An Estimated Business Cycle Model With Stage-Specific Technological Change

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Abstract

This paper develops and estimates a business-cycle model where firms operate at different processing stages while being vertically integrated through an input-output linkage. Producers face a stage-specific Calvo-probability of reoptimizing nominal prices in each period. Nominal wages are determined by staggered contracts. The model includes stage-specific technology shocks and demand shocks. We show that exogenous variations in the pace of technology at the intermediate stage, not at the final stage, account for the bulk of business-cycle fluctuations. The model generates predictions consistent with the Dunlop-Tarshis observation of a weak cyclicality of real wages and its modern reincarnation of a near-zero correlation between hours and productivity.

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1 Introduction

Relying on structural vector autoregression (SVAR) models, a recent empirical literature has considerably undermined the importance of technology shocks as a source of business cycles.[1] Gali (1999), for example, finds that the technology-driven correlation between output and hours worked is slightly negative, contrary to the strong positive comovement of GDP and labor input observed during the postwar period in several industrialized countries. Also, Christiano, Eichenbaum and Vigfusson (2004) estimate that technology shocks explain 10 percent or less of the cyclical variance of output.[2] These findings cast serious doubts about macroeconomic theories in which exogenous changes in the pace of technology are an important source of cyclical variation in aggregate time series.[3] Thus, identifying the causes of business-cycle fluctuations still is highly controversial more than two decades after the influential work of Kydland and Prescott (1982).

In response to these findings, Chari, Kehoe and McGrattan (2005) have argued that the SVAR approach is not a useful guide to construct and evaluate dynamic general equilibrium models.[4] Another line of research followed by Fisher (2006), while not questioning the SVAR approach itself, has seek to demonstrate that investment-specific, as opposed to neutral, technological change matters for business cycles.[5] Yet, our paper proposes a different explanation, as well as new evidence of the importance of technological change for short-run fluctuations.

In most business cycle models, from standard real business cycle (RBC) models to dynamic general equilibrium (DGE) models that feature nominal rigidities and (or) real frictions, firms are assumed to operate at the final stage of production, hence facing technological change only at that stage. But in reality, several goods are produced through more than one stage. This simple fact forces some potentially important questions about the role of technological change as a source of fluctuations. First, can exogenous variations in the pace of technology at other than the final stage have an impact on aggregate fluctuations? Second, if the answer is affirmative, is this effect large quantitatively? Third, if it is found that technological change is significant source of cyclical fluctuations in a multi-stage production framework, can the same model's structure also remedy the anomalies that have plagued several models in which stochastic change in total factor productivity is a driving force?

Our paper offers positive answers to these three questions using an estimated, dynamic general equilibrium (DGE) model that has four main features. First, it embeds a multi-stage production and pricing structure. Firms produce differentiated goods at each stage and are linked *vertically* through an input-output linkage between processing stages. They face technological change which is specific to their processing stage. While similar in spirit, our framework differs from the models of Basu (1995) and Huang, Liu and Phaneuf (2004)

[1] See, e.g. Gali (1999), Basu, Fernald and Kimball (2004), Christiano, Eichenbaum and Vigfusson (2004), Francis and Ramey (2005), Fernald (2005), and Gali and Rabanal (2005), among others.

[2] See Christiano *et al.* (2004, Table 5). The results are those from the six-variable SVAR. The contribution of technology shocks to the cyclical variance of output is 10 percent with hours measured in levels, and 1.6 percent with hours in differences. The corresponding percentages for the cyclical variance of hours are 4.1 percent and 6.1 percent.

[3] Francis and Ramey (2005) even announce the death of the technology-driven real business cycle hypothesis.

[4] See, however, the rebutals in Gali and Rabanal (2005) and Christiano, Eichenbaum and Vigfusson (2006).

[5] However, Gali and Rabanal (2005) show that investment-specific technology shocks do not explain much of business-cycle fluctuations over the postwar period if hours or employment are specified in first differences rather than levels. Fisher (2006) finds that these shocks had a much stronger impact on aggregate fluctuations after 1982.

in which firms are linked through a *horizontal* roundabout input-output structure within a *single*, final stage of production. Our framework is more closely related to the model of Huang and Liu (2005) that features an input-output linkage between sectors in order to analyze the design and implementation of optimal monetary policy when there are several sources of nominal price rigidities.

Our model is made more tractable and easier to estimate by assuming that final output is produced through two processing stages: one stage of intermediate goods and one of finished goods. Firms at the intermediate stage use capital and labor to produce differentiated intermediate goods, while firms at the final stage utilize a composite of goods produced at the intermediate stage as an intermediate input, in addition to capital and labor, to produce finished goods.

Second, the model incorporates Calvo-type nominal price and wage contracts. Producers at each processing stage charge their own price and revise price-setting decisions upon receiving a stage-specific, random signal allowing them to change their price. Households have differentiated labor skills and their preferences are subject to a shock, which is often interpreted as a disturbance to the expectational IS curve in a new keynesian context. In some recent work, this type of shock has been identified as the main source of employment and output fluctuations during the postwar period [Hall (1997) and Gali and Rabanal (2005)].[6] Households face in each period a constant probability that their nominal wages can be changed.

Third, our framework includes real frictions in the form of costs that firms have to pay to adjust capital and hours worked. The role of capital adjustment costs is stressed in Kim (2000) and Christiano, Eichenbaum and Evans (2005). Bordo, Erceg and Evans (2000) offer evidence showing that labor adjustment costs and sluggish nominal wage adjustment have contributed to the severity of the Great Depression in the face of monetary shocks, while the evidence in Ambler, Guay and Phaneuf (2006) shows that they have played an essential role in shaping U.S. postwar business cycle dynamics.

The fourth and final ingredient of our model is a monetary policy rule that sets short-term nominal interest rates in response to variations in finished-good inflation and output produced at the final stage (both measured in deviations from their steady-state values). The rule includes both interest rate smoothing [e.g. Rotemberg and Woodford (1999) and Clarida, Gali and Gertler (2000)], and serial correlation in the unsystematic intervention of the monetary authority. It has sometime been argued that the evidence of highly positive coefficients on the lagged interest rate in estimated rules could simply reflect serially correlated errors corresponding to the Fed's reaction to factors not included in the policy rule [see for example Rudebusch (2002)].

The structural parameters of the model and various second moments of the data are estimated with postwar, quarterly time series for the U.S. economy using a maximum likelihood procedure. Our main findings are as follows. First, the key structural parameters of the model, including the share of intermediate input entering the production of finished goods and the parameters governing nominal contracts and real frictions, are economically meaningful and statistically significant. Our estimates reveal that the share of labor input in the production of finished goods is about 2/3 at both stages, while the share of intermediate input in the production of finished goods is about 0.24. Nominal price contracts at the final stage are somewhat shorter than their counterpart at intermediate stage, lasting on average 2.9 quarters compared to

[6] Ireland (2004) also finds that preference shocks had a strong impact on the variability of inflation.

3.3 quarters. The average duration of nominal wage contracts, however, is higher at 6.5 quarters.

Second, intermediate-stage technology shocks account for the bulk of economic fluctuations in the postwar U.S. economy contributing, for example, to 31, 52 and 62 per cent of the variance of the one, four and eight-quarters ahead forecast errors in real GDP. The vertical integration of stages greatly magnifies the effects of intermediate-stage technology shocks. As the share of intermediate inputs into the production of finished goods grows, the final and intermediate stages become more vertically integrated, so that a change in the pace of technology at the intermediate stage generates a larger impact on final output and a smaller change in relative prices. We also show that nominal rigidities play a significant role in determining the real effects of technology shocks. A notable aspect of the dynamics implied by intermediate-stage technology shocks is that they generate hump-shaped impulse responses in several aggregate variables, including output, employment, consumption and investment.[7] Meanwhile, technology at the final stage explains only a small fraction of the cyclical variance of output, a finding which is broadly consistent with the SVAR evidence of Christiano *et al.* (2004). Still, with their effects combined, technology shocks explain a large fraction of fluctuations in total hours and output.

Third, monetary policy shocks contribute significantly to output fluctuations in the very short-run, but their effect declines rapidly at longer horizons. However, monetary shocks account for a high percentage of the volatility of finished-goods inflation (explaining more than 70 percent of its variance decomposition at a horizon of one to forty quarters). Technology shocks explain a non negligible fraction of the variability of finished-goods inflation. Preference shocks only have a small impact on cyclical output and almost no effect on the variability of inflation.

