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Abstract: This paper studies the macroeconomic effects of implicit government guarantees of the obligations of government-sponsored enterprises. We construct a model with competitive housing and mortgage markets in which the government provides banks with insurance against aggregate shocks to mortgage default risk. We use this model to evaluate aggregate and distributional impacts of this government subsidy of owner-occupied housing. Preliminary findings indicate that the subsidy leads to higher equilibrium housing investment, higher mortgage default rates, and lower welfare. The welfare effects of this policy vary substantially across members of the population with different economic characteristics.

JEL classification: E21, G11, R21

Key words: housing, mortgage market, default risk

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1 Introduction

With close to 70% the United States displays one of the highest home ownership ratios in the world. Part of the attractiveness of owner-occupied housing stems from a variety of subsidies the government provides to homeowners. Apart from direct subsidies to low-income households via the U.S. Department of Housing and Urban Development (HUD) programs, three important indirect subsidies exist. The first - and most well known - is the fact that mortgage interest payments (of mortgages up to $1 million) are tax-deductible. Second, the implicit income from housing capital (i.e. the imputed rental-equivalent) is not taxable, while other forms of capital income (e.g. interest, dividend and capital gains income) are being taxed. Gervais (2001) addresses the adverse effects of these two subsidies within a general equilibrium life-cycle model.

The third subsidy arises from the special structure of the US mortgage market. A large fraction of conventional conforming home mortgages in the US are being sold in the market with a guarantee provided by Government Sponsored Enterprises (GSEs) or purchased from individual banks for the GSEs' own portfolios. A formidable summary of the institutional details surrounding GSEs can be found in Frame and Wall (2002a) and (2002b). The three most important GSE are the two privately owned and publicly traded companies Fannie Mae (Federal National Mortgage Association) and Freddie Mac (Federal Home Loan Mortgage Association), and the FHLB (Federal Home Loan Bank system), a public and non-profit organization. According to Frame and Wall (2002a), citing a 2001 study by the Congressional Budget Office (CBO), Fannie Mae and Freddie Mac alone had a share of 39% of all home mortgages and a share of 71% among fixed-rate conforming mortgages.

The close link of GSEs to the federal government creates the impression that the government provides a guarantee to GSEs shielding them from aggregate risks, most notably aggregate credit risk which lowers their refinancing cost to below what private institutions would have to pay. The purpose of this paper is to quantify the macroeconomic and distributional effects of this subsidy; our paper is - to our knowledge - the first attempt to do so within a structural dynamic general equilibrium model.

According to Frame and Wall, GSEs enjoy an array of government benefits, for example a line of credit with the Treasury Department and very importantly a special status of GSE-issued debt. In particular, GSE securities can serve as substitutes to government bonds for transactions between public entities that normally require to be done in Treasuries. The Federal Reserve System also accepts GSE debt as a substitute for Treasuries in their portfolio of repurchase agreements. While no written federal guarantee for GSE debt exists, market participants view the special status of GSE debt as an indication of an implicit guarantee making them almost as safe as Treasury bills. The perception of a federal guarantee is further fueled by the sheer size of the GSE mortgage portfolio amounting to about 3 trillion dollars, 2.4 trillion dollars of which coming from the larger two GSEs, Fannie Mae and Freddie Mac. Insolvency of any one or both of these companies, say, due to an adverse shock in the real estate market that increases aggregate mortgage delinquency, will cause major disruptions in the financial system, which is why market participants consider housing GSEs to be too large to fail. Finally, two previous government bailouts of housing GSEs - Fannie Mae in the early 1980s and one of the smaller housing GSEs in the late 1980s - are further evidence that a bailout is likely should housing GSEs get into financial trouble.

The implicit federal guarantee is more than mere perception; most importantly, it is reflected in interest rates GSEs pay when borrowing. GSEs can borrow at rates only marginally higher than the Treasury but about 40 basis points lower than private companies without a government guarantee, according to the Congressional Budget Office CBO (2001). This is despite the fact that GSEs are highly leveraged entities with an equity cushion of only about 3% of their obligations, much lower than the 8.45% in the thrift industry (figures taken from Frame and Wall (2002a)).

A lively discussion ensued about how much of the subsidy is actually passed on to homeowners. Passmore et. al. (2004) argue that GSEs reduce mortgage rates by only about 0.07 percentage points, while the rest of the subsidy goes to GSE shareholders, raising doubts about the GSEs' self-proclaimed aim of making housing more affordable. Blinder (2004) on the other hand defends the GSEs and argues that they indeed pass on essentially the entire subsidy to homeowners. Besides making a methodological and theoretical contribution, our paper gives a new angle at this discussion. Specifically, we conduct the following thought experiment: Suppose the entire subsidy is indeed passed on to homeowners, just as the GSEs and Blinder (2004) claim, then what is the economic impact, especially on households of different income and wealth holdings? In our study we focus solely on an interest rate subsidy provided in the form of an implicit government guarantee, that is, we study the usefulness of a subsidy separately from any other programs, such as affordable housing programs run by the GSEs.

In order to assess the macroeconomic and distributional effects of this subsidy we construct a heterogeneous agent general equilibrium model in which households can default on their mortgages. Aggregate mortgage delinquency rates are impacted by aggregate shocks to the housing and labor market. Our aim is to compare two economies, one in which the aggregate risk is priced into mortgages and one economy in which the government offers a tax-financed bailout in case of a bad aggregate shock, that is, the aggregate delinquency risk is not priced into mortgages. In this way we capture the essence of the guarantee structure that GSEs enjoy without the need to explicitly model the behavior of these institutions. As a first step towards this goal in this paper we analyze the macroeconomic and distributional consequences of a tax-financed direct subsidy to mortgage interest rates. In our thought experiment, as well as in an approach that models aggregate uncertainty explicitly the government subsidizes home ownership by reducing effective mortgage interest rates in exactly the same way. Thus we feel that our abstraction from aggregate uncertainty reduces the numerical complexity of the model without losing the main effects and insights for the question at hand. Evidently, an explicit study with aggregate uncertainty has to confirm this conjecture.

Our preliminary results can be described as follows. First, the subsidy leads to an increase in household investment in housing assets and an increase in the construction of real estate. Using a steady state utilitarian social welfare functional we find that the aggregate welfare implications of the subsidy are mildly positive, in the order of 0.1% of consumption equivalent variation. The results also suggest that households with low wealth prefer to live in an economy without subsidy while high wealth households benefit strongly from it, indicating adverse distributional effects of the reform.[1]

The remainder of the paper is organized as follows. Section 2 introduces the model and defines equilibrium in an economy with a housing and mortgage market. Section 3 characterizes equilibria. Section 4 describes the calibration of an economy without aggregate uncertainty

[1] Gruber and Martin (2003) also study the distributional effects of the inclusion of housing wealth in a general equilibrium model, but do not address the role of government housing subsidies for this question.

and with a direct subsidy on mortgage interest rates. Section 5 details the numerical results comparing two steady states in economies with and without a mortgage interest subsidy. Section 6 concludes the paper.

2 The Model

The endowment economy is populated by a continuum of measure one of infinitely lived households, a continuum of competitive banks and a continuum of housing construction companies. Households face idiosyncratic endowment and housing depreciation shocks. In addition there may be aggregate shocks affecting endowments and housing depreciation. In what follows we will immediately proceed to describing the economy recursively, thereby skipping the (standard) sequential formulation of the economy.

2.1 Households

Households have endowment of the perishable consumption good given by yz. The aggregate part of endowments $z \in Z$ follows a finite state Markov chain with transition probabilities $\pi(z'|s)$ and unique invariant distribution $\Pi(z)$. The idiosyncratic part of endowments $y \in Y$ follows a finite state Markov chain with transition probabilities $\pi(y'|y, z', z)$ and unique invariant distribution $\Pi_z(y)$. That is, the distribution over idiosyncratic income shocks is allowed to depend on the aggregate state of the economy.

Households derive period utility $U(c, h)$ from consumption and housing services h, which can be purchased at a price p_l (relative to the numeraire consumption good). In addition to consumption and housing services the household can purchase two types of assets, one period bonds b' and houses g'. The price of bonds is denoted by P_b and the price of houses by P_h. Whereas households cannot short-sell bonds, they can borrow against their real estate property. Let by m' denote the size of their mortgage, and by P_m the receipt of resources (the consumption good) for each unit of mortgage issued and to be repaid tomorrow. These receipts will be determined in equilibrium by competition of banks, and will depend on the characteristics of households as well as the size of the mortgage m' and size of the collateral g'. Houses depreciate stochastically; let $F_{\delta,z',y'}(\delta')$ denote the cumulative distribution function of the depreciation rate δ' tomorrow, which has support $D = [\underline{\delta}, \bar{\delta}]$ and may depend on the realized depreciation rate δ today as well as on the endowment realization of the household (z', y'). Households possess the option of defaulting on their mortgages, at the cost of losing their housing collateral. They will choose to do so whenever

$$m' > P_h(1 - \delta')g'$$

If there is a government bailout guarantee, then the government obtains general tax revenues by levying proportional taxes τ on endowments. It will use the receipts from these taxes to bail out part of the mortgages that private households have defaulted on. Finally let a denote cash at hand, that is, after tax endowment plus receipts from all assets brought into the period.

