

## Some Recent Developments and Directions in Seasonal Adjustment

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**Abstract:** This article describes recent developments in software, diagnostics and research for seasonal adjustment. These include: software merging X-12-ARIMA and SEATS; improved diagnostics for SEATS derived from finite rather than infinite filters; a diagnostic showing the extent to which the trend delays the current business cycle information more than the seasonal adjustment; four new kinds of models for seasonal adjustment, including frequency-specific Airline models; and, finally, some methodological developments proposed for X-11 method seasonal adjustments. (*JEL* C87, C82, C84)

**Key Words:** Time series; X-12-ARIMA; SEATS; TSW; RunX12; trends; moving holiday effects; sliding spans; spectrum; phase delay; RegComponent models; sampling error, seasonal heteroskedasticity, uncertainty measures.

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### 1. Introduction.

This article provides an overview of some recent areas of development in seasonal adjustment. The emphasis is on areas connected to current and planned versions of

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SEATS and X-12-ARIMA for the combination program being developed at the U.S. Census Bureau with the support of the Bank of Spain, see Monsell, Aston and Koopman (2003). This program, tentatively named X-13-SEATS, offers the user the seasonal adjustment methods of both programs with improved and expanded diagnostics for the model-based seasonal adjustments. For this program, Sections 2-4 cover enhancements to regARIMA modeling capabilities, X-12-ARIMA diagnostics for SEATS adjustments, and improved and new diagnostics for SEATS made available by the signal extraction matrix formulas of Bell and Hillmer (1988) and McElroy and Sutcliffe (2005). Section 5 summarizes the features of four new classes of models for seasonal adjustment or trend estimation presented in Bell (2004), Aston, Findley, Wills and Martin (2004), Proietti (2004) and Wildi (2004) which might influence the future evolution of X-13-SEATS or other software. Section 6 briefly summarizes recent methodological developments directed toward enhancing the X-11 seasonal adjustment methodology. Some important topics, such as recent research on methods for trend estimation, receive little mention or none because of limitations of the author's experience and expertise.

### *1.1 Utility software and documents*

Before beginning the discussion of more technical developments in Sec. 2, we call attention to some rather recent software and documents that will be useful to a broad range of users of the current software as well as users of future versions. We start with software. There is now a fully menu-driven version of TRAMO-SEATS with somewhat reduced capabilities, the TSW program (Caporello and Maravall, 2004) distributed by the Bank of Spain, which is completely integrated into the Microsoft Windows operating

system environment. For X-12-ARIMA and X-13-SEATS, the RunX12 program of Feldpausch (2003) distributed by the Census Bureau is a Windows interface program that enables the user to edit or run existing .spc (command) files and metafiles, and invoke a (user-specifiable) text editor to display the output files.

For the user who needs to model complex holiday effects for which the regressors provided by X-13-ARIMA are inadequate, there is an auxiliary program named GenHol (Monsell, 2001). This program generates regressor matrices (and the .spc file commands needed to use them) for holiday effects that are different in (up to three) separate time intervals surrounding the holiday, as happens, for example, when there is an Easter holiday effect on Easter Sunday and Monday in addition to (and perhaps different) from the holiday's effect in an interval preceding the holiday. The general holiday effect model, presented in Findley and Soukup (2000), follows the simplest model of Bell and Hillmer (1983) in treating the effect of the holiday within an interval as constant. Lin and Liu (2003) describe the steps of the development and successful application within X-12-ARIMA of regressors from GenHol to the modeling and adjustment of three Chinese lunar holidays for a variety of Taiwanese time series. The regressor produced by GenHol for an interval will need to be modified if the daily effect of the holiday in an interval is linearly increasing (or decreasing) or quadratic rather than constant, see Zhang, McLaren and Leung (2003) for examples.

On the topic of documentation to help users of the seasonal adjustment programs, Maravall and Sánchez (2001) and Maravall (2002) give examples showing how to use the diagnostics of TRAMO-SEATS to make decisions concerning adjustment options. Findley and Hood (2000) do the same for X-12-ARIMA, and some of their analyses are

applicable to SEATS adjustment decisions based on diagnostics that are made available by X-13-SEATS (see sec. 3 below). Finally, the book by Ladiray and Quenneville (2001), French and Spanish versions of which are available from [www.census.gov/ts/papers/](http://www.census.gov/ts/papers/), provides a very clear, complete, and elaborately illustrated presentation of the X-11 seasonal adjustment methodology of X-12-ARIMA and X-13-SEATS and of those of its diagnostics that were inherited from the X-11-ARIMA program (Dagum 1980), the foundation on which X-12-ARIMA was built.

## **2. RegARIMA Modeling Capabilities of X-13-SEATS**

X-13-SEATS is an enhanced version of X-12-ARIMA containing the latest version of SEATS and having time series modeling capabilities of TRAMO that were not available in version 2.10 of X-12-ARIMA, see Monsell, Aston and Koopman (2003). These new capabilities include an automatic regARIMA model selection procedure patterned very closely after the procedure of TRAMO (Gómez and Maravall 2000, with additional details provided to us by Victor Gómez and summarized in Monsell 2002). There are also additional regressors, such as the seasonal outlier regressors of TRAMO, see Kaiser and Maravall (2003) and Bell (1983). Only the general intervention regressor of TRAMO might not be fully implemented at the time of the release of X-13-SEATS to statistical offices and central banks for testing and evaluation in late 2005.

In the automatic model identification procedures of X-13-SEATS patterned after TRAMO's, there are certain differences from TRAMO's procedures in the method of estimation of ARMA parameters, in the criterion for use of the log transformation, in the thresholds used to include outlier regressors, in the default models for trading day and

Easter holiday effects, and in the criteria that determine when trading day, holiday, or user-defined regressors are included in the model. As a result, X-13-SEATS and TRAMO make identical regARIMA model choices only about one-fourth of the time. In the Appendix, we give more details about these differences and about studies that have been done to compare the automatic modeling procedure of X-13-SEATS with TRAMO's. For transformation and regressor choice, the studies suggest that the procedure of X-13-ARIMA is modestly better (for U.S. series, at least). Some recent changes were made in the procedure for the selection of the differencing orders and a study will soon be done to determine if it now generally chooses the same differencings as TRAMO's procedure.

### **3. Diagnostics of X-13-SEATS Inherited from X-12-ARIMA**

The spectrum is a fundamental diagnostic for detecting the need for, or inadequacy of, seasonal and trading day adjustments, because these effects are basically periodic with known periods. The spectrum can also guide regARIMA modeling decisions in various ways, as we will illustrate. X-13-SEATS produces log spectral plots of the (usually differenced and log-transformed) original and seasonally adjusted series and of the irregular component. (See Findley, Monsell, Bell, Otto and Chen 1998 and Soukup and Findley 1999, 2001 for details concerning the spectrum estimator and trading day frequencies.)

Fig. 1 is the spectral plot of the SEATS irregular component of monthly U.S. Exports of Other Agricultural Material (Non-Manufactured) showing a peak at the highest seasonal frequency, six cycles per year, associated with period two months. This suggests that SEATS' seasonal adjustment has not adequately removed all seasonal effects.

Further support for this interpretation will be obtained in subsec. 4.2 from a modification of a SEATS diagnostic. For SEATS' adjustments, residual seasonality results from an inadequate model, which in our experience often arises from the use of too long a data span for the estimation of model parameters when the properties of the data are changing.

Fig. 2 is a spectral plot of the (differenced and log transformed) original series from January, 1992 through September, 2001 of U.S. Shipments of Defense Communications Equipment (Shipments for short) from the Census Bureau's monthly Manufacturers' Shipments, Inventories and Orders Survey. There are small-to-moderate peaks at four of the seasonal frequencies and a large seasonal peak at the frequency associated with quarterly movements. These peaks confirm that the series is seasonal but also suggest that its seasonality, being different in character at the quarterly seasonal frequency, is of a type that cannot be well modeled by standard seasonal ARIMA models. A new type of model, called a frequency-specific Airline model, is introduced in Section 5, where its canonical seasonal adjustment for this series is compared with that of the Airline model, which models all seasonal frequency components in the same way.