Fourth, the estimated benchmark model has major implications for key second moments of aggregate time series. It does well in reproducing the relative magnitude of fluctuations in several aggregate variables predicting, for example, that hours worked are slightly less volatile than output and that average labor productivity fluctuates much less than real GDP.[8] Also, assuming that the price index of finished goods is roughly approximated by the consumer price index (or the GDP deflator), while the price index of intermediate goods corresponds to the producer price index, the model predicts that inflation at the lower stage of production is about twice as volatile as inflation at the higher stage, just as in the data.

Fifth, the model also succeeds well in explaining some key comovements between variables. For example, it predicts that the correlation between hours and output is both positive and high, while identifying intermediate-stage technology shocks as the main source of that comovement. At the same time, it predicts a persistent decline in hours following a positive final-stage technology shock, consistent with the SVAR evidence in Gali (1999). The intuition for the different responses of hours depending on the source of technological change is the following. The effects of intermediate-stage technology shocks, unlike those of final-stage technology shocks, are propagated through the vertical input-output linkage between stages. Furthermore, at each stage, prices are a markup over the marginal cost. The marginal cost of firms operating at the

[7] King, Plosser and Rebelo (1988) show that the standard technology-driven, neoclassical growth model fails to generate hump-shaped impulse responses and is unable to produce positive serial correlation in output, investment and employment growth over short horizons.

[8] Many models where technology shocks are a main driving force have problems explaining these facts simultaneously [see Hansen (1985), and Hansen and Wright (1992)].

intermediate stage is composed of a rigid wage index and a flexible rental rate on capital. The marginal cost of firms producing at the final stage has an additional rigid component in the form of a rigid price index of the intermediate input entering the production of finished goods. It follows that, in response to an intermediate-stage technology shock, hours respond more like what a RBC model would predict, while following a final-stage technology shock, hours adjust more like in the sticky-price model of Gali (1999).

Perhaps more significant, however, is the model's ability to overcome some important anomalies that have been associated with technology-driven business cycle models. They pertain to the weak cyclical pattern of real wages observed in reality, often referred to as the Dunlop-Tarshis observation, which has been reinterpreted recently by Christiano and Eichenbaum (1992) as the near-zero correlation between the average labor productivity and total hours worked. Most business cycle models where technology shocks are the main driving force predict highly procyclical real wages and a high, positive correlation between hours and productivity. The benchmark model predicts a weak procyclicality of real wages and a correlation between hours and productivity which is close to zero, just as in the data. Interestingly, while the intermediate-stage technology shock triggers a positive correlation between hours and productivity, this correlation is negative conditional on the final-stage technology shock and of an order of magnitude which is broadly consistent with the technology-driven correlation of hours and productivity generated by SVAR models in Gali (1999).

The paper is organized as follows. Section 2 provides a description of our two-stage model with nominal rigidities. Section 3 discusses some estimation issues, data and calibration. Section 4 presents our estimation results and analyzes our main findings. Section 5 offers concluding remarks.

2 The model

The economy is inhabited by a large number of infinitely lived households endowed with differentiated labor skills. Households have preferences defined over expected streams of consumption goods, real balances and leisure. They face in each period a constant probability that their nominal wages are reoptimized. Differentiated goods are produced both at the intermediate and final processing stages. The two stages are vertically integrated, firms at the final stage using a composite of goods produced at the intermediate stage as an input. Exogenous technological change is specific to each processing stage. Producers at any given stage can change nominal prices according to a stage-specific probability that allows them to do so. Physical capital and hours worked are costly to adjust.

2.1 The Households

We assume a continuum of households indexed by i, with $i \in [0, 1]$ denoting a specific labor skill. Household $i's$ preferences are described by the following expected utility function:

$$U(i)_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma}{\gamma - 1} \kappa_t \log \left(C_t(i)^{\frac{\gamma-1}{\gamma}} + b^{\frac{1}{\gamma}} \left(\frac{M_t(i)}{P_{y,t}} \right)^{\frac{\gamma-1}{\gamma}} \right) - \mu \frac{N_t(i)^{1+\eta}}{1+\eta} \right], \tag{1}$$

where β is a discount factor, $C_t(i)$ is real consumption, $M_t(i)/P_{y,t}$ denotes real money balances, $M_t(i)$ is the nominal money stock, $P_{y,t}$ is the price index of finished goods, and $N_t(i)$ is hours worked; γ, b, μ and η are positive structural parameters with γ and η representing, respectively, the constant elasticity of substitution between consumption and real balances, and the inverse of the labor supply elasticity. In each period, the representative household's total time endowment is normalized to one.

The preference shock, κ_t, has the time-series representation:

$$\log(\kappa_t) = \rho_\kappa \log(\kappa_{t-1}) + \varepsilon_{\kappa,t}, \tag{2}$$

where $\varepsilon_{\kappa,t}$ is a serially uncorrelated independent and identically distributed process with a mean-zero and a standard error σ_κ.

The household $i's$ budget constraint is

$$\begin{aligned} & C_t(i) + I_t(i) + CAC_t(i) + \frac{M_t(i)}{P_{y,t}} + \frac{B_{t+1}(i)}{P_{y,t}} \\ = \; & \frac{W_t(i)}{P_{y,t}} N_t(i) + \frac{Q_t}{P_{y,t}} K_t(i) + \frac{M_{t-1}(i)}{P_{y,t}} + R_{t-1}\frac{B_t(i)}{P_{y,t}} + \frac{D_{y,t}(i)}{P_{y,t}} + \frac{D_{z,t}(i)}{P_{y,t}} + \frac{T_t(i)}{P_{y,t}}, \end{aligned} \tag{3}$$

where $I_t(i)$ is real investment, $CAC_t(i)$ is the real adjustment cost of physical capital, $B_{t+1}(i)$ is bonds carried in period $t+1$, $W_t(i)$ is the nominal wage rate, Q_t is the nominal rental rate of capital, $K_t(i)$ is the stock of physical capital, R_{t-1} is the gross nominal interest rate between period $t-1$ and period t, $D_{y,t}(i)$ and $D_{z,t}(i)$ are the nominal dividends paid to the household by firms producing at the final stage and firms producing at the intermediate stage, respectively, and $T_t(i)$ is a lump-sum nominal transfer received from the monetary authority.

The cost of adjusting the physical stock of capital is given by the function

$$CAC_t(i) = \frac{\varphi_k}{2}\left(\frac{K_{t+1}(i)}{K_t(i)} - 1\right)^2 K_t(i), \tag{4}$$

where $\varphi_k > 0$ is the capital-adjustment cost parameter.

The capital stock evolves according to

$$K_{t+1}(i) = (1-\delta)K_t(i) + I_t(i), \tag{5}$$

where δ is the rate of depreciation of physical capital.

Aggregate labor input, N_t, is a composite of all differentiated labor skills:

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where σ represents the elasticity of substitution between labor skills. Labor demand for skill i is,

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\sigma} N_t, \tag{7}$$

where W_t is the wage rate of the composite skill which given by

$$W_t = \left(\int_0^1 W_t(i)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \tag{8}$$

The household i chooses $C_t(i)$, $M_t(i)$, $B_{t+1}(i)$, $K_{t+1}(i)$ and $W_t(i)$, when nominal wages can be adjusted, that maximize the expected discounted sum of utility flows, subject to the budget constraint and the firms' labor demand for skill i. The first-order conditions for this problem are:

$$\frac{\kappa_t C_t(i)^{\frac{-1}{\gamma}}}{C_t(i)^{\frac{\gamma-1}{\gamma}}+b_t^{\frac{1}{\gamma}}\left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{\gamma-1}{\gamma}}}=\lambda_t(i), \tag{9}$$

$$\frac{\kappa_t b_t^{\frac{1}{\gamma}}\left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{-1}{\gamma}}}{C_t(i)^{\frac{\gamma-1}{\gamma}}+b_t^{\frac{1}{\gamma}}\left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{\gamma-1}{\gamma}}}=\lambda_t(i)\left(1-\frac{1}{R_t}\right), \tag{10}$$

$$\begin{aligned}&\beta E_t\frac{\lambda_{t+1}(i)}{\lambda_t(i)}\left[q_{t+1}+1-\delta+\varphi_k\left(\frac{K_{t+2}(i)}{K_{t+1}(i)}-1\right)\frac{K_{t+2}(i)}{K_{t+1}(i)}-\frac{\varphi_k}{2}\left(\frac{K_{t+2}(i)}{K_{t+1}(i)}-1\right)^2\right]\\ =\;&1+\varphi_k E_t\left(\frac{K_{t+1}(i)}{K_t(i)}-1\right),\end{aligned} \tag{11}$$

$$\lambda_t(i)=\beta R_t E_t\left(\frac{\lambda_{t+1}(i)}{\pi_{y,t+1}}\right). \tag{12}$$

where $\lambda_t(i)$ is the nonnegative Lagrange multiplier associated with the budget constraint, $q_t = Q_t/P_{y,t}$, and $\pi_{y,t+1}$ is the rate of inflation of finished goods.

