Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What is Important and What is Not

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Abstract

Central puzzles in international macroeconomics are why fluctuations of the real exchange rate are so volatile with respect to other macroeconomic variables, and the contradiction of efficient risk-sharing. Several theoretical contributions have evaluated alternative forms of pricing under nominal rigidities along with different asset markets structures to explain real exchange dynamics. In this paper, we use a Bayesian approach to estimate a standard two-country New Open Economy Macroeconomics (NOEM) using data for the United States and the Euro Area, and perform model comparisons to study the importance of departing from the law of one price and complete markets assumptions. Our results can be summarized as follows. First, we find that the baseline model does a good job in explaining real exchange rate volatility, but at the cost of implying too high volatility in output and consumption. Second, the introduction of incomplete markets allows the model to match the volatilities of all real variables best. Third, introducing sticky prices in local currency pricing (LCP) improves the fit of the baseline model, but not by as much as by introducing incomplete markets. Fourth, the combination of LCP and incomplete markets does not perform best, because it overpredicts real exchange rate volatility. Finally, we show that monetary shocks have played a minor role in explaining the behavior of the real exchange rate, while both demand and technology shocks have had some importance.

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I. INTRODUCTION

Most puzzles in international macroeconomics are related to real exchange rate dynamics. Fluctuations in the real exchange rates are very volatile and persistent, when compared to other real variables. In addition, there is clear evidence of lack of consumption risk-sharing across countries, which contradicts the assumption of complete markets. In order to replicate these features of the data, the New Open Economy Macroeconomics (NOEM) literature has incorporated either nominal rigidities, alternative structures of assets markets, or both. Pricing assumptions of imports goods are assumed to be either Producer Currency Pricing (PCP), where the law of one price holds and there is perfect pass-through; or Local Currency Pricing (LCP), where the pass-through is zero in the short run. The underlying asset markets structure assumes that either agents have access to complete markets to insure their wealth against idiosyncratic and country-specific shocks, or they do not.

Following this line of research, a recent paper by Chari, Kehoe and McGrattan (2002, hereafter CKM) attempts to explain the volatility and persistence of the real exchange rate by constructing a model with sticky prices and local currency pricing. Their main finding is that monetary shocks and complete markets, along with a high degree of risk aversion and price stickiness of one year are enough to account for real exchange rate volatility, and to less extent for its persistence. However, their model finds it difficult to account for the observed negative correlation between real exchange rates and relative consumption across countries, a fact that they labeled the *consumption-real exchange rate anomaly*. In addition, CKM argue that wealth effects coming from an incomplete markets assumption are too small to explain that anomaly.

In this paper, we use a Bayesian approach to estimate and compare two-country NOEM models under different assumptions of imports goods pricing and asset markets structures, thereby testing some of the key implications of CKM. Unlike them, we find that monetary policy shocks have a minor role in explaining real exchange rate volatility, and that both demand and technology shocks have had some importance. Using the Bayes factor to compare between competing alternatives, we find that what turns out to be crucial to explain real exchange rate dynamics and the exchange rate-consumption anomaly is the introduction of incomplete markets with stationary net foreign asset positions. Somewhat surprisingly, we find that in a complete markets set up, the introduction of LCP improves the fit of the model, while when incomplete markets are allowed for, LCP actually worsens the overall fit, because introducing the two assumptions at the same time delivers too high real exchange rate volatility and worsens its correlation with the ratio of relative consumptions.

In addition, the main contributions of our paper with respect to the existing literature are the following. First, we focus on the relationship between relative consumptions and the real exchange rate by introducing data on consumption for the two economic areas in the estimation. Second, while our model is quite rich in shocks (we need nine shocks because we try to explain nine variables), we have left aside uncovered interest-rate parity (UIP)-type shocks, which tend to explain a large fraction of real exchange rate variability. We do so because under complete markets these shocks, at least conceptually, should not be included and also because we want to study more carefully the role of "traditional" shocks (technology, demand, monetary and so on) in explaining real exchange rate fluctuations.[2] Third, we believe this is the first paper to evaluate the merits of the incomplete markets assumption with stationary net foreign assets in the NOEM model. Fourth, when we introduce some type of rigidity in imports prices that causes a deviation from the law of one price, we use sticky prices in local currency prices (LCP, as in CKM) rather than producer currency prices (PCP, as in Monacelli, 2005). We proceed this way to closely evaluate the findings in CKM and also to match the available evidence that exchange rate pass-through in the U.S. and the euro area tends to be very low. Last, but not least, we perform an in-sample forecast exercise and find that the preferred model does a good job in forecasting compared to the other NOEM models, but is still far away from the performance of a VAR.

The literature on estimating NOEM models in the spirit of Galí and Monacelli (2005) and CKM has grown greatly in recent years, following the Bayesian methodology that was used in their closed economy counterparts.[3] Thus, Lubik and Schorfheide (2003) estimate small open economy models using data for Australia, New Zealand, Canada and the U.K. and focus on whether the monetary policy rules in these countries have targeted the nominal exchange rate. Justiniano and Preston (2004) also estimate and compare small open economy models with an emphasis on the consequences of introducing imperfect pass-through. Adolfson et al. (2005) estimate a medium-scale (15 variable) small open economy model for the euro area, while Lubik and Schorfheide (2005) and Batini et al. (2005) estimate a small-scale two-country model using U.S. and euro area data.

The paper is organized as follows. In the next section we outline the baseline model, and we describe the LCP and the incomplete markets extensions. In section 3 we explain the data and the econometric strategy. The estimation results can be found in section 4. First, we present the parameter estimates of the baseline model. Then, we analyze the parameter estimates of all the extensions along with the second moments implied by each model. We select our preferred model based on the comparison of Bayes factors, and analyze its dynamics by studying the impulse response functions. Finally, we evaluate the importance of shocks through variance decompositions. In section 5 we conclude.

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[2] Our benchmark model, unlike the International Real Business Cycle (IRBC) literature, always includes nominal rigidities, because we want to evaluate the relative importance of monetary shocks in explaining real exchange rate fluctuations.

[3] Some examples are Rabanal and Rubio-Ramirez (2005), and Galí and Rabanal (2004) for the United States, and Smets and Wouters (2003) for the Euro Area, and Rabanal (2004a) for both.

II. THE MODEL

In this section we introduce a stochastic two country New Open Economy Macroeconomics (NOEM) model that we will use to analyze real exchange rate dynamics.[4] Since the model is now fairly standard, we only present the main features here, and refer the reader to an appendix available upon request for a full version of the model. We first present a baseline model with complete markets and where the law of one price holds, in the spirit of Clarida, Galí and Gertler (2002), Benigno and Benigno (2003) and Galí and Monacelli (2005).

In order to obtain a better fit to the data, we incorporate the following assumptions: home bias, habit formation in consumption, and staggered price setting a la Calvo (1983) with backward looking indexation. Afterwards, we introduce the two main extensions we are interested in comparing, namely incomplete markets and sticky prices in the price of imported goods in local currency.

The model has nine shocks: a world technology shock that has a unit root, and country-specific stationary technology, monetary, demand and preference shocks. We assume that there are two countries, home and foreign, of equal size. We present the preferences, technology and optimality conditions for the home country households and firms, obtaining the conditions for the foreign country is going to be simply a matter of notation. The convention will be to use asterisks (*) to denote the same variable or parameter in the foreign country, except where noted (i.e. if *C* is consumption in the home country, *C** is consumption in the foreign country), and so on.

A. Households

In each country there is a continuum of households in the unit interval indexed by *j*. Households in the home country obtain utility from consuming the final good (C_t^j) and disutility from supplying hours of labor (N_t^j). Their lifetime utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t G_t [\log(C_t^j - bC_{t-1}) - \frac{\left(N_t^j\right)^{1+\gamma}}{1+\gamma}]. \tag{1}$$

E_0 denotes the rational expectations operator using information up to time $t=0$. $\beta \in [0,1]$ is the discount factor. The utility function displays *external* habit formation. $b \in [0,1]$ denotes the importance of the habit stock, which is last period's aggregate consumption. $\gamma>0$ is inverse elasticity of labor supply with respect to the real wage. G_t is a preference shock that follows an AR(1) process in logs:

[4] This type of model that has been the *workhorse* of the NOEM literature after Obstfeld and Rogoff (1995).

$$\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g . \quad (2)$$

C_t denotes the consumption of the final good, which is a CES aggregate of consumption bundles of home and foreign goods:

$$C_t \equiv \left[(1-\delta)^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} (C_{F,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (3)$$

while the consumption of the final good in the foreign country is:

$$C_t^* \equiv \left[(1-\delta^*)^{\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} + (\delta^*)^{\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} .$$

The parameter $1-\delta$ is the fraction of home-produced goods in the consumer basket, and denotes the degree of home bias in consumption. Its analogous in the foreign country is δ^*. Consumption bundles of home and foreign produced goods are also CES aggregates:

$$C_{H,t} \equiv \left\{ \int_0^1 \left[c_t(h) \right]^{\frac{\varepsilon-1}{\varepsilon}} dh \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \text{ and } C_{F,t} \equiv \left\{ \int_0^1 \left[c_t(f) \right]^{\frac{\varepsilon-1}{\varepsilon}} df \right\}^{\frac{\varepsilon}{\varepsilon-1}} \quad (4)$$

There is a continuum of intermediate goods producers in the home country, indexed by $h \in [0,1]$ and the same amount in the foreign country, indexed by $f \in [0,1]$, with an elasticity of substitution of $\varepsilon > 1$.

Therefore, $c_t(h)$ denotes consumption by a home country household of a home-produced good, and $c_t(f)$ denotes consumption by a home country household of a foreign-produced good. Similarly, $C_{H,t}$ denotes aggregate consumption of home-produced goods by a home-country household, and $C_{F,t}$ denotes aggregate consumption bundle of foreign-produced goods by a home-country household.

The price level of the consumption aggregate in the home country is:

$$P_t \equiv \left[(1-\delta)(P_{H,t})^{1-\theta} + \delta (P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (5)$$

where,

$$P_{H,t} \equiv \left\{ \int_0^1 \left[p_t(h) \right]^{1-\varepsilon} dh \right\}^{\frac{1}{1-\varepsilon}}, \text{ and } P_{F,t} \equiv \left\{ \int_0^1 \left[p_t(f) \right]^{1-\varepsilon} df \right\}^{\frac{1}{1-\varepsilon}} \tag{6}$$

Therefore, $p_t(h)$ is the price in the home country of home-produced goods, in home-country currency. $p_t(f)$ denotes the price in the home country of foreign goods in home country currency.

The price level in the foreign country is:

$$P_t^* \equiv \left[(1-\delta^*)(P_{H,t}^*)^{1-\theta} + \delta^* (P_{F,t}^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

In the baseline setup, prices are set in the producer country currency (i.e. PCP). The law of one price holds, thus $p_t(h) = S_t p_t^*(h)$, and $p_t(f) = S_t p_t^*(f)$, where S_t is the nominal exchange rate, expressed as units of home-country currency in terms of a unit of foreign currency, $p_t^*(f)$ denotes the price in the foreign country of foreign-country goods in foreign-country currency, and the same applies to $p_t^*(h)$ (i.e. it is the price in the foreign country of home country goods in foreign country currency). From this assumption, $P_{H,t} = S_t P_{H,t}^*$, and $P_{F,t} = S_t P_{F,t}^*$.

Note, however, that purchasing power parity (PPP) does not necessarily hold because of the presence of home-bias in preferences. Therefore, the home-bias assumption allows to generate real exchange rate dynamics in a model with only tradable goods. Finally, we define the terms of trade as the price of imported goods from abroad relative to the price of the exported goods abroad, such that $T_t = P_{F,t} / S_t P_{H,t}^*$. Given that the law of one price holds the terms of trade can be expressed as $T_t = P_{F,t} / P_{H,t}$. From previous definitions we can express the real exchange rate as a function of the terms of trade:

$$Q_t = \frac{S_t P_t^*}{P_t} = \left[\frac{(1-\delta^*) + \delta^* T_t^{1-\theta}}{(1-\delta) + \delta T_t^{1-\theta}} \right]^{\frac{1}{1-\theta}} \tag{7}$$

Optimality conditions deliver the following demand functions for each type of aggregate consumption bundle:

$$C_{H,t} = (1-\delta) \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, \quad C_{F,t} = \delta \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, \tag{8}$$

and for each intermediate good:

$$c_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \quad c_t(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}. \tag{9}$$

B. Asset Market Structure, the Budget Constraint, and the Consumer's Optimizing Conditions

It is assumed that consumers have access to complete markets at the country level and at the world level, which means that consumer's wealth is insured against country specific and world shocks, and hence all consumers face the same consumption-savings decision.[5] Thus we model complete markets by assuming that households have access to a complete set of state contingent nominal claims which are traded domestically and internationally.[6]We represent the asset structure by assuming a complete contingent one-period nominal bond denominated in home currency[7]. Hence, households in the home country maximize their utility subject to the following budget constraint:

$$C_t^j = \frac{W_t N_t^j}{P_t} + \frac{E_t\left\{\xi_{t,t+1} B_{t+1}^j\right\} - B_t^j}{P_t} + \Pi_t^j(i), \tag{10}$$

where W_t is the nominal wage, and B_{t+1}^j is a nominal random payoffs of the portfolios purchased in domestic currency at t. with $\xi_{t,t+1}$ being the stochastic discount factor.[8] The last term of the right hand side of the previous expression denotes the profits from the monopolistically competitive intermediate goods producers firms, which are ultimately owned by households in each country. We further let $R_t > 1$ denote the gross nominal yield of one period risk-free discount bond in domestic currency such that $E_t\left\{\xi_{t,t+1}\right\} = 1/R_t$ is the price of that bond.

