Fractional Integration and Applied Macroeconomics

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Deciding Between $I(0)$ and $I(1)$ in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power

- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked
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Fractional Integration (FI) Approach to Time Series

- Standard $I(0)/I(1)$ Framework:

$$(1 - L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k$ is either $I(0)$ or $I(1)$, so $x_t$ is $I(0)$ or $I(1)$

- **Encompassing $I(d)$ Framework:**

$$(1 - L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does $x_t$
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Characterizing FI

\[(1 - L)^d x_t = \epsilon_t\]

\[\Rightarrow \left(1 - dL + \frac{d(d - 1)}{2!} L^2 - \frac{d(d - 1)(d - 2)}{3!} L^3 \ldots \right) x_t = \epsilon_t\]

Hyperbolic Decay of Autocorrelogram \(v/s\) Exponential Decay of Standard I(0) Processes \(\Rightarrow\) Long Memory

- \(d = 0 \Rightarrow I(0)\) Stationary. It includes white noise, ARMA...
- \(0 < d < 0.5 \Rightarrow I(d)\) Covariance Stationary (also mean reverting)
- \(1 > d \geq 0.5 \Rightarrow I(d)\) Non-Stationary, but mean-reverting
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 FI Literature

**Early Work:**
- Motivation: Robinson (1978)
- Derivation of Long-Memory Models: Granger (1980), Granger and Joyeaux (1980)

**Estimation:**
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Talk Summary

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results

2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
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3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications
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Identification of the Technology Shock

Big Question in Macro: Effect of the Technology Shock on Hours Worked at Business Cycle Frequencies

- Standard VAR with Long-Run Restrictions (Blanchard-Quah (1989))

\[
\begin{bmatrix}
\Delta x_t \\
\Delta i n_t
\end{bmatrix} =
\begin{bmatrix}
C^{11}(L) & C^{12}(L) \\
C^{21}(L) & C^{22}(L)
\end{bmatrix}
\begin{bmatrix}
\varepsilon^x_t \\
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\end{bmatrix}
\]

- \( x_t \) is productivity and \( n_t \) hours worked
- Christiano, Eichenbaum and Evans (2003) assume that \( i = 0 \), whereas Galí and Rabanal (2004) assume that \( i = 1 \). A major difference in responses emerges
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Multivariate FI VAR

Structural Form

\[ ADY_t = \nu_t \]
\[ \nu_t = G\nu_{t-1} + \varepsilon_t \]

\[ D = \begin{pmatrix}
(1 - L)^{d_1} & 0 \\
0 & (1 - L)^{d_2}
\end{pmatrix} \]

Estimable Reduced-Form

\[ DY_t = \zeta_t \]
\[ \zeta_t = F\zeta_{t-1} + \xi_t \]
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Multivariate FI VAR

Structural Form 12 parameters
Reduced-Form 9 parameters

⇒ Standard Identification Technique

- Normalize the variances of structural shocks to one
- Zero Long-run effect of Hours Shock on Productivity

The impulse responses in first differences are exactly the same as those implied by those in levels
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Empirical Analysis

- U.S. Quarterly Data (1948-2004): Hours Worked, Labor Productivity
- 2 Datasets and Sets of Estimates: CEV (Total Hours), GR (Non-farm Business Hours)
Hours: Order of Integration, Small-Sample

![Graphs showing GR and CEV trends](image)
FI Impulse Responses

GR

CEV

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Break-Date Tests

- Reveal the Turning Points of the Macroeconomy
- Associated ex-post to Historical Events
- Could Arise Through Either Changes in Shocks or Propagation
- Some Literature:
  - First Work: Chow (1960), Quandt (1960)
  - Multiple Breaks, Univariate Models: Kim, Madala (1991)
  - Multiple Breaks, Multivariate Models: Qu and Perron (2007)
Standard Approach

- In Multivariate $I(0)/I(1)$ Frameworks

\[ Y_t = \Phi Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim (0, \Sigma) \]

