On The User Cost and Home Ownership*

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Abstract

In this paper we study the determinants of housing tenure choice and the differences in the cost of one unit of housing across households in a heterogenous agent model with household specific uninsurable earning shocks and housing prices that may change over time. We build a model economy in which households obtain housing services either through a rental market or home ownership. We calibrate our model to match some stylized facts of the U.S. economy. To analyze the impact of taxation on tenure choice, we model a tax system that mimics that of the U.S. economy in a stylized way. The model is able to replicate the observed patterns in home ownership rates across households in the U.S. and explains them solely on the basis of the differences between the user cost and rental market prices.

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1 Introduction

This paper studies the determinants of housing tenure choice and the differences in the cost of one unit of housing across households in a heterogenous agent model with household specific uninsurable earning shocks and housing prices that may change over time.

Housing services can either be purchased in the rental market or derived from home ownership. For homeowners, a house serves the dual role of a consumption good and an investment. Houses are a very particular type of consumption good because of durability: current residential expenditures yield housing services for many periods. This implies that home ownership may help smooth consumption of housing services when there is rental price risk. House purchases are also a very important investment decision. According to the 1998 *Survey of Consumer Finances*, 32 percent of household wealth takes the form of primary residences. The fraction of wealth held as houses, however, varies significantly. The poorest 80 percent of households hold, on average, 83 percent of their wealth as primary residences, whereas this percent goes down to 21 for the richest 20 percent of the population (See Díaz and Luengo-Prado 2002).

Thus, households buy houses not only because they provide smooth housing services but also because the return on housing capital may be high. As an investment good, houses have some unique characteristics. The heterogeneity of housing results in a variety of search and transactions costs (real estate brokers' fees, legal costs, etc.), that make houses an illiquid asset. Moreover, home purchases can be partially financed after satisfying a downpayment requirement and the accumulated equity may serve as collateral for loans. Also, investment in primary residences have tax advantages that other types of investment do not have: mortgage (and home equity loan) interest payments are deductible from the income tax base and imputed rents on housing are not taxed.

When analyzing the determinants of tenure choice, each of these considerations has to be taken into account. Renters purchase housing services in the market and the cost they face is the rental price of housing. For homeowners, the cost of using and owning one unit of housing in a given period is known in the literature as the *user cost*, defined in a similar way to the user cost of capital from the Neoclassical theory of investment (see, for example Poterba 1984). This cost is part opportunity cost (the forgone after-tax return of housing equity on alternative assets), part out-of-pocket expenses (mortgage interest payments, maintenance costs, local housing taxes, etc.) and part value variation (depreciation and capital losses associated to house price fluctuations). When the user cost of owner occupied housing is lower than the rental price of housing services, households would prefer to purchase houses instead of renting and wealth (liquidity) constraints are likely to be the main deterrence from home ownership. It has been argued that this is the case in the U.S. (see for example Rosen 1979).

In this paper we analyze the determinants of household tenure choice in a partial equilibrium model economy populated by infinitely lived agents in which household earnings are subject to uninsurable idiosyncratic risk. We assume that house prices follow an exogenously given stochastic process independently distributed from the household specific earning shock. We also assume that the housing rental price tracks the changes in house prices. In our economy households can save in the form of deposits, risk free assets, and houses, which are subject to price risk. Individuals must satisfy a downpayment when purchasing houses but they can also use accumulated housing equity as collateral for other loans. For simplicity, no other form of credit is allowed. In any period, the value of a household's outstanding debt cannot be greater than a certain fraction of the value of the house owned. Every period households can either purchase housing services in the rental market at the given rental price or obtain housing services through home ownership. We allow for non-perfect substitutability between rental and owner-occupied housing services.[1] Moreover, house changes are subject to transaction costs. To analyze the impact of taxation on tenure choice, we model a tax system that mimics in a stylized way that of the U.S. economy. Owner occupied housing services are not part of the income tax base and mortgage interest payments (as well as home equity loans) are deductible. We impose no restriction either in the size of rental services nor the size of houses. Differences in the size of the services consumed arise endogenously as a function of the a household's user cost, which is largely affected by tax treatment. The preferential tax treatment on housing has been analyzed elsewhere (see for example Poterba 1984, **?**, **?**, Gervais 2002). To our knowledge, this is the first paper to have an endogenous tenure choice without imposing any restriction on the size of the services that can be rented in the market nor the size of the houses that can be purchased, in a dynamic framework.[2]

We find that the tax structure is key to understanding the household's portfolio choice. When the return on deposits and the mortgage interest rate are equal and mortgage interest payments are fully deductible from the tax base, households that are not liquidity constrained are indifferent about the volume of deposits and mortgages that they hold and they only care about their net position in both assets. Liquidity constrained households do not hold deposits and only have debt. In contrast, when the return to deposits is lower than the mortgage interest rate or mortgage interest payments are not fully deductible, households do not want to simultaneously hold deposits and debt.

Next, we derive the user cost as the marginal rate of substitution between housing services and nondurable consumption. We show that the user cost increases with the after-tax interest rate paid on financial assets (the opportunity cost of housing equity), the mortgage interest rate, housing depreciation, maintenance costs and local housing taxes. It decreases with tax subsidies to home ownership and expected house price appreciation. If the rental price is higher than some threshold level only households that cannot satisfy the down payment decide to rent. In contrast, if the

[1] It can be argued that one can more easily adjust owned property to one's taste.

[2] Of the studies mentioned before, only **?** and Gervais (2002) have an endogenous tenure choice. While the first study uses a static framework, Gervais (2002) imposes a maximum size on the housing services that can be purchased in the market. Households that desire to consume a higher amount are forced to buy a house.

rental price is sufficiently low, all households prefer renting to purchasing a house. As a result, the home ownership rate depends on the difference between the user cost of owner occupied housing and the rental price of housing services. Thus, our model may be used to understand the observed differences in home ownership rates across countries.[3]

We then calibrate our model economy so that the home ownership rate in the model is roughly 67 percent, as in the U.S. economy, and report the results of several experiments. We find that in the absence of uncertainty, the initial distribution of wealth and the discount rate determines the households' tenure choice. The intuition is the following. If there is no uncertainty, the investment motive for holding houses disappears. If the user cost is lower than the rental price, households may consider buying a home for consumption motives. Purchasing a house involves accumulating a down payment. If a household has the wealth required to satisfy the downpayment requirement, it purchases the house and benefits from the lower user cost afterwards. If not, the household is not willing to save for the down payment, giving up valuable consumption today, and rents. If the discount rate is sufficiently low, households do save for the downpayment and purchase houses. With uncertainty, the model predicts that the value of the user cost for liquidity constrained agents is lower the higher the level of household wealth. Thus, wealthy households are homeowners and renters are concentrated among the less wealthy households. Finally, we try to asses by how much the user cost of housing is below market rents, an important point to gain understanding on how the cost of owned housing compares to rental prices.