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[5] Baxter and Crucini (1993) have used the same assumption in an IRBC model in order to explain the saving-investment correlation.

[6] Given this assumption it is not necessary to characterize the current account dynamics in order to determine the equilibrium allocations.

[7] Given that markets are complete at the international level, the currency denomination of securities does not matter.

[8] Therefore $\xi_{t,t+1}$ is a price of one unit of nominal consumption of time $t+1$, expressed in units of nominal consumption at t, contingent on the state at $t+1$ being s_{t+1} ,and given any state s_t in t. If we define the value of the portfolio at the end of the period as A_t, the complete market assumptions implies that there exists a unique discount factor $\xi_{t,t+1}$ of a portfolio with the property that the price in period t of the portfolio with random value B_{t+1}^j is $A_t = E_t\left\{\xi_{t,t+1} B_{t+1}^j\right\}$.

The first order conditions for labor supply and consumption/savings decisions are as follows:

$$\frac{W_t}{P_t} = (C_t - bC_{t-1})(N_t)^{\gamma}, \tag{11}$$

and

$$\beta \frac{G_{t+1}}{G_t} \frac{(C_t - bC_{t-1})}{(C_{t+1} - bC_t)} \frac{P_t}{P_{t+1}} = \xi_{t,t+1}. \tag{12}$$

Notice that by taking expectations to the above equation across all possible states, and by using the fact that $E_t \{\xi_{t,t+1}\} = 1/R_t$ we can obtain the traditional Euler equation.

Moreover combining equation (12) with the intertemporal efficiency condition in the foreign country we obtain that under complete markets it follows that the ratio of marginal utilities of the two countries equalizes:

$$Q_t = \nu \frac{(C_t - bC_{t-1})}{(C_t^* - b^* C_{t-1}^*)} \frac{G_t^*}{G_t}, \tag{13}$$

where ν is a constant that depends on initial conditions (see CKM, and Galí and Monacelli, 2005). Notice that the risk-sharing condition differs with respect to the one in CKM because of the presence of both preference shocks and habit persistence.

C. Intermediate Goods Producers and Pricesetting

For each country, there is a continuum of intermediate goods producers, each producing a type of good that is an imperfect substitute of the others, that face a Calvo (1983)-type restriction when setting their prices. Also, there is backward looking indexation to last period's inflation rate. For each country, we denote by α and α^* the probabilities of the Calvo lottery (i.e. the probability that prices remain fixed for a given period), and as ω and ω^* the parameter that reflects indexation to last period's inflation rate.

In the home country, intermediate goods producers have the following production function:

$$Y_t(h) = A_t X_t N_t(h). \tag{14}$$

The aggregate world technology process has a unit root, as in Galí and Rabanal (2004) and Ireland (2004):

$$\log(A_t) = g + \log(A_{t-1}) + \varepsilon_t^a. \tag{15}$$

Hence, real variables in both countries grow at a rate *g*. The home country's country-specific technology shock evolves as an AR(1) in logs.

$$\log(X_t) = \rho_x \log(X_{t-1}) + \varepsilon_t^x. \tag{16}$$

First, from cost minimization, we obtain that the real marginal cost of production is:

$$MC_t(h) = \frac{W_t}{P_t}\frac{1}{A_t}\frac{1}{X_t} \tag{17}$$

Second, the overall demand for an intermediate good produced in h is:

$$Y_t^d(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\left[(1-\delta)C_t + (1-\delta^*)C_t^* Q_t^\theta\right] \tag{18}$$

Hence, whenever intermediate-goods producers are allowed to reset their price, they maximize the following profit function, which discounts future profits by the probability of not being able to reset their prices every period:

$$\underset{P_t(h)}{Max}\, E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \left\{\left[\frac{P_{t,t+k}(h)}{P_{t+k}} - MC_{t+k}(h)\right] Y_{t,t+k}^d(h)\right\}. \tag{19}$$

where $P_{t,t+k}(h)$ is the price prevailing at $t+k$ assuming that the firm last reoptimized at time t, and whose evolution will depend on whether the firm indexes to last period's inflation rate or to the steady-state rate of inflation, and $Y_{t,t+k}^d(h)$ the associated demand. $\xi_{t,t+k}$ is the $k$periods ahead stochastic discount factor.

The evolution of the aggregate consumption bundle price produced in the home country is:

$$P_{H,t} = \left\{(1-\alpha)(\hat{P}_{H,t})^{1-\varepsilon} + \alpha\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\omega} \overline{\Pi}_H^{1-\omega}\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}. \tag{20}$$

where $\hat{P}_{H,t}$ is the optimal price.[9]

[9] In order to have trend inflation, we assume, as in Yun (1996), that the remaining fractions $1-\omega$ and $1-\omega^*$ index their price to the unconditional inflation rate of each country's GDP deflator, $\overline{\Pi}_H$ and $\overline{\Pi}_F^*$.

D. Closing the Model

In order to close the model, we need a monetary policy rule for each country, which is a monetary policy rule of the Taylor-type, and the market clearing condition for home and foreign produced goods. We also introduce an exogenous demand shock for each country (η_t, η_t^*) that can account for government purchases, as well as trade with third countries that is not reflected in the model:

$$Y_{H,t} = \left[(1-\delta)C_t + (1-\delta^*)C_t^* Q_t^\theta\right]\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} + \eta_t. \tag{21}$$

And the analogous expression for the foreign country is:

$$Y_{F,t}^* = \left[\delta C_t + \delta^* C_t^* Q_t^\theta\right]\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} + \eta_t^*. \tag{22}$$

E. Symmetric Equilibrium

Since we have assumed a world technology shock that grows at a rate *g*, output, consumption, real wages, and the level of exogenous demand in the two economies grow at that same rate. In order to render these variables stationary, we divide them by A_t. Real marginal costs, hours, inflation, interest rates, the real exchange rate and the terms of trade are stationary.

F. Dynamics

We obtain the model's dynamics by taking a linear approximation to the steady state values. We denote by lower case variables percent deviations from steady state values. Lower case variables denote percent (log linear) deviation from the steady state with zero inflation. We denote with a variable with a tilde those variables that have been normalized by the level of technology (i.e. $\tilde{c}_t = c_t - a_t$). Also, we impose a symmetric home bias, such that $\delta = (1-\delta^*)$.

The consumption Euler equations are:

$$b\Delta c_t = -(1+g-b)(r_t - E_t\Delta p_{t+1}) + (1+g)E_t\Delta c_{t+1} + (1+g-b)(1-\rho_g)g_t, \tag{23}$$

$$b^*\Delta c_t^* = -(1+g-b^*)(r_t^* - E_t\Delta p_{t+1}^*) + (1+g)E_t\Delta c_{t+1}^* + (1+g-b^*)(1-\rho_g^*)g_t^*. \tag{24}$$

where Δ denotes the first difference operator. These equations are in terms of consumption growth, which is stationary, and not detrended consumption growth. In addition, equation (23) modifies the standard Euler equation by allowing for external habit formation (indexed by the parameter b) and a preference shock g_t.

Log-linearizing the optimal price-setting conditions delivers domestic inflation dynamics in each country:

$$\Delta p_{H,t} = \gamma_b \Delta p_{Ht-1} + \gamma_f E_t \Delta p_{Ht+1} + \kappa[\widetilde{\omega}_t - x_t + \delta t_t)], \tag{25}$$

$$\Delta p^*_{F,t} = \gamma^*_b \Delta p^*_{F,t-1} + \gamma^*_f E_t \Delta p^*_{F,t+1} + \kappa^*[\widetilde{\omega}^*_t - x^*_t - (1-\delta^*) t_t]. \tag{26}$$

where each real wage is deflated by each country's CPI, and where the backward and forward looking components are respectively $\gamma_b \equiv \omega/(1+\beta\omega)$, $\gamma_f \equiv \beta/(1+\beta\omega)$, and $\kappa \equiv (1-\alpha\beta)(1-\alpha)/[(1+\beta\omega)\alpha]$ for the home country, and $\gamma^*_b \equiv \omega^*/(1+\beta\omega^*)$, $\gamma^*_f \equiv \beta/(1+\beta\omega^*)$, and $\kappa^* \equiv (1-\alpha^*\beta)(1-\alpha^*)/[(1+\beta\omega^*)\alpha^*]$ for the foreign country.

The labor supply schedules are given by:

$$\widetilde{\omega}_t = \gamma n_t + \left[\frac{1+g}{1+g-b}\right]\widetilde{c}_t - \frac{b}{1+g-b}\widetilde{c}_{t-1} + \frac{b}{1+g-b}\varepsilon^a_t, \tag{27}$$

and

$$\widetilde{\omega}^*_t = \gamma^* n^*_t + \left[\frac{1+g}{1+g-b^*}\right]\widetilde{c}^*_t - \frac{b^*}{1+g-b^*}\widetilde{c}^*_{t-1} + \frac{b^*}{1+g-b^*}\varepsilon^a_t. \tag{28}$$

The risk sharing condition delivers the following relationship between consumption in the two countries, the preference shocks, and the real exchange rate:

$$q_t = \left[\frac{(1+g)\widetilde{c}_t - b\widetilde{c}_{t-1}}{1+g-b}\right] - \left[\frac{(1+g)\widetilde{c}^*_t - b^*\widetilde{c}^*_{t-1}}{1+g-b^*}\right] - \left(g_t - g^*_t\right) + \left(\frac{1+g}{1+g-b} - \frac{1+g}{1+g-b^*}\right)\varepsilon^a_t. \tag{29}$$

As in CKM, the real exchange rate depends on the ratio of marginal utilities of consumption, which in our case include the habit stock in each country, and the preference shocks. Note that the innovation to world growth enters as long as the effect on the ratio of marginal utilities is different in the two countries, due to differences in the habit formation parameters.[10]

[10] Stockman and Tesar (1995) add taste shocks in a two-sector (tradable and non tradable) two-country model similar to the one considered in this paper. Their results suggest that taste shocks are important to account for some international comovements between consumption and prices.

The real exchange rate and the terms of trade are linked as follows.[11]

$$q_t = (1-2\delta)t_t, \tag{30}$$

while the dynamic evolution of the terms of trade in terms of domestic inflations and the nominal exchange rate is:

$$\Delta t_t = \Delta s_t + \Delta p^*_{F,t} - \Delta p_{H,t}. \tag{31}$$

The market clearing conditions for the goods produced in the two countries are, at home:

$$\tilde{y}_{H,t} = \theta\left[\frac{2\delta(1-\delta)}{1-2\delta}\right]q_t + (1-\delta)\tilde{c}_t + \delta\tilde{c}^*_t + \eta_t, \tag{32}$$

and in the foreign country:[12]

$$\tilde{y}^*_{F,t} = -\theta\left[\frac{2\delta(1-\delta)}{1-2\delta}\right]q_t + \delta\tilde{c}_t + (1-\delta)\tilde{c}^*_t + \eta^*_t. \tag{33}$$

The production functions are simply:

$$\tilde{y}_{H,t} = x_t + n_t, \tag{34}$$

and

$$\tilde{y}^*_{F,t} = x^*_t + n^*_t, \tag{35}$$

while the Taylor rules react to domestic inflation and domestic output growth in each country:

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[11] The home bias assumption implies a positive comovement between the real exchange rate and TOT which is consistent with the data. Thus, in this model the real exchange rate inherits the properties of the terms of trade. Note than when $\delta = 1/2$ the real exchange rate is constant. In our one-sector economy the degree of home bias is crucial to account for the volatility of the real exchange rate, the larger the degree of home bias (smaller δ) the larger the volatility of the real exchange rate, although the implied real exchange rate volatility is always smaller than the one of the terms of trade. In a model with non-tradable goods this proportionality is broken down so that the real exchange rate will depend upon to the relative price of tradable to non tradable goods across countries.

