Fractional Integration and Applied Macroeconomics

Luis Gil-Alana[1] Antonio Moreno[2]

[1]Quantitative Methods Department
Universidad de Navarra

[2]Economics Department
Universidad de Navarra

September 19, 2007, Bank of Spain

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Deciding Between I(0) and I(1) in Applied Work

- Not always an Easy Choice
  - Box-Jenkins Methodology ... Not Definitive
  - Dickey-Fuller and Related Tests... Lack of Power
- But a very Important One
  - Very Different Implications for Estimation, Prediction, Analysis... Example: VAR v/s VECM
  - Controversial Variables: Inflation, Interest Rates, Hours Worked

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$$\Rightarrow k \text{ is either I(0) or I(1), so } x_t \text{ is I(0) or I(1)}$$

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$$\Rightarrow k \in \mathcal{R}, \text{ so does } x_t$$

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k$ is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow$ k is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1 - L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow$ k is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1 - L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow$ k is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow$ k is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Fractional Integration (FI) Approach to Time Series

- Standard I(0)/I(1) Framework:

$$(1-L)^k x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k$ is either I(0) or I(1), so x_t is I(0) or I(1)

- Encompassing $I(d)$ Framework:

$$(1-L)^d x_t = \epsilon_t, \quad \epsilon_t \sim I(0)$$

$\Rightarrow k \in \mathcal{R}$, so does x_t

Characterizing FI

$$(1-L)^d x_t = \epsilon_t$$

$$\Rightarrow \left(1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 \dots\right) x_t = \epsilon_t$$

Hyperbolic Decay of Autocorrelogram v/s Exponential Decay of Standard I(0) Processes $\Rightarrow$ Long Memory

- $d = 0 \Rightarrow$ I(0) Stationary. It includes white noise, ARMA...
- $0 < d < 0.5 \Rightarrow$ I(d) Covariance Stationary (also mean reverting)
- $1 > d \geq 0.5 \Rightarrow$ I(d) Non-Stationary, but mean-reverting
- $d \geq 1 \Rightarrow$ I(d) Non-Stationary, non-mean-reverting

Characterizing FI

$$(1-L)^d x_t = \epsilon_t$$

$$\Rightarrow \left(1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 \ldots\right) x_t = \epsilon_t$$

Hyperbolic Decay of Autocorrelogram v/s Exponential Decay of Standard I(0) Processes $\Rightarrow$ Long Memory

- $d = 0 \Rightarrow$ I(0) Stationary. It includes white noise, ARMA...
- $0 < d < 0.5 \Rightarrow$ I(d) Covariance Stationary (also mean reverting)
- $1 > d \geq 0.5 \Rightarrow$ I(d) Non-Stationary, but mean-reverting
- $d \geq 1 \Rightarrow$ I(d) Non-Stationary, non-mean-reverting

Characterizing FI

$$(1-L)^d x_t = \epsilon_t$$

$$\Rightarrow \left(1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 \ldots\right) x_t = \epsilon_t$$

Hyperbolic Decay of Autocorrelogram v/s Exponential Decay of Standard I(0) Processes $\Rightarrow$ Long Memory

- $d = 0 \Rightarrow$ I(0) Stationary. It includes white noise, ARMA...
- $0 < d < 0.5 \Rightarrow$ I(d) Covariance Stationary (also mean reverting)
- $1 > d \geq 0.5 \Rightarrow$ I(d) Non-Stationary, but mean-reverting
- $d \geq 1 \Rightarrow$ I(d) Non-Stationary, non-mean-reverting

FI Literature

- Early Work:
  - Motivation: Robinson (1978)
  - Derivation of Long-Memory Models: Granger (1980), Granger and Joyeaux (1980)
- Estimation:
  - Parametric: Sowell (1992), Robinson (1994)
  - Semi-Parametric: Robinson (1995), Phillips and Shimotsu (2005)
  - Multivariate: Gil-Alana (2003), Nielsen (2005)
  - Co-integration: Robinson-Hualde (2003)

FI Literature

- Early Work:
  - Motivation: Robinson (1978)
  - Derivation of Long-Memory Models: Granger (1980), Granger and Joyeaux (1980)
- Estimation:
  - Parametric: Sowell (1992), Robinson (1994)
  - Semi-Parametric: Robinson (1995), Phillips and Shimotsu (2005)
  - Multivariate: Gil-Alana (2003), Nielsen (2005)
  - Co-integration: Robinson-Hualde (2003)

