Strategic Complementarities and Optimal Monetary Policy

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In this paper, we show that strategic complementarities–such as firm-specific factors or quasi-kinked demand–have crucial implications for the design of monetary policy and for the welfare costs of output and inflation variability. Recent research has mainly used log-linear approximations to analyze the role of these mechanisms in amplifying the real effects of monetary shocks. In contrast, our analysis explicitly considers the nonlinear properties of these mechanisms that are relevant for characterizing the deterministic steady state as well as the second-order approximation of social welfare in the stochastic economy. We demonstrate that firm-specific factors and quasi-kinked demand curves yield markedly different implications for the welfare costs of steady-state inflation and inflation volatility, and we show that these considerations have dramatic consequences in assessing the relative price distortions associated with the Great Inflation of 1965-1979.

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1 Introduction

The New Keynesian literature has emphasized the role of strategic complementarities–also referred to as "real rigidities"–in reducing the sensitivity of prices with respect to marginal cost and thereby amplifying the real effects of monetary disturbances.[1] Several forms of strategic complementarity–including firm-specific factors, intermediate inputs, and quasi-kinked demand–have observationally equivalent implications for the first-order dynamics of aggregate inflation. However, there has been relatively little analysis of the nonlinear characteristics of these mechanisms that may be relevant for determining the steady-state properties of the economy and for assessing the welfare costs of stochastic fluctuations.

In this paper, we show that the specific formulation of strategic complementarity has crucial implications for the design of monetary policy and for the welfare costs of output and inflation variability. In conducting this analysis, we formulate a dynamic general equilibrium model that incorporates both quasi-kinked demand and firm-specific factors. We follow Kimball (1995) in specifying a generalized aggregator function that allows for a non-constant elasticity of demand while nesting the Dixit-Stiglitz aggregator as a special case. In addition, our specification of the production function encompasses a general degree of firm-specificity of both capital and labor, that is, the proportion of variable vs. fixed inputs of each factor used by each individual firm. In calibrating the overall degree of real rigidity, we consider several distinct combinations of the structural parameters that yield the same slope of the New Keynesian Phillips Curve (NKPC) and then proceed to determine the extent to which these alternative calibrations influence the nonlinear properties of the model.

Our steady-state analysis shows that quasi-kinked demand and firm-specific inputs have markedly different implications for the costs of deterministic inflation and for the degree to which the optimal steady-state inflation rate under the Ramsey policy differs from that of the Friedman rule.[2] In doing so we derive a non-linear expression for the evolution

[1] Following Kimball (1995), many authors have analyzed the implications of strategic complementarities for equilibrium inflation dynamics, such as Woodford (2003, 2005), Altig, Christiano, Eichenbaum and Linde (2005), and Dotsey and King (2005a,b). Most of these mechanisms are reminiscent from the literature on nominal and real rigidities originated with the seminal work of Ball and Romer (1990) and surveyed by Blanchard (1990) and Blanchard and Fisher (1989).

[2] We follow Khan, King and Wolman (2003), and more recently Schmitt-Grohe and Uribe (2005 a,b),

of the relative price distortion and average markup under each source of strategic complementarities. The different nature of the strategic linkage among firm's incentive to changes price is at the core of the asymmetric results that we emphasize in this paper. Thus, if the source of real rigidity is coming from the presence of quasi-kinked demand, the effects of negative inflation tends to dramatically shrink the profits of non-adjusting firms by moving consumers demand away from its products to other. If, on the contrary, there is a fraction of fixed factors, then the higher the steady state inflation the higher are the output costs associated with the presence of price dispersion.

To characterize the welfare implications of real rigidities in the stochastic economy, we follow the linear-quadratic approach of Woodford (2003) in deriving the second-order approximation of conditional expected household welfare.[3] For any given combination of nominal and real rigidities, we find that the welfare costs of inflation variability are *an order of magnitude smaller* when the real rigidity arises from quasi-kinked demand rather than firm-specific factors. Thus, the characteristics of optimal monetary policy also depend crucially on the particular form of real rigidity.

The final stage of our analysis gauges the welfare costs of the Great Inflation by using the observed time series for U.S. inflation to construct the corresponding sequence of relative price distortions under alternative assumptions about the form of strategic complementarity. Given a moderate degree of nominal rigidity (namely, an average duration of 2-1/2 quarters between price changes), we determine the degree of factor specificity or quasi-kinked demand needed to match the estimated slope of the NKPC. In the case of quasi-kinked demand, the high and volatile inflation of 1965-79 only generates a modest degree of inefficiency arising from relative price dispersion. In contrast, the case of firm-specific factors yields dramatically higher welfare costs: in this case, the Great Inflation generates relative price distortions that reduce the level of aggregate output by 10 percent or more.

Levin and Lopez-Salido (2004), and Levin et al. (2005) in using Lagrangian methods to obtain the first-order conditions of the underlying Ramsey problem to compute optimal long-run policy under commitment in distorted economies.

[3] Woodford (2003, 2005) uses second-order approximations to characterize the welfare implications of firm-specific inputs but does not consider the case of quasi-kinked demand.

Before proceeding further it is useful to briefly examine the NKPC under the assumption of Calvo-style staggered price setting,

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \gamma\kappa_p mc_t, \tag{1}$$

where π_t is the inflation rate and mc_t is the logarithmic deviation of real marginal cost from its steady-state value. Notice that the slope of the NKPC is expressed as the product of two coefficients: κ_p reflects the degree of nominal rigidity, and γ reflects the degree of strategic complementarity in price-setting behavior. When the value of κ_p is calibrated using microeconomic evidence suggesting relatively frequent price adjustment, then a small value of γ (corresponding to a high degree of real rigidity) is needed to account for the low estimated slope of the NKPC. In a nutshell, our analysis indicates that alternative forms of strategic complementarity may yield the same value of γ but have markedly different implications for monetary policy and welfare.[4]

The remainder of this paper is organized as follows. Sections 2 and 3 describe our specifications for quasi-kinked demand and firm-specific inputs, respectively, elaborating on the nonlinear characteristics as well as the implications for the degree of real rigidity in price-setting behavior. In Section 4 describes how do we calibrate the degree of real rigidities. Section 5 evaluates the costs of steady-state inflation associated with these forms of strategic complementarity. Section 6 uses linear-quadratic methods to characterize the social welfare function and the properties of optimal monetary policy in the stochastic economy. Section 7 considers the extent to which these strategic complementarities have markedly different implications regarding the costs of the Great Inflation. Section 8 concludes. Finally, in the appendix A we present the details on how to calibrate the curvature of the demand curve and we relate it with the preceding literature; in Appendix B we present the key derivations of the paper.

[4]Some papers have emphasized how to damp fluctuations in marginal costs through 'elastic supply' mechanisms. Among those are the possibility that the firms can adjust its capacity utilization, the existence of an elastic labor supply (Dotsey and King (2005b)). Alternatively, allowing for sticky price and sticky nominal wages also tend to generate persistent responses in real marginal costs in response to nominal shocks (see, e.g., Christiano, Eichenbaum and Evans (2005)). We do not consider these mechanisms in this paper.

2 Quasi-kinked Demand

In this section, we describe an economy where the production of final goods requires a continuum of differentiated goods, indexed by a unit interval, and a single monopolistic competitor produces each type of these differentiated goods. In order to generate strategic complementarities in price-setting, we begin with a Kimball-type of household preference for differentiated goods and then move onto a production function emphasizing the role played by firm-specific fixed-capital.[5]

2.1 Demand structure

The representative household seeks to maximize $E_0 \sum_{t=0}^{\infty} \beta^t \; U_t$, where $\beta \in (0,1)$ is the discount factor. The household's utility in period t has the form

$$U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} \; - \; \chi_0 \frac{N_t^{1+\chi}}{1+\chi} \; + \; \nu_0 \frac{(\frac{M_t}{P_t})^{1-\nu}}{1-\nu} \tag{2}$$

where C_t is an aggregator of the quantities of the different goods consumed by households that it will be defined later, N_t denotes hours worked, and $\frac{M_t}{P_t}$ denotes its real balances, and the parameter $\sigma > 0$ captures risk aversion attitudes; $\chi_0 > 0$, and $\chi \geq 0$ is the inverse of the Frisch labor supply elasticity; and finally, $\nu_0 \geq 0$, and $\nu > 0$ is related to the semielasticity of real balances to (gross) nominal interest rates. Later on will become clear why do we allow for money balances to directly influence household utility.[6]

We assume that the economy is populated by a continuum of monopolistically competitive firms producing differentiated intermediate goods. These goods are then used as inputs by a (perfectly competitive) firm producing a *single final (consumption) good*. Following Kimball (1995) we assume that each firm faces an endogenous demand elasticity that dampens its incentive to raise its price in response to an increase in its marginal cost of production.

Formally, the final good is produced by a representative, perfectly competitive, firm with the following general technology $\int_0^1 G(\widetilde{Y}_t(j)) \; dj = 1$, where $\widetilde{Y}_t(j) = \frac{Y_t(j)}{Y_t}$, and $Y_t(j)$ is

[5] In the Appendix we also describe how the model works once we allow for the existence of intermediate (materials) inputs in the production of differentiated goods.

[6] The cashless economy corresponds to the limiting case in which ν_0 becomes arbitrarily small. In the Appendix we describe the fairly standard first order conditions associated to this problem.

Figure 1: Quasi-Kinked Demand

the quantity of intermediate good j used as an input. The function G satisfies that $G' > 0$, $G'' < 0$, and $G(1) = 1$. The final good firm chooses input demands $Y_t(j)$ to maximize profits, subject to the previous technological constraint. [7]

While these general assumptions are sufficient for obtaining a first order approximation, our analysis requires a specific choice of functional form for the aggregator, G. Thus, following Dotsey and King (2004), we consider the following aggregator:

$$G(\widetilde{Y}) = \frac{\phi}{1+\psi}\left[(1+\psi)\widetilde{Y} - \psi\right]^{\frac{1}{\phi}} - \left[\frac{\phi}{1+\psi} - 1\right] \tag{3}$$

where the composite parameter $\phi = (\epsilon(1+\psi))/(\epsilon(1+\psi)-1)$, and the elasticity parameter $\epsilon > 1$.

The parameter ψ determines the degree of curvature of the firm's demand curve. When $\psi = 0$, the demand curve exhibits constant elasticity, as in the Dixit-Stiglitz formulation. When $\psi < 0$, each firm faces a quasi-kinked demand curve; in effect, consumers have a satiation level of demand for each good, so that a drop in its relative price only stimulates a small increase in demand, while a rise in its relative price generates a large drop in demand. In this paper, we will focus on non-negative values of this parameter; however, it is interesting to note that when $\psi > 0$, consumers have a subsistence level of demand for each good, implying that pricing decisions exhibit *strategic substitutability.*

Given expression (3) the solution of the firm problem yields the set of demand schedules given by

$$\widetilde{Y}_t(j) = \frac{1}{1+\psi}\left[\widetilde{P}_t(j)^{-\epsilon(1+\psi)}\lambda_t^{\epsilon(1+\psi)} + \psi\right] \tag{4}$$

where $\widetilde{P}_t(j) = \frac{P_t(j)}{P_t}$, P_t is the aggregate price level and $P_t(j)$ corresponds to the intermediate goods price; and the Lagrange multiplier $\lambda_t = \left(\int_0^1 \widetilde{P}_t(j)^{1-\epsilon(1+\psi)}\, dj\right)^{\frac{1}{1-\epsilon(1+\psi)}}$. After imposing a zero profit condition, then the aggregate price index can be written as follows:

[7] See for details Woodford (2003), Dotsey and King (2005a), Eichenbaum and Fisher (2004), and Klenow and Willis (2005).

$1 = \frac{1}{1+\psi}\left(\int_0^1 \widetilde{P}_t(j)^{1-\epsilon(1+\psi)}\, dj\right)^{\frac{1}{1-\epsilon(1+\psi)}} + \frac{\psi}{1+\psi}\int_0^1 \widetilde{P}_t(j)\, dj$. Finally, the market-clearing condition implies $Y_t = C_t$.