The remainder of the paper is organized as follows. Section 2 introduces our dynamic model. Sections 3 and 4 present some theoretical results on household composition and portfolio choice. The calibration and the simulation results are presented in section 5.

2 The Model Economy

We consider an economy populated by a continuum of households of measure one that live forever in a partial equilibrium model with housing capital and collateralized liquidity constraints.

2.1 Preferences and endowments

Households derive utility from the consumption of a nondurable good, c, and from housing services, s. We write the per period utility as $u(c,s)$, and lifetime utility as $\sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$, where β is the time discount factor. Households are endowed with e_t efficiency units of labor at period t. These units of labor are drawn from the set $e \in E = \{e_1, ..., e_{n_e}\}$.

[3]For instance, in 1999 the home ownership rate in the US was 67% versus 43% in Germany, 68% in the UK or 83% in Spain. Data from the U.S. Census and "Housing Statistics in the European Union 2002", Department of Housing of the Direction General of Planning, Housing and Heritage of the Walloon Region of Belgium.

2.2 Market arrangements

Households hold residential assets $h_t \geq 0$, deposits, $d_t \geq 0$, and mortgage debt and home equity loans, $m_t \geq 0$. Deposits earn a return r_t^d, whereas debt carries an interest payment at the rate r_t^m. We assume that households buy the stock of houses that will render services at period $t+1$ at the end of period t. We denote by q_t the price of one unit of residential stock at period t in terms of nondurable consumption.

We assume that whenever households buy a house they must satisfy a minimum down payment requirement equal to θ. On the other hand, houses serve as collateral for loans (through home equity loans or refinancing) up to a loan to value ratio $(1-\theta)$. For simplicity, we assume that only collateralized credit is available. This means that at any given period household debt must satisfy:[4]

$$m_{t+1} \leq (1-\theta)\, q_t\, h_{t+1}. \tag{1}$$

Therefore, there will be a link between the value of a household's outstanding debt and the market value of its property. This constraint implies that household's net worth is always non-negative, greater than or equal to $\theta\, q_t\, h_t$. It also means that whenever there is a fall in house prices and households do not change their house, they are forced to decrease their collateral debt to equal the fraction $(1-\theta)$ of their residence value. Thus, households take all the capital gains and losses associated to changes in house prices.[5]

We assume that selling a house is costly. A fraction of its value is lost when sold, which may reflect brokerage fees. This makes houses a less liquid asset than deposits. Owning a house entails paying a maintenance cost equal to the fraction δ^h of the value of the house.[6] Thus, we assume that whenever there is change in the stock (not its value) the household is selling its house and pays an adjustment cost, ϱ, proportional to the value of the stock.

Owning a house of size h delivers a flow of housing services equal to h. As an alternative to purchasing a house, households can rent housing services from a financial institution. The price of housing services is denoted by r^h. We assume that services of owner occupied houses and rental services may not be perfect substitutes.

[4]We abstract from income requirements when purchasing houses. Many lenders follow the rule of thumb of "3 times income" for mortgages. However, we do not believe this assumption is restrictive as the empirical literature finds that wealth constraints are more important than income constraints. See for example Linneman, Megbolugbe, Watcher, and Cho (1997) or Quercia, McCarthy, and Watcher (2000).

[5]In reality, the burden of downward property prices is shared by financial institutions and households. We adopt this approach for computational reasons and leave the more complicated case for future research.

[6]We assume for simplicity that if maintenance is done, the house does not depreciate.

2.3 The government

We assume that the government taxes income and allows tax deductions for the interest payments on mortgages and home equity loans. It also imposes a local tax on housing. However, imputed housing rents are tax-free. The entire proceeds from taxation are used to finance government expenditures that do not affect individuals at the margin.

2.4 The structure of uncertainty

Households face idiosyncratic risk in their endowments of efficiency labor units and aggregate risk in house prices. The earnings process is Markov with transition matrix $\pi_{e,e'}$. The house price shocks are also Markov with transition matrix $\pi_{q,q'}$. We assume that uncertainty in house prices does not affect the evolution of household's earnings. Moreover, households know the evolution of interest rates and the market wage. The rental price of housing may vary over time.[7]

2.5 The household's problem

The problem that a household solves is:

$$
\begin{aligned}
& E_0 \max_{c_t, s_t, d_{t+1}, m_{t+1}, h_{t+1}} && \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + \gamma u\left(\phi\, s_t + h_t\right) \right] \\
& \text{s. t.} && c_t + r_t^h s_t + d_{t+1} - m_{t+1} + q_t\, h_{t+1} + L_t\, q_t \varrho \left(1 - \delta^h\right) h_t \leq \\
& && w_t e_t + \left(1 + r_t^d\right) d_t - \left(1 + r_t^m\right) m_t + q_t\, \left(1 - \delta^h - \tau_h\right) h_t - \tau_{y_t^\tau}\left(y_t^\tau\right), \\
& && 0 \leq m_{t+1} \leq q_t\, \left(1 - \theta\right) h_{t+1}, \\
& && c_t \geq 0,\; s_t \geq 0,\; d_{t+1} \geq 0,\; m_{t+1} \geq 0,\; h_{t+1} \geq 0, \\
& && y_t^\tau = w_t e_t + r_t^d d_t - \tau_m r_t^m\, m_t.
\end{aligned}
\tag{2}
$$

The variable L_t indicates that the adjustment cost is paid only when the household changes its housing stock:

[7] For our theoretical results we assume that rent is known by households. We relax this assumption in the simulations.

$$L_t = \begin{cases} 1 & \text{if } h_{t+1} \neq h_t \text{, provided } h_t > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Note the timing of the model: consumption of the nondurable good takes place after depreciation of the stocks, whereas services of owned houses are obtained before the stock depreciates. We have denoted taxable income as y^τ. The income tax function $\tau_{y^\tau}(.)$ can vary with the level of income. Notice that mortgage interest payments are deductible from the tax base at the rate $\tau_m \in [0, 1]$. The amount of housing services consumed satisfies $\phi s_t + h_t$, where $\phi \leq 1$. This means that rental housing services and services of owner occupied housing may not perfect substitutes. In particular $\phi < 1$ could be justified on the grounds that it is easier for households to adjust the owned property to their particular taste.[8]

3 The composition of a household's portfolio

In this section we analyze the conditions under which households maintain deposits and debt simultaneously under different environment conditions.

3.1 No spread and full deductability

Proposition 1. *Assume that there is no spread between the return on deposits and the mortgage rate, $r_t^d = r_t^m$, and that mortgage interest payments are fully deductible, $\tau_m = 1$. Then, there are two types of households. Those who are liquidity constrained hold only debt. Those who are not, may hold both deposits and debt as they are only concerned with their net position $d_t - m_t$.*

Proof: See Appendix A.