[12] The assumption of demand shocks is also convenient empirically, because the two-country world budget constraint assumes an identity, namely that $y_{H,t}+y_{F,t}*=c_t+c_t*$ which is rejected by the data.

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\gamma_p \Delta p_{H,t} + (1-\rho_r)\gamma_y \Delta y_{H,t} + z_t \tag{36}$$

and

$$r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_r^*)\gamma_p^* \Delta p_{F,t}^* + (1-\rho_r^*)\gamma_y^* \Delta y_{F,t}^* + z_t^*. \tag{37}$$

The CPI Inflation rates are:

$$\Delta p_t = (1-\delta)\Delta p_{H,t} + \delta \Delta p_{F,t}^* + \delta \Delta s_t \tag{38}$$

and

$$\Delta p_t^* = (1-\delta^*)\Delta p_{H,t}^* - (1-\delta^*)\Delta s_t + \delta^* \Delta p_{F,t}^*. \tag{39}$$

III. Extensions to the Baseline Model

A. Incomplete Markets with Stationary Net Foreign Assets

In this section we extend the framework allowing for an incomplete asset market structure. We assume that home-country households are able to trade in two nominal riskless bonds denominated in domestic and foreign currency, respectively. These bonds are issued by home-country residents in the domestic and foreign currency to finance their consumption. Home-country households face a cost of undertaking positions in the foreign market[13]. For simplicity we further assume that foreign residents can only allocate their wealth in bonds denominated in foreign currency[14]. In each country, firms are assumed to be completely owned by domestic residents, and profits are distributed equally across households.

The real budget constraint of home-country households is now given by:

$$C_t + \frac{B_t}{P_t R_t} + \frac{S_t B_t^*}{P_t R_t^* \phi\left(\frac{S_t B_t^*}{P_t}\right)} = \frac{W_t N_t^j}{P_t} + \frac{B_{t-1}}{P_t} + \frac{S_t B_{t-1}^*}{P_t} + \int_0^1 \Pi_t^j(i)di, \tag{40}$$

where the $\phi(.)$ function depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as a given by the domestic household.[15]

[13] This cost allows to achieve stationarity in the net foreign asset position. See Schmitt-Grohe and Uribe (2001) and Kollman (2002) for applications in small open economies, and Benigno (2001) and Selaive and Tuesta (2003a) for applications in two-country models. Heathcote and Perri (2002) have used the same transaction cost in a two-country IRBC model.

[14] It is straightforward to allow foreign residents to allocate their wealth in domestic bonds also, but for tractability we set the model in this way.

[15] In order to achieve stationarity $\phi(.)$ has to be differentiable and decreasing in a neighborhood of zero. We further assume that $\phi(.)$ equals zero when B_t^*=0.

We further assume that the initial level of wealth is the same across households belonging to the same country. This assumption combined with the fact that households within a country work for firms sharing the profits in equal proportion, implies that within a country all households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption.

The conditions characterizing the allocations of domestic and foreign consumption are:

$$\frac{G_t}{(C_t - bC_{t-1})} = \beta E_t \left[\frac{G_{t+1}}{(C_{t+1} - bC_t)} \frac{R_t P_t}{P_{t+1}} \right], \tag{41}$$

$$\frac{G_t^*}{(C_t^* - b^* C_{t-1}^*)} = \beta E_t \left[\frac{G_{t+1}^*}{(C_{t+1}^* - b^* C_t^*)} \frac{R_t^* P_t^*}{P_{t+1}^*} \right], \tag{42}$$

and

$$\frac{G_t}{(C_t - bC_{t-1})} = \phi\left(\frac{S_t B_t^*}{P_t}\right) \beta E_t \left[\frac{G_{t+1}}{(C_{t+1} - bC_t)} \frac{R_t^* P_t S_{t+1}}{P_{t+1} S_t} \right]. \tag{43}$$

Equations (41) and (42) correspond to the Euler equations to the home and foreign countries, respectively. Equation (43) represents home-country households' Euler equation derived by optimizing holdings of the nominal bond denominated in foreign currency. The equilibrium budget constraint becomes[16]

$$\frac{S_t B_t^*}{P_t R_t^* \phi\left(\frac{S_t B_t^*}{P_t}\right)} = \frac{S_t B_{t-1}^*}{P_t} + \left(\frac{P_{H,t}}{P_t}\right) Y_{H,t} - C_t \tag{44}$$

Dynamics

Under incomplete markets the net foreign asset (NFA) position becomes a new state variable. In addition we have to modify the risk sharing condition which holds in expected first difference and which depends on the NFA position and preference shocks. The risk-sharing condition under incomplete markets reads:

[16] In a two-country world bonds must be zero in net supply. Thus we can drop one of the asset accumulation equations since one of the asset stocks is redundant. See Baxter and Crucini (1995).

$$E_t\left(q_{t+1}-q_t\right)=\left[\frac{(1+g)E_t\Delta c_{t+1}-b\Delta c_t}{1+g-b}\right]-\left[\frac{(1+g)E_t\Delta c^*_{t+1}-b^*\Delta c^*_t}{1+g-b^*}\right]+(1-\rho_g)g_t-(1-\rho^*_g)g^*_t$$
$$+\chi b_t \tag{45}$$

where $\chi=-\phi'(0)Y_H$ and $b_t=\left(\frac{S_tB^*_t}{P_t}\right)Y_H^{-1}$, which substitutes equation (29) in section II.F.

In addition, the log-linear dynamics of the net foreign asset position are given by:

$$\beta b_t=b_{t-1}+\delta\left[\frac{2\theta(1-\delta)-1}{1-2\delta}\right]q_t-\delta\left(\tilde{c}_t-\tilde{c}^*_t\right). \tag{46}$$

B. Local Currency Pricing by Intermediate Goods Producers

We assume price stickiness in each country's imports prices in terms of local currency pricing. This assumption allows us to generate deviations from the law of one price at the border, unlike Monacelli (2005). In particular, exchange rate movements generate ex-post deviations from the law of one price. Under this scenario the pass-through is zero.[17] Importantly, under the assumption of local currency pricing, even without home bias it is possible to generate real exchange rate fluctuations.

Each firm chooses a price for the domestic market and a price for the foreign market under the same conditions of the Calvo lottery as above. In this case, the overall demand for an intermediate good produced in *h* at home and abroad, respectively, is given by:

$$Y^d_t(h)=(1-\delta)\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}C_t, \tag{47}$$

and

$$Y^{d^*}_t(h)=\delta\left(\frac{P^*_t(h)}{P^*_{H,t}}\right)^{-\varepsilon}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\theta}C^*_t. \tag{48}$$

Hence, whenever domestic intermediate-goods producers are allowed to reset their price, they maximize the following profit function:

$$\underset{P_t(h)}{Max}\,E_t\sum_{k=0}^{\infty}\alpha^k\xi_{t,t+k}\left\{\left[\frac{P_{t,t+k}(h)+P^*_{t,t+k}(h)S_{t+k}}{P_{t+k}}-MC_{t+k}(h)\right]\left[Y^d_{t,t+k}(h)+Y^{d*}_{t,t+k}(h)\right]\right\}. \tag{49}$$

[17] See CKM (2002), Bergin (2004), and Beningo, G. (2004).

where $P_{t,t+k}(h)$ and $P^*_{t,t+k}(h)$ are prices of the home good set at home and abroad prevailing at $t+k$ assuming that the firm last reoptimized at time t, and whose evolution will depend on whether the firm indexes to last period's inflation rate or to the steady-state rate of inflation. $Y^d_{t,t+k}(h)$ and $Y^{d*}_{t,t+k}(h)$ are the associated demands for good h in each country.

The evolution of the price sub-indexes is given by:

$$P_{H,t} = \left\{ (1-\alpha)(\hat{P}_{H,t})^{1-\varepsilon} + \alpha \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\omega} \overline{\Pi}_H^{1-\omega} \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}. \tag{50}$$

$$P^*_{H,t} = \left\{ (1-\alpha)(\hat{P}^*_{H,t})^{1-\varepsilon} + \alpha \left[P^*_{H,t-1} \left(\frac{P^*_{H,t-1}}{P^*_{H,t-2}} \right)^{\omega} \overline{\Pi}^{*1-\omega}_H \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}. \tag{51}$$

where $\hat{P}_{H,t}$ and $\hat{P}^*_{H,t}$ denote the optimal prices. Similar equations hold for the foreign country.[18] To obtain the log-linear dynamics, we first need to define the following relative prices:

$$t_t \equiv p_{F,t} - p_{H,t}, \text{and } t^*_t \equiv p^*_{H,t} - p^*_{F,t}. \tag{52}$$

These ratios represent the relative price of imported good in terms of the good produced domestically expressed in local currency.[19]

Dynamics

The following new equations arise with respect to the baseline (PCP) case. The inflation equations are:

$$\Delta p_{H,t} = \gamma_b \Delta p_{Ht-1} + \gamma_f E_t \Delta p_{Ht+1} + \kappa[\tilde{\omega}_t - x_t + \delta t_t)], \tag{53}$$

[18] Pricing-to-market behavior arises if we assumed different nominal contracts (i.e. price stickiness might be different across countries for the same good).

[19] Note that if the law of one price holds, $t_t = -t^*_t$, but now it is no longer the case.

$$\Delta p^*_{H,t} = \gamma_b \Delta p^*_{H,t-1} + \gamma_f E_t \Delta p^*_{H,t+1} + \kappa[\tilde{\omega}_t - x_t - (1-\delta)t^*_t - q_t)], \tag{54}$$

$$\Delta p^*_{F,t} = \gamma^*_b \Delta p^*_{F,t-1} + \gamma^*_f E_t \Delta p^*_{F,t+1} + \kappa^*[\tilde{\omega}^*_t - x^*_t + \delta t^*_t], \tag{55}$$

$$\Delta p_{F,t} = \gamma^*_b \Delta p_{F,t-1} + \gamma^*_f E_t \Delta p_{F,t+1} + \kappa^*[\tilde{\omega}^*_t - x^*_t - (1-\delta)t_t + q_t], \tag{56}$$

The CPI inflation rates now become:

$$\Delta p_t = (1-\delta)\Delta p_{H,t} + \delta \Delta p_{F,t} \tag{57}$$

and

$$\Delta p^*_t = \delta \Delta p^*_{H,t} + (1-\delta)\Delta p^*_{F,t} \tag{58}$$

The market-clearing conditions are:

$$\tilde{y}_{H,t} = (1-\delta)\theta\delta\left(t_t - t^*_t\right) + (1-\delta)\tilde{c}_t + \delta\tilde{c}^*_t + \eta_t \tag{59}$$

and

$$\tilde{y}^*_{F,t} = -(1-\delta)\theta\delta\left(t_t - t^*_t\right) + \delta\tilde{c}_t + (1-\delta)\tilde{c}^*_t + \eta^*_t \tag{60}$$

These linear equations substitute equations (25), (26), (32), (33), (38) and (39) in section II.F. Equations (30) and (31) no longer hold.

C. Incomplete Markets and Sticky Prices in Local Currency Pricing

Under incomplete markets and local currency pricing, all equations are given by those in section II.F and III.B. In this case the behavior of the real exchange rate is the same than under incomplete markets (equation 45 in section III.A) but the NFA dynamics is given by:

$$\beta b_t = b_{t-1} + (\theta - 1)\delta(1-\delta)\left(t_t - t^*_t\right) + \delta q_t - \delta\left(\tilde{c}_t - \tilde{c}^*_t\right), \tag{61}$$

which substitutes (46) in section III.A.

IV. Estimation and Model Comparison

In this section, we describe the data that we use to estimate the two-country model for the United States and the euro area, and some of its main features. Also, we explain the Bayesian methodology that we use to estimate the parameters of each model, as well as to compare the different versions of the NOEM model. An appealing feature of the Bayesian approach is that additional information about the model's parameters (i.e. micro-data evidence, features of the first moments of the data) can be introduced via the prior distribution.

A. Data

Data sources for the United States are as follows (pnemonics are in parenthesis as they appear in the Haver USECON database): we use quarterly real GDP (GDPH), the GDP deflator (DGDP), real consumption (CH), and the 3-month T-bill interest rate (FTB3) as the relevant short-run interest rate. Since we want to express real variables in per capita terms, we divide real GDP and consumption by total population of 16 years and over (LN16).