In Figure 1 we plot the *log of relative demand* for alternative values of ψ; for this purpose, we calibrate the demand elasticity parameter $\epsilon = 7$, which yields a markup of 16 percent in the steady state with zero inflation.[8] Of course, the demand curve is log-linear when $\psi = 0$, corresponding to the Dixit-Stiglitz formulation. The value of $\psi = 2$ falls in the lower end of the range considered by Eichenbaum and Fisher (2004); in this case, the demand curve exhibits quite strong curvature. Finally, $\psi = -8$ implies a very high degree of curvature approaching that of a truly kinked demand curve. Thus, the presence of quasi-kinked demand implies that a drop in the firms relative price only stimulates a small increase in demand, while a rise in its relative price generates a large drop in demand. That is, consumers will costesly move away from relative expensive goods but do not run into inexpensive ones.

The production function for a typical intermediate goods firm j is given by:

$$Y_t(j) = A_t\ K_t(j)^{\alpha}\ N_t(j)^{1-\alpha} \tag{5}$$

where A_t represents an exogenous total factor productivity shifter, $K_t(j)$ and $N_t(j)$ represent the capital and labor services hired by firm j, and the parameter α represents the short run elasticity of output to capital. In this section, we implicitly assume that both inputs can be perfectly reallocated across firms, so the model corresponds to the one considered by Erceg, Henderson and Levin (2000) and Christiano, Eichenbaum and Evans (2005). In the next section we extend further the model in such a way that the capital stock is fixed at the firm level.

We now turn to the comparison of the demand curve specified in (4) with the standard Dixit-Stiglitz type of preferences. The prototypical demand curves under the Dixit-Stiglitz type can be derived by setting $\psi = 0$, which in turn implies that the multiplier $\lambda_t = 1$ in equation (4), so that the elasticity of demand is constant across firms, and it is determined by the elasticity of substitution among differentiated goods. Under the quasi-kinked de-

[8] As noted in the Appendix, the calibration of the curvature of demand depends crucially on the assumption about the steady state markup.

mand expression (4), the demand elasticities of differentiated goods vary with their relative demands. Formally, it can be easily shown that the elasticity of demand for good j, denoted by $\eta(\widetilde{Y}_j)$, can be written as follows:

$$\eta(\widetilde{Y}_j) = \epsilon \, \left(1 + \psi - \psi \widetilde{Y}_j^{-1}\right). \tag{6}$$

In the absence of a production subsidy, the desired markup of individual firms is given as $\mu(\widetilde{Y}_j) \equiv \frac{\eta(\widetilde{Y}_j)}{\eta(\widetilde{Y}_j)-1}$, and depends on the firm's relative demand, $\widetilde{Y}_j$. In general, in a non-zero steady state inflation, and under $\psi < 0$, the elasticity $\eta(\widetilde{Y})$ is decreasing in the relative demand. Hence, a increase in nominal demand that increases marginal costs will tend to reduce firm's desired markup so reducing the incentives to increase prices in response to the changes in demand.

Notice that for $\psi = 0$, the previous expression corresponds to the standard Dixit-Stiglitz demand function, $\mu(1) = \mu = \frac{\epsilon}{\epsilon - 1}$, where the desired markup is constant and a function of the parameter ϵ. Notwithstanding, once we departure from the constant elasticity of demand, by calibrating the steady state markup is not enough to pin down the degree of curvature of the demand function, ψ.[9] We will turn to this issue in section 2.4.

2.2 The Firm's Price-Setting Decision

Because of the presence of market power, intermediate firms are assumed to set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1 - \xi$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1 - \xi$ of producers reset their prices, while a fraction ξ keep their prices unchanged.[10]

Given the assumption of perfect factor mobility across firms, all firms have the same real marginal cost, which is given by the ratio of the real wage to the marginal product of

[9] The reason is that the calibration of μ only involves second order derivatives of G, while higher order derivatives will be necessary to understand the implications of the curvature of demand for price adjustment. See below for details.

[10] We do not assume an indexation clause for those firms that can not reoptimize its price. This is in line with recent with micro evidence for the U.S. (Bils and Klenow, 2004) and various European countries (Alvarez et al. (2006)).

Table 1: Quasi-Kinked Demand and Price-Setting Behavior

$\widetilde{P}_t^* = \left(\frac{\phi}{1+\tau_p}\right) \frac{Z_{2t}}{Z_{1t}} + \left(\frac{\psi\phi}{\epsilon(1+\psi)}\right) \frac{Z_{3t}}{Z_{1t}} \left(\widetilde{P}_t^*\right)^{1+\epsilon(1+\psi)}$
$Z_{1t} = E_t\{\ \beta\xi\Pi_{t+1}^{\epsilon(1+\psi)-1}\ Z_{1t+1}\ \} + Y_t^{1-\sigma}\lambda_t^{\epsilon(1+\psi)}$
$Z_{2t} = E_t\{\ \beta\xi\Pi_{t+1}^{\epsilon(1+\psi)}\ Z_{2t+1}\ \} + Y_t^{1-\sigma}\lambda_t^{\epsilon(1+\psi)} MC_t$
$Z_{3t} = E_t\{\ \beta\xi\Pi_{t+1}^{-1}\ Z_{3t+1}\ \} + Y_t^{1-\sigma}$

labor, i.e. $MC_t = w_t N_t/(1-\alpha)Y_t$. In the next section we analyze the effects of relaxing this assumption.

Table 1 indicates the first-order conditions for each firm that resets its price contract in a given period t. The optimal price is denoted by P_t^*, and the relative price $\widetilde{P}_t^* = \frac{P_t^*}{P_t}$. The firm's optimal price depends on the aggregate gross inflation rate $\Pi_t = P_t/P_{t-1}$, aggregate real marginal cost MC_t, and aggregate demand Y_t. The stochastic variables Z_{1t}, Z_{2t}, and Z_{3t} are described by recursive expressions in the table. Note that the term $(1+\tau_p)$ represents a production tax when $\tau_p > 0$ or a subsidy when $\tau_p < 0$.

2.3 Relative Price Distortions

The labor inputs of individual firms are linearly aggregated to obtain a measure of the aggregate labor, i.e. $N_t = \int_0^1 N_t(j)dj$. Substituting individual gross production function into the definition of the aggregate labor, we have a production relation between the aggregate output and labor:[11]

$$Y_t = (\frac{A_t}{\Delta_t}) K_t^{\alpha} N_t^{1-\alpha}$$

where the measure of relative price distortion, denoted by Δ_t, can be written as follows:

$$\Delta_t = \frac{1}{1+\psi} \int_0^1 (\lambda_t^{\epsilon(1+\psi)} \widetilde{P}_t(j)^{-\epsilon(1+\psi)} + \psi) dj. \tag{7}$$

[11] The aggregate variables are the sum of homogeneous capital and labor, i.e. $K_t = \int_0^1 K_t(j)dj$, $N_t = \int_0^1 N_t(j)dj$. Goods market equilibrium requires that $Y_t(j) = C_t(j)$, for all $j \in [0,1]$, and $Y_t = \int_0^1 Y_t(j)dj$.

Table 2: Quasi-Kinked Demand and Relative Price Distortions

$\Delta_t \equiv \frac{1}{1+\psi}\Delta_{2,t}^{\frac{\epsilon(1+\psi)}{1-\epsilon(1+\psi)}}\Delta_{1,t} + \frac{\psi}{1+\psi}$
$1 = \frac{1}{1+\psi}\Delta_{2,t}^{\frac{1}{1-\epsilon(1+\psi)}} + \frac{\psi}{1+\psi}\Delta_{3,t}$
$\Delta_{1,t} = (1-\xi)(\widetilde{P}_t^*)^{-\epsilon(1+\psi)} + \xi \Pi_t{}^{\epsilon(1+\psi)}\Delta_{1,t-1}$
$\Delta_{2,t} = (1-\xi)(\widetilde{P}_t^*)^{1-\epsilon(1+\psi)} + \xi \Pi_t{}^{\epsilon(1+\psi)-1}\Delta_{2,t-1}$
$\Delta_{3,t} = (1-\xi)\widetilde{P}_t^* + \xi \Pi_t{}^{-1}\Delta_{3,t-1}$

As emphasized by Goodfriend and King (1997), the relative price distortion results in a missallocation of aggregate output across alternative uses of goods, so that it appears as a technological shifter that reduces aggregate output.

In Table 2 we describe the main components of the previous definition of price dispersion. In particular, as described in the first row of the table, the previous expression for Δ_t can be written as a non-linear function of two different weighted average measures of price dispersions, $\Delta_{1,t} \equiv \int_0^1 \widetilde{P}_t(j)^{-\epsilon(1+\psi)}\, dj$ and $\Delta_{2,t} \equiv \int_0^1 \widetilde{P}_t(j)^{1-\epsilon(1+\psi)}\, dj$. Notice also that the Lagrange multiplier λ_t can be expressed as a function of the $\Delta_{2,t}$ measure of dispersion as follows: $\lambda_t = \Delta_{2,t}^{\frac{1}{1-\epsilon(1+\psi)}}$. The second row of the table corresponds to the relationship between two measures of dispersion that comes from the definition of aggregate prices, where we introduce a new measure of price dispersion, $\Delta_{3,t} \equiv \int_0^1 \widetilde{P}_t(j)\, dj$. Finally, the last three rows show that, following Yun (1996), the Calvo-type of staggered price setting allows us to write the three measures of relative price distortion in a recursive form.

Notice that the case of Dixit-Stiglitz constant elasticity of demand corresponds to $\psi = 0$, Hence, $\Delta_{2,t} = \lambda_t = 1$, $\Delta_t = \Delta_{1,t}$, and expression (7) corresponds to the standard equation linking aggregate inflation and the relative price of the newly set prices (see e.g., Schmitt-Grohé and Uribe (2005a, b)).

2.4 Implications for Real Rigidities

As shown in the Appendix a log linear approximation to the price equation of this model corresponds to (1). In particular, the frequency of price adjustment ξ and the exogenous discount factor β determine the degree of nominal rigidity:

$$\kappa_p = \frac{(1-\xi)(1-\xi\beta)}{\xi}$$

Furthermore, the degree of real rigidity γ can be expressed as follows:

$$\gamma = \frac{1}{1-\mu\psi}$$

where μ is the steady-state markup at zero inflation. Notice that $\psi < 0$ implies that the parameter $\gamma < 1$, and the magnitude of γ declines with the absolute value of ψ.

3 Firm-Specific Marginal Costs

We now consider the implications of assuming that each firm has a fixed allocation of capital rather than being able to obtain any desired amount on an aggregate rental market. In this case, the firm's real marginal cost (deflated by the aggregate price index) may differ from the average real marginal cost, and we denote the ratio as $\widetilde{MC}_t(j) = MC_t(j)/MC_t$. For ease of presentation this section assumes a Dixit-Stiglitz demand structure ($\psi = 0$), but we will subsequently consider the general model with both quasi-kinked demand and firm-specific capital, and the equations for the general case may be found in the Appendix.

3.1 The Determination of Marginal Costs

We extend the production function considered in the previous section so that, for any firm j, it can be written as follows[12]

$$Y_t(j) = A_t\ \overline{K}^{\alpha_{fk}}\ K_t(j)^{\alpha_{vk}}\ \overline{N}^{\alpha_{fl}}\ N_t(j)^{\alpha_{vl}} \tag{8}$$

where $\alpha_{fk} > 0$, $\alpha_{vk} > 0$, $\alpha_{fl} > 0$, $\alpha_{vl} > 0$, and $\alpha_{fk} + \alpha_{vk} + \alpha_{fl} + \alpha_{vl} = 1$. Notice that $\alpha_f = \alpha_{fl} + \alpha_{fk}$ represents the total fraction of input factors (capital stock and labor) that

[12] The case of intermediate inputs as an additional source of strategic complementarity is analyzed in the Appendix.

remains fixed at the firm level, $\overline{K}$ and $\overline{N}$. In particular, if $\alpha_{fl} = \alpha_{fk} = 0$, the production function (8) corresponds to the one considered in the previous section (expression (5)).