This proposition says that with full deductability and no spread, households that are not constrained are indifferent between equity and debt financing of their houses. As a matter of fact, they could always refinance their mortgage.

3.2 Spread or partial deductability

Proposition 2. *If there is a spread between the return on deposits and the mortgage rate ($r_t^m > r_t^d$), or mortgage interest payments are not fully deductible ($\tau_m < 1$), then households do not hold simultaneously deposits and debt.*

[8] Given the preferential tax treatment, households will choose either owning or renting and not both simultaneously.

Proof: See Appendix A.

This proposition says that with less than full deductability or interest spread households always prefer equity to debt financing of their houses. In other words, there is a complete segmentation of households: those who hold debt do not hold deposits and vice versa. Obviously, we may see households that do not hold either deposits nor mortgages.

4 The determinants of the user cost

In this section we analyze the determinants of the user cost and tenure choice decisions. We derive the *marginal* user cost as the marginal rate of substitution between housing services and nondurable consumption. First, we consider the simple case with no uncertainty or adjustment costs. We then consider the case with uncertainty.

4.1 The relationship between user cost, house prices, and rental prices

In this section we abstract from uncertainty in prices, earnings, and adjustment costs to provide a simple definition of the user cost.

Let us define cash-on-hand as after-tax total resources:

$$x_t = w_t e_t + \left(1 + r_t^d\right) d_t - \left(1 + r_t^m\right) m_t + q_t \left(1 - \delta^h - \tau_h\right) h_t - \tau_{y_t^\tau}\left(y_t^\tau\right). \tag{3}$$

There are two possible regions delimited by the price:

$$\overline{r}_{t+1} = \phi \left[\left(1 + \widehat{r}_t^d\right) q_t - q_{t+1}\left(1 - \delta^h - \tau_h\right)\right], \tag{4}$$

where $\widehat{r}_t^d$ denotes the after-tax return on deposits, $\left[1 - \tau'_{y^\tau}\left(y_t^\tau\right)\right] r_t^d$. Let us consider two possible scenarios.

No spread and full deductability

Proposition 3. *If the rental price at period $t+1$ satisfies $r_{t+1}^h > \overline{r}_{t+1}$, then all households that are not constrained prefer buying to renting. For the constrained households, there exists a level of cash-on-hand above which buying is preferred to renting and* vice versa. *If $r_{t+1}^h \leq \overline{r}_{t+1}$ all households prefer renting.*

Proof.

In this proof we use the first order conditions for the household problem presented in Appendix A.

Consider a household that decides to hold a house next period and is not liquidity constrained. Expression (17) can be written as

$$q_t = \beta \frac{u'(c_{t+1})}{u'(c_t))} \left[\gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})} + q_{t+1} \left(1 - \delta^h - \tau_h\right) \right], \tag{5}$$

Using expressions (14) and (15) we can write an expression for the *user cost*,

$$\gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})} = \left(1 + \widehat{r}_t^d\right) q_t - q_{t+1} \left(1 - \delta^h - \tau_h\right), \tag{6}$$

which tells us that the household's user cost is a function of the variations in house prices, maintenance costs and taxes. For the household to be indifferent between purchasing and renting, this user cost times ϕ should be equal to the rental price of housing. However, since the rental prices satisfies $r^h_{t+1} > \overline{r}_{t+1}$, the user cost of a non liquidity constrained household is lower than the rental price. Hence, the household strictly prefers purchasing a house to renting its services in the market and sets $s_{t+1} = 0$.

Now, let us think of a household that is liquidity constrained. This, by Lemma 2, implies that the household holds no deposits. Combining (16) and (17) we obtain:

$$q_t = (1-\theta)\, q_t + \frac{\beta\, u'(c_{t+1})}{u'(c_t)} \left[\gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})} + q_{t+1} \left(1 - \delta^h - \tau_h\right) - (1-\theta) q_t \left(1 + \left(1 - \tau_m \tau'_{y^\tau}(y_t^\tau)\right) r_t^m\right) \right]. \tag{7}$$

Let $\widehat{r}_t^m = \left(1 - \tau_m \tau'_{y^\tau}(y_t^\tau)\right) r_t^m$ be the after tax mortgage rate. Note that user cost can be written as:

$$\gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})} = \left[\frac{\theta}{\frac{\beta\, u'(c_{t+1})}{u'(c_t)}} q_t + (1-\theta)(1 + \widehat{r}_t^m)\, q_t - q_{t+1} \left(1 - \delta^h - \tau_h\right) \right]. \tag{8}$$

Since the liquidity constraint is binding we have that:

$$\frac{\theta}{\frac{\beta\, u'(c_{t+1})}{u'(c_t)}} + (1-\theta)\,(1 + \widehat{r}_t^m) > 1 + \widehat{r}_t^d.$$

The lower the level of cash-on-hand the lower the intertemporal marginal rate of substitution, $\frac{\beta\, u'(c_{t+1})}{u'(c_t)}$, and higher the user cost. Thus, there exists a level of cash-on-hand, x_t^*, below which the user cost is higher than the rental price. In such a case, the household prefers renting to buying. Therefore, only liquidity constrained households rent houses and do not purchase them.

When $r^h_{t+1} \geq \overline{r}_{t+1}$ it is easy to check all households prefer renting to purchasing a house. $\square$

Spread or partial deductability

When there is no spread and there is full deductability, there are only two types of households: those that are not liquidity constrained and hold deposits and those that are liquidity constrained and do not hold deposits. When there is spread or there is partial deductability, households want to repay their debts as fast as possible. This implies that there will be households that hold no deposits, have debt but are not liquidity constrained.

Proposition 4. *If the rental price at period $t+1$ satisfies $r^h_{t+1} > \overline{r}_{t+1}$ then all households that hold deposits prefer buying to renting. Those that hold debt but are not liquidity constrained prefer buying to renting if the after tax interest of mortgages is not too big. For those that are liquidity constrained, there exists a level of cash-on-hand above which they prefer buying.*

Proof.

Case 1. Consider first the households that hold deposits. These households, by Proposition 2, are not liquidity constrained. If the rental price satisfies $r^h_{t+1} > \overline{r}_{t+1}$ they strictly prefer buying to renting by Proposition 3. The user cost for this type of individuals is given in expression (6).

