Open Market Operations, Eligible Securities, and Macroeconomic Stabilization

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Abstract
This paper examines the role of collateralized money supply for short-run macroeconomic effects of monetary policy. We apply a simple sticky price model, where the central bank supplies money in exchange for securities that are discounted with the nominal interest rate. The central assumption is that only government bonds are eligible as collateral. If they are dominated in rate of return by private debt, there exists a liquidity premium and money injections are associated with a liquidity effect. When the central bank sets the nominal interest rate, the price level and the allocation are uniquely determined, regardless whether prices are perfectly flexible or not. Finally, we consider the case where the central bank sets the nominal interest rate in order to maximize social welfare in a discretionary way. Inflation is less volatile when money supply is unconstrained, while output fluctuations are smaller under a binding money market constraint. When households are very risk avers, social welfare can be raised by a collateralized money supply if the stock of eligible securities is constant.

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1 Introduction

Central banks in most industrial countries conduct monetary policy mainly via open market operations, where money is supplied in exchange for securities discounted with a short-run nominal interest rate. Hence, the costs of money acquisition depend on the current discount rate and the availability of collateral. In macroeconomic theory, however, is has often been claimed that open market operations are irrelevant in the sense that they are equivalent to lump-sum money transfers (see Wallace, 1981, Sargent and Smith,1987, or Eggerston and Woodford, 2003).[2] In this paper asset exchanges in open market operations are examined in a simple macroeconomic model. We particularly focus on the case where only government bonds are eligible as collateral in open market operations. When this collateral constraint is binding, there exists a liquidity premium for public debt and money injections are associated with a liquidity effect. Further, a binding collateral constraint in the money market leads to an uniquely determined price level and allocation when the central bank controls the interest rate. Moreover it is shown that the introduction of a collateral constraint can reduce macroeconomic fluctuations and even lower welfare losses when households are very risk avers.

The analysis is conducted in an infinite horizon model with identical and infinitely lived households, which demand money due to a cash-in-advance constraint. Money is supplied in open market operations up to the discounted value of eligible securities. Households' financial wealth comprises claims on other households and government bonds carried over from the previous period. We take into account that real world central banks typically restrict the set of eligible securities to assets with high credit quality. For example, the current asset acquisition of the US Federal Reserve can be summarized as a "Treasuries-only" policy (see Broaddus and Goodfriend, 2001).[3] Accordingly, we assume that only government bonds can be used as collateral for money in open market operations. We further assume that households internalize this money market constraint when they decide on their optimal plan. Then there exists a rational expectations equilibrium where private debt yields a higher interest than public debt and the money market restriction is binding. In this case, the outstanding stock of government bonds relates to the amount of money supplied in open market operations and there is a liquidity premium on public debt.[4]

In order to facilitate comparisons with many recent macroeconomic studies on monetary policy, we allow for prices to be set by monopolistically competitive (retail) firms in a staggered way. When there is no collateral constraint, the reduced set of linearized

[2] Dupor (2001) examines the role of open market operations, which are specifed as 'holding fiscal policy constant in the face of a government asset exchange' (see Sargent and Smith, 1987). For this case, he shows that open market operations are not irrelevant, since fiscal policy is non-Ricardian.

[3] To be more precise, the US Federal Reserve exclusively accepts securities issued by the Treasury, federal agencies, as well as acceptances and bank bills, which meet high credit quality standards (see Meulendyke, 1998). See Kopcke (2002) for the asset acquisition policy of central banks in other industrialized countries,

[4] The latter result relates to the solution of risk-free rate puzzle in Bansal and Coleman (1996).

equilibrium conditions is isomorphic to the standard New Keynesian model, as applied in Clarida et al. (1999). In case there is a binding constraint on eligible securities the model exhibits substantial differences. In particular, a monetary injection then reduces the nominal discount rate, such that the model generates a liquidity effect, which can hardly be reproduced in models with an unconstrained money market.[5] Since the current stock of eligible securities is limited by fiscal policy, a rise in the supply of money is accompanied by a decline in the nominal interest rate.

For the case where the central bank is assumed to control the nominal interest rate (discount) rate, a binding money market constraint further leads to different determinacy properties. For an unrestricted money supply, the model behaves conventionally, i.e., an interest rate peg leads to an indetermined price level under flexible prices and to multiple equilibria under sticky prices (see Kerr and King, 1996, and Benhabib et al., 2001).[6] If, however, there is a binding collateral constraint, interest rate policy is always associated with an uniquely determined price level and an unique rational expectations equilibrium.[7] In particular, equilibrium uniqueness does not require interest rate policy to satisfy the well-known Taylor-principle. Instead, the central bank can already stabilize the economy by controlling the nominal interest rate rather than being compelled to adjust the *real* interest rate. Yet, it should refrain from adjusting the interest rate in an extreme way when the supply of government bonds is not fixed. Macroeconomic stability then requires monetary policy to account for the evolution of public debt.

While the former results show that a binding collateral constraint in open market operations matters for the macroeconomic effects of monetary policy, they do not explain why an optimizing central bank imposes a money market constraint. As for example shown by Woodford (2003), a central bank that aims to maximize social welfare can uniquely implement its optimal plan by an appropriately designed state-contingent interest rate feedback rule. Thus, it seems that there is no reason for a central bank to collateralize money supply, if it aims to minimize welfare reducing macroeconomic fluctuations (regardless it faces fundamental or non-fundamental shocks). Yet, a central bank is in reality hardly able to credibly commit itself to a once-and-for-all policy. Hence, it can reasonably be argued that a central bank at best aims to implement a plan under *discretionary* optimization. Such a time-consistent policy plan is however known to be suboptimal with regard to macroeconomic stabilization, since a discretionary monetary policy does not account for private sector expectations on future policy actions. By introducing a collateral constraint

[5] With frictionless asset markets the nominal interest rate tends to increase with money supply due to higher expected inflation. This "liquidity puzzle" can be solved by allowing for limited asset market participation (see Lucas, 1990, Fuerst, 1992, and Alvarez et al., 2002),

[6] This result relates to the view that a monetary policy regime with "no effective limit to the quantity of money", which corresponds to the *real bills doctrine*, is prone to non-uniqueness of prices and equilibria, as shown by Sargent and Wallace (1982), McCallum (1986), or Smith (1988).

[7] Given that tax policy is assumed to ensure government solvency, these findings do not relate to determinacy results in Woodford (1994) or Benhabib et al. (2001) where the primary deficit is exogenous.

in open market operations, a central bank can mechanically steer expectations about the scope of macroeconomic responses to future policy actions, even if it acts under discretion (and therefore in a purely forward-looking way).

To examine to the effect of a constrained money supply on macroeconomic stabilization, we compute variances of inflation and output under cost-push shocks. The interest rate is set in a way that implements the central bank's plan under discretionary optimization for an unconstrained money supply. We thereby follow Woodford (2003) and use that social welfare can be approximated by a quadratic loss function, which depends on the variances of inflation and the output-gap. While inflation fluctuations are smaller under an unconstrained money supply, we find that the variance of the output-gap is reduced under a binding collateral constraint. The reason is that the predetermined stock of eligible securities introduces a history dependence, by which responses of aggregate demand to macroeconomic shocks are smoothed out. If the degree of households' risk aversion is sufficiently high, welfare losses can even be smaller under a constrained money supply. This, however, requires that the stock of eligible securities is held constant in nominal terms.