Absent the consideration of material inputs, and relative to the assumption of common factor markets, the presence of a fixed factor of production (capital) at the firm level will generate short run decreasing return in other factors (i.e. labor). This will imply that equilibrium wages will vary across firms, and so marginal costs. In the absence of perfect *reallocation* of factors across firms; the firm's marginal is increasing in its own output, where the elasticity of marginal cost to output (given the real wages) depend upon the existence of short run returns to scale in the variable factors. Formally, the deviations of firm's marginal costs from the (average) norm will become an increasing function of the deviation of the firm's output relative to the average, i.e.

$$\widetilde{MC}_t(j) = \widetilde{Y}_t(j)^{\frac{1-\alpha_f}{\alpha_f}} \tag{9}$$

with decreasing returns to labor, firms that maintain a high relative output will face a lower relative marginal cost than the average. Thus, the existence of fixed factors –because of the existence of local labor and capital markets– implies that price adjusters trying to undercut others to boots its own demand would also raise own's marginal costs. This is the nature of the real rigidity that induces the adjusters to have less incentive to price up.

3.2 The Firm's Price-Setting Decision

Limiting the possibility of reallocation of capital across firms change the representation of the optimal price contract of the firms that are allowed to changes its price at time t. In particular, the profit maximization condition (first row of Table 1) has the following form:

$$\left(\widetilde{P}_t^*\right)^{1+\epsilon\frac{\alpha_f}{1-\alpha_f}} = \left(\frac{\epsilon}{(1+\tau_p)(\epsilon-1)}\right)\frac{Z_{2t}}{Z_{1t}} \tag{10}$$

It is interesting to note that this expression differs to the one presented in the first row of Table 1 in two respects. First, the variables Z_{1t} and Z_{2t} corresponds to the ones of the previous model under $\psi = 0$. Second, the existence of firm specific factors matters for the left hand side of the previous expression. Hence, it is possible to explicitly solve for the variable $\widetilde{P}_t^*$ but it appears raised to the power $1+\frac{\epsilon\alpha_f}{1-\alpha_f}$, which absent the restriction of factor mobility, $\alpha_f = 0$, corresponds to the baseline model usually considered in the literature.

3.3 Relative Price Distortions

For convenience, we normalize firm-specific capital $\overline{K} = 1$, firm-specific labor $\overline{N} = 1$, and the aggregate stock of variable capital, $\int K_t(j)dj = 1$. Thus, the relation between aggregate output and the variable labor input can be expressed as follows:

$$Y_t = (\frac{A_t}{\Delta_t})\ N_t^{\alpha_{vl}}$$

where Δ_t is a new measure of time t relative price distortion which follows the following low of motion

$$\Delta_t^{\frac{1}{1-\alpha_f}} = (1-\xi)(\widetilde{P}_t^*(j))^{-\frac{\epsilon}{1-\alpha_f}} + \xi \Pi_t^{\frac{\epsilon}{1-\alpha_f}} \Delta_{t-1}^{\frac{1}{1-\alpha_f}} \tag{11}$$

Notice that the previous expression becomes the prototypical model considered in several papers under the assumption of $\alpha_f = 0$.

3.4 Implications for Real Rigidities

Under the assumption of fixed factors at the firm level, the first-order aggregate price dynamics continue to be described by the *NKPC* given in equation 1. The nominal rigidity coefficient κ_p is the same as defined in Section 2.4, but the real rigidity coefficient γ is now expressed as follows:

$$\gamma = \frac{1}{1+\epsilon\frac{\alpha_f}{1-\alpha_f}}.$$

It is worth noting that the higher the elasticity of demand, ϵ, and the higher is the elasticity of output to the fraction of fixed inputs, i.e. α_f, the lower is the pass-through coefficient from marginal costs to prices. In particular, under the assumption that $\alpha_f = \frac{1}{3}$, and $\epsilon = 7$, then $\gamma = 0.22$, so that the presence of a fixed factor implies that prices will respond around a 0.22 per cent to an 1 per cent increase in the marginal costs. While, assuming a higher elasticity, $\epsilon = 11$, and a lower elasticity to the fixed factor, say $\alpha_f = \frac{1}{2}$, the value of γ is reduced to 0.08.

4 Calibration

Table 3 describes the baseline parameter values that we use to calibrate the model. Much of these values closely follow those recently estimated (see, for instance, Levin et al. (2005)) and they are also in line with most of business cycle literature. We calibrate the model so that each period corresponds to a quarter, thus we set the discount factor $\beta = 0.99$. We allow for a moderate amount of nominal stickiness, i.e. the probability of changing prices and ξ is set equal to 0.6, which implies that prices are fixed slightly longer than two consecutive quarters (see e.g. Bils and Klenow (2004)).

When we allow for both quasi-kinked demand and firm-specific inputs, the slope coefficient of the *NKPC* takes the following form:

$$\gamma = \frac{1}{1 - \mu\psi + \epsilon\frac{\alpha_f}{1-\alpha_f}}$$

where $\psi < 0$.

In the baseline Calvo model studied in Woodford (2003) and many others, the parameter $\gamma = 1$, so that assuming $\xi = 0.6$ it implies a value for the slope of the *NKPC* of 0.27, which is higher than the estimates in the literature (see, e.g. Gali,Gertler and Lopez-Salido (2001), Sbordone (2002), and more recently Eichenbaum and Fisher (2004)). In general, those authors find that the estimates for the slope coefficient range between 0.03 to 0.05 (see e.g. Woodford (2005) for a recent discussion on these values.) Hence, in order to match the aggregate estimates with the micro evidence on price stickiness, it is necessary a low value for the parameter γ. We will now turn to see how the different strategic complementarities can be set as to fit a certain amount of real rigidities.

In this paper we assume that the slope coefficient is equal to 0.025, then the required amount of pass-through from marginal costs to prices, γ is around 0.09. We also assume that the steady state markup is around 16% (in particular we set $\epsilon = 7$ which implies that $\mu = 1.16$). Under these assumptions, the required curvature parameter, ψ, to obtain such a value for the pass-through coefficient is $\psi = -8$.[13] If the only source of strategic

[13]In the Appendix we show how to relate the curvature parameter with the existing literature on quasi-kinked demand.

Table 3: Calibrated Parameter Values

Parameter	Description	Value
β	Discount Factor	0.99
σ	Risk Aversion	1
χ^{-1}	(Frisch) Labor Supply Elasticity	1
α	Output elasticity to capital	0.33
ϵ	Price Elasticity of Demand	7
ξ	Probability of Changing Prices	0.60
ν	Inverse of Money Demand Elasticity	12

Figure 2: Quasi-Kinked Demand and Steady-State Inflation

complementarity comes from the existence of fixed factors at the firm level, then in order to match the value of γ we need a value for $\alpha_f = 0.58$. If we combine both frictions, then reducing α_f to 0.5 implies that we only required much smaller amount of curvature for the demand coefficient, i.e. $\psi = -2$, to get the required degree of pass-through.

5 The Costs of Steady-State Inflation

In this section, we solve for the non-linear steady state of the models to compare the implications of the two types of strategic complementarities for the costs of steady state inflation through its effects on average markup and the relative price distortion. We will use the Dixit-Stiglitz model as a reference model that helps in clarifying the distinct implications of alternative strategic complementarities on both the average markup and the relative price distortions. As noted before, the calibration we use to compare the non-linear implications of the models is such that all of them generate the same slope of the NKPC, hence they are observationally equivalent in terms of the first order approximation of inflation dynamics.

In Figure 2 we plot the average markup and the relative price distortion as a function of the steady state inflation in the model with quasi-kinked demand functions calibrated for the two values of ψ, -2 and -8, discussed in the previous section. The first interesting feature of this deviation from the standard Dixit-Stiglitz preferences is the strong asymmetry of inflation induced on both markup and relative price distortions. The higher the non-linearity in the demand function, the higher the asymmetry of negative and positive inflation on both average markup and relative price distortion. Instead of the Dixit-Stiglitz model, the existence of non-zero steady state inflation reduces the average markup.

Secondly, the existence of steady state deflation tends to reduce the a stronger decrease in the average markup relative to economies with positive inflation rate. As noticed by King and Wolman (1999), in the Dixit-Stiglitz model the average markup is minimized at zero steady state inflation markup (which corresponds to the constant desired markup, i.e. $\mu = \frac{\epsilon}{\epsilon - 1}$, given our calibration is 1.16 in the Figure). Nevertheless, under quasi-kinked demand, the presence of steady state inflation translate in an asymmetric way into the desired markup of the firms adjusting prices, so that average markup is dramatically reduced under deflation more than it is under positive inflation. The reason is also apparent from the right panel, which shows how the steady-state inflation influences the magnitude of relative price distortions. In the quasi-kinked demand environment, the presence of steady state deflation induces a higher cost in terms of relative price distortions than positive inflation. This is the side effects of the asymmetric demand functions, since the presence of deflation tends to increase the relative price of firms adjusting prices so consumers move immediately away from those price setters generation a higher output costs.

To see this, let us consider the standard model with economy-wide factor markets and constant elasticity of demand. In this case, for empirically –either positive or negative– inflation rates, the firms adjusting prices have strong incentives to do so to capture the demand of its competitors. The presence of quasi-kinked demand implies that a drop in the firms relative price only stimulates a small increase in demand, while a rise in its relative price generates a large drop in demand. That is, consumers will costesly flee from relative expensive goods but do not flock into inexpensive ones. Suppose, for instance, that the economy is facing a steady state positive inflation. On the one hand, the relative price of the non-adjusting firm's reduces dramatically without generating much gains in terms of relative demand (i.e. relative demand becomes relative inelastic if relative price is below the equilibrium). On the other hand, given the reduction in the relative price of the non-adjusting

Figure 3: Firm-Specific Inputs and Steady-State Inflation

firms, there are low incentives for adjusters to change prices, so to gain some relative demand they have to reduce the desired markup which tends to reduce the economy-wide (average) markup.

The effects of negative steady state inflation tends to dramatically shrink the profits of non-adjuster by moving consumers demand away from its products to other; this translates the relative price dispersion into higher output costs and generates a fall in the desired markup of firms so making the economy more competitive. The adjusting firms have low incentive to undercut others since they will face a lower elastic demand without boosting its own sales, hence they reduce their desired markups to avoid further reductions in their relative sales, which also tends to reduce the average markup (see the left panel of Figure 2). Overall, steady state deflation induces higher relative price distortions and tend to lower the average markup, while positive inflation generates, in equilibrium, less relative price dispersion.

In the previous set up, factor inputs can be costlessly reallocated across firms so that they can adjust their marginal cots in responses to steady state inflation. Figure 3 corresponds to the same exercise in the model with fixed factors, where we plot the average markup and relative price distortions for alternative values of the short run elasticity of output with respect to fixed factors, i.e. α_f. The figure makes it clear that, relative to the quasi-kinked demand model, this mechanism has sharply different effects on both the average markup and the relative price distortion factor. It should be noted that we set the curvature parameter $\psi = 0$ in order to isolate the pure effects of the presence of fixed factors, so the demand side of the model is identical to the one with Dixit-Stiglitz CES aggregator. Later we will discuss the implications of both frictions operating at the same time.

As can be seen from this figure, the most noticeable feature is the asymmetry induced in both the average markup and the relative price distortions by the presence of positive inflation rate. The higher is the share of fixed factors in the production function, the higher

Figure 4: Combining Quasi-Kinked Demand and Firm-Specific Inputs

Figure 5: Comparing the Alternative Specifications

are the costs generated by positive inflation rates. As in the baseline model, the relative price distortion is minimized at zero steady state inflation. In a situation of positive steady state inflation, if there is a fraction of fixed factors, then price adjusters trying to undercut prices to boost their demands would also raise their own marginal costs. Hence, a positive steady state inflation boosts the output costs of relative price distortions, while negative steady state inflation leads to smaller costs, given the upward slope of the firms's marginal curve.