Case 2. Consider first a non liquidity constrained household that holds debt and assume that it optimally chooses to rent housing services. Expression (17) can be written as

$$q_t = \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})}\right] + \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} q_{t+1} \left(1 - \delta^h - \tau_h\right)\right]. \tag{9}$$

Using expressions (14) and (15) we can write the expression above as:

$$q_t = \frac{1}{1+\widehat{r}^m_t} \left[\frac{1}{\phi} r^h_{t+1} + q_{t+1}\left(1 - \delta^h - \tau_h\right)\right], \tag{10}$$

The rental price satisfies $r^h_{t+1} > \overline{r}_{t+1}$. Thus, if the interest rate on mortgages is much larger than the return on deposits, this expression may hold with strict inequality, being the left hand side, q_t greater than the right hand side. That is, the cost of buying would be higher than its returns and, therefore, the household is better off renting. If the interest on mortgages is not too high, then these households would be better off buying. Thus, their user cost depends on the spread and the tax deduction and is given by the expression:

$$\gamma \frac{u'(\phi\, s_{t+1} + h_{t+1})}{u'(c_{t+1})} = \left(1+\widehat{r}^m_t\right) q_t - q_{t+1}\left(1 - \delta^h - \tau_h\right). \tag{11}$$

Case 3. Consider a household that is liquidity constrained. By Proposition 3 there exists a level of cash-on-hand, x_t^*, below which the user cost is higher than the rental price. In such a case, the household prefers renting to buying. For those who buy, its user cost is given by the expression (7). □

Calculating the average user cost the traditional way

So far we have derived the *marginal* user as the marginal rate of substitution between owned housing services and nondurable consumption. Its value depends on several parameters and varies across households because of the liquidity constraint. Let us now derive the *average* user cost as it is done in much of the literature. Suppose an individual purchases h_{t+1} units of housing at the end of period t. The value of the purchase would be $q_t h_{t+1}$. Let the household finance the purchase of the house. In our notation, m_{t+1} is the mortgage obtained while $q_t h_{t+1} - m_{t+1}$ is the equity purchased by the downpayment. There is an opportunity cost to holding equity in terms of the forgone return in financial assets, which after tax is $\widehat{r}^d$. On the other hand, the household carries debt, m_{t+1}, which involves an interest payment at the rate r^m. In period $t+1$, the household must also pay maintenance costs, local housing and adjustment costs if moving. Moreover, the house may appreciate or depreciate depending on price dynamics.

Combining all these elements we can write the (per unit) user cost as:

$$uc_{t+1} = (1+\widehat{r}_t^d)q_t\left(1 - \frac{m_{t+1}}{q_t h_{t+1}}\right) + (1+\widehat{r}_t^m)q_t\frac{m_{t+1}}{q_t h_{t+1}} - q_{t+1}[1 - \delta^h - \tau^h - L_{t+1}\varrho(1-\delta^h)].$$

or

$$uc_{t+1} = (1+\widehat{r}_t^d)(1-\mathcal{M})q_t + (1+\widehat{r}_t^m)\mathcal{M}q_t - q_{t+1}(1 - \delta^h - \tau^h - L_{t+1}\varrho(1-\delta^h)] \tag{12}$$

where $\mathcal{M}$ is the loan to value ratio at t prices, $m_{t+1}/(q_t h_{t+1})$. Equation (12) is simply a weighted average of equations (6) and (11), once adjustment costs are included. Given $\mathcal{M}$, the user cost is higher, the higher housing prices when the house was purchased, q_t, the higher the after-tax return on deposits $\widehat{r}^d$, the higher the after-tax mortgage rate, $\widehat{r}^m$, and the higher the maintenance costs, local taxes and moving costs (if moving). The user cost is lower, the higher the house prices in period $t+1$. Equation (12) applies only to non-liquidity constraint agents, though. We must look at equation (8) for those who are liquidity constrained. Given the rental price, the price in this equation is the one that determines those households that buy or rent at the margin.[9]

If the household is not moving, with full deductability, $\tau_m = 1$ and no interest spread, $r_t^m = r_t^d$, the expression simplifies even further to:

[9]Note that the lower β or the higher θ, the higher the proportion of people who would be constrained.

$$uc_{t+1} = (1 + \widehat{r}_t^d)q_t - q_{t+1}[1 - \delta^h - \tau^h],$$

as in equation (6). In this case, the average user cost for a non-liquidity constrained agent does not depend on the loan to value ratio, $\mathcal{M}$.

4.2 User cost, earnings uncertainty and adjustment costs

Here we extend the previous analysis to the case in which there is uncertainty in earnings, house prices and adjustment cost in houses. As in the previous case, there are two possible regions delimited by the price $\overline{r}_{t+1}$, for all t, shown in expression (4).

No spread and full deductability

Proposition 5. *If the rental price at period $t+1$ satisfies $r_{t+1}^h > \overline{r}_{t+1}$ then all households which are not liquidity constrained prefer buying to renting. For those that are liquidity constrained there exists a level of cash-on-hand above which they prefer buying.*

Proof: See Appendix B.

Proposition 6. *If the rental price at period $t+1$ satisfies $r_{t+1}^h \leq \overline{r}_{t+1}$ all households prefer renting to owning a house.*

Proof: See Appendix B.

Spread or partial deductability

When there is a spread or there is partial deductability, households want to repay their debts as fast as they can. This implies that there will be households that hold no deposits, have debt but are not liquidity constrained.

Proposition 7. *If the rental price at period $t+1$ satisfies $r_{t+1}^h > \overline{r}_{t+1}$ then all households that hold deposits prefer buying to renting. Those that hold debt but are not liquidity constrained prefer buying to renting if the after tax interest of mortgages is not too large. For those that are liquidity constrained, there exists a level of cash-on-hand above which they prefer buying.*

Proof: See Appendix B.

Table 1: THE EARNINGS PROCESS

$e \in \{e_1, e_2, e_3\} =$	$\{1.00, 5.29, 46.55\}$		
$\pi_{e,e'} =$	0.96500	0.00347	0.000333
	0.03937	0.95000	0.010625
	0.00000	0.08300	0.917000
Stationary distribution			
$\pi_e^* =$	0.4983	0.4429	0.05870

5 Taxes, down payments and patterns of home ownership

In order to illustrate our previous results, we simulate a partial equilibrium model and vary some key parameters to evaluate their effects on home ownership rates. We first describe the calibration and then report and discuss the results.

5.1 Calibration

Earnings

With respect to the process for earnings, we choose an AR(1) in the logarithm of labor income that we approximate using the Markov discrete chain describe in Table 1. This process generates a Gini coefficient for earnings of 0.60- roughly the Gini coefficient for the U.S. economy- and has been constructed to match the Lorenz curves of the U.S distributions for earnings and total wealth.[10]

Preferences and Technology

For preferences over consumption of the nondurable goods and housing services, we choose $\left(\frac{c^{1-\sigma}}{1-\sigma} + \gamma\frac{(\phi s+h)^{1-\sigma}}{1-\sigma}\right)$ similar to Farr and Luengo-Prado (2001). This specification implies that services from owner occupied housing are proportional to the housing stock on a one-to-one basis. Households obtain higher services from owned houses than from rented ones. We set ϕ so that the ownership rate in our model matches that of the U.S. economy, 67 percent.