2.1.1 Nominal wage setting

At the beginning of each period, the nominal wage has probability $1-d_w$ of being reoptimized. The first-order condition with respect to $W_t(i)$ is

$$\widetilde{W}_t(i)=\frac{\sigma}{\sigma-1}\frac{E_t\sum_{q=0}^{\infty}(\beta d_w)^q N_{t+q}(i)^{\eta+1}}{E_t\sum_{q=0}^{\infty}(\beta d_w)^q N_{t+q}(i)\lambda_{t+q}(i)\frac{1}{P_{y,t+q}}}. \tag{13}$$

At the symmetric equilibrium, the aggregate nominal wage obeys the following recursive equation:

$$W_t=\left[d_w W_{t-1}^{1-\sigma}+(1-d_w)\widetilde{W}_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{14}$$

where $\widetilde{W}_t$ is the optimal or average wage of workers allowed to revise nominal wages at time t.

2.2 Firms and Stages of Production

Monopolistically competitive firms produce differentiated goods at both stages and are linked by a vertical input-output structure. They are also price-setters at each stage. Nominal prices at the final stage have probability d_y of survival in each period, while the probability that prices at the intermediate stage remain unchanged in each period is d_z.

2.2.1 Final stage

Final output, Y_t, is a composite of the differentiated finished goods, $Y_t(j)$, $j \in [0,1]$ denoting a type of finished good,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta_y - 1}{\theta_y}} dj \right)^{\frac{\theta_y}{\theta_y - 1}},$$

where θ_y is the elasticity of substitution between differentiated finished goods.

Given prices $P_{y,t}$ and $P_{y,t}(j)$, the finished-good–producing firm j maximizes its profits choosing the production of finished goods, $Y_t(j)$. It solves the following problem

$$\max_{Y_t(j)} P_{y,t} \left(\int_0^1 Y_t(j)^{\frac{\theta_y - 1}{\theta_y}} dj \right)^{\frac{\theta_y}{\theta_y - 1}} - \int_0^1 P_{y,t}(j) Y_t(j) dj.$$

Profit maximization leads to the following first–order condition for the demand of finished good j

$$Y_t(j) = \left(\frac{P_{y,t}(j)}{P_{y,t}} \right)^{-\theta_y} Y_t, \tag{15}$$

where the price index of finished goods is

$$P_{y,t} = \left(\int_0^1 P_{y,t}(j)^{1-\theta_y} dj \right)^{\frac{1}{1-\theta_y}}.$$

2.2.2 Intermediate stage

Intermediate output, Z_t, is a composite of differentiated goods produced at the intermediate stage, $Z_t(l)$, $l \in [0,1]$ denoting a type of intermediate good

$$Z_t = \left(\int_0^1 Z_t(l)^{\frac{\theta_z - 1}{\theta_z}} dl \right)^{\frac{\theta_z}{\theta_z - 1}},$$

where $\theta_z \in (1, \infty)$ is the elasticity of substitution between differentiated intermediate goods.

Profit maximization leads to the following first–order condition for the demand of intermediate good l

$$Z_t(l) = \left(\frac{P_{z,t}(l)}{P_{z,t}} \right)^{-\theta_z} Z_t, \tag{16}$$

where $P_{z,t}$ corresponds to the price index of differentiated intermediate goods and $P_{z,t}(l)$ is the price of intermediate good l. The price index of differentiated goods is given by

$$P_{z,t} = \left(\int_0^1 P_{z,t}(l)^{1-\theta_z} dl \right)^{\frac{1}{1-\theta_z}}.$$

2.2.3 Finished–good–producing firm

Producing a finished good j requires the use of labor $N_{y,t}(j)$, capital $K_{y,t}(j)$, and a composite of intermediate goods, $Z_t(j)$. Firms utilize a constant returns to scale (CRS) technology

$$Y_t(j) = Z_t(j)^{\phi} \left[A_{y,t} K_{y,t}(j)^{\alpha_y} N_{y,t}(j)^{1-\alpha_y}\right]^{1-\phi}, \tag{17}$$

where α_y is the final-stage share of capital and and ϕ is the share of intermediate goods entering the production of finished goods.

The final-stage productivity shock, $A_{y,t}$, follows a log-difference stationary process

$$\log(A_{y,t}) = (1-\rho_{A,y})\log(A_y) + \rho_{A,y}\log(A_{y,t-1}) + \varepsilon_{y,t}, \tag{18}$$

where $\varepsilon_{y,t}$ is a mean–zero, iid normal process that is independent, with a standard error σ_y.

Adjusting the final-stage labor input is costly. Labor adjustment costs are measured as a proportional loss of final output:

$$LAC_{y,t}(j) = \frac{\varphi_y}{2}\left(\frac{N_{y,t}(j)}{N_{y,t-1}(j)} - 1\right)^2 Y_t, \qquad \varphi_y \geqslant 0, \tag{19}$$

where φ_y is the final-stage labor adjustment-cost parameter.

Firms are price-takers in the markets for inputs and monopolistic competitors in the markets for products. At each processing stage, nominal prices are chosen optimally in a randomly staggered fashion. At the beginning of each period, a fraction (1− d_y) of final-stage producers can change their prices.

The maximization problem for the finished-good producing firm j is :

$$\max_{\{K_{y,t}(j),N_{y,t}(j),Z_t(j),P_{y,t}(j)\}} E_t \sum_{q=0}^{\infty} (\beta d_y)^q \frac{\lambda_{t+q}}{\lambda_t}\frac{D_{y,t+q}(j)}{P_{t+q}},$$

subject to:

$$D_{y,t}(j) = P_{y,t}(j)Y_t(j) - Q_t K_{y,t}(j) - W_t N_{y,t}(j) - P_{z,t}Z_t(j) - P_{y,t}LAC_{y,t}(j),$$

$$Y_t(j) = \left(\frac{P_{y,t}(j)}{P_{y,t}}\right)^{-\theta_y} Y_t,$$

and

$$Y_t(j) = Z_t(j)^{\phi}\left[A_{y,t}K_{y,t}^{\alpha_y} N_{y,t}(j)^{1-\alpha_y}\right]^{1-\phi}.$$

The first–order conditions for this maximization problem are:

$$\begin{aligned} w_t &= (1-\alpha_y)(1-\phi)\zeta_{y,t}(j)\frac{Y_t(j)}{N_{y,t}(j)} - \varphi_y \frac{Y_t}{N_{y,t-1}(j)}\left(\frac{N_{y,t}(j)}{N_{y,t-1}(j)} - 1\right) + \\ &\quad \beta\varphi_y E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{N_{y,t}(j)}\frac{N_{y,t+1}(j)}{N_{y,t}(j)}\left(\frac{N_{y,t+1}(j)}{N_{y,t}(j)} - 1\right), \end{aligned} \tag{20}$$

$$q_t = \alpha_y(1-\phi)\zeta_{y,t}(j)\frac{Y_t(j)}{K_{y,t}(j)}, \tag{21}$$

$$p_{z,t} = \phi\zeta_{y,t}(j)\frac{Y_t(j)}{Z_t(j)}, \tag{22}$$

where $w_t = W_t/P_{y,t}$ is the real wage, $p_{z,t} = P_{z,t}/P_{y,t}$ is the relative price of the intermediate input, and $\zeta_{y,t}(j)$ is firm $j's$ real marginal cost .

2.2.4 Finished-good price determination

The first-order condition with respect to $P_{y,t}(j)$ is:

$$\tilde{P}_{y,t}(j) = \frac{\theta_y}{\theta_y - 1}\frac{E_t\sum_{q=0}^{\infty}(\beta d_y)^q\frac{\lambda_{t+q}}{\lambda_t}\zeta_{y,t}(j)Y_{t+q}(j)}{E_t\sum_{q=0}^{\infty}(\beta d_y)^q\frac{\lambda_{t+q}}{\lambda_t}Y_{t+q}(j)\frac{1}{P_{y,t+q}}}. \tag{23}$$

At the symmetric equilibrium, the aggregate price of finished goods is

$$P_{y,t} = \left[d_y P_{y,t-1}^{1-\theta_y} + (1-d_y)\tilde{P}_{y,t}^{1-\theta_y}\right]^{\frac{1}{1-\theta_y}}, \tag{24}$$

where $\tilde{P}_{y,t}$ is the optimal or average price of finished-good-producing firms allowed to change their prices at time t.