The remainder is organized as follows. Section 2 develops the model. In section 3 examine the relation between money supply and interest rates, and the determination of the price level for perfectly flexible prices. In section 4 we assess the impact of constrained money supply on macroeconomic stabilization. Section 5 concludes.

2 The model

Identical and infinitely lived household-firm units are endowed with government bonds, money, and claims on other households carried over from the previous period. They produce a wholesale good employing labor from all households. Aggregate uncertainty is due to monetary policy shocks and cost-push shocks, which are realized at the beginning of the period. Then goods are produced and asset markets open, where households can trade without restrictions. Money demand is induced by a cash constraint for purchases of consumption goods. The central bank supplies money exclusively via open market operations. There, the supplied amount of money equals the discounted value of interest bearing assets, which are deposited at the central bank.[8] Then the goods market opens. After goods have been traded, households can repurchase the securities from the central bank. The remaining amount of money is carried over to the next period. There exists a nominal rigidity, which is introduced by monopolistically competitive retail firms that differentiate the wholesale goods and set their prices in a staggered way.[9]

[8] Equivalently, it can be assumed that financial intermediaries or traders engage in open market operations on the behalf of the households.

[9] As a consequence, the log-linear approximation of the model nests the standard New Keynesian model presented in Clarida et al. (1999).

Households Lower (upper) case letters denote real (nominal) variables. There is an infinite number of time periods t ($t = 0, 1, 2, ...$) and a continuum of perfectly competitive household-firm units distributed uniformly over $[0, 1]$. In each period t a household $j \in [0, 1]$ consumes a composite good c and supplies working time $l_{jt} = \int_0^1 l_{jt}^k dk$ to household-firm units, where l_{jt}^k denotes the working time of household j supplied to household k. It produces a wholesale good x_{jt} with the technology $x_{jt} = \int_0^1 l_{kt}^j dk$, and sells the wholesale good to retail firms charging a price P_{jt}^w per unit. Household j is assumed to maximize the expected value of the discounted stream of utility stemming from consumption and leisure,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt}, l_{jt}), \quad \beta \in (0, 1), \tag{1}$$

where β denotes the subjective discount factor. The instantaneous utility function u is assumed to be strictly increasing in consumption c, strictly decreasing in working time l, strictly concave, twice continuously differentiable with respect to both arguments. It further satisfies the usual Inada conditions, and is additively separable.

We separate the household problem into a intratemporal and an intertemporal part. In the *intratemporal* part, households decide on production and on the composition of consumption. Profit maximizing leads to the following demand for labor $l_{kt}^j : P_t w_t = P_{jt}^w = P_t^w$, where P_t denotes the aggregate price level and w_t the real wage rate. Let c_{jt} be consumption of a composite good which is defined as a CES aggregate of differentiated goods y_{it}^j, which are bought from retailers indexed with $i \in [0, 1] : c_{jt}^{\frac{\epsilon_t - 1}{\epsilon_t}} = \int_0^1 y_{it}^{j\,\frac{\epsilon_t - 1}{\epsilon_t}} di$, where $\epsilon_t > 1$ is the elasticity of substitution between any two retail goods. We allow this elasticity to vary exogenously over time. It will serve as a so-called cost-push shock, which will be relevant in the last part of the paper where the issue of macroeconomic stabilization is examined. Let P_{it} denote the price of the retail good y_{it} and let the price of the composite good P_t be given by $P_t^{1-\epsilon_t} = \int_0^1 P_{it}^{1-\epsilon_t} di$. Minimizing costs for purchasing a unit of the composite good leads to the following demand for the retail good y_{it}^j :

$$y_{it}^j = (P_{it}/P_t)^{-\epsilon_t} c_{jt}. \tag{2}$$

The *intertemporal* part unfolds as follows. Note that the index j is – except for the supply side variables – disregarded, for convenience, as households are (otherwise) identical. At the beginning of period t households are endowed with financial wealth A_{t-1}, which comprises government bonds holdings B_{t-1}, claims on other households D_{t-1}, and money holdings $M_{t-1}^H : A_{t-1} = B_{t-1} + D_{t-1} + M_{t-1}^H$. Both interest bearing assets are assumed to be nominally state contingent leading to a payoff equal to $R_t^d D_{t-1}$ and $R_t B_{t-1}$ in period t. This assumption is introduced to deliver the conventional specification of the consumption Euler equation.[10]

[10] This feature will particularly be helpful to simplify the analysis of the model.

Before agents trade in assets or goods, the aggregate shocks arrive, goods are produced, and wages are credited on checkable accounts at financial intermediaries. Then households enter the assets market, where they can trade with other households and the treasury in an unrestricted way. After the asset market is closed, households can participate in open market operations, where they can exchange interest bearing assets B_t^c for money I_t. The amount of money traded in open market operations I_t equals the discounted value B_t^c/R_t :

$$I_t = \frac{B_t^c}{R_t}. \tag{3}$$

Thus, the exchange (repo) rate in open market operations equals the gross nominal interest rate on government bonds. The condition (3) is assumed to hold for all types of open market operations, namely outright sale/purchases as well as repurchase agreements. Money traded via repurchase agreements, which is denoted by $M_t^R \geq 0$, is only held until the end of the period, when repurchase agreements are settled. Hence, M_t^R is a flow variable and is the counterpart of securities temporarily deposited at the central bank. Changes in money thus satisfy $I_t = M_t^R + M_t^H - M_{t-1}^H$.

After households have traded with the central bank, they enter the goods market. Here, they rely on cash balances as means of payment. Cash balances consist of the total amount of money $M_t = M_t^H + M_t^R$, i.e., money under outright sales/purchases M_t^H and held under repurchase agreements M_t^R, and checkable non-interest bearing accounts at a financial intermediary. These accounts consist of the individual labor income $P_t w_t l_{jt}$ net of wage outlays for the own firm $P_t w_t \int_0^1 l_{kt}^j dk$. Hence, purchases of goods are subject to the following liquidity constraint:

$$P_t c_t \leq M_t + \left[P_t w_t l_{jt} - P_t w_t \int_0^1 l_{kt}^j dk \right]. \tag{4}$$

The constraint (4) differs from more conventional cash-in-advance constraint by the term in the square brackets, which is introduced to avoid the cash-credit good distortion between consumption and leisure.[11] Applying a standard cash-in-advance constraint causes the nominal interest rates to distort the household's decision for consumption and labor supply. While the main results in this paper are not affected by this distortion, it would exacerbate the comparison with conventional models, given that the nominal interest rate would then distort aggregate supply.[12]

Households receive cash from sales of the wholesale good x_{jt} to retail firms and from retail firms' profits $P_t \int_0^1 \omega_{it} di$, and have to pay a lump sum tax $P_t \tau_t$. After the goods market is closed, inside money M_t^R is used by the households to repurchase securities from

[11] This specification closely follows Jeanne (1998).