Quantitatively, these relative price distortions associated with positive inflation rates produce non negligible output cost. In particular, a 3% steady state inflation rate generates an output loss of nearly one percent, while a 3% annual deflation is only about half as costly.

In Figure 4, we plot the effects of the calibration of the model combining both fixed factors (assuming that $\alpha_f = 0.5$) and quasi-kinked demand (assuming in such a $\psi = -2$). In the figure we plot the model assuming $\psi = -2$, which corresponds to the one discussed above with fixed factors alone. It is clear that the joint effects of both friction flips the costs of inflation on relative price distortions. In particular, now the presence of quasi-kinked demand and the fact that the firms setting price can not perfectly adjust its factors in response to inflation makes deflation much more costly than positive inflation. In particular, a negative steady state inflation of -3% (somehow closest to the one associated with the Friedman's rule) will generate important output costs due to the amount of relative price distortion that the model imposes on the price setting firms. In addition, the benefits of both positive and negative inflation on average markup are more balanced, i.e. the average markup curve is more symmetric around the zero steady state inflation. Finally, Figure 5 compares the four specifications using calibrations that imply the same magnitude of real rigidity, γ.

6 Optimal Policy in the Stochastic Economy

In this section, we discuss implications of strategic complementarities for the optimal policy when the economy is subject to exogenous random shocks. In so doing, we derive the second-order approximation to the utility function of the representative household, following the linear-quadratic approach of Woodford (2003). Besides, we do not include monetary distortions to create incentive for holding fiat money in this section (i.e. we set $\nu_0 = 0$ in expression (2)).

6.1 Characterizing the Optimal Policy Problem

Before proceeding, notice that the deterministic steady-state equilibrium achieves the first-best allocation in the presence of the fiscal policy to eliminate the distortion associated with the monopolistic competition. Given that the steady state is Pareto optimal, we can characterize the first-order approximation of the optimal policy from optimizing the second-order approximation of the social welfare function subject to the first-order approximation of equilibrium conditions.[14] Moreover, the log deviation of the real marginal cost from its steady-state level is proportional to the log deviation of output from its first-best level, while their proportionality is the weight of the output gap in the second-approximation to the social welfare function, denoted by λ_x.

As a result, substituting $mc_t = \lambda_x \, x_t$ into the NKPC specified in the introduction (expression (1)), we have an expression of the Phillips curve equation in terms of output gap:

$$\pi_t = \beta E_t[\pi_{t+1}] + (\lambda_x \kappa_p \gamma) x_t. \tag{12}$$

It is noteworthy that the parameter κ_p $(= (1-\xi)(1-\xi\beta)/\xi)$ is associated with the average frequency of price changes under the Calvo pricing, while degree of real rigidity is measured by $\gamma = (1 - \mu\psi + \epsilon \frac{\alpha_f}{1-\alpha_f})^{-1}$.

Having described the constraint of the optimal policy problem, we turn to the second-order approximation to the social welfare function. It is shown in the appendix that the

[14] Woodford (2003) includes the second-order approximation to the social welfare when the steady state is distorted, while Benigno and Woodford (2005) discuss the optimal policy when the steady state achieves the Ramsey allocation.

second-order approximation to the social welfare function can be written as follows:

$$\sum_{t=0}^{\infty} \beta^t E_0[\lambda_\pi \frac{\pi_t^2}{2} + \lambda_x \frac{x_t^2}{2}], \tag{13}$$

where λ_x and λ_π are weights for output gap and inflation, respectively. The weight on output gap can be written as $\lambda_x = \sigma + (\chi + 1 - \alpha_{vl})/\alpha_{vl}$, though the magnitude of α_{vl} is affected by the presence of fixed labor inputs, given that $\alpha_{vk} + \alpha_{vl} = 1 - \alpha_{fl} - \alpha_{fk}$. However, λ_π has different expressions depending on sources of strategic complementarities: either $\lambda_\pi = \epsilon/(\kappa_p \gamma)$ in the case of fixed factors inputs or $\lambda_\pi = \epsilon/\kappa_p$ in the case of kinked demand curves.

Given the second-order approximation to the social welfare function and the first-order approximation of equilibrium conditions, the optimal policy from timeless perspective can be written as $\epsilon\pi_t = -x_t + x_{t-1}$ in the case of fixed-inputs, and $\epsilon\gamma\pi_t = -x_t + x_{t-1}$ in the case of kinked demand curves. We thus find that the optimal responses of inflation to changes in output gap depend on sources of strategic complementarities. However, since there is no mechanism that generates trade-offs between inflation and output gap, the optimal inflation rate under timeless perspective becomes zero regardless of sources of strategic complementarities. Hence, in the next section, we incorporate cost-push shocks into the Phillips curve equation to create short-run trade-offs between inflation and output, following Clarida, Gali and Gertler (1999, 2001).

6.2 Fixed-Inputs at the Firm Level

We begin with the case of fixed-inputs at the firm level and then move onto the case of kinked demand curves. When state-contingent commitment is feasible, the social planner chooses its state-contingent plan on $\{\pi_t, x_t\}_{t=0}^{\infty}$ in order to minimize

$$\sum_{t=0}^{\infty} \beta^t E_0[\frac{\lambda_\pi}{2}\pi_t^2 + \frac{\lambda_x}{2}x_t^2 + \omega_{ct}(\pi_t - \lambda_x \kappa_p \gamma x_t - \beta E_t[\pi_{t+1}] - u_t)], \tag{14}$$

where u_t represents an i.i.d. exogenous cost-push shock[15], ω_{ct} represents the Lagrange multiplier for the Phillips curve in the optimization problem under commitment and $\omega_{c-1} = 0$.

[15] Although we do not make it explicit in the previous section, u_t can take place when tax rates are subject to exogenous variations.

Combining the first-order necessary conditions then yields the optimal policy rule that links targets:

$$\begin{aligned} \epsilon\pi_t &= -x_t + x_{t-1} \qquad \text{for } t \geq 1, \\ \epsilon\pi_0 &= -x_0. \end{aligned} \tag{15}$$

Under discretion, the social planner can not make any binding commitment over its future policy actions, so that it has to take as given the public's expectations about the future. Hence, the optimization problem under discretion turns out to be

$$\min_{\pi_t,\ x_t} \{ \frac{\lambda_\pi}{2}\pi_t^2 + \frac{\lambda_x}{2}x_t^2 + \omega_{dt}(\pi_t - \lambda_x \kappa_p \gamma x_t - \beta E_t[\pi_{t+1}] - u_t) \}, \tag{16}$$

where ω_{dt} is the Lagrange multiplier for the Phillips curve in the optimization problem under discretion. The first-order conditions can be combined to yield

$$\epsilon\pi_t = -x_t. \tag{17}$$

It follows from (15) and (17) that the introduction of strategic complementarities through the fixed-capital at the firm level does not affect the optimal ratio of inflation to output gap under both of discretion and commitment. As a result, we can find that the welfare level under the optimal policy is not affected by the introduction of strategic complementarities through the fixed-inputs at the firm level, up to the first-order approximation of the optimal policy with the second-order approximation of the welfare.

6.3 Kinked Demand Curves

Having described the optimal policies in the case of fixed factors at the firm level, we solve the optimal policy problems when we allow for only kinked demand curves without having the fixed-capital at the firm level. Notice that the case of kinked demand curves corresponds to $\gamma = 1/(1-\mu\psi)$ and $\lambda_\pi = \epsilon/\kappa_p$. Given these definitions of parameters, we solve the optimal policy problems similar with those in the previous section. As a result, the optimal policy under commitment can be written as

$$\begin{aligned} \epsilon\gamma\pi_t &= -x_t + x_{t-1} \qquad \text{for } t \geq 1, \\ \epsilon\gamma\pi_0 &= -x_0. \end{aligned} \tag{18}$$

The optimal policy under discretion is given by

$$\epsilon\gamma\pi_t = -x_t. \tag{19}$$

It then follows that the introduction of kinked demand curves reduces the optimal response of inflation to output, as opposed to the case of the fixed factors at the firm level. The reason for this is that the introduction of kinked demand curves (through a change from the Dixit-Stiglitz preference to Dotsey-King type preference) affects the trade-off between inflation and output gap in the constraint, while it does not have any influence on the trade-off between inflation and output gap in the objective function of the social planner.

Furthermore, substituting the efficiency condition (19) into the social period loss function (13), we can find that the period loss function at the optimum under discretion turns out to be

$$\frac{1}{2}\frac{\epsilon(1+\epsilon\gamma(\lambda_x\kappa_p\gamma))}{\kappa_p}\pi_t^2. \tag{20}$$

It thus follows from (20) that in the case of kinked demand curves, the optimal loss becomes smaller if inflation variability is the same. But it does not mean that the optimal loss under the same cost-push shock becomes smaller. For the strategic complementarities generated by the introduction of kinked demand curves increases the optimal response of the aggregate inflation rate to the same size of cost-push shocks. In order to see this, notice that substituting (19) into the Phillips curve specified in (12), solving the resulting linear difference equation, and then putting the resulting solution into the period loss function yields

$$\frac{1}{2}\frac{\epsilon}{\kappa_p(1+\epsilon\gamma(\lambda_x\kappa_p\gamma))}u_t^2. \tag{21}$$

As a result, we can see that the optimal loss under the same cost-push shock can be increased when the degree of real rigidity rises with the introduction of kinked-demand curves.

7 Costs of the Great Inflation

In this section, we use explicit functional forms of relative price distortion under each source of strategic complementarities to construct time-series of relative price distortions, taking as given a set of observed time-series of inflation rates and initial values of relative price distortions. Specifically, we measure relative price distortions that are implied by different sources of strategic complementarities under the assumption that the government in the model achieves the same set of inflation rates observed in the U.S. economy.

Figure 6: The Evolution of the Distortion Factor (%)

7.1 Dixit-Stiglitz Preferences

Consider the standard Dixit-Stiglitz aggregator, which leads to the following relationship between relative price distortion and inflation under the Calvo pricing:

$$\Delta_t = (1-\xi)\left(\frac{1-\xi\Pi_t^{\epsilon-1}}{1-\xi}\right)^{\frac{\epsilon}{\epsilon-1}} + \xi\Pi_t^{\epsilon}\Delta_{t-1}. \tag{22}$$

This means that if we have an observed set of the aggregate inflation rate, denoted by $\{\pi_t\}_{t=0}^{T}$,[16] it is possible to construct a set of relative price distortions $\{\Delta_t\}_{t=0}^{T}$ using equation (22) given an initial value of relative price distortion, Δ_{-1}. Besides, the initial value of relative price distortion is set to be $\Delta_{-1} = \Delta$ whose value is measured at the long-run average inflation rate in the periods before the inflation series begin.

In order to give a concrete idea about the construction of the time-series of relative price distortion, we proceed with the measure of relative price distortion (22). As a benchmark choice of parameter values, values of parameters ξ and ϵ are, respectively, set to be $\xi = 0.6$ and $\epsilon = 7$, though various sets of parameter values will be used. Specifically, $\xi = 0.60$ means that firms fix prices on average for 2.5 quarters, while $\epsilon = 7$ implies that the steady state markup, defined as the ratio of price to marginal cost, equals 17 percent, because the steady state markup is $\frac{\epsilon}{\epsilon-1}$. Furthermore, the sample in this section covers the quarterly data on non-farm business sector inflation rate over the period 1947:1 - 2005:3. In addition, a sample average of non-farm business sector inflation rate over the period 1947:1 - 1959:4 is used to compute a steady state value of relative price distortion:

$$\Delta = \frac{1-\xi}{1-\xi(1+\pi)^{\epsilon}}\left(\frac{1-\xi(1+\pi)^{\epsilon-1}}{1-\xi}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{23}$$

The initial value of the measure of relative price distortion is then set to be $\Delta_{-1} = \Delta$.