2.2.5 Intermediate–good–producing firm

The intermediate firm l rents capital, $K_{z,t}(l)$ and hires workers, $N_{z,t}(l)$. It uses a CRS technology to produce intermediate good $Z_t(l)$

$$Z_t(l) = A_{z,t}K_{z,t}(l)^{\alpha_z}N_{z,t}(l)^{1-\alpha_z}, \tag{25}$$

where α_z is the share of capital at the intermediate stage.

The intermediate-stage productivity shock, $A_{z,t}$, follows a log-difference stationary process

$$\log(A_{z,t}) = (1-\rho_{A,z})\log(A_z) + \rho_{A,z}\log(A_{z,t-1}) + \varepsilon_{z,t}, \tag{26}$$

where $\varepsilon_{z,t}$ is a mean–zero, iid normally distributed process with a standard error σ_z.

Firms at the intermediate stage must pay a cost to vary the labor input. The adjustment-cost function, $LAC_{z,t}(l)$, is

$$LAC_{z,t}(l) = \frac{\varphi_z}{2}\left(\frac{N_{z,t}(l)}{N_{z,t-1}(l)} - 1\right)^2 Z_t, \qquad \varphi_z \geqslant 0, \tag{27}$$

where φ_z is the intermediate-stage labor adjustment-cost parameter.

Firm l solves the following maximization problem:

$$\max_{\{K_{z,t}(l),N_{z,t}(l),P_{z,t}(l)\}} E_t\sum_{q=0}^{\infty}(\beta d_z)^q\frac{\lambda_{t+q}}{\lambda_t}\frac{D_{z,t+q}(l)}{P_{y,t+q}},$$

subject to:

$$D_{z,t}(l) = P_{z,t}(l)Z_t(l) - Q_t K_{z,t}(l) - W_t N_{z,t}(l) - P_{z,t}LAC_{z,t}(l),$$
$$Z_t(l) = \left(\frac{P_{z,t}(l)}{P_{z,t}}\right)^{-\theta_z} Z_t,$$

and

$$Z_t(l) = A_{z,t}K_{z,t}(l)^{\alpha_z} N_{z,t}(l)^{1-\alpha_z}.$$

The first–order conditions for this maximization problem are:

$$\begin{aligned} w_t &= (1-\alpha_z)\zeta_{z,t}(l)\frac{Z_t(l)}{N_{z,t}(l)} - \varphi_z \frac{p_{z,t}Z_t}{N_{z,t-1}(j)}\left(\frac{N_{z,t}(l)}{N_{z,t-1}(l)} - 1\right) + \\ & \beta\varphi_z E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{p_{z,t+1}Z_{t+1}}{N_{z,t}(j)}\frac{N_{z,t+1}(l)}{N_{z,t}(l)}\left(\frac{N_{z,t+1}(l)}{N_{z,t}(l)} - 1\right), \end{aligned} \tag{28}$$

$$q_t = \alpha_z \zeta_{z,t}(l)\frac{Z_t(l)}{K_{z,t}(l)}. \tag{29}$$

where $\zeta_{z,t}(l)$ is firm $l's$ real marginal cost.

2.2.6 Intermediate-stage price determination

The first-order condition with respect to $P_{z,t}(l)$ is

$$\tilde{P}_{z,t}(l) = \frac{\theta_z}{\theta_z - 1}\frac{E_t \sum_{q=0}^{\infty}(\beta d_z)^q \frac{\lambda_{t+q}}{\lambda_t}\zeta_{z,t}(l)Z_{t+q}(l)}{E_t \sum_{q=0}^{\infty}(\beta d_z)^q \frac{\lambda_{t+q}}{\lambda_t}Z_{t+q}(l)\frac{1}{P_{z,t+l}}}. \tag{30}$$

At the symmetric equilibrium, the aggregate price of intermediate-stage goods is

$$P_{z,t} = \left[d_z P_{z,t-1}^{1-\theta_z} + (1-d_z)\tilde{P}_{z,t}^{1-\theta_z}\right]^{\frac{1}{1-\theta_z}}, \tag{31}$$

where $\tilde{P}_{z,t}$ is the optimal or average price of intermediate-good-producing firms allowed to change their prices at time t.

2.3 Monetary Policy

Monetary policy sets the short-term nominal interest rate, R_t, in response to finished-good inflation, $\pi_{y,t}$, and output produced at the final stage, y_t, measured in deviations from their steady-state values. The rule includes both an interest-rate smoothing term and an autocorrelated policy shock. The policy rule is

$$\log\left(\frac{R_t}{R^*}\right) = \rho_R\left(\frac{R_{t-1}}{R^*}\right) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_{y,t}}{\pi_y^*}\right) + \rho_y \log\left(\frac{y_t}{y^*}\right)\right] + v_t, \tag{32}$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t}. \tag{33}$$

The variables π_y^* and y^* represent steady-state values of $\pi_{y,t}$ and y_t, respectively, while R^* is the steady-state, gross nominal interest rate, and $\varepsilon_{v,t}$ is a mean–zero, iid normally distributed process with a standard error σ_v.

2.4 Closing the Model

At the symmetric equilibrium, the market-clearing conditions are:

$$K_t = K_{y,t} + K_{z,t}, \tag{34}$$

$$N_t = N_{y,t} + N_{z,t}, \tag{35}$$

$$Y_t = C_t + I_t + CAC_t + LAC_{y,t} + LAC_{z,t}, \tag{36}$$

and

$$M_t - M_{t-1} = T_t. \tag{37}$$

2.5 Equilibrium

An equilibrium consists, for $k = y, z$, in a set of allocations $\{C_t, N_t, B_t, m_t, K_{t+1}, Y_t, I_t, Z_t, K_{k,t}, N_{k,t}, \pi_{k,t}, \pi_{w,t}, p_{z,t}, w_t, \zeta_{k,t,} q_t, R_t\}_{t=0}^{\infty}$ (where $m_t \equiv M_t/P_{y,t}$) that satisfies the following conditions: (i) the household's allocations solve its utility maximization problem; (ii) each finished-good producer's allocations and price solve its profit maximization problem taking the wage and all prices but its own as given; (iii) each intermediate-good producer's allocations and price solve its profit maximization problem; and (iv) all markets clear.

3 Estimation Methodology, Data and Calibration

3.1 Estimation Procedure

The model is solved through log-linearization of its equilibrium conditions around a symmetric steady state where all variables are constant. It is assumed that the steady-state, finished-good, gross rate of inflation is equal to one. The linearized system leads to the following state space representation:

$$\begin{aligned} \mathcal{X}_t &= \mathbf{A}\mathcal{X}_{t-1} + \mathbf{B}\boldsymbol{\epsilon}_t, & (38) \\ \mathcal{Y}_t &= \mathbf{C}\mathcal{X}_t, & (39) \end{aligned}$$

where the vector $\mathcal{X}_t$ keeps track of the model's predetermined and exogenous variables, and the vector $\mathcal{Y}_t$ includes the remaining endogenous variables. The Kalman filter is used to evaluate the likelihood function, $\mathcal{L}(Y^T|\Theta)$, associated with the state-space solution. Prior to the estimation, we define the following vector of observable:

$$\mathcal{Z}_t = \begin{bmatrix} \hat{c}_t & \hat{y}_t & \widehat{R}_t & \hat{\pi}_{y,t} & \hat{y}_t - \hat{n}_t & \hat{w}_t \end{bmatrix}'$$

which consists of real consumption, output, the nominal interest rate, the rate of finished-good inflation, the average productivity of labor, and the real wages, all expressed as percentage deviations from their own steady-state values.