The individual state of a household consists of $s = (a, \delta, y)$, which reduces to $s = a$ in the case where idiosyncratic endowments and housing depreciation are *iid*. Let the cross-sectional distribution over individual states be given by μ; the aggregate state of the economy then consists of (z,μ). The dynamic programming problem of a household reads as

$$v(s,z,\mu) = \max_{c,h,b',m',g'\geq 0} \left\{ U(c,h) + \beta \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y,z',z) \int_{\underline{\delta}}^{\bar{\delta}} v(s',z',\mu') dF_{\delta,z',y'}(\delta') \right\} \quad (1)$$

s.t.

$$c + b'P_b(z,\mu) + hP_l(z,\mu) + g'P_h(z,\mu) - m'P_m(s,g',m',z,\mu)) = a + g'P_l(z,\mu)$$

$$a'(\delta',y',m',g',z',\mu') = b' + \max\{0, P_h(z',\mu')(1-\delta')g' - m')\} + (1-\tau(z',\mu'))z'y'$$

with $\mu' = T(z,z',\mu)$. Note that the budget constraint implies the timing convention that newly purchased real estate g' can immediately be rented out in the same period. The function T describes the aggregate law of motion.

2.2 The Real Estate Construction Sector

Firms in the real estate construction sector act competitively and face the linear technology

$$I = A_h C_h$$

where I is the output of houses of a representative firm, C_h is the input of the consumption good and A_h is a technological constant, measuring the amount of consumption goods required to build one house. For now we assume that this technology is reversible, that is, real estate companies can turn houses back into consumption goods using the same technology. Thus the problem of a representative firm reads as

$$\max_{I,C_h} P_h(z;\mu)I - C_h \quad (2)$$

s.t.

$$I = A_h C_h$$

Thus the equilibrium house price necessarily satisfies

$$P_h(z;\mu) = \frac{1}{A_h}.$$

2.3 The Banking Sector

We assume that the risk free interest rate on one-period bonds r_b is exogenously given; one may interpret our economy as a small open economy. Thus $P_b = \frac{1}{1+r_b}$ is exogenously given as well, which is equal to the refinancing costs of the banking sector. In addition we assume that issuing mortgages is costly; let r_w be the percentage real resource cost, per unit of mortgage issued, to the bank. This cost captures screening costs, administrative costs as well as maintenance costs of the mortgage (such as preparing and mailing a quarterly mortgage balance). As a consequence, the effective net cost of the banking sector for financing one dollar of mortgage, equals $r_b + r_w$.

Mortgage receipts P_m for a mortgage of size m' against real estate of size g' are determined by perfect competition in the banking sector, which implies that banks make zero expected profits for *each mortgage* they issue (as in Chatterjee et al. (2002)). Banks take account of the fact that household may default on their mortgage, in which case the bank recovers the collateral value of the house, which we assume to be a fraction $\gamma \leq 1$ of the value of the real estate. For ease of exposition we assume that the cost of mortgage generation is paid not when the mortgage is issued, but when it repaid, which implies that households defaulting on their mortgage payments also default on paying for the cost of generating the mortgage. Since this cost is fully priced into the mortgage, this is equivalent to assuming that the resource cost of mortgage issue is due at the receipt of the mortgage, but makes notation less cumbersome.

In order to define a typical banks' problem we first have to characterize the optimal default choice of a household. The cut-off level of depreciation, above which a household defaults on her mortgage is given as follows.. Define as $\kappa' = \frac{m'}{g'}$ the leverage (for $g' > 0$) of a mortgage m' backed by real estate g'. Then if the default cut-off $\delta^*(m', g', z', \mu')$ is in the interior of $D = [\underline{\delta}, \bar{\delta}]$ it is given by

$$m' = (1 - \delta^*(m', g', z', \mu'))P_h(z', \mu')g'$$

and thus explicitly

$$\delta^*(m', g', z', \mu') = \begin{cases} \underline{\delta} & \text{if } 1 - \frac{\kappa'}{P_h(z',\mu')} < \underline{\delta} \\ 1 - \frac{m'}{g' P_h(z',\mu')} = 1 - \frac{\kappa'}{P_h(z',\mu')} & \text{if } 1 - \frac{\kappa'}{P_h(z',\mu')} \in [\underline{\delta}, \bar{\delta}] \\ \bar{\delta} & \text{if } 1 - \frac{\kappa'}{P_h(z',\mu')} > \bar{\delta} \end{cases}$$

Now we first note that our previous assumptions imply that $P_h(z', \mu') = P_h = \frac{1}{A_h}$. Thus $\delta^*(m', g', z', \mu') = \delta^*(m', g') = \delta^*(\kappa')$, where, as before, κ' is a function of (s, z, μ).

Evidently a household that obtains a mortgage $m' > 0$ without collateral, i.e. with $g' = 0$ defaults for sure. The receipt for this mortgage thus necessarily has to equal 0 as well, i.e. $P_m(s, g' = 0, m', z, \mu) = 0$. For other types of mortgages (m', g') with $m' > 0$ and $g' > 0$, the banks' problem is to choose the price $P_m(s, g', m', z, \mu)$ to maximize

$$\begin{aligned} &\max_{P_m(s,g',m',z,\mu)} \left[\begin{array}{c} -m' P_m(s, g', m', z, \mu) + \left(\frac{1}{1+r_b+r_w}\right) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) * \\ \left\{ m' F_{\delta,z',y'}(\delta^*(\kappa')) + \gamma P_h(z', \mu') g' \int_{\delta^*(\kappa')}^{\bar{\delta}} (1 - \delta') dF_{\delta,z',y'}(\delta') \right\} \end{array} \right] \\ = \; m' &\max_{P_m(s,g',m',z,\mu)} \left[\begin{array}{c} -P_m(s, g', m', z, \mu) + \left(\frac{1}{1+r_b+r_w}\right) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) * \\ \left\{ F_{\delta,z',y'}(\delta^*(\kappa')) + \frac{\gamma P_h(z',\mu')}{\kappa'} \int_{\delta^*(\kappa')}^{\bar{\delta}} (1 - \delta') dF_{\delta,z',y'}(\delta') \right\} \end{array} \right] \end{aligned} \quad (3)$$

In the presence of a government bailout, the government effectively subsidizes mortgages, in forms to be specified below.

2.4 The Government

As stated above the government levies endowment taxes $\tau(z, \mu)$ on households to subsidize mortgages. Subsidies take the form of direct interest rate subsidies.[2]

Define the interest rate on a mortgage with characteristics (m', g') as

$$r_m(s, g', m', z, \mu) = \frac{1}{P_m(s, g', m', z, \mu)} - 1$$

[2]Other forms of mortgage subsidies can be easily mapped into these interest rate subsidies.

where $P_m(s, g', m', z, \mu)$ is the mortgage pricing function without subsidy. Define as $\hat{r}_m(s, g', m', z, \mu)$ and $\hat{P}_m(s, g', m', z, \mu)$ the corresponding entities with subsidy. Since the subsidy is a mortgage interest rate subsidy we model it as

$$\hat{r}_m(s, g', m', z, \mu) = r_m(s, g', m', z, \mu) - sub(s, g', m', z, \mu)$$

and thus

$$\hat{P}_m(s, g', m', z, \mu) = \frac{P_m(s, g', m', z, \mu)}{1 - sub(s, g', m', z, \mu) * P_m(s, g', m', z, \mu)} \geq P_m(s, g', m', z, \mu)$$

The total subsidy for a mortgage of characteristics (s, g', m', z, μ) is thus

$$\begin{aligned} Sub(s, g', m', z, \mu) &= m' \left(\hat{P}_m(s, g', m', z, \mu) - P_m(s, g', m', z, \mu) \right) \\ &= m' P_m(s, g', m', z, \mu) \left(\frac{sub(s, g', m', z, \mu) P_m(s, g', m', z, \mu)}{1 - sub(s, g', m', z, \mu) P_m(s, g', m', z, \mu)} \right) \end{aligned}$$

and the total economy-wide subsidy is

$$Aggsub(z, \mu) = \int Sub(s, g', m', z, \mu) d\mu$$

Thus taxes have to satisfy

$$\begin{aligned} \tau(z, \mu) \int zy d\mu &= Aggsub(z, \mu) \\ \tau(z, \mu) &= \frac{Aggsub(z, \mu)}{z\bar{y}_z} \end{aligned} \tag{4}$$

where $\bar{y}_z$ is average (aggregate) endowment if the aggregate state of the economy is z.

2.5 Equilibrium

We are now ready to define a Recursive Competitive Equilibrium. Let $S = R_+ \times D \times Y$ denote the individual state space and $\mathcal{M}$ the space of finite measures over the measurable space $(S, \mathcal{S})$, where $\mathcal{S} = \mathcal{B}(R_+) \times \mathcal{B}(D) \times \mathcal{P}(Y)$ and $\mathcal{B}$ is the Borel σ-algebra and $\mathcal{P}$ is the power set, so that $\mathcal{S}$ is a well-defined σ-algebra over S.

Definition 1 *Given a government subsidy policy* $sub : S \times R_+ \times R_+ \times Z \times \mathcal{M} \to R$, *a* ***Recursive Competitive Equilibrium*** *are value and policy functions for the households,* $v, c, h, b', m', g' : S \times Z \times \mathcal{M} \to R$, *policy functions for the real estate construction sector* $I, C_h : Z \times \mathcal{M} \to R$, *pricing functions* $P_l, P_h, P_b : Z \times \mathcal{M} \to R$, *mortgage pricing functions* $P_m, \hat{P}_m : S \times R_+ \times R_+ \times Z \times \mathcal{M} \to R$, *a government tax policy* $\tau : Z \times \mathcal{M} \to R$ *and an aggregate law of motion* $T : Z \times Z \times \mathcal{M} \to \mathcal{M}$ *such that*

1. *(Household Maximization) Given prices* $P_l, P_h, P_b, \hat{P}_m$ *and government policies the value function solves* (1) *and* c, h, b', m', g' *are the associated policy functions.*

2. *(Real Estate Construction Company Maximization) Given P_h, policies I, C_h solve* (2).