(Figs. 1-2 near here)

X-13-SEATS also includes model comparison diagnostics, such as the out-of-sample forecast error diagnostics of X-12-ARIMA discussed in Findley et al. (1998) and Findley (2005), and diagnostics of the stability of seasonal adjustment and trend estimates, such as the revisions history diagnostics and sliding spans diagnostics, see Findley et al. (1998) and Findley, Monsell, Shulman and Pugh (1990). For the sliding spans diagnostics of SEATS estimates, a new criterion for choosing the span length is used, see Findley, Wills, Aston, Feldpausch and Hood (2003). The span length is determined by the

ARIMA model's seasonal moving average parameter  $\Theta$ , which generally determines the effective length of the seasonal adjustment filter. Because SEATS filters can have effective lengths much greater than X-11 filters, this modified criterion only permits standard sliding spans comparisons to be made when  $|\Theta| < 0.685$  with monthly series of length thirteen years, a significant limitation, and research is needed to find more versatile measures of sliding spans' instability for model-based adjustments. However, for a large variety of simulated series of this length whose estimated  $\Theta$  satisfies  $|\Theta| < 0.685$ , Feldpausch, Hood and Wills (2004) show that the sliding spans statistics (and, to a slightly lesser extent revision histories) are much better than the other diagnostics of TRAMO and SEATS at detecting models that yield quite inaccurate seasonal adjustments.

#### **4. New or Improved Diagnostics of X-13-SEATS for SEATS Estimates**

It is in the area of diagnostics that the technology of SEATS seems most conspicuously out-of-date and, in some cases, flawed. This is mainly due to the fact that most diagnostics provided are calculated as though an infinite or bi-infinite time series were available. Often the diagnostics obtained this way fail to convey valuable information contained in the corresponding diagnostics associated with the actual length of the time series. Finite-sample analogues of the SEATS diagnostics can be obtained from the signal extraction matrix formulas of Bell and Hillmer (1988) and McElroy and Sutcliffe (2005), which provide the coefficients of the finite filters for frequency domain diagnostics of subsec. 4.1 below and also the covariance matrices needed for the diagnostics and tests for over- or underestimation discussed in subsec. 4.2.

#### 4.1 Gain and phase-delay functions of concurrent trend and seasonal adjustment filters

For seasonal adjustment and trend estimates, the filter diagnostics we now discuss can suggest the extent of the tradeoff between smoothness and the delay or exaggeration of business cycle components in the estimates. Findley and Martin (2005) give examples involving competing seasonal adjustment. Here we will show how the diagnostics can detect when a model's trend might be less informative for recent data than its seasonal adjustment, due to filter properties.

Using powers of the backshift operator  $BY_t = Y_{t-1}$ , a linear filter whose application to a time series  $Y_t$  results in an output series  $Z_t = \sum_j c_j Y_{t-j}$  can be expressed as the function  $C(B) = \sum_j c_j B^j$ , i.e.,  $Z_t = C(B)Y_t$ . For monthly data, the *transfer function* of the filter is the generally complex-valued periodic function defined by  $C(e^{-i\frac{2\pi}{12}\lambda}) = \sum_j c_j e^{-i\frac{2\pi}{12}j\lambda}$ ,  $-6 \leq \lambda \leq 6$ , when  $\lambda$  is in units of cycles per year. Its amplitude function,  $G(\lambda) = |C(e^{-i\frac{2\pi}{12}\lambda})|$ , is called the *gain function* of the filter. For example, for the  $j$ -step delay filter  $B^j$ , the transfer function is  $e^{-i\frac{2\pi}{12}j\lambda}$ , so the gain function is constant with value 1. For the seasonal sum filter,  $\delta_s(B) = \sum_{k=0}^{11} B^k$ , the transfer function is

$$\delta_s(e^{-i\frac{2\pi}{12}\lambda}) = \frac{1 - e^{-i\frac{2\pi}{12}12\lambda}}{1 - e^{-i\frac{2\pi}{12}\lambda}} = \left( \frac{\sin \pi\lambda}{\sin \frac{\pi}{12}\lambda} \right) e^{-i\frac{2\pi}{12}5.5\lambda} \quad (1)$$

and the gain function is  $|\sin \pi \lambda / \sin \frac{\pi}{12} \lambda|$  (except at  $\lambda = 0$  where their value is value 12).

For seasonal adjustment and trend filters,  $C(e^{-i\frac{2\pi}{12}\lambda}) = 0$  at the seasonal frequencies

$\lambda = \pm 1, \dots, \pm 5, 6$ , see Appendix C of Findley and Martin (2005), and  $C(1) = \sum_j c_j = 1$ .

A real-valued function  $\phi(\lambda)$  that is defined whenever  $C(e^{-i\frac{2\pi}{12}\lambda}) \neq 0$  and has the property that  $C(e^{-i\frac{2\pi}{12}\lambda}) = \pm G(\lambda) e^{i\frac{2\pi}{12}\phi(\lambda)}$  holds for such  $\lambda$  is a *phase function* of the filter.

Here  $\pm G(\lambda)$  denotes a function that is real-valued, even (i.e. such that  $\pm G(-\lambda) = \pm G(\lambda)$ ),

and nonnegative at  $\lambda = 0$ , and whose absolute value is  $G(\lambda)$  for all  $\lambda$ . Some values can

be negative. An example is  $\sin \pi \lambda / \sin \frac{\pi}{12} \lambda$  in (1), which changes sign at  $\lambda = \pm 1, \dots, \pm 5$ .

Thus, from (1),  $\phi_{\delta_S}(\lambda) = -5.5\lambda$  is a phase function for  $\delta_S(B) = \sum_{k=0}^{11} B^k$  which is a continuous function of  $\lambda$ .

In general, when  $\phi(\lambda)$  is a continuous phase function for a filter with transfer function

$C(e^{-i\frac{2\pi}{12}\lambda})$  such that  $C(1) > 0$  and  $\phi(0) = 0$ , then for every  $\lambda \neq 0$  for which  $C(e^{-i\frac{2\pi}{12}\lambda}) \neq 0$ , the

value of the function  $\tau(\lambda) = -\phi(\lambda)/\lambda$  is interpreted as the (time) *delay*, or the *advance* if

$\tau(\lambda)$  is negative, induced by the filter on the frequency component of  $Y_t$  with frequency

$\lambda$ . (Subsec. A.5 of Findley and Martin (2005) justifies this interpretation of  $\tau(\lambda)$  for

stationary  $Z_t$ . Some numerical experiments like those of Findley, 2001 for the phase

function are needed to clarify the properties of  $\tau(\lambda)$  for nonstationary data.) We call

$\tau(\lambda)$  the *phase delay* function. (Rabiner and Gold 1975, p. 80 use this term for  $-\tau(\lambda)$ ,

which Wildi 2004, p. 50 calls the time shift.) For example, the filter  $B^j$  has a constant

phase delay of  $j$  months, and  $\delta_S(B) = \sum_{k=0}^{11} B^k$  has the constant phase delay  $\tau_{\delta_S}(\lambda) = 5.5$ .

Delay properties are the most useful information conveyed by the phase function, and the phase delay function reveals delay properties more directly than the phase.

Fig. 3, whose phase delays were obtained by the method of Appendix D of Findley and Martin (2005), shows the squared gain and phase delay functions of the filters that produce the SEATS concurrent seasonal adjustment and trend from the Airline model

$$(1 - B)(1 - B^{12})Y_t = (1 - \theta B)(1 - \Theta B)a_t \quad (2)$$

with  $\theta = 0.8$  and  $\Theta = 0.4$  for a series of length 109. Note that the phase delay of the trend filter is roughly twice that of the seasonal adjustment filter and is between two and four months over most of  $0 < \lambda \leq 0.5$  cycles/month, the frequency interval associated with trend and business cycle movements with periods two years or more. Consequently the trend could have substantial timing distortions of many business cycle components.

(Fig. 3 near here)

Also, the squared gains of both filters are greater than one over much of this interval, indicating that any business cycle components at such frequencies will be exaggerated in the filter output, see Appendix A of Findley and Martin (2005). Where both exceed one, the trend filter squared gain is usually substantially greater than the seasonal adjustment filter squared gain, indicating correspondingly greater exaggeration. This phenomenon and the greater phase delay are generally connected to the greater smoothing of the trend filter (i.e., the greater suppression as indicated by smaller values of the squared gain at higher frequencies).

The squared gains and phase delays (not shown) of the infinite concurrent filters of the same model (calculated using formulas of Bell and Martin, 2004) are indistinguishable from those of Fig. 3, because of the rapid decay of the filter coefficients. Findley and

Martin (2005) provide a variety of examples in which the squared gains and phase delays of the series-length filters implicitly used by SEATS differ in important ways from the corresponding function of infinite length series, and they discuss sources of such differences. They also argue that phase delays should only be plotted over  $[0,1)$  for seasonal adjustment and trend filters, because delays of higher frequency components are not of interest for business cycle analyses. That is, five-sixths of the phase-delay plot of Fig. 3 is visual “noise” whose presence results in a lack of resolution over  $[0, 1)$ . (They show in Appendix D that concurrent seasonal adjustment and trend filter phase delays tend to 5.5 at the frequency  $\lambda = 6$  because the filters contain  $\delta_s(B)$ .)