Talk Summary

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Identification of the Technology Shock

- Big Question in Macro: Effect of the Technology Shock on Hours Worked at Business Cycle Frequencies
- Standard VAR with Long-Run Restrictions (Blanchard-Quah (1989))
- $$\begin{bmatrix} \triangle x_t \\ \triangle^i n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^n \end{bmatrix}$$

  x_t is productivity and n_t hours worked
- Christiano, Eichenbaum and Evans (2003) assume that $i = 0$, whereas Galí and Rabanal (2004) assume that $i = 1$. A major difference in responses emerges

Identification of the Technology Shock

- Big Question in Macro: Effect of the Technology Shock on Hours Worked at Business Cycle Frequencies
- Standard VAR with Long-Run Restrictions (Blanchard-Quah (1989))
- $$\begin{bmatrix} \triangle x_t \\ \triangle^i n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^n \end{bmatrix}$$
  x_t is productivity and n_t hours worked
- Christiano, Eichenbaum and Evans (2003) assume that $i = 0$, whereas Galí and Rabanal (2004) assume that $i = 1$. A major difference in responses emerges

Identification of the Technology Shock

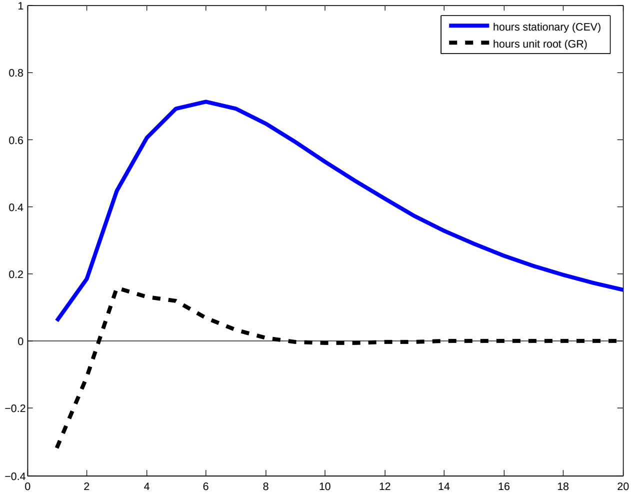
- Big Question in Macro: Effect of the Technology Shock on Hours Worked at Business Cycle Frequencies
- Standard VAR with Long-Run Restrictions (Blanchard-Quah (1989))
- $$\begin{bmatrix} \triangle x_t \\ \triangle^i n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^n \end{bmatrix}$$
  x_t is productivity and n_t hours worked
- Christiano, Eichenbaum and Evans (2003) assume that $i = 0$, whereas Galí and Rabanal (2004) assume that $i = 1$. A major difference in responses emerges

Identification of the Technology Shock

- Big Question in Macro: Effect of the Technology Shock on Hours Worked at Business Cycle Frequencies
- Standard VAR with Long-Run Restrictions (Blanchard-Quah (1989))
- $$\begin{bmatrix} \triangle x_t \\ \triangle^i n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^n \end{bmatrix}$$

  x_t is productivity and n_t hours worked
- Christiano, Eichenbaum and Evans (2003) assume that $i = 0$, whereas Galí and Rabanal (2004) assume that $i = 1$. A major difference in responses emerges

CEV (2003) v/s GR(2004)



Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Multivariate FI VAR

Structural Form

$$
\begin{aligned}
ADY_t &= \nu_t \\
\nu_t &= G\nu_{t-1} + \varepsilon_t
\end{aligned}
$$

$$
D = \begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix}
$$

Estimable Reduced-Form

$$
\begin{aligned}
DY_t &= \zeta_t \\
\zeta_t &= F\zeta_{t-1} + \xi_t
\end{aligned}
$$

Multivariate FI VAR

Structural Form

$$\begin{aligned} ADY_t &= \nu_t \\ \nu_t &= G\nu_{t-1} + \varepsilon_t \end{aligned}$$

$$D = \begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix}$$

Estimable Reduced-Form

$$\begin{aligned} DY_t &= \zeta_t \\ \zeta_t &= F\zeta_{t-1} + \xi_t \end{aligned}$$