3. *(Bank Maximization) Given P_h, P_b, the function P_m solves* (3)

4. *(Small Open Economy Assumption) The function P_b is exogenously given by*

$$P_b(z, h) = \frac{1}{1 + r_b}$$

where r_b is the exogenously given fixed world risk free interest rate

5. *(Government Budget Balance) The tax rate function τ satisfies* (4), *given the functions $m', P_m, \hat{P}_m, sub$.*

6. *(Market Clearing in Rental Market) For all (μ, z)*

$$\int g'(s, z, \mu) d\mu = \int h(s, z, \mu) d\mu$$

7. *(Aggregate Law of Motion) The aggregate law of motion T is generated by the exogenous Markov processes π and the policy functions m', g', b'*

When we derive the welfare consequences of removing the mortgage interest subsidy, we measure aggregate welfare via a Utilitarian social welfare function in the steady state, defined as

$$\mathcal{WEL} = \int v(s)\mu(ds)$$

3 Theoretical Results

In this section we state theoretical properties of our model the use of which makes the computation of the model easier. These results consist of a characterization of the mortgage interest rate, a partial characterization of the solution to the household maximization problem and, finally, bounds on the equilibrium rental price $P_l(z, h)$.

3.1 Mortgage Interest Rates

From equation (3) and the fact that competition requires profits for all mortgages issued in equilibrium to be zero we immediately obtain a characterization of equilibrium mortgage payoffs as

$$\begin{aligned} P_m(s, g', m', z, \mu) &= \left(\frac{1}{1 + r_b + r_w}\right) \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) * \\ &\quad \left\{ F_{\delta,z',y'}(\delta^*(m', g', z', \mu')) + \frac{\gamma P_h(z', \mu')}{\kappa'} \int_{\delta^*(m',g',z',\mu')}^{\bar{\delta}} (1 - \delta') dF_{\delta,z',y'}(\delta') \right\} \\ &= \frac{\sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z)}{1 + r_b + r_w} \left\{ F_{\delta,z',y'}(\delta^*(\kappa')) + \frac{\gamma}{A_h \kappa'} \int_{\delta^*(\kappa')}^{\bar{\delta}} (1 - \delta') dF_{\delta,z',y'}(\delta') \right\} \\ &= P_m(s, z, \kappa'(s)) \end{aligned}$$

with implied interest rates

$$r_m(s, z, \kappa'(s)) = \frac{1}{P_m(s, z, \kappa'(s))} - 1$$

We note the following facts:

1. Besides the aggregate state variables the only information determining mortgage interest rates are the individual states δ, y and the leverage of the mortgage $\kappa' = \frac{m'}{g'}$. If income and depreciation shocks are *iid*, then $P_m(s, \kappa'(s)) = P_m(\kappa'(s), z)$ and mortgages are priced exclusively based on leverage and aggregate conditions. Furthermore, if aggregate shocks are *iid*, then mortgage prices and interest rates only depend on the leverage of the mortgage chosen by the household, $\kappa'(s)$.

2. $P_m(s, z, \kappa'(s))$ is decreasing in κ', strictly so if the household defaults with positive probability. Thus mortgage interest rates are increasing in leverage κ'.

3. Households that repay their mortgage with probability one have $\delta^*(\kappa') = \bar{\delta}$ and thus $P_m(s, z, \kappa') = \left(\frac{1}{1+r_b+r_w}\right)$, i.e. they can borrow at the rate $r_b + r_w$.

4. Since for all $\delta' > \delta^*(\kappa')$ we have $\gamma P_h \kappa'(1-\delta') < 1$, households that do default with positive probability tomorrow receive $P_m(s, z, \kappa'(s)) < \left(\frac{1}{1+r_b+r_w}\right)$ today, that is, they borrow with a risk premium $r_m(s, z, \kappa'(s)) > r_b + r_w$.

3.2 Simplification of the Household Problem

In the household problem define as

$$\begin{aligned} u(c; P_l) &= \max_{\tilde{c}, h \geq 0} U(\tilde{c}, h) \\ & \quad s.t. \\ \tilde{c} + P_l(z, \mu) h &= c \end{aligned}$$

Then the above problem can be rewritten as

$$v(s, z, \mu) = \max_{c, b', m', g' \geq 0} \left\{ u(c; P_l(z, \mu)) + \beta \sum_{z'} \pi(z'|z) \sum_{y'} \pi(y'|y, z', z) \int_{\underline{\delta}}^{\bar{\delta}} v(s', z', \mu') dF_{\delta, z', y'}(\delta') \right\}$$

$$\begin{aligned} \text{s.t.} \qquad & c + b' P_b + g' \left[P_h - P_l(z, \mu)\right] - m' P_m(s, z, \kappa') = a \\ a'(\delta', h', m', g', z', \mu') &= b' + \max\{0, P_h(1-\delta')g' - m'\} + (1 - \tau(z', \mu')) z' y' \\ \mu' &= T(z, z', \mu) \end{aligned}$$

For future reference, in the absence of aggregate uncertainty and with individual shocks being *iid* the individual state variables collapse to just cash at hand a' and the problem becomes

$$\begin{aligned}
v(a) &= \max_{c,b',m',g'\geq 0}\left\{u(c;P_l)+\beta\sum_{y'}\pi(y')\int_{\underline{\delta}}^{\bar{\delta}}v(a')dF(\delta')\right\} \\
\text{s.t.} &\quad c+b'P_b+g'\left[P_h-P_l\right]-m'P_m(\frac{m'}{g'})=a \\
a'(\delta',y',m',g') &= b'+\max\{0,P_h(1-\delta')g'-m')\}+(1-\tau)y'
\end{aligned}$$

3.3 Endogenous Borrowing Limit

We now want to show that it is never strictly beneficial for a household to obtain a mortgage with higher leverage than that level which will lead to default for sure. We will carry out the discussion in the next two subsections for the case without government bailout policy; the analysis goes through unchanged with government policy, mutatis mutandis. Remember that by construction $P_h(z',\mu')=P_h=\frac{1}{A_h}$. Define the leverage that leads to certain default by the smallest number $\bar{\kappa}$ such that

$$\begin{aligned}
\delta^*(\bar{\kappa}) &= \underline{\delta} \\
\bar{\kappa} &= (1-\bar{\delta})P_h=\frac{1-\underline{\delta}}{A_h}
\end{aligned}$$

Now we rewrite the budget constraint as

$$\begin{aligned}
c+b'P_b+g'\left[P_h-P_l(z,\mu)-\frac{m'}{g'}P_m(s,z,\frac{m'}{g'})\right] &= a \text{ or} \\
c+b'P_b+g'\left[P_h-P_l(z,\mu)-\kappa'P_m(s,z,\kappa')\right] &= a \text{ or} \\
c+b'P_b+g'P(s,\kappa',z,\mu) &= a
\end{aligned}$$

where

$$P(s,\kappa',z,\mu)=P_h-P_l(z,\mu)-\kappa'P_m(s,\kappa',z)$$

is the is down payment per unit of real estate purchased, net of rental income. With this definition the total down payment for a house of size g' is given by $g'P(s,\kappa',z,\mu)$

For all $\kappa'\geq\bar{\kappa}$ we have

$$\begin{aligned}
\kappa'P_m(s,\kappa',z) &= \left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)* \\
&\quad \left\{\kappa'F_{\delta,z',y'}(\underline{\delta})+\gamma P_h\int_{\underline{\delta}}^{\bar{\delta}}(1-\delta')dF_{\delta,z',y'}(\delta')\right\} \\
&= \left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)\gamma P_h\int_{\underline{\delta}}^{\bar{\delta}}(1-\delta')dF_{\delta,z',y'}(\delta') \\
&= \left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)\gamma P_h(1-E_{\delta,z',y'}(\delta')) \\
&= \bar{\kappa}P_m(s,\bar{\kappa},z,\mu)
\end{aligned}$$

and thus leveraging further does not bring extra revenues today and does not change resources obtained tomorrow (since the household defaults for sure and thus loses all real estate).[3] That is, the household faces an endogenous effective borrowing constraint of the form

$$\begin{aligned} \kappa' &\leq \bar{\kappa} \text{ or} \\ m' &\leq \left[\frac{1-\delta}{A_h}\right] g' \end{aligned}$$

One can interpret $1-\bar{\kappa}$ as the minimum down payment requirement in this economy.

3.4 Bounds on the Equilibrium Rental Price of Housing

3.4.1 An Upper Bound

Evidently for all admissible choices of the household it has to be the case that $P(s,\kappa',z,\mu) \geq 0$, otherwise the household can obtain positive cash flow today by buying a house; the default option on the mortgage guarantees that the cash flow from the house tomorrow is non-negative. Thus, the absence of this arbitrage in equilibrium requires $P(s,\kappa',z,\mu) \geq 0$. Therefore in particular

$$P(s,\kappa'=\bar{\kappa},z,\mu) = P_h - P_l(z,\mu) - \bar{\kappa}P_m(s,\kappa'=\bar{\kappa},z) \geq 0$$

But

$$\begin{aligned} P(s,\bar{\kappa},z,\mu) &= P_h - P_l(z,\mu) - \bar{\kappa}P_m(s,\bar{\kappa},z) \\ &= P_h - P_l(z,\mu) - \left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)\gamma P_h(1-E'_{\delta,z',y'}(\delta')) \\ &\geq 0 \\ P_l(z,\mu) &\leq P_h - \left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)\gamma P_h(1-E_{\delta,z',y'}(\delta')) \end{aligned}$$

which places an upper bound on the equilibrium rental price.