Matrix filter formulas from Bell and Hillmer (1988) such as (3) below and from McElroy and Suttcliffe (2005) are currently being programmed into X-13-SEATS so that the coefficients of series-length concurrent and central filters, their graphs, and the graphs of the values of their squared gains and phase delays can be produced. The analogous calculations for X-11 filters with forecast extension are more cumbersome to produce (e.g. see Appendix B of Findley and Martin 2005) and various strategies for providing them are being considered.

*Remark.* Strictly interpreted, the phase delay of the concurrent filter provides information about the delays in a series consisting of concurrent estimates, which is not the kind of series produced by statistical offices. However, using  $T_{N|M}$  to denote the trend estimate at time  $N$  obtained from data  $Y_t, 1 \leq t \leq M$ , some limited analyses kindly provided by Donald Martin for filters from the Airline model used for Fig. 3 indicate that if the phase delay is approximately four at a business cycle frequency for the filter that produces the

concurrent trend  $T_{N|N}$ , then it is approximately three at this frequency for the filter that produces  $T_{N|N+1}$ , two for the filter that produces  $T_{N|N+2}$ , ..., and less than one and tending to zero for the filters that produce  $T_{N|M}$  for  $M \geq N + 4$ , with a similar pattern of decrease holding for other business cycle phase delay values. These results suggest that concurrent phase delays are usefully predictive. Somewhat analogously, the squared gains of the filters that produce  $T_{N|M}$  for  $M > N$  evolve only slowly away from the shape of the squared gain of the concurrent filter as  $M$  increases, suggesting that this function is also predictive.

#### *4.2 Bias-correcting modifications of SEATS' diagnostic for over- or underestimation and associated test statistics*

We start by presenting a precise concept for an example of a diagnostic type of SEATS for detecting over- or underestimation of the irregular component and of the stationary transforms of the seasonal, trend and seasonally adjusted series from a decomposition  $Y_t = S_t + T_t + I_t$ ,  $1 \leq t \leq N$ . This diagnostic suite is one of the few in SEATS for the component estimates that does not assume the estimated regARIMA model for  $Y_t$  is correct. The concept we present leads to a finite-sample-based diagnostic that does not suffer from the strong bias (toward indicating underestimation) that can occur with SEATS' diagnostics with series of moderate lengths. We also present an associated test statistic of Findley, McElroy and Wills (2005) for testing whether overestimation or underestimation is statistically significant.

Here we consider only the diagnostic for the irregular component  $I_t$  modeled as white noise, because its formulas are simpler. McElroy and Suttcliffe (2005) provide the basic formulas required for the stationary transforms of the other components.

First we have to address a scaling issue. The calculations that derive the canonical models for the seasonal, trend and irregular components from the ARIMA model for  $Y_t$  produce models whose innovations variances are given in units of this model's innovation variance  $\sigma_a^2$  (i.e. as though this variance were one). We use  $\sigma_t^2 / \sigma_a^2$  to denote the variance specified for  $I_t$  in this manner. With  $\delta(B)$  denoting the differencing operator of the ARIMA model for  $Y_t$ , e.g.  $(1-B)(1-B^{12})$  in (2), let  $\Sigma_w$  denote the autocovariance matrix of order  $N-d$  of the differenced series  $W_t = \delta(B)Y_t, d+1 \leq t \leq N$  specified by this ARIMA model but calculated as though  $\sigma_a^2$  were equal to one. Finally, let  $\Delta$  denote the  $(N-d) \times N$  matrix which transforms  $[Y_1, \dots, Y_N]'$  to  $W = [W_{d+1}, \dots, W_N]'$ . With this notation, the vector  $\hat{I} = [\hat{I}_1, \dots, \hat{I}_N]'$  of SEATS estimates of the irregulars has the formula

$$\hat{I} = \frac{\sigma_t^2}{\sigma_a^2} \Delta' \Sigma_w^{-1} W, \quad (3)$$

see (4.3) of Bell and Hillmer (1988).

Now we can state what we take to be the basic concept of SEATS' diagnostic: investigate the sign (and, in an unspecified way, also the size of) the difference between the sample mean square  $\overline{\hat{I}^2} = N^{-1} \sum_{t=1}^N \hat{I}_t^2$  and its expected value  $N^{-1} \sum_{t=1}^N E \hat{I}_t^2$ . Here  $E$  denotes expectation calculated for quadratic forms of the  $W_t = \delta(B)Y_t, d+1 \leq t \leq N$  as if

the autocovariances of the  $W_t$  specified by the estimated ARIMA model for the series  $Y_t$  were correct.

The formula (3) immediately yields a formula for  $\Sigma_{\hat{I}/\sigma_a}$ , the covariance matrix of  $\hat{I}$  in units of  $\sigma_a^2$ , namely  $\Sigma_{\hat{I}/\sigma_a} = (\sigma_t^2 / \sigma_a^2)^2 \Delta' \Sigma_W^{-1} \Delta$ . For  $1 \leq t \leq N$ , we denote the  $t$ -th diagonal entry of this matrix by  $\sigma_t^2 / \sigma_a^2$ . To obtain  $E\hat{I}_t^2$ , we multiply  $\sigma_t^2 / \sigma_a^2$  by the bias-corrected maximum likelihood estimate of  $\sigma_a^2$  due to Ansley and Newbold (1981),  $\hat{\sigma}_a^2 = c_N (N-d)^{-1} W' \Sigma_W^{-1} W$ , with bias correction factor  $c_N = (N-d)/(N-d-n_{coeffs})$ , where  $n_{coeffs}$  denotes the number of estimated ARMA coefficients, estimated via exact maximum likelihood estimation. Thus, with  $tr$  denoting matrix trace, i.e., the sum of the diagonal entries,  $N^{-1} \sum_{t=1}^N E\hat{I}_t^2 = \hat{\sigma}_a^2 (N^{-1} tr \Sigma_{\hat{I}/\sigma_a})$ . Our basic diagnostic is therefore

$$\tau = \overline{\hat{I}^2} - \hat{\sigma}_a^2 (N^{-1} tr \Sigma_{\hat{I}/\sigma_a}). \quad (4)$$

By analogy with SEATS and Marvall (2003), *underestimation* is indicated when  $\tau < 0$ , and *overestimation* is indicated when  $\tau > 0$ , where, reformulating the description in Marvall (2003), underestimation means inadequate suppression, and overestimation means excessive suppression, of the complementary component to the one being estimated, e.g., inadequate or excessive suppression of  $S_t + T_t$  when estimating  $I_t$ .

The diagnostic provided by SEATS is effectively  $\tau^{SEATS} = \overline{\hat{I}^2} - \hat{\sigma}_a^2 (\sigma_{WK}^2 / \sigma_a^2)$ , where  $\sigma_{WK}^2 / \sigma_a^2$  denotes the variance of the Wiener-Kolmogorov estimator of  $I_t$  from bi-infinite data from the ARIMA model for  $Y_t$  in units of  $\sigma_a^2$ . In Proposition 1 of Findley, McElroy and Wills (2005), it is shown that  $\sigma_{WK}^2 / \sigma_a^2 > \sigma_t^2 / \sigma_a^2$  holds for all  $t$  when the model has a

moving average component. Consequently, SEATS' diagnostic has a bias toward indicating underestimation. For series lengths  $N = 72,144$ , the bias is shown in Tables 1 and 3 of Findley, McElroy and Wills (2005) to strongly compromise the performance of SEATS' diagnostic. This is particularly the case relative to the performance of a modification  $\tau^{(2)}$  of (4) obtained by using averages of  $E\hat{I}_t^2$  and  $E\hat{I}_t^2$  over the more restricted range  $13 \leq t \leq N - 12$ , see subsec. 3.2 and subsec. A.3 of this reference where approximate standard deviations for  $\tau$  and  $\tau^{(2)}$  are derived for near-Gaussian data, e.g.

$$\hat{\sigma}_\tau = \frac{\sqrt{2}\hat{\sigma}_a^2}{N} \left\{ \text{tr}\Sigma_{\hat{I}/\sigma_a}^2 - \frac{2c_N - c_N^2}{N - d} \left( \text{tr}\Sigma_{\hat{I}/\sigma_a} \right)^2 \right\}^{1/2} \text{ for } \tau. \text{ These yield test statistics, } \bar{\tau} = \bar{\tau} / \sigma_\tau$$

and the analogously defined  $\bar{\tau}^{(2)}$ , which can be compared to the standard normal distribution for one-sided tests the significance of over- or underestimation. For example, for the Export series whose spectrum in Fig. 1 has a peak at the seasonal frequency six cycles year, indicating residual seasonality in the irregular component, the value of  $\bar{\tau}^{(2)}$  is  $-2.177$ , which has a  $p$ -value of .013 indicating significant underestimation of the irregular component. (The value of  $\bar{\tau}$  is  $-0.546$ . Note that calculation of the trace of  $\Sigma_{\hat{I}/\sigma_a}^2$ , and therefore of  $\bar{\tau}$ , requires the full autocovariance matrix  $\Sigma_{\hat{I}/\sigma_a}$ , and  $\bar{\tau}^{(2)}$  requires all but its first and last twelve rows and columns, more autocovariances than can be obtained from standard state space algorithms.)