[12] By avoiding the cash-credit good distortion, the model to nest the standard New Keynesian model for $\eta_t = 0$.

the central bank. Household j's budget constraint is given by

$$D_t + B_t + M_t^H + (R_t - 1)\left(M_t^R + M_t^H - M_{t-1}^H\right) + P_t c_t + P_t \tau_t \tag{5}$$
$$\leq R_t B_{t-1} + R_t^d D_{t-1} + M_{t-1}^H + P_t w_t l_{jt} - P_t w_t \int_0^1 l_{kt}^j dk + P_t^w x_{jt} + P_t \int_0^1 \omega_{it} di.$$

The central assumption is that money acquisition is subject to a collateral constraint. Given that asset acquisition of central banks is often restricted to a set of high credit quality securities, we impose, accordingly, a money market constraint by which only government bonds are accepted as collateral:

$$B_t^c \leq B_t. \tag{6}$$

Such a restriction on the asset acquisition of a central bank is commonly justified by the aim to avoid credit risk in its portfolio and effects on credit allocation (see Meyer, 2001). It actually imposes an upper bound on the supply of money, given by the discounted value of total government bonds held by private sector. In the case where the central bank sets the interest rate, the constraint (6) can be viewed as the main difference between a money supply regime, as for example applied by the US Federal Reserve in the recent past and the so-called 'real bills doctrine' (see Friedman and Schwartz, 1963).[13]

It is further assumed that households are aware of the fact that their access to cash is restricted by their holdings of government bonds. This restriction would be irrelevant when they can issue private debt with an interest rate not higher than the interest rate on government bonds. However, as the monetary authority (directly or indirectly) controls the latter, a positive spread $R_t^d > R_t$ cannot generally be ruled out. Combining (3) with (6) the money market constraint reads

$$M_t^R + M_t^H - M_{t-1}^H \leq B_t / R_t. \tag{7}$$

Maximizing (1) subject to the constraints for the goods market (4), the asset market (5), the money market (7), a non-negativity constraint on money held under repurchase agreements, $M_t^R \geq 0$, and a no-Ponzi-game condition $\lim_{i\to\infty} E_0 A_{t+i} \prod_{v=1}^{i} 1/R_{t+v}^d \geq 0$, for a given initial value of total nominal wealth $A_{-1} > 0$ leads to the following first order conditions for consumption, leisure, holdings of private and public debt, and holdings of

[13] According to the real bills doctrine, the nominal interest rate is held at its target, while money is supplied in exchange for short-term commercial *bills* that are intended to finance *real* transactions and is, thus, a potentially unbounded way. See McCallum (1986) for the relation between an interest rate peg and the real bills doctrine, and for further references.

money, M_t^R and M_t^H :

$$u_{ct} = \lambda_t + \psi_t, \quad (8)$$

$$-u_{lt} = u_{ct} w_t, \quad (9)$$

$$\lambda_t = \beta E_t \left[R_{t+1}^d \lambda_{t+1} / \pi_{t+1} \right], \quad (10)$$

$$\lambda_t = \beta E_t \left[R_{t+1} \lambda_{t+1} / \pi_{t+1} \right] + \eta_t, \quad (11)$$

$$\xi_t = (R_t - 1)\lambda_t + R_t \eta_t - \psi_t, \quad (12)$$

$$\psi_t = R_t \lambda_t - \beta E_t \left[R_{t+1} \lambda_{t+1} / \pi_{t+1} \right] + R_t \eta_t - \beta E_t \left[R_{t+1} \eta_{t+1} / \pi_{t+1} \right], \quad (13)$$

where $\pi_t = P_t / P_{t-1}$ denotes the inflation rate, λ the shadow price of wealth, ψ the multiplier on the goods market constraint (4), and η the multiplier on the money market constraint (7). The equations (12) and (13) give the first order conditions for M^R and M^H, where ξ denotes the multiplier on the non-negativity constraint on M^R. The household's optimum is further characterized by the constraints (4), (5), and (7),

$$\xi_t \geq 0, \ \xi_t M_t^R = 0, \quad (14)$$

$$\eta_t \geq 0, \ \eta_t \left[B_t - R_t M_t \right] = 0, \quad (15)$$

$$\psi_t \geq 0, \ \psi_t \left[M_t P_t^{-1} + w_t l_{jt} - w_t \int_0^1 l_{kt}^j dk - c_t \right] = 0, \quad (16)$$

the budget constraint (5) holding with equality, $\lambda_t \geq 0$, and the transversality condition, $\lim_{i \to \infty} E_0 a_{t+i} \prod_{v=1}^{i} \pi_{t+v} / R_{t+v}^d = 0$.

Retailer There is a monopolistically competitive retail sector with a continuum of retail firms indexed with $i \in [0,1]$ (owned by the households). Each retail firm buys a quantity x_{jt}^i of the wholesale good produced by household j at price P_t^w. To minimize distortions induced by liquidity constraints, we assume that retail firms can purchase the wholesale good on credit. We assume that a retailer is able to differentiate the wholesale good without further costs. The differentiated retail good $y_{it} = \int_0^1 x_{jt}^i dj$ is then sold at a price P_{it}. We assume that retailers set their prices according to Calvo's (1983) staggered price setting model. The retailer changes its price when it receives a signal, which arrives in a given period with probability $(1 - \phi)$, where $\phi \in [0, 1)$. A retailer who does not receive a signal adjusts its price by the steady state aggregate inflation rate $\overline{\pi}$, such that $P_{it} = \overline{\pi} P_{it-1}$. A retailer who receives a price change signal in period t chooses a price $\widetilde{P}_{it}$ to maximize the expected sum of future discounted profit streams given by $E_0 \sum_{v=0}^{\infty} (\beta\phi)^v \left[\lambda_{t+v} / (\lambda_t P_{t+v}) \right] P_t \widetilde{\omega}_{it+v}$, where $P_t \widetilde{\omega}_{it+v}$ denotes profits in period $t+v$ for own prices not being adjusted after period $t : P_t \widetilde{\omega}_{it+v} = (\widetilde{P}_{it} y_{it+v} - P_{t+v}^w \int_0^1 x_{jt+v}^i dj)$. Note that retailer exhibit the same discount factor as households. For this one might think of randomly drawn households that are the managers of retail firms. Maximizing the expected sum of discounted profits subject to the demand $y_{it} = (P_{it}/P_t)^{-\epsilon_t} c_t$, taking the price P_t^w of the wholesale good, the aggregate final goods price index P_t and the initial price level P_{-1} as given, yields the following first-order condition for $\widetilde{P}_{it}$

$$\widetilde{P}_{it} = \frac{\epsilon_t}{\epsilon_t - 1} \frac{E_0 \sum_{v=0}^{\infty} (\beta\phi)^v x_{t+v} P_{t+v}^{\epsilon_t} \pi^{-\epsilon_t v} P_{t+v}^w}{E_0 \sum_{v=0}^{\infty} (\beta\phi)^v x_{t+v} P_{t+v}^{\epsilon_t} \pi^{(1-\epsilon_t)v}}, \tag{17}$$

where $x_{t+v} = \int_0^1 x_{jt+v} dj$. Using the pricing rule for the remaining fraction ϕ of the firms ($P_{it} = \overline{\pi} P_{it-1}$), the price index for the final good P_t evolves recursively over time. In a symmetric equilibrium the price level satisfies $P_t^{1-\epsilon_t} = \phi \left(\overline{\pi} P_{t-1}\right)^{1-\epsilon_t} + (1-\phi) \widetilde{P}_t^{1-\epsilon_t}$, which can be rewritten as:$1 = \phi \left(\overline{\pi}/\pi_t\right)^{1-\epsilon_t} + (1-\phi) [\widetilde{P}_t/P_t]^{1-\epsilon_t}$.