Data for the Euro area as a whole come from the Fagan, Henry and Maestre (2001) dataset. This dataset is a synthetic dataset constructed by the Econometric Modeling Unit at the European Central Bank, and should not be viewed as an "official" series. We extract from that database real consumption (PCR), real GDP (YER), the GDP deflator (YED), and short-term interest rates (STN). The euro zone population series is taken from Eurostat. Since it consists of annual data, we transform it to quarterly frequency by using linear interpolation.

Finally, we construct the real exchange rate series by taking the nominal exchange rate in terms of euros per U.S. dollar and multiplying it by the ratio of consumer price indices. The "synthetic" euro/U.S. dollar exchange rate prior to the launch of the euro in 1999 also comes from Eurostat, while the US CPI comes from the Haver USECON database (PCU) and the euro area CPI comes from the Fagan, Henry and Maestre data base (HICP).

Our sample period goes from 1973:1 to 2003:4, at quarterly frequency, which is when the euro area data set ends. Figure 1 plots the series that we use for the estimation. To compute per capita output and consumption growth rates, and inflation, we take logs and first differences of per capita output and consumption, and the GDP deflator respectively. We divide the short term interest rate by four to obtain its quarterly equivalent. We also take logs and first differences of the euro/dollar real exchange rate.

Table 1 presents some relevant statistics. Interestingly, the raw data show that per capita output growth rates in the United States and the euro area are not that different (0.48 versus 0.47), while per capita consumption and output in the euro area grow at the same rate (0.47). Consumption growth in the U.S. displays a higher sample mean growth rate (0.53) which is not surprising given current recent trends. Interestingly, growth rates in the euro area are less volatile than in the U.S. The real exchange rate displays a small appreciating trend mean during the sample period, and is much more volatile than any other series.

Interest rates and inflation rates display high persistence, and so do all real variables when they are HP-filtered. Out of the HP-filtered statistics, we would highlight the usual result that the real exchange rate is much more volatile than any other series. Interestingly, only consumption in the euro area displays some noticeable correlation with the real exchange rate (-0.26), while the correlation of output in Europe, and output and consumption in the U.S. with the real exchange rate is basically zero.

Finally, it is worth noting that the correlation between consumptions is smaller than between outputs (0.33 versus 0.47), although the size of the two correlations are smaller than using shorter sample periods (ending in the early 1990s), like Backus, Kehoe and Kydland (1992).[20] Related to what we said in the previous paragraph, the correlation of relative output with the real exchange rate is fairly small.[21] On the other hand, the correlation between the real exchange rate and relative consumptions across countries is negative (-0.17) which contradicts efficient risk-sharing.

The convention we adopt from now on is that the home country is the euro area, and the foreign country is the United States, such that the real exchange rate consists of the nominal exchange rate in euros per U.S. dollar, converted to the real exchange rate index by multiplying it by the U.S. CPI and dividing it by the Euro area CPI.

B. Parameter Estimation

Applying Bayes' rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. To implement the Bayesian estimation method, we simply need to be able to evaluate numerically the prior and the likelihood function. The likelihood function is evaluated using the state-space representation of the law of motion of the model, and the Kalman filter. Then, we use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters.[22]

Let ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the two countries of the model. The vector of observable variables consists of $x_t = \{\Delta y_t, \Delta c_t, r_t, \Delta p_{H,t}, \Delta y_t^*, \Delta c_t^*, r_t^*, \Delta p_{F,t}^*, \Delta q_t\}$. The assumption of a world technology shock with a unit root makes the real variables stationary in the model in first differences. Hence, we use consumption and output growth per country, which are stationary in the data and in the model. The relationship between the transformed variables in the model (normalized by the level of technology) and the first-differenced variables is as follows:

$$\tilde{c}_t = \tilde{c}_{t-1} + \Delta c_t - \varepsilon_t^a, \tilde{y}_t = \tilde{y}_{t-1} + \Delta y_t - \varepsilon_t^a, \tilde{c}_t^* = \tilde{c}_{t-1}^* + \Delta c_t^* - \varepsilon_t^a, \text{and } \tilde{y}_t^* = \tilde{y}_{t-1}^* + \Delta y_t^* - \varepsilon_t^a.$$

We introduce the real exchange rate in first differences, while inflation and the nominal interest rate in each country enter in levels. The main advantage of this methodology is that we use the first-difference filter to make variables stationary, rather than relying on linear,

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[20] Heathcote and Perri (2004) document that in recent years the U.S. economy has become less correlated with the rest of the world.

[21] All the facts related to the U.S. economy are very similar to the ones presented in CKM.

[22] See the Appendix for some details on the estimation.

quadratic or HP-filter trends. We demean all variables. We denote by $L(\{x_t\}_{t=1}^{T} \mid \psi)$ the likelihood function of $\{x_t\}_{t=1}^{T}$.

Priors

Table 2 shows the prior distributions for the model's parameters, that we denote by Π(ψ). For the estimation, we decide to fix only two parameters. The first one is the steady-state growth rate of the economy. Based on the evidence presented in section III.A, we set $g=0.005$, which implies that the world growth rate of per capita variables is about 2 percent per year. In order to match a real interest rate in the steady state of about 4 percent per year, we set the discount factor to $\beta=0.995$. For reasonable parameterizations of these two variables the parameter estimates will not change significantly. For the remainder of parameters, gamma distributions are used as priors when non-negativity constraints are necessary, and uniform priors when we are mainly interested in estimating fractions or probabilities. Normal distributions are used when more informative priors seem to be necessary.

Unlike other two-country model papers (i.e. Lubik and Schorfheide, 2005; and CKM), we do not impose that the parameter values should be the same in the two countries. However, we do use the same prior distributions for parameters across countries. We use normal distributions for the coefficients of habit formation and inverse elasticity of labor supply with respect to the real wage, centered at conventional values in the literature (0.7 and 1, respectively). We truncate the habit formation parameter to be between 0 and 1, which on the upper bound it would be six standard deviations away from the prior mean. We assume that the average duration of price contracts has a prior mean of 3 in the two countries, following empirical evidence reported in Taylor (1999). In this case, a gamma distribution is used.[23] The prior on the fraction of price setters that follow a backward looking indexation rule is less informative and takes the form of a uniform distribution between zero and one.

The parameters that incorporate the open economy features to the model take the following distributions. The parameter δ, which captures the implied home bias, has a prior distribution with mean 0.2 and standard deviation 0.03, which is in accordance with values suggested by Heathcote and Perri (2002) and CKM. The elasticity of substitution between home and foreign goods (θ) is source of controversy. We center it at a value of 1.5 as suggested by CKM, but with a large enough standard deviation to accommodate other feasible parameters, even those below one.[24] Finally, the parameter χ, that measures the elasticity of the risk

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[23] Because we want to keep the probability of the Calvo lottery between 0 and 1, the prior distribution is specified as average duration of price contracts minus one: $D=1/(1-\theta_p)-1$.

[24] Trade studies typically find values for the elasticity of import demand to respect to price (relative to the overall domestic consumption basket) in the neighborhood of 5 to 6, see Trefter and Lai (1999). Most of the NOEM models consider values of 1 for this elasticity which arises from the assumption of Cobb-Douglas preferences in aggregate consumption.

premium with respect to the net foreign asset position, is assumed to have a gamma distribution with mean 0.02 and standard deviation 0.014, following the evidence in Selaive and Tuesta (2003a and 2003b).

For the coefficients of the interest rate rule, we center the coefficients to the values suggested by Rabanal (2004b) who estimates rules with output growth for the United States. Hence, γ_p has a prior mean of 1.5, and γ_y has a prior mean of 1. The same values are used for the monetary policy rule in the Euro area, and we use uniform priors for the autoregressive processes between zero and one. We also truncate the prior distributions of the Taylor rule coefficients such that the models deliver a unique, stable solution.

Regarding the priors on the shocks of the model, we use also uniform priors on the autoregressive coefficients of the six AR(1) shocks, with a support between 0 and 0.96. This truncation is imposed because we want to examine how far can the models go in replicating persistence endogenously. We choose gamma distributions for the priors on the standard deviations of the shocks, to stay in positive reals. The prior means are chosen to match previous studies. For instance, the prior mean for the standard deviation of all technology shocks is set to 0.007, while the prior mean of the standard deviation of the monetary shocks comes from estimating the monetary policy rules using OLS. At the same time, the standard deviation of the prior reflects the uncertainty over these parameters.

Drawing from the Posterior and Model Comparison

We implement the Metropolis-Hastings algorithm to draw from the posterior. The results are based on 250,000 draws from the posterior distribution. The definition of the marginal likelihood for each model is as follows:

$$L(\{x_t\}_{t=1}^{T}) = \int_{\psi\in\Psi} L(\{x_t\}_{t=1}^{T} \mid \psi)\Pi(\psi)d\psi \tag{62}$$

The marginal likelihood averages all possible likelihoods across the parameter space, using the prior as a weight. Multiple integration is required to compute the marginal likelihood, making the exact calculation impossible. We use a technique known as modified harmonic mean to estimate it.[25]

Then, for two different models (A and B), the posterior odds ratio is

$$\frac{P(A \mid \{x_t\}_{t=1}^{T})}{P(B \mid \{x_t\}_{t=1}^{T})} = \frac{\Pr(A)L(\{x_t\}_{t=1}^{T} \mid \text{model} = A)}{\Pr(B)L(\{x_t\}_{t=1}^{T} \mid \text{model} = B)}.$$

[25] See Fernández-Villaverde and Rubio-Ramírez (2004).

If there are $m \in M$ competing models, and one does not have strong views on which model is the best one (i.e. $Pr(A)=Pr(B)=1/M$) the posterior odds ratio equals the Bayes factor (i.e. the ratio of marginal likelihoods).

V. RESULTS

In this section we present our results. First, we present the posterior estimates of a closed economy vis-à-vis the four specifications considered for open economy models: complete and incomplete markets under either PCP or LCP. Second, we perform model comparison by evaluating the marginal likelihood for each model. Third, we compute the standard deviations and correlations of each model at the mode posterior values. Fourth, we discuss the dynamics of our preferred model by analyzing the importance of the structural shocks for real exchange rate fluctuations. Finally, we look at the one-step ahead in-sample forecast performance of all models, and compare their performance to VARs.

A. Posterior Distributions for the Parameters

In Table 3 we present the means and standard deviations of the posterior parameters of all the models. In order to have a benchmark for the open economy estimates, we first provide the results from estimating each country as a closed economy. For the closed economy specification we assume that within each country agents have preferences only to home produced goods ($\delta=\theta=0$), and in addition they are not allowed to trade bonds internationally. Also, the real exchange rate is dropped from the set of observed variables.

In the second and third columns of Table 3 we report the mean and standard deviation of the posterior distributions of the parameters for the euro area and US, treating each of them as a closed economy. Overall, our estimates are in the line of previous contributions. The average duration of price contracts implied by the point estimate of the price stickiness parameters are above four (5.94) and six quarters (7.09), respectively. The implied proportion of firms that index their prices to the inflation rate are 0.06 and 0.09 percent for the euro area and U.S. respectively, which, as in Galí and Rabanal (2004), suggests that with highly correlated shocks the pure forward-looking model seems to be valid. The habit formation parameters are around 0.5 and 0.6 in the euro area and the U.S. respectively, which are in line with the values found by Smets and Wouters (2003) and slightly above to the ones of Galí and Rabanal (2004) for the U.S. economy. The estimates of the monetary policy rule parameters are similar to what it is usually assumed in the literature. Thus, the estimated coefficient over inflation and output are 1.62 and 1.10 for the euro area and 1.85 and 0.92 for the U.S. respectively. Our results also suggest a high degree of interest rate smoothing (0.87 and 0.83 for the euro area and U.S. respectively). Finally, the estimated processes of the shocks suggest that all of them but the productivity shock in the euro area are highly autocorrelated.

Our benchmark open economy model is the one that assumes complete markets and PCP. The results are displayed in columns 4 and 5 of Table 3. Interestingly, the results change in important ways with respect to the closed economy case. First, the proportion of firms that index their prices to the lagged inflation rate increases to almost one in the euro area, while inflation remains almost purely forward looking in the United States. The average duration of price contracts decreases for the euro area to 4.77 and increases significantly for the United States to 14.74, which is a fairly large number.[26] The habit persistence parameters increase in the euro area (to 0.78) and the U.S. (to 0.69). Estimates of the Taylor rule for the U.S. obtained from the two-country model are more or less the same to the one obtained from the closed economy counterpart. However, we observe significant changes in the euro area, thus the estimated coefficient on inflation rises from 1.59 to 2.24 and the one on output decreases from 1.08 to 0.07. The degree of interest rate smoothing for each block presents minor changes with respect to the closed economy estimations.

