Is the Elasticity of Intertemporal Substitution Constant?*

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Abstract

This paper shows that the CRRA specification of intertemporal preferences (implying a constant elasticity of intertemporal substitution, or EIS), imposes surprising limitations on within period budget allocations. Consequently, the constant EIS assumption can be tested with demand data. In fact, the parameter of the CRRA utility function is pinned down completely by the shape of Engel curves; if preferences are CRRA then the EIS could in principle be estimated without any variation in the interest rate. The fact that a price elasticity can be estimated without variation in the relevant price illustrates just how strong the constant EIS assumption is. The constant EIS assumption is rejected by demand data. We also show analagous results can be obtained for more general HARA preferences.

Keywords: CRRA, elasticity of intertemporal substitution, demand systems

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1 Introduction

Optimizing models of the intertemporal allocation of consumption are the work-horses of modern macroeconomics and public finance. Almost always, such models assume the CRRA form for within period utility (also called the power or isoelastic form). The reason is that, in combination with additivity over time, this gives homothetic (intertemporal) preferences and this homotheticity is of considerable analytic convenience (for example, it allows for the analysis of steady states in growth models). These assumptions imply that the elasticity of intertemporal substitution (EIS), and its inverse, fluctuation (risk) aversion, are constant: rich and poor agents are equally averse to proportional fluctuations in consumption. This may have significant implications, for example when evaluating the costs of business cycles or evaluating a policy change in a dynamic general equilibrium model with heterogeneous agents.

There is a large empirical literature that attempts to estimate the EIS, which is a key macroeconomic parameter. This turns out to be a difficult problem, because data on consumption growth is noisy, because we have limited variation in the intertemporal price (the interest rate) and because the relationship between the intertemporal price and consumption growth is mediated by uncertainty and liquidity constraints. With very few exceptions, this literature also assumes that the EIS is constant. The small number of papers that explore whether the EIS varies with the level of consumption (or wealth) seem to reject the constant EIS hypothesis (Blundell, Browning and Meghir, 1994, Atkeson and Ogaki, 1996, and Attanasio and Browning, 1996). However, these rejections tend not to be statistically strong, exactly for the reasons just listed.

The goal of this paper is to highlight a surprising limitation that the assumption of a constant EIS imposes on the nature of the allocation of expenditure across goods within a period. In particular, the CRRA form for preferences (over total expenditure in each period) that delivers a constant EIS requires that within period preferences must be from the PIGL/PIGLOG class. These preferences correspond to rank 2 demand systems.[1] This restriction on the shapes of Engel curves is contradicted by a substantial body of empirical evidence on demands. Further, if we make the standard assumptions of additive inter-temporal preferences and a constant EIS, and we allow that agents

[1] The rank of a demand system is the rank of the matrix of coefficients on income terms, or equivalently the dimension of the space spanned by Engel curves, Lewbel (1991).

consume both luxuries and necessities (so that within period preferences are non-homothetic) then the parameter of the CRRA utility function is pinned down completely by the shape of Engel curves and the EIS can be estimated without any variation in the intertemporal price. The fact that a price elasticity can be estimated without variation in the relevant price illustrates just how strong the constant EIS assumption is.

The connection between intra- and inter-temporal allocation developed in this paper provides for a much more powerful test of the constant EIS assumption (and one that, at least implicitly, has already been carried out.) It both supports the results of the papers that test the constant EIS assumption with consumption growth data (Blundell, Browning and Meghir, 1994, Atkeson and Ogaki, 1996, and Attanasio and Browning, 1996) and provides and an explanation for those results: the EIS cannot be constant because the within period budget allocations of rich and poor households differ in a complicated way.

Our main results are covered in the next section. Section 3 provides a very brief empirical illustration. Section 4 extends our analysis to the more general class of HARA preferences. Section 5 concludes.