Public sector The public sector consists of a fiscal and a monetary authority. The monetary authority supplies money in open market operations in exchange for securities and transfers the seigniorage to the fiscal authority. The budget constraint of the central bank is given by

$$M_t^H + (R_t - 1)\left[M_t^R + M_t^H - M_{t-1}^H\right] = M_{t-1}^H + P_t \tau_t^c,$$

where τ^c denotes transfers to the fiscal authority. We consider two monetary policy regimes, which differ with regard to the choice of the instrument. The first regime is characterized by the central bank controlling the supply of money $\mu_t = M_t/M_{t-1}$. For the second regime, which is analyzed in the last part of the paper, we assume that the central bank controls the nominal interest rate R_t.

The fiscal authority issues one period bonds earning a gross nominal interest rate R_t, collects lump-sum taxes τ from the households, and receives the transfer τ^c from the monetary authority:

$$R_t B_{t-1} = B_t + P_t \tau_t^c + P_t \tau_t. \tag{18}$$

Hence, interest rate payments on public debt are the only source of expenditures for the fiscal authority. The fiscal authority is assumed to finance a constant fraction ϑ of interest rate payments on outstanding debt by taxes and transfers from the central bank:[14]

$$P_t \tau_t + P_t \tau_t^c = \vartheta (R_t - 1) B_{t-1}, \quad \vartheta \in (0, 1]. \tag{19}$$

Thus, the feedback parameter ϑ governs the share of government expenditures covered by tax receipts. Using (19) to eliminate taxes in the budget constraint (18) leads to the following equation which describes the evolution of nominal government debt

$$B_t = \left[(1 - \vartheta)(R_t - 1) + 1\right] B_{t-1}, \tag{20}$$

or in real terms $b_t = [(1-\vartheta)(R_t - 1) + 1] b_{t-1}/\pi_t$, where $b_t = B_t/P_t$. Hence, a higher

[14] A similar fiscal policy specification which further allows for a so-called non-Ricardian fiscal regime ($\vartheta = 0$) can be found in Benhabib et al. (2001).

value for the fiscal policy parameter ϑ reduces the growth rate of government bonds. Put differently, a high value for ϑ (close to one) leads to a stable supply of government bonds. In the subsequent analysis we will focus, for convenience, on the case where money is not used as a store of value by households, $M_t^H = 0$. In this case, (20) with $\vartheta > 0$ implies that solvency of the public sector is guaranteed, i.e., that $\lim_{i\to\infty}[B_{t+i}+M_{t+i}]\prod_{v=1}^{i} 1/R_{t+v} = 0$ is always satisfied.[15]

In what follows we restrict our attention to the case of a binding goods market constraint, which reads $c_t = m_t = M_t/P_t$ in equilibrium. For this, the nominal interest rate on government bonds R_t will be assumed to be larger than one such that $\psi_t > 0$.

3 Monetary policy under flexible prices

In this section we examine the role of the money market constraint for the relation of money supply and interest rates. For this we focus on the simplifying case where the elasticity of substitution ϵ_t is constant $\epsilon_t = \epsilon$ and prices are perfectly flexible ($\phi = 0$). It should be noted that monetary policy is neutral in this case. In the first parts of this section we discuss the collateral constraint and the existence of a liquidity effect. In the final part we show that the price level is determined under a binding collateral constraint, when the central bank pegs the interest rate.

3.1 Collateralized money supply

In equilibrium money serves as the single means of payment in the goods market (see 4). Thus, for purchases of the consumption good households either have to carry over money from one period to the other, or they can hold money under repurchase agreements. When there is no collateral constraint in the money market, households are actually indifferent between both types of money holdings. The reason is that the foregone interest from holding money instead of debt exactly equals the additional cost of money acquisition under repurchase agreements. Accordingly, the multiplier ξ_t on the non-negativity constraint $M_t^R \geq 0$ is zero for $\eta_t = 0$. This can easily be seen from both first order conditions for money holdings (12) and (13) combined with the first order condition for government bonds (11). Hence, without loss of generality we can assume that households only hold money under repurchase agreements in this case ($\eta_t = 0$).[16]

When there exists a collateral constraint in the money market (7), the model features two fundamentally different versions depending on whether the constraint is binding or not. When public and private debt are eligible, the collateral constraint would become

[15] Hence, our specification differs from the one in Dupor (2001), where open market operations are defined as government asset exchanges associated with a constant tax policy (see also Sargent and Smith, 1987, or Schreft and Smith, 1998).

[16] This propery crucially relies on the assumption that government bonds are nominally state contingent. If it is assumed that the interest rate payments are risk-free, $R_{t-1}B_{t-1}$, the multiplier on money holdings under repurchase agreements would not be equal to zero.

irrelevant, since money can also be acquired in exchange for securities that can be issued by households. Even if open market operations are restricted by (6), the collateral constraint would be irrelevant as long as households' government bonds holdings are sufficiently large such that $B_t \geq B_t^c$ always holds. If government bonds earn the same interest as private debt, $R_t = R_t^d$, households can borrow to invest in government bonds without any additional costs. In contrast, when the interest rate on government bonds is smaller than the interest rate on private debt, this strategy becomes costly and households are willing to economize on the holdings of government bonds.

When only government bonds are eligible, there is a liquidity premium indicated by $\eta_t > 0$, such that an interest rate spread $R_t^d > R_t$ can exist in equilibrium. Combining (10) and (11) shows that the multiplier η_t is positive if the households expect the (weighted) spread between future interest rates on private debt and public debt to be positive.

$$\eta_t = \beta E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \left(R_{t+1}^d - R_{t+1} \right) \right]. \tag{21}$$

In this case, the money market constraint (7) is binding and households only hold government bonds up to the desired amount of money times the current interest rate, $B_t = R_t M_t$. When the collateral constraint is binding, households are in general not indifferent between both types of money holdings. Yet, in order to simply the subsequent analysis we assume for the remainder of the paper that households only hold money temporarily.

Assumption 1 *Money is exclusively held under repurchase agreements,* $M_t^H = 0 \; \forall t > 0$, *and the initial value of money held by the households is equal to zero,* $M_0^H = 0$, *such that* $I_t = M_t = M_t^R$ *and* $\xi_t = 0 \; \forall t$.

In what follows it will be shown that a binding money market constraint, $\eta_t > 0$, affects the equilibrium behavior of government bonds, interest rates, money, and consumption. Money and, thus, consumption are then linked to the real value of government bonds by $c_t R_t = b_t$. If, however, the money market constraint is not binding, the amount of securities traded in open market operations B_t^c is not directly linked to the total stock of public debt. This case corresponds to the conventional specification of monetary business cycle models, where there is no money market restriction. Changes in money supply via open market operations are then equivalent to lump-sum injections of money.

3.2 Liquidity effects

While the empirical evidence for a liquidity effect is mixed,[17] it is often viewed as the basic mechanism by which a central bank controls the short-run interest rate via money supply adjustments. Yet, a liquidity effect can hardly be generated in macroeconomic

[17] See Eichenbaum (1992), Hamilton (1997), or Bernanke and Mihov (1998) for empirical evidence in support of a liquidity effect in US data.

models with frictionless asset market trade. However, a liquidity effect can emerge if one considers a limited asset market participation, as shown by Lucas (1990), Fuerst (1992), or Alvarez et al. (2002). Since the money market constraint (7) implies an inverse relation between money supply and the nominal interest rate, our model is also able to generate a liquidity effect. Evidently, this requires the money market constraint to be binding.