Multivariate FI VAR

Structural Form

$$\begin{aligned} ADY_t &= \nu_t \\ \nu_t &= G\nu_{t-1} + \varepsilon_t \end{aligned}$$

$$D = \begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix}$$

Estimable Reduced-Form

$$\begin{aligned} DY_t &= \zeta_t \\ \zeta_t &= F\zeta_{t-1} + \xi_t \end{aligned}$$

Multivariate FI VAR

Structural Form

$$
\begin{aligned}
ADY_t &= \nu_t \\
\nu_t &= G\nu_{t-1} + \varepsilon_t
\end{aligned}
$$

$$
D = \begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix}
$$

Estimable Reduced-Form

$$
\begin{aligned}
DY_t &= \zeta_t \\
\zeta_t &= F\zeta_{t-1} + \xi_t
\end{aligned}
$$

Multivariate FI VAR

Structural Form 12 parameters
Reduced-Form 9 parameters

⇒ Standard Identification Technique

- Normalize the variances of structural shocks to one
- Zero Long-run effect of Hours Shock on Productivity

The impulse responses in first differences are exactly the same as those implied by those in levels

Multivariate FI VAR

Structural Form 12 parameters
Reduced-Form 9 parameters

⇒ Standard Identification Technique

- Normalize the variances of structural shocks to one
- Zero Long-run effect of Hours Shock on Productivity

The impulse responses in first differences are exactly the same as those implied by those in levels

Multivariate FI VAR

Structural Form 12 parameters
Reduced-Form 9 parameters

$\Rightarrow$ Standard Identification Technique

- Normalize the variances of structural shocks to one
- Zero Long-run effect of Hours Shock on Productivity

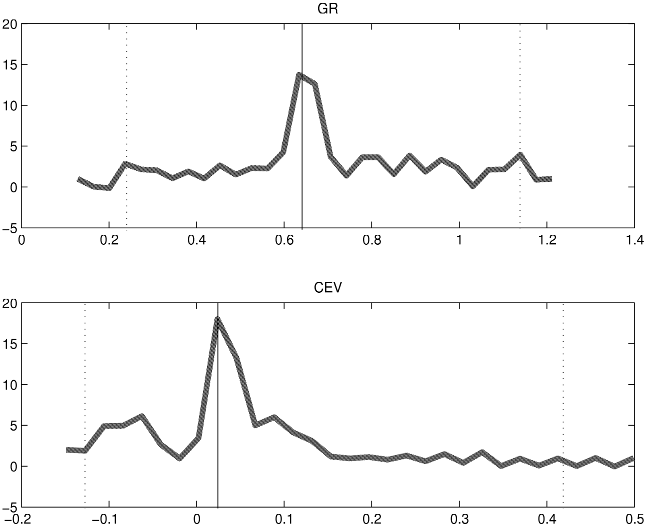
The impulse responses in first differences are exactly the same as those implied by those in levels

Talk Outline

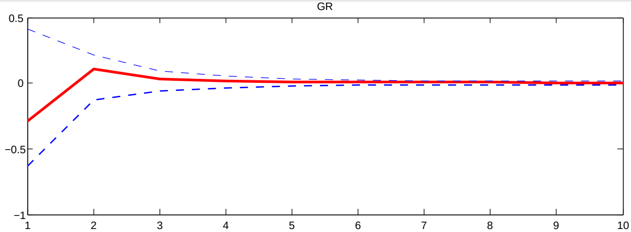
Empirical Analysis

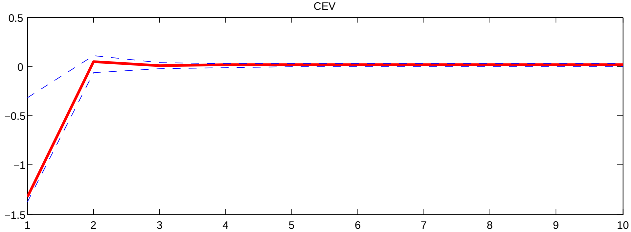
- U.S. Quarterly Data (1948-2004): Hours Worked, Labor Productivity
- 2 Datasets and Sets of Estimates: CEV (Total Hours), GR (Non-farm Business Hours)
- Estimation: Following Gil-Alana (2003) Multivariate Tests