Since the model is driven by four structural shocks, it would in principle limit to four the number of observed variables required in the estimation to avoid stochastic singularity. However, the number of variables used to estimate the structural parameters of the model can be increased by adding measurement errors [see also Altug (1989), Sargent (1989), McGrattan (1994), Hall (1996), and Ireland (2004)]. Hence, the model is augmented with a vector of two measurement errors, e_t. The system of equations for the selected variables is

$$\mathcal{Z}_t = \mathbf{K} \begin{pmatrix} \mathcal{X}_t \\ \mathcal{Y}_t \end{pmatrix} + \mathbf{L} \begin{pmatrix} \epsilon_t \\ e_t \end{pmatrix} \tag{40}$$

where the matrices $\mathbf{K}$ and $\mathbf{L}$ are obtained after selecting the appropriate variables in $\mathcal{X}_t$, $\mathcal{Y}_t$, and the vector of errors. The measurement errors, which are assumed to be independent from the structural shocks, follow the autoregressive process:

$$e_{t+1} = \mathbf{M} e_t + \upsilon_t \tag{41}$$

$$E\left(\upsilon_t \upsilon_t'\right) = \Sigma_\upsilon \tag{42}$$

where the two matrices $\mathbf{M}$ and Σ_υ are diagonal.

3.2 Data

The model is estimated with U.S. quarterly data for the period 1960Q1 to 2004Q4. The nominal interest rate is the 3-month Treasury Bill Rate. The rate of finished-good inflation is measured by the quarterly rate of change of the consumer price index. Consumption is measured by real personal consumption expenditures for non durables and services. Output is measured by real GDP. The real wage is the ratio of the nonfarm business sector compensation to the consumer price index. All series, except the nominal interest rate, are seasonally adjusted. Consumption, output and hours are all converted into per capita terms, dividing by the civilian population. All series, except the rates of interest rate and the inflation rate, are logged and detrended using the HP filter.

3.3 Calibration

When estimating relatively large structural models by maximum likelihood, it is sometime difficult to obtain sensible estimates of all the structural parameters, either because some parameters are not easily identifiable or because the optimization algorithm fails to locate the maximum due the complexity of the objective function, so that the algorithm breaks down. To deal with this issue, some parameters can be calibrated prior to estimation. First, the subjective discount rate, β, is set to 0.995, implying an annual real interest rate of 2 percent in the steady state. The weight on leisure in the utility function, μ, is calibrated so that the representative household spends about one third of its total time working in the steady state. The depreciation rate of physical capital is chosen to be 0.025. The parameters θ_y and θ_z are set to 8, which yields steady-state markups of 14 percent, consistent with several estimates in the literature [see, for example, Basu 1995 and Huang, Liu and Phaneuf (2004)]. The elasticity of substitution between differentiated labor skills, σ, is set at 6, which is consistent with the microeconomic evidence produced by Griffin (1992) and the macroeconomic evidence obtained by Ambler, Guay and Phaneuf (2006).

4 Empirical Results

4.1 Parameter Estimates (Benchmark Model)

Table 1 summarizes the point estimates of the structural parameters of three different models. We use the label *benchmark model* for one that includes all the theoretical ingredients previously described. When estimating the benchmark model, we seek to estimate the following group of structural parameters $\{\rho_{A,z}, \rho_{A,y}, \rho_\kappa, \rho_\upsilon, \sigma_{A,z}, \sigma_{A,y}, \sigma_\kappa, \sigma_\upsilon, b, \gamma, \eta, \alpha_z, \phi, \alpha_y, d_z, d_y, d_w, \varphi_k, \varphi_z, \varphi_y, \rho_R, \rho_\pi, \rho_y\}$. We also estimate a model, labeled *Model I*, that features only one stage of production (the final stage), sticky prices at the final stage and sticky wages. Hence, when estimating Model I, the following parameter restrictions are imposed: $\rho_{A,z} = \sigma_{A,z} = \alpha_z = \phi = d_z = \varphi_z = 0$. This version contains three structural shocks instead of four, and is estimated using a vector of three measurement errors instead of two. A third version of the model, labeled *Model II*, features two processing stages, perfectly flexible nominal prices at the two stages and perfectly flexible nominal wages. Model II is estimated under the following parameter restrictions: $d_z = d_y = d_w = 0$. Model II, like the benchmark model, has four structural shocks.

In general, the structural parameters of the benchmark model are estimated quite precisely. The point estimate of γ is 0.0701, which implies an interest elasticity of money demand equal to -0.0754, consistent with the evidence in Ireland (2003) and Kim (2000). The parameter b, determining the relative importance of consumption with respect to real balances is 0.0744. The point estimate $\eta = 0.8831$, implies a labor supply elasticity of 1.13, consistent with the evidence presented in Mulligan (1999).

Our point estimates of $\rho_{A,y}$, $\rho_{A,z}$, $\sigma_{A,y}$ and $\sigma_{A,z}$ suggest that the intermediate-stage technology shock is somewhat more persistent than its final-stage counterpart and also has a slightly larger innovation. Our point estimate of the capital's share in intermediate-stage output, α_z, is 0.3407. The point estimates of the final-stage technology parameters are $\phi = 0.2416$ and $\alpha_y = 0.13$. These point estimates imply a share of labor in final-stage output of about 2/3.

Our point estimate of d_y, which is 0.6561, means that the average duration of final-stage price contracts is 2.9 quarters. With a point estimate $d_z = 0.6992$, price contracts at the intermediate stage last, on average, 3.3 quarters. These estimates suggest a moderate amount of price stickiness at both stages. Wage contracts, however, last longer on average than price contracts, with a point estimate of d_w of 0.8461 corresponding to a duration of 6.5 quarters.

The average duration of price contracts implied by our two-stage model is consistent with the evidence reported in Christiano, Eichenbaum and Evans (2005), which shows that nominal prices are reoptimized once every 2.5 quarters on average.[9] However, our evidence suggests that prices are changed much less frequently than in the sticky-price models of Gali and Gertler (1999) and Eichenbaum and Fisher (2004)

[9] Christiano *et al.* (2005) estimate a one-stage model featuring sticky nominal prices, sticky nominal wages, and several other real frictions. In their model, macroeconomic fluctuations are driven solely by monetary shocks.

where firms reoptimize prices once every six quarters, or in the model of Smets and Wouters (2003) where prices are adjusted once every nine quarters. Bils and Klenow (2004) and Golosov and Lucas (2003), based on microeconomic evidence, argue that firms reoptimize prices somewhat more frequently than our point estimates suggest.[10]

The estimated capital-adjustment-cost parameter, $\varphi_k = 9.5827$, is statistically significant and, as we show later, allows a reasonable match of the volatility of investment in the model. The point estimates of the labor-adjustment-cost parameters are $\varphi_y = 5.7406$ and $\varphi_z = 3.3746$, suggesting that it is less costly to adjust labor at the intermediate stage than at the final stage.

Looking at the parameters of the variables entering the policy rule, the point estimate of ρ_π, the response of the nominal interest rate to the finished-good rate of inflation, is 1.4702, which is close to 1.5, the value reported in Taylor (1993). The parameter ρ_y, which accompanies final output, is both close to zero and statistically insignificant. We do not find evidence of interest-rate smoothing, with a point estimate of ρ_R of 0.0918 which is statistically insignificant. Our point estimate of ρ_v of 0.1571, suggests a weak persistence in the unsystematic intervention of the monetary authority.

4.2 Sources of Business Cycles

How much does each type of shock contribute to aggregate fluctuations? We answer this question by generating forecast error variance decomposition of several variables using the estimated benchmark model. Table 2 reports the results at the infinite horizon. The intermediate-stage technology shock ε_z explains 72 per cent of the variance of final output, 45 per cent of the variance of total hours, 67 per cent of the variance of consumption and 81 per cent of the variance of investment. This shock also explains 76 per cent of the variance of intermediate-stage hours and 84 per cent of the cyclical variability of intermediate-stage output. Notice that, while ε_z contributes 37 per cent of the variance of final-stage hours, it does explain a high percentage of the variance of final output. Thus, the vertical integration of stages, through the use of Z_t by finished-good-producing firms as an intermediate input, substantially magnifies the effect of ε_z on the variability of final output.

The policy shock ε_v explains a modest 15 per cent of the variability in final output, and a larger proportion of the volatility of total hours with 22 per cent. However, the policy shock is an important determinant of inflation variability, accounting for as high as 71 per cent of the total volatility of finished-good inflation and 89 per cent of the volatility of intermediate-good inflation. Still, technology shocks ε_z and ε_y explain a non negligible fraction of about 1/4 of the variance of finished-good inflation once their effects are combined. The preference shocks explain a small percentage of variability in all variables.