2 The Intratemporal Implications of a Constant EIS

We consider an agent that has an additive inter-temporal utility function of the form:

$$\sum \beta^t u(x_t)$$

where $u(x_t)$ is the "felicity" function that captures the utility derived from per period "consumption", x_t. Of course, households consume many goods. Therefore, we interpret $u(x_t)$ as an indirect utility function derived over total expenditure within the period and within period prices. This interpretation follows from two-stage budgeting which holds because intertemporal preferences are additive. We should therefore write $u(x_t; p_t)$, where p_t is a vector of prices of different goods. This interpretation has been adopted by a number of papers that simultaneously examine inter- and intra-period allocation (Blundell, Browning, and Meghir, 1994, and Attanasio and Weber, 1995, among others).

An alternative interpretation would treat $u(x_t)$ as a direct utility function defined over the composite consumption good, x_t. This relies either on (Hicks) composite commodity arguments (which require constant relative prices) or on the assumption of within-period homotheticity. We do not consider either to be credible. The assumption of constant relative prices is contrary to everyday experience and is particularly difficult to defend for an open economy (movements in gasoline prices are a good counter example.) More formally, the fact that demand systems - including price responses - can be estimated on aggregated data (Deaton and Muellbauer, 1980a, is a classic example) is itself evidence of substantial variation in relative prices. The alternative assumption of within period homotheticity implies that there are neither luxuries nor necessities, which, as Deaton (1992) notes "contradicts both common sense and more than a hundred years of empirical research". It is perhaps worth noting that homotheticity over goods is rejected not just by micro data but also in aggregate data (see, for example, Deaton and Muellbauer, 1980a).

We define the elasticity of intertemporal substitution (EIS) as the derivative of log total expenditure with respect to the log of the intertemporal price (that is, the interest rate) holding within-period relative prices and the discounted marginal utility of expenditure constant.[2] It is well known in this intertemporally additive setup this quantity is the inverse of the coefficient of relative risk (or fluctuation) aversion:

$$EIS = -\frac{\partial \log x_t}{\partial r_t} = -\frac{u_x\left(x_t; p_t\right)}{u_{xx}\left(x_t; p_t\right) x_t} \tag{1}$$

The EIS is constant when this expression is independent of relative prices and of total expenditure.

Remark 1 *The indirect utility function defined over total expenditure within the period has a CRRA utility representation, so that the EIS is constant, if and only if within period preferences take one of the following two forms:*

$$u = a(p)\frac{x^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + b(p) \qquad \theta \neq 1 \tag{2}$$

$$u = a\left(p\right)\log\left(x\right) + b\left(p\right) \qquad \theta = 1$$

These are of the PIGL and PIGLOG class, Muellbauer (1975, 1976).[3]

[2] Both total expenditure and the interest rate are nominal. An alternative would be to relate expenditure deflated by a single price index to the real interest rate. As Gorman (1959) shows, deflation by a single price index requires within period homotheticity.

[3] Note also that PIGL/PIGLOG preferences are a subset of the Generalized Gorman Polar Form (Gorman, 1959):

Sufficiency follows directly from repeated differentiation of these utility functions with respect to total expenditure and substitution into equation (1). To see necessity, note that assuming that a constant EIS implies

$$-\frac{u_{xx}}{u_x} = \frac{k}{x}$$

where k is a constant term (independent of p and x). We can use this expression to solve for u_x by integrating to give

$$\log u_x = -k \log x + \log a$$

$$u_x = ax^{-k}$$

giving the general form for utility

$$u(x) = \frac{1}{1-k} a(p) x^{1-k} + b(p)$$

where we have allowed the constants of integration to depend on prices.

The point of this remark is to note that the indirect utility functions resulting from the assumption of CRRA utility correspond to well known demand systems.[4]

The immediate question that this remark raises is whether it is possible to take any monotonic transform of $u(\cdot)$ to generate a different (but still constant) EIS while leaving within period demands unchanged. Of course this means that the transformation must preserve the CRRA form while changing the parameter which determines the EIS. Proposition 1 states that, in general, this is not possible. An implication of this is that if the EIS is constant, its value is pinned down by the shapes of Engel curves.