Hours: Order of Integration, Small-Sample



FI Impulse Responses





Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Break-Date Tests

- Reveal the Turning Points of the Macroeconomy
- Associated ex-post to Historical Events
- Could Arise Through Either Changes in Shocks or Propagation
- Some Literature:
  - First Work: Chow (1960), Quandt (1960)
  - Multiple Breaks, Univariate Models: Kim, Madala (1991)
  - Single Break, Multivariate Models: Bai, Lumsdaine and Stock (1998)
  - Multiple Breaks, Multivariate Models: Qu and Perron (2007)

Standard Approach

- In Multivariate I(0)/I(1) Frameworks

$$Y_t = \Phi Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim (0, \Sigma)$$

- Detects Breaks in Φ, Σ, e.g. Bai, Lumsdaine and Stock (1998)
- Advantages:
  - Provides Distribution for the Break-Dates
  - Precision Increases with System Dimension
- Disadvantages:
  - Only 1 break (New paper tackle this: Qu and Perron (2007))
  - Orders of Integration Restricted to I(0), I(1)

Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

New FI Break-date Procedure: 2-Regime, n-variable Setup

$$
\begin{aligned}
D^a Y_t &= U_t, \quad t = 1, 2, \ldots, T_k - 1 \\
D^b Y_t &= U_t, \quad t = T_k, \ldots, T
\end{aligned}
$$

$$
D^i = \begin{pmatrix} (1-L)^{d_1^i} & 0 & 0 & \ldots & 0 \\ 0 & (1-L)^{d_2^i} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & (1-L)^{d_n^i} \end{pmatrix} \quad i = a, b
$$

$$
\begin{aligned}
U_t &= \Phi^a U_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T_k - 1 \\
U_t &= \Phi^b U_{t-1} + \varepsilon_t, \quad t = T_k, \ldots, T
\end{aligned}
$$

New FI Break-date Procedure: 2-Regime, n-variable Setup

- For a given Partition T_b and a Grid of Values $(d^a_{10}, \ldots, d^a_{n0}$ and $d^b_{10}, \ldots, d^b_{n0})$:

$$\sum_{t=1}^{T_b-1} \sum_{j=1}^{n} [\varepsilon_{jt}]^2 + \sum_{t=T_b}^{T} \sum_{j=1}^{n} [\varepsilon_{jt}]^2$$

- Residual Square Minimization across Partitions:

$$RSS(T_b) = \arg \min_{i,\ldots,l} RSS(T_b; d^a_{1i}, \ldots, d^b_{nl})$$

- The Estimated Structural Break:

$$\hat{T}_k = \arg \min_{s=1,\ldots,m} RSS(T_s)$$

Advantages

- Flexibility: Allows for breaks in Φ, Σ and D.
- Permits estimation of Two Propagation Channels:
  - Short-Run Dynamics (Φ)
  - Medium and Long-Run (FI) Dynamics (D)
- Can Accommodate More than 2 Regimes.
- Can Include Deterministic Regressors.

Disadvantages

- It does not yield confidence intervals for the breaks
- But Monte-Carlo Study Reveals that the Procedure Performs Quite Well for large samples ("almost a 100%" probability of having the true break into the $T_k \pm 4$ closed interval)

Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Empirical Analysis

- U.S. Data (1948-2005): CPI Inflation, Unemployment rate, 3-month T-Bill Rate
- 3 Frequencies: Monthly, Quarterly, Annual
- 3 Models: $I(0)$ (Sup-Wald), $I(d)$, $I(d) + I(0)$
- 2-Regime Framework

Summary, $I(d)$ Models Add...

- Structural Breaks around 1973
- Breaks around 1980,81

Reduced-Form Interpretation

Explain Structural Changes in terms of changes in Propagation (d, VAR coefficients (Φ)) and Shocks (standard deviation of structural shocks (Σ), identified via Recursive scheme)

- **Monthly:** Increase in d and short-term persistence around 1973, decrease of d after 1980. Shocks decline after 1980.
- **Quarterly:** Important drop in d after 1980. Big drop in supply shocks after 1980.
- **Annual:** Changes in VAR coefficients crucial to understand the 1973 and 1981 structural breaks. Shocks increase after 1973, drop after 1981.