In our benchmark case, $r_t^m = r_t^d = 0.0463$, for all t and $w = 0.887$.[11] We choose $\sigma = 3$ and calibrate γ to match the ratio of nondurable goods to investment in housing C/I_H. We set the depreciation rate $\delta^h = 0.0426$.[12]

[10] See Castañeda, Díaz-Giménez, and Ríos-Rull (2001), and Díaz, Pijoan-Mas, and Ríos-Rull (2003) for a discussion on this calibration choice.

[11] These numbers are from Díaz and Luengo-Prado (2002) who calibrated a general equilibrium version of this model.

[12] The number is obtained using data from the Bureau of Economic Analysis: *National Income an Product Accounts* for flows and *Fixed Reproducible Tangible Wealth in the United States* for stocks.

Housing Prices

We assume that housing prices follow the AR(1) process described in Table 2. We model price changes as an aggregate shock (i.e. it affects all households simultaneously), independently distributed from the household-specific earnings shock.[13]

Table 2: THE PROCESS FOR HOUSE PRICES

$e \in \{q_1, q_2, q_3\} =$	$\{0.9, 1.0, 1.1\}$
$\pi_{q,q'} =$	$\begin{bmatrix} 0.650 & 0.350 & 0.000 \\ 0.025 & 0.950 & 0.025 \\ 0.000 & 0.350 & 0.650 \end{bmatrix}$

Housing Rental Prices

We assume that rents track housing prices. In particular,

$$r_t^h = q_t(r^d + \delta^h + \tau^h).$$

This condition could be seen as a competitive equilibrium condition for landlords if maintenance costs and local housing taxes were deductible for a landlord. Interest forgone in equity plus maintenance and tax costs should equal rent.

Taxation

Our benchmark is a proportional tax system:

$$\tau_{y_t^\tau}(y_t^\tau) = \tau_y y_t^\tau.$$

Recall that taxable income is defined as $y_t^\tau = w_t e_t + r_t^d d_t - \tau_m r_t^m m_t$. We set $\tau_y = 0.225$ and the mortgage tax deduction equal to $\tau_m = 1$, which delivers a government spending to output ratio of roughly 20%.

We also try a non-distortionary progressive taxation scheme with a flat rate and a minimum exempt, $y_{\min}$:

$$\tau_{y_t^\tau}(y_t^\tau) = \tau_y(y_t^\tau - y_{\min}).$$

We set $\tau_y = 0.35$ and $y_{\min} = 1.9$ to obtain the same government spending to output ratio.

[13]The assumed process is fairly “ad-hoc” and we plan to recalibrate it to match the appropriate data.

Table 3: Benchmark Parameters

Utility			Housing				Interest and Wages			Taxes		
β	σ	γ	δ^h	θ	ϱ	ϕ	r^d	r^m	w	τ_y	τ_m	τ_h
0.8	3	0.3	0.043	0.3	0.05	0.75	0.0463	0.0463	0.8878	0.225	1	0

Both schemes can be accommodated easily in the model and can shed light on how mortgage interest deductions affect household decisions under different degrees of progressiveness of the tax system. Table 3 summarizes the key parameters.

5.2 Simulations Results

In order to illustrate our previous results, we solve the household problem to obtain optimal policy functions for both assets, h and d, and simulate an economy of $9,000$ heterogenous households for different variations of the key parameters.[14] We first discuss the model with no uncertainty. Then, we present some descriptive statistics about the benchmark case. Lastly, we perform different experiments to gain some intuition on the key determinants of home ownership.

5.2.1 No uncertainty

We first solve the model with no uncertainty to gain some intuition on the consumption aspect of housing. Agents know their income profile (they are either poor, middle class or rich forever) and house prices stay constant at their median value, $q = 1$. We use the parameters from the benchmark case (no spread, full deductability). The user cost for an agent who is not liquidity constrained given in equation (6) is well below the rental price.

Interestingly, we find that what agents do depends crucially on initial conditions and the discount factor. For $\beta = 0.8$, our benchmark value, agents do not purchase houses unless they are given enough initial wealth to satisfy the downpayment requirement. In the absence of uncertainty, there is no need for precautionary savings since a certain stream of income is guaranteed. Therefore, agents do not want to put down the money and enjoy the lower user cost, given the high discount rate, and the sacrifice in current consumption that this entails. Moreover, when they do buy, agents keep on refinancing their mortgages period after period, always keeping a loan to value ratio equal to $(1 - \theta)$. We also simulated the model for a lower discount rate, $\beta = 0.9$. In this case, agents purchase houses even when they start with zero assets and also refinance all periods.

In summary, with no uncertainty, the relevant user cost is given by equation (8), which depends on β. If households are not wealth constrained and do no discount the future greatly, they purchase homes and enjoy the lower user cost of housing but they do not accumulate more equity on the

[14]Appendix C describes the solution technique.

Table 4: WEALTH DISTRIBUTION AND WEALTH COMPOSITION

	Quintiles (Share (%))					
	1st	2nd	3rd	4th	5th	Gini
EARNINGS	3.53	3.53	11.62	18.7	62.62	0.602
U.S. Data	*-0.16*	*4.01*	*12.98*	*22.93*	*60.24*	*0.609*
TOTAL WEALTH	0.00	0.69	5.96	9.99	83.36	0.783
U.S. Data 1	*-0.29*	*1.35*	*5.14*	*12.37*	*81.40*	*0.797*
U.S. Data 2	*0.23*	*1.87*	*5.62*	*12.55*	*79.71*	*0.784*
HOUSES	0.00	2.01	13.16	18.99	65.84	0.665
U.S. Data 1	*1.51*	*5.83*	*15.73*	*24.35*	*52.57*	*0.635*
U.S. Data 2	*0.79*	*6.00*	*16.05*	*24.62*	*52.52*	*0.634*
DEPOSITS	0.00	0.13	2.86	6.12	90.90	0.847
U.S. Data 1	*-0.41*	*1.06*	*3.30*	*9.04*	*86.97*	*0.855*
U.S. Data 1	*0.30*	*1.63*	*3.97*	*9.39*	*84.67*	*0.837*
HOUSING SERVICES	41.89	35.64	0.00	0.00	22.47	0.775
NONDURABLE	3.19	4.88	12.18	17.81	61.94	0.594
WEALTH COMPOSITION	–	97.93	71.48	57.19	28.66	
U.S. Data 1	–	*140.31*	*99.57*	*63.96*	*21.00*	
U.S. Data 2	–	*99.57*	*88.60*	*60.87*	*20.45*	

All variables ordered by total wealth but earnings.
U.S. Data from the *1998 Survey of Consumer Finances.* Deposits includes all financial assets. *U.S. Data 1*, raw data. For *U.S. Data 2* only non-collateral (mortgage and home equity) debt is considered. Household wealth is household net worth plus all debt minus mortgage and home equity loans.

house than is required. Otherwise, they rent. Let us now introduce uncertainty in the model.