Proposition 1 *Suppose that intertemporal preferences have the CRRA form, and within-period preferences are non-homothetic, so that the indirect utility function, $u(x;p)$ has the PIGL form, (2). There is no transformation of these preferences, $v = F(u(x;p))$, such that: (i) within-period demands are unchanged by $F(\cdot)$; (ii) v has the CRRA form (a constant EIS) and (iii) the (constant) power parameter (and hence EIS) in v is different from the (constant) power parameter (EIS) in u.*

$u(x) = F[x/\alpha(p)] + \beta(p)$ where F is a monotone, increasing function.

[4] PIGL/PIGLOG preferences have the convenient property of allowing exact nonlinear aggregation (over agents). See Muellbauer (1975, 1976) or Deaton and Muellbauer (1980a, 1980b).

Proof. The proof is by contradiction, and proceeds in three steps.

Suppose that such an $F(\cdot)$ does exist.

1. Demands are unchanged if and only if F is monotone in u and independent of prices: $F_u > 0$ and $F_p = 0$.

2. v has the power utility form and the power parameter (and hence EIS) is different from u if and only if:
$$\frac{F_{uu} u_x x}{F_u} = k \ , \qquad k \neq 0. \tag{3}$$
This follows from using the chain rule in differentiating v to give
$$-\frac{v_{xx} x}{v_x} = -\frac{u_{xx} x}{u_x} - \frac{F_{uu} u_x x}{F_u} = \frac{1}{\theta} - k$$
where θ is the power parameter (EIS) defined by the function u. Note that linear transformations of course preserve both the power utility functional form and the value of the elasticity of substitution ($k = 0$), so $F(\cdot)$ must be nonlinear ($F_{uu} \neq 0$).

3. Given $u = a(p) \frac{x^{1-1/\theta}}{1-1/\theta} + b(p)$, we know $u_x = a(p) x^{-\frac{1}{\theta}}$.Substituting this into equation (3) gives,
$$\frac{F_{uu}}{F_u} a(p) x^{1-\frac{1}{\theta}} = k \tag{4}$$
Note that $x = c(u,p)$ where $c(u,p)$ is the cost function corresponding to the within period indirect utility function $u(x,p)$,
$$c(u,p) = \left(\frac{u - b(p)}{a(p)} \left(1 - \frac{1}{\theta}\right) \right)^{\frac{\theta}{\theta-1}} .$$
Substituting back into equation (4), gives
$$\frac{F_{uu}}{F_u} (u - b(p)) \left(1 - \frac{1}{\theta}\right) = k$$
Defining Φ and Γ to be constants of integration, this differential equation has the general solution:
$$F(u,p) = \Phi \frac{1}{k \frac{1}{1-\frac{1}{\theta}} + 1} (u - b(p))^{k \frac{1}{1-\frac{1}{\theta}} + 1} + \Gamma$$
However, this $F(\cdot)$ is a function of p: $F_p \neq 0$, which is a contradiction (of 1).

■

To understand this proposition, consider taking a power transform of within period utility:

$$v = \frac{1}{1-\gamma}\left(a(p)\frac{x^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + b(p)\right)^{1-\gamma}$$

This gives the general expression for the EIS as

$$\frac{1}{EIS} = \gamma\left[\frac{a(p)\,x^{1-\frac{1}{\theta}}}{a(p)\,x^{1-\frac{1}{\theta}} + b(p)}\right] + \frac{1}{\theta}.$$

The EIS will in general depend on x and p. There are two special cases:

1. If $b(p) = 0$, the EIS does not depend on x or p and can vary without changing intratemporal demands. However this requires within period homotheticity (no luxuries or necessities).

2. If within period preferences are PIGLOG (so that all shares are linear in log expenditure), we can write

$$v = \frac{1}{1-\gamma}\left(Exp\left[a(p)\ln x + b(p)\right]\right)^{1-\gamma}$$

   In this case the EIS is given by

$$\frac{1}{EIS} = \gamma a(p) - a(p) + 1$$

   and is independent of x but dependent on p. This is the functional form used in Attanasio and Weber (1995).