SEATS does not have a test statistic for  $\tau^{SEATS}$  and its analogues. Maravall (2003) illustrates how such a test statistic could be obtained, but the consequences of the bias of  $\tau^{SEATS}$  shown in Findley, McElroy and Will (2005) indicate that such tests will often be unreliable. These results, those of Findley and Martin (2005), and the availability of the needed finite-filter formulas like those just presented have led us to conclude that all of

the diagnostics of SEATS that depend on Wiener-Kolmogorov (i.e. infinite-data-based) coefficients or autocovariances should be replaced by quantities appropriate to the length of the time series to which SEATS procedures are being applied. This is a goal the U.S. Census Bureau is pursuing for X-13-SEATS.

## 5. New Time Series Models for Seasonal Adjustment

### 5.1. The RegComponent Models of Bell

Bell (2004) describes methods and associated software for estimating the various model parameters and unobserved components when observed data  $Y_t, 1 \leq t \leq N$  are assumed to have the form

$$Y_t = x_t' \beta + \sum_{i=1}^m h_{it} z_{it},$$

where  $x_t'$  is a row vector of known regression variables,  $\beta$  is the vector of constant regression coefficients,  $h_{it}$  for  $i = 1, \dots, m$  are series of known constants called “scale factors” (often  $h_{it} = 1$ ), and  $z_{it}$  for  $i = 1, \dots, m$  are independent unobserved component series following ARIMA models,  $\phi_i(B)\delta_i(B)z_{it} = \theta_i(B)\varepsilon_{it}$  (Sometimes, as in the first example below, a regARIMA model is used for an unobserved component.) Such models for  $Y_t$  are called *RegComponent models*, or simply (ARIMA) *Component models* when no regressors occur. RegARIMA models are the special case with  $m = 1$ ,  $h_{it} = 1$ . Bell (2004) shows how the various types of seasonal “structural” models of Harvey (1989) can be formulated as RegComponent models and also gives other important examples, two of which we present below. Bell and Martin (2004b) show how the time-varying trading-day effects model of Harvey (1989) can be formulated as a RegComponent model

and they compare its performance to that of the time-varying trading day model considered in Bell (2004).

### 5.1.1 Seasonal adjustment of a time series with sampling error.

For the U.S. Monthly Retail Trade Survey series of (unbenchmarked estimates) of Sales of Drinking Places from September, 1977 through October, 1989, let  $Y_t$  denote the logarithm of the survey estimates,  $y_t$  the corresponding true population quantities, and  $e_t$  the sampling error in  $Y_t$  as an estimate of  $y_t$ . The RegComponent model for  $Y_t$  considered in Bell (2004) is, in our notation,

$$Y_t = y_t + e_t$$

$$(1-B)(1-B^{12})(y_t - x_t' \hat{\beta}) = (1-0.23B)(1-0.88B^{12})b_t$$

$$\sigma_b^2 = 3.97 \times 10^{-4}$$

$$(1-0.75B)(1-0.66B^3)(1-0.71B^{12})e_t = (1+0.13B)c_t$$

$$\sigma_c^2 = 0.93 \times 10^{-4}$$

where  $x_t'$  consists of trading day regressors. The model for  $e_t$  was developed given knowledge of the survey design and its parameters were estimated using estimates of sampling error autocovariances. With these parameters fixed, the estimated parameters of the regARIMA model for  $y_t$  were obtained from Bell's REGCMPNT program (which will be made available soon). The value of  $\hat{\beta}$  is not of concern here. From this model, a canonical seasonal decomposition of the model for  $y_t - x_t' \hat{\beta}$  yields a RegComponent model  $Y_t = x_t' \hat{\beta} + S_t^{(1)} + N_t^{(1)} + e_t$ . From this, the state space smoothing algorithm of the REGCMPNT program produces an estimate  $\hat{N}_t^{(1)}$  of the nonseasonal component of the

unobserved sampling-error-free series  $y_t$ . This can be compared to the nonseasonal component estimate  $\hat{N}_t^{(2)}$  obtained from the alternative RegComponent model  $Y_t = x_t'\beta + S_t^{(2)} + N_t^{(2)}$  arising from the canonical decomposition of the directly estimated regARIMA model for  $Y_t$ ,

$$(1-B)(1-B^{12})(Y_t - x_t'\beta) = (1-0.29B)(1-0.56B^{12})a_t$$

$$\sigma_a^2 = 6.37 \times 10^{-4}.$$

Fig. 4 presents the graphs of the resulting nonseasonal components estimates  $\exp(\hat{N}_t^{(1)})$  and  $\exp(\hat{N}_t^{(2)})$  of the original data. The series  $\exp(\hat{N}_t^{(1)})$  is smoother than  $\exp(\hat{N}_t^{(2)})$ . Fig. 12.1 of Bell (2004, p. 273) shows that its signal extraction standard errors are larger than those of  $\exp(\hat{N}_t^{(2)})$  at all times  $t$ , as one would expect, because  $\hat{N}_t^{(1)}$  attempts to suppress both  $S_t^{(1)}$  and  $e_t$  when estimating  $N_t^{(1)}$ .

(Fig. 4 near here)

#### 5.1.2. Modeling seasonal heteroskedasticity.

For the series of logarithms of U.S. Northeast region monthly total housing starts from August, 1972 to March, 1989, Bell (2004) notes that standard ARIMA modeling procedures without outlier identification led to the ARIMA model  $(1-B)(1-B^{12})Y_t = (1-.64B)(1-.58B^{12})a_t$  with  $\hat{\sigma}_a^2 = .050$ . However, automatic outlier detection in X-12-ARIMA with a critical value of 3.0 leads to a regARIMA model,  $(1-B)(1-B^{12})(Y_t - x_t'\tilde{\beta}) = (1-.55B)(1-1.00B^{12})a_t$  with  $\hat{\sigma}_a^2 = .024$ , whose regressors model four additive outliers for Januarys and five for Februarys, due presumably to the effects of unusually bad or unusually good winter weather on construction activity in these months. The value 1.00 of the seasonal moving average parameter in the second

model indicates the seasonal pattern of the data is fixed, whereas the coefficient .58 of the first model suggests that it is rather variable.

Bell (2004) then presents a Component model without regressors that includes an observation error component for Januarys and Februarys and requires only two more parameters than the initial Airline model instead of nine more,

$$\begin{aligned} Y_t &= z_{1t} + h_{2t}z_{2t} \\ (1-B)(1-B^{12})z_{1t} &= (1-.54B)(1-1.00B^{12})b_t \\ z_{2t} &= c_t, \hat{\sigma}_c = .12, \hat{\sigma}_b = .023, \end{aligned}$$

where  $b_t$  and  $c_t$  are mutually independent white noise processes, and where

$$h_{2t} = \begin{cases} 1, & t \sim \text{January or February} \\ 0, & \text{all other months.} \end{cases}$$

The model for  $z_{1t}$  is almost identical to the model obtained for the outlier corrected data and leads to quite similar fixed seasonal factors and seasonal adjustments with moderate exceptions for January and February. This third model is more appealing, because it avoids the instabilities associated with the use of a fixed and somewhat arbitrarily chosen critical value for outlier detection (not to mention instability associated with the estimation of nine regression coefficients in this case). It also leads to seasonal factor standard errors that are significantly larger for January and February than for the other months, as seems appropriate, something the other models do not show.

Trimbur (2005) will provide further results for Bell's approach to seasonal heteroskedasticity together with the results of his study of a different, structural model-based approach of Proietti (2004), some initial results from which we present next.

Whereas X-11 based software has always had an approach to dealing with seasonal heteroskedasticity that is often effective (see Findley and Hood, 2000 for some

examples), no versatile approach has been established for model-based seasonal adjustment.