Thus

$$\begin{aligned} P_l(z,\mu) &\leq P_h - \left(\frac{\gamma P_h}{1+r_b+r_w}\right)(1-E_{\delta,y,z}(\delta')) \\ &= P_h * \left[\frac{r_b+r_w+\gamma E_{\delta,y,z}(\delta)+1-\gamma}{1+r_b+r_w}\right] \end{aligned}$$

If $\gamma = 1$, this condition simply states that the rental price P_l cannot be larger that the user cost of housing $\frac{r_b+r_w+E_{\delta,y,z}(\delta')}{1+r_b+r_w}P_h$.

[3] The household is obviously indifferent between choosing $\kappa' = \bar{\kappa}$ and $\kappa' > \bar{\kappa}$; from here on we resolve any indifference of the household by assuming that in this case he chooses $\kappa' = \bar{\kappa}$.

3.4.2 A Lower Bound

Housing is an inherently risky asset. Since households are risk averse, for them to purchase the housing asset the expected return of housing at zero leverage has to be at least as high as the risk free interest rate. This implies

$$\left(\frac{1}{1+r_b+r_w}\right)\sum_{z'}\pi(z'|z)\sum_{y'}\pi(y'|y,z',z)P_h\int_{\underline{\delta}}^{\bar{\delta}}(1-\delta')dF_{\delta,z',y'}(\delta') \geq P_h - P_l(z,\mu)$$

Remembering that $P_h = \frac{1}{A_h}$ yields

$$\begin{aligned}\left(\frac{1}{1+r_b+r_w}\right)P_h(1-E_{\delta,z,y}(\delta')) &\geq P_h - P_l(z,\mu) \text{ or}\\ P_l(z,\mu) &\geq P_h\left[\frac{r_b+r_w+E_{\delta,z,y}(\delta')}{1+r_b+r_w}\right]\end{aligned}$$

which states that the rental price of housing cannot be smaller than the (expected) user cost of housing in equilibrium (otherwise nobody would invest in housing, which cannot be an equilibrium given strictly positive demand for housing services by consumers).[4]

In summary, what these theoretical results buy us, besides being interesting in its own right, is a simplified household problem, a concise characterization of the high-dimensional equilibrium mortgage interest rate function and bounds for the equilibrium rental price, the only endogenous price to be determined in our analysis.

4 Calibration

4.1 Technology

Summary

Parameter	Interpretation	Value	Target
A_h	Technology Const. in Housing Constr.	1.0	none (normalized)
π	Transition Matrix for Income	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
y	Income States	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
μ_δ	Depreciation	0.0199	$E(\delta) = 0.0148$
σ_δ	Std. Dev. of Depreciation	0.10	OFHEO volatility
$\bar{\delta}$	Upper Bound on Depreciation	0.3429	$1-\exp\{-\mu_\delta - 4\sigma_\delta\}$
$\underline{\delta}$	Lower Bound on Depreciation	−0.4624	$1-\exp\{-\mu_\delta + 4\sigma_\delta\}$
γ	Foreclosure Technology	0.78	Pennington and Cross (2004)

[4] Without aggregate uncertainty and $\gamma = 1$ we thus immediately obtain that the rental price of housing P_l equals its user cost $P_h\left[\frac{r_b+r_w+E_{\delta,z,y}(\delta')}{1+r_b+r_w}\right]$. In fact, what happens in this equilibrium is that households purchase houses, leverage such that they default for sure tomorrow and the houses end up in the hand of the banks. Since these are risk-neutral, default is fully priced into the mortgage and banks receive the full (depreciated) value of the house, banks rather than households (which are risk averse) should and will end up owning the real estate.

Foreclosure technology The default technology parameter γ has been estimated by Pennington-Cross (2004) who looks at the sales revenue from foreclosed houses and compares it to a market price constructed via the OFHEO repeat sales index. He finds that on average the loss is 22%. The loss varies only slightly depending on the age of the loan, between 20% for loans 16-20 months old to 26% for loans up to 10 months old, so it is safe to assume that in the model $\gamma = 0.78$ for all loans.

The depreciation process The Office of Federal Housing Enterprise Oversight (OFHEO) models house prices as a diffusion process and estimates within-state and within-region annual house price volatility. The technical details can be found in the paper by Calhoun (1996). The ballpark figure for the eight census regions is $9-10\%$ volatility in the years 1998-2004. We use the upper bound $\sigma_\delta = 0.10$ to account for the fact that nationwide volatility is slightly higher than the within-region volatility. Assume that $(1-\delta)$ is log-normally distributed, that is, $\log(1-\delta) \sim N\left(-\mu_\delta, \sigma_\delta^2\right)$. The average depreciation for residential housing according to the Bureau of Economic Analysis was 1.48% between 1960 and 2002 (standard deviation 0.05%), computed as consumption of fixed capital in the housing sector (Table 7.4.5) divided by the capital stock of residential housing. Since the mean of the log-Normal is $\exp\left\{-\mu_\delta + \frac{1}{2}\sigma_\delta^2\right\}$, we set $\mu_\delta = \frac{1}{2}\sigma_\delta^2 - \log 0.9852 \approx 1.99\%$ in order to match the average depreciation of 1.48%.

In the program we have to truncate the support for δ to $[\delta_l, \delta_h]$. We draw $\log(1-\delta)$ from a range of ± 4 standard deviations around μ_δ. This makes sure that the moments of the simulated truncated distribution are indistinguishable from theoretical moments and also the probabilities of drawing from the far right tail of the distribution - depreciation rates high enough to trigger default - are close enough to their theoretical values:

	Analytical	Truncated
$E(\delta)$	0.0148	0.0149
$St.dev.$	0.1000	0.0999
$P(\delta \geq 0.20)$	0.021055	0.021024
$P(\delta \geq 0.25)$	0.003705	0.003674
$P(\delta \geq 0.30)$	0.000379	0.000347

Housing Technology We normalize the housing construction constant to $A_h = 1.0$, and thus the price of one unit of housing to unity.

Income process For a continuous state $AR(1)$ process of the form

$$\begin{aligned} \log y' &= \rho \log y + (1-\rho^2)^{0.5}\varepsilon \\ E(\varepsilon) &= 0 \\ E\left(\varepsilon^2\right) &= \sigma_e^2 \end{aligned} \tag{5}$$

we can calculate the unconditional standard deviation to be σ_e and the one-period autocorrelation (persistence) to be ρ. Estimates for ρ range from $[0.53, 1]$ where the lower number is somewhat of an outlier. We choose $\rho = 0.98$. The estimates for the standard deviation range from 0.2 to 0.4, so we pick $\sigma_\varepsilon = 0.3$.

We approximate the continuous state AR(1) with a 5 state Markov chain using the procedure put forth by Tauchen and Hussey (1991). We get the five labor productivity shocks $y \in \{0.3586, 0.5626, 0.8449, 1.2689, 1.9909\}$ and the following transition matrix:

$$\Pi = \begin{bmatrix} 0.7629 & 0.2249 & 0.0121 & 0.0001 & 0.0000 \\ 0.2074 & 0.5566 & 0.2207 & 0.0152 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0152 & 0.2207 & 0.5566 & 0.2074 \\ 0.0000 & 0.0001 & 0.0121 & 0.2249 & 0.7629 \end{bmatrix}$$

which generates the stationary distribution $(0.190722, 0.206633, 0.205290, 0.206633, 0.190722)$ and average labor productivity of one.

4.2 Preferences

For the utility function we start with a CES functional form:

$$u(c, h) = (1 - \beta) \frac{(\theta c^{\nu} + (1 - \theta) h^{\nu})^{\frac{1-\sigma}{\nu}} - 1}{1 - \sigma}$$

Notice that the first order conditions in the intratemporal optimization problem yield the condition

$$\frac{h}{c} = \left(P_l \frac{\theta}{1 - \theta} \right)^{\frac{1}{\nu - 1}}$$

which implies that in steady state θ and ν cannot be pinned down at the same time. We therefore assume for now that $\nu = 0$, and therefore:

$$u(c, h) = \frac{c^{\theta(1-\sigma)} h^{(1-\theta)(1-\sigma)} - 1}{1 - \sigma}$$

which allows us to easily calibrate θ to the share of housing vs. non-housing consumption.

Summary

Parameter	Interpretation	Value	Target
σ	Risk Aversion	2.60	Bond portfolio shares
β	Time Discount Factor	0.90	Net Worth/Income
θ	Share Parameter on Nondur. Cons.	0.86	Exp. Share in Data

Details The risk aversion and time discount parameters are calibrated to match targets in the data using the benchmark economy without aggregate uncertainty. We use data from the Survey of Consumer Finances and restrict our attention to only bonds and net real estate, that is real estate holdings net of mortgages, and compute the bond share and net worth to income ratio as a) the unrestricted mean over all households, b) the restricted mean of all households having a net worth smaller than 50 times median income[5] and c) the mean within the median net worth bin using 25 equally-sized bins along household net worth. The results are reported below:

	unrestricted mean	restricted mean	median bin
Bond Share	0.4473	0.3854	0.2832
Net worth / income	2.7733	2.2666	1.2137

One can see from this table that bond shares and net worth ratios are affected a lot by extremely high net worth households. Since we will have trouble matching the skewness of the wealth distribution we decided to match the moments at the median household. Using $\sigma = 2.6$ and $\beta = 0.9$ generates a bond share of around 26% and a net worth to income ratio of about 1.40 which as good an approximation to the data as we can obtain for now.[6]

The share of housing in total consumption θ is set to generate a realistic share of housing in total consumption which has been steady at 14% over the last 40 years with a standard deviation of only about 0.5%. Hence, we set $\theta = 0.86$.