If neither of these special cases holds, any nonlinear transformation results in an EIS that depends on x (and p). Thus if we want to have a constant EIS, we must have $\gamma = 0$ and a single curvature parameter, θ, controls both intertemporal and intratemporal allocation.

Corollary 1 *If intertemporal preferences are additive, the EIS is constant (the felicity function is CRRA), and within-period preferences are not homothetic, then the EIS is identified by the curvature of Engel curves.*

Proposition 1 demonstrates that if the EIS is constant, the representation of intra-temporal preferences given by (2) is not an arbitrary normalization. We can use Roy's identify to derive the budget share equation for good j from the utility function (2):

$$w_j = -\frac{a_j}{a(p)(1-\frac{1}{\theta})} - \frac{b_j}{a(p)} x^{\frac{1}{\theta}-1} \qquad \theta \neq 1 \tag{5}$$

$$w_j = -\frac{a_j}{a(p)} \log(x) - \frac{b_j}{a(p)} \qquad \theta = 1 \tag{6}$$

where a_j and b_j are the derivatives of $a(p)$ and $b(p)$ with respect to the price of good j, p_j.

As shown in equation (5), each member of the PIGL/PIGLOG family has a different curvature for Engel curves, determined by the power parameter θ. If the EIS is constant, and within period preferences are non-homothetic, then the same parameter θ is the EIS. One Engel curve is sufficient to identify the EIS. The fact that this restriction does not hold in the homothetic case makes intuitive sense: with homothetic preferences, $b(p) = 0$, Engel curves are linear (for any value of θ), and hence are not informative about the curvature of the indirect utility function.

There is a substantial recent literature (Ludvigson and Paxson, 2001, Carroll, 2001, Attanasio and Low, 2004, Browning and Alan, 2003) documenting the difficulties associated with estimating the EIS from data on consumption growth and the intertemporal price (Euler equation estimation). One response to these problems has been to move to structural estimation, as in Gourinchas and Parker (2002). Structural estimation brings its own difficulties, including that fact that the environment in which agents operate is completely - and correctly - specified. However, all of the above papers *assume* that intertemporal preferences have the additive-over-time, CRRA form. Thus it seems that this literature has been going to considerable effort to estimate a model parameter, which, under the maintained assumptions of the model, could be estimated much more simply and easily.

However, the crucial point is that PIGL/PIGLOG (and hence CRRA) implies testable restrictions on the shapes of Engel curves. It has not been common, in the literature, to test PIGL/PIGLOG preferences against more general alternatives.[5] On the other hand, PIGL/PIGLOG preferences are at most rank 2 (the homothetic form being rank 1) and the literature contains numerous tests of demand system rank. For example, Lewbel (1991), Banks, Blundell and Lewbel (1997) and Donald (1997) provide nonparametric tests of rank, and Lewbel (2004) describes a parametric approach using a rational rank four demand system that nests rational rank three polynomial demands. The

[5] More commonly, a particular member of this class - usually a parameterization of PIGLOG - is tested against a non-PIGL/PIGLOG parametric demand system which nests only that particular member of the class. See for example, Banks, Blundell and Lewbel (1997).

typical finding of this literature is that demands are at least rank 3, and possibly rank 4. In other words, there is a strong consensus that rank 2 demand systems (including PIGL/PIGLOG) provide an inadequate representation of intra-period allocation, which implies that the EIS can not be constant.

In terms of equation (5), a PIGL/PIGLOG specification of preferences can be tested in two ways: first, one can test whether the curvature of individual Engel curves is adequately captured by equation (5). That is, for each good, we can test equation (5) against more general specifications. Second, estimates of equation (5) for different goods should all give the same value of θ.