Table 3 focuses on the forecast error variance decompositions of final-stage output and finished-good inflation at shorter horizons of one to forty quarters. Technology shocks account for the bulk of short-run output fluctuations. The intermediate-stage technology shock ε_z clearly represents the main driving force at business cycle frequencies (of say, one to twelve quarters). It explains 31 per cent of the one-quarter

[10] It is difficult, however, to establish a direct comparison between our evidence and theirs. For example, Bils and Klenow (2004) examine the frequency of price changes for 350 categories of goods and services covering about 70 percent of consumer spending between 1995 and 1997.

ahead forecast error in final output, and this percentage rises to 52, 62 and 66 per cent of the four-, eight-, and twelve-quarter ahead forecast error variance, respectively. Once their effects are combined, ε_z and ε_y contribute to 47 per cent, 59, 68 and 72 per cent of the variability of final output at the same horizons. The policy shock nonetheless exerts a significant short-run impact on the variability of final output, accounting for 51, 36, 25 and 20 per cent of the cyclical variance of output at similar horizons. Also, it is the dominant source of inflation variability. Finally, the preference shocks explain 8 per cent or less of the total volatility of final output at all horizons.

4.3 The Effects of Intermediate-Stage vs Final-Stage Technology Shocks

The estimated benchmark model generates rich predictions concerning the macroeconomic effects of technology shocks. Figure 1 displays the impulse response functions of the estimated benchmark model (solid lines) to a positive one percent intermediate-stage technology shock. Several aspects of these dynamic responses are worth noticing. A positive technology shock ε_z has a substantial, positive, cumulative impact on final output, total investment and total hours worked. Of course, ε_z has a stronger cumulative impact on intermediate-stage output Z and employment N_z. Also, it generates persistent, *hump-shaped* responses of final output, consumption, investment and hours worked, often seen as desirable characteristics a modern business cycle model should produce to be able to account for postwar business-cycle dynamics [see for example Cogley and Nason (1995)]. Also, following a positive ε_z, the relative price of intermediate input, p_z, declines persistently, with an initial response of -0.45 per cent and a peak response of about -1.5 per cent in the fifteenth quarter. The rate of inflation of intermediate goods declines persistently, with an initial response of -0.2 per cent. The rate of inflation of finished goods, which is not directly affected by the exogenous change in the pace of technology at the intermediate stage, initially increases by 0.22 per cent, and declines slightly after three periods. The real wages decrease on impact by 0.18 per cent, then rise in the fifth period, and reach a maximum increase of about 0.57 per cent before slowly returning to the pre-shock level. The nominal interest, which is linked to the rate of inflation of finished goods through the estimated Taylor rule, rises on impact and then begins to fall in the third period.

Figure 2 displays the impulse responses to a one percent final-stage technology shock. Notice first that in response to a positive technology shock ε_y, the relative price of intermediate input persistently rises, with an initial increase of 0.35 per cent, and a peak response of 0.55 per cent in the fourth quarter. Intermediate-stage output and hours worked decline. Unlike ε_z which boosts hours at the same stage, ε_y generates a *persistent, hump-shaped decline* in final-stage hours. Specifically, the initial response of total hours is -0.15 per cent, and the peak response is -0.75 per cent in the fifth quarter. The fall in hours generated by ε_y is broadly consistent with the SVAR evidence of Gali (1999) suggesting that a technology improvement leads to a persistent fall in per capita hours. Basu, Fernald and Kimball (2004) obtain a similar result with technology shocks measured by a "purified Solow residual" that controls for non-technological factors that may affect measured total factor productivity. Here, the effects of ε_y, unlike those of ε_z, are not amplified through the vertical input-output linkage. Also, prices at both stages are a markup over marginal cost. At the intermediate-stage, the marginal cost is composed of the rigid wage index and the flexible rental rate of capital. At the final stage, an additional rigid element enters the composition of the marginal cost in the form of the rigid price index of the intermediate input. Hence, the effect of ε_y on hours is quite similar to that predicted by Gali's (1999) sticky-price model.

Also, following a positive ε_y, the rate of inflation of finished goods initially drops by 0.28 per cent, while wage inflation decreases only by 0.06 per cent. Therefore, finished-good inflation is more responsive to final-stage technology shocks than wage inflation. Interestingly, this prediction is consistent with the evidence presented by other researchers. For example, Basu, Fernald and Kimball (2004) providence evidence showing that wage inflation is almost unresponsive to their measure of technology shock, whereas price inflation falls. Using a VAR-identified measure of technology shock, Liu and Phaneuf (2006) also find that price inflation decreases more than wage inflation following a technology improvement.

How important is the role played by the vertical integration of stages and nominal rigidities in these findings? To answer this question, we first look at the impulse responses generated by our estimated model after two sets of restrictions have been successively imposed on some parameter values: $\phi = 0.01$ (weak vertical integration of stages) and $d_w = d_y = d_z = 0$ (no nominal rigidities). Later, we reestimate a version of the model with only one stage of production (the final stage) and nominal rigidities, and another version with two stages without nominal rigidities, and compare these alternative models more formally to the benchmark model.

The impulse responses to ε_z and ε_y corresponding to each of these scenarios are also presented in Figures 1 and 2. With a weak vertical integration of stages, the real effects of technology-shock ε_z are concentrated mostly at the intermediate stage of production. Both Z and N_z still rise significantly. However, with a small share of intermediate input into the production of finished goods, the boost in intermediate-stage output is not transmitted to the final stage, which is almost insulated from the lower stage, and hence final output is almost unaffected by ε_z. Total hours increase more than final output as a result of the significant rise in intermediate-stage hours. Hence, the input-output linkage between stages represents a mechanism that strongly propagates the effects of intermediate-stage technology shocks at the final stage. The effects of final-stage technology shocks on final output, total hours, consumption and investment are only slightly affected by changes in ϕ.

Nominal rigidities also are important. Again, this can be seen from Figures 1 and 2. Without nominal rigidities, a technology improvement taking place at the final stage is not followed by a decline in final-stage hours, but rather by a modest rise. N_z, however, continues to fall. Therefore, after an initial small increase, total hours are more or less constant following a positive ε_y. Nominal rigidities have a stronger impact on the real effects of ε_z. Without nominal rigidities, the two-stage model behaves very much like a RBC model where labor supply is not very elastic. Thus hours, and by extension output, increase less without nominal rigidities, much like a RBC model with divisible labor would predict. Labor adjustment costs are important to obtain hump-shaped impulse responses of final output and total hours worked following an intermediate-stage technology shock (results not reported)

4.4 How Well Does the Benchmark Model Account for Business Cycle Statistics?

This is an important question since in the literature that focuses on stochastic dynamic general-equilibrium models an important criterion of evaluating a model's performance is to look at a fairly comprehensive set of business-cycle facts within a single model. Table 4 reports the business-cycle statistics in the data and those generated from the estimated benchmark model. The relative volatility of consumption, measured by its standard deviation relative to that of real GDP, is 0.51 in the data and 0.83 in the benchmark model. The relative volatility of investment is 2.87 in the data and 2.64 in the model. The model generates a sufficiently high volatility of hours worked, with a relative volatility of 0.93 in the model compared to 0.85 in the data. The model correctly predicts that real wages and the average productivity of labor are both significantly less volatile than real GDP. The relative volatility of the real wages is 0.64 in the data and 0.80 in the model, while the relative volatility of labor productivity is 0.57 in the data compared to 0.50 in the model. The relative volatility of final-good inflation to intermediate-good inflation, assuming that it is approximated in the data by the relative volatility of CPI-inflation to PPI-inflation, is 0.55 according to the model and 0.52 in the data. Thus, the benchmark model seems to do well in reproducing the relative magnitude of fluctuations in several detrended aggregate time series.

Turning to the comovements of macroeconomic variables, we see that the benchmark model correctly predicts that consumption, investment and hours worked are all highly correlated with output. The benchmark model also implies that labor productivity is mildly procyclical, with a correlation of 0.52 in the data and 0.39 in the model. CPI-inflation and PPI-inflation are highly correlated in the data at 0.75, while the model predicts a correlation of 0.81.

One of the most interesting predictions of the benchmark model pertains to the weak procyclicality of real wages found in the data, often referred to as the Dunlop-Tarshis observation, which has been reinterpreted recently by Christiano and Eichenbaum (1992) as the near-zero correlation between average labor productivity and hours worked. Gomme and Greenwood (1995) discuss at length the difficulty encountered by standard RBC models to account for the weak procyclical movement in real wages. Also, several authors have tried to reduce the correlation between average productivity and hours. Christiano and Eichenbaum (1992) incorporate a government spending shock into an otherwise RBC model with the aim of shifting the labor supply curve along the labor demand curve, so that in principle the correlation between hours and productivity can be reduced. They show that this correlation is at best lowered to 0.58. Braun (1994) and McGrattan (1994) also add shocks to the tax rates on capital and labor, increasing the number of disturbances that can potentially shift the labor supply curve. This helps reduce the correlation of hours and productivity, but at the cost of a significant reduction in the contribution of technology shocks to employment and output fluctuations at business cycle frequencies. Bénassy (1995) and Cho, Cooley and Phaneuf also (1997) show that combining technology and monetary shocks in a dynamic general equilibrium model that features sticky nominal wages can fix this correlation.