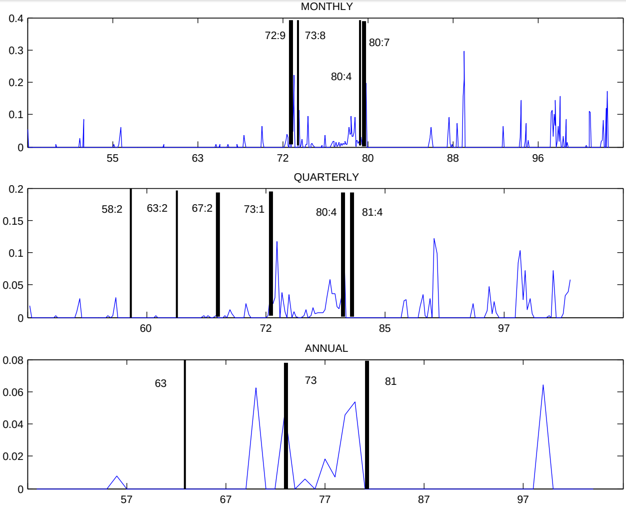
Structural Interpretation I: Oil Shocks

Plot Structural Breaks Against O_t

$$O_t = \max \begin{cases} 0 \\ X_t \end{cases} \quad t = 1, 2, \ldots$$

where X_t is the percentage point difference between the current oil price and the maximum price during the previous year.

Structural Interpretation I: Oil Shocks



Structural Interpretation II: Monetary Policy

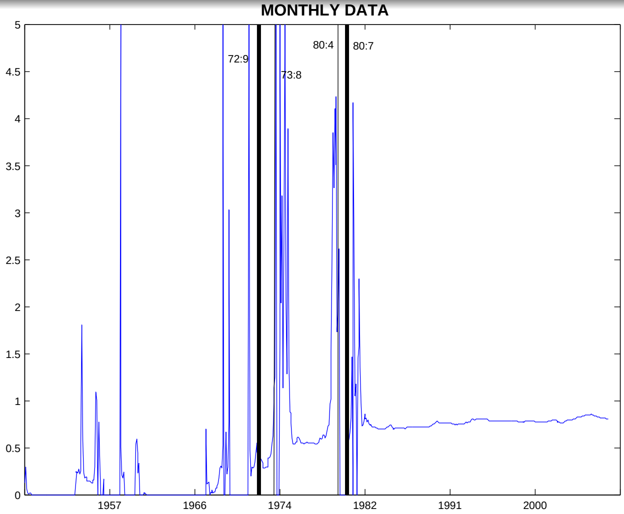
Popular View: Monetary Policy was lax during the 1970s and it only became stabilizing after 1980.

Test through rolling Monetary Policy rules following Taylor (1993)

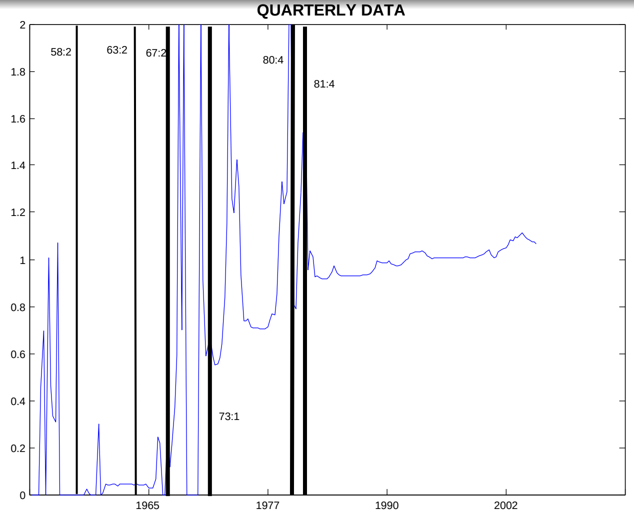
$$i_t = \rho i_{t-1} + (1-\rho)(\bar{\imath} + \beta(\pi_t - \bar{\pi}) + \gamma u_t)$$

Rolling (increasing windows with 1 additional observation) estimations yield β-time-varying series. It captures the increase in the aggressive stance of the US Fed against inflation

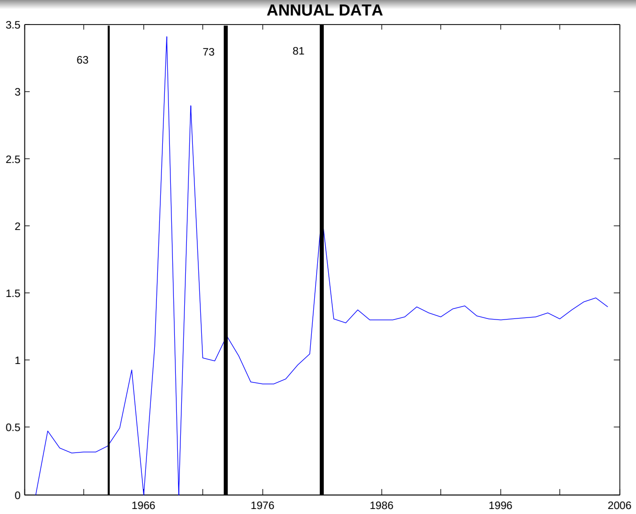
Structural Interpretation II: Monetary Policy