This approach to testing the CRRA assumption has considerable advantages over tests using consumption growth. Evidence from data on consumption growth is against a constant EIS (Blundell, Browning and Meghir, 1994, Atkeson and Ogaki, 1996, and Attanasio and Browning, 1996). However, these tests face the same difficulties as Euler equation estimation. They use limited variation in the intertemporal price and noisy consumption growth data to assess not just how consumption growth responds to the intertemporal price (as in the usual Euler equation estimation exercise) but also how that relationship varies with the level of consumption. It is perhaps not surprising then that the results of these tests are often suggestive but not strongly statistically significant (as in Blundell, Browning and Meghir, 1994). In contrast, tests of the PIGL specification based on the curvature of Engel curves are powerful simply because we have so much household level budget data and so much variation in total expenditure.[6]

Extensive searching uncovered two precedents for our results in little-known sources. First, Muellbauer (1987) considers estimation of the intertemporal parameters from information on demands. He shows that our remark follows from results in Gorman (1959), but does not state or prove proposition 1. Second, in a 1985 working paper that has just appeared as Browning (2005), Browning provides a proof of the equivalence of PIGL preferences and a constant EIS. Neither Browning nor Muellbauer consider the potential value of this relationship for testing the CRRA assumption, although in fairness to both authors they were writing before the microeconometric evidence we now have on the rank of demand systems became available. Further, they do not consider the extension to HARA preferences that we develop below.

[6] Another approach to testing CRRA is based on asset prices: see Ait-Sahalia and Lo (2000).

3 Empirical Illustration

As a brief illustration of the results of the previous section, we estimate Engel curves with micro data on expenditures from the 1997 and 1998 UK Family Expenditure Survey. To avoid (unobserved) within-period price variation, we focus on households in London and the South East (as in Banks, Blundell and Lewbel, 1997). We also focus on a restricted set of family types: couples with and without children.

As shown in equation (5), we can write the budget share equation for good j:

$$w_j = -\frac{a_j}{a(p)(1-\frac{1}{\theta})} - \frac{b_j}{a(p)}\, x^{\frac{1}{\theta}-1} \qquad \theta \neq 1 \tag{7}$$

If the assumption of power utility is valid, the curvature from θ should capture all the curvature in Engel curves and the estimate of the parameter θ should be the same for any Engel curve. Table 1 reports, for four different goods, the estimate of θ (the EIS), a confidence interval for that estimate, the implied coefficient of relative risk aversion, and test of equation (5). Equation (7) was estimated by nonlinear least squares. It is worth stressing that although we imposed that θ must be constant across households, we did not restrict the degree of heterogeneity entering via the slope or intercept coefficients. The specific parametric alternative that we tested against is the general HARA share equation developed in the next section.[7]

Table 1: Estimates and Tests of Constant EIS Engel Curves

Commodity	θ (EIS)	Het. Robust Confidence Interval	Implied Coef. RRA	Test of Functional Form (Het. Consistent T-test)
Food	1.72	[1.31,2.14]	0.58	-0.23
Clothing	1.43	[0.47,2.40,]	0.70	-1.31
Fuel and Light	2.69	[1.50,3.89]	0.37	-1.91
Leisure Services	0.56	[0.47,0.65]	1.80	-0.27

We begin, in the first row, with the Food Engel curve. The estimate of θ is 1.7, which is higher than EIS estimates obtained from linearized Euler equation estimation. For example, Attanasio and

[7] We considered a number of other parametric alternatives. These all gave similar results.

Weber (1995) estimate the EIS to be 0.67, while. Gourinchas and Parker (2002) report a range of (implied) EIS estimates of $\theta = 0.7$ to $\theta = 2.0$, based on their structural estimation strategy.

The 2nd, 3rd and 4th rows of Table 1 report similar estimates and tests for clothing, for fuel and light and for leisure services. For clothing we obtain an estimate of θ similar to that from the food Engel curve. The estimate based on the fuel and light Engel curve is a bit higher. Both of these estimates are less precise than the estimate based on the food Engel curve. For leisure services, we estimate a θ which is substantially lower, around 0.5, and very precisely estimated.