Figure 6 reports constructed series of relative price distortion over the period 1960:1 - 2005:3. It demonstrates that the measure of relative price distortion rises in early 1970s,

[16] In the previous section, π_t denotes the logarithmic difference between price levels at period t and $t-1$. In this section, when we construct the time-series of relative price distortion, we define π_t as the change rate of price levels at period t and $t-1$, so that $\pi_t = \frac{P_t - P_{t-1}}{P_t}$.

reaches its peak around 1975 and then declines. The size of relative price distortion at its peak is around 2 % in terms of quarterly real output under the set of parameter values specified above. Besides, relative price distortion shows large declines after 1982, while it is stable around 1990s.

It is worthwhile to mention that the measure of relative price distortion specified in (22) depends mainly on the inflation rate. In addition, when the long-run average inflation rate is set equal to zero, the steady-state relative price distortion disappears. The central bank can therefore adjust the level of relative price distortion by controlling the rate of inflation. This in turn implies that the sample average of relative price distortion can be interpreted as representing the cost of inflation.

Furthermore, traditional works on the welfare costs of inflation has focused on the size of the deadweight loss under a money demand curve that occurs because of inflation, as can be seen in the works of Bailey (1956) and Lucas (1987, 2000). The costs of relative price distortion, however, are not associated with the frictions that make households voluntarily hold real money balances. The findings explained above thus indicate that staggered price-setting can be an independent and significant source of the welfare costs of inflation.

7.2 Implications of Real Rigidities

Having discussed the output costs of inflation based on the standard Dixit-Stiglitz preference, we move onto the cases of kinked demand curves and fixed production inputs. In so doing, we choose values of relevant parameters in order to match the estimated slope coefficient of the Phillips curve. As noted earlier, this implies that the coefficient to measure the size of fixed factors inputs is $\alpha_f = 0.58$ in the first case, while the curvature parameter of demand curves is set to be $\psi =$ -8 in the second case. Given these parameter values, we compute time-series of relative price distortion taking as given the observed time-series of aggregate inflation rate. We do this to show that these two sources of strategic complementarities have different welfare implications, even though they produce the same first-order dynamics of the aggregate inflation rate.

The results from these experiments can be summarized as follows. Above all, different sources of strategic complementarities share similar time patterns of relative price distortion during the whole sample period. Specifically, measures of relative price distortion rise in early 1970s and then decline after 1982. Thus, they show their peaks around 1975.

But their magnitudes are dramatically different. In particular, when we derive quasi-kinked demand curves using the Dotsey and King aggregator, relative price distortion becomes small relative to the case of the standard Dixit-Stiglitz preferences. In contrast, allowing for fixed-inputs at the firm level raises the relative price distortion. As shown in Figure 6, the peak relative price distortion under quasi kinked demand curves is less than 0.25 % of quarterly real output, whereas it becomes slightly less than 30 % in the case of fixed-inputs.

8 Conclusions

Many papers have studied the role of real rigidities to match the aggregate response of inflation to marginal cost. But, in all this literature, the non-linear implications of the underlying mechanism have not been further explored. This issue is of central importance to understand the monetary policy implications of micro-founded New Keynesian models. What are the different implications for the welfare costs of steady-state inflation and inflation volatility of alternative sources of real rigidities? Our analysis corroborates that alternative real rigidities lead to very different implications for the welfare costs of steady state inflation and inflation variability, although they might have identical implications for the first order dynamics of the inflation rate.

Under the presence of quasi-kinked demand, there is a strategic link between firms' marginal revenues and the incentive for price adjustment. The presence of local factor markets –fixed factor inputs–, induces a strategic link between the firm's marginal costs and the incentive for price adjustment. This different nature of the strategic linkage among firm's incentive to changes price is at the core of the asymmetric results for monetary policy that we find in this paper.

More generally, this paper puts forward the need to carefully examine mechanisms (embedded into New Keynesian dynamic general equilibrium models) observationally equivalent up to first order, which may yield sharply different conclusions for monetary policy once the non-linearities implied by the models are accounted for.

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Appendix A: The Calibration of Quasi-Kinked Demand

The purpose of this appendix is to overview how the existent literature has proceeded in calibrating demand functions with non-constant elasticity and to relate it with the specification presented in the main text (e.g. Dotsey and King (2005)). To do this, we start by relating the curvature of the demand function, i.e. the parameter ψ, with the degree of pass-through from marginal costs to prices. In the absence of other sources of strategic complementarities Woodford (2003, 2005) and Eichenbaum and Fisher (2005) have shown that *the degree of pass-through from a rise in the marginal costs to prices*, γ, takes the following equivalent form:

$$\gamma = \frac{1}{\eta\epsilon_\mu + 1} = \frac{1}{\frac{\epsilon_p}{\eta-1} + 1} \tag{24}$$

where ϵ_μ denotes the *elasticity of markup, μ, with respect to changes in relative demand, $\widetilde{Y}$, evaluated at the steady state*: $\epsilon_\mu = \frac{\partial\mu}{\partial\widetilde{Y}}\frac{\widetilde{Y}}{\mu}$; and ϵ_p denotes *the percent change in the elasticity of demand, η, due to a one percent change in the relative price,$\widetilde{P}$*: $\epsilon_p = \frac{\partial\eta}{\partial\widetilde{P}}\frac{\widetilde{P}}{\eta}$. The latter is used by Eichenbaum and Fisher (2005), while the former is used by Kimball (1995) and Woodford (2003, 2005), respectively.[17] From expression (24) it follows that, $\eta\epsilon_\mu = \frac{\epsilon_p}{\eta-1}$. At zero steady state inflation, the elasticity $\eta = \epsilon$, and hence we pind down the value of the elasticity by setting a value for the gross markup, i.e. $\mu \equiv \frac{\epsilon}{\epsilon-1}$. Then, under the Dotsey and King (2004) specification, it follows that, $\epsilon_p = -\epsilon\psi$, which implies that: $\psi = -\frac{\epsilon_p}{\epsilon} = -(\epsilon-1)\epsilon_\mu < 0$. This allows to write the expression for γ as a function of the steady state markup and the curvature parameter ψ, i.e.

$$\gamma = \frac{1}{1-\mu\psi}.$$

The previous expression can be used to infer the curvature of the demand function that matches *a certain degree of pass-through from a rise in the marginal costs to prices*: $\psi = -\frac{1-\gamma}{\mu\gamma}$. Hence, for an given steady state markup, the lower the passthrough the higher the absolute value of ψ, i.e. the higher the degree of curvature; and, for a given degree of passthrough, the lower the steady state markup (i.e. the higher the elasticity of demand) the higher the degree of curvature.

Kimball (1995), Woodford (2003, 2005), and Eichenbaum and Fisher (2005) implicitly calibrate the curvature of the demand function by setting the value of the parameter γ. All of them assume a value for the –zero inflation– steady state markup $\mu = 1.1$–i.e. 10%– which implies a value for $\eta = \epsilon = 11$. Under this assumption, Kimball calibration sets a value for $\epsilon_\mu = 4.28$, which implies a value for $\gamma = 0.021$, and corresponds to a extremely high degree of curvature, $\psi = -42.8$. Woodford (2005) calibrates the pass-through parameter $\gamma = 0.5$, and Eichenbaum and Fisher (2005) and Coenen and Levin (2005) calibrate γ between 0.23 and 0.5 (this corresponds to values for ϵ_p ranging between 10 to 33). These numbers imply that, for a 10% steady state markup, the degree of curvature ψ will vary between -0.9, and -3, respectively. Notice that, holding the same degree of passthrough (i.e. $\epsilon_p = 10,\ 33$) but assuming a higher steady state markup of 20% $-\epsilon = 6$–, requires a higher curvature of the demand, i.e. ψ ranges between -1.7 and -5.5.

[17] Coenen and Levin (2005) refer to this coefficient as *the relative slope of the demand elasticity around its steady-state, i.e.* $\epsilon_p = 1+\eta+\eta\frac{G'''}{G''}$, which is related to the curvature of the demand function since it involves the second and third derivatives of the aggregator G. Klenow and Willis (2005) refer to this expression as the *super-elasticity*, or the rate of change of the elasticity.

Related Literature Instead of the previous work, Chari, Kehoe and Mcgrattan (2000) and Bergin and Feenstra (2000) emphasize a direct measure of the curvature of demand through its implications for *the percent reduction in relative demand -market share- due to a one percent increase in the relative price*, ζ. To do that, these authors compute a second order Taylor expansion of the quasi-kinked relative demand, $\widetilde{Y}_t$, around the relative price, $\widetilde{P}_t(j)$. This yields to

$$\widetilde{Y}_t(j) \cong 1 - \eta(\widetilde{P}_t(j) - 1) + \frac{\eta\vartheta}{2}(\widetilde{P}_t(j) - 1)^2 \tag{25}$$

where the parameter $\vartheta = \frac{G'\widetilde{Y}''}{\widetilde{Y}'}$, is evaluated at the steady state. It is easy to find a relationship among the two elasticities (ϵ_p and ϵ_μ) and the parameter ϑ. To see this, we first take a log-linear approximation approximation to the elasticity of demand, $\eta(\widetilde{Y})$. This yields to $\widehat{\eta}(\widetilde{Y}) \simeq -e_y \,\widehat{\widetilde{Y}}$, where$\widehat{\ }$ represents log-deviation respect to the steady state, and $e_y = \frac{(1+\eta-\vartheta)}{\eta}$. From this follows that the parameter $\vartheta = 1 + \eta(1 - e_y)$, where $e_y = \frac{\partial\eta}{\partial\widetilde{Y}}\frac{\widetilde{Y}}{\eta}$, and can easily be related to ϵ_μ through the following expression $e_y = (\eta - 1)\epsilon_\mu$. Hence, in a zero inflation steady state, $e_y = (\epsilon - 1)\epsilon_\mu$, which corresponds to the curvature parameter, $-\psi$, under the Dotsey and King (2004) quasi-kinked demand specification.

From expression (25) it is possible to obtain *the percent reduction in relative demand -market share- due to a* **1** *percent increase in the relative price* is given by $\zeta = -\eta(.01) + \frac{\eta\vartheta}{2}(.01)^2$. Hence, it is now clear how to translate the calibration of Woodford (2005) and Eichenbaum and Fisher (2005) into *the percent reduction in relative demand -market share- due to a* **1** *percent increase in the relative price.* Hence, assuming a value for $\epsilon_p = 10$ and $\eta = 11$ (i.e. $\gamma = 0.5$), then $\epsilon_\mu = \frac{\epsilon_p}{(\eta-1)\eta} = \frac{10}{10*11} = \frac{1}{11} = 0.09$ and $e_y = (11-1)\frac{1}{11} = 0.9$, $\vartheta = 1 + 11(1 - 0.9) = 2.1$; which implies that $\zeta = -11(.01) + \frac{11*2.1}{2}(.01)^2 \simeq -0.108$. That is, this curvature of demand implies that a ***one*** *percent increase in the relative price* results in a 10.8 *percent reduction in relative demand.* Under the assumption of $\epsilon_p = 30$, then $\epsilon_\mu = \frac{3}{11}$, $e_y = \frac{30}{11}$, and $\vartheta = 1 + 11 * (1 - 30/11) = -21$, and we can obtain that $\zeta = -11(.01) - \frac{11*21}{2}(.01)^2 \simeq -0.12$, i.e. *the percent reduction in relative demand -market share- due to a **1** percent increase in the relative price* results in a 12 *percent*.