5.2.2 The benchmark case

Our benchmark calibration produces an ownership ratio of 66.5%.[15] Table 4 summarizes the distribution of household wealth, deposits, houses, nondurable, housing services and wealth composition (houses over total wealth) by wealth quintiles, as well as Gini coefficients for our simulations. We present numbers for our simulations and counterparts from the 1998 *Survey of Consumer Finances.*

The Gini coefficient for our calibrated earnings process is 0.6. Note that deposits (financial assets) and total wealth are more concentrated than earnings, as in the data.[16] In our simulations, renters are mainly "wealth poor". Some of the newly rich do rent too, while they accumulate a down payment.[17] The simulated data replicate satisfactorily the patterns of wealth composition in the U.S.[18]

[15]This is an average calculated over 100 periods for 100 different independent samples of 9,000 households.
[16]These are the numbers for period 50 in one simulation with 9,000 households.
[17]These are very few – 0.7% of the population– and all had the highest earnings shock. They do not want to buy a house smaller than their optimal size and move later on to avoid the adjustment cost.
[18]Readers should take into account that our calibration is still preliminary.

Table 5: HOME OWNERSHIP UNDER DIFFERENT SCENARIOS

	Home Ownership (%)	s.d.
Benchmark	66.5	0.00359
No adjustment costs ($\varrho = 0$)	69.7	0.00442
Higher adjustment costs ($\varrho = 0.1$)	62.8	0.00719
No uncertainty in housing prices	64.4	0.00215
i.i.d. earnings shocks	64.2	0.00962
Spread ($r^m - r^d = 0.01$)	66.5	0.00378
No deductability ($\tau_m = 0$)	66.4	0.00381
Lower interest	65.8	0.00578
Lower downpayment ($\theta = 0.05$)	95.8	0.00944
Better rental ($\phi = 1$)	28.7	0.02886
Progressive taxation ($\tau_y = 0.35$, $y_{\min} = 1.9$)	82.5	0.03200
Progressive taxation and spread	59.1	0.00388
Progressive taxation and no deductability	58.0	0.00371
Lower discount rate ($\beta = 0.9$)	98.9	0.00347
Lower discount rate and better rental ($\beta = 0.9$, $\phi = 1$)	68.3	0.02131

Averages over 100 periods for 100 different independent samples of 9,000 households. s.d. is the standard deviation over the samples.
i.i.d shocks: the probabilities of the shocks are 0.4983, 0.4429 and 0.05870 respectively.

5.2.3 Changes in the parameters and home ownership

We next change some of the parameters of our benchmark calibration and report the associated changes in home ownership rates. Table 5 summarizes the results. We change one parameter at a time unless otherwise indicated. Remember that when deciding tenure choice, a household compares his corresponding user cost with the rental price divided by the parameter, ϕ, which represents rental services "quality" with respect to the owner occupied service.

In the table we can observe several interesting patterns. Eliminating the adjustment costs will increase home ownership by 4.8%. Doubling the adjustment costs reduces home ownership by 5.6%.

When there is no uncertainty in house prices or the earnings process is i.i.d. the ownership rate is slightly lower. No uncertainty in house prices implies that the precautionary motivation for purchasing houses is lower, specially since we assume that the rental price follows that of house prices. When shocks are i.i.d, the persistence in the earnings process is lower and the probability of receiving the highest shock is very small and purely transient. The earnings variations that a household face is much smaller than in the benchmark case. Therefore, the precautionary motive for buying a house is lower and, hence, the home ownership rate decreases.

The existence of spread or the elimination of the deductability of mortgage interest payments results in a negligible reduction in the ownership rate. Recall that we are considering proportional taxation. We will return to this issue shortly.

A decrease in the interest rate also decreases the ownership rate. The reason is that a decrease in the interest rate, despite the fact that it decreases the user cost, also decreases the household's saving rate and the market rent, so fewer households accumulate the required downpayment.

We can see substantial changes in home ownership rates when the down payment decreases. We observe a 44% increase in home ownership when the required down payment goes down to 5%. The "quality" of the rental market is also essential. If rental and owner occupied housing services were perfect substitutes, home ownership will decrease by 57%.

In our benchmark case, we calibrate a proportional tax system. Let us consider a progressive system as the one described in Section 5.1. This system is a proportional tax system with an important deduction. Thus, poor households may receive transfers. Households that were liquidity constrained in the benchmark economy and had a user cost higher than the rental price of housing now receive a subsidy that eases the liquidity constrained and lowers their user cost. Hence, the ownership rate increases. But also, home ownership is more attractive to rich households, who pay the higher marginal rates, because of the mortgage deduction. In this case, introducing a positive spread between the return on deposits and the mortgage rate reduces drastically the ownership rate. This is because the user cost of individuals who hold debt increases substantially with spread or no deductability.

Finally, a lower discount rate (larger β) implies that households care more about the future and are more concerned with future fluctuations in consumption. Hence they increase their savings and buy more houses to shield their consumption path against price and earnings fluctuations. Nevertheless the last row indicates that the results are very sensitive to the degree of substitution between rental housing and owner occupied housing.

5.2.4 Calculating the per unit cost of housing

We have seen that when households decide about tenure choice, since they face uncertainty, they compare an expected user cost to the expected market rent net of service quality differences. However, the actual cost of housing can be calculated *ex-post*, once changes in prices and earnings are realized.

Let us calculate an average *ex-post* per unit cost of housing for the households in our benchmark economy. Those households who rent are assigned the rental price, while for those households who owned we calculate the average user cost given in equation (12).

We simulate 100 samples of 9,000 household each, that we observe for 50 periods. The average per unit cost of housing that we obtained is 0.08488, with a standard deviation of 0.00064. What if we assigned all households the rental price instead? Then, our average would be 0.08894 with a standard deviation of 0.00068, a 4.78% higher.

6 Final comments

This paper presents a dynamic model of heterogenous agents that accommodates both a consumption and an investment motive for home ownership as well as endogenous tenure choice. Houses are modelled as illiquid assets that can be partially financed. The framework is used to study household portfolio composition and tenure choice comparing prices of rental and owner-occupied housing services. Our simulations show the importance of liquidity constraints, the discount factor and the quality of the rental service in determining home ownership rates. Interestingly, with progressive taxation, eliminating the mortgage interest rate deductability reduces home ownership by 30%.