### *5.2 Proietti's seasonal specific structural models*

Proietti (2004) has developed a structural-model-based approach to modeling seasonal heteroskedasticity in which a season-specific noise process, analogous to Bell's  $h_{2t}, z_{2t}$ , is added to the process governing the trend changes from one period to the next. Another such noise process can be added to the process governing the trend slope changes. We refer the reader to Proietti's paper for details. Thomas Trimbur kindly provided Fig. 5 and the results we now summarize for eleven U.S. construction series published by the Census Bureau. These results are taken from a preliminary version of Trimbur (2005). These series are ones whose X-12-ARIMA seasonal adjustments are obtained by specifying different seasonal filter lengths for some calendar months, the main X-11 procedure for dealing with calendar month heteroskedasticity.

Because the X-11 procedure addresses heteroskedasticity in the detrended series (the "SI" ratios) whereas Proietti models heteroskedasticity in the trend, it is not surprising that the log likelihood ratios comparing Proietti's heteroskedastic and homoskedastic local linear trend models suggest that only three of the eleven series are heteroskedastic in Proietti's sense. The properties of the trend estimates of Proietti's models that were observed for these three series are illustrated in the graphs of the trends of two of the series shown in Fig. 5, which also include the X-12-ARIMA final trends (table D 12). For both series, the trends from the two models have similar shapes but the trends from the heteroskedastic model always have larger values than those from the homoskedastic

model. These larger values are closer to the values of the X-12-ARIMA trend, which is always the smoothest of the trends. Particularly for the Total Building Permits series (top), the model-based trends are more erratic and prone to local peaks or troughs, in January or February. These features would make this trend unacceptable to many users. No such movements occur in the X-12-ARIMA trends. Thus, Proietti's new models yield an alternative trend to the X-12-ARIMA trend that might be acceptable for at most two of the eleven series. Hence they do not provide a satisfactory model-based alternative to the X-11 approach to seasonal heteroskedasticity for these series. Also, the lower level of the trends from Proietti's homoskedastic model might be a deficiency related to the fact that twelve month sums of log seasonal factors are not required to have mean zero for either model.

(Fig. 5 near here)

### 5.3. Frequency-specific generalized Airline models.

Aston, Findley, Wills and Martin (2004) consider special cases of the model

$$\begin{aligned}
 (1-B)^2 \left( \sum_{j=0}^{11} B^j \right) Y_t = \\
 (1-B)^2 (1+B) \prod_{j=1}^5 \left( 1 - 2 \cos\left(\frac{2\pi j}{12}\right) B + B^2 \right) Y_t = \\
 (1-\theta B)(1-b_0 B) \left[ (1+b_6 B) \prod_{j=1}^5 \left( 1 - 2b_j \cos\left(\frac{2\pi j}{12}\right) B + b_j^2 B^2 \right) \right] \varepsilon_t.
 \end{aligned} \tag{5}$$

This eight-coefficient model is the most general form of what we call a *frequency-specific Airline model*. It has an individual coefficient for each moving average factor that corresponds to a seasonal unit root factor of the differencing operator on the left. We call

these six moving average factors *seasonal frequency factors*. Box-Jenkins Airline models (2) with  $\Theta \geq 0$  are the special case  $b_j = \Theta^{1/12}$ ,  $0 \leq j \leq 6$ . Aston et al. (2004) conclude that the generalized Airline models of the form (5) which are most useful for seasonal adjustment are those in which only two of the coefficients  $b_j, 0 \leq j \leq 6$  are distinct. The other models have too great a tendency to have estimated coefficients equal to one in situations in which the evidence for such a value is not strong.

Here we single out those three-coefficient frequency-specific models, denoted 5-1(3) models, in which five seasonal frequency factors have one coefficient and the remaining seasonal frequency factor has its individual coefficient, for example,

$$(1-B)^2 \left( \sum_{j=0}^{11} B^j \right) Y_t = (1-aB)(1-c_1B) \times \left\{ (1+c_2B) \prod_{j=1}^5 \left( 1 - 2c_1 \cos\left(\frac{2\pi j}{12}\right) B + c_1^2 B^2 \right) \right\} a_t.$$

The situation in which a series (perhaps after taking logs and a first difference) has a spectrum with a strong seasonal peak only at one of the seasonal frequencies, as in Fig. 2 for the Shipments series, is the most easily identified one for which a 5-1 (3) frequency-specific model is a natural alternative to the Airline model, and this situation is not particularly rare, as Wildi (2004) also observes. Conventional seasonal ARIMA models like the Airline model, which use the same coefficient  $\Theta^{1/12}$  for each frequency, are ill suited to such series.

For the Shipments series, the quarterly-effect frequency, 4 cycles/year, is the one associated with the  $c_2$  coefficient because this frequency has the dominant spectral peak in Fig. 2. The  $c_1$  coefficient estimate is 0.9718, close to the twelfth root of the estimated

Airline model seasonal coefficient  $\Theta$  ( $\sqrt[12]{0.6667} = 0.9668$ ). However, the  $c_2$  coefficient estimate is 0.8086 ( $= \sqrt[12]{0.0781}$ ). This smaller value results in the concurrent squared gains of the 5-1 (3) model in Fig. 6 having wider “troughs” at the 4 cycles/year frequency than the Airline model’s filters (and similarly for the squared gains of the symmetric filter, see Findley, McElroy and Wills, 2005). The wider troughs indicate more suppression of variability around this quarterly frequency. As a result, for this series, the canonical seasonal adjustment of the 5-1(3) model is generally smoother than that of the Airline, see Fig. 7.

(Figs. 6-7 near here)

There are six 5-1(3) models corresponding the available choices of season frequency factors for  $c_2$ . Analogously, there are fifteen so-called 4-2(3) models in which  $c_2$  is assigned to two seasonal frequency factors and  $c_1$  to the other four. Aston et al. (2004) find this to be type of three-coefficient models most frequently preferred over the airline model by their generalization of Akaike’s AIC that seeks to account for the increase in uncertainty in selecting among numerous frequency-specific models and the Airline model. Finally, there are twenty so-called 3-3(3) models in which the  $c_1$  and  $c_2$  are each assigned to three seasonal moving average factors. This type does not appear to be as useful at the other two.

Estimation of the frequency-specific Airline models and the calculation of their canonical ARIMA model-based seasonal adjustments were done with a version of the state space modeling software discussed in Aston and Koopman (2003). This software is being further developed to have the capabilities of Bell’s REGCMPNT program.

#### *5.4 Wildi's ZPC-QMP Filters*

The monograph of Wildi (2004) provides a radically different approach to model-based trend estimation (and, secondarily, seasonal adjustment) which is very stimulating, also because some new and substantial auxiliary theoretical results on frequency domain techniques for nonstationary series are obtained. It can be argued that the long-run behavior of any “true” ARIMA process (meaning cases like the airline model (2) with  $\theta = 1$  or  $\Theta = 1$  are excluded) is unlike that of any persistently seasonal economic time series because each calendar month's seasonal factors will change direction over time, but Wildi emphasizes how especially ill-suited such models seem to be for the long index series he considers, whose values are constrained to satisfy  $-100 \leq Y_t \leq 100$ . He proposes a new class of “zero-pole-combination quasi-minimum- phase” (abbreviated ZPC-QMP) filters with ARMA transfer functions.

The case of primary interest in Wildi (2004) is that of one-sided ZPC-QMP filters for concurrent trend estimates. Their seventeen parameters are estimated by minimizing a frequency domain expression, calculated from the periodogram of the time series of interest or its differences, which estimates the mean squared difference (or revision error) between the concurrent filter's output and the output of a prescribed symmetric trend filter for the given series. Constraints on the magnitude of the phase-delay (or time-shift) of the filter can also be imposed on the estimation criterion.

If the prescribed symmetric filter is appropriate, this would seem to be more application-appropriate criterion than maximum likelihood estimation of an ARIMA model, which is related to minimizing the model's mean squared one-step-ahead forecast

error of the original seasonal time series. Estimation of Wildi's models turns out to often be difficult and delicate, but for the set of series he considers, they generally yield better trends, e.g. for turning point estimation, than the maximum likelihood based trend estimates he investigates.

Wildi's approach seems quite impractical for statistical offices with a large number of time series, or with short series, but it may give indications of important limitations of current methods and indications of how some new methods that will be useful for statistical offices could be found.

## **6. Recent Developments for X-11 Filter Seasonal Adjustments.**

Here we call attention to two noteworthy recent research directions related to seasonal adjustments from X-11 filters.

### *6.1 Shrinkage methods for X-11 seasonal factors.*

We start with the discussion paper of Miller and Williams (2004) which summarizes a study of the application to X-11 seasonal factors of a "global" and a "local" shrinkage procedure developed by the authors. It shows, through a forecasting study with a large test set of series frequently used by academic forecasters, that the shrinkage methods lead to seasonal adjustments from which better forecasts are obtained from certain methods that forecast seasonal time series by (i) seasonally adjusting, (ii) forecasting the seasonally adjusted series, and (iii) reseasonalizing these forecasts.