Bergin and Feenstra (2000) uses a different strategy. They assume that the aggregator, G, is a translog function, so the expression for he relative demand of good i takes the following particular form:

$$\widetilde{Y}_i = \left[\frac{1 + (\eta - 1)\log\left(\frac{\overline{P}_{-i}}{P_i}\right)}{P_i}\right] P_t \tag{26}$$

where $\overline{P}_{-i}$ represents the average price set by competitors, and the own-price elasticity of demand, i.e. $\eta(\widetilde{Y}_i) = 1 - \frac{\phi}{s(\widetilde{Y}_i)}$, where s represents the expenditure share of good i, and ϕ is a constant. Then, it follows that $e_p = \frac{\partial\eta}{\partial\widetilde{Y}}\frac{\widetilde{Y}}{\eta} = \frac{(\eta-1)^2}{\eta}$. They calibrate the steady state markup equal to 50%, which implies a value of $\eta = 3$. From expression (26) if follows that this is the only parameter that determines the curvature of the demand. Hence, a 2% rise in the relative price of good i from steady state leads to a fall in demand of: 1-(1+(3-1)*log(1/1.02))/1.02=1-0.942=0.058, i.e slightly more than a 5%; and accordingly a rise in the relative prices of 3% leads to a fall of 8.7%. Notice that Bergin and Feenstra (2000) sets e_y=4/3, ϵ_μ=2/3 or ϵ_p=4. This is implies a value for γ=0.33, and in terms of the Dotsey and King (2005) it corresponds to ψ=$-$1.33.

Appendix B: Key Derivations

Households Optimality Conditions The flow-budget constraint at period t of the representative household can be written as

$$C_t + E_t[Q_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] + M_{t+1} = \frac{B_t + M_t}{P_t} + \frac{W_t}{P_t}N_t + \Phi_t - T_t,$$

where B_{t+1} denotes a portfolio of nominal state contingent claims in the complete contingent claims market, $Q_{t,t+1}$ is the stochastic discount factor for computing the real value at period t of one unit of consumption goods at period $t+1$, W_t is the nominal wage rate, T_t is the real lump-sum tax, and Φ_t is the real dividend income. The first-order conditions for consumption and labor supply can be combined to yield

$$\chi_0 C_t^{\sigma} N_t^{\chi} = \frac{W_t}{P_t}. \tag{27}$$

The optimization condition for bond holdings is

$$Q_{t,t+1} = \beta(\frac{C_t}{C_{t+1}})^{\sigma}. \tag{28}$$

Hence, if R_t represents the risk-free (gross) nominal rate of interest at period t, the absence of arbitrage at an equilibrium gives the following Euler equation:

$$\beta E_t[R_t(\frac{C_t}{C_{t+1}})^{\sigma}\frac{P_t}{P_{t+1}}] = 1. \tag{29}$$

The money-bonds portfolio allocation decision is given by

$$\nu_0\left(\frac{M_t}{P_t}\right)^{-\nu} = (R_t - 1)C_t^{-\sigma} \tag{30}$$

Profit Maximization Conditions

Marginal Costs under Fixed Inputs The production function of an individual firm is given by

$$Y_t(j) = A_t K_f^{\alpha_{fk}} K_{vt}(j)^{\alpha-\alpha_{fk}} N_f^{\alpha_{fl}} N_{vt}(j)^{1-\alpha-\alpha_{fl}},$$

where $K_f(j)$ and $L_f(j)$ are fixed capital and labor, $K_{vt}(j)$ and $L_{vt}(j)$ are variable capital and labor, α_{fk} and α_{fl} are output elasticities of fixed capital and labor. Notice that letting α_f denote the sum of output elasticities of fixed inputs, we have $\alpha_f = \alpha_{fk} + \alpha_{fl}$. In addition, the output elasticity of variable labor can be written as: $\alpha_{vl} = 1 - (\alpha + \alpha_{fl})$. We assume that amounts of fixed capital and labor respectively are the same across firms. It is also assumed that there are perfectly competitive factors markets for variables inputs in which their prices are fully flexible.

Given the functional forms of production function specified above, minimizing total costs of obtaining variable inputs leads to the following optimization condition:

$$\frac{K_{vt}(j)}{N_{vt}(j)} = \frac{\alpha - \alpha_{fk}}{1 - \alpha - \alpha_{fl}}\frac{W_t}{R_t},$$

where W_t is the real wage at period t and R_t is real rental rate at period t. It then follows that the aggregate variable capital and labor can be written as

$$\frac{W_t}{R_t} = \frac{1-\alpha-\alpha_{fl}}{\alpha-\alpha_{fk}} \frac{K_{vt}}{N_{vt}},$$

where the aggregate variable capital and labor are defined as

$$K_{vt} = \int_0^1 K_{vt}(j)\ dj: \quad N_{vt} = \int_0^1 N_{vt}(j)\ dj.$$

In order to derive cost function of variable inputs in terms of individual output, notice that substituting the cost-minimization condition specified above into production function and then rearranging, we have

$$Y_t(j) = A_t Z_t N_{vt}(j)^{1-\alpha_f}.$$

where Z_t is defined

$$Z_t = (\frac{\alpha-\alpha_{fk}}{1-\alpha-\alpha_{fl}} \frac{W_t}{R_t})^{\alpha-\alpha_{fk}} K_f^{\alpha_{fk}} N_f^{\alpha_{fl}}.$$

Here, $Z_t(j)$ does not depend on price and output decisions at period t of individual firms. Meanwhile, substituting the cost-minimization condition specified above into the total-cost of obtaining variable inputs

$$W_t N_{vt} + R_t K_{vt} = \frac{1-\alpha_f}{1-(\alpha+\alpha_{fl})} W_t N_{vt}(j)$$

As a result, the cost-function of variable inputs can be written as

$$TC_{vt}(j) = V_t Y_t(j)^{\frac{1}{1-\alpha_f}},$$

where $V_t(j)$ is defined as

$$V_t = \frac{(1-\alpha_f)}{1-(\alpha+\alpha_{fl})} W_t (A_t Z_t)^{-\frac{1}{1-\alpha_f}}.$$

Given the cost-function defined above, the marginal cost at period t can be written as

$$MC_{vt}(j) = (\frac{V_t}{1-\alpha_f}) Y_t(j)^{\frac{\alpha_f}{1-\alpha_f}}.$$

Intertemporal Profit Maximization When firms are able to change prices in period t, their expected discounted sum of profits can be written as

$$\max_{\{P_t^*\}} \sum_{k=0}^{\infty} \beta^k E_t[Q_{t,t+k}(TR_{t+k} - TC_{t+k})],$$

where TR_{t+k} denotes the total revenue at period $t+k$, and TC_{t+k} denotes the total cost at period $t+k$, and $(1+\tau_p)$ is a fiscal production subsidy factor. The total revenue at period $t+k$ is

$$TR_{t+k} = \frac{1+\tau_p}{1+\psi} (\frac{P_t^*}{P_{t+k}})^{1-\epsilon(1+\psi)} \lambda_{t+k}^{\epsilon(1+\psi)} Y_{t+k} + \frac{\psi(1+\tau_p)}{1+\psi} \frac{P_t^*}{P_{t+k}} Y_{t+k}.$$

The partial differentiations of total revenues and costs can be written as

$$\frac{\partial TR_{t+k}(j)}{\partial P_t^*} = \{\frac{(1+\tau_p)(1-\epsilon(1+\psi))}{1+\psi}(\frac{P_t^*}{P_{t+k}})^{-\epsilon(1+\psi)}\lambda_{t+k}^{\epsilon(1+\psi)} + \frac{\psi(1+\tau_p)}{1+\psi}\}\frac{Y_{t+k}}{P_{t+k}},$$

$$\frac{\partial TC_{t+k}}{\partial P_t^*} = -\epsilon MC_{t+k}^*(\frac{P_t^*}{P_{t+k}})^{-(\epsilon(1+\psi)+1)}\lambda_{t+k}^{\epsilon(1+\psi)}\frac{Y_{t+k}}{P_{t+k}},$$

where MC_{t+k}^* is defined as

$$MC_{t+k}^* = (\frac{V_{t+k}}{1-\alpha_f})Y_{t+k}^{*\frac{\alpha_f}{1-\alpha_f}},\ Y_{t+k}^* = \frac{1}{1+\psi}((\frac{P_t^*}{P_{t+k}})^{-\epsilon(1+\psi)}\lambda_{t+k}^{\epsilon(1+\psi)} + \psi)Y_{t+k}. \tag{31}$$

The profit maximization condition can be written as

$$-\frac{\epsilon(1+\tau_p)}{\phi}(\tilde{P}_t^*)^{-\epsilon(1+\psi)}Z_{1t} + \frac{\psi(1+\tau_p)}{1+\psi}(\tilde{P}_t^*)^{-(\epsilon(1+\psi)+1)}Z_{2t} + \epsilon Z_{3t} = 0. \tag{32}$$

where $\widetilde{P}_t^* = \frac{P_t^*}{P_t}$ and Z_{1t}, Z_{2t} and Z_{3t}, respectively, are written as

$$Z_{1t} = \sum_{k=0}^{\infty}(\beta\xi)^k E_t[Y_{t+k}^{1-\sigma}(\frac{P_{t+k}}{P_t})^{\epsilon(1+\psi)-1}\lambda_{t+k}^{\epsilon(1+\psi)}], \tag{33}$$

$$Z_{2t} = \sum_{k=0}^{\infty}(\beta\xi)^k E_t[Y_{t+k}^{1-\sigma}(\frac{P_{t+k}}{P_t})^{\epsilon(1+\psi)}\lambda_{t+k}^{\epsilon(1+\psi)}MC_{t+k}^*], \tag{34}$$

$$Z_{3t} = \sum_{k=0}^{\infty}(\beta\xi)^k E_t[Y_{t+k}^{1-\sigma}(\frac{P_t}{P_{t+k}})]. \tag{35}$$

Recursive Representations of Profit Maximization Conditions When MC_{t+k}^* depends on $\tilde{P}_t^*$, we cannot have a closed-form recursive representation of Z_{2t} unless α_f takes a set of special values. However, if there are no firm specific inputs, we have the following recursive representation of the first-order conditions.

$$Z_{1t} = E_t\{\beta\xi\Pi_{t+1}^{\epsilon(1+\psi)-1}Z_{1t+1}\} + Y_t^{1-\sigma}\lambda_t^{\epsilon(1+\psi)}, \tag{36}$$

$$Z_{2t} = E_t\{\beta\xi\Pi_{t+1}^{\epsilon(1+\psi)}Z_{2t+1}\} + Y_t^{1-\sigma}\lambda_t^{\epsilon(1+\psi)}MC_t, \tag{37}$$

$$Z_{3t} = E_t\{\beta\xi\Pi_{t+1}^{-1}Z_{3t+1}\} + Y_t^{1-\sigma}. \tag{38}$$

Relative Price Distortion Having shown that the production function of an individual firm can be written as $N_{vt}(j)$=$(Y_t(j)/(A_tZ_t))^{1/(1-\alpha_f)}$, a linear aggregation of both sides of this equation leads to

$$Y_t = \frac{A_tZ_t}{\Delta_t}N_{vt}^{1-\alpha_f}, \tag{39}$$

where Δ_t is defined as

$$\Delta_t = (\int_0^1(\frac{Y_t(j)}{Y_t})^{\frac{1}{1-\alpha_f}}dj)^{1-\alpha_f}. \tag{40}$$

While Δ_t is a measure of relative price distortion, substituting the demand function of an individual firm into the definition of relative price distortion leads to

$$\Delta_t = (\left(\frac{1}{1+\psi}\right)^{\frac{1}{1-\alpha_f}} \int_0^1 (\lambda_t^{\epsilon(1+\psi)} \tilde{P}_t(j)^{-\epsilon(1+\psi)} + \psi)^{\frac{1}{1-\alpha_f}} dj)^{1-\alpha_f}. \tag{41}$$

In order to obtain closed-form solution, we set $\alpha_f = 1/2$. As a result, we have

$$\Delta_t^2 = \frac{1}{(1+\psi)^2} (\lambda_t^{2\epsilon(1+\psi)} \int_0^1 \tilde{P}_t(j)^{-2\epsilon(1+\psi)} dj + 2\psi\lambda_t^{\epsilon(1+\psi)} \int_0^1 \tilde{P}_t(j)^{-\epsilon(1+\psi)} dj + \psi^2). \tag{42}$$