In our analysis we have abstracted from several issues. First, financial assets are riskless. Therefore, the model cannot capture well the behavior of households who hold stocks. Second, households accumulate wealth only for precautionary motives therefore abstracting from life-cycle considerations. Thus, we cannot analyze issues such as the effects of the housing tax treatment or the financial conditions on the age of purchasing the first home. Lastly, households cannot hold negative equity on housing. This assumption implies that if the price of houses falls, households are forced to decrease their outstanding debt. Therefore, in our model economy, households who get indebted to purchase houses face greater risk than in reality and the volume of mortgages in our model is lower than what we would see in the data. Allowing for negative home equity imposes an important computational burden (one more state variable). We plan to address these issues in future research.

Appendices

A The household's portfolio

If we solve the household's problem shown in expression (2) we obtain the following first order conditions,

$$c_t : \beta^t u'(c_t) - \lambda_t = 0, \text{ for all } t, \tag{13}$$

$$s_t : \phi\, \gamma\, \beta^t u'(\phi\, s_t + h_t) - \lambda_t\, r_t^h + \varphi_t^s = 0, \tag{14}$$

$$d_{t+1} : -\lambda_t + E_t \lambda_{t+1} \left\{ (1 + r_{t+1}^d) - \tau'_{y^\tau}(y^\tau)\, r_{t+1}^d \right\} + \varphi_t^d = 0, \text{ for all } t, \tag{15}$$

$$m_{t+1} : \lambda_t - E_t \lambda_{t+1} \left\{ (1 + r_{t+1}^m) - \tau'_{y^\tau}(y^\tau) \tau_m\, r_{t+1}^m \right\} + \varphi_t^m - \mu_t = 0, \text{ for all } t, \tag{16}$$

$$\begin{aligned} h_{t+1} : \quad & E_t \gamma\, \beta^{t+1} u'(\phi\, s_{t+1} + h_{t+1}) - \lambda_t\, q_t + \\ & E_t \lambda_{t+1}\, q_{t+1} \left[1 - \delta^h - \tau_h - \varrho \left(1 - \delta^h\right)\, L_{t+1} \right] + (1 - \theta)\, \mu_t\, q_t + \varphi_t^h = 0, \text{ for all } t. \end{aligned} \tag{17}$$

where λ_t is the multiplier of the budget constraint, φ_t^d and φ_t^m are the multipliers of the non-negativity constraints for deposits and mortgages, respectively, and μ_t is the multiplier associated to the liquidity constraint shown in (1).

Lemma App. 1. *The liquidity constraint and the non negativity constraint on mortgages cannot bind simultaneously.*

Proof. Straightforward. □

A.1 No spread and full deductability

Lemma App. 2. *The non negativity constraint on deposits and mortgages cannot bind simultaneously.*

Proof. We prove it by contradiction. Let us assume that $\varphi_t^d > 0$ and $\varphi_t^m > 0$. If $\varphi_t^d > 0$, then by (15) we have that $-\lambda_t + E_t \lambda_{t+1} \left\{ (1 + r_{t+1}^d) - \tau'_{y^\tau}(y^\tau)\, r_{t+1}^d \right\} < 0$. In (16) it implies that $\mu_t > 0$, violating Lemma 1. Therefore, both non negativity constraints cannot bind at the same time. □

Lemma App. 3. *The non negativity constraint on mortgages is never binding,* $\varphi_t^m = 0$.

Proof. If $\varphi_t^d > 0$ then Lemma 2 ensures that $\varphi_t^m = 0$. If $\varphi_t^d = 0$, then we have that $\varphi_t^m = \mu_t$. If $\varphi_t^m > 0$ this implies that $\mu_t > 0$, which contradicts Lemma 1. □

Proof of Proposition 1.

Proof. Adding expressions (15) and (17) we obtain $\varphi_t^d + \varphi_t^m - \mu_t = 0$. By Lemma 3 this expression becomes $\varphi_t^d = \mu_t$. Thus, if the liquidity constraint is binding the household holds no deposits and $m_{t+1} = (1 - \theta)\, q_t\, h_{t+1}$. If the liquidity constraint is not binding only the difference $d_{t+1} - m_{t+1}$ matters. Hence we can set $m_{t+1} = 0$ and the result follows. □

A.2 Spread or partial deductability

Proof of Proposition 2.

Proof. Adding expressions (15) and (17) we obtain:

$$E_t\, \lambda_{t+1} \left[\left(1 - \tau'_{y^\tau}(y_t^\tau)\right) r_t^d - \left(1 - \tau'_{y^\tau}(y_t^\tau)\, \tau_m\right) r_t^m \right] + \varphi_t^d + \varphi_t^m - \mu_t = 0.$$

Notice that since we assume that $r_t^d \leq r_t^m$, $\tau'_{y^\tau}(y_t^\tau) < 1$, and $\tau_m \leq 1$, the expression inside the brackets is negative. Then it must be that $\varphi_t^d + \varphi_t^m - \mu_t > 0$, which implies that $\varphi_t^d + \varphi_t^m > 0$. Thus, households do not hold simultaneously positive amounts of deposits and debt. Specifically, liquidity constrained households do not hold deposits. □

B The determinants of the user cost

Full deductability and no spread

Proof of Proposition 5.

Proof.

Case 1. Let us consider first a non liquidity constrained household and let us assume that it optimally decides to rent a house and not to buy. Expression (17) can be written as:

$$q_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \gamma \frac{u'(\phi\, s_{t+1})}{u'(c_{t+1})} \right] + E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} q_{t+1} \left(1 - \delta^h - \tau_h\right) \right], \tag{18}$$

Using expressions (14) and (15) taking into account that consumption at period $t+1$ is independent of house prices and that the rental price of period $t+1$ is known, we can write the expression above as:

$$q_t = E_t \frac{1}{1+\widehat{r}_t^d} \left[\frac{1}{\phi} r_{t+1}^h + q_{t+1} \left(1 - \delta^h - \tau_h\right) \right], \tag{19}$$

But since the rental price satisfies $r_{t+1}^h > \overline{r}_{t+1}$, this expression cannot hold; in fact, the left hand side, q_t, is lower than the right hand side. This leads to a contradiction. Thus, it implies that the household does not rent and buy simultaneously and that:

$$E_t \frac{\phi\, \gamma\, u'(s_{t+1})}{u'(c_{t+1})} < r_{t+1}^h,$$

That is, the household's user cost is lower than the housing rental price. The households strictly prefers buying to renting.