Findley, Wills and Monsell (2004) investigate whether seasonal adjustment after application of the shrinkage methods to the seasonal factors improves some of the X-12-

ARIMA Q2, M7, M8, M10 and M11 quality statistics (see Ladiray and Quenneville 2001). They find that the “local” shrinkage method of Miller and Williams, whose statistical motivation is less clear than that of their “global” method, increases the number of multiplicatively seasonal series with acceptable values of these statistics (defined for this study as being any value less than 1.10) from a little over 200 to almost 250. The shrinkage smoothes the evolution from year to year of the seasonal factors for many of the calendar months. However, for the few series examined in detail, the resulting seasonally adjusted series had seasonal peaks in their spectra that did not occur without shrinkage. This suggests that shrinkage of the seasonal factor by this method led to residual seasonality, a significant problem for official seasonal adjustments if not for forecasters. The global shrinkage method provided no useful improvement to the set of diagnostics. Thus, it remains to be seen whether shrinkage methods can be used to improve X-11 filter-based seasonal adjustments in a way that is useful to producers of official statistics.

## *6.2 Uncertainty measures*

The second research direction of note is the development of methods to provide standard errors for X-11 filter based seasonal adjustments, statistics that have always been available for model-based method adjustments but not for X-11 adjustments. We are referring to the methods of Pfeffermann (1994) and its further development described in Scott, Sverchkov and Pfeffermann (2004) and to the quite different method of Bell and Kramer (1999). These methods can take account of sampling error in the data through the use of time series models of the sampling error like the model of subsec. 5.1.1 and the

models of Nguyen, Bell and Gomish (2002) and Scott et al. (2004).

The Pfeffermann approach described in Scott et al. (2004) accounts for time series uncertainty beyond sampling error in a way that the Bell and Kramer approach does not, by modeling the irregular component of the unobserved sampling-error-free series with a moving average model of order  $0 \leq q \leq 3$ , assuming at least one such model can be found that is not incompatible with the sampling-error autocovariances and the autocovariances of the X-11 irregulars.

Going beyond uncertainty due to sampling error, the approach of Bell and Kramer (1999) extends the approach of Wolter and Monsour (1981) to addresses uncertainty associated with ARIMA forecast extension before application of X-11 filters in a way that the Pfeffermann approach does not. When sampling error is negligible or not modeled, the Bell and Kramer mean square errors are estimates of the square of the seasonal adjustment revisions that will occur when future data become available, making possible the application of more symmetric filters. Such revision estimates (and the standard error estimates mentioned previously and also those of ARIMA model-based procedures) do not account for the effect of revisions to the unadjusted data, for example, revisions due to late reporting or benchmarking.

As an alternative or complement to the procedures mentioned above, simple descriptive statistics calculated from several years of published revisions over time to originally published adjusted values have several useful properties. (1) They reflect the effects of revisions to the unadjusted data as well as the effects of applying different asymmetric filters. (2) They describe an intelligible basic type of empirical uncertainty, namely the kinds of changes a data user will see over time. (3) They have the very

important property that they can be provided for indirectly seasonally adjusted series, e.g. sums of seasonally adjustment component series. (Univariate time series model-based methods like that of SEATS cannot provide plausible standard errors for indirect adjustments, nor can the univariate methods of Pfeffermann and Bell and Kramer.) Finally, (4) they are technically easy to produce (after the historical data are assembled).

Of course, such descriptive statistics suffer from two fundamental limitations that must be made clear to users: they ignore both the uncertainty due to sampling error (which is small in many high level aggregates) and the uncertainty due to the signal extraction error that remains even when enough future data become available that further revisions are negligible. For these reasons, they cannot be used to establish the statistical significance of, say, an increase in the seasonally adjusted series from one month to the next. (But the value of such significance tests is debatable for several reasons. For example, it has been noted that the null hypothesis of no change is seldom plausible for economic time series. See Smith 1978 for other inadequacies.) Within these limitations, empirical revision statistics may provide useful indications of basic uncertainty for seasonal adjustments where currently none are provided, and, in the case of indirectly adjusted aggregates, where more satisfactory measures have yet to be established.

Some branches of the Census Bureau are undertaking the preparations necessary to enable production of robust descriptive statistics on an ongoing basis for the revisions of some of their published (direct and indirect) seasonally adjustments or seasonally adjusted month-to-month changes. This is being done in a spirit of experimentation to investigate how such statistics could be used by consumers of seasonally adjusted data.

## **7. Some Concluding Remarks.**

Most of this article has been related to model-based seasonal adjustment of one kind or another. This is a relatively young area in which there is much interest because of the seeming simplicity and the analyzability of models and also because some of the progress of model development in other areas of statistics (e.g. Bayesian methods) can be expected to stimulate progress in model-based seasonal adjustment. This is therefore the area of seasonal adjustment in which the most significant progress can be expected to occur in the next years, in the development of better direct or indirect diagnostics for model inadequacies, in the development and use of new models and new model fitting criteria, and also in more effective uses of existing model types. For example, for the problem of data spans too long for modeling with a single model or coefficient vector, the approach of BAYSEA (Akaike, 1980) could be explored. This involves estimating a model of specified type (or sometimes a set of models, see Akaike and Ishiguro, 1983) over a shorter data window which is advanced by steps of length one year through the available data. It can also involve a method of averaging the results from different windows or different models like that illustrated in sec. 6 of Akaike and Ishiguro (1983). The Australian Bureau of Statistics is investigating a somewhat analogous approach to estimating time-varying trading day effects.

Finally, the important issue of staff training for model-based seasonal adjustment must be mentioned. The greater variety of seasonal adjustment filters available from SEATS than from X-12-ARIMA can lead to worse results from an inadequately trained user of the software who makes poor option choices and doesn't recognize this fact. (It is not enough to check that the p-value of a model's Ljung-Box statistic at lag 24 is greater

than 0.05. For example, models that omit significant trading day and outlier effects can have this property.) The X-13-SEATS program is an important development, not only because of its stronger suite of diagnostics but also because the user can always compare a SEATS adjustment with an X-11 adjustment. Conversely, with this software a user of the X-12-ARIMA approach can gradually become familiar with the alternatives made available by model-based adjustment. Thus X-13-SEATS will be an important training tool, but inadequate by itself. We expect that statistical agencies and central banks who wish to make broad use of model-based seasonal adjustments will find it necessary to make a considerable and sustained effort, even more than with X-12-ARIMA, to insure that enough sufficiently trained staff are available to maintain adjustment quality when already trained staff move on.

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### **Appendix: Differences between the automatic modeling procedure of X-13-SEATS and TRAMO.**

The most important automatic regARIMA model selection procedure differences from TRAMO are that X-13-SEATS uses: (1) exact maximum likelihood coefficient estimates where TRAMO uses Hannan-Rissanen estimates or approximate maximum likelihood

estimates; (2) larger critical values for inclusion of outlier regressors (based on theoretical results of Ljung 1991); (3) a theoretical value derived in Findley et al. (1998) rather than an estimated coefficient for the length-of-month/leap-year effect component of multiplicative trading day models; (4) candidate intervals of lengths 1, 8 and 15 for the Easter effect regressor instead of an interval of length. Also, (5) Akaike's AIC is used instead of individual coefficient t-statistics for inclusion of trading day and holiday regressors.

McDonald-Johnson and Hood (2001) investigate the automatic outlier procedure of X-13-SEATS (and X-12-ARIMA) and consider the effects of lower critical values like those used by TRAMO but find them disadvantageous more often than they are advantageous. McDonald-Johnson, Nguyen, Hood and Monsell (2002) study other aspects of the automatic regARIMA modeling procedure and discuss differences from TRAMO's procedure. The results of these and other unpublished studies done at the Census Bureau mildly favor the automatic transformation and trading day and holiday effect decision procedures of X-13-SEATS and show that TRAMO selects differencings and seasonal differencings slightly more often than X-13-SEATS, whereas the X-13-SEATS has a mild tendency to choose ARIMA models with more coefficients. Some improvements have been made recently to the way in which X-12-SEATS decides on differencing operators but a large-scale study to see if the decisions are now the same as TRAMO's more often has not yet been done.