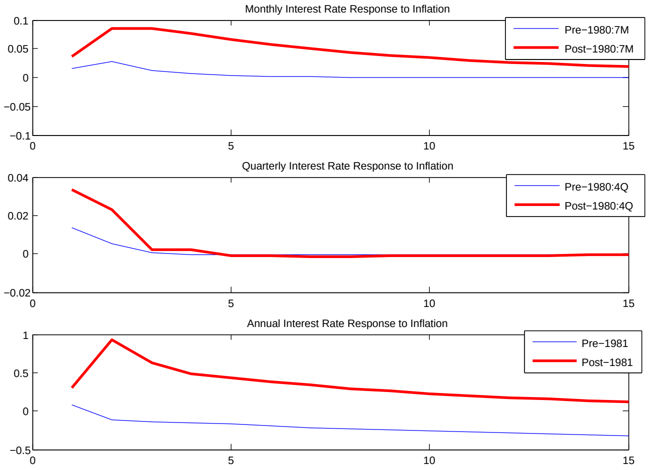
Structural Interpretation II: Monetary Policy



Structural Interpretation II: Monetary Policy



Structural Interpretation II: Interest Rate Response to Supply Shock



Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

Term Premium and Long-Term Rates

- The Term Premium is the extra return required of a long-term bond, relative to a short-term bond.
- It tells us about current and future risks (inflation, fiscal, financial, business cycles...)
- Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- Variations in long-term rates can be explained by changes in expectations and/or term premium

Term Premium and Long-Term Rates

- The Term Premium is the extra return required of a long-term bond, relative to a short-term bond.
- It tells us about current and future risks (inflation, fiscal, financial, business cycles...)
- Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- Variations in long-term rates can be explained by changes in expectations and/or term premium

Term Premium and Long-Term Rates

- The Term Premium is the extra return required of a long-term bond, relative to a short-term bond.
- It tells us about current and future risks (inflation, fiscal, financial, business cycles...)
- Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- Variations in long-term rates can be explained by changes in expectations and/or term premium

Term Premium and Long-Term Rates

- The Term Premium is the extra return required of a long-term bond, relative to a short-term bond.
- It tells us about current and future risks (inflation, fiscal, financial, business cycles...)
- Standard decomposition based on the Expectations Hypothesis + Time Varying Term Premium:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- Variations in long-term rates can be explained by changes in expectations and/or term premium

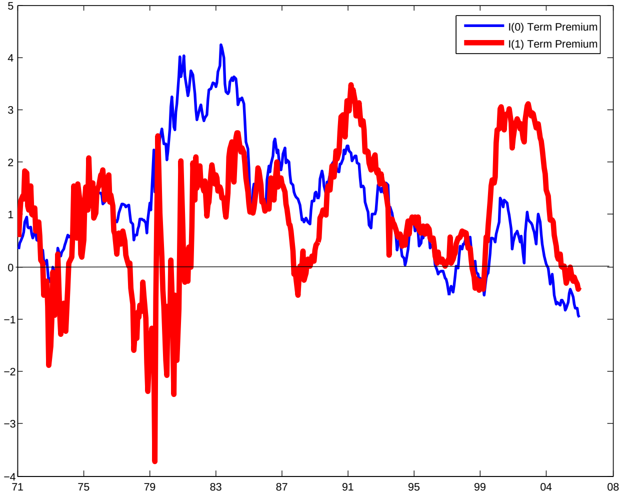
Measuring the Term Premium

- It critically depends on the assumption you make about the stochastic process (Cochrane, Piazessi (2006)).
- Predictions of short-term rates critically depend on whether you assume that the short-term rate is $I(0)$ or $I(1)$.

Measuring the Term Premium

- It critically depends on the assumption you make about the stochastic process (Cochrane, Piazessi (2006)).
- Predictions of short-term rates critically depend on whether you assume that the short-term rate is $I(0)$ or $I(1)$.