The persistence and volatility of all shocks increases greatly when the model tries to match the behavior of the real exchange rate. Except for the monetary and the demand shocks, the standard deviation of the shocks doubles or triples with respect to the closed economy estimates. Also, the autocorrelation of technology shocks in the United States and of preference shocks in the euro area increases to 0.96, which is the upper bound allowed for in the estimation. Thus, there is a tension in the model between matching a volatile real exchange rate and the less volatile output and consumption series. Below, we examine how well the models match the second moments of the data[27].

We turn to analyze the estimated parameters that are critical in NOEM models and which are key in shaping the real exchange rate dynamics. In particular, the implied degree of home bias captured by $1-\delta$, the intratemporal elasticity of substitution between tradable goods across countries, θ, and the real exchange-rate elasticity with respect to the stock of foreign debt, χ, that arises from the incomplete markets assumption. In our benchmark NOEM model we find that the implied degree of home bias towards home goods is 0.87 which is below 0.984, the value used by CKM (2002) and Heathcote and Perri (2002). The baseline two-country model estimates a very small elasticity θ, close to zero. This result can be somewhat expected from equations (32) and (33); because output and consumption are relatively much less volatile than the real exchange rate. Another main reason to expect such low elasticity is that the real exchange rate displays close-to-zero correlations with consumption and output in each country. Note that, our prior distribution was centered at a value of 1.5, so the data clearly provide evidence that the value is much smaller[28]. The larger this parameter the larger

[26] This result comes from the assumption of a production function that is linear in labor input. If we assumed, as Galí, Gertler and López-Salido (2001) that the production function is concave in labor, we would obtain smaller average price durations. The same would happen if we introduced firm-specific capital, as in Altig et al. (2005).

[27] In our estimation we assume that shocks are orthogonal. However, the world aggregate shock indirectly adds some form of spillovers. Baxter and Crucini (1995) highlight the importance of the structure of shocks for international asset market structures in IRBC models. In particular, they find that if shocks are stationary and with substantial spillovers both complete and incomplete markets perform similarly. But if shocks are very persistent without spillovers, adding incomplete markets changes significantly the prediction of IRBC models.

[28]On the other hand, Batini et al. (2005) allow for different elasticities of substitution of home and foreign goods for the United States and the euro area, and find that this elasticity is above one in both countries under complete markets.

the expenditure-switching effect of a tradable productivity shock. Under PCP the law of one price holds so we should attain the largest expenditure switching effect. A value close to zero for θ is neglecting this transmission mechanism.

When we relax the PCP assumption (columns 6 and 7) allowing for deviations from the law of one price by using local currency pricing (LCP), the results only change marginally. The parameter θ stays close to zero. Two changes are worth mentioning. First, the implied degree of home-bias in preferences drops from 0.87 to 0.77 in the euro area, and second, the proportion of firms that index their prices to the inflation rate drops from 0.93 to 0.41. Given the poor fitting regarding θ, our estimation does not favor the complete asset market structure.

In columns 8 to 11 of Table 3 we present the estimates of the model under incomplete markets with both PCP and LCP, respectively. There are some important differences to highlight with respect to the models with complete markets. First, the estimates of θ increase significantly, with point estimates of 0.45 and 0.91 under PCP and LCP, respectively. The intuition for this result can be seen from the law of motion of the NFA position. A real depreciation has to lead to a positive income effect to avoid having an explosive NFA position. For that to happen, θ has to be in the neighborhood of ½ under PCP and 1 under LCP. This implicit restriction pushes the value of the elasticity up, although in both cases it stays around the lowest possible value that would deliver stable dynamics.

Given the low estimated values of the elasticity of substitution between tradable goods in complete market models compared to incomplete market models (which are in contrast with both macro and micro empirical evidence), our results give support for the latter asset market structure. Therefore, the degree of financial integration (or lack thereof) is central in understanding the international transmission of business cycles and real exchange rate dynamics.

Table 3 also provides an estimate for χ, which represents the debt elasticity premium. We find values of 0.007 and 0.0129 under both PCP and LCP, respectively. These values are larger to the ones found by Bergin (2004) for the G7 countries (0.0038) and smaller to the ones obtained by Lane and Milesi-Ferreti (2001) from a panel of OECD countries (0.0254). Selaive and Tuesta (2003a and 2003b), by using GMM, estimate a risk-sharing condition similar to ours, with estimates in a range between 0.004 and 0.071 for a sample of OECD countries. From the above results it seems that the data give support for an incomplete asset market structure with a stationary net foreign asset position.

It is worth noting that the volatility of the shocks affecting the U.S. economy becomes much smaller under incomplete markets, and even for the productivity shock they are half of the value under a closed economy set up. For the case of the euro area, the estimated volatility of the shocks is smaller, although the reduction is not as important as in the U.S. case. Finally, the estimates of the Taylor rule for the euro area become closer to what was obtained under a closed economy. Hence, it seems that the introduction of incomplete markets does help improve the internal dynamics of the model by requiring smaller shocks.

B. Model Comparison

The last row of Table 3 shows the marginal likelihood of the four open economy models.[29] While introducing incomplete markets is always better than complete markets (either under PCP or LCP), and while LCP improves the fit of the model under complete markets, the model that ranks highest is the incomplete markets model with PCP. The model that ranks second is the incomplete markets model with LCP, while the two models with complete markets rank worse.

The differences are very important. In all cases the (log) differences are of similar magnitude, and such differences would imply "decisive" evidence for the model with highest log marginal likelihood, using the Bayesian model comparison language (Kass and Raftery, 1995). For instance the difference between the log-marginals of the first and the second model is about 36. This means that we would need a prior that favors the second model over the first by a factor of $3.9*10^{15}$ in order to accept it after observing the data. Since this is a large number, we conclude that the incomplete market model with PCP outperforms the incomplete market model with LCP, which in turn outperforms the two models with complete markets.

C. Second Moments

To understand why the model with more features, that *a priori* should be the "best model", does not rank best in terms of the Bayes factor comparison, in this subsection we present some selected second moments. In all the models, the evaluation is done at the mode of the posterior distribution. Table 4 presents some selected second moments implied by our estimations and are compared with those in the actual data.[30]

We find that the baseline model does a good job in explaining real exchange rate volatility and persistence, but at the cost of implying too high volatility and persistence in output and consumption in both countries, and too high volatility of interest rates and inflation in the euro area. Extending the benchmark complete markets model, by allowing for deviations from the law of one price, gives a slightly better fit of the rest of the real variables although delivers a slightly less volatile and persistent real exchange rate. The introduction of incomplete markets allows the model to match the volatilities of all real variables best. This is a consequence of the size of all shocks being smaller in this case, as we discussed above.

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[29] The marginal likelihood of the closed economies is not computed because of the different observed variables used in the estimation (we dropped the real exchange rate in that case).

[30] All model-based standard deviations and autocorrelations of nominal variables are based on simulating the model at the posterior mode, to be consistent with the observable counterpart. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 with 124 periods at the posterior mode and applying the HP filter.

However, while the model fits all U.S. variables and the nominal interest rate and inflation in the euro area fairly well, it still predicts that output and consumption volatility in the euro area is twice as much as we observe in the data. Adding sticky prices in local currency pricing to the incomplete markets model results is an overprediction of the real exchange rate volatility (the standard deviations rises from 4.90 to 6.83 percent, while in the data it is 4.59 percent), and a mild worsening of other features of the data. Hence, this is why this model, which is the one analyzed by CKM, does not rank best using the Bayes factor.

The bottom panel of Table 4 shows some selected cross-correlations of the HP-filtered data of real variables. We focus on these because they are typically central in international business cycle analysis. In terms of consumption and output correlations across countries both the PCP and LCP complete market models perform quite well, but they fail at explaining the correlation between the real exchange rate and relative consumptions across countries. In the data this correlation is negative (-0.17) and we obtain positive values (0.03 and 0.20 for the PCP and LCP models, respectively). CKM (2002) refer to this discrepancy between the models and the data as *the consumption real exchange rate anomaly*. CKM find hard to explain this anomaly even when introducing incomplete markets. Remarkably, once we extend the models allowing for incomplete markets we get closer to the data without affecting other moments significantly. In particular, we obtain a negative correlation between the relative consumption across-countries and the real exchange rate (-0.37). Once again, extending the incomplete markets model allowing for deviations from the law of one price give us a correlation closed to zero, worsening the fit with respect to the incomplete market and PCP model.

This discussion of second moments provides some more intuition and shows how introducing different assumptions about the asset markets structure and the pricing of imports prices affects the behavior of the models. Clearly, Table 4 shows that each model matches a particular moment of the data better than the others. The advantage of the Bayesian approach to model comparison is that it is a likelihood-based method: all the implications of each model for fitting the data are contained in the likelihood function. The good news is that the model that ranks highest using the marginal likelihood criterion seems to deliver the best fit to most features of the data.

D. Real Exchange Rate Dynamics and Importance of the Shocks

In this subsection, we investigate what is the importance of the shocks for explaining the real exchange dynamics. To focus our presentation, we perform this exercise only for the "preferred" model, which is the incomplete markets model with PCP and where the law of one price holds. Table 5 reports the contribution of each shock to standard deviation of the observable variables in the model.[31] In terms of real exchange rate dynamics, the shock that

[31] Lubik and Schorfheide (2005) add error terms to either the UIP or the PPP equations to assess the degree of model mispecification in explaining real exchange rate dynamics.

explains most of its variance is the demand shock. It explains above 49.2 percent of the variance and can be interpreted as government purchases or trade with other countries. The second largest component are the country-specific technology shocks which explain 35.5 percent of the variance of the real exchange rate.[32] Interestingly, the estimated model shows that real exchange rate fluctuations have fairly little to do with either monetary or preference shocks (9.0 and 5.8 percent, respectively).[33] In terms of persistence, only the world technology shock and the preference shocks are able to generate a persistent response of the real exchange rate, but as we argued they have trouble explaining its variability. Monetary policy shocks on their own cannot account for real exchange rate persistence, which is only 0.49 under these shocks. Our results suggest that, unlike CKM, it is very difficult that monetary shocks and sticky prices with LCP might help to account for the real exchange rate volatility. However, monetary shocks have some importance at explaining domestic inflation rates (14.4 and 14.9 percent for the euro area and U.S., respectively).

Regarding the negative correlation between relative consumptions and the RER, a model with either monetary or world demand shock being the only driving force, delivers positive correlations, of 0.79 and 0.42, respectively. Our results suggest that in order to simultaneously account for the real exchange rate volatility and the *consumption-real exchange rate anomaly* we need technology, preference and demand shocks to be included.

Next, we proceed to better understand real exchange rate dynamics by analyzing the posterior impulse response functions. In what follows, given the major importance of both technology and demand shocks in explaining the real exchange rate volatility, we will focus our analysis on these two shocks.[34] We plot the responses of relative consumption, consumption, output, real exchange rates and the net foreign asset position.

Figures 2 and 3 display the effects of one standard deviation of euro area and U.S. technology shocks, respectively. The effects of technology shocks are broadly similar in both economies, but they have more persistent effects in the case of the U.S., because the persistence parameter is much higher. A euro area positive technology shock expands both output and consumption in the euro area, it also expands consumption in the U.S., but reduces U.S. output. It takes around 12 quarters for output in both countries to return to their steady state value. The technology shock also generates a persistent real exchange rate

[32] Note, however, that this is an upper bound for the importance of technology shocks. Since the model does not have capital in the production function or sticky wages, then price markup, wage markup and temporary technology shocks have the same effect and cannot be separately identified.

[33] Faust and Rogers (2003) find that monetary shocks explain a small share of the volatility in the nominal exchange rate. Clarida and Galí (1994) in an earlier contribution find that demand shocks explain most of the variance in the real exchange rate fluctuations.

[34] Once again, we use the mode of the posterior distribution of the model's parameters to plot the impulse responses. The impulse responses to monetary, preference and world technology shocks are available upon request.

depreciation which is consistent with a decrease in domestic prices (worsening in the terms of trade) due to an improvement in productivity. Because of this worsening, we observe that the euro area net foreign asset position deteriorates, and then goes back to its the steady state value very slowly. The model also predicts an increase in relative consumptions. U.S. technology shocks imply a similar pattern, but with higher persistence and interestingly also makes the real exchange rate and relative consumptions to move in opposite directions.

