Y}_F(s^t) = \frac{Y_F(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{Y}_F^*(s^t) = \frac{Y_F^*(s^t)}{A^*(s^{t-1})}$ , and  $\widehat{K}(s^{t-1}) = \frac{K(s^{t-1})}{A(s^{t-1})}$ ,  $\widehat{K}^*(s^{t-1}) = \frac{K^*(s^{t-1})}{A^*(s^{t-1})}$ ,  $\widehat{Y}(s^t) = \frac{Y(s^t)}{A(s^{t-1})}$ ,  $\widehat{Y}^*(s^t) = \frac{Y^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{C}(s^t) = \frac{C(s^t)}{A(s^{t-1})}$ ,  $\widehat{C}^*(s^t) = \frac{C^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{X}(s^t) = \frac{X(s^t)}{A(s^{t-1})}$ ,  $\widehat{X}^*(s^t) = \frac{X^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{W}(s^t) = \frac{W(s^t)}{A(s^{t-1})}$ ,  $\widehat{W}^*(s^t) = \frac{W^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{D}(s^t) = \frac{D(s^t)}{A(s^{t-1})}$ ,  $\widehat{D}^*(s^t) = \frac{D^*(s^t)}{A^*(s^{t-1})}$ ,  $\widehat{\lambda}(s^t) = \lambda(s^t) A(s^{t-1})^{1-\gamma(1-\sigma)}$ , and  $\widehat{\lambda}^*(s^t) = \lambda^*(s^t) A^*(s^{t-1})^{1-\gamma(1-\sigma)}$ . Then, the stationary first order conditions are

$$U_C(s^t) = \widehat{\lambda}(s^t), \quad (39)$$

$$\frac{U_L(s^t)}{U_C(s^t)} = \widehat{W}(s^t), \quad (40)$$

$$\left(\frac{A(s^t)}{A(s^{t-1})}\right)^{1-\gamma(1-\sigma)} \widehat{\lambda}(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \widehat{\lambda}(s^{t+1}) (R(s^{t+1}) + 1 - \delta), \quad (41)$$

$$\widehat{K}(s^t) = (1 - \delta) \widehat{K}(s^{t-1}) \frac{A(s^{t-1})}{A(s^t)} + \widehat{X}(s^t) \frac{A(s^{t-1})}{A(s^t)}, \quad (42)$$

$$U_{C^*}(s^t) = \widehat{\lambda}^*(s^t), \quad (43)$$

$$\frac{U_{L^*}(s^t)}{U_{C^*}(s^t)} = \widehat{W}^*(s^t), \quad (44)$$

$$\left(\frac{A^*(s^t)}{A^*(s^{t-1})}\right)^{1-\gamma(1-\sigma)} \widehat{\lambda}^*(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \widehat{\lambda}^*(s^{t+1}) (R^*(s^{t+1}) + 1 - \delta), \quad (45)$$

$$\widehat{K}^*(s^t) = (1 - \delta) \widehat{K}^*(s^{t-1}) \frac{A^*(s^{t-1})}{A^*(s^t)} + \widehat{X}^*(s^t) \frac{A^*(s^{t-1})}{A^*(s^t)}, \quad (46)$$

$$\bar{Q}(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}/s^t) \frac{\hat{\lambda}(s^{t+1})}{\hat{\lambda}(s^t)} \left( \frac{A(s^{t-1})}{A(s^t)} \right)^{1-\gamma(1-\sigma)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} - \Phi'(D(s^t)), \quad (47)$$

$$\sum_{s^{t+1}} \pi(s^{t+1}/s^t) \left[ \frac{\hat{\lambda}^*(s^{t+1})}{\hat{\lambda}^*(s^t)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} \frac{RER(s^t)}{RER(s^{t+1})} \left( \frac{A(s^t)}{A(s^{t-1})} \frac{A^*(s^{t-1})}{A^*(s^t)} \right)^{1-\gamma(1-\sigma)} - \frac{\hat{\lambda}(s^{t+1})}{\hat{\lambda}(s^t)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} \right] = -\frac{\Phi'(D(s^t))}{\beta}, \quad (48)$$

$$\widehat{W}(s^t) = (1-\alpha) \tilde{P}_H(s^t) \widehat{K}(s^{t-1})^\alpha L(s^t)^{-\alpha} \left( \frac{A(s^t)}{A(s^{t-1})} \right)^{1-\alpha}, \quad (49)$$