For none of the goods that we consider do we reject the functional form of equation (7) against a more flexible alternative (at a 5% level of statistical significance.) However, even an informal examination of the four estimates of θ and their confidence intervals reveals that the restriction that θ must be common across goods is strongly rejected by the data. Thus the data is incompatible with the constant EIS hypothesis. This illustrates that the power of this demand based test of the constant EIS hypothesis derives from this restriction that θ be common across goods.

4 Extension to HARA Preferences

HARA is the general class of utility functions which includes CRRA. It is defined by risk tolerance being an affine transform of total expenditure. Other members of the HARA class are quadratic, negative exponential, and translated power.[8] Within the HARA class, both increasing and decreasing relative risk (or fluctuation) aversion is possible. Many important results in finance rest on the assumption of HARA preferences, such as the two-fund separation theorems (See for example Eeckhoudt, Gollier and Schlesigner, 2005).[9]

Here we show the restrictions on within period utility implied by this more general specification of preferences. In particular, analogously to Remark 1 and Proposition 1, we show that assuming intertemporal preferences are additive with utility functions of the HARA class implies demand

[8] Translated power is simply CRRA defined over a translation of total expenditure:

$$u(x) = \frac{1}{\theta - 1} \ (\gamma + \theta x)^{1-\frac{1}{\theta}}$$

The translation γ is sometimes interpreted as subsistence consumption.

[9] Just as additivity plus the CRRA functional form gives homothetic intertemporal preferences, additivity plus HARA preferences gives quasi-homothetic preferences over time periods or states (Pollack, 1971). Inter-temporal expansion paths (and Engel curves for periods or states) are linear but not through the origin. This linearity is crucial to aggregation results (just as in the intra-temporal case).

functions which are of at most rank 3.

HARA implies

$$-\frac{u_x}{u_{xx}} = \gamma + \theta x.$$

Solving for u_x

$$u_x = A(p)(\gamma + \theta x)^{-\frac{1}{\theta}}$$

$$u = \frac{1}{\theta - 1} A(p)(\gamma + \theta x)^{1-\frac{1}{\theta}} + B(p)$$

Proposition 1 (above) holds for HARA demands, and hence its corollaries do too. To see this, define

$$Q = \frac{\gamma}{\theta} + x$$

Then the definition of HARA preferences and the corresponding indirect utility function can be written:

$$-\frac{u_x}{u_{xx}} = \theta Q.$$

$$u = \frac{1}{\theta - 1} A(p)(\theta Q)^{1-\frac{1}{\theta}} + B(p)$$

and the proof to proposition 1 given above can be applied directly (noting that $u_Q = u_x$ because $\frac{\partial Q}{\partial x} = 1$).

Thus, under the HARA functional form, the EIS is still identified by the curvature of Engel curves (as long as within period preferences are not homothetic), and the assumption of HARA preferences can be tested with budget data.

Using Roy's identity,

$$x_j = -\frac{A_{p_j}\left(\frac{1}{\theta-1}\right)(\gamma + \theta x)^{1-\frac{1}{\theta}} + B_{p_j}}{A(p)(\gamma + \theta x)^{-\frac{1}{\theta}}}$$

Thus share equations generated from an indirect utility function from the HARA class have the form:

$$w_j = \frac{x_j}{x} = \frac{\theta}{1-\theta}\frac{A_{p_j}}{A(p)} + \frac{1}{1-\theta}\gamma\frac{A_{p_j}}{A(p)}x^{-1} + \frac{B_{p_j}}{A(p)}\frac{(\gamma + \theta x)^{\frac{1}{\theta}}}{x} \tag{8}$$

Note that these demands are most rank 3.[10]