Figures 4 and 5 presents the impulse response to the euro area and U.S. demand shocks, respectively. A positive demand shock in the euro area generates an increase in output and a decrease in consumption. Consequently, as expected, we observe a persistent NFA accumulation in the euro area along with a RER euro-dollar appreciation. It takes several periods for consumption in the euro area to recover, such that the euro area NFA slowly decreases to its steady state level. The strong euro appreciation causes consumption to decline in the U.S. but output to rise. A positive demand shock in the U.S. generates the exact opposite result. Remarkably, the impulse responses generated by the demand shocks will imply in both cases a negative co-movement between the real exchange rate and relative consumptions as it is observed in the data. By comparing the reaction of the RER vis-à-vis relative consumptions under both technology and demand shocks, it can be see that demand shocks are crucial to explain the real exchange rate behavior.

E. Forecasts

Table 6 presents the mean squared errors (MSE) for one-step ahead in-sample forecasts of all models. We also estimated VARs with the same nine observable variables, with up to 6 lags. All DSGE models perform poorly when trying to forecast the real exchange rate. The preferred model with incomplete markets and PCP is the only one of the four that can beat a random walk with drift, that would have a MSE of 4.59 percent. A VAR with just one lag does not perform better, and it is only after including 4 lags in the VAR that the MSE for forecasting the real exchange rate drops significantly. Second, the forecasting performance of the five DSGE models is quite similar, with no single model standing out as a best forecaster of all variables. Finally, structural models can claim some victory in forecasting several variables (consumption and interest rates in both areas, and inflation in the U.S.), even when the VAR includes 6 lags.[35]

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[35] Using net foreign asset position data would enrich the real exchange rate dynamics, as shown by Lane and Milesi-Ferretti (2001) since it would be useful to evaluate recent trends in the international investment positions across borders. Thus, we also estimated the incomplete market models by using the quarterly U.S. net foreign asset position as an observed variable. However, the estimated models do not change significantly and forecast in-sample evaluations do not perform better than models without NFA as an observable. Results are available upon request.

VI. Concluding Remarks

In this paper we have estimated a two-country NOEM model for the U.S. and the euro area with a particular focus on the implications for real exchange rate dynamics. We have used a Bayesian approach to estimate the models' parameters and to compare a baseline two-country model with complete markets and producer currency pricing with two main extensions, namely incomplete markets and sticky prices in imported goods with local currency pricing.

Our results suggest that the complete markets assumption implies too little importance to bilateral trade across areas. In particular, we obtain a very low estimated parameter for the elasticity of substitution between home and foreign goods, which is somehow expected from the correlation between the real exchange rate, consumption and output. Our close-to-zero estimate implies that the expenditure-switching effect of a real devaluation as a transmission mechanism is negligible. By contrast, the estimation gives empirical support to the incomplete markets assumption by highlighting the importance of the debt-premium elasticity.

We find that the baseline model with complete markets and law of one price performs well in explaining real exchange rate dynamics, but at the cost of implying too large volatilities in other real variables, especially in the euro area. The extension with incomplete markets in which the law of one price holds performs best at fitting the data. In particular, this model is able to simultaneously account for the real exchange rate volatility and persistence along with the negative correlation between the real exchange rate and relative consumptions. Interestingly, a model with both incomplete markets and sticky imports prices in local currency does not perform best, but it still outperforms models with complete markets. We show that both demand and technology shocks have played a major role in explaining the behavior of the real exchange rate, while monetary shocks have not, hence contradicting the hypothesis of CKM that the latter can be a main source of real exchange rate fluctuations.

There are some interesting avenues for future research, some of which we are exploring in ongoing work. We believe that the failure of the LCP assumption could be due to the fact that we are not explicitly using imports price series. Hence, if we want to explore the implications of these types of models for aggregate data, some other form of deviations from the law of one price should be explored. A promising line of research consists in incorporating distribution services in two-sector (with tradable and nontradable goods) two-country models. For instance, Selaive and Tuesta (2003a) have shown that models with non tradable goods help to explain the *consumption-real exchange rate anomaly* by capturing wealth effects transferred from a tradable shock to the non-tradable sector.[36] In related work

[36] Other papers that incorporate two-sector models without distribution services are Beningo and Thoenissen (2004) and Ghironi and Melitz (2005), and mostly rely on the Balassa-Samuelson effect to explain the consumption real exchange rate anomaly. In the second paper, the mechanism relies on aggregate productivity shocks rather than sector specific shocks.

Corsetti, Dedola and Leduc (2004) have shown that distribution services can help to account for the real exchange rate-consumption correlation by lowering the import demand elasticity. Campa and Goldberg (2004) provide evidence that deviations from the law of one price at the border due to the presence of distribution services helps explain a lower exchange rate pass-through at the consumer level than at the producer level. Finally, analyzing the out-of-sample forecasting performance of competing NOEM models, along the lines of the exercise performed by Del Negro et al. (2004) in the closed economy, and exploiting the information content of the net foreign asset position (Lane and Milesi-Ferreti, 2001) would help clarify the role of these models for policy formulation and analysis.

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APPENDIX: THE LIKELIHOOD FUNCTION AND THE METROPOLIS-HASTINGS ALGORITHM

The law of motion and the likelihood function

Let ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the two countries of the model, d_t be the vector of all endogenous variables (state and forward looking), z_t the vector of exogenous variables (i.e. shocks), and ε_t the vector of innovations. x_t is the vector of the nine observable variables that will enter the likelihood function.

The system of equilibrium conditions and the process for the exogenous shocks can be written as a second-order difference equation

$$
\begin{aligned}
&A(\psi)E_t d_{t+1} = B(\psi)d_t + C(\psi)d_{t-1} + D(\psi)z_t, \\
&z_t = N(\psi)z_{t-1} + \varepsilon_t, \qquad E(\varepsilon_t \varepsilon_t') = \Sigma(\psi).
\end{aligned}
$$

We use standard solution methods for linear models with rational expectations (as in Uhlig, 1999) to write the law of motion in state-space form. The *transition* and *measurement* equations are:

$$
\begin{aligned}
&d_t = F(\psi)d_{t-1} + G(\psi)z_t, \\
&z_t = N(\psi)z_{t-1} + \varepsilon_t, \qquad E(\varepsilon_t \varepsilon_t') = \Sigma(\psi).
\end{aligned}
$$

and

$$x_t = Hd_t$$

Let $y_t = [d_t', z_t']'$ be the vector of all variables, endogenous and exogenous. The evolution of the system can be rewritten as

$$y_t = \tilde{A}y_{t-1} + \tilde{B}\xi_t, \quad \text{where } E(\xi_t\xi_t') = I, \tilde{B} = \tilde{C}\Sigma^{1/2}, \text{ and } \varepsilon_t = \Sigma^{1/2}\xi_t.$$

and

$$x_t = \tilde{D}y_t$$

The $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ matrices are functions of *F,G,N,* and *Σ*. The $\tilde{D}$ matrix contains zeros everywhere, and a one in each row to select the variable of interest from the vector of all variables y_t. We can evaluate the likelihood function of the observable data conditional on the parameters $L(\{x_t\}_{t=1}^{T} \mid \psi)$, by applying the Kalman filter recursively as follows.

Define the prediction error as

$$v_t = x_t - x_{t|t-1} = x_t - \tilde{D}y_{t|t-1}.$$

whose mean squared error is

$$K_t = \tilde{D}P_{t|t-1}\tilde{D}',$$

where $x_{t|t-1}$ is the conditional expectation of the vector of observed variables using information up to *t-1*, and $P_{t|t-1} = E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})']$.

The updating equations are:

$$y_t = y_{t|t-1} + P_{t|t-1}\tilde{D}'K_t^{-1}v_t, \text{ and } P_t = P_{t|t-1} - P_{t|t-1}\tilde{D}'K_t^{-1}\tilde{D}P_{t|t-1}.$$

And the prediction equations are:

$$y_{t+1|t} = \tilde{A}y_t, \text{ and } P_{t+1|t} = \tilde{A}P_t\tilde{A}' + \tilde{C}\Sigma\tilde{C}'.$$

Then, the log-likelihood function is equal to

$$L_t = -\frac{1}{2}\sum_{t=1}^{T}\{n\log(2\pi) + \log[\det(K_t)] + v_t^{'}K_t^{-1}v_t\}.$$

where n is the size of the vector of observable variables x.

Note that the log-likelihood function has to be computed recursively. To initialize the filter, we set $y_0 = x_0 = 0$, and we set P_0 as the solution to the nonlinear system of equations $P = \tilde{A}P\tilde{A}' + \tilde{C}\Sigma\tilde{C}'$.

Drawing from the Posterior

To obtain a random draw of size *N* from the posterior distribution, a random walk Markov Chain using the Metropolis-Hastings algorithm is generated. The algorithm is implemented as follows:

1. Start with an initial value (ψ^0). From that value, evaluate the product $L(\{x_t\}_{t=1}^{T} \mid \psi^0)\Pi(\psi^0)$.

2. For each *i*:

$$\begin{cases} \psi^{i} = \psi^{i-1} \text{ with probability } 1\text{-}R \\ \psi^{i} = \psi^{i,*} \text{ with probability } R \end{cases}$$

where $\psi^{i,*} = \psi^{i-1} + v^{i}$, v^{i} follows a multivariate Normal distribution, and $R = \min\{1, \frac{L(\{x_t\}_{t=1}^{T} \mid \psi^{i,*})\Pi(\psi^{i,*})}{L(\{x_t\}_{t=1}^{T} \mid \psi^{i-1})\Pi(\psi^{i-1})}\}$.

The idea for this algorithm is that, regardless of the starting value, more draws will be accepted from the regions of the parameter space where the posterior density is high. At the same time, areas of the posterior support with low density (the tails of the distribution) are less represented, but will eventually be visited. The variance-covariance matrix of v^{i} is proportional to the inverse Hessian of the posterior mode and the constant of proportionality is specified such that the random draw has some desirable time series properties.

In all cases, the acceptance rates were between 20 and 30 percent, and the autocorrelation functions of the parameters decay fairly fast. We used two methods to simulate the posterior that delivered the same result (with very small numerical differences). First, we simulated the posterior 250,000 times taking as initial value the prior mean, and updating the Hessian of the posterior every 25,000 draws. We discarded all the values from that chain, and from the last value, generated a second chain of size 250,000, updating the Hessian each 25,000 draws. The second method involved finding the posterior mode using standard optimization algorithms to be used as initial value. Then, we generated a chain of 250,000 draws, updating the Hessian every 25,000 draws.