Hence, it is possible to write the aggregate dispersion measure Δ_t as the following non-linear combination of different definition of price dispersions,

$$\Delta_t^2 = \frac{1}{(1+\psi)^2} \Delta_{2,t}^{\frac{2\epsilon(1+\psi)}{1-\epsilon(1+\psi)}} \widetilde{\Delta}_{1,t} + \frac{2\psi}{(1+\psi)^2} \Delta_{2,t}^{\frac{\epsilon(1+\psi)}{1-\epsilon(1+\psi)}} \Delta_{1,t} + \frac{\psi^2}{(1+\psi)^2} \tag{43}$$

where $\Delta_{1,t}$ and $\tilde{\Delta}_{1,t}$ are, respectively, defined as

$$\Delta_{1,t} = \int_0^1 \tilde{P}_t(j)^{-\epsilon(1+\psi)} dj; \quad \tilde{\Delta}_{1,t} = \int_0^1 \tilde{P}_t(j)^{-2\epsilon(1+\psi)} dj.$$

Under the Calvo pricing, evolution equations of these measures are given by

$$\Delta_{1,t} = (1-\xi)(\tilde{P}_t^*)^{-\epsilon(1+\psi)} + \xi \Pi_t^{\epsilon(1+\psi)} \Delta_{1,t-1}. \tag{44}$$

$$\tilde{\Delta}_{1,t} = (1-\xi)(\tilde{P}_t^*)^{-2\epsilon(1+\psi)} + \xi \Pi_t^{2\epsilon(1+\psi)} \tilde{\Delta}_{1,t-1}. \tag{45}$$

Aggregate Production Function and Marginal Cost of Production Notice that $Y_t = (A_t Z_t/\Delta_t) N_{vt}^{1-\alpha_f}$ may not reflect the true effect of the aggregate labor input on the aggregate output because Z_t depends on the aggregate labor input. It follows from the definition of Z_t that we have

$$Z_t = K_f^{\alpha_{fk}} K_v^{\alpha-\alpha_f} N_f^{\alpha_{fl}} N_{vt}^{-(\alpha-\alpha_{fk})}. \tag{46}$$

As a result, the aggregate production function can be written as

$$Y_t = B_1 \frac{A_t}{\Delta_t} N_{vt}^{1-\alpha-\alpha_{fl}}, \quad B_1 = K_f^{\alpha_{fk}} K_v^{\alpha-\alpha_f} N_f^{\alpha_{fl}}. \tag{47}$$

In addition, the aggregate unit-cost of production is given by

$$V_t = \frac{(1-\alpha_f)B_2}{1-(\alpha+\alpha_{fl})} W_t A_t^{-\frac{1}{1-\alpha_f}} N_{vt}^{\frac{\alpha-\alpha_{fk}}{1-\alpha_f}}, \tag{48}$$

where B_2 is defined as

$$B_2 = K_f^{-\frac{\alpha_{fk}}{1-\alpha_f}} N_f^{-\frac{\alpha_{fl}}{1-\alpha_f}} K_v^{-\frac{\alpha-\alpha_{fk}}{1-\alpha_f}}. \tag{49}$$

Substituting the aggregate production function into the unit cost of production, we have

$$V_t = B_3 W_t A_t^{-\frac{1}{1-(\alpha+\alpha_{fl})}} (Y_t \Delta_t)^{\frac{\alpha-\alpha_{fk}}{(1-\alpha_f)(1-\alpha-\alpha_{fl})}}, \tag{50}$$

where B_3 is

$$B_3 = \frac{(1-\alpha_f) B_2 B_1^{-\frac{\alpha-\alpha_{fk}}{(1-\alpha-\alpha_{fl})(1-\alpha_f)}}}{1-(\alpha+\alpha_{fl})}.$$

The aggregate real marginal cost is defined as

$$MC_t = \frac{V_t Y_t^{\frac{\alpha_f}{1-\alpha_f}}}{1-\alpha_f}. \tag{51}$$

Substituting the unit-cost into the definition of the aggregate marginal cost, we have

$$MC_t = \frac{B_3}{1-\alpha_f} W_t A_t^{-\frac{1}{1-\alpha-\alpha_f}} \Delta_t^{\frac{\alpha-\alpha_{fk}}{(1-\alpha_f)(1-\alpha-\alpha_{fl})}} Y_t^{\frac{\alpha-\alpha_{fl}}{1-\alpha-\alpha_{fl}}}. \tag{52}$$

Then, MC^*_{t+k} is given by

$$MC^*_{t+k} = MC_{t+k} \{ \frac{1}{1+\psi} ((\frac{P^*_t}{P_{t+k}})^{-\epsilon(1+\psi)} \lambda_{t+k}^{\epsilon(1+\psi)} + \psi) \}^{\frac{\alpha_f}{1-\alpha_f}}. \tag{53}$$

Steady-State Equilibrium Conditions

Relative Price Distortion The definition of the aggregate price level can be written as

$$\tilde{P}^* = \{ \frac{1}{1+\psi} (\frac{1-\xi}{1-\xi \Pi^{\epsilon(1+\psi)-1}})^{\frac{1}{1-\epsilon(1+\psi)}} + \frac{\psi}{1+\psi} (\frac{1-\xi}{1-\xi \Pi}) \}^{-1} \tag{54}$$

When $\alpha_f = 1/2$, the relative price distortion is given by

$$\Delta = (\frac{1}{(1+\psi)^2} \Delta_2^{\frac{2\epsilon(1+\psi)}{1-\epsilon(1+\psi)}} \widetilde{\Delta}_1 + \frac{2\psi}{(1+\psi)^2} \Delta_2^{\frac{\epsilon(1+\psi)}{1-\epsilon(1+\psi)}} \Delta_1 + \frac{\psi^2}{(1+\psi)^2})^{1/2} \tag{55}$$

where Δ_1, $\tilde{\Delta}_1$, and Δ_2 are, respectively, given by

$$\Delta_1 = \frac{1-\xi}{1-\xi \Pi^{\epsilon(1+\psi)}} (\tilde{P}^*)^{-\epsilon(1+\psi)}, \tag{56}$$

$$\tilde{\Delta}_1 = \frac{1-\xi}{1-\xi \Pi^{2\epsilon(1+\psi)}} (\tilde{P}^*)^{-2\epsilon(1+\psi)}. \tag{57}$$

$$\Delta_2 = \frac{1-\xi}{1-\xi \Pi^{\epsilon(1+\psi)-1}} (\tilde{P}^*)^{1-\epsilon(1+\psi)} \tag{58}$$

The steady-state relationship between inflation and relative price distortion can be obtained by combining these relations.

Profit Maximization We continue to assume that $\alpha_f = 1/2$. We can use the profit maximization condition for firms to compute the relationship between marginal cost and inflation. The profit maximization condition at the steady state equilibrium can be written as

$$-\frac{\epsilon(1+\tau_p)}{\phi}(\tilde{P}^*)^{-\epsilon(1+\psi)}Z_1 + \frac{\psi(1+\tau_p)}{1+\psi}(\tilde{P}^*)^{-(\epsilon(1+\psi)+1)}Z_2 + \epsilon Z_3 = 0, \tag{59}$$

where Z_1, Z_2 and Z_3 are defined as

$$Z_1 = \frac{Y^{1-\sigma}\lambda^{\epsilon(1+\psi)}}{1-(\alpha\xi)\Pi^{\epsilon(1+\psi)-1}},$$

$$Z_2 = \frac{MCY^{1-\sigma}\lambda^{\epsilon(1+\psi)}}{1+\psi}\left(\frac{\lambda^{\epsilon(1+\psi)}}{1-(\alpha\xi)\Pi^{2\epsilon(1+\psi)}} + \frac{1}{1-(\alpha\xi)\Pi^{\epsilon(1+\psi)}}\right),$$

$$Z_3 = \frac{Y^{1-\sigma}\lambda^{\epsilon(1+\psi)}}{1-(\alpha\xi)\Pi}.$$

Specifically, MC can be obtained by solving the following equation:

$$\frac{\epsilon\tilde{P}^*(1-(\alpha\xi)\Pi^{\epsilon(1+\psi)})}{\phi(1-(\alpha\xi)\Pi^{\epsilon(1+\psi)-1})} = \frac{\psi MC}{(1+\psi)^2}\left(\frac{\lambda^{\epsilon(1+\psi)}(1-(\alpha\xi)\Pi^{\epsilon(1+\psi)})}{1-(\alpha\xi)\Pi^{2\epsilon(1+\psi)}}+1\right)+\frac{\epsilon(\tilde{P}^*)^{\epsilon(1+\psi)+1}(1-(\alpha\xi)\Pi^{\epsilon(1+\psi)})}{(1+\tau_p)(1-(\alpha\xi)\Pi)}. \tag{60}$$

Second-Order Approximations to Utility Function

Quasi-Kinked Demand Function (Dotes and King's (2005) Aggregator) In this section, we do not assume that households are farmers and yeomen. Instead, households are trading labor services in factors markets. Given market transactions of labor services, we derive a quadratic loss function from utility functions of households by combining utility function with aggregate production function. Since we use the assumption that households are homogenous, it is important to relate relative price distortion to inflation so as to include inflation term in the derived quadratic loss function.

Relative Price Distortion We describe how relative price distortion evolves over time under the aggregator of Dotesy and King (2005). First, relative price distortion is expressed in terms of two price distortions as follows:

$$\Delta_t = \frac{1}{1+\psi}\Delta_{2t}^{\frac{\epsilon(1+\psi)}{1-\epsilon(1+\psi)}}\Delta_{1t} + \frac{\psi}{1+\psi}. \tag{61}$$

In particular, Δ_{2t} is a non-linear function of the Lagrange multiplier for the consumer's cost-minimization to produce composite goods, which is defined as

$$\Delta_{2t} = \int_0^1 \tilde{P}_t(i)^{1-\epsilon(1+\psi)}di, \tag{62}$$

where $p_t(i)$ is the relative price at period t of firm i. In addition, Δ_{1t} is an average of a non-linear function of individual relative prices, which is defined as

$$\Delta_{1t} = \int_0^1 \tilde{P}_t(i)^{-\epsilon(1+\psi)}di. \tag{63}$$

It is clear for their definitions that Δ_{1t} and Δ_{2t} can be interpreted as measures of relative price dispersions.[18]

Furthermore, applying the Calvo pricing rule to each measure of relative price dispersion leads to the following law of motion for each measure of relative price dispersion:

$$\Delta_{1t} = (1-\xi)(\tilde{P}_t^*)^{-\epsilon(1+\psi)} + \xi\Pi_t^{\epsilon(1+\psi)}\Delta_{1t-1}, \tag{64}$$

$$\Delta_{2t} = (1-\xi)(\tilde{P}_t^*)^{1-\epsilon(1+\psi)} + \xi\Pi_t^{\epsilon(1+\psi)-1}\Delta_{2t-1}, \tag{65}$$

where $\tilde{P}_t^*$ is the relative price of a new contract price at period t. We also have the following measure of relative price dispersion:

$$\Delta_{3t} = (1-\xi)\tilde{P}_t^* + \xi\Pi_t^{-1}\Delta_{3t-1}, \tag{66}$$

where Δ_{3t} is defined as

$$\Delta_{3t} = \int_0^1 \tilde{P}_t(i)di. \tag{67}$$

Second-Order Approximation to Measures of Relative Price Distortion Solving equation (66) for p_t^* and then substituting the resulting equation into equations (64) and (65) respectively, we can relate inflation to measures of relative price dispersion as follows:

$$\Delta_{1t} = (1-\xi)(\frac{\Delta_{3t} - \xi\Pi_t^{-1}\Delta_{3t-1}}{1-\xi})^{-\epsilon(1+\psi)} + \xi\Pi_t^{\epsilon(1+\psi)}\Delta_{1t-1}, \tag{68}$$

$$\Delta_{2t} = (1-\xi)(\frac{\Delta_{3t} - \xi\Pi_t^{-1}\Delta_{3t-1}}{1-\xi})^{1-\epsilon(1+\psi)} + \xi\Pi_t^{\epsilon(1+\psi)-1}\Delta_{2t-1}. \tag{69}$$