$I(0)$ v/s $I(1)$ Term Premiums



Greenspan's Conundrum

"Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience, which suggests that, other things being equal, increasing short-term interest rates are normally accompanied by a rise in longer term yields... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum."

Alan Greenspan, February 2005

Bernanke at the Crossroads

"What does the historically unusual behavior of long-term yields imply for the conduct of monetary policy? ... To the extent that the decline in long rates can be traced to a decline in the term premium ... the effect is financially stimulative and argues for greater monetary policy restraint... However, if the behavior of long-term yields reflects current or prospective economic conditions, the implications for policy may be quite different indeed, quite the opposite."

Ben Bernanke, March 2006

Talk Outline

1. Technology Shocks and Hours Worked
   - The Controversy
   - FI Approach, Advantages
   - Results
2. Structural Break Procedure in US Data, An FI Approach
   - Motivation
   - FI Approach
   - Results
3. U.S. Term Premium on Treasury Bonds
   - Specification of the Bond Term Premium
   - FI Approach
   - Implications

FI Term Premium

- Endogenously determine the Term Premium, Get Rid of Specification Errors.
- Let the data determine the true order of integration of the short-rate process.
- $$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

FI Term Premium

- Endogenously determine the Term Premium, Get Rid of Specification Errors.
- Let the data determine the true order of integration of the short-rate process.
-

$$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

FI Term Premium

- Endogenously determine the Term Premium, Get Rid of Specification Errors.
- Let the data determine the true order of integration of the short-rate process.
- $$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

Estimation of d

- $$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

- $$\begin{aligned} \hat{d} &= 0.84 \\ \hat{\rho} &= 0.48 \end{aligned}$$

- The interest rate is non-stationary, but mean-reverting

Estimation of d

- $$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

- $$\begin{aligned} \hat{d} &= 0.84 \\ \hat{\rho} &= 0.48 \end{aligned}$$

- The interest rate is non-stationary, but mean-reverting

Estimation of d

- $$\begin{aligned} (1-L)^d(i_t-\mu) &= \epsilon_t \\ \epsilon_t &= \rho\epsilon_{t-1}+\xi_t \end{aligned}$$

- $$\begin{aligned} \hat{d} &= 0.84 \\ \hat{\rho} &= 0.48 \end{aligned}$$

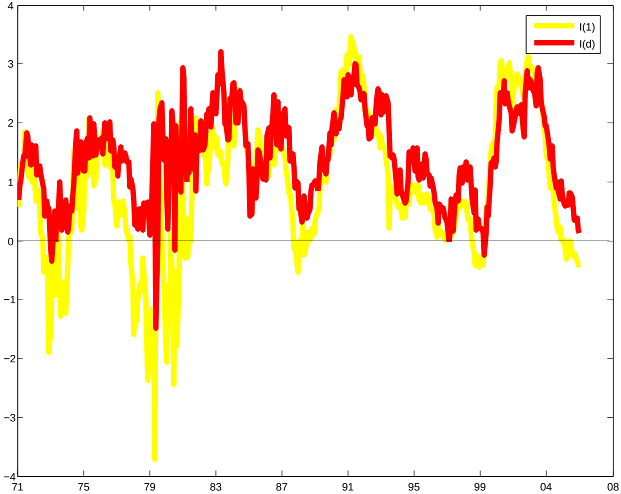
- The interest rate is non-stationary, but mean-reverting

Talk Outline

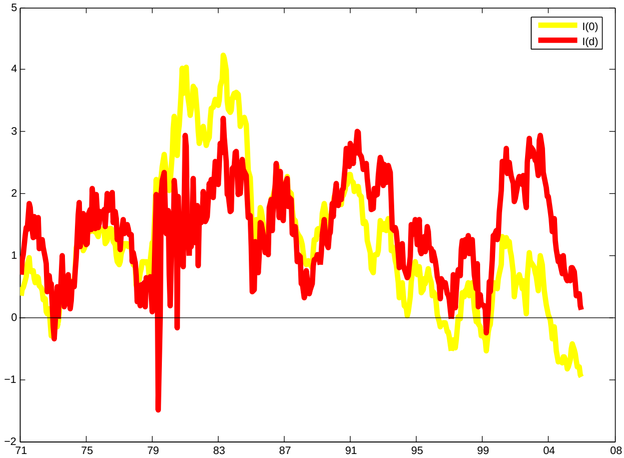
Empirical Analysis

- U.S. Monthly Data (1961-2006) for the 1-year T-Bond rate and 10-year T-Bond rate
- Source: Gurkaynak, Sack and Swanson (2006) Dataset
- Estimation: Following Robinson's (2004) Parametric Method