- Detects Breaks in $\Phi, \Sigma$, e.g. Bai, Lumsdaine and Stock (1998)
- Advantages:
  - Provides Distribution for the Break-Dates
  - Precision Increases with System Dimension
- Disadvantages:
  - Only 1 break (New paper tackle this: Qu and Perron (2007))
  - Orders of Integration Restricted to $I(0), I(1)$
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New FI Break-date Procedure: 2-Regime, n-variable Setup

\[ D^a Y_t = U_t, \quad t = 1, 2, \ldots, T_k - 1 \]
\[ D^b Y_t = U_t, \quad t = T_k, \ldots, T \]

\[ D^i = \begin{pmatrix}
(1 - L)^{d_1^i} & 0 & 0 & \ldots & 0 \\
0 & (1 - L)^{d_2^i} & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & (1 - L)^{d_n^i}
\end{pmatrix} \quad i = a, b \]

\[ U_t = \Phi^a U_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T_k - 1 \]
\[ U_t = \Phi^b U_{t-1} + \varepsilon_t, \quad t = T_k, \ldots, T \]
New FI Break-date Procedure:
2-Regime, n-variable Setup

- For a given Partition $T_b$ and a Grid of Values $(d_{10}^a, \ldots, d_{n0}^a$ and $d_{10}^b, \ldots, d_{n0}^b)$:
  \[
  \sum_{t=1}^{T_b-1} \sum_{j=1}^{n} [\varepsilon_{jt}]^2 + \sum_{t=T_b}^{T} \sum_{j=1}^{n} [\varepsilon_{jt}]^2
  \]

- Residual Square Minimization across Partitions:
  \[
  RSS(T_b) = \arg\min_{i, \ldots, l} RSS(T_b; d_{1i}^a, \ldots, d_{nl}^b)
  \]

- The Estimated Structural Break:
  \[
  \hat{T}_k = \arg\min_{s=1, \ldots, m} RSS(T_s)
  \]
Advantages

- Flexibility: Allows for breaks in $\Phi$, $\Sigma$ and $D$.

- Permits estimation of Two Propagation Channels:
  - Short-Run Dynamics ($\Phi$)
  - Medium and Long-Run (FI) Dynamics ($D$)

- Can Accommodate More than 2 Regimes.

- Can Include Deterministic Regressors.
Disadvantages

- It does not yield confidence intervals for the breaks

- But Monte-Carlo Study Reveals that the Procedure Performs Quite Well for large samples ("almost a 100%" probability of having the true break into the $T_k \pm 4$ closed interval)
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Empirical Analysis

- U.S. Data (1948-2005): CPI Inflation, Unemployment rate, 3-month T-Bill Rate
- 3 Frequencies: Monthly, Quarterly, Annual
- 3 Models: $I(0)$ (Sup-Wald), $I(d)$, $I(d) + I(0)$
- 2-Regime Framework
Summary, $I(d)$ Models Add...

- Structural Breaks around 1973
- Breaks around 1980,81
Reduced-Form Interpretation

Explain Structural Changes in terms of changes in Propagation ($d$, VAR coefficients ($\Phi$)) and Shocks (standard deviation of structural shocks ($\Sigma$), identified via Recursive scheme)


Structural Interpretation I: Oil Shocks

Plot Structural Breaks Against $O_t$

$$O_t = \max \begin{cases} 0 & \text{if } t = 1, 2, \ldots \text{ until a break occurs} \\ X_t & \text{otherwise} \end{cases}$$

where $X_t$ is the percentage point difference between the current oil price and the maximum price during the previous year.
Structural Interpretation I: Oil Shocks

MONTHLY

QUARTERLY

ANNUAL

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Popular View: Monetary Policy was lax during the 1970s and it only became stabilizing after 1980.