Case 2. Consider a household that is liquidity constrained and decides not to buy and to purchase rental services. Lemma 2 implies that the non negativity constraint on deposits is binding. Combining (17) and (16) we obtain that:

$$\begin{aligned} q_t = (1-\theta)\, q_t + E_t \tfrac{\beta\, u'(c_{t+1})}{u'(c_t)} \Bigg[\gamma \tfrac{u'(\phi\, s_{t+1})}{u'(c_{t+1})} + q_{t+1} \left(1 - \delta^h - \tau_h\right) \\ -(1-\theta) q_t \left(1 + \left(1 - \tau_m \tau'_{y^\tau}(y_t^\tau)\right) r_t^m\right) \Bigg] \end{aligned} \tag{20}$$

If this household is indifferent between buying and renting,

$$q_t = E_t \frac{1}{1+\widetilde{r}_t^d} \left[\frac{1}{\phi} r_{t+1}^h + q_{t+1} \left(1 - \delta^h - \tau_h\right) \right], \tag{21}$$

where

$$\frac{1}{1+\widetilde{r}_t^d} = \frac{\beta\, E_t\, u'(c_{t+1}) / u'(c_t)}{\theta + (1-\theta) \left(1 + \widehat{r}_t^d\right) \beta\, E_t\, u'(c_{t+1}) / u'(c_t)}.$$

Since the household is liquidity constrained,

$$\frac{1}{1+\widetilde{r}_t^d} < \frac{1}{1+\widehat{r}_t^d}.$$

Notice that $1 + \widetilde{r}_t^d$ is the household's specific discount factor and it is greater the lower the level of cash-on-hand. Recall that $r_{t+1}^h > \overline{r}_{t+1}$, shown in expression (4). Thus, there might exist a level of cash-on-hand x_t^* below which the left hand side of (21) is greater than its right hand side. In that case, the household prefers renting to buying. Therefore, only liquidity constrained households rent services and do not purchase houses. □

Spread and partial deductability

Proof of Proposition 7.

Proof.

Case 1. Consider first the households that hold deposits. These households, by Proposition **??**, are not liquidity constrained. If the rental price satisfies $r_{t+1}^h > \overline{r}_{t+1}$ they strictly prefer buying to renting by Proposition 3.

Case 2. Consider first a non liquidity constrained household that holds debt and assume that it optimally chooses to rent housing services. Expression (17) can be written as:

$$q_t = E_t \left[\beta \, \frac{u'(c_{t+1})}{u'(c_t)} \, \gamma \, \frac{u'(\phi \, s_{t+1} + h_{t+1})}{u'(c_{t+1})} \right] + E_t \left[\beta \, \frac{u'(c_{t+1})}{u'(c_t)} \, q_{t+1} \left(1 - \delta^h - \tau_h\right) \right]. \tag{22}$$

Using expressions (14) and (15) we can write the expression above as

$$q_t = E_t \, \frac{1}{1 + \widehat{r}_t^m} \left[\frac{1}{\phi} r_{t+1}^h + q_{t+1} \left(1 - \delta^h - \tau_h\right) \right], \tag{23}$$

The rental price satisfies $r_{t+1}^h > \overline{r}_{t+1}$. Thus, if the interest rate on mortgages is much larger than the return on deposits, this expression may hold with strict inequality, being the left hand side, q_t greater than the right hand side. That is, the cost of buying would be higher than its returns and, therefore, the household is better off renting. If the interest on mortgages is not too high, then these households would be better off buying. Thus, their user cost depends on the spread and the tax deduction.

Case 3. Consider a household that is liquidity constrained. By Proposition 5 there exists a level of cash-on-hand, x_t^*, below which the user cost is higher than the rental price. In such a case, the household prefers renting to buying. □

C Computational Method

Solving the household problem as stated in section 2.5 is challenging. There are five state variables, (d, m, h, q, e), and five controls, (c, s, d', m', h'), and the adjustment costs prevent us from using standard techniques such Euler equation iteration. We use a finite state approximation to solve the problem instead.

First, we take advantage of the theoretical results about portfolio composition. When there is spread or no full deductability, households prefer equity to debt financing. Thus, they do not accumulate deposits until mortgages are paid for. In order to reduce the number of state variables in the problem, we define financial assets as $a = d - m$. Note that with full segmentation, $a < 0$ as long as the household is paying the mortgage and $a > 0$ afterwards. Therefore this variable contains the information that we need. In the case with no spread and full deductability, households are only concerned with their net financial position $a = d - m$, and we choose the particular equilibrium in which we force agents to pay their debts before they accumulate deposits.

The liquidity constraint, in terms of a becomes $a_{t+1} > -(1-\theta)q_t h_{t+1}$, and the budget constraint is:

$$\begin{aligned} & c_t + r_t^h s_t + a_{t+1} + q_t\, h_{t+1} + L_t\, q_t \varrho\, (1 - \delta^h)\, h_t \leq \\ & w_t e_t + \left(1 + r_t^d\right) \max\{a_t, 0\} + (1 + r_t^m) \min\{a_t, 0\} + q_t \left(1 - \delta^h - \tau_h\right) h_t - \tau_{y_t^\tau}(y_t^\tau). \end{aligned}$$

We then replace the continuous state variables, a and h with the finite sets, $\mathcal{A} = \{a_1, \ldots, a_{N_a}\}$ and $\mathcal{H} = \{h_1, \ldots, h_{N_h}\}$, where N_h and N_a are the number of points in the grid for h and a respectively. For a given tuple $(a_t, h_, q_t, e_t)$ of state variables in period t, the household chooses (a_{t+1}, h_{t+1}).

We solve the household problem using value function iteration and speed up convergence by using a standard policy function accelerator (see Judd 1998). We obtained the optimal policy functions for a and h, $g^a(a, h, q, e)$ and $g^h(a, h, q, e)$, that we use in the simulations.[19]

Note that the problem in formulated in such a way that control variables today (a_{t+1}, h_{t+1}) are next period's states. The other two control variables (c_t, s_t) can be obtained as a residual. When $h_t \neq 0$, households do not purchase housing services, $s_t = 0$, and c_t can be obtained from the budget constraint, once a_{t+1} and h_{t+1} are chosen. When $h_t = 0$, the household must purchase housing services and thus leftover resources -after choosing a_{t+1} and h_{t+1}- are divided between c_t and s_t according to the first order conditions. In particular $c_t = (r_t^h/\phi\gamma)^{\frac{1}{\sigma}} s_t$. We make agents choose services from the grid $\mathcal{H}$, though.

[19] We do not interpolate between grid points since we cannot warrantee the concavity of the program. The liquidity constraint is implemented using penalty functions.

Choosing the grids carefully is specially important when using this technique and some trial and error is required to find appropriate upper bounds. We set $h_{N_h} = 70$ and $a_{N_a} = 200$. With respect to the lower bounds, $h_1 = 0$ but $a_1 < 0$. In our benchmark case we use a convex grid for h and set $N_h = 30$. The grid for a is also convex and includes sufficient negative values to accommodate financing of the durable according to the liquidity constraint for all possible prices for housing. Then $N_a = N_q * (N_h - 1) + n$, where N_q is the number of possible housing prices. We set $n = 100$, thus $N_a = 187$. These numbers implied solving the household's problem for 50,490 $(30 \times 187 \times 3 \times 3)$ points at each iteration.

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