# Spectrum of the SEATS Irregular Component of Export

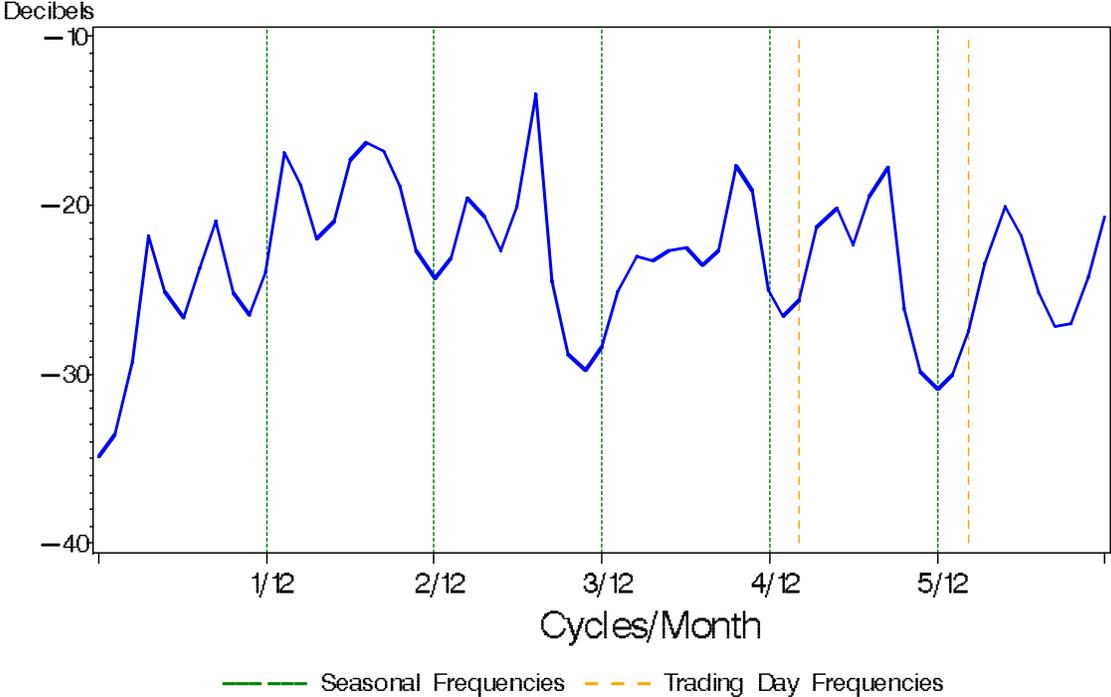


Fig.1. Log spectrum of the irregular component of U.S. Exports of Agricultural Materials (Non-Manufactured). Note the peak at 6 cycles/year suggesting the presence of a residual component with period two months.

Spectrum of Diff'd Log Shipments of Defense Commun. Equip.

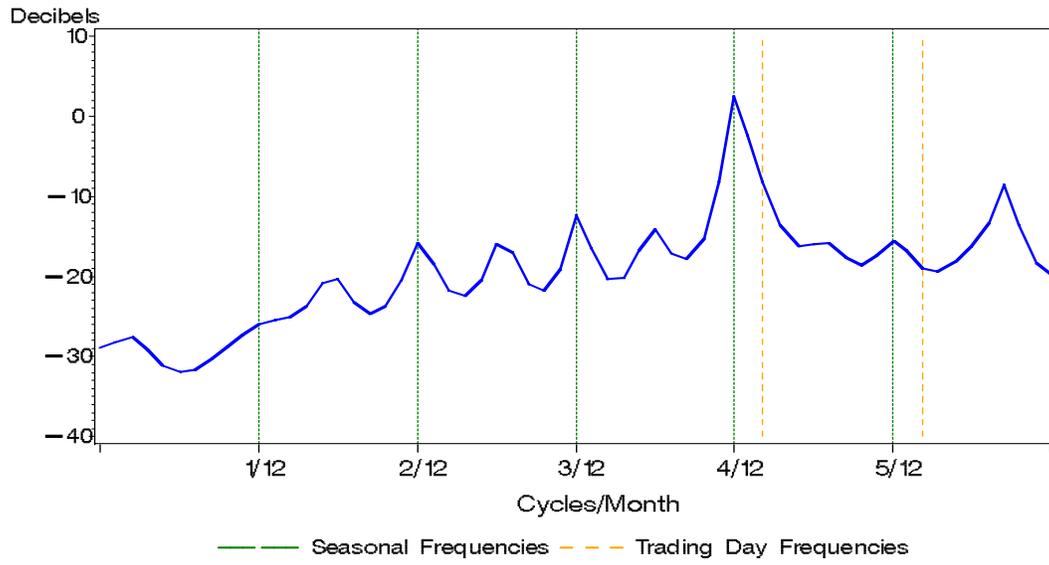


Fig. 2. Log spectrum of (first differences of logs of) Shipments showing a dominant peak at four cycles/year.

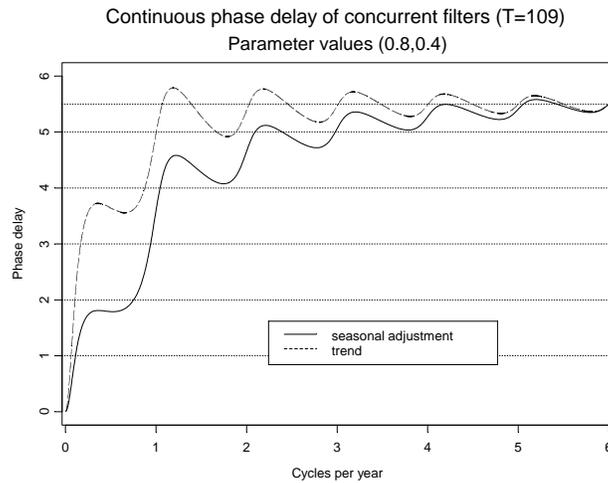
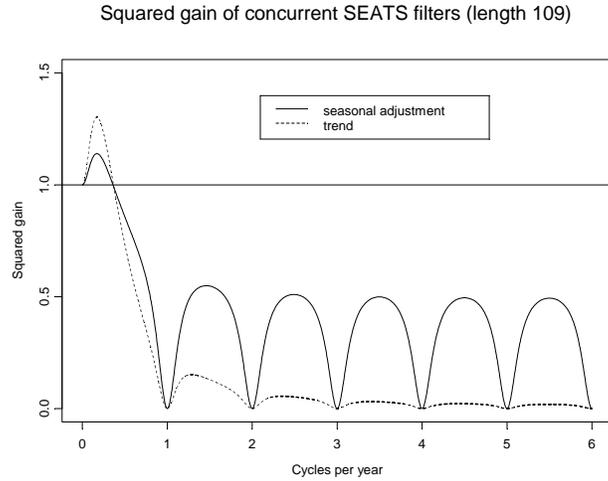


Fig. 3. Squared gains and phase delays of concurrent SEATS seasonal adjustment and trend filters of length 109 for the Airline model with  $\theta = 0.8$  and  $\Theta = 0.4$ . Note that the phase delay of the trend filter is roughly twice that of the seasonal adjustment filter and between two and four months over most of the frequency interval  $[0, 1]$ . Consequently the trend could have large timing distortions for many business cycle components, substantially larger than those of the seasonal adjustment.

### U.S. Retail Sales of Drinking Places, 9/77 - 10/89 Estimated Nonseasonal Components

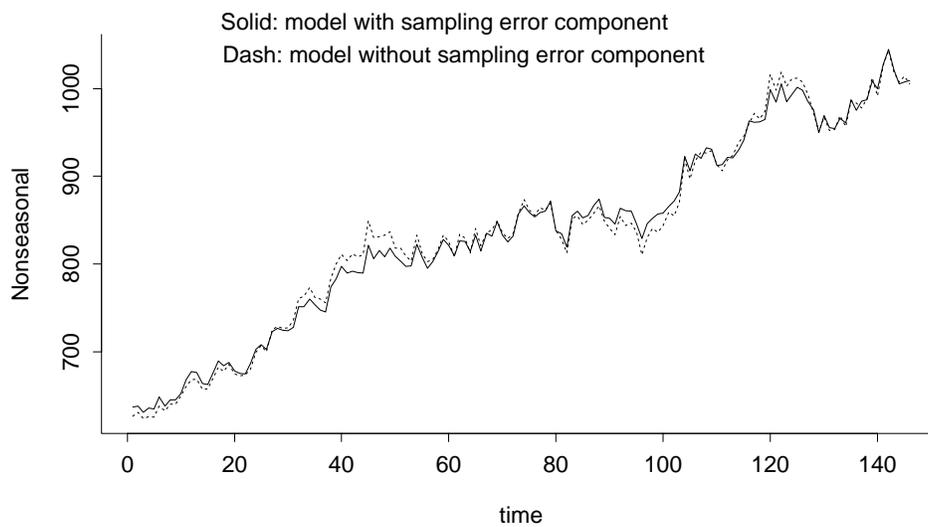


Fig. 4. The graph of the nonseasonal component estimate of the unobserved sampling-error-free series, denoted  $\exp(\hat{N}_t^{(1)})$  in the text, is noticeably less smooth than the graph of nonseasonal component estimate, denoted  $\exp(\hat{N}_t^{(2)})$  in the text, from the observed data model without a sampling error component.