4.3 Policy and Markets

Summary

Parameter	Interpretation	Value	Target
sub	Implicit Interest Rate Subsidy	40 BP	Passmore
r_b	Risk Free Interest Rate on Bonds	0.01	1 year real return on TIPS

Details On the interest rate subsidy we take the view that the pass-through is 100% to make the case for the GSEs as positive as possible. The subsidy is then chosen to match the estimated implicit interest rate differential of around 40 basis points.

The risk-free interest rate is set to the real return on risk-free government bonds with maturity equal to the length of model period, that is, probably a year. 1% is a reasonable estimate.

5 Results

In this section we document results from our thought experiment, that is, we compare steady states of economies with and without a mortgage interest rates subsidy of 40 basis points. Table I summarizes the main macroeconomic aggregates

[5] This would eliminate the top 0.93% of the wealth distribution.

[6] The problem with matching the two emppricial statistics with the two preference parameters is the following. The time discount factor β is effective in controlling the wealth to output ration in the model, so reducing β further below 0.9 allows us to match that statistic. Unfortunately β exerts strong influence also on the bond portfolio share that drops further as β declines (households with little savings hold no or very little bonds, see below for an explanation). While higher risk aversion σ favors bonds as the safe asset, increasing σ is not too helpful because it increases precautionary saving and hence the welath-output ratio, counteracting the effect of lowering β.

Table 1: Consequences of Removing the Subsidy

Variable	Subsidy	No Subsidy	Difference
P_l	0.033911	0.036008	+6.1838%
H	4.396227	4.163278	-5.2988%
M	3.100073	2.517359	-18.7968%
Default share	0.001715	0.001297	-24.3732%
median net worth	1.4048	1.3812	-1.6800%
Wealth Gini	0.4761	0.4737	-0.0024
$Sub/\bar{y}$	0.012276	0.0000	-0.012276
$\mu(g' > 0)$	0.9370	0.9422	0.5550%
$\mu(g' > h)$	0.4007	0.3978	-0.7237%
EV^{SS}	-0.8733	-0.8742	-0.065% c.e.[7]

We see that removing the subsidy decreases the equilibrium housing stock and rental demand H by about 5% and increases the rental price by over 6%. Households use far fewer mortgages in the absence of the subsidy partially due to less housing consumption, but mainly due to lower leverage. As a result, the subsidy has a very strong effect on mortgage default rates, which are substantially lower without the subsidy, as most households are less leveraged. The overall size of the subsidy and thus the tax rate to finance it is quite substantial at about 1.3% of average income. This is due to the fact that all mortgages that are taken out in this economy receive the subsidy.

Removing the subsidy reduces median net worth by about 1.7% and also reduces the wealth Gini coefficient by about a quarter point. The effect on the tenure decision depends on the definition of home ownership. The share $\mu(g' > 0)$ of households that have a positive level of housing actually increases by removing the subsidy. Note however, that some of these household consume more housing services than they own houses, which we would call renters in our model. The share of households with bigger house than housing services consumption, $\mu(g' > h)$, on the other hand, is higher with the subsidy than without, by about 0.7%. In this sense the housing subsidy increases the fraction of households than own the house they live in.[8]

In order to provide some intuition for our aggregate results it is instructive to investigate individual portfolio allocation and consumption decisions, disaggregated by current income and cash at hand (i.e. wealth). The integration over the stationary distribution over these individual policy functions then determines the aggregate quantities in this economy. Figure 1 displays the asset cdf for the five different income shocks $z_1, ..., z_5$ and indicates that removing the subsidy decreases mass at both the lower and upper end of the distribution, consistent with the observation that it also reduces the wealth Gini coefficient.

Figure 2 plots the demand for real estate g', as a function of individual household characteristics. We observe that a household with given income and cash at hand demands less real estate without the subsidy, indicating that the higher rental price without subsidy is more than offset by higher borrowing rates for all households. In addition, the removal of the subsidy does not significantly alter the asset cutoff point at which households start accumulating housing as-

[7] Computed as Consumption Equivalent, that is $(EV_{no\ subs}/EV_{subs})^{1/(1-\sigma)}$

[8] Note, however, that nothing links the housing stock a household owns to the housing services she consumes. This need not be the same physical house, although it is convenient for the interpretation of our results to make that association.

sets. This explains why the measure of households with strictly positive housing assets actually increases if we eliminate the subsidy. Now remember that the effect on the stationary cash at hand distribution was that there are fewer households with very low assets without the subsidy. Thus some households are pushed, without the subsidy, to higher cash at hand positions where they start saving $g' > 0$.

Removing the subsidy has even stronger effects on the decision of whether and how high a mortgage to take out. Without the subsidy, not only are mortgages lower for all household types, as figure 3 indicates. In addition, mortgages drop even more than housing, resulting in a decline in housing leverage, as documented in figure 4.[9] The lower housing leverage explains the sharp drop in the fraction of default that we observed in the aggregate statistics.

Somewhat less obvious is the strong impact of the subsidy on the portfolio allocation of households. This can be seen from figure 6, which plots the bond share in households' portfolios, as a function of the households' characteristics. Without the subsidy, households uniformly hold a smaller fraction of their wealth in bonds. This indicates that a lot of bond demand is driven by the fact that the subsidy reduces borrowing rates substantially. With the subsidy households simply take out higher mortgages for the same size house (see the increase in leverage documented above) and invest part of these additional funds in bonds. Note that this investment strategy obviously does not entail an arbitrage opportunity. While, even after taking into account default premia, borrowing rates on mortgages may be lower than the risk free interest rate with the subsidy, investment into housing that is needed to obtain mortgages entails substantial housing price risk. This limits a households' willingness to engage in the mortgage borrowing cum bond investment strategy.[10]

Another notable feature of the bond portfolio shares is that, with and without subsidy, the bond share in households' portfolios is increasing in wealth. While this may sound nonintuitive at first (poorer households put a larger share of their wealth into the risky, rather than the safe asset), it is in line with recent work on portfolio choice behavior (see Cocco et al. (2003) or Haliassos and Michaelides (2001)). These authors have argued that it should be households with high cash at hand that hold a higher share of their portfolio in the save asset. Households with high net worth tend to be people with high financial relative to human wealth (the present discounted value of future labor income). As such, these households expect to finance their current and future consumption primarily with capital income, whereas low cash-at-hand people tend to rely mostly on their labor income. Thus it is relatively more important for the high cash at hand people not to be exposed to a lot of financial asset return risk. In fact, since labor income shocks and housing price shocks are uncorrelated in our model, housing is a good asset for hedging labor income risk (of course the bond is even better in this regard, but has a lower expected return).

We now turn to a discussion of the welfare consequences of the reform. In terms of aggregate welfare, removing the subsidy reduces steady state welfare measured as consumption equivalent by a modest 0.065 of a percent: household consumption (of both nondurables and housing services) in the steady state without the subsidy has to be increased by this percentage in all states of the world and for all households, such that a household to be born into the steady state

[9] There is an increase in leverage for z_1 households with very low assets, though this happens at extremely low real estate holdings.

[10] As we showed above, there is an endogenous borrowing constraint for mortgage borrowing anyhow, which would limit the extent of exploiting this strategy even if there were an arbitrage opportunity.

without the subsidy is indifferent to being born into the steady state with the subsidy. Notice, however, that the steady state welfare comparison may bias the results in favor of the subsidy. The steady state without the subsidy has lower steady state wealth and housing stock. Thus along the transition from the status quo (the steady state with subsidy) towards the steady state without subsidy a part of that higher wealth and housing stock can be consumed. Thus it is entirely possible that once the transition dynamics and the welfare effects from that transition dynamics are explicitly calculated, a reversal of the welfare conclusion obtained in our results is obtained.

Figure 7 sheds some light on who (that is, households with which characteristics) benefits from the subsidy. The figure plots the consumption equivalent gain for households with different income and cash at hand. The same proviso about ignoring the welfare effects along the transition apply, as before. Therefore this plot should only be understood as a thought experiment of asking the following question: in which economy would someone with state (a, z) prefer to start her life; an economy with or without subsidy. Our quantitative results suggest that households with low wealth prefer to live in an economy without subsidy while high wealth households benefit strongly from it.[11] Also notice that for a given cash at hand level lower income households benefit more from the subsidy, which is quite intuitive because they carry less of the income tax burden. Of course, since the wealth distribution conditional on income has a much lower mean for z_1 households, low income households obviously benefit the least from the subsidy.

6 Conclusions

We constructed a model with competitive housing and mortgage markets where the government provides banks with insurance against aggregate shocks to mortgage default risk. We used this model to evaluate aggregate and distributional impacts of this implicit government subsidy to owner-occupied housing. Our main findings are that the subsidy policy leads to more mortgages issued and a higher housing stock as well as more mortgage delinquencies. The subsidy mostly benefits high income and mostly high wealth households. The aggregate welfare effect of the subsidy is ambiguous so far. In steady state the subsidy generates higher welfare, though this is partially due to a significantly higher housing stock. An explicit characterization of the transition path induced by eliminating the subsidy is needed to obtain a definite answer as to whether the subsidy is indeed welfare improving or not. We defer this to future research.