The benchmark model correctly predicts that the real wages are weakly procyclical and that the correlation between hours worked and labor productivity is close to zero. Specifically, the correlation between the real wages and output is 0.37 in the data and 0.25 in the model, while the correlation between hours and productivity is -0.05 in the data and -0.12 in the model. Notice that, at the same time, the model captures the high correlation between labor productivity and the real wages (0.67 in the data *vs* 0.85 in the model) and the slightly negative correlation between hours and the real wages (-0.01 in the data *vs* -0.1888 in the model).

Why does the benchmark model succeed in accounting for these critical comovements? Consider for example the correlation between hours and productivity. We have established that the benchmark model implies very different effects of technology shocks ε_z and ε_y on total hours worked. Table 5 reports the correlations between hours and productivity conditional on different types of shocks in the benchmark model. The correlation between hours and productivity conditional on technology shock ε_z is mildly positive at 0.51. However, this correlation is strongly negative at -0.83 conditional on technology shock ε_y. Hence, the correlation between hours and productivity is negative at -0.025 conditional on both technology shocks ε_z and ε_y, while it is -0.05 in the data. Therefore, technology shocks trigger a near-zero correlation between hours and productivity, and still represent the main driving force behind employment and output fluctuations at business cycle frequencies.

4.5 Alternative Models

This subsection presents the estimation results of two alternative versions of the model. As stated before, Model I is a final-stage model that features a single technology shock, one source of nominal price rigidity and sticky nominal wages. Model II has two stages of production, two technology shocks, perfectly flexible prices at each stage, and perfectly flexible wages. Thus, Model I is close to existing, modern new keynesian models, while Model II is more in the spirit of RBC models. The estimated structural parameters for the alternative models are presented in Table 1.

Looking at the results reported under the label "*Model I*", we see that there are a few major changes in some of the parameter estimates. The point estimate of d_w is now 0.9250, implying an excessively high average duration of nominal wage contracts of 13.3 quarters. The point estimate of d_y is 0.7325, meaning that nominal price contracts last on average 3.74 quarters. The other significant changes in the parameter estimates concern the monetary policy rule. Again, as in the benchmark model, we find little evidence of interest rate smoothing. But the point estimate of ρ_v is now much higher at 0.62. This is also the case for ρ_π which is estimated at 2.13, meaning that monetary policy seems to be significantly more accommodative according to Model I than to the benchmark model. More importantly, using the likelihood ratio test (see bottom of Table 1), the benchmark model is decisively preferred to Model I.

The business-cycle statistics generated by Model I and presented in Table 4, are those one would expect from a sticky-wage model, which is not surprising given that the estimated Calvo-probability d_w is much higher than d_y. Therefore, Model I implies that hours worked are much more volatile than output, which is usual in sticky-wage models. The relative volatility of consumption is even higher than in the benchmark model, and the relative volatility of investment is too low. More importantly, real wages are strongly countercylical, with a correlation of real wages and output of about -0.69. The correlation between hours and productivity, at -0.66, is now far from the near-zero correlation observed in the data.

Model II, which is closer in spirit to the RBC model, does not perform well either and, in fact, have many of the anomalies of standard RBC models. The relative volatility of hours is very low. The relative volatility of real wages is much too high. Real wages and productivity are strongly procyclical. Productivity and hours are highly correlated. Once more, based on the likelihood ratio test, the benchmark model is strongly preferred to Model II.

5 Conclusion

This paper proposes a new type of business-cycle model where different processing stages are vertically integrated through an input-output linkage. Firms face technological change which is specific to their processing stage. Our model stands in stark contrast with a wide range of dynamic general equilibrium models in which firms are assumed to operate only at the final stage. The vertical integration of stages, combined with nominal rigidities, has rich business-cycle implications.

Empirically, intermediate-stage technology shocks are the main source of cyclical fluctuations. Furthermore, they generate hump-shaped impulse responses in several aggregate variables. However, unlike several other types of models where technology shocks are important, the present model has the ability to overcome some important business-cycle anomalies. A notable implication of the two-stage model lies in its ability to account for the Dunlop-Tarshis observation that there is no strong cyclical pattern in real wages, and its modern reincarnation of a near-zero correlation between average labor productivity and total hours worked.

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Table 1: Parameter Estimation Results

	Benchmak Model		Model I		Model II	
Parameter	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
$\rho_{A,y}$	0.8716	0.0177	0.8573	0.0090	0.8524	0.0145
$\rho_{A,z}$	0.9600	0.0711	$---$	$---$	0.9600	0.0014
ρ_v	0.1571	0.0335	0.6232	0.0366	0.3177	0.0302
ρ_κ	0.9512	0.0171	0.9108	0.0137	0.7188	0.0445
$\sigma_{A,y}$	0.0181	0.0007	0.0187	0.0010	0.0123	0.0008
$\sigma_{A,z}$	0.0197	0.0060	$---$	$---$	0.0086	0.0003
σ_v	0.0232	0.0020	0.0059	0.0003	0.0079	0.0002
σ_κ	0.0133	0.0006	0.0101	0.0010	0.0052	0.0002
ρ_R	0.0918	0.0767	0.0000	$---$	0.0363	0.0597
ρ_π	1.4702	0.0793	2.1285	0.0574	0.9984	0.0013
ρ_y	-0.0050	0.0060	-0.0122	0.0039	-0.0153	0.0020
d_w	0.8461	0.0079	0.9250	0.0313	$---$	$---$
d_y	0.6561	0.0256	0.7325	0.0063	$---$	$---$
d_z	0.6992	0.0539	$---$	$---$	$---$	$---$
φ_k	9.5827	0.6927	11.1243	0.2791	7.4139	0.5207
φ_y	5.7406	1.8588	2.4015	0.2979	5.9127	0.8304
φ_z	3.3746	1.1554	$---$	$---$	1.7069	0.5428
ϕ	0.2416	0.1312	$---$	$---$	0.4954	0.0245
b	0.0744	0.0389	0.2521	0.0595	0.1792	0.0198
γ	0.0701	0.1537	0.2974	0.0450	0.1131	0.0215
α_y	0.1300	0.0128	0.2564	0.0229	0.1333	0.0520
α_z	0.3407	0.0461	$---$	$---$	0.6110	0.0298
η	0.8831	0.4621	0.7120	0.3003	1.3040	0.0659
	$\mathcal{L} = 3567.40$		$\mathcal{L}_I = 3506.73$		$\mathcal{L}_{II} = 3387.33$	

Benchmark Model: Two-stage model with nominal rigidities; Model I: One-stage model with nominal rigidities; Model II: Two-stage model with flexible wages and prices

$\mathcal{L}$ denotes the maximized value of the log likelihood function. Then, the likelihood ratio statistic for the null hypothesis that the benchmark model is preferred to model I is equal to $2(\mathcal{L} - \mathcal{L}_I)$ that has a $\chi^2(4)$ distribution which gives a $p - value = 0.9999$.