We apply second-order approximation to each measure of relative price dispersion around the deterministic steady state with zero inflation rate. In doing so, we define a set of new variables as follows:

$$\delta_{1t} = \log\Delta_{1t},\ \delta_{2t} = \log\Delta_{2t},\ \pi_t = \log\Pi_t. \tag{70}$$

As results of second-order approximation to Δ_{1t} and Δ_{2t}, we have

$$\delta_{1t} = -\epsilon(1+\psi)\delta_{3t} + \epsilon(1+\psi)\xi\delta_{3t-1} + \xi\delta_{1t-1} + \frac{\psi_1}{2}\pi_t^2. \tag{71}$$

$$\delta_{2t} = -(\epsilon(1+\psi)-1)\delta_{3t} + (\epsilon(1+\psi)-1)\xi\delta_{3t-1} + \xi\delta_{2t-1} + \frac{\psi_2}{2}\pi_t^2. \tag{72}$$

where ψ_1 and ψ_2 are defined as

$$\psi_1 = \xi\epsilon(1+\psi)(\frac{\epsilon(1+\psi)+1}{1-\xi} + \epsilon(1+\psi)-3),\ \psi_2 = \xi(\epsilon(1+\psi)-1)(\frac{\epsilon(1+\psi)}{1-\xi} + \epsilon(1+\psi)-4). \tag{73}$$

It also follows from (61) that log-deviation of relative price distortion can be expressed in terms of two measures of relative price dispersion:

$$\delta_t = -\frac{\epsilon}{\epsilon(1+\psi)-1}\delta_{2t} + \frac{1}{1+\psi}\delta_{1t}. \tag{74}$$

[18] It is noteworthy that the aggregator of Dixit and Stiglitz (1977) leads to $\Delta_t = \Delta_{1t}$ and $\Delta_{2t} = 1$, while $\psi = 0$ corresponds to the case of Dixit and Stiglitz (1977).

Substituting (71) and (72) into (74), we can obtain an approximated law of motion for relative price distortion of the form:

$$\delta_t = \xi\delta_{t-1} + \frac{\xi\epsilon}{2(1-\xi)}\pi_t^2. \tag{75}$$

Since $\psi = 0$ leads to the case of Dixit and Stiglitz (1977), it follows from (75) that the effect of the aggregator of Dotes and King (2005) on the approximated law of motion for relative price distortion is summarized in the coefficient of inflation term.

Elasticity of Quasi-Kinked Demand The demand curve facing firm i at period t can be written as

$$\widetilde{Y}_t(i) = \frac{1}{1+\psi}((\frac{\lambda_t}{\tilde{P}_t(i)})^{\epsilon(1+\psi)} + \psi), \tag{76}$$

where $\widetilde{Y}_t(j) = \frac{Y_t(j)}{Y_t}$ is the relative demand at period t of firm i and λ_t is the Lagrange multiplier of each consumer's cost-minimization problem to choose differentiated goods. We can see that the elasticity of demand is affected by the amount of relative demand. The steady state markup is given by

$$\mu = \frac{\epsilon}{\epsilon - 1}. \tag{77}$$

It is noteworthy that since kinked demand curve requires a negative value of ψ, parameter $\epsilon(1+\psi)$ can be negative for a certain range of negative values for ψ if we impose a restriction of $\xi > 1$. Given the calibration of kinked demand curve discussed above, the second-order approximation of relative price distortion can be rewritten as[19]

$$\delta_t = \xi\delta_{t-1} + \frac{\xi\epsilon}{2(1-\xi)}\pi_t^2. \tag{78}$$

Deriving Quadratic Loss Function (Non-Distorted Deterministic Steady State) The utility function for consumption can be written as

$$\frac{C_t^{1-\sigma} - 1}{1-\sigma} = \frac{(C_t^*)^{1-\sigma}X_t^{1-\sigma} - 1}{1-\sigma}, \tag{79}$$

where C_t^* denotes the first-best level of consumption at period t and X_t is the ratio of the current-period level of output to its first-best level. In the following, I focus on second-order Taylor expansions around the steady state with constant prices, in which there is no distortion so that $C = C^*$. The second-order Taylor expansion to the utility function for consumption is given by

$$\tfrac{C_t^{1-\sigma}}{1-\sigma} = \tfrac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}[(X_t - 1) - \sigma\tfrac{(X_t-1)^2}{2}] + O(||\zeta||^3), \tag{80}$$

[19] We now compare our specification of relative price distortion (78) with that of Woodford (2003, p 399 proposition 6.6). Specifically, the dispersion measure of Woodford is defined as cross-section variance of logarithms of relative prices. Because of this difference, ξ does not show up in the law of motion for the dispersion measure used in Woodford. However, it is possible to obtain the same form of loss function as in Woodford (2003) if we maintain the same order of approximation especially in terms of relative price distortion. Hence, it is worthwhile to mention the approximation order of the price-dispersion measure used in Woodford (2003). Specifically, the order of approximation in Woodford is $O(||\delta_{t-1}^{\frac{1}{2}}, \varphi, \tilde{\zeta}||^3)$. For this reason, we maintain the same order of approximation used in Woodford (2003) in terms of relative price distortion, though our definition differs from that of Woodford.

where $||\zeta||$ is a bound on the amplitude of exogenous shocks and $O(||\zeta||^3)$ denotes the order of approximation residual.

Following Woodford (2003), it is possible to express the second-order approximation to X_t in terms of its logarithmic deviation from steady state value as follows:

$$X_t - 1 = x_t + \frac{1}{2}x_t^2 + O(||\zeta||^3), \tag{81}$$

where output gap, denoted by x_t $(= \log X_t)$, is defined as logarithmic deviations of X_t from their steady state value. Substituting (81) into (80) and then rearranging, a quadratic approximation can be written as

$$\tfrac{C_t^{1-\sigma}}{1-\sigma} = \tfrac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}(x_t + \tfrac{(1-\sigma)x_t^2}{2}) + t.i.p. + O(||\zeta||^3), \tag{82}$$

where $t.i.p.$ collects terms that are independent of monetary policy. Besides, a second-order Taylor expansion to the utility function of labor can be written as a second-order Taylor expansion to the utility function of labor can be written as

$$\frac{(\frac{\Delta_t Y_t}{A_t})^{1+\chi}}{1+\chi} = \frac{N^{1+\chi}}{1+\chi} + N^{1+\chi}[(X_t - 1) + (\Delta_t - 1) + \frac{\chi(X_t-1)^2}{2}] + O(||\zeta||^3). \tag{83}$$

Similarly, substituting (81) into (83) and then rearranging yields

$$\frac{N_t^{\chi}}{1+\chi} = \frac{N^{1+\chi}}{1+\chi} + N^{1+\chi}[x_t + \frac{1+\chi}{2}x_t^2 + \delta_t] + t.i.p. + O(||\delta^{\frac{1}{2}}, \zeta||^3), \tag{84}$$

where the order of approximation residual is $O(||\delta^{\frac{1}{2}}, \zeta||^3)$.

We now discuss the role of distorting labor income tax or employment subsidy in deriving a quadratic loss function. Note that the marginal rate of substitution between consumption and labor at the steady state is given by

$$C^{1-\sigma} = N^{1+\varphi}. \tag{85}$$

Subtracting (84) from (82) and then setting the equality of (85) in the resulting equation, we can obtain a quadratic approximation to the instantaneous utility function of the representative household:

$$u_t = u^o - v[\delta_t + (\sigma+\chi)\frac{x_t^2}{2}] + t.i.p. + O(||\delta^{\frac{1}{2}}, \zeta||^3), \tag{86}$$

where $v = C^{1-\sigma}$, u_t is the instantaneous utility level at period t, and u^o is the steady-state instantaneous welfare level.

Next, I turn to the approximation of the measure of relative price distortion. A second-order Taylor expansion is

$$\delta_t = \xi\delta_{t-1} + \frac{\xi\epsilon}{2(1-\xi)}\pi_t^2 + O(||\delta^{\frac{1}{2}}, \zeta||^3). \tag{87}$$

Integrating forward from an initial value of δ_{-1} yields

$$\delta_t = \xi^{t+1}\delta_{-1} + (\frac{\xi\epsilon}{2(1-\xi)})\sum_{k=0}^{t}\xi^{t-k}\pi_k^2 + O(||\delta, \zeta||^3). \tag{88}$$

It follows from (88) that a discounted sum of logarithms of relative price distortions can be written as

$$\sum_{t=0}^{\infty}\beta^t\delta_t = \frac{\xi\epsilon}{(1-\xi)(1-\xi\beta)}\sum_{t=0}^{\infty}\beta^t\frac{\pi_t^2}{2} + \frac{\xi\delta_{-1}}{1-\xi\beta} + O(||\delta^{\frac{1}{2}},\zeta||^3). \tag{89}$$

As a result, substituting (89) into (88), we derive a quadratic loss function of the central bank from the utility function of the representative household:

$$-v\sum_{t=0}^{\infty}\beta^t E_0[\frac{\xi\epsilon}{(1-\xi)(1-\xi\beta)}\frac{\pi_t^2}{2} + (\sigma+\chi)\frac{x_t^2}{2}]. \tag{90}$$

Second-Order Approximation under Fixed Factors Inputs In this section, we proceeds with the Dixit-Stiglitz aggregator. In the presence of fixed factors inputs, the relative price distortion can be written as

$$\Delta_t = \{(1-\xi)(\frac{1-\xi\Pi_t^{\epsilon-1}}{1-\xi})^{\frac{-\epsilon}{(1-\epsilon)(1-\alpha_f)}} + \xi\Pi_t^{\frac{\epsilon}{1-\alpha_f}}\Delta_{t-1}^{\frac{1}{1-\alpha_f}}\}^{1-\alpha_f}. \tag{91}$$

Thus, the second-order Taylor approximation to the relative price distortion leads to

$$\delta_t = \xi\delta_{t-1} + \frac{\xi\epsilon}{2(1-\xi)\gamma}\pi_t^2 + O(||\delta^{\frac{1}{2}},\zeta||^3), \tag{92}$$

where γ is defined as

$$\gamma = \frac{1}{1+\epsilon\frac{\alpha_f}{1-\alpha_f}}. \tag{93}$$

In the similar way as we did for (88), we can show that a discounted sum of logarithms of relative price distortions is

$$\sum_{t=0}^{\infty}\beta^t\delta_t = \frac{\xi\epsilon}{(1-\xi)(1-\xi\beta)\gamma}\sum_{t=0}^{\infty}\beta^t\frac{\pi_t^2}{2} + \frac{\xi\delta_{-1}}{1-\xi\beta} + O(||\delta^{\frac{1}{2}},\zeta||^3). \tag{94}$$

Besides, since the presence of fixed factor inputs leads to

$$\frac{N_t^{\chi}}{1+\chi} = \frac{N^{1+\chi}}{1+\chi} + N^{1+\chi}[x_t + \frac{1+\chi-\alpha_{vl}}{2\alpha_{vl}}x_t^2 + \delta_t] + t.i.p. + O(||\delta^{\frac{1}{2}},\zeta||^3), \tag{95}$$

we have the following second-order approximation of the utility function:

$$u_t = u^o - v[\delta_t + (\sigma + \frac{1+\chi-\alpha_{vl}}{2\alpha_{vl}})\frac{x_t^2}{2}] + t.i.p. + O(||\delta^{\frac{1}{2}},\zeta||^3). \tag{96}$$

As a result, we derive a quadratic loss function of the central bank from the utility function of the representative household as follows:

$$-v\sum_{t=0}^{\infty}\beta^t E_0[\frac{\xi\epsilon}{(1-\xi)(1-\xi\beta)\gamma}\frac{\pi_t^2}{2} + (\sigma + \frac{\chi+1-\alpha_{vl}}{\alpha_{vl}})\frac{x_t^2}{2}]. \tag{97}$$