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[11] Notice that the welfare effect is actually reversed for z_4, z_5 and very low cash at hand levels, but this happens in a region of the state space with zero measure. Remember that cash at hand includes today's income in addition to assets. The portions of the $z_2, ..., z_5$ functions that are increasing occur in the region where cash at hand is smaller than today's income, but since assets can never be negative housholds will, by definition, never end up in that region of the state space.

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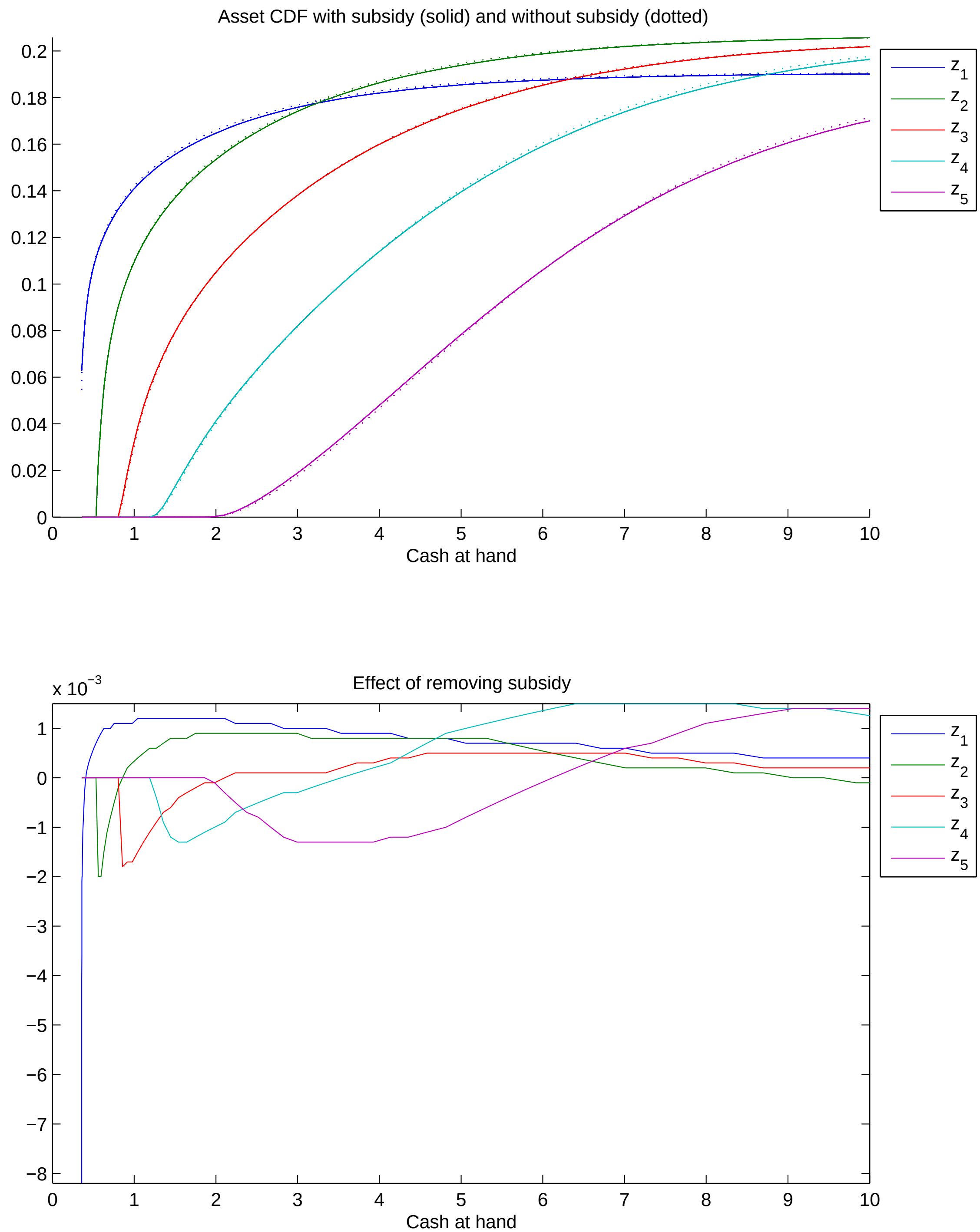