Table 2: Benchmark Model: Variance Decomposition (Infinite Horizon)

Variable	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
Y_t	5.12	72.38	14.76	7.74
Z_t	3.86	84.22	11.46	0.46
C_t	4.83	67.03	14.54	13.60
I_t	5.46	80.69	13.35	0.50
N_t	19.64	44.91	21.87	13.57
$N_{y,t}$	19.48	37.36	22.94	20.23
$N_{z,t}$	10.39	76.19	11.54	1.88
w_t	12.93	70.06	14.41	2.60
$\frac{Y_t}{N_t}$	47.89	48.48	1.98	1.64
$\pi_{y,t}$	14.67	10.25	71.58	3.50
$\pi_{z,t}$	0.70	7.80	89.51	1.99

Table 3: Benchmark Model: Variance Decomposition (Different Horizons)

Final-goods sector output (Y_t)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	15.64	30.95	50.63	2.78
4	6.52	52.07	35.81	5.60
8	5.80	62.26	24.71	7.22
12	6.29	65.69	20.20	7.82
20	6.00	68.48	17.38	8.13
40	5.36	71.21	15.45	7.98

Intermediary-goods sector output (Z_t)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	0.41	20.41	78.43	0.75
4	3.61	46.07	49.18	1.14
8	6.21	61.42	31.27	1.09
12	6.57	68.64	23.86	0.93
20	5.66	75.98	17.64	0.71
40	4.35	82.13	12.99	0.53

Final-goods sector inflation ($\pi_{y,t}$)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	12.09	6.73	77.98	3.19
4	14.99	6.49	74.84	3.67
8	14.39	8.10	73.97	3.52
12	14.51	9.45	72.59	3.44
20	14.73	9.92	71.89	3.44
40	14.69	10.08	71.70	3.50

Intermediary-goods sector inflation ($\pi_{z,t}$)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	0.11	2.25	95.71	1.94
4	0.62	4.49	92.83	2.06
8	0.67	6.67	90.65	2.01
12	0.67	7.43	89.90	1.99
20	0.68	7.54	89.79	1.99
40	0.70	7.68	89.63	1.99

Table 4: Second-Order Unconditional Moments in the Benchmark and Alternative Models

Moments	Post-war US data	Benchmark Model	Model I	Model II
$\frac{std(C)}{std(Y)}$	0.5062 (0.0204)	0.8334	0.9104	0.7642
$\frac{std(I)}{std(Y)}$	2.8681 (0.0836)	2.6380	2.2057	2.1711
$\frac{std(N)}{std(Y)}$	0.8543 (0.0611)	0.9277	1.3100	0.2184
$\frac{std(w)}{std(Y)}$	0.6372 (0.0712)	0.7965	0.8293	1.0218
$\frac{std(Y/N)}{std(Y)}$	0.5677 (0.0405)	0.4965	0.6780	0.8115
$Corr(Y, C)$	0.9105 (0.2345)	0.9909	0.9875	0.9615
$Corr(Y, I)$	0.9630 (0.2645)	0.9341	0.8378	0.9287
$Corr(Y, N)$	0.8192 (0.1860)	0.8700	0.8612	0.8909
$Corr(Y, Y/N)$	0.5188 (0.1856)	0.3886	−0.1891	0.9925
$Corr(N, Y/N)$	−0.0535 (0.1033)	−0.1163	−0.6619	0.8287
$Corr(Y, w)$	0.3721 (0.1804)	0.2472	−0.6873	0.9710
$Corr(Y/N, w)$	0.6727 (0.1705)	0.8506	0.7519	0.9629
$Corr(N, w)$	−0.0115 (0.1572)	−0.1888	−0.9138	0.8683
$Corr(\pi_y, \pi_z)$	0.5200 (0.1622)	0.5500	− − −	− − −

Table 5: Second-Order Conditional Moments in the Benchmark Model

Moments	Post-war US data	Benchmark Model				
		All shocks	ε_y	ε_z	Supply shocks	Demand shocks
$\frac{std(C)}{std(Y)}$	0.5062 (0.0204)	0.8334	0.8090	0.8020	0.8025	0.9320
$\frac{std(I)}{std(Y)}$	2.8681 (0.0836)	2.6380	2.7228	2.7854	2.7813	2.0698
$\frac{std(N)}{std(Y)}$	0.8543 (0.0611)	0.9277	1.8165	0.7308	0.8467	1.1644
$\frac{std(w)}{std(Y)}$	0.6372 (0.0712)	0.7965	1.2652	0.7836	0.8242	0.6927
$\frac{std(Y/N)}{std(Y)}$	0.5677 (0.0405)	0.4965	1.5179	0.4064	0.5536	0.1993
$Corr(Y, C)$	0.9105 (0.2345)	0.9909	0.9951	0.9948	0.9948	0.9877
$Corr(Y, I)$	0.9630 (0.2645)	0.9341	0.9691	0.9691	0.9691	0.8057
$Corr(Y, N)$	0.8192 (0.1860)	0.8700	0.5493	0.9366	0.8329	0.9946
$Corr(Y, Y/N)$	0.5188 (0.1856)	0.3886	0.0015	0.7765	0.5325	−0.7931
$Corr(N, Y/N)$	−0.0535 (0.1033)	−0.1163	−0.8348	0.5065	−0.0250	−0.8523
$Corr(Y, w)$	0.3721 (0.1804)	0.2472	−0.2865	0.6248	0.5257	−0.8916
$Corr(Y/N, w)$	0.6727 (0.1705)	0.8506	0.9208	0.9528	0.8771	0.8428
$Corr(N, w)$	−0.0115 (0.1572)	−0.1888	−0.9272	0.3252	0.0473	−0.9099

Figure 1: Impulse Responses to an Intermediate-Stage Technology Shock

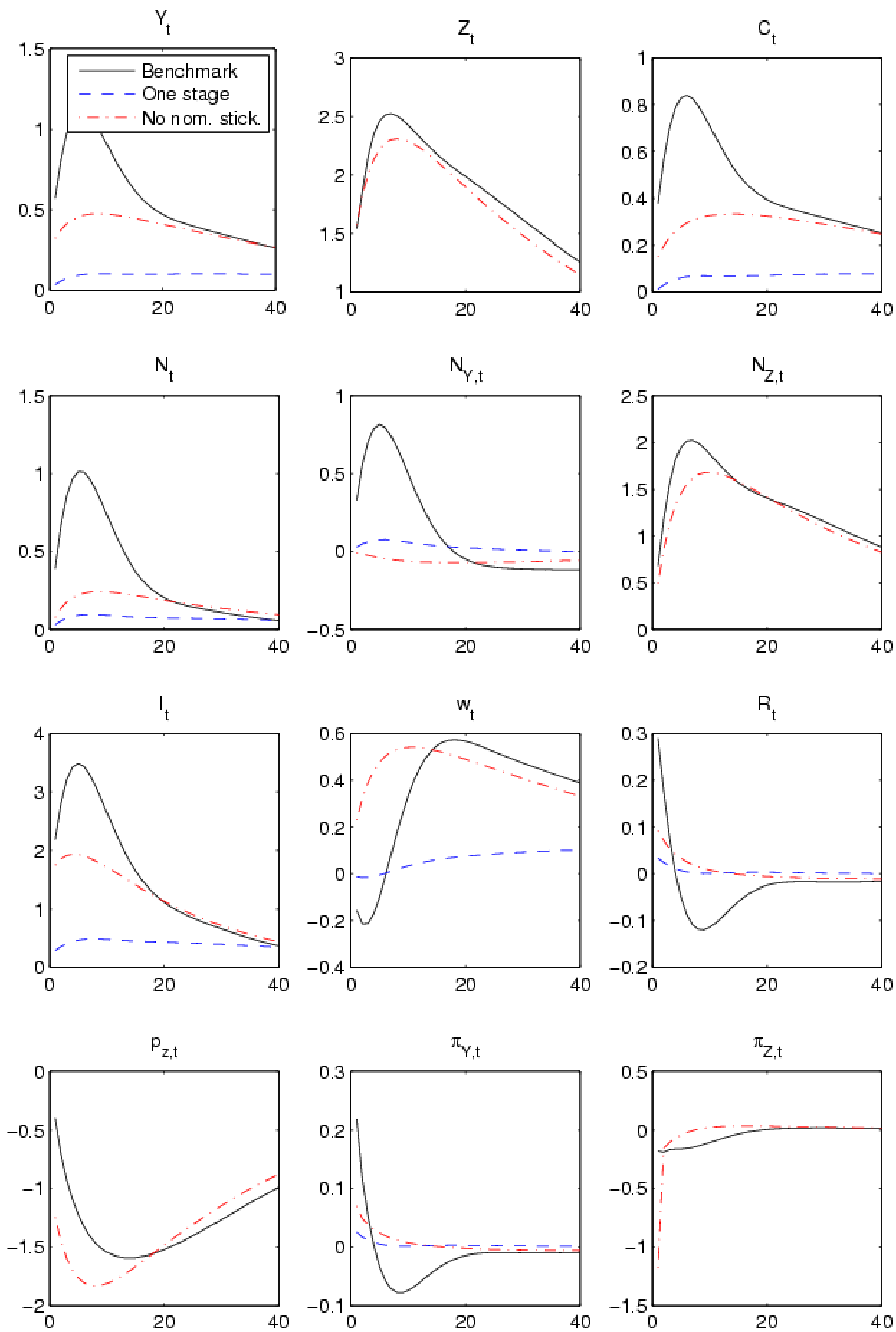

Figure 2: Impulse Responses to a Final-Stage Technology Shock