Figure 1. Series used for the Estimation of the Two-Country Model

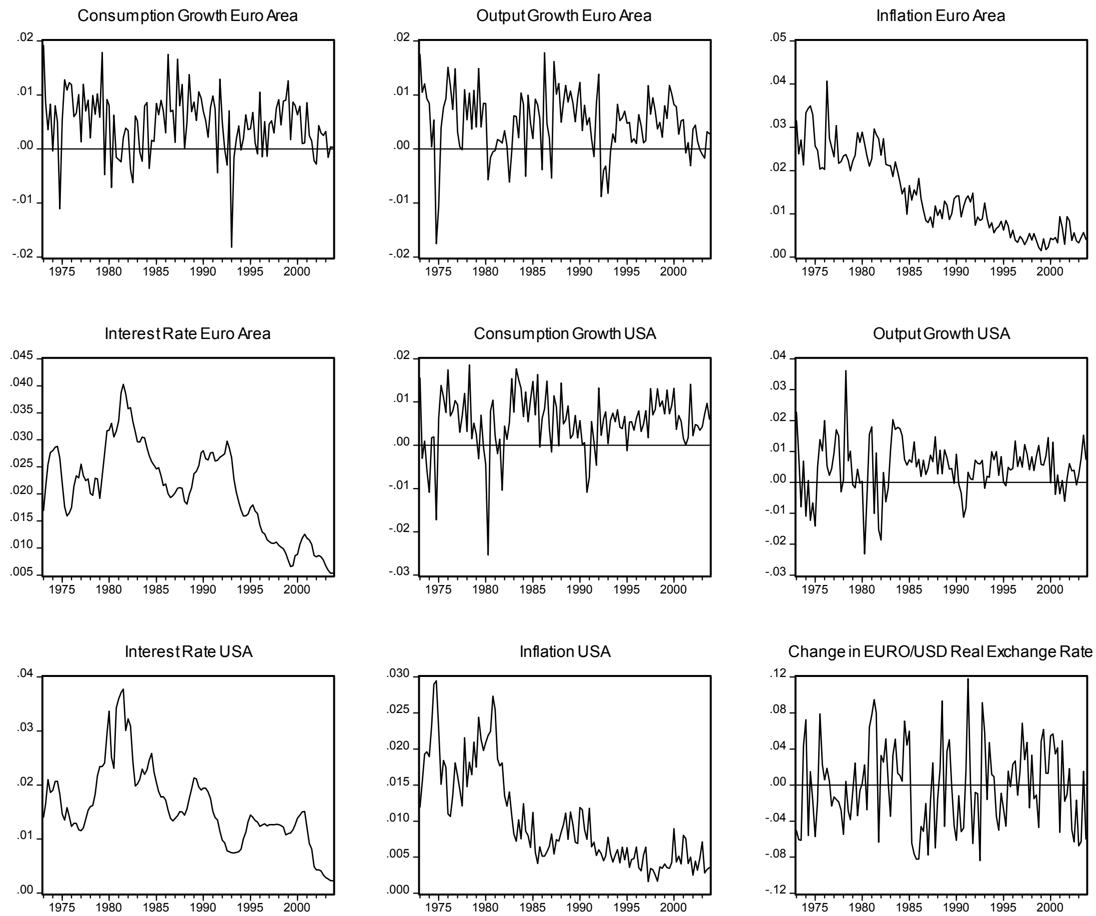

Figure 2: Impulse-Response to a euro area technology shock

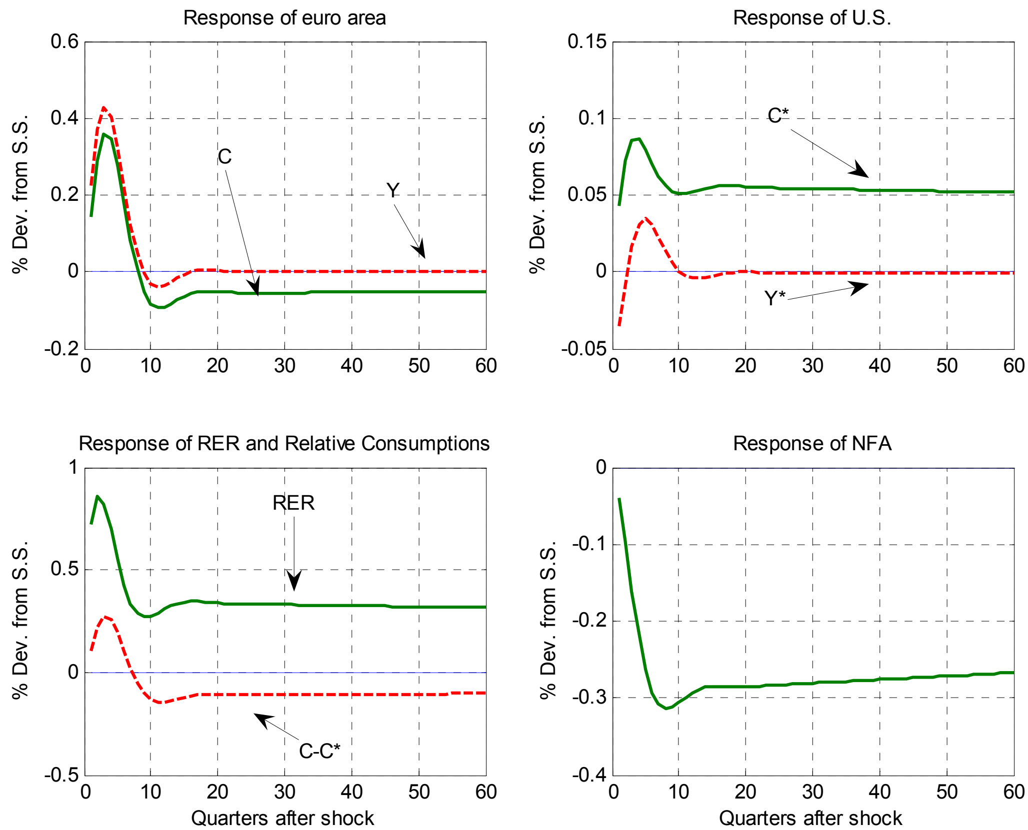

Figure 3: Impulse Response to a U.S. technology shock

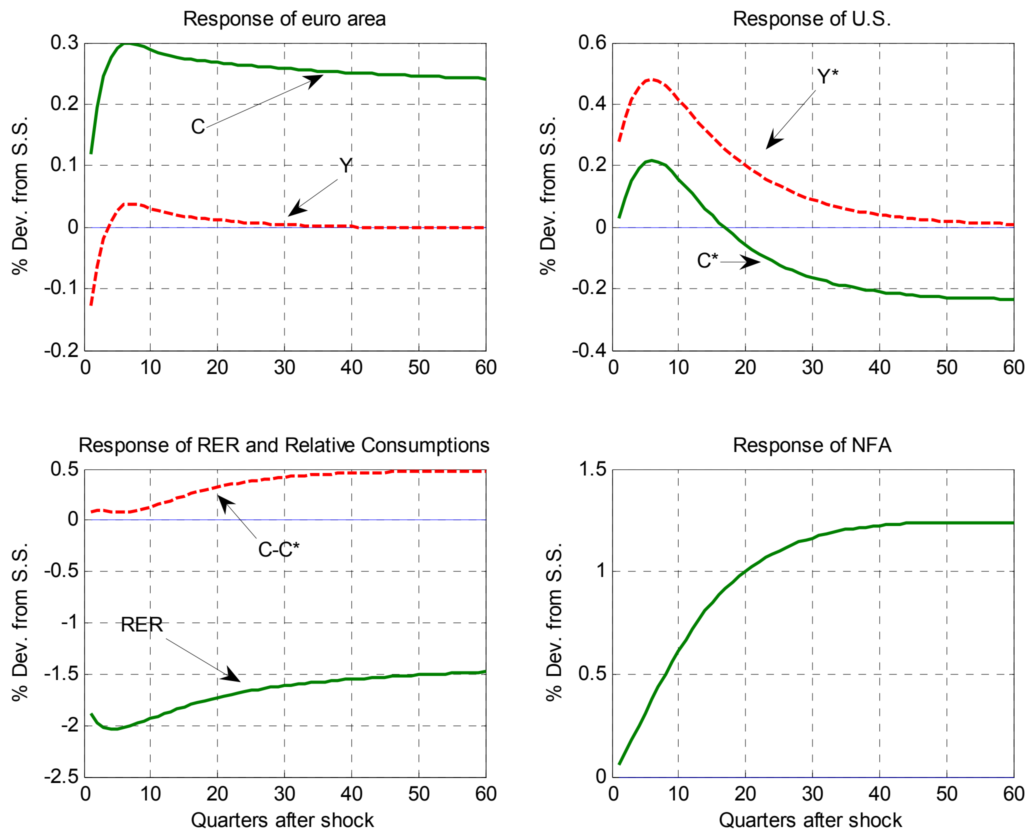

Figure 4: Impulse-Response to a euro area demand shock

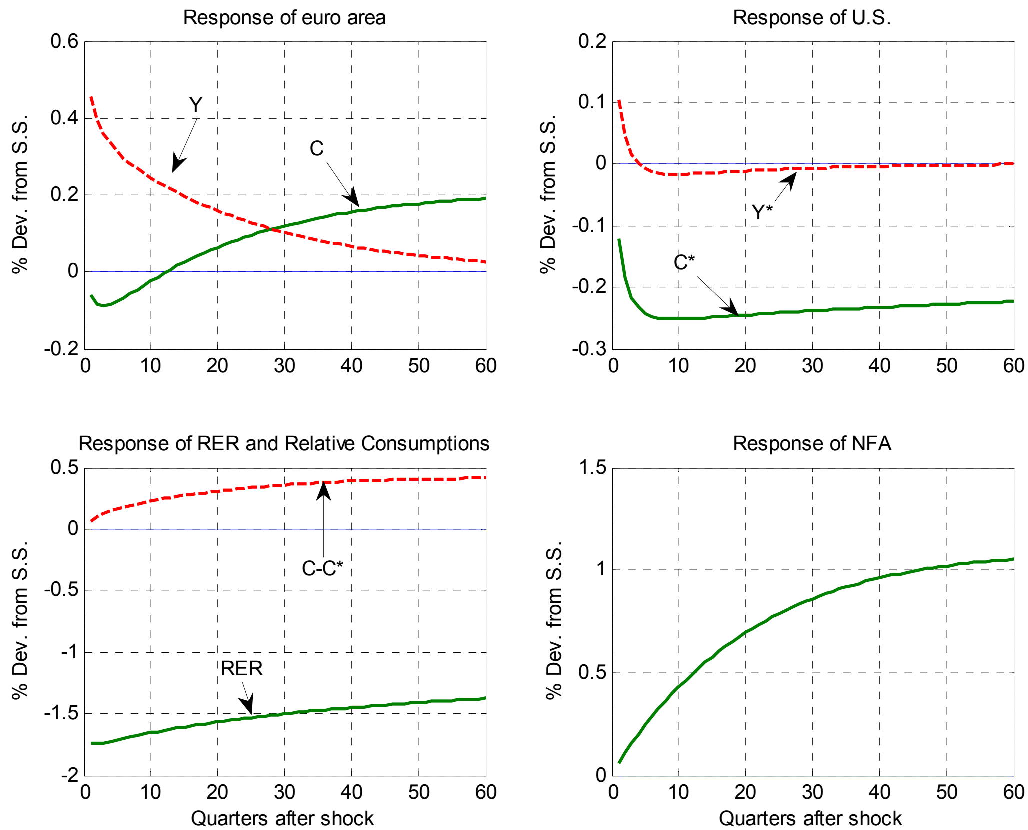

Figure 5: Impulse Response to a U.S. demand shock

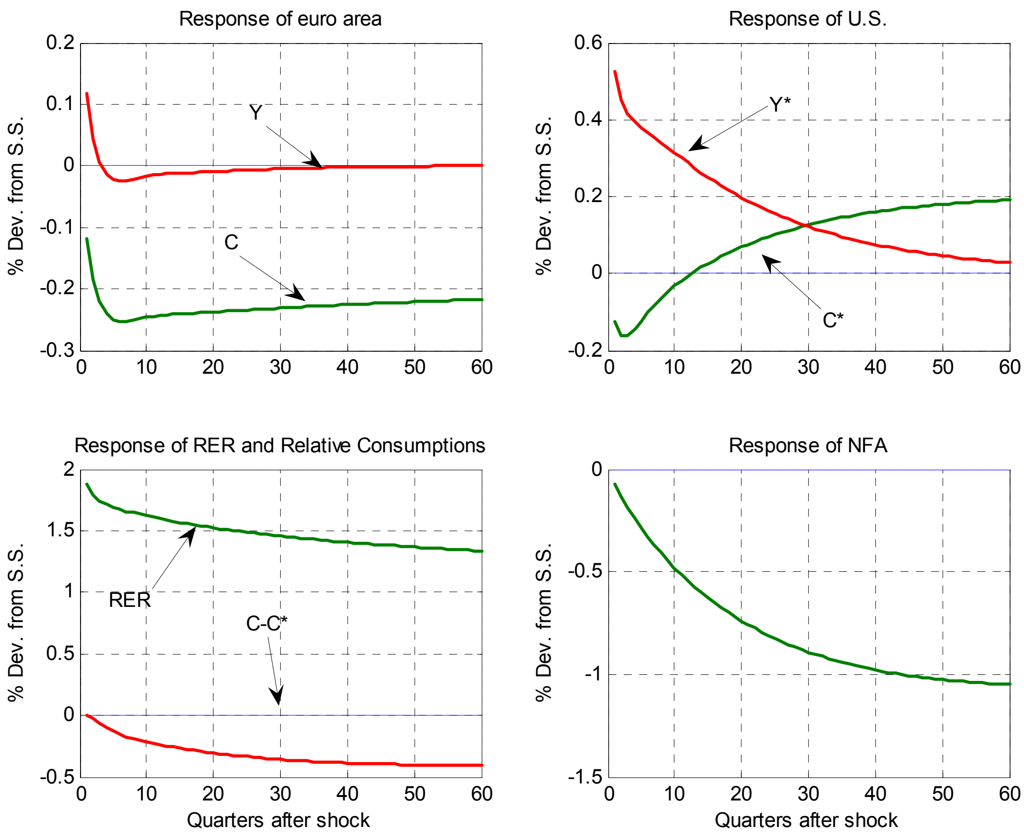

Table 1: Properties of the Data for the United States and the Euro Area

Raw Data, Quarterly Growth Rates					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Mean	0.47	0.47	0.53	0.48	-0.14
Std. Dev.	0.57	0.58	0.67	0.85	4.59
Raw Data, Quarterly Rates					
	Interest Rate Euro	Inflation Euro	Interest Rate USA	Inflation USA	
Mean	2.08	1.44	1.59	1.00	
Std. Dev.	0.83	0.93	0.73	0.67	
First Autocorr.	0.96	0.89	0.94	0.90	
HP-Filtered Data					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Std. Dev.	0.91	1.01	1.28	1.58	7.83
Corr. with RER	-0.26	-0.06	-0.02	-0.08	1.00
First Autocorr.	0.84	0.86	0.87	0.87	0.83
	Consumption Euro, USA	Output Euro, USA	Relative Cons., RER	Relative Outputs, RER	
Other Correlations	0.33	0.47	-0.17	0.04	

Note: Relative variables are the ratio between the euro area variable and its US counterpart.