$$R(s^t) = \alpha \tilde{P}_H(s^t) \widehat{K}(s^{t-1})^{\alpha-1} L(s^t)^{1-\alpha} \left( \frac{A(s^t)}{A(s^{t-1})} \right)^{1-\alpha}, \quad (50)$$

$$\widehat{W}^*(s^t) = (1-\alpha) \tilde{P}_F^*(s^t) \widehat{K}^*(s^{t-1})^\alpha L^*(s^t)^{-\alpha} \left( \frac{A^*(s^t)}{A^*(s^{t-1})} \right)^{1-\alpha}, \quad (51)$$

$$R^*(s^t) = \alpha \tilde{P}_F^*(s^t) \widehat{K}^*(s^{t-1})^{\alpha-1} L^*(s^t)^{1-\alpha} \left( \frac{A^*(s^t)}{A^*(s^{t-1})} \right)^{1-\alpha}, \quad (52)$$

$$\widehat{Y}_H(s^t) = \omega \tilde{P}_H(s^t)^{-\theta} \widehat{Y}(s^t), \quad (53)$$

$$\widehat{Y}_F(s^t) = (1-\omega) \left( \tilde{P}_F^*(s^t) RER(s^t) \right)^{-\theta} \widehat{Y}(s^t) \frac{A(s^{t-1})}{A^*(s^{t-1})}, \quad (54)$$

$$\widehat{Y}_H^*(s^t) = (1-\omega) \left( \frac{\tilde{P}_H(s^t)}{RER(s^t)} \right)^{-\theta} \widehat{Y}^*(s^t) \frac{A^*(s^{t-1})}{A(s^{t-1})}, \quad (55)$$

$$\widehat{Y}_F^*(s^t) = \omega \tilde{P}_F^*(s^t)^{-\theta} \widehat{Y}^*(s^t). \quad (56)$$

$$\widehat{C}(s^t) + \widehat{X}(s^t) = \widehat{Y}(s^t), \quad (57)$$

$$\widehat{C}^*(s^t) + \widehat{X}^*(s^t) = \widehat{Y}^*(s^t), \quad (58)$$

$$\widehat{Y}(s^t) = \left[ \omega^{\frac{1}{\theta}} \widehat{Y}_H(s^t)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_F(s^t)^{\frac{\theta-1}{\theta}} \left( \frac{A^*(s^{t-1})}{A(s^{t-1})} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (59)$$

$$\widehat{Y}^*(s^t) = \left[ \omega_F^{\frac{1}{\theta}} \widehat{Y}_F^*(s^t)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \widehat{Y}_H^*(s^t)^{\frac{\theta-1}{\theta}} \left( \frac{A(s^{t-1})}{A^*(s^{t-1})} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (60)$$

$$\widehat{Y}_H(s^t) + \widehat{Y}_H^*(s^t) = \widehat{K}(s^{t-1})^\alpha \left( \widehat{L}(s^t) \frac{A(s^t)}{A(s^{t-1})} \right)^{1-\alpha}, \quad (61)$$

$$\widehat{Y}_F(s^t) + \widehat{Y}_F^*(s^t) = \widehat{K}^*(s^{t-1})^\alpha \left( \widehat{L}^*(s^t) \frac{A^*(s^t)}{A^*(s^{t-1})} \right)^{1-\alpha}, \quad (62)$$

$$\widehat{D}(s^t) + \widehat{D}^*(s^t) \frac{A^*(s^{t-1})}{A(s^{t-1})} = 0, \quad (63)$$

and

$$\begin{aligned} \widetilde{P}_H(s^t) \overline{Q}(s^t) \widehat{D}(s^t) &= \widetilde{P}_H(s^t) \widehat{Y}_H^*(s^t) - \widetilde{P}_F^*(s^t) RER(s^t) \widehat{Y}_F(s^t) \frac{A^*(s^{t-1})}{A(s^{t-1})} \\ &+ \widetilde{P}_H(s^t) \widehat{D}(s^{t-1}) \frac{A(s^{t-2})}{A(s^{t-1})} - \widetilde{P}_H(s^t) \frac{\Phi(D(s^t))}{A(s^{t-1})}. \end{aligned} \quad (64)$$

Finally, the productivity shocks do not need to be normalized. Also, note that our functional form  $\Phi[D(s^t)] = \frac{\phi}{2} A(s^{t-1}) \left[ \frac{D(s^t)}{A(s^{t-1})} \right]^2$  implies that  $\Phi[\widehat{D}(s^t)]/A(s^{t-1})$  and  $\Phi'[\widehat{D}(s^t)]$  are stationary. This is important to make normalized equations (47) to (48) stationary.