Suppose that the central bank exogenously controls the supply of money via open market operations. The growth rate, $\mu_t = m_t\pi_t/m_{t-1}$, is assumed to satisfy

$$\mu_t = \overline{\mu}^{1-\rho}\mu_{t-1}^{\rho}\exp(\varepsilon_t), \quad \rho \in [0,1), \tag{22}$$

where the innovations ε_t have an expected value equal to zero and are serially uncorrelated. It should be noted that money supply is specified in terms of the growth rate to facilitate comparisons. Given that money is actually a flow variable under assumption 1, it might be more intuitive to specify money supply in levels.

Under flexible prices the real wage rate is constant and equals the inverse of the retailers' markup, $w_t = P_t^w/P_t = \frac{\epsilon-1}{\epsilon}$. Hence, consumption is uniquely pinned down by (9) and $c_t = l_t$, and is therefore constant, $c_t = c$. Since the binding goods market constraint implies, $m_t = c_t$, the inflation rate equals the growth rate of money, $\pi_t = \mu_t$. If the money market constraint is *not* binding, $\eta_t = 0$, the first order conditions (12), which reads $\psi_t = (R_t - 1)\lambda_t$, and (8) imply $u_{ct} = R_t\lambda_t$. Together with the first order condition for government bonds (11), one gets to the consumption Euler equation

$$u_c(c_t) = \beta R_t E_t\left[u_c(c_{t+1})/\pi_{t+1}\right]. \tag{23}$$

Given that consumption is constant, the nominal interest rate satisfies $R_t = \mu_t^{\rho}(\overline{\mu}^{1-\rho}/\beta)$. Thus, if $\rho > 0$ the current nominal interest rate rises with a money growth shock due to higher expected inflation.

Now suppose that money market constraint is binding, $\eta_t > 0$. Then, the stock of government bonds relates to the supply of money and thus to consumption expenditures, $b_t = c_t R_t$. Using the bond supply equation (20) and that consumption is constant, leads to the following relation between money supply and the nominal discount rate R :

$$R_t = \frac{\vartheta}{\mu_t R_{t-1}^{-1} - 1 + \vartheta}. \tag{24}$$

According to (24) a rise in the money growth rate μ_t is associated with a decline in the nominal interest rate R_t. It should be noted that the strictly positive feedback parameter, $\vartheta > 0$, leads to a bounded supply of eligible securities and is therefore responsible for the price of money, i.e., the nominal discount rate, to decline in response to a money injection.

Proposition 1 *A money injection leads to liquidity effect only if the money market constraint is binding.*

Notably, the nominal interest rate declines in response to a monetary injection, regardless whether the money growth rate is serially correlated or not.

3.3 Price level determination

When the central bank controls the money growth rate the price level can be determined in equilibrium, regardless whether the money market constraint is binding or not. If however the central bank exogenously controls the interest rate rather than the money growth rate, the price level can usually not be determined (see, e.g., Sargent and Wallace, 1975). In contrast to the case where the central bank sets the money growth policy, there is no nominal anchor that would allow to determine the price level under an exogenous interest rate policy. In this model, this conclusion has to be qualified, since the stock of nominal public debt might serve as a nominal anchor. To be more precise, the price level can be determined if the money market constraint is binding.[18]

Proposition 2 *Suppose that the cash constraint is binding and that the central bank pegs the nominal interest rate $R_t = R$. Then the price level is (in)determined if the money market constraint is (not) binding.*

Proof. Initial wealth is predetermined and satisfies $A_{-1} > 0$ and $A_t = B_t$ in equilibrium. Hence, it evolves, by (20), according to $A_t = \alpha^t A_0$, where $\alpha = (1 - \vartheta)R + \vartheta > 0$. When $\eta_t = 0$, an interest rate peg determines the inflation rate by $\pi = R\beta$, while the growth rate of real financial wealth is given by $a_t/a_{t-1} = \alpha/(R\beta)$. The current value of real wealth and the price level are indeterminated. For $\eta_t > 0 \Rightarrow m_t = a_t/R$ and $m_t = c_t$, real wealth equals $a_t = a = Rc$, such that the price level is determined by: $P_t = A_t/a_t = \alpha^t A_0/(cR)$. ■

The reason why the price level can be determined when the money market constraint binds relies on the property that government bonds provide liquidity services. Hence, in equilibrium one can determine the current real value of financial wealth which is predetermined in nominal terms. This mechanism closely relates to the determination of the price level in Canzoneri and Diba's (2005) model, where government bonds directly enter a goods market constraint. To summarize, a restriction on the supply of money can resolve the problem of price level indeterminacy, which occurs when the nominal interest rate is pegged and money supply is unbounded.

4 Monetary policy under imperfectly flexible prices

We now turn to the case where prices are not perfectly flexible, which is widely viewed as the more realistic case. Specifically we assume that there exist a non-zero fraction $\phi > 0$

[18] It should be noted that the price level is determined, though the fiscal authority guarantees government solvency (by $\vartheta > 0$).

of firms that do not adjust prices in an optimizing way. In order to obtain a tractable form we log-linearize the model at the steady state. Since we want to assess the role of the money market constraint, we are interested in a steady state where the latter is binding.

4.1 Steady state

Let a variable with a bar denote the particular steady state value. The steady state is characterized by constant values for c, π, a, m, R^d, and R given by: $u_c(\overline{c})/[-u_l(\overline{c})] = \epsilon/(\epsilon-1)$, $\overline{m}=\overline{c}$, $\overline{\pi}=\mu$, and $\overline{R^d}=\overline{\pi}/\beta$, regardless whether the money market constraint is binding or not. If the money market constraint is binding, $\overline{\eta}>0$, the steady state satisfies

$$\overline{\pi} = \overline{R}(1-\vartheta)+\vartheta \quad \text{and } \overline{a}=\overline{c}\overline{R}. \tag{25}$$

Otherwise ($\overline{\eta}=0$), the interest rate satisfies $\overline{R}=\overline{R}^d$. The existence of a steady state with a binding money market constraint, $\overline{R}<\overline{R}^d \Rightarrow \overline{\eta}>0$, requires the central bank to choose a small average money growth rate $\overline{\mu}$ or a small average interest rate $\overline{R}$, as well as a sufficiently large value for the feedback parameter ϑ. The steady state conditions for binding constraints in the money and the goods market ($\overline{R}>1 \Rightarrow \overline{\psi}>0$), are presented in the following proposition.

Proposition 3 *Suppose that the share of tax financing is sufficiently large such that $\vartheta > 1-\beta$. Then there exists a steady state with binding constraints in the money and the goods market if the central bank either chooses i.) an average money growth rate satisfying $\overline{\mu}\in(\beta,\widetilde{\mu})$, where $\widetilde{\mu}=\vartheta\beta/[\vartheta-(1-\beta)]>1$, or ii.) an average nominal interest rate satisfying $\overline{R}\in(1,\widetilde{R})$, where $\widetilde{R}=\vartheta/[\vartheta-(1-\beta)]>1/\beta$.*

The condition on bond issuance, $\vartheta>1-\beta$, which is hardly restrictive, ensures that the intervals for $\overline{\mu}$ and $\overline{R}$ are non-empty. The upper (lower) bounds $\widetilde{\mu}$ and $\widetilde{R}$ then guarantee the existence of a steady state with binding constraints in the goods market and in the money market. In what follows we will (implicitly) use that when public policy satisfies the conditions in proposition 3, both constraints can be binding, $\psi_t>0$ and $\eta_t>0$ (provided that the support of aggregate shocks is sufficiently small).