Table 2: Prior Distributions of the Model's Parameters

	Distribution	Mean	Standard Deviation
β	-	0.995	-
g	-	0.005	-
b	Normal	0.70	0.05
b^*	Normal	0.70	0.05
γ	Normal	1.00	0.25
γ^*	Normal	1.00	0.25
Average Price Duration Home	Gamma	3.00	1.42
Average Price Duration Foreign	Gamma	3.00	1.42
ω	Uniform(0,1)	0.50	0.29
ω^*	Uniform(0,1)	0.50	0.29
δ	Normal	0.20	0.03
θ	Normal	1.50	0.25
χ	Gamma	0.02	0.014
ρ_r	Uniform(0,1)	0.50	0.29
γ_y	Normal	1.00	0.20
γ_p	Normal	1.50	0.25
ρ_r^*	Uniform(0,1)	0.50	0.29
γ_y^*	Normal	1.00	0.20
γ_p^*	Normal	1.50	0.25
$\rho_x, \rho_x^*, \rho_g, \rho_g^*, \rho_\eta, \rho_\eta^*$	Uniform(0,0.96)	0.48	0.28
$\sigma_x, \sigma_x^*, \sigma_a$	Gamma	0.007	0.003
σ_g, σ_g^*	Gamma	0.010	0.005
σ_z, σ_z^*	Gamma	0.004	0.002
$\sigma_\eta, \sigma_\eta^*$	Gamma	0.010	0.005

Table 3: Posterior Distributions

	Two Closed Economies		Complete Markets, PCP		Complete Markets, LCP		Incomplete Markets, PCP		Incomplete Markets, LCP	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
b	0.57	0.04	0.78	0.02	0.72	0.03	0.62	0.03	0.64	0.03
b^*	0.61	0.04	0.69	0.04	0.68	0.04	0.55	0.04	0.54	0.03
γ	1.09	0.23	1.25	0.21	1.47	0.20	1.18	0.20	0.79	0.23
γ^*	1.00	0.21	0.86	0.25	1.02	0.23	1.16	0.20	1.03	0.23
Avg. Prices Euro	5.94	0.89	4.77	0.46	6.29	0.62	4.28	0.39	4.12	0.47
Avg. Prices USA	7.09	0.92	14.74	1.68	12.66	1.36	5.74	0.63	4.95	0.63
ω	0.06	0.07	0.93	0.06	0.41	0.17	0.94	0.06	0.84	0.13
ω^*	0.09	0.08	0.04	0.04	0.06	0.05	0.09	0.07	0.09	0.08
δ	-	-	0.13	0.02	0.23	0.02	0.12	0.01	0.06	0.01
θ	-	-	0.04	0.03	0.01	0.01	0.44	0.00	0.91	0.01
χ	-	-	-	-	-	-	0.0070	0.0041	0.0129	0.0088
ρ_r	0.87	0.02	0.88	0.01	0.87	0.01	0.89	0.01	0.88	0.01
γ_y	1.08	0.16	0.07	0.06	0.16	0.10	1.04	0.16	0.98	0.17
γ_p	1.59	0.13	2.24	0.15	2.03	0.13	1.90	0.15	1.71	0.16
ρ_r^*	0.82	0.02	0.85	0.02	0.85	0.01	0.81	0.02	0.82	0.02
γ_y^*	0.91	0.15	1.24	0.13	1.34	0.14	1.16	0.14	1.01	0.15
γ_p^*	1.81	0.17	1.67	0.13	1.71	0.13	1.85	0.13	1.86	0.15
ρ_x	0.80	0.27	0.63	0.04	0.69	0.04	0.17	0.08	0.39	0.07
ρ_x^*	0.93	0.02	0.96	0.002	0.96	0.002	0.92	0.02	0.96	0.002
ρ_g	0.91	0.02	0.96	0.001	0.96	0.001	0.93	0.02	0.93	0.02
ρ_g^*	0.82	0.05	0.84	0.03	0.89	0.03	0.87	0.03	0.88	0.03
ρ_η	0.93	0.03	0.89	0.04	0.86	0.04	0.96	0.00	0.93	0.02
ρ_η^*	0.93	0.03	0.94	0.02	0.93	0.04	0.95	0.01	0.94	0.01
σ_x (in %)	1.93	0.45	4.45	0.52	4.27	0.48	4.00	0.55	4.23	0.54
σ_x^* (in %)	1.91	0.29	4.68	0.43	3.32	0.33	0.92	0.20	0.62	0.09
σ_g (in %)	2.32	0.29	5.72	0.40	4.78	0.31	3.64	0.45	3.41	0.49
σ_g^* (in %)	2.16	0.24	2.73	0.27	3.07	0.30	2.13	0.26	2.17	0.27
σ_z (in %)	0.20	0.02	0.23	0.02	0.22	0.02	0.20	0.01	0.19	0.01
σ_z^* (in %)	0.23	0.02	0.22	0.02	0.22	0.02	0.25	0.02	0.23	0.02
σ_a (in %)	0.70	0.02	2.58	0.21	2.03	0.17	1.39	0.12	1.44	0.14
σ_η (in %)	0.46	0.03	0.43	0.03	0.44	0.03	0.67	0.05	0.50	0.03
σ_η^* (in %)	0.69	0.05	0.67	0.04	0.66	0.04	0.89	0.06	0.70	0.05
Log-Marginal			3981.6		4033.2		4106.8		4070.9	

Table 4: Selected Second Moments in the Data and in the Models

	Euro Area				United States				
Std. Deviations (in percent)	Cons. Growth	Output Growth	Int. Rate	Inflation	Cons. Growth	Output Growth	Int. Rate	Inflation	RER Growth
Data	**0.57**	**0.58**	**0.83**	**0.93**	**0.67**	**0.85**	**0.73**	**0.67**	**4.59**
Two Closed Economies	0.73	0.81	0.59	0.74	0.85	1.00	0.57	0.56	-
Complete, PCP	1.30	1.27	1.43	1.31	1.24	1.27	0.63	0.79	4.96
Complete, LCP	1.30	1.18	1.10	0.90	1.17	1.16	0.77	0.72	5.32
Incomplete, PCP	1.04	1.07	0.83	1.11	0.83	0.98	0.61	0.62	4.90
Incomplete, LCP	1.03	1.11	0.82	1.22	0.86	0.97	0.60	0.63	6.83

Autocorrelations	Cons.	Output	Int. Rate	Inflation	Cons.	Output	Int. Rate	Inflation	RER
Data	**0.84**	**0.86**	**0.96**	**0.89**	**0.87**	**0.87**	**0.94**	**0.90**	**0.83**
Two Closed Economies	0.87	0.85	0.91	0.73	0.88	0.85	0.89	0.77	-
Complete, PCP	0.92	0.91	0.95	0.82	0.91	0.89	0.89	0.85	0.77
Complete, LCP	0.90	0.89	0.93	0.71	0.91	0.88	0.91	0.84	0.71
Incomplete, PCP	0.87	0.85	0.94	0.79	0.88	0.83	0.87	0.72	0.72
Incomplete, LCP	0.87	0.86	0.93	0.78	0.87	0.83	0.88	0.72	0.70

Other Correlations	C, C*	C, Y	Y,Y*	C*,Y*	C-C*,Q	Y-Y*,Q
Data	**0.33**	**0.81**	**0.47**	**0.85**	**-0.17**	**0.04**
Complete, PCP	0.32	0.97	0.48	0.91	0.03	0.09
Complete, LCP	0.21	0.93	0.60	0.87	0.20	0.13
Incomplete, PCP	0.53	0.86	0.60	0.76	-0.37	0.09
Incomplete, LCP	0.53	0.90	0.44	0.78	0.04	0.26

Note: all model-based standard deviations and autocorrelations of nominal variables are based on simulating the model at the posterior mode, to be consistent with the observable counterpart. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 with 124 periods at the posterior mode and applying the HP filter.

Table 5: Contributions of the Shocks to Selected Second Moments in the Preferred Model

	Euro Area				United States				
Percent Variance due to Each Shock	Cons. Growth	Output Growth	Int. Rate	Inflation	Cons. Growth	Output Growth	Int. Rate	Inflation	RER Growth
Monetary shocks	10.1	14.7	2.9	14.4	10.5	14.4	5.3	14.9	9.0
Country tech. shocks	12.7	16.2	2.9	24.7	2.8	12.9	2.2	11.4	35.5
World tech. shocks	31.3	33.6	0.9	15.8	48.5	31.8	0.1	33.6	0.6
Preference shocks	41.8	19.3	90.9	44.7	31.8	16.8	79.2	37.4	5.8
Demand shocks	4.1	16.2	2.5	0.4	6.4	24.2	13.1	2.6	49.2
Autocorrelations due to each shock	Cons.	Output	Int. Rate	Inflation	Cons.	Output	Int. Rate	Inflation	RER
Data	**0.84**	**0.86**	**0.96**	**0.89**	**0.87**	**0.87**	**0.94**	**0.90**	**0.83**
Monetary shocks	0.77	0.71	0.45	0.87	0.77	0.70	0.44	0.72	0.49
Country tech. shocks	0.89	0.83	0.82	0.53	0.91	0.84	0.88	0.67	0.73
World tech. shocks	0.93	0.93	0.92	0.85	0.92	0.93	0.83	0.71	0.86
Preference shocks	0.78	0.78	0.97	0.89	0.81	0.76	0.92	0.74	0.85
Demand shocks	0.88	0.62	0.87	0.94	0.87	0.63	0.80	0.97	0.69
Other Correlations due to each shock	C,C*	C,Y	Y,Y*	C*,Y*	C-C*,Q	Y-Y*,Q			
Data	**0.33**	**0.81**	**0.47**	**0.85**	**-0.17**	**0.04**			
Monetary shocks	0.33	0.97	0.01	0.96	0.79	0.94			
Country tech. shocks	0.72	0.75	-0.03	0.61	-0.24	0.90			
World tech. shocks	0.99	1.00	0.99	1.00	0.42	0.86			
Preference shocks	-0.30	0.93	0.38	0.82	-0.80	-0.40			
Demand shocks	0.25	-0.10	0.35	-0.35	-0.74	-0.97			

Note: all model-based standard deviations and autocorrelations of nominal variables are based on simulating the model at the posterior mode, to be consistent with the observable counterpart. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 with 124 periods at the posterior mode and applying the HP filter.

Table 6: Mean Squared Errors of One Period Ahead Forecasts (in percent)

	Euro Area				United States				
	Cons.	Output	Int.Rate	Inflation	Cons.	Output	Int.Rate	Inflation	RER
Two Closed Economies	0.46	0.53	0.09	0.21	0.38	0.73	0.11	0.14	-
Complete, PCP	0.33	0.52	0.19	0.38	0.35	0.72	0.11	0.08	4.59
Complete, LCP	0.34	0.53	0.14	0.35	0.39	0.75	0.11	0.08	4.95
Incomplete, PCP	0.45	0.59	0.11	0.43	0.42	0.73	0.11	0.18	4.56
Incomplete, LCP	0.43	0.59	0.11	0.31	0.41	0.71	0.12	0.17	4.76
VAR(1)	0.83	0.82	0.47	0.62	0.93	1.03	0.49	0.54	4.63
VAR(2)	0.49	0.47	0.22	0.30	0.57	0.66	0.23	0.25	4.03
VAR(4)	0.44	0.41	0.20	0.27	0.48	0.52	0.19	0.22	3.70
VAR(6)	0.38	0.35	0.10	0.21	0.40	0.45	0.13	0.17	3.24