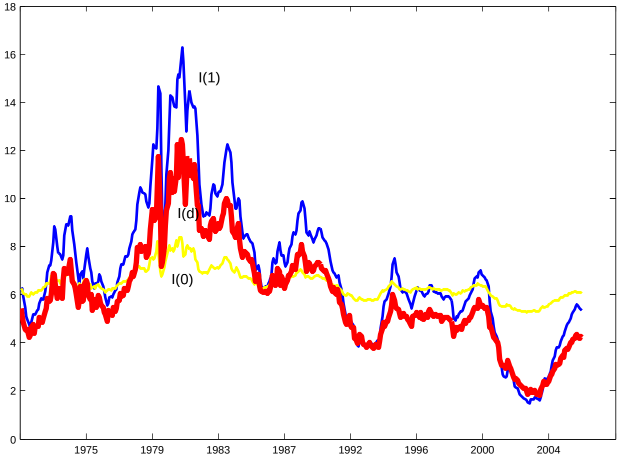
Term Premium: $I(d)$ v/s $I(1)$



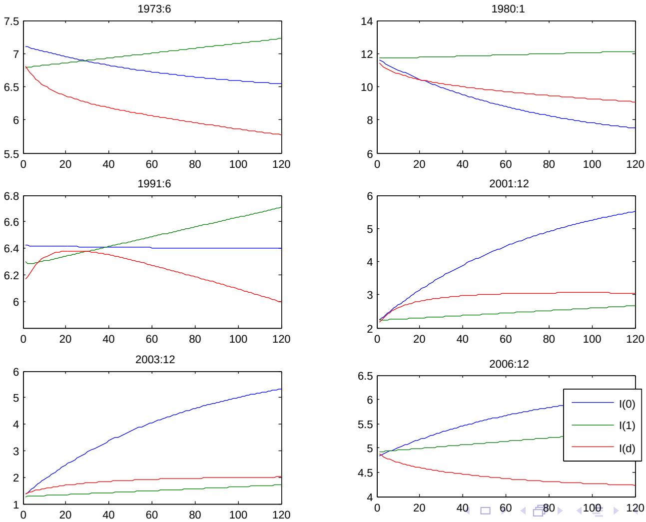
Term Premium: $I(d)$ v/s $I(0)$



Long-Run Prediction Comparison: 1-year rate, 9 years out



Historical Long-Run Prediction



I(d) Analysis of Term Premiums across Specifications

Recall that Term Premium Identification is done based on the General Model:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- $I(0)$ Model: $\hat{d}(tp_{t,n})$=0.60
- $I(1)$ Model: $\hat{d}(tp_{t,n})$=0.62
- $I(d)$ Model: $\hat{d}(tp_{t,n})$=0.44

$\Rightarrow$ Only Under the $I(d)$ model, the Term Premium is Stationary, unlike in the standard $I(0)/I(1)$ models

I(d) Analysis of Term Premiums across Specifications

Recall that Term Premium Identification is done based on the General Model:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- $I(0)$ Model: $\hat{d}(tp_{t,n})$=0.60
- $I(1)$ Model: $\hat{d}(tp_{t,n})$=0.62
- $I(d)$ Model: $\hat{d}(tp_{t,n})$=0.44

$\Rightarrow$ Only Under the $I(d)$ model, the Term Premium is Stationary, unlike in the standard $I(0)/I(1)$ models

I(d) Analysis of Term Premiums across Specifications

Recall that Term Premium Identification is done based on the General Model:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

- $I(0)$ Model: $\hat{d}(tp_{t,n})$=0.60
- $I(1)$ Model: $\hat{d}(tp_{t,n})$=0.62
- $I(d)$ Model: $\hat{d}(tp_{t,n})$=0.44

$\Rightarrow$ Only Under the $I(d)$ model, the Term Premium is Stationary, unlike in the standard $I(0)/I(1)$ models

Future Work

- Macroeconomic Foundations for FI
- Fractional Co-Integration in Multivariate Systems

Future Work

- Macroeconomic Foundations for FI
- Fractional Co-Integration in Multivariate Systems