Relative to CRRA utility, this extra degree of rank means that there may be more scope to rationalize HARA intertemporal preferences with within period demand patterns. In practice, the two members of the HARA class which generate rank 3 demands are translated power and quadratic. The latter imply certainty equivalent behaviour (no precautionary saving motive). Thus, if we wish to assume HARA preferences, but rule out quadratic utility (because of the substantial empirical evidence of precautionary behaviour), then evidence of demands being (at least) rank 3 can only be accommodated by translated power utility. Translated power utility exhibits decreasing relative risk aversion, or equivalently an EIS that rises with wealth (the rich are less averse to proportional fluctuations in consumption and more inclined to move consumption across time to take advantage of the rate of return). Ogaki and Zhang (2001) show that the translated power utility function implies substantially different behaviour from CRRA. Complete risk sharing is rejected using CRRA but not when preferences are specified as translated power.[11]

Note also that the term in equation (8) that gives the extra degree of flexibility goes to zero as total within period expenditure grows. As total expenditure grows, translated power utility converges to CRRA, implying that the demands of the rich should be very close to rank 2. One way out of this might be to assume that the translation (γ) is determined by an external reference point (as in models of "external habits"). Such a model would imply (see equation (8)) that budget shares depend - in potentially testable ways - on both total expenditure and the excess of total expenditure over the reference point.

[10] Lewbel and Perraudin (1995) show that there is a correspondence between the rank of demands and the degree of fund separation: for example, two -fund separation is consistent with rank 2 demands. Note that this is a very different result from ours: Lewbel and Perraudin are refering to the rank of (general) preferences over states and treating total consumption in each state as a single commodity.

[11] In standard risksharing models, the consumption growth of an agent (household or group) is related to aggregate consumption growth by that agent's relative risk aversion. Thus, under full insurance with CRRA, the consumption growth of each agent is equal to aggregate consumption growth and so the cross-section variance of consumption is unaffected by aggregate shocks. By contrast, full insurance with HARA preferences allows for changes in the cross-sectional variance of consumption in response to an aggregate shock.

5 Discussion

This paper has shown that assumptions about the form of inter-temporal preferences impose restrictions on within period allocations. Our main result demonstrated that within period demands must be in the PIGL/PIGLOG class if they are to be consistent with a constant EIS (and hence consistent with CRRA utility). This means that the assumption of a constant EIS requires that within period demands be at most Rank 2, a restriction which is typically rejected by demand data. To emphasize the restrictiveness of the constant EIS assumption, we showed that when intertemporal preferences are additive, the EIS is constant, and within period preferences are non-homothetic, then the EIS can be estimated from a single Engel curve (without data on consumption growth or variation the inter-temporal price). The connection between intra- and intertemporal allocation is not limited to the CRRA case. For example, the broader class of HARA (intertemporal) preferences are consistent with a particular form of intratemporal demands which are at most rank 3.

More generally our results reflect the fact that all behavioral responses are governed by the curvature of the (indirect) utility function. Thus they are similar in spirit to Deaton (1974) who showed that additivity over goods implied a connection between (intratemporal) price and income elasticities, and to Deaton (1992) and Browning and Crossley (2000) who note that additivity over time *and* goods implies that the intertemporal substitution elasticities of particular goods are proportional to their income elasticities.

Our analysis shows that the assumption of the power (or CRRA) form for the felicity function is inconsistent with well documented features of the micro-data on intra-temporal allocation. Our analysis supports the results of Blundell, Browning and Meghir (1994) and Attanasio and Browning (1995) who find evidence that the EIS is not constant. Our paper provides a further explanation for those results: the EIS cannot be constant because the within period allocations of rich and poor households differ in a complicated way.

An obvious question is how wrong is the constant EIS assumption. In a statistical sense, our empirical results amount to a large rejection of the overidentifying restrictions implied by this functional form. It is more difficult to give an economic answer. The difficulty arises because with a general specification of the felicity function, we must use data on consumption growth and variation in the intertemporal price to characterize the EIS (and how the EIS varies with the level of consumption). As discussed above, this is a difficult task. We do know that relaxing the CRRA assumption can significantly change our answers to substantive questions. For example, Ogaki and Zhang (2001) show that it is important to allow for declining relative risk aversion when testing the full risk-sharing hypothesis.

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