Test through rolling Monetary Policy rules following Taylor (1993)

\[ i_t = \rho i_{t-1} + (1 - \rho)(\bar{i} + \beta(\pi_t - \bar{\pi}) + \gamma u_t) \]

Rolling (increasing windows with 1 additional observation) estimations yield $\beta$-time-varying series. It captures the increase in the aggressive stance of the US Fed against inflation.
Structural Interpretation II: Monetary Policy
Structural Interpretation II: Monetary Policy

ANNUAL DATA


0 0.5 1 1.5 2 2.5 3 3.5

63 73 81

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Structural Interpretation II: Interest Rate Response to Supply Shock

- Monthly Interest Rate Response to Inflation
- Quarterly Interest Rate Response to Inflation
- Annual Interest Rate Response to Inflation

Pre−1980:7M
Post−1980:7M
Pre−1980:4Q
Post−1980:4Q
Pre−1981
Post−1981

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The **Term Premium** is the extra return required of a long-term bond, relative to a short-term bond.

- It tells us about current and future risks (inflation, fiscal, financial, business cycles...)
- Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

\[
i^n_t = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}
\]

- Variations in long-term rates can be explained by changes in expectations and/or term premium
Term Premium and Long-Term Rates

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Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + t\rho_{t,n}$$

Variations in long-term rates can be explained by changes in expectations and/or term premium
Measuring the Term Premium

- It critically depends on the assumption you make about the stochastic process (Cochrane, Piazzesi (2006)).

- Predictions of short-term rates critically depend on whether you assume that the short-term rate is $I(0)$ or $I(1)$.
Measuring the Term Premium

- It critically depends on the assumption you make about the stochastic process (Cochrane, Piazessi (2006)).

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$I(0) \text{ v/s } I(1)$ Term Premiums

Graph showing the comparison between $I(0)$ and $I(1)$ term premiums from 1971 to 2008.
“Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience, which suggests that, other things being equal, increasing short-term interest rates are normally accompanied by a rise in longer term yields... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.”

Alan Greenspan, February 2005
“What does the historically unusual behavior of long-term yields imply for the conduct of monetary policy? ... To the extent that the decline in long rates can be traced to a decline in the term premium ... the effect is financially stimulative and argues for greater monetary policy restraint... However, if the behavior of long-term yields reflects current or prospective economic conditions, the implications for policy may be quite different indeed, quite the opposite.”

Ben Bernanke, March 2006
Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results

2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results

3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications
Endogenously determine the Term Premium, Get Rid of Specification Errors.

Let the data determine the true order of integration of the short-rate process.

\[(1 - L)^d (i_t - \mu) = \epsilon_t\]

\[\epsilon_t = \rho \epsilon_{t-1} + \xi_t\]
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The interest rate is non-stationary, but mean-reverting
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Empirical Analysis

- U.S. Monthly Data (1961-2006) for the 1-year T-Bond rate and 10-year T-Bond rate
- Source: Gurkaynak, Sack and Swanson (2006) Dataset
Term Premium: $I(d)$ v/s $I(1)$
Term Premium: $I(d)$ vs $I(0)$
Long-Run Prediction Comparison: 1-year rate, 9 years out
Historical Long-Run Prediction

- 1973:6
- 1980:1
- 1991:6
- 2001:12
- 2003:12
- 2006:12

Graphs showing the historical long-run prediction for different years, with lines indicating different scenarios or models.
I(d) Analysis of Term Premiums across Specifications

Recall that Term Premium Identification is done based on the General Model:

\[ i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n} \]

- \( I(0) \) Model: \( \hat{d}(tp_{t,n}) = 0.60 \)
- \( I(1) \) Model: \( \hat{d}(tp_{t,n}) = 0.62 \)
- \( I(d) \) Model: \( \hat{d}(tp_{t,n}) = 0.44 \)

⇒ Only Under the \( I(d) \) model, the Term Premium is Stationary, unlike in the standard \( I(0)/I(1) \) models
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Future Work

- Macroeconomic Foundations for FI
- Fractional Co-Integration in Multivariate Systems
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