4.2 Local dynamics

Due to the assumption of imperfectly flexible price adjustments, the model features a dynamic aggregate supply constraint. Log-linearizing (17) and $1=\phi\,(\overline{\pi}/\pi_t)^{1-\epsilon_t}+(1-\phi)\,[\widetilde{P}_t/P_t]^{1-\epsilon_t}$, the evolution of the inflation rate can be summarized by the following constraint, i.e., the so-called marginal cost based Phillips curve:[19] $\widehat{\pi}_t=\chi\widehat{mc}_t+\widehat{\varkappa}_t+\beta E_t\widehat{\pi}_{t+1}$, where $\chi=(1-\phi)(1-\beta\phi)\phi^{-1}>0$, $\widehat{\varkappa}_t=\chi\cdot\widehat{\iota}_t$ and $\iota_t=\epsilon_t/(\epsilon_t-1)$, and $mc_t=P_t^w/P_t(=w_t)$ denotes the retailers' real marginal costs. The equilibrium of the log-linear approximation to the model at a steady state with $\overline{R}>1$, $\sigma=-\overline{u}_c/(\overline{u}_{cc}\overline{c})\geq 1$, and $\upsilon=\overline{u}_l/(\overline{u}_{ll}\overline{l})\geq 0$,

[19] See, for example, Yun (1996) for the derivation of this constraint.

can the be summarized as follows: A *rational expectations equilibrium* of the log-linear approximation to the sticky price model at the steady state with $\overline{R} > 1$, and $\mu \geq 0$ is a set of sequences $\{\widehat{y}_t, \widehat{m}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{a}_t\}_{t=0}^{\infty}$ satisfying $\widehat{m}_t = \widehat{c}_t$,

$$\widehat{y}_t = \begin{cases} \widehat{a}_t - \widehat{R}_t & \text{if} \quad \eta_t > 0 \\ E_t\widehat{y}_{t+1} - (\widehat{R}_t - E_t\widehat{\pi}_{t+1})/\sigma & \text{if} \quad \eta_t = 0 \end{cases}, \tag{26}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \gamma_1\widehat{y}_t + \widehat{\varkappa}_t, \tag{27}$$

$$\widehat{a}_t = \widehat{a}_{t-1} + \gamma_2\widehat{R}_t - \widehat{\pi}_t \qquad \text{if} \quad \eta_t > 0, \tag{28}$$

where $\gamma_1 = \chi(\sigma + \upsilon) > 0$, $\gamma_2 = (\overline{\pi} - \vartheta)/\overline{\pi} \in [0, 1)$, and the transversality condition, for a monetary policy, a sequence $\{\widehat{\varkappa}_t\}_{t=0}^{\infty}$ and a given initial value $a_{-1} = A_{-1}/P_{-1} > 0$. Note that uncertainty is now due to cost-push shocks $\varkappa_t$. The cost-push shock is assumed to satisfy $\widehat{\varkappa}_t = \rho_{\varkappa}\widehat{\varkappa}_{t-1} + \varepsilon_t$, where $\rho_{\varkappa} \in [0, 1)$ and the innovations are normally distributed with mean zero and a constant variance, $\varepsilon_t \sim N(0, var_e)$.

The equilibrium conditions (26)-(28) reveal that real financial wealth, i.e., the real value of government bonds outstanding, only affects consumption and inflation in the case where the collateral constraint is binding ($\eta_t > 0$). Otherwise ($\eta_t = 0$), the equilibrium sequences of consumption, inflation, real balances, and the nominal interest rate are unaffected by real wealth, since they can solely be determined by (26)-(27) and a monetary policy.[20] Put differently, when the money market constraint is not binding, the evolution of public debt is irrelevant for the allocation, i.e., Ricardian equivalence applies.

In what follows we consider a monetary policy regime that satisfies a simple feedback rule for the nominal interest rate. In particular, interest rate setting is considered to depend on the realizations of the current inflation rate $R_t = \pi_t^{\rho_\pi}\kappa_\rho$, such that

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t, \quad \text{where } \rho_\pi \geq 0. \tag{29}$$

Since prices are not perfectly flexible, the stabilization of inflation is a welfare enhancing policy strategy. In the subsequent section we will use that an optimal monetary policy plan (under discretion) can be implemented by an interest rate feedback rule satisfying (29). For now, we will not further restrict the value of the feedback parameter ρ_π. Instead we want to examine conditions for the policy parameter ρ_π and ϑ which ensure the existence of exactly one stable set of equilibrium sequences.[21]

Proposition 4 *Suppose that the central bank sets the nominal interest rate according to (29). The rational expectations equilibrium with a binding money market constraint is locally stable and uniquely determined if and only if* $\rho_\pi < \overline{\rho_\pi}$, *where* $\overline{\rho_\pi} = 1 + \vartheta[(1 - \vartheta)R]^{-1} > 1$. *The equilibrium sequences are then non-oscillatory.*

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[20] The equations (26)-(27) together with an interest rate feedback rule are also known as the standard New Keynesian model, which is for example used in Clarida et al. (1999).

[21] Given that the model exhibits one predetermined variable (a_{t-1}), equilibrium stability and uniqueness requires the existence of exactly one stable eigenvalue.

Proof. When the interest rate is set according to (29), the deterministic version of the model with $\eta_t > 0$ can be reduced to $(1 + \gamma_1 \rho_\pi) \widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \gamma_1 \widehat{a}_t$ and $\widehat{a}_t = \widehat{a}_{t-1} + (\gamma_2 \rho_\pi - 1) \widehat{\pi}_t$. Its characteristic polynomial reads

$$f(X) = X^2 - [(\gamma_1 \rho_\pi + 1) - \gamma_1 (\gamma_2 \rho_\pi - 1) + \beta] \beta^{-1} X + (\gamma_1 \rho_\pi + 1) \beta^{-1}. \tag{30}$$

Apparently, $f(0)$ equals $f(0) = (1 + \gamma_1 \rho_\pi) / \beta > 1$, implying that there cannot be two stable roots. Further, $f(1)$ is given by $f(1) = \gamma_1 (\gamma_2 \rho_\pi - 1) / \beta$. Thus, in order to exhibit a stable root between zero and one the policy parameter have to satisfy $\rho_\pi < 1/\gamma_2 = \overline{\pi}/(\overline{\pi} - \vartheta)$. Then model exhibits one stable and one unstable eigenvalue, implying that there exists exactly one stable solution. If $\rho_\pi \geq 1/\gamma_2$, both eigenvalues exhibit a real part larger than one, since the slope at $X = 1$ is negative $f'(1) = -\beta^{-1}\{\gamma_1[(1 - \gamma_2)\rho_\pi + 1] + (1 - \beta)\} < 0$. Hence, equilibrium indeterminacy cannot occur, while, using $\overline{\pi} = (1 - \vartheta)R + \vartheta$, stability prevails if and only if $\rho_\pi < [((1 - \vartheta)R + \vartheta]/[(1 - \vartheta)R]$. ■

According to proposition 4, the model with a binding money market constraint is in any case associated with a unique rational expectation equilibrium. Hence, in contrast to the case where the money market constraint is not binding, the Taylor-principle ($\rho_\pi > 1$) is neither necessary nor sufficient for real determinacy. In the latter case ($\eta_t = 0$), $\rho_\pi > 1$ avoids a non-fundamentally induced rise in expected inflation to induce a decline in the real interest rate that would lead to a rise in current consumption and, thus, in inflation, which would cause the initial expectation to become self-fulfilling. When the money market constraint is binding, there is another mechanism which rules out sunspot equilibria. A rise in inflation leads to a decrease in real financial wealth by (28). Since condition (26) implies consumption to rise with real wealth, aggregate demand declines, given that the nominal interest rate is non-decreasing in inflation (see 29). Hence, the aggregate demand response tends to lower current inflation by the aggregate supply constraint (27), such that inflation expectations cannot be self-fulfilling.