Figure 1: Distribution

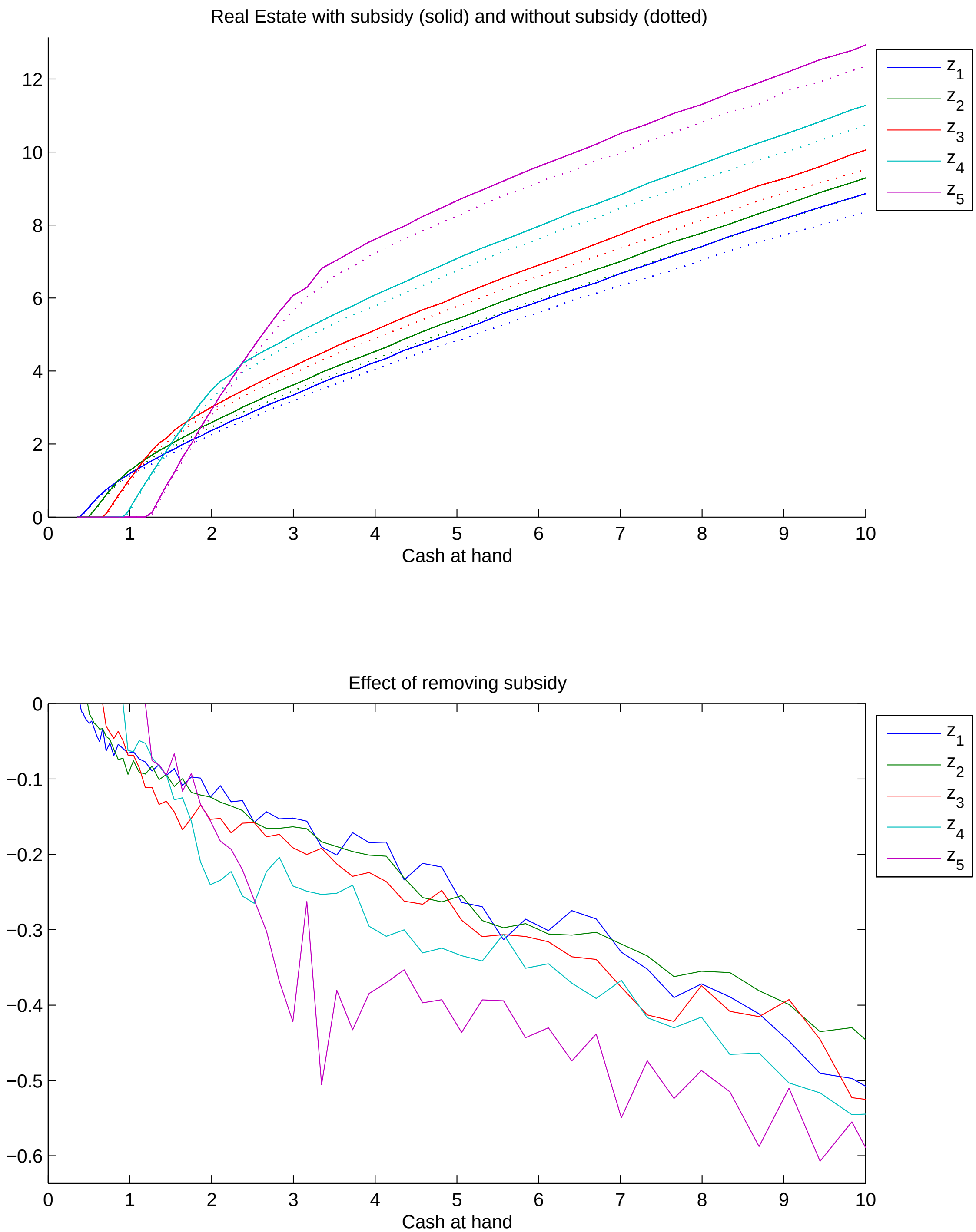


Figure 2: Policy Functions

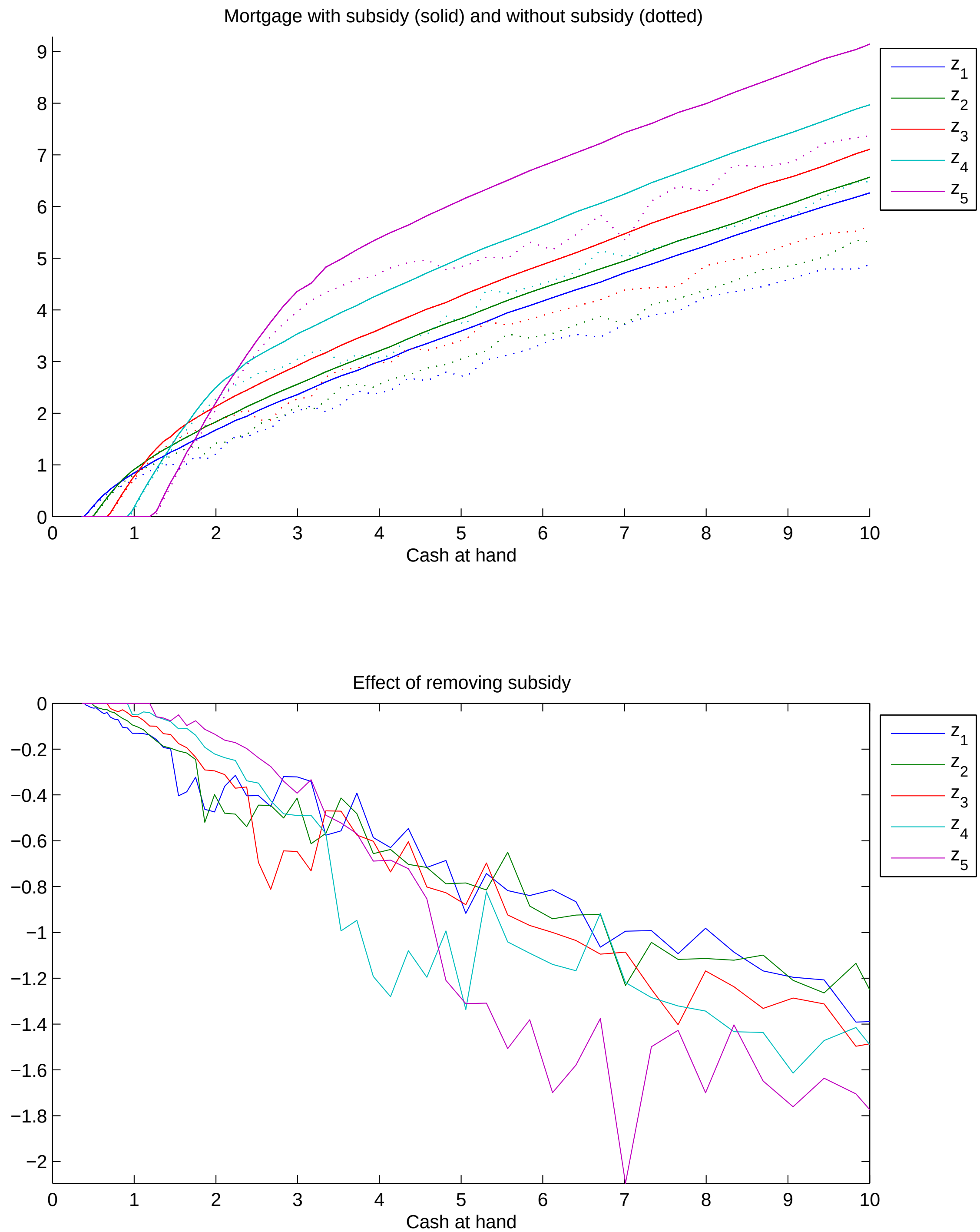
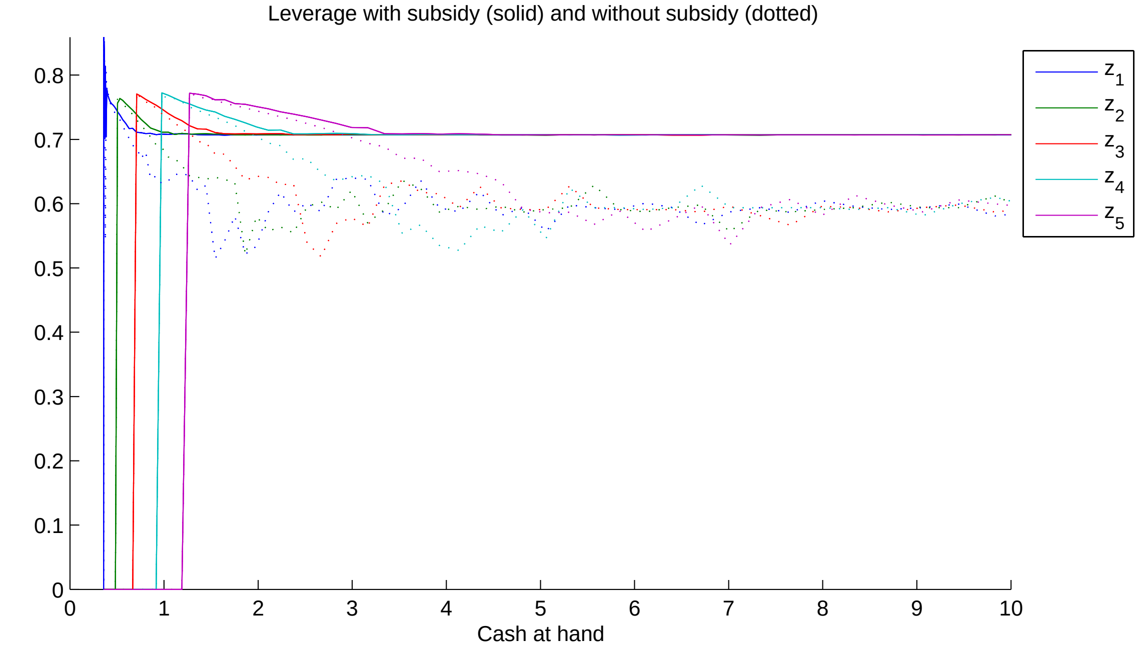


Figure 3: Policy Functions (cont'd)



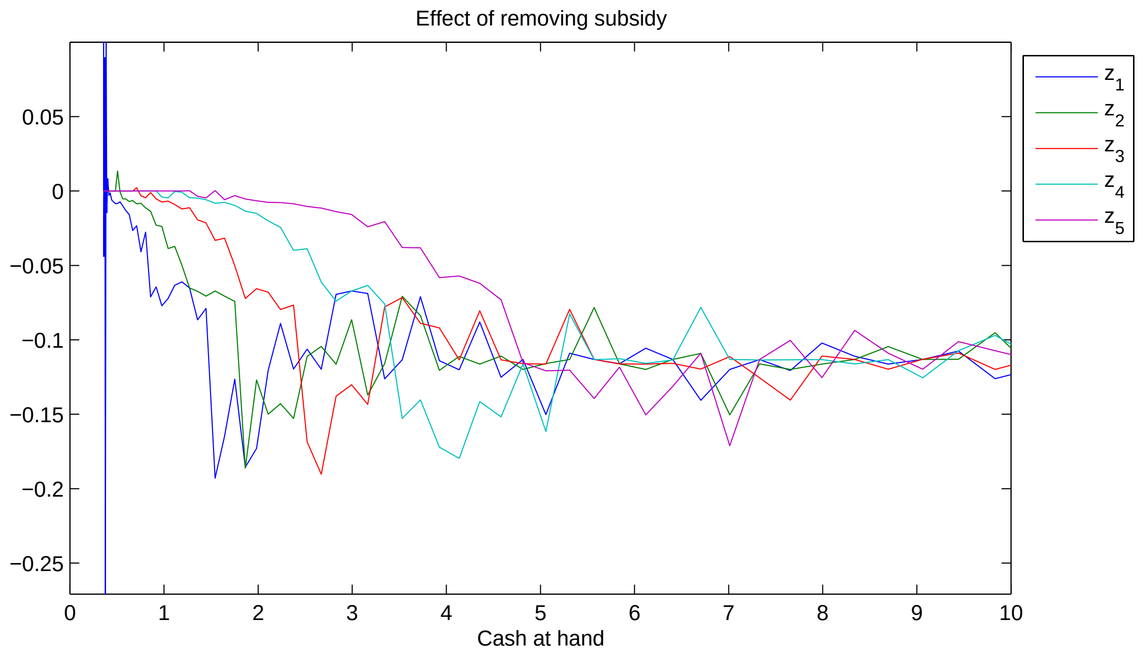


Figure 4: Policy Functions (cont'd)