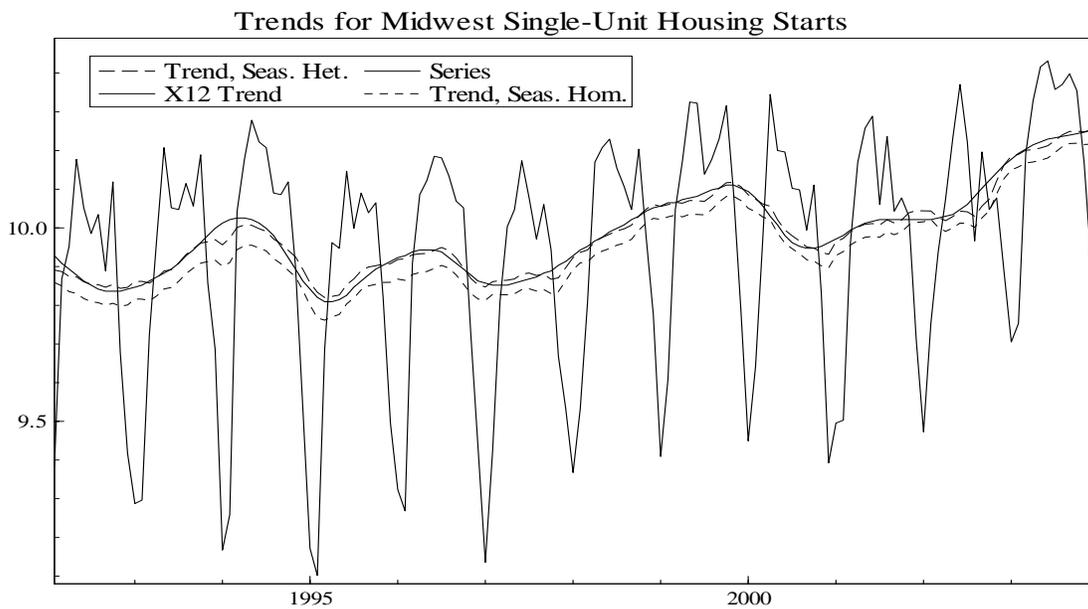
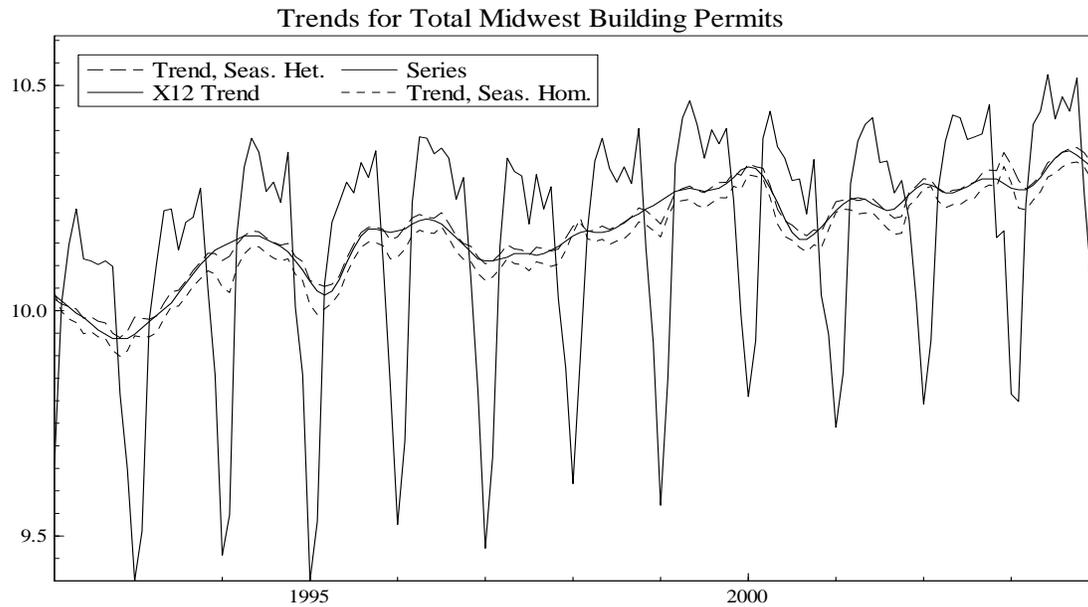


Fig. 5. Logs of the original series and three alternative trends for the U.S. series of Midwest-region Total Building Permits (above) and Single Unit Housing Starts (below). The X-12-ARIMA trends, from seasonal adjustments with longer filters for some winter months, are generally smoother than the trends from Proietti's heteroskedastic model with extra noise sources for these months and also than the trends from the homoskedastic structural model, which have smaller values than the other trends. For Permits, the model-based trends have unappealing sharp peaks or troughs in winter months.

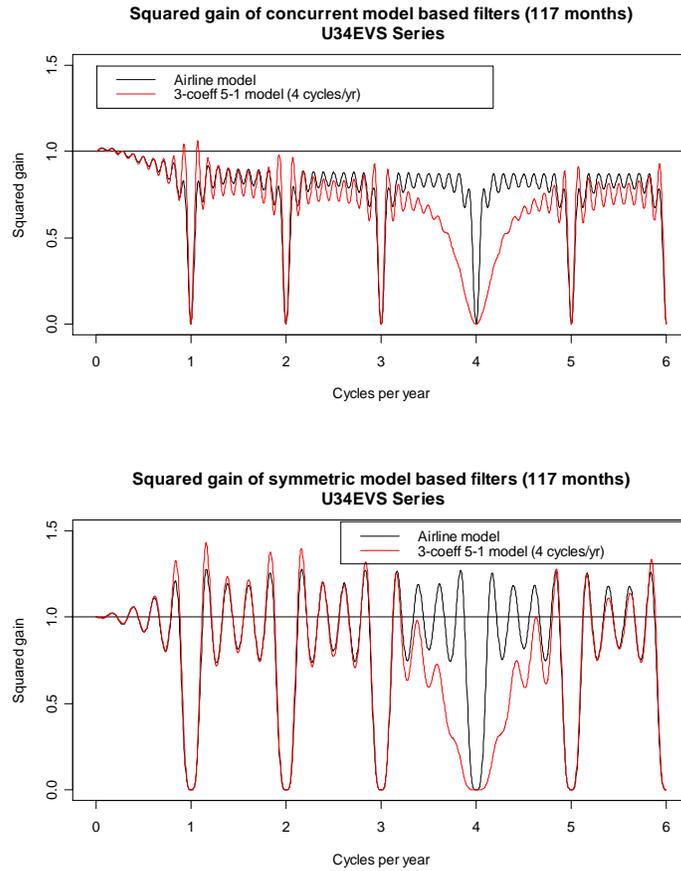


Fig. 6. Squared gains of the concurrent (above) and central (below) model-based filters of length 117 for Shipments from the Airline model and the frequency-specific model for Shipments. Note the latter model's substantially greater suppression of quarterly seasonal components.

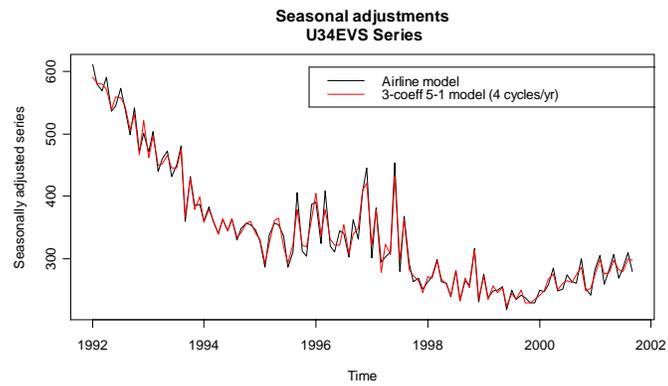


Fig. 7. Airline and 5-1(3) model-based seasonal adjustment of Shipments. The 5-1(3) model has frequency 4 cycles/year associated with the  $c_2$  coefficient and its filters provide a mostly smoother seasonal adjustment due to greater suppression of the strong frequency components close to this frequency, see Fig. 4.