To get an intuition for the potentially destabilizing effect of bond supply (ϑ), consider that the central bank chooses a high inflation elasticity ρ_π and that inflation rises due to a cost push shock. If ϑ is high enough, the real value of public debt will be reduced by higher prices. If, however, the the stock of nominal debt is not very stable (low ϑ), then the associated rise in the nominal interest rate R_t can lead to higher real debt and thus to a rise in the real value of eligible securities held by the households. Households can therefore raise their consumption expenditures, since the money market constraint is eased. The rise in aggregate demand then feeds inflation by (27) such that the initial inflationary impulse is enhanced. Concisely, a highly aggressive interest rate policy might lead to an explosive equilibrium when the fiscal feedback ϑ is too small. The upper bound $\overline{\rho_\pi}$ given in proposition 4 further reveals that it is sufficient for stability (and uniqueness) if the stock of eligible securities is constant ($\vartheta = 1 \Rightarrow \overline{\rho_\pi} = \infty$) or if the central bank sets the nominal interest rate in a passive way ($\rho_\pi < 1$).

4.3 Macroeconomic stabilization

The previous analysis has already shown that there might be a good reason for a central bank to impose a collateral constraint in open market operations. By inducing a link between money supply and government bonds, it can avoid equilibrium multiplicity that arises under a passive policy ($\rho_\pi < 1$) if money supply is unrestricted. Yet, a stationary evolution of the economy requires a sufficiently stable supply of nominal public debt, i.e. a sufficiently large feedback parameter ϑ.

In this section we aim at disclosing if there might be a further reason for a central bank to impose a money market constraint. In particular, we want to assess the stabilization properties of monetary policy under both scenarios, $\eta_t = 0$ and $\eta_t > 0$. For this purpose, we disregard any differences with regard to their long-run macroeconomic consequences. Thus, we assume that the values of relevant variables are identical in the steady state under both scenarios and that the steady state is undistorted. Following Woodford (2003) we therefore assume that the average mark-up distortion has been eliminated by some unspecified wage subsidy and that prices are stable in the long-run. Hence, we have to restrict the fiscal policy parameter ϑ to be equal to one, which implies a constant supply of nominal public debt and therefore a steady state inflation rate equal to one (see 25). Finally, we assume that the central bank aims to maximize social welfare in a discretionary way. Hence, we realistically assume that the central bank cannot credibly commit itself to a once-and-for-all policy.

Suppose that the money market constraint is not binding. As shown by Woodford (2003), social welfare can then be approximated by a quadratic loss function

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \approx \overline{U} - \Upsilon \cdot E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\hat{\pi}_t^2 + \frac{\gamma_1}{\epsilon} \widehat{y}_t^2 \right]. \tag{31}$$

where $\Upsilon > 0$ and $\widehat{y}_t$ measures the output-gap (provided that shocks are distortionary). The central bank maximizes this approximated welfare measure subject to the private sector equilibrium conditions taking expectations as given. Its plan under discretionary optimization can then be summarized by (27), $\widehat{y}_t = E_t \widehat{y}_{t+1} - (\widehat{R}_t - E_t \widehat{\pi}_{t+1})/\sigma$, and $\widehat{\pi}_t = -\frac{1}{\epsilon} \widehat{y}_t$ $\forall t$ (see Woodford, 2003). The solution to this plan is uniquely determined and can be implemented by an interest rate feedback rule satisfying (29) and

$$\rho_\pi = \rho_\pi^*, \text{ where } \rho_\pi^* = (1 - \sigma\epsilon)\, \rho_\varkappa + \sigma\epsilon > 1. \tag{32}$$

Now consider that the central bank imposes the money market constraint. When it is binding, the equilibrium under an interest rate feedback rule satisfying (32) will be different from the one under an unconstrained money supply. Evidently, the two (uniquely determined) equilibria exhibit different dynamic properties and thus different implications for macroeconomic stabilization. In what follows we compare social welfare between both equilibria under an interest rate policy that is optimal for the case where money supply is unconstrained, i.e., for an interest rate feedback rule satisfying (29) and (32).

To simplify the analysis we assume that the economy is in its steady state in the initial period. As implied by (31), it is then sufficient to compare the weighted sum of the unconditional variances of inflation and output, $L = var_\pi + \frac{\gamma_1}{\epsilon} var_y$, where var_x denotes the variance of a generic variable x conditional on the information in the initial period. Since the unconditional variances, which are derived in the appendix, cannot be assessed in an analytical way, we apply a numerical analysis. For this we use some standard parameter values: $\sigma = 2$ (5), $\upsilon = 2$ (0), $\beta = 0.99$, $\phi = 0.8$, $\epsilon = 6$, and $\rho_\varkappa = 0.8$. The computed variances and losses are related to the variance of the cost-push shock for convenience and are given in table 1.

As can be seen from table 1, the inflation variance is always smaller under an unconstrained money supply, while the output variance is smaller when there is a binding money market constraint. The computed values for the weighted sum of the variances, i.e., the relative losses $L/var_\varkappa$, show that social welfare is higher under an unconstrained money supply if the degree of households' risk aversion σ is moderate ($\sigma = 2$). This result might be viewed as not very surprising given that interest rate policy has been designed to maximize social welfare under an unconstrained money supply. Yet, when the degree of risk aversion is raised from 2 to 5, welfare losses are always higher under a binding money market constraint. Thus, the central bank can raise social welfare by constraining money supply in cases where households are extremely unwilling to substitute consumption over time. In contrast, the intertemporal substitution elasticity of labor ($1/\upsilon$) is less important for the welfare comparison.

The main difference between both cases is the existence of an endogenous state variable under binding money market constraint (namely a_t) causes the equilibrium sequences to be history dependent. In contrast the equilibrium under an unconstrained money supply is entirely forward looking, given that monetary policy under discretion does not account for its impact on private sector expectations. Due to the history dependence induced by the collateral constraint, expectations about future macroeconomic aggregates are changed. In particular, the sequence of aggregate demand can be stabilized under a constant supply of eligible securities. As a consequence, the output variance is always smaller when the collateral constraint is binding, since the relevance of an additional state variable helps to smooth out responses of aggregate demand to cost-push shocks. Thus, the history dependent conduct of public policy contributes to the stabilization of real activity. This mechanism actually relates to the fact that an optimal policy under commitment (in a timeless perspective) is able to enhance welfare by manipulating private sector expectations via a history dependent monetary policy (see Woodford, 2003). In a similar way, the introduction of a collateral constraint in open market operations can help to steer expectations in a favorable way by restricting the scope of macroeconomic responses to future actions of discretionary monetary policy.