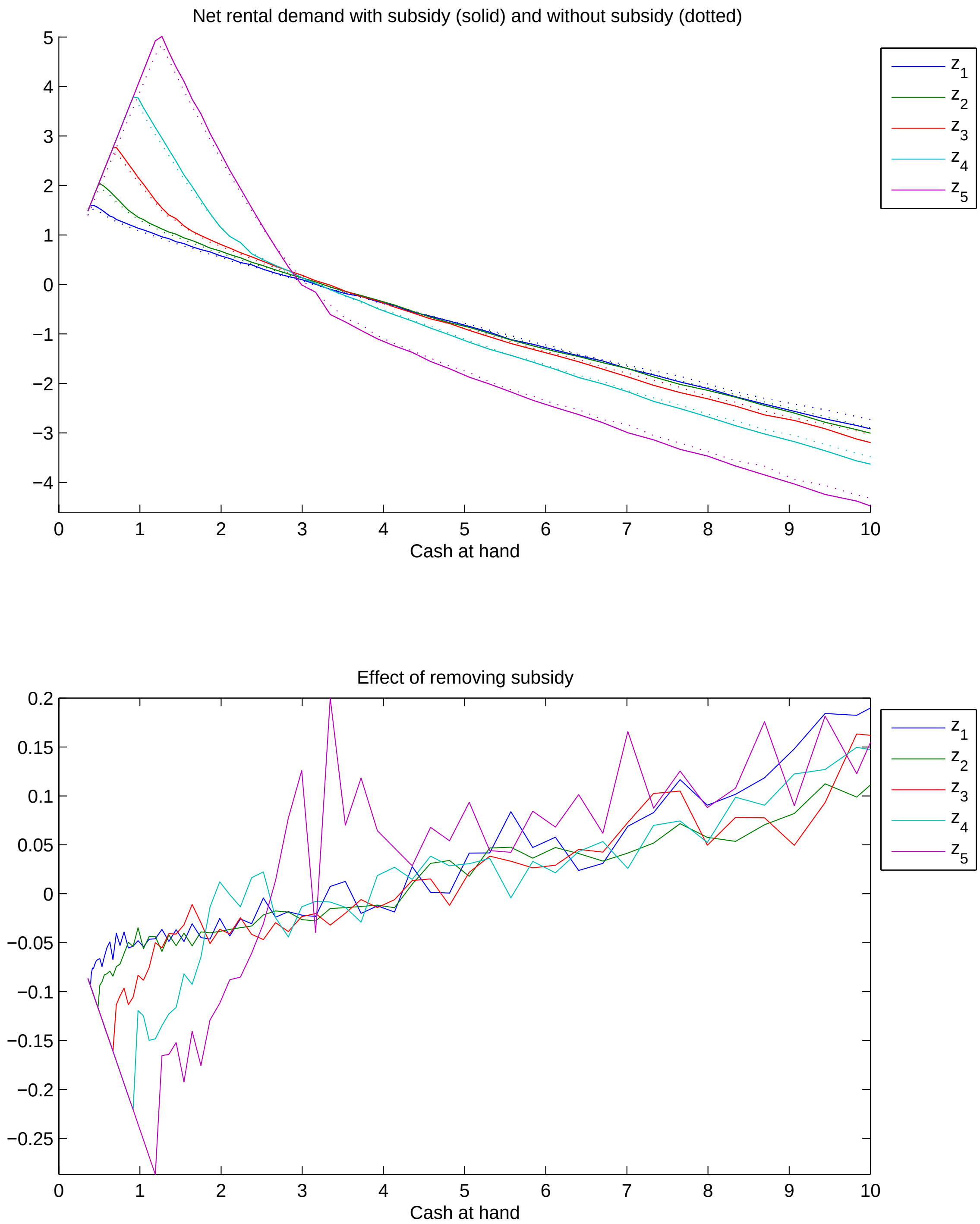


Figure 5: Policy Functions (cont'd)

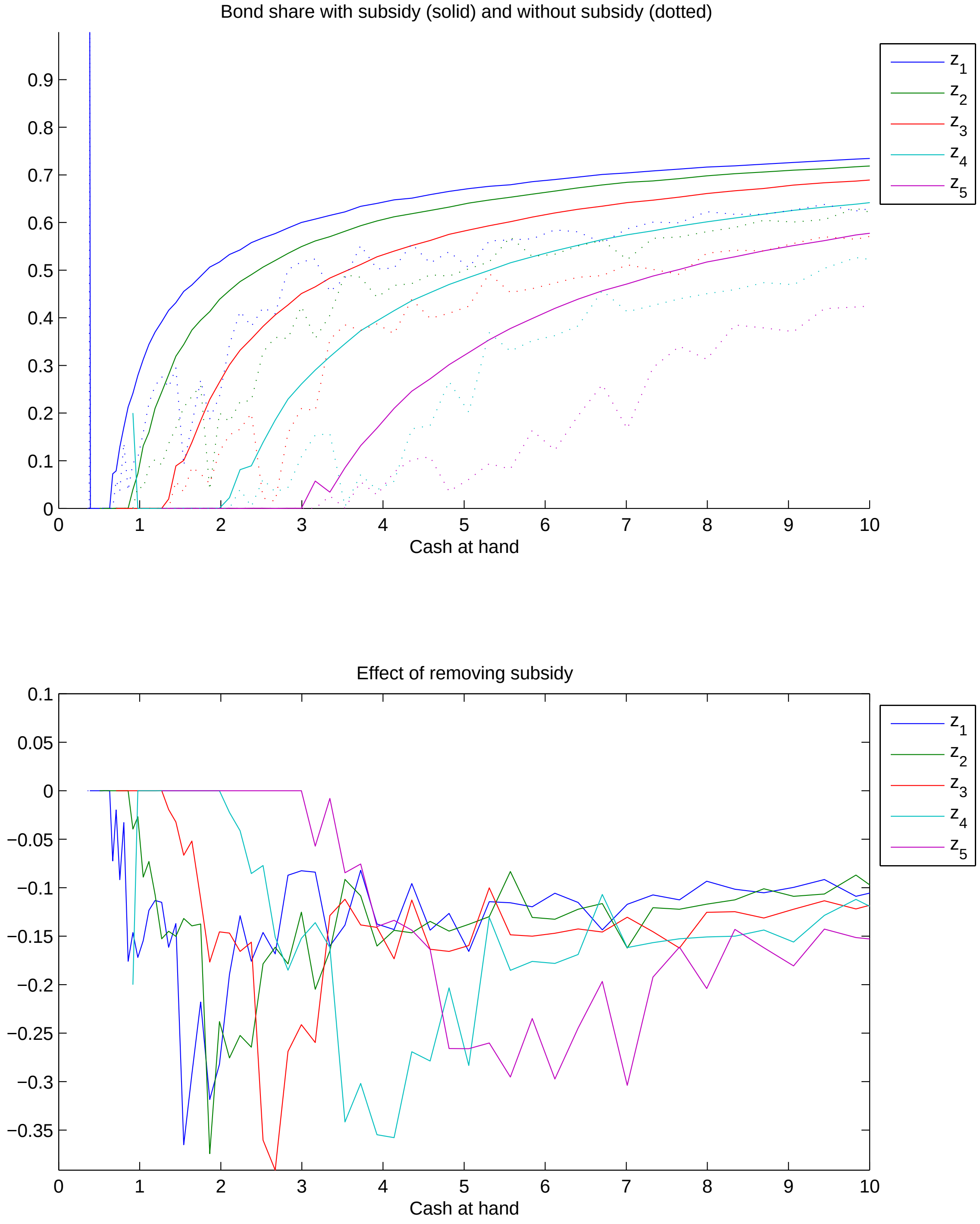


Figure 6: Policy Function (cont'd)

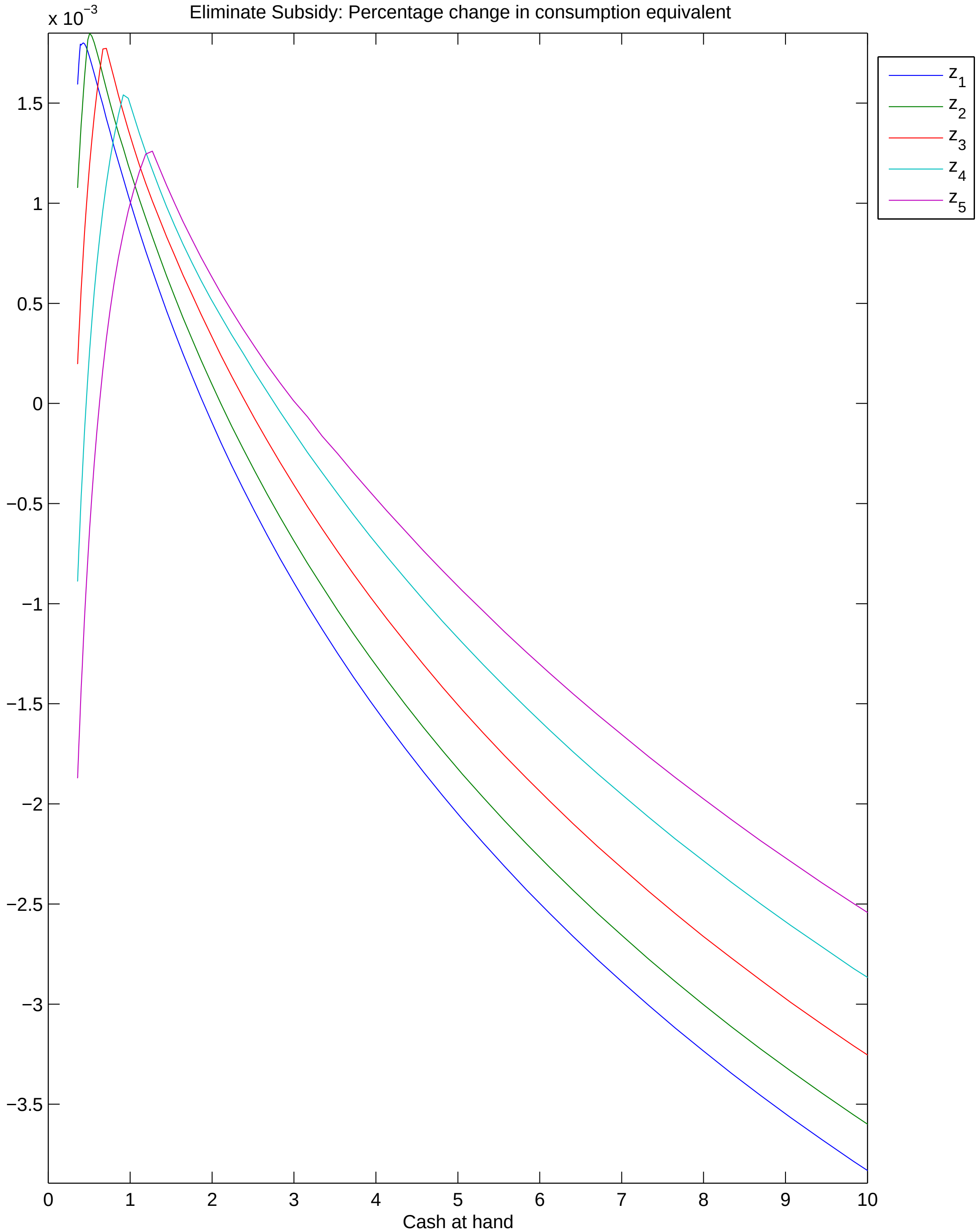


Figure 7: Welfare