Table 1 Variances under constrained and unconstrained money supply

	Parameter values			Constrained Money Supply			Unconstrained Money Supply		
	σ	υ	ρ_π^*	$\frac{var_y}{var_\kappa}$	$\frac{var_\pi}{var_\kappa}$	$\frac{L}{var_\kappa}$	$\frac{var_y}{var_\kappa}$	$\frac{var_\pi}{var_\kappa}$	$\frac{L}{var_\kappa}$
$\vartheta = 1$									
	2	2	2.1	10.1	1.17	1.51	19.6	0.54	1.22
	2	0	2.1	25.6	2.83	3.27	67.0	1.86	3.02
	5	2	3.9	5. 58	0.26	0.59	6.85	0.19	0.61
	5	0	3.9	9.90	0.45	0.88	12.9	0.36	0.92
$\vartheta = 0.8$									
	2	2	2.1	10.5	1.59	1.95#	19.6	0.54	1.22
	5	2	3.9	6.17	0.37	0.75#	6.85	0.19	0.61

Note: # marks a weighted sum of variances which cannot be interpreted as welfare losses.

The last two rows show that the weighted variances tend to be larger under a binding money market constraint, if the supply of eligible securities is not constant, $\vartheta = 0.8$. In this case, inflation has to rise whenever the supply of public debt is higher than in the steady state. As a consequence, the inflation and output variances are larger than under $\vartheta = 1$. It should be noted that the loss function can hardly been used as a proxy for social welfare under $\vartheta < 1$, since prices grow on average. (Accordingly, the values for $\frac{L}{var_\xi}$ are marked with #).

5 Conclusion

Are open market operations, or to be more precise, collateral constraints therein, irrelevant for macroeconomic dynamics? In this paper it is shown that, when money is the counterpart of discounted securities deposited at the central bank, macroeconomic effects of monetary policy depend on whether the set of eligible securities is constrained or not. In accordance with the practice of many central banks, a constraint for open market operations is introduced, by which only government bonds are accepted as collateral. When this money market constraint is binding, an otherwise standard sticky price model exhibits an equilibrium with a liquidity premium for public debt. Given that the supply of eligible securities is not unbounded, money is inversely related to the nominal interest rate and a cash injection is associated with a liquidity effect.

A collateral constraint is further shown to facilitate the determination of the price level and the allocation for interest rate policies, which are associated with indeterminacies when money supply is unconstrained. The reason for this property is that the stock of eligible securities provides a nominal anchor that allows to pin down the price level under an interest rate peg and perfectly flexible prices. When prices are sticky, the predetermined stock of public debt serves as an equilibrium selection criterion which avoids real indeterminacy under a passive interest rate policy. These results might already be viewed as a rationale for central banks to impose a restriction on eligible securities. Yet, given that central banks can avoid indeterminacies by state contingent interest rate policies, it remains unclear why there actually exist tight restrictions on the set of securities that can be used as collateral in open market operations.

To find an answer for this question, the final part of the paper compares the macroeconomic stabilization properties of monetary policy under a constrained and an unconstrained money supply. Using an interest rate feedback rule that implements the optimal policy plan of a central bank acting under discretion, we find that the inflation variance is always smaller under an unconstrained money supply. The output variance is however smaller under a constrained money supply, which can even lead to an increase social welfare if households are very risk avers. By establishing a link between the supply of money and the predetermined stock of eligible securities, monetary policy becomes history dependent, such that macroeconomic responses to aggregate shocks are smoothed out. Expectations about the macroeconomic effects of future policy actions can then be steered even by a discretionary monetary policy. Yet, in order to be welfare enhancing collateral constraints have to be associated with a constant stock of eligible securities.

Appendix to the sticky price case

With a simple interest rate feedback rule satisfying (29), the version with a binding money market constraint ($\eta_t > 0$) can be reduced to the following set of equilibrium conditions in inflation and real wealth:

$$(1 + \gamma_1 \rho_\pi)\, \widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \gamma_1 \widehat{a}_t + \widehat{\varkappa}_t,$$
$$\widehat{a}_t = \widehat{a}_{t-1} + (\gamma_2 \rho_\pi - 1)\, \widehat{\pi}_t.$$

As shown in proposition 4, the fundamental solution, which takes the form $\widehat{a}_t = \delta_1 \widehat{a}_{t-1} + \delta_2 \widehat{\varkappa}_t$ and $\widehat{\pi}_t = \delta_3 \widehat{a}_{t-1} + \delta_4 \widehat{\varkappa}_t$ is the unique stable solution for this model if ϑ is sufficiently small. Inserting this solution into the two equilibrium conditions leads to the following conditions for the unknown solution coefficients

$$\delta_3 = \frac{\delta_1 - 1}{\gamma_2 \rho_\pi - 1},\ \delta_4 = -\frac{1}{(\gamma_2 \rho_\pi - 1)(\gamma_1 + \beta \delta_3) + \beta \rho_\varkappa - (\gamma_1 \rho_\pi + 1)},\ \delta_2 = (\gamma_2 \rho_\pi - 1)\, \delta_4,$$

where the eigenvalue $\delta_1 \in (01,)$ is the single stable root of the characteristic equation $f(X) = 0$ (see 30). The output solution takes the form $\widehat{y}_t = \delta_5 \widehat{a}_{t-1} + \delta_6 \widehat{\varkappa}_t$. Using that $\widehat{y}_t = \widehat{a}_t - \rho_\pi \widehat{\pi}_t$ holds under a binding collateral constraint, we further get

$$\delta_5 = \delta_1 - \rho_\pi \frac{\delta_1 - 1}{\gamma_2 \rho_\pi - 1} \quad \text{and } \delta_6 = ((\gamma_2 - 1)\, \rho_\pi - 1)\, \delta_4.$$

When the money market constraint is not binding, the solution takes the form $\widehat{\pi}_t = \delta_4^* \widehat{\varkappa}_t$ and $\widehat{y}_t = \delta_6^* \widehat{\varkappa}_t$. Inserting these solutions into the equilibrium condition (26) for $\eta_t = 0$ and (27), and using (29) gives

$$\delta_4^* = \frac{\sigma (1 - \rho_\varkappa)}{\gamma_1 (\rho_\pi - \rho_\varkappa) + (1 - \beta \rho_\varkappa)(1 - \rho_\varkappa)\, \sigma} \quad \text{and } \delta_6^* = -\frac{\rho_\pi - \rho_\varkappa}{\sigma (1 - \rho_\varkappa)} \delta_4^*.$$

Setting ρ_π equal to $\rho_\pi^* = (1 - \sigma \epsilon)\, \rho_\varkappa + \sigma \epsilon$, leads to $\delta_6^* = -\epsilon \delta_4^*$. Hence, an interest rate feedback rule satisfying $\widehat{R}_t = \rho_\pi^* \widehat{\pi}_t$ is consistent with the fundamental solution to the central bank's plan under discretionary optimization, which is characterized by $\widehat{y}_t = -\epsilon \widehat{\pi}_t$. Since $\rho_\pi^* > 1$, the plan is then uniquely implemented.

The unconditional variances for output var_y and inflation var_π can be computed, using the fundamental solutions derived above. For the case of a constrained money supply they are given by

$$var_\pi = \left(\frac{\delta_3^2 \delta_2^2}{1 - \delta_1^2} + \delta_4^2 \right) var_\varkappa \quad \text{and} \quad var_y = \left(\frac{\delta_5^2 \delta_2^2}{1 - \delta_1^2} + \delta_6^2 \right) var_\varkappa.$$

where $var_\varkappa = (1 - \rho_\varkappa^2)^{-1} var_e$. When money supply is unconstrained the variances are $var_\pi = (\delta_4^*)^2 var_\varkappa$ and $var_y = (\delta_6^*)^2 var_\varkappa$.

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